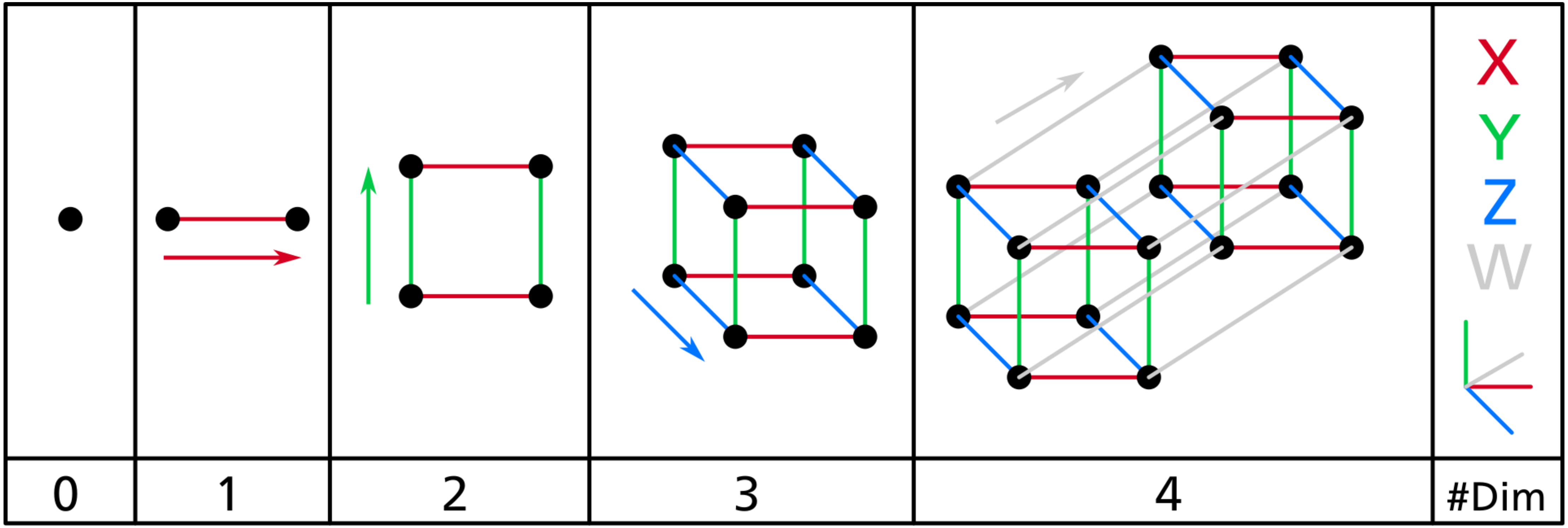
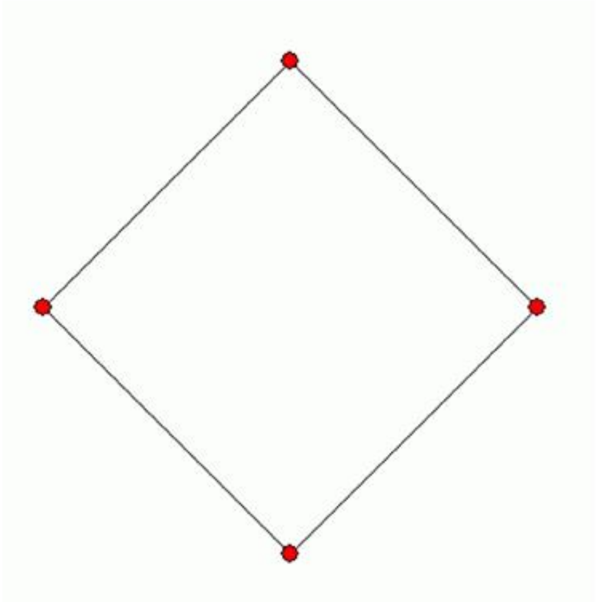
Geometrical Intuitions

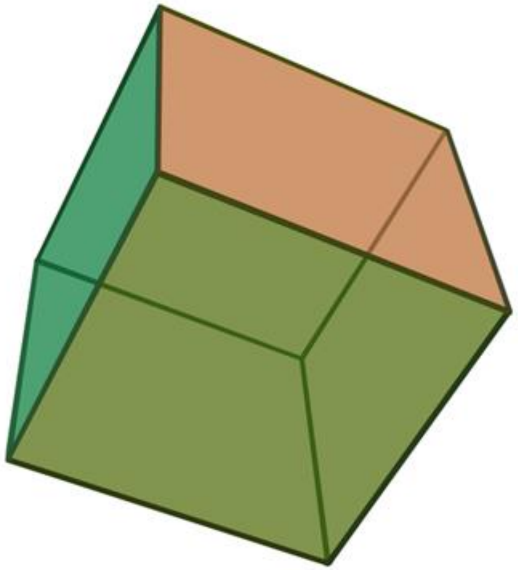
KochiTechGroup

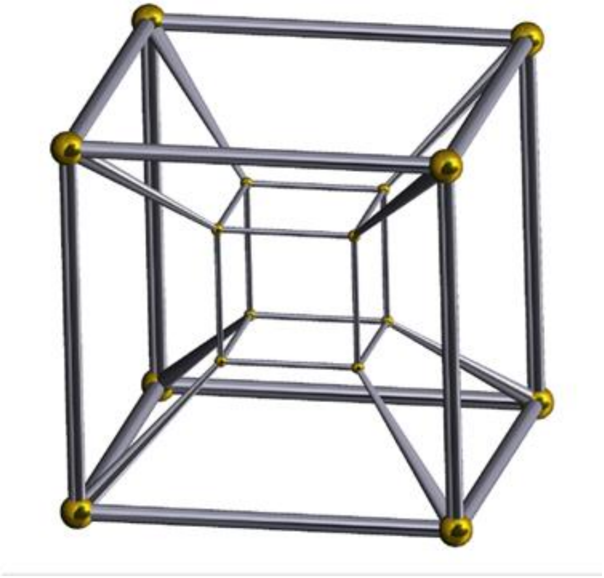
Dimensions!



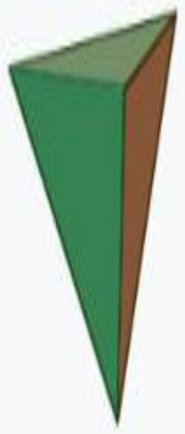
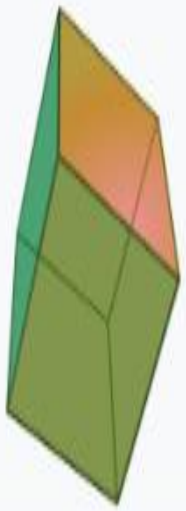
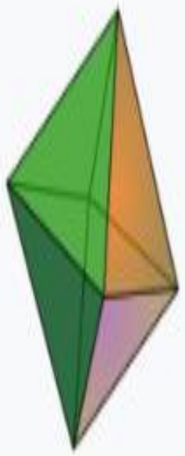
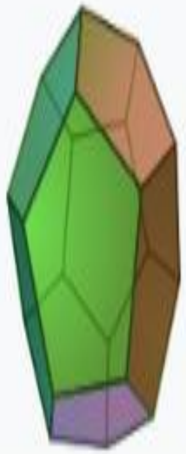
Square (Rhombus),Cube,Tesseract



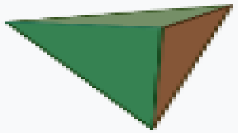
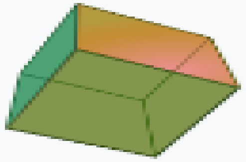
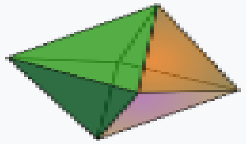
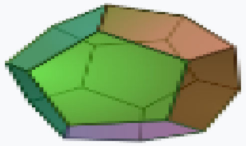
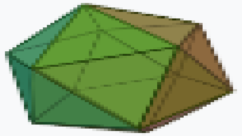




