Mathematical Literacy – Part 1

Write a Program to Print From 1 to N?

```
//---------------- O(n) algorithm
int sum_from_one_to_n_lineartime( int n ) {
    int sum = 0,j;
    for( j=0; j<= n; ++j) { sum += j; }
    return sum;
}
```

Is there any Constant Time Algorithm?

Constant Time Algorithm

```
//------------------ O(1) algorithm
int sum_from_one_to_n_constanttime( int n ) { return (n*(n+1)) >> 1; }
```

The Whole Program

```
#include <stdio.h>
#include <stdlib.h>
//----------------- O(n) algorithm
int sum_from_one_to_n_lineartime( int n ) {
      int sum = 0,j;
      for( j=0; j<= n; ++j) { sum += j; }
      return sum;
}
//------------------- O(1) algorithm
int sum_from_one_to_n_constanttime( int n ) { return (n*(n+1)) >> 1; }
//-------------------- User Entry Point
int main( int argc , char **argv ){
      int n = ( argc > 1 ) ? atoi(argv[1]) : 10;
      printf("%d\t%d\n",sum_from_one_to_n_lineartime(n),
                              sum_from_one_to_n_constanttime(n));
      fflush(stdout);
}
```

Same Program with Large Number Arithmetic

```java
import java.math.BigInteger;
import java.util.Scanner;

public class OneToN
{
   // Returns Products of First  N
   static BigInteger Product(int N) {
      BigInteger f = new BigInteger("1");
      for (int i = 2; i <= N; i++)
         f = f.multiply(BigInteger.valueOf(i));
            return f;
   }
    // Returns Sigma of First  N
   static BigInteger Sigma(int N) {
      BigInteger f = new BigInteger("0");
      for (int i = 1; i <= N; i++)
         f = f.add(BigInteger.valueOf(i));
      return f;
   }

   // Returns Sigma of First  N WithoutLoop
   static BigInteger SigmaWithOutLoop(int N) {
      BigInteger f = BigInteger.valueOf(N);
      // N*(N+1)/2
      f = f.multiply(f.add( BigInteger.valueOf(1))).divide(new BigInteger("2"));
      return f;
   }
```

```java
 // Driver method
   public static void main(String args[]) throws Exception
   {
      int N = 200;

      int len = args.length;
            if(len==0) {
                              System.out.println("No args");
                              return;
            }

       N = Integer.parseInt(args[0]);
            System.out.println(Product(N));
       System.out.println(Sigma(N));
            System.out.println(SigmaWithOutLoop(N));

   }
}
```

From Where I can get the Code Listing?

- https://github.com/praseedpai/ElementaryMathForProgrammingSeries/blob/master/AlgebraNArith/OneToN/oneton.cpp

- https://github.com/praseedpai/ElementaryMathForProgrammingSeries/blob/master/AlgebraNArith/OneToN/OneToN.java

Introducing Summation Notation (Sigma)

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\mu = \frac{\sum x}{n}$$

$$\mathbf{E}[X] = \sum_{i=1}^{k} x_i\, p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k.$$

$$\mathbf{E}[X] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5.$$

$$\mathbf{E}[X] = \mu.$$

Introducting Product Notation (PI)

$$\left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}} = \sqrt[n]{a_1 a_2 \cdots a_n}.$$

Some mathematical Shortcuts!

(i) $$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

(ii) $$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(iii) $$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

(iv) $$1 + 3 + 5 + \ldots + (2n - 1) = n^2$$

Mathematical Induction

Mathematical Induction

Step 1: Prove the result is true for $n = 1$ (or whatever the first term is)

Step 2: Assume the result is true for $n = k$, where k is a positive integer (or another condition that matches the question)

Strong Mathematical Induction

- Let $P(n)$ be a property that is defined for all integers n, and let a and b be fixed integers with $a \leq b$. Suppose the following two statements are true:

1. Basic Step: $P(a)$, $P(a+1)$, …, and $P(b)$ are all true.
2. Inductive Step: For any integer $k \geq b$, if $P(i)$ is true for all integers i from a through k, then $P(k + 1)$ is true.

