

# **School of Computing**

# CSE306 - DESIGN & ANALYSIS OF ALGORITHMS LABORATORY

### List of Exercises

- 1. Programs on Comparison Sorts Merge Sort, Quick Sort and Heap Sort
- 2. Programs on Linear Sorting techniques Counting Sort, Radix Sort and Bucket Sort
- 3. Applications of Divide and Conquer approach Maximum Subarray problem
- 4. Applications of Dynamic Programming Matrix Chain Multiplication
- 5. Applications of Dynamic Programming Longest Common Subsequence
- 6. Applications of Greedy Technique Text data compression using Huffman Codes
- 7. Programs on Graphs Topological Sort of Directed Acyclic Graph
- 8. Programs on Graphs Minimum Spanning Tree using Prim's and Kruskal's algorithms
- 9. Programs on Graphs Single Source Shortest paths using Bellman-Ford algorithm
- 10. Programs on Graphs All-Pairs Shortest Paths using Floyd-Warshall algorithm
- 11. String Matching with Rabin-Karp algorithm
- 12. String Matching Knuth-Morris-Pratt (KMP) algorithm

### **General Instructions:**

For every exercise,

- ➤ Input should be generated as random numbers and it should be stored as a separate file.
- > The generated file has to give as input for the program.
- > Output has to be written as separate file.
- > Graph has to be drawn with respect to size of the input and time taken.

### **Evaluation Procedure:**

#### **Lab Performance Assessment**

S. No	Metrics	Marks
1	Student's Performance	5
2	Output Verification	2
3	Record	3
4	Total	10

#### **Internal Mark Assessment**

S. No	Metrics	Marks
1	Model - I	15
2	Model - II	15
3	Lab Performance Assessment	20
4	Total	50

### Ex. No: 01 Comparison Sorting

**Objective:** To analyze the comparison sorting algorithms by counting the number of swaps performed

**Pre-Lab Exercise:** Implementation knowledge on insertion sort

#### 1.Merge Sort

```
MERGE-SORT(A, p, r)
1 if p < r
2
       q = \lfloor (p+r)/2 \rfloor
3
       MERGE-SORT(A, p, q)
4
       MERGE-SORT(A, q + 1, r)
5
       Merge(A, p, q, r)
Merge(A, p, q, r)
 1 \quad n_1 = q - p + 1
 2 n_2 = r - q
 3 let L[1..n_1 + 1] and R[1..n_2 + 1] be new arrays
 4 for i = 1 to n_1
 5
        L[i] = A[p+i-1]
 6 for j = 1 to n_2
 7
        R[j] = A[q+j]
   L[n_1+1]=\infty
9 R[n_2 + 1] = \infty
10 i = 1
11
   j = 1
12 for k = p to r
13
        if L[i] \leq R[j]
14
            A[k] = L[i]
            i = i + 1
15
        else A[k] = R[j]
16
17
            j = j + 1
```

### 2.Quick Sort

```
Quicksort(A, p, r)

1 if p < r

2 q = \text{Partition}(A, p, r)

3 Quicksort(A, p, q - 1)

4 Quicksort(A, q + 1, r)
```

```
Partition(A, p, r)
1
   x = A[r]
   i = p - 1
3
   for j = p to r - 1
4
       if A[j] \leq x
5
            i = i + 1
            exchange A[i] with A[j]
6
7
   exchange A[i + 1] with A[r]
   return i+1
3.Heap Sort
HEAPSORT(A)
   BUILD-MAX-HEAP(A)
   for i = A.length downto 2
3
      exchange A[1] with A[i]
4
      A.heap-size = A.heap-size - 1
5
      Max-Heapify(A, 1)
BUILD-MAX-HEAP(A)
   A.heap-size = A.length
2 for i = |A.length/2| downto 1
3
      Max-Heapify(A, i)
Max-Heapify(A, i)
    l = LEFT(i)
     r = Right(i)
    if l \leq A. heap-size and A[l] > A[i]
 4
         largest = l
 5
     else largest = i
     if r \le A.heap-size and A[r] > A[largest]
 6
 7
         largest = r
    if largest \neq i
 8
 9
         exchange A[i] with A[largest]
         Max-Heapify(A, largest)
10
```

