Basic Signal Processing

Module 3

Introduction & Classification of signals, Basic operations on Signals, System classification and properties

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Dept of Electronics & Communication
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Department of Electronics and Communication

Video links

Module 3: Introduction & Classification of signals, Basic operations on Signals, System classification and properties

Sl. No	Topic	URL
1	Definition of signal and system	https://www.youtube.com/watch?v=up55tuwestg
2	Definition of signal and system (animation video)	https://www.youtube.com/watch?v=ljsvOGlygz0-
3	Representation of signals	https://www.youtube.com/watch?v=ulq2D1F1_DI
4	Elementary signals	https://www.youtube.com/watch?v=RwQNnlLWWcU
5	Basic impulse function	https://www.youtube.com/watch?v=gGHfj-B-w60
6	Basic Unit step function	https://www.youtube.com/watch?v=jQ1xtgpJ-VU
7	Basic Exponential function	https://www.youtube.com/watch?v=fXjQjkUg2ug
8	Basic sinusoidal function	https://www.youtube.com/watch?v=6WgEazflJec
9	Basic operations on discrete time signals	https://www.youtube.com/watch?v=6rNcXqYCRd4
10	Basic operations on discrete time signals	https://www.youtube.com/watch?v=uFjPxA5b6c0
11	System classification	https://www.youtube.com/watch?v=npsZ2SBeVpA
12	System properties	https://www.youtube.com/watch?v=KWXEwYCoL2U-

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Module 3

Introduction and Classification of Signals

Definition of Signal and systems with examples, Elementary Signals / Functions: Exponential, Sinusoidal, Step, impulse and ramp functions

Basic operation on Signals: Amplitude Scaling, Addition, Multiplication, time scaling, time Shift and time reversal

Expression of triangular, rectangular and other waveforms in terms of elementary signals.

System Classification and Properties: Linear-nonlinear, Time variant - invariant, causal- non causal, Static-dynamic stable-unstable, invertible.

Signals

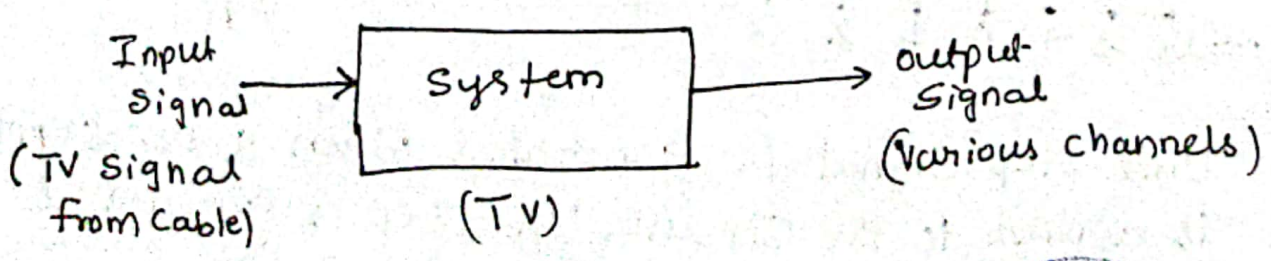
A Function of one or more independent variables which contain some information is called Signal

Eg:- Radio Signal, TV signal, Telephone Signal, Computer Signals etc.
Pressure Signal, Sound Signal, Traffic Signal etc.

Systems

A System is a set of elements or functional block that are connected together and produces an output in response to an input Signal.

Eg:- Amplifiers, Oscillators, TV Transmitters, Receivers etc.



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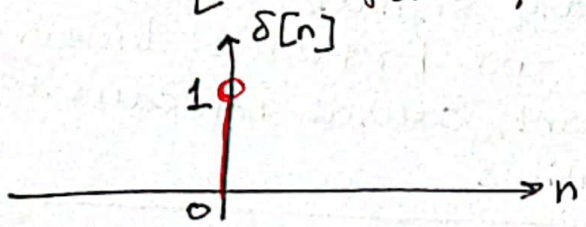
Elementary Signals

1) Unit Sample Signal $\delta[n]$

Definition : Amplitude of unit sample is '1' at $n=0$ and it has zero value at all other values of n.

$$\delta[n] = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

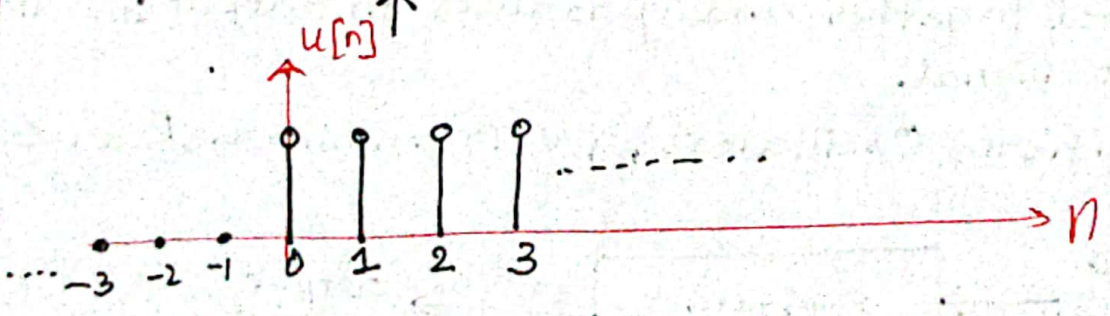


2) Unit Step Signal $u[n]$

Definition : The unit step signal has amplitude of '1' for positive values of independent variable. And it has amplitude of '0' for negative values of independent variable.

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$u[n] = \{0, 0, \underset{\uparrow}{1}, 1, 1, 1, 1, \ldots\ldots\}$$



Unit step signal is generated when a DC supply is applied to the circuit. Unit step is generated when a switch is closed at $t=0$

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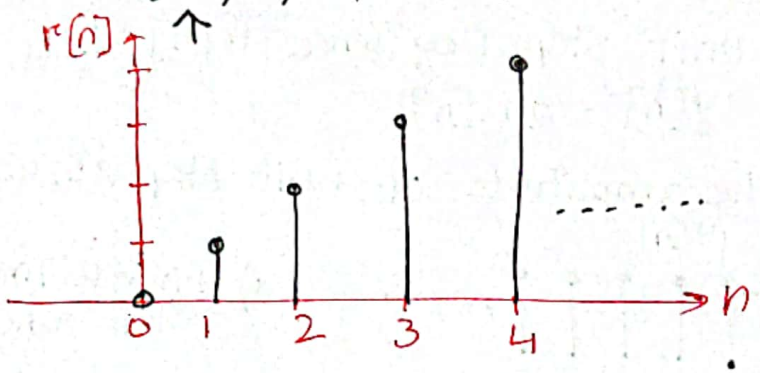
3) Unit Ramp Signal $r[n]$

Definition: The Amplitude of every sample increases linearly with its number (n) for positive Values of 'n'

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$= n\,u[n]$$

$$r(n) = \{0, 1, 2, 3, 4, 5, \ldots\}$$



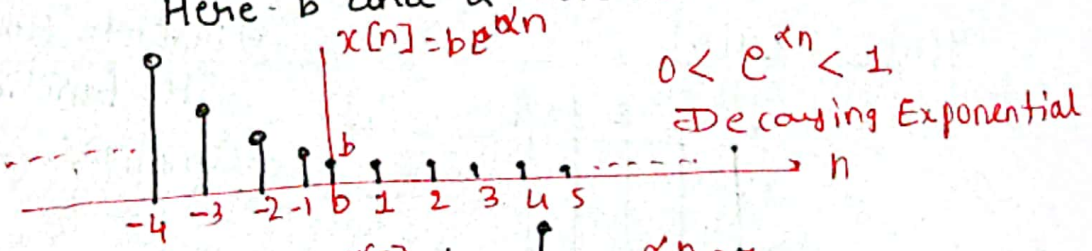
Significance: It indicates constant current charging of the capacitor

4) Real Exponential Signal

Definition: It is exponentially growing or decaying Signal

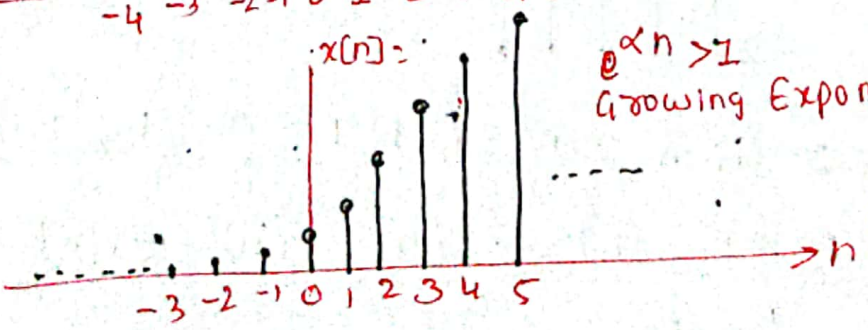
$$x[n] = b e^{\alpha n}$$

Here b and α are real



$0 < e^{\alpha n} < 1$

Decaying Exponential



$e^{\alpha n} > 1$

Growing Exponential

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i) Population growth as a function of generation

ii) Return on investment as a function of day, month or quarter.

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5) Sinusoidal Signals

The General forms of a continuous-time sinusoidal signal is $x[n] = A\cos(\omega n + \phi)$

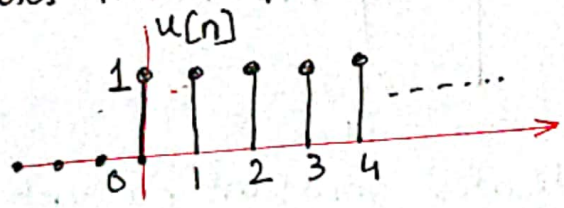
Basic Operation on Signals

1) **Amplitude Scaling**

Consider the unit step, sequence $u[n]$. Let,

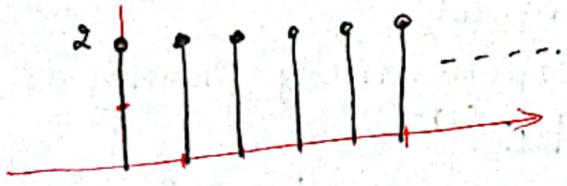
$$y[n] = 2u[n]$$

This increases the amplitude of unit step sequence by '2'



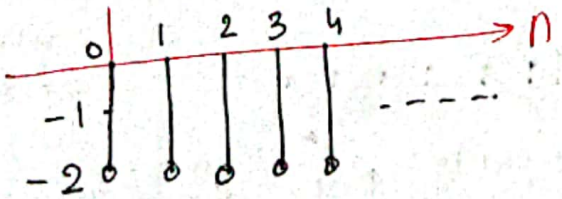
a) Discrete Time unit Step function

$y[n] = 2u[n]$



b) Discrete time unit step function with amplitude 2

$y[n] = -2u[n]$



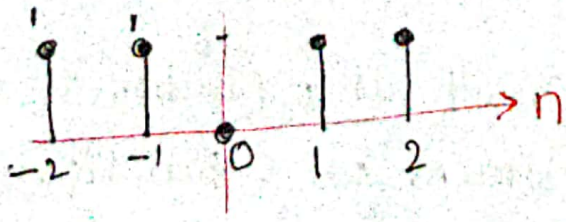
c) Discrete time unit step function with amplitude −2

Example 1 :- Given $x[n] = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$

Sketch (i) $x[n]$ (ii) $-x[n]$ (iii) $2x[n]$ (iv) $-\frac{1}{2}x[n]$

Soln:-

(i) $x[n]$



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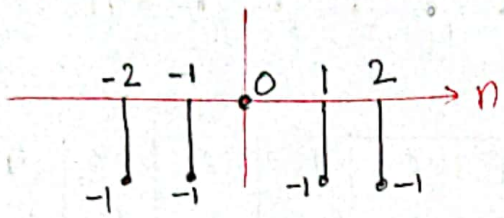
(ii) $-x[n]$

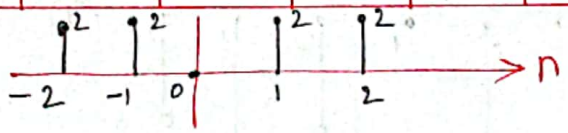
Multiply Each Amplitude by -1

n	-2	-1	0	1	2
x[n]	1	1	0	1	1
-x[n]	-1	-1	0	-1	-1

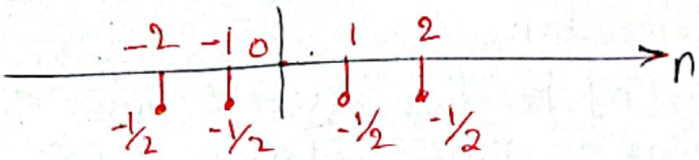


iii) $2x[n]$ Multiply each amplitude by 2

n	-2	-1	0	1	2
x[n]	1	1	0	1	1
2x[n]	2	2	0	2	2



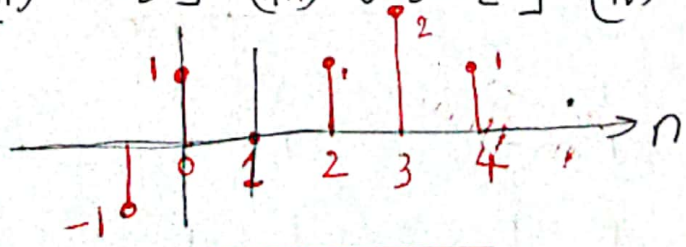
(iv) $-\frac{1}{2}x[n]$ Multiply each amplitude by $-1/2$



Example 2 : Given $x[n] = \{-1, \underset{\uparrow}{1}, 0, 1, 2, 1\}$

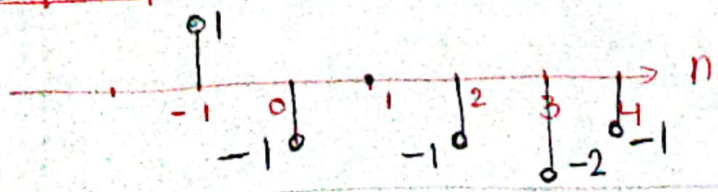
Sketch (i) $x[n]$ (ii) $-x[n]$ (iii) $0.5x[n]$ (iv) $-2x[n]$

Solns:- (i) $x[n]$



ii) $-x[n]$

n	-1	0	1	2	3	4
x[n]	-1	1	0	1	2	1
-x[n]	1	-1	0	-1	-2	-1



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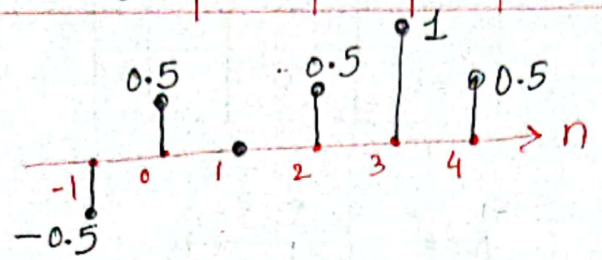
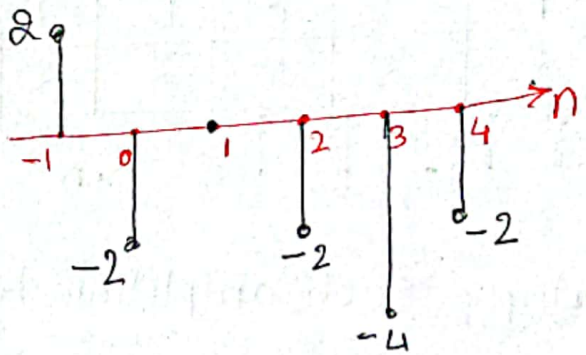
iii) $0.5\,x[n]$

n	-1	0	1	2	3	4
$x[n]$	-1	1	0	1	2	1
$0.5\,x[n]$	-0.5	0.5	0	0.5	1	0.5



iv) $-2\,x[n]$

n	-1	0	1	2	3	4
$x[n]$	-1	1	0	1	2	1
$-2\,x[n]$	2	-2	0	-2	-4	-2



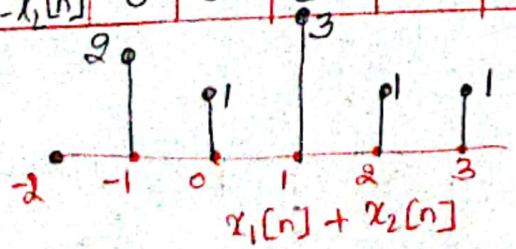
Addition and Subtraction

Let $x_1[n]$ and $x_2[n]$ be two discrete time signals. Their addition is given as $y[n] = x_1[n] + x_2[n]$

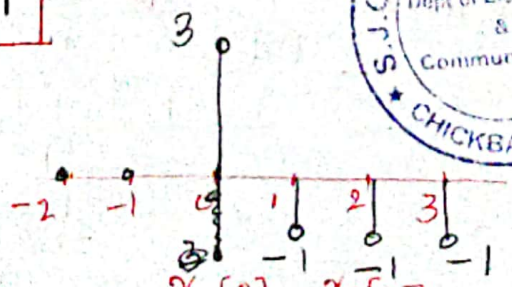
Similarly subtraction is defined as, $y[n] = x_1[n] - x_2[n]$

Example 1 $x_1[n] = \{1, \underset{\uparrow}{2}, 1\}$, $x_2[n] = \{1, \underset{\uparrow}{-1}, 2, 1, 1\}$

n	-2	-1	0	1	2	3
$x_1[n]$	0	1	2	1	0	0
$x_2[n]$	0	1	-1	2	1	1
$x_1[n]+x_2[n]$	0	2	1	3	1	1
$x_1[n]-x_2[n]$	0	0	3	-1	-1	-1



$x_1[n] + x_2[n]$



$x_1[n] - x_2[n]$

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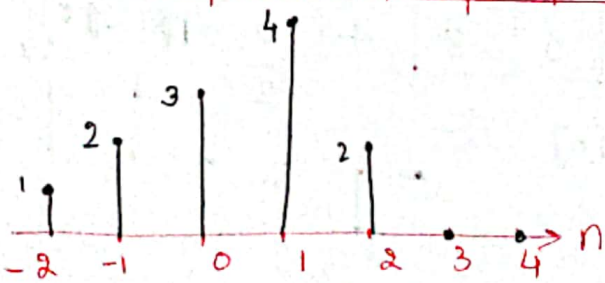
Example Given $x[n] = \{1, \underset{\uparrow}{1}, 2, 1\}$ & $y[n] = \{1, 1, \underset{\uparrow}{2}, 2, 1\}$

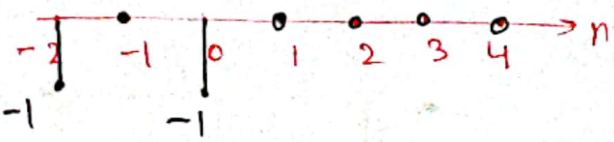
Obtain (i) $x[n] + y[n]$ (ii) $x[n] - y[n]$ (iii) $2x[n] - y[n]$

Soln:-

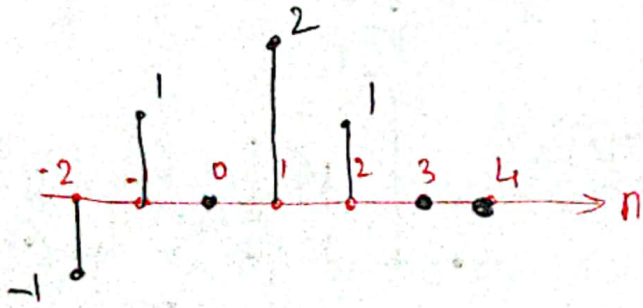
n	-2	-1	0	1	2	3	4
$x[n]$	0	1	1	2	1	0	0
$y[n]$	1	1	2	2	1	0	0
$x[n]+y[n]$	1	2	3	4	2	0	0
$x[n]-y[n]$	-1	0	-1	0	0	0	0
$2x[n]$	0	2	2	4	2	0	0
$2x[n]-y[n]$	-1	1	0	2	1	0	0



i) Sketch of $x[n]+y[n]$



ii) Sketch of $x[n]-y[n]$



iii) Sketch of $2x[n]-y[n]$

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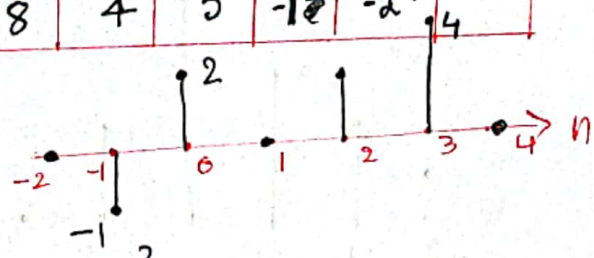
Given $x[n] = \{1, \underset{\uparrow}{2}, 1, 1, 2\}$ & $y[n] = \{-2, \underset{\uparrow}{0}, -1, 1, 2\}$ sketch

i) $x[n] + y[n]$ (ii) $x[n] - y[n]$ (iii) $y[n] - x[n]$ (iv) $2x[n] - 3y[n]$

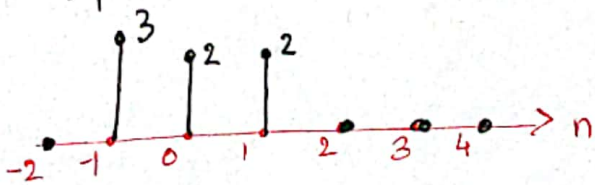
Soln:

n	-2	-1	0	1	2	3	4
$x[n]$	0	1	2	1	1	2	0
$y[n]$	0	-2	0	-1	1	2	0
$x[n] - y[n]$	0	3	2	2	0	0	0
$x[n] + y[n]$	0	-1	2	0	2	4	0
$y[n] - x[n]$	0	-3	-2	-2	0	0	0
$2x[n]$	0	2	4	2	2	4	0
$3y[n]$	0	-6	0	-3	3	6	0
$2x[n] - 3y[n]$	0	8	4	5	-1	-2	0

