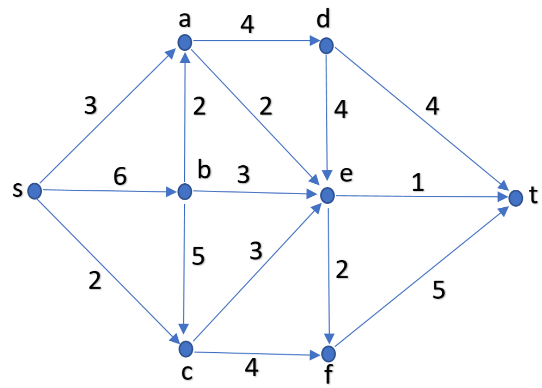
Algorithms - CS5800 - Summer 2, 2017 - Assignment 5

1. Apply the Ford-Fulkerson algorithm on the following flow network to find a maximal flow and a minimal cut.



   Explain the steps of your work.
   For the last iteration of the algorithm:

   (a) Draw the network with the capacities of each edge and the values of the flow on each edge (before the last iteration is performed).

   (b) Draw the residual network.

   (c) Apply the BFS algorithm on the residual network to find a path from s to t.

   (d) Explain how to define the "improved flow".

   (e) Draw the residual network for the improved flow.

   (f) Show a minimal cut of the (original) flow network.

2. Let $G = (V, E)$ be a flow network such that the capacity of each edge is either 5 or 4.
   Prove or disprove: The value of a maximal flow for G can't be 11.

3. The teams in the NBA chooses new players each year in a process called "the draft". Each year there are n teams $t_1, ..., t_n$ and $2n$ players.
   The league wants to change the rules of the draft so that each team will give a list of players that it is willing to get and some algorithm will match 2 players for each team, out of the list of players the team is willing to get.
   For a team t_i let us denote by A_{t_i} the set of players that the team is willing to get.
   Show that a necessary and sufficient condition for it to be possible to give each team 2 players out of the list of players it is willing to get, is that:

$$\left| \bigcup_{i \in I} A_{t_i} \right| \geq 2|I|$$

   for any subset $I \subseteq \{1, ..., n\}$.

4. Let $G = (V, E)$ be a flow network with capacity $c : E \to \mathbb{R}_{\geq 0}$.
For 2 s-t cuts $(S, \overline{S})$ and $(S', \overline{S'})$, we define their intersection to be $(S \cap S', \overline{S \cap S'})$.
Show that if $(S, \overline{S})$ and $(S', \overline{S'})$ are both minimal cuts, then their intersection $(S \cap S', \overline{S \cap S'})$ is also a minimal cut.

5. Optional (A bonus might be given for a good and elegant solution)
Let $G = (V, E)$ be a flow network with integer capacities:

$$c : E \to \mathbb{N}$$

Find an algorithm that finds a minimal cut $(S, \overline{S})$ of G with minimal number of edges of the form (u, v) with $u, \in S$, and $v \in \overline{S}$.