Course Title:	Mathematics I	Semester:	1	Date:	18.07.2022
Subject Module:	1	Subject Code:	05BH0101	Faculty:	Keshavi Mehta

UNIT 1 : SET THEORY

COURSE CONTENT :

1) Introduction

2) Methods of describing a set

3) Types of Sets (Null Set, Singleton Set, Finite Set, Infinite Set, Equal Set, Equivalent Set, Subset, Proper Subset, Power Set, Universal Set)

4) Operation on Sets (Union, Intersection, Difference, Symmetric Difference, Complement of a set)

5) Algebra of Sets (Commutative Laws, Associative Laws, Distributive Laws)

6) De Morgan's Laws

7) Venn Diagrams

8) Cardinality of sets

9) Cartesian Product of two sets

➢ A Set :

A set is a collection of elements without repetition. A set can be finite or infinite. Following are some examples of sets.

A = {1, 2, 3}, B = {a, b, c}, C = {-1, 0, 1} etc.

The above sets are all finite sets. We say that a member/element 'belongs to' that set, and this is denoted using the symbol ∈.

Thus, mathematically we write

1, 2, 3 ∈ A
a, b, c ∈ B
-1, 0, 1 ∈ C

SET :- A set is a any collection of distinct objects of our thoughts.

➢ Some Standard Set Notations :

- **Natural numbers (Counting numbers) :**

  N = {1, 2, 3, 4, 5, …….}

- **Integers :**

  Z = {……, -2, -1, 0, 1, 2, …….}

- **Rational numbers (Fractions) :**

  The rational numbers cannot be listed as above, but are defined by their property.

  $$Q = \left\{ x / x = \frac{p}{q}, \text{where } p,\ q \in Z, q \neq 0 \right\}$$

  -3, 0, 1/3, -5/2 etc. are some examples of rational numbers.

- **Irrational numbers :**

  The irrational numbers are those which are not rational. That is,

  $$I = \left\{ x / x \neq \frac{p}{q}, \text{where } p,\ q \in Z, q \neq 0 \right\}$$

  $\sqrt{2}, \sqrt{3}, \pi$ etc. are irrational numbers.

- **Real numbers :**

  The union of the rational and the irrational numbers is called the set of real numbers, denoted by R. Thus mathematically, **R = Q ∪ I**

Methods to denote a set :

- **Listing method :**
  In this method we simply list out all the members of the set separated by a comma enclosed within a pair of curly brackets.
  As, $A = \{1, 2, 3, 4, 5\}$ Listing method :- In this method a set is described by listing elements seprated by comma within braces {}.

- **Property method :**
  In this method, instead of Explicitly mentioning each member of the set, we use their mathematical property using some standard set notations and some algebraic conditions.
  The above set A shown in 'Listing method' can be demonstrated using 'Property method' as,
  $A = \{x / x \in N,\ x \le 5\}$ Property method :- In this method a set is described by characterizing property.

Null Set or Empty Set :

A set which has no elements is called an empty set or a null set, and is denoted using { } or φ(phi). Thus, if A is an empty set, we write A = { } or A = φ

(Note that A = {φ} is not a correct notation for a null set.)

Null set :- A set in which there is no element is called null set.

Singleton Set :

A set having only one element is called singleton set.

Examples :

A = {2}

B = {$x \in$ N / x is an even prime number}

C = {x / x is least positive integer} = {1}

D = {x / x is a perfect square of an integer $60 < x < 70$}

Finite Set :

A set is called finite set if it is either null set or its elements can be counted by natural numbers or process of listing terminates at a certain natural number.

Examples : A = {$x \in$ N / $1 < x < 100$}

Finite set :- If total number of element in a set can be counted by natural number that it is called finite set.

B = {a, e, i, o, u}

- **Cardinality or Order of a Finite Set :**

The total number of element in a finite set is called Cardinality or Order of a finite set. It is denoted by n **(A)** or **|A|**.

Examples :

A = {a, e, i, o, u}

Here n (A) = 5

B = $\{x \in Z \,/\, -4 < x < 4\}$

Thus, B = {-3, -2, -1, 0, 1, 2, 3}

Here n (B) = 7

➢ **Infinite Set :**

If the elements of a set cannot be counted in a finite number, then the set is called an infinite set.

Infinite set :- If total number of element in a set can not be counted by natural number that it is called infinite set.

Examples :

A = {1, 2, 3, 4, …}

B = $\{x \,/\, x$ is an even natural number$\}$

Also, N, Z, Q, R, Z^+, Q^+, R^+ all are infinite sets.

➢ **Subset of a Set :**

For any two sets A and B, if all the elements of set A are also present in set B, then we say that,set A is a subset of set B. And we denote this as $A \subset B$. Consider the following example.

If A = {1, 2, 3} and B = {0, 1, 2, 3, 4} then $A \subset B$.

- **Notes :**

A null set is a subset of every set.

Any set can be regarded as a subset of itself.

Thus, any non - empty set has atleast two subsets; the null set and the set itself.

Also, if A is a subset of B, then B is called a 'superset' of A.

- **Union and intersection of sets :**

  - **Union of sets :**
    Consider two sets A and B. Then the union of sets A and B is a set which contains all the elements of set A and set B, and this is denoted as $A \cup B$.

  - **Intersection of sets :**
    Consider two sets A and B. Then the intersection of sets A and B is a set which contains only the elements which are common to both sets A and B, and this is denoted as $A \cap B$.

    Example : Consider the following sets.
    A = {0, 1, 2, 3}, B = {2, 3, 4, 5} and C = {6, 7}

    Here, observe that

    $A \cup B = \{0, 1, 2, 3, 4, 5\}$
    $A \cap B = \{2, 3\}$
    $B \cap C = \{\ \}$

NOTE :

For the above defined sets, observe the inclusion relation

$$\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R}$$

$$\mathbf{I} \subset \mathbf{R}$$

$$\mathbf{Q} \cap \mathbf{I} = \{\ \}$$

- **Equal Sets :**

Two sets A and B are said to be equal sets if every element of set A is also an element of set B and every element of set B is an element of set A.

Notation : A = B

Examples :

(1) A = {-1, 1}, B = {$x \in \mathbf{Z}$ / $x^2 - 1 = 0$}

Here A = B

(2) C = {2, 3, 4, 5}, D = {$x \in \mathbf{N}$ / $1 < x < 6$}

Here C = D

➢ Equivalent Sets :

Two finite sets A and B are said to be equivalent sets if they have same number of elements. In other words, two finite sets A and B are said to be equivalent sets if n (A) = n (B).

Notation: $A \equiv B$

Examples :

(1) A = {a, e, i, o, u} and B = {1, 2, 3, 4, 5}

Here n (A) = n (B) = 5.

Therefore $A \equiv B$

(2) A = {$x \in N$ / x is a factor of 4} and B = {$x \in N$ / x is a factor of 9}

Here A = {1, 2, 4} and B = {1, 3, 9}

So, n (A) = n (B) = 3

Therefore $A \equiv B$

Note : Two equal sets are always equivalent, but two equivalent sets may not be equal sets.

➢ Power set of a set :

The set of all subsets of a given set is known as the 'power set' of that set.

Notation : The power set of a set A is denoted as P(A).

