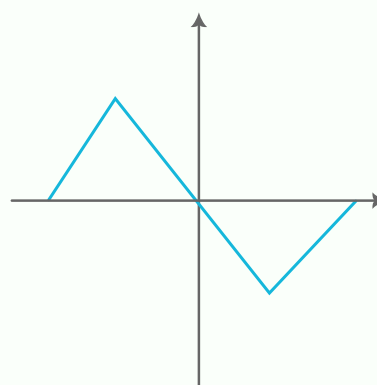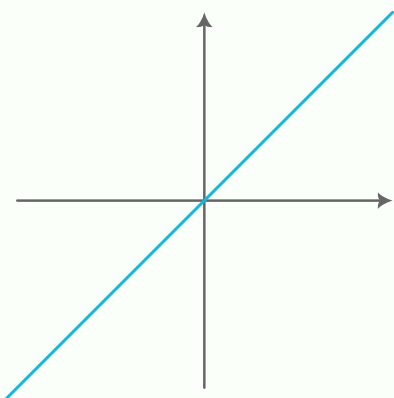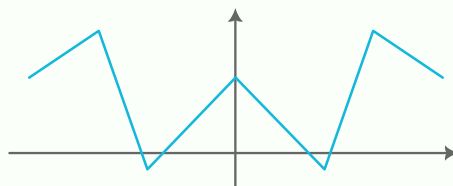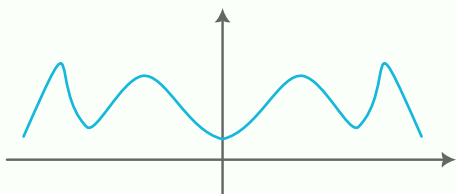# GATE 2025: Homework #2

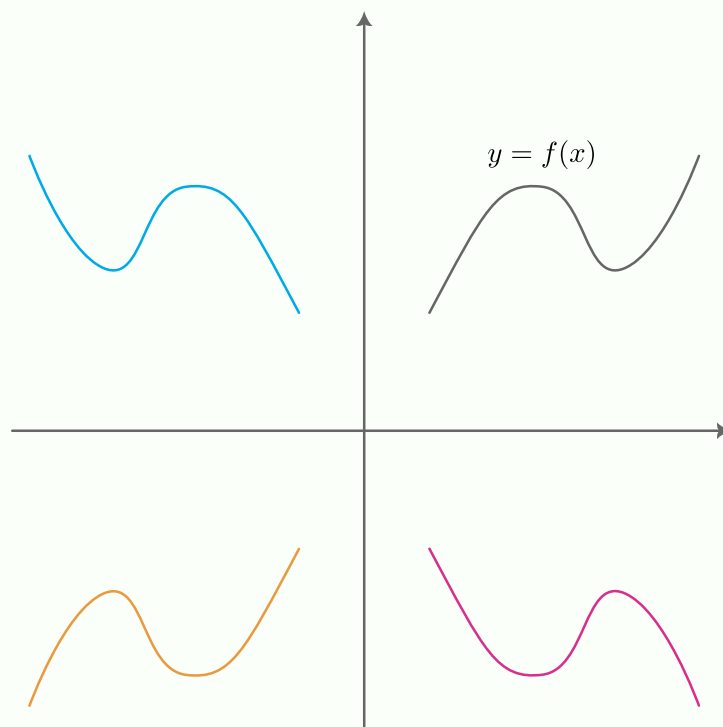Based on Limits

*Dr. Sachchidanand Prasad*

## Problem 1

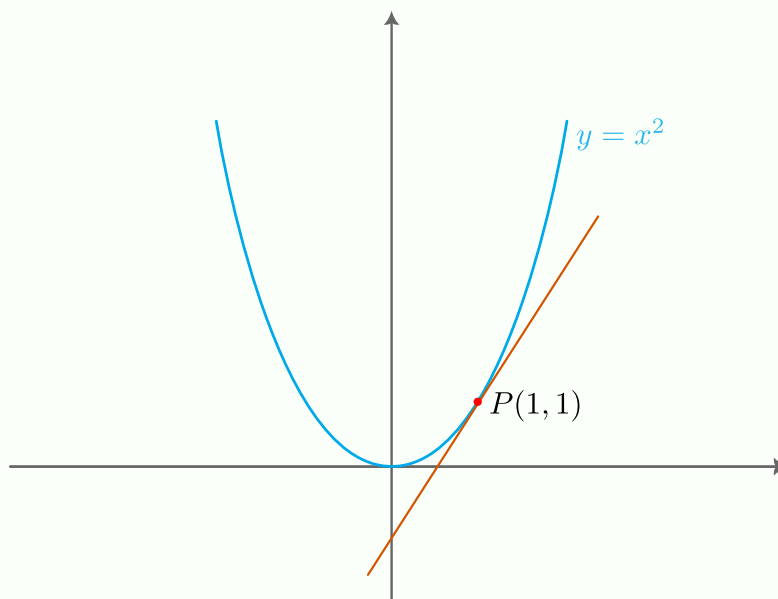Identify if the given function is even or odd.



