Algebraic Topology I

(MTH566)

Quiz 3

Wednesday, 4th February 2026

Name: ______________________________

Roll Number: ______________________________

Obtained Marks: ____________________ **/10**

EXAMINATION INSTRUCTIONS

1. This is a **Closed Book Examination**.
2. Answer all questions in the space provided on subsequent pages.
3. Show all necessary working steps clearly and legibly.
4. State any theorems or results used. Only results discussed in lectures may be used without proof.
5. **Duration:** 25 minutes.

- -

Good Luck!

Problem Set

Problem 1

The goal of this problem is to prove Brouwer fixed point theorem for one dimension. Let $\mathbb{D}^1 = [-1,1]$. Our goal is to prove that any continuous function $f : \mathbb{D}^1 \to \mathbb{D}^1$ has a fixed point.

(i) Let $f(-1) = a$ and $f(1) = b$. Can we assume that $a > -1$ and $b < 1$? What if either does not hold?

(ii) Let $\Delta = \{(x,x) : x \in \mathbb{D}^1\}$. It enough to show that $\Delta \cap$ ___ is non-empty. Fill in the blank with a reasoning.

(iii) Draw pictures of $\mathbb{D}^1 \times \mathbb{D}^1$ and Δ? Mark points $(-1,a)$ and $(1,b)$.

(iv) Complete the proof by assuming $\Delta \cap$ ___ $= \varnothing$ and get a contradiction.

$\boxed{0.5 + 0.5 + 1 + 2 = 4}$

Problem 2

Let $(G,\cdot)$ be a group. Show that G gives rise to a category $\mathscr{G}$ with single object, say $*$, such that

$$\mathrm{Ob}(\mathscr{G}) = \{*\} \text{ and } \mathrm{Hom}(*,*) = G.$$

$\boxed{3}$

Problem 3

Recall that for any space X, the *cone* is defined by

$$\frac{X \times [0,1]}{X \times \{0\}}.$$

Show that CS^1 is homeomorphic to $\mathbb{D}^2 = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$.

$\boxed{3}$

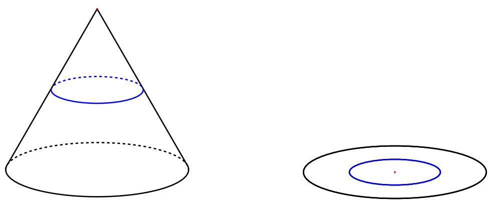

Figure 1: A picture solution

SOLUTION SPACE

Write your solution from the next page.

Begin Your Solution

Solution (continued)