Assignment-1

EE:1205 Signals and System
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I. Question 12.7-15

A 100μF capacitor in series with a 40Ω resistance is connected to a 110V, 60Hz supply.
(a) What is the maximum current in the circuit?
(b) What is the time lag between the current maximum and the voltage maximum?

II. Solution

Capacitance of the capacitor:

$$C = 100\mu F = 100 \times 10^{-6} \tag{1}$$

Resistance of the resistor:

$$R = 40\Omega \tag{2}$$

Supply voltage:

$$V = 110V \tag{3}$$

(a) Frequency of oscillations:

$$v = 60Hz \tag{4}$$

Angular frequency:

$$\omega = 2\Pi v = 2\pi \times 60 \tag{5}$$

Peak voltage:

$$V_0 = V\sqrt{2} = 110\sqrt{2}V \tag{6}$$

For an RC circuit,we have the relation for impedance as:

$$Z = R^2 + \frac{1}{\omega^2 C^2} \tag{7}$$

Maximum current is given as:

$$I_0 = \frac{V_0}{Z} \tag{8}$$

From (7), (8)

$$I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \tag{9}$$

$$I_0 = \frac{V_0}{\sqrt{40^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}}} \tag{10}$$

$$I_0 = \frac{100\sqrt{2}}{\sqrt{1600 + \frac{10^8}{(120\pi)^2}}} \tag{11}$$

$$I_0 = 3.24 \tag{12}$$

(b) In a capacitor circuit,the voltage lags behind the current by a phase angle of ϕ . This angle is given by the relation :

$$tan\phi = \frac{1}{\omega CR} \tag{13}$$

$$tan\phi = \frac{1}{120\pi \times 10^{-4} \times 40} \tag{14}$$

$$\phi = tan^{-1}(0.6635) = 33.56\,\text{deg} = \frac{33.56\pi}{180} rad \tag{15}$$

$$\therefore Time\ lag = \frac{\phi}{\omega} \tag{16}$$

$$Time\ lag = \frac{33.56\pi}{180 \times 120\pi} \tag{17}$$

$$Time\ lag = 1.55 \times 10^{-3} s = 1.55ms \tag{18}$$

Hence,the time lag between maximum current and maximum voltage is $1.55ms$.