Example : Write the power set of set A = {a, b}

Subsets of A = ∅, {a, b}, {a}, {b}

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Example : Write power set of a set B = {$x \in N$ / x is a factor of 9}

Here B = {1, 3, 9}

$P(B) = \{\emptyset, \{1\}, \{3\}, \{9\}, \{1, 3\}, \{1, 9\}, \{3, 9\}, \{1, 3, 9\}\}$

Example : Write power set of a set C = {$x \in N$ / x is a factor of 8}

Here C = {1, 2, 4, 8}

P(C) = $\{\emptyset$, {1}, {2}, {4}, {8}, {1, 2}, {1, 4}, {1, 8}, {2, 4}, {2, 8}, {4, 8}, {1, 2, 4}, {1, 2, 8}, {1, 4, 8}, {2, 4, 8}, {1, 2, 4, 8}$\}$

No. of Elements (n)	**Set**	**Subsets**	**No. of subsets = 2^n**
0	A = { }	{ }	$2^0 = 1$
1	A = {a}	{ }, {1}	$2^1 = 2$
2	A = {a, b}	{ }, {a}, {b}, {a, b}	$2^2 = 4$
3	A = {a, b, c}	{ }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}	$2^3 = 8$
4	A = {a, b, c, d}	{ }, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}	$2^4 = 16$

Note : Observe that, for a set with n members, its power set will have 2^n members

i.e., if n (A) = n, then n{P(A)} = 2^n

Exercise :

(1) Write power set of a set A = {***x*** **ϵ N /** ***x*** **is a factor of 25**}

(2) Write all proper subsets of set A = {***x*** **ϵ N /** ***x*** **is a factor of 15**}

Example : Find total number of subsets of a set A = {***x*** **∈ N /** ***x*** **is a factor of 4**}**. Also write all subsets of set A**.

Solution : Here A = {1, 2, 4}

So, n (A) = 3

Total number of subsets of A = $2^3 = 8$

Also, P(A) = $\{\emptyset, \{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{1, 2, 4\}\}$

Example : Find total number of subsets of the set A = $\{x \in N / x$ is a factor of 27$\}$.

Solution : Here A = {1, 3, 9, 27}

So, n (A) = 4

Total number of subsets of set A = $2^4 = 16$

Also, P(A) = $\{\emptyset, \{1, 3, 9, 27\}, \{1\}, \{3\}, \{9\}, \{27\}, \{1, 3\}, \{1, 9\}, \{1, 27\}, \{3, 9\}, \{3, 27\},$

$\{9, 27\}, \{1, 3, 9\}, \{1, 3, 27\}, \{1, 9, 27\}, \{3, 9, 27\}\}$

- **Universal Set :**

A set which is the superset of all the sets under consideration is known as Universal Set and is denoted as U.

- **Disjoint Sets :**

Two sets A and B are called disjoint sets if they have no elements in common, i.e., their intersection is an empty set. Mathematically, if $A \cap B = \phi$, then we say that A and B are disjoint sets.

Example : If A = {-1, 0, 1} and B = {2, 4, 6, 8, 10}, then A and B are disjoint.

- **Complement of a Set :**

The complement of a set is a set all the member of the Universal set, which are not present in the given set.

Notation : The complement of a set A is denoted as A’ or $A^{C.}$

Example : Let $\mathbb{U}$ = {1, 2, 3, 4, 5, 6, 7, 8} and A = {2, 4, 6, 8}. Then find A’.

Here $\mathbb{U}$ = {1, 2, 3, 4, 5, 6, 7, 8} and A = {2, 4, 6, 8}

Therefore A’ = {1, 3, 5, 7}

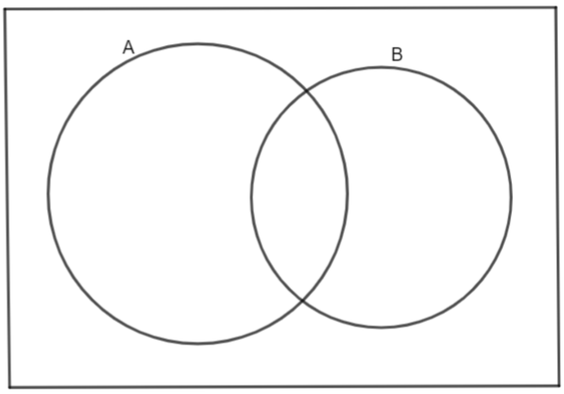
- **Venn Diagrams :**

A Venn Diagram is a pictorial representation of sets where the Universal set is shown as the interior of a rectangle, and all other sets are demonstrated as the interiors of circles within the rectangle.

Let us consider the following cases for two sets A and B. And then represent them in Venn diagrams.

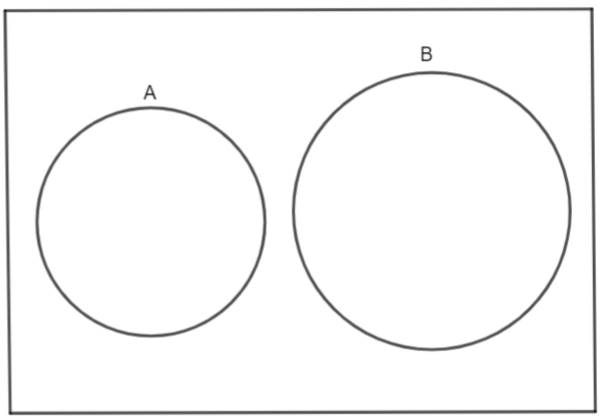
(i) Intersecting Sets ($A \cap B \neq \phi$ **) :**

U



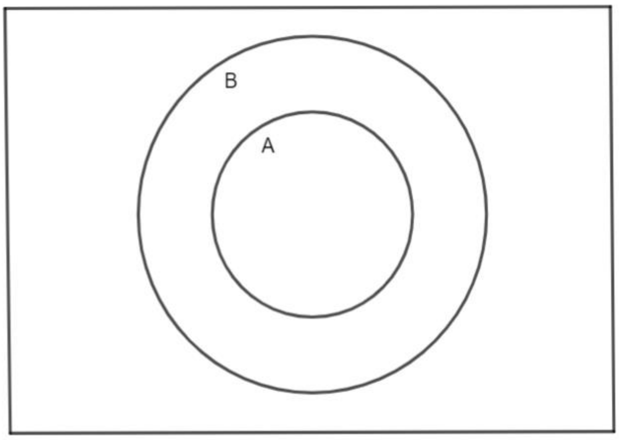
(ii) Disjoint Sets ($A \cap B = \phi$ **) :**

U



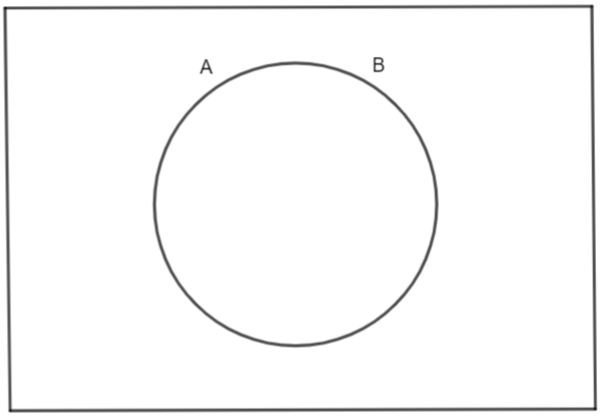
(iii) A is a subset of B ($A \subset B$ **) :**

