Error correction Codes-Linear Block codes

Error Correcting Codes

- Linear Block Codes
- Convolution Codes

* Linear Block Codes.
- In this parity Bits and message bits have linear combinat^{n}
- i.e., resultant code word

=> Let us consider block of data,
- contains **k** bits in each block.
- These bits are mapped with block
  - which has **n** bits in each block.
- Here,

$$n > k$$

-> The ratio k/n is the Code rate
- code rate is denoted by 'r'
- value $r < 1$

→ n−k bits are added as parity bits.

- Parity bit help in error detection and error correction.
- also helps in locating data.

In transmitted data,

- left most bits of code word is 'message'
- right most bits of codeword correspond to 'parity bits'

Structure of code word

K-bits	n-K bits
message	Parity

⟵ n bits ⟶

Known as systematic code

* Error detection & Error Correction Capability of LBC

⇒ Identify d_{min} $\quad$ ($\because d_{min}$ = Minimum Hamming distance)

Error detection capacity of LBC

$$\Rightarrow \quad d_{min} \geq s+1 \qquad (\because s = \text{error detect}^n \text{ capacity})$$

Error correction capacity of LBC

$$\Rightarrow \quad d_{min} \geq 2t+1 \qquad (\because t = \text{error correction capacity})$$

Example.

If minimum Hamming distance of LBC is 3. Find LBC error detection and error correction capacity.

Sol:- Given, $d_{min} = 3$

- for error detection

$\rightarrow d_{min} \geq s+1$

$3 \geq s+1$

$2 \geq s$

LBC can detect 2 bit errors

- for error correction

$\rightarrow d_{min} \geq 2t+1$

$3 \geq 2t+1$

$2 \geq 2t$

$1 \geq t$

$\therefore$ It can correct 1 bit error.

Qu 1 For (6,3) code, the generator matrix G is

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Find (a) All corresponding code vectors

(b) Minimum Hamming distance, d_{min}

(c) Error detection and error correction capacity

(d) Parity check matrix

(e) Find error if received code (100011)

Sol,

Given, $G = [I_K : P]$ — Identity Matrix (I_K), Parity matrix (P)

$$G = \begin{bmatrix} 1&0&0&1&0&1 \\ 0&1&0&0&1&1 \\ 0&0&1&1&1&0 \end{bmatrix}$$

$$I_K = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 1&0&1 \\ 0&1&1 \\ 1&1&0 \end{bmatrix}$$

So, code word = [information] [Generator Matrix]

$$[C] = [i][G]$$

$$= [m : P_c]$$

ie m = message, P_c = Parity of code word

$$\Rightarrow [P_c] = [i_m][P]$$

ie., i_m = Information of message

message bit = 6, Parity bit = $\underline{\underline{3}}$

$$\Rightarrow [P_0 \, P_1 \, P_2] = [i_0 \, i_1 \, i_2] \begin{bmatrix} 1&0&1 \\ 0&1&1 \\ 1&1&0 \end{bmatrix}$$

$P_0 = i_0 \oplus i_2$

$P_1 = i_1 \oplus i_2$

$P_2 = i_0 \oplus i_1$

C	i_0	i_1	i_2	P_0	P_1	P_2	Weight(w)
c_1	0	0	0	0	0	0	0
c_2	0	0	1	1	1	0	3
c_3	0	1	0	0	1	1	3
c_4	0	1	1	1	0	1	4
c_5	1	0	0	1	0	1	3
c_6	1	0	1	0	1	1	4
c_7	1	1	0	1	1	0	4
c_8	1	1	1	0	0	0	3

XOR

0 0 → 0
0 1 → 1
1 0 → 1
1 1 → 0

$d_{min} = \underline{\underline{3}}$

Ans (a) $c_1, c_2, c_3 \dots c_8$

Ans (b) $d_{min} = 3$

Ans (c) $s = 2, \; t = 1$

(d) Parity check matrix

$$H = [P^T : I_{n-k}]$$

Parity check matrix

P transpose

Information

We have $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $\quad P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$I_{n-k} = I_{6-3} = I_3 = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Identity metrix with 3 row

$$H = [P^T : I_3] = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(e) Find error in code (100011)

Error syndrome = [Receive Signal] $[H^T]$

$$[S] = [Y][H^T]$$

Given $[Y] = [100011]$, $H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ ES, $[S] = [100011] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

error, $e = [001000]$

$Y = [100011]$

Information,

$X = e + Y$

$= \boxed{101011}$

Qu 2 A generator matrix of $(6,3)$ linear block code is given as

$(6,3)$: $n = 6$, $k = 3$

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$p = n - k = 3$

(i) Find code word for the message 011

(ii) Decode the received signal 101101.

Sol.

Message	Parity
k	$n-k$

$\longleftarrow n \longrightarrow$

Given,

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [I_K : P] \Rightarrow I_K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Code = [information] [Generator Matrix]

$$[C] = [i][G]$$
$$= [m : P_c]$$

Parity bits is 3 bits

$$[P_c] = [i_m][P]$$

$$[P_0 \; P_1 \; P_2] = [i_0 \; i_1 \; i_2] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$P_0 = i_0 \oplus i_1$$
$$P_1 = i_0 \oplus i_1 \oplus i_2$$
$$P_2 = i_0 \oplus i_2$$

(i) Give message is 011

$\therefore$ $i_0 = 0$, $i_1 = 1$, $i_2 = 1$

So,
$$P_0 = 0 \oplus 1 = 1$$
$$P_1 = 0 \oplus 1 \oplus 1 = 0$$
$$P_2 = 0 \oplus 1 = 1$$

Code word $= [i_0 i_1 i_2 \; P_0 P_1 P_2]$

$= [011101]$

(ii) Received Signal, $Y = 101101$

Error Syndrome, $S = YH^T$

Parity check Matrix, $H = [P^T : I_3]$

$$= \begin{bmatrix} 110 & 100 \\ 111 & 010 \\ 101 & 001 \end{bmatrix}$$

$P = \begin{matrix} 111 \\ 110 \\ 011 \end{matrix}$

$P^T = \begin{matrix} 110 \\ 111 \\ 101 \end{matrix}$

$$H^T = \begin{bmatrix} 111 \\ 110 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix}$$

$1 \rightarrow 1 \oplus 1$

$2 \rightarrow 1 \oplus 1 = 0$

$3 \rightarrow 1 \oplus 1 \oplus 1 = 1$

$$S = YH^T = [101101] \begin{bmatrix} 111 \\ 110 \\ 011 \\ 100 \\ 010 \\ \boxed{001} \end{bmatrix} = \underline{001}$$

error $= 000001$

$Y = 101101$ (contains error)

Correct message $= \underline{\underline{101100}}$.