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NON PARAMETRIC TEST



CHAPTER OUTLINE

After studying this chapter, students will be able to understand the:

- ❖ Parametric Vs. non-parametric test, Needs of applying non-parametric tests,
- ❖ One-sample test, Run test, Binomial test, Kolmogorov-Smirnov test
- ❖ Two independent sample test, Median test, Kolmogorov-Smirnov test, Wilcoxon Mann Whitney test, Chi-square test
- ❖ Paired-sample test: Wilcoxon signed rank test, Cochran's Q test, Friedman two way analysis of variance test, Kruskal Wallis test
- ❖ Problems and illustrative examples related using software.

Most of the hypothesis testing procedures so far such as Z test, t test, F test are based upon the assumption that the random samples are selected from a normal population. If this is true, these methods can extract all the information that is available in sample and they usually give the best possible precision. Parametric tests depends on parameters, viz., mean or proportion or standard deviation of the population from which sample is taken.

In practice there are many circumstances in which sample are selected from non normal population. In such case, we can have no assumptions about parameters or normality about the population. In such special cases, Parametric are inevitable and are used for testing of hypothesis. Many non parametric procedures are based upon ranked data or even categorical data. For such data no parametric tests are available.

Non parametric tests are often used in place of their parametric counterparts when certain assumptions about the underlying population are in weak state. However, if the sample size is large enough most non parametric tests can be viewed as (the usual normal theory based procedures applied to ranks- make this statement more clear).

The following table shows a comparative view of some normal distribution based tests (that is parametric tests) and its non parametric counterparts.

Normal Distribution based Test	Corresponding non Parametric Test	Purpose of Test
t test for independent samples	Mann Whitney U test	Compare two independent samples
Paired t test	Wilcoxon matched pair signed rank test	Compare dependent samples
One way ANOVA	Kruskalwallis H test	Compare three or more groups
Two way ANOVA	Friedman test	Compare groups classified by two factors

Advantages of Non-parametric Test

- Can be applied to qualitative data (rank, ordinal, categorical data) as well as quantitative data.
- For relatively small sample it is the only possible test.
- It is simple to understand, quicker an easier to apply.
- It is less time consuming.
- It needs no assumption about the population from which sample is selected.
- It has greater range of applicability because of milder assumption.
- It does not require complicated sampling theory.

Disadvantages of Non-parametric Test

- It can not be used to estimate the parameters of population.
- These tests are less reliable and less powerful than parametric tests.
- These tests are less efficient than parametric tests.
- In these test many times lot of information are discarded or unused.
- Lot of tables are needed for tests.

Uses of non parametric test

- (i) When data size is small.
- (ii) When assumptions of parametric procedure are not satisfied.
- (iii) When quick or preliminary data analysis is required.
- (iv) When data are in weak scaled.
- (v) When data show highly skewed nature (large positive/negative skewness).
- (vi) When basic question of interest is distribution free in nature.

Measurement Scale

The measurement is the process of assigning number codes to observations with or without true numerical meaning. Measurement scales are used to categorize and/or quantify variables. Any statistical data must have measurement. There are four measurement scale in which any data is measured.

- (i) Nominal scale
- (ii) Ordinal scale
- (iii) Interval scale
- (iv) Ratio scale

Nominal Scale

It is a process of assigning numbers or symbols to events such that the numbers assigned have no numerical meaning. It is the weakest scale. It is numeric in name only because it does not satisfy ordinary arithmetic properties such as addition, subtraction, multiplication, division etc. e.g. Numbers on football player's jerseys, numbers assigned to sex, numbers assigned to marital status.

Ordinal Scale

It is system of assigning numbers to events such that numbers assigned have grading to events. It is rank order of the events. In comparing one is higher or more than the other ordinal scale is used. e.g. grading system in government job, preference to different cold drinks such as Coca-Cola, Sprite, Fanta, Pepsi.

Interval Scale

It is extended form of ordinal scale in which distance (interval) between two objects are exactly known. In this scale the numerals with quantitative meaning are associated to the objects. In this scale ratio of any two interval is independent of unit of measurements and zero point(origin).e.g. temperature recorded.

Ratio Scale

It is extended form of interval scale including a true zero point as it's origin. In this case ratio of any two points is independent of unit of measurement but not of zero point. The numbers associated to ratio scale are true numbers with true zero. e.g. heights recorded, weights recorded.

Difference between Parametric and Non Parametric test

Parametric test	Non parametric test
1. It specifies certain condition about parameter of the population from which sample is selected.	1. It does not specify certain condition about parameter of the population from which sample is selected.
2. It is used in testing of hypothesis and estimation of parameters.	2. It is used in testing of hypothesis but not in estimation of parameters.
3. Mostly it is used in data measured in interval and ratio scale.	3. It is used in data measured in nominal and ordinal scale.
4. It is most powerful.	4. It is less powerful.
5. It requires complicated sampling technique.	5. It does not require complicated sampling technique.

Assumptions of Non-parametric Test

Non parametric tests do not depend upon assumption about parameter of the population from which sample is selected. However, following basic and weak assumptions are made in the non parametric tests.

- (i) The sample observations are independent.
- (ii) The variable under study is continuous.
- (iii) Sample p.d.f. is continuous.
- (iv) Lower order moments exists (mean and variance).

One Sample Test

Run Test

It is non parametric test used to determine the randomness of the selected samples.

Run is a set of identical or related symbols contained between two different symbols or none at all. In sequence HHHTTHHTHHHTT number of run is 8.

Let us consider a random sample of size n is selected from non normal population. Let $x_1, x_2, x_3, \dots, x_n$ be samples.

Different steps in the test are

Problem to test

H_0 : Sample observations are in random order

H_1 : Sample observations are not in random order.

First find the median of sample and assign a symbol (say A) for x_i if $x_i > M_d$, assign next symbol (say B) for x_i if $x_i < M_d$ and omit if $x_i = M_d$ (called tie)

Count the number of runs (r) of the symbols, number of observations of symbol A denoted by n_1 and number of observations of symbol B denoted by n_2 .

For small sample size ($n_1, n_2 \leq 20$)

Test statisticNumber of runs (r)**Level of significance**Let α be the level of significance. Usually we take $\alpha = 0.05$ unless we are given.**Critical value**Critical or tabulated value t_c and \bar{r} is obtained from table according to the level of significance α , degree of freedom n_1 and n_2 and alternative hypothesis.**Decision**Accept H_0 at α level of significance if $r \in (t_c, \bar{r})$, reject otherwise.For large sample size (n_1 or $n_2 > 20$)In case of large sample size r is approximately normally distributed with mean $\mu_r = \frac{2n_1 n_2}{n_1 + n_2} + 1$ And variance $\sigma_r^2 = \frac{2n_1 n_2 (2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}$ **Test statistic**

$$Z = \frac{r - \mu_r}{\sigma_r} \sim N(0, 1)$$

Level of significanceLet α be the level of significance. Usually we take $\alpha = .05$ unless we are given.**Critical value**Critical or tabulated value of Z is obtained from table according to the level of significance, and alternative hypothesis.**Decision**Reject H_0 at α level of significance if $|Z| > Z_{\text{tabulated}}$, accept otherwise.Note: For α other than 5% even if sample size is small use Z test in which $Z = \frac{|r - \mu_r| - 0.5}{\sigma_r}$.

Example 1: In 15 toss of a coin the following sequence of heads (H) and tails (T) is obtained
 H T T T H T H H H T T H H T

Test at 0.05 level of significance whether the sequence is random.

Solution:Here, Number of heads (n_1) = 8, Number of tails (n_2) = 7, Number of runs (r) = 8, Level of significance (α) = 0.05**Problem to test** H_0 : The sequence is in random order H_1 : The sequence is not in random order**Test statistic**

$$r = 8$$

Critical value

At $\alpha = 0.05$ level of significance, the critical value for $n_1=8$ and $n_2 = 7$ degree of freedom are $t_{\alpha/2} = 2.22$ and $t = 1.3$ for two tailed test.

Decision

$r = 8 \in (t = 4, t = 13)$, accept H_0 at 0.05 level of significance.

Conclusion

The sequence of H and T are in random order.

Example 2: The following is the arrangement of defective (d) and non defective (n) pieces of keyboard produced in the given order by a certain machine:

n n n n n d d d d n n n n n n n n n n d d d d d d d d n n n d d d d n n n

Test the randomness at the 0.01 level of significance.

Solution:

Here, number of n (n_1) = 25

Number of d (n_2) = 13 number of runs (r) = 11

$$\mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1 = \frac{2 \times 25 \times 13}{25 + 13} + 1 = 8.105$$

$$\begin{aligned}\sigma_r^2 &= \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)} \\ &= \frac{2 \times 25 \times 13(2 \times 25 \times 13 - 25 - 13)}{(25 + 13)^2(25 + 13 - 1)} \\ &= \frac{397800}{53428} = 7.44\end{aligned}$$

$$\sigma_r = \sqrt{7.44} = 2.72.$$

Problem to test

H_0 : Arrangement is in random order

H_1 : Arrangement is not in random order.

Test statistic

$$Z = \frac{r - \mu_r}{\sigma_r} = \frac{11 - 8.105}{2.72} = 1.064$$

Critical value

At $\alpha = 0.01$ level of significance critical value is $Z_{\text{tabulated}} = Z_{\alpha/2} = 2.576$.

Decision

$Z = 1.064 < Z_{\text{tabulated}} = 2.576$, accept H_0 at 0.01 level of significance

Conclusion

The defective and non defective pieces produced by machine are in random order.

Example 3: The height (in inches) of 16 BSC CSIT students are as follows:

68.2, 71.6, 69.3, 71.6, 70.4, 65.0, 63.6, 64.7, 65.3, 64.2, 67.6, 68.6, 66.8, 68.9, 66.8, 70.1

Test whether the order of the heights is random or not? Use 5% level of significance.

Solution:

Here, $n = 16$, to find run first of all find median. To find the median, arranging data in ascending order

63.6, 64.2, 64.7, 65.0, 65.3, 66.8, 66.8, 67.7, 68.2, 68.6, 68.9, 69.3, 70.1, 70.4, 71.6, 71.6

$$Md = \frac{(n+1)}{2}^{\text{th}} \text{ item} = \frac{16+1}{2} = 8.5^{\text{th}} \text{ item}$$

Hence, median = $(8^{\text{th}} \text{ item} + 9^{\text{th}} \text{ item})/2 = (67.7 + 68.2)/2 = 67.95$

Let the number greater than 67.95 is denoted by A and number less than 67.95 is denoted by B, then the arrangement of given sample are

A AAAA B BBBB A B A B A

Here, number of A (n_1) = 8, number of B (n_2) = 8, number of run (r) = 7

Problem to test

H_0 : Order of heights are in random order

H_1 : Order of heights are not in random order.

Test statistic

$$r = 7$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then critical value for $n_1 = 8$ and $n_2 = 8$ are $L = 4$ and $\bar{r} = 14$

Decision

$r = 7 \in (L = 4, \bar{r} = 14)$, accept H_0 at 0.05 level of significance.

Conclusion

The order of heights are in random order.

Example 4: The following are the no. of emails arrived within one hour in a communication room: 46, 58, 60, 56, 70, 66, 48, 54, 62, 41, 39, 52, 45, 62, 53, 69, 65, 65, 67, 76, 52, 52, 59, 59, 67, 51, 46, 61, 40, 43, 42, 77, 67, 63, 59, 63, 63, 72, 57, 59, 42, 56, 47, 62, 67, 70, 63, 66, 69 and 73. Given that median is 59.5

Test the randomness at the 0.05 level of significance.

Solution:

Here, Sample size (n) = 50, Median (Md) = 59.5, Level of significance (α) = 0.05

Assign A for number greater than 59.5 and assign B for number less than 59.5 then arrangement of sample becomes

B B A B A A B B A B BBB A B A A A A A A B BBBB A A A A A A A

Number of B (n_1) = 25, Number of A (n_2) = 25, Number of run (r) = 20

$$\mu_r = \frac{2n_1 n_2}{n_1 + n_2} + 1 = \frac{2 \times 25 \times 25}{25 + 25} + 1 = 26$$

$$\sigma_r^2 = \frac{2n_1 n_2 (2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)} = \frac{2 \times 25 \times 25 (2 \times 25 \times 25 - 25 - 25)}{(25 + 25)^2 (25 + 25 - 1)} = 12.2,$$

$$\sigma_r = \sqrt{12.2} = 3.5$$

Problem to test

H_0 : Emails are in random order

H_1 : Emails are not in random order.

Test statistic

$$Z = \frac{T - \mu_r}{\sigma_r} = \frac{20 - 26}{3.5} = -1.71$$

Critical value

At $\alpha = 0.05$ level of significance, critical value is $Z_{\text{abulated}} = Z_{\alpha/2} = 1.96$.

Decision

$|Z| = 1.71 < Z_{\text{abulated}} = 1.96$, accept H_0 at 0.05 level of significance.

Conclusion

The sample is in random order.

Binomial Test

It is non parametric test used for data present in either nominal or ordinal scale. It is used to test whether the binomial population has two distinct groups of two equal numbers of outcomes or not.

Let us consider a sample of size n is selected from binomial population (dichotomized population)

Let $x_1, x_2, x_3, \dots, x_n$ be independent sample of size n selected from binomial population having probability mass function $P(x) = c(n,x) p^x (1-p)^{n-x}$ such that probability associated with each trial is equal. x is number of observations having certain characteristic and $n - x$ is number of observations having next characteristic then $p = \frac{x}{n}$

Different steps in the test are

Problem to test

$H_0: P = P_0$

$H_1: P \neq P_0$ (two tailed) or $H_1: P > P_0$ (one tail) or $H_1: P < P_0$ (one tail)

Let n_1 and n_2 be the number of observations belonging to two groups from n samples.

$$X_0 = \min(n_1, n_2)$$

Small sample size ($n \leq 25$)

Test statistic

$$X_0 = \min(n_1, n_2)$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Using the binomial distribution $p = \text{Prob}(X \leq x_0) = \sum_{x=0}^{x_0} C(n, x) p^x (1-p)^{n-x} = \sum_{x=0}^{x_0} C(n, x) \left(\frac{1}{2}\right)^n$

Decision

Accept H_0 at α level of significance if $p > \alpha$ for one tailed test and $2p > \alpha$ for two tailed test, reject otherwise

Large sample size ($n > 25$)

For large sample size, x_0 is normally distributed with mean np and variance npq

Test statistic

$$Z = \frac{x_0 - \mu}{\sigma} = \frac{x_0 - np}{\sqrt{npq}}$$

Since x_0 is discrete so that continuity correction is made as

$$Z = \frac{(x_0 + 0.5) - np}{\sqrt{npq}}, \text{ use } + 0.5 \text{ if } x_0 < np \text{ and use } - 0.5 \text{ if } x_0 > np$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value $Z_{\text{tabulated}}$ is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $Z > Z_{\text{tabulated}}$. Accept otherwise.

Example 5: The following are defective (D) and non defective (N) electronic items produced in the given order by a certain machine:

N N D D N D N N D D N D D N N D N N N

Test whether defective and non defective are equally produced or not. Use Binomial test at the 0.01 level of significance. yesai le Binomial test

Solution:

Here number of N (n_1) = 13

Number of D (n_2) = 9

Problem to test

$$H_0: P = \frac{1}{2}$$

$$H_1: P \neq \frac{1}{2}$$

Test statistics

$$X_0 = \min \{n_1, n_2\} = \min (13, 9) = 9$$

Level of significance

$$\alpha = 0.01$$

Critical value

$$P = \sum_{x=0}^{X_0} C(n, x) \left(\frac{1}{2}\right)^n = \sum_{x=0}^9 C(n, x) \left(\frac{1}{2}\right)^n = 0.262$$

$$2P = 2 \times 0.262 = 0.524$$

Decision

$$2P = 0.524 > \alpha = 0.01$$

Accept H_0 at 0.01 level of significance.

Conclusion

Defective and non defective are produced equally.

Example 6: Out of 50 students willing to express opinion of laptop 30 expressed preferences to brand Dell and 20 for brand Lenovo. Use binomial test to test hypothesis that both brand of laptop are equally popular

Solution:

Here number of female students prefer Dell laptop (n_1) = 30

Number of female students prefer Lenovo laptop (n_2) = 20

Problem to test

$$H_0 : P = \frac{1}{2}$$

$$H_1 : P \neq \frac{1}{2}$$

Test statistics

$$X_0 = \min \{n_1, n_2\} = \min (20, 30) = 20$$

$$np = 50 \times \frac{1}{2} = 25$$

$$\begin{aligned} Z &= \frac{(x_0 + 0.5) - np}{\sqrt{npq}} \\ &= \frac{(20 + 0.5) - 25}{\sqrt{12.5}} \\ &= \frac{-4.5}{\sqrt{12.5}} \\ &= -1.27 \end{aligned}$$

Level of significance

Let $\alpha = 0.05$

Critical value

$Z_{\alpha/2} = 1.96$

Decision

$$|Z| = 1.27 < Z_{\alpha/2} = 1.96$$

Accept H_0 at 0.05 level of significance.

Conclusion

Both brand of laptop are equally popular

Kolmogorov Smirnov Test

Kolmogorov Smirnov one sample test is a test of goodness of fit. It is alternate to chi square test for goodness of fit when sample size is small.

Let $x_1, x_2, x_3, \dots, x_n$ be random sample of size n from population having distribution function $F(x)$.

Let us consider n observations of random variable x is classified into k classes with their respective frequencies

Different steps in the test are;

Problem to test

H_0 : Samples come from population with distribution $F_0(x)$

H_1 : Samples do not come from population with distribution $F_0(x)$

Obtain the observed relative frequency F_e and expected relative frequency F_0 respectively. Then find the absolute deviation of F_e and F_0 i.e., $|F_e - F_0|$

Test statistic

$$D_1 = \text{Max } |F_e - F_0|$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value $D_{n,\alpha}$ is obtained according to the level of significance α and sample size n

Decision

Reject H_0 at α level of significance if $D_1 \geq D_{n,\alpha}$ accept otherwise.

Example 7: The number of laptop in 10 different department are given below. Test whether the laptops are uniformly distributed over the entire office use Kolmogorov smirnov test.

yesaike Kolmogorov

Department No.	1	2	3	4	5	6	7	8	9	10
No of laptop	8	10	9	12	15	7	5	12	13	9

Solution:

Her expected frequency for each class = Sum of frequencies/No of class

Department no.	No of laptop	Observed cf (f_{0i})	Observed relative freq (F_0)	Expected freq (f_e)	Expected cf (f_{ei})	Expected relative freq (F_e)	$ F_e - F_0 $
1	8	8	8/100	10	10	10/100	2/100
2	10	18	18/100	10	20	20/100	2/100
3	9	27	27/100	10	30	30/100	3/100
4	12	39	39/100	10	40	40/100	1/100
5	15	54	54/100	10	50	50/100	4/100
6	7	61	61/100	10	60	60/100	1/100
7	5	66	66/100	10	70	70/100	4/100
8	12	78	78/100	10	80	80/100	2/100
9	13	91	91/100	10	90	90/100	1/100
10	9	100	100/100	10	100	100/100	0/100

Problem to test H_0 : The disease infected plants are uniformly distributed over the entire area. H_1 : The disease infected plants are not uniformly distributed over the entire area.**Test statistic**

$D_0 = \text{Max } |F_e - F_0| = 4/100 = 0.04$

Critical valueLet 5% be the level of significance then critical value is $D_{100,0.05} = \frac{1.36}{\sqrt{100}} = 0.136$.**Decision** $D_0 = 0.04 < D_{100,0.05} = 0.136$, accept H_0 at 5% level of significance.**Conclusion**

The disease infected plants are uniformly distributed over the entire area.

Example 8: A random sample of 20 volume based internet connected have following speed of internet connection in mps:

2.7, 2.9, 3.0, 3.1, 2.8, 3.0, 2.9, 3.0, 2.6, 3.1, 3.2, 3.1, 3.0, 2.9, 3.3, 3.0, 2.8, 2.9, 3.0, 2.9

Apply the Kolmogorov Smirnov test for testing that the internet speed are equally distributed.

Solution:Expected frequency for each class = $\Sigma f_0 / \text{no of class} = 20/8 = 2.5$

Speed	Tally bar	No of internet connection	cf_0	F_0	Expected Freq (f_e)	cf_e	F_e	$ F_e - F_0 $
26		1	1	1/20	2.5	2.5	2.5/20	1.5/20
27		1	2	2/20	2.5	5	5/20	3/20
28		2	4	4/20	2.5	7.5	7.5/20	3.5/20
29		5	9	9/20	2.5	10	10/20	1/20
30		6	15	15/20	2.5	12.5	12.5/20	2.5/20
31		3	18	18/20	2.5	15	15/20	3/20
32		1	19	19/20	2.5	17.5	17.5/20	1.5/20
33		1	20	20/20	2.5	20	20/20	0/20

Problem to test H_0 : Internet speeds are equally distributed. H_1 : Internet speed is not equally distributed.**Test statistic**

$D_0 = \text{Max } |F_e - F_0| = 3.5/20 = 0.175$

Critical valueLet 5% be the level of significance then critical value is $D_{20, 0.05} = 0.294$ **Decision** $D_0 = 0.175 < D_{20, 0.05} = 0.294$, accept H_0 at 5% level of significance.**Conclusion**

Children of different heights are equally enrolled.

**EXERCISE**

- What do you mean by non parametric test? Write down advantages of non parametric test over parametric test.
- Differentiate between parametric test and non parametric test.
- Discuss assumptions of non parametric test.
- Write down steps of non parametric test.
- Describe the function and procedure of run test.
- What is binomial test? Why is it used?
- Describe the method of binomial test.
- Discuss process of kolmogorov Smirnov test.
- In 31 toss of a coin the following sequence of heads (H) and tails (T) is obtained.
HTTHTHHHTHH TTHTHTHHHTHTTHHTH
Test at 0.05 level of significance level whether the sequence is random. Ans: r = 23, sig.

10. A random sample of 15 adults living in a small town is selected to estimate the proportion of voting favoring a certain candidate for Mayer. Each individual was also asked if he or she was a college graduate. By letting Y and N designate the response of yes and no to the education question, the following sequence was obtained N NNN Y Y N Y Y N Y N NNN. Use the run test to determine if the sequence supports the condition that the sample was selected at random.
- Ans: $r = 7$, insig.
11. In production line touch screen are inspected periodically for defectives. The following is sequence of defective item D and non defective item N produced by production line. D D N N N D N N D D N N N N D D D N N D N N N D N D. Use run test with a significance level 0.05 to determine whether the defectives are occurring at random order.
- Ans: $r = 13$, insig.
12. The following are the number of IT students absent from a college on 24 consecutive college days: 29, 25, 31, 28, 30, 28, 33, 31, 35, 29, 31, 33, 35, 28, 36, 30, 33, 26, 30, 28, 32, 31, 38 and 27. Test for randomness at the 0.01 level of significance.
- Ans: $z = 0$, insig.
13. The height (in inches) of 15 football players are as follows: 65.3, 67.7, 68.4, 71, 70.2, 67, 69.8, 64, 67.6, 71.5, 64.4, 63.6, 66.8, 70.4, 69. Test whether the order of the heights is random or not. Use 1% level of significance.
- Ans: $z = 0$, insig.
14. The following are defective (D) and non defective (N) electric cable produced in the given order by a certain machine in a manufacturing industry:
D N D D N D N D N N D N D N N D N D N N N
Test whether defective and non defective cables are equally produced. Use Binomial test at the 5% level of significance.
- Ans: $p = 0.345$, insig.
15. 100 workers of sales house willing to express opinion of cell phones, 40 expressed preferences to brand Nokia and 60 for brand Oppo. Are both brand of cell phone equally popular at 5% level of significance. Use Binomial test
- Ans: $p = 1.9$, insig.
16. In a certain computer hardware manufacturing industry six different types of machines are working to cut pieces of wires. The number of wires of unequal length recorded in a day is as follows:

Machine	1	2	3	4	5	6
No of wire	2	0	4	8	5	11

Do these data provide sufficient evidence that the six machines equally cut the wires of unequal length? Apply Kolmogorov Smirnov test at 5% level of significance.

$$\text{Ans: } D_0 = 0.3, \text{ sig.}$$

17. The number of virus infected computers of five different capacity of hard disk is given below.

Capacity of hard disk (GB)	500	320	1000	2000	4000
No of virus infected	11	15	20	3	1

Test whether the computers of five hard disk are uniformly infected using Kolmogorov Smirnov test.

$$\text{Ans: } D_0 = 0.333, \text{ insig.}$$

18. A game consists of four pairs of color cards. Twenty chimpanzees of same age were taught the matching game of color cards for a specified period of time. At the end of the training 4 pairs of color cards are given to each of the chimpanzee for matching. The result were as follows:

Matched set	0	1	2	3	4
Frequency	1	0	5	7	7

Does chimpanzees recognize colors? Use Kolmogorov Smirnov test at 5% level of significance.

$$\text{Ans: } D = 0.35, \text{ insig.}$$

Two Independent Sample Test

Median Test

It is non parametric test used to test the significance difference between two independent distribution.

