Vector Spaces

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Vector Spaces

$V = (x_1, x_2 \cdots x_6)$ $\quad V \in \mathbb{R}^6$

$\mathbb{R}^1 \rightarrow$ Straight line

$\mathbb{R}^2 \Rightarrow$ all possible points in the x-y plane ..

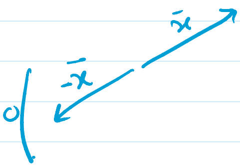
Properties of Vector Spaces

$\boxed{V \Rightarrow \text{Vector space}}$

$\underline{x}, \bar{y}$

① $\bar{x} + \vec{0} = \vec{x}$ for all $\vec{x} \in V$

② Additive Inverse: For all $\vec{x} \in V$, $\vec{x} + (-\vec{x}) = 0$



③ Multiplicative Indentity: $1 \cdot x = x$

④ Vectors are commutative: $\bar{x}, \bar{y} \in V$, $\bar{x} + \bar{y} = \bar{y} + \bar{x}$

⑤ Vectors are associative: $\bar{z}, \bar{x}, \bar{y} \in V$, $(\bar{x} + \bar{y}) + \bar{z} = \bar{x} + (\bar{y} + \bar{z})$

⑥ Distributive Prop: $\alpha(\bar{x} + \bar{y}) = \alpha\bar{x} + \alpha\bar{y}$

$(\alpha + \beta) \cdot \bar{x} = \alpha\bar{x} + \beta\bar{x}$

$\alpha, \beta \Rightarrow$ Scalars

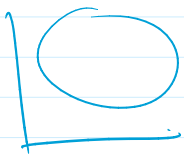
$\alpha, \beta \in \mathbb{R}$.

⑦ Vectors can be said to be linearly independent, if

$\alpha_1 \bar{V}_1 + \alpha_2 \bar{V}_2 + \cdots + \alpha_n \bar{V}_n = 0$

implies that $\alpha_1 = \alpha_2 \cdots = \alpha_n = \underline{0}$

Subspaces : $S \subseteq V$

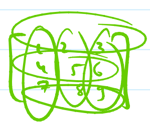


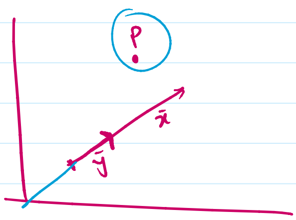
→ $\vec{0} \in S$

→ $\bar{x}, \bar{y} \in S$, and $\bar{x} + \bar{y} \in S$, then it implies that S is closed under addition.

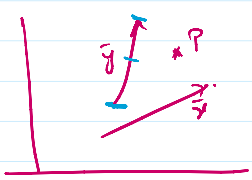
→ $\bar{x} \in S$ and $\alpha \in \mathbb{R}$, $\alpha\bar{x} \in S$, implies S is closed under scaler multiplication.

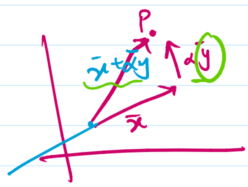
$R_4 = R_1 + 2R_2$



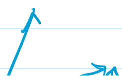


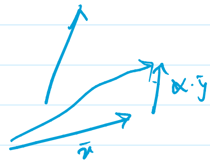
$\bar{x}, \bar{y}$ linearly dep





$\bar{x}$ & $\bar{y}$ linearly indep.





