QUESTION 2: SPLINE FITTING

Problem Description:

Find an optimal spline that fits the river contour such that it minimizes the least square error between model and measurement points. These points are obtained by ginput.

Objective Function:

- Spline/Model definition: It consist of M piecewise cubic polynomial P segments, and points in each segment can be represented as $x(t)$ and $y(t)$.
    - $x(t) = a_3t^3 + a_2t^2 + a_1t + a_0$
    - $y(t) = b_3t^3 + b_2t^2 + b_1t + b_0$
- We are interested in minimizing the error between the model and measurement points.
    - **Objective Function:** $\boldsymbol{Minimize}(\,||\boldsymbol{x(t)} - \boldsymbol{x_m}|| + ||\boldsymbol{y(t)} - \boldsymbol{y_m}||)$

Where x(t) and y(t) represent points from model, x_m and y_m are my measured points from ginput.

Constraint:

We want to ensure continuity between the end points (end of one polynomial must be the start of other). Also, we must look at corner conditions such that first and second derivative at these points are equal between two polynomial segments.

- **Constraint 1**: $P_i(x_i(t), y_i(t)) = P_{i+1}(x_i(t), y_i(t))$ (not required- in class discussion)
- **Constraint 2:** $\frac{d}{dt}P_i(x_i(t), y_i(t)) = \frac{d}{dt}P_{i+1}(x_i(t), y_i(t))$
- **Constraint 3:** $\frac{d^2}{dt^2}P_i(x_i(t), y_i(t)) = \frac{d^2}{dt^2}P_{i+1}(x_i(t), y_i(t))$

Procedure:

We first get the points from the river figure using matlab- ginput command. Here I have taken 150 points for the whole river contour. I must divide this entire river contour into multiple segments, for convenience I have used 50 segments and each segment has 3 points each. Also, we must sample t (0<t<1), and I have used this same sample for all the segments. The coefficients for the polynomial are as follows $a_3, a_2, a_1, a_0, b_3, b_2, b_1, b_0$. We basically have to optimize these variables such that the least squared error between model and measured points are minimised. Since the constraints defined above depends on adjacent polynomial we cannot independently optimize this but has to be done globally. The variables of $x(t)$ and $y(t)$ (i.e $a_3, a_2, a_1, a_0, b_3, b_2, b_1, b_0$) must be optimised globally.

In Matlab this can be represented as (m segments)

```
variables a3(m) a2(m) a1(m) a0(m) b3(m) b2(m) b1(m) b0(m)

minimize(norm(Dx - p(:,1),2)+norm(Dy - p(:,2),2))-objective function

Dx represents x(t)and Dy represents y(t). Also, p represents measured points.
```

At first, I create the t matrix which consist of sampled values between 0 and 1, and coefficient matrix (formed by variables as above). From this I generate the points obtained from model- basically perform matrix multiplication of t with coefficient matrix. Once I obtain this, I use it in my objective function along with the constraint defined above.

In my solution I have considered 150 measurement points and have divided my contour in to 50 equal segments each having 3 points each for better efficiency.

Output of Solver:

