NUMERICAL ANALYSIS FINAL PROJECT REPORT

PORTFOLIO ALLOCATION USING PRINCIPAL COMPONENT ANALYSIS

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Introduction:

Principal Component Analysis (PCA) is a statistical technique that reduces the dimensionality of data by finding its most significant features. In asset allocation, PCA helps identify underlying patterns in asset returns, aiding in portfolio diversification and risk management by capturing correlations among assets efficiently. In this project we explored:

- Comparing performance and variance captured between PCA self-made & PCA sklearn libraries [1].
- Evaluating all PCAs with min variance weightage portfolio using Sharpe ratio and yearly return

Methodology:

We start with 7 years(2016 Jan - 2023 Oct) of data picking 10 stocks(which are **Nvdia, Tesla, Lockheed Martin, Apple, Amazon, Costco, J&J, JP Morgan, Target, Exxon Mobil**) from NYSE across various sectors. We split them into training and test sets (75/25%). We developed a common covariance matrix which will be plugged into individual strategies to calculate weights, which are applied on the testing dataset.

Principal Component Analysis (PCA):

The PCA is defined as an orthogonal linear transformation on a real inner product space that transforms the data to a new coordinate system such that the greatest variance by some scalar projection of the data comes to lie on the first coordinate (called the first principal component), the second greatest variance on the second coordinate, and so on [2]. PCA is generally used for dimensionality reduction by projecting each data point onto few principal components to obtain lower-dimensional data, while preserving as much of the data's variation as possible. The Singular Value Decomposition (SVD) of a rectangular data matrix is a powerful method in analyzing the data structure and the relationship between rows and columns. SVD has been applied in many methods one of them is PCA [8].

Since PCA stands to capture major variance, we wanted to see an improvement in performance from minimum variance portfolio allocation strategy. Prior to that we have compared PCA using inbuilt sklearn libraries versus PCA without any libraries (from scratch) to validate the model.

$$A^T A = U \ x \ \sum x \ V^T$$

Where $U, V - Principal\ Component\ Matrix$

$\sum - \ Diagonal\ Matrix$

$A - [aij], where\ aij\ is\ the\ normalized\ of\ stock\ j\ at\ time\ i$

$A^T A - Covariance\ Matrix$

Minimum Variance Weighted Portfolio:

The minimum variance portfolio is constructed by optimizing asset weights to minimize portfolio volatility. It utilizes historical data on asset returns and correlations to determine the ideal allocation [3]. The process involves finding the mix of assets that offers the lowest overall portfolio risk, irrespective of individual asset returns, aiming for the highest diversification benefit. The portfolio is very important point for efficient frontier representing equal balance between risk and return.

Metrics to measure performance of portfolio:

Sharpe ratio measures the risk-adjusted return of an investment or portfolio.

$$Sharpe\ Ratio\ = \frac{\mathrm{E}[\mathrm{R_a}]}{\sigma_a}$$

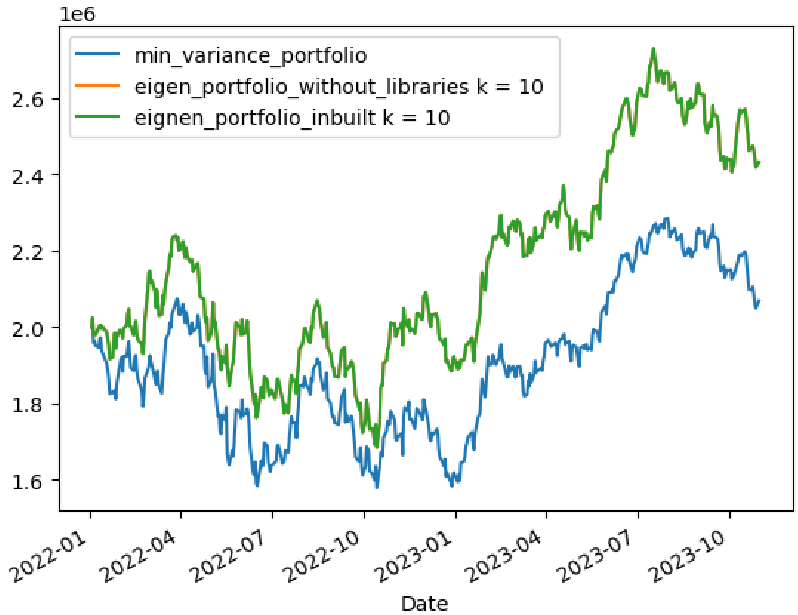
Yearly Return is percentage increase or decrease from the investment at start of the year.