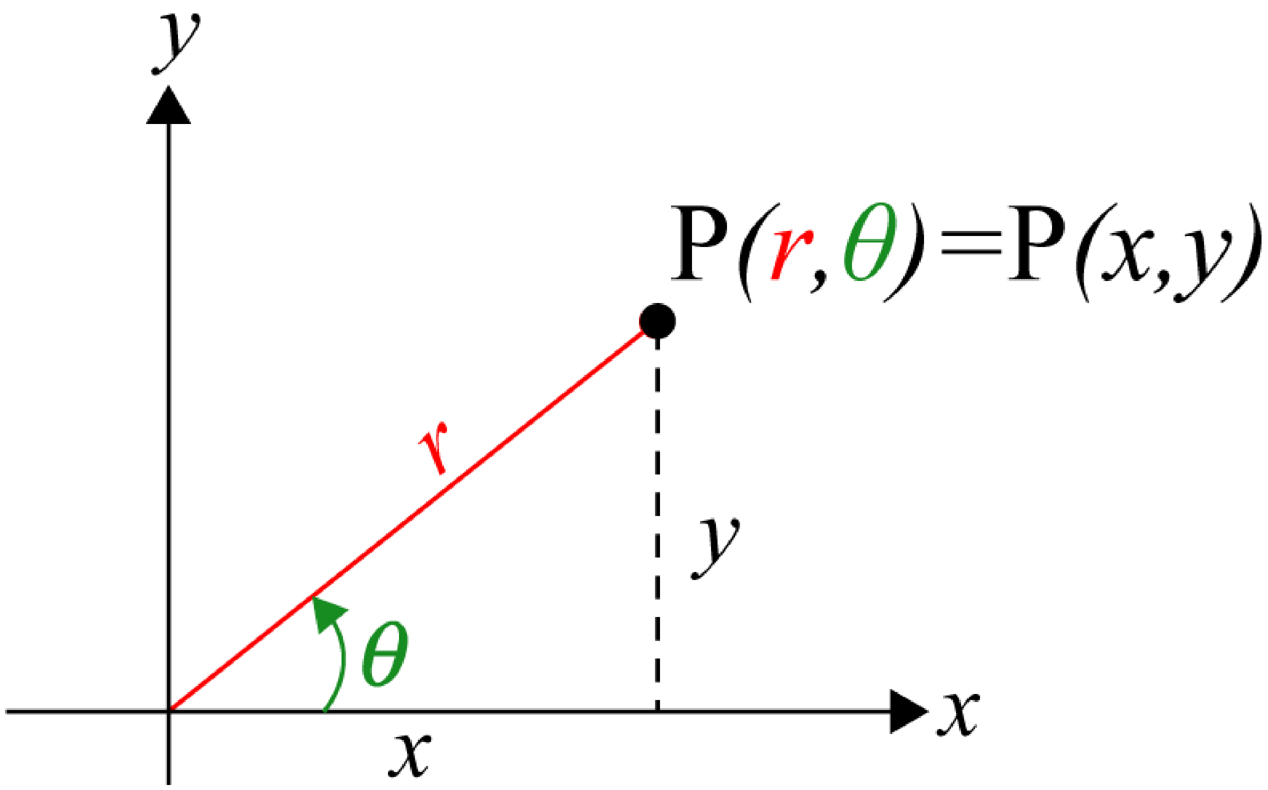
Platonic Solids

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
				

$$V - E + F = 2$$

Polyhedron		Vertices	Edges	Faces
Regular tetrahedron		4	6	4
cube		8	12	6
Regular octahedron		6	12	8
dodecahedron		20	30	12
icosahedron		12	30	20

