

Mathematical Literacy – Part 3

A Challenge from my Blog

📅 Sunday, December 01, 2013

A Rs. 1,00,00,000 Offer from me , If you are able to find a triplets (3 #'s satisfying a mathematical property)

When I learned to write computer programs, finding Pythagorean triplets was one of the first programs written by me. One can easily find lot of triplets which satisfy Pythagorean triangle equation ,
 $x^2 + y^2 = z^2$.

Some examples are , $4^2 + 3^2 = 5^2$
 $12^2 + 5^2 = 13^2$

The challenge is to find out triplets for any number n , provided $n > 2$ which satisfies the equation
 $x^n + y^n = z^n$.

If you are able to find a integer solution, you will get this reward !

Pythagorean Triplets in Python

```
from math import sqrt
n = int(10)+1
print("-----\n")
for a in range(1,n):
    for b in range(a,n):
        c_square = a**2 + b**2
        c = int(sqrt(c_square))
        if ((c_square - c**2) == 0):
            print(a, b, c, "\n")
```

Words which gave Mathematicians Sleepless nights for Centuries

The Celebrated French Mathematician wrote the following in one of his book.

"It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain."

What essentially, he told was

$$x^3 \neq y^3 + y^3 \text{ or } x^4 \neq y^4 + z^4,$$

in general, $x^n + y^n \neq z^n$ for $n > 2$

We are familiar with Pythagoras Theorem, which states that,

"In a right triangle, hypotenuse (z) squared is equivalent to base (x) squared added to opposite side(y)"

