

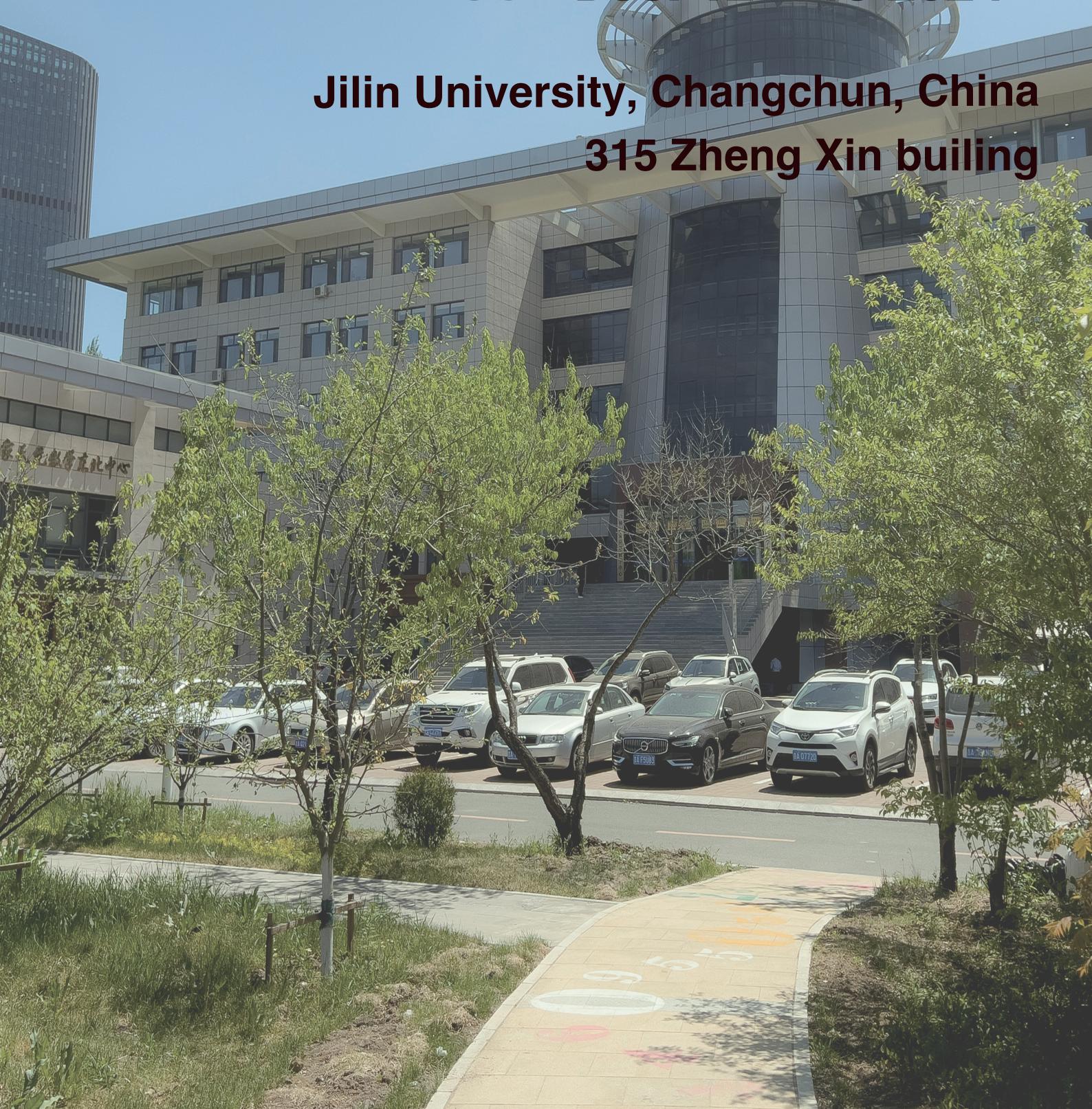
# **International Conference**

# **on Differential Geometry**

# **and Topology**

**Dec. 12 ~ 15 2024**

**Jilin University, Changchun, China**  
**315 Zheng Xin builing**



# About

## International Conference on Differential Geometry and Topology

The Conference on differential geometry and topology will be held from 13th December to 15th December 2024 at the School of Mathematics, Jilin University. The venue of the conference will be *315 Zheng Xin* building.

The Conference Program of the ICDGT-2024 will have a mini course on *K*-theory by Prof. Thomas Schick from the University of Göttingen.

