Co-ordinate Systems

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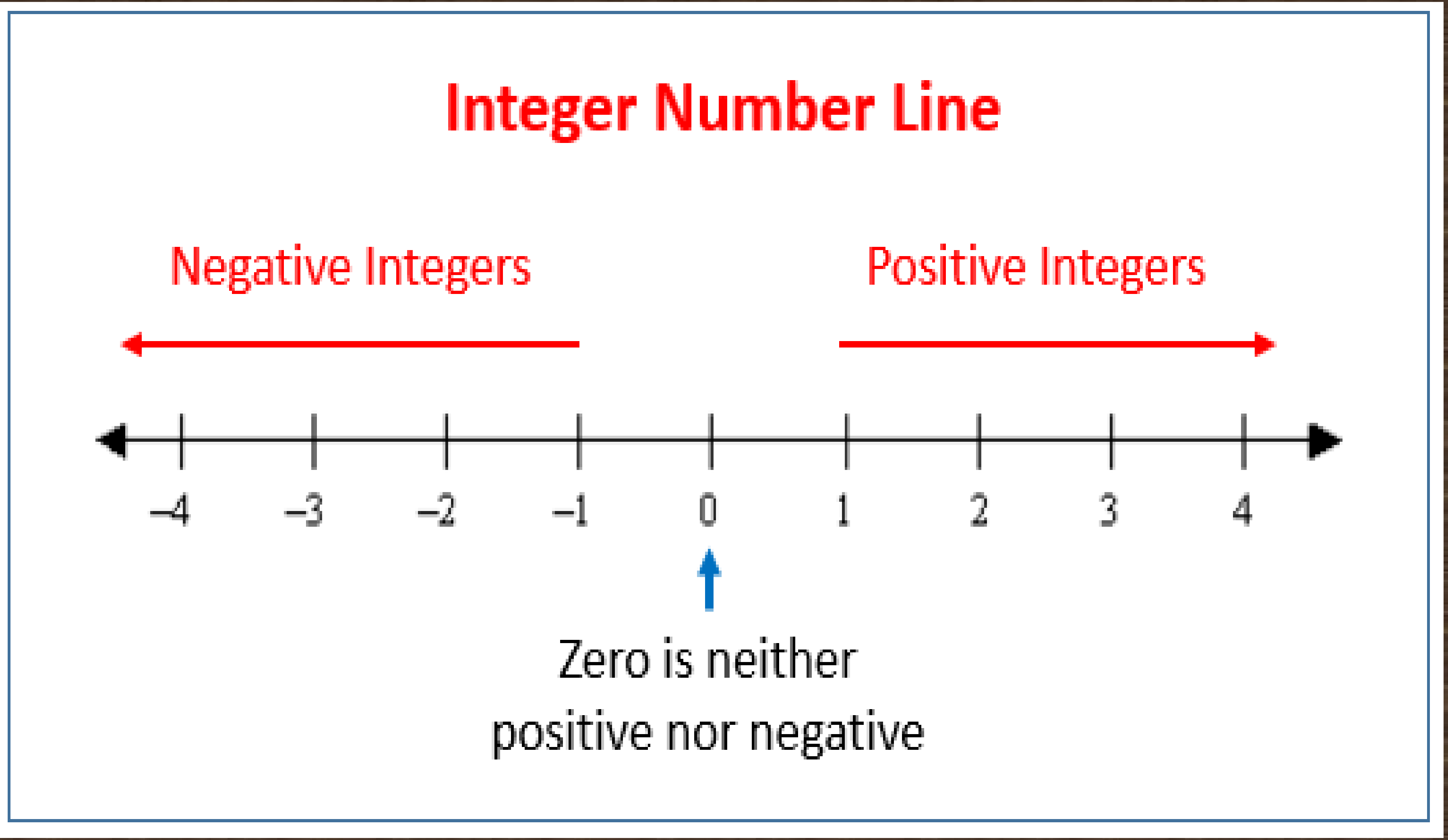
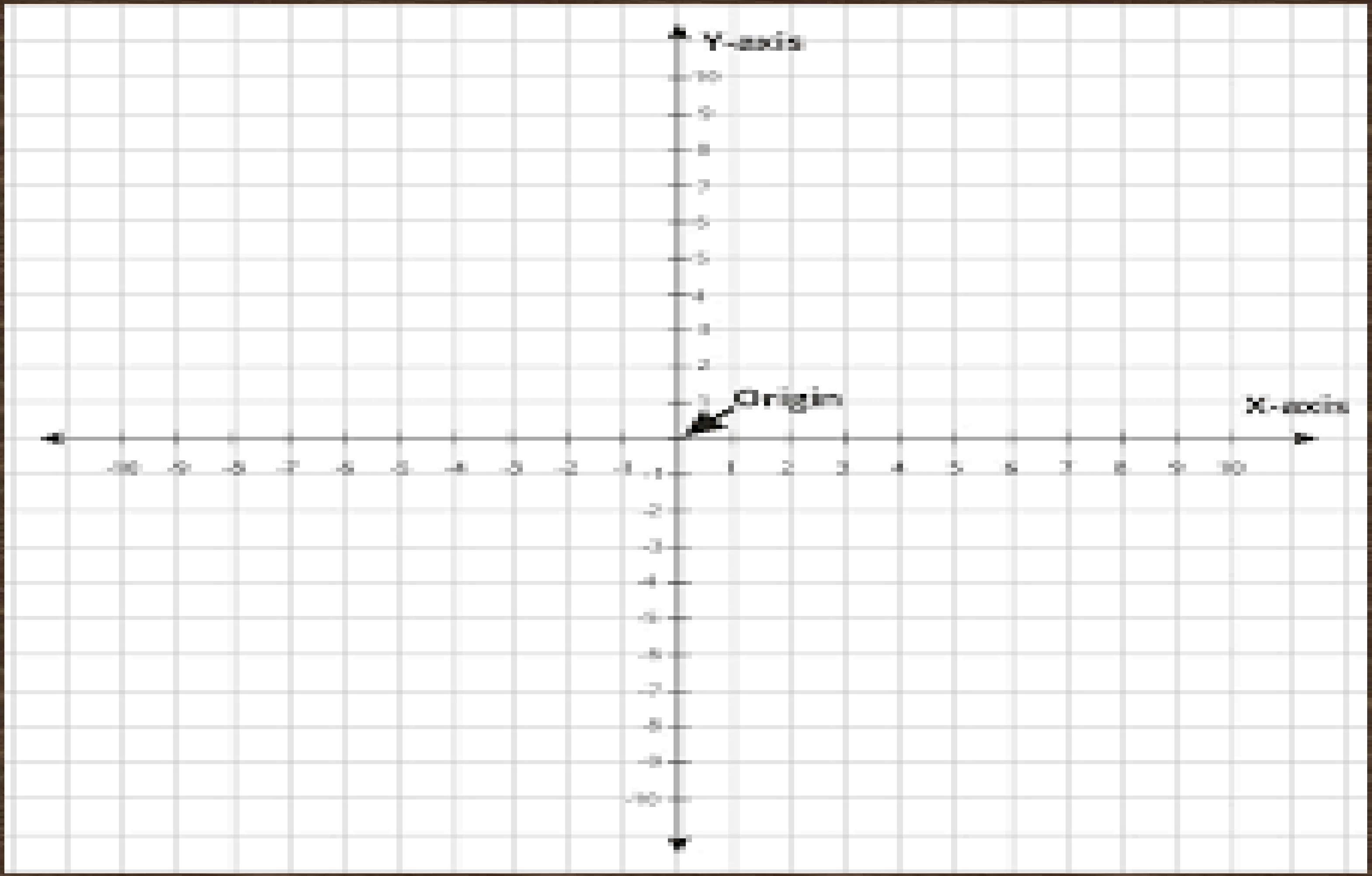
