Mathematical Literacy – Part 2

What is “e”?

```
e = ( 1 + 1/n ) ^n ,  as n -> INF ( infinity )

Imagine the functional relationship , f(n ) = ( 1 + 1/n)^n

Then,

    f(1) = f(1 + 1/1)^1 = 2
    f(3) = f(1 + 1/3)^3 = 2.37037
    f(4) = f(1 + 1/4)^4 = 2.44141
    f(5) = f(1+  1/5)^5 = 2.48832
                  ..
                  ..
    f(1000) = f(1 + 1/1000)^1000
= 2.7169239322358924573830881219476

    f(5000) = f(1 + 1/5000)^5000
= 2.7180100501018540468342171061063

    f(50000) = f( 1 + 1/50000)^50000
= 2.7182546461391027996571232576133

The value slowly converges to 2.71828 as n approaches INF.
```

Compute E using a Simple C/C++ program

```
// e => ( n + (1/n))^n as lim n => INFINITY
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

int main( int argc , char **argv ){
  if ( argc == 1 ) { return 0; }
  int n = atoi(argv[1]);
  // Compute the Kernel ( 1 + 1/n);
  double nucleus = 1.0 + (1.0 /(double)n);
  // Compute EXP for one
  double exp_one   = exp(1.0);
  // Compute  nucleus ^ n
  double exp_brute = pow(nucleus,(double)n);
  // Spit the result on to the console
  printf("CRT exp = %g\t BRUTE exp = %g\n",exp_one,exp_brute);
}
```

From Where I can get the Above Program?

- https://github.com/praseedpai/ElementaryMathForProgrammingSeries/blob/master/AlgebraNArith/Exponent/Exp.cpp

The Dump of the Result

```
G:\BarCampReport>Test 1000000
CRT exp = 2.71828       BRUTE exp = 2.71828
G:\BarCampReport>Test 1
CRT exp = 2.71828       BRUTE exp = 2
G:\BarCampReport>Test 10
CRT exp = 2.71828       BRUTE exp = 2.59374
G:\BarCampReport>Test 100
CRT exp = 2.71828       BRUTE exp = 2.70481
G:\BarCampReport>Test 1000
CRT exp = 2.71828       BRUTE exp = 2.71692
G:\BarCampReport>Test 5000
CRT exp = 2.71828       BRUTE exp = 2.71801
G:\BarCampReport>Test 50000
CRT exp = 2.71828       BRUTE exp = 2.71825
G:\BarCampReport>Test 100000
CRT exp = 2.71828       BRUTE exp = 2.71827
G:\BarCampReport>Test 1000000
CRT exp = 2.71828       BRUTE exp = 2.71828
```

How To Find Nth Root of a Number?

$$a\text{^}(1/n) = \text{antilog}((1/n)*\log(a))$$
$$= \exp((1/n)*\log(a))$$

```
public static double NthRoot(double num, double  n){
        return Math.Exp((1 / n) * Math.Log(num));
}
```

From Where I can get the Above Program?

- https://github.com/praseedpai/ElementaryMathForProgrammingSeries/blob/master/AlgebraNArith/NthQuad/QuadNth.cs

Application of Log and Exp in Geometric Mean

Geometric Mean

$$\left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}} = \sqrt[n]{a_1 a_2 \cdots a_n}.$$

When $a_1, a_2, \ldots, a_n > 0$

$$\left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}} = \exp\left[\frac{1}{n}\sum_{i=1}^{n} \ln a_i\right]$$

additionally,

$$\left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}} = (-1)^m \exp\left[\frac{1}{n}\sum_{i=1}^{n} \ln|a_i|\right]$$

where m is the number of negative numbers.

Unusual Effectiveness of Mathematics in Finance!

Why mathematics is "unusually" Effective in Finance ?

Now a days , most social sciences are using mathematical tools for analysis. At least , mathematics is a good representational tool for ideas in these sciences.

Let me illustrate , why mathematical equations (differential equations) are used to interpret financial phenomena, using a simple problem , given below.

If a person is getting a hike of R% year on year. What will be the Time (T) for doubling his salary ?

This can be put in as a algebraic expression , as given below (How much time it takes one rupee to become two , given R and T ?)

1x(1+ R) ^ T = 2

This can be simplified as

(1 + R)^T = 2

The above assumes that compounding is done annually. Why cannot we do it monthly ? In the case , The formula gets modified into

(1 + R/12)^12*T = 2

(1 + 1/N) ^N = e (base of natural logarithm) as N => Infinity

You can read more about e here. The value of e is 2.71828.

The above formula becomes

e^RT = 2

Taking natural logarithms on both sides of the equation , the equation becomes

log(e^RT) = log(2)

This can be written as R*T*log(e) = log(2)

Since log(e) is 1 , as log and exp (e) are inverse operations

R*T = log(2)

T = log(2) /R

As log(2) is 0.693 The equation becomes , T = 0.693/R

0.693 is 69.3% , is approximately , 70%. Thus the rule of 70 or 72 , came into the existence.

Rule of 70 revisited!