Let $x_1, x_2, x_3, \dots, x_{n_1}$ and $y_1, y_2, y_3, \dots, y_{n_2}$ be two independent samples of sizes n_1 and n_2 respectively be selected from continuous populations with unknown medians Md_1 and Md_2 respectively.

Different steps in the test are

Problem to test

$$H_0: Md_1 = Md_2$$

$$H_1: Md_1 \neq Md_2 \text{ (Two tailed test)} \text{ or } H_1: Md_1 > Md_2 \text{ (One tailed right)} \text{ or } H_1: Md_1 < Md_2 \text{ (one tailed left)}$$

Small sample size ($n_1 \leq 10, n_2 \leq 10$)

Combine n_1 and n_2 such that $n = n_1 + n_2$ and obtain median of n observation. Find number of observations in $x_i \leq Md$ and denote by a .

Test statistic

$$P(A=a) = \frac{c(n_1, a) c(n_2, k-a)}{c(n_1 + n_2, k)} : a = 0, 1, 2, \dots, \min(n_1, k), k = \frac{n_1 + n_2}{2} = \frac{n}{2}$$

Level of significance

Let α be the level of significance. Generally we fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value is given by $p = P(A \geq a)$

Decision

Accept H_0 at α level of significance if $p > \alpha$ for one tailed and $2p > \alpha$ for two tailed test, reject otherwise.

Large sample size ($n_1 > 10, n_2 > 10$)

Combine n_1 and n_2 such that $n = n_1 + n_2$ and obtain median of n observation. Find number of observations in $x_i \leq Md$ and denote by

a. Find number of observations in $y_i \leq Md$ and denote by b .

Also find the number of observations in $x_i > Md$ and $y_i > Md$ and denote by c and d respectively.

	No. of obs. $\leq Md$	No. of obs. $> Md$	Total
Sample x	a	c	$a+c$
Sample y	b	d	$b+d$
Total	$a+b$	$c+d$	$N=a+b+c+d$

Test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)} \sim \chi^2_{(1)}$$

If any cell frequency is less than 5 then

$$\text{Corrected } \chi^2 = \frac{N(|ad - bc| - \frac{N}{2})^2}{(a+c)(b+d)(a+b)(c+d)} \sim \chi^2_{(1)}$$

Level of significance

Let α be the level of significance. Generally we fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance for 1 degree of freedom critical value is $\chi^2_{\alpha(1)}$

Decision

Reject H_0 at α level of significance if $\chi^2 > \chi^2_{\alpha(1)}$, accept otherwise.

Example 1: Two independent samples are given below;

Sample I	10	11	8	8	14		
Sample II	9	12	13	9	15	9	17

Test whether the two samples have come from the same population with respect to their medians. Use median test at 0.05 level of significance.

Solution:

Here, to find the median of combined group, arranging data in ascending order,
8, 8, 9, 9, 9, 10, 11, 12, 13, 14, 15, 17.

$$n = 12, Md = \frac{(n+1)}{2}^{\text{th}} \text{ item} = \frac{(12+1)}{2} = 6.5^{\text{th}} \text{ item.}$$

$$\text{Hence median} = \frac{10+11}{2} = 10.5$$

Now no of observations in first sample less than or equal to median (a) = 3

First sample size (n_1) = 5

$$\text{Second sample size } (n_2) = 7, k = \frac{n_1 + n_2}{2} = \frac{(5+7)}{2} = 6$$

Let Md_1 and Md_2 be median of I population and II population respectively.

Problem to test

H_0 : There is no significant difference between median of I population and median of II population ($Md_1 = Md_2$)

H_1 : There is significant difference between median of I population and II population ($Md_1 \neq Md_2$)

Test statistic

$$P(A=a) = \frac{c(n_1, a) c(n_2, k-a)}{c(n_1 + n_2, k)} = \frac{c(5, a) c(7, 6-a)}{c(12, 6)}, a = 0, 1, 2, 3, 4, 5$$

Critical value

$$\begin{aligned} P &= P(A \geq a) = P(A \geq 3) = \sum \frac{c(5, a) c(7, 6-a)}{c(12, 6)} \\ &= \frac{c(5, 3) c(7, 6-3)}{c(12, 6)} + \frac{c(5, 4) c(7, 6-4)}{c(12, 6)} + \frac{c(5, 5) c(7, 6-5)}{c(12, 6)} \\ &= 462/9 = 24 = 0.5 \end{aligned}$$

Decision

$2P = 1 > \alpha = 0.05$, accept H_0 at 0.05 level of significance.

Conclusion

Two samples have come from same population with respect to median.

Example 2: An IQ test was given to a random sample of 15 male and 20 female students of a university. Their scores were recorded as follows;

Male: 56, 66, 62, 81, 75, 73, 83, 68, 48, 70, 60, 77, 86, 44, 72

Female: 63, 77, 65, 71, 74, 60, 76, 61, 67, 72, 64, 65, 55, 89, 45, 53, 68, 73, 50, 81

Use median test to determine whether IQ of male and female students is same in the university. (Given that the median of combined sample = 68)

Solution:

Here Number of male (n_1) = 15, Number of female (n_2) = 20, $N = n_1 + n_2 = 15 + 20 = 35$,

a(no. of obs. of male \leq Md) = 7, b(no. of obs. of female \leq Md) = 12

c(no. of obs. of male $>$ Md) = 8, d(no. of obs. of female $>$ Md) = 8

The 2×2 contingency table is

	Sample I	Sample II	Total
No. of obs. \leq Md	7 (a)	12 (b)	19
No. of obs. $>$ Md	8 (c)	8 (d)	16
	15	20	35

Let Md_1 and Md_2 be median IQ of male and female respectively.

Problem to test

H_0 : There is no significant difference between IQ of male and female ($Md_1 = Md_2$)

H_1 : There is significant difference between IQ of male and female. ($Md_1 \neq Md_2$)

Test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)} = \frac{35(7 \times 8 - 12 \times 8)^2}{15 \times 20 \times 19 \times 16} = 0.61$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then critical value at 0.05 level of significance with 1 degree of freedom is $\chi^2_{(1)} = 3.84$.

Decision

$\chi^2 = 0.61 < \chi^2_{(1)} = 3.84$, accept H_0 at 0.05 level of significance.

Conclusion

IQ of male and female is same in the university.

Two Sample Kolmogorov Smirnov Test

pg: 302 n 303

It is non parametric test used to test whether two independent samples are from same population or not.

Let $x_1, x_2, x_3, \dots, x_{n_1}$ and $y_1, y_2, y_3, \dots, y_{n_2}$ be independent samples of size n_1 and n_2 drawn from continuous population having distribution function $F(x)$ and $F(y)$ respectively.

Different steps in the test are

Problem to test

$$H_0: F(x) = F(y)$$

$$H_1: F(x) \neq F(y) \text{ (two tailed)} \text{ or } H_1: F(x) > F(y) \text{ (one tail)} \text{ or } H_1: F(x) < F(y) \text{ (one tail)}$$

Obtain cumulated distribution function of x and y separately after arranging in order.

$$F(x) = \frac{k_1}{n_1}, \text{ where } k_1 \text{ is observed cumulative frequency of } x$$

$$F(y) = \frac{k_2}{n_2}, \text{ where } k_2 \text{ is observed cumulative frequency of } y$$

Small sample test ($n_1 = n_2 < 40$, n_1 and $n_2 \leq 20$ for $n_1 \neq n_2$)

Test statistic

$$D_0 = \text{maximum} \{ |F(x) - F(y)| \}$$

Level of significance

Let α be level of significance

Critical value

At α level of significance critical value for n_1 and n_2 is

$$D_{n_1, n_2, \alpha}$$

Decision

Reject H_0 at α level of significance if $D_0 \geq D_{n_1, n_2, \alpha}$

Accept otherwise

Large sample test ($n_1 = n_2 > 40$, n_1 and $n_2 > 20$ for $n_1 \neq n_2$)

Test statistic

$$D_0 = \text{maximum} \{ |F(x) - F(y)| \text{ for two tail test} \}$$

$$\chi^2 = 4D_0^2 \frac{n_1 n_2}{n_1 + n_2} \text{ follows chi square distribution with 2 degree of freedom. For one tail test}$$

Level of significance

Let α be the level of significance

Critical value

At α level of significance critical value is

$$D_\alpha = 1.36 \sqrt{\frac{n_1 + n_2}{n_1 n_2}} \text{ for two tail with } \alpha = 5\%$$

$$\chi^2_{\alpha/2} \text{ for one tail.}$$

Decision

Reject H_0 at α level of significance if $D_0 \geq D_\alpha$ for two tail

$$\text{if } \chi^2 > \chi^2_{\alpha/2} \text{ for one tail}$$

Accept otherwise.

Example 3: Life in years of two types of cells used in laptop are given below:

Cell X	4	6	5	6	3	4	5	3	5
Cell Y	2	5	4	3	4	2	4	3	5

Test whether life of two brands of cells are same or not? Use Kolmogorov Smirnov test at 0.05 level of significance.

Solution:

Problem to test

$$H_0: F(x) = F(y)$$

$$H_1: F(x) \neq F(y)$$

Combined ordered life	Frequency for x	Frequency for Y	F(x)	F(y)	F(x) - F(y)
2	0	2	0/9	2/9	2/9
3	2	2	2/9	4/9	2/9
4	2	3	4/9	7/9	3/9
5	3	2	7/9	1	2/9
6	2	0	1	1	0
Total	9	9			

Test statistics

$$D = \text{maximum } \{|F(x) - F(y)|| = 3/9$$

Critical value

At $\alpha = 0.05$ and $n_1 = 9, n_2 = 9$ from K-S table

$$D_{n_1, n_2, \alpha} = 5/9$$

Decision

$$D = 3/9 < D_{n_1, n_2, \alpha} = 5/9$$

Accept H_0 at 0.05 level of significance.

Conclusion

Two brands of cell X and Y are same.

Example 4: Given below represents monthly income distribution of employees in a hardware company and software company

Income (000 rs)	Number of employees in hardware company	Number of employees in software company
20 - 30	6	12
30 - 40	10	18
40 - 50	11	16
50 - 60	13	12
60 - 70	25	10
70 - 80	15	12
80 - 90	10	10

Do the income distribution support that income of employees in Hardware Company is more than income of employees in Software Company? Use Kolmogorov Smirnov test.

Solution:

Let x be salary of employees in hardware company and y be salary of employees in software company

Problem to test:

$$H_0 : H_0; F(x) = F(y)$$

$$H_1 : F(x) > F(y)$$

Income (000 Rs)	Number of employees in hardware company (x)	Number of employees in software company (y)	$F(x)$	$F(y)$	$ F(x) - F(y) $
20 - 30	6	12	6/100	12/90	66/900
30 - 40	10	18	16/100	30/90	156/900
40 - 50	11	16	27/100	46/90	57/900
50 - 60	13	12	40/100	58/90	220/900
60 - 70	25	10	65/100	68/90	95/900
70 - 80	20	12	85/100	80/90	35/900
80 - 90	15	10	1	1	0
Total	100	90			

Test statistic

$$D_0 = \text{maximum } |F(x) - F(y)| = \frac{220}{900} = \frac{11}{45}$$

$$\chi^2 = 4D_0^2 \frac{n_1 n_2}{n_1 + n_2} = 4 \times \left(\frac{11}{45}\right)^2 \frac{100 \times 90}{100 + 90} = 11.32$$

Critical value

Let $\alpha = 0.05$ then critical value is $\chi_{\alpha/2}^2 = 5.99$.

Decision

$$\chi^2 = 11.32 > c^2_{\alpha/2} = 5.99.$$

Reject H_0 at $\alpha = 0.05$ level of significance.

Conclusion

Income of employees in Hardware Company is more than income of employees in Software Company.

Example 5: Life time of cell phones branded A and B recorded from a repair centre is given below, determine if there is any significant difference between two brand cell phones using Kolmogorov Smirnov test at 5% level of significance.

Life of cell phone in years	Cell phone A	Cell phone B
0 - 2	8	5
2 - 4	12	7
4 - 6	16	31
6 - 8	10	12
8 and above	4	5

solution:

Let cell phone A = x and cell phone B = y

Problem to test

$H_0: F(x) = F(y)$

$H_1: F(x) \neq F(y)$

Life of cell phone in years	Cell phone A	Cell phone B	F(x)	F(y)	$ F(x) - F(y) $
0 - 2	8	5	8/50	5/60	23/300
2 - 4	12	7	20/50	12/60	60/300
4 - 6	16	31	36/50	43/60	1/300
6 - 8	10	12	46/50	55/60	1/300
8 and above	4	5	1	1	0
Total	50	60			

Test statistic

$$D_0 = \text{maximum} (|F(x) - F(y)|) = \frac{60}{300} = 0.2$$

Critical value

$$\text{At } \alpha = 0.05 \text{ critical value is } D_a = 1.36 \sqrt{\frac{n_1 + n_2}{n_1 n_2}} = 1.36 \sqrt{\frac{50 + 60}{50 \times 60}} = 0.26.$$

Decision

$$D_0 = 0.2 < D_a = 0.26$$

Accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between two brand cell phones.

Mann Whitney U Test

pg: 313

It is non parametric test used to determine whether two independent samples have been drawn from populations with same distribution.

Let us consider two independent samples of sizes n_1 and n_2 are drawn from continuous populations with unknown medians Md_1 and Md_2 respectively.

Let $x_1, x_2, x_3, \dots, x_{n_1}$ and $y_1, y_2, y_3, \dots, y_{n_2}$ be independent samples of size n_1 and n_2 .

Different steps in the test are

Problem to test

$H_0: Md_1 = Md_2$

$H_1: Md_1 \neq Md_2$ (two tailed) or $H_1: Md_1 > Md_2$ (one tailed right)

or $H_1: Md_1 < Md_2$ (one tailed left)

Combine n_1 and n_2 such that $n_1 + n_2 = n$ and rank these n observations in ascending order. If two or more observations are equal then assign average rank and is called tied. Sum the ranks of sample of sizes n_1 and n_2 separately to get R_1 and R_2 . If two sample sizes are unequal then smaller one is n_1 . Obtain U_1 and U_2 as $U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$ and $U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$

such that $n_1 n_2 = U_1 + U_2$. Finally get $U_0 = \min \{U_1, U_2\}$

Small sample size ($n_1 \leq 10, n_2 \leq 10$)

Test statistic

$$U_0 = \min \{U_1, U_2\}$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance, we obtain critical value from Mann Whitney table as

$$p = \text{Prob}(U \leq U_0)$$

Alternately

At α level of significance, we obtain critical value from Mann Whitney table as

$$U_{\text{tabulated}} = U_{\alpha(n_1, n_2)}$$
 for two tail and $U_{\alpha/2(n_1, n_2)}$ for one tail test.

Decision

Accept H_0 at α level of significance if $p > \alpha$ for one tailed test and $2p > \alpha$ for two tailed test, reject otherwise.

Alternately

Accept H_0 at α level of significance if $U_0 > U_{\text{tabulated}}$ reject otherwise.

Large sample size ($n_1 > 10, n_2 > 10$)

For large sample size U_0 is approximately normally distributed with mean $\mu_u = \frac{n_1 n_2}{2}$ and variance $\sigma_u^2 = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$.

Test statistic

$$Z = \frac{U_0 - \mu_u}{\sigma_u} = \frac{U_0 - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}} \quad \text{it is used even if tied occurs within sample. If tied occurs}$$

between samples then standard deviation is corrected as $\sigma_u = \frac{n_1 n_2}{n(n-1)} \left\{ \frac{n^3 - n}{12} - \frac{\sum t_i^3 - t_i}{12} \right\}$
 $t_i = \text{number of times } i^{\text{th}} \text{ rank repeated between samples.}$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value Z_{table} is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at a level of significance if $Z > Z_{\text{table}}$. Accept otherwise.

Example 8: The heart beating rate of 5 vegetarians and 5 non vegetarians are recorded below:

Vegetarians	56	67	82	60	75
Non vegetarians	53	42	75	58	65

Is the mean heart beating rate of non vegetarians significantly high. Use Mann Whitney U test.

Solution:

Vegetarians	Ranks	Non vegetarians	Ranks
56	3	53	2
67	7	42	1
82	10	75	8.5
60	5	58	4
75	8.5	65	6
	$R_1 = 33.5$		$R_2 = 21.5$

Here, Sample size of vegetarian (n_1) = 5

Sample size of Non vegetarian (n_2) = 5

Sum of ranks of vegetarian (R_1) = 33.5

Sum of ranks of non vegetarian (R_2) = 21.5

$$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1 = 5 \times 5 + \frac{5 \times 6}{2} - 33.5 = 6.5$$

$$U_2 = n_1 n_2 - U_1 = 5 \times 5 - 6.5 = 18.5$$

$$U_0 = \min[U_1, U_2] = 6.5$$

Let Md_1 and Md_2 be median heart beat rate of vegetarians and non vegetarians respectively.

Problem to test

H_0 : There is no significant difference between heart beating rate of vegetarian and non vegetarian ($Md_1 = Md_2$)

H_1 : Heart beating rate of non vegetarian is significantly high than vegetarian ($Md_1 < Md_2$)

Test statistic

$$U_0 = 6.5$$

Critical value

Let $\alpha = 0.05$ be the level of significance then critical value is $p = 0.111$

Decision

$p = 0.111 > \alpha = 0.05$, accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between heart beating rate of vegetarians and non vegetarians.

Example 7: Test the hypothesis of no difference between the ages of male and female employees of a certain IT company, using the Mann-Whitney U test for the sample data below. Use $\alpha = 0.1$.

Male	35	43	26	44	40	42	33	38	25	26
Female	30	41	34	31	36	32	25	47	28	24

Solution:

Male	Ranks	Female	Ranks
35	12	30	7
43	18	41	16
26	4.5	34	11
44	19	31	8
40	15	36	13
42	17	32	9
33	10	25	2.5
38	14	47	20
25	2.5	28	6
26	4.5	24	1
	$R_1 = 116.5$		$R_2 = 93.5$

Here, Sample size of male (n_1) = 10

Sample size of female (n_2) = 10

Sum of ranks of male (R_1) = 116.5

Sum of ranks of female (R_2) = 93.5

$$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1 = 10 \times 10 + \frac{10 \times 11}{2} - 116.5 = 38.5$$

$$U_2 = n_1 n_2 - U_1 = 10 \times 10 - 38.5 = 61.5$$

$$U_0 = \min(U_1, U_2) = 38.5$$

Let Md_1 and Md_2 be median age of male and female employees respectively.

Problem to test

H_0 : There is no significant difference between age of male and female employee ($Md_1 = Md_2$)

H_1 : There is significant difference between age of male and female employee ($Md_1 \neq Md_2$)

Test statistic

$$U_1 = 38.5$$

Critical value

Let $\alpha = 0.10$ be the level of significance then critical value is $U_{\text{unstated}} = U_{(n_1, n_2)} = 27$

Decision

$$U_1 = 38.5 > U_{(n_1, n_2)} = 27, \text{ accept } H_0 \text{ at } 0.10 \text{ level of significance.}$$

Conclusion:

There is no significant difference between ages of males and female employees.

Example 8: Comparing 2 kinds of emergency flares, a consumer testing service obtained the following burning times (rounded to the nearest tenth of a minute):

Brand C	19.4	21.5	15.3	17.4	16.8	16.6	20.3	22.5	21.3	23.4	19.7	21.0
Brand D	16.5	15.8	24.7	10.2	13.5	15.9	15.7	14.0	12.1	17.4	15.6	15.8

Use Mann Whitney U test at the 0.01 level of significance to check whether it is reasonable to say that the population of burning times of the two kinds of flares are identical.

Solution:

Brand C	Rank	Brand D	Rank
19.4	16	16.5	11
21.5	21	15.8	8.5
15.3	5	24.7	24
17.4	14.5	10.2	1
16.8	13	13.5	3
16.6	12	15.9	10
20.3	18	15.7	7
22.5	22	14.0	4
21.3	20	12.1	2
23.4	23	17.4	14.5
19.7	17	15.6	6
21.0	19	15.8	8.5
	$R_1 = 200.5$		$R_2 = 99.5$

Here,

Sample size of brand C ($n_1 = 12$)Sample size of brand D ($n_2 = 12$)Sum of ranks of brand C ($R_1 = 200.5$)Sum of ranks of brand D ($R_2 = 99.5$)

$$U_1 = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - R_1 = 12 \times 12 + \frac{12 \times 13}{2} - 200.5 = 21.5$$

$$U_2 = n_1 n_2 - U_1 = 12 \times 12 - 21.5 = 122.5$$

$$U_{\text{min}} = \min[U_1, U_2] = \min[21.5, 122.5] = 21.5$$

$$A_e = \frac{n_1 n_2}{2} = \frac{12 \times 12}{2} = 72$$

Here tied occurs between groups, hence

$$\sigma_0 = \frac{n_1 n_2}{n(n-1)} \left| \frac{n^2 - n}{12} - \frac{\sum t_i^2 - ti}{12} \right| = \frac{12 \times 12}{24(24-1)} \left\{ \frac{24^2 - 24}{12} - \frac{2^2 - 2}{12} \right\} = 0.2608 \times 1149.5 = 299.76$$

$$\therefore \sigma_0 = 17.3$$

Problem to test:

H_0 : Population burning time of two kinds of flares are identical.

H_1 : Population burning time of two kinds of flares are not identical.

Test statistic:

$$Z = \frac{U_0 - \mu_0}{\sigma_0} = \frac{21.5 - 72}{17.3} = -2.91$$

Critical value:

At $\alpha = 0.01$ level of significance critical value is $Z_{\text{tabulated}} = Z_{\alpha/2} = 2.58$

Decision:

$|Z| = 2.91 > Z_{\text{tabulated}} = 2.58$, reject H_0 at 0.01 level of significance.

Conclusion:

Burning time of two kind of flares are different.

Example 9: The following are the scores which random samples of students from 2 groups obtained on a random coding test.

Group I	73	82	39	68	91	75	89	67	50	86	57	65	70
Group II	51	42	36	53	88	59	49	66	25	64	18	76	74

Use Mann Whitney U test at the 0.05 level of significance to test whether or not students from the two groups can be expected to score equally well on the test

Solution:

Group I	Rank	Group II	Rank
73	18	51	8
82	22	42	5
39	4	36	3
68	16	53	9
91	26	88	24
75	20	59	11
89	25	49	6
67	15	66	14
50	7	25	2
86	23	64	12
57	10	18	1
65	13	76	21
70	17	74	19
	R ₁ = 216		R ₂ = 135

Here, sample size for group I (n_1) = 13

Sample size for group II (n_2) = 13

Sum of ranks of group I (R_1) = 216

Sum of ranks of group II (R_2) = 135

$$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1 = 13 \times 13 + \frac{13 \times 14}{2} - 216 = 44$$

$$U_2 = n_1 n_2 - U_1 = 13 \times 13 - 44 = 125$$

$$U_0 = \min\{U_1, U_2\} = \text{Min } \{44, 125\} = 44$$

$$\mu_0 = \frac{n_1 n_2}{2} = \frac{13 \times 13}{2} = 84.5$$

$$\sigma_0 = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} = \sqrt{\frac{13 \times 13 (13 + 13 + 1)}{12}} = \sqrt{380.25} = 19.5$$

Let Md_1 and Md_2 be median scores of group I and group II respectively.

Problem to test

H_0 : Students from two groups score equally ($Md_1 = Md_2$)

H_1 : Students from two groups do not score equally ($Md_1 \neq Md_2$)

Test statistic

$$Z = \frac{U_0 - \mu_0}{\sigma_0} = \frac{44 - 84.5}{19.5} = -2.07$$

Critical value

At $\alpha = 0.05$ level of significance, critical value is $Z_{\text{tabulated}} = Z_{0.025} = 1.96$.

Decision

$|Z| = 2.07 > Z_{0.025} = 1.96$, reject H_0 at 0.05 level of significance.

Conclusion

Students from two minority groups do not score equally well

Chi Square Test for Goodness of Fit

It is test used to test the significant difference between observed frequencies (O_i) and expected frequencies (E_i).

Let us consider n observations of random variable x is classified into k classes with their respective frequencies.

Different steps in the test are:

Problem to test

$H_0: O_i = E_i$

$H_1: O_i \neq E_i$

Test statistic

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} \sim \chi^2(k-1)$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value $\chi^2_{\alpha(k-1)}$ is obtained according to the level of significance α and degree of freedom $(k-1)$.

Decision

Reject H_0 at α level of significance if $\chi^2 > \chi^2_{\alpha(k-1)}$, accept otherwise.

Note

If any cell frequencies are less than 5 then combine the frequencies of adjoining cells till the resulting cell frequency is greater than or equal to 5.