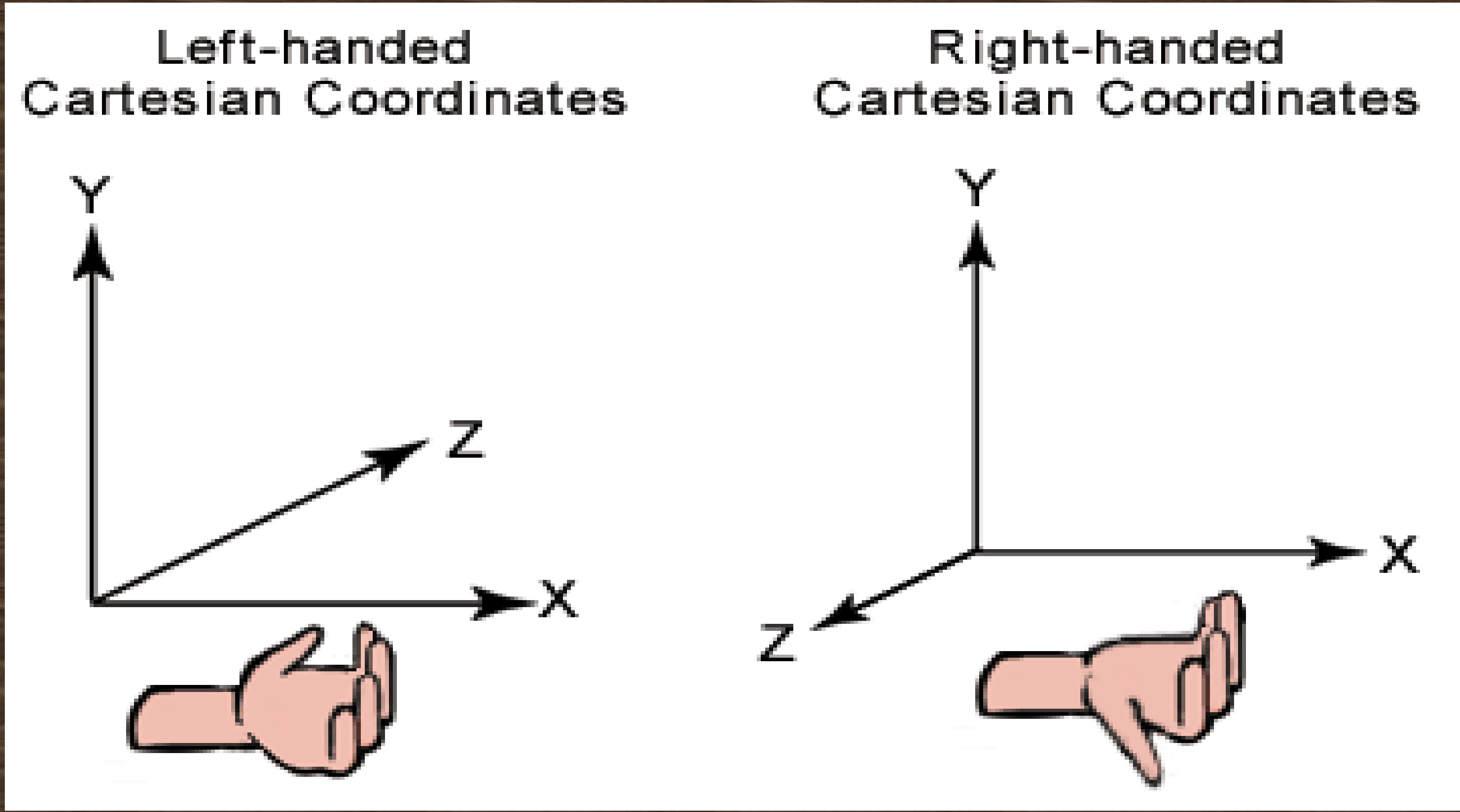
Number Line