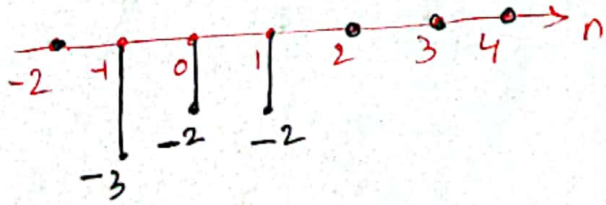
sketch of $x[n] + y[n]$



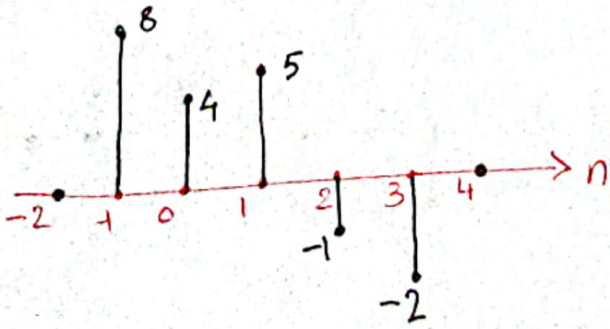
Sketch of $x[n] - y[n]$



Sketch of $y[n] - x[n]$



Sketch of $2x[n] - 3y[n]$



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Multiplication

Let $x[n]$ and $y[n]$ be two discrete time signals. They are Multiplied as $z[n] = x[n] \cdot y[n]$

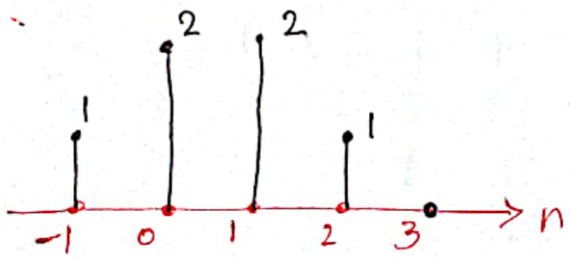
Example 1: Given $x[n] = \{1, \underset{\uparrow}{2}, 1, 1\}$ & $y[n] = \{1, \underset{\uparrow}{1}, 2, 1, 2\}$

Sketch i) $x[n] \cdot y[n]$ (ii) $2 \cdot x[n] \cdot y[n]$.

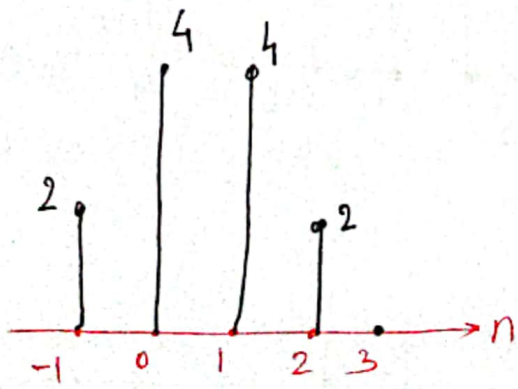
Soln:-

n	-1	0	1	2	3
$x[n]$	1	2	1	1	0
$y[n]$	1	1	2	1	2
$x[n] \cdot y[n]$	1	2	2	1	0
$2 \cdot x[n] \cdot y[n]$	2	4	4	2	0

Sketch of $x[n] \cdot y[n]$



Sketch of $2 \cdot x[n] \cdot y[n]$



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Time Scaling

Two type of time Scaling

1. Time Compression : The time axis is Compressed
   For example $y[n] = x[2n]$
   Here $y[n]$ will be Compressed in time

2. Time Expansion: The time axis is expanded
   For Example $y[n] = x\left[\frac{n}{2}\right]$. Here $y[n]$ will be expanded in time.

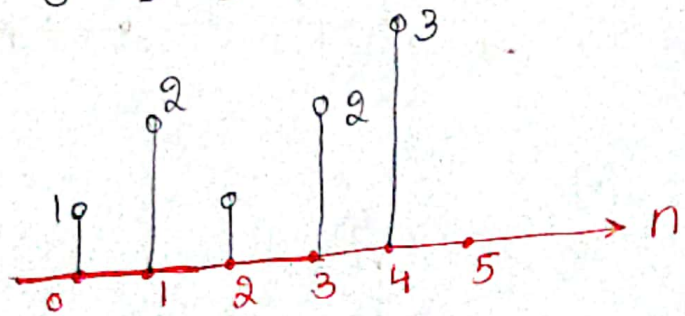
Compression of Discrete Time Signal

Consider the discrete time signal $x[n] = \{\underset{\uparrow}{1}, 2, 1, 2, 3\}$

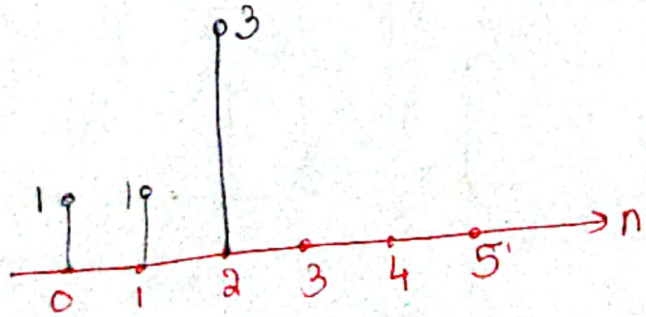
Sketch $y[n] = x[2n]$

n	-1	0	1	2	3	4
$x[n]$	0	1	2	1	2	3
$2n$	-2	0	2	4	6	8
$x[2n]$	0	1	1	3	0	0

$x[n]$



$y[n] = x[2n]$



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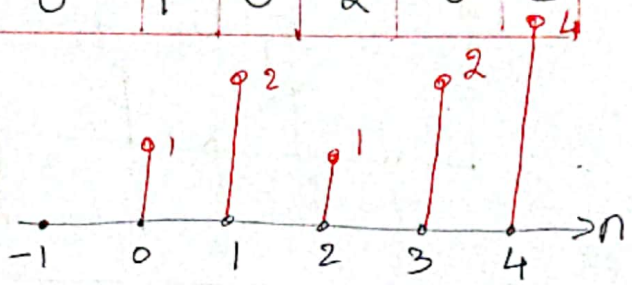
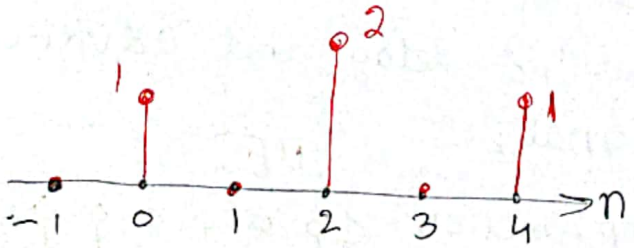
Sketch $y[n] = x\left[\frac{n}{2}\right]$

n	-1	0	1	2	3	4
$x[n]$	0	1	2	1	2	3
$n/2$	$-1/2$	0	$1/2$	1	1.5	2
$x\left[\frac{n}{2}\right]$	0	1	0	2	0	1

$x[n]$



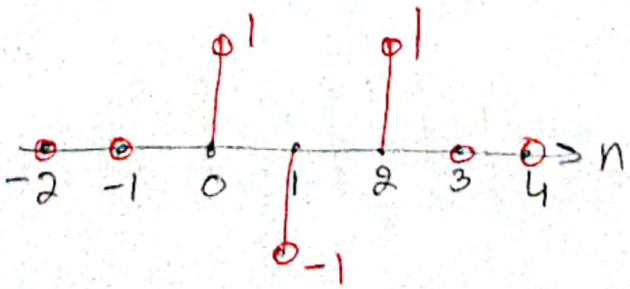
$x[n/2]$



① Given $x[n] = \{2, \underset{\uparrow}{1}, 1, -1, 2, 1\}$. Sketch $x[2n]$, $x\left[\frac{n}{2}\right]$

a) $x[2n]$

n	-2	-1	0	1	2	3	4	5
$x[n]$	0	2	1	+1	-1	2	1	0
$2n$	-4	-2	0	2	4	6	8	10
$x[2n]$	0	0	1	-1	1	0	0	0



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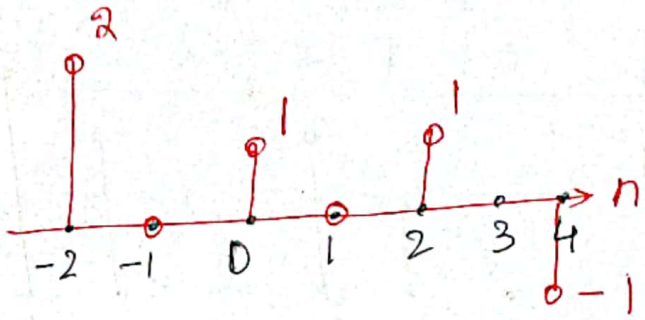
b) $x\left[\frac{n}{2}\right]$

n	-2	-1	0	1	2	3	4
$x[n]$	0	2	1	1	-1	2	0
$n/2$	-1	$-1/2$	0	$1/2$	1	$1\frac{1}{2}$	2
$x\left[\frac{n}{2}\right]$	2	0	1	0	1	0	-1



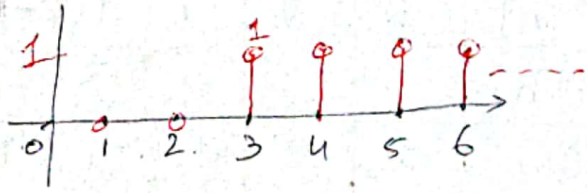
Time Shift

Figure shows the delay and advance operations on Unit Step signal.

Unit Step Function

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

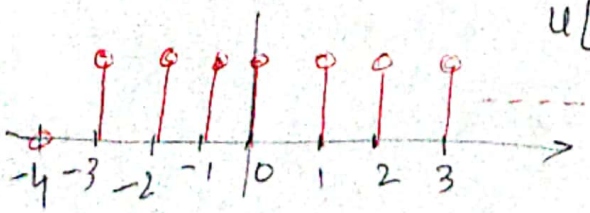
Unit Step function delayed by 3 units



$$u(n-3) = \begin{cases} 1 & \text{for } n \geq 3 \\ 0 & \text{for } n < 3 \end{cases}$$

$\boxed{\text{Delay means Shift Right}}$

Unit Step Function advanced by 3 units



$$u[n+3] = \begin{cases} 1 & \text{for } n \geq -3 \\ 0 & \text{for } n < -3 \end{cases}$$

$\boxed{\text{Advance means Shift left}}$

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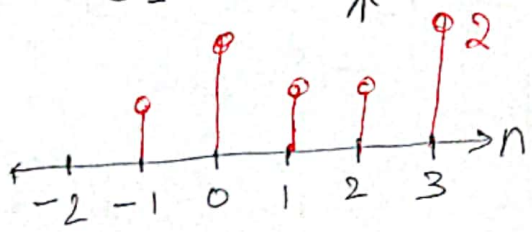
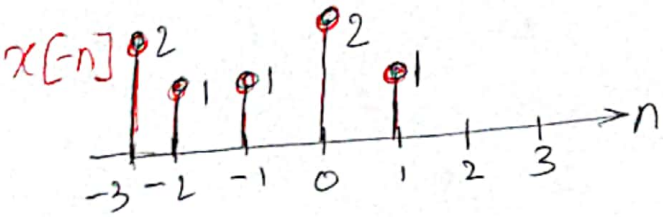
Time Reversal / Time folding

The Time folding is used in Convolution. Consider the Continuous time signal $x[n]$. Then its folded signal can be obtained by replacing n by $-n$

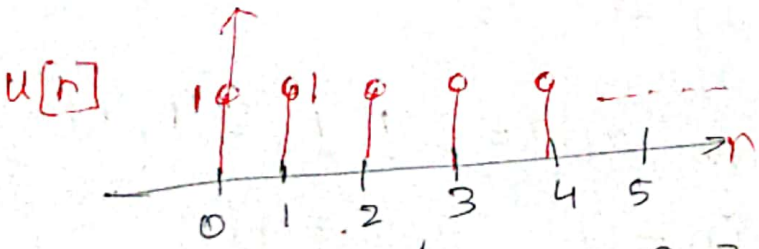
i.e $y[n] = x[-n]$

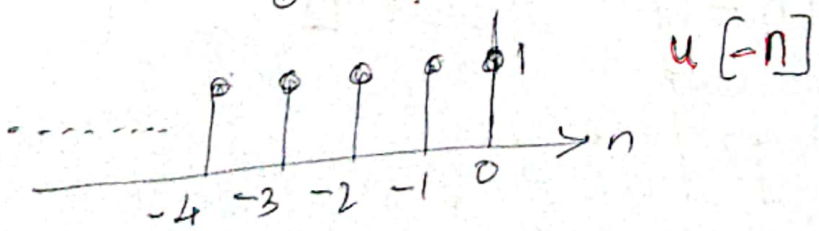
Example 1: Given $x[n] = \{1, \underset{\uparrow}{2}, 1, 1, 2\}$ sketch $x[-n]$





Example 2:- Sketch $u[-n]$





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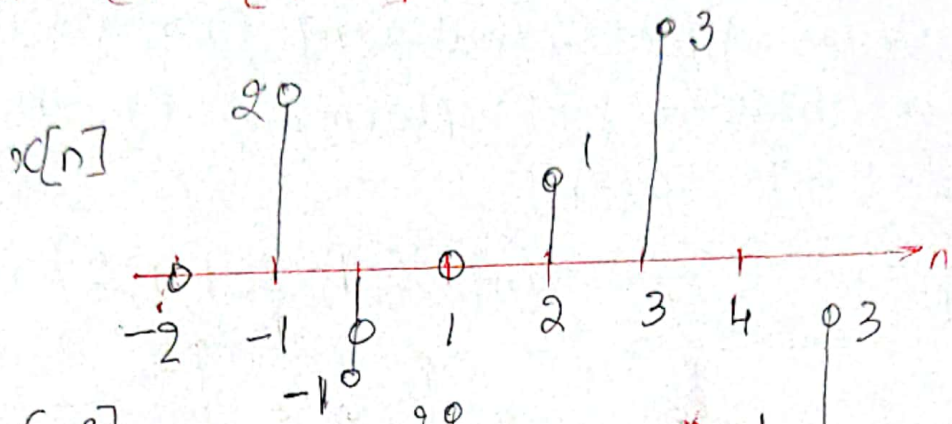
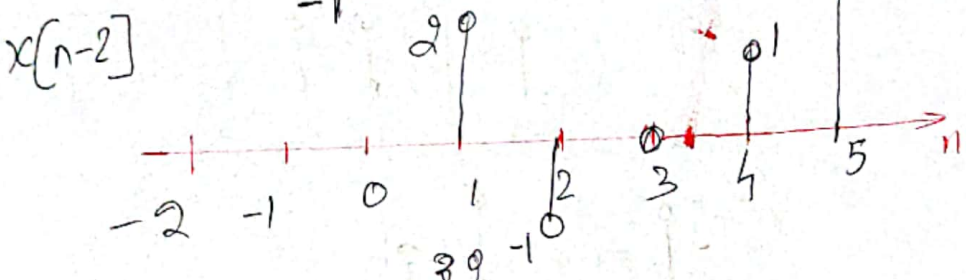
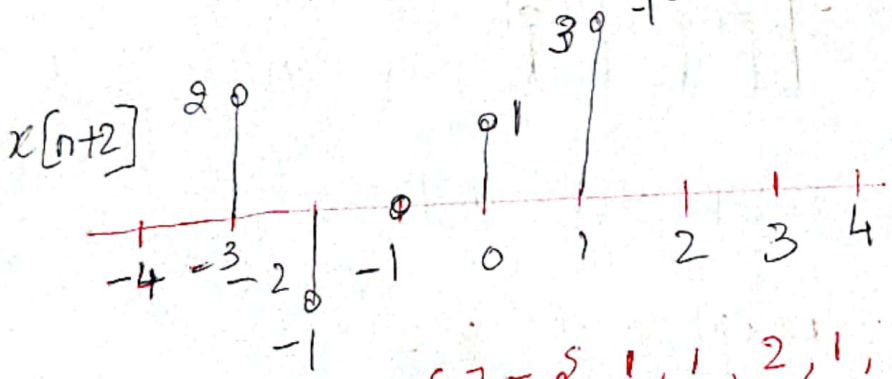
Example 1 :- Given $x[n] = \{2, \underset{\uparrow}{-1}, 0, 1, 3\}$

Sketch (i) $x[n-2]$ (ii) $x[n+2]$

Soln:-

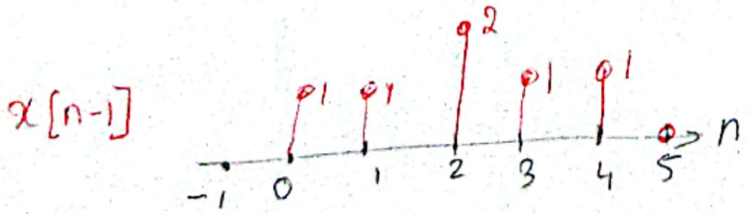


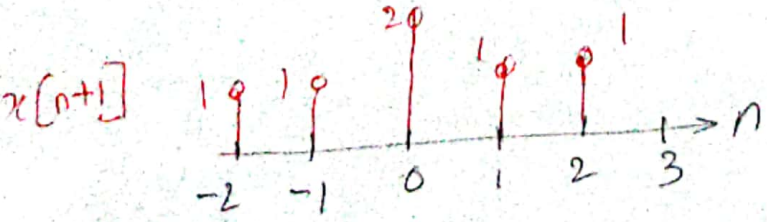




Example 2:- Given $x[n] = \{1, \underset{\uparrow}{1}, 2, 1, 1\}$

Sketch i) $x[n-1]$ ii) $x[n+1]$





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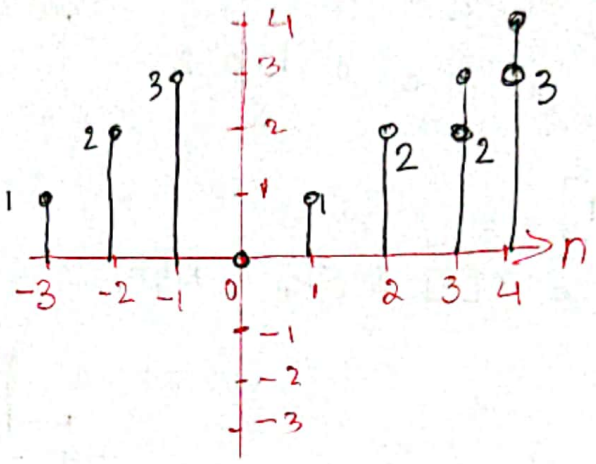
Example Given $x[n] = \begin{cases} n+4 & -3 \le n \le -1 \\ n & 0 \le n \le 2 \\ n-1 & 3 \le n \le 4 \end{cases}$

(i) Sketch a) $x[n]$, b) $x[-n]$, c) $x[2n]$ d) $x[n/2]$ e) $x[n-1]$ f) $x[n+4]$

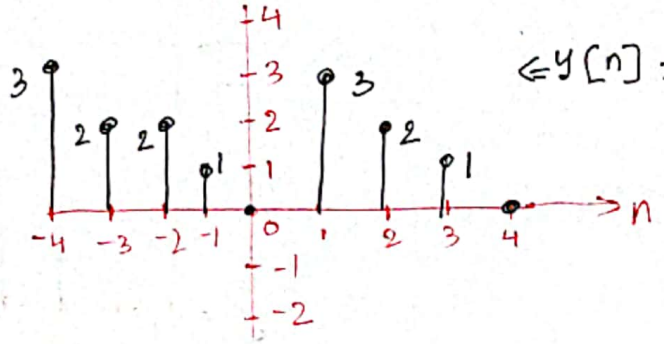
Soln:- a) Sketch $x[n]$

$x[-3] = -3+4 = 1$
$x[-2] = -2+4 = 2$
$x[-1] = -1+4 = 3$
$x[0] = 0$
$x[1] = 1$
$x[2] = 2$
$x[3] = 3-1 = 2$
$x[4] = 4-1 = 3$



b) $x[-n] = y[n]$

$y[-3] = x[3] = 2$
$y[-2] = x[2] = 2$
$y[-1] = x[1] = 1$
$y[0] = x[0] = 0$
$y[1] = x[-1] = 3$
$y[2] = x[-2] = 2$
$y[3] = x[-3] = 1$
$y[4] = x[-4] = 0$
$y[-4] = x[4] = 3$

