

PS 2 Part 1 Find Eigenvalues of A

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

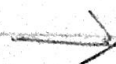
λ is an eigenvalue of A iff
 $A\vec{v} = \lambda\vec{v}$ for some non zero \vec{v} .

then, $R\mathbb{I}_n\vec{v} = A\vec{v} = \vec{0}$ iff
 $\vec{0} = (\lambda\mathbb{I}_n - A)\vec{v}$.

So, $(\lambda\mathbb{I}_n - A)\vec{v} = \vec{0}$ iff
 $\det(\lambda\mathbb{I}_n - A) = 0$.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}^{\mathbb{R}^3}$ and $\lambda\mathbb{I}_3 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

So, $(\lambda\mathbb{I}_3 - A) = \begin{bmatrix} (\lambda-1) & -2 & -3 \\ 0 & (\lambda-4) & -5 \\ 0 & 0 & (\lambda-6) \end{bmatrix}$



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And,

$$\det(\lambda I_3 - A) =$$

$$\begin{vmatrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-4 & -5 \\ 0 & 0 & \lambda-6 \end{vmatrix} = \begin{vmatrix} \lambda-1 & (-2) \\ 0 & (\lambda-4) \\ 0 & 0 \end{vmatrix}$$

$$= \left[(\lambda-1)(\lambda-4)(\lambda-6) \right] + (-2)(-5)(0) + (-3)(0)(0)$$

$$\left[(-2)(0)(\lambda-6) \right] - \left[(\lambda-1)(-5)(0) \right] - \left[(3)(\lambda-4)(0) \right]$$

$$= (\lambda-1)(\lambda-4)(\lambda-6)$$

$$= (\lambda-1)(\lambda^2 - 10\lambda + 24)$$

$$= \lambda^3 - 10\lambda^2 + 24\lambda - \lambda^2 + 10\lambda - 24$$

$$= \lambda^3 - 11\lambda + 39\lambda - 24 = p(\lambda)$$

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PS #2 Part 2 Eigen vectors

If λ is an eigenvalue, then

$$\lambda^3 - 11\lambda^2 + 34\lambda - 24 = 0$$

Possible roots are factors of 24.

Through trial and error we find $P(6) = 0$.

$$(6)^3 - 11(6)^2 + 34(6) - 24 = 0.$$

Then,

$$\begin{array}{r} \lambda^2 - 5\lambda + 4 \\ (\lambda - 6) \overline{) \lambda^3 - 11\lambda^2 + 34\lambda - 24} \\ \underline{-(\lambda^3 - 6\lambda^2)} \\ -5\lambda^2 + 34\lambda \\ \underline{-(5\lambda^2 - 30\lambda)} \\ -0 \end{array}$$

$$\begin{array}{r} 4\lambda - 24 \\ \underline{4\lambda - 24} \\ 0 + 0 \end{array}$$

$$\text{Thus, } \lambda^2 - 5\lambda + 4 = 0$$

$$\neq 0 + (\lambda - 4)(\lambda - 1)$$

$$\Rightarrow \lambda^2 - \lambda - 4\lambda + 4 \Rightarrow \lambda^2 - 5\lambda + 4$$

$$\text{So, } \lambda = 1, \lambda = 4, \text{ and } \lambda = 6.$$

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PS # 2 Part 2

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Plugging in $\lambda = 1, 4, 6$ to find
eigenvectors,
into

$$(\lambda I_3 - A) = \begin{bmatrix} (\lambda - 1) & -2 & -3 \\ 0 & (\lambda - 4) & -5 \\ 0 & 0 & (\lambda - 6) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

and reduce to rref $= \vec{0}$

we get

$$\text{span} \begin{bmatrix} .57 & .55 & 1 \\ .79 & .43 & 0 \\ .31 & .0 & 0 \end{bmatrix}$$

$\lambda = 1, 4, 6$

P. 3

vectors