U



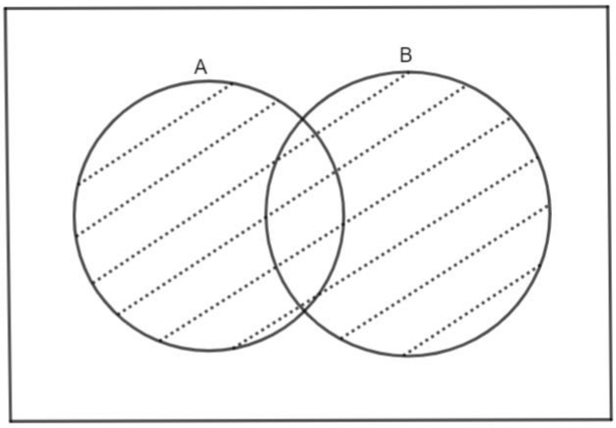
(iv) A and B are equal (A = B) :

U

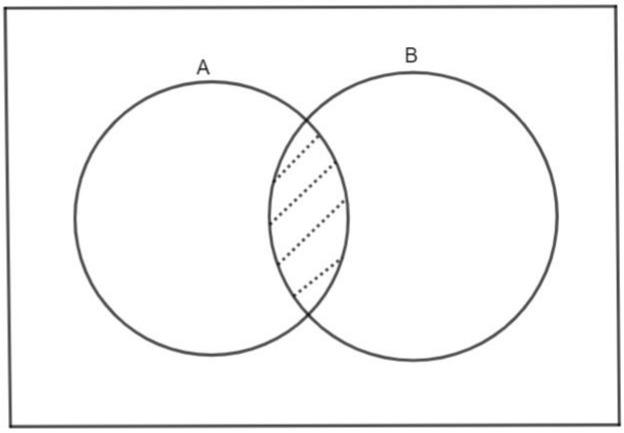


- ## Demonstrating set operations using Venn Diagrams :

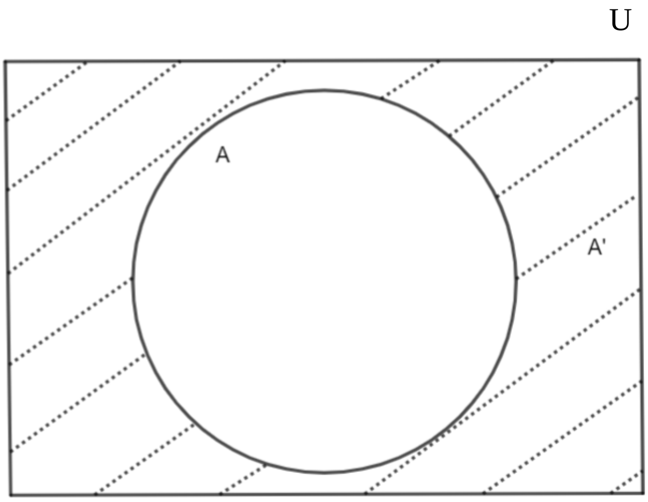
(i) Union of sets A and B (A $\cup$ B) :



(ii) Intersection of A and B (A $\cap$ B) :



(iii) Complement of a set A (i.e. A') :

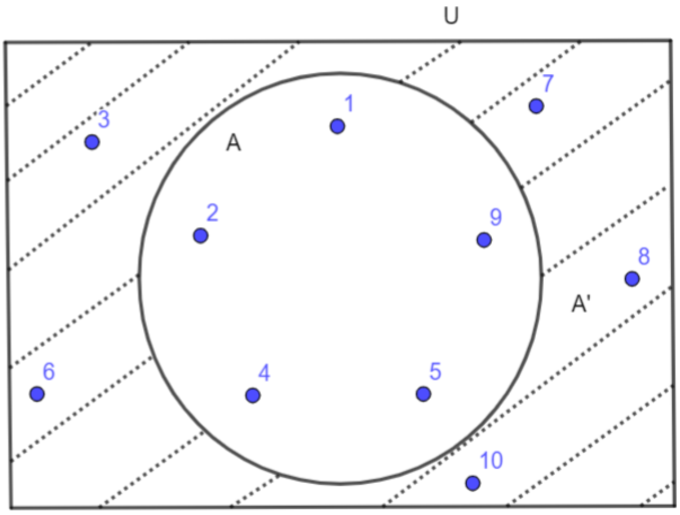


Example : Let $\mathbb{U}$ = {1, 2, 3, …, 8, 9, 10} and A = {1, 2, 4, 5, 9}. Then, find A' and represent it using a Venn Diagram.

Solution : Here A = {1, 2, 4, 5, 9}

Thus, A' = {3, 6, 7, 8, 10}

Venn Diagram for A' is shown below.



A'

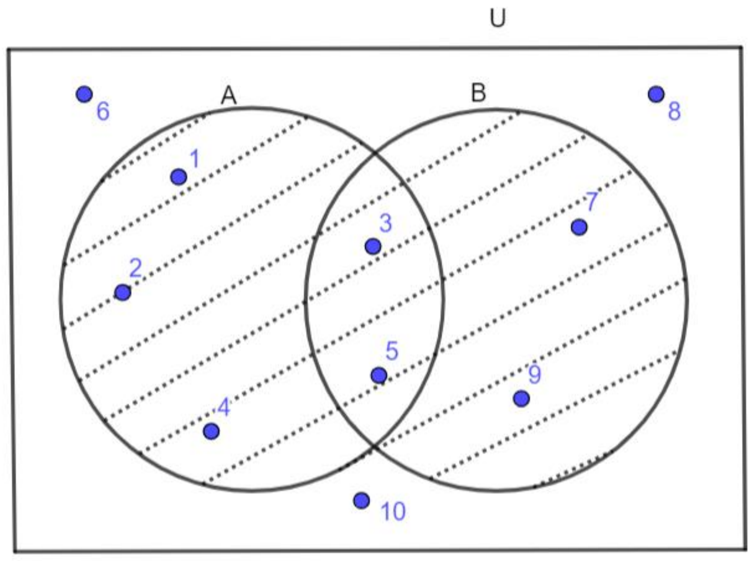
Example : Let 𝕌 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 3, 4, 5} and

B = {3, 5, 7, 9}. Then find A ∪ B and A ∩ B. And represent their Venn diagrams.

Solution : Here A = {1, 2, 3, 4, 5} and B = {3, 5, 7, 9}

Therefore, A ∪ B = {1, 2, 3, 4, 5, 7, 9}

Venn diagram for A U B is shown below.



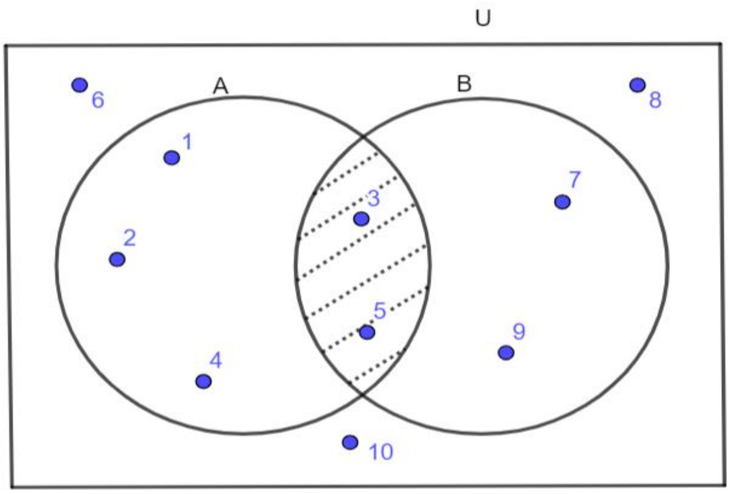
A U B

Again, A = {1, 2, 3, 4, 5} and B = {3, 5, 7, 9}

Therefore A ∩ B = {1, 2, 3, 4, 5} ∩ {3, 5, 7, 9}

= {3, 5}

Venn diagram for A ∩ B is shown below.