Algebraically, It can be written as,

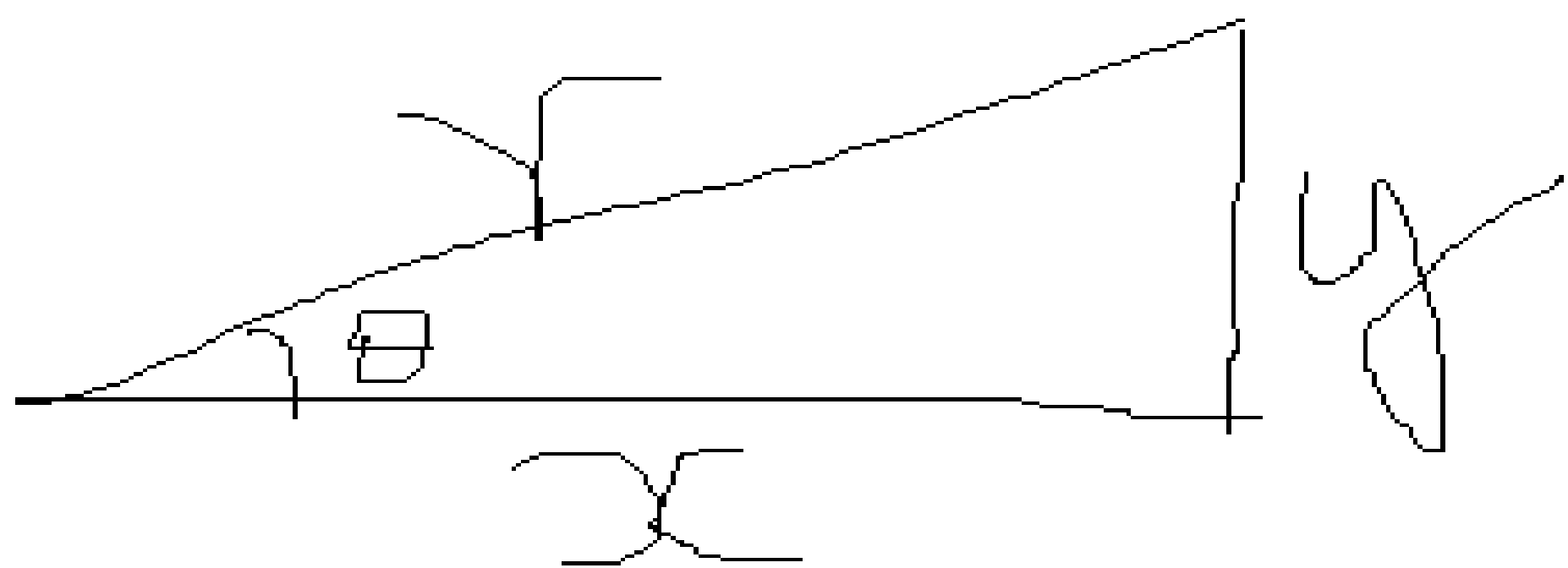
$$z^2 = x^2 + y^2.$$

Diophantine equations are generalization of Pythagoras theorem where factors (x,y,z) are integral numbers. What Fermat essentially told us was , **For a Diophantine equation with powers greater than 2, there is no solutions.**

Pythagorean Triplets and Testing FLT

```
#include <stdio.h>
int main() {
float x , y , z ;
for( x = 1.0 ; x <= 100; x=x+1.0 )
  for( y = 1.0 ; y <= 100 ; y=y+1.)
