Practice in 1st-order predicate logic – with answers.

1. Mary loves everyone. [assuming D contains only humans]

   $\forall x$ **love** (**Mary**, x)

2. Mary loves everyone. [assuming D contains both humans and non-humans, so we need to be explicit about 'everyone' as 'every *person*']

   $\forall x$ (**person**(x) $\rightarrow$ **love** (**Mary**, x))

3. No one talks. [assume D contains only humans unless specified otherwise.]

   $\neg\exists x$ **talk**(x) **or equivalently,** $\forall x \neg$ **talk**(x)

4. Everyone loves himself.

   $\forall x$ **love** (x, x)

5. Everyone loves everyone.

   $\forall x \forall y$ **love** (x, y)

6. Everyone loves everyone except himself. (= Everyone loves everyone else.)

   $\forall x \forall y(\neg\, x = y \rightarrow$ **love** (x, y)) **or** $\forall x \forall y(\, x \neq y \rightarrow$ **love** (x, y))

7. Every student smiles.

   $\forall x$ (**student**(x) $\rightarrow$ **smile**(x))

8. Every student except George smiles.

   $\forall x$ ((**student**(x) & $x \neq$ **George**) $\rightarrow$ **smile**(x))

9. Everyone walks or talks.

   $\forall x$ (**walk** (x) $\vee$ **talk** (x))

10. Every student walks or talks.

    $\forall x$ (**student**(x) $\rightarrow$ (**walk** (x) $\vee$ **talk** (x)))

11. Every student who walks talks.

    $\forall x$ ((**student**(x) & **walk** (x)) $\rightarrow$ **talk** (x))) **or**

    $\forall x$ (**student**(x) $\rightarrow$ (**walk** (x) $\rightarrow$ **talk** (x)))

12. Every student who loves Mary is happy.

    $\forall x$ ((**student**(x) & **love** (x, **Mary**)) $\rightarrow$ **happy** (x)))

13. Every boy who loves Mary hates every boy who Mary loves.

    $\forall x$((**boy**(x) & **love** (x, **Mary**)) $\rightarrow \forall y$((**boy**(y) & **love**(**Mary**, y))$\rightarrow$ **hate** (x,y)))

14. Every boy who loves Mary hates every other boy who Mary loves.

    (So if John loves Mary and Mary loves John, sentence 13 requires that John hates himself, but sentence 14 doesn't require that.)

    $\forall x$((**boy**(x) & **love** (x, **Mary**)) $\rightarrow \forall y$((**boy**(y) & **love**(**Mary**, y) & $y \neq x$) $\rightarrow$ **hate** (x,y)))

Homework #1, with answers.

1. Everyone loves Mary.

   $\forall x$ **love** (x, **Mary**)

2. John does not love anyone. (Not ambiguous, but there are two equivalent and equally good formulas for it, one involving negation and the existential quantifier, the other involving negation and the universal quantifier. Give both.)

   $\neg\exists x$ **love**(**John**, x) **or equivalently,** $\forall x\neg$ **love**(**John**, x)

3. Everyone who sees Mary loves Mary.

   $\forall x$ (**see** (x, **Mary**) $\rightarrow$ **love** (x, **Mary**))

4. Everyone loves someone. (Ambiguous)

   (i) $\forall x\exists y$ **love** (x, y) (For every person x, there is someone whom x loves.)

   (ii) $\exists y\forall x$ **love** (x, y) (There is some person y whom everyone loves, i.e. everyone loves some one specific person.)

5. Someone loves everyone. (Ambiguous)

   (i) $\exists x\forall y$ **love** (x, y) (There is some person x who loves everyone.)

   (ii) $\forall y\exists x$ **love** (x, y) (For every person y, there is someone who loves them – i.e., no one is totally unloved.)

6. Someone walks and talks.

   $\exists x$(**walk** (x) & **talk** (x))

7. Someone walks and someone talks.

   ($\exists x$ **walk** (x) & $\exists x$ **talk** (x)) **or** ($\exists x$ **walk** (x) & $\exists y$ **talk** (y))

Because neither quantifier is inside the scope of the other – i.e. their scopes are independent – it doesn't matter whether we use different variables here or use the same variable twice. But if one quantifier is inside the scope of the other, then it matters a great deal. When one quantifier is inside the scope of another, as in questions 4 and 5 above, always give them different variables!

