is605 Assignment 5

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March 10, 2015

Problem Set 1.

```
A <- matrix(c(1,1,1,1,0,1,3,4), nrow = 4, ncol = 2)
A
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    1    1
## [3,]    1    3
## [4,]    1    4
```

```
Atrans <- t(A)
Atrans
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    1    1
## [2,]    0    1    3    4
```

```
# Compute A^TA
AtA <- Atrans%*%A
AtA
```

```
##      [,1] [,2]
## [1,]    4    8
## [2,]    8   26
```

```
b <- matrix(c(0,8,8,10), nrow = 4, ncol = 1)
b
```

```
##      [,1]
## [1,]    0
## [2,]    8
## [3,]    8
## [4,]   10
```

```
# Compute A^Tb
Atb <- Atrans%*%b
Atb
```

```
##      [,1]
## [1,]   26
## [2,]   72
```

```r
# Compute Least Squares Approximate Solution
least_sq_approx_sol <- solve(AtA)%*%Atb
least_sq_approx_sol
```

```
##      [,1]
## [1,]  2.5
## [2,]  2.0
```

```r
# ||b - Ax^hat||
b-(A%*%least_sq_approx_sol)
```

```
##      [,1]
## [1,] -2.5
## [2,]  3.5
## [3,] -0.5
## [4,] -0.5
```

```r
# applying the distance formula to the results we get a least sq error of:
sqrt((-1.15)^2+(3.05)^2+(-4.55)^2+(2.65)^2)
```

```
## [1] 6.193
```

```r
p <- matrix(c(1,5,13,17))
Atp <- Atrans%*%p
Atp
```

```
##      [,1]
## [1,]   36
## [2,]  112
```

```r
# Compute Least Squares Exact Solution
least_sq_exact_sol <- solve(AtA)%*%Atp
p-(A%*%least_sq_exact_sol)
```

```
##            [,1]
## [1,]  0.000e+00
## [2,] -8.882e-16
## [3,] -3.553e-15
## [4,] -3.553e-15
```

```r
# applying the distance formula to the results we get a least sq error of:
sqrt((0)^2+(-8.881784e-16)^2+(-3.552714e-15)^2+(-3.552714e-15)^2)
```

```
## [1] 5.102e-15
```

```r
# above we see that the error goes to 0 when using p instead of b.
e <- b - p
e
```

```
##      [,1]
## [1,]   -1
## [2,]    3
## [3,]   -5
## [4,]   -7
```

```
# here we can see that e is orthogonal to p
e %*%t(p)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -1   -5  -13  -17
## [2,]    3   15   39   51
## [3,]   -5  -25  -65  -85
## [4,]   -7  -35  -91 -119
```

```
# here we can see that e is orthogonal to the first column of A
e %*% t(A[1])
```

```
##      [,1]
## [1,]   -1
## [2,]    3
## [3,]   -5
## [4,]   -7
```

```
# here we can see that e is orthogonal to the second column of A
e %*% t(A[2])
```

```
##      [,1]
## [1,]   -1
## [2,]    3
## [3,]   -5
## [4,]   -7
```

Problem Set 2.