$$Returns = \frac{P_{t+1} - P_t}{P_t}$$

Implementation:

	sharpe	yearly_ret
min_variance_portfolio	0.196091	1.016992
eigen_portfolio_without_libraries k = 10	0.593967	1.102808
eignen_portfolio_inbuilt k = 10	0.593967	1.102808

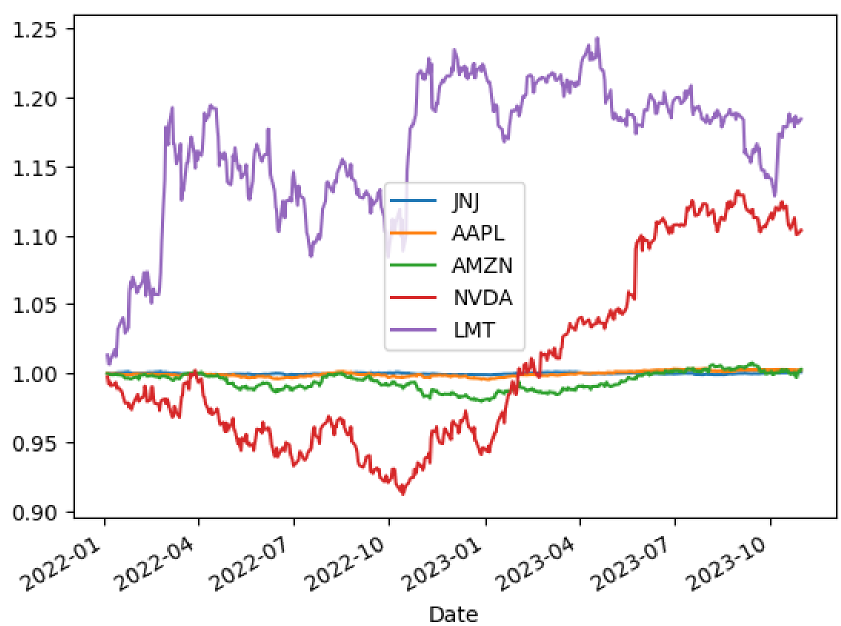
It can be observed that, that in both metrics PCA strategies outperform minimum variance strategy. In yearly returns it is almost 10% improvement over the minimum variance strategy. Adding to that we have observed that the PCA we built works fine without any errors.



The portfolio value graph suggests same thing that PCA strategy continues to give higher returns than minimum variance and ends ups at profit of around half a million dollars. However, we could observer that both PCA and minimum variance seem to follow similar ups and downs with magnitudes for most of the testing period.

Discussion:

We presented the top 5 most relevant stock that resulted in eigen values over the period. NVIDIA seem to influence less than 1 for whole year 2022 while Amazon and Apple were stable.



While the PCA strategy seem to outperform the minimum variance, it may not be a better strategy overall. Considering following ideas to be confirmed:

- Although we measured the variance from eigen values we cannot understand the direction of the variance.
- We need deeper understanding of market movement in each month to attribute the common ups and downs better.
- Impact of LMT on the PCA strategy is huge considering the actual market cap of the company. Major returns for investing in firms like LMT could be less frequent to validate a strategy.
- We considered the stocks to be completely independent from each other for this strategy however we are aware that some sectors have significant correlation.
- Adding limitations to amount to be invested in one stock would have shown the performance of model better.

Code:

We incorporated some of the ideas from [3] [9] in our code.

