Is 605 Assignment 3

PS 2 Part 1 Find Eigenvalues of A

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

λ is an eigenvalue of A iff
$A\vec{v} = \lambda\vec{v}$ for some non zero $\vec{v}$.

Then, $\lambda I_n \vec{v} - A\vec{v} = \vec{0}$ iff
$\vec{0} = (\lambda I_n - A)\vec{v}$.

So, $(\lambda I_n - A)\vec{v} = \vec{0}$ iff
$\det(\lambda I_n - A) = 0$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}^{3}_{n} \quad \text{and} \quad \lambda I_3 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

So, $(\lambda I_3 - A) = \begin{bmatrix} (\lambda-1) & -2 & -3 \\ 0 & (\lambda-4) & -5 \\ 0 & 0 & (\lambda-6) \end{bmatrix}$

$\longrightarrow$

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Ps 2 Part 1 Eigenvalues

And,

$\det(\lambda I_3 - A) =$

$$\begin{vmatrix} (\lambda-1) & -2 & -3 \\ 0 & (\lambda-4) & -5 \\ 0 & 0 & (\lambda-6) \end{vmatrix} \begin{matrix} (\lambda-1) & (-2) \\ 0 & (\lambda-4) \\ 0 & 0 \end{matrix}$$

$= \{[(\lambda-1)(\lambda-4)(\lambda-6)] + (-2)(-5)(0) + (-3)(0)(0)]$

$- \{[(-2)(0)(\lambda-6)] - \{(\lambda-1)(-5)(0)\} - [(3)(\lambda-4)(0)]\}$

$= (\lambda-1)(\lambda-4)(\lambda-6)$

$= (\lambda-1)(\lambda^2-10\lambda+24)$

$= \lambda^3 - 10\lambda^2 + 24\lambda - \lambda^2 + 10\lambda - 24$

$= \lambda^3 - 11\lambda + 34\lambda - 24 = p(\lambda)$

$\Rightarrow$

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PS #2 Part 2 Eigenvectors

If λ is an eigenvalue, then

$\lambda^3 - 11\lambda^2 + 34\lambda - 24 = 0$

Possible roots are factors of 24.
Through trial and error we find $P(6) = 0$.
$(6)^3 - 11(6)^2 + 34(6) - 24 = 0$.

Then,

$$
\begin{array}{r l}
 & \lambda^2 - 5\lambda + 4 \\
(\lambda - 6) \big) & \lambda^3 - 11\lambda^2 + 34\lambda - 24 \\
 & -\ \lambda^3 - 6\lambda^2 \\
 & \quad -5\lambda^2 + 34\lambda \\
 & \quad -5\lambda^2 + 30\lambda \\
 & \quad\ 0 \quad 4\lambda - 24 \\
 & \qquad\quad 4\lambda - 24 \\
 & \qquad\quad 0 + 0
\end{array}
$$

Thus, $\lambda^2 - 5\lambda + 4 = 0$
$(\lambda - 4)(\lambda - 1)$
$\Rightarrow \lambda^2 - \lambda - 4\lambda + 4 \Rightarrow \lambda^2 - 5\lambda + 4$

So, $\lambda = 1$, $\lambda = 4$, and $\lambda = 6$.

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PS # 2 Part 2

p 4

Plugg in $\lambda = 1, 4, 6$ to Find Eigenvectors,

into

$$(\lambda I_3 - A) = \begin{bmatrix} (\lambda-1) & -2 & -3 \\ 0 & (\lambda-4) & -5 \\ 0 & 0 & (\lambda-6) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \vec{0}$$

and reduce to rref

we get Span $\left\{ \begin{bmatrix} .51 \\ .79 \\ .31 \end{bmatrix}, \begin{bmatrix} .55 \\ .83 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

$\lambda = 1, 4, 6$

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