Cartesian and Polar Coordinates



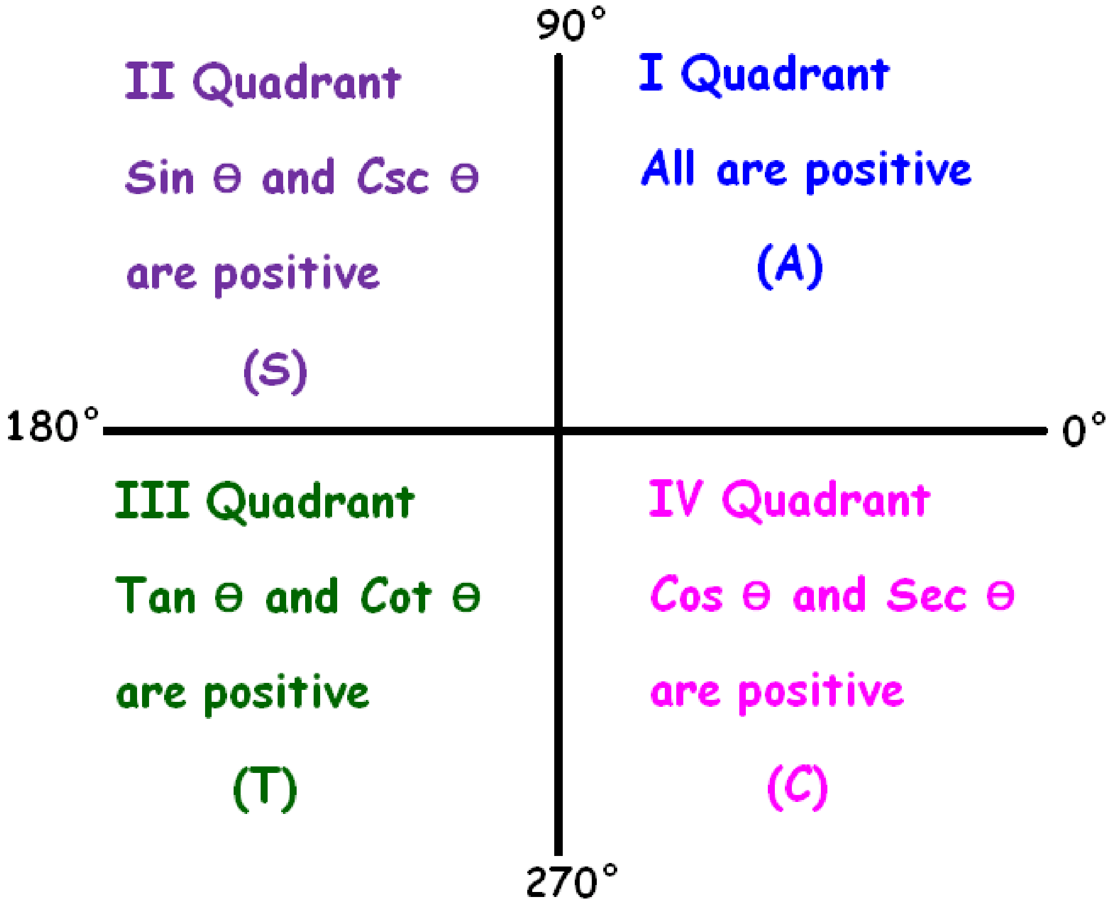
$$x = r\cos\theta$$

$$y = r\sin\theta$$

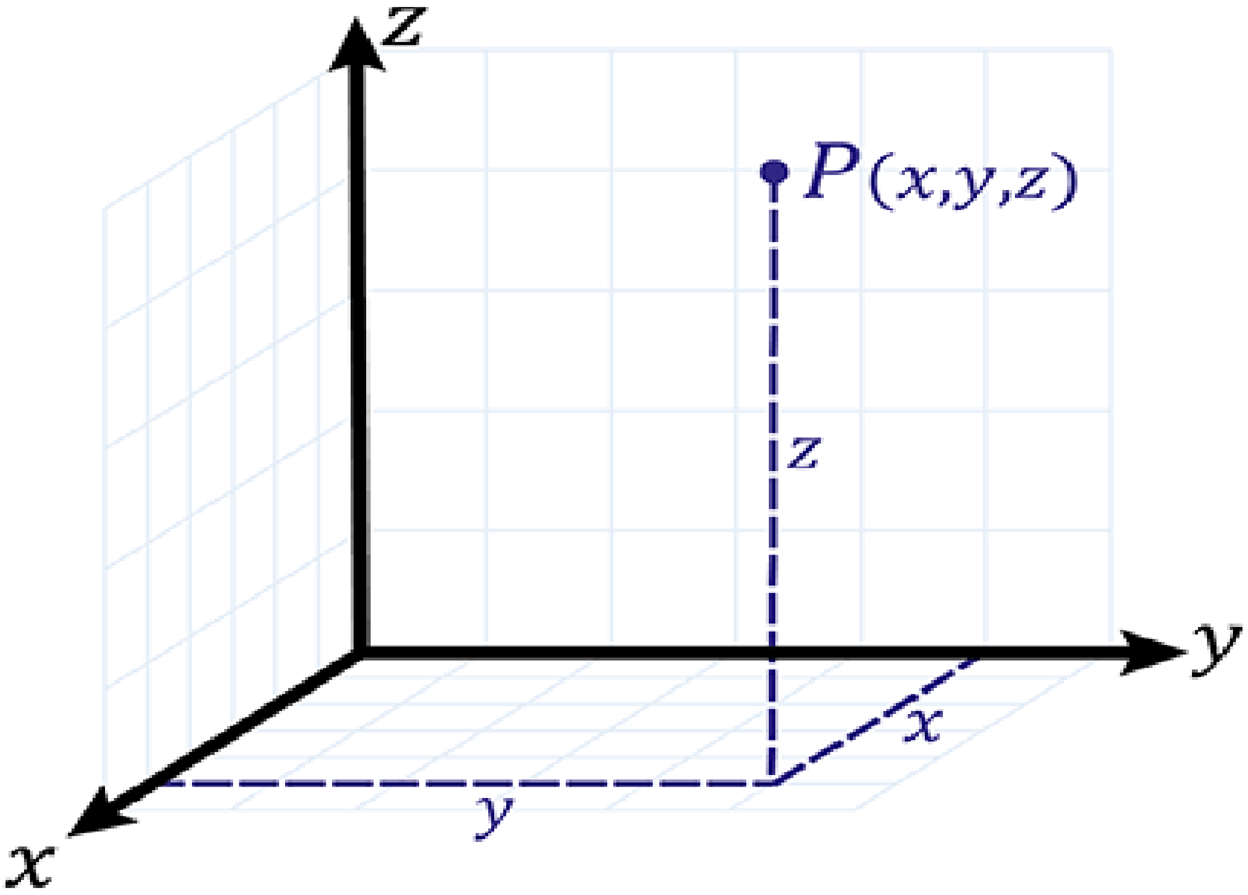
$$x^2 + y^2 = r^2$$

$$\tan\theta = \frac{y}{x}$$

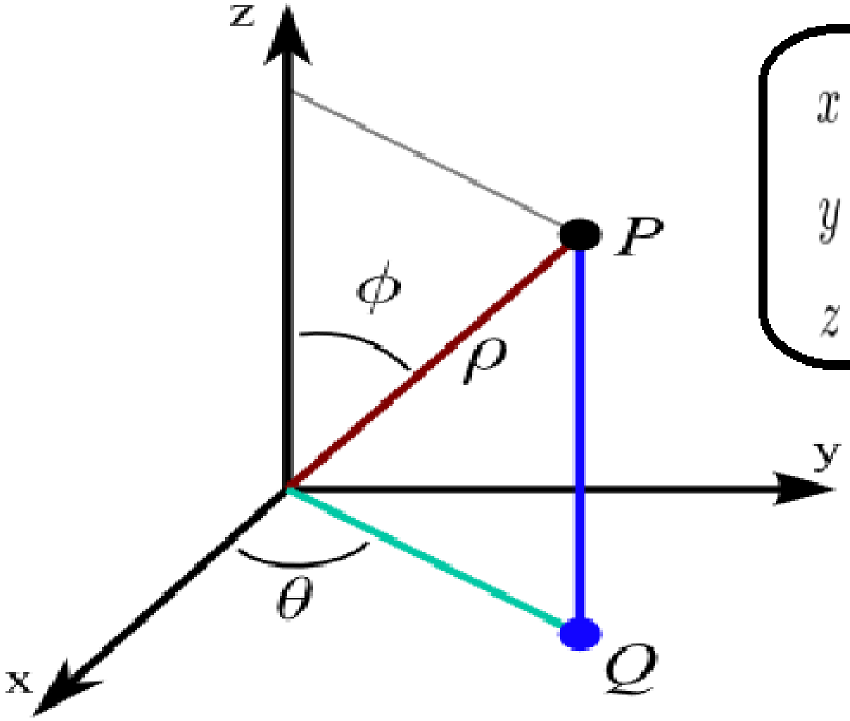
All Students Take Calculus or All Silver Tea Cups



3D Cartesian Co-odinates



Spherical Coordinates



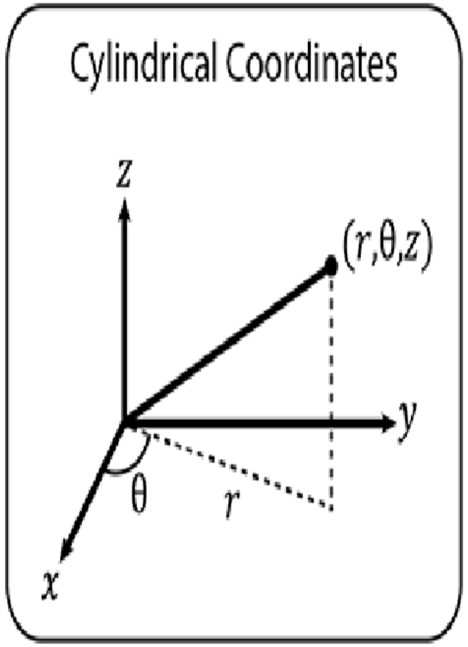
$$
\begin{aligned}
x &= r\sin\theta\cos\phi \\
y &= r\sin\theta\sin\phi \\
z &= r\cos\theta
\end{aligned}
$$

Spherical To Cartesian

$$
\begin{aligned}
r &= \sqrt{x^2+y^2+z^2} \\
\cos\theta &= \frac{z}{\sqrt{x^2+y^2+z^2}} \\
\tan\phi &= \frac{y}{x}
\end{aligned}
$$

