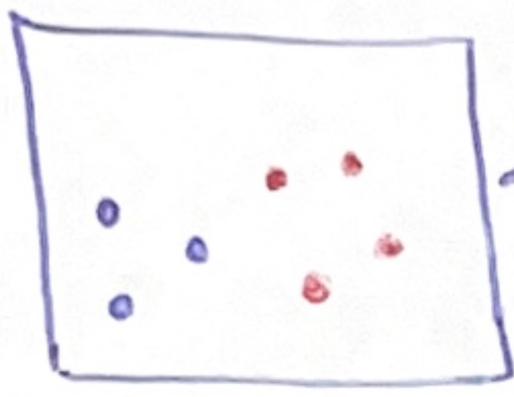
Baye's Theorem

1. Conditional Probability → $P(A|B) = \frac{P(A \cap B)}{P(B)}$
2. Independent Events
3. Dependent Events

A given B

example (Dependent Events)



E1

$P(A) = 3/7$ (Prob of getting 1 blue object)

E2

$P(B|A) = \cancel{2}^{1}/\cancel{6}3$ (Prob of getting another blue object)

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{3/7 * 1/3}{3/7} = \frac{1/7}{3/7} = 1/3$$

Baye's Theorem (Derivation)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$P(A|B) \cdot P(B) = P(A \cap B)$

$P(B|A) \cdot P(A) = P(B \cap A)$

Since: $P(A \cap B) = P(B \cap A)$

$P(A|B) * P(B) = P(B|A) \cdot P(A)$

$$\boxed{P(A|B) = \frac{P(B|A) * P(A)}{P(B)}}$$

Posterior

Likelihood

Prior Probability

Baye's Theorem

Marginal Probability

Naive Bayes Classifier

Given Dataset $X = (x_1, x_2, x_3, \cdots x_n)$

$y = \{y\}$

Baye's Theorem: $P(A|B) = \dfrac{P(B|A) * P(A)}{P(B)}$

$$P(y|(x_1, x_2 \ldots x_n)) = \frac{P(x_1|y)\, P(x_2 y) \ldots P(x_n|y) * P(y)}{P(x_1) \cdot P(x_2) \cdots P(x_n)}$$

$$= \frac{P(y) * \prod_{i=1}^{n} P(x_i|y)}{\underbrace{P(x_1)\, P(x_2) \cdots P(x_n)}_{\text{constant}}}$$

$$P(y|(x_1, x_2 \cdots x_n)) \propto P(y) \cdot \prod_{i=1}^{n} P(x_i|y)$$

$$y = \operatorname{argmax} \left[P(y) * \prod_{i=1}^{n} P(x_i|y) \right]$$

↳ Takes the highest value for classification.

Example

Outlook

	Yes	No	P(Y)	P(N)
Sunny	2	3	2/9	3/5
Overcast	4	0	4/9	0/5
Rainy	3	2	3/9	2/5
Tot:	9	5		

Temperature

	Yes	No	P(Y)	P(N)
Hot	2	2	2/9	2/5
Mild	4	2	4/9	2/5
Cold	3	1	3/9	1/5
Tot.	9	5		

Play

	Y	P( )
Yes	9	9/14
No	5	5/14
	14	

To Find: Today (Sunny, Hot) ⇒ Play / No Play?

$$\text{Play}(\text{Yes} \mid \text{Today}) = P(\text{Sunny} \mid \text{yes}) \cdot P(\text{Hot} \mid y) \cdot P(\text{Yes})$$

$$= 2/9 * 2/9 * 9/14 = 0.031$$

$$\text{Play}(\text{No} \mid \text{Today}) = P(\text{Sunny} \mid \text{No}) \cdot P(\text{Hot} \mid \text{No})\ P(\text{N})$$

$$= 3/5 * 2/5 * 5/14 = 0.0857$$

Normalize $\boxed{\dfrac{\text{Numericals}}{\Sigma(\text{all Nume})}}$

$$P(Y) = \frac{0.031}{0.031 + 0.0857} = 0.27$$

$$(N) = 1 - 0.27 = \boxed{0.73}$$

Arg Max: No Play