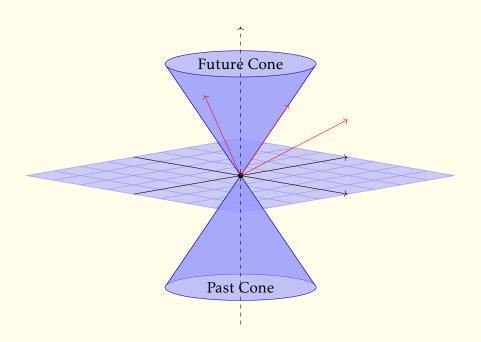
LECTURE NOTES

Lorentzian and Semi-Riemannian Geometry



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Course information

- Course name: Lorentzian and semi-Riemannian Geometry
- Instructor: Sachchidanand Prasad
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- Office 2.110 (Institute for Numerical and Applied Mathematics, Lotzestraße, 37073 Göttingen)
- **Classroom** : SZ (Mathematisches Institut)
- **Time** : Friday 12:15 13:45
- Course webpage : Link to the course website
- References:
 - [1] Global Lorentzian Geometry, by John K. Beem and Paul E. Ehrlich.
 - [2] Semi-Riemannian Geometry with Applications to Relativity, by Barrett O'Neill.
 - [3] Techniques of Differential Topology in Relativity, by Roger Penrose.
- Evaluation: The evaluation will be done by presentation of a project. A paper will be assigned to a group of students and they will present it in the class. The project will be evaluated based on the presentation and the report (a detailed writeup of the project) submitted.
- **Papers**: The following papers/projects will be assigned to the students for presentation:
 - ► Existence of Lorentzian Metric on a Smooth Manifold [O'N83]
 - ► Wrapped products [O'N83]
 Study of the wrapped product of Lorentzian manifolds.
 - ▶ Paracompactness of Lorentzian Manifolds
 A smooth Hausdorff manifold admitting a Lorentzian metric is paracompact.
 A Condition for Paracompactness of a Manifold[Mar73]
 K. B. Marathe

References

▶ Timelike Cut Locus

Study of the timelike cut locus in space-time geometry.

The Space-Time Cut Locus [BE79]

J. K. Beem and P. E. Ehrlich

▶ Null Cut Locus

Exploration of the null cut locus in Lorentzian geometry.

The Space-Time Cut Locus[BE79]

J. K. Beem and P. E. Ehrlich

▶ Morse-Index Theorem for Null Geodesics

A theorem relating the Morse index to null geodesics in space-time.

A Morse Index Theorem for Null Geodesics[Bee75]

J. K. Beem

Comparison Theorems in Lorentzian Geometry

Cut points, conjugate points, and their role in comparison theorems.

Cut Points, Conjugate Points, and Lorentzian Comparison Theorems [BE76]

J. K. Beem and P. E. Ehrlich

▶ Geodesic completeness

Geodesic completeness in submanifolds of Minkowski space.

Geodesic Completeness of Submanifolds in Minkowski Space [BE80]

J. K. Beem and P. E. Ehrlich

References

- [BE76] John K. Beem and Paul E. Ehrlich. Cut Points, Conjugate Points, and Lorentzian Comparison Theorems. *Mathematical Proceedings of the Cambridge Philosophical Society*, 79(1):73–83, 1976.
- [BE79] John K. Beem and Paul E. Ehrlich. The Space-Time Cut Locus. *General Relativity and Gravitation*, 11(2):89–103, 1979.
- [BE80] John K. Beem and Paul E. Ehrlich. Geodesic Completeness of Submanifolds in Minkowski Space. *General Relativity and Gravitation*, 12(10):897–908, 1980.
- [Bee75] John K. Beem. A Morse Index Theorem for Null Geodesics. *Duke Mathematical Journal*, 42(4):707–714, 1975.
- [Mar73] K. B. Marathe. A Condition for Paracompactness of a Manifold. *Journal of Differential Geometry*, 7(3-4):571–573, 1973.
- [O'N83] Barrett O'Neill. Semi-Riemannian Geometry with Applications to Relativity, volume 103 of Pure and Applied Mathematics. Academic Press, San Diego, 1983.