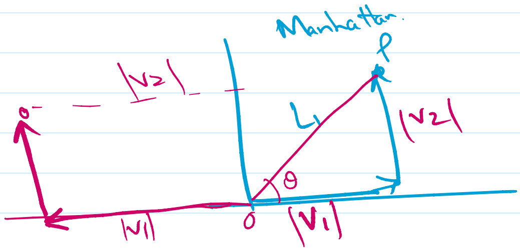
$v = [v_1 \ v_2 \ \cdots \ v_n]$

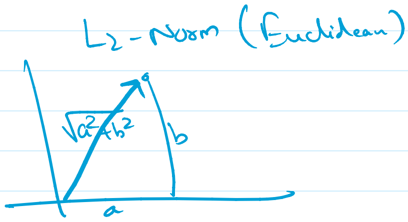
Vector Norm

p-Norm : $L_p = \left(\Sigma |v_j|^p \right)^{1/p}$

$p=1$ L_1-Norm (LASSO) ; $L_1 = |v_1| + |v_2| + \cdots + |v_n|$

$p=2$ L_2-Norm (Ridge) : $L_2 = \sqrt{v_1^2 + v_2^2 + \cdots v_n^2}$

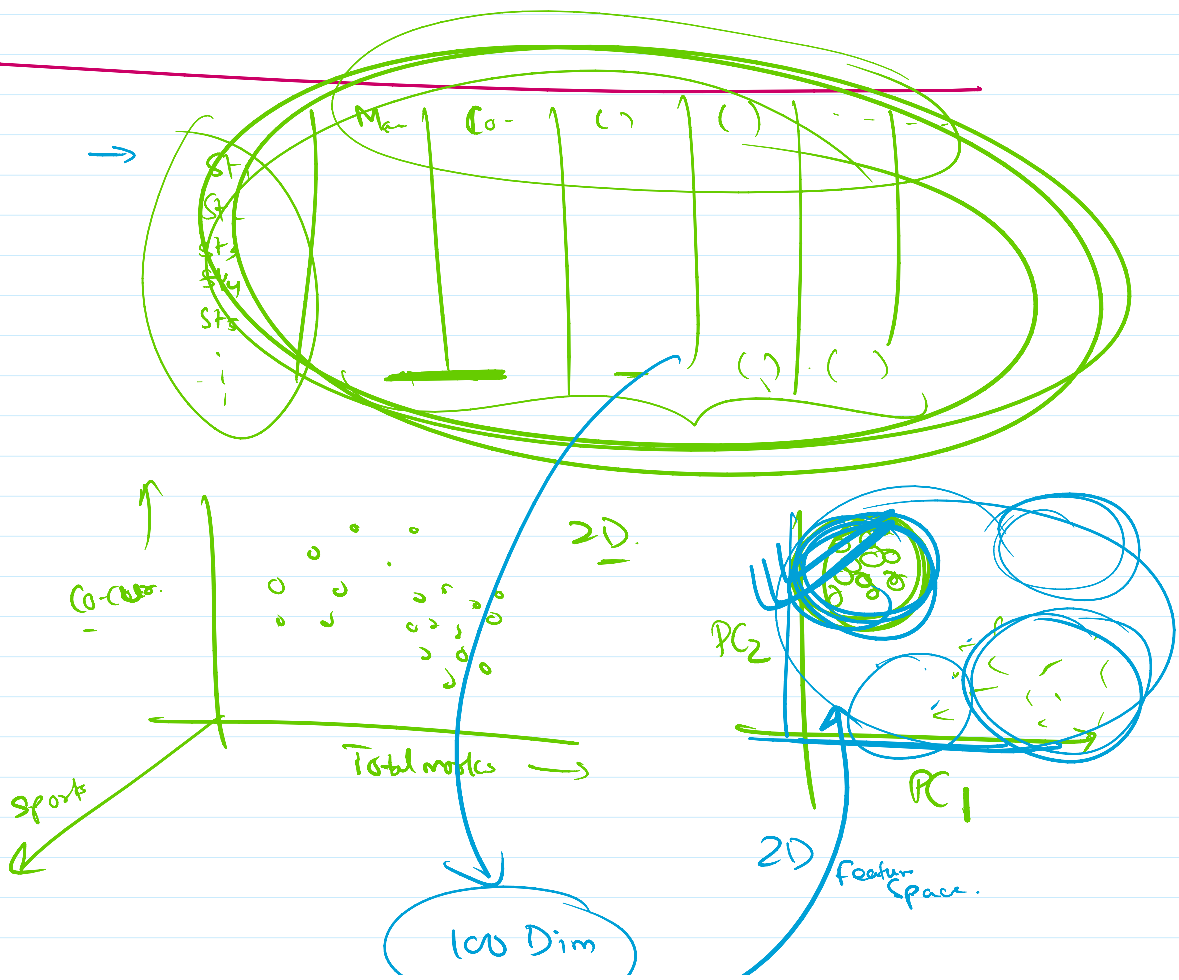




$v_1 = v \cos\theta$

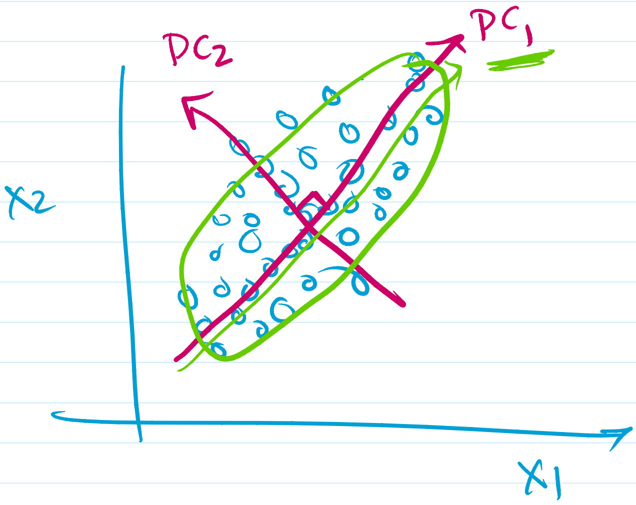
$v_2 = v \cdot \sin\theta$

PCA →



space.

100 Dim



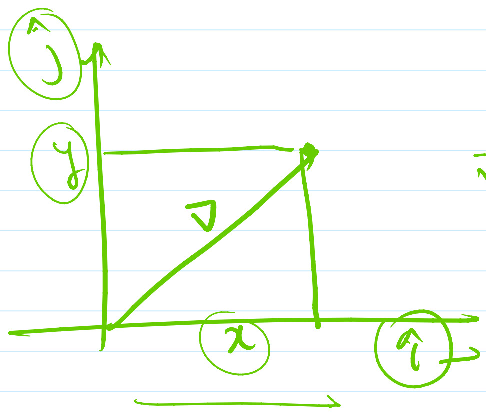
2D

each PC ⇒ unit vector

eigen.val. , eig. vect

10 → [−2.3 , 5.12] ⇒ PC1

2 → [−1.1 , 2.3] ⇒



$\bar{v} = x\hat{i} + y\hat{j}$