Lecture Notes on Sets and Maps

September 28, 2024

Sets

$$
\begin{aligned}
\mathbb{N} &= \{0, 1, 2, \dots\} && \text{natural numbers} \\
\mathbb{Z} &= \{\dots, -2, -1, 0, 1, 2, \dots\} && \text{integers} \\
\mathbb{Q} &= \left\{\frac{k}{n} \mid k, n \in \mathbb{N}, n \neq 0\right\} && \text{rationals} \\
\mathbb{R} &= && \text{real numbers} \\
\mathbb{C} &= \{a + ib \mid a, b \in \mathbb{R}\} && \text{complex numbers} \\
& \quad i = \text{imaginary unit} \\
& \quad \text{characterized by } i^2 = -1. \\
\mathbb{R}^d &= \{(a_1, \dots, a_d) \mid a_i \in \mathbb{R}\} && \text{d-dimensional space} \\
\mathbb{R}^\infty &= \{(a_0, a_1, a_2, \dots) \mid a_i \in \mathbb{R}\} && \text{space of infinite sequences of reals}
\end{aligned}
$$

Operations on Sets

Let A, B be sets.

$$
\begin{aligned}
A \cup B &= \{c \mid c \in A \text{ or } c \in B\} && \textit{union} \\
A \cap B &= \{c \mid c \in A \text{ and } c \in B\} && \textit{intersection} \\
A \backslash B &= \{c \in A \mid c \notin B\} && \textit{difference} \\
A \Delta B &= A \backslash B \cup B \backslash A \\
&= (A \cup B) \backslash (A \cap B) && \textit{symmetric difference}
\end{aligned}
$$

Indexed Families of Sets

Let $(A_\alpha)_{\alpha \in I}$ be an *indexed family of sets(index set I).*

EX.

1. $I = \mathbb{N}$,

$$A_\alpha := \{\alpha, \alpha + 1\}$$

2. $I = \mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$,

$$A_\alpha := [\alpha, \infty)$$

Union:

$$\bigcup_{\alpha\in I} A_\alpha = \{a \mid a \in A_\alpha \text{ for some } \alpha \in I\}$$
$$= \{a \mid \exists \alpha \in I \text{ such that } a \in A_\alpha\}$$

Intersection:

$$\bigcap_{\alpha\in I} A_\alpha = \{a \mid a \in A_\alpha \text{ for every } \alpha \in I\}$$
$$= \{a \mid a \in A_\alpha \text{ for all } \alpha \in I\}$$

Maps

Map= function= mapping(transformation)

Let A, B be sets. A mapf

$$f : A \to B$$
$$a \mapsto f(a)$$

is an assignment: to $\forall a \in A$, f assigns a unique $b \in B$ which is called the valueof f ata and is denoted by $f(a)$.

***Note**: f assigns to $\forall a \in A$ a value, but there may exists $b \in B$ st $f(a) \neq b$ $\forall a \in A$.*

A is called the domain(of definition) of f. B is the target spaceof f.

If $A' \subset A$, then the map f'

$$f' : A' \to B$$
$$a \mapsto f'(a) = f(a)$$

is called the restrictionof f to A' (denoted by $f' = f|_{A'}$).

EX.

1. $f : \mathbb{R} \to \mathbb{R} \to f = \sin$ *fet.*

$$x \mapsto sin(x)$$

2. $f : \mathbb{R}\backslash\{0\} \to \mathbb{R}$

$$x \mapsto y^x$$

3. $g : \mathbb{R} \to \mathbb{R}$

$$x \mapsto \begin{cases} \frac{1}{x} & x \neq 0 \\ 2 & x = 0 \end{cases}$$

4. $f : V \to$ *set of subspaces of* $V = S$

$$v \mapsto span(v)$$

5. $D : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$

$$p(x) \mapsto D : p(x) = p'$$

derivative