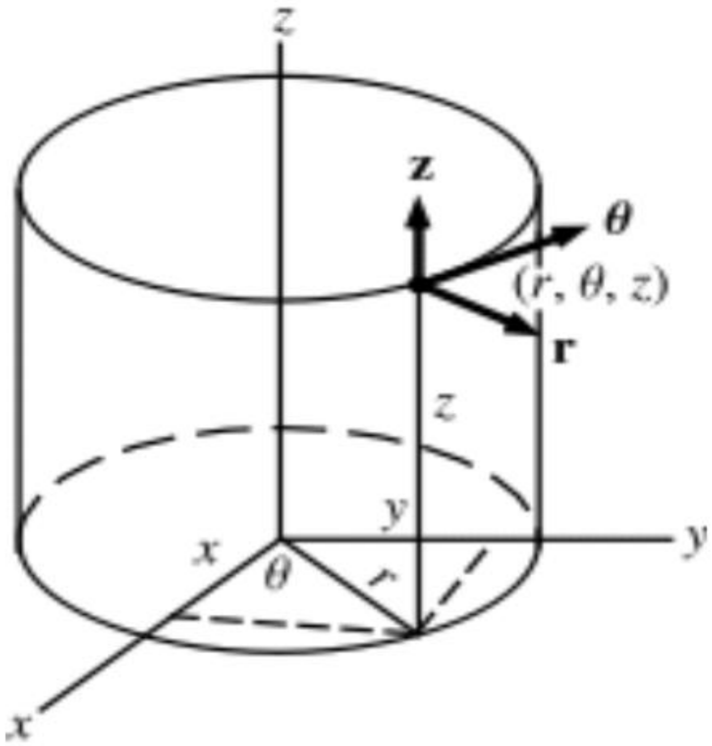
Cartesian to Spherical

Cylindrical Co-ordinates

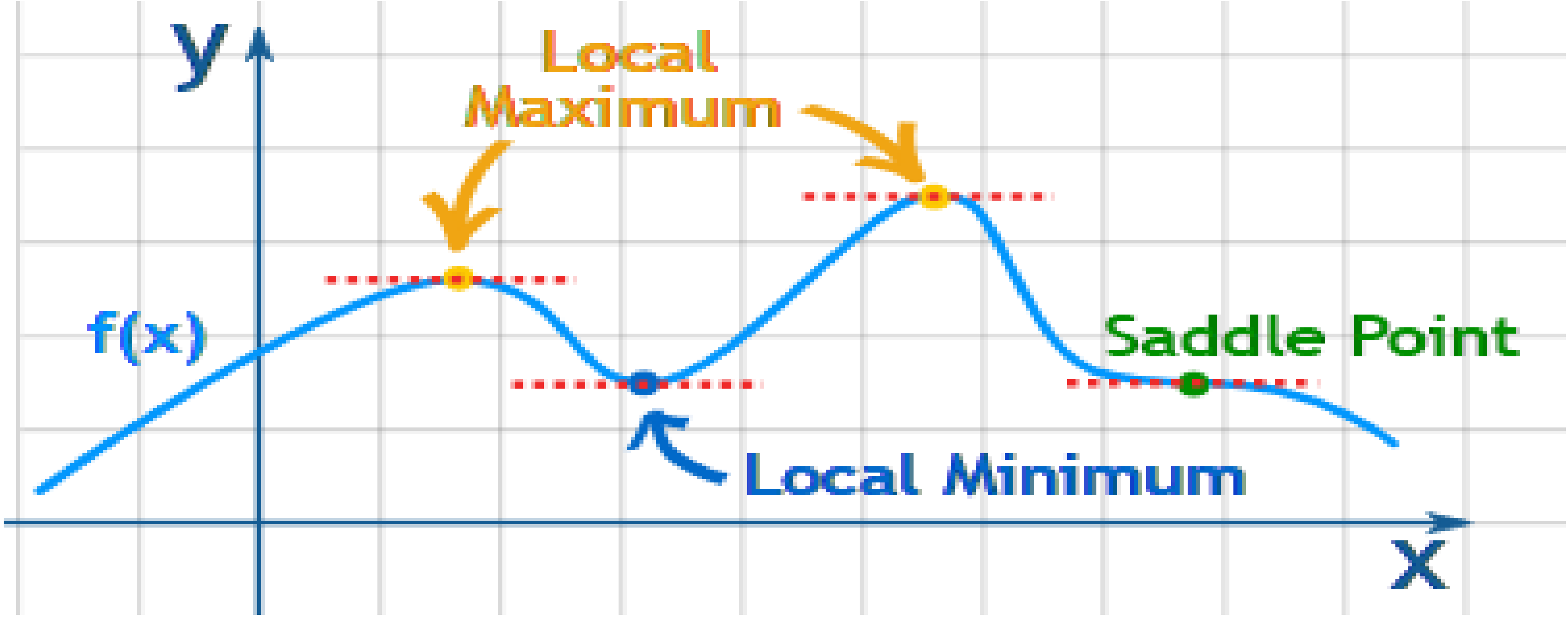


$$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x} \quad \text{and} \quad z = z$$

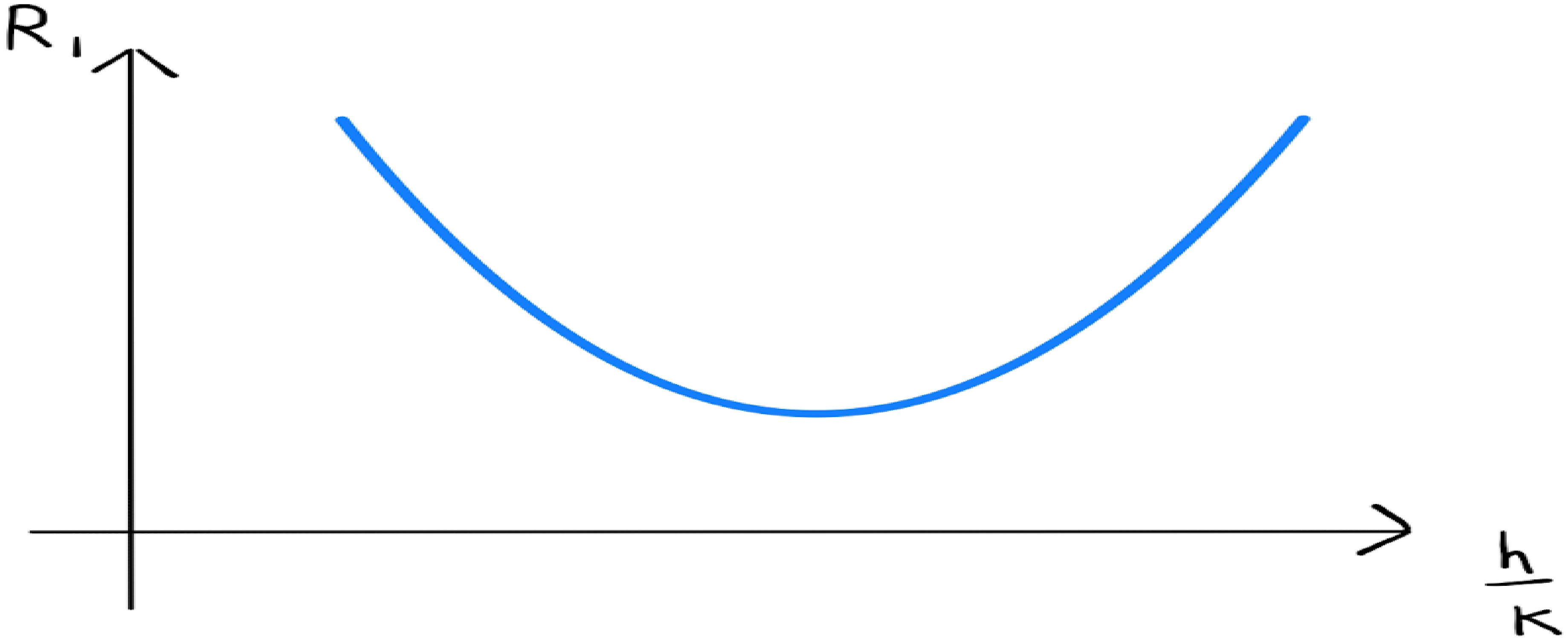
$$x = r\cos\theta, \quad y = r\sin\theta \quad \text{and} \quad z = z$$



