Sets, Operations, Maps

Sets

$$
\begin{aligned}
\mathbb{N} &= \{0, 1, 2, ...\} \qquad \textit{natural numbers} \\
\mathbb{Z} &= \{-2, -1, 0, 1, 2, ...\} \qquad \textit{integers} \\
\mathbb{Q} &= \left\{\frac{k}{n} | k, n \in \mathbb{N}, n \neq 0\right\} \qquad \textit{rationals} \\
\mathbb{R} &= \qquad \textit{real numbers} \\
\mathbb{C} &= \{a + ib | a, b \in \mathbb{R}\} \qquad \textit{complex} \\
& \quad i = \textit{imaginary unit} \\
& \quad \textit{characterized by} \qquad i^2 = -1. \\
\mathbb{R}^d &= \{(a_1....a_d) | a_i \in \mathbb{R}\} \qquad \textit{d-dimensional space} \\
\mathbb{R}^\infty &= \{(a_0, a_1, a_2.....) | a_i \in \mathbb{R}\} \\
& \quad \textit{space of infinite sequences of reals}
\end{aligned}
$$

Operations

Let A, B be sets.

$$
\begin{aligned}
A \cup B &= \{c | c \in A \quad \text{or} \quad c \in B\} \qquad \textit{union} \\
A \cap B &= \{c | c \in A \quad \text{and} \quad c \in B\} \qquad \textit{intersection} \\
A \backslash B &= \{c \in A | c \notin B\} \qquad \textit{difference} \\
A \Delta B &= A \cup B \backslash \{x \in A : x \in B\} \qquad \textit{symmetric difference} \\
&= (A \cup B) \cap A \cap B
\end{aligned}
$$

Indexed Family of Sets

Let $(A_\alpha)_{\alpha \in I}$ be an indexed family of sets (index set I).

- $I = \mathbb{N}$, $A_\alpha := \{\alpha, \alpha + 1\}$
- $I = \mathbb{R}^+ = \{x \in \mathbb{R} | x > 0\}$, $A_\alpha := [\alpha, \infty)$

$$
\begin{aligned}
\bigcup_{\alpha \in I} A_\alpha &:= \{a | a \in A_\alpha \quad \text{for some} \quad \alpha \in I\} \\
&= \{a | \exists \alpha \in I \quad \text{such that} \quad a \in A_\alpha\} \\
\bigcap_{\alpha \in I} A_\alpha &= \{a | a \in A_\alpha \quad \text{for every} \quad \alpha \in I\} \\
& \quad \text{for all } \alpha \in I
\end{aligned}
$$

Maps

Map = *function* = *mapping* = *transformation* Let A, B be sets. A *map* f

$$\begin{aligned} f : A &\to B \\ a &\mapsto f(a) \end{aligned}$$

is an *assignment*: to $\forall a \in A$ f assigns a unique $b \in B$, which is called the *value of f at a* and is denoted by $f(a)$.
Note: f assigns to $\forall a \in A$ a *value*, but there may exists $b \in B$ s.t $f(a_1) = f(b) \quad \forall a_1 \in A$.

- A is called the *domain* (of definition) of f.
- B is called the *target space* of f.

If $A' \subset A$, then the map f'

$$\begin{aligned} f' : A' &\to B \\ a &\mapsto f'(a) = f(a) \end{aligned}$$

is called the *restriction of f to A'*
(denoted by $f' = f|_{A'}$).

Examples

1. $f : \mathbb{R} \to \mathbb{R}, \qquad x \mapsto \sin(x) \qquad \Rightarrow f = \sin \qquad$ *f.ct.*
2. $f : \mathbb{R}^{1203} \to \mathbb{R}, \qquad x \mapsto y^x$
3. $g : \mathbb{R} \to \mathbb{R}, \qquad x \mapsto \begin{cases} \frac{1}{x} & x \neq 0 \\ 20 & x = 0 \end{cases}$
4. $f : V \to$ *set of subspaces of* $V = \mathbb{R}^3 \qquad v \mapsto$ *span*(v)
5. $D : P(\mathbb{R}) \to P(\mathbb{R}), \qquad p(x) \mapsto p'(x) = p' \qquad$ *derivative*