Example 10: The following table gives the number of air crafts accidents that occurred during seven days of the week. Find whether the accidents are uniformly distributed over the week.

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	16	8	12	11	9	14

Solution:

Here, $n = 14 + 16 + 8 + 12 + 11 + 9 + 14 = 84$, $k = 7$

Days	No. of accidents(O_i)	P_i	$E_i = NP_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Sun	14	1/7	12	4	0.333
Mon	16	1/7	12	16	1.333
Tue	8	1/7	12	16	1.333
Wed	12	1/7	12	0	0
Thu	11	1/7	12	1	0.0833
Fri	9	1/7	12	9	0.75
Sat	14	1/7	12	4	0.333
Total	$N = \sum O_i = 84$				$\sum \frac{(O_i - E_i)^2}{E_i} = 4.166$

Problem to test

H_0 : Accidents are uniformly distributed over the week.

H_1 : Accidents are not uniformly distributed over the week.

Test statistic

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} = 4.166$$

Critical value

Let $\alpha = 0.05$ be the level of significance then critical value for $k-1 = 6$ degree of freedom at 0.05 level of significance is $\chi^2_{0.05(6)} = 12.59$

Decision:

$\chi^2 = 4.166 < \chi^2_{0.05(6)} = 12.59$, accept H_0 at 0.05 level of significance.

Conclusion

Accidents are uniformly distributed over the week.

Example 11: In an experiment on pea breeding, Mendal obtained the following frequencies of seeds: 315 round and yellow, 101 wrinkle and yellow, 108 round and green, 32 wrinkle and green in total of 556. Theory predicts that the frequencies should be in ratio 9:3:3:1 respectively. Set up the proper hypothesis and test it at 10% level of significance.

Solution:

Seeds	Frequencies (O_i)	P_i	$E_i = N P_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
Round and yellow	315	9/16	312.75	5.0625	0.01618
Wrinkle and yellow	101	3/16	104.25	10.5625	0.10131
Round and green	108	3/16	104.25	14.0625	0.13489
Wrinkle and green	32	1/16	34.75	2.75	0.07913
Total	$N = \sum O_i = 556$				0.3315

Here $k = 4$

Problem to test

H_0 : There is no significant difference between theory and experiment.

H_1 : There is significant difference between theory and experiment.

Test statistic

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

$$= 0.3315$$

Critical value

At 0.10 level of significance and 3 degree of freedom, critical value is $\chi^2_{0.05(3)} = 4.166$

Decision: $\chi^2 = 0.3315 < \chi^2_{0.05(3)} = 4.166$, accept H_0 at 10% level of significance.

Conclusion: There is no significant difference between theory and experiment.

Example 12: A die is thrown 132 times with the following results:

Number turned up	1	2	3	4	5	6
Frequency	16	20	25	14	29	28

Is the die unbiased?

Solution:

No. turned up	Observed freq.(O _i)	P _i	Expected freq(E _i) = NP _i	(O _i - E _i) ²	(O _i - E _i) ² /E _i
1	16	1/6	22	36	36/22
2	20	1/6	22	4	4/22
3	25	1/6	22	9	9/22
4	14	1/6	22	64	64/22
5	29	1/6	22	49	49/22
6	28	1/6	22	36	36/22
Total	N = ΣO _i = 132				9

Here k = 6

Problem to test

H₀: Die is unbiased (O_i = E_i)

H₁: Die is biased (O_i ≠ E_i)

Test statistic

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} = 9$$

Critical value

Let α = 5% be the level of significance, then critical value for 5 degree of freedom at 0.05 level of significance is $\chi^2_{0.05(5)} = 11.07$.

Decision

$\chi^2 = 9 < \chi^2_{0.05(5)} = 11.07$, accept H₀ at 0.05 level of significance.

Conclusion

The die is unbiased.

Example 13: Test whether binomial distribution fits the following data:

x	0	1	2	3	4
f	28	62	46	10	4

Solution:

Problem to test

H₀: Binomial distribution fits the data

H₁: Binomial distribution does not fit the data,

x	f	fx
0	28	0
1	62	62
2	46	92
3	10	30
4	4	16
	$\Sigma f = 150$	$\Sigma fx = 200$

$N = 150, n = 4$

$\bar{x} = \frac{\sum fx}{N} = \frac{200}{150} = \frac{4}{3}$

$\bar{x} = np$

$\sigma^2 = \frac{4}{3} = 4p$

$\text{or } p = \frac{1}{3} \text{ and } q = \frac{2}{3}$

x	O _i	P _i = c(n,x)p ^x q ^{n-x}	E _i = Np _i	(O _i - E _i) ²	(O _i - E _i) ² / E _i
0	28	C(4,0) (1/3) ⁰ (2/3) ⁴ = 0.1975	29.63 ≈ 30	4	0.13
1	62	C(4,1) (1/3) ¹ (2/3) ³ = 0.3951	59.26 ≈ 59	9	0.15
2	46	C(4,2) (1/3) ² (2/3) ² = 0.2963	44.44 ≈ 44	4	0.09
3	10	C(4,3) (1/3) ³ (2/3) ¹ = 0.0988	14.81 ≈ 15	9	0.529
4	4	C(4,4) (1/3) ⁴ (2/3) ⁰ = 0.0123	1.85 ≈ 2	17	
					$\Sigma (O_i - E_i)^2 / E_i = 0.899$

Test statistic

$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} = 0.899$

Critical value

Let 0.05 be the level of significance then critical value for 2 degree of freedom is $\chi^2_{0.05, 2} = 5.911$ (here 2 more df is reduced because of estimation of parameter p and combination of two classes)

Decision

$\chi^2 < \chi^2_{0.05, 2}$, accept H_0 at 0.05 level of significance.

Conclusion

Binomial distribution fits the data.

Example 14: The number of cyber attack per month at a certain websites was checked for months and the results shown are as follows:

X (no of cyber attack)	0	1	2
f	22	18	10

Assuming the observations are independent, test the hypothesis that the random variable has poisson distribution.

Solution:

x	f	fx
0	22	0
1	18	18
2	10	20
	$\Sigma f = 50$	$\Sigma fx = 38$

$$N = 50$$

$$\bar{X} = \frac{\Sigma fx}{N} = \frac{38}{50} = 0.76 \quad \lambda = \bar{X} = 0.76$$

Problem to test

H_0 : The random variable has poisson distribution.

H_1 : The random variable has not poisson distribution.

x	O _i	P _i = $e^{-\lambda} \lambda^x / x!$	E _i = Np _i	(O _i - E _i) ²	(O _i - E _i) ² / E _i
0	22	$e^{-0.76} (0.76)^0 / 0! = 0.4676$	23.38 ≈ 23	1	0.0434
1	18	$e^{-0.76} (0.76)^1 / 1! = 0.3554$	17.77 ≈ 18	0	0
2	10	$e^{-0.76} (0.76)^2 / 2! = 0.1350$	6.75 ≈ 7	9	1.285
					$\Sigma (O_i - E_i)^2 / E_i = 1.3284$

Test statistic

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} = 1.3284$$

Critical value

Let 0.05 be the level of significance then critical value for 1 degree of freedom is $\chi^2_{0.05(1)} = 3.841$
 (Here one more df is reduced due to estimation of parameter λ)

Decision

$\chi^2 = 1.3284 < \chi^2_{0.05(1)} = 3.841$, accept H_0 at 0.05 level of significance.

Conclusion

Random variable has poisson distribution.

Chi Square Test for Independence of Attributes

The characteristics which are capable of being measured qualitatively but not quantitatively are called attributes. This test is used to find any association or not between the attributes.

Let us consider a sample of size n is taken from population of unknown distribution. The observations are classified into attributes say A and B into $A_1, A_2, A_3, \dots, A_r$ and $B_1, B_2, B_3, \dots, B_c$ classes respectively. Let O_{ij} be the observed frequency of $(i, j)^{th}$ class.

The arrangement of observed frequencies O_{ij} in $r \times c$ contingency table is present as

B_j	B_1	B_2	B_l	B_c	Total(O_i)
A_i	O_{11}	O_{12}	O_{1j}	O_{1c}	$O_{1.}$
A_1	O_{21}	O_{22}	O_{2j}	O_{2c}	$O_{2.}$
A_2	O_{31}	O_{32}	O_{3j}	O_{3c}	$O_{3.}$
A_l	O_{rl}	O_{r2}	O_{rj}	O_{rc}	$O_{r.}$
Total($O_{.j}$)	$O_{.1}$	$O_{.2}$	$O_{.j}$	$O_{.c}$	N

Different steps in the test are

problem to test

H_0 : Attributes A and B are independent.

H_1 : Attributes A and B are dependent.

Test statistic

$$\chi^2 = \sum_{i=1}^{r-1} \sum_{j=1}^{c-1} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \sim \chi^2_{(r-1)(c-1)}$$

$$E_{ij} = (O_{i.} \times O_{.j})/N$$

level of significance

let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

At a level of significance and $(r-1)(c-1)$ degree of freedom the critical value is $\chi^2_{\alpha, (r-1)(c-1)}$

Decision

Reject H_0 at α level of significance if $\chi^2 > \chi^2_{\alpha, (r-1)(c-1)}$

When attributes A and B are arranged into two subgroups then the 2×2 contingency table will be

B_j	B_1	B_2	Total
A_i	a	b	a+b
A_1	c	d	c+d
Total	a+c	b+d	N

Different steps in the test are

Problem to test:

H_0 : Attributes A and B are independent.

H_1 : Attributes A and B are dependent.

Test statistic

$$\chi^2 = \frac{N(ad - bc)^2}{(a + c)(b + d)(a + b)(c + d)} \sim \chi^2_{(1)}$$

If any cell frequency is less than 5 then the test statistic is

$$\chi^2_{\text{corrected}} = \frac{N(|ad - bc| - \frac{N}{2})^2}{(a + c)(b + d)(a + b)(c + d)} \sim \chi^2_{(1)}$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance and 1 degree of freedom the critical value is $\chi^2_{\alpha(1)}$

Decision

Reject H_0 at α level of significance if $\chi^2 > \chi^2_{\alpha(1)}$

Example 15: In an experiment to study the dependence of hypertension on smoking habits, the following data were taken on 180 individuals:

	No smokers	Moderate smokers	Heavy smokers
Hypertension	21	36	36
No hypertension	48	26	19

Test the hypothesis that presence or absence of hypertension is independent of smoking habit.

Solution:

	No smokers (B ₁)	Moderate smokers (B ₂)	Heavy smokers (B ₃)	O _i
Hypertension (A ₁)	21	36	36	93
No hypertension (A ₂)	48	26	19	93
O _j	69	62	55	N=186

Now

Group	O _{ij}	E _{ij} = (O _i × O _j) / N	(O _{ij} - E _{ij})	(O _{ij} - E _{ij}) ² / E _{ij}
A ₁ B ₁	21	(93 × 69) / 186 = 34.5	-13.5	5.2826
A ₁ B ₂	36	(93 × 62) / 186 = 31	5	0.8064
A ₁ B ₃	36	(93 × 55) / 186 = 27.5	8.5	2.6272
A ₂ B ₁	48	(93 × 69) / 186 = 34.5	13.5	5.2826
A ₂ B ₂	26	(93 × 62) / 186 = 31	-5	0.8064
A ₂ B ₃	19	(93 × 55) / 186 = 27.5	-8.5	2.6272
Total				17.4324

Problem to test

- H₀: Hypertension is independent of smoking habit.
 H_a: Hypertension is dependent of smoking habit.

Test statistic

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 17.4324$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then critical value for (2-1)(3-1) = 2 degree of freedom at 0.05 level of significance is $\chi^2_{0.05(2)} = 5.99$

Decision

$\chi^2 = 17.4324 > \chi^2_{0.05(2)} = 5.99$, reject H₀ at 5% level of significance.

Conclusion

Presence or absence of hypertension depends upon the smoking habit.

Example 16: A tobacco company claims that there is no relationship between smoking and lung ailments. To investigate the claims random sample of 300 males in age group of 40 to 50 is given medical test. The observed sample result are tabulated below:

	Lung ailment	No lung ailment	Total
Smokers	75	105	180
No smokers	25	95	120
Total	100	200	300

On the basis of this information, can it be concluded that smoking and lung ailment are independent?

Solution:

	Lung ailment(B)	No lung ailment(B)	Total
Smokers (A)	75 = a	105 = b	180 = a + b
No smokers(a)	25 = c	95 = d	120 = c + d
Total	100 = a + c	200 = b + d	300 = N

Problem to test

- H₀: Smoking and lung ailment are independent

- H_a: Smoking and lung ailment are dependent

Test statistic

$$\begin{aligned} \chi^2 &= \frac{N(ad - bc)^2}{(a + c)(b + d)(a + b)(c + d)} \\ &= \frac{300(75 \times 95 - 105 \times 25)^2}{100 \times 200 \times 180 \times 120} = 14.063 \end{aligned}$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then critical value for 1 degree of freedom at 0.05 level of significance is $\chi^2_{0.05(1)} = 3.84$.

Decision

$\chi^2 = 14.063 > \chi^2_{0.05(1)} = 3.84$, reject H_0 at 0.05 level of significance.

Conclusion

Smoking and lung ailment are dependent.

Example 17: In an experiment on immunization of cattle from tuberculosis, the following results were obtained.

	Affected	Not affected
Inoculated	2	10
Not inoculated	6	6

Examine whether the vaccine has an effect in controlling the disease.

Solution:

	Affected	Not affected	Total
Inoculated	2 = a	10 = b	12 = a+b
Not inoculated	6 = c	6 = d	12 = c+d
Total	8 = a+c	16 = b+d	N=24

Problem to test

H_0 : The vaccine has no effect in controlling the disease.

H_1 : The vaccine has effect on controlling the disease.

Test statistic

$$\begin{aligned} \chi^2_{\text{corrected}} &= \frac{N((ad - bc) - \frac{N}{2})^2}{(a+c)(b+d)(a+b)(c+d)} \\ &= \frac{24(|12 - 60| - 12)^2}{8 \times 16 \times 12 \times 12} = 1.687 \end{aligned}$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then for 1 degree of freedom at 0.05 level of significance critical value is $\chi^2_{0.05(1)} = 3.84$

Decision

$\chi^2 = 1.687 < \chi^2_{0.05(1)} = 3.84$, accept H_0 at 0.05 level of significance.

Conclusion

The vaccine has no effect in controlling the disease.

Example 18: The table given below shows the data obtained during outbreak of smallpox:

	Attacked	Not attacked
Vaccinated	31	469
Not vaccinated	185	1315

Test the effectiveness of vaccination in preventing the attack from smallpox.

Solution:
Here,

	Attacked(B)	Not attacked (β)	Total (O_i)
Vaccinated (A)	31	469	500
Not vaccinated (α)	185	1315	1500
Total ($O_{\cdot j}$)	216	1784	2000

Group	O_i	$E_i = (O_{\cdot i} \times O_{\cdot j})/N$	$(O_i - E_i)$	$(O_i - E_i)^2/E_i$
AB	31	54	-23	9.796
A β	469	446	23	1.186
α B	185	162	23	3.265
α β	1315	1338	-23	0.395
Total				14.642

Problem to test

H_0 : Vaccination is not effective in preventing the attack from smallpox

H_1 : Vaccination is effective in preventing the attack from smallpox.

Test statistic

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 14.642$$

Critical value

Let $\alpha = 0.05$ be the level of significance, then for $(r-1)(c-1) = 1$ degree of freedom at 0.05 level of significance, the critical value is $\chi^2_{0.05(1)} = 3.84$

Decision

$\chi^2 = 14.642 > \chi^2_{0.05(1)} = 3.84$, reject H_0 at 0.05 level of significance.

Conclusion

Vaccination is effective in preventing the attack from smallpox.



EXERCISE

- What do you mean by median test? Describe the procedure of median test.
- Describe Kolmogorov Smirnov two sample test.
- Describe the function and procedure of Mann Whitney U test.
- Describe function and procedure of chi square test for goodness of fit.
- Discuss the rationale and method of chi square test for independence of attributes.
- Differentiate between Chi square test for goodness of fit and Kolmogorov Smirnov one sample test.

7. The following are the performance score of 10 person under two software training X and Y.

X	46	45	32	42	39	48	49	30	51	34
Y	44	40	59	47	55	50	47	71	43	55

Use median test to test the effectiveness of two training.

Ans: (0.328, insig)

8. The length of life in kilowatt hours of some type of electronic Neon tube and Helium tube made by two manufacturers were as follows.

Tube	Length of life													
Neon	96	238	24	200	7	108	76	140	39	165	61	25	41	99
Helium	11	125	47	20	34	101	25	68	17	59	178	30	83	75

Compare by using Median test at 5% level of significance, the median lives of the electronic tubes made by manufacturers Neon and Helium.

Ans: (2.14, insig)

9. The same C programming papers were marked by two teachers A and B. The final marks were recorded as follows:

Teacher A	73	89	82	43	80	73	66	45	93	36	77	60
Teacher B	88	78	91	48	85	74	77	31	78	62	76	71

Using median test at 5% level of significance to determine if the marks distributions of two teachers differ significantly.

Ans: (0.66, insig)

10. A quality controller wishes to determine whether there is a difference in outcome between two different tools of software I and II. The following data shows the outcome of two different tools. Can the controller conclude that a difference exists? Use median test at 5% level of significance.

Software I	24.0	16.7	22.8	19.8	18.9	
Software II	23.2	19.8	18.1	17.6	20.2	17.8

Ans: ($p=0.6$, insig)

11. Life in years of two types of electric motor used in irrigation of farms are given below:

Motor M	3	6	7	4	3	5	5	3	4
Motor N	4	3	5	6	3	2	4	7	6

Is there any significant difference between two types of motor? Use Kolmogorov Smirnov test at 0.05 level of significance.

Ans: $D_0 = 0.15$ insig

12. Amount of time required to design website by software developers A and B are found as follows:

Time (hrs)	0 - 4	4 - 8	8 - 12	12 - 16	16 - 20
Number of website designed by A	2	7	12	5	4
Number of websites designed by B	6	9	8	4	3

Does A take more time than B to design website? Use Kolmogorov Smirnov test at 5% level of significance.

Ans: $D_0 = 0.1$ insig

13. Two independent samples of 26 junior programming and 25 senior programming smokers selected from a software company used to smoke following number of cigarettes per day

Number of cigarettes	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14
Number of junior programmer	7	6	4	2	2	3	2
Number of senior programmer	5	4	6	3	4	2	1

Using Kolmogorov Smirnov test identify if there is any significance between junior and senior programmer? Use 0.05 level of significance.

$$\text{Ans: } D_0 = 0.14, \text{ insig.}$$

14. Two groups of date managers, one group consisting of trained ones, another group not trained have the following number of correction required.

Trained	78	64	75	45	82
Untrained	110	70	53	51	

Use Mann Whitney U test to test if there is a difference between the two average number of correction of trained and untrained data manager.

$$\text{Ans: } U_0 = 9, \text{ insig.}$$

15. The nicotine contents of two brands of cigarettes, measured in milligrams was found to be as follows:

Brand A	2.1	4.0	6.3	5.4	4.8	3.7	6.1	3.3		
Brand B	4.1	0.6	3.1	2.5	4.0	6.2	1.6	2.2	1.9	5.4

If there any significance difference between two brands of cigarettes. Use Mann-whitney U-test.

$$\text{Ans: } p=0.0729, \text{ insig.}$$

16. A farmer wishes to determine whether there is a difference in yields between two different varieties of wheat I and II. The following data shows the production of wheat per unit area using the two varieties. Can the farmer conclude at significance level 0.01 that a difference exists?

Wheat I	15.9	15.3	16.4	14.9	15.3	16.0	14.6	15.3	14.5	16.6	16.0
Wheat II	16.4	16.8	17.1	16.9	18.0	15.6	18.1	17.2	15.4		

Use Mann Whitney U test.

$$\text{Ans: } z=-2.89, \text{ sig.}$$

17. Two independent random samples of unemployed men and women are drawn and the ages of 4 unemployed women and 5 unemployed men are recorded as follows.

Women	60	63	36	44	
Men	53	39	22	23	24

Do the data present sufficient evidence to conclude that there is a difference in the average age of unemployed men and women? Use Mann Whitney U test at $\alpha = 0.05$. ($p = 0.055$, insig.)

18. The following are the number of minutes it took a sample of 13 men and 12 women to complete the application form for a position:
- | | | | | | | | | | | | | | |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| Men | 16.5 | 20.0 | 17.0 | 19.8 | 18.5 | 19.2 | 19.0 | 18.2 | 20.8 | 18.7 | 16.7 | 18.1 | 17.8 |
| Women | 18.6 | 17.8 | 18.3 | 16.6 | 20.5 | 16.3 | 19.3 | 18.4 | 19.7 | 18.8 | 19.9 | 17.6 | 17.5 |

Use Mann Whitney U test at the 0.05 level of significance to test the hypothesis that the two samples come from identical population against the alternative that two populations are not identical.

Ans: $z = -0.108$, insig

19. A die was rolled for 60 times and observed the following outcomes:

Side	1	2	3	4	5	6	Total
Number of times observed	8	9	13	7	15	8	60

Is the die fair? Test the hypothesis at 5% level of significance.

Ans: (3.2, insig)

20. A certain chemical plant processes sea water to collect sodium chloride and magnesium. From scientific analysis, sea water is known to contain sodium chloride, magnesium and other elements in the ratio of 62:4:34. A sample of 200 tons of sea water resulted in 130 tons of sodium chloride and 6 tons of magnesium. Are these data consistent with the scientific model at 5% level of significance?

Ans: (1.025, insig)

21. A group of 150 college students were asked to indicate their most liked mobile brand among six different well known film actors viz., A, B, C, D, E and F in order to ascertain their relative popularity. The observed frequency data were as follows:

Brand	A	B	C	D	E	F	Total
Frequencies	24	20	32	25	28	21	150

Test at 5% level of significance whether all brands are equally popular.

Ans: (4, insig)

22. Genetic theory states that children having one parent of blood type A and other of blood type B will always be one of three types; A, AB, B and the proportion of three types will on an average be as 1:2:1. A report states that out of 300 children having one A parent and other B parent, 30% were found to be type A, 45% type AB and remainder type B. Test the hypothesis at 5% level of significance.

Ans: (4.5, insig)

23. A publishing house got a 500 page book composed for printing. Before final printing the first draft was sent for proof reading. The proof reader detected the number of misprints on each pages tabulated as follows. Test whether poisson distribution fits the data.

No. of misprint page	0	1	2	3	4	5
No. of pages	221	167	70	30	7	5

Ans: (13.32, sig)

24. In 50 random sample of a manufactured mice, the number of samples containing defective mice is noted below:

No. of defective mice	0	1	2	3	4	5
Frequencies	4	13	17	12	3	1

Can the binomial distribution be a good with $p = 0.30$?

Ans: (11.11, sig)

25. The number of telephone calls received per day over a period of 100 days is shown in the table below.

Number of calls	0	1	2	3
Number of days	45	25	20	10

Carry out chi square test to test the hypothesis that the number of calls received per day has a Poisson distribution at 5% level of significance. Ans: (8.01, sig.)

26. The distribution of persons according to sex and blood groups are given below:

Sex	Blood group			
	O	A	B	AB
Male	100	40	45	10
Female	110	35	55	5

Is there any association between sex and blood group? Ans: (3.607, insig.)

27. A random sample of 200 married men, all retired was classified according to education and number of children.

Education	Number of children		
	0 - 1	2 - 3	Over 3
Elementary	14	37	32
Secondary	19	42	17
College	12	17	10

Test the hypothesis at the 0.05 level of significance, that the number of children is independent of the level of education attained by the father. Ans: (8.04, insig.)

28. Test whether the color of son's eyes is associated with that of the father's at 5% level of significance using the data available in the following table.

Father's eye color	Son's eye color	
	Not light	Light
Not light	230	148
Light	151	471

Ans: (133.32 sig.)

29. 88 workers of a IT company were interviewed during a sample survey for their smoking habit. Classification of respondents according to their gender and their smoking habit are found as

Habit	Sex	
	Male	Female
Smoker	40	33
Non-smoker	3	12

Do the smoking tea habit is associated with gender. Ans: 4.71, sig.

30. Out of a sample of 120 persons in a village, 76 were administered a new drug for preventing influenza and out of them 24 persons were attacked by influenza. Out of those who were not administered the new drug, 12 persons were not affected by influenza. Is the new drug effective in controlling influenza, test at 5% level of significance. Ans: 19.54, sig.

31. The following table gives the result of flower color and type of leaf.

Flower color	Type of leaf	
	Flat	Curled
Pink	3	22
Red	9	11

Test whether the flower color is independent of flatness of leaf or not.

Ans: 4.61, sig.

Paired Sample Test

Wilcoxon Matched Pair Signed Rank Test

It is non parametric test used to compare two populations for which observations are paired. It is based upon magnitude and direction of difference between observations within each pair of related random samples.

