Sets and Maps

1 Sets

$$\begin{aligned}
\mathbb{N} &= \{0,1,2,...\} \text{ *natural numbers*} \\
\mathbb{Z} &= \{-2,-1,0,1,2,...\} \text{ *integers*} \\
\mathbb{Q} &= \left\{\frac{k}{n}|k,n\in\mathbb{N}, n\neq 0\right\} \text{ *rationals*} \\
\mathbb{R} &= \text{ *real numbers*} \\
\mathbb{C} &= \{a+ib|a,b\in\mathbb{R}\} \text{ *complex numbers*} \\
&\text{*i = imaginary unit*} \\
&\text{*characterized by i}^2\text{=-1*} \\
\mathbb{R}^d &= \{(a_1...a_d)|a_i\in\mathbb{R}\} \text{ *d-dimensional space*} \\
\mathbb{R}^\infty &= \{(a_0,a_1,a_2,...)|a_i\in\mathbb{R}\} \text{ *space of infinite sequences of reals*}
\end{aligned}$$

2 Set Operations

$$\begin{aligned}
A\cup B &= \{c|c\in A \text{ or } c\in B\} \text{ *union*} \\
A\cap B &= \{c|c\in A \text{ and } c\in B\} \text{ *intersection*} \\
A\backslash B &= \{c\in A|c\notin B\} \text{ *difference*} \\
A\Delta B &= A\cup B\backslash(A\cap B) \text{ *symmetric difference*}
\end{aligned}$$

3 Families of Sets

Let $(A_\alpha)_{\alpha\in I}$ be an *indexed family of sets* (index set I).

$$\begin{aligned}
\text{EX: } (1)\;\; & I=\mathbb{N} \\
& A_\alpha := \{\alpha,\alpha+1\} \\
(2)\;\; & I=\mathbb{R}^+ = \{x\in\mathbb{R}|x>0\} \\
& A_\alpha := [\alpha,\infty)
\end{aligned}$$

$$\begin{aligned}
\text{union: } \bigcup_{\alpha\in I} A_\alpha &= \{\alpha|\alpha\in A_\alpha \text{ for some } \alpha\in I\} \\
&= \{\alpha|\exists\alpha\in I \text{ such that } \alpha\in A_\alpha\} \\
\text{intersection: } \bigcap_{\alpha\in I} A_\alpha &= \{\alpha|\alpha\in A_\alpha \text{ for every } \alpha\in I\} \\
&= \{\alpha|\forall\alpha\in I \text{ we have } \alpha\in A_\alpha\}
\end{aligned}$$

4 Maps

Maps in general.
Map = function = mapping = transformation
Let A, B be *sets* ($+\emptyset$). A *map* f:

$$\begin{aligned} A &\to B \\ a &\mapsto f(a) \end{aligned}$$

is an *assignment*: to $\forall a \in A$ f assign a unique $b \in B$ which is called the *value of f at a* and is denoted by $f(a)$.
Note: f assigns to $\forall a \in A$ a value, but there may exists $b \in B$ st $f(a_1) \neq b \ \forall a \in A$.
A is called the *domain* (of definition) of f
B is the *target space* of f

5 Restriction of a Map

If $A' \subset A$, then the map f' (0.15

$$\begin{aligned} f' : A' &\to B \\ a &\mapsto f'(a) = f(a) \end{aligned}$$

is called the *restriction of f to A'* (denoted by $f' = f|_{A'}$)
EX: (1) $f : \mathbb{R} \to \mathbb{R}$ -¿ f = sin fct.

$$\begin{aligned} (2)\ f : \mathbb{R}\backslash\{0\} &\to \mathbb{R} \\ x &\mapsto \frac{1}{x} \\ (3)\ g : \mathbb{R} &\to \mathbb{R} \\ x &\mapsto \begin{cases} \frac{1}{x} & x \neq 0 \\ 2 & x = 0 \end{cases} \\ (4)\ f : V &\text{ set of subspaces of V = S} \\ v &\mapsto \text{span}(v) \\ (5)\ D : P(\mathbb{R}) &\to P(\mathbb{R}) \text{ derivative} \\ p(x) &\mapsto p'(x) = p' \end{aligned}$$