Install Libraries

```
[1]  pip install yfinance
```

```
[2]  import pandas as pd
     import numpy as np
     import yfinance as yf
     yf.pdr_override() # <== that's all it takes :-)
     from pandas_datareader import data as pdr
```

```
[3]  from matplotlib import pyplot as plt
     import seaborn as sns
     import scipy.cluster.hierarchy as sch
     import scipy.spatial.distance as ssd
     from scipy.optimize import minimize
     import warnings
     from collections import OrderedDict
     import datetime as dt
     from sklearn.decomposition import PCA
```

Pick stocks and time period

```
[4] ticks = ['TSLA','NVDA','AMZN','JNJ','AAPL','XOM','JPM','TGT','LMT','COST']
    total_stocks = len(ticks)
    start_date = '2016-01-01'
    end_date = '2023-10-31'

    # Training and Testing Periods
    train_start_date = '2016-01-01'
    train_end_date = '2022-01-01'

    test_start_date = '2022-01-01'
    test_end_date = '2023-10-31'

    trading_days = 252

    training_period = trading_days*5

    portfolio_current_value = 2000000 # 2 Million USD
```

Fetch data from Yahoo Finance

```
[5] price_data = pdr.get_data_yahoo(ticks,start=start_date,end=end_date)['Adj Close']
```

```
[*********************100%%**********************]  10 of 10 completed
```

Normalization of stock prices (Adjusted Close Price)

```
[6] daily_returns = price_data.pct_change()
    daily_returns.head()
```

Date	AAPL	AMZN	COST	JNJ	JPM	LMT	NVDA	TGT	TSLA	XOM
2016-01-04	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2016-01-05	-0.025059	-0.005024	0.002445	0.004180	0.001729	0.023685	0.016064	0.017539	0.000090	0.008521
2016-01-06	-0.019570	-0.001799	-0.009254	-0.005055	-0.014436	-0.002886	-0.041350	-0.010155	-0.019648	-0.008320
2016-01-07	-0.042205	-0.039058	-0.022909	-0.011655	-0.040440	-0.019942	-0.039645	-0.002700	-0.015477	-0.016006
2016-01-08	0.005288	-0.001464	-0.017504	-0.010683	-0.022399	-0.001782	-0.021466	-0.033838	-0.021563	-0.020202

Splitting data into Training(75%) and Testing Dataset (25%)

```
[7] training_returns = daily_returns[daily_returns.index <= train_end_date]
    testing_returns  = daily_returns[daily_returns.index > train_end_date]

    print("training length:",len(training_returns),"testing length:",len(testing_returns))
```

```
training length: 1511 testing length: 459
```

Covariance matrix from training normalized returns

```
[8] covariance_returns = training_returns.cov()
```

PCA with inbuilt libraries

```
[9] pca = PCA()
    pca.fit(covariance_returns)
    factor_loading = pca.components_
```

PCA with inbuilt libraries

```
[9] pca = PCA()
    pca.fit(covariance_returns)
    factor_loading = pca.components_
    print("The matrix of Eigen Vectors has the shape: %s"%str(factor_loading.shape))

    eigen_vals = pca.explained_variance_
    print("The eigen values of the matrix: %s"% str(eigen_vals))
```

```
The matrix of Eigen Vectors has the shape: (10, 10)
The eigen values of the matrix: [1.64572782e-07 4.39225995e-08 1.32084507e-08 7.92752110e-09
 3.83564960e-09 2.23461245e-09 1.52414227e-09 1.26570250e-09
 7.11227671e-10 5.73216771e-41]
```

```
[10]
    def eigen_portfolio_inbuilt_libraries(eigen_prices):
      ereturn = eigen_prices.pct_change()
      ecov = ereturn.cov()
      epca = PCA()
      epca.fit(ecov)
```

```
    efactor_loadings = epca.components_

    eigen_values = epca.explained_variance_
    return OrderedDict(zip(ticks,efactor_loadings[0]/np.sum(efactor_loadings[0])))
```

