Limits

Engineering Mathematics-I

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Calculus for single variable

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- Intermediate Form

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- Riemann Integration
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- Improper integral
- Beta and Gamma functions and their properties

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$$0 \times \infty \qquad \lim_{x\to\infty} \frac{2x}{x^3-1} \cdot \ln x$$

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∞^0	$\lim\limits_{x \to \frac{\pi}{2}} (\tan x)^{x - \frac{\pi}{2}}$

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Theorem (L'Hospital Rule)

For a limit $\lim_{x \to a} \frac{f(x)}{g(x)}$ *of the indeterminate form* $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ *exists or equals to* $\pm\infty$.

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3. $\displaystyle \lim_{x \to 0} \frac{\sec x - x}{\sin x} = 0$

4. $\displaystyle \lim_{x \to 1} \frac{x^3 - x^2 \ln x + \ln x - 1}{x^2 - 1}$

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6. $\lim\limits_{t \to 0} \left(t + \dfrac{1}{t} \right) \left((4 - t)^{3/2} - 8 \right)$

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6. $\lim_{x \to 0} 2x \tan\left(\frac{\pi}{2} - x\right)$

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=&\lim_{x\to\infty}\left[\left(x - \sqrt{x^2+x}\right)\times\frac{\left(x + \sqrt{x^2+x}\right)}{\left(x + \sqrt{x^2+x}\right)}\right]
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&=\lim_{x\to\infty}\frac{x^2-(x^2+x)}{\left(x+\sqrt{x^2+x}\right)}
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&= \lim_{x \to \infty} \frac{x^2 - (x^2 + x)}{\left(x + \sqrt{x^2 + x} \right)} = \lim_{x \to \infty} \frac{-x}{x + \sqrt{x^2 + x}}
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&=\lim_{x\to\infty}\frac{-x/x}{x/x+\left(\sqrt{x^2+x}\right)/x}=-\frac{1}{1+1}=-\frac{1}{2}
\end{aligned}$$

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We can also write this in terms of $\frac{\infty}{\infty}$.

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