A ∩ B

Example : Let 𝕌 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {*x* ∈ N / *x* is a factor of 10} and B = {*x* ∈ N / *x* is a factor of 8}. Then, find A ∪ B and represent its Venn Diagram:

Solution : A = {1, 2, 5, 10}, B = {1, 2, 4, 8}

Thus, $A \cup B = \{1, 2, 4, 5, 8, 10\}$

Venn Diagram : Try yourself.

Exercise :

(1) Let $\mathbb{U} = \{1, 2, 3, \ldots, 9, 10, 11\}$, $A = \{1, 2, 4, 5, 10\}$ and $B = \{2, 6, 9, 11\}$. Then determine the following.

(i) A’ and its Venn Diagram

(ii) B’ and its Venn Diagram

(iii) $A \cup B$ and its Venn Diagram

(iv) $A \cap B$ and its Venn Diagram

(2) Let $\mathbb{U} = \{1, 2, 3, \ldots, 9, 10, 11, 12\}$, $A = \{x \in N \,/\, x$ is factor of $12\}$ and

$B = \{x \in N \,/\, x$ is factor of $10\}$. Then, determine the following:

(i) A’ and its Venn Diagram

(ii) B’ and its Venn Diagram

(iii) $A \cup B$ and its Venn Diagram

(iv) $A \cap B$ and its Venn Diagram

Notes :

(i) $(A')' = A$

(ii) $\phi' = \mathbb{U}$ and $\mathbb{U}' = \phi$

(iii) For any set A, $A \cup A' = \mathbb{U}$ and $A \cap A' = \phi$

(iv) For any set A, $A \cup \phi = A$ and $A \cap \phi = \phi$

(v) For any set A, $A \cup \mathbb{U} = \mathbb{U}$ and $A \cap \mathbb{U} = A$

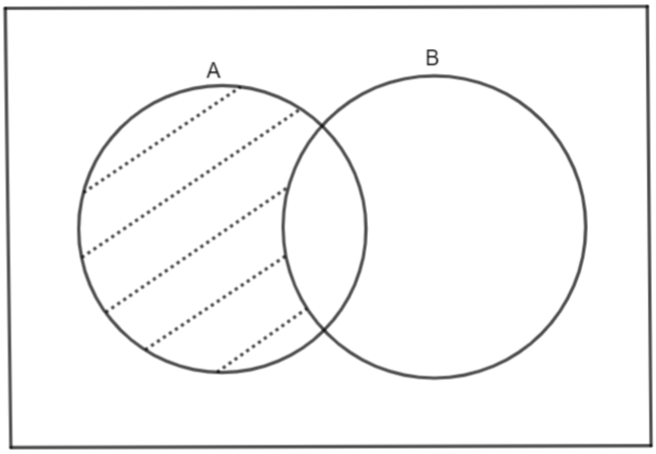
(vi) For sets A and B, if $A \subset B$, then $A \cup B = B$ and $A \cap B = A$

Example : $N \cup Z = Z$, $N \cap Q = N$ etc.

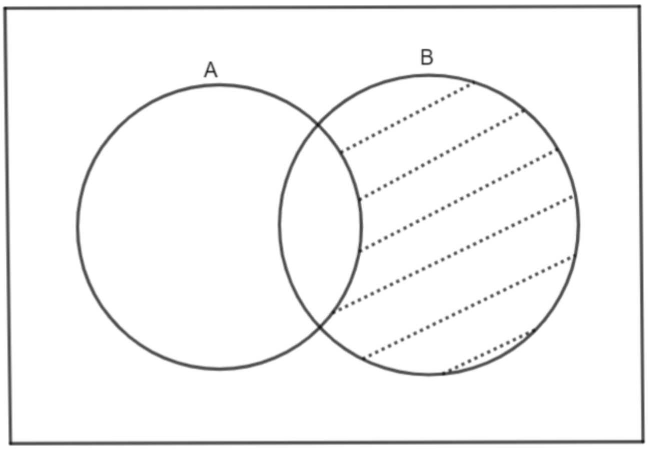
- **Difference of two sets :**

Let A and B be two sets. Then the 'Difference set' A – B is a set of all elements which belong to A but do not belong to B.

Venn Diagram for Difference set A – B :



Venn Diagram for Difference set B – A :



Example : A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 7}. Then, find A – B and B – A.

A – B = {1, 2, 3, 4, 5} – {2, 4, 6, 7} = {1, 3, 5}

B – A = {2, 4, 6, 7} – {1, 2, 3, 4, 5} = {6, 7}

Exercise :

For A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8, 9, 10}. Then find A – B and B – A.

.

➢ **Note :**

If A and B are disjoint sets, then A – B = A and B – A = B

Example : A = {1, 2, 3} B = {4, 5}

A – B = {1, 2, 3} – {4, 5} = {1, 2, 3} = A

B – A = {4, 5} – {1, 2, 3} = {4, 5} = B

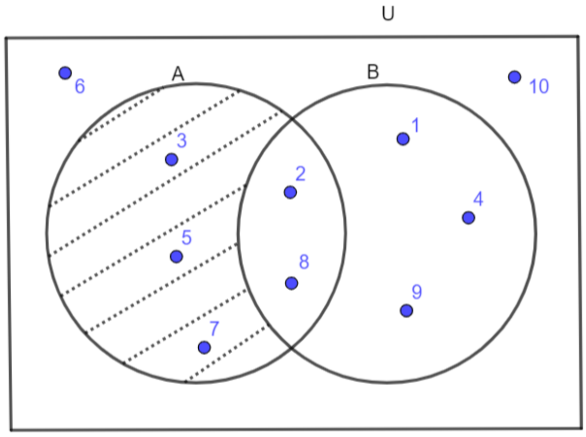
Example : Let 𝕌 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 3, 5, 7, 8} and B = {1, 2, 4, 8, 9}.

Then find A – B and B – A and draw their Venn Diagrams.

Solution : Here 𝕌 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 3, 5, 7, 8} and B = {1, 2, 4, 8, 9}

Now A – B = {2, 3, 5, 7, 8} – {1, 2, 4, 8, 9} = {3, 5, 7}

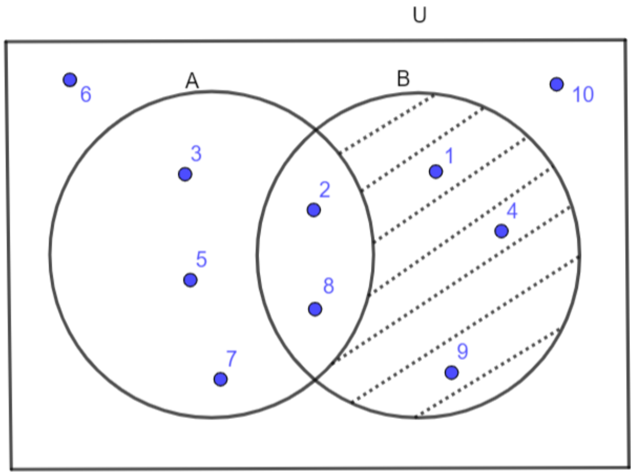
Venn diagram for A – B :



A - B

And B – A = {1, 2, 4, 8, 9} – {2, 3, 5, 7, 8} = {1, 4, 9}

Venn diagram for B – A :



B - A

➢ Symmetric Difference of Two Sets :

Let A and B are two sets. A set containing all those element which belongs to set A but do not belongs to set B or all those elements which belongs to set B but do not belongs to set A is called symmetric difference of two sets.

