Matrix Groups: Homework 1 Solution

Based on review of linear algebra

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Problem 1

Let T be a linear transformation from V to W. Show that $T(\mathbf{0}) = 0$.

Solution

Given that $T : V \to W$ is a linear transformation.

$$0 = T(\mathbf{0}) = T(\mathbf{0} + \mathbf{0}) = T(\mathbf{0}) + T(\mathbf{0}) \Rightarrow T(\mathbf{0}) = 0.$$

Problem 2

Describe geometrically the action of T on the square whose vertices are at $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.

1. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$
2. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$
3. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ y \end{pmatrix}$
4. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y \end{pmatrix}$

Solution

1. Reflection about x-axis.

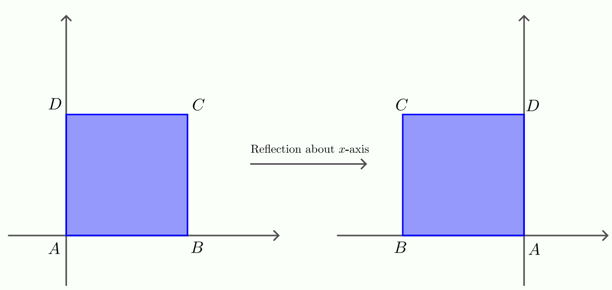

Figure 1: Reflection about the x-axis

2. Reflection about the line $y = x$.

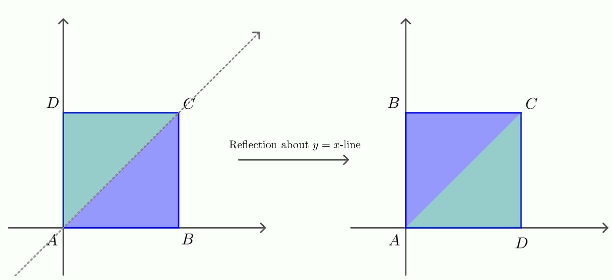

Figure 2: Reflection about the $y = x$-line

3. Stretch the square to the right by 2-times.

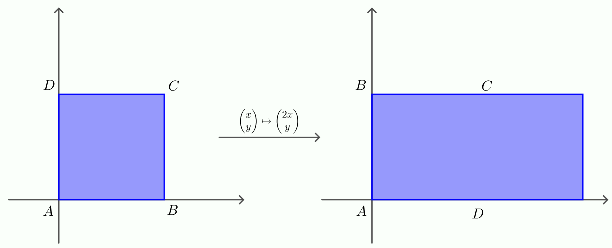

Figure 3: stretching the square.

Problem 3

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation defined by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y \\ 3x-z \\ 2y-3z \\ 4x+2y+z \end{pmatrix}.$$

Find the matrix of T with respect to the standard bases. Also find the matrix of T with respect to the bases

$$\{e_1+e_2, e_1-e_2, e_3\} \text{ and } \{e_1, e_2, e_3, e_4\}.$$

Solution