

MSE Loss in Scalar Notation
$$L = \frac{1}{N} \sum_{i=1}^N (\hat{y}_i - y_i)^2$$

$\hat{y}_i \Rightarrow$ Predictions from Model

$y_i \Rightarrow$ Actual values

$\hat{y} - y \Rightarrow$ Error terms / Residuals.

$$\hat{y} = w_0 + w_1 x$$

$$L = \frac{1}{N} \sum \left[(w_0 + w_1 x) - y \right]^2$$

$$L = \frac{1}{N} \sum \left[w_0^2 + w_1^2 x^2 + 2w_0 w_1 x - y^2 \right]$$

Gradients of Loss fn w.r.t. model coeff.

$$\frac{\partial L}{\partial w_0} = \frac{1}{N} \sum [2w_0 + 2w_1 x] = 0$$

$$\frac{\partial L}{\partial w_1} = \frac{1}{N} \sum [2w_1 x^2 + 2w_0 x] = 0$$

$$\frac{\partial L}{\partial w_0} = \frac{1}{N} \sum [w_0 + w_1 x] \cdot 1 = 0 \quad \text{--- ①}$$

$$\frac{\partial L}{\partial w_1} = \frac{1}{N} \sum [w_0 + w_1 x] \cdot x = 0 \quad \text{--- ②}$$

$$\text{②} = x \cdot \text{①}$$

$$w_1^* = \frac{(\bar{xy}) - (\bar{x}) \cdot (\bar{y})}{(\bar{x^2}) - (\bar{x})^2}$$

$$(\bar{x^2}) - (\bar{x})^2 \rightarrow \text{mean of } x^2 \text{ values} - (\text{mean of } x \text{ values})^2$$

$\bar{xy} \Rightarrow$ means mean of the products of values of x & y

x	y	xy	x ²
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-

Loss fn in Vector Notation: $\hat{y} = XW$

$$L = \frac{1}{N} (\hat{y} - y)^T (\hat{y} - y)$$

$$L = \frac{1}{N} (Y - \hat{Y})^T (Y - \hat{Y})$$

$$= \frac{1}{N} [(XW - Y)^T (XW - Y)]$$

$$L = \frac{1}{2N} [W^T X^T \cdot XW - Y^T XW - W^T X^T Y + Y^T Y]$$

$$\frac{\partial L}{\partial W} = \begin{bmatrix} \dots \\ \dots \end{bmatrix} = 0$$

$$\Rightarrow (X^T X)W - X^T Y = 0$$

Model cell.
for Linear Reg.

$$W = (X^T X)^{-1} \cdot X^T Y \rightarrow \text{SLR}$$

$$\rightarrow \text{MLR}$$

Model and Algo.

$$\text{Model} = \text{Algo}(X, Y)$$

$$\left\{ \begin{array}{l} \text{Model}_1 = \text{Algo}_1(\text{Data}) \\ \text{Model}_2 = \text{Algo}_2(\text{Data}) \\ \vdots \end{array} \right\} \begin{array}{l} \text{Same data} \\ \text{Change the} \\ \text{Algo} \end{array}$$

$$\begin{array}{c} DT_1 \\ | \\ \text{Hyperparam.} \end{array} \quad \text{vs} \quad \begin{array}{c} DT_2 \\ | \end{array} \quad \text{vs} \quad \begin{array}{c} DT_3 \\ | \end{array} \quad \dots$$

$$y = f(x)$$

$$\downarrow$$

$$x = z \quad \text{Some operation on } x$$

$$y = \sin(x)$$

$$y = 2x^2 - 3x$$

$$y = \cos(x) \cdot \log(x)$$