Notation : A Δ B (A delta B)

A Δ B = (A ∪ B) – (A ∩ B)

OR A Δ B = (A – B) ∪ (B – A)

Consider A = { 1, 2, 3, 4, 5} and B = { 2, 5, 6 } then find A Δ B.

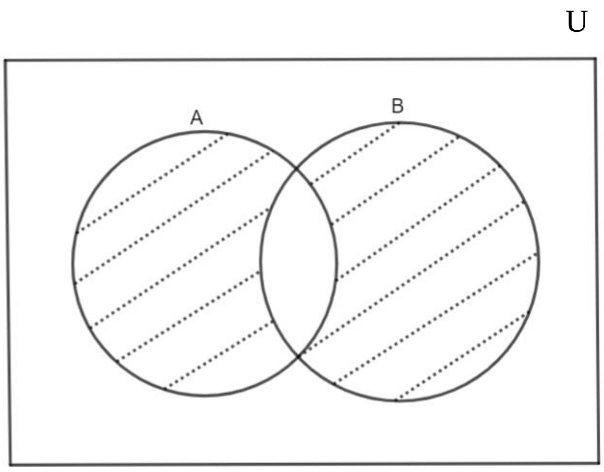
A Δ B = (A – B) ∪ (B – A)

So , A – B = { 1, 2, 3, 4, 5} – {2, 5, 6} = { 1, 3, 4}

And B – A = { 2, 5, 6 } – { 1, 2, 3, 4, 5 } = { 6}

Therefore, A Δ B = { 1, 3, 4} ∪ { 6} = {1, 3, 4, 6}

Venn diagram for Symmetric Differen+ce Set A Δ B :



A Δ B

Example : Let 𝕌 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 3, 5, 7, 8} and

B = {1, 2, 4, 8, 9}. Then find A Δ B and draw its Venn Diagram.

Solution : A Δ B = (A ∪ B) – (A ∩ B)

A ∪ B = {2, 3, 5, 7, 8} ∪ {1, 2, 4, 8, 9} = {1, 2, 3, 4, 5, 7, 8, 9}

A ∩ B = {2, 3, 5, 7, 8} ∩ {1, 2, 4, 8, 9} = {2, 8}

Therefore, A Δ B = (A ∪ B) – (A ∩ B)

= {1, 2, 3, 4, 5, 7, 8, 9} – {2, 8}

= {1, 3, 4, 5, 7, 9}

OR : A Δ B = (A – B) ∪ (B – A)

Now A – B = {2, 3, 5, 7, 8} – {1, 2, 4, 8, 9}

A – B = {3, 5, 7} and

B – A = {1, 2, 4, 8, 9} – {2, 3, 5, 7, 8}

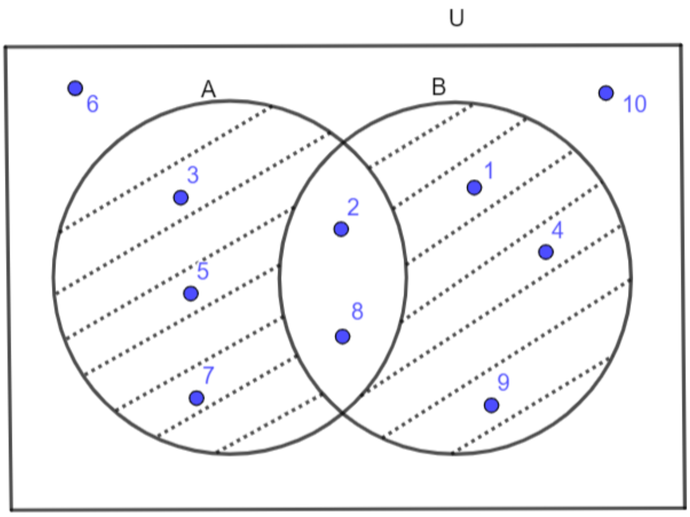
= {1, 4, 9}

Therefore A Δ B = (A – B) ∪ (B – A)

= {3, 5, 7} ∪ {1, 4, 9}

= {1, 3, 4, 5, 7, 9}

Venn Diagram for A Δ B :



A Δ B

Exercise : Let A = {1, 2, 3, 4, 5, 6} and B = {3, 2, 4, 7, 8, 9}, then determine A Δ B.

(**Solution :** A Δ B = {1, 5, 6, 7, 8, 9})

Examples :

Let 𝕌 = {*x* ∈ N / *x* is a factor of 48}, A = {*x* ∈ N / *x* is a factor of 24},

B = {*x* ∈ N / *x* is a factor of 12} and C = {*x* ∈ N / *x* is a factor of 8}.

Then find the following :

(1) A’ (2) B’ (3) C’

(4) A ∪ B and its Venn Diagram

(5) A ∩ B and its Venn Diagram

(6) A Δ B and its Venn Diagram

(7) P(C) (8) (B ∪ C)’ (9) B’ ∩ C’

Solution : Here 𝕌 = {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

A = {1, 2, 3, 4, 6, 8, 12, 24}, B = {1, 2, 3, 4, 6, 12} and C = {1, 2, 4, 8}

(1) A’ = {16, 48}

(2) B’ = {8, 16, 24, 48}

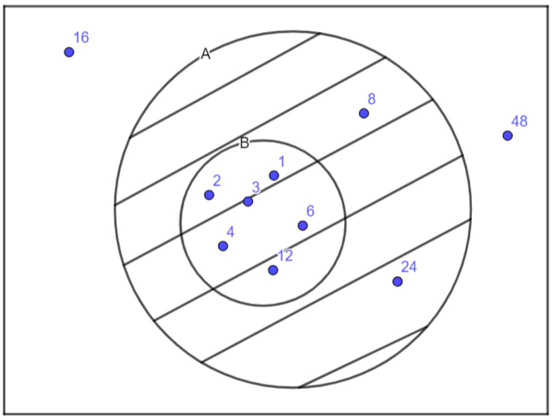
(3) C’ = {3, 6, 12, 16, 24, 48}

(4) A ∪ B = {1, 2, 3, 4, 6, 8, 12, 24} ∪ {1, 2, 3, 4, 6, 12}

= {1, 2, 3, 4, 6, 8, 12, 24}

Venn Diagram :

U

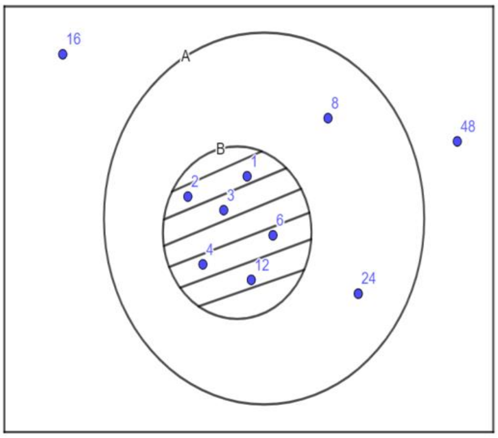


(5) A ∩ B = {1, 2, 3, 4, 6, 8, 12, 24} ∩ {1, 2, 3, 4, 6, 12}

= {1, 2, 3, 4, 6, 12}

Venn Diagram :