### Organizing Committee

Seongjeong Kim, Jilin University  
Sachchidanand Prasad, Jilin University  
Yunhe Sheng, Jilin University  
Xiao Wang, Jilin University

### Chairs

Thomas Schick, University of Göttingen  
Yunhe Sheng, Jilin University



# Timetable

## Friday, 13th December

Time		
8:30 – 9:00		Registration and Group photo
9:00 – 10:00	<b>Thomas Schick</b> University of Göttingen	<i>K</i> -theory, the basics
10:00 – 10:25		Coffee break
10:25 – 11:10	<b>Lipin Yuan</b> Hebei Normal University	On some properties of convex surfaces
11:10 – 11:15		Break
11:15 – 12:00	<b>Sachchidanand Prasad</b> Jilin University	On the cut and focal locus of a Finsler manifold
12:00 – 13:30		Lunch break
13:30 – 14:15	<b>Sandip Samanta</b> Indian Institute of Science Education and Research Kolkata	Spheres Bundle Over Spheres: Topological and Geometric View
14:15 – 14:20		Break
14:20 – 15:05	<b>Somnath Basu</b> Indian Institute of Science Education and Research Kolkata	On manifolds homeomorphic to the $n$ -sphere
15:05 – 15:30		Coffee break
15:30 – 16:15	<b>Miguel Angel Javaloyes</b> University of Murcia	Applications of Finsler Geometry to wildfire propagation models. The importance of focal and cut points
16:15 – 16:20		Coffee break
16:20 – 17:05	<b>Wilhelm Klingenberg</b> Durham University	Local non-injectivity of the exponential map at critical points in sub-Riemannian geometry
17:05 – 19:15		Dinner

## Saturday, 14th December

Time		
9:00 – 10:00	<b>Thomas Schick</b> University of Göttingen	<i>K</i> -theory and geometry
10:00 – 10:10		Break
10:10 – 11:10	<b>Thomas Schick</b> University of Göttingen	<i>K</i> -theory and higher index theory
11:10 – 11:35		Coffee break
11:35 – 12:20	<b>Shicheng Xu</b> Capital Normal University	Rigidity for Einstein manifolds under bounded covering geometry
12:20 – 13:50		Lunch break
13:50 – 14:35	<b>Jin-ichi Itoh</b> Sugiyama Jogakuen University	Cut loci for non-convex polyhedra
14:35 – 14:40		Break
14:40 – 15:25	<b>Byeorhi Kim</b> Pohang University of Science and Technology	On a classification of topological surfaces
15:25 – 15:50		Coffee break
15:50 – 16:35	<b>Tumpa Mahato</b> Indian Institute of Science Education and Research Pune	Parameterization of knotted surfaces arising from classical and welded knots
16:35 – 16:40		Break
16:40 – 17:25	<b>Rama Mishra</b> Indian Institute of Science Education and Research Pune	Geometry of knots in $\mathbb{RP}^3$
17:25 – 19:25		Dinner

# Sunday, 15th December

Time		
9:00 – 10:00	<b>Thomas Schick</b> University of Göttingen	<i>K</i> -theory and topological $T$ -duality
10:00 – 10:15		Coffee break
10:15 – 11:00	<b>Aritra Bhowmick</b> Indian Institute of Science Online	Fibration with a Section and H-Splitting of the Looped Total Space
11:00 – 11:05		Break
11:05 – 11:50	<b>Sai Rasmi Ranjan</b> Shiv Nadar University Online	Higher genus Maxfaces with arbitrarily many catenoid or planar ends
11:50 – 12:00		Break
12:00 – 12:45	<b>Pradip Kumar</b> Shiv Nadar University Online	Constructing Zero Mean Curvature Surfaces in Euclidean Space and in the Lorentz-Minkowski Space
12:45 – 14:15		Lunch
14:30 – 17:30		Free Discussion

# List of Abstracts – Talks

**Friday, 13th December**

## ***K-theory, the basics***

**Thomas Schick**, University of Göttingen

We introduce the  $K$ -theory groups of a  $C^*$ -algebra and explain its basic properties. In a nutshell, the group  $K_0(A)$  classifies  $A$ -modules (finitely generated projective) and  $K_1(A)$  classifies automorphisms of the trivial modules. These come up in numerous situations, and at the same time provide information about the structure of the algebra  $A$ . We give a glimpse at computation tools and at first applications.

