QUESTION 1: GUITAR

Problem Description:

The original guitar image is corrupted, the goal is to remove the global background shading without impacting the real image.

Objective Function:

To obtain $r(i,j)$ that minimizes the norm of difference between the model and measured intensity.

$$\boldsymbol{minimize}\ (|\boldsymbol{e_c(i,j)} - \boldsymbol{e_m(i,j)}|$$

Where, $e_c(i,j) = r(i,j) * e(i,j)$ (model)

$$e_m(i,j) = measured\ intensity$$

$$e(i,j) = true\ image$$

Constraint:

Consider $r(i,j) = ai + bj + c$, where a, b, c are global variables and i and j represents coordinates.

- **Constraint 1**: $ai + bj + c \geq 0$
- **Constraint 2:** $ai + bj + c \leq 1$

Also, given the upper left corner in the true image should be uniform, specifically e (i, j) = 255 for i < 50 and j < 250.

Procedure:

Input the given image using imread and convert it to double since the problem involves matrix multiplication. From the given information obtain the true image which is of just 249x49 dimension. Also obtain (i,j) matrix using meshgrid command. Design the model which is nothing but the matrix multiplication of r(i,j) and e(i,j). Build a convex optimization problem with a,b,c as variables subject to the constraint as defined above.

(in matlab):
```
minimize(norm(e.*(a*X'+b*Y'+c) - edm(1:249,1:49)))
    variables a b c
    subject to
                (a*X'+b*Y'+c)>=0
                (a*X'+b*Y'+c)<=1
```

Here X' represent matrix of all i values and Y' represents matrix of all j values obtained form meshgrid. Once you solve this optimization problem you will get a set of a,b,c values and with these values you can retrieve back the true guitar image.

```
for i=1:1:size(em,1)
    for j=1:1:size(em,2)
        e_new(i,j) = em(i,j)/(a*i+b*j+c);
    end
end
```

Where e_new is the new true image obtained after solving optimization problem. We can use imshow to display the image.

Output of Solver:

```
Calling SDPT3 4.0: 68953 variables, 4 equality constraints
   For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------
 num. of constraints =  4
 dim. of sdp    var  = 298,   num. of sdp  blk  =  1
 dim. of linear var  = 24402
*****************************************************************
   SDPT3: Infeasible path-following algorithms
*****************************************************************
 version  predcorr  gam  expon  scale_data
   HKM      1      0.000  1       0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|4.9e+03|3.0e+03|2.3e+11| 3.811870e+06  0.000000e+00| 0:0:01| chol  1  1
 1|0.989|0.989|5.3e+01|3.3e+01|2.5e+09| 3.811474e+06 -5.680150e+06| 0:0:03| chol  1  1
 2|0.999|0.992|7.4e-02|2.5e-01|2.6e+07| 3.788370e+06 -5.636188e+06| 0:0:03| chol  1  1
 3|1.000|1.000|1.8e-06|6.0e-04|2.9e+06| 2.227550e+06 -9.518603e+04| 0:0:03| chol  1  1
 4|0.994|0.885|9.6e-07|3.3e-04|6.1e+04| 4.377248e+04 -1.103253e+04| 0:0:03| chol  1  1
 5|1.000|0.183|1.1e-04|2.9e-04|2.1e+04| 1.057734e+04 -9.320831e+03| 0:0:03| chol  1  1
 6|1.000|0.897|5.3e-04|3.8e-05|1.0e+04| 9.402925e+03 -8.031754e+02| 0:0:03| chol  1  1
 7|0.898|0.769|5.6e-05|9.9e-06|3.6e+03| 3.233587e+03 -3.023790e+02| 0:0:03| chol  1  1
 8|1.000|0.907|2.1e-06|1.6e-06|1.1e+03| 1.025879e+03 -8.297068e+01| 0:0:03| chol  1  1
 9|1.000|0.821|4.6e-07|7.2e-07|2.7e+02| 2.489548e+02 -2.079631e+01| 0:0:03| chol  1  1
10|1.000|0.271|2.2e-08|6.2e-07|1.3e+02| 1.077688e+02 -1.779737e+01| 0:0:04| chol  1  1
11|0.837|1.000|3.1e-09|4.4e-09|3.5e+01| 1.696853e+01 -1.807303e+01| 0:0:04| chol  1  1
12|0.895|0.943|6.5e-10|8.9e-10|5.0e+00|-1.252538e+01 -1.753101e+01| 0:0:04| chol  1  1
13|0.942|0.976|1.8e-10|1.5e-10|1.5e+00|-1.601574e+01 -1.751597e+01| 0:0:04| chol  1  1
14|0.903|0.987|1.8e-11|3.9e-11|1.6e-01|-1.735995e+01 -1.751532e+01| 0:0:04| chol  1  1
15|0.896|0.977|8.0e-11|4.4e-12|3.0e-02|-1.748557e+01 -1.751530e+01| 0:0:04| chol  1  1
16|0.975|1.000|1.1e-10|5.3e-12|7.5e-03|-1.750775e+01 -1.751530e+01| 0:0:04| chol  1  1
17|1.000|1.000|1.8e-10|7.9e-12|9.6e-04|-1.751433e+01 -1.751530e+01| 0:0:04| chol  1  1
18|1.000|1.000|1.6e-10|1.2e-11|2.3e-05|-1.751527e+01 -1.751530e+01| 0:0:04| chol  1  1
19|1.000|1.000|1.1e-09|1.8e-11|1.1e-06|-1.751530e+01 -1.751530e+01| 0:0:04| chol  1  1
20|1.000|0.987|5.9e-10|2.2e-11|5.1e-08|-1.751530e+01 -1.751530e+01| 0:0:04|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 20
 primal objective value = -1.75152979e+01
 dual   objective value = -1.75152980e+01
 gap := trace(XZ)       = 5.10e-08
 relative gap           = 1.41e-09
 actual relative gap    = 1.28e-09
 rel. primal infeas (scaled problem)   = 5.95e-10
 rel. dual     "        "       "      = 2.23e-11
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.0e+00, 1.8e+01, 3.2e+02
 norm(A), norm(b), norm(C) = 5.9e+06, 2.0e+00, 3.3e+04
 Total CPU time (secs)  = 4.22
 CPU time per iteration = 0.21
 termination code       =  0
 DIMACS: 5.9e-10  0.0e+00  2.9e-09  0.0e+00  1.3e-09  1.4e-09
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +17.5153
```

Figure:

Before Solving

After Solving