Answer 4:

Logic –

Modify dp for finding longest common subsequence (LCS) of two strings to find LCS of three strings.

{let X be the first string with length l, Y be second string with length m and Z be third string with length n}

LCS3(X[:l], Y[:m], Z[:n])

Start comparing characters from the end of strings.

Then depending on this outcome, problem will be divided into 2 cases.

1. Characters from all strings matches

In this case, algorithm will append character in the list of common subsequences (CS) and then run LCS3 on the remaining string

*if (X[l] == Y[m] == Z[n])*

*cs.append(X[l])*

*cs.append(LCS3(X[:l-1], Y[:m-1], Z[:n-1]))*

1. Characters from all or two strings do not match.

In this, again three situations arise,

1. Decrement length of string one by removing last character from input string and run LCS3
2. Decrement length of string two by removing last character from input string and run LCS3
3. Decrement length of string three by removing last character from input string and run LCS3
4. Return the list with maximum length out of above 3 and append it in the original CS list

*if ( X[l] != Y[m] || Y[m] != Z[n] || X[l] != Y[n] )*

*getListWithMaximumLength (*

*LCS3(X[:l-1], Y[:m], Z[:n]),*

*LCS3(X[:l], Y[:m-1], Z[:n]) ,*

*LCS3(X[:l], Y[:m], Z[:n-1])*

*)*

Base cases for this algorithm, they will be as follows –

1. If length of any one or all string becomes less than 0 then return the CS.

*if ( l < 0 || m < 0 || n < 0 )*

*return cs*

Recurrence –

LCS3(X[0:l], Y[0:m], Z[0:n]) =

Correctness –

Recurrence that we have taken into consideration is correct.

Proof –

Proof by induction.

Base Case :

Let, l, m and n are length of string one, two and three respectively.

Either l < 0 or m < 0 or n < 0

This is trivial case, if l < 0; then value of X[l] is undefined thus, common subsequence for all 3 strings at

that time will be an empty string.

Similar case will arise if any of the ‘m’ or ‘n’ is less than 0.

Induction Step :

Suppose the recurrence holds true till indices l + m + n – 1;

Let us consider we have to compare strings X[0:l]; Y[0:m] & Z[0:n].

Now there will be two conditions either last character of all strings matches or they don’t.

X

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |

Y Z

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |

In case X[l] == Y[m] == Z[n] == , then according to our algorithm the longest common subsequence of X, Y and Z will be: – ‘’ + longest common subsequence of X[0:l-1], Y[0:m-1], Z[0:n-1].

We will try to prove this by contradiction. Suppose the longest common subsequence does not include then this means that there exists a common subsequence that does not end with however, this is not possible as the last character of every string is Thus, the longest common subsequence must have in it. Hence the recurrence that we got from our algorithm is correct.

Now suppose, last characters of all strings are not matching.

Let`s say Z[n] != , then it might be the case that is second last character in Z, thus we need to check longest common subsequence of X[0:l], Y[0:m], Z[0:n-1]. Similarly for the case where Y[n] != longest common subsequence can be found by analyzing X[0:l], Y[0:m-1], Z[0:n]. Also, we can write same argument for X. Thus longest common subsequence will be string with maximum length out of LCS3(X[0:l], Y[0:m], Z[0:n-1]) and LCS3(X[0:l], Y[0:m-1], Z[0:n]) and LCS3(X[0:l-1], Y[0:m], Z[0:n]). This is exactly same recurrence returned by algorithm.

Hence from base case and induction step we can say that our recurrence is correct.

Algorithm –

|  |
| --- |
| ArrayList<char> cs = new LinkedList<char>(max (l, m, n));  ArrayList<char> LCS3(String X, String Y, String Z) {  int l = X.length() - 1;  int m = Y.length() - 1;  int n = Z.length() - 1;          /\* Following steps build cs[l+1][m+1][n+1] in bottom up fashion.             Note that L[i][j][k] contains length of LCS of X[0..i-1] and Y[0..j-1]  and Z[0.....k-1]          \*/  String[][][] cs = new String[l+1][m+1][n+1];            for (int i=0; i<=l; i++) {              for (int j=0; j<=m; j++) {                  for (int k=0; k<=n; k++) {                      if (i == 0 || j == 0|| k == 0)                      cs[i][j][k] = "";                      else if (X.charAt(i) == Y.charAt(j)  &&  X.charAt(i)==Z.charAt(k))                      cs[i][j][k] = cs[i-1][j-1][k-1].append(X.charAt(i));                      else                      cs.[i][j][k] =                      getListWithMaximumLength (                      cs[i-1][j][k],                      cs[i][j-1][k],                      cs[i][j][k-1]);                  }              }          }            /\* L[l][m][n] contains length of LCS for X[0..l-1] and Y[0..m-1] and Z[0..n-1]\*/          return cs[l][m][n];  }  ArrayList<char> getListWithMaximumLength (ArrayList<char>A, ArrayList<char>B, ArrayList<char>C) {  if (A.length() > B.length()  &&  A.length() > C.length())          return A;  if (B.length() > A.length()  &&  B.length() > C.length())          return B;  if (C.length() > A.length() &&  C.length() > B.length())          return c;  } |

Answer 5:

Logic –

Let i and j represents the lower and upper bound on the number of coins remaining and

DP(i, j) represents the maximum value user can collect from coins between i and j and

Vi represents the value of the coin at the ith position.

Logic for this algorithm is to maximize the value of all coins collected by first user and minimize the

values collected by second. As there are two ways for user to collect the coins, the function will have two conditions and it will return the maximum value out of them.

Recurrence –

DP(i, j)

Correctness –

Algorithm –

|  |
| --- |
| public int[][] maximizeProfit(int[] arr) {  // Returns optimal maximum cash first player can collect from given 'n' coins  // let us assume that 'n' is even, so that every user gets a fair chance        // Create a table to store solutions of subproblems     int[][] val = new int[n][n];      for (int interval = 0; interval < n; ++interval)  // Fill matrix val using above recursive formula.  // map is filled from diagonal to map[0][n-1] which is the result.      {          for (int i = 0, j = interval; j < n; ++i, ++j)          {              int firstChoseFirst =  arr[i];              int firstChoseLast  =  arr[j];              int bothUsersChoseFirst =  0;              int bothUsersChoseLast =  0              int firstChoseFirstSecondChoseLast =  0;              int firstChoseLastSecondChoseFirst =  0;                if ((i+2) <= j)              bothUsersChoseFirst = val[i+2][j];                if ((i+1) <= (j-1))              firstChoseFirstSecondChoseLast =  val[i+1][j-1;                firstChoseLastSecondChoseFirst = firstChoseFirstSecondChoseLast;                if (i <= (j-2))              bothUsersChoseLast  =   val[i][j-2];              val[i][j] = Math.max(              firstChoseFirst              + Math.min( bothUsersChoseFirst,              firstChoseFirstSecondChoseLast),              firstChoseLast              + Math.min( firstChoseLastSecondChoseFirst,              bothUsersChoseLast)              );          }      }      return val[0][n-1];  } |