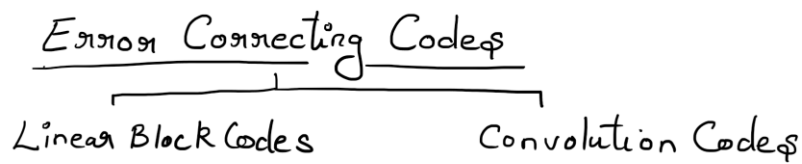


## Error correction Codes-Linear Block codes



### \* Linear Block Codes.

- In this parity Bits and message bits have linear combinat?
- i.e., resultant code word

=> Let us consider block of data,

- contains k bits in each block.
- These bits are mapped with block
  - which has n bits in each block.
- Here ,  
 $n > k$

-> The ratio  $k/n$  is the Code rate

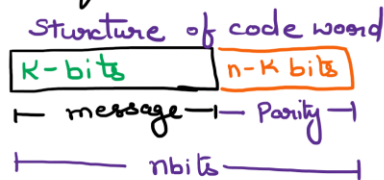
- code rate is denoted by ' $\gamma$ '
- value  $\gamma < 1$

→  $n-k$  bits are added as parity bits.

- Parity bit help in error detection and error correction.
- also helps in locating data.

In transmitted data,

- left most bits of code word is 'message'.
- right most bits of codeword correspond to 'parity bits'.



Known as  
systematic  
code

### \* Error detection & Error Correction Capability of LBC

⇒ Identify  $d_{\min}$  ( $\because d_{\min}$  = Minimum Hamming distance)

Error detection capacity of LBC

$$\Rightarrow d_{\min} \geq s+1 \quad (\because s = \text{error detection capacity})$$

Error correction capacity of LBC

$$\Rightarrow d_{\min} \geq 2t+1 \quad (\because t = \text{error correction capacity})$$

### Example

If minimum Hamming distance of LBC is 3. Find LBC error detection and error correction capacity.

Sol:- Given,  $d_{\min} = 3$

- for error detection

$$\rightarrow d_{\min} \geq s+1$$

$$3 \geq s+1$$

$$2 \geq s$$

LBC can detect 2 bit errors

- for error correction

$$\rightarrow d_{\min} \geq 2t+1$$

$$3 \geq 2t+1$$

$$2 \geq 2t$$

$$1 \geq t$$

$\therefore$  It can correct 1 bit error.

Q1 For (6,3) code, the generator matrix  $G$  is

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Find (a) All corresponding code vectors

(b) Minimum Hamming distance,  $d_{\min}$

(c) Error detection and error correction capacity

(d) Parity check matrix

(e) Find error if received code (100011)

Sol, Given,  $G = \left[ \begin{array}{c|c} \text{Identity Matrix} & \text{Parity matrix} \\ \hline I_K & P \end{array} \right]$   $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$\underline{I_K} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and  $\underline{P} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

So, code word = [information] [Generator Matrix]

$[C] = [u][G]$

$= [m: P_c]$

ie  $m = \text{message}$   
 $P_c = \text{Parity of code word}$

$\Rightarrow [P_c] = [u_m][P]$

ie,  $u_m = \text{Information of message}$   
message bit = 6, Parity bit = 3

$\Rightarrow [P_0 P_1 P_2] = [u_0 u_1 u_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$P_0 = u_0 \oplus u_2$

$P_1 = u_1 \oplus u_2$

$P_2 = u_0 \oplus u_1$

C	$u_0$	$u_1$	$u_2$	$P_0$	$P_1$	$P_2$	Weight(w)
$C_1$	0	0	0	0	0	0	0
$C_2$	0	0	1	1	1	0	3
$C_3$	0	1	0	0	1	1	3
$C_4$	0	1	1	1	0	1	4
$C_5$	1	0	0	1	0	1	3
$C_6$	1	0	1	0	1	1	4
$C_7$	1	1	0	1	1	0	4
$C_8$	1	1	1	0	0	0	3

XOR

$0 \oplus 0 \rightarrow 0$   
 $0 \oplus 1 \rightarrow 1$   
 $1 \oplus 0 \rightarrow 1$   
 $1 \oplus 1 \rightarrow 0$

$d_{\min} = \underline{\underline{3}}$

Ans(a)  $C_1, C_2, C_3 \dots C_8$

Ans(b)  $d_{\min} = 3$

Ans(c)  $S = 2, t = 1$

(d) Parity check matrix

$$H = \begin{bmatrix} P^T & I_{n-k} \end{bmatrix}$$

Parity check matrix
P transpose
Information

We have  $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$        $P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$I_{n-k} = I_{6-3} = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Identity matrix with } 3 \times 3 \text{ rows}$$

$$H = [P^T : I_3] = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(e) Find error in code (100011)

$$\text{Error syndrome} = [\text{Receive Signal}] [H^T]$$

$$[S] = [Y] [H^T]$$

Given  $[Y] = [100011]$  ,  $H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore ES, [S] = [100011] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{error, } e = [001000]$$

$$Y = [100011]$$

Information,

$$X = e + Y$$

$$= \boxed{101011}$$

Q2 A generator matrix of  $(6, 3)$  linear block code is given

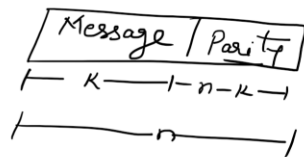
as  $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$p = n - k$   
 $= 3$

(i) Find code word for the message 011

(ii) Decode the received signal 101101.

sol.



Given,

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [I_k : P] \Rightarrow I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\text{Code} = [\text{information}] [\text{Generator Matrix}]$$

$$[c] = [i] [G]$$

$$= [m : P_c]$$

Parity bits is 3 bits

$$[P_c] = [i_m] [P]$$

$$[P_0, P_1, P_2] = [i_0, i_1, i_2] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$P_0 = i_0 \oplus i_1$$

$$P_1 = i_0 \oplus i_1 \oplus i_2$$

$$P_2 = i_0 \oplus i_2$$

(i) Give message is 011

$$\therefore i_0 = 0, i_1 = 1, i_2 = 1$$

$$\text{So, } P_0 = 0 \oplus 1 = 1$$

$$P_1 = 0 \oplus 1 \oplus 1 = 0$$

$$P_2 = 0 \oplus 1 = 1$$

$$\text{Code word} = [i_0, i_1, i_2, P_0, P_1, P_2]$$

$$= [0, 1, 1, 0, 1]$$

(ii) Received Signal,  $Y = 101101$

Error Syndrome,  $S = YH^T$

Parity check Matrix,  $H = [P^T : I_3]$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = YH^T = [101101] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underline{001}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

1  $\rightarrow 1 \oplus 1$   
2  $\rightarrow 1 \oplus 1 = 0$   
3  $\rightarrow 1 \oplus 1 \oplus 1 = 1$

$$\text{error} = 000001$$

$$Y = 101101 \text{ (contains error)}$$

$$\text{Correct message} = \underline{\underline{101100}}.$$