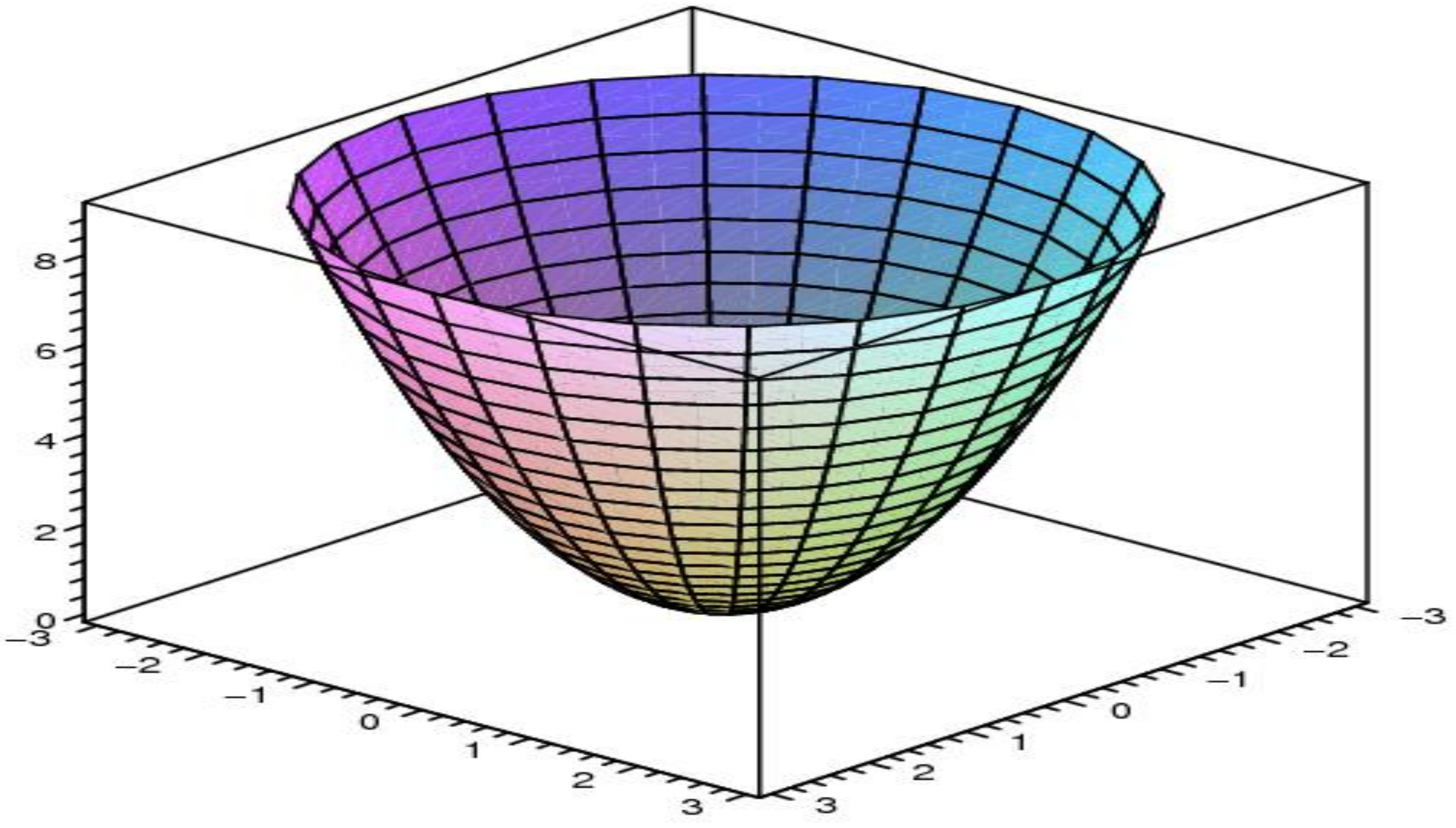
Maxima,Minima and Saddle Point



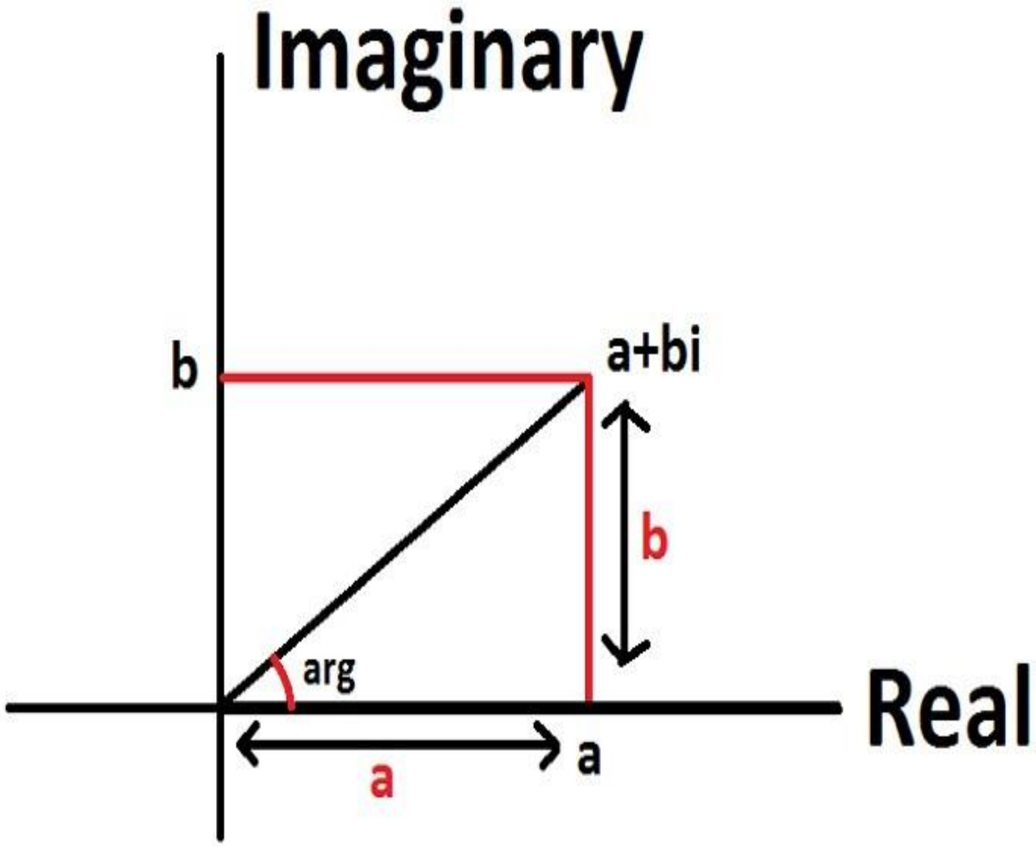
Parabolic Curve , Maxima and Minima

