Analog 12.7

EE:1205 Signals and System
Indian Institute of Technology, Hyderabad

Prashant Maurya
EE23BTECH11218

Question 15: A 100μF capacitor in series with a 40Ω resistance is connected to a 110V, 60Hz supply.
(a) What is the maximum current in the circuit?
(b) What is the time lag between the current maximum and the voltage maximum?

Solution

Symbol	Value	Description
V	110 V	Voltage Supplied
ν	$60Hz$	Frequency
R	40 Ω	Resistance
C	100 μF	Capacitance
ω	$2\pi\nu$	Angular Frequency
ϕ	$tan^{-1}\frac{1}{\omega CR}$	Phase Angle
I_0	$\frac{V_0}{Z}$	Max Current
V_0	V $\sqrt{2}$	Peak Voltage
Z	$\sqrt{R^2+\frac{1}{\omega^2C^2}}$	Impedance
$H(s)$	$\frac{V(s)}{I(s)}$	Transfer Function

TABLE 1: Given Parameters

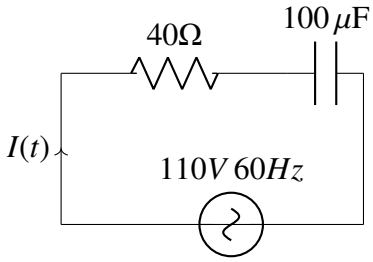

Fig. 1: RC Circuit

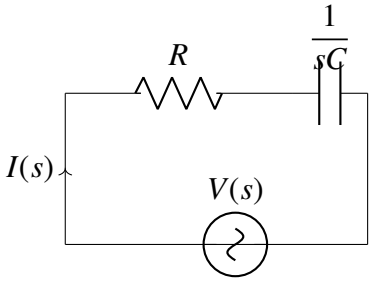

Fig. 2: RC Circuit

$$H(s) = R + \frac{1}{sC} \tag{1}$$

$$\implies H(j\omega) = R + \frac{1}{j\omega C} \tag{2}$$

$$= \sqrt{R^2 + \frac{1}{\omega^2C^2}} \tag{3}$$

$$= \sqrt{40^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}} \tag{4}$$

$$= 48 \tag{5}$$

$$\therefore I(s) = \frac{V(s)}{H(s)} \tag{6}$$

$$= \frac{110}{48} \tag{7}$$

$$= 2.29 \tag{8}$$

(a) Angular frequency:

$$\omega = 2\Pi\nu = 2\pi \times 60 \tag{9}$$

Current is given as:

$$\implies I = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2C^2}}} \tag{10}$$

$$= \frac{110}{\sqrt{1600 + \frac{10^8}{\omega^2}}} \tag{11}$$

For maximum current

$$\frac{dI_0}{d\omega} = 0 \tag{12}$$

$$\implies \frac{V_0}{\omega^3 c^2 \sqrt{R^2 + \frac{1}{(\omega^2 c^2)^3}}} = 0 \tag{13}$$

$$\omega = \infty \tag{14}$$

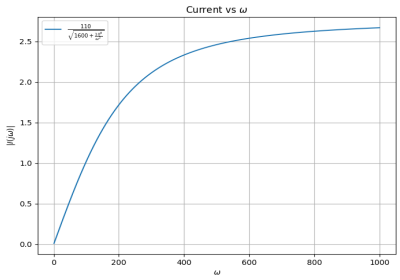

Fig. 3: Current vs ω

Maximum current at $\omega = 120\pi$:

$$\implies I_0 = \frac{V_0}{\sqrt{40^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}}} \tag{15}$$

$$= 3.24 \tag{16}$$

(b) In a capacitor circuit,the voltage lags behind the current by a phase angle of ϕ .
This angle is given by the relation:

$$tan\phi = \frac{1}{\omega CR} \tag{17}$$

$$= \frac{1}{120\pi \times 10^{-4} \times 40} \tag{18}$$

$$\implies \phi = \frac{33.56\pi}{180} rad \tag{19}$$

$$\therefore Time\ lag = \frac{\phi}{\omega} \tag{20}$$

$$= \frac{33.56\pi}{180 \times 120\pi} \tag{21}$$

$$= 1.55ms \tag{22}$$

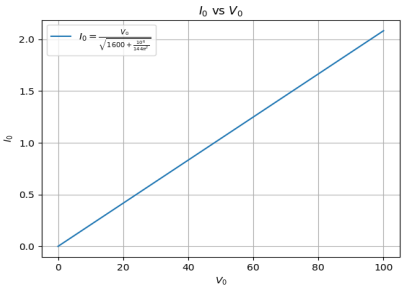

Fig. 4: I_0 vs V_0

(c) Plot of Impedance vs Angular Frequency

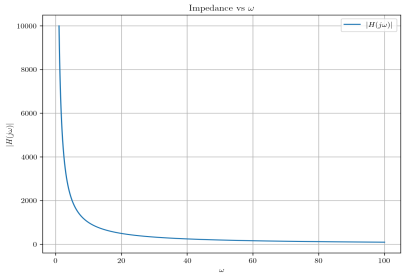

Fig. 5: Impedance vs ω