## Problem 2

Look at the figure below. The graph of the function $y = f(x)$ is given. Identify the other graphs.

## Problem 3

Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.



## Problem 4

In the following problems find the average rate of change of the function over the given intervals.

1. $f(x) = x^3 + 1$

   a. $[2, 3]$          b. $[-1, 1]$

2. $g(x) = x^2 - 2x$

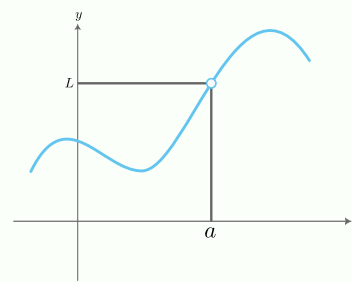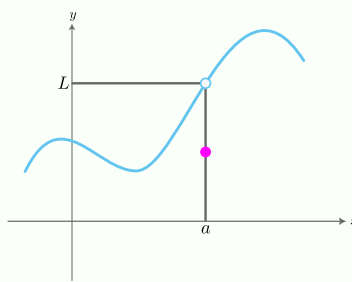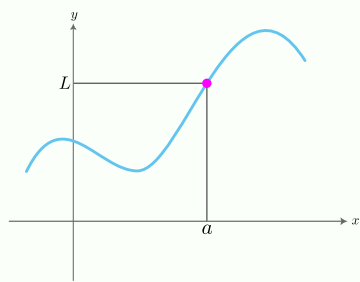   a. $[1, 3]$          b. $[-2, 4]$

3. $h(t) = 2 + \cos t$

   a. $[0, \pi]$          b. $[-\pi, \pi]$

## Problem 5

In the figures below, find the limit as $x$ approacher to $a$, that is, $\lim\limits_{x \to a} f(x)$.

## Problem 6

In the figures below, find the following limits.



$y = f(x)$
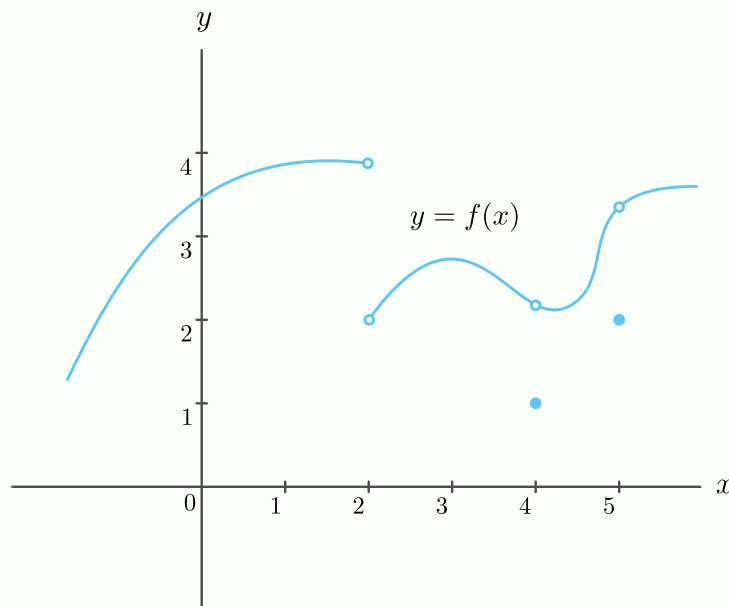
(i) $\lim\limits_{x \to 2^-} f(x)$

(ii) $\lim\limits_{x \to 2^+} f(x)$

(iii) $\lim\limits_{x \to 2} f(x)$

(iv) $\lim\limits_{x \to 4^-} f(x)$

(v) $\lim\limits_{x \to 4^+} f(x)$

(vi) $\lim\limits_{x \to 4} f(x)$

(vii) $\lim\limits_{x \to 5^-} f(x)$

(viii) $\lim\limits_{x \to 5^+} f(x)$

(ix) $\lim\limits_{x \to 5} f(x)$

## Problem 7

Consider the folloiwng graphs.

On each of the above graphs, find the following limits (if exist).

$$\lim_{x \to a^+} f(x), \quad \lim_{x \to a^-} f(x) \quad \text{and} \quad \lim_{x \to a} f(x)$$