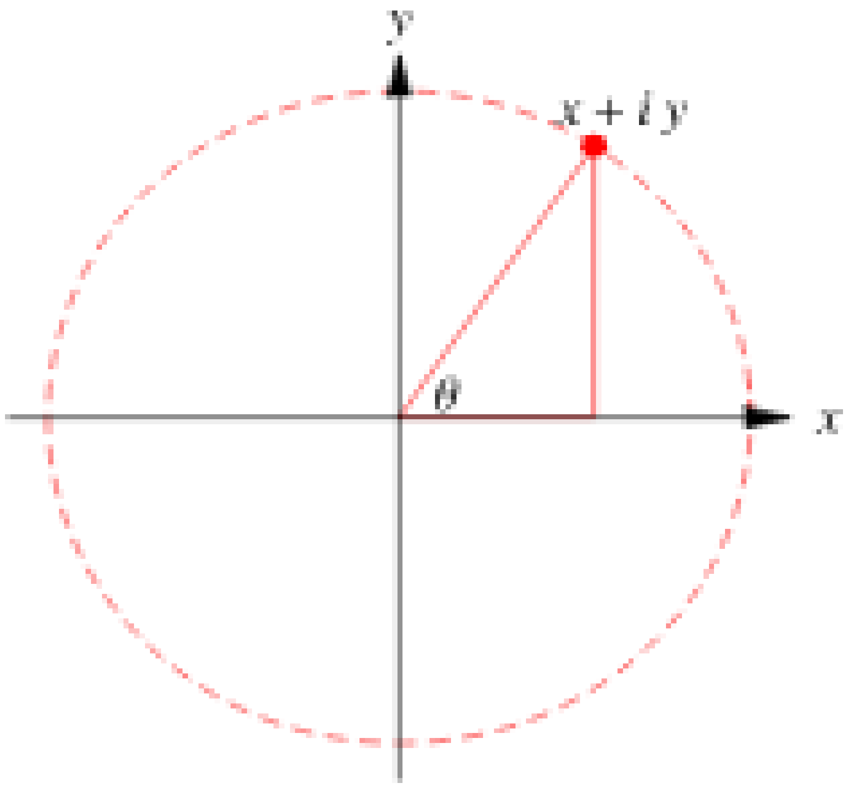


Function of two variables and Extremum

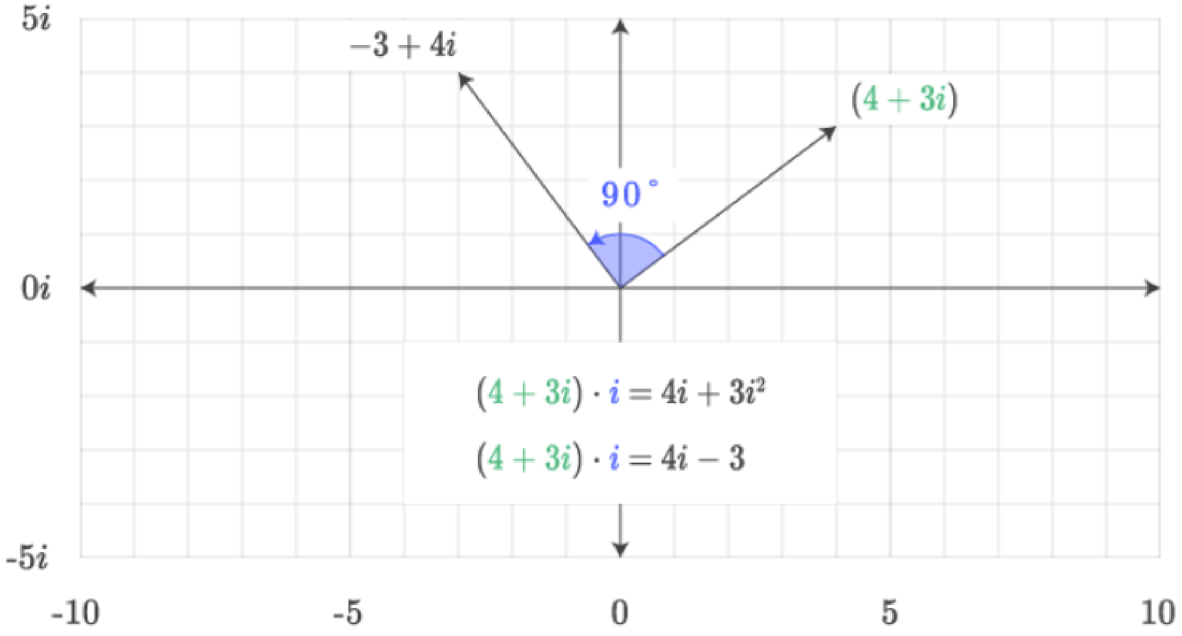


Argand Diagram (for Complex Numbers)

