Algebraic Topology I: Homework #6

Based on homotopy

Dr. Sachchidanand Prasad

Problem 1

Give the definitions of:

i) Homotopic maps.
ii) A contractible space.
iii) Homotopy equivalence between two topological spaces.

Explain briefly how (ii) is a special case of (iii).

Problem 2

Let X be a topological space. Show that any continuous function $f : X \to X$ is null homotopic if and only if X is contractible.

Problem 3

Prove that every convex subset of $\mathbb{R}^n$ is contractible.

Problem 4

Show that $\mathbb{R} \setminus \{0\}$ is not contractible.

Problem 5

Let X be a topological space and $C(X)$ its cone. Prove that $C(X)$ is contractible.

Problem 6

Give an example of a space which is path connected but not contractible.

Problem 7

Let X and Y be topological spaces.

i) Prove that if X and Y are contractible, then $X \times Y$ is contractible.
ii) Is the converse true? Justify your answer with either a proof or a counterexample.

Problem 8

Show that the annulus

$$A = \{(x,y) \in \mathbb{R}^2 \mid 1 \leq x^2 + y^2 \leq 2\}$$

is homotopy equivalent to S^1.

Problem 9

Let $X_1 = \mathbb{R}^2 \setminus \{(0,0)\}$ and $X_2 = \mathbb{R} \setminus \{(0,0),(1,0)\}$. Is X_1 and X_2 are homotopic equivalent?

Problem 10

Define explicitly a homotopy equivalence between the following two subspaces of $\mathbb{R}^2$.

$$X = S^1 \cup (\{0\} \times [-1,1])$$
$$Y = (S^1 + (-2,0)) \cup ([-1,1] \times \{0\}) \cup (S^1 + (2,0)).$$