PCA without using libraries for k = 10 Principal components

```
[11] def mean_normalize(data):
         mean = np.mean(data, axis=0)
         normalized_data = data - mean
         return normalized_data, mean

     def compute_covariance_matrix(data):
         num_samples = data.shape[0]
         covariance_matrix = np.dot(data.T, data) / (num_samples - 1)
         return covariance_matrix

     def pca_without(data, num_components):
         # Step 1: Mean normalization
         normalized_data, mean = mean_normalize(data)
```

```
def pca_without(data, num_components):
    # Step 1: Mean normalization
    normalized_data, mean = mean_normalize(data)

    # Step 2: Compute the covariance matrix
    covariance_matrix = compute_covariance_matrix(normalized_data)

    # Step 3: Compute eigenvalues and eigenvectors of the covariance matrix
    eigenvalues, eigenvectors = np.linalg.eig(covariance_matrix)

    # Step 4: Sort eigenvalues and corresponding eigenvectors in descending order
    sorted_indices = np.argsort(eigenvalues)[::-1]
    eigenvalues = eigenvalues[sorted_indices]
    eigenvectors = eigenvectors[:, sorted_indices]

    #Step 4.1 Corrections to limit number of components
    eigenvalues = eigenvalues[:num_components]
    eigenvectors = eigenvectors.T
    eigenvectors = eigenvectors[:num_components]
    # Calculate the variance captured by each principal component as a percentage of t

    variance_captured = np.sum(eigenvalues)
    return eigenvectors, eigenvalues, mean, variance_captured
```

```python
def eigen_portfolio_without_libraries(prices):
  ereturn = prices.pct_change()

  ecov = ereturn.cov()

  eigenvectors,eigenvalues,  mean, variance_captured = pca_without(ecov, num_components=10)

  efactor_loadings = eigenvectors

  eigen_values = eigenvalues

  explained_variance = variance_captured

  return OrderedDict(zip(ticks,efactor_loadings[0]/np.sum(efactor_loadings[0])))
```

Minimum Variance Portfolio using Optimization

```python
def min_variance_portfolio(prices):
  min_var_returns = prices.pct_change()
  cov_matrix = min_var_returns.cov()
  num_assets = len(cov_matrix)
  # Define the objective function to minimize (variance of the portfolio)
  def portfolio_variance(weights):
      return np.dot(weights, np.dot(cov_matrix, weights))

  def weight_cons(weights):
    return np.sum(weights)-1

  bounds_lim = [(0, 1) for x in range(len(min_var_returns.columns))] # change to (-1, 1)
  init = [1/len(min_var_returns.columns) for i in range(len(min_var_returns.columns))]
  constraint = {'type': 'eq', 'fun': weight_cons}

  optimal = minimize(fun=portfolio_variance, x0=init, bounds=bounds_lim,
                     constraints=constraint, method='SLSQP')

  return OrderedDict(zip(ticks,optimal['x']))
```

```
[14] # Metrics - Portfolio Level
     def sharpe_ratio(portfolio_returns):
       return portfolio_returns.mean()/portfolio_returns.std()*np.sqrt(trading_days)

     def yoy_return(portfolio_returns):
       number_of_years = len(portfolio_returns.index.year.unique())
       return np.power((portfolio_returns+1).cumprod().iloc[-1],1/number_of_years)
```

Defining function to back test the performance of each strategy for selected time period

```
[15] # Portfolio backtesting - Testing period with different strategies
     def daily_portfolio_value(prices,alloc_func = eigen_portfolio_inbuilt_libraries):


       #Creating a dataframe with index for testing period
       daily_port_alloc = pd.DataFrame(index=prices.loc[test_start_date:].index,columns=prices.columns)
       #daily_port_variance = pd.DataFrame(index=prices.loc[test_start_date:].index,columns='variance')


       #identify the first day of each unique month in daily_port_alloc's index and store those days in rebal
       rebalance_days = daily_port_alloc[~daily_port_alloc.index.to_period('m').duplicated()].index
```

```
     #Create a dataframe with only rebalance days so that updates weights are added later on
     port_alloc_weights = pd.DataFrame(index =rebalance_days,columns=prices.columns)
     current_portfolio_current_value = portfolio_current_value # 2 million USD
     share_allocation = None #default

     for day in rebalance_days:
       if share_allocation is not None:
          current_portfolio_current_value = (prices.loc[day]*share_allocation).sum()
       allocation = alloc_func(prices[day-dt.timedelta(days=training_period):day])   #Taking 1511 Days prio
       port_alloc_weights.loc[day] = allocation #Adding weights calculated in the prior step according to t
       share_allocation = (pd.Series(allocation)*current_portfolio_current_value)/prices.loc[day] #multiply
       daily_port_alloc.loc[day] = share_allocation # assigning the values to rebalancing days

     daily_port_alloc = (daily_port_alloc.ffill()).dropna() #applying same share allocation to the whole pe
     portfolio_each_asset_daily_val = prices.loc[daily_port_alloc.index]*daily_port_alloc
     portfolio_daily_val = (portfolio_each_asset_daily_val).sum(axis=1)
     portfolio_daily_return = portfolio_daily_val.pct_change()
     return {
         'sharpe': sharpe_ratio(portfolio_daily_return),
         'yearly_ret': yoy_return(portfolio_daily_return),
         'portfolio_vals':portfolio_daily_val,
         'portfolio_each_asset_daily_val':portfolio_each_asset_daily_val}

     return portfolio_daily_val
```