```
setwd("~/Desktop")
auto_data <- read.csv("auto.csv", header = FALSE)
head(auto_data)
```

```
##    V1  V2   V3   V4 V5
## 1 307 130 3504 12.0 18
## 2 350 165 3693 11.5 15
## 3 318 150 3436 11.0 18
## 4 304 150 3433 12.0 16
## 5 302 140 3449 10.5 17
## 6 429 198 4341 10.0 15
```

```
AD <- data.frame(auto_data)
disp <- AD$V1
horsepower <- AD$V2
weight <- AD$V3
```

```r
accel <- AD$V4
mpg <- AD$V5

# Create A Matrix with First 4 Col Variables
Adf <- data.frame(disp, horsepower, weight, accel)
A_matrix <- as.matrix(Adf)
b_mpg <- (mpg)

# Compute the Best Fitting Solution
res <- lm(A_matrix ~ b_mpg)

# Compute ANOVA
anova(res)
```

```
## Analysis of Variance Table
##
##              Df Pillai approx F num Df den Df Pr(>F)
## (Intercept)   1  0.994    15172      4    387 <2e-16 ***
## b_mpg         1  0.707      233      4    387 <2e-16 ***
## Residuals   390
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Compute Error and Summary
summary(res)
```

```
## Response disp :
##
## Call:
## lm(formula = disp ~ b_mpg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -183.20  -37.50   -3.19   40.02  224.69
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  447.501      9.949    45.0   <2e-16 ***
## b_mpg        -10.795      0.403   -26.8   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 62.1 on 390 degrees of freedom
## Multiple R-squared:  0.648,  Adjusted R-squared:  0.647
## F-statistic:  719 on 1 and 390 DF,  p-value: <2e-16
##
##
## Response horsepower :
##
## Call:
## lm(formula = horsepower ~ b_mpg)
##
## Residuals:
```

```
##    Min     1Q Median     3Q    Max
## -64.89 -15.72  -2.09  13.11  96.95
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  194.476      3.873    50.2   <2e-16 ***
## b_mpg         -3.839      0.157   -24.5   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 24.2 on 390 degrees of freedom
## Multiple R-squared:  0.606,  Adjusted R-squared:  0.605
## F-statistic:  600 on 1 and 390 DF,  p-value: <2e-16
##
##
## Response weight :
##
## Call:
## lm(formula = weight ~ b_mpg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1346.8  -325.9   -23.8   318.1  1355.6
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5101.11      75.49    67.6   <2e-16 ***
## b_mpg         -90.57       3.06   -29.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 472 on 390 degrees of freedom
## Multiple R-squared:  0.693,  Adjusted R-squared:  0.692
## F-statistic:  879 on 1 and 390 DF,  p-value: <2e-16
##
##
## Response accel :
##
## Call:
## lm(formula = accel ~ b_mpg)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.128 -1.726 -0.224  1.472  8.697
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  12.0330     0.4007   30.03   <2e-16 ***
## b_mpg         0.1496     0.0162    9.23   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.5 on 390 degrees of freedom
## Multiple R-squared:  0.179,  Adjusted R-squared:  0.177
```

```
## F-statistic: 85.2 on 1 and 390 DF,  p-value: <2e-16
```

```
# Diff in Error
aggregate(A_matrix ~ mpg, auto_data, function(x) c(M = mean(x), SE = sd(x)/sqrt(length(x))))
```

```
##      mpg  disp.M disp.SE horsepower.M horsepower.SE weight.M weight.SE
## 1    9.0 304.000      NA      193.000            NA  4732.00        NA
## 2   10.0 333.500  26.500      207.500         7.500  4495.50    119.50
## 3   11.0 374.250  24.854      187.000        14.107  4419.00    281.55
## 4   12.0 394.500  17.295      185.000         9.606  4786.50     98.15
## 5   13.0 353.000   8.725      158.350         4.815  4254.45     99.77
## 6   14.0 359.526  12.942      167.947         6.866  4171.58     86.75
## 7   14.5 351.000      NA      152.000            NA  4215.00        NA
## 8   15.0 326.250  14.195      140.438         9.079  3805.25     94.60
## 9   15.5 344.600  16.576      153.400        12.106  4129.20     60.56
## 10  16.0 309.692  17.346      141.462        10.592  