1) a. Schema $R(a, b, c, d, e)$

$ab \rightarrow c, \; ce \rightarrow d, \; a \rightarrow e$

$\{a, b\}^+ = \{a, b, c, d, e\}$

$\{c, e\}^+ = \{c, e, d\}$

$\{a\}^+ = \{a, e\}$

$\{a, b\}$ is the key

So a, b are prime attributes.

c, d, e are non-prime attributes.

$\{a, b\} \; \{c, e\}$ Satisfies the conditions but $a \rightarrow e$ doesn't satisfies because a is the proper subset of $\{a, b\}$

The Given Relational Schema is not in 2NF.

b. $R(w, x, y, z)$

$w \rightarrow z, \; yz \rightarrow x, \; wz - y$

$\{w\}^+ = \{w, z, y, x\}$

$\{y, z\}^+ = \{y, z, x\}$

$\{w, z\}^+ = \{w, z, y, x\}$

$\{w\}^+ = \{w, x, y, z\}$

$\{w\}$ is the key, as key is having only one value it doesn't have proper Subsets.

Given Relational Schema is in 2NF.

2) (a) $R(a, b, d, e)$

$F = \{ab \to c, de \to c, b \to d\}$

$ab \to c$

$de \to c$

$b \to d$

$\{a, b\}^+ = \{a, b, c, d\}$

$\{d, e\}^+ = \{d, e, c\}$

$\{b\}^+ = \{b, d\}$

Given relational schema is not in BCNF

b)

Take $b \to d$

$\{b\}^+ = \{b, d\}$

$R_1 = \{b, d\}$

$R_2 = R - \{\{b\}^+ - b\}$

$R_2 = (a, b, c, e)$

R_1 is in BCNF as it holds only two attributes.

Now take $R_2 (a, b, c, e)$

R_2 is not in BCNF, Now R_2 holds $ab \to c$ only take $ab \to c$

$\{a, b\}^+ = \{a, b, c\}$

$R_3 (a, b, c)$

$R_4 (a, b, e)$

Now, R_3 is in BCNF and R_4 doesn't hold only relation in functional dependencies.

Now final schema is

$\quad R_1(b, d) \quad R_3(a, b, c) \quad R_4(a, b, e)$

If we take $ab \rightarrow c$ in the first phase.

$\quad R(a, b, c, d, e)$

$\{a, b\}^+ = \{a, b, c, d\}$

$R_1(a, b, c, d)$

$R_2(a, b, c)$

then $R_1(a, b, c, d)$ is not in BCNF.

Now take $b \rightarrow d$

$R_3(b, d)$

$R_4(a, b, c)$

R_3 is in BCNF as it has only 2 variables.

R_4 is in BCNF as it satisfies $ab \rightarrow c$.

Now take $R_2(a, b, e)$ and this doesn't satisfy any functional dependency so take $R_2(a, b, e)$

Now final Schema is

$\quad R_2(a, b, c)$

$\quad R_3(b, d)$

$\quad R_4(a, b, e)$

3.(a) $R(e, t, h, r, s, g)$

$c \to t$, $hr \to c$, $hr \to t$, $ht \to r$, $hs \to v$, $ch \to v$, $cs \to g$

Now, $c \to t$, $hr \to c$, $hr \to t$, $ht \to r$, $ch \to r$, $cs \to g$ doesn't satisfy the condition of 3NF, so it is not in 3NF.

$hs \to r$ it is in 3NF.

When a functional dependency $A \to B$ is given for relationship schema R then :-

i) A is in superkey for R
ii) B is contained in a key for R.

So $hs \to r$ functional dependency

$\{h, s\}^+ = \{h, s, r, t, c, g\}$ becomes the Super key. So it satisfies 3NF.

other functional dependencies does not contain Super keys also right side attribute are non prime attributes.

(b) (i) $\{h, s\}$ is the key for given R

ii) Canonical Cover F.

$c \to t$

$hr \to c$, $hr \to t$ therefore $hr \to ct$

$ht \to r$

$hs \to r$

$ch \to r$

$cs \to g$

So, final Cover is

c → t $\{c\}^+ = \{c, t\}$

hr → ct $\{h, r\}^+ = \{h, r, c, t\}$

ht → r $\{h, t\}^+ = \{h, t, r, t\}$

hs → r $\{h, s\}^+ = \{h, s, r, t, c, g\}$

ch → r

cs → g $\{c, h\} = \{c, h, r, t\}$

               $\{c, s\} = \{c, s, g, t\}$

Remove of left side of a funtional dependency.

c → t   (no redundancy)

hr → ct

ht → r

hs → r

ch → r   i.e., ch → r and hr → c are redudnat as $\{h, r\}^+ \supseteq \{ch\}^+$

So, Remove ch → r

cs → g is not redudnot.

Now. Remove rignt side of a functional dependeny to remove
redundany

c → t   So, final schema is

hr → ct becomes hr → c as c → t

ht → r $R_1 (c, t)$

hs → r $R_2 (h, r, c)$

cs → g $R_3 (h, t, r)$

                                        $R_4 (h, s, R)$

                                        $R_5 (c, s, g)$ //