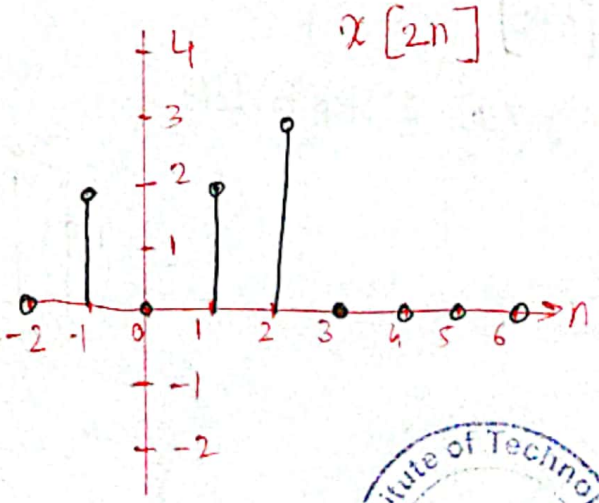


$\Leftarrow y[n] = x[-n]$

c)

n	-3	-2	-1	0	1	2	3	4
$x[n]$	1	2	3	0	1	2	2	3
$2n$	-6	-4	-2	0	2	4	6	8
$x[2n]$	0	0	2	0	2	3	0	0

$x[2n]$



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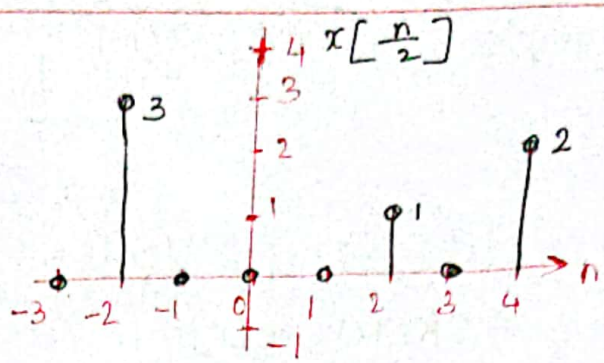
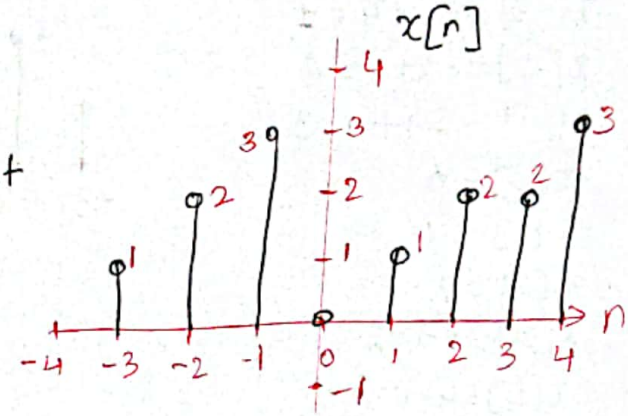
d) $x[\frac{n}{2}]$

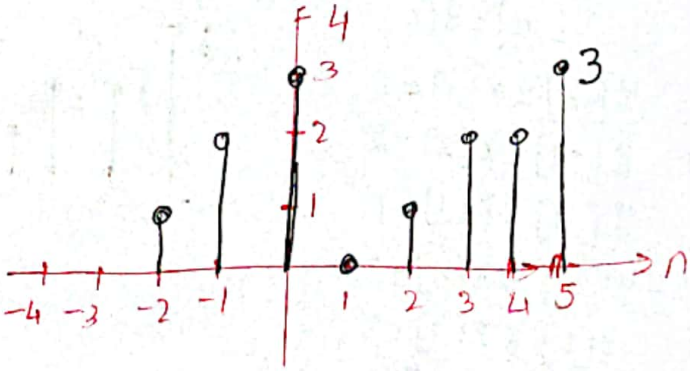
n	-3	-2	-1	0	1	2	3	4
$x[n]$	1	2	3	0	1	2	2	3
$n/2$	$-3/2$	-1	$-1/2$	0	$1/2$	1	$3/2$	2
$x[\frac{n}{2}]$	0	3	0	0	0	1	0	2



e) $x[n-1]$

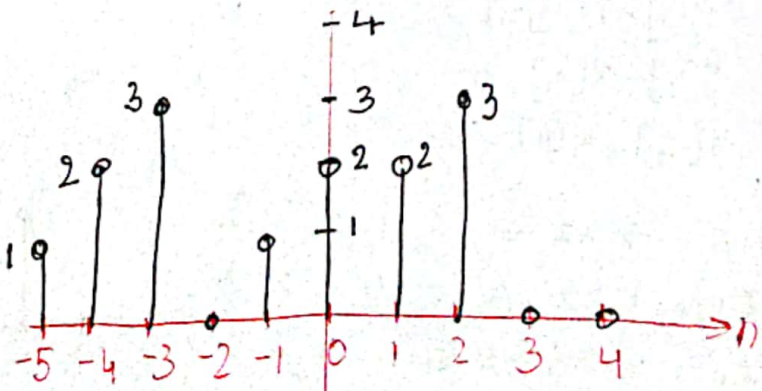
Advance $x[n]$ 1 Step to Right





f $x[n+2]$

Delay $x[n]$ 2 Step to Left



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Precedence Rule

To sketch $y[n] = x[an+b]$

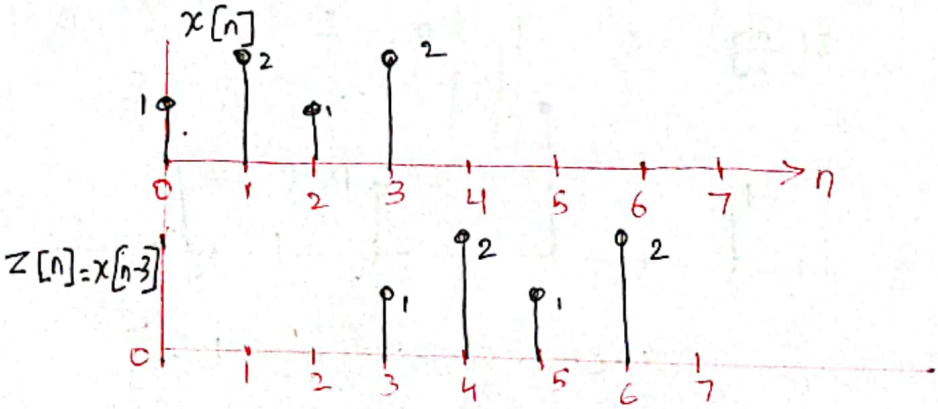
- Time shift $x[n]$ to get $z[n] = x[n+b]$
- Time scale $z[n]$ to get $z[an] = x[an+b]$

Example 1 : Given $x[n] = \{\underset{\uparrow}{1}, 2, 1, 2\}$

Sketch a) $x[2n-3]$ b) $x[\frac{n}{2}+3]$

Soln:- (i) $x[2n-3]$

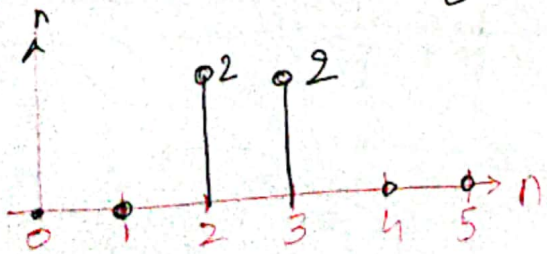
a) Time shift $x[n]$ by a factor 3



b). Sketch $z[2n]$

n	-2	-1	0	1	2	3	4	5	6	7
$z[n]$	0	0	0	0	0	1	2	1	2	0
$2n$	-4	-2	0	2	4	6	8	10	12	14
$z[2n]$	0	0	0	0	2	2	0	0	0	0

Sketch of $x[2n-3]$



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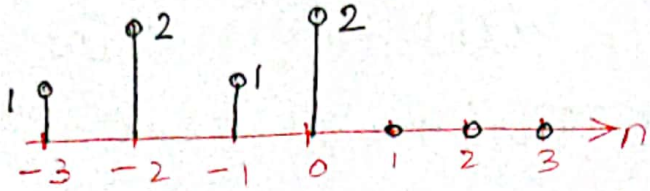
(ii) $x\left[\frac{n}{2}+3\right]$

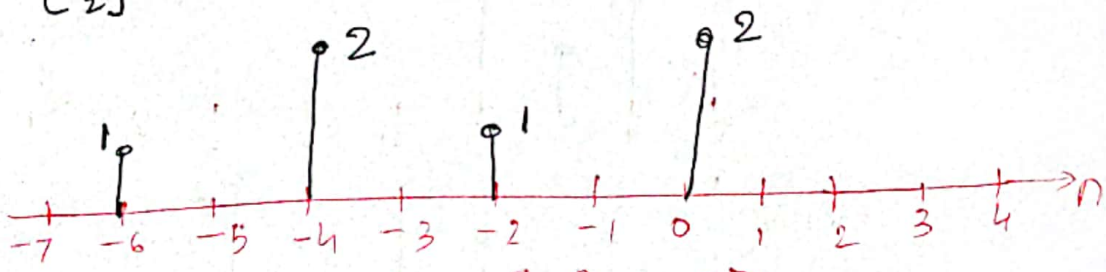
a) Time Shift $x[n]$ by a factor of 3



$z[n] = \{1, 2, 1, \underset{\uparrow}{2}, 0, 0, 0\}$

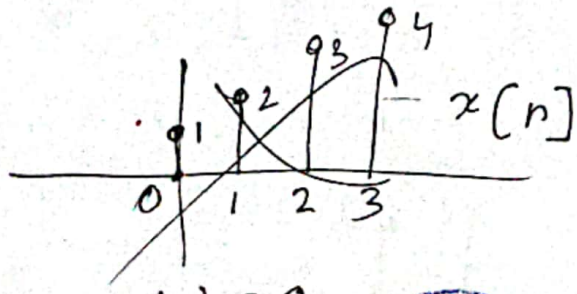
b) Sketch $z\left[\frac{n}{2}\right]$

n	-3	-2	-1	0	1	2	3	4	5	6	7
$z[n]$	1	2	1	2	0	0	0	0	0	0	0
$n/2$	-3/2	-1	-1/2	0	1/2	1	3/2	2	5/2	3	7/2
$z\left[\frac{n}{2}\right]$	0	1	0	2	0	0	0	0	0	0	0

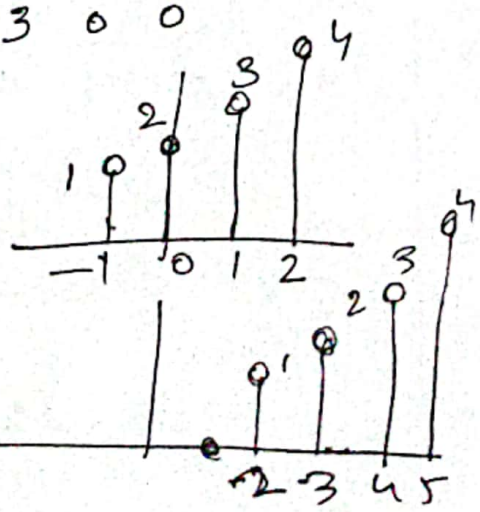


Sketch of $x\left[\frac{n}{2}+3\right]$

n	-3	-2	-1	0	1	2	3	4
$x[n]$	0	0	1	2	3	4	0	0
$2n-3$	-9	-7	-5	-3	-1	1	3	5
$x[2n-3]$	0	0	0	0	1	3	0	0



$-1+3=2$



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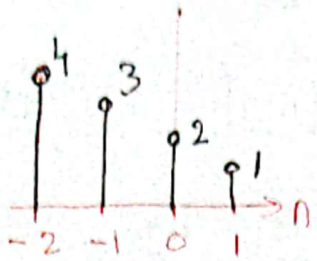
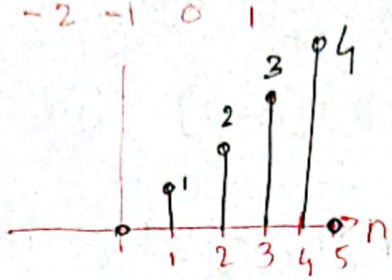
(2) Given $x[n] = \{1, \underset{\uparrow}{2}, 3, 4\}$

Sketch i) $x[-n]$ ii) $x[n-2]$ iii) $x[2n-3]$ iv) $x[\frac{n}{2}+3]$

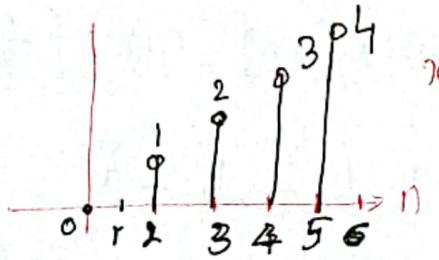
i) Sketch $x[-n]$



ii) $x[n-2]$

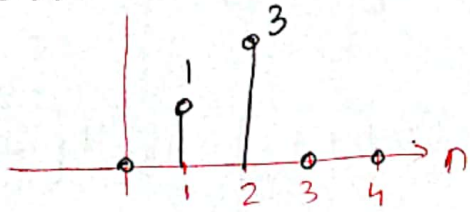


iii) $x[2n-3]$

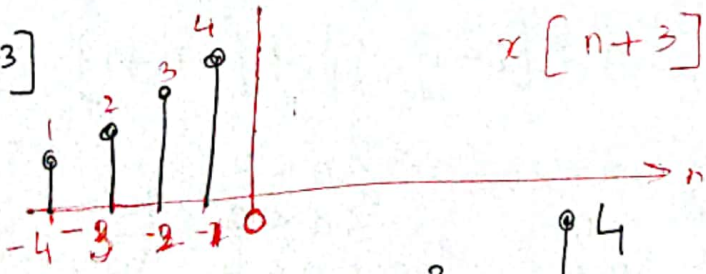


$x[n-3]$

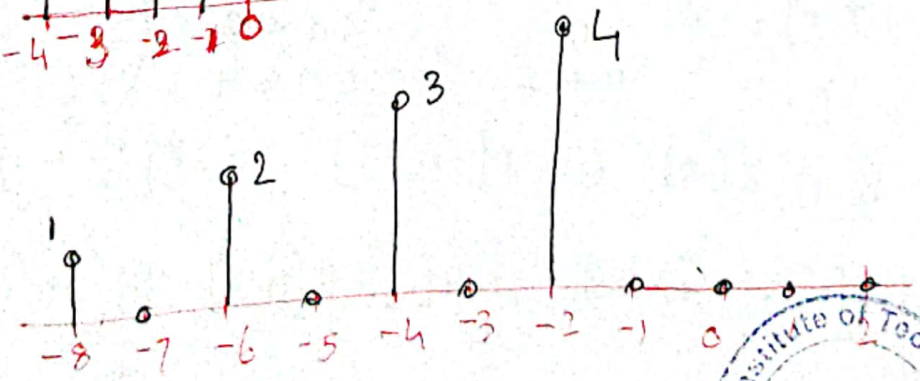
Scale by factor 2



iv) $x[\frac{n}{2}+3]$



$x[n+3]$



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Exercise Problems

a) Given $x[n] = \{1, \underset{\uparrow}{1}, 1, 2\}$

Sketch (i) $x[-n]$ (ii) $x[n-1]$ (iii) $x[n+1]$

b) Given $x[n] = \{1, 2, \underset{\uparrow}{3}, 4\}$

Sketch (i) $x[-n]$ (ii) $2x[n-2]$ (iii) $\frac{1}{3}x[n+1]$

c) Given $x[n] = \{1, -1, 1, \underset{\uparrow}{-1}, 1, -1\}$

Sketch (i) $x[-n]$ (ii) $x[2n]$ (iii) $x\left[\frac{n}{3}\right]$

d) Given $x[n] = \{-1, 1, 2, \underset{\uparrow}{1}, 2\}$

Sketch (i) $x[-n]$ (ii) $x[2n+1]$ (iii) $x\left[\frac{n}{3}+2\right]$ (iv) $x\left[\frac{n}{3}-3\right]$

e) Sketch i) $u[-n]$ (ii) $u[n+1]$ iii) $u[2n+1]$ (iv) $u[2n-3]$

f) Sketch i) $r[n]$ ii) $r[n-1]$ (iii) $r[-n+1]$ (iv) $r\left[\frac{n}{2}-2\right]$

g) Given $x[n] = \begin{cases} n-2 & -3 \le n < 0 \\ n+1 & 0 \le n < 2 \\ n+2 & 2 \le n \le 4 \end{cases}$

Sketch i) $x[-n]$ (ii) $x[-n+1]$ iii) $x\left[\frac{n}{2}+1\right]$ iv) $x[2n+2]$

h) Given $u[n]$; Sketch i) $u[n]-u[n-3]$ ii) $2u[n]-u[2n-1]$

i) Sketch i) $r[n]-r[n+1]$ (ii) $2r[n-1]+3r[2n+1]$

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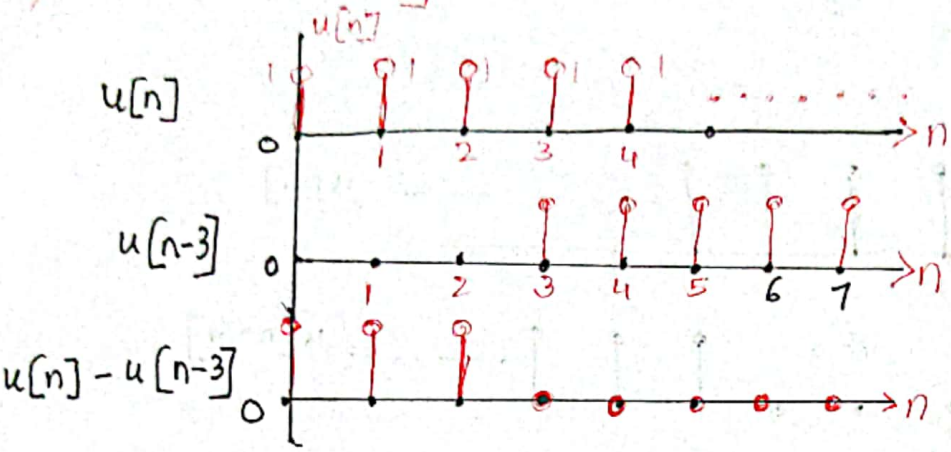
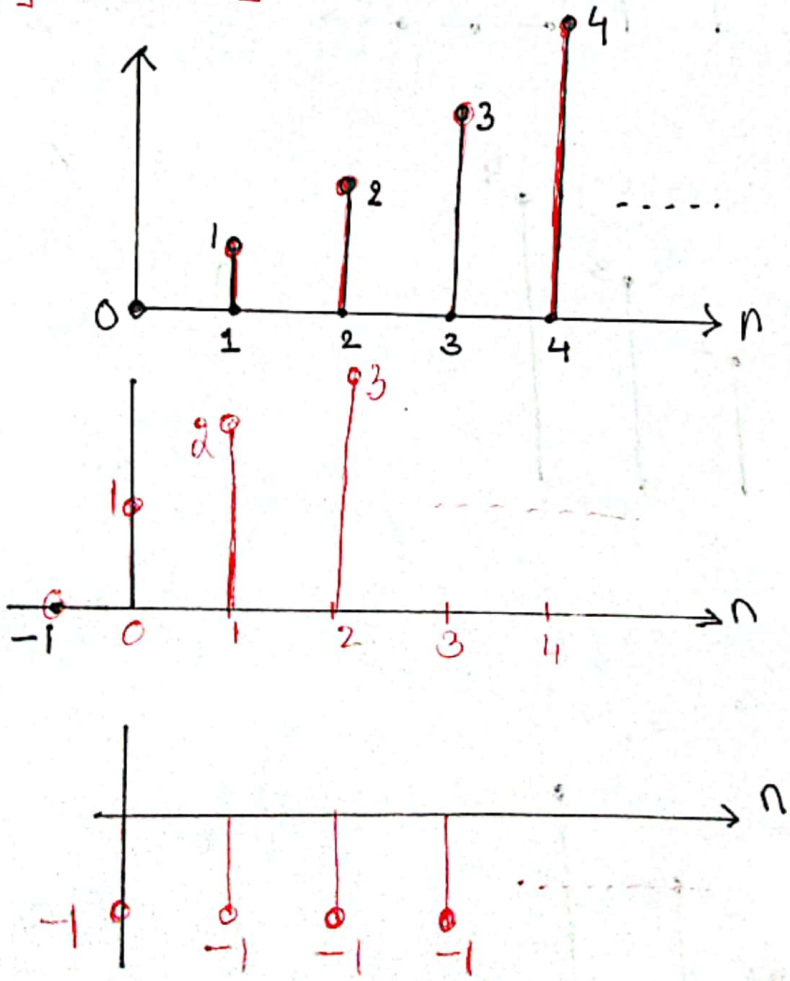
Rectangular waveform in terms of Elementary Signals

i) $u[n] - u[n-3]$



ii) $r[n] - r[n+1]$



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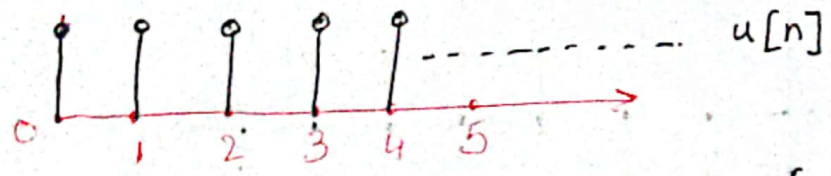
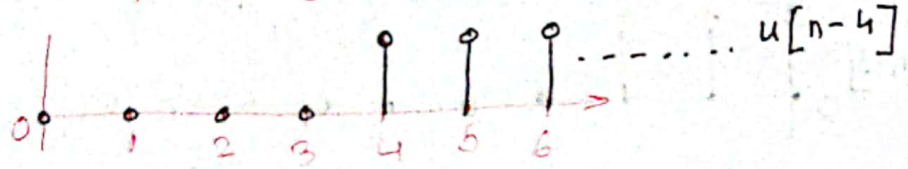
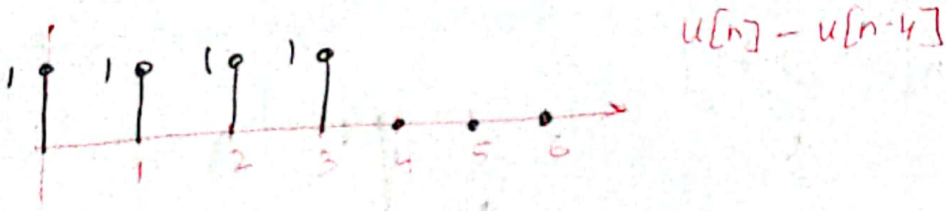
1) ~~Rectangular~~ W.