### **Plan**

- Long exact sequence of an extension
- Matrix and compact operator stability
- Product structure
- Bott periodicity
- Sample calculations (Calkin index sequence)
- A glimpse at the Elliott classification program

## **On some properties of convex surfaces**

**Liping Yuan**, Hebei Normal University

A convex body  $K$  in  $\mathbb{R}^d$  is a compact convex set with interior points. A convex surface in  $\mathbb{R}^3$  is the boundary of a convex body in  $\mathbb{R}^3$ . In this talk we will investigate some properties of convex surfaces in  $\mathbb{R}^3$ .

## **On the cut and focal locus of a Finsler manifold**

**Sachchidanand Prasad**, Jilin University

In this talk, we explore key aspects of Finsler geometry with a focus on the structure of the cut and focal loci. We begin by revisiting fundamental concepts in Finsler geometry before defining the cut locus and illustrating examples in Riemannian manifolds. The discussion culminates with a proof of a special case of the generalized Klingenberg lemma for Finsler manifolds, specifically for  $N$ -geodesic loops, where  $N$  is a closed submanifold of a Finsler manifold  $M$ . This is a joint work with Aritra Bhowmick.

## **Spheres Bundle Over Spheres: Topological and Geometric View**

**Sandip Samanta**, Indian Institute of Science Education and Research Kolkata

I will introduce the brace product for a fibration admitting a section and then specialize to spheres bundle over spheres. Then we classify these bundles using brace product upto rational homotopy equivalence. Also we will discuss a geometric view through Morse function in some special case of spheres bundle over spheres.

## **On manifolds homeomorphic to the $n$ -sphere**

**Somnath Basu**, Indian Institute of Science Education and Research Kolkata

We shall discuss Reeb's Theorem and basic differential topology of Morse functions. This was used by Milnor to prove the existence of exotic spheres in 7 dimensions. We shall propose a generalization of Reeb's Theorem and discuss a proof of it. This is joint work with Sachchidanand Prasad.

## **Applications of Finsler Geometry to wildfire propagation models. The importance of focal and cut points**

**Miguel Angel Javaloyes**, University of Murcia

We will first show how Finsler metrics appear as a tool to solve the time-independent Zermelo problem, or more generally, the problem of finding the shortest trajectory in time when the velocity is prescribed at any direction, namely, the velocity is a function of the direction. These findings can be applied to wildfire propagation models as the velocity of the fire in every direction is prescribed, namely, it depends on the wind, the slope, the vegetation, humidity. Indeed, the new firefront is obtained by computing the orthogonal geodesics to the initial firefront, and focal and cut points will indicate places where fire comes from various directions, with an increasing danger for firefighters. When the velocity depends also on time, we will see that Zermelo problem can be solved by considering Finsler spacetimes. It turns out that the shortest trajectories are the projections to  $M$  of lightlike geodesics in the non-relativistic spacetime  $\mathbb{R} \times M$ , where the first coordinate is the absolute time. So the propagation of the fire can be obtained computing the orthogonal lightlike geodesics to the firefront.

## **Local non-injectivity of the exponential map at critical points in sub-Riemannian geometry**

**Wilhelm Klingenberg**, Durham University

In joint work with Samuel Borza, we give a verison of the Theorem of Morse and Littauer.

# Saturday, 14th December

## *K*-theory, and geometry

**Thomas Schick**, University of Göttingen

We look at the classical topological  $K$ -theory of spaces introduced by Atiyah. These involve vector bundles (and their automorphisms). We discuss how this relates to operator  $K$ -theory. As a famous application, we study how topological  $K$ -theory enters the study of the index of an elliptic differential operator and formulate the ground-breaking Atiyah-Singer index theorem.

### Plan

- Vector bundle  $K$ -theory
- Serre-Swan theorem
- Elliptic differential operators and their topological and analytical index
- Atiyah-Singer index theorem

## *K*-theory and higher index theory

**Thomas Schick**, University of Göttingen

We venture further into index theory, where we now bring in interesting (non-commutative)  $C^*$ -algebras, in particular  $C^*$ -algebras associated to the fundamental group. We show how this can be used to give deep information about the non-existence of Riemannian metrics of positive scalar curvature.

