GATE 2025: Homework #2

Based on Limits

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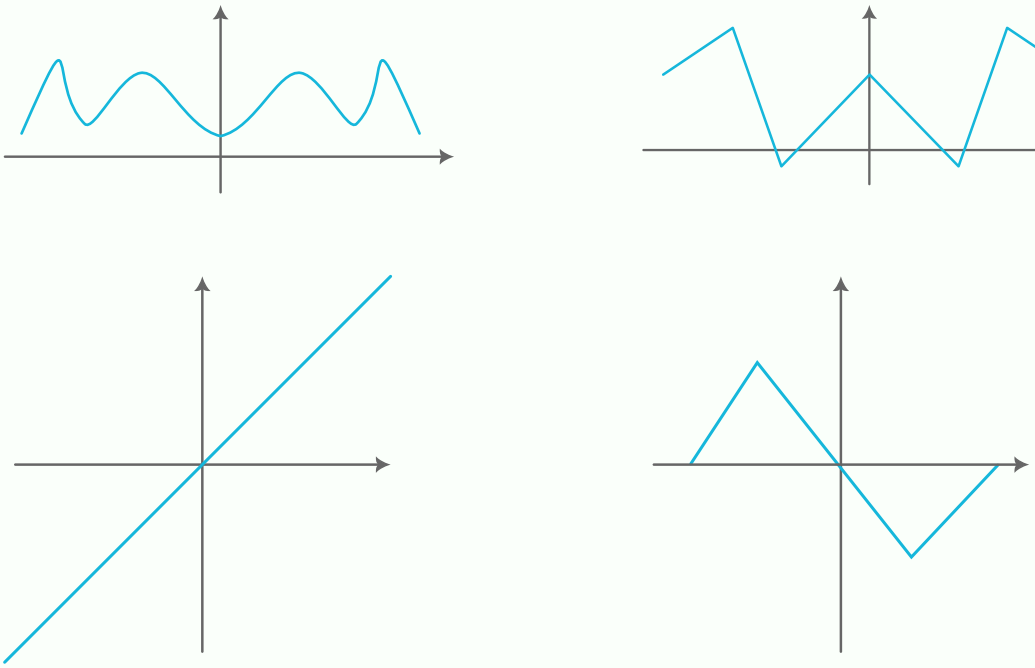
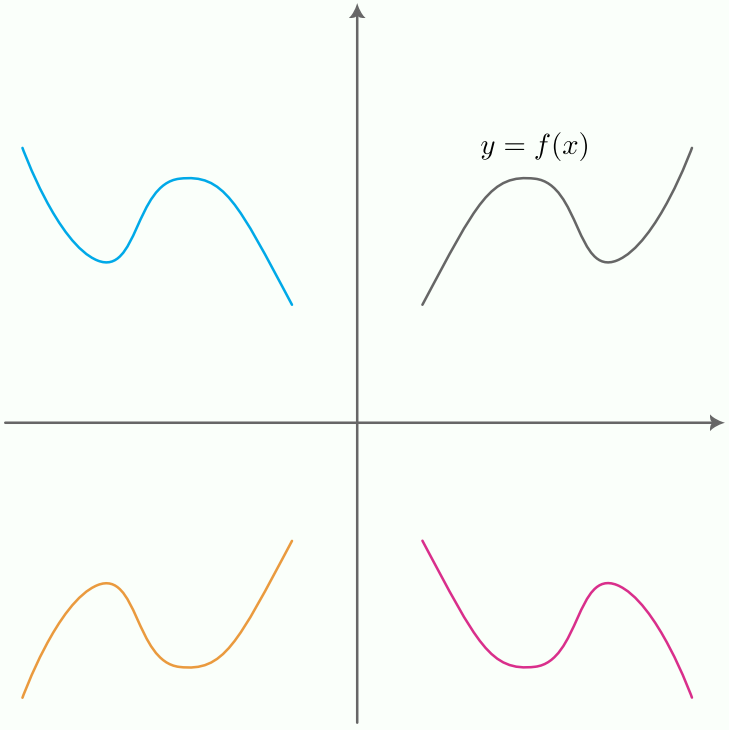
Problem 1

Identify if the given function is even or odd.



