Sets

$\mathbb{N} = \{0, 1, 2, \dots\}$ natural numbers

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ integers

$\mathbb{Q} = \{ \frac{k}{n} \mid k, n \in \mathbb{N}, \ n \neq 0 \}$

rationals

$\mathbb{R}$ = real numbers

$\mathbb{C} = \{ a + ib \mid a, b \in \mathbb{R} \}$ = complex

i = "imaginary unit"
characterized by $i^2 = -1$.

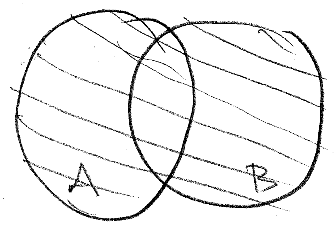
$\mathbb{R}^d = \{ (a_1, \dots, a_d) \mid a_i \in \mathbb{R} \}$ d-dimensional space

$\mathbb{R}^\infty = \{ (a_0, a_1, a_2, \dots) \mid a_i \in \mathbb{R} \}$
space of <u>infinite sequences</u> of reals

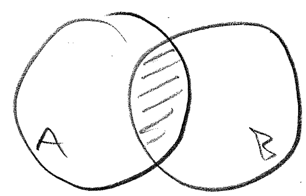
A, B sets. Operations:

$A \cup B = \{ c \mid c \in A \text{ or } c \in B \}$ union

$A \cap B = \{ c \mid c \in A \text{ and } c \in B \}$ intersection



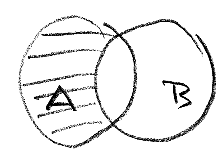
$A \cup B$



$A \cap B$

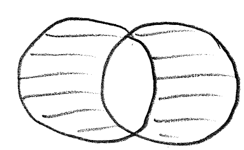
$A \setminus B = \{ c \in A \mid c \notin B \}$:

"difference"



$A \setminus B$

$A \Delta B = A \setminus B \cup B \setminus A$:



$= (A \cup B) \setminus A \cap B$ etc.

"symmetric difference"

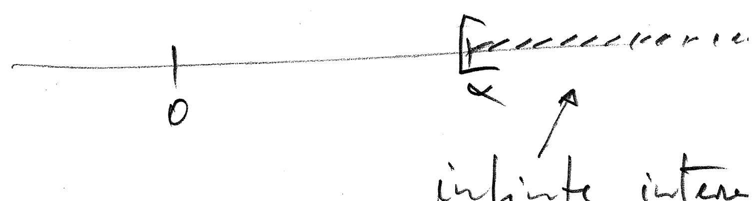
Let $(A_\alpha)_{\alpha \in I}$ be an indexed family of sets (index set I).

EX ① $I = \mathbb{N}$,

$$A_\alpha \overset{\text{definition}}{:=} \{\alpha, \alpha+1\}$$

② $I = \mathbb{R}^+ = \{x \in \mathbb{R} \mid x \geq 0\}$

$$A_\alpha := [\alpha, \infty)$$



Union: $\displaystyle\bigcup_{\alpha \in I} A_\alpha := \{a \mid a \in A_\alpha \text{ for some } \alpha \in I\}$

$$= \{a \mid \exists\, \alpha \in I \text{ such that } a \in A_\alpha\}$$

Intersectn $\displaystyle\bigcap_{\alpha \in I} A_\alpha = \{a \mid a \in A_\alpha \underbrace{\text{fore every } \alpha \in I}_{\forall \alpha \in I}\}$.

<u>Maps</u> in general.

Map = function = mapping (= transformation)

Let A, B be sets $(\neq \emptyset)$. A map f

$$f : A \longrightarrow B$$
$$\quad\;\; a \longmapsto f(a)$$

is an <u>assignment</u>: to $\forall a \in A$ f assigns a unique $b \in B$ which is called the <u>value of f at a</u> and is denoted by $f(a)$.

<u>Note</u>: f assigns to <u>$\forall\, a \in A$</u> a value, but there may exists $b \in B$ st $f(a) \neq b$ $\forall a \in A$.

<u>A</u> is called <u>the domain</u> (of definition) <u>of f</u>

<u>B</u> is — " — the <u>target space</u> of f.

If $A' \subseteq A$, then the <u>map f'</u>

$$\begin{aligned} f' : A' &\longrightarrow B \\ \rotatebox{90}{$\in$}\ \ & \\ a &\longmapsto f'(a) := f(a) \end{aligned}$$

is called the <u>restriction of f to A'</u>

(denoted by $f' = f\restriction_{A'}$)

<u>EX</u>: ① $f : \mathbb{R} \to \mathbb{R}$ → $f = \sin$ fct.
$x \mapsto \sin(x)$

② $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$
$x \mapsto 1/x$

③ $g : \mathbb{R} \to \mathbb{R}$
$$x \longmapsto \begin{cases} 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

④ $f : V \longrightarrow$ set of subspaces of $V = \mathcal{S}$
$\vec{v} \longmapsto \operatorname{span}(\vec{v})$

⑤ $D : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$ <u>derivative</u>.
$p(x) \longmapsto \frac{d}{dx} p(x) = p'$