### Ex. No: 2 Linear Sorting Techniques

**Objective:** To compare the Non-comparison sorting algorithms with comparison sorting algorithms with respect to the space time trade off

**Pre-Lab Exercise:** Simple program for time, space trade off

### 1. Counting Sort

```
COUNTING-SORT (A, B, k)

1 let C[0..k] be a new array

2 for i = 0 to k

3 C[i] = 0

4 for j = 1 to A.length

5 C[A[j]] = C[A[j]] + 1

6 //C[i] now contains the number of elements equal to i.

7 for i = 1 to k

8 C[i] = C[i] + C[i - 1]

9 //C[i] now contains the number of elements less than or equal to i.

10 for j = A.length downto 1

11 B[C[A[j]]] = A[j]

12 C[A[j]] = C[A[j]] - 1
```

Note: Each element in the n-element array 'A' has nonnegative integer no larger than 'k' and 'B' is the sorted output array

### 2. Radix Sort

```
RADIX-SORT(A, d)

1 for i = 1 to d

2 use a stable sort to sort array A on digit i
```

Note: Each element in the n-element array 'A' has 'd' digits, where digit '1' is the lowest-order digit and digit 'd' is the highest-order digit.

#### 3. Bucket Sort

```
BUCKET-SORT(A)

1 let B[0..n-1] be a new array

2 n = A.length

3 for i = 0 to n - 1

4 make B[i] an empty list

5 for i = 1 to n

6 insert A[i] into list B[\lfloor nA[i] \rfloor]

7 for i = 0 to n - 1

8 sort list B[i] with insertion sort

9 concatenate the lists B[0], B[1], \ldots, B[n-1] together in order
```

Note: Each element in the n-element array 'A' satisfies 0<A[i]<1

# Ex. No: 03 Divide and Conquer approach- Maximum Subarray problem

**Objective:** To compare the divide conquer approach of maximum subarray problem with its brute force approach

**Pre-Lab Exercise:** Program to solve maximum subarray problem using brute force approach

# 1.Solving the Maximum Subarray problem using Divide and Conquer

```
FIND-MAXIMUM-SUBARRAY (A, low, high)
 1 if high == low
        return (low, high, A[low])
                                            // base case: only one element
 3 else mid = |(low + high)/2|
        (left-low, left-high, left-sum) =
             FIND-MAXIMUM-SUBARRAY (A, low, mid)
         (right-low, right-high, right-sum) =
 5
             FIND-MAXIMUM-SUBARRAY (A, mid + 1, high)
 6
        (cross-low, cross-high, cross-sum) =
             FIND-MAX-CROSSING-SUBARRAY (A, low, mid, high)
 7
        if left-sum > right-sum and left-sum > cross-sum
             return (left-low, left-high, left-sum)
        elseif right-sum \ge left-sum and right-sum \ge cross-sum
10
             return (right-low, right-high, right-sum)
11
        else return (cross-low, cross-high, cross-sum)
```

```
FIND-MAX-CROSSING-SUBARRAY (A, low, mid, high)
```

```
left-sum = -\infty
 2 \quad sum = 0
 3 for i = mid downto low
       sum = sum + A[i]
4
 5
       if sum > left-sum
6
            left-sum = sum
 7
            max-left = i
8 right-sum = -\infty
9
   sum = 0
10 for j = mid + 1 to high
        sum = sum + A[j]
        if sum > right-sum
12
            right-sum = sum
13
14
            max-right = j
15 return (max-left, max-right, left-sum + right-sum)
```

# Ex. No: 04 Dynamic Programming - Matrix Chain Multiplication

**Objective:** To compare the dynamic programming approach of matrix chain multiplication problem with its brute force approach