Integer Number Line
Negative Integers
Positive Integers
–4 –3 –2 –1 0 1 2 3 4
Zero is neither
positive nor negative

2D Cartesian Co-ordinate System

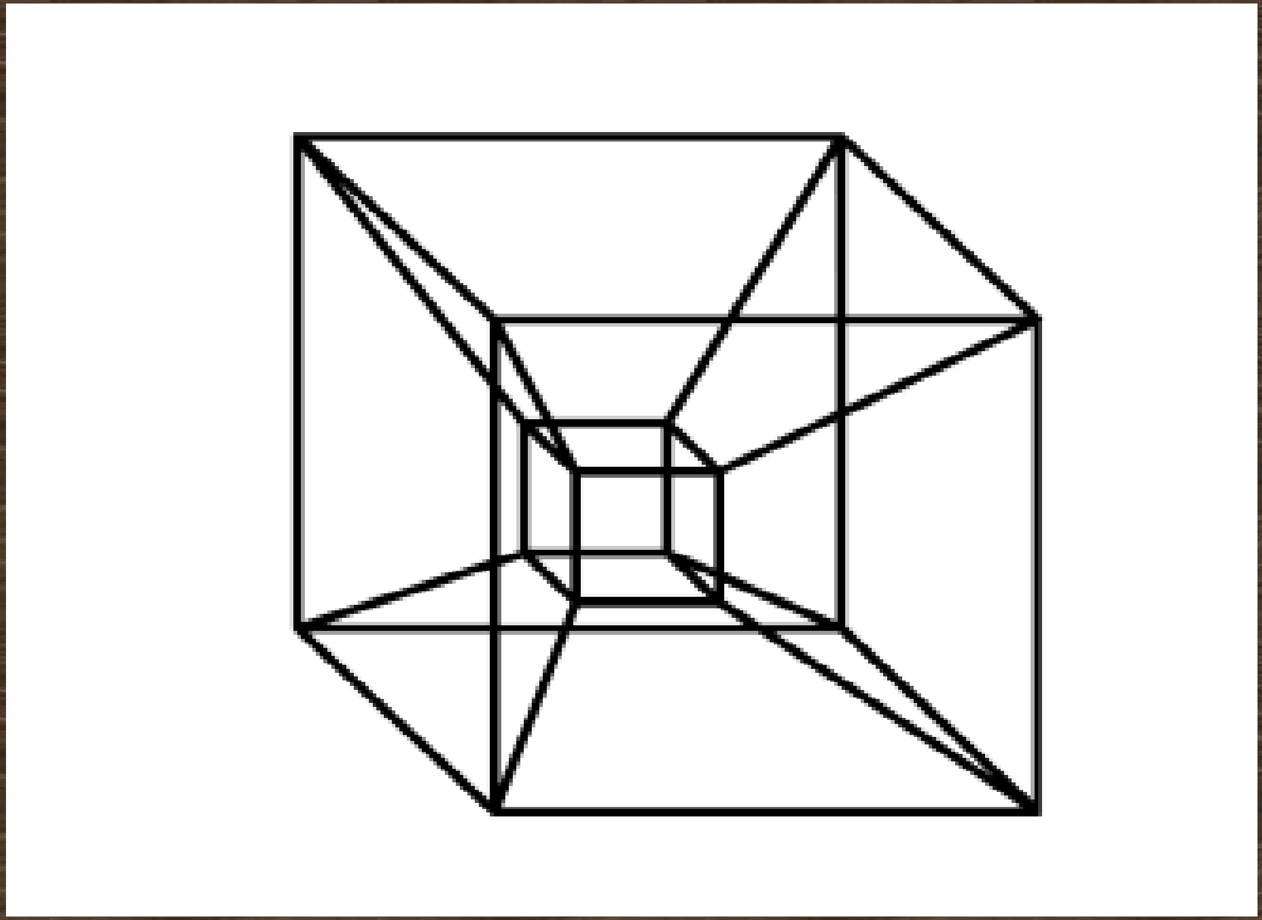


In 3D, Where should Z go?

