Complex Variables: Homework #1

Based on algebra of complex numbers

Dr. Sachchidanand Prasad

Problem 1

Let $\mathbb{C}$ denotes the set of all complex numbers. Then show the following.

1. Addition and product operations on $\mathbb{C}$ are commutative. That is, for any $z_1, z_2 \in \mathbb{C}$, we have
$$z_1 + z_2 = z_2 + z_1 \quad \text{and} \quad z_1 \cdot z_2 = z_2 \cdot z_1$$
2. Addition and product operations on $\mathbb{C}$ are associative. That is, for any $z_1, z_2, z_3 \in \mathbb{C}$, we have
$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3) \quad \text{and} \quad (z_1 \cdot z_2) \cdot z_3 = z_1 \cdot (z_2 \cdot z_3)$$

Problem 2

Represent the following complex numbers in the form of $a + \iota b$, where a and b are real numbers.

1. $\frac{1}{3+4\iota}$
2. $\frac{3+5\iota}{2-7\iota}$
3. $\frac{1}{\iota}$
4. $\frac{1}{x+\iota y}$, where $x^2 + y^2 = 7$.
5. $(1+\iota)^5$.

Problem 3

Let
$$z_1 = 2 + 3\iota, \quad z_2 = 3\iota, \quad z_3 = 3 - 4\iota \text{ and } z_4 = 1 - \iota.$$
Simplify the following.

1. $\frac{z_1 + z_2 \cdot z_3}{z_4}$.
2. $z_1 \cdot z_2 \cdot z_3 \cdot z_4$.
3. $\frac{\frac{z_1}{z_2}}{z_3} - z_4$.

Problem 4

Look at the following figure and write the corresponding complex number. Each grid shows one unit. For example, the complex number corresponding to the point $(2, 2)$ will be $2 + 2\iota$.

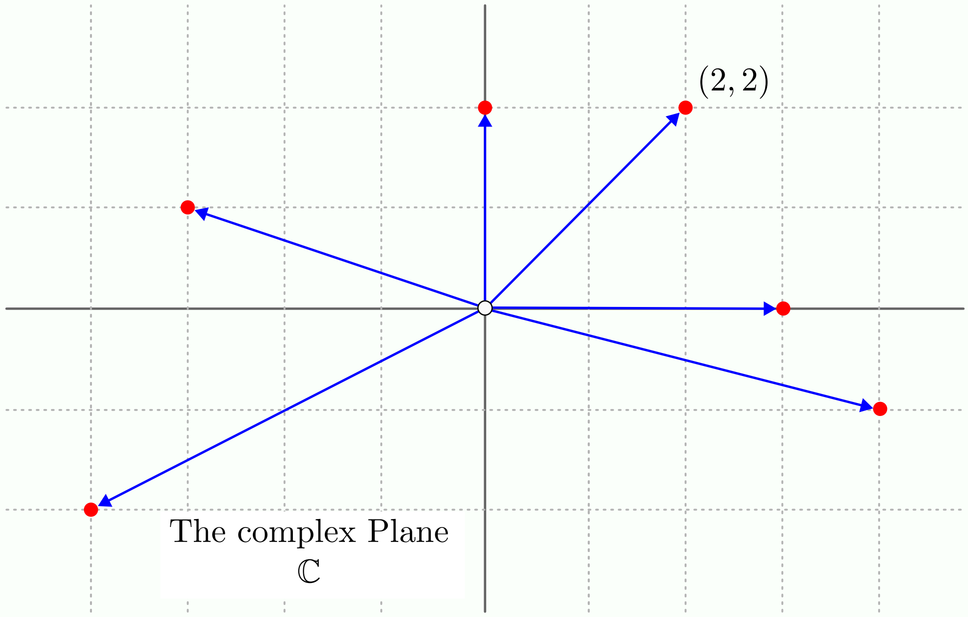

Figure 1: The complex plane

Problem 5

Geometrically demonstrate the following.

- Sum of two complex numbers.
- Product of complex numbers.

Problem 6

We want to understand the geometric meaning of difference of two complex numbers. Answer the following steps to understand the geometric meaning of difference of two complex numbers, say $z_1 - z_2$.

- Draw the complex number z_1 and z_2. It is an arbitrary choice, your drawing maybe different from your friends' drawing.
- Draw the complex number $-z_2$.
- Use the previous problem to draw the complex number $z_1 + (-z_2)$.