Let us consider two related samples of size n drawn from continuous populations with unknown medians Md_1 and Md_2 respectively.

Let $x_1, x_2, x_3, \dots, x_n$ and $y_1, y_2, y_3, \dots, y_n$ be two related samples of size n .

Different steps in the test are

Problem to test

$$H_0: Md_1 = Md_2$$

$H_1: Md_1 \neq Md_2$ (two tailed) or $H_1: Md_1 > Md_2$ (one tailed right) or $H_1: Md_1 < Md_2$ (one tailed left)

Find $d_i = y_i - x_i$ or $x_i - y_i$ for each pair of observations (x_i, y_i) , $i = 1, 2, 3, \dots, n$. Rank d_i irrespective of sign in ascending order but omit $d_i = 0$. If two or more d_i are equal then assign the average rank and is called tied. In such case corrected sample size $n_c = n - t$, t is number of tied occurred. Assign sign to the ranks with respect to the sign of d_i . Sum the ranks of + sign and - sign separately to get $S(+)$ and $S(-)$ respectively. Finally get $T = \min \{S(+), S(-)\}$.

Small sample size ($n \leq 25$)

Test statistic

$$T = \min \{S(+), S(-)\}$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

At α level of significance, we obtain critical value from Wilcoxon Matched pair signed rank test table

$T_{\alpha, n}$ where n is corrected sample size after omitting $d_i = 0$.

Decision

Reject H_0 at α level of significance if $T \leq T_{\alpha, n}$ accept otherwise.

Large sample size ($n > 25$)

For large sample size sampling distribution of T is approximately normally distributed with mean μ_T and variance σ_T^2 .

$$\mu_T = \frac{n(n+1)}{4} \text{ and } \sigma_T^2 = \frac{n(n+1)(2n+1)}{24} \text{ where } n \text{ is corrected sample size if } d_i = 0.$$

Test statistic

$$Z = \frac{T - \mu_T}{\sigma_T} = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \sim N(0, 1) \quad (\text{Here } n = n_c)$$

Level of Significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical Value

Critical value $Z_{\text{tabulated}}$ is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $Z > Z_{\text{tabulated}}$. Accept otherwise.

Example 1: The number of server manufactured from each of two companies A and B was recorded daily for a period of 10 days with the following results;

Days	1	2	3	4	5	6	7	8	9	10
A	172	165	206	184	174	142	190	169	161	200
B	201	179	159	192	177	170	182	179	169	210

Assuming both companies produced same daily output, test the hypothesis that there is no difference between median defectives for two companies against the alternative hypothesis that company B produced more defective than company A using Wilcoxon Matched pairs signed rank test.

Solution:

Days	Company A(x_i)	Company B(y_i)	$d_i = y_i - x_i$	R_i
1	172	201	29	9
2	165	179	14	7
3	206	159	-47	-10
4	184	192	8	3
5	174	177	3	1
6	142	170	28	8
7	190	182	-8	-3
8	169	179	10	5.5
9	161	169	8	3
10	200	210	10	5.5

Here, $n = 10$, Sum of ranks of + sign [$S(+)$] = $9+7+3+1+8+5.5+3+5.5 = 42$, Sum of ranks of - sign [$S(-)$] = $10+3=13$

Let Md_A and Md_B be median defective of company A and B respectively.

Problem to test

H_0 : There is no significant difference in median defect of company A and company B ($Md_A = Md_B$)

H_1 : There is significant increase in median defective in company B than company A ($Md_A < Md_B$)

Test statistic

$$T = \min\{S(+), S(-)\}$$

$$= \min\{42, 13\} = 13$$

Critical value

Let $\alpha = 5\%$ be the level of significance the critical value is $T_{0.05(10)} = 8$

Decision

$$T = 13 > T_{0.05(10)} = 8$$

Accept H_0 at 5% level of significance.

Conclusion

There is no difference between median defectives for two companies A and B.

Example 2: The scores under two conditions X and Y obtained by the respondents are given below:

X	12	16	8	6	4	8
Y	7	12	17	5	12	11

Use Wilcoxon Matched pair signed rank test to test the difference between scores under condition X and Y.

Solution:

X	Y	$d = Y - X$	R
12	7	-5	-4
16	12	-4	-3
8	17	9	6
6	5	-1	-1
4	12	8	5
8	11	3	2

Here, $n = 6$, Sum of ranks of + sign [$S(+)$] = $6+5+2 = 13$, Sum of ranks of - sign [$S(-)$] = $4+3+1 = 8$

Problem to test

H_0 : Distribution of X = Distribution of Y

H_1 : Distribution of X \neq Distribution of Y

Test statistic

$$T = \min\{S(+), S(-)\} = \min\{13, 8\} = 8$$

Critical value

Let $\alpha = 5\%$ be the level of significance then critical value is $T_{0.05(6)} = 0$

Decision

$$T = 8 > T_{0.05(6)} = 0$$

Accept H_0 at 5% level of significance.

Conclusion

There is no difference between scores under condition X and Y.

Example 3: The following data gives the additional hours of sleep gained by 31 patients in an experiment to test the effect of a drug. Do these data give evidence that the drug produces additional hours of sleep?

Sleep hours gained by patients are: 0.5, 0.7, 0.1, -0.2, 1.2, 1.5, -2, 4, 0.1, 3.4, 3.7, 1.1, 0.8, -0.8, 1.3, 2.7, -3.4, -1.9, 3.4, 0.1, 0.6, 2.3, 0.1, 2.7, -0.9, 3.1, 2.0, 1.2, 1.2, 1.8, 1.0

Solution:

Additional hours sleep (d_i)	Rank
0.5	6
0.7	8
0.1	2.5
-0.2	-5
1.2	15
1.5	18
-2	-21.5
4	31
0.1	2.5
3.4	28
3.7	30
-1.1	-13
0.8	9.5
-0.8	-9.5
1.3	17
2.7	24.5
-3.4	-28
-1.9	-20
3.4	28
0.1	2.5
0.6	7
2.3	23
0.1	2.5
2.7	24.5
-0.9	-11
3.1	26
2.0	21.5
1.2	15
1.2	15
-1.8	-19
1.0	12

Here, $n = 31$

$$\text{Sum of - sign ranks } [S(-)] = 5 + 21.5 + 13 + 9.5 + 28 + 20 + 11 + 19 = 127$$

$$\text{Sum of + sign ranks } [S(+)] = S_n - S(-) = \frac{n(n+1)}{2} - S(-) = \frac{31(1+31)}{2} - 127 = 369$$

$$T^* = \min [S(+), S(-)] = \min \{ 369, 127 \} = 127$$

Problem to test

H_0 : drug does not produce additional hours of sleep

H_1 : drug produces additional hours of sleep

Test statistic

$T = 127$

$$\mu_T = \frac{n(n+1)}{4} = \frac{31(31+1)}{4} = 248$$

$$\sigma_T^2 = \frac{n(n+1)(2n+1)}{24} = \frac{31(31+1)(62+1)}{24} = 2604$$

$$\Rightarrow \sigma_T = 51.029$$

$$\text{Hence, } Z = \frac{T - \mu_T}{\sigma_T} = \frac{127 - 248}{51.029} = -2.37$$

Critical value

Let $\alpha = 5\%$ be the level of significance then critical value is $Z_{0.05} = 1.96$

Decision

$|Z| = 2.37 > Z_{0.05} = 1.96$, reject H_0 at 5% level of significance.

Conclusion

Drug produces additional hours of sleep to the patients.

Cochran Q Test

It is non parametric test used for more than two related samples. It is used to test significant difference in frequencies or proportions of three or more related samples.

Let us consider k treatments ($k > 2$) applied to the set of n objects dichotomized as Yes(Y) or No(N), Presence(P) or Absence(A), Pass(P) or Fail(F), Accept(A) or Reject (R), Increase (I) or Decrease (D) etc.

Treatments	Objects				
	1	2	3	J	N
T_1	Y	N	Y	Y	N
T_2	N	Y	N	Y	Y
T_3	Y	N	Y	N	N
.					
.					
.					
T_l	N	Y	N	Y	Y
T_k	Y	N	Y	N	Y

Different steps in the test are

Problem to test

H_0 : All the treatments are equally effective

H_1 : All the treatments are not equally effective.

Sum all the Y_i (Positive) according to treatment to get R_i (Row wise) and according to objects to get C_j (Column wise), $i = 1, 2, 3, \dots, k$ and $j = 1, 2, 3, \dots, n$. Then get

$$\sum_{i=1}^k R_i^2 - \sum_{j=1}^n C_j^2$$

Test statistic

$$Q = \frac{(k-1)[\sum R_i^2 - (\sum R_i)^2]}{K^2 C_j^2 - \sum C_j^2} \sim \chi^2$$

level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

table pg:296

$\chi^2_{\alpha/2}$ is the critical value obtained according to level of significance and degree of freedom, decision

Reject H_0 at α % level of significance if $Q > \chi^2_{\alpha/2}$, accept otherwise.

Example 4: Five laptop users were asked for the acceptability of four brands for daily use. The response of acceptability (A) and rejection (R) are given below;

Users	Brands			
	Alfa	Beta	Gamma	Delta
H_1	A	R	A	R
H_2	R	A	A	R
H_3	R	A	R	A
H_4	A	R	R	R
H_5	A	A	R	A

Test whether there is any significant difference between brands with respect to acceptability
Solution:

Upstick Brands	Users					R_i	R_i^2
	H_1	H_2	H_3	H_4	H_5		
Alfa	A	R	R	A	A	3	9
Beta	R	A	A	R	A	3	9
Gamma	A	A	R	R	R	2	4
Delta	R	R	A	R	A	2	4
C_j	2	2	2	1	3	$\sum R_i = \sum C_j = 10$	$\sum R_i^2 = 26$
Q	4	4	4	1	9	$\sum C_j^2 = 22$	

Here,

Number of brands (k) = 4, Number of housewives (n) = 5

Problem to test

H_0 : There is no significant difference between brands
 H_1 : There is at least one significant difference between brands.

Test statistic

$$Q = \frac{(k-1) \left\{ k \sum_{i=1}^k R_i^2 - \left(\sum_{i=1}^k R_i \right)^2 \right\}}{k \sum_{j=1}^n C_j - \sum_{j=1}^n C_j^2} = \frac{(4-1) [4 \times 26 - (10)^2]}{4 \times 10 - 22} = \frac{12}{18} = 0.666$$

Critical value

Let 5% be the level of significance then critical value is $\chi^2_{0.05(3)} = 7.81$.

Decision

$Q = 0.666 < \chi^2_{0.05(3)} = 7.81$, accept H_0 at 5% level of significance.

Conclusion:

There is no significant difference between brands according to acceptability.

Kruskal Wallis H Test

It is also called Kruskal Wallis one way ANOVA test

It is test used to test the significant difference between location of three or more independent populations.

Let us consider k independent samples of size n_i such that $\sum n_i = n$ drawn from continuous population with unknown medians Md_1, Md_2, \dots, Md_k respectively.

Let $x_{11}, x_{12}, x_{13}, \dots, x_{1n_1}$ be samples of size n_1 . $x_{21}, x_{22}, x_{23}, \dots, x_{2n_2}$ be sample of size n_2 . $x_{31}, x_{32}, x_{33}, \dots, x_{3n_3}$ be sample of size n_3 $x_{k1}, x_{k2}, x_{k3}, \dots, x_{kn_k}$ be sample of size n_k respectively.

Samples	Blocks				n
	1	2	3	j	
1	x_{11}	x_{12}	x_{13}	x_{1j}	$x_1 n_1$
2	x_{21}	x_{22}	x_{23}	x_{2j}	$x_2 n_2$
3	x_{31}	x_{32}	x_{33}	x_{3j}	$x_3 n_3$
.					
i	x_{i1}	x_{i2}	x_{i3}	x_{ij}	$x_i n_i$
.					
k	x_{k1}	x_{k2}	x_{k3}	x_{kj}	$x_k n_k$

Different steps in the test are

Problem to test

$$\therefore M_1 = M_2 = M_3 = \dots, M_k$$

At least one M_{di} is different $i = 1, 2, 3, \dots, k$

Combine n_1, n_2, n_3, \dots and n_k such that $n_1 + n_2 + n_3 + \dots + n_k = n$ and rank these n observations in ascending order. If two or more observations are equal then assign average rank and is called tied. Sum the ranks of sample of sizes n_1, n_2, n_3, \dots and n_k separately to get R_1, R_2, \dots, R_k .

χ^2 statistic

$$H_0: \frac{12}{n(n+1)} \sum \frac{n_i^2}{n_i} - 3(n+1) \sim \chi^2(k-1)$$

occurs then corrected test statistic is

$$H = \frac{\frac{12}{n(n+1)} \sum \frac{R_i^2}{n_i} - 3(n+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n^3 - n}}, \quad t_i = \text{number of times } i^{\text{th}} \text{ rank is repeated}$$

Level of significance

Let α be the level of significance .Generally fix $\alpha = 0.05$ unless we are given

Critical value

For $n_i < 5$ and $k = 3$, critical value p is obtained from Kruskal Wallis table.

For $n > 5$ and $k > 3$, critical value is $\chi^2_{\alpha(k-1)}$

Decision

Accept H_0 at α level of significance if $p > \alpha$, reject otherwise for $n \leq 5$ and $k=3$.

Reject H_0 at α level of significance if $H > \chi^2_{\alpha} (k-1)$, accept otherwise for $n_i > 5$ and $k > 3$.

Example 5: A bacteriologist was interested to study the number of plankton organisms inhabiting the lake water. He made hauls of water from three lakes each and the following results were obtained.

Lake	Number of plankton organism				
Phewa	12	19	16		
Rara	4	8	3	2	3
Taudaha	14	12	20	12	

Do the data provide substantial evidence to conclude significant variation between lake water?

Use Kruskal-Wallis test at 0.05 level of significance

Solutions

Sample size of Phewa (n_1) = 3

Sample size of Rara (n_2) = 5

Sample size of Taudha (n_3) = 4

Total sample size (n) = $n_1 + n_2 + n_3 = 3 + 5 + 4 = 12$

No of times rank 2.5 is repeated (t_1) = 2

No of times rank 7 is repeated (t_2) = 3

$$\sum (t_i^3 - t_i) = (2^3 - 2) + (3^3 - 3) = 6 + 24 = 30$$

R_1 , R_2 , R_3 be sum of ranks for Phewa, Rara and Taudaha respectively.

Let Md_1 , Md_2 and Md_3 be median of Phewa, Rara and Taudaha lake respectively.

Problem to test

H_0 : There is no significant variation between lake water. ($Md_1 = Md_2 = Md_3$)

H_1 : There is at least one significant variation between lake water. (At least one Mdi is different, $i = 1, 2, 3$)

Test statistic

$$H = \frac{\frac{12}{n(n+1)} \sum \frac{R_i^2}{n_i} - 3(n+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n^3 - n}} = \frac{\frac{12}{12(12+1)} \times 612.5 - 3(12+1)}{1 - \frac{30}{12^3 - 12}} = \frac{8.12}{0.98} = 8.285$$

Level of significance

$\alpha = 0.05$ is the level of significance.

Critical value

From Kruskal Wallis table critical value is $p = 0.01$

Decision

$P = 0.01 < \alpha = 0.05$, reject H_0 at 0.05 level of significance.

Conclusion

There is at least one significant variation between lake water.

Example 6: The following are the numbers of misprints counted on pages selected at random from three editions of a book

Edition I	4	10	2	6	4	12
Edition II	8	5	13	8	8	10
Edition III	7	9	11	2	14	7

Use Kruskal Wallis H test at the 0.05 level of significance to test the null hypothesis that the samples come from identical populations.

Solution:

Date	No. of misprints						R_i	R_i^2/n_i
April 11	4	10	2	6	4	12		
Rank	3.5	13.5	1.5	6	3.5	16	44	322.66
April 18	8	5	13	8	8	10		
Rank	10	5	17	10	10	13.5	65.5	715.041
April 25	7	9	11	2	14	7		
Rank	7.5	12	15	1.5	18	7.5	61.5	630.375
Total								1668.076

Sample size of April 11 (n_1) = 6Sample size of April 18 (n_2) = 6Sample size of April 25 (n_3) = 6Total sample size (n) $n = n_1 + n_2 + n_3 = 6 + 6 + 6 = 18$ No of times rank 1.5 is repeated (t_1) = 2, No of times rank 3.5 is repeated (t_2) = 2No of times rank 7.5 is repeated (t_3) = 2, No of times rank 10 is repeated (t_4) = 3No of times rank 13.5 is repeated (t_5) = 2

$$\sum(t_i^3 - t_i) = (2^3 - 2) + (2^3 - 2) + (2^3 - 2) + (3^3 - 3) + (2^3 - 2) = 48$$

 R_1, R_2, R_3 be sum of ranks for April 11, April 18 and April 25 respectively.Let Md_1, Md_2 and Md_3 be median misprint of April 11, April 18 and April 25 respectively.**Problem to test** H_0 : Populations are identical. ($Md_1 = Md_2 = Md_3$) H_1 : Populations are not identical. (At least one Md_i is different, $i = 1, 2, 3$)**Test statistic**

$$H = \frac{\frac{12}{n(n+1)} \sum \frac{R_i^2}{n_i} - 3(n+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n^3 - n}}$$

$$= \frac{\frac{12}{18(18+1)} \times 1668.076 - 3(18+1)}{1 - \frac{48}{18^3 - 18}}$$

$$\approx \frac{58.528 - 57}{0.9917} = 1.5407$$

Critical valueCritical value at 0.05 level of significance for 2 degree of freedom is $\chi^2_{0.05(2)} = 5.99$.**Decision** $H = 1.54 < \chi^2_{0.05(2)} = 5.99$, accept H_0 at 0.05 level of significance.**Conclusion**

The samples come from identical population.

Friedman F Test

It is also called Friedman two way ANOVA test.

It is test used to test the significant difference between location of three or more independent populations.

Let us consider k independent samples of size n each.

Let x_{ij} be the observations classified into k rows (treatments) and n columns(blocks) such that there are $N = n \times k$ observations.

Samples	Blocks				n
	1	2	3	j	
1	x_{11}	x_{12}	x_{13}	x_{1j}	x_{1n}
2	x_{21}	x_{22}	x_{23}	x_{2j}	x_{2n}
3	x_{31}	x_{32}	x_{33}	x_{3j}	x_{3n}
.					
i	x_{i1}	x_{i2}	x_{i3}	x_{ij}	x_{in}
.					
k	x_{k1}	x_{k2}	x_{k3}	x_{kj}	x_{kn}

Different steps in the test are

Problem to test

$$H_0: M_{d1} = M_{d2} = M_{d3} = \dots, M_{dk}$$

$$H_1: \text{At least one } M_{di} \text{ is different } i = 1, 2, 3, \dots, k.$$

Rank k sample observations for each block separately from 1 to k , in ascending order. If two or more observations are same then assign average rank which is also called tied. Obtain sum of ranks for each sample to get R_i , $i = 1, 2, 3, \dots, k$

Test statistic

$$F_r = \frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)$$

If tied occurs then corrected test statistic is

$$F_r = \frac{\frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)}{1 - \frac{\sum (t_i^3 - t_i)}{n(k^3 - k)}} \quad t_i = \text{number of times } i\text{th rank is repeated.}$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

For $2 \leq n \leq 9$ and $k = 3$, also $2 \leq n \leq 4$ and $k = 4$ critical value p is obtained from Friedman probability table.

For $n \geq 5$ and $k > 3$, critical value is $\chi^2_{\alpha(k-1)}$

Decision

Accept H_0 at α level of significance if $p > \alpha$, reject otherwise for $2 \leq n \leq 9$ and $k = 3$, also $2 \leq n \leq 4$ and $k = 4$.

Reject H_0 at α level of significance if $H > \chi^2_{\alpha(k-1)}$, accept otherwise for $n \geq 5$ and $k > 3$.

Example 7: A survey was conducted in four hospitals in a particular city to obtain the number of babies born over a 12 months period. This time period was divided into four seasons to test the hypothesis that the birth rate is constant over all the four seasons. The results of the survey were as follows:

Hospital	No of births			
	Winter	Spring	Summer	Fall
A	92	72	94	77
B	15	16	10	17
C	58	71	51	62
D	19	26	20	18

Analyze the data using Friedman two way ANOVA test.

Solution:

Hospital	No of births in Hospital								R_i	R_i^2
	A	Rank	B	Rank	C	Rank	D	Rank		
Winter	92	3	15	2	58	2	19	2	9	81
Spring	72	1	16	3	71	4	26	4	12	144
Summer	94	4	10	1	51	1	20	3	9	81
Fall	77	2	17	4	62	3	18	1	10	100

$$\Sigma R_i^2 = 406$$

Here,

$$\text{Number of hospitals (n)} = 4$$

$$\text{Number of seasons (k)} = 4$$

Let Md_1, Md_2, Md_3, Md_4 be median of winter, spring, summer and fall seasons respectively.

Problem to test

H_0 : The birth rate is constant over all four seasons. ($Md_1 = Md_2 = Md_3 = Md_4$)

H_1 : The birth rate is not constant over all four seasons (At least one Md_i is different, $i = 1, 2, 3, 4$)

Test statistic

$$F_r = \frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)$$

$$= \frac{12 \times 406}{4 \times 4 \times 5} - 3 \times 4 \times 5 = 0.9$$

Level of significance

Let $\alpha = 5\%$ be the level of significance.

Critical value

The critical value is $p = P(F_r > 0.9) = 0.9$ for $n = 4$ and $k = 4$

Decision

$P = 0.9 > \alpha = 0.05$, accept H_0 at 5% level of significance.

Conclusion: Birth rate is constant over all four seasons.

Example 8: An investigator wants to study the scores of 3 matched groups under 5 conditions, each group contains five subjects, one being assigned to each of the five conditions. Let the score obtained are given in the following table. Is there any significant difference between three groups.

Group	Conditions				
	I	II	III	IV	V
A	9	8	5	1	7
B	6	7	5	2	8
C	9	7	5	2	6

Solution:

Ranking the different conditions separately

Group	Conditions					R_i	R^2
	I	II	III	IV	V		
A	2.5	3	2	1	2	10.5	110.25
B	1	1.5	2	2.5	3	10	100
C	2.5	1.5	2	2.5	1	9.5	90.25
					300.5		

Here,

Number of conditions (n) = 5

Number of groups (k) = 3

Number of times rank 2.5 is repeated in I (t_1) = 2

Number of times rank 1.5 is repeated in II (t_2) = 2

Number of times rank 2 is repeated in III (t_3) = 3

Number of times rank 2.5 is repeated in IV (t_4) = 2

Let Md_A , Md_B , Md_C be median of group A, B and C respectively.

Problem to test

H_0 : Scores of 3 matched groups is same ($Md_A = Md_B = Md_C$)

H_1 : Scores of 3 matched groups is different (At least one Md_i is different, $i = A, B, C$)

Test statistic

$$F_r = \frac{\frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)}{1 - \frac{\sum (t_i^3 - t_i)}{n(k^3 - k)}} = \frac{\frac{12 \times 300.5}{5 \times 3 \times 4} - 3 \times 5 \times 4}{1 - \frac{[(2^3 - 2) + (2^3 - 2) + (3^3 - 3) + (2^3 - 2)]}{5(3^3 - 3)}} = \frac{60.1 - 60}{1 - 0.35} = 0.153$$

Level of significance

Let $\alpha = 5\%$ be the level of significance

Critical value

Critical value is $p = P(F_r > 0.153) = 0.4$ for $n=5$ and $k=3$

Decision

$p = 0.4 > \alpha = 0.05$, accept H_0 at 5% level of significance.

Conclusion

There is no significant difference between scores of three matched group.

Example 9: Three different advertising media T.V., Radio and News paper are being compared to study their effectiveness in promoting sales of WaiWai noodles. Each advertising media is exposed for specified period of time and sales (000 package) from 10 stores located at different areas are recorded.

Advertising Media	Stores									
	A	B	C	D	E	F	G	H	I	J
T.V.	20	21	15	12	14	17	21	16	20	18
Radio	7	9	11	12	10	10	14	12	8	7
News Paper	8	6	11	12	9	6	8	10	8	6

Are three advertising media equally effective, use Friedman two way ANOVA test.

Solution:

Ranking the sales from different stores separately

Advertising Media	Stores										R_i	R_i^2
	A	B	C	D	E	F	G	H	I	J		
T.V.	3	3	3	2	3	3	3	3	3	3	29	841
Radio	1	2	1.5	2	2	2	2	2	1.5	2	18	324
News Paper	2	1	1.5	2	1	1	1	1	1.5	1	13	169
										$\Sigma R_i^2 = 1334$		

Here,

Number of stores (n) = 10

Number of advertising media (k) = 3

Number of times rank 1.5 is repeated in store C (t_1) = 2

Number of times rank 2 is repeated in store D (t_2) = 3

Number of times rank 1.5 is repeated in store I (t_3) = 2

Let Md_A , Md_B , Md_C be medians of advertising media TV, Radio and Newspaper respectively.