Creating a dictionary to tabulate various strategies and storing the results similarly

```
[16] strategy_list = {'min_variance_portfolio': min_variance_portfolio,
                      'eigen_portfolio_without_libraries k = 10 ':eigen_portfolio_without_libraries,
                      'eignen_portfolio_inbuilt k = 10 ':eigen_portfolio_inbuilt_libraries,
                      }


     strategy_results = { key: daily_portfolio_value(price_data,func) for key,func in strategy_list.items()}
     res = pd.DataFrame(strategy_results).T
```

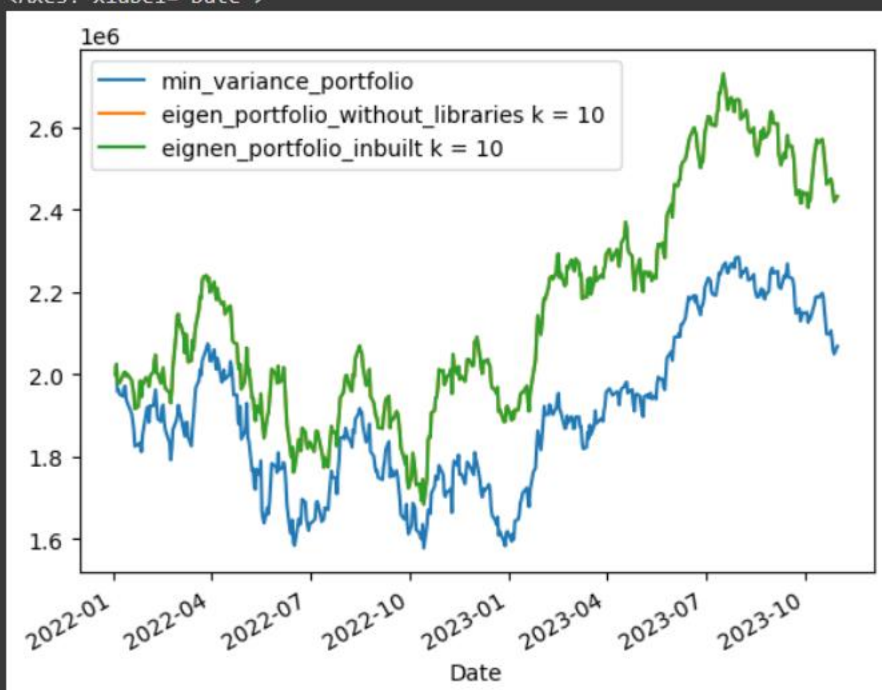
```
[17] return_frame = pd.DataFrame(index=res['portfolio_vals']['min_variance_portfolio'].index)
     for key in res['portfolio_vals'].keys():

       return_frame[key] =res['portfolio_vals'][key]
```

Portfolio Value for each strategy : x axis - time period. y axis - value in Millions USD

```
return_frame.plot()
```

