First name:____________ Last name:____________

College of Computer & Information Science — CS5800

Northeastern University — Algorithms

Fall 2017 — October 23 2017

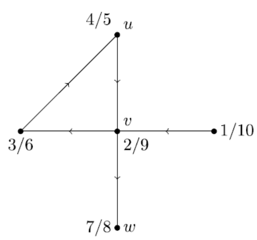
Quiz 2

Instructions:

- This quiz is closed book and closed notes. Please use both sides of the page.
- Please write clearly and legibly. Grading will be based on both clarity and correctness.

1. **(4 points)**
   Prove or disprove the following proposition:
   If $G = (V, E)$ is a directed graph, and $(u, v), (v, w) \in E$ are two edges in G (G has the edges $u \to v \to w$), then whenever we apply the DFS algorithm on G we get $d(w) < f(u)$. (Recall that $d(v)$ denotes the discovery time of v when applying the DFS algorithm. Similarly, $f(v)$ denotes the finishing time of v when applying the DFS algorithm.)

   Answer: The claim is incorrect. The following graph with the DFS execution described in the picture is a counter example.



In this graph we have the edges (u, v) and (v, w) but for the DFS execution that is described in the picture we get $d(w) > f(u)$.

2. **(6 points)**
For a directed graph $G = (V, E)$, we say that $v_0 \in V$ is a root if for any vertex $u \in V$ there exists a path from v_0 to u in the graph G ($v_0 \overset{G}{\rightsquigarrow} u$ in G).
Give an algorithm that takes as input a directed ***acyclic*** graph $G(V, E)$ and returns the set of all the roots of G. State the worst-case running time of your algorithm in terms of the number of vertices and number of edges of G.

Your grade for this question will be determined on the basis of the correctness of your algorithm and its efficiency, given by its worst-case running time. Partial credit may be given for non-optimal algorithms provided they are correct and well explained.

Answer: Since G is acyclic we can use the finishing times of the DFS algorithm to find a topological sorting of G. Let $\phi : V \to \mathbb{N}$ be a topological sort of G. Clearly, the fact that G is acyclic also means that it can have only one root. If G has a root v_0 then ϕ must get its minimal value on v_0. It follows that it is enough to check if all the vertices are reachable from the vertex v_0 for which $\phi(v_0)$ is minimal.

A possible algorithm is:

(a) Find a topological sorting of G (using the DFS algorithm).

(b) Check if all the vertices are reachable from the vertex v_0 that has minimal value of the topological sorting (this can be done using DFS or BFS for example). If all the vertices are reachable return $\{v_0\}$, else return Φ.

Clearly the complexity of this algorithm is linear $O(n + m)$.