Problem to test

H_0 : Three advertising media are equally effective ($Md_A = Md_B = Md_C$)
 H_1 : Three advertising media are not equally effective (At least one Md_i is different, $i = A, B, C$)

Test statistic

$$Fr = \frac{\frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n(k^3 - k)}}$$

$$= \frac{\frac{12 \times 1334}{10 \times 3 \times 4} - 3 \times 10 \times 4}{1 - \frac{[(2^3 - 2) + (3^3 - 3) + (2^3 - 2)]}{10(3^3 - 3)}} = \frac{133.4 - 120}{1 - 0.15} = 15.76$$

Critical value

Let $\alpha = 5\%$ be the level of significance then critical value is $\chi^2_{0.05(2)} = 5.99$

Decision

$F_r = 15.76 > \chi^2_{0.05(2)} = 5.99$, reject H_0 at 5% level of significance

Conclusion

Three advertising media T.V., Radio and News Paper are not equally effective in sales of WaiWai noodle.



EXERCISE

- What is wilcoxon matched pair signed rank test? Why is it superior to sign test?
- Discuss the rationale and method of Wilcoxon Matched pair signed rank test.
- Describe rationale and procedure of Cochran Q test.
- Discuss the function and procedure of Kruskal Wallis H test.
- What do you mean by Friedman two way ANOVA test? Describe process of the test.
- Differentiate between Kruskal Wallis One way ANOVA test and Friedman Two way ANOVA test.
- The weight (kg) of 5 people before they stopped smoking are as follows:

Before	66	80	69	52	75
After	71	82	68	56	73

Use wilcoxon Matched pairs signed rank test for paired observations to test the hypothesis at 0.05 level of significance that giving up smoking has no effect on a person's weight against the alternative hypothesis that one's weight increases if he or she quits smoking.

Ans: $T=3.5$, insig

- To evaluate a speed of reading course, a group of 10 subjects were asked to read two comparable articles one before the course and one after the course. Their scores on reading test are as follows;

Before course(X)	57	80	64	70	90	59	76	98	70	83
After course(Y)	60	90	62	70	95	58	80	99	75	94

Test whether the course is beneficial using the Wilcoxon Matched pairs signed rank test at 5% level of significance.

Ans: $T=4.5$, sig

Seven prospective graduate students took a test twice with the following scores.

First attempt	470	530	610	440	600	590	580
Second attempt	510	550	600	490	585	620	598

Test whether there is significant difference between scores in first attempt and second attempt using Wilcoxon Matched pair signed rank test.

Ans: $T=3$, insig.

The following are the number of artifacts dug up by two archaeologists at an ancient cliff dwelling on 30 days.

Arch. X	1	0	2	3	1	0	2	2	3	0	1	1	4	1	2
Arch. Y	0	0	1	0	2	0	0	1	1	2	0	1	2	1	1
Arch. X	1	3	5	2	1	3	2	4	1	3	2	0	2	4	1
Arch. Y	0	2	2	6	0	2	3	0	2	1	0	1	0	1	5

Do these data present sufficient evidence to support the research hypothesis that archaeologist X is better than Y. Use Wilcoxon Matched pairs signed rank test.

Ans: $z = 1.86$, sig.

Four objective questions are given to 5 students and the results of correct answer(1) and wrong answers(0) are arranged in the following table.

Objective Questions	Students				
	1	2	3	4	5
Q ₁	1	0	0	1	1
Q ₂	0	1	1	0	1
Q ₃	1	1	1	0	0
Q ₄	0	0	1	0	1

Apply Cochran q test for testing the hypothesis that there is no significant difference between four objective questions with respect to the correct answers.

Ans: $Q=0.529$, insig.

Three diets P, Q and R are fed to 9 buffaloes, each diet for one month and the result of increasing (I) and decreasing (D) of milk given by different buffaloes are given in the following table.

Diet	Buffaloes								
	1	2	3	4	5	6	7	8	9
P	I	D	I	D	I	I	D	D	I
Q	I	I	I	D	I	I	D	I	D
R	I	D	D	D	I	D	I	I	I

Test whether the three diets are equally effective or not using Cochran Q test at $\alpha = 5\%$.

Ans: $Q=0.33$, insig.

Following are the final examination marks of three group of students who were taught computer by three different methods;

Method I	94	88	91	74	87	97	
Method II	85	82	79	84	61	72	80
Method III	89	67	72	76	69		

Are all three methods equally effective? Use H test at 0.05 level of significance.

Ans: $H=6.67$, sig.

An agricultural experiment was conducted to compare the yield of wheat by using three types of chemical fertilizer nitrogen(N), phosphorous(P) and potash(K). Twelve plots of equal size were selected at random and divided into three groups of four each and planted wheat. Each group was randomly selected and the fertilizer was applied in the plots under the identical conditions. Then the yield of wheat recorded were given in the following table.

Chemical fertilizer		
N	P	K
122	81	80
80	80	82
138	79	65
121	65	58

Test whether the three types of fertilizers are equally effective or not. Use Kruskal Wallis test at 0.05 level of significance.

Ans: $P=0.008$, insig.

15. For the following scores of 3 groups, apply Kruskal Wallis H test to test the hypothesis that the three groups are not significantly different:

Group	Scores				
	96	128	83	61	101
A	96	128	83	61	101
B	82	124	132	135	109
C	115	149	166	147	

Ans: $p=0.01$, sig

16. An experiment designed to compare three preventive methods against corrosion yielded the following maximum depths of pits (in thousandths of an inch) in pieces of wire subjected to the respective treatments:

Method I	77	54	67	74	71	66	
Method II	60	41	59	65	62	64	
Method III	49	52	69	47	56		52

Use the 0.05 level of significance to test the hypothesis that the three samples come from identical population. Use Kruskal Wallis H test.

Ans: $H=6.66$, sig

17. The following data represent the operating times in hours for 3 types of scientific pocket calculators before a charge is required.

Calculator A	4.9	6.1	4.3	4.6	5.3		
Calculator B	5.5	5.4	6.2	5.8	5.5	5.2	
Calculator C	6.4	6.8	5.6	6.5	6.3	6.6	4.8

Use Kruskal Wallis test, at the 0.01 level of significance to test the hypothesis that the operating times for all three calculators are equal.

Ans: $H=10.4$, sig

18. A researcher wants to compare the teaching standard of three English medium schools on the basis of performance of the student's final examination scores. The percentage of passers in I to IV grade in the schools are presented in the following table.

	Grade			
	I	II	III	IV
Alpha	89	98	70	80
Sigma	45	76	40	55
Gamma	20	58	35	67

Test the performances of the schools with respect to pass percentage using Friedman's test.

Ans: $p=0.042$, sig

19. A survey was conducted in four Hospitals in a particular city to obtain the number of babies born over a 12 months period divided into four seasons to test the hypothesis that the birth rate is constant over all four seasons. The result of the survey are as follows:

Hospitals	Number of births			
	Winter	Spring	Summer	Fall
A	92	112	94	77
B	9	11	10	15
C	58	71	51	62
D	19	26	19	18

Analyze the data using Friedman's test.

Ans: $p = 0.928$, insig

20. The scores of 3 matched groups under the six conditions are given below

Group	Condition					
	I	II	III	IV	V	VI
A	9	5	2	5	6	7
B	6	4	3	4	6	5
C	5	1	3	3	6	5

Apply the Friedman two way ANOVA test to identify if there is significantly difference in variation between matched groups. Use 5% level of significance.

Ans: 0.184 , insig

Using Software for Non parametric test:

Run test

In 30 toss of a coin the following sequence of heads (H) and tails (T) is obtained.

H T T H T H H H T H H T T H T H T H T H T H T H T H T H T

Test at 0.05 level of significance level whether the sequence is random.

Using Excel

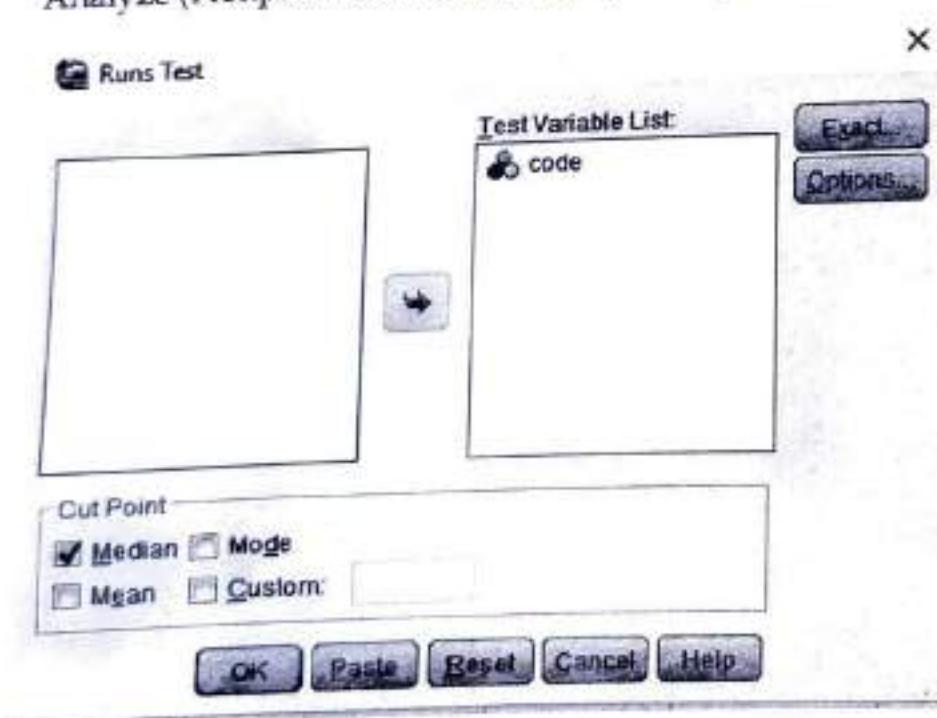
	A	B	C
1	outcome	code	runs
2	H	1	1
3	T	0	2
4	T	0	2
5	H	1	3
6	T	0	4
7	H	1	5
8	H	1	5
9	H	1	5
10	T	0	6
11	H	1	7
12	H	1	7
13	T	0	8
14	T	0	8
15	H	1	9
16	T	0	10
17	H	1	11
18	T	0	12
19	H	1	13
20	H	1	13
21	T	0	14
22	H	1	15
23	T	0	16
24	T	0	16
25	H	1	17
26	T	0	18
27	H	1	19
28	H	1	19
29	T	0	20
30	H	1	21
31	T	0	22

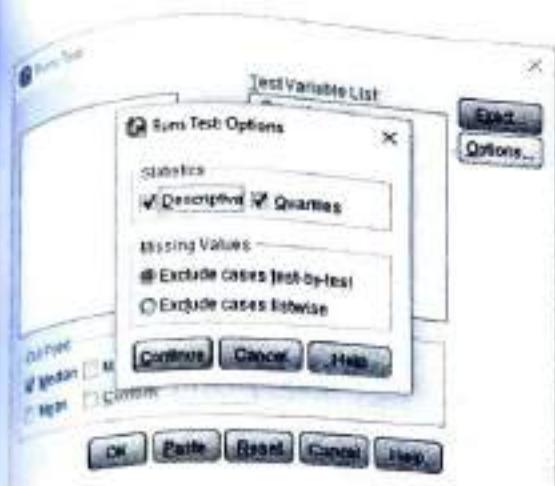
A	B	C	D
34 cases	symbol	value	formula
35 no of observation	n	30	=COUNT(B2:B31)
36 no of head	n1	16	=COUNTIF(B2:B31,1)
37 no of tail	n2	14	=COUNTIF(B2:B31,0)
38 no of runs	R	22	=MAX(C2:C31)
39 mean	E(R)	15.93333	=(2*C36*C37)/(C36+C37))+1
40 $\mu_r = \frac{2n_1 n_2}{n_1 + n_2} + 1$	$\sigma_r^2 = \frac{2n_1 n_2 (2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}$		
41			
42			
43 variance	V(R)	7.174866	=(2*C36*C37*(2*C36*C37-C36-C37))/(((C36+C37)^2)*(C36+C37-1))
44			
45 test hypothesis			H0: sample observation is random.
46 Null Hypothesis			
47 Alternative hypothesis			H1: sample observation is not random.
48			
49 test statistic			
50 z statistic	z	2.26487	=(C38-C39)/SQRT(C43)
51 level of significance	α	0.05	
52 tabulated z for two tailed test	$z_{\alpha/2}$	1.959964	=NORMSINV(1-C51/2)
53 p value	p	0.01176	=1-NORMSDIST(C50)

A	B	C	D
54			
55 Decision:			
56 Significant approach			=IF(ABS(C50)<C52,"There is no reason to reject Null Hypothesis H0","H0 is rejected")
57 H0 is rejected			
58 p-value approach			=IF(C51<C53,"It is insignificant","It is significant")
59 It is significant			

Using SPSS

Analyze\Nonparametric tests\Legacy Dialogs\Runs



**Outputs****NPar Tests**

activate

Descriptive Statistics								
	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles		
						25th	50th (Median)	75th
code	30	.53	.507	0	1	.00	1.00	1.00

Runs Test

code	
TestValue*	1
Cases < TestValue	14
Cases >= TestValue	16
Total Cases	30
Number of Runs	22
Z	2.078
Asymp. Sig. (2-tailed)	.038

a. Median

Using STATA**Run test using stata**

Recode the data such that the head = 1 and tail = 2 in variable n1code.

Then use syntax

Runtest n1code

Output will be as follows;

```
- runtest n1code
|| n1code <= 1) = 16
|| n1code > 1) = 14
      obs = 30
      N(runs) = 22
      z = 2.26
      Prob>|z| = .02
```

Binomial test

Test whether the coin is unbiased from following observations.

Tail	Tail	Head	Head	Tail	Head	Tail	Head	Head	Head	Tail
Head	Head	Head	Tail	Head	Head	Tail	Tail	Tail	Head	Head
Tail	Tail	Tail	Tail	Head	Tail	Head	Tail	Tail	Tail	Tail
Tail	Tail	Head	Tail	Tail	Tail	Head	Head	Tail	Tail	Tail
Tail	Head	Tail	Tail	Head	Tail	Head	Tail	Tail	Tail	Tail

Using SPSS

Analyze\Nonparametric tests\Legacy Dialogs\Binomial

The screenshot shows the SPSS interface with the "Binomial Test" dialog box open. In the background, the "Data View" window displays a list of 18 observations, each consisting of two columns: "obs outcomes" and "code". The "obs outcomes" column lists alternating "Tail" and "Head" values, while the "code" column lists alternating "Tail" and "Head" values. The "Binomial Test" dialog box has the following settings:

- Test Variable List:** Contains the variable "code".
- Test Proportion:** Set to 0.50.
- Define Dichotomy:**
 - Get from data
 - Cut point: [empty field]
- Buttons:** OK, Paste, Reset, Cancel, Help.



Outputs

Non Parametric Tests

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	25%	Percentiles 50th (Median)	75th
code	50	.40	.495	0	1	.00	.00	1.00

Binomial Test

	Category	N	Observed Prop	Test Prop.	Exact Sig. (2-tailed)
code	Group 1 0 Tail	30	.60	.50	.203
code	Group 2 1 Head	20	.40	.50	.203
	Total	50	1.00		

Since $p=0.203 > 0.05$, the null hypothesis of equality of proportion of head and tail can be accepted.

Using STATA

```
bitest code == 0.5
```

```
bitest code == 0.5
```

Variable	N	Observed k	Expected k	Assumed p	Observed p
code	50	20	25	0.50000	0.40000

$\Pr(k \geq 20) = 0.940540$ (one-sided test)

$\Pr(k \leq 20) = 0.101319$ (one-sided test)

$\Pr(k \leq 20 \text{ or } k \geq 30) = 0.202639$ (two-sided test)

Since $p=0.203 > 0.05$, the null hypothesis of equality of proportion of head and tail can be accepted.

Kolmogorov Smirnov one sample

The number of disease infected tomato plants in 10 different plots of equal size are given below. Test whether the disease infected plants are uniformly distributed over the entire area use Kolmogorov Smirnov test.

Plot no.	1	2	3	4	5	6	7	8	9	10
No. of infected plants	8	10	9	12	15	7	5	12	13	9

Using excel

A	B	C
4	Plot no.	No of infected plants
5	1	8
6	2	10
7	3	9
8	4	12
9	5	15
10	6	7
11	7	5
12	8	12
13	9	13
14	10	9
15		
16		

17 Problem to test:

18 H₀: The disease infected plants are uniformly distributed over the entire area.19 H₁: The disease infected plants are not uniformly distributed over the entire area.

A	B	C	D	E	F	G	H	
	No of infected plants		Observed relative freq (F _O)	Expected freq (F _E)	Expected relative freq(F _E)	F _O - F _E		
21	Plot no.	c (cf)		Expected cf (cef)		difference		
22	1	8	8	0.08	10	10	0.1	0.02
23	2	10	18	0.18	10	20	0.2	0.02
24	3	9	27	0.27	10	30	0.3	0.03
25	4	12	39	0.39	10	40	0.4	0.01
26	5	15	54	0.54	10	50	0.5	0.04
27	6	7	61	0.61	10	60	0.6	0.01
28	7	5	66	0.66	10	70	0.7	0.04
29	8	12	78	0.78	10	80	0.8	0.02
30	9	13	91	0.91	10	90	0.9	0.01
31	10	9	100	1.00	10	100	1	0
32								
33	Test statistic	value	formula					
34	D ₀ =	0.04	=MAX(H22:H31)					
35	n		=COUNT(B22:B31)					
36	B	0.05						
37	Critical value							
38	Let 5% be the level of significance then critical value is							
39	D _{0.05}	0.40925	from Ks test D table					

A	B	C	D	E
41	decision			
42				

43 Significant approach

44 There is no reason to reject Null Hypothesis H₀

use Kolmogorov Smirnov test.

Test for normal distribution

weight	19.5	20	26.9	27.1	28.1	30	31.6	32.7	34.4	37.2
	37.5	37.9	38	38.4	38.6	38.8	38.9	40.1	41.6	42.6
	42.9	45	45.2	45.5	46.5	46.8	47.3	48.1	48.3	48.4
	48.8	49	49.1	49.3	49.4	49.5	49.9	50.4	51.8	54.4
	54.9	55.3	55.6	57.3	57.4	57.5	58.7	58.8	59.7	59.9

using excel

A	B	C	D	E	F
weight	rank	expected	(rank-1)/n	actual	difference
19.5	1	0.02	0	0.007859	0.007859
20	2	0.04	0.02	0.008969	0.011031
26.9	3	0.06	0.04	0.044763	0.004763
27.1	4	0.08	0.06	0.046624	0.013376
28.1	5	0.1	0.08	0.056876	0.023124
30	6	0.12	0.1	0.081156	0.018844
31.6	7	0.14	0.12	0.10708	0.01292
32.7	8	0.16	0.14	0.128072	0.011928
34.4	9	0.18	0.16	0.165842	0.005842
37.2	10	0.2	0.18	0.242198	0.062198
37.5	11	0.22	0.2	0.251377	0.051377
37.9	12	0.24	0.22	0.263896	0.043896
38	13	0.26	0.24	0.267074	0.027074
38.4	14	0.28	0.26	0.279979	0.019979
38.6	15	0.3	0.28	0.286543	0.006543
38.8	16	0.32	0.3	0.293179	0.006821
38.9	17	0.34	0.32	0.296524	0.023476
40.1	18	0.36	0.34	0.337946	0.002054
41.5	19	0.38	0.36	0.392564	0.032564
42.6	20	0.4	0.38	0.430271	0.050271
42.9	21	0.42	0.4	0.441725	0.041725
45	22	0.44	0.42	0.522732	0.102732
45.2	23	0.46	0.44	0.53045	0.09045
45.5	24	0.48	0.46	0.542006	0.082006
46.5	25	0.5	0.48	0.580216	0.100216

[part of data is shown]

A B C D E F G

55 Problem to test:

H0: weights are normally distributed over the entire area.

H1: weights are not normally uniformly distributed over the entire area.

56

n 50

mean 44.412

st dev 10.3139

62

63 Test Statistics

value formula

D 0.102732 =MAX(F2:F51)

n 50 =COUNT(A2:A51)

a 0.05

64 Critical value

65 Let 5% be the level of significance then critical value is

70 D_(5%,5) 0.18845 From table

71

72 decision

73

74 Significant approach

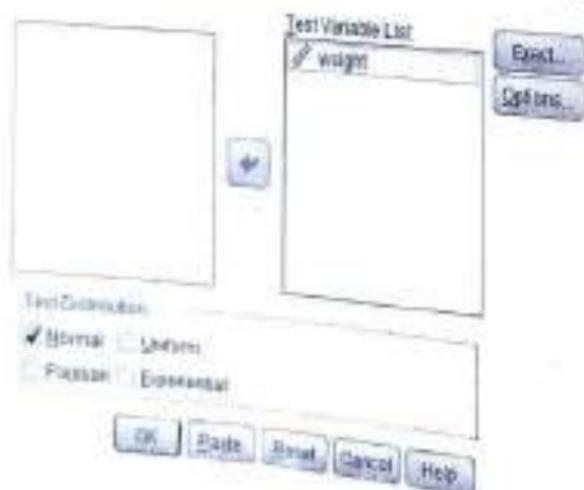
75 There is no reason to reject Null Hypothesis H0

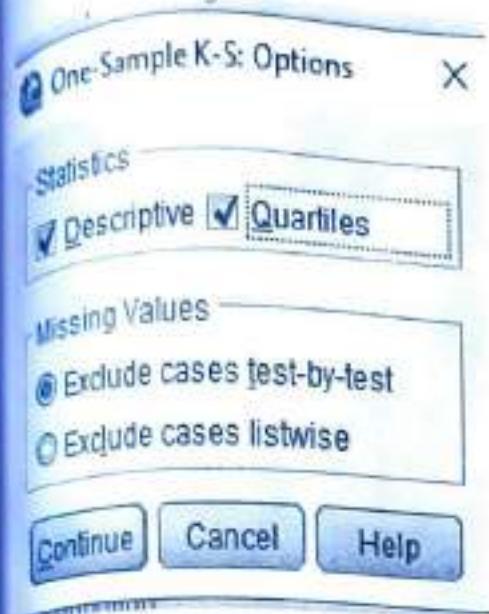
Using SPSS

Analyze\Nonparametric tests\Legacy Dialogs\1 sample K-S

One-sample Kolmogorov-Smirnov Test

X





NPar Tests

[Dataset1]

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles		
						25th	50th (Median)	75th
weight	50	44.4120	10.31390	19.50	59.90	37.9750	46.6500	50.7500

One-Sample Kolmogorov-Smirnov Test

weight		
n		50
Test Parameters ^{a,b}	Mean	44.4120
	Std. Deviation	10.31390
Most Extreme Differences	Absolute	.103
	Positive	.067
	Negative	-.103
Test Statistic		.103
Asymp. Sig. (2-tailed)		.200 ^{c,d}

^a Test distribution is Normal.^b Calculated from data.^c Ulefs Significance Correction.^d This is a lower bound of the true significance.

Mann-Whitney U test

Small sample

Test the hypothesis of no difference between the ages of male and female employees of a certain company, using the Mann-Whitney U test for the sample data below. Use $\alpha = 0.1$.