<Axes: xlabel='Date'>



Final Result

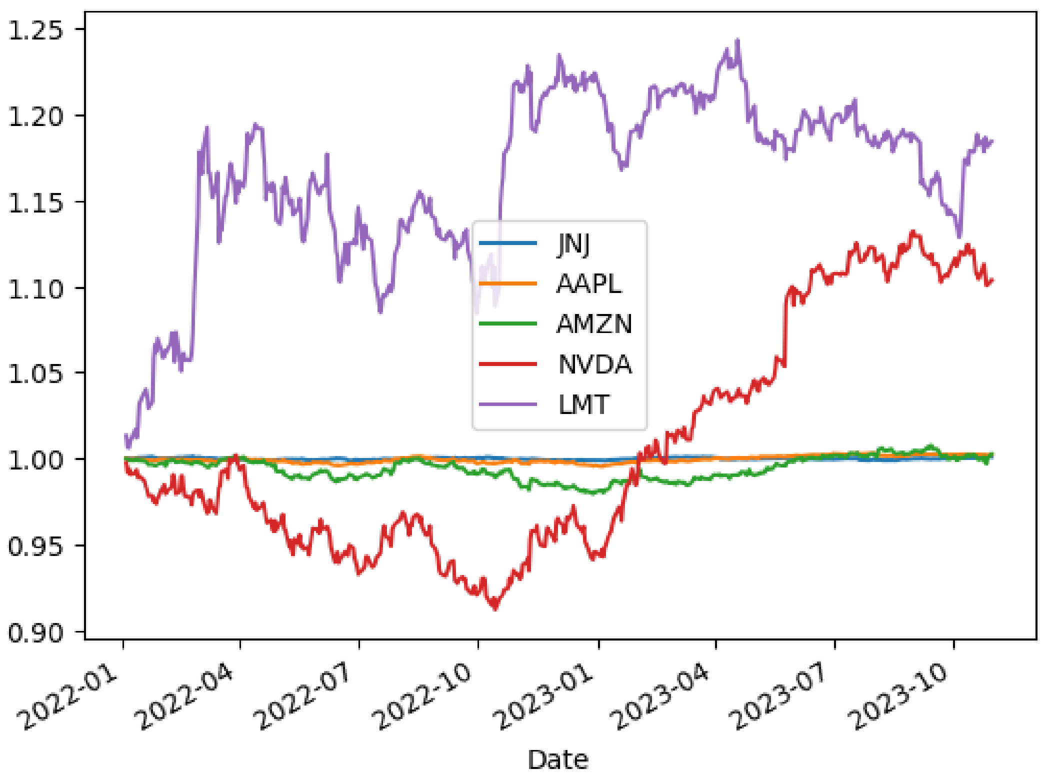
```
[19] res[['sharpe','yearly_ret']]
```

	sharpe	yearly_ret
min_variance_portfolio	0.196091	1.016992
eigen_portfolio_without_libraries k = 10	0.593968	1.102808
eignen_portfolio_inbuilt k = 10	0.593968	1.102808

Finding top 5 influencing stocks

```
[21] df = res.loc['eigen_portfolio_without_libraries k = 10 ']['portfolio_each_asset_daily_val']
     df.iloc[-1].sum()/df.iloc[0].sum()

     #attribution for top 5 winners
     ret_attr = (price_data.loc[df.index].pct_change()*(df.div(df.sum(axis=1),axis=0)).shift(1)+1).cumprod()
     top_five = ret_attr.iloc[-1].sort_values().iloc[-5:].index
     ret_attr[top_five].plot()
```



References:

[1] https://scikit-learn.org/stable/modules/generated/sklearn.decomposition.PCA.html

[2] https://link.springer.com/book/10.1007/b98835#keywords

[3] https://github.com/zhuodannychen/Portfolio-Optimization

[4] https://en.wikipedia.org/wiki/Principal_component_analysis

[5] https://en.wikipedia.org/wiki/Singular_value_decomposition

[6] https://www.youtube.com/watch?v=vAmlFBpipT8&ab_channel=QuantPy

[7] https://github.com/Doj-i/NYU_Machine_Learning_in_Finance/blob/master/Portfolio%20Construction%20using%20PCA/Eigen-portfolio%20construction%20using%20Principal%20Component%20Analysis%20(PCA)_ML2_ex3.ipynb

[8]https://www.researchgate.net/publication/319381431_Adjustable_Robust_Singular_Value_Decomposition_Design_Analysis_and_Application_to_Finance

[9] https://github.com/10mohi6/portfolio-backtest-python