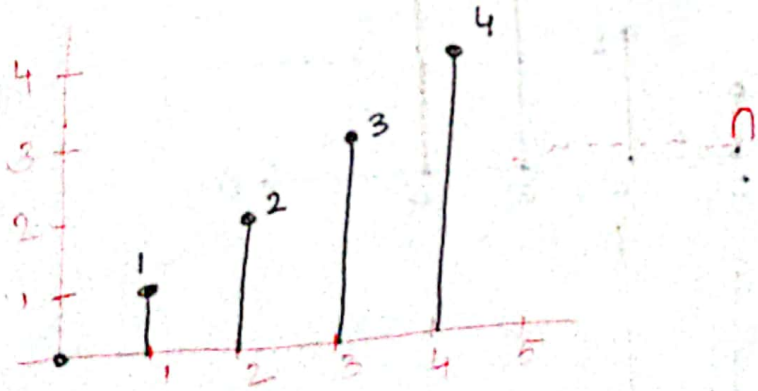
2) Triangular wave form in terms of elementary signal

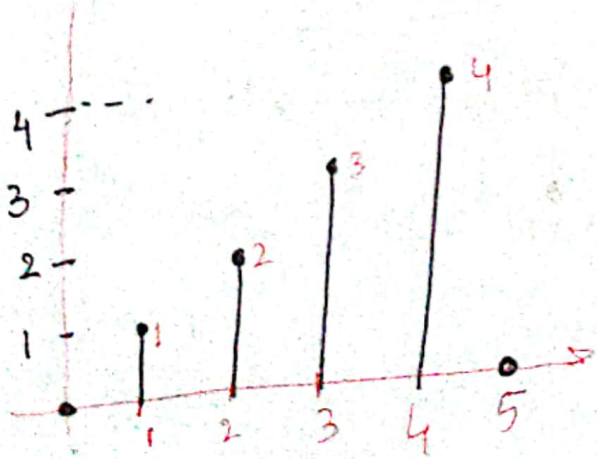
(i) Sketch $x[n] = n[u[n] - u[n-4]]$











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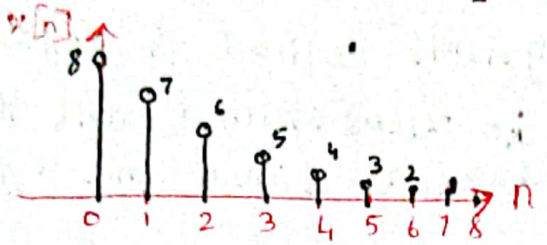
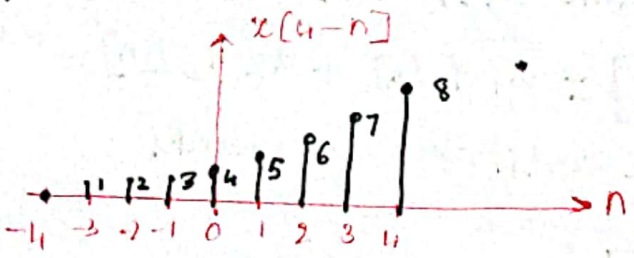
1) Given the Signal $x[n] = (8-n)[u[n] - u[n-8]]$. Determine and sketch i) $y_1[n] = x[4-n]$ (ii) $y_2[n] = x[2n-3]$

Soln:

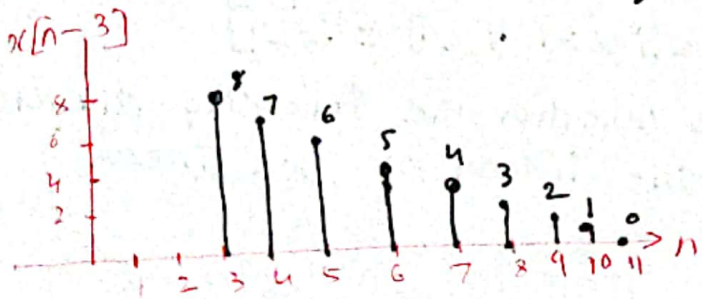
i) Here note that $u[n] - u[n-8] = 1$ for $n = 0$ to 7.

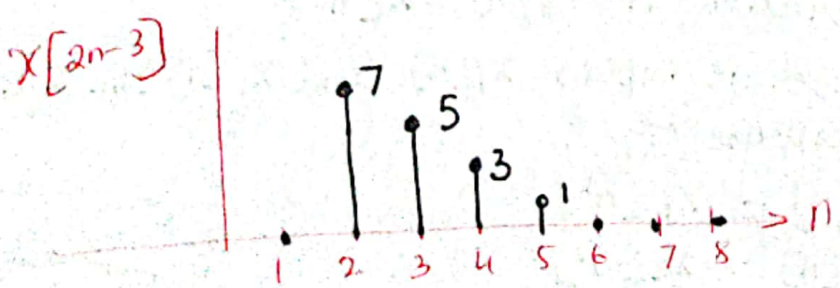


$x[4-n]$ is obtained by folding $x[n]$ and then right shifting by 4 samples



The Signal $x(2n-3)$ is obtained in two steps. In the first step $x[n-3]$ is obtained. In the Second step $x[n-3]$ is time scaled by two samples to get $x[2n-3]$





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System Classification and Properties

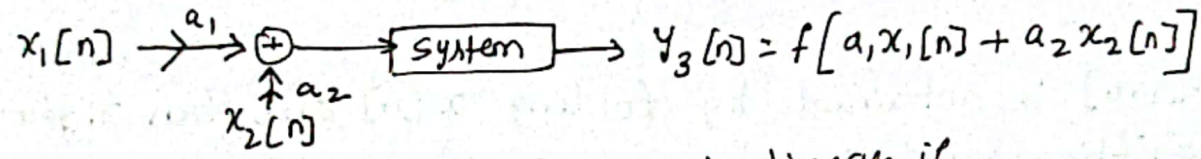
i) Linear and Non Linear.

A system is said to be linear if it satisfies the Superposition Principle.

Consider the two systems defined as follows

$y_1[n] = f[x_1[n]]$ i.e $x_1[n]$ is input and $y_1[n]$ is output

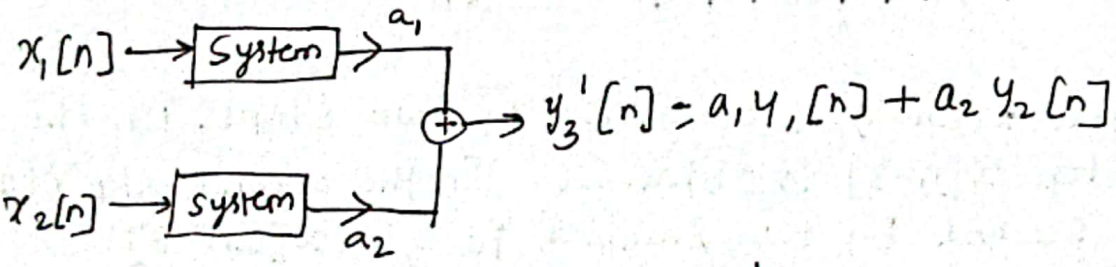
$y_2[n] = f[x_2[n]]$ i.e $x_2[n]$ is input and $y_2[n]$ is output



Then the discrete time System is linear if,

$$f[a_1x_1[n] + a_2x_2[n]] = a_1y_1[n] + a_2y_2[n]$$

Here a_1 and a_2 are arbitrary constants



For a System to be linear $y_3[n] = y_3'[n]$

Example 1: Determine whether the following discrete time systems are linear or non linear

i) $y[n] = x[n^2]$

Soln:- Given Equation is $y[n] = x[n^2]$

For the two seperate inputs $x_1[n]$ and $x_2[n]$ the System produces the response of,

$$y_1[n] = x_1[n^2]$$
$$y_2[n] = x_2[n^2]$$

The Response of the System to the linear Combination of $x_1[n]$ and $x_2[n]$ will be

$$y_3[n] = f[a_1x_1[n] + a_2x_2[n]]$$

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since the linear systems satisfy additive property; the above equation can be written as

$$y_3[n] = f\{a_1 x_1[n]\} + f\{a_2 x_2[n]\}$$

The linear systems also satisfy scaling property. Hence we can write above equation as

$$y_3[n] = a_1 x_1[n^2] + a_2 x_2[n^2] \quad \text{--- (1)}$$

Now the response of the system due to linear combination of two outputs will be,

$$y_3'[n] = a_1 y_1[n] + a_2 y_2[n]$$

$$y_3'[n] = a_1 x_1[n^2] + a_2 x_2[n^2] \quad \text{--- (2)}$$

On comparing (1) & (2), we get

$$y_3[n] = y_3'[n]$$

Hence the system is Linear.

2) Check for Linearity $y[n] = x^2[n]$

Soln:- Let $x_1[n] = x[n]$ $\therefore y_1[n] = x_1^2[n]$

Let $x_2[n] = x[n]$ $\therefore y_2[n] = x_2^2[n]$

Let $x_3[n] = \alpha x_1[n] + \beta x_2[n]$

$$\therefore y_3[n] = x_3^2[n] = [\alpha x_1[n] + \beta x_2[n]]^2$$

$$y_3[n] = \alpha^2 x_1^2[n] + \beta^2 x_2^2[n] + 2\alpha\beta x_1[n] x_2[n]$$

$$\neq \alpha x_1^2[n] + \beta x_2^2[n]$$

Hence System is Non linear

3) $y[n] = \log x[n]$

Let $x_1[n] = x[n]$ $\therefore y_1[n] = \log x_1[n]$

Let $x_2[n] = x[n]$ $\therefore y_2[n] = \log x_2[n]$

Let $x_3[n] = \alpha x_1[n] + \beta x_2[n]$

$$y_3[n] = \log[x_3[n]]$$

$$= \log[\alpha x_1[n] + \beta x_2[n]]$$

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$$\neq \alpha \log[x_1[n]] + \beta \log[x_2[n]]$$

$$\neq \alpha y_1[n] + \beta y_2[n]$$

Hence System is Non linear

3) $y[n] = x[n] + x[n-1]$

Let $x_1[n] = x[n] + x[n-1]$ $\therefore y_1[n] = x_1[n] + x_1[n-1]$

Let $x_2[n] = x[n] + x[n-1]$ $\therefore y_2[n] = x_2[n] + x_2[n-1]$

Let $x_3[n] = \alpha x_1[n] + \beta x_2[n]$

$$= \alpha x_1[n] + \alpha x_1[n-1] + \beta x[n] + \beta x[n-1]$$

$$y_3[n] = x_3[n] + x_3[n-1]$$

$$= \alpha x_1[n] + \beta x_2[n] + \alpha x_1[n-1] + \beta x_2[n-2]$$

$$= \alpha x[n] + \alpha x[n-1] + \beta x[n] + \beta x[n-1] + \alpha x_1[n-1] + \beta x_2[n-1]$$

$$+ \beta x_2[n-1] + \beta x_2[n-2]$$

$$\neq \alpha y_1[n] + \beta y_2[n]$$

Hence System is Non-Linear.

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Time invariant - Variant

A Time invariant system is one for which a time shift or delay of the input signal cause a corresponding shift in the output signal.

If the input signal $x[n]$ is shifted in time by 'n_0' seconds then the corresponding output for time invariant systems is given by $y[n-n_0] = H[x[n-n_0]]$

Procedure to Check for invariance

Step 1: Apply an i/p $x[n] = x_1[n]$. Find output $y_1[n]$

Step 2: Shift output by 'n_0' units

Step 3: Apply new input $x_2[n] = x_1[n-n_0]$. Find the output $y_2[n]$

Step 4: If $y_2[n] = y_1[n-n_0]$ system is Time invariant

---

(1) Check for invariance $y[n] = x[n]\cos\omega_0 n$

Soln:- $x[n] \rightarrow \boxed{H(\cdot)} \rightarrow y[n] = x[n]\cos\omega_0 n$

Then $x_1[n] \rightarrow \boxed{H(\cdot)} \rightarrow y_1[n] = x_1[n]\cos\omega_0 n$

Shifting output by 'n_0' seconds

$y_1[n-n_0] = x_1[n-n_0]\cos\omega_0[n-n_0]$

Let $x_1[n-n_0] \rightarrow \boxed{H(\cdot)} \rightarrow y_2[n] = x_1[n-n_0]\cos\omega_0 n$

Clearly $y_2[n] \neq y[n-n_0]$

Hence system is "Time Variant.

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(2) $y[n] = 2e^{x[n]}$

Soln:- $x[n] \Rightarrow \boxed{H(\cdot)} \longrightarrow y[n] = 2e^{x[n]}$

if $x_1[n] \rightarrow \boxed{H(\cdot)} \longrightarrow y_1[n] = 2e^{x_1[n]}$

Shifting output by 'n_0' seconds

$$y_1[n-n_0] = 2e^{x_1[n-n_0]}$$

Let $x_2[n] = x_1[n-n_0] \longrightarrow \boxed{H(\cdot)} \longrightarrow y_2[n] = 2e^{x[n-n_0]}$

$y_2[n] = y_1[n-n_0]$ Hence System is Time Invariant.

---

(3) $y[n] = x[n] + 2x[n-1]$

Soln: $x[n] \longrightarrow \boxed{H(\cdot)} \longrightarrow y[n] = x[n] + 2x[n-1]$

If $x_1[n] \rightarrow \boxed{H(\cdot)} \longrightarrow y_1[n] = x_1[n] + 2x_1[n-1]$

Shifting output by 'n_0' unit

$$y_1[n-n_0] = x_1[n-n_0] + 2x_1[n-n_0-1]$$

Let $x_2[n] \Rightarrow x_1[n-n_0] \longrightarrow \boxed{H(\cdot)} \longrightarrow y_2[n] = x_1[n-n_0] + 2x_1[n-n_0-1]$

$$y_2[n] = y_1[n-n_0]$$

Hence system is Time Invariant.

---

(4) $y[n] = x[-n]$

Soln: $x[n] \longrightarrow \boxed{H(\cdot)} \longrightarrow y[n] = x[-n]$

if $x_1[n] \longrightarrow \boxed{H(\cdot)} \longrightarrow y_1[n] = x_1[-n]$

Shifting output by 'n_0' unit

$$y_1[n-n_0] = x_1[-n-n_0]$$

Let $x_2[n] = x_1[n-n_0] \rightarrow \boxed{H(\cdot)} \rightarrow y_2[n] = x[-(n-n_0)]$

$$= x[-n+n_0]$$

$y_2[n] \neq y_1[n-n_0]$ Hence system is Time Variant.

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Causal and Non Causal

A System is one for which the output at any time depends only on present and past inputs but not on future inputs. In a non-Causal System the output at any time depends on future values with or without present and past values of inputs.

Eg: Causal systems are

$$y[n] = 2x[n] + x[n-1] + x[n-2]$$

$$y[n] = 2x[n] + 3x[n-2]$$

Non Causal Systems are

$$y[n] = x[n^2]$$

$$y[n] = x[n] + x[n+1] + x[n-2]$$

Eg: Check for Causality

a) $y[n] = x[n] + x[n-1]$

for $n=-1$ $\quad y[-1] = x[-1] + x[-2]$

for $n=0$ $\quad y[0] = x[0] + x[-1]$

for $n=1$ $\quad y[1] = x[1] + x[0]$

For all values of n, the output depends on present and past values of input. Therefore the system is causal.

b) $y[n] = x[n^2]$

for $n=-1$ $\quad y[-1] = x[1]$

for $n=0$ $\quad y[0] = x[0]$

for $n=1$ $\quad y[1] = x[1]$

for $n=2$ $\quad y[2] = x[4]$

The output at $n=-1$ & $n=2$ depends on future value $x[n]$. Therefore the system is non-Causal

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Static and Dynamic Systems (Systems with memory or Without Memory)

Definition : The Continuous time system is said to be Static or (memoryless) if its output depends upon the Present input only.

The discrete time systems can also be static or dynamic If output of the discrete time systems depends upon the present input sample only, then it is called Static or memoryless

For Example $y[n] = 10x[n]$

or $y[n] = 15x^2[n] + 10x[n]$

A System is said to be dynamic if the output depends upon the past values of inputs also. for example,

$$y[n] = x[n] + x[n-1]$$

$$y[n] = \sum_{k=0}^{4} x[n-k]$$

Stable and Unstable Systems

Definition; When every bounded input produces bounded output, then the System is called Bounded Input Bounded output (BIBO) Stable.

The input is said to be bounded if there exists some finite number M_x such that,

$$|x[n]| \leq M_x < \infty$$

Similarly the output is said to be bounded if there exists some finite number M_y such that

$$|y[n]| \leq M_y < \infty$$

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If the System produces unbounded output for bounded input, then it is unstable

① Determine whether the following discrete time Systems are stable or not?

i) $y[n] = x[n] + x[n-1] + y[n-1]$

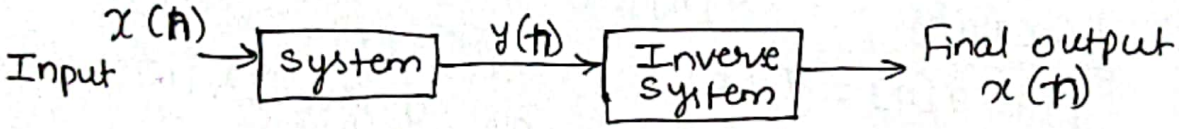
In the above equation $x[n]$ and $x[n-1]$ are present and previous inputs. $y[n-1]$ is the previous output. As long as $x[n]$ is bounded $x[n-1]$ will also be bounded. Hence $y[n]$ will also be bounded. Hence the System is BIBO Stable.

(ii) $y[n] = r^n x[n]$, $r > 1$

Here $r > 1$, Hence as $n \to \infty$ $r^n \to \infty$. Therefore $y[n]$ will be unbounded even if $x[n]$ is bounded. Hence this system is unstable.

Invertibility and Inverse System.

A System is said to be invertible if there is unique output for every unique input.



If the system is invertible, there exists an inverse System. If these two systems are cascaded as shown in figure then final output is same as input.

