Model Question Paper-II with effect from 2022

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Third Semester B.E Degree Examination
Transform Calculus, Fourier Series and Numerical Techniques
(21MAT31)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

		Module -1	**Marks**
Q.01	a	Find the Laplace transform of $(i)\ e^{-3t} sin5t\ cos3t \qquad (ii)\ \frac{1-e^t}{t}$	**06**
	b	Find the Laplace transform of the square–wave function of period a given by $$f(t) = \begin{cases} 1, & 0 < t < a/2 \\ -1, & a/2 < t < 2 \end{cases}$$	**07**
	c	Using convolution theorem find the inverse Laplace transform of $\frac{1}{(s^2+1)(s^2+9)}$	**07**
		OR	
Q.02	a	Using unit step function , find the Laplace transform of $$f(t) = \begin{cases} \cos t, & 0 \le t \le \pi \\ \cos 2t, & \pi \le t \le 2\pi \\ \cos 3t, & t \ge 2\pi \end{cases}$$	**06**
	b	Find the inverse Laplace transform of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$	**07**
	c	Solve by using Laplace transform techniques $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x(0) = 2$ and $x'(0) = -1$	**07**
		Module-2	
Q. 03	a	Find a Fourier series to represent $f(x) = x^2 \quad in \quad -\pi \le x \le \pi$	**06**
	b	Obtain the half range cosine series for $f(x) = x\,sinx$ in $(0,\ \pi)$ and hence show that $\frac{\pi-2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots \infty$	**07**
	c	The following table gives the variation of periodic current over a period. <table><tr><td>*t sec*</td><td>0</td><td>T/6</td><td>T/3</td><td>T/2</td><td>2T/3</td><td>5T/6</td><td>T</td></tr><tr><td>*A amp*</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr></table>Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic.	**07**
		OR	
Q.04	a	Find the Fourier series expansion of $f(x) = 2x - x^2\ , \quad in\ (\ 0,\ 3)$	**6**
	b	Obtain half-range sine series for	**07**

<table>
<tr><td></td><td></td><td>$f(x) = \begin{cases} kx\,, & 0 \le x \le \frac{l}{2} \\ k(l-x)\,, & \frac{l}{2} \le x \le l \end{cases}$</td><td></td></tr>
<tr><td></td><td>c</td><td>Expand y as a Fourier series up to first harmonic if the values of y given by
<table>
<tr><td>x</td><td>0°</td><td>30°</td><td>60°</td><td>90°</td><td>120°</td><td>150°</td><td>180°</td><td>210°</td><td>240</td><td>270</td><td>300</td><td>330</td></tr>
<tr><td>y</td><td>1.80</td><td>1.10</td><td>0.30</td><td>0.16</td><td>1.50</td><td>1.30</td><td>2.16</td><td>1.25</td><td>1.30</td><td>1.52</td><td>1.76</td><td>2.00</td></tr>
</table></td><td>07</td></tr>
<tr><td colspan="3">Module-3</td><td></td></tr>
<tr><td>Q. 05</td><td>a</td><td>Find the Fourier transform of $f(x) = \begin{cases} 1\,, & |x| \le 1 \\ 0\,, & |x| > 1 \end{cases}$
Hence evaluate $\int_0^\infty \frac{sinx}{x} dx$</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Find the Fourier cosine and sine transforms of e^{-ax}</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Find the Z-transforms of $(i)\ (n+1)^2$ and $(ii)\ \sin(3n+5)$</td><td>07</td></tr>
<tr><td colspan="3">OR</td><td></td></tr>
<tr><td>Q. 06</td><td>a</td><td>Find the Fourier transform of $e^{-a^2x^2}, a > 0$. Hence deduce that it is self reciprocal in respect of Fourier series</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Find the inverse z –transform of $\frac{2z^2+3z}{(z+2)(z-4)}$</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Using z-transformation, solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, $u_0 = 0\,, u_1 = 1$</td><td>07</td></tr>
<tr><td colspan="3">Module-4</td><td></td></tr>
<tr><td>Q. 07</td><td>a</td><td>Classify the following partial differential equations
(i) $u_{xx} + 4u_{xy} + 4u_{yy} - u_x + 2u_y = 0$
(ii) $x^2u_{xx} + (1-y^2)u_{yy} = 0\,, -1 < y < 1$
(iii) $(1+x^2)u_{xx} + (5+2x^2)u_{xt} + (4+x^2)u_{tt} = 0$
(iv) $y^2u_{xx} - 2yu_{xy} + u_{yy} - u_y = 8y$</td><td>10</td></tr>
<tr><td></td><td>b</td><td>Find the values of $u(x,t)$ satisfying the parabolic equation $u_t = 4u_{xx}$ and the boundary conditions $u(0,t) = 0 = u(8,0)$ and $u(x,0) = 4x - \frac{x^2}{2}$ at the points $x = i : i = 0,1,2, \dots ,8$ and $t = \frac{j}{8} : j = 0,1,2,3,4$.</td><td>10</td></tr>
<tr><td colspan="3">OR</td><td></td></tr>
<tr><td>Q. 08</td><td>a</td><td>Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0) = \sin \pi x\,, 0 \le x \le 1$ $u(0,t) = u(1,t) = 0$, Carry out computations for two levels, taking $h = \frac{1}{34}\, and\ k = \frac{1}{36}$</td><td>10</td></tr>
<tr><td></td><td>b</td><td>The transverse displacement u of a point at a distance x from one end and at any time t of a vibrating string satisfies the equation $u_{tt} = 25\, u_{xx},$ with the boundary conditions $u(x,t) = u(5,t) = 0$ and the initial conditions $u(x,0) = \begin{cases} 20x\,, & 0 \le x \le 1 \\ 5(5-x), & 1 \le x \le 5 \end{cases}$ and $u_t(x,0) = 0$. Solve this equation numerically up to $t = 5$ taking $h = 1, k = 0.2$.</td><td>10</td></tr>
</table>

