# Algebraic Topology I

(MTH566)

## Quiz 2

*Wednesday, 21st January 2026*

**Name:** ____________________

**Roll Number:** ____________________

**Obtained Marks:** ______________ **/10**

## EXAMINATION INSTRUCTIONS

1. This is a **Closed Book Examination**.
2. Answer all questions in the space provided on subsequent pages.
3. Show all necessary working steps clearly and legibly.
4. State any theorems or results used. Only results discussed in lectures may be used without proof.
5. **Duration:** 25 minutes.

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*Good Luck!*

# Problem Set

## Problem 1

Let $\mathbb{R}$ be endowed with the standard topology. Define an equivalence relation $\sim$ on $\mathbb{R}$ by $x \sim y \iff x - y \in \mathbb{Z}$. What is the quotient space $\mathbb{R}/\sim$?

$\boxed{4}$

## Problem 2

Recall a quotient map between two topological spaces $X$ anf $Y$ is a map $q : X \to Y$ such that $U$ is open in $Y$ if and only if the inverse image $q^{-1}(U)$ is open in $X$. As noticed, we can assume that the quotient map is surjective.

(i) Let $q : X \to Y$ be a continuous map. Show that if there is a continuous map $f : Y \to X$ such that $q \circ f$ is identity on $Y$, then $q$ is a surjective quotient map.

(ii) If $A \subseteq X$, a *retraction* of $X$ onto $A$ is a continuous map $r : X \to A$ such that $r(a) = a$ for every $a \in A$. Show that a retraction is a quotient map.

$\boxed{3 + 2}$

## Problem 3

Consider the following quotient space $[0,1]^2/\sim$

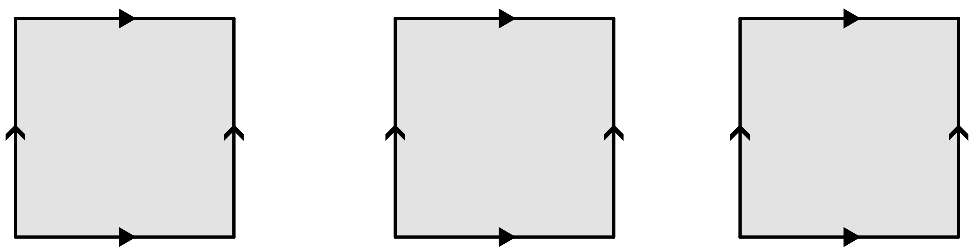

Figure 1: Draw open sets

(i) Draw a proper open neighbourhood of $(0,0)$.

(ii) Draw a proper open neighbourhood of $\left(\frac{1}{3}, 1\right)$.

(iii) Draw a proper open neighbourhood of $\left(0, \frac{1}{2}\right)$.

$\boxed{0.5 + 0.5 + 0.5 = 1.5}$

## SOLUTION SPACE

*Write your solution from the next page.*

## Begin Your Solution

**Solution** (continued)