Assignment-1

EE:1205 Signals and System
Indian Institute of Technology, Hyderabad

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I. Question 12.7-15

A 100μF capacitor in series with a 40Ω resistance is connected to a 110V, 60Hz supply.
(a) What is the maximum current in the circuit?
(b) What is the time lag between the current maximum and the voltage maximum?

II. Solution

Symbol	Value	Description
V	110 V, 60Hz	Voltage Supplied
R	40 Ω	Resistance
C	100 μF	Capacitance
ω	$2\pi\nu$	Angular Frequency
ϕ	$tan^{-1}\frac{1}{\omega CR}$	Phase Angle
I_0	$\frac{V_0}{Z}$	Max Current
V_0	V $\sqrt{2}$	Peak Voltage
Z	$\sqrt{R^2+\frac{1}{\omega^2C^2}}$	Impedance
$H(s)$	$H(s)=\frac{V(s)}{I(s)}$	Transfer Function

TABLE 1: Given Parameters

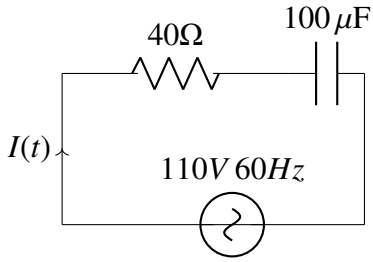

Fig. 1: RC Circuit

Capacitance of the capacitor:

$$C = 100\mu F = 100\times10^{-6} \tag{1}$$

Resistance of the resistor:

$$R = 40\Omega \tag{2}$$

Supply voltage:

$$V = 110V \tag{3}$$

(a) Frequency of oscillations:

$$\nu = 60Hz \tag{4}$$

Angular frequency:

$$\omega = 2\Pi\nu = 2\pi\times60 \tag{5}$$

Peak voltage:

$$V_0 = V\sqrt{2} = 110\sqrt{2}V \tag{6}$$

For an RC circuit,we have the relation for impedance as:

$$Z = R^2 + \frac{1}{\omega^2C^2} \tag{7}$$

Maximum current is given as:

$$I_0 = \frac{V_0}{Z} \tag{8}$$

From (7), (8)

$$\implies I_0 = \frac{V_0}{\sqrt{R^2+\frac{1}{\omega^2C^2}}} \tag{9}$$

$$\implies I_0 = \frac{V_0}{\sqrt{40^2+\frac{1}{(120\pi)^2\times(10^{-4})^2}}} \tag{10}$$

$$\implies I_0 = 3.24 \tag{11}$$

(b) In a capacitor circuit,the voltage lags behind the current by a phase angle of ϕ . This angle is given by the relation :

$$tan\phi = \frac{1}{\omega CR} \tag{12}$$

$$\implies tan\phi = \frac{1}{120\pi \times 10^{-4} \times 40} \tag{13}$$

$$\implies \phi = \frac{33.56\pi}{180} rad \tag{14}$$

$$\therefore Time\ lag = \frac{\phi}{\omega} \tag{15}$$

$$\implies Time\ lag = \frac{33.56\pi}{180 \times 120\pi} \tag{16}$$

$$\implies Time\ lag = 1.55ms \tag{17}$$

Hence,the time lag between maximum current and maximum voltage is $1.55ms$.

(c) Transfer Function is defined as:

$$H(s) = \frac{V(s)}{I(s)} \tag{18}$$

From (18)

$$H(s) = R + \frac{1}{sC} \tag{19}$$

$$\implies H(j\omega) = R + \frac{1}{j\omega C} \tag{20}$$

$$\implies |H(j\omega)| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \tag{21}$$

$$\implies |H(j\omega)| = \sqrt{40^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}} \tag{22}$$

$$\implies |H(s)| = 48 \tag{23}$$

$$\therefore I(s) = \frac{V(s)}{H(s)} \tag{24}$$

$$\implies I(s) = \frac{110}{48} \tag{25}$$

$$\implies I(s) = 2.29 \tag{26}$$

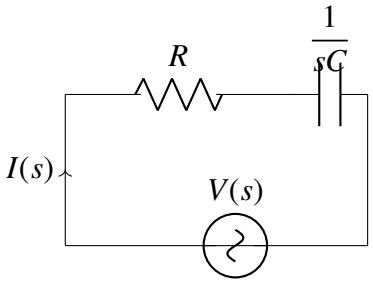

Fig. 2: RC Circuit

(d) Plot of Impedance vs Angular Frequency

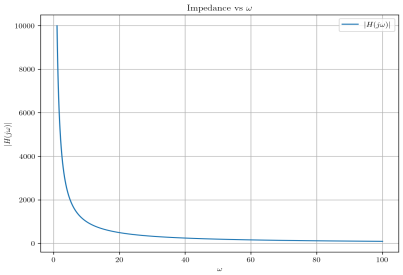

Fig. 3: Impedance vs ω

(e) Plot of Current vs Angular Frequency

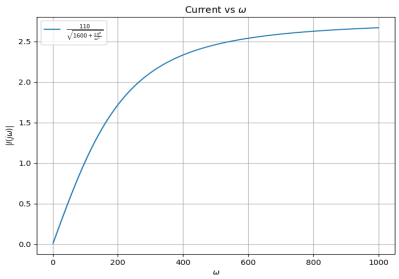

Fig. 4: Current vs ω