GATE 2021 BM

EE:1205 Signals and System
Indian Institute of Technology, Hyderabad

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Question 5: Let $X(j\omega)$ denotes the Fourier transform of $x(t)$. If

$$X(j\omega) = 10e^{-j\pi f}\left(\frac{\sin(\pi f)}{\pi f}\right) \tag{1}$$

then $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,d\omega =$ _____ . (where $\omega = 2\pi f$)

(A) 10π

(B) 100

(C) 10

(D) 20π

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Solution

$$Arect\left(\frac{t}{\tau}\right) \longleftrightarrow A\tau sinc(f\tau) \tag{2}$$

$$x(t) \longleftrightarrow X(j\omega) \tag{3}$$

$$x(t-a) \longleftrightarrow e^{-j\omega a}X(j\omega) \tag{4}$$

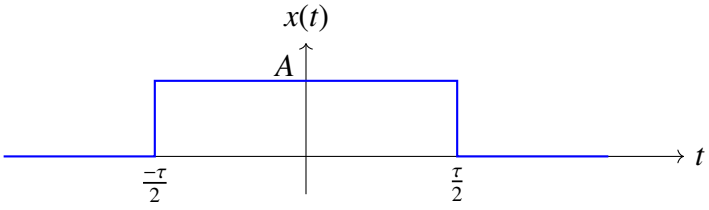

Fig. 1

Now , Fourier Transform of $x(t)$ is

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,e^{j\omega t}d\omega \tag{5}$$

For $x = 0$,

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,d\omega \tag{6}$$

On comparing , we get $A = 10$ and $\tau = 1$,

$$10rect(t) \longleftrightarrow 10sinc(f) \tag{7}$$

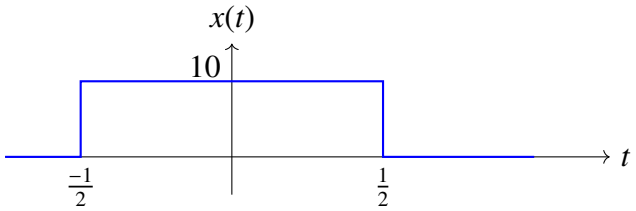

Fig. 2

$$10rect\left(t-\frac{1}{2}\right) \longleftrightarrow 10e^{-j2\pi f\times\frac{1}{2}}sinc(f) \tag{8}$$

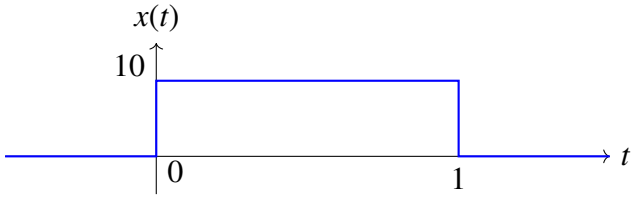

Fig. 3

From the above figure, $x(0)$ is 10.
Hence, the correct option is (C).