3983.85    122.38
## 11  16.2 163.000      NA      133.000            NA  3410.00        NA
## 12  16.5 289.667  60.834      146.000        17.776  4051.67    168.73
## 13  16.9 350.000      NA      155.000            NA  4360.00        NA
## 14  17.0 259.286  19.562      123.571         6.789  3628.14    126.39
## 15  17.5 287.200  13.818      126.000         9.925  3777.60    187.27
## 16  17.6 263.500  38.500      107.000        22.000  3595.00    130.00
## 17  17.7 231.000      NA      165.000            NA  3445.00        NA
## 18  18.0 224.412  14.544      102.941         4.062  3144.18     98.00
## 19  18.1 280.000  22.000      129.500         9.500  3307.50    102.50
## 20  18.2 318.000      NA      135.000            NA  3830.00        NA
## 21  18.5 286.667  36.667      119.333        15.720  3703.33    123.30
## 22  18.6 225.000      NA      110.000            NA  3620.00        NA
## 23  19.0 186.250  18.384       96.917         2.369  2994.33    118.24
## 24  19.1 225.000      NA       90.000            NA  3381.00        NA
## 25  19.2 267.667  21.365      125.000        11.547  3521.67     52.39
## 26  19.4 275.000  43.000      115.000        25.000  3472.50    262.50
## 27  19.8 200.000      NA       85.000            NA  2990.00        NA
## 28  19.9 260.000      NA      110.000            NA  3365.00        NA
## 29  20.0 172.667  19.442       99.778         3.609  2901.56    144.72
## 30  20.2 233.500  24.047      100.500        12.874  3215.00    133.87
## 31  20.3 131.000      NA      103.000            NA  2830.00        NA
## 32  20.5 218.667   9.493      100.000         2.887  3336.67     90.84
## 33  20.6 228.000   3.000      107.500         2.500  3370.00     10.00
## 34  20.8 200.000      NA       85.000            NA  3070.00        NA
## 35  21.0 166.714  16.414       91.000         5.014  2621.71    112.48
## 36  21.1 134.000      NA       95.000            NA  2515.00        NA
## 37  21.5 144.000  45.081      111.667         1.667  2855.00    198.05
## 38  21.6 121.000      NA      115.000            NA  2795.00        NA
## 39  22.0 166.300  17.108       93.500         3.911  2770.70    109.89
## 40  22.3 140.000      NA       88.000            NA  2890.00        NA
## 41  22.4 231.000      NA      110.000            NA  3415.00        NA
## 42  22.5 232.000      NA       90.000            NA  3085.00        NA
## 43  23.0 155.778  26.040       89.000         6.263  2740.67    167.30
## 44  23.2 156.000      NA      105.000            NA  2745.00        NA
## 45  23.5 173.000      NA      110.000            NA  2725.00        NA
## 46  23.7  70.000      NA      100.000            NA  2420.00        NA
## 47  23.8 151.000      NA       85.000            NA  2855.00        NA
## 48  23.9 189.500  70.500       93.500         3.500  2912.50    507.50
```

```
## 49  24.0 124.818   8.488       91.182        3.168  2510.82     85.48
## 50  24.2 146.000      NA      120.000           NA  2930.00        NA
## 51  24.3 151.000      NA       90.000           NA  3003.00        NA
## 52  24.5 124.500  26.500       74.000       14.000  2452.00    288.00
## 53  25.0 121.250   8.389       90.100        4.540  2457.40     83.68
## 54  25.1 140.000      NA       88.000           NA  2720.00        NA
## 55  25.4 175.500   7.500       96.500       19.500  3215.00    315.00
## 56  25.5 131.000   9.000       92.500        3.500  2527.50    227.50
## 57  25.8 156.000      NA       92.000           NA  2620.00        NA
## 58  26.0 105.214   5.018       76.643        4.747  2206.00     56.65
## 59  26.4 140.000      NA       88.000           NA  2870.00        NA
## 60  26.5 140.000      NA       72.000           NA  2565.00        NA
## 61  26.6 250.500  99.500       94.500       10.500  3180.00    545.00
## 62  26.8 173.000      NA      115.000           NA  2700.00        NA
## 63  27.0 115.889   8.089       84.556        