Sets

$$
\begin{aligned}
\mathbb{N} &= \{0, 1, 2, ...\} && \text{natural numbers} \\
\mathbb{Z} &= \{..., -2, -1, 0, 1, 2, ...\} && \text{integers} \\
\mathbb{Q} &= \left\{ \frac{k}{n} \,\middle|\, k, n \in \mathbb{N}, n \neq 0 \right\} && \text{rationals} \\
\mathbb{R} &= && \text{real numbers} \\
\mathbb{C} &= \{a + ib \mid a, b \in \mathbb{R}\} && \text{complex numbers}
\end{aligned}
$$

where *i* is the *imaginary unit* characterized by $i^2 = -1$.

$$
\begin{aligned}
\mathbb{R}^d &= \{(a_1, a_2, ..., a_d) \mid a_i \in \mathbb{R}\} && \text{d-dimensional space} \\
\mathbb{R}^\infty &= \{(a_0, a_1, a_2, ...) \mid a_i \in \mathbb{R}\} && \text{space of infinite sequences of reals}
\end{aligned}
$$

Set Operations

Let A and B be sets:

$$
\begin{aligned}
A \cup B &= \{c \mid c \in A \text{ or } c \in B\} && \text{union} \\
A \cap B &= \{c \mid c \in A \text{ and } c \in B\} && \text{intersection} \\
A \setminus B &= \{c \mid c \in A \text{ and } c \notin B\} && \text{difference} \\
A \Delta B &= (A \cup B) \setminus (A \cap B) && \text{symmetric difference}
\end{aligned}
$$

Indexed Families of Sets

Let $(A_\alpha)_{\alpha \in I}$ be a family of sets indexed by the set I:

$$
\begin{aligned}
\bigcup_{\alpha \in I} A_\alpha &= \{a \mid a \in A_\alpha \text{ for some } \alpha \in I\} \\
\bigcap_{\alpha \in I} A_\alpha &= \{a \mid a \in A_\alpha \text{ for every } \alpha \in I\}
\end{aligned}
$$

Maps

Let A and B be sets. A *map* $f : A \to B$ is an *assignment* that assigns to each element a in A a unique element $f(a)$ in B, which is called the *value* of f at a.

Note: f assigns to each $a \in A$ a value, but there may exist $b \in B$ such that $f(a) \neq b$ for some $a \in A$.

A is called the *domain* of f, and B is called the *target space* of f.

If $A' \subseteq A$, then the *restriction* of f to A' is denoted by $f' = f|_{A'}$ and defined as $f' : A' \to B$, $f'(a) = f(a)$ for all $a \in A'$.

Examples

1. $f : \mathbb{R} \to \mathbb{R}$, $x \to \sin(x)$ (f is the sine function).
2. $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$, $x \to \frac{1}{x}$ (f is the reciprocal function).
3. $g : \mathbb{R} \to \mathbb{R}$, $x \to \begin{cases} x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ (g is a piecewise function).
4. $f : V \to \{\text{set of subspaces of } V\}$, $v \to \operatorname{span}(v)$ (f assigns the span of a vector).
5. $D : P(\mathbb{R}) \to P(\mathbb{R})$, $p(x) \to p'(x)$ (D is the derivative of a polynomial).