GATE 2025: Homework #1

Based on Functions

Dr. Sachchidanand Prasad

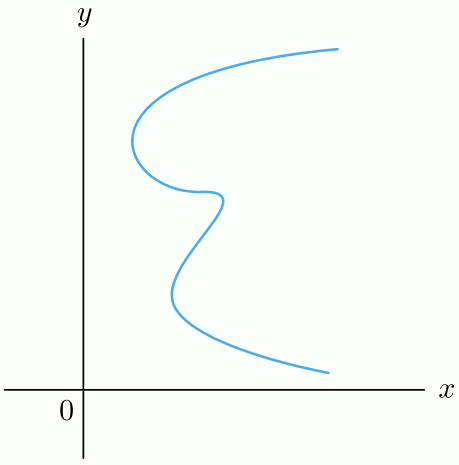
Problem 1

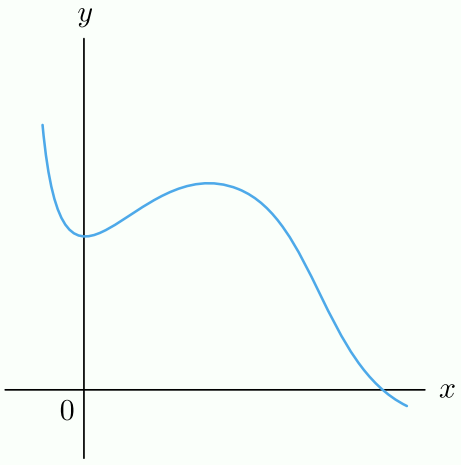
Find the domain and range of each functions.

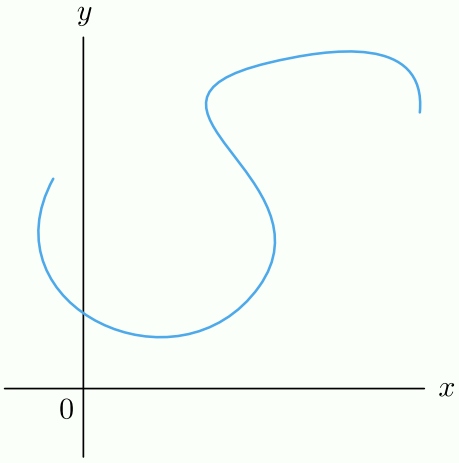
1. $f(x) = 1 + x^2$
2. $g(t) = \frac{2}{t^2 - 16}$
3. $h(s) = \sqrt{x^2 - 3x}$
4. $p(x) = \frac{4}{3 - x}$
5. $s(x) = \sqrt{x^2 + 1}$

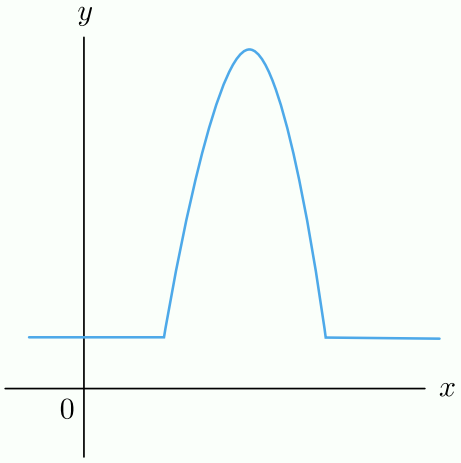
Problem 2

Which of the following are graphs of functions of x?









Problem 3

Express the area and perimeter of an equilateral triangle as a function of the triangle's side length x.

Problem 4

Consider the point (x, y) lying on the graph of the line $2x + 4y = 5$. Let ℓ be the distance from the point (x, y) to the origin $(0, 0)$. Write ℓ as a function of x.

Problem 5

Find the domain of each functions.

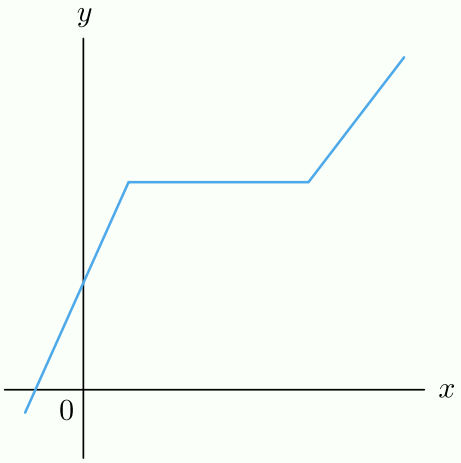
1. $f(x) = \frac{x+3}{4-\sqrt{x^2-9}}$.
2. $g(t) = \frac{t}{|t|}$.
3. $h(x) = \sqrt{1-x^2}$.
4. $s(t) = \sqrt{-t}$.

