Discrete 11.9.5

EE:1205 Signals and System
Indian Institute of Technology, Hyderabad

Prashant Maurya
EE23BTECH11218

Question 29: A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8th set of letter is mailed.

Parameter	Description	Value
$x(0)$	First Term	4
r	Common Ratio	4
n	Total terms	8
$s(n)$	Sum of n terms of GP	$r^n x(0)u(n)$
m	No of poles	2

TABLE 1: Given Parameters

$$x(n) = x(0)\, r^n u(n) \tag{1}$$

On taking Z transform

$$X(z) = \frac{x(0)}{1 - rz^{-1}} \quad |z| > |r| \tag{2}$$

$$= \frac{4}{1 - 4z^{-1}} \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$\implies Y(z) = X(z)\, U(z) \tag{5}$$

$$= \frac{4}{(1 - 4z^{-1})} \frac{1}{(1 - z^{-1})} \quad |z| > |r| \cap |z| > |1| \tag{6}$$

Using contour integration to find the inverse Z-transform:

$$\implies y(7) = \frac{1}{2\pi j} \oint_C Y(z)\ z^6\, dz \tag{7}$$

$$= \frac{1}{2\pi j} \oint_C \frac{4}{(1 - 4z^{-1})(1 - z^{-1})}\, dz \tag{8}$$

$$= \frac{4}{3} \left(\frac{1}{2\pi j} \oint_C \frac{z^9}{z - 4}\, dz - \frac{1}{2\pi j} \oint_C \frac{z^9}{z - 1} \right) dz \tag{9}$$

We know that

$$\implies R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z) \right) \tag{10}$$

For first contour integral,

$$R_1 = \frac{1}{(1-1)!} \lim_{z \to a} \left((z - a)\, f(z) \right) \tag{11}$$

$$= r^{n+1} \tag{12}$$

For second contour integral,

$$R_2 = \frac{1}{(1-1)!} \lim_{z \to a} \left((z - a)\, f(z) \right) \tag{13}$$

$$= 1 \tag{14}$$

The sum of n terms of a GP is given by :

$$s(n) = \frac{x(0)}{r - 1} (R_1 - R_2) \tag{15}$$

$$= 87380 \tag{16}$$

$$\tag{17}$$

$$\therefore \text{Total amount spent on postage} = 87380 \times 0.5 \tag{18}$$

$$= Rs.\ 43690 \tag{19}$$

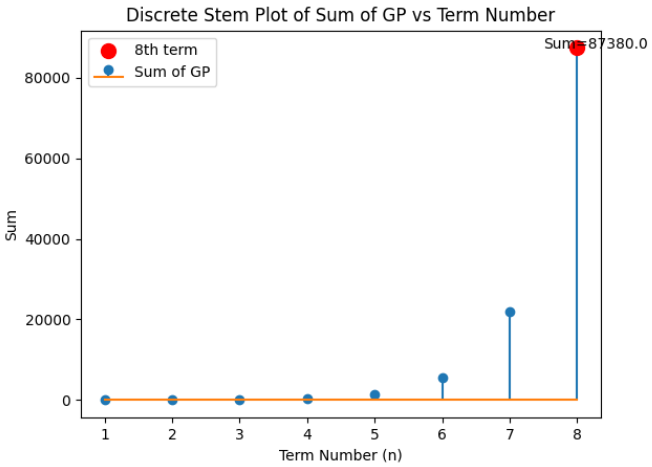

Fig. 1: Plot of $x(n)$ *vs* n