① Determine whether the following systems are invertible?

(i) $y(t) = 10x(t)$

For this System the inverse system will be

$$w(t) = \frac{1}{10} y(t)$$

Hence if the two Systems are cascaded, input is first multiplied by 10. Then the output of first system is divided by 10. Therefore original input is obtained back.

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$$x(n) \rightarrow \boxed{\text{System}} \xrightarrow{y(n)=10x(n)} \boxed{\text{Inverse system}} \xrightarrow{w(n)=\frac{1}{10}y(n)=\frac{1}{10}\times 10x(n)} x(n)$$

(ii) $y(n) = x^2(n)$

This system squares the input. Hence inverse system should take square root. i.e.,

$$w[n] = \sqrt{y[n]}$$
$$= \sqrt{x^2[n]}$$
$$= \pm x[n]$$

Thus two outputs are possible i.e $x[n]$ or $-x[n]$

This means there is no unique output for unique input. Hence this system is not invertible.

Example problems

① Check for (i) Linearity (ii) Causality (iii) Time-Invariance (iv) Stability

a) $y[n] = e^{x[n]}$

(i) Linearity

$y[n] = e^{x[n]}$

Let $x[n] = x_1[n]$ $\therefore$ o/p $y_1[n] = e^{x_1[n]}$

Let $x[n] = x_2[n]$ $\therefore$ o/p $y_2[n] = e^{x_2[n]}$

Let $x[n] = x_3[n] = a\,x_1[n] + b\,x_2[n]$

$$y_3[n] = e^{x_3[n]} = e^{a x_1[n] + b x_2[n]}$$
$$= e^{a x_1[n]} + e^{b x_2[n]}$$
$$\neq a\,y_1[n] + b\,y_2[n]$$

Hence system is non-linear.

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ii) Causal

Given $y[n] = e^{x[n]}$ output depends only on present value of input hence the system is causal.

iii) Time Invariant

$$x_1[n] \longrightarrow \boxed{H(\cdot)} \longrightarrow y_1[n] = e^{x_1[n]}$$

Shifting output by 'n_0' units

$$y_1[n-n_0] = e^{x_1[n-n_0]}$$

Let $x[n] = x_2[n] = x_1[n-n_0]$

$$\longrightarrow \boxed{H(\cdot)} \longrightarrow y_2[n] = e^{x_1[n-n_0]}$$

$$y_2[n] = y_1[n-n_0]$$

Hence the system is time - invariant.

(iv) Stable

Given $y[n] = e^{x[n]}$. Let $|x[n]| \leq M_x$ finite

If $|x[n]|$ is finite the $|e^{x[n]}|$ is also finite

$$\therefore |y[n]| = |e^{x[n]}| \leq M_y$$

$\therefore$ System is stable.

b) $y[n] = \cos(x[n])$

Soln: 1. A system is static if its output depends only upon only the present input sample. Here since $y[n]$ depends upon cosine of $x[n]$. i.e present input sample, the system is static.

2. For two seperate inputs the system produces the response of,

$$y_1[n] = T\{x_1[n]\} = \cos[x_1[n]]$$

$$y_2[n] = T\{x_2[n]\} = \cos[x_2(n)]$$

The Response of the system to linear combination of two inputs will be,

$$y_3[n] = T\{a_1 x_1[n] + a_2 x_2[n]\} = \cos[a_1 x_1[n] + a_2 x_2[n]]$$

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The linear combination of two outputs will be,

$y_3'[n] = a_1 y_1[n] + a_2 y_2[n] = a_1 \cos[x_1[n]] + a_2 \cos[x_2[n]]$

Clearly $y_3[n] \neq y_3'[n]$. Hence system is non-linear

3. The system is said to be shift invariant or time invariant if its characteristics do not change with shift of time origin

The given system is, $y[n] = T\{x[n]\} = \cos[x(n)]$

Let us delay the input by k samples. Then output will be,

$y(n,k) = T\{x(n-k)\} = \cos[x(n-k)]$

Now let us delay the output $y[n]$ given by 'k' samples i.e $y(n-k)$. This is equivalent to replacing n by n-k

i.e $y(n-k) = \cos[x(n-k)]$

We observe that, $y(n,k) = y(n-k)$

This shows that the system is shift invariant.

4. The system is said to be causal if output depends upon past and present inputs only. The output is given as,

$$y[n] = \cos[x(n)]$$

Here observe that n^{th} sample of output depends upon n^{th} sample of input $x[n]$. Hence the system is a causal system.

5. For any bounded value of $x[n]$ the cosine function has bounded value. Hence $y[n]$ has bounded value. Therefore the system is said to be BIBO stable.

Thus the given system is

$\boxed{\text{Static, non-linear, shift invariant, Causal \& Stable}}$

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3) $y(n) = |x(n)|$

1. The output is equal to magnitude of present input sample. Hence the system does not need memory storage There the system is Static

2. The given system equation is

$$y[n] = T\{x(n)\} = |x(n)|$$

For two seperate inputs $x_1[n]$ & $x_2[n]$ the System has the response of,

$$y_1[n] = T\{x_1[n]\} = |x_1(n)|$$
$$y_2[n] = T\{x_2[n]\} = |x_2(n)|$$

The Response of the system to linear combination of two inputs $x_1[n]$ and $x_2[n]$ will be,

$$y_3[n] = T\{a_1 x_1[n] + a_2 x_2[n]\}$$
$$= |a_1 x_1(n) + a_2 x_2(n)|$$

Now the linear combination of two outputs will be,

$$y_3'[n] = a_1 y_1[n] + a_2 y_2[n] = a|x_1(n)| + a_2|x_2(n)|$$

Here observe that $y_3(n) \neq y_3'(n)$. Hence the System is nonlinear.

3. Delaying the input by 'k' samples, output will be,

$$y(n,k) = T\{x(n-k)\} = |x(n-k)|$$

and the delayed output will be, $y(n-k) = |x(n-k)|$

Since $y(n,k) = y(n-k)$, the system is shift invariant.

4. The System equation is $y[n] = |x(n)|$. The output depends upon present input Hence the System is causal.

5. From the given equation it is clear that as long as $x[n]$ is bounded, $y[n]$ will be bounded. Hence the system is stable.

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Exercise Questions

1. Check whether the following ~~continu~~ Discrete time systems are Time invariant or not.

   a) $y[n] = n\,x[n]$ b) $y[n] = \sin[x(n)]$

2. Check whether the following discrete time systems are linear or non-linear

   a) $y[n] = n\,x[n]$ b) $y[n] = 2\,x^3[n]$

3. Determine if the system described by the following equations are causal or non causal

   a) $y[n] = x[n] + \frac{1}{x[n-1]}$ b) $y[n] = x[n^2]$

4. For the discrete time systems are given below

   (i) $y[n] = x(-n+2)$ (ii) $y[n] = x[n]\,u[n]$

   (iii) $y[n] = x[2n]$ (iv) $y[n] = x[n] + n\,x[n+1]$

   (v) $y[n] = n\,x[n]$

   Check whethere these systems are:

   1. Static or Dynamic
   2. Linear or Non-Linear
   3. Shift invariant or Shift ~~i~~ovariant
   4. Causal or non-Causal
   5. Stable or Unstable.

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Basic Signal Processing

Module 4

Time Domain representation of LTI System, LTI system properties in terms of impulse response

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Video links

Module 4 :Time Domain representation of LTI System, LTI system properties in terms of impulse response

Sl. No	Topic	URL
1	Impulse response	https://www.youtube.com/watch?v=plor6ywAXXg
2	Impulse response problem	https://www.youtube.com/watch?v=lO7Fl09tA-A
3	Convultion Sum	https://www.youtube.com/watch?v=hl2T2adZWFI
4	Convultion Sum	https://www.youtube.com/watch?v=A6qR7UhLvng-
5	Convolution Sum using graphical method.	https://www.youtube.com/watch?v=JQl_2zLmQ1k
6	Convolution Sum using graphical method	https://www.youtube.com/watch?v=YE7MFuwmL7o
7	LTI system properties in terms of impulse response	https://www.youtube.com/watch?v=QsUWKfoWZAk
8	Interconnection of System in terms of Impulse Response	https://www.youtube.com/watch?v=_EMZIogZrfM
9	Memoryless, causal, stable	https://www.youtube.com/watch?v=ypBfqKmEzNI
10	Invertibility and deconvolution	https://www.youtube.com/watch?v=3Efz6BUK924
11	Step response problem	https://www.youtube.com/watch?v=0co5IfcDx_A&t=322s
12	Step Response example problem	https://www.youtube.com/watch?v=0co5IfcDx_A

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Module-4

Time domain representation of LTI System

Impulse Response, convolution sum. Computation of convolution sum using graphical method for unit step, and unit step and exponential, exponential and exponential, unit step and rectangular, and rectangular & rectangular

LTI System properties in terms of impulse response:

System interconnection, Memoryless, Causal, Stable, Invertible and Deconvolution and step response

---

LTI systems are the systems that satisfy both linearity and Time invariance properties.

The Impulse response of a LTI system is the output of the system due to an impulse input.

The impulse response characterizes the behaviour of any LTI System and hence it is considered as the basic property of all LTI system.

Convolution Sum

consider a discrete time LTI system with input $x[n]$ and output $y[n]$

$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n]$$

$$y[n] = T\{x[n]\}$$

We have $x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$

because any discrete time signal can be considered as a weighted sum of time shifted impulses.

$$y[n] = T\left\{\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]\right\}$$

Using Linearity property of LTI Systems

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot T\{\delta[n-k]\}$$

$$\delta[n] \longrightarrow \boxed{T} \longrightarrow h[n] \text{ (Impulse Response)}$$

By Time Invariance property, we can write

$$h[n-k] = T\{\delta(n-k)\}$$

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]} \Rightarrow y[n] = x[n] * h[n]$$

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Computation of Convolution Sum

① Find the Convolution sum of $x[n] = \{1, \underset{\uparrow}{2}, 3\}$ & $h[n] = \{2, \underset{\uparrow}{3}, 5\}$

Soln:- Size of $x[n] = m = 3$ $\qquad y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

Size of $h[n] = n = 3$

Size of $y[n] = L = m+n-1 = 3+3-1 = 5$

Start value of n of $y[n]$ = start value of n of $x[n]$ + start value of n of $h[n]$

$= -1 + (-1) = -2$

$$y[n] = \sum_{k=-1}^{1} x[k]\, h[n-k]$$

$n=-2$ $\quad y[-2] = \sum_{k=-1}^{1} x[k]\, h[n-k]$

$= x[-1]\,h[-2+1] + x[0]\,h[-2] + x[1]\,h[-2-1]$

$= x[-1]\,h[-1] + x[0]\,h[-2] + x[1]\,h[-3]$

$= 1\times 2 + 2\times 0 + 3\times 0 = 2$

$n=-1$ $\quad y[-1] = x[-1]\,h[-1+1] + x[0]\,h[-1] + x[1]\,h[-2]$

$= 1\times 2 + 2\times 2 + 3\times 0 = 6$

$n=0$ $\quad y[0] = x[-1]\,h[1] + x[0]\,h[0] + x[1]\,h[-1]$

$= 1\times 5 + 2\times 2 + 3\times 2 = 15$

$n=1$ $\quad y[1] = x[-1]\,h[2] + x[0]\,h[1] + x[1]\,h[0]$

$= 1\times 0 + 2\times 5 + 3\times 2 = 16$

$n=2$ $\quad y[2] = x[-1]\,h[3] + x[0]\,h[2] + x[1]\,h[1]$

$= 0 + 0 + 3\times 5 = 15$

$y[n] = x[n] * h[n] = \{2, 6, \underset{\uparrow}{15}, 16, 15\}$

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2) Convolve the following two sequences $x[n]$ and $h[n]$ to get $y[n]$

$x[n] = \{\underset{\uparrow}{1}, 1, 1, 1\}$ & $h[n] = \{\underset{\uparrow}{2}, 2\}$

Soln:- Size of $x[n] = m = 4$ $\qquad y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

Size of $h[n] = n = 2$

Size of $y[n] = L = m+n-1 = 4+2-1 = 5$

Start value of $y[n]$ = Start value of n of $x[n]$ + Start value of n of $h[n]$

$= 0 + 0 = 0$

$$y[n] = \sum_{k=0}^{3} x[k]\, h[n-k]$$

$n=0$ $\quad y[0] = \sum_{k=0}^{3} x[k]\, h[-k] = x[0]h[0] + x[1]h[-1] + x[2]h[-2] + x[3]h[-3]$

$= 1\times 2 + 1\times 0 + 1\times 0 + 1\times 0$

$= 2$

$n=1$, $y[1] = \sum_{k=0}^{3} x[k]\, h[1-k] = x[0]h[1] + x[1]h[0] + x[2]h[-1] + x[3]h[-2]$

$= 1\times 2 + 1\times 2 + 1\times 0 + 1\times 0$

$= 2 + 2$

$= 4$

$n=2$, $y[2] = \sum_{k=0}^{3} x[k]\, h[2-k] = x[0]h[2] + x[1]h[1] + x[2]h[0] + x[3]h[-1]$

$= 1\times 0 + 1\times 2 + 1\times 2 + 1\times 0$

$= 2+2$

$= 4$

$n=3$, $y[3] = \sum_{k=0}^{3} x[k]\, h[3-k] = x[0]h[3] + x[1]h[2] + x[2]h[1] + x[3]h[0]$

$= 1\times 0 + 1\times 0 + 1\times 2 + 1\times 2$

$= 4$

$n=4$, $y[4] = \sum_{k=0}^{3} x[k]\, h[4-k] = x[0]h[4] + x[1]h[3] + x[2]h[2] + x[3]h[1]$

$= 1\times 0 + 1\times 0 + 1\times 0 + 1\times 2$

$= 2$

$n=5$, $y[5] = \sum_{k=0}^{3} x[k]\, h[5-k] = x[0]h[5] + x[1]h[4] + x[2]h[3] + x[3]h[2]$

$= 1\times 0 + 1\times 0 + 1\times 0 + 1\times 0$

$= 0$

$\therefore\ y[n] = \{\underset{\uparrow}{2}, 4, 4, 4, 2, 0\}$

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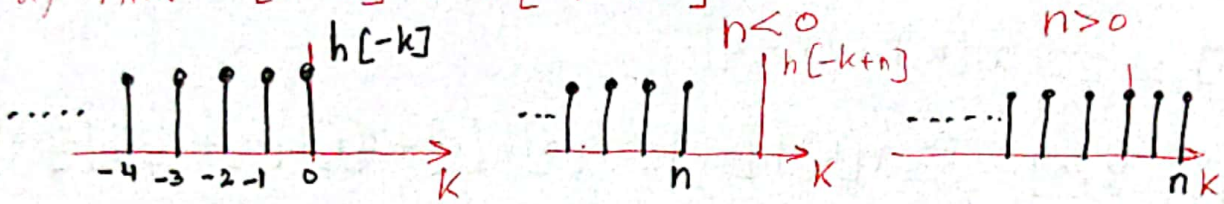
Computation of Convolution Sum using Graphical Method for Unit Step and unit Step.

1) Find the Convolution Sum of $x[n] = u[n]$ & $h[n] = u[n]$

Soln:- $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

a) Find $h[n-k] = h[-k+n]$



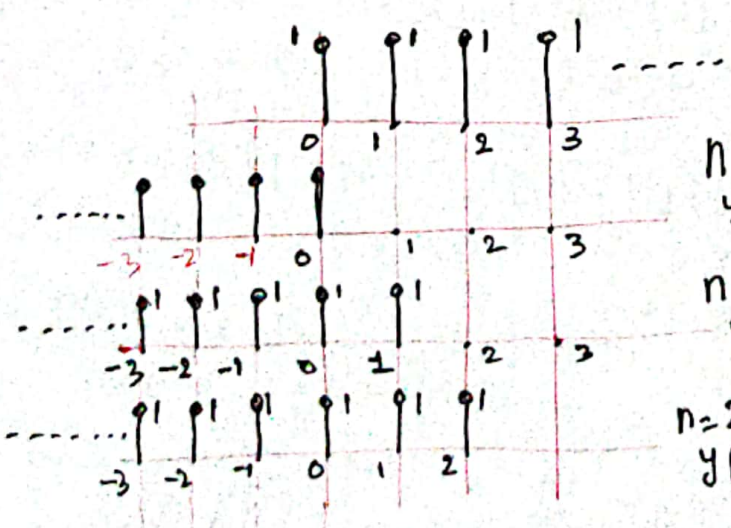
$$h[k] = \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$$

$$h[n-k] = \begin{cases} 1 & \text{for } n-k \geq 0 \\ 0 & \text{for } n-k < 0 \end{cases} = \begin{cases} 1 & \text{for } k \leq n \\ 0 & \text{for } k > n \end{cases} = u[n-k]$$

i) $n < 0$ $\quad y[n] = x[k] * h[n-k] = \sum_{k=0}^{n} u[k]\, u[n-k]$

$$= 0$$

(ii) $n \geq 0$ $\quad y[n] = \sum_{k=0}^{n} u[k]\, u[n-k] = \sum_{k=0}^{n} 1 = n+1$



$n=0$ $\quad u[-k]$
$y[n] = 1 \times 1 = 1$

$n=1$ $\quad u[1-k]$
$y[n] = 1\times1 + 1\times1 = 2$

$n=2$ $\quad u[2-k]$
$y[n] = 1\times1 + 1\times1 + 1\times1 = 3$

$\therefore\ y[n] = \{0, 0, 0, \underset{\uparrow}{1}, 2, 3, 4, 5, \ldots\}$

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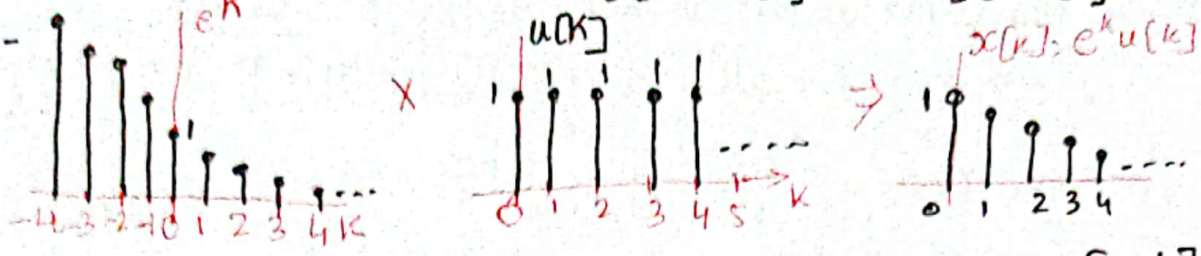
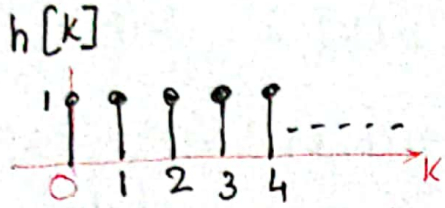
Computation of Convolution Sum using graphical method for unit step and Exponential.

(i) Find the Convolution Sum of $x[n]=e^{n}u[n]$ and $h[n]=u[n]$

Soln:-



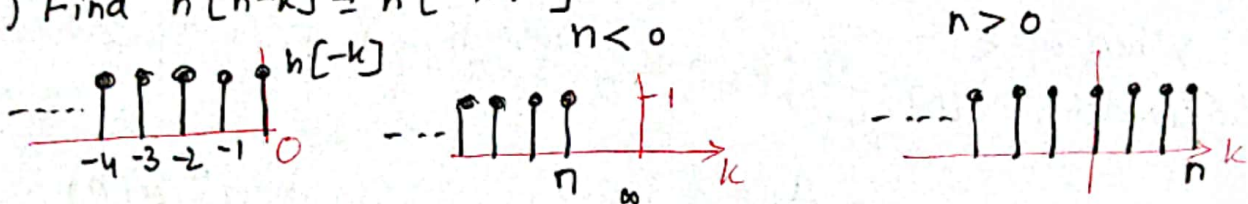


$$= \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$$

$$h(n-k) = u[n-k] = \begin{cases} 1 & \text{for } n-k \geq 0 \\ 0 & \text{for } n-k < 0 \end{cases} = \begin{cases} 1 & \text{for } k \leq n \\ 0 & \text{for } k > n \end{cases}$$

(i) To find $x[k] * h[n-k]$

a) Find $h[n-k] = h[-k+n]$



(i) $n<0$ $\quad y[n] = x[k] * h[n-k] = \sum_{k=0}^{\infty} x[k]\, h[n-k]$

$$= \sum_{k=0}^{\infty} e^{k} u[k]\, u[n-k]$$

$$= \sum_{k=0}^{n} \alpha^{k} \cdot 1 \quad = 0$$

(ii) $n \geq 0$ $\quad y[n] = \sum_{k=0}^{n} e^{k} = \frac{1-e^{n+1}}{1-e}$ for $n \geq 0$

$$\therefore y[n] = \begin{cases} 0 & \text{for } n < 0 \\ \frac{1-e^{n+1}}{1-e} & \text{for } n \geq 0 \end{cases}$$

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3 Computation of convolution sum using graphical method for ~~Unit step and~~ exponential & exponential

① Find the convolution of two exponential sequences $h[n] = a^n u[n]$ for all n, $x[n] = b^n u[n]$ for all n

i) When $a \neq b$ (ii) When $a = b$

Soln:- i) When $a \neq b$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{\infty} b^k u[k]\, a^{n-k} u[n-k]$$

$$= \sum_{k=0}^{\infty} b^k a^{n-k} \quad \text{since } u(k).u(n-k) = 1 \text{ for } 0 \le k \le n$$

$$= a^n \sum_{k=0}^{n} (ba^{-1})^k = \frac{b^{n+1} - a^{n+1}}{b-a}$$

(ii) When $a = b$

Consider, $y[n] = \sum_{k=0}^{n} b^k a^{n-k} = \sum_{k=0}^{\infty} a^k . a^n . a^{-k}$ since $b = a$

$$= a^n \sum_{k=0}^{n} 1 = a^n.(n+1) = (n+1)\, a^n u(n)$$

4) Computation of convolution sum using graphical method for Unit step & rectangular.

