Analog 12.7

EE:1205 Signals and System

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Question 15: A 100μF capacitor in series with a 40Ω resistance is connected to a 110V, 60Hz supply.
(a) What is the maximum current in the circuit?
(b) What is the time lag between the current maximum and the voltage maximum?

Solution

Symbol	Value	Description
V	110 V	Voltage Supplied
ν	$60Hz$	Frequency
R	40 Ω	Resistance
C	100 μF	Capacitance
ω	$2\pi\nu$	Angular Frequency
ϕ	$tan^{-1}\frac{1}{\omega CR}$	Phase Angle
I_0	$\frac{V_0}{Z}$	Max Current
V_0	$V\sqrt{2}$	Peak Voltage
Z	$\sqrt{R^2+\frac{1}{\omega^2C^2}}$	Impedance
$H(s)$	$\frac{V(s)}{I(s)}$	Transfer Function

TABLE 1: Given Parameters

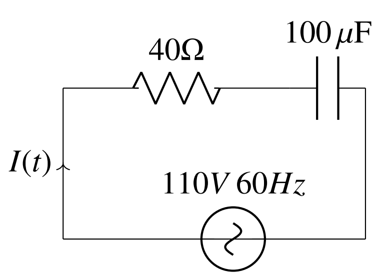

Fig. 1: RC Circuit

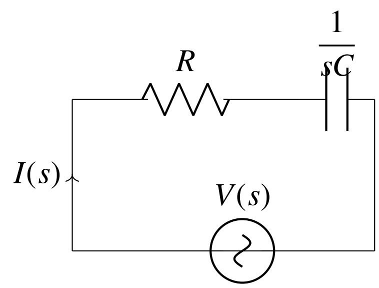

Fig. 2: RC Circuit

(a) V_{out} across capacitor,

$$V_{out} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}}V_{in} \tag{1}$$

$$\frac{V_{out}}{V_{in}} = H(s) \tag{2}$$

$$\implies H(s) = \frac{1}{1+sRC} \tag{3}$$

$$= \frac{1}{\sqrt{1+(\omega RC)^2}}e^{-tan^{-1}\frac{1}{(\omega RC)}} \tag{4}$$

On taking fourier transform of $H(s)$,

$$V_{out} = \frac{110}{\sqrt{1+(\omega RC)^2}}cos\left(\omega t - tan^{-1}\frac{1}{(\omega RC)}\right) \tag{5}$$

For current across circuit,

$$\implies I = C\frac{dV_{out}}{dt} \tag{6}$$

$$= \frac{110\omega C}{\sqrt{1+(\omega RC)^2}}sin\left(\omega t - tan^{-1}(\omega RC)\right) \tag{7}$$

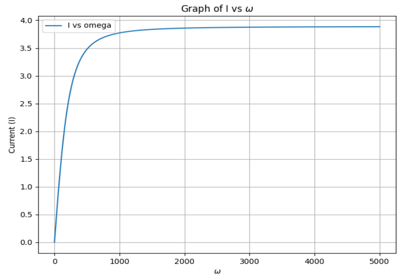

Fig. 3: Current vs ω

Maximum current in the circuit ,

$$\implies I_0 = \frac{110\omega C}{\sqrt{1 + (\omega RC)^2}} \tag{8}$$
$$= 3.24\, A \tag{9}$$

(b) In a capacitor circuit,the voltage lags behind the current by a phase angle of ϕ .

$$\implies \phi = tan^{-1} \frac{1}{(\omega RC)} \tag{10}$$
$$= \frac{33.56\pi}{180 \times 120\pi} \tag{11}$$

$$\phi = \frac{33.56\pi}{180} rad \tag{12}$$

$$\implies Time\ lag = \frac{\phi}{\omega} \tag{13}$$
$$= \frac{33.56\pi}{180 \times 120\pi} \tag{14}$$
$$= 1.55 ms \tag{15}$$

(c) Plot of Impedance vs Angular Frequency

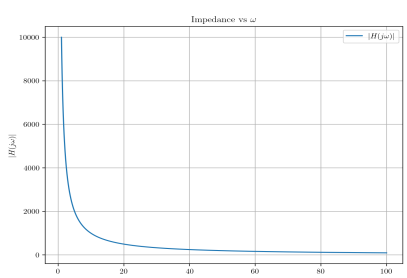

Fig. 4: Impedance vs ω