    for( z = 1.0 ; z <=100; z=z+1. ){
      if ( x*x + y*y == z*z )
        printf(" triplets %d\t\t%d\t\t%d\n",
               (long)x,(long)y,(long)z);
      if ( x*x*x + y*y*y == z*z*z )
        printf("cube =%d\t\t%d\t\t%d\n",
               (long)x,(long)y,(long)z);
    }
}
```


A wonderful proof



$$\sin \theta = \frac{y}{r}$$
$$\cos \theta = \frac{x}{r}$$

$$y = r \sin \theta$$

$$x = r \cos \theta$$

$$x^2 + y^2 = r^2$$

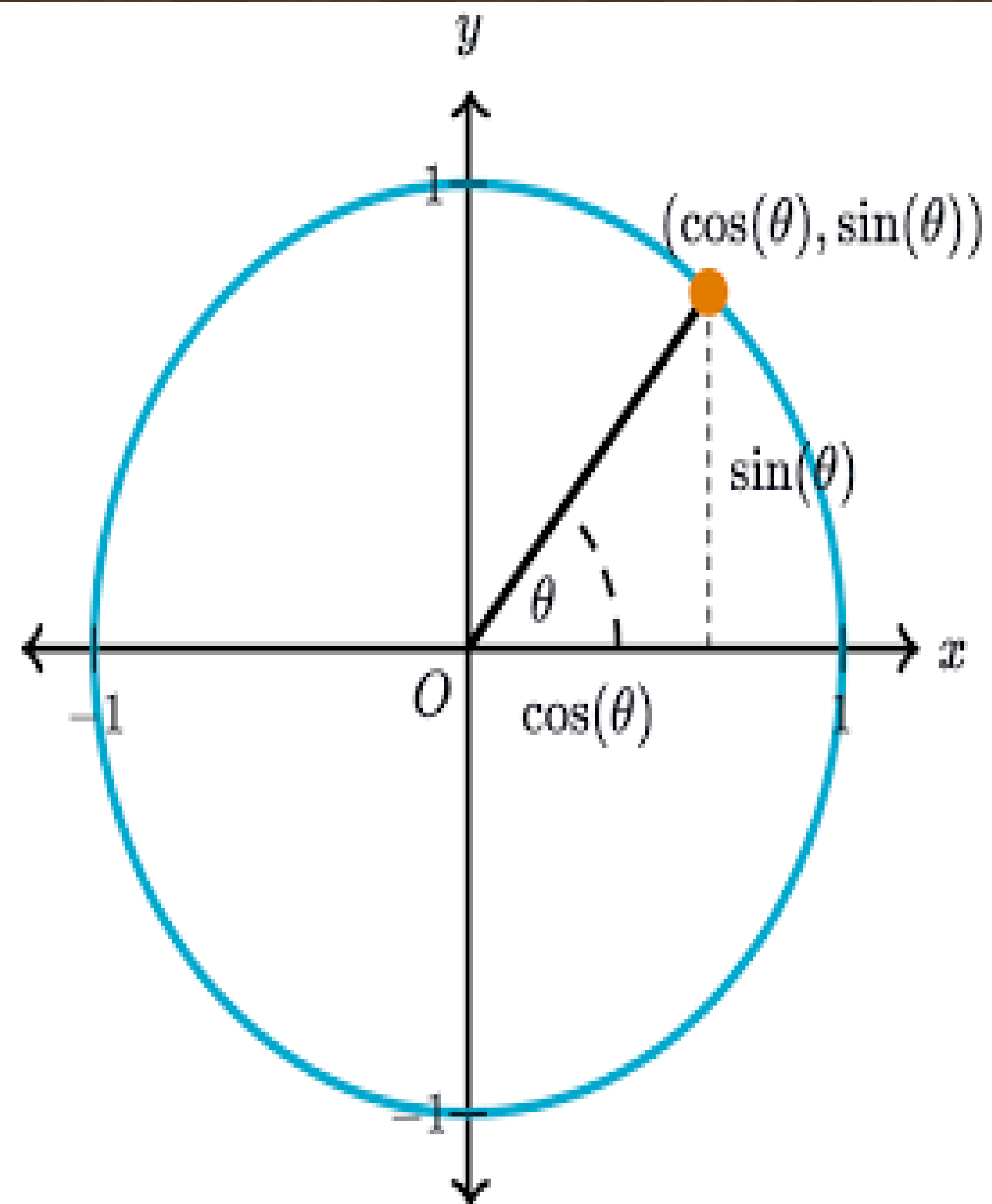
$$r^2 \sin^2 \theta + r^2 \cos^2 \theta = r^2$$