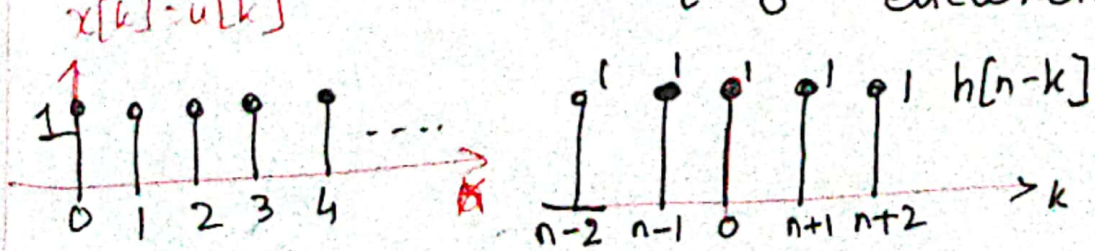
① Find the convolution of $x[n] = u[n]$ & $h[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

Soln:- $y[n] = \sum_{k=-\infty}^{\infty} x[n]\, h[n-k]$

$x[k] = u[k]$ $\quad h[k] = \begin{cases} 1 & -2 \le k \le 2 \\ 0 & \text{elsewhere} \end{cases}$

$$h[n-k] = \begin{cases} 1 & -2 \le n-k \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore h[n-k] = \begin{cases} 1 & n-2 \le k \le n+2 \\ 0 & \text{elsewhere} \end{cases}$$



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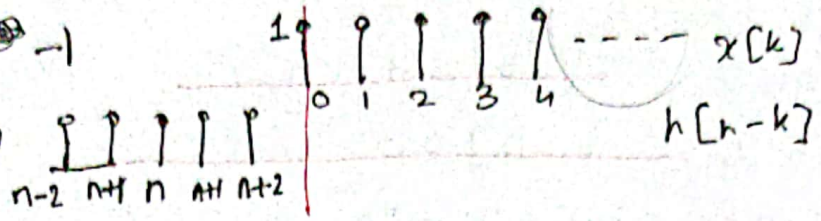
① $n+2 \leq -1$

$n \leq -3$



No overlap $\therefore$ $y[-3] = 0$

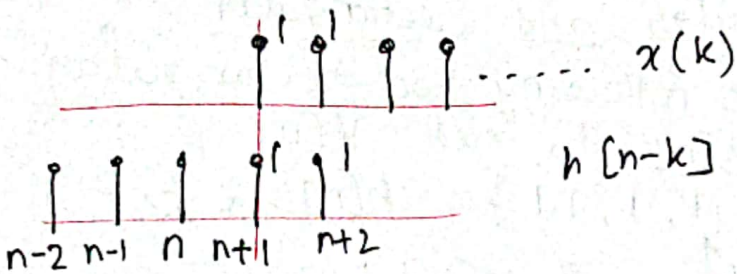
② $n+2 = 0$

$n = -2$

$y[-2] = 0$

3) $n+2 = 1$

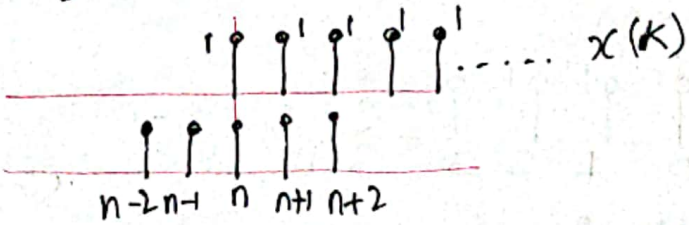
$n = -1$



$y[-1] = 1\times1 + 1\times1 = 2$

4) $n+2 = 2$

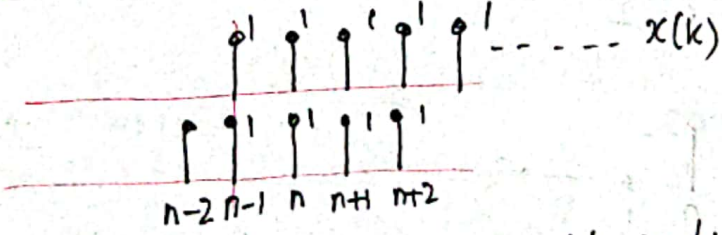
$n = 0$



$y[0] = 1\times1 + 1\times1 + 1\times1 = 3$

5) $n+2 = 3$

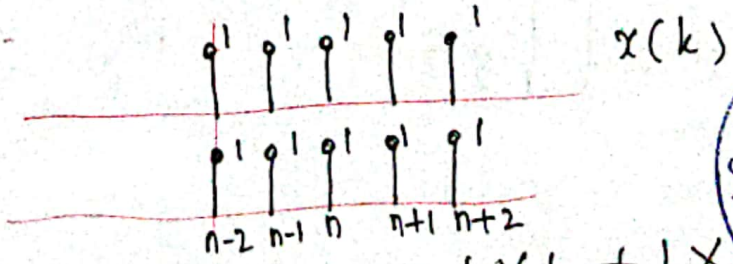
$n = 1$



$y[1] = 1\times1 + 1\times1 + 1\times1 + 1\times1 = 4$

6) $n+2 = 4$

$n = 2$



$y[2] = 1\times1 + 1\times1 + 1\times1 + 1\times1 + 1\times1$

$= 5$

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Similarly

for $n=3$ $\quad y[3]=4$

$n=4$ $\quad y[4]=3$

$n=5$ $\quad y[5]=2$

$n=6$ $\quad y[6]=1$

$n\geq 7$ $\quad y[7]=0$

$$\therefore y[n]=\{0,0,0,1,2,\underset{\uparrow}{3},4,3,2,1,0,0,\ldots\}$$

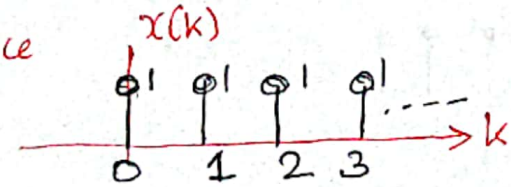
Computation of Convolution sum using graphical method for rectangular and rectangular

① Convolve the following two rectangular sequences $x[n]$ and $h[n]$ to get $y[n]$

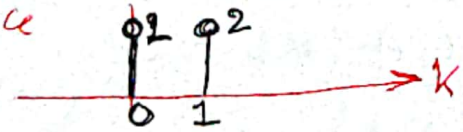
$x[n]=\{\underset{\uparrow}{1},1,1,1\}$ $\quad h[n]=\{\underset{\uparrow}{2},2\}$

Soln:- $\quad y[n]=\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

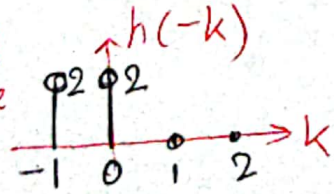
a) Sequence $x(k)$



b) Sequence $h(k)$



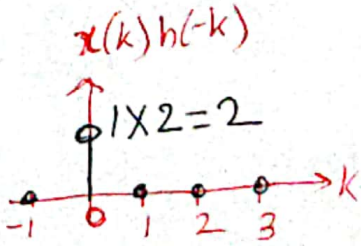
(c) Folded Sequence $h(-k)$



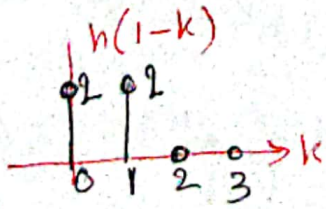
d) $\sum x(k)h(-k)$

$=0+2+0+0+0$

$=2$



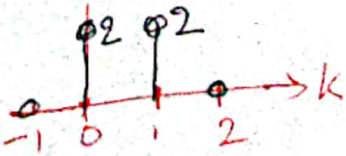
e) $h(-k)$ shifted to right by on Sample



f) $\sum x(k)h(1-k)$

$=0+2+2+0+0$

$=4$



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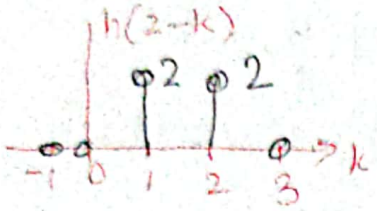
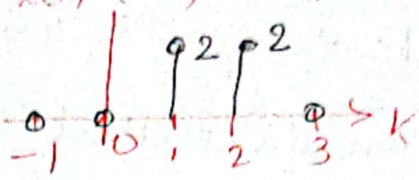
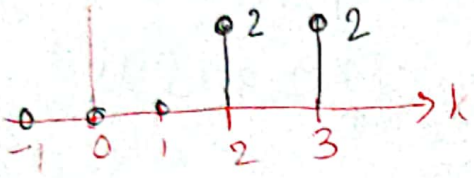
g) h(-k) shifted to right by two samples



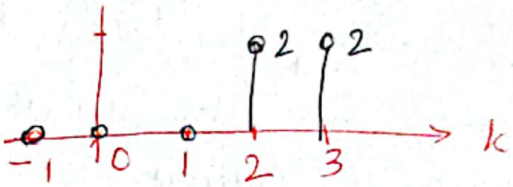
(h) $\sum x(k)\,h(2-k)$
$= 0+2+2+0$
$= 4$



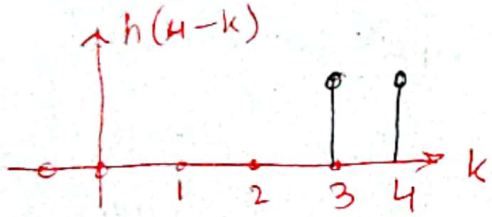
i) h(-k) shifted to right by 3 samples



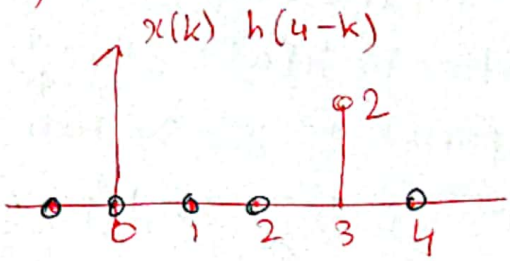
j) $\sum x(k)\,h(3-k)$
$= 4$



k) h(-k) shifted to right by four samples



l) $\sum x(k)\,h(4-k) = 2$



Thus the sequence y[n] becomes as shown below

$$y[n] = \{ \cdots 0, 0, \underset{\uparrow}{2}, 4, 4, 4, 2, 0, 0, 0, \cdots \}$$

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Excercise Problems

(1) Determine the convolution of two given sequences
$x[n] = \{1, \underset{\uparrow}{2}, 3, 4\}$ and $h[n] = \{1, \underset{\uparrow}{5}, 1\}$
(i) Formula method (ii) Graphical Method

(2) The impulse response of a LTI system is given by $h[n] = \{1, \underset{\uparrow}{2}, 1, -1\}$. Determine the response of the system for the input and $x[n] = \{\underset{\uparrow}{1}, 2, 3, 1\}$ and sketch the output

(3) The impulse response of a linear time invariant system is $h[n] = \{1, \underset{\uparrow}{-1}, 1, -1\}$. Determine the response of the system to the input signal
$x[n] = \{1, \underset{\uparrow}{2}, 3, 1\}$

(4) Perform the convolution of two finite sequences using graphical method only
$x[n] = \{-1, 1, \underset{\uparrow}{0}, 1, -1\}$, $h[n] = \{1, \underset{\uparrow}{2}, 3\}$

(5) The impulse response of the discrete LTI system is given by, $h[n] = u[n+1] - u[n-4]$. The system is excited by the input signal $x[n] = u[n] - 2u[n-2] + u[n-4]$
Obtain the response of the system $y[n] = x[n] * h[n]$ and plot the same.

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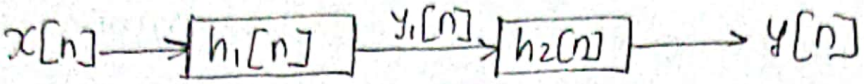
LTI System properties in terms of impulse response

System Interconnection

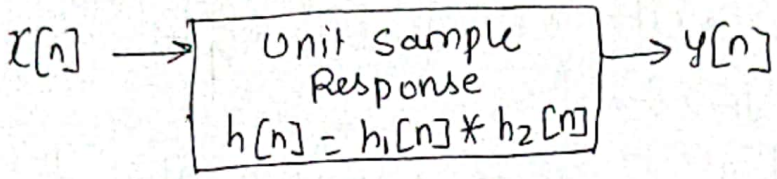
Cascade connection of Systems



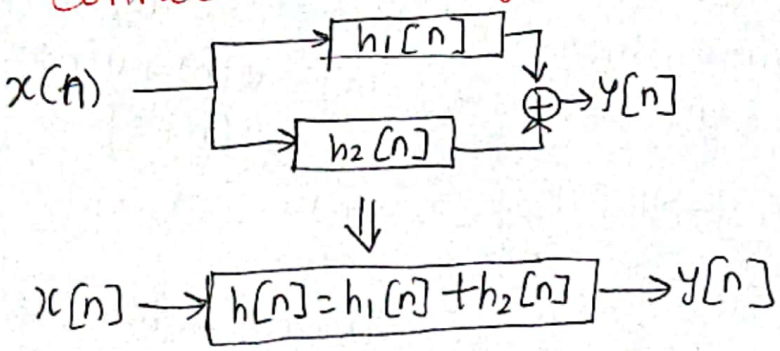
For this system we can write

$$y[n] = x[n] * h[n]$$

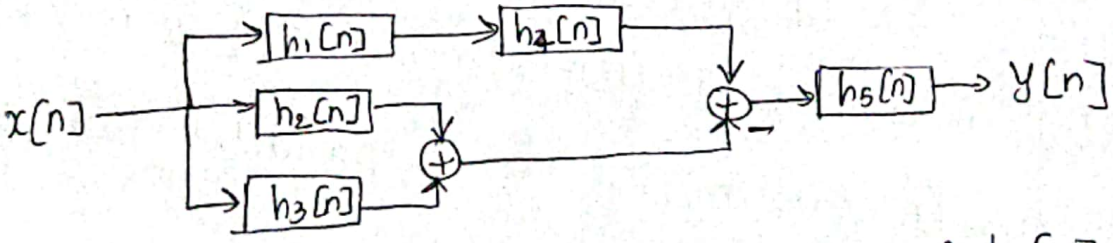
where $h[n] = h_1[n] * h_2[n]$



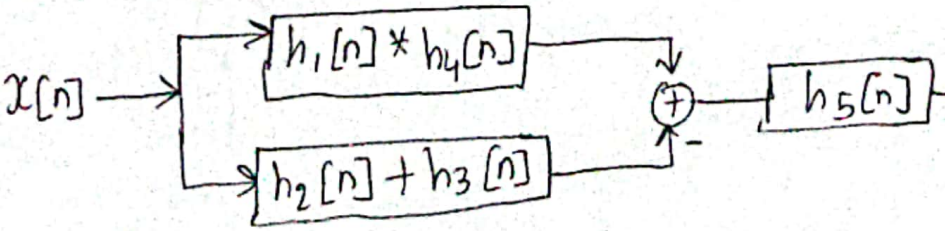
Parallel connection of Systems



① For the interconnection of the system shown. obtain the overall impulse response in terms of impulse responses of individual subsystems



Soln: Let us combine cascade connection of $h_1[n]$ and $h_4[n]$. Similarly combine parallel Connection of $h_2[n]$ and $h_3[n]$. This is shown below



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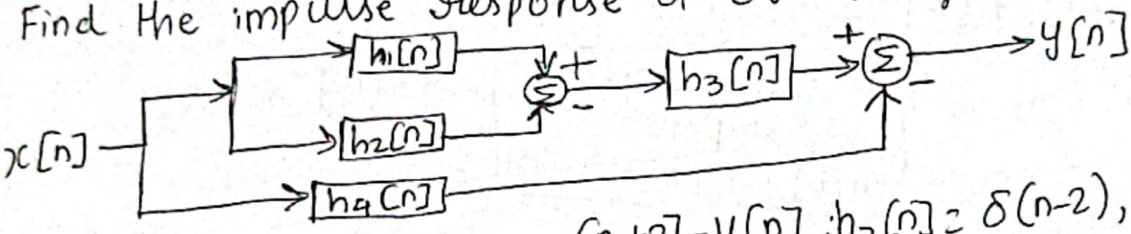
Now let us combine parallel combination of the two systems

$$x[n] \longrightarrow \boxed{[h_1[n] * h_4[n]] - [h_2[n] + h_3[n]]} \longrightarrow \boxed{h_5[n]} \longrightarrow y[n]$$

We can write the overall impulse response as,

$$h[n] = \{ h_1[n] * h_4[n] - [h_2[n] + h_3[n]] \} * h_5[n]$$

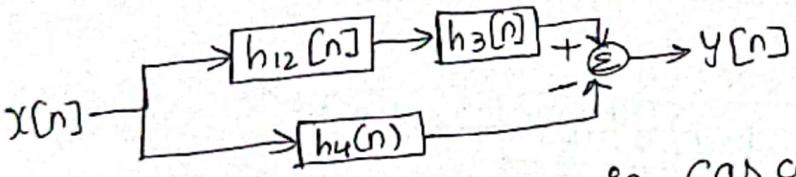
② Find the impulse response of overall system shown



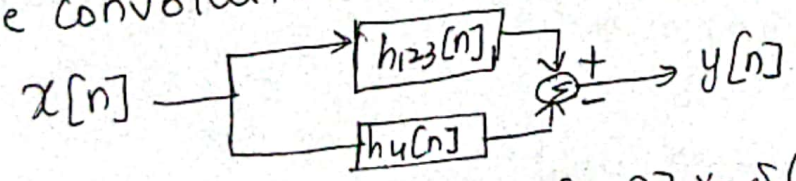
where $h_1[n] = u[n]$, $h_2[n] = u[n+2] - u[n]$; $h_3[n] = \delta(n-2)$, $h_4[n] = \alpha^n u[n]$

soln:- In the figure, $h_1[n]$ and $h_2[n]$ are in parallel, hence we have, $h_{12}[n] = h_1[n] + h_2[n] = u[n] + u[n+2] - u[n]$

$$= u[n+2]$$



Now $h_{12}[n]$ and $h_3[n]$ are in cascade. Here there will be convolution of $h_{12}[n]$ and $h_3[n]$ i.e



$$h_{123}[n] = h_{12}[n] * h_3[n] = u[n+2] * \delta[n-2] = u[n-2+2] = u[n]$$

Here $h_{123}[n]$ and $h_4[n]$ are in parallel but sign of $h_4[n]$ is negative

Hence, $h_{123}[n] = h_{123}[n] - h_4[n]$

$$= u[n] - \alpha^4 u[n]$$

$$= (1 - \alpha^4) u[n]$$

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System Properties in terms of Impulse Response.

1) An LTI System is $\boxed{\text{Causal}}$ if and only if $\boxed{h[n] = 0 \text{ for } n < 0}$

2) The System is said to be $\boxed{\text{stable}}$ if it produces bounded output for every bounded input.
Thus bounded input $x[n]$ produces bounded output $y[n]$ in the LTI System only if,

$$\boxed{\sum_{k=-\infty}^{\infty} |h(k)| < \infty}$$

3) Condition for Unit Sample response of $\boxed{\text{Memoryless}}$ or Static System is $\boxed{h(n) = 0}$ for $n \neq 0$.
Under the above Condition, the unit sample response will be of the form of unit impulse i.e $\boxed{h[n] = c\,\delta[n]}$

1) The input and output relationship of a discrete time LTI System is $y[n] = x[n+1] + 5x[n] - 7x[n-1] + 4x[n-2]$
Find i) The impulse response of the system
ii) Whether the System is stable and causal.

