Calculating the state space of a Single pile Nim game:

For the sake of this implementation, we will consider a single pile game in which has the following inputs:

n = 10
k = 3

The initial state of the board will be empty: **. . . . .**

The 10 dots in the above representation means the empty board or the initial state.
Note that the board will only be filled incrementally from the first position and cannot skip an empty position.

In our game, x will always make the first move, hence the first state will be fixed to x : **x . . . .**

As we move the next position on the board, a total of 4 states can be associated with filling this position, this can be given as:

. . . .
x. . . . .
xx. . . . .
xo. . . . .

Thus, the sum of all the possible states of the first two positions on the board is 1 + 1 + 2 = 4.

If we continue this process for all the 10 positions on the board we get the total number of possible states on the board to be 2^10 = 1024.

This is an over estimate as these states also considers some of the illegal moves which consider k > 3 in a given turn:
Ex. **x x x x . . . . . .** is an illegal move as k exceeds 3.

These illegal moves are calculated as follows:

For n = 4, there can be only one state which has an illegal move where all the positions are **x,** note that the first position is always x. Similarly, for n = 5, we have 3 illegal moves.

After this state, the board becomes somewhat symmetric for rest of the positions and the number of illegal moves can be calculated by 2^3. 2^4, 2^5, and so on for the next layers.

Thus the total number of illegal moves sums up to 1 + 3 + 8 + 16 + 32 +64 +128 = **252**.

Therefore, the final state space of or (10,3) Nim game is 772.