Logarithm Soup

$$\log_{10}(x) = \frac{\ln(x)}{\ln(10)} \qquad \text{or} \qquad \log_{10}(x) = \frac{\log_2(x)}{\log_2(10)}$$

Rule of 70

$$\begin{aligned}
(e^r)^p &= 2 \\
e^{rp} &= 2 \\
\ln e^{rp} &= \ln 2 \\
rp &= \ln 2 \\
p &= \frac{\ln 2}{r} \\
p &\approx \frac{0.693147}{r}
\end{aligned}$$

An Application of Base 2 Probability!

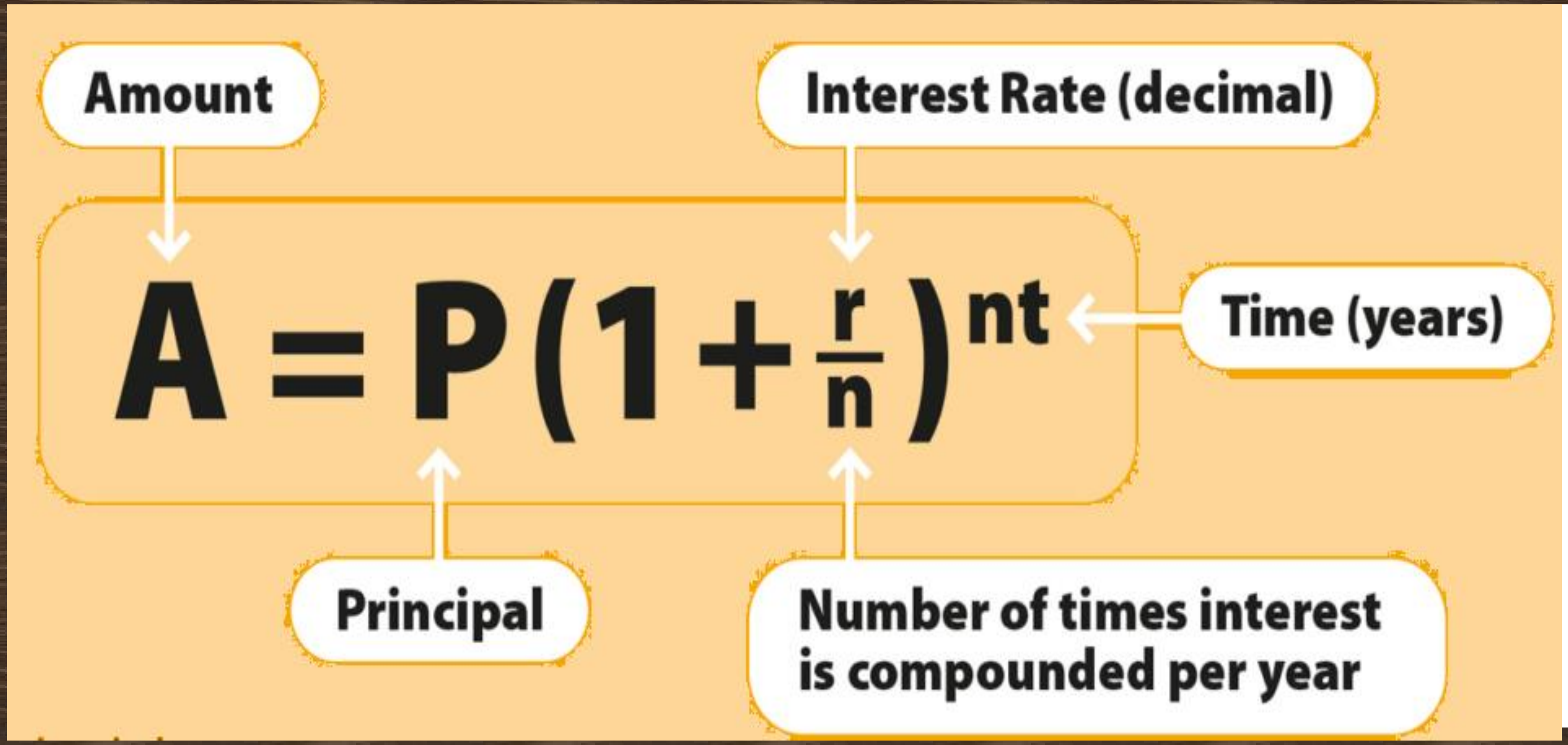
Claude Shannon was right

My elder son has got habit of screaming when I try to punish him or deny something which he feels is due to him. It has put me in an embarrassing state at times. Currently, I am staying in a rented house and I am careful while reprimanding him. He has screamed in the current place at times.

Last week, when he screamed, I just went to the sit out and peeked around. No one was taking notice. I told myself, "More frequent the message, less the information contained in it". I understood that Claude Shannon was dead right in saying that "Information content of any message is the negative logarithm (base 2) of the probability of the message"

The message (scream) was so frequent (probable) and neighbors felt it inconsequential(no additional information)

Discrete/Continuous Period



$$P(t) = P_0 e^{rt}.$$

Future Value / Present Value

$$\text{Future value} = \text{Present value} * e^{rt}$$

$$\text{Present value} = \text{Future value} * e^{-rt}$$

```
public static double FV(double present_value,
                        double rate, double period) {
        return present_value * Math.Exp(rate * period);
}

public static double PV(double future_value,
                        double rate, double period) {
        return future_value * Math.Exp(-(rate * period));
}
```

Q&A

- If any!