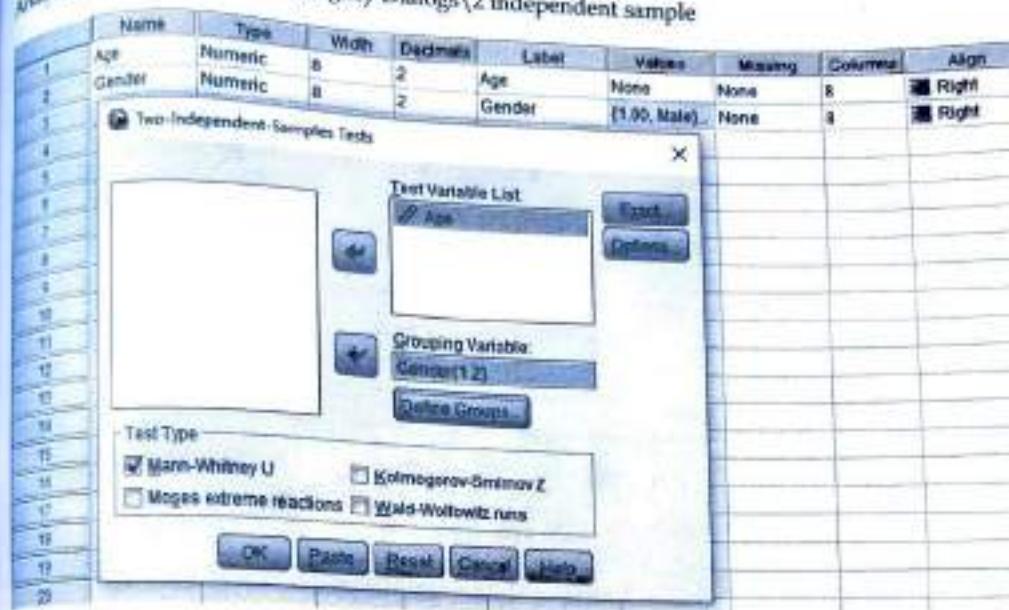
Male	35	43	26	44	40	42	33	38	25	26
Female	30	41	34	31	36	32	25	47	28	24

Using excel

	A	B	C
4			
5	Age	Gender	rank
6	24	2	1
7	25	1	2.5
8	25	2	2.5
9	26	1	4.5
10	26	1	4.5
11	28	2	6
12	30	2	7
13	31	2	8
14	32	2	9
15	33	1	10
16	34	2	11
17	35	1	12
18	36	2	13
19	38	1	14
20	40	1	15
21	41	2	16
22	42	1	17
23	43	1	18
24	44	1	19
25	47	2	20
26			

A	B	C	D	E	F	G	H
27	symbol	value	formula				
28	Sample size of male	n_1	10 =COUNTIF(B6:B25,1)				
29	Sample size of female	n_2	10 =COUNTIF(B6:B25,2)				
30	Sum of ranks of male	R_1	116.5 =SUMIF(B6:B25,1,C6:C25)				
31	Sum of ranks of female	R_2	93.5 =SUMIF(B6:B25,2,C6:C25)				
32	level of significance	α	0.01				
33	$U_1 = \frac{n_1(n_1 + 1)}{2} - R_1$	38.5	=C28*C29+(C28*(C28+1)/2)-C30				
34							
35	U_2	$n_1n_2 - U_1$	61.5 =C28*C29-C33				
36							
37	U_0	$\min(U_1, U_2)$	38.5 =MIN(C33,C35)				
38							
39	Let Md_1 and Md_2 be median age of male and female employees respectively.						
40	Problem to test						
41	H_0 : There is no significant difference between age of male and female employee ($Md_1 = Md_2$)						
42	H_1 : There is significant difference between age of male and female employee ($Md_1 \neq Md_2$)						
43							
44	Test statistic						
45	U_0	38.5					
46	Critical value						
47	Let $\alpha = 0.10$ be the level of significance then critical value is						
48	$U_{\alpha/2} = n_1n_2$		27 From table				
49							
50	Decision						
51	$U_0 = 38.5 > U_{\alpha/2} = 27$		accept H_0 at 0.10 level of significance.				

Analyze\Nonparametric tests\Legacy Dialogs\2 independent sample



Outputs

Mann-Whitney Test

Ranks

Gender	Gender	N	Mean Rank	Sum of Ranks
Age	1.00 Male	10	11.65	116.50
Age	2.00 Female	10	9.35	93.50
	Total	20		

Test Statistics^a

Age	Age
Mann-Whitney U	38.500
Wilcoxon W	93.500
Z	- .870
Asymp. Sig. (2-tailed)	.384
Exact Sig. (2-tailed) [Sig.]	.383 ^b

a. Grouping Variable: Gender
Gender

b. Not corrected for ties.

Using STATA

ranksum Age, by(Gender)

ranksum Age, by (Gender)			
Two-sample Wilcoxon rank-sum (Mann-Whitney) test			
Gender	obs	rank sum	expected
Male	10	116.5	105
Female	10	93.5	105
combined	20	210	210
unadjusted variance		175.00	
adjustment for ties		-0.26	
adjusted variance		174.74	

Ho: Age (Gender==Male) = Age (Gender==Female)

z = 0.870

Prob > |z| = 0.3843

Result: Analysis result shows the p-value for Z test is 0.3843. This indicates that our null hypothesis of no difference in male female, is accepted.

Large sample

The following are the scores which random samples of students from 2 minority groups obtained on a current event test:

Minority Group I	73	82	39	68	91	75	89	67	50	86	57	65	70
Minority Group II	51	42	36	53	88	59	49	66	25	64	18	76	74

Use Mann Whitney U test at the 0.05 level of significance to test whether or not students from the two minority groups can be expected to score equally well on the test.

	A	B	C	D	E
			rank		
			=RANK.AVG(A7:A32,B7:B32,1)		
		1	18		
	71	1	22		
	92	1	4		
	95	1	36		
	86	1	26		
	99	1	25		
	75	1	25		
	99	1	15		
	77	1	7		
	50	1	23		
	88	1	23		
	75	2	10		
	93	2	19		
	79	2	17		
	92	2	8		
	72	2	5		
	76	2	3		
	95	2	9		
	88	2	24		
	59	2	11		
	49	2	6		
	73	2	14		
	75	2	3		
	90	2	12		
	78	2	3		
	76	2	21		
	74	2	19		

A	B	C	D	E	F	G	H
	symbol	value	formula				
36	sample size of male	n ₁	=COUNTIF(B7:B32,1)				
37	sample size of female	n ₂	=COUNTIF(B7:B32,2)				
38	Sum of ranks of male	R ₁	=SUMIF(B7:B32,1,C7:C32)				
39	Sum of ranks of female	R ₂	=SUMIF(B7:B32,2,C7:C32)				
40	level of significance	α	0.05				
41	$U_1 = \frac{n_1(n_1 + 1)}{2} - R_1$		=C28^C29+(C28*(C28+1)/2)-C30				
42	$U_2 = n_1n_2 - U_1$		=C28^C29-C33				
43							
44	min[U ₁ , U ₂]		=MIN(C33,C35)				

45 Let M_{d1} and M_{d2} be median age of male and female employees respectively.

46 Problem to test:

47 H₀: There is no significant difference between age of male and female employee ($M_{d1} = M_{d2}$)

48 H₁: There is significant difference between age of male and female employee ($M_{d1} \neq M_{d2}$)

	A	B	C	D	E	F	G	H	I
51.									
52.									
53. mean		μ_1							
54.									
55. std. dev.		σ_1							
56.									
57. Test statistic									
58.									
59.									
60.									
61. Critical value									
62.									
63. Decision									
64.									
65.									
66.									

$\mu_u = \frac{n_1 n_2}{2}$
 $\sigma_u = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$

$Z = \frac{C45 - C53}{C55}$
 $Z_{\alpha/2} = 1.959964 = \text{NORMSINV}(1-C40/2)$

$|Z| = 2.07 > 2.05 = 1.96$, reject H_0 at 0.05 level of significance.

Using SPSS

Analyze\Nonparametric tests\Legacy Dialogs\

	scores	group	var						
1	73.00	1.00							
2	82.00	1.00							
3	39.00	1.00							
4	66.00	1.00							
5	91.00	1.00							
6	75.00	1.00							
7	89.00	1.00							
8	57.00	1.00							
9	50.00	1.00							
10	86.00	1.00							
11	57.00	1.00							
12	65.00	1.00							
13	70.00	1.00							
14	51.00	2.00							
15	42.00	2.00							
16	36.00	2.00							
17	53.00	2.00							
18	88.00	2.00							
19	59.00	2.00							
20	49.00	2.00							
21	66.00	2.00							
22	25.00	2.00							
23	64.00	2.00							
24	18.00	2.00							
25	76.00	2.00							
..							

Two-Independent-Samples Tests

Test Variable List: scores

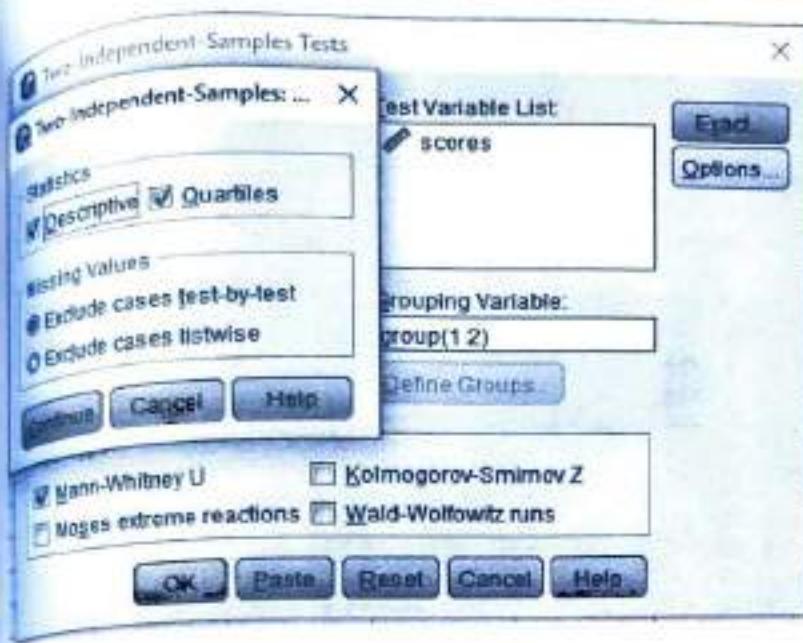
Grouping Variable: group(1 2)

Test Type:

Mann-Whitney U Kolmogorov-Smirnov Z

Moses extreme reactions Wald-Wolfowitz runs

OK **Paste** **Reset** **Cancel** **Help**



Outputs

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	25th	Percentiles 50th (Median)	75th
scores	26	62.0385	19.45761	18.00	91.00	49.7500	65.5000	75.2500
group	26	1.5000	.50990	1.00	2.00	1.0000	1.5000	2.0000

Mann-Whitney Test

Ranks

group	N	Mean Rank	Sum of Ranks
1	13	16.62	216.00
2	13	10.38	135.00
Total	26		

Test Statistics^a

	scoring
Mann-Whitney U	44.000
Wilcoxon W	135.000
Z	-2.077
Asymp. Sig. (2-tailed)	.038
Exact Sig. (2-tailed) (approx.)	.039 ^b

^a Grouping Variable: group
^b Not corrected for ties.

Using STATA

ranksum scores, by(group)

ranksum scores, by(group)			
Two-sample Wilcoxon rank-sum (Mann-Whitney) test			
group	obs	rank sum	expected
1	13	216	175.5
2	13	135	175.5
combined	26	351	351
unadjusted variance		380.25	
adjustment for ties		0.00	
adjusted variance		380.25	
No. scores(group==1) = scores(group==2)			
= 2.077			
Prob > z = 0.0378			

Result: Analysis result shows the p-value for Z test is 0.0378. This indicates that our null hypothesis of no difference in male female, is rejected.

Use chi-square test of independence to determine association between type of place of residence (V102) with source of drinking water(V113) and type of toilet facility(V116) using NPBR7HFL data.

Using Excel

	A	B	C		D	E
			V102 Type of place of residence	Total		
1		Observed Frequency				
2			1 Urban	2 Rural		
3						
4	V113	11 Piped into dwelling				
5	Source of		958	124		1082
6	drinking	12 Piped to yard/plot				
7	water		3306	2523		5809
8		13 Piped to neighbor				
9			283	173		456
10		14 Public tap/standpipe				
11			2730	2818		5548
12		21 Tube well or borehole				
13			6051	3937		9988
14		31 Protected well				
15			148	38		186
16		32 Unprotected well				
17			269	32		301
18		41 Protected spring				
19			254	136		390
20		42 Unprotected spring				
21			233	209		442
22		43 River/dam/lake/ponds/stream/canal/irrigation channel				
23			458	283		741
24		51 Rainwater				
25			9	0		9
26		61 Tanker truck				
27			39	0		39
28		62 Cart with small tank				
29			3	0		3
30		71 Bottled water				
31			265	6		271
32		97 Not a dejure resident				
33		Total				
34			423	240		663
35			15509	10519		26028

	C	D	E	F	G	H	I	J	K	L
	H102 Type of place of residence		Total							
	Urban	Rural								
1) Residential dwelling	645	457	1082							
2) Apartment/flat	3021	2586	5607							
3) House/flat/number	272	164	436							
4) Hotel, dormitory	3386	2242	5628							
5) House and/or bungalow	5851	4837	9688							
6) Private dwell.	111	75	186							
7) Apartment/flat	179	122	301							
8) Residential spring	255	159	394							
9) Uninhabited spring	253	179	432							
10) Residential/pond/river/sea/canal/lake/impound channel	442	299	741							
11) Residential	5	4	9							
12) Temple/mausoleum	29	10	39							
13) Church/chapel	2	1	3							
14) Residential	161	110	271							
15) Not a regular resident	399	288	587							
Total	15559	10518	26077							

$$\begin{aligned} &= \frac{[(A-B)(C-D)-1]^2}{C+D} \\ &= \frac{[15559(10518)-1]^2}{15559+10518} \\ &= \frac{152191^2}{26077} = 224.444 \\ &= \frac{5.132377^2}{5.132377} = 25.3354 \\ &= \frac{0.632374^2}{0.632374} = 0.09485 \\ &= \frac{301.125^2}{301.125} = 109.291 = 147.8789 \\ &= \frac{1.040588^2}{1.040588} = 1.65758 = 1.45525 \\ &= \frac{12.13137^2}{12.13137} = 12.4483 = 18.3806 \\ &= \frac{41.39988^2}{41.39988} = 41.8811 = 66.06447 \\ &= \frac{1.938804^2}{1.938804} = 2.00243 = 2.064304 \\ &= \frac{3.287359^2}{3.287359} = 3.18121 = 5.303161 \\ &= \frac{0.577524^2}{0.577524} = 0.634281 = 0.95695 \\ &= \frac{1.805394^2}{1.805394} = 2.469387 = 3.07275 \\ &= \frac{10.02778^2}{10.02778} = 10.0823 = 13.78133 \\ &= \frac{0.283933^2}{0.283933} = 0.418623 = 0.422205 = 1.212405 \\ &= \frac{55.77006^2}{55.77006} = 56.50816 = 66.38794 = 51.85111 \\ &= \frac{1.986770^2}{1.986770} = 2.013312 = 1.976883 = 2.314676 \end{aligned}$$

P>0.05

	C	D	E	F	G	H	I
Here							
No of rows	r						
No of columns	c						
No of observation	N _{rc}						
Level of significance	α						
Degrees of freedom	(r-1)*(c-1)						

Test Hypothesis:H₀: Type of residence and source of Drinking water are independentH₁: Type of residence and source of drinking water are not independent

Test Statistics

$$\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

for Yates corrected cases

$$\chi^2 = \frac{\sum (|O_i - E_i|)^2}{E_i}$$

for no Yates correction

calculated chi square

$$992.6774914 = \text{SUM}(H24:I38)$$

$$1003.508 = \text{SUM}(K24:L38)$$

$$2.8E-205 = \text{CHISQ.TTEST}(C4:D18,C24:D38)$$

p value

$$5.8268E-203 = \text{CHISQ.DIST.RT}(F56,D46)$$

$$2.8E-205 = \text{CHISQ.DIST.RT}(F56,D46)$$

23.68479

Tabulated Chi square

decision:

significant approach

Reject H₀=IF(F56<F60, "there is no reason to reject null hypothesis H₀", "Reject H₀")

p value approach

=IF(F56>D45, "it is insignificant", "It is significant")

It is significant

Using SPSS

Count		Crosstab		
		V102 Type of place of residence		Total
		1 Urban	2 Rural	
V113 Source of drinking water	11 Piped into dwelling	958	124	1082
	12 Piped to yard/plot	3386	2523	5909
	13 Piped to neighbour	283	173	456
	14 Public tap/standpipe	2730	2818	5548
	21 Tube well or borehole	6051	3937	9988
	31 Protected well	148	38	186
	32 Unprotected well	269	32	301
	41 Protected spring	254	136	390
	42 Unprotected spring	233	209	442
	43 River/dam/lake/ponds/irrigation canal/migration channel	458	283	741
	51 Rainwater	9	0	9
	61 Tanker truck	39	0	39
	62 Cart with small tank	3	0	3
	71 Bottled water	285	6	271
	97 Not a de jure resident	423	240	663
Total		15509	10519	26028

Chi Square test of type of residence vs source of drinking water

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)
Pearson Chi-Square	1003.508*	14	.000
Likelihood Ratio	1174.554	14	.000
Linear-by-Linear Association	90.020	1	.000
N of Valid Cases	26028		
a. 3 cells (10.0%) have expected count less than 5. The minimum expected count is 1.21.		Double-click to activate	

V102 Type of place of residence * V116 Type of toilet facility

Crosstab

V116 Type of toilet facility	V102 Type of place of residence		
	1 Urban	2 Rural	Total
11 Flush to piped sewer system	595	30	625
12 Flush to septic tank	8919	4766	13685
13 Flush to pit latrine	2226	2094	4320
14 Flush to somewhere else	44	21	65
15 Flush, don't know where	3	0	3
21 Ventilated Improved Pit latrine (VIP)	569	663	1232
22 Pit latrine with slab	479	533	1012
23 Pit latrine without slab/open pit	168	138	306
31 No facility/bushfield	1817	1870	3687
41 Composting toilet	259	162	421
96 Other	7	2	9
97 Not a dejure resident	423	240	663
Total	15509	10519	26028

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)
Pearson Chi-Square	954.042*	11	.000
Likelihood Ratio	1050.446	11	.000
Linear-by-Linear Association	45.574	1	.000
N of Valid Cases	26028		

*3 cells (12.5%) have expected count less than 5. The minimum expected count is 1.21.

Using STATA

Chi-square type of place of residence (V102) vs Source of drinking water (V113)

Merge V113 V102, chi2

tabulate V113 V102, chi2

Source of drinking water	Type of place of residence		Total
	Urban	Rural	
Piped into dwelling	556	124	1,082
Piped to yard/plot	3,386	2,523	5,909
Piped to neighbor	183	173	456
Public tap/standpipe	2,730	2,018	5,548
Tube well or borehole	6,051	3,537	9,588
Protected well	148	38	186
Unprotected well	289	32	321
Protected spring	254	136	390
Unprotected spring	233	209	442
River/dam/lake/pond/	458	263	721
Bearwater	9	0	9
Tanker truck	39	0	39
Cart with small tank	3	0	3
Bottled water	268	6	274
Not a de jure resident	423	240	663
Total	15,509	10,813	26,022

Pearson chi2(14) = 1.0e+63 Pr = 1.000

Chi-square type of place of residence (V102) vs Type of toilet facility (V116)

tab V116 V102, chi2

tab V116 V102, chi2

Type of toilet facility	Type of place of residence		Total
	Urban	Rural	
Flush to piped sewer	555	30	625
Flush to septic tank	9,919	4,768	13,685
Flush to pit latrine	2,226	2,094	4,320
Flush to somewhere else	44	21	65
Flush, don't know where	3	0	3
Ventilated Improved P	649	643	1,222
Pit latrine with slab	479	439	1,012
Pit latrine without s	168	138	306
No facility/bush/fiel	1,817	1,870	3,687
Composting toilet	259	162	421
Other	7	2	9
Not a de jure resident	423	240	663
Total	15,509	10,813	26,022

Pearson chi2(11) = 954.0416 Pr = 0.008

Median Test

for Small sample size ($n \leq 10, n_2 \leq 10$)

The below shows one week growth(in cm) of maize plant from two different localities (Sample I)

Step III:

Sample I	10	11	8	8	14		
Sample II	9	12	13	9	15	9	17

Test whether the two samples have come from the same population with respect to their medians.

Use median test at 0.05 level of significance.

using Excel

A	B	C	D	E	F	G	H
Sample I	10	11	8	8	14		
Sample II	9	12	13	9	15	9	17

	sample	value
1	1	10
1	1	11
1	1	8
1	1	8
1	1	14
2	1	9
2	1	12
2	1	13
2	1	9
2	1	15
2	1	9
2	1	17

	symbol	value	formula
total observation	n	12	
median	MD	10.5	
sample size 1	n1	5	
sample size 2	n2	7	
	k	9	

A	B	C	D	E	F	G	H	I	J
---	---	---	---	---	---	---	---	---	---

non values less than median									
in Sample	8	7	3						

Hence $Md_1 = 8$ and $Md_2 = 9$ be median of 1 population and 2 population respectively.

Probability test

Hence there is significant difference between median of 1 population and median of 2 population ($Md_1 - Md_2$)

Hence sign test difference between median of all population and known median ($Md_1 - Md_0$)

Test statistic

$$P(X_{(k)} < M_d) = \frac{C}{n!} \cdot \frac{(n-k)!}{(n-k)!}$$

level of significance

0.05

Critical value

0.05

Decision

Reject H₀

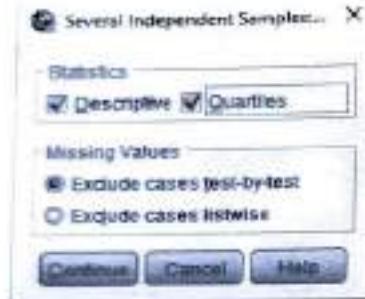
Conclusion

There is significant difference between median of 1 population and 2 population.

Using SPSS

Analyze\Non parametric test\Legacy Dialogs\k-independent samples

The screenshot shows two overlapping dialog boxes. The top box is titled "Kruskal-Wallis H Test for Several Independent Samples". It contains a "Test Variable List" with "value" selected, a "Grouping Variable" set to "sample120", and a "Test Type" section where "Kruskal-Wallis H" is checked. The bottom box is titled "Several Independent Samples..." and includes sections for "Statistics" (with "Descriptive" checked), "Missing Values" (with "Exclude cases test-by-test" checked), and buttons for "Continue", "Cancel", and "Help".



Outputs

[DataSet1] D:\Desktop\11111111111111\stat2_pr1\md small.sav

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	25th	50th (Median)	75th
value	12	11.2500	2.95804	8.00	17.00	9.0000	10.5000	13.250
sample	12	1.5033	.51493	1.00	2.00	1.0000	2.0000	2.000

Median Test

Frequencies

	sample	sample	
	1.00	2.00	
value	> Median	2	4
	= Median	3	3

Test Statistics^a

	value	value
N		12
Median		10.5000
Exact Sig.		1.000

a. Grouping

Variable:

sample sample

Answer as the p value associated with the median test is about 1 it indicates that null hypothesis of no difference is accepted for the given data.

using STATA

median value, by(sample) medianties(below)

median value, by (sample) medianties(below)

median test

Greater than the median	sample		Total
	1	2	
no	3	3	6
yes	2	4	6
Total	5	7	12

Pearson chi2(1) = 0.3429 Pr = 0.558

Continuity corrected:

Pearson chi2(1) = 0.0000 Pr = 1.000

Large sample size ($n_1 > 10, n_2 > 10$)

An IQ test was given to a randomly selected 15 male and 20 female students of a university. Their scores were recorded as follows:

Male: 56, 66, 62, 81, 75, 73, 83, 68, 48, 70, 60, 77, 86, 44, 72

Female: 63, 77, 65, 71, 74, 60, 76, 61, 67, 72, 64, 65, 55, 89, 45, 53, 68, 73, 50, 81

Use median test to determine whether IQ of male and female students is same in the university
 (Given that the median of combined sample = 68)

Using EXCEL

A	B	C	D	E
5	scores	Gender		
6		56 Male		
7		66 Male		
8		62 Male		
9		81 Male		
10		75 Male		
11		73 Male		
12		83 Male		
13		68 Male		
14		48 Male		
15		70 Male		
16		60 Male		
17		77 Male		
18		86 Male		
19		44 Male		
20		72 Male		
21		63 Female		
22		77 Female		
23		65 Female		
24		71 Female		
25		74 Female		
26		60 Female		
27		76 Female		
28		51 Female		
29		67 Female		
30		72 Female		
31		64 Female		
32		65 Female		
33		55 Female		
34		83 Female		
35		45 Female		
36		53 Female		
37		58 Female		
38		73 Female		
39		50 Female		
40		81 Female		

A	B	C	D	E
Table using pivot table and grouping by 25 since minimum value is 44				
Count of scores	Male	Female	Grand Total	
Row Labels	7	12	19	
44-58	8	8	16	
59-93	15	20	35	
Grand Total				

A	B	C	D	E
2x2 contingency table for below and above median values from pivot table				
	Male	Female	Grand Total	
Row Labels	7	12	19	
44-58	8	8	16	
59-93	15	20	35	
Grand Total				

A	B	C	D	E
Here				
no. of male	n1	15	=B52	
no. of female	n2	20	=C52	
Total sample size	N=n1+n2	35	=D52	
no. of obs. of male ≤ Md	a	7	=B50	
no. of obs. of female ≤ Md	b	12	=C50	
no. of obs. of male > Md	c	8	=B51	
no. of obs. of female > Md	d	8	=C51	

Let Md_1 and Md_2 be median IQ of male and female respectively.