## Problem 8

Consider the following graph of some function $y = f(x)$.
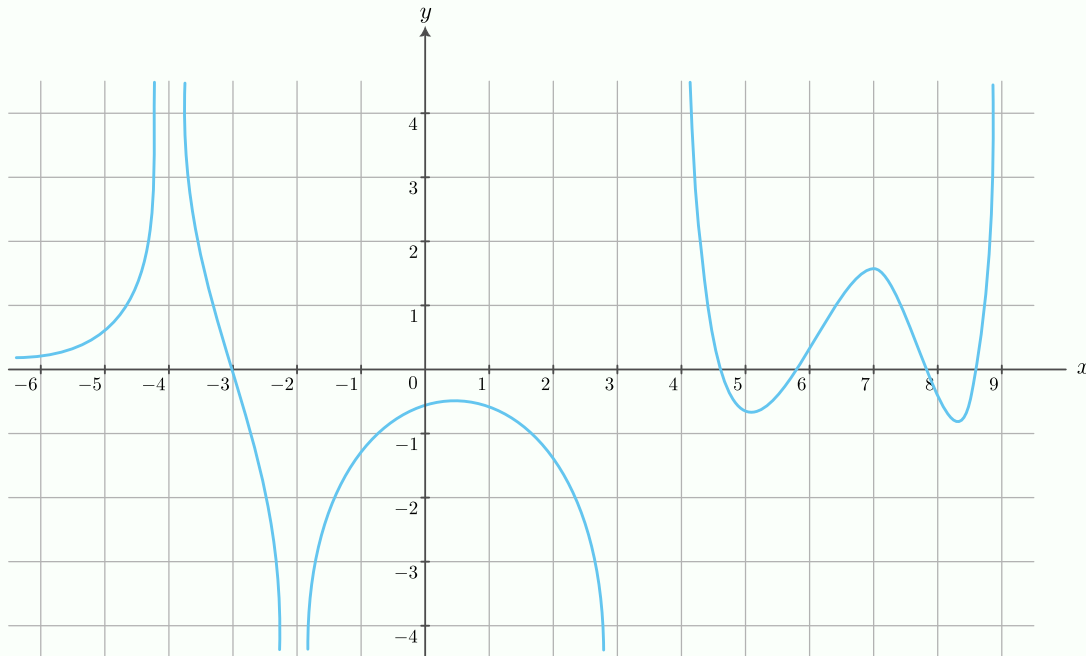


Find the folloiwng limits.

(i) $\lim_{x \to -4} f(x)$

(ii) $\lim_{x \to -3} f(x)$

(iii) $\lim_{x \to -2} f(x)$

(iv) $\lim_{x \to 3} f(x)$

(iv) $\lim_{x \to 9} f(x)$

(vi) $\lim_{x \to -\infty} f(x)$

## Problem 9

Sketch the graph of the function and use it to determine the values of $a$ for which $\lim\limits_{x \to a} f(x)$ exists.

$$f(x) = \begin{cases} 1 + x & \text{if, } x < -1 \\ x^2 & \text{if, } -1 \le x \le 1 \\ 2 - x & \text{if, } x \ge 1. \end{cases}$$

## Problem 10

Compute the following limits.

(i) $\lim\limits_{t \to 0} \dfrac{\sqrt{t^2 + 9} - 3}{t^2}$.

(ii) $\lim\limits_{x \to 0} \dfrac{|x|}{x}$.

(iii) $\lim\limits_{x \to 0} x^2 \sin\left(\dfrac{1}{x}\right)$.

(iv) $\lim\limits_{x \to 0} \dfrac{t^2 - 9}{2t^2 + 7t + 3}$.

(v) $\lim\limits_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t}\right)$.

(vi) $\lim\limits_{t \to 0} \dfrac{\sqrt{1 + t} - \sqrt{1 - t}}{t}$.

(vii) $\lim\limits_{x \to 0} \dfrac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$.

(viii) $\lim\limits_{x \to 0} \dfrac{\tan x - \sin x}{x^3}$.

(ix) $\lim\limits_{x \to 0} f(x)$, where $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \ne 0 \\ 2, & x = 0. \end{cases}$

(x) $\lim\limits_{x \to 0} \dfrac{x \cos x - \log(1 + x)}{x^2}$.

(xi) $\lim\limits_{x \to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right)$.

(xii) $\lim\limits_{x \to 0} x \cot x$.

## Problem 11

Find the $\lim\limits_{x \to 1} g(x)$, where

$$g(x) = \begin{cases} x + 1 & \text{if } x \ne 1 \\ \pi & \text{if } x = 1. \end{cases}$$

## Problem 12

Find

$$\lim_{x \to 0} \frac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)}.$$

## Problem 13

If

$$\alpha = \lim_{x \to 0} x \sin\left(\frac{1}{x}\right),$$

then find the value of $2024^\alpha$.

## Problem 14

Compute

$$\lim_{x \to \infty} \left(\frac{1}{\sin x} - \frac{1}{\tan x}\right).$$

## Problem 15

If

$$k = \lim_{\theta \to 0} \frac{\sin \theta}{\theta},$$

then what is the value of $k^{2024}$.

## Problem 16

Find the limiting value of the ratio of the sum of square of $n$ natural numbers to $n$ natural numbers

## Problem 17

Determine if the following limit exists. If yes, the find the value of the limit.

$$\lim_{x \to \infty} \frac{x^3 - \cos x}{x^2 + \sin^2 x}$$

## Problem 18

Find the values of $a$ and $b$ such that

$$\lim_{x \to 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 2.$$