U



(6) A Δ B = (A – B) ∪ (B – A)

Now A – B = {1, 2, 3, 4, 6, 8, 12, 24} - {1, 2, 3, 4, 6, 12}

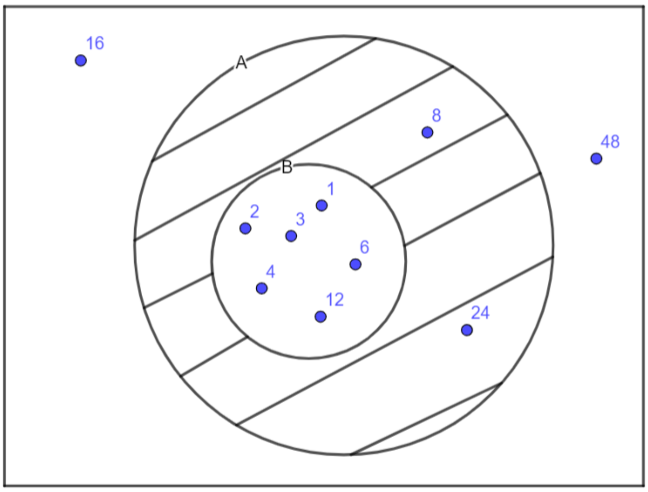
= {8, 24}

And B – A = {1, 2, 3, 4, 6, 12} – {1, 2, 3, 4, 6, 8, 12, 24} = ∅

Therefore A Δ B = {8, 24} ∪ ∅ = {8, 24}

Venn Diagram :

U



(7) **P(C)**

Here C = {1, 2, 4, 8}

Therefore P(C) = {∅, {1}, {2}, {4}, {8}, {1, 2}, {1, 4}, {1, 8}, {2, 4}, {2, 8}, {4, 8}, {1, 2, 4}, {1, 2, 8}, {1, 4, 8}, {2, 4, 8}, {1, 2, 4, 8}}

(8) **(B ∪ C)’**

B ∪ C = {1, 2, 3, 4, 6, 12} ∪ {1, 2, 4, 8} = {1, 2, 3, 4, 6, 8, 12}

(B ∪ C)’ = {16, 24, 48}

9) **B’ ∩ C’** = {8, 16, 24, 48} ∩ {3, 6, 12, 16, 24, 48}

= {16, 24, 48}

Example : Let 𝕌 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 4, 5}, B = {6, 7, 9, 10} and C = {1, 2, 9, 10}. Then find the following :

(1) A’ (2) B’ (3) C’ (4) A ∪ B and its Venn Diagram (5) A ∩ B and its Venn Diagram (6) A Δ B and its Venn Diagram (7) Verify (B ∪ C)’ = B’ ∩ C’

Solution : Here 𝕌 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A = {1, 2, 4, 5}, B = {6, 7, 9, 10} and C = {1, 2, 9, 10}

(1) A’ = {3, 6, 7, 8, 9, 10}

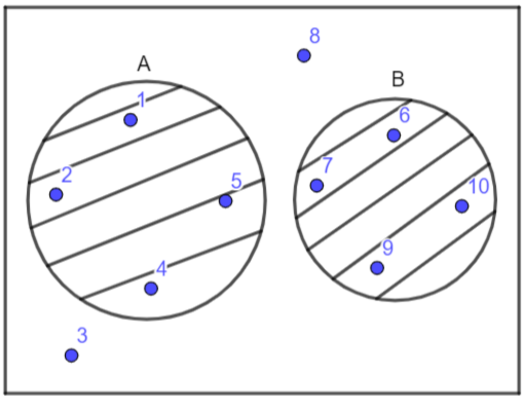
(2) B’ = {1, 2, 3, 4, 5, 8}

(3) C’ = {3, 4, 5, 6, 7, 8}

(4) A ∪ B = {1, 2, 4, 5} ∪ {6, 7, 9, 10} = {1, 2, 4, 5, 6, 7, 9, 10}

Venn Diagram :

U

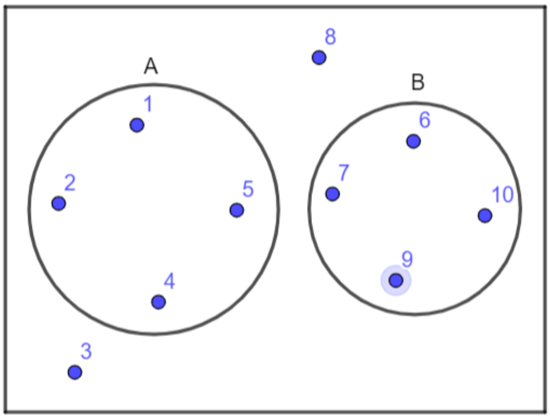


A ∪ B

(5) A ∩ B = {1, 2, 4, 5} ∩ {6, 7, 9, 10} = ∅

Venn Diagram :

U



A ∩ B

(6) A Δ B = (A – B) ∪ (B – A)

Now A – B = {1, 2, 4, 5} – {6, 7, 9, 10}

= {1, 2, 4, 5} = A

And B – A = {6, 7, 9, 10} – {1, 2, 4, 5}

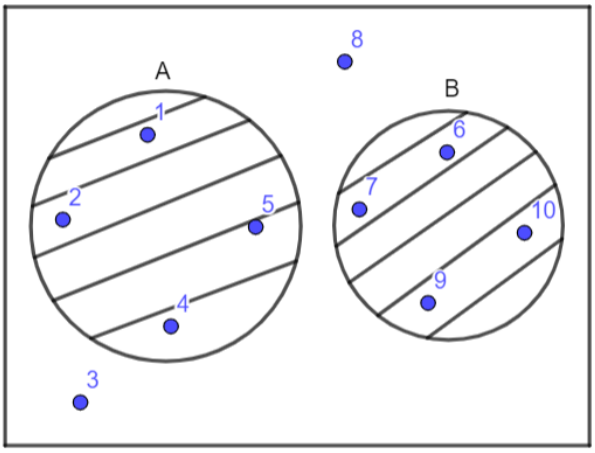
= {6, 7, 9, 10} = B

Therefore A Δ B = (A – B) ∪ (B – A) = A ∪ B = {1, 2, 4, 5} ∪ {6, 7, 9, 10}

= {1, 2, 4, 5, 6, 7, 9, 10}

Venn Diagram :

U



(7) Verify (B ∪ C)' = B' ∩ C'

L.H.S. = (B ∪ C)'

Now, B ∪ C = {6, 7, 9, 10} ∪ {1, 2, 9, 10}

= {1, 2, 6, 7, 9, 10}

Therefore (B ∪ C)' = {3, 4, 5, 8}

R.H.S = B' ∩ C'

= {1, 2, 3, 4, 5, 8} ∩ {3, 4, 5, 6, 7, 8}

= {3, 4, 5, 8}

Thus, (B ∪ C)' = B' ∩ C'

Example : Let 𝕌 = {11, 12, 13, 14, 15, 16, 17, 18,19, 20}, A = {11, 13, 15, 18},

B = {12, 14, 16, 17, 18}, C = {18, 19, 20}. Then find the following:

