# GATE 2021 BM

## EE:1205 Signals and System
## Indian Institute of Technology, Hyderabad

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**Question 5:** Let $X(j\omega)$ denotes the Fourier transform of $x(t)$. If

$$X(j\omega) = 10e^{-j\pi f}\left(\frac{\sin(\pi f)}{\pi f}\right) \tag{1}$$

then $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,d\omega =$ _____ . (where $\omega = 2\pi f$)
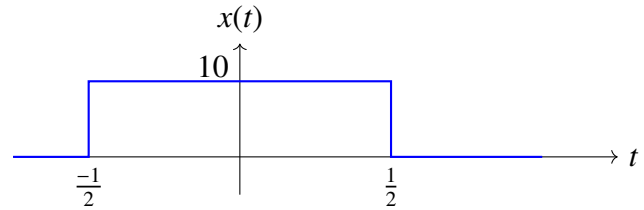
(A) $10\pi$

(B) $100$

(C) $10$

(D) $20\pi$

GATE 2021

**Solution**

$$A\,rect\left(\frac{t}{\tau}\right) \longleftrightarrow A\tau\,sinc(f\tau) \tag{2}$$

$$x(t) \longleftrightarrow X(j\omega) \tag{3}$$

$$x(t-a) \longleftrightarrow e^{-j\omega a}X(j\omega) \tag{4}$$



Fig. 1

From $X(j\omega)$, we can have $x(t)$ as

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,e^{j\omega t}d\omega \tag{5}$$

For $x = 0$,

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,d\omega \tag{6}$$

On comparing, we get $A = 10$ and $\tau = 1$,

$$10\,rect(t) \longleftrightarrow 10\,sinc(f) \tag{7}$$



Fig. 2

$$10\,rect\left(t - \frac{1}{2}\right) \longleftrightarrow 10e^{-j2\pi f \times \frac{1}{2}}\,sinc(f) \tag{8}$$
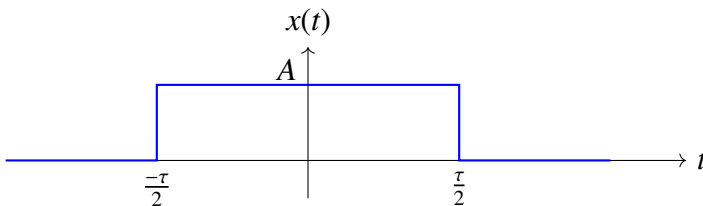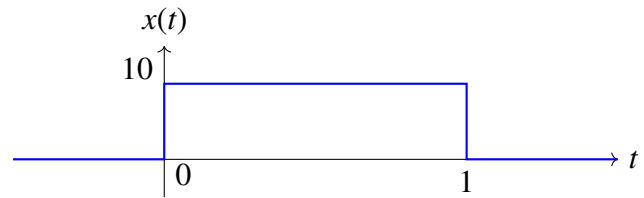


Fig. 3

From the above figure, $x(0)$ is 10.
Hence, the correct option is (C).