A	B	C	D	E	F	G	H
Problem to test							
H0: There is no significant difference between IQ of male and female ($Md_1 = Md_2$)							
H1: There is significant difference between IQ of male and female. ($Md_1 \neq Md_2$)							
Test statistic	$N(ad - bc)^2$						
$\chi^2 = \frac{N(ad - bc)^2}{(a + c)(b + d)(a + b)(c + d)}$	0.614035088						
level of significance	α	0.05					
degrees of freedom	$(r-1)(c-1)$						
Critical value							
Tabulated chi-square	tab $\chi^2 0.05, (1)$	3.8415	=CHISQ.INV.RT(C73, C74)				
p-value	P	0.4333	=CHISQ.DIST.RT(B71, C74)				
Decision							
Significant approach							
There is no reason to reject null hypothesis H_0							
Value approach							
It is insignificant.							
Decision							
Significant approach							
There is no reason to reject null hypothesis H_0							
Value approach							
It is insignificant.							

Using SPSS

Analyze\Non parametric test\Legacy Dailogs\k-independent samples

	scores	Gender	var	var	var	var	var	var
13	86.00	Male						
14	44.00	Male						
15	72.00	Male						
16	63.00	Female						
17	77.00	Female						
18	65.00	Female						
19	71.00	Female						
20	74.00	Female						
21	60.00	Female						
22	76.00	Female						
23	61.00	Female						
24	67.00	Female						
25	72.00	Female						
26	64.00	Female						
27	65.00	Female						
28	55.00	Female						
29	89.00	Female						
30	45.00	Female						
31	53.00	Female						
32	68.00	Female						
33	73.00	Female						
34	50.00	Female						
35	81.00	Female						

Tests for Several Independent Samples

Test Variable List: scores

Grouping Variable: Gender(1 2)

Define Range...

Test Type

Kruskal-Wallis H Median
 Jonckheere-Terpstra

OK Paste Reset Cancel Help

Several Independent Samples... X

Statistics

Descriptive Quartiles

Missing Values

Exclude cases test-by-test
 Exclude cases listwise

Continue Cancel Help

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	25th	Percentiles 50th (Median)	75th
scores scores	35	67.1429	11.31408	44.00	89.00	60.0000	68.0000	75.0000
Gender Gender	35	1.57	.502	1	2	1.00	2.00	2.00

Median Test**Frequencies**

	Gender	Gender
	1 Male	2 Female
scores scores	> Median	8
<= Median	7	12

Test Statistics^a

	scores	scores
N		35
Median		68.0000
Chi-Square		.614
df		1
Asymp. Sig.		.433
Yates' Continuity Correction	Chi-Square	.194
df		1
Asymp. Sig.		.659

^a Grouping Variable: Gender Gender

Answer: As the asymptotic significance related with the Chi-square test is found to be 0.659 this indicates that the null hypothesis of no difference is accepted for the given data.

Using STATA

median scores, by(Gender) medianties(below)

- median scores, by(Gender) medianties(below)

Median test

Greater than the median	Gender		Total
	Male	Female	
no	7	12	19
yes	8	8	16
Total	15	20	35

Pearson chi2(1) = 0.6140 Pr = 0.433

Continuity corrected:

Pearson chi2(1) = 0.1943 Pr = 0.659

170 □ Statistics - II

Small Sample ($n < 25$)

Use Wilcoxon Matched pair signed rank test to drugs in suppressing pain from following data.

Patient No.	Drug A	Drug B	Patient No.	Drug A	Drug B
1	6.5	3.5	11	5.4	5.5
2	3.7	3.7	12	4.0	4.1
3	3.0	4.7	13	5.7	4.1
4	6.7	5.0	14	3.9	4.2
5	6.2	5.6	15	3.6	3.7
6	6.7	4.3	16	4.9	4.1
7	6.1	5.4	17	3.9	5.4
8	4.3	5.8	18	5.8	3.7
9	5.5	4.3	19	4.9	4.1
10	6.8	4.3	20	4.9	4.1

Using Excel

A	B	C	D	E	F	G	H	I
Patient No.	Drug A	Drug B	difference	Sign	difference	Rank	Signed Rank	tied count list
					=IF(F3="","", =IF(D3>0, -IF(OR(B3="",,RANK.AVG 1,IF(D3<0,C3="",D3=0),{F3,\$F\$3:\$F =IF(F3="","",">0,"",COUNTIF(G: G22,G3))))			=IF(H3="","", UNTIL(SGS2,G2,G3) G22,G3)))
1	6.5	3.5	3.0	1	3	19	19	1
2	3.7	3.7	0.0					
3	3.9	4.7	-0.8	-1	0.8	7	-7	1
4	6.7	5.0	1.7	1	1.7	15	15	1
5	5.2	5.6	0.6	1	0.6	5	5	1
6	6.7	4.3	2.4	1	2.4	17	17	1
7	6.1	5.4	0.7	1	0.7	6	6	1
8	4.3	5.8	-1.5	-1	1.5	12	-12	1
9	5.5	4.3	1.2	1	1.2	11	11	1
10	6.8	4.3	2.5	1	2.5	18	18	1
11	5.4	5.5	-0.1	-1	0.1	1.5	-1.5	1
12	4.0	4.1	-0.1	-1	0.1	1.5	-1.5	1
13	5.7	4.1	1.6	1	1.6	14	14	1
14	3.9	4.2	-0.3	-1	0.3	4	-4	1
15	3.6	3.7	-0.1	-1	0.1	3	-3	3
16	4.9	4.1	0.8	1	0.8	9	9	1
17	3.9	5.4	-1.5	-1	1.5	13	-13	1
18	5.8	3.7	2.1	1	2.1	16	16	1
19	4.9	4.1	0.8	1	0.8	9	9	1
20	4.9	4.1	0.8	1	0.8	9	9	1

A	B	C	D	E	F	G
Sample Size of Drug A	n1	Value		Formula		
Sample Size of Drug B	n2			=COUNT(B3:B22)		
no of case with zero difference				=COUNT(C3:C22)		
sum of positive ranks	S(+)			=COUNTIF(D3:D22,0)		
sum of negative ranks	S(-)			=SUMIF(E3:E22,>"&0,H3:H22)		
count of positive ranks	n+			=ABS(SUMIF(E3:E22,<"&0,H3:H22))		
count of negative ranks	n-			=COUNTIF(H3:H22,>"&0)		
effective sample size	n'			=COUNTIF(H3:H22,<"&0)		
Since sample size, n < 25				19 =C31-C32		
Test statistic	T					
$T = \min [S(+), S(-)]$.				=MIN(C33:C34)		
Level of significance	α			0.05		

2 Critical value

At α level of significance, we obtain critical value from Wilcoxon Matched pair signed rank test table

$T_{\alpha, n} = T_{0.05, 19}$

From Wilcoxon Matched pair signed rank test table

Note: $T_{\alpha, n}$ where n is corrected sample size after omitting $d_i = 0$.

4 Decision

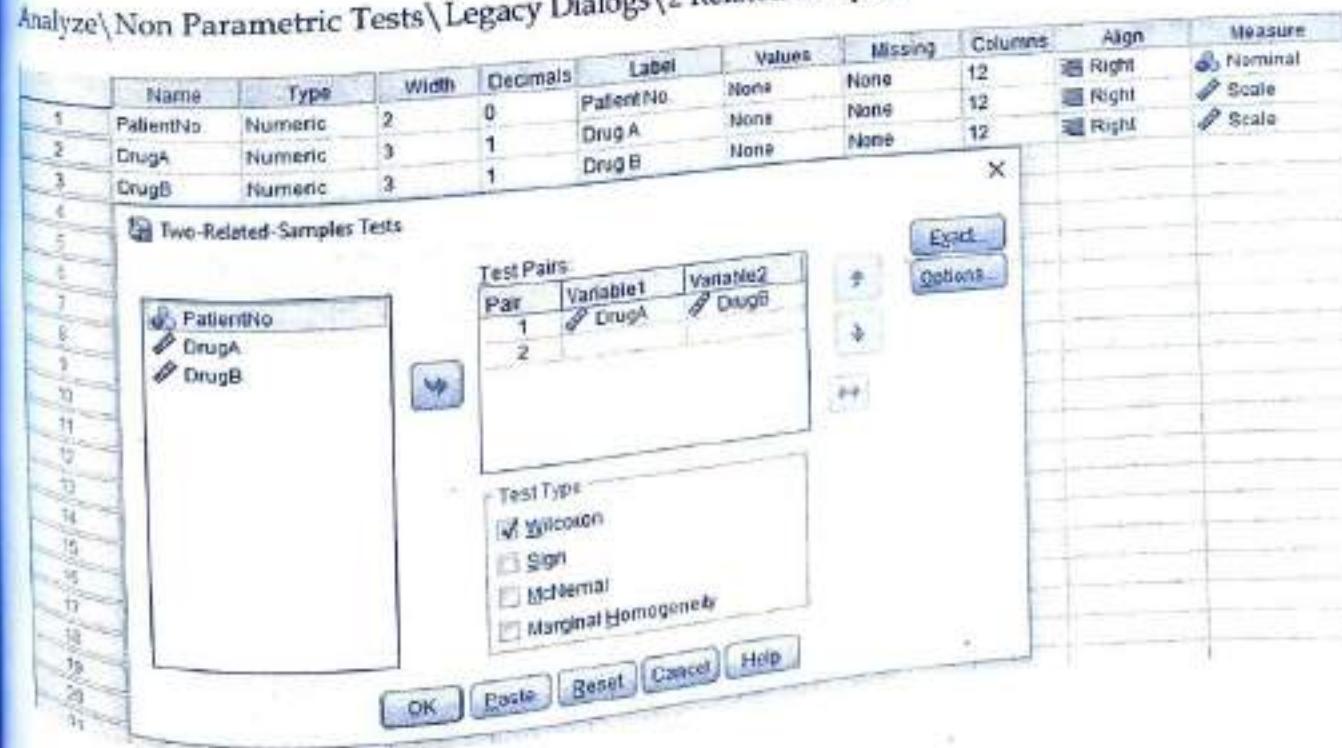
H_0 is rejected

Note:

Reject H_0 at α level of significance if $T < T_{\alpha, n}$, accept otherwise

Using SPSS

Analyze\Non Parametric Tests\Legacy Dialogs\2 Related Samples





Outputs

NPar Tests

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	25th	Percentiles 50th (Median)	75th
DrugA Drug A	20	5.170	1.1154	3.6	6.8	3.925	5.150	6.175
DrugB Drug B	20	4.480	.7157	3.5	5.8	4.100	4.250	5.300

Wilcoxon Signed Ranks Test

Ranks

	N	Mean Rank	Sum of Ranks
DrugB Drug B - DrugA Drug A	Negative Ranks	12 ^a	12.21
	Positive Ranks	7 ^b	6.21
	Ties	1 ^c	
	Total	20	

a. DrugB Drug B - DrugA Drug A

b. DrugB Drug B + DrugA Drug A

c. DrugB Drug B = DrugA Drug A

Test Statistics^a

DrugB Drug B
- DrugA Drug
A

Z	-2.076 ^b
Asymp. Sig. (2-tailed)	.038

a. Wilcoxon Signed Ranks Test

b. Based on positive ranks.

Since $p < 0.05$, it is significant.

Using STATA

`signrank DrugA=DrugB`

`. signrank DrugA=DrugB`

`Wilcoxon signed-rank test`

sign	obs	sum ranks	expected
positive	12	160	104.5
negative	7	49	104.5
zero	1	1	1
all	20	210	210

unadjusted variance	717.50
adjustment for ties	-0.63
adjustment for zeros	-0.25
adjusted variance	716.63

$H_0: \text{DrugA} = \text{DrugB}$

$z = 2.073$

Prob $> |z| = 0.0382$

Since, $p < 0.05$, it is significant.

large sample size ($n > 25$)

For large sample size sampling distribution of T is approximately normally distributed with mean μ_T and variance σ_T^2

$$\mu_T = \frac{n(n+1)}{4}$$

$\sigma_T = \sqrt{\frac{n(n+1)}{4}}$ and $\sigma_T^2 = \frac{n(n+1)(2n+1)}{24}$, where n is corrected sample size if $d_i = 0$.

Test statistic

$$Z = \frac{T - \mu_T}{\sigma_T} = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \sim N(0, 1) \quad (\text{Here } n = n_c)$$

Level of significance

Let α be the level of significance. Generally fix $\alpha = 0.05$ unless we are given.

Critical value

Critical value $Z_{\text{tabulated}}$ is obtained from table according to level of significance and alternative hypothesis.

Decision

Reject H_0 at α level of significance if $Z > Z_{\text{tabulated}}$. Accept otherwise.

Use Wilcoxon Matched pair signed rank test to determine the equality of oxygen level of patients in ICU on the day of admission and 7 days after admission from following data.

Patient No	Day1	Day7	Patient No	Day1	Day7	Patient No	Day1	Day7
1	39.32	64.88	21	61.68	62.28	41	64.08	65.0
2	70.59	67.20	22	71.56	72.80	42	60.34	73.2
3	53.92	74.77	23	75.84	44.88	43	37.35	66.1
4	73.46	72.39	24	46.03	46.65	44	60.93	58.75
5	45.16	58.17	25	32.74	58.93	45	66.48	79.0
6	72.21	61.86	26	69.85	55.90	46	46.74	64.9
7	47.35	82.36	27	42.24	59.98	47	58.38	85.6
8	72.32	52.09	28	45.49	83.25	48	74.70	83.0
9	43.60	65.38	29	58.86	81.93	49	49.50	64.9
10	35.69	76.27	30	72.18	75.00	50	75.66	69.5
11	73.25	71.53	31	48.12	56.99	51	73.74	47.9
12	54.22	80.13	32	50.11	78.41	52	63.07	69.1
13	42.50	50.80	33	71.18	69.54	53	64.88	44.2
14	46.85	69.35	34	64.92	67.39	54	51.60	73.0
15	46.17	73.07	35	71.45	59.49	55	39.14	66.7
16	46.89	84.32	36	51.67	78.51	56	59.49	68.0
17	71.03	67.15	37	75.67	67.74	57	51.75	55.0
18	64.79	68.41	38	40.13	82.34	58	39.06	58.8
19	34.68	59.60	39	52.48	52.68	59	75.43	81.0
20	57.02	54.47	40	53.50	43.92			

Using EXCEL

A	B	C	D	E	F	G	H	I
Patient No	Day1	Day7	difference	Sign	difference	Rank	Signed Rank	Tied count list
						=IF(F3="")	=IF(H3="", "", I)	
						, "", RANK,	F(COUNTIF(\$G\$2:G2, G3)>0,"	
						1, IF(D3>0, =IF(OR(B3="",	Avg(F3:\$	
						, C3="", D3=0), F\$3:\$F\$6	\$2:G2, G3)>0,"	
						, -1, "")))	"", ABS(D3))	=IF(F3="", "", ", COUNTIF(G3:I
							E3*G3))	:G22, G3)))
1	39.32	64.88	-25.56	-1	25.56	44	-44	1
2	70.59	67.20	3.39	1	3.39	13	13	1
3	53.92	74.77	-20.85	-1	20.85	38	-38	1
4	73.46	72.39	1.07	1	1.07	5	5	1
5	45.16	58.17	-13.01	-1	13.01	29	-29	1
6	72.21	61.86	10.35	1	10.35	26	26	1
7	47.35	82.36	-35.01	-1	35.01	55	-55	1
8	72.32	52.09	20.23	1	20.23	36	36	1
9	43.60	65.38	-21.78	-1	21.78	40	-40	1
10	35.69	76.27	-40.58	-1	40.58	58	-58	1
11	73.25	71.53	1.72	1	1.72	8	8	1
12	54.22	80.13	-25.91	-1	25.91	45	-45	1
13	42.50	50.80	-8.30	-1	8.3	20	-20	1
14	46.85	69.35	-22.50	-1	22.5	41	-41	1
15	46.17	73.07	-26.90	-1	26.9	49	-49	1
16	46.89	84.32	-37.43	-1	37.43	56	-56	1
17	71.03	67.15	3.88	1	3.88	16	16	1
18	64.79	68.41	-3.62	-1	3.62	14	-14	1
19	34.68	59.60	-24.92	-1	24.92	43	-43	1
20	57.02	58.87	1.85	1	1.85	11	11	1

Only part of data is shown

A	B	C	D	E	F	G	H
54	HO: there is no significant difference between oxygen level						
55	HA: there is significant difference between oxygen level						
56							
57							
58							
59							
60							
61							
62							
63							
64							
65							
66							
67							
68	Sample Size of Drug A	Symbol	Value	Formula			
69	n1		59	=COUNT(B3:B22)			
70	Sample Size of Drug B	n2		59	=COUNT(C3:C22)		
71	No of case with zero difference			0	=COUNTIF(D3:D22, 0)		
72	Sum of positive ranks	S(+)		388	=SUMIF(E3:E22, ">0, H3:H22)		
73	Sum of negative ranks	S(-)		1382	=ABS(SUMIF(E3:E22, "<0, H3:H22))		
74	Count of positive ranks	n+		17	=COUNTIF(H3:H22, ">0")		
75	Count of negative ranks	n-		42	=COUNTIF(H3:H22, "<0")		
76	Effective sample size	n'		59	=C31-C32		
77		T		388	=MIN(C71:C72)		
78	Since Sample Size n>25						
79	For large sample size sampling distribution of T is approximately normally distributed with mean						
80	μ_T and variance σ_T^2						
81							
82	mean	Symbol	Value	Formula			
83	μ_T			885 =C75*(C75+1)/4	$\frac{n(n+1)}{4}$		
84	Variance	σ_T^2		17552.5 =(C75*(C75+1)*(2*C75+1))/(24)	$\frac{n(n+1)(2n+1)}{24}$		

85
86 Test Statistics
87 $Z =$

$$\frac{T - \mu_T}{\sigma_T}$$

$$\sqrt{\frac{n(n+1)(2n+1)}{24}} \sim N(0,1)$$

$$-3.75134406 = (C76-C82)/SQRT(C84)$$

90 cal Z

91 level of significance
92 tabulated Z for Two tailed test
93 tabulated Z for one tailed test

a

0.05

$Z_{\alpha/2}$

$$1.959963985 = NORM.S.INV(1-C92/2)$$

Z_α

$$1.644853627 = NORM.S.INV(1-C92)$$

94
95 pvalue
96 Two tailed
97 One tailed

$$0.000175889 = 2 * NORMSDIST(C90)$$

$$8.79446E-05 = NORMSDIST(C90)$$

100 Decision:

101 Significant approach for two tailed test

102 Null hypothesis H_0 is rejected

103

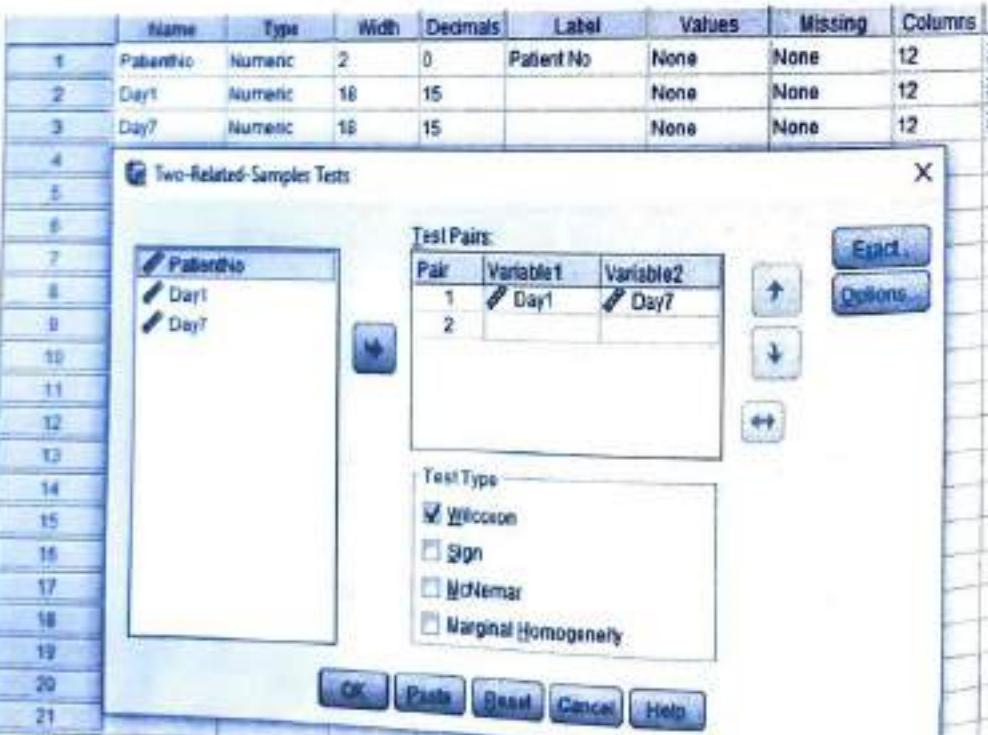
104 p value approach

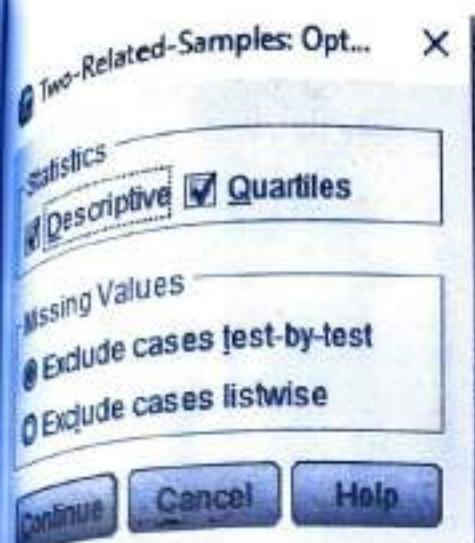
105 It is significant

106

Using SPSS

Analyze\Non Parametric Tests\Legacy Dialogs\2 Related Samples





Outputs

Wilcoxon Signed Ranks Test

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	25th	50th (Median)	75th	Percentiles
Day1	59	56.93288136	13.00609659	32.74000000	75.84000000	45.17000000	57.02000000	71.03000000	
Day7	59	66.66932203	11.19511514	43.92000000	85.63100000	59.86000000	67.20000000	75.00000000	

Wilcoxon Signed Ranks Test

Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
Day7 - Day1	Negative Ranks	17 ^a	22.82	388.00
	Positive Ranks	42 ^b	32.90	1382.00
	Ties	0 ^c		
	Total	59		

a. Day7 < Day1

b. Day7 > Day1

c. Day7 = Day1

Test Statistics^a

Day7 - Day1	
Z	-3.751 ^b
Asymp. Sig. (2-tailed)	.000

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

Since $p < 0.05$, it is significant.

Using STATA

signrank Day1=Day7

. signrank Day1=Day7

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	17	389	885
negative	42	1382	885
zero	0	0	0
all	59	1770	1770

unadjusted variance 17552.50

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 17552.50

Ho: Day1 = Day7

z = -3.751

Prob > |z| = 0.0002

Since $p < 0.05$, it is significant.

Cochran Q test

Five housewives were asked for the acceptability of four brands of lipsticks for daily use. The response of acceptability (A) and rejection (R) are given below;

House wives	Lipstick Brands			
	Alfa	Beta	Gamma	Delta
H ₁	A	R	A	R
H ₂	R	A	A	R
H ₃	R	A	R	A
H ₄	A	R	R	R
H ₅	A	A	R	A

Test whether there is any significant difference between brands with respect to acceptability

Solution: Using Excel

Cochran's Q test is a non-parametric statistical test to verify whether k treatments have identical effects. It is the nonparametric counterpart of the two-way ANOVA in RBD. The data for the test are binary that needed to be coded 0, 1 for analysis. In this test the null hypothesis is acceptability of four different lipstick brands are same among housewives.

A	B	C	D	E	F	G
	Lipstick Brands					
House wives	Alfa	Beta	Gamma	Delta		
H ₁	A	R	A	R		
H ₂	R	A	A	R		
H ₃	R	A	R	A		
H ₄	A	R	R	R		
H ₅	A	A	R	A		
	Lipstick Brands					
House wives	Alfa	Beta	Gamma	Delta	C _j	C _j ²
H ₁	1	0	1	0	2	4
H ₂	0	1	1	0	2	4
H ₃	0	1	0	1	2	4
H ₄	1	0	0	0	1	1
H ₅	1	1	0	1	3	9
R _i	3	3	2	2	10	22
R _i ²	9	9	4	4	26	

A	B	C	D	E	F	G	H
19				formula			
20			4	=COUNT(B11:E11)			
21	Number of brands	k					
22	success	1					
23	Number of housewives	n	5	=COUNT(B11:B15)			
24		ΣR_i	10				
25		ΣR_i^2	26				
26		ΣC_j	10				
27		ΣC_j^2	22				
28							

29 Problem to test

30 H_0 : There is no significant difference between brands31 H_1 : There is at least one significant difference between brands.