(i) A U B and its Venn Diagram (ii) B U C and its Venn Diagram

(iii) P(A) (iv) P(C) (v) A' – B' (vi) B' – C' (vii) A Δ B (viii) C Δ B (ix) B ∩ C

Solution :

(i) A U B = {11, 13, 15, 18} U {12, 14, 16, 17, 18} = {11, 13, 15, 18, 12, 14, 16, 17}

(iii) P(A) = {{ }, {11}, {13}, {15}, {18}, {11, 13}, {11, 15}, {11, 18}, {13, 15}, {13, 18}, {15, 18}, {11, 13, 15}, {11, 13, 18}, {13, 15, 18}, {11, 15, 18}, {11, 13, 15, 18}}

(v) A' = {12, 14, 16, 17, 19, 20} and B' = {11, 13, 15, 19, 20}

C' = {11, 12,13, 14, 15, 16, 17}

A' – B' = {12, 14, 16, 17, 19, 20} – {11, 13, 15, 19, 20} = {12, 14, 16, 17}

B' – A' = {11, 13, 15}

(vi) B' – C' = {11, 13, 15, 19, 20} – {11, 12, 13, 14, 15, 16, 17} = {19, 20}

(vii) A Δ B = (A – B) ∪ (B – A)

A – B = {11, 13, 15, 18} – {12, 14, 16, 17, 18} = {11, 13, 15}

B – A = {12, 14, 16, 17, 18} – {11, 13, 15, 18} = {12, 14, 16, 17}

A Δ B = {11, 13, 15} ∪ {12, 14, 16, 17} = {11, 13, 15, 12, 14, 16, 17}

(viii) C Δ B = **(**C – B) ∪ **(**B – C)

C – B = {18, 19, 20} – {12, 14, 16, 17, 18} = {19, 20}

B – C = {12, 14, 16, 17, 18} – {18, 19, 20} = {12, 14, 16, 17}

C Δ B = **(**C – B) ∪ **(**B – C)

= {19, 20} ∪ {12, 14, 16, 17}

= {19, 20, 12, 14, 16, 17}

➢ Algebras Of Sets

(1) Commutative Laws :

(i) Union is commutative : A ∪ B = B ∪ A

Let A = {1, 2, 3} and B = {3, 4, 5}

L.H.S. = A ∪ B

= {1, 2, 3} ∪ {3, 4, 5}

= {1, 2, 3, 4, 5}

R.H.S. = B ∪ A

= {3, 4, 5} ∪ {1, 2, 3}

= {1, 2, 3, 4, 5}

Thus, A ∪ B = B ∪ A

i.e., Union is commutative.

(ii) Intersection is commutative : A ∩ B = B ∩ A

Let A = {1, 2, 3} and B = {3, 4, 5}

L.H.S. = A ∩ B

= {1, 2, 3} ∩ {3, 4, 5}

= {3}

R.H.S. = B ∩ A

= {3, 4, 5} ∩ {1, 2, 3}

= {3}

Thus, A ∩ B = B ∩ A

i.e., Intersection is commutative

(2) Associative Laws :

(i) Union is associative : A ∪ (B ∪ C) = (A ∪ B) ∪ C

Consider A = {1, 2, 3, 4}, B = {2, 3, 5, 6} and C = {2, 4, 7}

L.H.S. = A ∪ (B ∪ C)

= {1, 2, 3, 4} ∪ [{2, 3, 5, 6} ∪ {2, 4, 7}]

= {1, 2, 3, 4} ∪ {2, 3, 4, 5, 6, 7}

= {1, 2, 3, 4, 5, 6, 7}

R.H.S. = (A ∪ B) ∪ C

= [{1, 2, 3, 4} ∪ {2, 3, 5, 6}] ∪ {2, 4, 7}

= {1, 2, 3, 4, 5, 6} ∪ {2, 4, 7}

= {1, 2, 3, 4, 5, 6, 7}

Thus, A ∪ (B ∪ C) = (A ∪ B) ∪ C

i.e., Union is associative.

(ii) Intersection is associative : A ∩ (B ∩ C) = (A ∩ B) ∩ C

Consider A = {1, 2, 3, 4}, B = {2, 3, 5, 6} and C = {2, 4, 7}

L.H.S. = A ∩ (B ∩ C)

= {1, 2, 3, 4} ∩ [{2, 3, 5, 6} ∩ {2, 4, 7}]

= {1, 2, 3, 4} ∩ {2}

= {2}

R.H.S. = (A ∩ B) ∩ C

= [{1, 2, 3, 4} ∩ {2, 3, 5, 6}] ∩ {2, 4, 7}

= {2, 3} ∩ {2, 4, 7}

= {2}

Thus, A ∩ (B ∩ C) = (A ∩ B) ∩ C

i.e., Intersection is associative.

(3) Distributive Laws :

(i) Union is distributive over intersection : A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Consider A = {1, 2, 3, 4}, B = {2, 4, 5, 6}, C = {4, 5, 7}

L.H.S. = A ∪ **(**B ∩ C**)**

= {1, 2, 3, 4} ∪ [{2, 4, 5, 6} ∩ {4, 5, 7}]

= {1, 2, 3, 4} ∪ {4, 5}

= {1, 2, 3, 4, 5}

R.H.S. = **(**A ∪ B**)** ∩ **(**A ∪ C**)**

= [{1, 2, 3, 4} ∪ {2, 4, 5,6}] ∩ [{1, 2, 3, 4} ∪ {4, 5, 7}]

= {1, 2, 3, 4, 5, 6} ∩ {1, 2, 3, 4, 5, 7} = {1, 2, 3, 4, 5}

Thus, A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

i.e., Union is distributive over intersection.

(ii) Intersection is distributive over union : A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Consider A = {1, 2, 3, 4}, B = {2, 4, 5, 6}, C = {4, 5, 7}

L.H.S. = A ∩ **(**B ∪ C**)**

= {1, 2, 3, 4} ∩ [{2, 4, 5, 6} ∪ {4, 5, 7}]

= {1, 2, 3, 4} ∩ {2, 4, 5, 6, 7}

= {2, 4}

R.H.S. = **(**A ∩ B**)** ∪ **(**A ∩ C**)**

= [{1, 2, 3, 4} ∩ {2, 4, 5, 6}] ∪ [{1, 2, 3, 4} ∩ {4, 5, 7}]

= {2, 4} ∪ {4}

= {2, 4}

Thus, A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

i.e., Intersection is distributive over union.