3.150  2390.11    130.11
## 64  27.2 131.667   6.566       84.000        7.506  2660.00    270.62
## 65  27.4 121.000      NA       80.000           NA  2670.00        NA
## 66  27.5 134.000      NA       95.000           NA  2560.00        NA
## 67  27.9 156.000      NA      105.000           NA  2800.00        NA
## 68  28.0 112.800   6.266       84.500        2.088  2349.10     70.49
## 69  28.1 141.000      NA       80.000           NA  3230.00        NA
## 70  28.4 151.000      NA       90.000           NA  2670.00        NA
## 71  28.8 173.000      NA      115.000           NA  2595.00        NA
## 72  29.0  95.000   6.676       70.125        4.665  2078.88     77.17
## 73  29.5  97.500   0.500       69.500        1.500  1980.00    155.00
## 74  29.8 111.500  22.500       76.000       14.000  2278.00    433.00
## 75  29.9  98.000      NA       65.000           NA  2380.00        NA
## 76  30.0 107.714   9.314       73.143        2.604  2295.57    165.99
## 77  30.5  97.500   0.500       70.500        7.500  2120.50     69.50
## 78  30.7 145.000      NA       76.000           NA  3160.00        NA
## 79  30.9 105.000      NA       75.000           NA  2230.00        NA
## 80  31.0  89.571   7.111       69.429        4.191  2091.00    151.91
## 81  31.3 120.000      NA       75.000           NA  2542.00        NA
## 82  31.5  93.500   4.500       69.500        1.500  2017.50     27.50
## 83  31.6 120.000      NA       74.000           NA  2635.00        NA
## 84  31.8  85.000      NA       65.000           NA  2020.00        NA
## 85  31.9  89.000      NA       71.000           NA  1925.00        NA
## 86  32.0 101.500  12.361       73.833        5.474  2125.67    124.19
## 87  32.1  98.000      NA       70.000           NA  2120.00        NA
## 88  32.2 108.000      NA       75.000           NA  2265.00        NA
## 89  32.3  97.000      NA       67.000           NA  2065.00        NA
## 90  32.4 107.500   0.500       73.500        1.500  2320.00     30.00
## 91  32.7 168.000      NA      132.000           NA  2910.00        NA
## 92  32.8  78.000      NA       52.000           NA  1985.00        NA
## 93  32.9 119.000      NA      100.000           NA  2615.00        NA
## 94  33.0  95.667   4.667       60.000        7.000  1926.67    131.67
## 95  33.5 111.333  20.185       81.000        5.859  2192.00    185.83
## 96  33.7 107.000      NA       75.000           NA  2210.00        NA
## 97  33.8  97.000      NA       67.000           NA  2145.00        NA
## 98  34.0 110.000   2.000       79.000        9.000  2320.00     75.00
## 99  34.1  88.500   2.500       66.500        1.500  1980.00      5.00
## 100 34.2 105.000      NA       70.000           NA  2200.00        NA
## 101 34.3  97.000      NA       78.000           NA  2188.00        NA
## 102 34.4  98.000      NA       65.000           NA  2045.00        NA
```

```
## 103 34.5 105.000      NA       70.000           NA  2150.00        NA
## 104 34.7 105.000      NA       63.000           NA  2215.00        NA
## 105 35.0  97.000  25.000       78.500        9.500  2056.50    443.50
## 106 35.1  81.000      NA       60.000           NA  1760.00        NA
## 107 35.7  98.000      NA       80.000           NA  1915.00        NA
## 108 36.0 107.333   7.792       74.833        4.339  2110.83     76.91
## 109 36.1  94.500   3.500       63.000        3.000  1800.00      0.00
## 110 36.4 121.000      NA       67.000           NA  2950.00        NA
## 111 37.0  98.333  10.477       75.000        8.544  2144.67    145.38
## 112 37.2  86.000      NA       65.000           NA  2019.00        NA
## 113 37.3  91.000      NA       69.000           NA  2130.00        NA
## 114 37.7  89.000      NA       62.000           NA  2050.00        NA
## 115 38.0 137.250  41.714       70.500        4.924  2275.00    249.10
## 116 38.1  89.000      NA       60.000           NA  1968.00        