### Plan

- Group  $C^*$ -algebras and Baum-Connes conjecture
- Rosenberg obstruction to positive scalar curvature
- Gromov-Lawson-Rosenberg conjecture

## **Rigidity for Einstein manifolds under bounded covering geometry**

**Shicheng Xu**, Capital Normal University

We prove three rigidity results for Einstein manifolds with bounded covering geometry. (1) any almost flat manifold  $(M, g)$  must be flat if it is Einstein, i.e.  $\text{Ric} = Lg$  for some real number  $L$ . (2) A compact Einstein manifold with a non-vanishing and almost maximal volume entropy is hyperbolic. (3) A compact Einstein manifold admitting a uniform local rewinding almost maximal volume is isometric to a space form. This is a joint work with Cuifang Si.

## **Cut loci for non-convex polyhedra**

**Jin-ichi Itoh**, Sugiyama Jogakuen University

I will give a definition of cut locus of a 2-dimensional polyhedra which is not necessary to be convex and its fundamental properties. The cut locus is a strong deformation retract punctured polyhedron and give a handle decomposition of the polyhedron. This is a joint work with T. Yoshiyasu.

## **On volumes of hyperbolic right-angled polyhedra**

**Byeorhi Kim**, POSTECH

In 2019, D. Gabai introduced the Light Bulb Theorem, offering a partial solution to the classification of embeddings of 2-spheres in 4-manifolds. Since then, the theorem has been adapted and extended to yield results for other 2-manifolds. In this talk, we explore the classification of surfaces in 4-manifolds, building on the recent developments that have emerged as a continuation of Gabai's work.

## **Parameterization of knotted surfaces arising from classical and welded knots**

**Tumpa Mahato**, Indian Institute of Science Education and Research Pune

The main objects of the talk are knotted surfaces in four dimensional space. Although we study knotted surfaces using diagrams and braids, visualizing is also very important to understand these abstract mathematical objects. Therefore, parameterizing these embeddings of 2-manifolds using elementary functions becomes crucial not only for computing invariants but also to provide a machinery to visualize and interact with these objects. In this talk, we will provide a concrete parameterization of a few classes of knotted surfaces.

Moreover, we will briefly discuss the non-triviality of a specific class of surface knots called ribbon torus knots by using its connection with welded knots by S. Satoh's *Tube map*.

## **Geometry of knots in $\mathbb{RP}^3$**

**Rama Mishra**, Indian Institute of Science Education and Research Pune

We discuss many geometric and intrinsic properties of knots and links inside real projective 3-space.

# Sunday, 15th December

## **$K$ -theory and topological $T$ -duality**

**Thomas Schick**, University of Göttingen

In the last lecture, we discuss another application of  $K$ -theory, this time inspired by mathematical physics:  $T$ -dual space-times have isomorphic twisted  $K$ -theory. We briefly introduce the physics idea of  $T$ -duality and the concept of twisted  $K$ -theory. At the end, we have a glimpse at equivariant  $K$ -theory and an equivariant improvement of  $T$ -duality.

### **Plan**

- $T$ -dual space-times
- Twisted  $K$ -theory
- The  $T$ -duality transform
- Equivariant  $K$ -theory
- Equivariant  $T$ -duality

## **Fibration with a Section and H-Splitting of the Looped Total Space**

**Aritra Bhowmick**, Indian Institute of Science, Bengaluru

A principal  $G$ -bundle admitting a section is always trivial. However, this does not hold for a general fibration with a section. To any such fibration, I. M. James introduced a certain product involving the homotopy groups of the fiber and the base space, known as the James brace product. In the first part of this talk, we shall see when the vanishing of the James brace product implies that the fibration is indeed trivial.

The space of based loops in a given space is a prototypical example of an H-space. Any fibration with a section becomes trivial when it is looped once. This means that the loop space of the total space is homotopy equivalent to the product of the loop spaces of the base and the fiber. A natural question arises: when is this an equivalence of H-spaces, i.e., an H-splitting? In the second part of this talk, we shall introduce a generalization of the James brace product, and identify the vanishing of this generalized brace product as the obstruction for the H-splitting of a fibration with section after looping. We shall provide an example where the generalized brace products do not vanish, even though the James brace products vanish identically.

This is a joint work with S. Basu and S. Samanta.