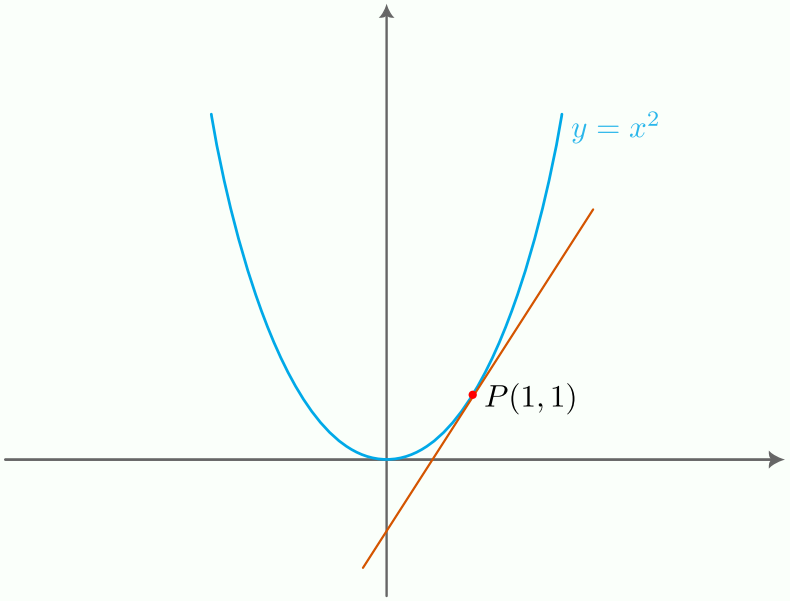
Problem 2

Look at the figure below. The graph of the function $y = f(x)$ is given. Identify the other graphs.



Problem 3

Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.



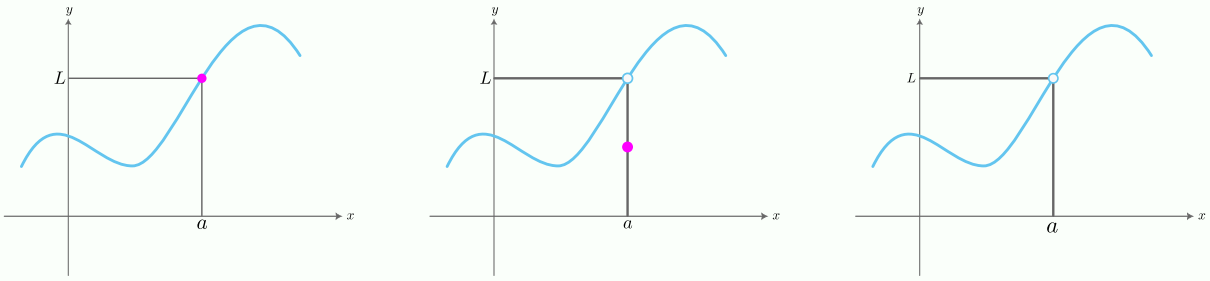
Problem 4

In the following problems find the average rate of change of the function over the given intervals.

1. $f(x) = x^3 + 1$
   a. $[2, 3]$ b. $[-1, 1]$
2. $g(x) = x^2 - 2x$
   a. $[1, 3]$ b. $[-2, 4]$
3. $h(t) = 2 + \cos t$
   a. $[0, \pi]$ b. $[-\pi, \pi]$

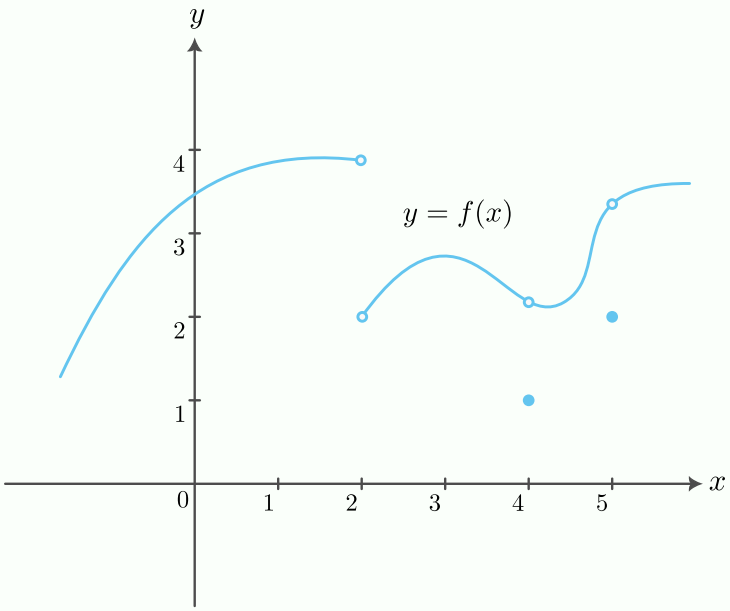
Problem 5

In the figures below, find the limit as x approacher to a, that is, $\lim_{x \to a} f(x)$.



Problem 6

In the figures below, find the following limits.



