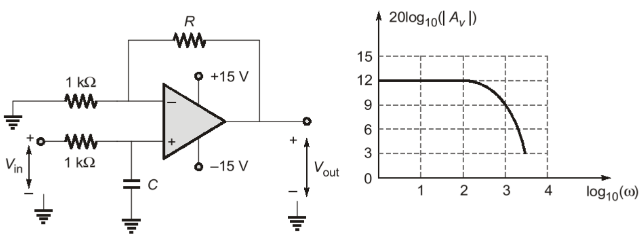
GATE 2022 EC

EE:1205 Signals and System

Indian Institute of Technology, Hyderabad

Prashant Maurya
EE23BTECH11218

Question 42: A circuit with an ideal OPAMP is shown. The Bode plot for the magnitude (in dB) of the gain transfer function $(A(j\omega)) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ of the circuit is also provided (here, ω is the angular frequency in rad/s). The values of R and C are



(A) $R = 3k\Omega$, $C = 1\mu F$

(B) $R = 1k\Omega$, $C = 3\mu F$

(C) $R = 4k\Omega$, $C = 1\mu F$

(D) $R = 3k\Omega$, $C = 2\mu F$

Solution

Parameter	Description	Value
R	Resistance	?
C	Capacitance	?
R_1	Resistance	1000
ω_{dB}	Cut-off frequency	1000
A_V	Gain Transfer	$\frac{V_{out}}{V_{in}}$

TABLE 1: Given Parameters

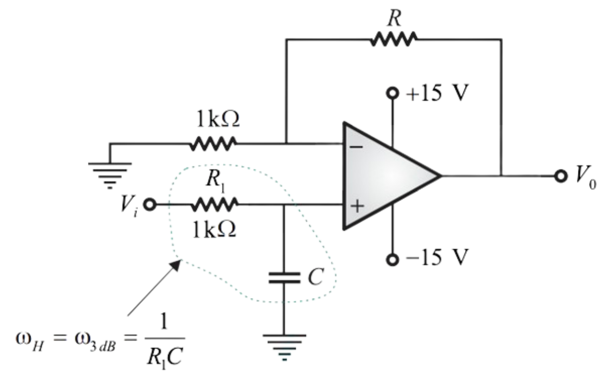


Fig. 1: Active Low Pass Filter

$$V_{IN} - R_1 i_1 - V_C = 0 \tag{1}$$

$$\frac{dV_i(t)}{dt} - R_1\frac{di_1}{dt} - \frac{i}{C} = 0 \tag{2}$$

$$i_1(s) = \frac{V_{IN}(s)\, s}{sR_1 + \frac{1}{C}} \tag{3}$$

$$V_C - Ri_2 - V_0 = 0 \tag{4}$$

$$\frac{i_1}{C} - R\frac{di_2}{dt} - \frac{d(V_0(t))}{dt} = 0 \tag{5}$$

$$\frac{i_1(s)}{C} - sRi_2(s) = sV_0(s) \tag{6}$$

From (3) and (6),

$$\frac{sV_{IN}(s)}{sR_1C + 1} - sRi_2(s) = sV_0(s) \tag{7}$$

$$0 - V_C = i_2 R_1 \tag{8}$$

$$-\frac{i_1(s)}{C} = sR_1 i_2(s) \tag{9}$$

From (3), (7) and (9) ,

$$-\frac{V_{IN}(s)}{R_1^2Cs + R_1} = \frac{V_{IN}(s)}{RsR_1C + R} - \frac{V_0(s)}{R} \tag{10}$$

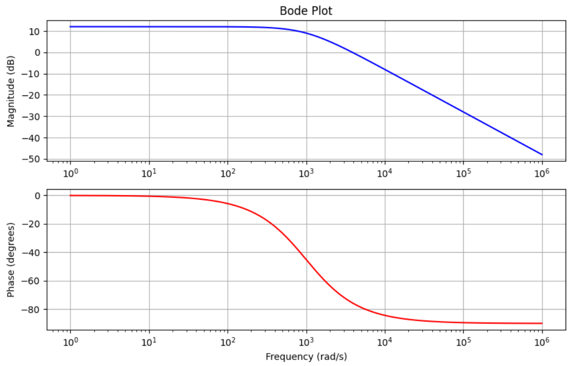


Fig. 2: bode plot

The 3-dB frequency from bode magnitude plot,

$$\implies \omega_{3dB} = 1000\ rad/sec \tag{11}$$

$$\omega_{3dB} = \frac{1}{R_1 C} \tag{12}$$
$$\implies C = 1\mu F \tag{13}$$

$$\implies A(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} \tag{14}$$
$$= \frac{1 + 10^{-3}R}{1 + sC10^3} \tag{15}$$
$$|A(s)| = \frac{1 + 10^{-3}R}{\sqrt{1 + \omega^2 10^{-6}}} \tag{16}$$
$$\tag{17}$$

A_V at low frequency,

$$|A_V| = 1 + 10^{-3}R \tag{18}$$
$$1 + 10^{-3}R = 10^{\frac{3}{5}} \tag{19}$$
$$R = 3k\Omega \tag{20}$$

Hence, The correct option is (A).