## **Higher genus Maxfaces with arbitrarily many catenoid or planar ends**

**Shiv Nadar University**, Sai Rasmi Ranjan Mohanty

Maximal surfaces in 3-dimensional Lorentz-Minkowski space arise as solutions to the variational problem of local area maximizing among the spacelike surfaces. These surfaces are zero mean curvature surfaces, and maximal surfaces with singularities are called generalized maximal surfaces. Maxfaces are a special class of these generalized maximal surfaces where singularities appear at points where the tangent plane contains a light-like vector. I will present the construction of a new family of maxfaces of high genus that are embedded outside a compact set and have arbitrarily many catenoid or planar ends. The surfaces look like spacelike planes connected by small necks. Among the examples are maxfaces of the Costa-Hoffman-Meeks type. More specifically, the singular set form curves around the waists of the necks. In generic and some symmetric cases, all but finitely many singularities are cuspidal edges, and the non-cuspidal singularities are swallowtails evenly distributed along the singular curves. This work is conducted in collaboration with Dr. Hao Chen, Ms. Anu Dhochak, and Dr. Pradip Kumar, and is accessible at <https://arxiv.org/pdf/2402.11965>.

## **Constructing Zero Mean Curvature Surfaces in Euclidean Space and in the Lorentz-Minkowski Space**

**Pradip Kumar**, Shiv Nadar University

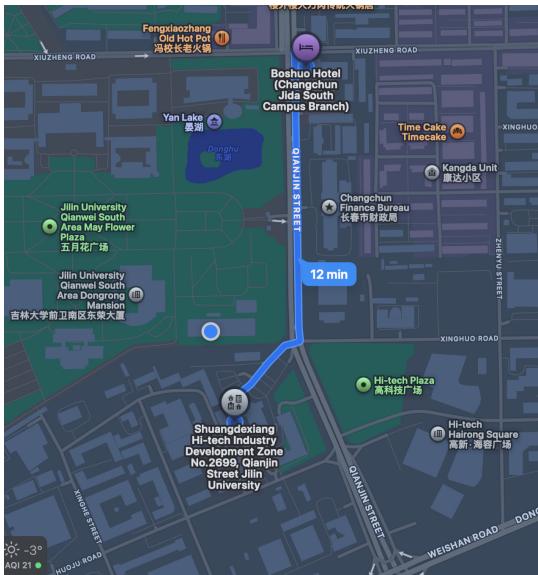
Similar to minimal surfaces in  $\mathbb{R}^3$ , maximal surfaces are zero-mean curvature immersions in Lorentz-Minkowski space. These surfaces arise as solutions to the variational problem of locally maximizing the area among spacelike surfaces. In this talk, we will define minimal surfaces in Euclidean space and maximal surfaces in Lorentz-Minkowski space. We will demonstrate how calculus on Teichmuller space aids us in constructing these maximal and minimal surfaces. In particular, we will show the construction of new higher-genus maximal surfaces with Enneper end. To address the period problem, we will apply Wolf and Weber's method. This is a joint work with Rivu Bardhan and Indranil Biswas.

# List of Participants

Name	Affiliation	Country
Somnath Basu	Indian Institute of Science Education and Research Kolkata	India
Aritra Bhowmick	Indian Institute of Science Bengaluru	India
Bohui Chen	Sichuan University	China
Bingzhe Hou	Jilin University	China
Shuai Hou	Jilin University	China
Jianxun Hu	Sun Yat-sen University	China
Jin-ichi Itoh	Sugiyama Jogakuen University	Japan
Miguel Angel Javaloyes	University of Murcia	Spain
Byeorhi Kim	Pohang University of Science and Technology	South Korea
Seongjeong Kim	Jilin University	China
Wilhelm Klingenberg	Durham University	United Kingdom
Pradip Kumar	Shiv Nadar University	India
Jianya Liu	Shandong University	China
Qingping Liu	China University of Mining and Technology	China
Tumpa Mahato	Indian Institute of Science Education and Research Pune	India
Rama Mishra	Indian Institute of Science Education and Research Pune	India
Sachchidanand Prasad	Jilin University	China
Sai Rasmii Ranjan	Shiv Nadar University	India
Sandip Samanta	Indian Institute of Science Education and Research Kolkata	India
Thomas Schick	University of Göttingen	Germany
Yunhe Sheng	Jilin University	China
Yucai Su	Tongji University	China
Rong Tang	Jilin University	China
Xiao Wang	Jilin University	China
Shicheng Xu	Capital Normal University	China

Name	Affiliation	Country
Xiaoping Xu	University of Chinese Academy of Sciences	China
Liping Yuan	Hebei Normal University	China
Yimu Zhang	Jilin University	China
Youjin Zhang	Tsinghua University	China
Jian Zhou	Tsinghua University	China

# Useful Information



## Hotel Address

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2426 Qianjin Street, Chaoyang District  
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