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Question) Derivative the Gradient Descent of Logistic Regression.

$$J(h_\theta) = -y \cdot \log(h_\theta) - (1-y) \cdot \log(1-h_\theta)$$

$$= y^{(i)} \frac{1}{h_\theta(x^{(i)})} \frac{\partial}{\partial \theta_j} h_\theta(x^{(i)}) + (1-y^{(i)}) \frac{1}{1-h_\theta(x^{(i)})} \frac{\partial}{\partial \theta_j} (1-h_\theta(x^{(i)}))$$

$$= y^{(i)} \frac{}{}$$

- lemma 1 : $\frac{d}{dx} \ln(x) = \frac{1}{x}$

  lemma 2 : $\frac{d}{dx} \ln(f(x)) = \frac{1}{f(x)} \cdot f(x)'$

$$-y^{(i)} \log\left(h^{(i)}\right) - (1-y^{(i)}) \log\left(1-h^{(i)}\right)$$

$$\frac{\partial}{\partial h^{(i)}} \left(-y^{(i)} \log\left(h^{(i)}\right) - (1-y^{(i)}) \log\left(1-h^{(i)}\right)\right)$$

The chain Rule, we get:

$$= -y^{(i)} \frac{1}{h^{(i)}} - \left(1-y^{(i)}\right) \frac{1}{1-h^{(i)}} (-1)$$

$$= \frac{y^{(i)}}{h^{(i)}} + \frac{1-y^{(i)}}{1-h^{(i)}}$$

Let compute the derivative of $h^{(i)}$

$$\frac{\partial h^{(i)}}{\partial \theta_j} = h^{(i)}\left(1-h^{(i)}\right) x_j^{(i)}$$

$$\frac{\partial}{\partial \theta_j} \left(-y^{(i)} \log\left(h^{(i)}\right) - (1-y^{(i)}) \log\left(1-h^{(i)}\right)\right)$$

$$= \left(-\frac{y^{(i)}}{h^{(i)}} + \frac{1-y^{(i)}}{1-h^{(i)}}\right) \cdot h^{(i)}\left(1-h^{(i)}\right) x_j^{(i)}$$

$$= \left(h^{(i)} - y^{(i)}\right) x_j^{(i)}$$