

Discrete 11.9.2

EE:1205 Signals and System
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Question-2 : Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Solution:

Parameter	Description	Value
$x(0)$	First Term	105
d	Common Difference	5
n	Total terms	179
$x(178)$	Last Term	995
m	No of poles	3

TABLE 1: Given Parameters

$$x(n) = (105 + 5n)(u(n)) \quad (1)$$

On taking Z transform

$$X(z) = \frac{x(0)}{(1 - z^{-1})} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (2)$$

$$= \frac{105}{1 - z^{-1}} + \frac{5z^{-1}}{(1 - z^{-1})^2} \quad (3)$$

$$\Rightarrow X(z) = \frac{105 - 100z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (4)$$

$$y(n) = x(n) * u(n) \quad (5)$$

$$\Rightarrow Y(z) = X(z) U(z) \quad (6)$$

$$= \frac{105 - 100z^{-1}}{(1 - z^{-1})^2} \frac{1}{(1 - z^{-1})} \quad (7)$$

$$= \frac{105 - 100z^{-1}}{(1 - z^{-1})^3} \quad |z| > 1 \quad (8)$$

Using contour integration to find the inverse Z-transform:

$$\Rightarrow y(178) = \frac{1}{2\pi j} \oint_C Y(z) z^{177} dz \quad (9)$$

$$= \frac{1}{2\pi j} \oint_C \frac{(105 - 100z^{-1}) z^{177}}{(1 - z^{-1})^3} dz \quad (10)$$

We can observe that there is only a 3 times repeated pole at $z = 1$,

$$\Rightarrow R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (11)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{(105 - 100z^{-1}) z^{180}}{(z-1)^3} \right) \quad (12)$$

$$= \frac{1}{2} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (105z^{180} - 100z^{179}) \quad (13)$$

$$= 98450 \quad (14)$$

$$\therefore y(178) = 98450 \quad (15)$$

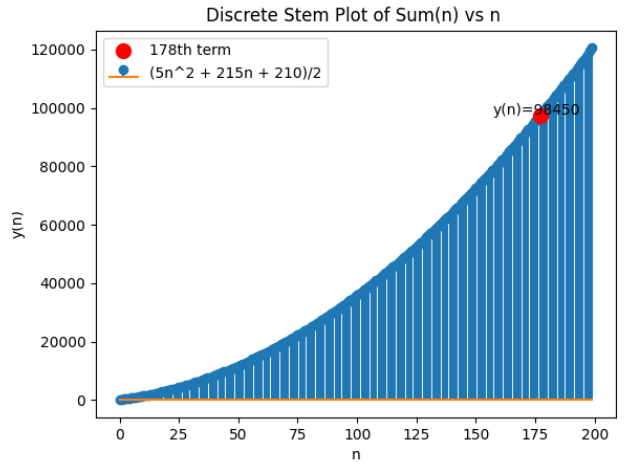


Fig. 1: Plot of $x(n)$ vs n