32 Test statistic

$$33 \quad Q = \frac{(k-1) \left[k \sum_{i=1}^k R_i^2 - (\sum_{i=1}^k R_i)^2 \right]}{k \sum_{j=1}^n C_j - \sum_{j=1}^n C_j^2} \sim \chi_{k-1}^2$$

$$34 \quad Q = 0.6666667 = [(C21 \cdot 1) * (C21 * C25 - (C24)^2)] / (C21 * C26 - C27)$$

$$35 \quad \text{level of significance} \quad \alpha \quad 0.05$$

$$36 \quad \text{degrees of freedom} \quad k-1 \quad 3$$

$$37 \quad \text{Critical value} \quad \chi_{(0.95)}^2 \quad 7.8147279$$

$$38 \quad p \text{ value} \quad p \quad 0.8810148$$

$$39 \quad$$

A	B	C	D	E	F	G	H
43 Decision							
44 Significant Approach							
45 There if no reason to reject null hypothesis H_0 .				=IF(C37 < C40, "There if no reason to reject null hypothesis H_0 .", "Reject H_0 ")			
46							
47 p value Approach							
48 It is in significant.				=IF(C41 > C38, "It is in significant.", "It is significant.")			
49 Conclusion:							
50 There is no significant difference between brands according to acceptability.							
51							

Answer: Here the analysis shows the p-value associated with the test statistics Q is 0.88 that indicates acceptance of the null hypothesis of no difference.

Solution: Using SPSS

Analyze\Nonparametric test\Legacy Dialogs\k Related Samples

	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure
1	Housewives	Numeric	2	0	House wives	{1, H1}..	None	16	Right	Nominal
2	Alfa	Numeric	8	0	Alfa	{1, Accept}..	None	8	Right	Nominal
3	Beta	Numeric	8	0	Beta	{1, Accept}..	None	8	Right	Nominal
4	Gamma	Numeric	8	0	Gamma	{1, Accept}..	None	8	Right	Nominal
5	Delta	Numeric	8	0	Delta	{1, Accept}..	None	8	Right	Nominal

Tests for Several Related Samples

Housewives

Test Variables:

- Alfa
- Beta
- Gamma
- Delta

Exact Statistics

Test Type

Friedman Kendall's W Cochran's Q

OK Cancel Help

Outputs

Cochran Test

		Frequencies	
		Value	
		0	1
Alfa	Alfa	2	3
Beta	Beta	2	3
Gamma	Gamma	3	2
Delta	Delta	3	2

Test Statistics

N	5
Cochran's Q	.667*
df	3
Asymp. Sig.	.881

a. 1 is treated as a success.

Kruskal Wallis H test

For non-repeated ranks

Following are the scores obtained by trainees in 3 different categories. Test whether 3 categories have performed equally.

categories	scores									
A	68	65	92	82	62	64	68	92	86	64
B	93	86	73	87	76	85	67	79	75	75
C	95	72	85	70	80	80	78	85	72	90

Using Excel

	A	B	C	D	E	F
6	categories	scores	rank			
7			=RANK.AVG(B8:B22,B8:B22,1)			
8	1	67	9			
9	1	77	14			
10	1	79	15			
11	1	72	13			
12	1	64	7			
13	2	55	3			
14	2	50	1			
15	2	58	5			
16	2	57	4			
17	2	54	2			
18	3	66	8			
19	3	70	12			
20	3	69	11			
21	3	62	6			
22	3	68	10			
23			120			
24			=SUM(C8:C22)			
25						
26	test statistics					
27	H0:	there is no significant difference between median score of three categories				
28	H1:	there is significant difference between median score of three categories				

A	B	C	D	E	F	G	H
cases		code	symbol	value	formula		
# of categories			k		3 =MA((A8:A22)		
# of categories			1 n1		5 =COUNTIF(\$A\$8:\$A\$22,B34)		
# of categories			2 n2		5 =COUNTIF(\$A\$8:\$A\$22,B35)		
# of categories			3 n3		5 =COUNTIF(\$A\$8:\$A\$22,B36)		
# of observations			n		15 =COUNT(A8:A22)		
# sum of rank in			1 R1		58 =SUMIF(\$A\$8:\$A\$22,B38,\$C\$8:\$C\$22)		
# sum of rank in			2 R2		15 =SUMIF(\$A\$8:\$A\$22,B39,\$C\$8:\$C\$22)		
# sum of rank in			3 R3		47 =SUMIF(\$A\$8:\$A\$22,B40,\$C\$8:\$C\$22)		

For nottied observations

$$\frac{12}{n(n+1)} \sum_{i=1}^k \frac{R_i^2}{n_i} - 3(n+1)$$

$$9.98 = (|12/(D37^2*D37)| * (|D38^2/D34|*D39^2/D35)*(|D40^2/D36|) - 3*(D37+1))$$

level of significance

α 0.05

degrees of freedom

$k-1$ 2 =D33-1

critical value

$X_{\alpha,k-1}^2$ 5.99146 =CHISQ.INV(1-D49,D50)

p-value

p 0.99319 =CHISQ.DIST(D45,D50,TRUE)

Decision

Significant approach

=IF(D45<D51, "There is no reason to reject null hypothesis H0", "Reject Null Hypothesis H0")

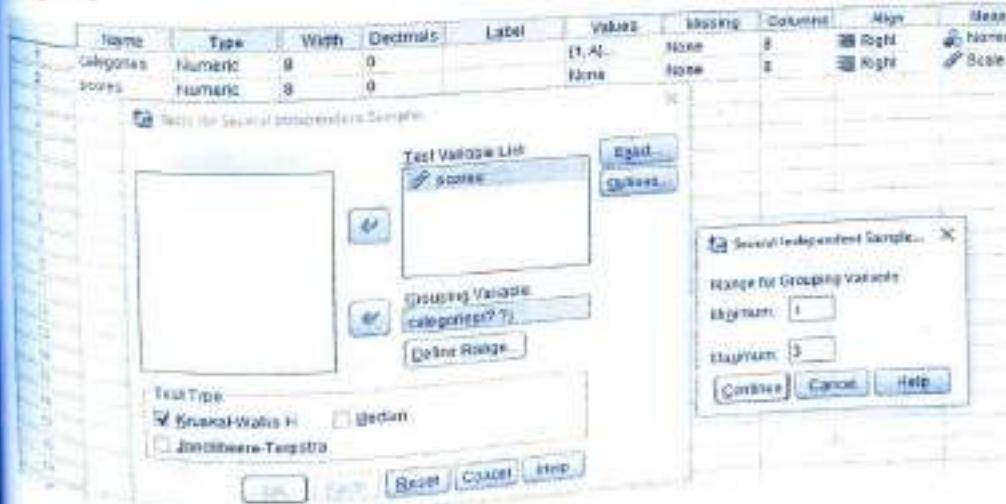
Reject Null Hypothesis H0

p-value approach

=IF(D52>D49, "it is insignificant", "it is significant")

it is insignificant

Using SPSS



Outputs

	N	Descriptive Statistics					Percentiles	
		Mean	Std. Deviation	Minimum	Maximum	25th	50th (Median)	75th
scores	15	64.53	9.493	50	79	67.00	66.00	70.80
categories	15	2.00	.845	1	3	1.00	2.00	3.80

Kruskal-Wallis Test**Ranks**

categories	N	Mean Rank
category A	5	11.60
category B	5	3.00
category C	5	9.40
Total	15	

Test Statistics^{a,b}

scoring	
Kruskal-Wallis H	9.980
d.f.	2
Asymp. Sig.	.007

a. Kruskal-Wallis Test

Using STATA

kwallis scores, by(categories)

kwallis scores, by(categories)

Kruskal-Wallis equality-of-populations rank test

categories	Obs	Rank Sum
A	5	58.00
B	5	15.00
C	5	47.00

chi-squared = 9.980 with 2 d.f.

probability = 0.0068

chi-squared with ties = 9.980 with 2 d.f.

probability = 0.0068

Answer: As we found that P value for Chi-square test with tie is 0.0068 which is smaller than 0.05 we can conclude that the scores among three categories are significantly different as null is rejected.

For repeated ranks

Following are the scores obtained by trainees in 3 different categories. Test whether 3 categories are equally performed.

categories	scores									
1	68	65	92	82	62	64	68	92	86	64
2	93	86	73	87	76	85	67	79	75	75
3	95	72	85	70	80	80	78	85	72	90

Using Excel

A	B	C	D	E
categories	scores	rank		
1	68	6.5	=RANK.AVG(B4:B37,B4:B37,1)	
1	65	4		
1	92	27.5		
1	82	19		
1	62	1		
1	64	2.5		
1	68	6.5		
1	92	27.5		
1	86	23.5		
1	64	2.5		
2	93	29		
2	86	23.5		
2	73	11		
2	87	25		
2	76	14		
2	85	21		
2	67	5		
2	79	16		
2	75	12.5		
2	75	12.5		
3	95	30		
3	72	9.5		
3	85	21		
3	70	8		
3	80	17.5		
3	80	17.5		
3	78	15		
3	85	21		
3	72	9.5		
3	90	26		
		455		
			=SUM(C4:C37)	

41 test statistics
42 H0: there is no significant difference between median score of three categories

43 H1: there is significant difference between median score of three categories
44

45

46

47 sum

	code	symbol	value	formula
48 no of categories	t			=MAX(A8:A37)
49 no of categories	t1			=COUNTIF(\$A\$8:\$A\$37,849)
50 no of categories	t2			=COUNTIF(\$A\$8:\$A\$37,850)
51 no of categories	t3			=COUNTIF(\$A\$8:\$A\$37,851)
52 no of observations	n			=COUNT(A8:A37)
53 sum of rank in	R1		120.5	=SUMIF(\$A\$8:\$A\$37,B53,\$C\$8:\$C\$37)
54 sum of rank in	R2		169.3	=SUMIF(\$A\$8:\$A\$37,B54,\$C\$8:\$C\$37)
55 sum of rank in	R3		175	=SUMIF(\$A\$8:\$A\$37,B55,\$C\$8:\$C\$37)
56				
57 no of repeated ranks	T1		23.13	=COUNTIF(\$C\$8:\$C\$37,857)
58 no of repeated ranks	T2		6.12	=COUNTIF(\$C\$8:\$C\$37,858)
59 no of repeated ranks	T3		9.13	=COUNTIF(\$C\$8:\$C\$37,859)
60 no of repeated ranks	T4		12.14	=COUNTIF(\$C\$8:\$C\$37,860)
61 no of repeated ranks	T5		17.15	=COUNTIF(\$C\$8:\$C\$37,861)
62 no of repeated ranks	T6		23.16	=COUNTIF(\$C\$8:\$C\$37,862)
63 no of repeated ranks	T7		27.17	=COUNTIF(\$C\$8:\$C\$37,863)
64 no of repeated ranks	T8		21.18	=COUNTIF(\$C\$8:\$C\$37,864)

65 $\sum T = \sum_{p=1}^k T_p = \sum_{i=1}^k (t_i^2 - t)$ 66 =SUMPRODUCT(D57:D64^3)-SUM(D57:D64)

67

68 For ties observations

69 formula $\frac{12}{n(n+1)} \sum_{i=1}^k \frac{n_i^2}{n_i} - 3(n+1)$ 70 2.328925 =([12/(D52*D52+D52)]*((D53^2/D49)+(D54^2/D50)-(D55^2/D51))-3*(D52+1))/(1-(D66/(D52*D53)))

71 $1 - \frac{\sum T}{n^2 - n}$

72

73

74

75 level of significance

alpha

0.05

76 degrees of freedom

df = D48-1

77 critical value

F_{0.05, k-1} = CHISQ.INV(1-D75, D76)

78 p value

p = 0.68791 = CHISQ.DIST(D71, D76, TRUE)

79

80 Decision

81 Significant approach

82 There is no reason to reject null hypothesis H0

=IF(D73<D77, "There is no reason to reject null hypothesis H0", "Reject Null Hypothesis H0")

83

84 p value approach

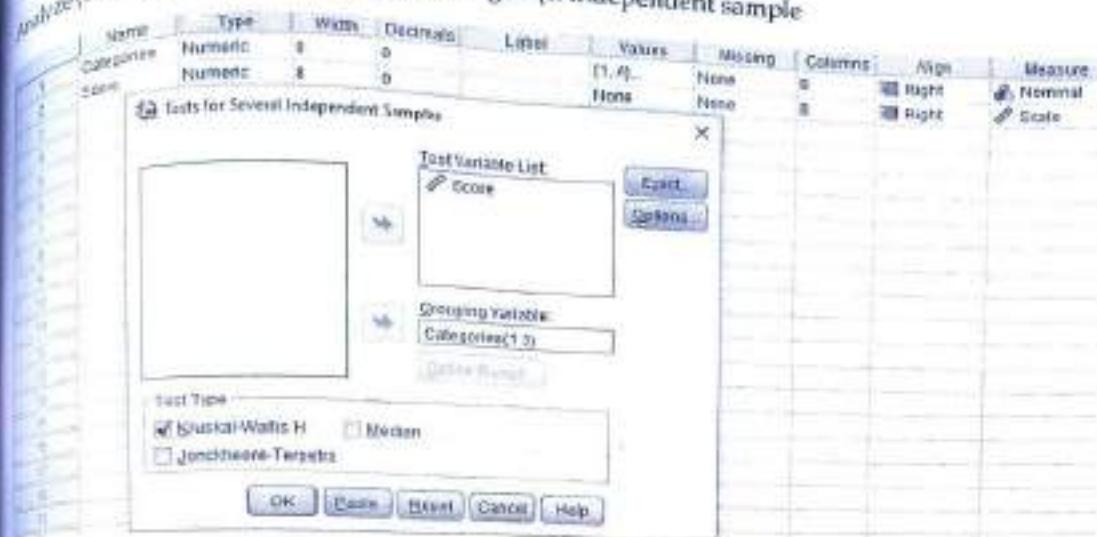
85 it is insignificant

=IF(D78>D75, "It is insignificant", "It is significant")

86

using SPSS

Analyze \ Non parametric test \ Legacy dialogue \ k independent sample



4 SPSS Tests

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	25th	Percentiles 50th (Median)	75th
Score	30	78.20	9.739	62	95	88.50	78.50	86.00
Categories	30	2.00	.830	1	3	1.00	2.00	3.00

Kruskal-Wallis Test

Ranks

Categories	N	Mean Rank
Score 1 A	10	12.05
2 B	10	16.95
3 C	10	17.50
Total	30	

Test Statistics^{a,b}

	Score
Kruskal-Wallis	2.329
df	2
Asymp. Sig.	.312

^a Kruskal-Wallis Test^b Grouping Variable: Categories

Using STATA

kwallis Score, by(Categories)

Kruskal-Wallis equality-of-populations rank test
Kwallis Score, by(Categories)

Category	Obs	Rank Sum
A	10	129.00
B	10	169.50
C	10	179.00

chi-squared = 2.323 with 2 d.f.

probability = 0.3530

chi-squared with ties = 2.329 with 2 d.f.

probability = 0.3121

Answer: As we found that P value with tie for the Chi-square test is 0.3121 which is larger than 0.05 we can conclude that the scores among three categories is not significantly different as null is accepted.

Friedman F test

A survey was conducted in four hospitals in a particular city to obtain the number of babies born over a 12 months' period. This time period was divided into four seasons to test the hypothesis that the birth rate is constant over all the four seasons. The results of the survey were as follows:

Hospital	No of births			
	Winter	Spring	Summer	Fall
A	92	72	94	77
B	15	16	10	17
C	58	71	51	62
D	19	26	20	18

Analyze the data using Friedman two way ANOVA test.
Using EXCEL

Here we assume that the depended variable (here birth) is normally distributed, which is the needed assumption for this test. Also, we perform Friedman test when we have one within-subject independent variable(hospital) with two or more levels(seasons) and a dependent variable. We use this test to determine if there is a difference in the birth by season, so the null hypothesis tested will be H0: birth is distributed homogeneously in all seasons vs H1: at least in some season it is different from other.

Hospital	No of births			
	Winter	Spring	Summer	Fall
A	92	72	94	77
B	15	16	10	17
C	58	71	51	62
D	19	26	20	18

A	B	C	D	E	F	G	H	I
ranks								
Hospital	Winter	Spring	Summer	Fall				
A	3	1	4	2				
B	2	3	1	4				
C	2	4	1	3				
D	2	4	3	1				
RI	9	12	9	10				
RI ²	81	144	81	100	406	=E15^2		
						=SUM(B16:E16)		

Test Hypothesis

H₀: The birth rate is constant over all four seasons.H₁: The birth rate is not constant over all four seasons.

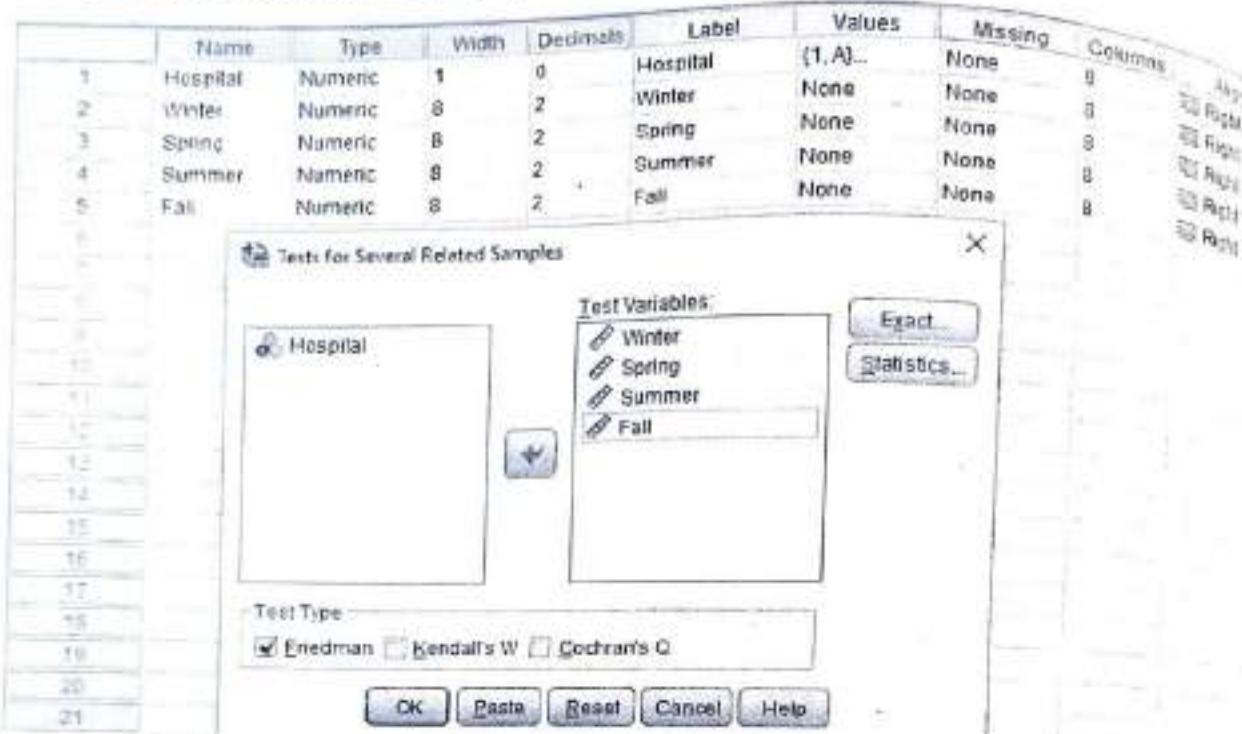
A	B	C	D	E	F	G	H
cases	symbols	value	formula				
sample size	n	4					
no of samples	k	4					

A	B	C	D	E	F	G	H
cases	symbols	value	formula				
sample size	n	4	=COUNT(B11:B14)				
no of samples	k	4	=COUNT(B11:E11)				
Test statistic	$F_r = \frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)$						
	F_r	0.9	= (12/(C25*C26*(C26+1)))*F16-3*C25*(C26+1)				
level of significance	α	0.05					
Critical value	$p=P(F_r > 0.9)$	0.9	from table				
decision	There is no reason to reject Null hypothesis H ₀						

Answer: As from the Friedman test the p-value of the test is 0.9 we conclude that the null hypothesis is correct. This indicates there is no seasonal effect found in the birth rate in the data.

Using SPSS

Analyze\Nonparametric test\Legacy Dialogs\k Related Samples

**Outputs****Friedman Test**

Ranks		Test Statistics ^a	
	Mean Rank	N	4
Winter	2.25	Chi-Square	.900
Spring	3.00	df	3
Summer	2.25	Asymp. Sig.	.825
Fall	2.50		

a. Friedman Test

Friedman test for repeated ranks

Three different advertising media T.V., Radio and Newspaper are being compared to study their effectiveness in promoting sales of WaiWai noodles. Each advertising media is exposed for specific period of time and sales (000 package) from 10 stores located at different areas are recorded.

Advertising Media	Stores								
	A	B	C	D	E	F	G	H	I
T.V.	20	21	15	12	14	17	21	16	20
Radio	7	9	11	12	10	10	14	12	8
News Paper	8	6	11	12	9	6	8	10	8

Are three advertising media equally effective, use Friedman two-way ANOVA test.
Using EXCEL

	B	C	D	E	F	G	H	I	J	K	L
	Stores										
Advertising Media	A	B	C	D	E	F	G	H	I	J	K
T.V.	20	21	15	12	14	17	21	16	20	18	
Radio	7	9	11	12	10	10	14	12	5	7	
News Paper	8	6	11	12	9	6	5	10	3	9	

	B	C	D	E	F	G	H	I	J	K	χ^2
Advertising Media	3	3	3	2	3	3	3	3	3	3	54.1
T.V.	1	2	1.5	2	2	2	2	2	1.5	2	32.4
Radio	2	1	1.5	2	1	1	1	1	1.5	2	13.9
News Paper											13.34

Test Hypothesis	
H0:	Three advertising media are equally effective.
H1:	Three advertising media are not equally effective.

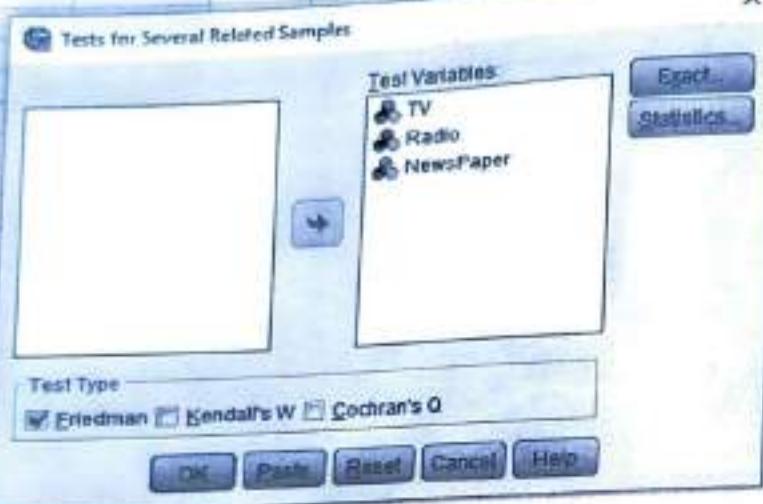
A	B	C	D	E	F	G	H	I	J	
cases	symbols	value	formula							
sample size	n		=COUNT(B11:B14)							
no of samples	k		=COUNT(B11:E11)							
	1.5 t1			2		repeats of C				
	2 t2			3		repeats of D				
	1.5 t3			2		repeats of I				
			$F_r = \frac{\frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)}{1 - \sum \frac{(t_i^2 - t_i)}{n(k^2 - k)}}$							
Test statistic										
			$F_r = 15.76471 = ((12/(C21*C22*(C22+1)))*M13-3*C21^4*(C22+1))/((1-(C23^3-C23)+(C24^3-C24)+(C25^3-C25))/(C21*(C22^3-C22)))$							
level of significance	α	0.05								
degrees of freedom	df			2 =C72-1						
Critical value	$\chi^2_{0.05}$	5.991465	=CHISQ.INV.RT(C32,C33)							
p value	p	0.000377	=CHISQ.DIST.RT(C30,C33)							
decision										
significant approach										
Reject H0										
P-value approach										
It is significant										

=IF(C34>C30, "There is no reason to reject null hypothesis H0", "Reject H0")
=IF(C35<C32, "It is insignificant", "It is significant")

Using SPSS

Analyze\Nonparametric test\Legacy Dialogs\k Related Samples

	Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align
1	Advertising	String	2	0	Advertising Me	{1, A}...	None	16	Left
2	TV	Numeric	8	2	TV	None	None	8	Right
3	Radio	Numeric	8	2	Radio	None	None	8	Right
4	NewsPaper	Numeric	8	2	News Paper	None	None	14	Right
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									



OUTPUTS

Friedman Test

Ranks	
	Mean Rank
TV T.V.	2.90
Radio Radio	1.80
NewsPaper News Paper	1.30

Test Statistics^a

N	10
Chi-Square	15.765
df	2
Asymp. Sig.	.000

a. Friedman Test

□□□