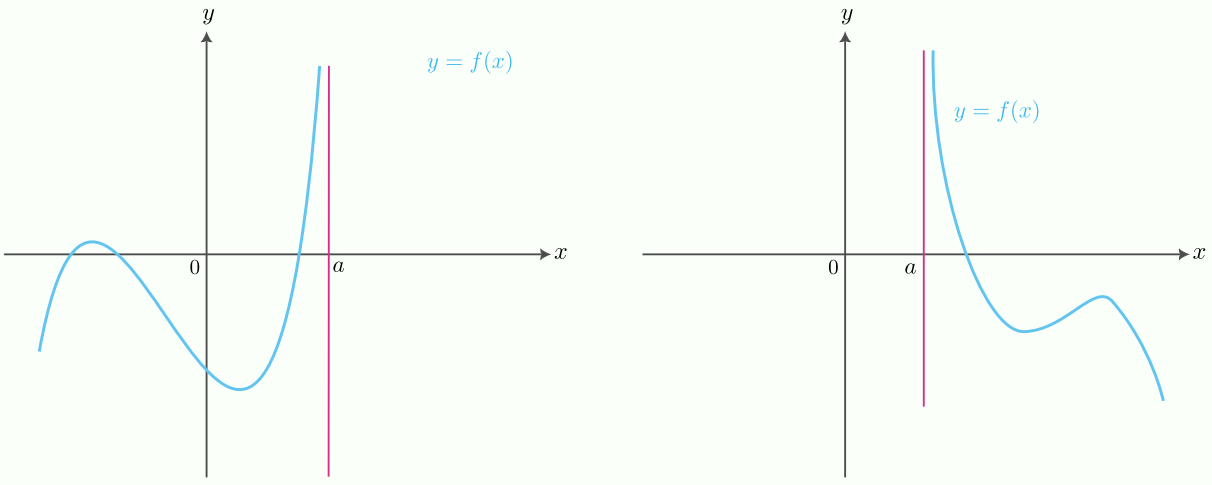
(i) $\lim_{x \to 2^-} f(x)$ (ii) $\lim_{x \to 2^+} f(x)$ (iii) $\lim_{x \to 2} f(x)$

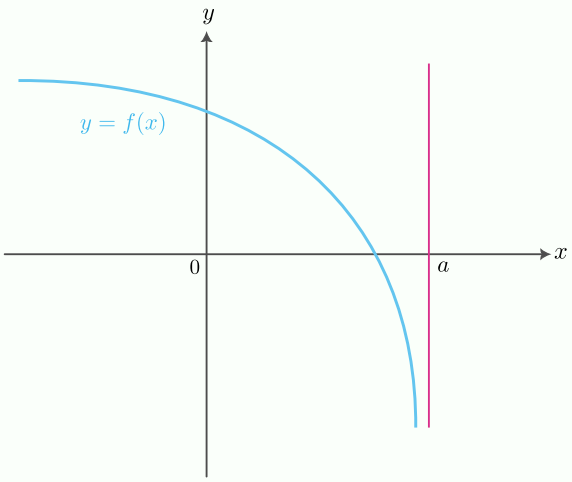
(iv) $\lim_{x \to 4^-} f(x)$ (v) $\lim_{x \to 4^+} f(x)$ (vi) $\lim_{x \to 4} f(x)$

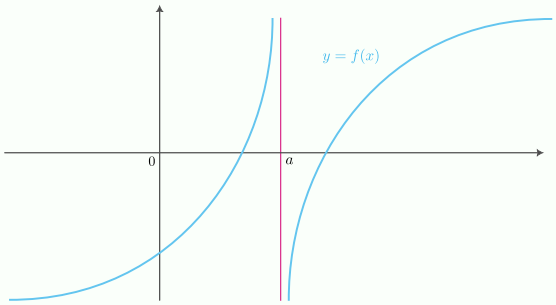
(vii) $\lim_{x \to 5^-} f(x)$ (viii) $\lim_{x \to 5^+} f(x)$ (ix) $\lim_{x \to 5} f(x)$

Problem 7

Consider the following graphs.