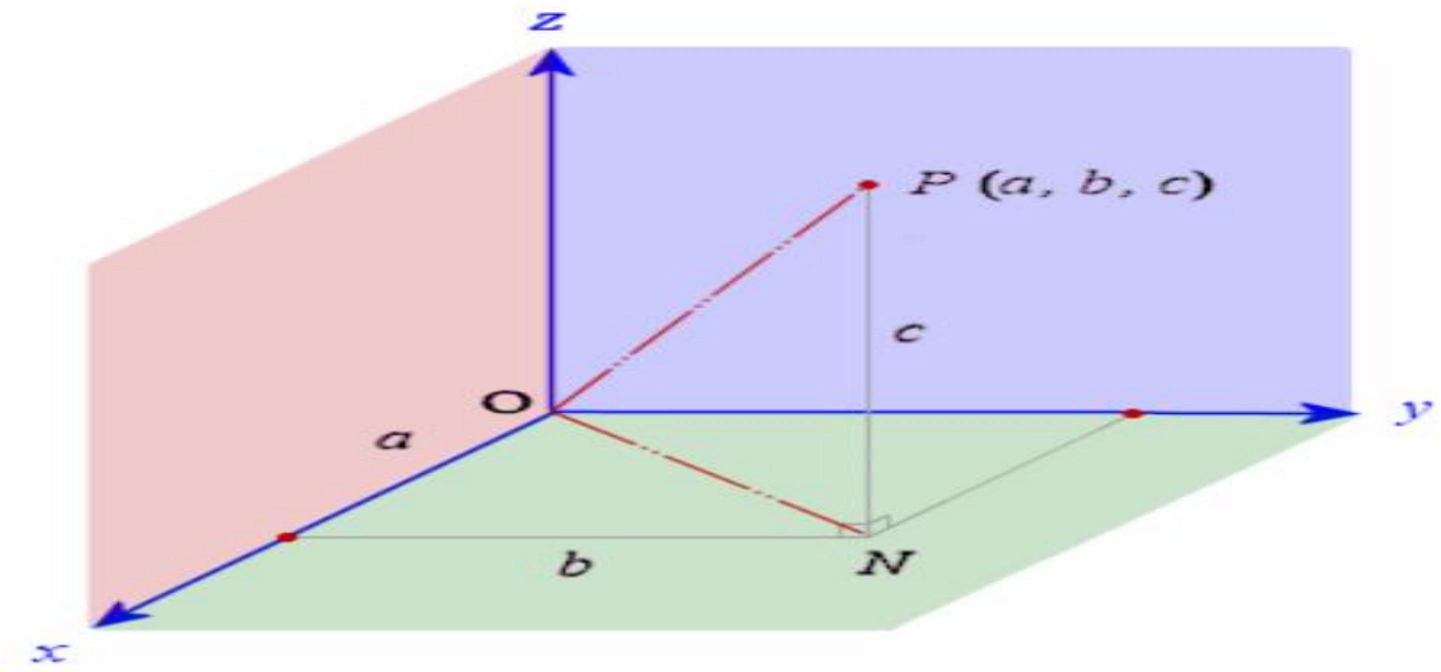




Rotation by 90 degree by multiplying with i



Pythagoras Theorem extends to 3D



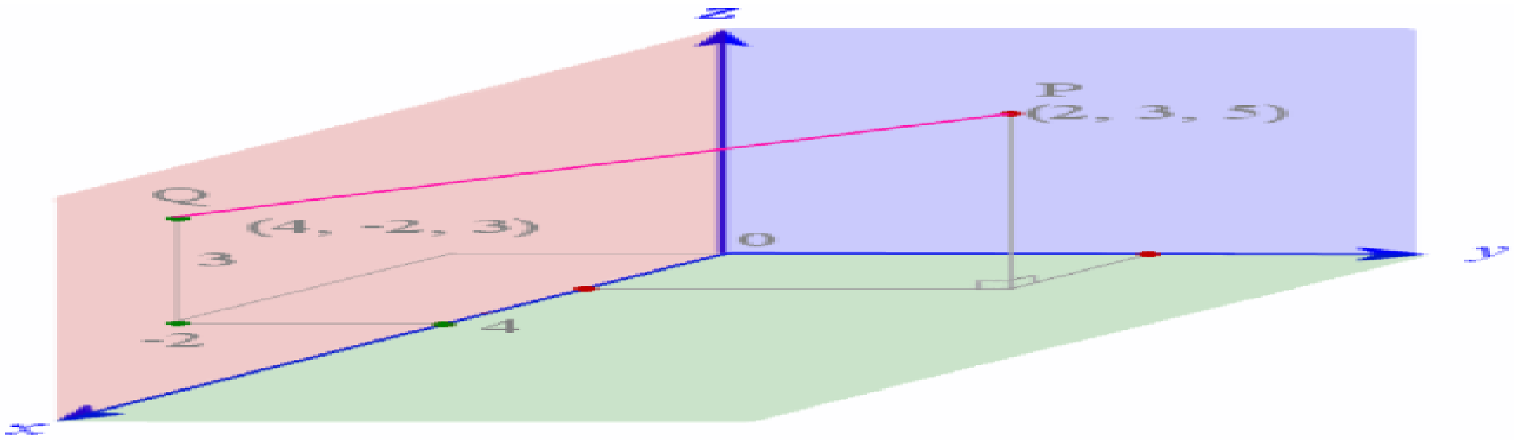
The distance from $(0, 0, 0)$ to the point $P\,(a, b, c)$ is given by:

$$\boxed{\text{distance } OP = \sqrt{a^2 + b^2 + c^2}}$$

Distance Between Two points (origin,included)

If we have point A (x_1, y_1, z_1) and another point B (x_2, y_2, z_2) then the distance AB between them is given by the formula:

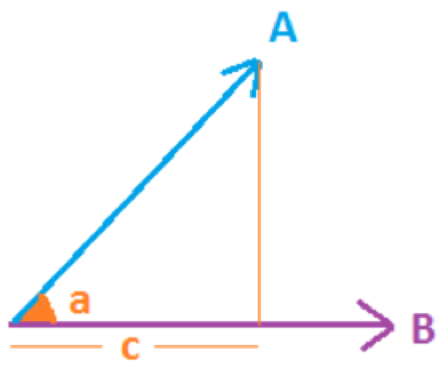
$$\text{distance } AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



Using the formula, we have:

$$\begin{aligned} \text{distance } PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(2 - 4)^2 + (3 - (-2))^2 + (5 - 3)^2} \\ &= 5.74 \text{ units} \end{aligned}$$

Dot Product



c = |A| . cos(a) *(projection of A on B)*

A . B = |A| |B| cos(a)

$$A \, . \, B = \sum_{i=1}^{N} a_i b_i$$

for unit vectors;

a = arccos(A . B)

Dot Product revisited

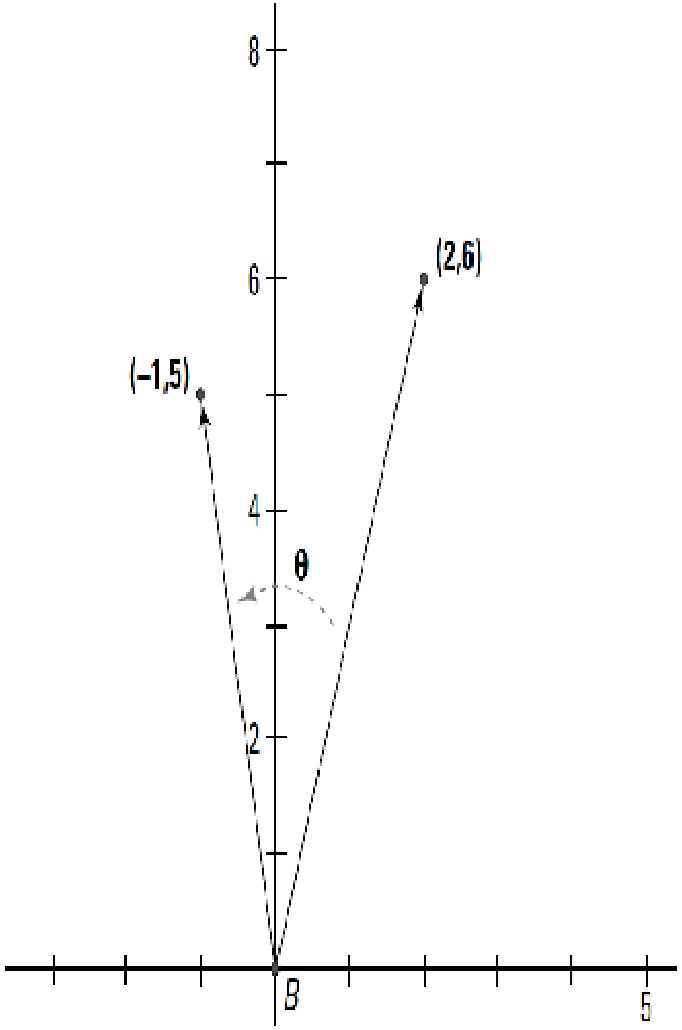
Considering the two 2D vectors:

$a = a_x\, i + a_y\, j$ and b$= b_x\, i + b_y\, j$

The scalar product:

$$a \cdot b = a_x\, b_x + a_y\, b_y$$

Angle between two vectors



$$\mathbf{u} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

$$\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 2 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \end{bmatrix} = 2(-1) + 6(5) = 28$$

$$\|\mathbf{u}\| = \sqrt{2^2 + 6^2} = \sqrt{40}, \quad \|\mathbf{v}\| = \sqrt{(-1)^2 + 5^2} = \sqrt{26}$$

$$\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \cdot \|\mathbf{v}\|} = \frac{28}{\sqrt{40} \cdot \sqrt{26}} = \frac{28}{\sqrt{1{,}040}} \approx 0.8682$$

Computing Sine and Cosine in Vectors

- **Dot-product:**

$$a \cdot b = \|a\| \, \|b\| \cos\theta \quad \Rightarrow \quad \cos\theta = \frac{a \cdot b}{\|a\|\|b\|}.$$

- **Cross-product magnitude:**

$$\|a \times b\| = \|a\| \, \|b\| \sin\theta \quad \Rightarrow \quad \sin\theta = \frac{\|a \times b\|}{\|a\|\|b\|}.$$

1. **Compute norms:** $\|a\| = \sqrt{a_x^2 + a_y^2 + a_z^2}, \quad \|b\| = \sqrt{b_x^2 + b_y^2 + b_z^2}.$
2. **Compute dot:** $a \cdot b = a_x b_x + a_y b_y + a_z b_z.$
3. **Compute cross:**

$$a \times b = \left(a_y b_z - a_z b_y,\ a_z b_x - a_x b_z,\ a_x b_y - a_y b_x\right).$$

4. **Extract cosine & sine:**

$$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|}, \quad \sin\theta = \frac{\|a \times b\|}{\|a\|\|b\|}.$$

Computing Cosine and Sine of Force vectors

Problem. Two forces act at a point:

$$F_1 = (2, 1, 2), \quad F_2 = (1, 3, 4).$$

Find the cosine and sine of the angle between them.

Solution.