**Pre-Lab Exercise:** Program to solve matrix chain multiplication problem using brute force approach

# 1. Matrix chain multiplication problem using dynamic programming

```
Algorithm Bottom_up_MCM(P)
1.n=p. length-1
2.for i=1 to n
3. m[i, i] = 0
4.\text{for } l=1 \text{ to } n-1
5. for i=1 to n-1
   j=i+1
      q=\infty for k=i to j-1
7.
8.
9.
           if q > m [i, k] + m [k+1, j] + P_{i-1}P_kP_j
10.
             q= m [i, k] + m [k+1, j] + P_{i-1}P_kP_j
11.
             s[i, j] = k
             m[i, j] = q
13. return(m,s)
```

```
Algorithm OPT_PAR(s,i,j)

1. if i==j then

2. print A<sub>i</sub>

3. else

4. print "("

5. OPT_PAR(s,i,s[i,j])

6. OPT_PAR(s,s[i,j]+1,j)

7. print ")"

8. return
```

# Ex. No: 05 Dynamic Programming - Longest Common Subsequence

**Objective:** To compare the dynamic programming approach of longest Common Subsequence (LCS) problem with its brute force approach

Pre-Lab Exercise: Solve LCS problem using brute force approach

### 1. problem using dynamic programming

```
LCS-LENGTH(X, Y)
 1 \quad m = X.length
 2 \quad n = Y.length
 3 let b[1..m, 1..n] and c[0..m, 0..n] be new tables
 4 for i = 1 to m
 5
         c[i, 0] = 0
 6 for j = 0 to n
 7
        c[0, j] = 0
 8 for i = 1 to m
 9
         for j = 1 to n
10
             if x_i == y_i
                  c[i, j] = c[i-1, j-1] + 1
11
                  b[i,j] = "
"
12
             elseif c[i - 1, j] \ge c[i, j - 1]
13
                  c[i,j] = c[i-1,j]
14
15
                  b[i,j] = "\uparrow"
             else c[i, j] = c[i, j - 1]
16
17
                  b[i,j] = "\leftarrow"
18 return c and b
```

```
PRINT-LCS (b, X, i, j)

1 if i == 0 or j == 0

2 return

3 if b[i, j] == \text{``\cdot'}

4 PRINT-LCS (b, X, i - 1, j - 1)

5 print x_i

6 elseif b[i, j] == \text{``\cdot'}

7 PRINT-LCS (b, X, i - 1, j)

8 else PRINT-LCS (b, X, i, j - 1)
```

# Ex. No: 06 Applications of Greedy Technique - Text data compression using Huffman Codes

**Objective:** To compare the variable length encoding with fixed length encoding

**Pre-Lab Exercise:** Program for fixed length encoding

```
HUFFMAN(C)

1 n = |C|

2 Q = C

3 for i = 1 to n - 1

4 allocate a new node z

5 z.left = x = Extract-Min(Q)

6 z.right = y = Extract-Min(Q)

7 z.freq = x.freq + y.freq

8 INSERT(Q, z)

9 return Extract-Min(Q) // return the root of the tree
```

## Ex. No: 07 Programs on Graphs -Topological Sort of Directed Acyclic Graph

**Objective:** Find the linear ordering of all vertices in Directed Acyclic Graph (DAG).

**Pre-Lab Exercise:** Knowledge on Depth first search algorithm.

TOPOLOGICAL-SORT(G)

- 1 call DFS(G) to compute finishing times  $\nu$ . f for each vertex  $\nu$
- 2 as each vertex is finished, insert it onto the front of a linked list
- 3 **return** the linked list of vertices

```
DFS(G)
1 for each vertex u \in G.V
       u.color = WHITE
3
       u.\pi = NIL
4 \quad time = 0
5 for each vertex u \in G.V
       if u.color == WHITE
7
           DFS-VISIT(G, u)
DFS-VISIT(G, u)
 1 time = time + 1
                                  /\!\!/ white vertex u has just been discovered
 2 \quad u.d = time
 3 u.color = GRAY
 4 for each v \in G.Adj[u]
                                  /\!\!/ explore edge (u, v)
 5
        if v.color == WHITE
 6
             v.\pi = u
             DFS-VISIT(G, v)
 8 u.color = BLACK
                                  // blacken u; it is finished
 9 time = time + 1
10 u.f = time
```

# Ex. No: 08 Programs on Graphs - Minimum Spanning Tree using Prim's and Kruskal's algorithms

**Objective:** Find the Minimum Spanning Tree by using Prim's and Kruskal's algorithms.

Pre-Lab Exercise: Knowledge on generic minimum spanning tree.

