

Assignment-1

EE:1205 Signals and System
Indian Institute of Technology, Hyderabad

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I. QUESTION 12.7-15

A $100\mu\text{F}$ capacitor in series with a 40Ω resistance is connected to a 110V , 60Hz supply.

- (a) What is the maximum current in the circuit?
(b) What is the time lag between the current maximum and the voltage maximum?

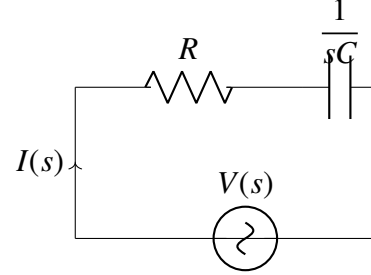


Fig. 2: RC Circuit

II. SOLUTION

Symbol	Value	Description
V	110V	Voltage Supplied
ν	60Hz	Frequency
R	40Ω	Resistance
C	$100\mu\text{F}$	Capacitance
ω	$2\pi\nu$	Angular Frequency
ϕ	$\tan^{-1} \frac{1}{\omega CR}$	Phase Angle
I_0	$\frac{V_0}{Z}$	Max Current
V_0	$V \sqrt{2}$	Peak Voltage
Z	$\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$	Impedance
$H(s)$	$\frac{V(s)}{I(s)}$	Transfer Function

TABLE 1: Given Parameters

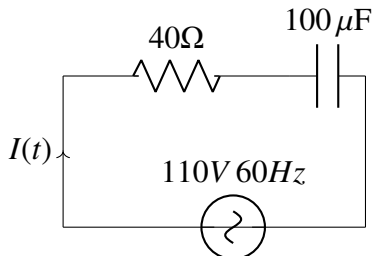


Fig. 1: RC Circuit

$$H(s) = R + \frac{1}{sC} \quad (1)$$

$$\Rightarrow H(j\omega) = R + \frac{1}{j\omega C} \quad (2)$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \quad (3)$$

$$\Rightarrow |H(j\omega)| = \sqrt{40^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}} \quad (4)$$

$$\Rightarrow |H(s)| = 48 \quad (5)$$

$$\therefore I(s) = \frac{V(s)}{H(s)} \quad (6)$$

$$\Rightarrow I(s) = \frac{110}{48} \quad (7)$$

$$\Rightarrow I(s) = 2.29 \quad (8)$$

(a) Angular frequency:

$$\omega = 2\pi\nu = 2\pi \times 60 \quad (9)$$

Current is given as:

$$I = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \quad (10)$$

$$\Rightarrow I = \frac{110}{\sqrt{1600 + \frac{10^8}{\omega^2}}} \quad (11)$$

For maximum current

$$\frac{dI_0}{d\omega} = 0 \quad (12)$$

$$\Rightarrow \frac{V_0}{\omega^3 c^2 \sqrt{R^2 + \frac{1}{(\omega^2 c^2)^3}}} = 0 \quad (13)$$

$$\Rightarrow \omega = \infty \quad (14)$$

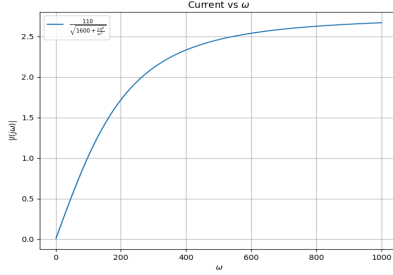


Fig. 3: Current vs ω

Maximum current at $\omega = 120\pi$:

$$\Rightarrow I_0 = \frac{V_0}{\sqrt{40^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}}} \quad (15)$$

$$\Rightarrow I_0 = 3.24 \quad (16)$$

(b) In a capacitor circuit, the voltage lags behind the current by a phase angle of ϕ .

This angle is given by the relation:

$$\tan\phi = \frac{1}{\omega CR} \quad (17)$$

$$\Rightarrow \tan\phi = \frac{1}{120\pi \times 10^{-4} \times 40} \quad (18)$$

$$\Rightarrow \phi = \frac{33.56\pi}{180} \text{ rad} \quad (19)$$

$$\therefore \text{Time lag} = \frac{\phi}{\omega} \quad (20)$$

$$\Rightarrow \text{Time lag} = \frac{33.56\pi}{180 \times 120\pi} \quad (21)$$

$$\Rightarrow \text{Time lag} = 1.55 \text{ ms} \quad (22)$$

Hence, the time lag between maximum current and maximum voltage is 1.55 ms.

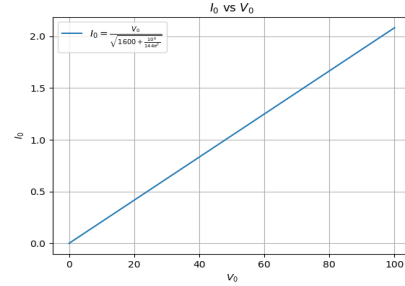


Fig. 4: I_0 vs V_0

(c) Plot of Impedance vs Angular Frequency

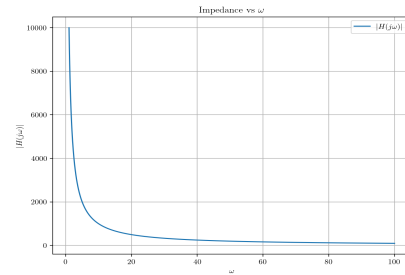


Fig. 5: Impedance vs ω