1. $\|F_1\| = \sqrt{2^2 + 1^2 + 2^2} = 3, \ \|F_2\| = \sqrt{1^2 + 3^2 + 4^2} = \sqrt{26}.$
2. Dot: $d = 2 \cdot 1 + 1 \cdot 3 + 2 \cdot 4 = 13. \Rightarrow \cos\theta = 13/(3\sqrt{26}) \approx 0.850.$
3. Cross:

$$F_1 \times F_2 = (-2, \ -6, \ 5), \quad \|F_1 \times F_2\| = \sqrt{(-2)^2 + (-6)^2 + 5^2} = \sqrt{65}.$$

$\Rightarrow \sin\theta = \sqrt{65}/(3\sqrt{26}) \approx 0.527.$

Computing Angles and Trig values

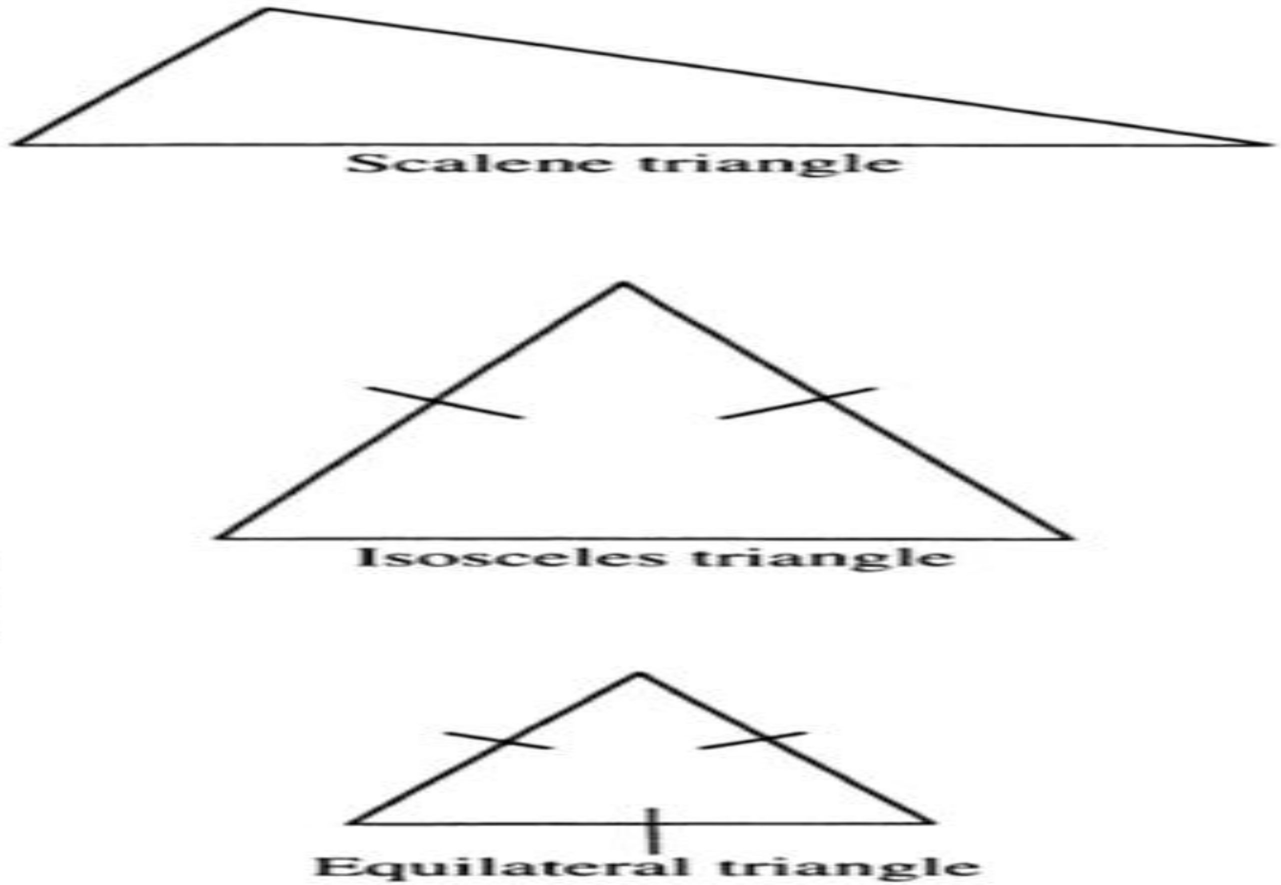
Problem. A parallelogram has adjacent edges $u = (4,2,0)$ and $v = (1,3,2)$. Compute the angle's cosine and sine.

Solution.

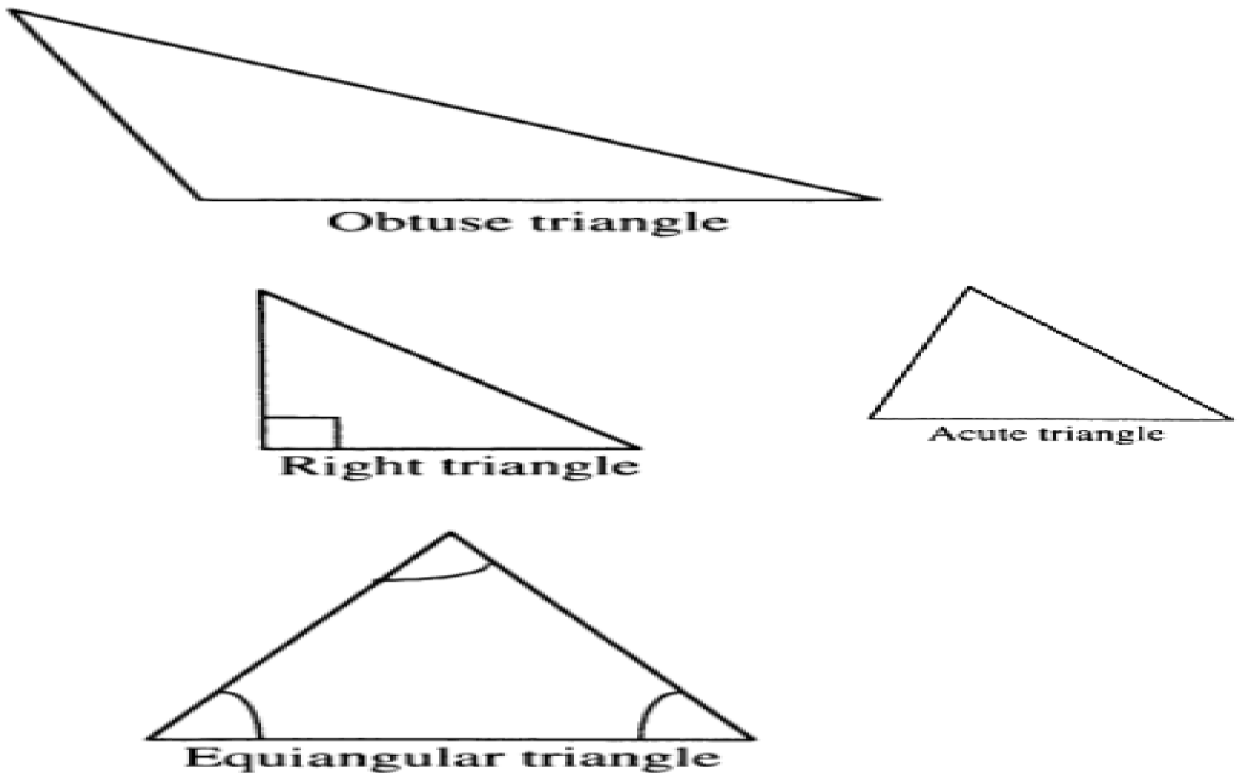
1. $\|u\| = \sqrt{4^2 + 2^2} = \sqrt{20},\ \|v\| = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{14}.$
2. Dot: $d = 4 \cdot 1 + 2 \cdot 3 + 0 \cdot 2 = 10. \Rightarrow \cos\theta = 10/(\sqrt{20}\,\sqrt{14}) \approx 0.598.$
3. Cross: $u \times v = (4, -8, 10),\ \|u \times v\| = \sqrt{4^2 + (-8)^2 + 10^2} = \sqrt{180}. \Rightarrow$

   $\sin\theta = \sqrt{180}/(\sqrt{20}\,\sqrt{14}) \approx 0.802.$

Triangle Taxonomy – By Side



Triangle Taxonomy by Angle



Questions

- If, any