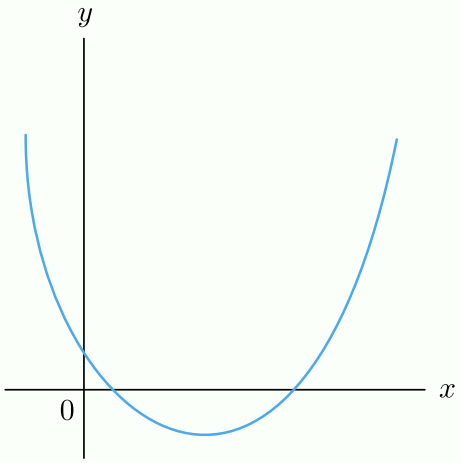
Problem 6

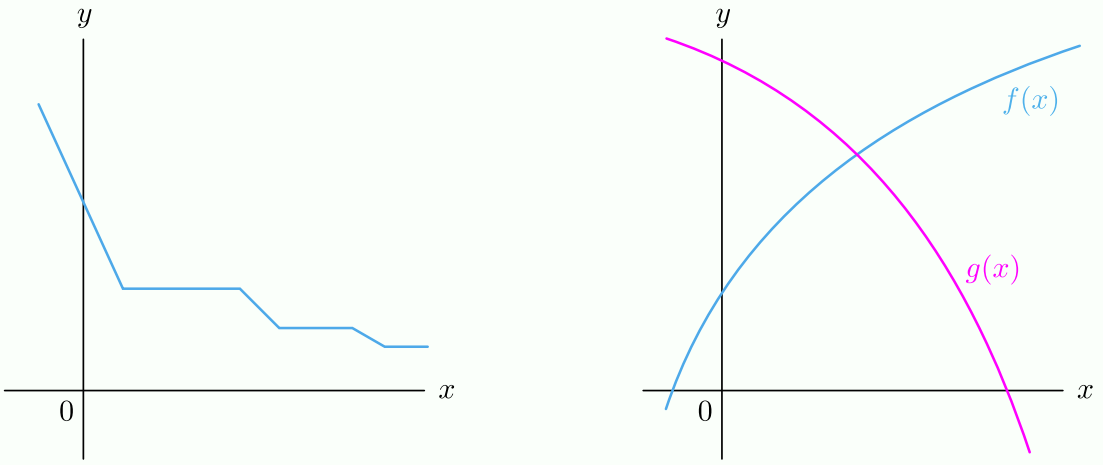
How many points are there in the range of a constant function $f : \mathbb{R} \to \mathbb{R}$?

Problem 7

Write if the functions are **increasing**, **decreasing**, **strictly inccreasing** or **strictly decreasing**.







Problem 8

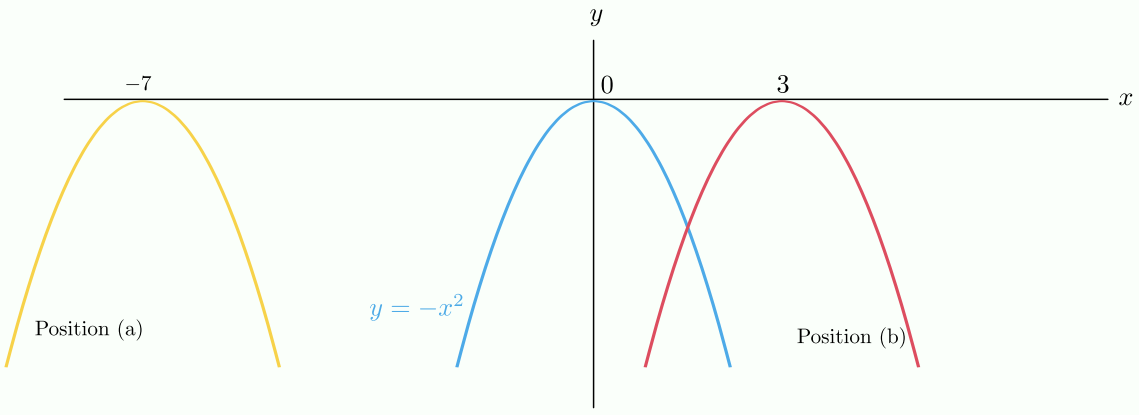
Write the function after the given transformations.

1. $f(x) = \sqrt{x}$.
   - Upward 4 units.
   - Right side 10 units.
2. $f(x) = \sin x + \tan x + e^{x^2}$.
   - Towards left 20 units.
   - Downward 5 units.
   - Towards right 20 units.
   - Upward 10 units.

Problem 9

The accompanying figure shows the graph of $y = -x^2$ shifted to two new positions. Write equations for the new graphs.

1.



2.