#### **Prim's Algorithm**

```
MST-PRIM(G, w, r)
     for each u \in G.V
 1
 2
         u.kev = \infty
 3
         u.\pi = NIL
 4 r.key = 0
 5 \quad Q = G.V
 6 while Q \neq \emptyset
 7
         u = \text{EXTRACT-MIN}(Q)
         for each v \in G.Adj[u]
 8
              if v \in Q and w(u, v) < v. key
 9
10
                   \nu.\pi = u
                   v.key = w(u, v)
11
```

#### Kruskal's Algorithm

```
MST-KRUSKAL(G, w)
1
  A = \emptyset
2 for each vertex v \in G.V
3
        MAKE-SET(\nu)
  sort the edges of G.E into nondecreasing order by weight w
5
   for each edge (u, v) \in G.E, taken in nondecreasing order by weight
6
        if FIND-SET(u) \neq FIND-SET(v)
7
            A = A \cup \{(u, v)\}\
8
            UNION(u, v)
9
   return A
```

# Ex. No: 09 Programs on Graphs - Single Source Shortest paths using Bellman-Ford algorithm

**Objective:** Computes shortest path from a single source vertex to all of the other vertices in a weighted digraph.

**Pre-Lab Exercise:** Graph representation as adjacency list and adjacency matrix.

```
BELLMAN-FORD(G, w, s)

1 INITIALIZE-SINGLE-SOURCE(G, s)

2 for i = 1 to |G.V| - 1

3 for each edge (u, v) \in G.E

4 RELAX(u, v, w)

5 for each edge (u, v) \in G.E

6 if v.d > u.d + w(u, v)

7 return FALSE

8 return TRUE
```

# Ex. No: 10 Programs on Graphs - All-Pairs Shortest Paths using Floyd-Warshall algorithm

**Objective:** Finding shortest paths between all pairs of vertices in a graph.

**Pre-Lab Exercise:** A recursive solution to the all-pairs shortest-paths problem.

n = W.rows $D^{(0)} = W$ **for** k = 1 **to** n4 let  $D^{(k)} = (d_{ij}^{(k)})$  be a new  $n \times n$  matrix **for** i = 1 **to** n**for** j = 1 **to** n

FLOYD-WARSHALL(W)

 $d_{ij}^{(k)} = \min \left( d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \right)$ 

# Ex. No: 11 String Matching - with Rabin-Karp algorithm

**Objective:** For a given text and patten, find out any valid shifts that are found in the text.

**Pre-Lab Exercise:** Implement naive string-matching algorithm.

```
RABIN-KARP-MATCHER (T, P, d, q)
 1 \quad n = T.length
 2 m = P.length
 3 \quad h = d^{m-1} \bmod q
   p = 0
 5 t_0 = 0
 6 for i = 1 to m
                                 // preprocessing
 7
        p = (dp + P[i]) \mod q
 8
        t_0 = (dt_0 + T[i]) \bmod q
    for s = 0 to n - m
 9
                                 // matching
10
        if p == t_s
            if P[1..m] == T[s+1..s+m]
11
12
                 print "Pattern occurs with shift" s
        if s < n - m
13
            t_{s+1} = (d(t_s - T[s+1]h) + T[s+m+1]) \bmod q
14
```

## Ex. No: 12 String Matching - Knuth-Morris-Pratt (KMP) algorithm

**Objective:** For a given text and patten, find out any valid shifts that are found in the text.

### COMPUTE-PREFIX-FUNCTION (P)

```
m = P.length
 1
 2
    let \pi[1..m] be a new array
3
    \pi[1] = 0
4
    k = 0
5
    for q = 2 to m
6
         while k > 0 and P[k+1] \neq P[q]
7
             k = \pi[k]
        if P[k+1] == P[q]
8
             k = k + 1
 9
         \pi[q] = k
10
11
    return \pi
```

### **Additional Exercises**

- 1. Programs on Strassen's algorithm Matrix Multiplication.
- 2. Applications of Dynamic Programming Rod Cutting.
- 3. Applications of Greedy Technique Activity Selection Problem.
- 4. Programs on Graphs Dijikstra's Algorithm.
- 5. All-Pairs Shortest Paths Johnson's algorithm for sparse graphs.