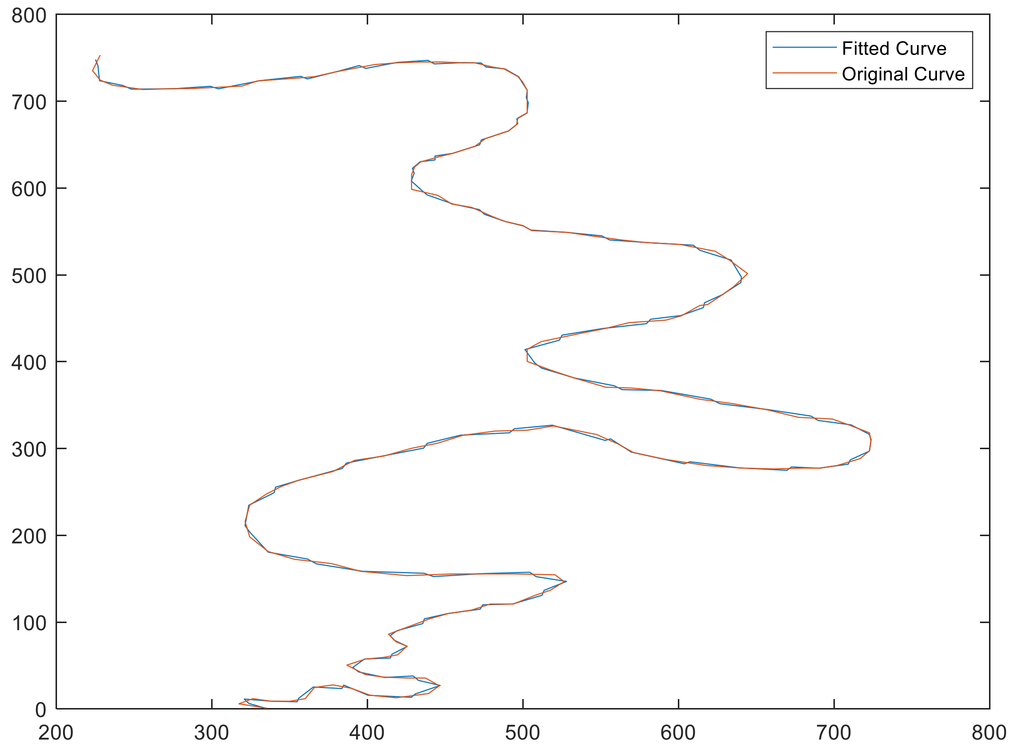
```
Calling SDPT3 4.0: 498 variables, 402 equality constraints
   For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

 num. of constraints = 402
 dim. of socp   var  = 302,   num. of socp blk  =  2
 dim. of free   var  = 196 *** convert ublk to lblk
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
   NT      1      0.000   1       0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|6.6e+00|3.1e+01|1.2e+08| 0.000000e+00  0.000000e+00| 0:0:00| chol  1  1
 1|1.000|0.977|8.8e-03|7.4e-01|5.8e+05|-5.455880e+01 -4.374734e+02| 0:0:00| chol  1  1
 2|1.000|0.942|7.8e-05|5.4e-02|8.3e+03|-6.428486e+01 -9.946058e+01| 0:0:00| chol  1  1
 3|0.974|1.000|2.6e-05|3.5e-03|1.1e+02|-8.788192e+01 -9.261919e+01| 0:0:00| chol  1  1
 4|0.989|0.985|5.8e-06|4.0e-04|2.1e+00|-8.958513e+01 -8.964154e+01| 0:0:00| chol  1  1
 5|0.987|0.975|7.4e-08|4.5e-05|4.6e-02|-8.961004e+01 -8.960967e+01| 0:0:00| chol  1  1
 6|0.986|0.948|1.3e-09|2.8e-06|8.8e-04|-8.961052e+01 -8.961016e+01| 0:0:00| chol  1  1
 7|0.990|0.947|9.0e-10|7.5e-08|1.3e-05|-8.961054e+01 -8.961052e+01| 0:0:01| chol  1  1
 8|0.990|0.929|3.8e-10|4.1e-09|3.1e-07|-8.961054e+01 -8.961054e+01| 0:0:01|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   =  8
 primal objective value = -8.96105388e+01
 dual   objective value = -8.96105374e+01
 gap := trace(XZ)       = 3.15e-07
 relative gap           = 1.75e-09
 actual relative gap    = -7.85e-09
 rel. primal infeas (scaled problem)   = 3.83e-10
 rel. dual     "        "       "      = 4.06e-09
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 2.5e+00, 7.9e+02, 9.9e+01
 norm(A), norm(b), norm(C) = 1.5e+02, 2.4e+00, 2.5e+02
 Total CPU time (secs)  = 0.52
 CPU time per iteration = 0.06
 termination code       =  0
 DIMACS: 4.6e-10  0.0e+00  2.1e-08  0.0e+00  -7.8e-09  1.7e-09
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +89.6105

>>
```

Figure:



River Contour