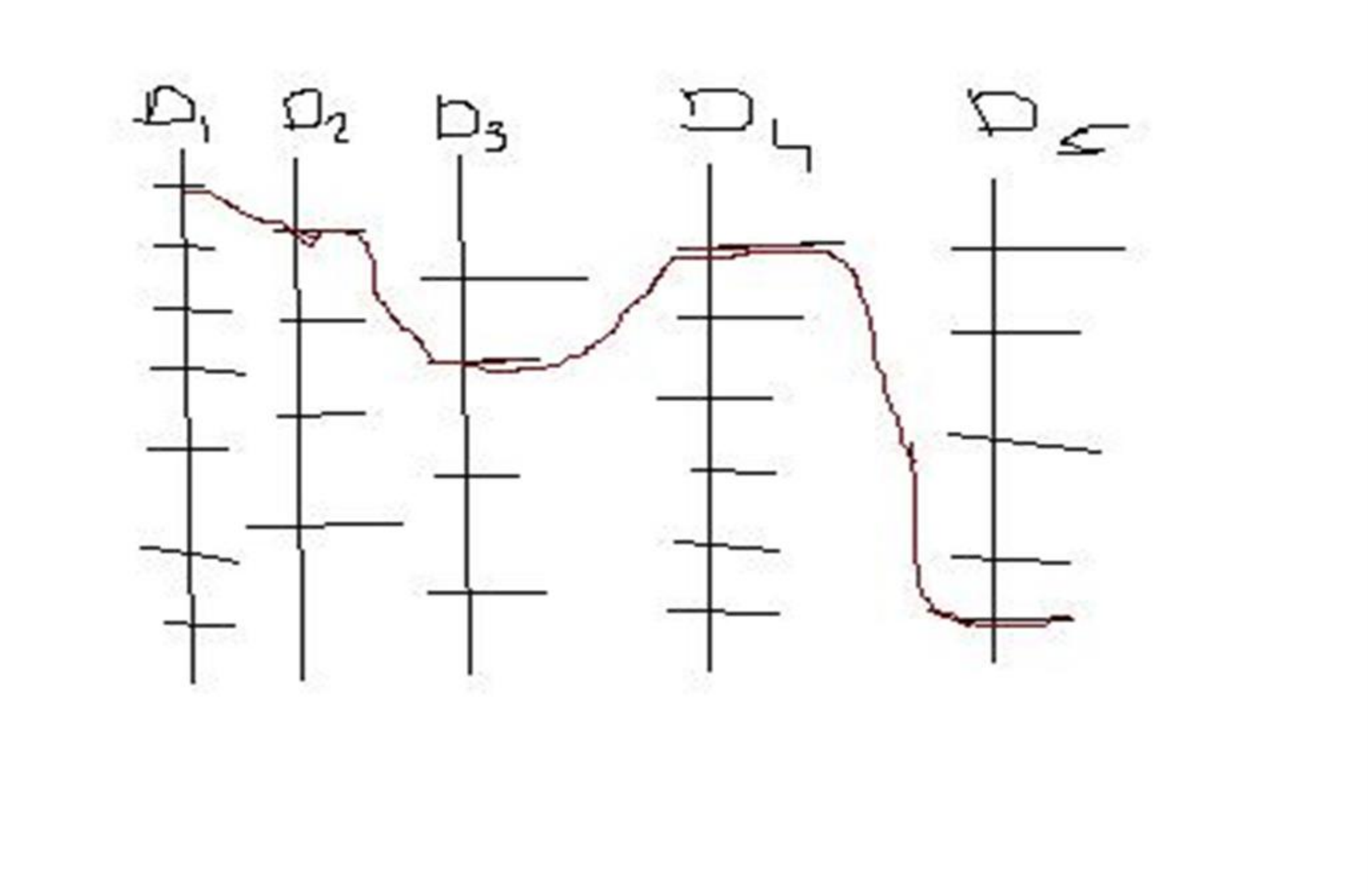


Right handed vs Left Handed System

Tesseract (4 Dimensional Cube)



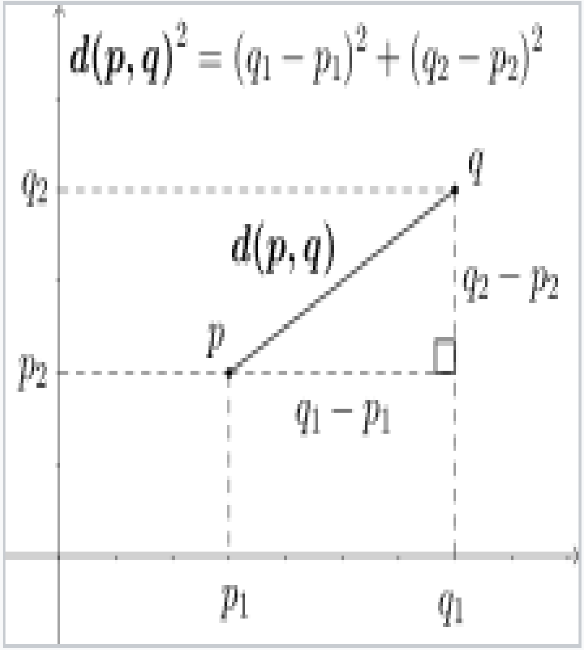
How to Visualize more than three dimensions?