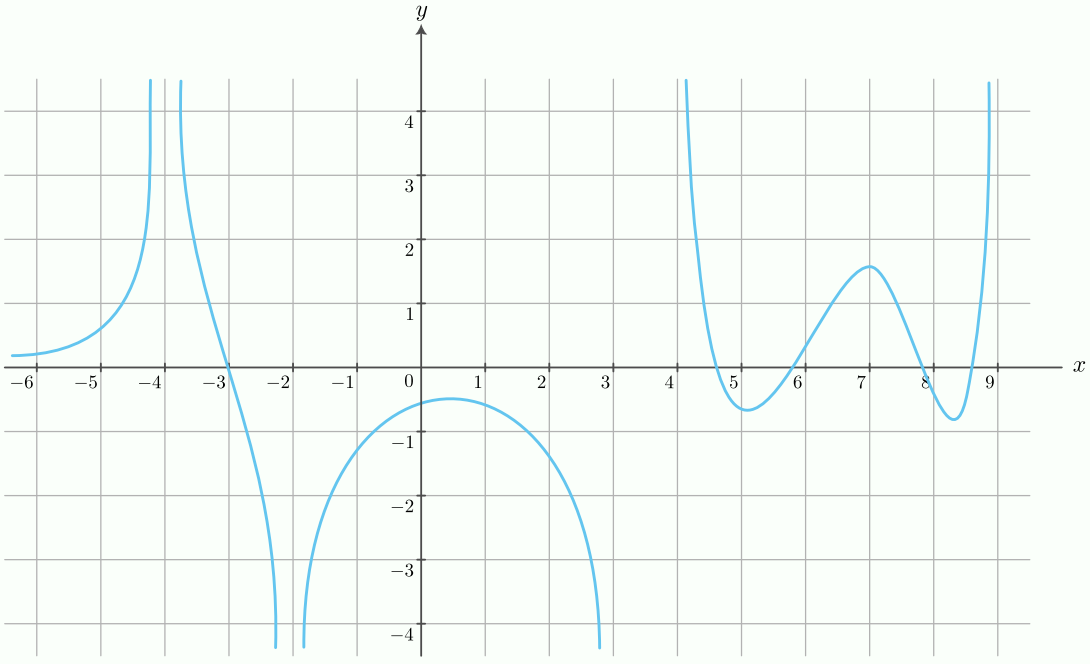


On each of the above graphs, find the following limits (if exist).

$$\lim_{x\to a^+} f(x), \quad \lim_{x\to a^-} f(x) \quad \text{and} \quad \lim_{x\to a} f(x)$$

Problem 8

Consider the following graph of some function $y = f(x)$.



Find the following limits.

(i) $\lim_{x\to -4} f(x)$ (ii) $\lim_{x\to -3} f(x)$ (iii) $\lim_{x\to -2} f(x)$

(iv) $\lim_{x\to 3} f(x)$ (iv) $\lim_{x\to 9} f(x)$ (vi) $\lim_{x\to -\infty} f(x)$

Problem 9

Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} 1+x \text{ if, } & x < -1 \\ x^2 \text{ if, } & -1 \le x \le 1 \\ 2-x \text{ if, } & x \ge 1. \end{cases}$$

Problem 10

Compute the following limits.

(i) $\lim_{t\to 0} \dfrac{\sqrt{t^2+9}-3}{t^2}$.

(ii) $\lim_{x\to 0} \dfrac{|x|}{x}$.

(iii) $\lim_{x\to 0} x^2 \sin\left(\dfrac{1}{x}\right)$.

(iv) $\lim_{x\to 0} \dfrac{t^2-9}{2t^2+7t+3}$.

(v) $\lim_{t\to 0}\left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right)$.

(vi) $\lim_{t\to 0} \dfrac{\sqrt{1+t}-\sqrt{1-t}}{t}$.

(vii) $\lim_{x\to 0} \dfrac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$.

(viii) $\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$.

(ix) $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0. \end{cases}$

(x) $\lim_{x\to 0} \dfrac{x\cos x - \log(1+x)}{x^2}$.

(xi) $\lim_{x\to 0}\left(\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right)$.

(xii) $\lim_{x\to 0} x\cot x$.

Problem 11

Find the $\lim_{x\to 1} g(x)$, where

$$g(x) = \begin{cases} x+1 & \text{if } x \neq 1 \\ \pi & \text{if } x = 1. \end{cases}$$

Problem 12

Find

$$\lim_{x \to 0} \frac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)}.$$

Problem 13

If

$$\alpha = \lim_{x \to 0} x \sin\left(\frac{1}{x}\right),$$

then find the value of 2024^{α}.

Problem 14

Compute

$$\lim_{x \to \infty} \left(\frac{1}{\sin x} - \frac{1}{\tan x}\right).$$

Problem 15

If

$$k = \lim_{\theta \to 0} \frac{\sin \theta}{\theta},$$

then what is the value of k^{2024}.

Problem 16

Find the limiting value of the ratio of the sum of square of n natural numbers to n natural numbers

Problem 17

Determine if the following limit exists. If yes, the find the value of the limit.

$$\lim_{x \to \infty} \frac{x^3 - \cos x}{x^2 + \sin^2 x}$$

Problem 18

Find the values of a and b such that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 2.$$