NA
## 117 39.0  86.000      NA       64.000           NA  1875.00        NA
## 118 39.1  79.000      NA       58.000           NA  1755.00        NA
## 119 39.4  85.000      NA       70.000           NA  2070.00        NA
## 120 40.8  85.000      NA       65.000           NA  2110.00        NA
## 121 41.5  98.000      NA       76.000           NA  2144.00        NA
## 122 43.1  90.000      NA       48.000           NA  1985.00        NA
## 123 43.4  90.000      NA       48.000           NA  2335.00        NA
## 124 44.0  97.000      NA       52.000           NA  2130.00        NA
## 125 44.3  90.000      NA       48.000           NA  2085.00        NA
## 126 44.6  91.000      NA       67.000           NA  1850.00        NA
## 127 46.6  86.000      NA       65.000           NA  2110.00        NA
##     accel.M accel.SE
## 1   18.5000       NA
## 2   14.5000   0.5000
## 3   12.3750   0.8004
## 4   12.0833   0.3745
## 5   12.9350   0.2889
## 6   12.2895   0.5344
## 7   12.8000       NA
## 8   13.5875   0.9461
## 9   13.1000   0.5541
## 10  14.3923   0.8082
## 11  15.8000       NA
## 12  14.0000   1.3868
## 13  14.9000       NA
## 14  15.2143   1.4355
## 15  14.6800   1.0302
## 16  15.0000   1.6000
## 17  13.4000       NA
## 18  15.3529   0.5732
## 19  13.1500   1.9500
## 20  15.2000       NA
## 21  16.0667   1.7333
## 22  18.7000       NA
## 23  16.2583   0.6806
## 24  18.7000       NA
## 25  15.8000   1.7776
## 26  15.2000   2.0000
## 27  18.2000       NA
## 28  15.5000       NA
```

```
## 29  16.1556   0.7524
## 30  15.9750   1.1665
## 31  15.9000       NA
## 32  17.4333   0.3930
## 33  16.2000   0.4000
## 34  16.7000       NA
## 35  16.5000   0.8309
## 36  14.8000       NA
## 37  13.9000   0.7767
## 38  15.7000       NA
## 39  15.8600   0.4956
## 40  17.3000       NA
## 41  15.8000       NA
## 42  17.6000       NA
## 43  16.9889   0.9495
## 44  16.7000       NA
## 45  12.6000       NA
## 46  12.5000       NA
## 47  17.6000       NA
## 48  18.5500   3.6500
## 49  15.4091   0.3709
## 50  13.8000       NA
## 51  20.1000       NA
## 52  19.0500   3.0500
## 53  16.1200   0.4599
## 54  15.4000       NA
## 55  16.3500   3.7500
## 56  15.6500   0.1500
## 57  14.4000       NA
## 58  16.7429   0.7081
## 59  18.1000       NA
## 60  13.6000       NA
## 61  17.7000   1.3000
## 62  12.9000       NA
## 63  16.5889   0.5741
## 64  18.4000   3.2130
## 65  15.0000       NA
## 66  14.2000       NA
## 67  14.4000       NA
## 68  16.2600   0.5598
## 69  20.4000       NA
## 70  16.0000       NA
## 71  11.3000       NA
## 72  16.6125   1.0127
## 73  14.4000   2.2000
## 74  15.4000   0.1000
## 75  20.7000       NA
## 76  16.6286   1.1633
## 77  15.5500   1.4500
## 78  19.6000       NA
## 79  14.5000       NA
## 80  17.6714   0.5532
## 81  17.5000       NA
## 82  16.7000   1.8000
```

```
## 83  18.3000       NA
## 84  19.2000       NA
## 85  14.0000       NA
## 86  16.3667   1.3911
## 87  15.5000       NA
## 88  15.2000       NA
## 89  17.8000       NA
## 90  16.9000   0.1000
## 91  11.4000       NA
## 92  19.4000       NA
## 93  14.8000       NA
## 94  16.3667   1.0837
## 95  15.3000   1.0817
## 96  14.4000       NA
## 97  18.0000       NA
## 98  17.4500   0.5500
## 99  15.6000   0.4000
## 100 13.2000       NA
## 101 15.8000       NA
## 102 16.2000       NA
## 103 14.9000       NA
## 104 14.9000       NA
## 105 16.5500   1.4500
## 106 16.1000       NA
## 107 14.4000       NA
## 108 15.5333   0.8393
## 109 15.4000   1.0000
## 110 19.9000       NA
## 111 17.5333   1.3132
## 112 16.4000       NA
## 113 14.7000       NA
## 114 17.3000       NA
## 115 15.7250   0.5344
## 116 18.8000       NA
## 117 16.4000       NA
## 118 16.9000       NA
## 119 18.6000       NA
## 120 19.2000       NA
## 121 14.7000       NA
## 122 21.5000       NA
## 123 23.7000       NA
## 124 24.6000       NA
## 125 21.7000       NA
## 126 13.8000       NA
## 127 17.9000       NA
```