$$r^2 (\sin^2 \theta + \cos^2 \theta)$$

$$\frac{r^2}{r^2} = \sin^2 \theta + \cos^2 \theta$$

$$1 = \sin^2 \theta + \cos^2 \theta$$

Trigonometry



Why do Software Libraries ask you to give angle in radians ?

Trigonometric functions are evaluated using a series (a variant of Taylor series you studied in Calculus) where the arguments are supposed to be in radians !. The following series can help you to evaluate Sine and Cosine.

$$\begin{aligned}\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}.\end{aligned}$$

One can define a Macro or function to convert Degrees to Radians

```
double DTOR( double angle ) {  
    return angle*3.14159/180.0  
}
```


How to Compute Sine Theta ?

```
#include <stdio.h>
#include <math.h>

double m_abs( double arg ) {
    if ( arg < 0.0 ) { arg = -1*arg;}
    return arg;
}

double m_dtor(double x ) { return ( x * 3.14159/180.0 ); }

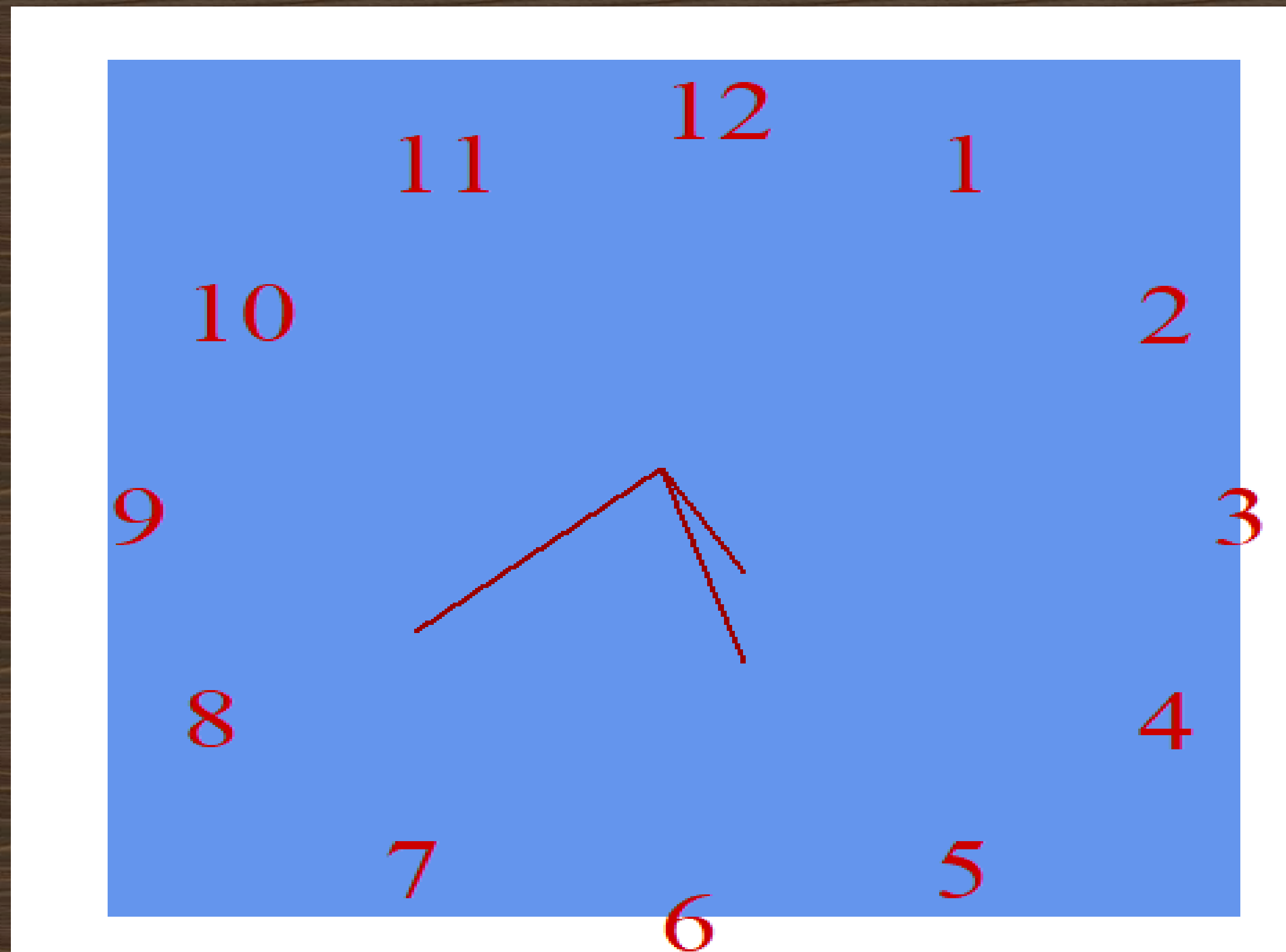
double m_sin(double arg ) {
    double term = arg;
    double i = 1;
    double x2 = arg*arg;
    double tsin = arg;
    while ( m_abs(term) > 0.0000001 ){
        i = i + 2;
        term = -term*x2/(i*(i-1));
        tsin = tsin + term;
    }
    return tsin;
}
```

```
double m_cos(double arg ) { return m_sin( (2*3.14159/4.0) - arg ); }

int main(int argc, char **argv) {
    double dval = (argc == 1 )? 45.0 : atof(argv[1]);
    printf("\n\n\nValue is %e\n\n",dval*3.14159/180.0);
    printf("my sine = %f\t\t\t\t\tmath.h sin = %f\n",
           m_sin(m_dtor(dval)),sin(m_dtor(dval)));
    printf("my cosine = %f\t\t\t\t\tmath.h cosine = %f\n",
           m_cos(m_dtor(dval)),cos(m_dtor(dval)));
}
```

$$\begin{aligned}\cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \\ \sin(x) &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}\end{aligned}$$

How To Render a Clock?



