Sets

$$
\begin{aligned}
\mathbb{N} &= \{0, 1, 2, \dots\} \quad \textit{natural numbers} \\
\mathbb{Z} &= \{\dots, -2, -1, 0, 1, 2, \dots\} \quad \textit{integers} \\
\mathbb{Q} &= \left\{ \frac{k}{n} \mid k, n \in \mathbb{N}, n \neq 0 \right\} \\
&\quad \textit{rationals} \\
\mathbb{R} &= \textit{ real numbers} \\
\mathbb{C} &= \{a + ib \mid a, b \in \mathbb{R}\} = \textit{ complex numbers} \\
&\quad i = \textit{imaginary unit} \\
&\quad \text{characterized by } i^2 = -1. \\
\mathbb{R}^d &= \{(a_1, \dots, a_d) \mid a_i \in \mathbb{R}\} \quad d\textit{-dimensional space} \\
\mathbb{R}^\infty &= \{(a_0, a_1, a_2, \dots) \mid a_i \in \mathbb{R}\} \\
&\quad \textit{space of infinite sequences of reals}
\end{aligned}
$$

Operations on Sets

Let A, B be *sets*.

$$
\begin{aligned}
A \cup B &= \{c \mid c \in A \text{ or } c \in B\} \quad \textit{union} \\
A \cap B &= \{c \mid c \in A \text{ and } c \in B\} \quad \textit{intersection} \\
A \backslash B &= \{c \in A \mid c \notin B\} \quad \textit{difference} \\
A \triangle B &= A \cup B \backslash (A \cap B) \quad \textit{symmetric difference}
\end{aligned}
$$

Indexed Families of Sets

Let $(A_\alpha)_{\alpha \in I}$ be an *indexed family of sets* (*index set* I).

Examples

1. $I = \mathbb{N}$,

$$
\begin{aligned}
A_\alpha &= \{\alpha, \alpha + 1\} \\
&\quad \textit{definition}
\end{aligned}
$$

2. $I = \mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$,

$$A_x = [x, \infty)$$

Union

$$
\begin{aligned}
\bigcup_{\alpha \in I} A_\alpha &= \{a \mid a \in A_\alpha \text{ for some } \alpha \in I\} \\
&= \{a \mid \exists \alpha \in I \text{ such that } a \in A_\alpha\}
\end{aligned}
$$

Intersection

$$
\begin{aligned}
\bigcap_{\alpha \in I} A_\alpha &= \{a \mid a \in A_\alpha \text{ for every } \alpha \in I\} \\
&= \{a \mid \forall \alpha \in I, a \in A_\alpha\}
\end{aligned}
$$

Maps in General

Map = *function* = *mapping* (*transformation*)

Let A, B be *sets* ($\neq \emptyset$). A *map* f

$$f : A \to B$$

$$a \mapsto f(a)$$

is an *assignment*: to $\forall a \in A$, f assigns a unique $b \in B$ which is called the *value of* f *at* a and is denoted by $f(a)$.

Note: f assigns to $\forall a \in A$ a *value*, but there may exists $b \in B$ st $f(a) \neq b\ \forall a \in A$.

A is called the *domain* (of *definition*) of f. B is the *target space* of f.

Restriction If $A' \subset A$, then the *map* f'

$$f' : A' \to B$$

$$a \mapsto f'(a) = f(a)$$

is called the *restriction of* f *to* A' (denoted by $f' = f|_{A'}$)

Examples

1. $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto sin(x)$, $f = sin$ *fct.*
2. $f : \mathbb{R}\backslash\{0\} \to \mathbb{R}$, $x \mapsto \frac{1}{x}$.
3. $g : \mathbb{R} \to \mathbb{R}$, $x \mapsto \begin{cases} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$.
4. $f : V \to \{\text{set of subspaces of V} = \mathcal{P}(V)\}$, $v \mapsto span(v)$.
5. $D : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$, $p(x) \mapsto p'(x) = p'$ *derivative.*