

is605 Assignment 3 (PS1 & PS2)

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PS1

PS1 Part 1.

The rank of the matrix below is 4. Since the reduced row echelon form of matrix A is a full rank (4 pivots) square matrix the rank is equal to the number of pivots in the reduced row echelon form of A and in this case the rank is 4. This means that there are 4 linearly independent rows in A and this is validated by the fact that $\det(A) \neq 0$.

```
require(Matrix)
```

```
## Loading required package: Matrix
```

```
## Warning: package 'Matrix' was built under R version 3.1.2
```

```
require(pracma)
```

```
## Loading required package: pracma
```

```
## Warning: package 'pracma' was built under R version 3.1.2
```

```
##
```

```
## Attaching package: 'pracma'
```

```
##
```

```
## The following objects are masked from 'package:Matrix':
```

```
##
```

```
##      expm, lu, tril, triu
```

```
A <- matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), nrow = 4, ncol = 4, byrow = TRUE)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```
det(A)
```

```
## [1] -9
```

```
# Verify the Solution
```

```
reducedA <- rref(A)
```

```
reducedA
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

```
rankMatrix(reducedA)
```

```
## [1] 4
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 8.882e-16
```

PS1 Part 2.

Given an $m \times n$ matrix where $m > n$, the maximum rank of the given matrix is $\leq \min(m, n)$. Since $m > n$, then the maximum rank of the given $m \times n$ matrix is $\leq n$.

Since the given matrix is assumed to be a non-zero matrix the minimum rank of the given matrix is 1.

PS1 Part 3.

The matrix B below only has 1 linearly independent row. R3 is $2 \times R1$ and therefore is eliminated when reduced to row echelon form. R2 is $3 \times R1$ and is also eliminated when reduced to row echelon form. Accordingly the rank of matrix B is 1 since the reduced row echelon form of B yields only 1 linearly independent row.

```
B <- matrix(c(1,2,1,3,6,3,2,4,2), nrow = 3, ncol = 3, byrow = TRUE)
# Check the Solution
reducedB <- rref(B)
reducedB
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    0    0    0
## [3,]    0    0    0
```

```
rankMatrix(reducedB)
```

```
## [1] 1
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 6.661e-16
```

PS2 Part 1 and Part 2 - See Attached PDF For Handwritten Solution.