Assignment-2

EE:1205 Signals and System
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I. QUESTION 11.9.2 - 2

Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

II. SOLUTION

Parameter	Description	Value
$x(0)$	First Term	105
d	Common Difference	5
n	Total terms	179
$x(178)$	Last Term	995

TABLE 1: Given Parameters

$$x(n) = (105 + 5n)(u(n)) \tag{1}$$

$$X(z) = \frac{x(0)}{(1-z^{-1})} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{2}$$

$$= \frac{105}{1-z^{-1}} + \frac{5z^{-1}}{(1-z^{-1})^2} \tag{3}$$

$$X(z) = \frac{105 - 100z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \tag{4}$$

$$y(n) = x(n) * u(n) \tag{5}$$

$$Y(z) = X(z)U(z) \tag{6}$$

$$= \frac{105 - 100z^{-1}}{(1-z^{-1})^2} \frac{1}{(1-z^{-1})} \tag{7}$$

$$= \frac{105 - 100z^{-1}}{(1-z^{-1})^3} \quad |z| > 1 \tag{8}$$

Using contour integration to find the inverse Z-transform:

$$\implies y(178) = \frac{1}{2\pi j} \oint_C Y(z) \, z^{177} \, dz \tag{9}$$

$$= \frac{1}{2\pi j} \oint_C \frac{(105 - 100z^{-1})z^{177}}{(1-z^{-1})^3} \, dz \tag{10}$$

We can observe that there is only a 3 times repeated pole at z=1,

$$\implies R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}}((z-a)^m f(z)) \tag{11}$$

$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2}\left((z-1)^3 \frac{(105 - 100z^{-1})z^{180}}{(z-1)^3}\right) \tag{12}$$

$$= \frac{1}{2} \lim_{z \to 1} \frac{d^2}{dz^2}(105z^{180} - 100z^{179}) \tag{13}$$
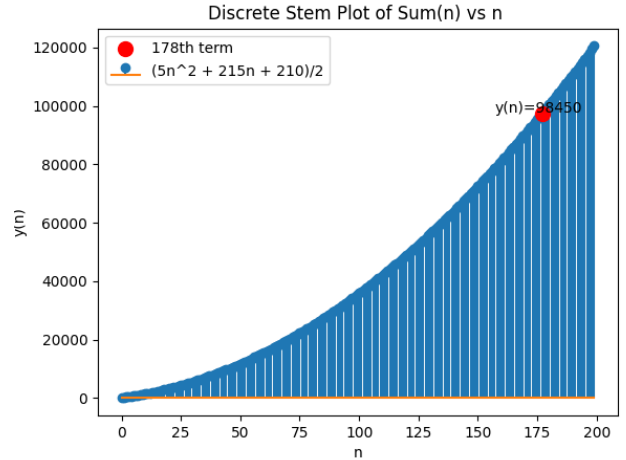
$$= 98450 \tag{14}$$

$$\therefore \boxed{y(178) = 98450} \tag{15}$$

Fig. 1: Plot of x(n) *vs* n