1 Set Theory

1.1 Basic Definitions

$$\begin{aligned} \mathbb{N} &= \{0, 1, 2, \ldots\} && \text{natural numbers} \\ \mathbb{Z} &= \{\ldots, -1, 0, 1, \ldots\} && \text{integers} \\ \mathbb{Q} &= \{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\} && \text{rationals} \\ \mathbb{R} &= \text{real numbers} \\ \mathbb{C} &= \{a + bi \mid a, b \in \mathbb{R}\} && \text{complex numbers} \end{aligned}$$

Interval notation: $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$
Set-builder notation: $\{x \mid P(x)\}$ where $P(x)$ is a property of x

1.2 Set Operations

$$\begin{aligned} A \cup B &= \{x \mid x \in A \text{ or } x \in B\} && \text{union} \\ A \cap B &= \{x \mid x \in A \text{ and } x \in B\} && \text{intersection} \end{aligned}$$

Venn diagrams are used to visualize set operations.

$$\begin{aligned} A \setminus B &= \{x \in A \mid x \notin B\} && \text{set difference} \\ A \triangle B &= (A \cup B) \setminus (A \cap B) && \text{symmetric difference} \end{aligned}$$

1.3 Properties

Let A_α be an indexed family of sets (where $\alpha \in I$).

$$\begin{aligned} \bigcup_{\alpha \in I} A_\alpha &= \{x \mid x \in A_\alpha \text{ for some } \alpha \in I\} \\ \bigcap_{\alpha \in I} A_\alpha &= \{x \mid x \in A_\alpha \text{ for every } \alpha \in I\} \end{aligned}$$

De Morgan's Laws:

$$\begin{aligned} \overline{\bigcup_{\alpha \in I} A_\alpha} &= \bigcap_{\alpha \in I} \overline{A_\alpha} \\ \overline{\bigcap_{\alpha \in I} A_\alpha} &= \bigcup_{\alpha \in I} \overline{A_\alpha} \end{aligned}$$

1.4 Functions

A *function* is a mapping or transformation from one set to another.
Let A, B be sets. $A \to B$ denotes a function from A to B.
Injective (one-to-one): If $f(x_1) = f(x_2)$, then $x_1 = x_2$.
Surjective (onto): For every $y \in B$, there exists $x \in A$ such that $f(x) = y$.
Bijective: Both injective and surjective.
If $f : A \to B$ is bijective, then there exists an inverse function $f^{-1} : B \to A$.

1.5 Composition of Functions

If $f : A \to B$ and $g : B \to C$, then the composition $g \circ f : A \to C$ is defined by $(g \circ f)(x) = g(f(x))$.