Soln:- The input and output relationship of LTI System is given by convolution as, $y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$

Let $k = -1$ to 2, $y[n] = \sum_{k=-1}^{2} h(k)\,x[n-k]$

$$\therefore y[n] = h(-1)\,x(n+1) + h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2)$$

Comparing above equation with given difference equation,

$$h(-1) = 1,\ h(0) = 5,\ h(1) = -7,\ h(2) = 4$$

or $h[n] = \{1, \underset{\uparrow}{5}, -7, 4\}$

OR $h[n] = \delta[n+1] + 5\delta[n] - 7\delta[n-1] + 4\delta[n-2]$

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Here $h[n] \neq 0$ for $n<0$. Hence system is not causal

$$\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=-1}^{2} |h(k)| = 1+5-7+4 = 3 < \infty$$

Since $h[n]$ is absolutely summable, the system is stable

2) The impulse response of the discrete time LTI system is given below $h[n] = n\left(\frac{1}{2}\right)^n u[n]$

i) Determine whether the system is stable or not

ii) Justify whether the system is causal

Soln:- i) To check stability

The discrete time system is stable if, $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

Let us check this condition for given system i.e,

$$\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=-\infty}^{\infty} k\left(\frac{1}{2}\right)^k u(k) = \sum_{k=0}^{\infty} k\left(\frac{1}{2}\right)^k$$

Using the standard formula $\sum_{k=0}^{\infty} k a^k = \frac{a}{(1-a)^2}$, if $|a|<1$

$$\sum_{k=-\infty}^{\infty} |h(k)| = \frac{1/2}{\left(1-\frac{1}{2}\right)^2} = 2$$

Thus, $\sum_{k=-\infty}^{\infty} |h(k)| = 2 < \infty$

Hence the system is stable.

ii) To check for causality

A LTI system is causal if, $h(n) = 0$ for $n<0$

$$h(n) = \begin{cases} n\left(\frac{1}{2}\right)^n & \text{for } n \geq 0 \\ 0 & \text{for } n<0 \end{cases}$$

Thus the system is causal since $h[n] = 0$ for $n<0$

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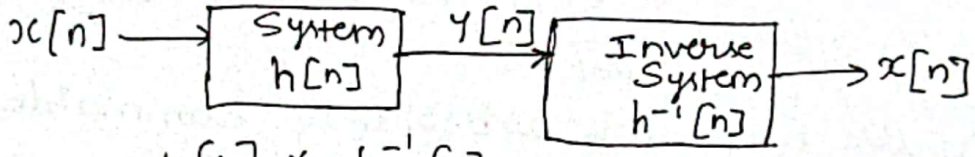
Invertible Systems and De-Convolution

Definition: A system is said to be invertible if the input of the system can be recovered from the output. If the impulse response of the system is $h[n]$, then impulse response of the inverse system is denoted by $h^{-1}[n]$.

The process of obtaining $x[n]$ back from $y[n]$ and $h[n]$ is called $\boxed{\text{de-Convolution}}$.



$$h[n] * h^{-1}[n] = \delta[n]$$

This is the condition for discrete time system to be invertible

Step Response

Expression for step Response of discrete time system is given by $\boxed{y[n] = \sum_{k=-\infty}^{n} h(k)}$

① By direct evaluation of Convolution sum, determine the step response of a discrete system whose unit impulse response $h[n] = (1/2)^{-n} u[-n]$. Sketch the response and hence verify whether the system is stable and causal.

Soln:- $h[n] = \left(\frac{1}{2}\right)^{-n} u(-n)$ & $x[n] = u(n)$ for Step Response

i) $y[n] = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{-k} u(-k)\, u(n-k)$

Here $u(-k) \cdot u(n-k) = \begin{cases} 1 & \text{for } k \le n \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore y[n] = \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^{-k} = \sum_{k=-\infty}^{\infty} 2^{k} = \frac{2^{n+1} - 2^{-\infty}}{2-1} \quad \text{for } n \le 0$$

$$y[n] = 2^{n+1} u[-n]$$

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ii) Stability and Causality

- Causality: since $h[n]$ contain the term $u[-n]$. This system is non causal
- Stability: Consider $\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{-k} u(-k)$

$$= \sum_{k=-\infty}^{0} \left(\frac{1}{2}\right)^{-k} \text{ since } u(-k)=1 \text{ for } k \leq 0$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{k} = \frac{1}{1-\frac{1}{2}} = 2 < \infty$$

$\therefore$ Impulse Response is absolutely summable. Hence this system is stable.

Excercise problems

1. For a system $h[n] = (0.99)^n u[n+3]$. Determine whether the corresponding system is Memoryless, Causal & Stable.

2. Find the step response of an LTI system, if Impulse response is $h[n] = n^2 u[n]$

3. Determine whether the described by $h[n] = (1/2)^n u[n]$ stable or causal.

4. Check whether the system whose impulse response is $h[n] = e^{-n} u[n-1]$ stable, memoryless and Causal.

5. Let $h[n] = 3\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^{n-1} u[n]$. If the input to this System is a unit step. Find $\lim_{n\to\infty} y[n]$ where $y[n] = h[n] * x[n]$

6. Find the Step Response $s[n]$ of an LTI System whose impulse response is defined by $h[n] = \begin{cases} \frac{1}{3} & 0 \leq n \leq 2 \\ 0 & \text{elsewhere} \end{cases}$

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Basic Signal Processing

Module 5

The Z-Transforms

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Module 5: Z-Transform

Sl. No	Topic	URL
1	Z- Transform	https://www.youtube.com/watch?v=mJgVOV9jRZU
2	ROC	https://www.youtube.com/watch?v=a05UJmw5wak&t=23s
3	Properties of ROC	https://www.youtube.com/watch?v=LyOQVvPVIuE-
4	Properties of Z-Transform part 1	https://www.youtube.com/watch?v=RYXlHkqqdh8-
5	Properties of Z-Transform part 2	https://www.youtube.com/watch?v=qYENYgbIDzg
6	Properties of Z-Transform part 3	https://www.youtube.com/watch?v=l1BhkFCqsRw-
7	Inverse Z-Transform by Partial fraction method	https://www.youtube.com/watch?v=ldpeNO642cs-
8	Inverse Z-Transform by Partial fraction method	https://www.youtube.com/watch?v=XRWaKZoJVZY
9	Causality and stability in Z-transform	https://www.youtube.com/watch?v=NClEFkAyMcA-
10	Causality and stability in Z-transform example	https://www.youtube.com/watch?v=uEl85xPyjZI
11	Transform analysis of LTI System in Z-transform	https://www.youtube.com/watch?v=8CRShpEn9tI
12	Transform analysis of LTI System in Z-transform	https://www.youtube.com/watch?v=3DL0xSF5H1c

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Module 5

The Z-Transform : Z Transform, Properties of the region of Convergence, Properties of the Z-Transform, Inverse Z-Transform by Partial Fraction, Causality and Stability, Transform analysis of LTI systems.

Z-Transform

The Z-Transform of a discrete time signal $x[n]$ is defined as the power series

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}}$$

Where z is a complex variable.

Region of Convergence (ROC)

The z-Transform is an infinite power series, it exists only for those values of z for which the above summation converges. Thus ROC of $X(z)$ is the set of all values of z, for which $X(z)$ attains a finite value.

① Find the Z-Transform of $x[n] = \{3, 2, \underset{\uparrow}{1}, 4, 5\}$

$$X(z) = \sum_{n=-2}^{2} x[n]\, z^{-n}$$

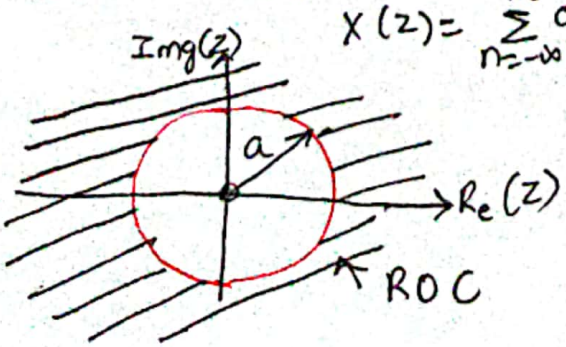
$$= 3z^2 + 2z + z^0 + 4z^{-1} + 5z^{-2}$$

ROC: All $|z|$ except $z = 0$ & $z = \infty$

② $x[n] = a^n u[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n} = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= \frac{1}{1 - az^{-1}} = \frac{z}{z-a}$$



ROC: $|z| > |a|$

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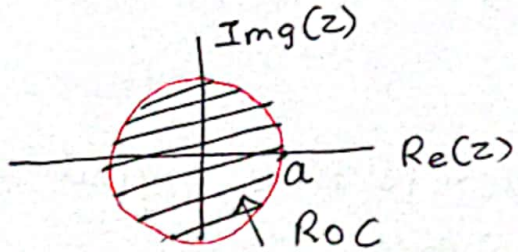
3) $x[n] = -a^n u[-n-1]$

$$X(z) = \sum_{n=-\infty}^{\infty} -a^n u[-n-1] z^{-n} = \sum_{n=-\infty}^{-1} -a^n z^{-n} = -\sum_{n=1}^{\infty} a^{-n} z^{n} = -\sum_{n=1}^{\infty} (a^{-1}z)^n$$

The sequence will converge if $|a^{-1}z| < 1$ i.e $|z| < |a|$

$$X(z) = -\frac{a^{-1}z}{1-a^{-1}z} \therefore X(z) = \frac{z}{z-a} \quad \text{ROC}: |z| < |a|$$



4) $x[n] = a^n u[n] + b^n u[-n-1]$, $a > b$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{-1} b^n z^{-n} + \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=1}^{\infty} b^{-n} z^{n} + \sum_{n=0}^{\infty} a^n z^{-n}$$

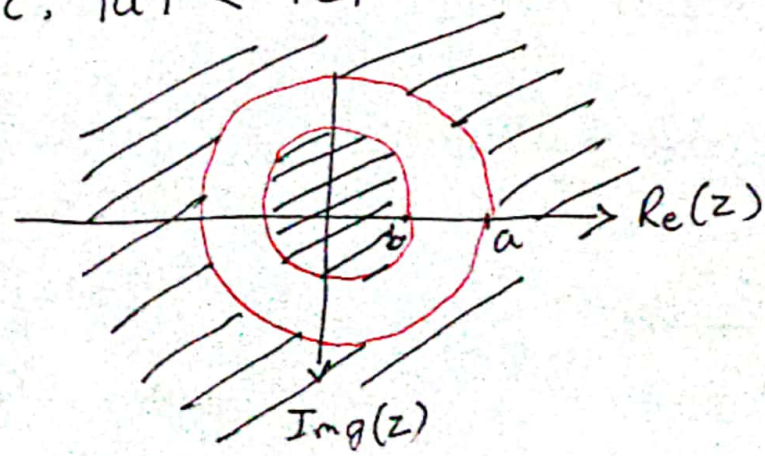
$$= \sum_{n=1}^{\infty} (b^{-1}z)^n + \sum_{n=0}^{\infty} (az^{-1})^n$$

1st sequence will converge if $|b^{-1}z| < 1$ i.e $|z| < b$

2nd sequence will converge if $|az^{-1}| < 1$, $|z| > |a|$

$$X(z) = \frac{b^{-1}z}{1-b^{-1}z} + \frac{1}{1-az^{-1}}$$

ROC: $|a| < |z| < |b|$



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Properties of ROC

1) The ROC of $X(z)$ consists of a ring in the Z-plane centered about the origin.
2) ROC does not contain any poles.
3) If $x[n]$ is a finite duration sequence, then the ROC is entire Z-plane except $z=0$ and/or $z=\infty$
4) If $x[n]$ is a right sided infinite sequence, then ROC is exterior to the circle.
5) If $x[n]$ is a left sided infinite sequence, then ROC is interior to the circle
6) If $x[n]$ is two sided infinite sequence, then ROC will consist of a ring in the Z-plane. This ROC will be the intersection of the ROC's of the components.

① Determine Z-Transform of following sequences

$x[n] = \{1, 2, 3, \underset{\uparrow}{4}, 5, 0, 7\}$

Soln: $x[-3]=1$, $x[-2]=2$, $x[-1]=3$, $x[0]=4$

$x[1]=5$, $x[2]=0$, $x[3]=7$

$$\therefore \quad X(z) = \sum_{n=-3}^{3} x[n] z^{-n}$$

$$= 1 \cdot z^{3} + 2z^{2} + 3z^{+1} + 4z^{0} + 5z^{-1} + 0 \cdot z^{-2} + 7z^{-3}$$

$$= z^{3} + 2z^{2} + 3z + 4 + \frac{5}{z} + \frac{7}{z^{3}}$$

$X(z) = \infty$ for $z=0$ & $z=\infty$

Hence ROC : Entire Z-plane except $z=0$ and $z=\infty$

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1) Obtain Z-Transform of $\delta[n]$

Solution: We know that $\delta[n] = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$

$$\therefore X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} \delta[n] z^{-n}$$

$$= 1 \cdot z^0 = 1$$

$$\therefore \boxed{\delta[n] \xleftrightarrow{z} 1}$$ ROC: Entire Z-plane

2) Obtain Z-Transform of $u[n]$

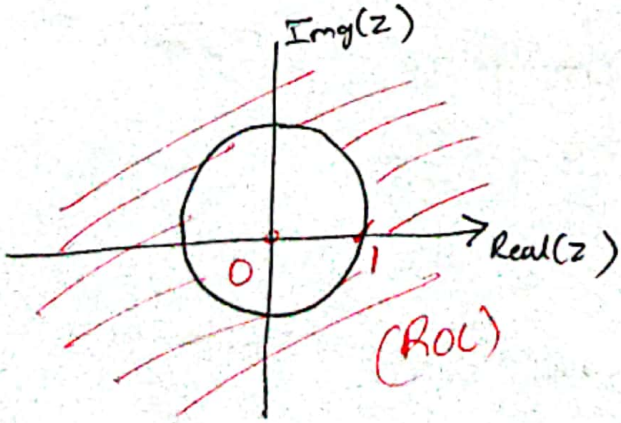
Soln:- Unit Step sequence, $u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} z^{-n} = \sum_{n=0}^{\infty} (z^{-1})^n = 1 + z^{-1} + (z^{-1})^2 + (z^{-1})^3 + \cdots$$

$$X(z) = \frac{z}{z-1} \; ; \; |z| > 1$$

$$\therefore u[n] \xleftrightarrow{z} \frac{z}{z-1}$$ ROC: $|z| > 1$



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Properties of the Z-Transform

① Linearity

It states that if $x_1[n] \xrightarrow{Z} X_1(z)$

& $x_2[n] \xrightarrow{Z} X_2(z)$

Then $\alpha x_1[n] + \beta x_2[n] \xrightarrow{Z} \alpha X_1(z) + \beta X_2(z)$

Proof: $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} [a_1 x_1[n] + a_2 x_2[n]] z^{-n}$

$$= \sum_{n=-\infty}^{\infty} a_1 x_1[n] z^{-n} + \sum_{n=-\infty}^{\infty} a_2 x_2[n] z^{-n}$$

$$= a_1 \sum_{n=-\infty}^{\infty} x_1[n] z^{-n} + a_2 \sum_{n=-\infty}^{\infty} x_2[n] z^{-n}$$

$$= a_1 X_1(z) + a_2 X_2(z)$$

② Time Shifting or Translation

If $x[n] \xrightarrow{Z} X(z)$

$x[n-k] \xrightarrow{Z} z^{-1} X(z)$

Proof: $Z\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-k) z^{-n}$

Let $n-k=m$. Hence $n=k+m$ and $m=-\infty$ to $+\infty$

i.e $Z\{x(n-k)\} = \sum_{m=-\infty}^{\infty} x(m) z^{-(k+m)}$

$$= \sum_{m=-\infty}^{\infty} x(m) z^{-k} z^{-m} = z^{-k} \sum_{m=-\infty}^{\infty} x(m) z^{-m}$$

$$= z^{-k} X(z)$$

③ Scaling in Z-Domain or Multiplication by Exponential

Let $x[n] \xrightarrow{Z} X(z)$, ROC: $r_1 < |z| < r_2$

then $a^n x[n] \xrightarrow{Z} X\left(\frac{z}{a}\right)$

Proof: $Z\{a^n x[n]\} = \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n} = \sum_{n=-\infty}^{\infty} x(n) (a^{-1} z)^{-n}$

$$= X(a^{-1} z)$$

$$= X\left(\frac{z}{a}\right)$$

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4) Time Reversal

Let $x[n] \xrightarrow{z} X(z)$ then $x(-n) \xrightarrow{z} X(z^{-1})$

Proof: $Z[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$

Put $n=-m$ $\therefore Z[x(-n)] = \sum_{m=-\infty}^{\infty} x(n)(z^{-1})^{-m}$

$= X(z^{-1})$

$\therefore x[-n] \xrightarrow{z} X(z^{-1})$

5) Differentiation in Z-Domain or Multiplication by a ramp

Let $x[n] \xrightarrow{z} X(z)$

then $n x(n) \xrightarrow{z} -z \frac{d}{dz} X(z)$

Proof:- $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$\frac{d}{dz} X(z) = \sum_{n=-\infty}^{\infty} \frac{d}{dz}[x[n] z^{-n}] = \sum_{n=-\infty}^{\infty} x[n] \frac{d}{dz} z^{-n}$

$= \sum_{n=-\infty}^{\infty} x[n](-n) z^{-n-1} = -\sum_{n=-\infty}^{\infty} n\, x[n] z^{-n} z^{-1}$

$= -z^{-1} \sum_{n=-\infty}^{\infty} [n x[n]] z^{-n} = -z^{-1} Z\{n\, x[n]\}$

$\therefore Z\{n\, x(n)\} = -z \frac{d}{dz} X(z)$

6) Convolution in Time Domain

Let $x_1[n] \xrightarrow{z} X_1(z)$

$x_2[n] \xrightarrow{z} X_2(z)$

$x_1[n] * x_2[n] \xrightarrow{z} X_1(z) . X_2(z)$

Proof: $x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$

$= \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k) \right] z^{-n}$

Interchanging orders of summation

$= \sum_{k=-\infty}^{\infty} x_1(k) \left\{ \sum_{n=-\infty}^{\infty} x_2(n-k) z^{-n} \right\}$.