8. Everyone who walks is calm.

   $\forall x$ (**walk**(x) $\rightarrow$ **calm**(x))

9. No one who runs walks. (Not ambiguous, but same note as for number 2.)

   (i) $\neg\exists x$ (**run** (x) & **walk** (x)) **or equivalently,**

   (ii) $\forall x$ (**run**(x) $\rightarrow$ $\neg$ **walk**(x))

10. Everyone who Mary loves loves someone who is happy.

    $\forall x$(**love** (**Mary**, x)$\rightarrow$ $\exists y$(**love**(x,y) & **happy**(y)))

11. If anyone cheats, he suffers.

    $\forall x$ (**cheat**(x) $\rightarrow$ **suffer**(x))

12. If anyone cheats, everyone suffers.

    $\forall x$ (**cheat**(x) $\rightarrow$ $\forall y$ **suffer**(y))

13. Anyone who loves everyone loves himself.

    $\forall x$($\forall y$ **love** (x,y)$\rightarrow$ (**love**(x,x))

note: NOT this: $\forall x\forall y$ (**love** (x,y)$\rightarrow$ (**love**(x,x)) What this one says is "Anyone who loves anyone loves himself" What the correct one says is IF you love everyone, THEN you love yourself. So the $\forall y$ quantifier has to be inside the scope of the $\rightarrow$.

14. Mary loves everyone except John. (For this one, you need to add the two-place predicate of identity, “=”. Think of “everyone except John” as “everyone who is not identical to John”.)

$\forall \boldsymbol{x}\ (\neg\ \boldsymbol{x} = \textbf{John} \rightarrow \textbf{love}\ (\textbf{Mary}, \boldsymbol{x}))$ **or equivalently**

$\forall \boldsymbol{x}\ (\boldsymbol{x} \neq \textbf{John} \rightarrow \textbf{love}\ (\textbf{Mary}, \boldsymbol{x}))$

15. Redo the translations of sentences 1, 4, 6, and 7, making use of the predicate **person,** as we would have to do if the domain D contains not only humans but cats, robots, and other entities.

1’. Everyone loves Mary.

$\forall \boldsymbol{x}\ (\textbf{person}(\boldsymbol{x}) \rightarrow \textbf{love}\ (\boldsymbol{x}, \textbf{Mary}))$

4’. Everyone loves someone. (Ambiguous)

(i) $\forall \boldsymbol{x}(\textbf{person}(\boldsymbol{x}) \rightarrow \exists \boldsymbol{y}(\textbf{person}(\boldsymbol{y})\ \&\ \textbf{love}\ (\boldsymbol{x}, \boldsymbol{y})))$ (For every person x, there is some person y whom x loves.)

(ii) $\exists \boldsymbol{y}(\textbf{person}(\boldsymbol{y})\ \&\ \forall \boldsymbol{x}(\textbf{person}(\boldsymbol{x}) \rightarrow \textbf{love}\ (\boldsymbol{x}, \boldsymbol{y})))$ (There is some person y whom every person x loves.)

6’. 6. Someone walks and talks.

$\exists \boldsymbol{x}(\textbf{person}(\boldsymbol{x})\ \&\ \textbf{walk}\ (\boldsymbol{x})\ \&\ \textbf{talk}\ (\boldsymbol{x}))$

Note: technically, we need more parentheses – either

$\exists \boldsymbol{x}(\textbf{person}(\boldsymbol{x})\ \&\ (\textbf{walk}\ (\boldsymbol{x})\ \&\ \textbf{talk}\ (\boldsymbol{x})))$ or

$\exists \boldsymbol{x}((\textbf{person}(\boldsymbol{x})\ \&\ \textbf{walk}\ (\boldsymbol{x}))\ \&\ \textbf{talk}\ (\boldsymbol{x}))$

But since it’s provable that & is associative, i.e. the grouping of a sequence of &’s doesn’t make any difference, it is customary to allow expressions like (p & q & r). And similarly for big disjunctions, ($p \vee q \vee r$). But not with $\rightarrow$!

7’. Someone walks and someone talks.

$(\exists \boldsymbol{x}\ (\textbf{person}(\boldsymbol{x})\ \&\ \textbf{walk}\ (\boldsymbol{x}))\ \&\ \exists \boldsymbol{x}(\textbf{person}(\boldsymbol{x})\ \&\ \textbf{talk}\ (\boldsymbol{x})))$ **or equivalently**

$(\exists \boldsymbol{x}\ (\textbf{person}(\boldsymbol{x})\ \&\ \textbf{walk}\ (\boldsymbol{x}))\ \&\ \exists \boldsymbol{y}\ (\textbf{person}(\boldsymbol{y})\ \&\ \textbf{talk}\ (\boldsymbol{y})))$

Note: both in the original 7 and in this 7’, it would be OK and customary to drop outermost parentheses, i.e. the very first left parenthesis and the very last right parenthesis may be dropped. (But no parentheses can be dropped in 6; they are not really “outermost”. Only when a pair of parentheses contains the entire formula can it be dropped under the “drop outermost parentheses” convention.