How To Convert Co-ordinate System?

```
const double PI = 3.14159;

void Transform(HWND m_hWnd, int *px , int *py){
    RECT _rect;
    ::GetClientRect(m_hWnd,&_rect);
    int width = (_rect.right-_rect.left)/2;
    int height = (_rect.bottom-_rect.top)/2;
    *px = *px + width;
    *py = height - *py;
}
```


The Render Routine

```
void CallPaintRoutine( HWND hwnd , HDC paintdc ){
    int x1 = -200,y1=-220,x2 = 210,y2=200;
    Transform(hwnd,&x1,&y1); Transform(hwnd,&x2,&y2);
    RECT rect = { x1,y1,x2,y2};
    HBRUSH hbr = CreateSolidBrush( RGB(100,149,237));
    FillRect(paintdc, &rect,hbr);
    HPEN hpen = CreatePen(PS_SOLID,2,RGB(153,0,0));
    SelectObject(paintdc,hpen);
    SYSTEMTIME systime;GetLocalTime(&systime);

    int fhour,fmin,fsec ;
    fhour = systime.wHour%12; fmin = systime.wMinute;
    fsec = systime.wMinute;

    fhour += fmin/60; fhour = fhour * 360 / 12;
    fmin = fmin * 360 / 60; fsec = fsec * 360 / 60;

    int xs = 120 * cos((-fsec * PI / 180.0)+ PI/2.0);
    int ys = 120 * sin((-fsec * PI / 180.0)+ PI/2.0);
    int xm = 100 * cos((-fmin * PI / 180.0)+ PI/2.0);
    int ym = 100 * sin((-fmin * PI / 180.0)+ PI/2.0);
    int xh = 60 * cos((-fhour * PI / 180.0)+ PI/2.0);
    int yh = 60 * sin((-fhour * PI / 180.0)+ PI/2.0);
```

```
    int x = 0,y=0;
    Transform(hwnd,&xh,&yh);Transform(hwnd,&x,&y);
    MoveToEx(paintdc,x,y,NULL);LineTo(paintdc,xh,yh);
    Transform(hwnd,&xm,&ym); MoveToEx(paintdc,x,y,NULL);
    LineTo(paintdc,xm,ym);Transform(hwnd,&xs,&ys);
    MoveToEx(paintdc,x,y,NULL);LineTo(paintdc,xs,ys);
    char *str[] = {"3","2","1","12","11","10","9","8","7","6","5","4"};
    HFONT hFont = CreateFont(48,0,0,0,FW_DONTCARE,FALSE,TRUE,FALSE,
        DEFAULT_CHARSET,OUT_OUTLINE_PRECIS,
        CLIP_DEFAULT_PRECIS,CLEARTYPE_QUALITY,
        VARIABLE_PITCH,TEXT("Times New Roman"));
    SelectObject(paintdc,hFont);
    SetTextColor(paintdc,RGB(204,0,0));
    SetBkMode(paintdc,TRANSPARENT);
    for(int i = 0;i < 12;i++){
        int x = 200 * cos(i * PI / 6),y= 200 * sin(i * PI / 6);
        Transform(hwnd,&x,&y);
        TextOut(paintdc,x,y,str[i],strlen(str[i]));
    }
}
```


Q&A

- If any!
- Source Code of the Clock Program Available @
<https://github.com/praseedpai/ElementaryMathForProgrammingSeries/blob/master/AlgebraNArith/AnalogClock/PSClock.cpp>
- Source Code of the SineCos from Scratch available @
<https://github.com/praseedpai/ElementaryMathForProgrammingSeries/blob/master/AlgebraNArith/SinCosScratch/SinCos.cpp>
- Source Code of the Pythagorean and FLT available @
<https://github.com/praseedpai/ElementaryMathForProgrammingSeries/blob/master/AlgebraNArith/Fermat/Pyth.cpp>