Module-5			
Q. 09	a	Using Runge –Kutta method of order four, solve $\frac{d^2y}{dx^2} = y + x\frac{dy}{dx}$ for x = 0.2 , Given that , $y(0) = 1\ , y'(0) = 0$	**06**
	b	Find the extremals of the functional $\int_{x_1}^{x_2}[y^2 + (y')^2 + 2ye^x]dx$	**07**
	c	Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity	**07**
OR			
Q. 10	a	Apply Milne's method to solve $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ at x = 0.4. given that $y(0) = 1\ , y(0.1) = 1.1103,\ y(0.2) = 1.2427\ ,\ y(0.3) = 1.399$ $y'(0) = 1\ , y'(0.1) = 1.2103, y'(0.2) = 1.4427, y'(0.3) = 1.699$	06
	b	Find the extremals of the functional $\int_{x_1}^{x_2}\frac{(y')^2}{x^3}dx$	**07**
	c	Find the curve on which the functional $\int_0^{\pi/2}[(y')^2 + 12xy]\,dx$ with $y(0) = 0\ and\ y(\pi/2) = 0$ can be extremised	**07**

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome				
Question		**Bloom's Taxonomy Level attached**	**Course Outcome**	**Program Outcome**
Q.1	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 02
	(c)	L2	CO 01	PO 02
Q.2	(a)	L2	CO 01	PO 02
	(b)	L2	CO 01	PO 02
	(c)	L2	CO 01	PO 02
Q.3	(a)	L2	CO 02	PO 02
	(b)	L2	CO 02	PO 02
	(c)	L3	CO 02	PO 02
Q.4	(a)	L2	CO 02	PO 02
	(b)	L2	CO 02	PO 02
	(c)	L2	CO 02	PO 02
Q.5	(a)	L2	CO 03	PO 02
	(b)	L2	CO 03	PO 02
	(c)	L2	CO 03	PO 02
Q.6	(a)	L2	CO 03	PO 02
	(b)	L2	CO 03	PO 02
	(c)	L3	CO 03	PO 02
Q.7	(a)	L1	CO 04	PO 01
	(b)	L2	CO 04	PO 02
Q.8	(a)	L2	CO 04	PO 02
	(b)	L3	CO 04	PO 02

Q.9	(a)	L2	CO 05	PO 01
	(b)	L2	CO 05	PO 02
	(c)	L3	CO 05	PO 02
Q.10	(a)	L2	CO 05	PO 01
	(b)	L2	CO 05	PO 02
	(c)	L2	CO 05	PO 02

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6