Algebraic Topology I: Homework #5

Based on paths and path connectedness

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Problem 1

Let X be a topological space. Define a relation $\sim$ on X by

$$x \sim y \Leftrightarrow \exists \text{ a path } \ \gamma : x \to y.$$

Show that $\sim$ is an equivalence relation on X.

Problem 2

Let X be a topological space. Recall that $\pi_0(X)$ denotes the colelction of path components of X (that is, the collection of equivalnece classes from Problem 1). The following are equivalent:

i) X is path connected.
ii) $\pi_0(X)$ is singleton.
iii) Any continuous function $f : \{0, 1\} \to X$ ha a continuous extension $F : [0, 1] \to X$.

Problem 3

i) For $n > 1$, $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^n$.
ii) The space $\mathbb{R} \times \{0\} \cup \{0\} \times \mathbb{R}$ is not homeomorphic to $\mathbb{R}$ or $\mathbb{R}^2$.

Problem 4

Show that the concatenation of paths is not associative. That is, if $\alpha_{[0,1]} : x \to y$, $\beta_{[0,1]} : y \to z$ and $\gamma_{[0,1]} : z \to w$, then it is not necessary that

$$(\alpha * \beta) * \gamma = \alpha * (\beta * \gamma).$$