A point can represent one dimension (number line) , a line can represent 2 dimension (a plane) and a cube three dimension (space) . when it comes to four and above , cartesian geometric depiction of dimension fails .

2.161 "There must be something identical in a picture and what it depicts , to enable one to be a picture of the other at all"

Norm or Length (in 2D,3D, N D)



Euclidean distance in R^2

Two dimensions

In the Euclidean plane, if **p** = (p_1, p_2) and **q** = (q_1, q_2) then the distance is given by

$$d(\mathbf{p}, \mathbf{q}) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}.$$

This is equivalent to the Pythagorean theorem.

Three dimensions

In three-dimensional Euclidean space, the distance is

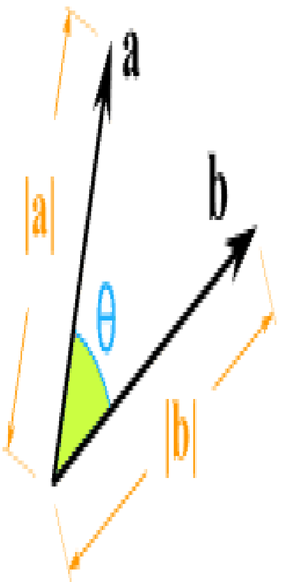
$$d(\mathbf{p}, \mathbf{q}) = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + (p_3 - q_3)^2}.$$

n dimensions [edit]

In general, for an *n*-dimensional space, the distance is

$$d(\mathbf{p}, \mathbf{q}) = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + \cdots + (p_i - q_i)^2 + \cdots + (p_n - q_n)^2} = \sqrt{\sum_{i=1}^{n} (p_i - q_i)^2}.$$

Dot Product (in 2D aka Inner Product)



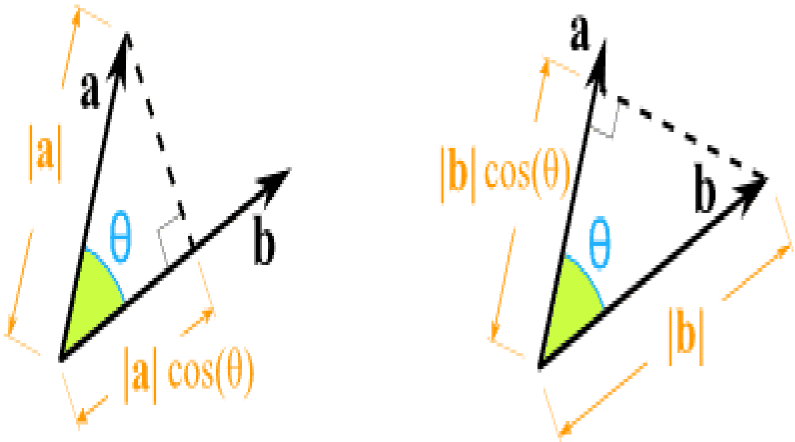
$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$$

Where:

$|\mathbf{a}|$ is the magnitude (length) of vector **a**

$|\mathbf{b}|$ is the magnitude (length) of vector **b**

θ is the angle between **a** and **b**



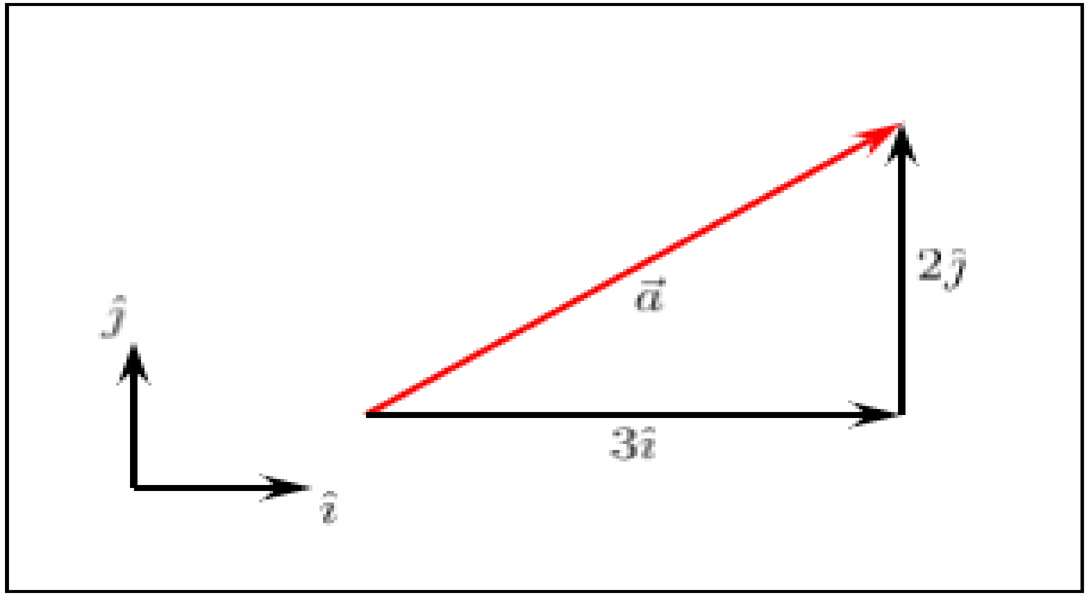
$$|\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta) = |\mathbf{a}| \times \cos(\theta) \times |\mathbf{b}|$$

2D Basis Vector

Components of a vector.

$$\vec{a} = a_1 \hat{\imath} + a_2 \hat{\jmath}$$

Writing a vector as the sum of scaled basis vectors. The scale factors are the components of the vector. Here $\vec{a} = 3\hat{\imath} + 2\hat{\jmath}$, so the components of $\vec{a}$ are $a_1 = 3$ and $a_2 = 2$. #rvv-fb



An important fact about the Dot Product

Thursday, July 17, 2014

Why DotProduct is term wise mutliplication?

V = (ax,ay) and U = (bx,by) , their DotProduct is U.V = (ax*bx , ay*by)

V = (ax*I , ay*J) , U = (bx*I,by*J) , I and J be unit vector on X and Y

Term-wise multiplication yields

U.V = (ax*bx*I*I + ax*by*I*J , ay*bx*J*I + ay*by*J*J)

As I*J=0 , I*I = 1 , J*J=1,J*I= 0 , the whole stuff reduces to

U.V= (ax*bx*1 + 0 , 0+ ay*by*1)

Therefore, U.V = (ax*bx , ay*by)

Find the Angle using Dot Product

Find the angle between two vectors **a** and **b**, given

$$\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \qquad \mathbf{b} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}.$$

$$\| \mathbf{a} \| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} \qquad \| \mathbf{b} \| = \sqrt{4^2 + 5^2 + 6^2} = \sqrt{77}.$$

Using $\quad \mathbf{a} \cdot \mathbf{b} = \| \mathbf{a} \| \cdot \| \mathbf{b} \| \cos\theta = x_a x_b + y_a y_b + z_a z_b$

$$\mathbf{a} \cdot \mathbf{b} = \sqrt{14}\sqrt{77} \cos\theta = 1 \times 4 + 2 \times 5 + 3 \times 6 = 32.$$

Using $\quad \theta = \cos^{-1}\left(\dfrac{x_a x_b + y_a y_b + z_a z_b}{\| \mathbf{a} \| \, \| \mathbf{b} \|} \right)$

$$\theta = \cos^{-1}\left(\frac{32}{\sqrt{14}\sqrt{77}} \right) = 12.9^\circ.$$

See whether two vectors are Perpendicular

Prove that two vectors are perpendicular, given that

$\mathbf{a} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ $\mathbf{b} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

For Vectors to be perpendicular, Dot Product should be Zero!

$$\mathbf{a} \cdot \mathbf{b} = (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) \cdot (4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = 0.$$

4i^2 + 2ij + 5ik + 12ij + 6j^2 + 15jk + -8ik + -4jk + - 10k^2
4i^2 + 14ij + 20ik + 6j^2 + 11jk -3ik - 10k^2
4 + 0 + 0 + 6 + 0 - 0 - 10 = 0

Q&A

- If any!