- **De Morgan's Laws :**

Let 𝕌 be the Universal set and A and B are any two subsets of 𝕌 then,

(i) (A ∪ B)' = A' ∩ B'

Consider 𝕌 = {1, 2, 3, 4, 5, 6, 7}, A = {1, 2, 3, 4} and B = {3, 4, 5}

A ∪ B = {1, 2, 3, 4} ∪ {3, 4, 5} = {1, 2, 3, 4, 5}

L.H.S. = (A ∪ B)'

= 𝕌 – (A ∪ B)

= {1, 2, 3, 4, 5, 6, 7} – {1, 2, 3, 4, 5}

= {6, 7}

A' = {5, 6, 7} and B' = {1, 2, 6, 7}

R.H.S. = A' ∩ B'

= {5, 6, 7} ∩ {1, 2, 6, 7}

= {6, 7}

Thus, (A ∪ B)' = A' ∩ B'

(ii) (A ∩ B)' = A' ∪ B'

Consider 𝕌 = {1, 2, 3, 4, 5, 6, 7}, A = {1, 2, 3, 4} and B = {3, 4, 5}

A ∩ B = {1, 2, 3, 4} ∩ {3, 4, 5} = {3, 4}

L.H.S. = (A ∩ B)'

= 𝕌 – (A ∩ B)

= {1, 2, 3, 4, 5, 6, 7} – {3, 4}

= {1, 2, 5, 6, 7}

A' = {5, 6, 7}, B' = {1, 2, 6, 7}

R.H.S. = A' ∪ B'

= {5, 6, 7} ∪ {1, 2, 6, 7}

= {1, 2, 5, 6, 7}

Thus, (A ∩ B)’ = A’ ∪ B’

Example : If A, B and C are three sets such that A ⊂ B then prove that **:** C – B ⊂ C – A

Solution : A = {1, 2, 3}, B = {1, 2, 3, 4, 5} and C = {1, 4, 6, 7}

L.H.S. = C – B

= {1, 4, 6, 7} – {1, 2, 3, 4, 5}

= {6, 7} ----- (1)

R.H.S. = C – A

= {1, 4, 6, 7} – {1, 2, 3}

= {4, 6, 7} ----- (2)

Now, {7, 6} ⊂ {4, 6, 7}

Thus, from (1) and (2), C – B ⊂ C – A

Example : If A, B and C are any three sets then prove the following**.**

A – (B ∪ C) = (A – B) ∩ (A – C)

Solution : Let A = {1, 2, 3, 4}, B = {1, 3, 5} and C = {1, 5, 6}

L.H.S. = A – **(**B ∪ C**)**

= {1, 2, 3, 4} – [{1, 3, 5} ∪ {1, 5, 6}]

= {1, 2, 3, 4} – {1, 3, 5, 6}

= {2, 4}

R.H.S. = (A – B) ∩ (A – C)

= [{1, 2, 3, 4} – {1, 3, 5}] ∩ [{1, 2, 3, 4} – {1, 5, 6}

= {2, 4} ∩ {2, 3, 4}

= {2, 4}

Thus, A – (B ∪ C) = (A – B) ∩ (A – C)

Example : If A, B and C are any three sets then prove the following**.**

$A - (B \cap C) = (A - B) \cup (A - C)$

Solution : $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ and $C = \{1, 5, 7\}$

L.H.S. $= A - (B \cap C)$

$= \{1, 2, 3, 4\} - [\{3, 4, 5, 6\} \cap \{1, 5, 7\}]$

$= \{1, 2, 3, 4\} - \{5\}$

$= \{1, 2, 3, 4\}$

R.H.S. $= (A - B) \cup (A - C)$

$= [\{1, 2, 3, 4\} - \{3, 4, 5, 6\}] \cup [\{1, 2, 3, 4\} - \{1, 5, 7\}]$

$= \{1, 2\} \cup \{2, 3, 4\}$

$= \{1, 2, 3, 4\}$

Thus, $A - (B \cap C) = (A - B) \cup (A - C)$

- <u>**Some Important Results on Cardinality of finite sets :**</u>

(1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Example : $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$

Thus, $A \cap B = \{3, 4\}$ and $A \cup B = \{1, 2, 3, 4, 5, 6\}$

$n(A \cup B) = 6$

$n(A) + n(B) - n(A \cap B) = 4 + 4 - 2 = 6$

(2) $n(A \cup B) = n(A) + n(B)$ if A and B are disjoint sets

Example : $A = \{1, 2, 3\}$ and $B = \{4, 5\}$ so $A \cup B = \{1, 2, 3, 4, 5\}$

$n(A \cup B) = 5$

$n(A) + n(B) = 3 + 2 = 5$

(3) $n(A - B) = n(A) - n(A \cap B)$

Example : $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 5, 7, 8\}$

$A - B = \{1, 2, 3, 4, 5\} - \{3, 5, 7, 8\} = \{1, 2, 4\}$, $A \cap B = \{3, 5\}$

$n(A - B) = 3$

n (A) – n (A ∩ B) = 5 – 2 = 3

n (B – A) = n (B) – n (A ∩ B)

- **Cartesian product of two sets : (Cross Product)**

Let A and B are two sets. A set containing all possible pairs (x, y) where $x \in$ A and $y \in$ B is called Cartesian product of A and B.

Notation : A × B

A × B = {(x, y) | $x \in$ A and $y \in$ B}

Note**:** (i) If A and B are different sets then A × B ≠ B × A but n (A × B) = n (B × A).

(ii) n (A × B) = n (B × A) = n (A) × n (B)

Example : Let A = {1, 2, 3} and B = {a, b, c, d}. Then find A × B and B × A.

A × B = {1, 2, 3} × {a, b, c, d}

= {(1, a), (1, b), (1, c), (1, d), (2, a), (2, b), (2, c), (2, d), (3, a), (3, b), (3, c), (3, d)}

B × A = {a, b, c, d} × {1, 2, 3}

= {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3), (d, 1), (d, 2), (d, 3)}

Example : Let A = {1, 2, 3, 4} and B= {1, 4, 5} then find

(i) A × B (ii) B × A (iii) (A × B) ∩ (B × A)

(i) A × B = {1, 2, 3, 4} × {1, 4, 5}

= {(1, 1), (1, 4), (1, 5), (2, 1), (2, 4), (2, 5), (3, 1), (3, 4), (3, 5), (4, 1), (4, 4), (4, 5)}

(ii) B × A = {1, 4, 5} × {1, 2, 3, 4}

= {(1, 1), (1, 2), (1, 3), (1, 4), (4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4)}

(iii) (A × B) ∩ (B × A) = {(1, 1), (1, 4), (4, 1), (4, 4)}

- **Multiple Choice Questions :**

1) Which of the following is a singleton set?

a) A = {$x \in$ N / 1 < x < 4}

b) A = {$x \in$ N / 1 < x < 2}

c) A = $\{x \in Q \,/\, 1 < x < 2\}$

d) None of these

2) Which of the following is a null set?

a) $\{x \in N \,/\, 1 < x < 2\}$

b) $\{x \in N \,/\, 1 \leq x < 2\}$

c) $\{x \in N \,/\, 1 < x \leq 2\}$

d) $\{x \in N \,/\, 1 \leq x \leq 2\}$

3) Which of the following is a finite set?

a) $\{x \in Q \,/\, 1 < x < 2\}$

b) $\{x \in R \,/\, 1 < x < 2\}$

c) $\{x \in N \,/\, 1 \leq x < 2\}$

d) None of these

4) Which of the following is an infinite set?

a) $\{x \in Q \,/\, 1 < x < 2\}$

b) $\{x \in N \,/\, x < 2\}$

c) $\{x \in N \,/\, x < 200\}$

d) $\{x \in Z \,/\, 1 < x < 100\}$

5) If A ⊂ B, then A – B = ____.

a) Ø

b) A

c) B

d) None of these

6) A ∩ A’ = ____

a) Ø

b) A

c) A’

d) None of these

7) A ∪ A’ = _____

a) ∅

b) A

c) A’

d) U

8) Let U = {a, e, i, o, u} and A = {a, i, e}. Then A’ = ___

a) {a, i, e}

b) {o, u}

c) {a, o, u}

d) {a, e, u}

9) If n (A) = 3 and n (B) = 4, then n (A × B) = ____.

a) 7

b) 27

c) 12

d) None of these

10) If n (A) = 4, then A has _____ subsets.

a) 16

b) 15

c) 8

d) 7