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| College of Computer & Information Science | CS5800 |
| Northeastern University | MS Algorithms |
| Fall 2017 | 11 September 2017 |

# Problem Set 1 (due Friday, September 22, 5:59 PM)

**Instructions:**

- The assignment is due at the time and date specified. Late assignments will not be accepted.
- We encourage you to attempt and work out all of the problems on your own. You are permitted to study with friends and discuss the problems; however, *you must write up you own solutions, in your own words.*
- If you do collaborate with any of the other students on any problem, please do list all your collaborators in your submission for each problem.
- Finding solutions to homework problems on the web, or by asking students not enrolled in the class (or the class staff) is strictly prohibited.
- We require that all homework submissions be neat, organized, and *typeset.* You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. If you need to draw any diagrams, however, you may draw them with your hand.

**1. (6 points) Instability of a local improvement algorithm**

As we discussed in class, there may be several approaches to finding a stable matching. One reasonable approach is the following. Suppose we start with an arbitrary matching, and then repeat the following step until there are no unstable pairs.

- If there exists unstable pairs $(m, w)$ and $(m', w')$ such that $m$ prefers $w'$ over $w$ and $w'$ prefers $m$ over $m'$, then replace the pairs by $(m, w')$ and $(m', w)$.

The above local improvement algorithm is fairly natural. But it does not work! Cycles may occur, especially if you choose the "wrong" unstable pairs to swap, causing the algorithm to loop forever. Consider the following preference lists with 3 women $A$ $B$, $C$, and 3 men $U$, $V$, and $W$.

| A | B | C | U | V | W |
|---|---|---|---|---|---|
| U | W | U | B | A | A |
| W | U | V | A | B | B |
| V | V | W | C | C | C |

For the above preference list, show that there exists a 4-step cycle in the local improvement algorithm, starting with the matching $\{(A, U), (B, V), (C, W)\}$.

**2. (6 points) Stability in competition** Chapter 1, Exercise 3, page 22.

**3. (6 points) Ordering functions**

Arrange the following functions in order from the slowest growing function to the fastest growing function. Briefly justify your answers. (*Hint:* It may help to plot the functions and obtain an estimate of their relative growth rates. In some cases, it may also help to express the functions as a power of 2 and then compare.)

$$\sqrt{n} \qquad n\sqrt{\lg n} \qquad 2^{\sqrt{\lg n}} \qquad (\lg n)^2$$

**4. (6 points)** Chapter 2, Exercise 6, page 68.

**5. (6 points) Properties of asymptotic notation**

Let $f(n)$, $g(n)$, and $h(n)$ be asymptotically positive and monotonically increasing functions. For each of the following statements, decide whether you think it is true or false and give a proof or a counterexample.

**(a)** If $f(n) = \Omega(h(n))$ and $g(n) = O(h(n))$, then $f(n) = \Omega(g(n))$.

**(b)** If $f(n) = O(g(n))$, then $f(n)^2$ is $O(g(n)^2)$.