Mathematical Induction – Example #1

Proposition. For any $n \in \mathbb{N}$, $0 + 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

Proof. Let $P(n)$ be the statement $0 + 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$. We give a proof by induction on n.

Base case: Show that the statement holds for the smallest natural number $n = 0$.

$P(0)$ is clearly true: $0 = \frac{0(0+1)}{2}$.

Inductive step: Show that for any $k \geq 0$, if $P(k)$ holds, then $P(k+1)$ also holds.

Assume the induction hypothesis that for a particular k, the single case $n = k$ holds, meaning $P(k)$ is true:

$$0 + 1 + \cdots + k = \frac{k(k+1)}{2}.$$

$$(0 + 1 + 2 + \cdots + k) + (k+1) = \frac{k(k+1)}{2} + (k+1).$$

$$\begin{aligned}\frac{k(k+1)}{2} + (k+1) &= \frac{k(k+1) + 2(k+1)}{2} \\ &= \frac{(k+1)(k+2)}{2} \\ &= \frac{(k+1)((k+1)+1)}{2}.\end{aligned}$$

Equating the extreme left hand and right hand sides, we deduce that:

Mathematical Induction – Example #2

$\sum_{i=1}^{n} i^2 = \frac{1}{6}\, n(n+1)(2n+1).$

$$
\begin{aligned}
\sum_{i=1}^{n+1} i^2 &= (n+1)^2 + \sum_{i=1}^{n} i^2 \\
&= (n+1)^2 + \frac{1}{6}\, n(n+1)(2n+1) \quad \textbf{(by the inductive hypothesis)} \\
&= \frac{1}{6}\,(n+1)[6(n+1) + n(2n+1)] \\
&= \frac{1}{6}\,(n+1)\left[6n + 6 + 2n^2 + n\right] \\
&= \frac{1}{6}\,(n+1)\left[2n^2 + 7n + 6\right] \\
&= \frac{1}{6}\,(n+1)(n+2)(2n+3).
\end{aligned}
$$

Mathematical Induction – Example #3

$$1^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

First, we show that this statement holds for $n = 1$.

$$1^3 = 1$$

$$\left(\frac{1(1+1)}{2}\right)^2 = 1$$

Suppose it's true for $n = k$. Then,

$$1^3 + \cdots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$$

$$\begin{aligned}
(1^3 + \cdots + k^3) + (k+1)^3 &= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 \\
&= (k+1)^2 \left(\left(\frac{k}{2}\right)^2 + (k+1)\right) \\
&= \frac{(k+1)^2(k^2 + 4k + 4)}{4} \\
&= \left(\frac{(k+1)(k+2)}{2}\right)^2
\end{aligned}$$

Mathematical Induction – Example #4

$1 + 3 + 5 + \ldots + (2n-1) = n^2$ for $n = 1, 2, \ldots$

Step 1 – For $n = 1, 1 = 1^2$, Hence, step 1 is satisfied.

Step 2 – Let us assume the statement is true for $n = k$.

Hence, $1 + 3 + 5 + \cdots + (2k-1) = k^2$ is true (It is an assumption)

We have to prove that $1 + 3 + 5 + \ldots + (2(k+1)-1) = (k+1)^2$ also holds

$$
\begin{aligned}
&1 + 3 + 5 + \cdots + (2(k+1)-1) \\
&= 1 + 3 + 5 + \cdots + (2k+2-1) \\
&= 1 + 3 + 5 + \cdots + (2k+1) \\
&= 1 + 3 + 5 + \cdots + (2k-1) + (2k+1) \\
&= k^2 + (2k+1) \\
&= (k+1)^2
\end{aligned}
$$

So, $1 + 3 + 5 + \cdots + (2(k+1)-1) = (k+1)^2$ hold which satisfies the step 2.

Hence, $1 + 3 + 5 + \cdots + (2n-1) = n^2$ is proved.

Q&A

- If any!