GATE 2025: Homework #2

Based on Limits

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Problem 1

Find

$$\lim_{x\to 0} \frac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)}.$$

Problem 2

If

$$\alpha = \lim_{x\to 0} x \sin\left(\frac{1}{x}\right),$$

then find the value of 2024^{α}.

Problem 3

Compute

$$\lim_{x\to\infty} \left(\frac{1}{\sin x} - \frac{1}{\tan x}\right).$$

Problem 4

If

$$k = \lim_{\theta\to 0} \frac{\sin\theta}{\theta},$$

then what is the value of k^{2024}.

Problem 5

Find the limiting value of the ratio of the sum of square of n natural numbers to n natural numbers

Problem 6

Determine if the following limit exists. If yes, the find the value of the limit.

$$\lim_{x\to\infty} \frac{x^3 - \cos x}{x^2 + \sin^2 x}$$