Since $x_2(n-k) \xrightarrow{z} z^{-k} X_2(z)$

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$$= \sum_{k=-\infty}^{\infty} x_1(k) \{ z^{-k} X_2(z) \}$$

$$= \left\{ \sum_{k=-\infty}^{\infty} x_1(k) z^{-k} \right\} X_2(z)$$

$$= X_1(z) X_2(z)$$

7) **Parseval's Relation**

Let $x_1[n] \xrightarrow{z} X_1(z)$ & $x_2[n] \xrightarrow{z} X_2(z)$

$$\sum_{n=-\infty}^{\infty} x_1[n] x_2^*[n] = \frac{1}{2\pi j} \oint X_1(v) X_2^*\left(\frac{1}{v^*}\right) v^{-1} dv$$

8) **Initial value Theorem**

Let $x[n] \xrightarrow{z} X(z)$, then

$$x(0) = \lim_{z \to \infty} X(z)$$

Proof: Z-Transform of a Causal sequence is given as

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}, \text{ since } x[n] = 0 \text{ for } n < 0$$

$$= x(0) + x(1) z^{-1} + x(2) z^{-2} + \cdots$$

$$\lim_{z \to \infty} X(z) = \lim_{z \to \infty} x(0) + \lim_{z \to \infty} x(1) z^{-1} + \lim_{z \to \infty} x(2) z^{-2} + \cdots$$

$$= x(0) + 0 + 0 + 0 + \cdots$$

$$x(0) = \lim_{z \to \infty} X(z)$$

9) **Final value Theorem**

If $x[n]$ is causal & $x[n] \longrightarrow X(z)$

then $\lim_{n \to \infty} x[n] = x[\infty] = \lim_{z \to 1} (1 - z^{-1}) X(z)$

Proof:- $Z\{x[n] - x[n-1]\} = \sum_{n=0}^{\infty} \{x[n] - x[n-1]\} z^{-n}$

$$= X(z) - X(z) z^{-1}$$

$$= X(z) [1 - z^{-1}]$$

Taking the limits $n \to \infty$ & $z \to 1$ on both the sides

$$\lim_{z \to 1} \lim_{n \to \infty} \sum_{n=0}^{\infty} \{x[n] - x[n-1]\} z^{-n} = \lim_{z \to 1} \lim_{n \to \infty} X(z)[1 - z^{-1}]$$

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$$\lim_{n\to\infty} \sum_{n=0}^{\infty} [x[n] - x[n-1]] = \lim_{z\to 1} X(z)[1-z^{-1}]$$

$$\lim_{n\to\infty} x[n] = x[\infty] = \lim_{z\to 1} [1-z^{-1}]\, X(z)$$

① Find Z-Transform of the following

$x[n] = u[-n]$

Soln:

$$x[n] \xrightarrow{Z} X(z)$$

$$x[-n] \xrightarrow{Z} X(z^{-1})$$

$$u[n] \xrightarrow{Z} \frac{z}{z-1} \quad \text{ROC}: |z| > 1$$

$$\therefore u[-n] \xrightarrow{Z} \frac{1}{1-z} \quad \text{ROC}: |z| < 1$$

② Determine Z-Transform of $x[n] = \sin(\Omega_0 n)\, u[n]$

Soln:-

$$\sin \Omega_0 n = \frac{e^{j\Omega_0 n} - e^{-j\Omega_0 n}}{2j}$$

$$X(z) = Z\left\{ \frac{e^{j\Omega_0 n} - e^{-j\Omega_0 n}}{2j} \right\} u[n]$$

$$= \frac{1}{2j}\left[Z\{e^{j\Omega_0 n} u(n)\} - Z\{e^{-j\Omega_0 n} u(n)\} \right]$$

$$= \frac{1}{2j}\left[\frac{z}{z - e^{j\Omega_0}} - \frac{z}{z - e^{-j\Omega_0}} \right]$$

$$= \frac{1}{2j}\left[\frac{(e^{j\Omega_0} - e^{-j\Omega_0}) z^{-1}}{1 - z^{-1}(e^{j\Omega_0} + e^{-j\Omega_0}) + z^{-2}} \right]$$

$$= \frac{1}{2j}\left[\frac{2j \sin\Omega_0 z^{-1}}{1 - z^{-1} 2\cos\Omega_0 + z^{-2}} \right]$$

$$= \frac{z^{-1} \sin\Omega_0}{1 - 2z^{-1}\cos\Omega_0 + z^{-2}}$$

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① Determine Z-Transform of $x[n] = (0.2)^n\{u[n] - u[n-4]\}$

Soln:- $x[n] = (0.2)^n\{u[n] - u(n-4)\}$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} (0.2)^n\{u[n] - u[n-4]\} z^{-n}$$

Here $u[n] - u[n-4] = \begin{cases} 1 & \text{for } 0 \le n \le 3 \\ 0 & \text{for } n \ge 4 \ \& \ n \le 1 \end{cases}$

$$X(z) = \sum_{n=0}^{3} (0.2)^n z^{-n} = \sum_{n=0}^{3} (0.2 z^{-1})^n$$

Here we use $\sum_{n=0}^{N} a^n = \frac{1 - a^{N+1}}{1-a} = \frac{a^{N+1} - 1}{a-1}$

$$\therefore X(z) = \frac{1 - (0.2 z^{-1})^4}{1 - 0.2 z^{-1}}$$

② Determine Z-Transform of $x[n] = n\left(\frac{1}{3}\right)^{n+3} u(n+3)$

Soln:- $x[n] = n\left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right)^3 u(n+3)$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} n\left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right)^3 u(n+3) z^{-n}$$

$$= \left(\frac{1}{3}\right)^3 \sum_{n=-\infty}^{\infty} n\left(\frac{1}{3}\right)^n u(n+3) z^{-n}$$

Here $u(n+3) = 1$ for $-3 \le n \le \infty$.

$$\therefore X(z) = \left(\frac{1}{3}\right)^3 \sum_{n=-3}^{\infty} n\left(\frac{1}{3}\right)^n z^{-n}$$

Let $X_1(z) = \sum_{n=-3}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} = \sum_{n=-3}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n = \frac{(3z)^4}{3z-1}$

We know that $n x[n] \xrightarrow{z} -z \frac{d}{dz} X(z)$

$$z\left\{\sum_{n=-3}^{\infty} n\left(\frac{1}{3}\right)^n z^{-n}\right\} = -z \frac{d}{dz}\left\{\frac{(3z)^4}{3z-1}\right\} = \frac{3z^4(4-9z)}{(3z-1)^2}$$

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3) If $x[n] = n^2 u[n]$ then find $X(z)$

Soln:- $n\,x[n] \xrightarrow{z} -z\frac{d}{dz} X[n]$

$$n^2 x[n] \xrightarrow{z} -z \frac{d}{dz}\left[-z \frac{d}{dz} X(z)\right]$$

Here $u[n] \xrightarrow{z} \frac{z}{z-1}$

$$\therefore n^2 u[n] = -z\frac{d}{dz}\left[-z \frac{d}{dz} \frac{z}{z-1}\right]$$

$$= \frac{z(z+1)}{(z-1)^3}$$

4) If $x[n] = n(n+1)u(n)$ find $X(z)$

Soln:- $x[n] = [n^2+n]\,u[n]$

$= n^2 u[n] + n\,u[n]$

$$n u[n] \xrightarrow{z} \frac{z}{(z-1)^2}$$

$$n^2 u[n] \xrightarrow{z} -z \frac{d}{dz} \frac{z}{(z-1)^2}$$

$$\xrightarrow{z} -z \frac{-(z+1)}{(z-1)^3}$$

$$\rightarrow \frac{z(z+1)}{(z-1)^3}$$

$$\therefore X(z) = \frac{z(z+1)}{(z-1)^3} + \frac{z}{(z-1)^2}$$

$$= \frac{2z^2}{(z-1)^3}$$

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Inverse z-Transform by Partial Fraction Method

① Find the IZT of $X(z) = \frac{z}{z^2-5z+6}$

ROC (i) $|z| > 3$ (ii) $|z| < 2$

Soln: $X(z) = \frac{z}{z^2-5z+6} = \frac{z}{(z-3)(z-2)}$

$$\frac{X(z)}{z} = \frac{1}{(z-3)(z-2)} = \frac{A}{z-3} + \frac{B}{z-2}$$

$$\therefore 1 = A(z-2) + B(z-3)$$

on Solving we get $A = 1$, $B = -1$

$$\frac{X(z)}{z} = \frac{1}{z-3} - \frac{1}{z-2}$$

$$X(z) = \frac{z}{z-3} - \frac{z}{z-2}$$

Taking Inverse z-Transform

$$x[n] = 3^n u[n] - 2^n u[n] \quad \text{if } |z| > 3$$

$$x[n] = -3^n u[-n-1] + 2^n u[-n-1] \quad \text{if } |z| < 2$$

② Find the IZT of $X(z) = \frac{z}{(z-1)(z-2)(z-3)}$

(i) $|z| > 3$ (ii) $|z| < 1$ (iii) $1 < |z| < 2$ (iv) $2 < |z| < 3$ (v) $1 < |z| < 3$

Soln:- $X(z) = \frac{z}{(z-1)(z-2)(z-3)}$

$$\frac{X(z)}{z} = \frac{1}{(z-1)(z-2)(z-3)} = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{z-3}$$

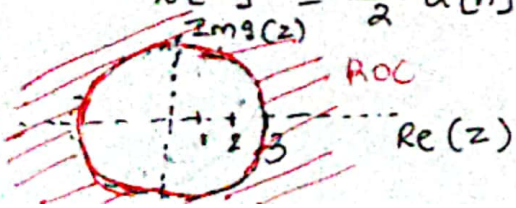
On Solving we get $A = \frac{1}{2}$, $B = -1$, $C = \frac{1}{2}$

Taking IZT

(i) ROC : $|z| > 3$

ROC is Exterior, it represents right sided sequence

$$x[n] = \frac{1}{2} u[n] - 2^n u[n] + \frac{1}{2} 3^n u[n]$$



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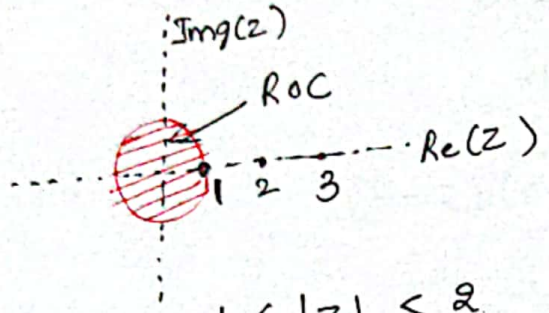
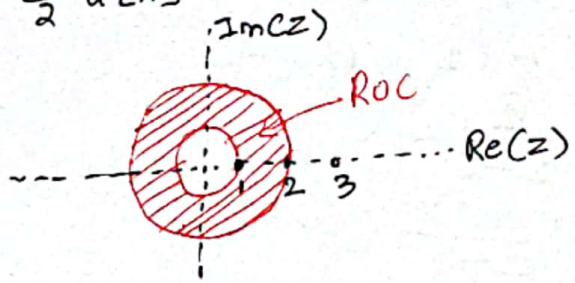
(ii) ROC : $|z| < 1$

$$x[n] = -\frac{1}{2}u[-n-1] + 2^n u[-n-1] - \frac{1}{2}3^n u[-n-1]$$



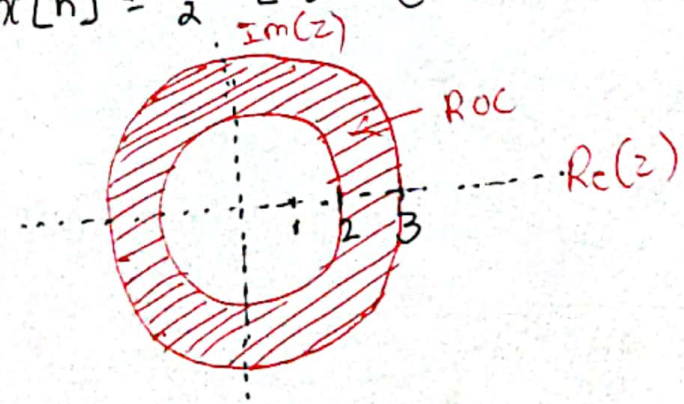
iii) ROC : $1 < |z| < 2$

$$x[n] = \frac{1}{2}u[n] + 2^n u[-n-1] - \frac{1}{2}3^n u[-n-1]$$



(iv) ROC : $2 < |z| < 3$

$$x[n] = \frac{1}{2}u[n] - (2)^n u[n] - \frac{1}{2}3^n u[-n-1]$$



(v) ROC : $1 < |z| < 3$

Not a Valid ROC as pole (Pole at $z = 2$) lies within ROC

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Causality and Stability interms of Z-Transform

LTI System is causal if and only if the ROC of the system function is exterior of a circle of radius $r<\infty$

LTI system is BIBO stable if and only if the ROC of the System function includes the unit circle.

A causal and stable system must have a system function that converges for

$|z| > r < 1$ for causal and stable system

1) A Discrete LTI system is characterized by the difference equation $y[n] = y[n-1] + y[n-2] + x[n-1]$. Find the system function $H(z)$ and indicate the ROC if the system (i) Stable (ii) Causal. Also determine the unit sample response of the stable system.

Soln: (i) ROC of system

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

Taking Z-Transform,

$$Y(z) = z^{-1} Y(z) + z^{-2} Y(z) + z^{-1} X(z)$$

$$Y(z)\left[1 - z^{-1} - z^{-2}\right] = z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}} = \frac{z}{z^2 - z - 1}$$

$$= \frac{z}{(z - 0.5 - \sqrt{1.25})(z - 0.5 + \sqrt{1.25})}$$

$$= \frac{z}{(z - 1.618)(z + 0.618)}$$

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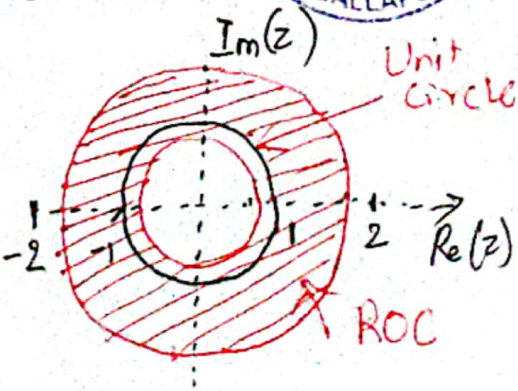
poles lie at : $P_1 = 1.618$, $P_2 = -0.618$

zeros lie at : $Z_1 = 0$



ROC for Stable System

ROC of Stable system includes unit circle

Hence ROC: $-0.618 < |z| < 1.618$

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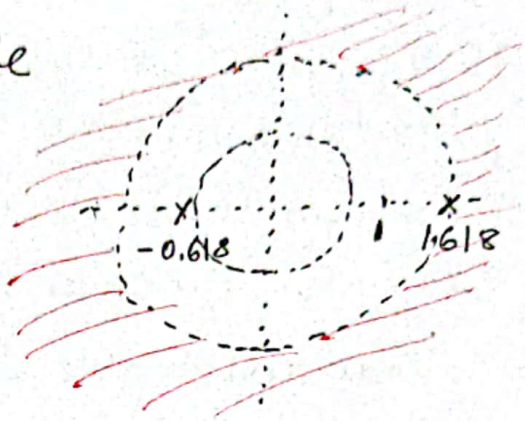
ROC for Causal System

For the causal system, ROC is outside of some circle of finite radius.

ROC : $|z| > 1.618$



Unit Sample Response of Stable System

The stable system is given as

$$H(z) = \frac{z}{(z-1.618)(z+0.618)}$$

ROC : $-0.618 < |z| < 1.618$

$$\frac{H(z)}{z} = \frac{1}{(z-1.618)(z+0.618)} = \frac{0.447}{(z-1.618)} - \frac{0.447}{(z+0.618)}$$

$$H(z) = \frac{0.447z}{z-1.1.618} - \frac{0.447z}{z+0.618}$$ with $|z| < 1.618$ & $|z| > 0.618$

$$\therefore h[n] = 0.447\left[-(1.618)^n u[-n-1] - (-0.618)^n u[n]\right]$$

Transform Analysis of LTI Systems

$$x[n] \longrightarrow \boxed{\text{LTI System}} \longrightarrow y[n] = x[n] * h[n]$$

$$y[n] = x[n] * h[n]$$

Taking Z-Transform on both sides

$$Y(z) = X(z)\,H(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

H(z) is called Transfer function of the system.

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To Obtain Difference Equation from H(z)

① Obtain the Difference Equation given

$$H(z) = \frac{1+3z^{-1}+4z^{-2}}{1-z^{-1}+2z^{-2}}$$

Soln; $H(z) = \frac{Y(z)}{X(z)} = \frac{1+3z^{-1}+4z^{-2}}{1-z^{-1}+2z^{-2}}$

$$Y(z)\left[1-z^{-1}+2z^{-2}\right] = X(z)\left[1+3z^{-1}+4z^{-2}\right]$$

$$Y(z) - Y(z)z^{-1} + 2Y(z)z^{-2} = X(z) - 3X(z)z^{-1} + 4X(z)z^{-2}$$

$$y[n] - y[n-1] + 2y[n-2] = x[n] + 3x[n-1] + 4x[n-2]$$

Unilateral Z-Transform

The Unilateral or one-sided z-Transform is evaluated for Causal sequence ($n \geq 0$)

Properties

The unilateral z-Transform satisfies the same properties as the bilateral z-Transform with one important exception the 'time shift property'

$$x[n] \longleftrightarrow X(z)$$

$$x[n-1] \overset{z}{\Rightarrow} x[-1] + X(z)z^{-1}$$

$$x[n-2] \overset{z}{\Rightarrow} x[-2] + x[-1]z^{-1} + X(z)z^{-2}$$

Solution of Difference Equation

① Consider the system described by the difference equation

$y[n] = 0.1\, y[n-1] + x[n]$, $n \geq 0$

if the input $x[n] = u[n]$ & $y[-1] = 2$. Find the output of the system

Soln:- $y[n] - 0.1\, y[n-1] = x[n]$

Applying Unilateral z Transform

$$Y(z) - 0.1\left[y(-1) + z^{-1}Y(z)\right] = X(z)$$

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$$x[n] = u[n] \longrightarrow \frac{z}{z-1} = X(z)$$

$$Y(z) - 0.1\left[2 + Y(z)z^{-1}\right] = \frac{z}{z-1}$$

$$Y(z)\left[1 - 0.1z^{-1}\right] = \frac{z}{z-1} + 0.2$$

$$Y(z)\left[\frac{z-0.1}{z}\right] = \frac{z}{z-1} + 0.2$$

$$Y(z)\left[\frac{z-0.1}{z}\right] = \frac{z + 0.2z - 0.2}{z-1}$$

$$\frac{Y(z)}{z} = \frac{1.2z - 0.2}{(z-0.1)(z-1)} = \frac{A}{z-1} + \frac{B}{z-0.1}$$

Solving for A & B, $A = 1.11$ & $B = 0.08$

$$Y(z) = \frac{1.11z}{z-1} + \frac{0.08z}{z-0.1}$$

Taking Inverse Z-Transform

$$y[n] = 1.11\,u[n] + 0.08\,(0.1)^n\,u[n]$$

(2) Determine the response of the system

$$3y[n] - 4y[n-1] + y[n-2] = x[n]$$

For the input signal $x[n] = \left(\frac{1}{2}\right)^n u[n]$ with $y(-1) = 1$ & $y(-2) = 2$

Soln:- $3y[n] - 4y[n-1] + y[n-2] = x[n]$

Taking unilateral Z-Transform on both sides

$$3Y(z) - 4\left[y(-1) + Y(z)z^{-1}\right] + y(-2) + y(-1)z^{-1} + Y(z)z^{-2} = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$3Y(z) - 4\left[1 + Y(z)z^{-1}\right] + 2 + z^{-1} + Y(z)z^{-2} = \frac{2}{2 - z^{-1}}$$

$$Y(z)\left[3 - 4z^{-1} + z^{-2}\right] = \frac{z^{-2} - 4z^{-1} + 6}{2 - z^{-1}}$$

$$\frac{Y(z)}{z} = \frac{z^2 - \frac{2}{3}z + \frac{1}{6}}{(z-1)(z-\frac{1}{3})(z-\frac{1}{2})} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{3}} + \frac{C}{z-\frac{1}{2}}$$

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On Solving for A, B, C, we get

$A = \frac{3}{2}$, $B = \frac{1}{2}$, $C = -1$

$$\frac{Y(z)}{z} = \frac{3}{2}\frac{1}{z-1} + \frac{1}{2}\frac{1}{z-\frac{1}{3}} + \frac{-1}{z-\frac{1}{2}}$$

$$Y(z) = \frac{3}{2}\frac{z}{z-1} + \frac{1}{2}\frac{z}{z-\frac{1}{3}} - \frac{z}{z-\frac{1}{2}}$$

Taking Inverse Z-Transform

$$y[n] = \frac{3}{2}u[n] + \frac{1}{2}\left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[n]$$

3 Obtain the response of the system described by the following equation $y[n] = 0.85y[n-1] + 0.15x[n]$ for the inputs, (i) $x[n] = u[n]$ (ii) $x[n] = (-1)^n u[n]$

Soln:-

$$y[n] = 0.85y[n-1] + 0.15x[n]$$

$$Y(z) = 0.85z^{-1}Y(z) + 0.15X(z)$$

$$Y(z)\left[1 - 0.85z^{-1}\right] = 0.15X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.15}{1-0.85z^{-1}} = \frac{0.15z}{z-0.85}$$

For step Response $X(z) = \frac{z}{z-1}$

$$Y(z) = H(z)X(z)$$

$$= \frac{0.15z}{(z-0.85)} \cdot \frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{0.15z}{(z-0.85)(z-1)} = \frac{A}{z-0.85} + \frac{B}{z-1}$$

$$\therefore \frac{Y(z)}{z} = \frac{-0.85}{z-0.85} + \frac{1}{z-1}$$

$$Y(z) = -\frac{0.85z}{z-0.85} + \frac{z}{z-1}$$

$$\therefore y[n] = -0.85(0.85)^n u[n] + u[n]$$

$$\boxed{y[n] = \left[1 - (0.85)^{n+1}\right]u[n]}$$

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(2) For $x[n] = (-1)^n u[n]$

$$X(z) = \frac{z}{z+1}$$

$$Y(z) = H(z)\, X(z)$$

$$Y(z) = \frac{0.15\, z}{z - 0.85} \cdot \frac{z}{z+1}$$

$$\frac{Y(z)}{z} = \frac{0.15 z}{(z-0.85)(z+1)} = \frac{A}{z-0.85} + \frac{B}{z+1}$$

On Solving $A = 0.0689$ & $B = 0.08108$

$$\therefore \frac{Y(z)}{z} = \frac{0.0689\, z}{z - 0.85} + \frac{0.08108}{z+1}$$

$$Y(n) = 0.0689\,(0.85)^n u[n] + 0.08108(-1)^n u[n]$$

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