Module - 1

Introduction

Chapter - 1

* Signals and Systems Definition
* Why to Study Signals, Systems? (with examples.)
* Classification of Signals
  - i) Continuous & discrete time Signals
  - ii) Even and Odd Signals
  - iii) Periodic & Non-periodic Signals
  - iv) Random & Deterministic Signals
  - v) Energy & Power Signals.

* Operation on Signals
  - i) Operation on Dependent Variables → Addition, Multiplication, Differentiation, integration
  - ii) Operation on independent Variables → Time Shift, Time Reversal, Reflection
* Elementary Signals
  - i) Ramp Signals ii) Step function iii) Impulse function
  - iv) Sinusoidal v) Exponential Sinusoidal
* Properties of System
  - i) Stability ii) Linearity iii) Time invariance
  - iv) Memory or Memory less v) Causal or Non Causal
  - vi) Inverse System

Signals :

"A signal is a function of one or more variables that conveys information on the nature of a physical phenomena".

"A signal is defined as any physical quantity that varies with time, space, or any other independent variable/variables.

Examples

* Electric voltage or current such as radio signal, TV signal, Telephone signal Etc...

* By listening to the heart beat of a patient, a doctor is able to diagnose the presence or absence of disease. The heart beat represents signal that convey information to the doctor about the state of health of the patient.

* Pressure signals, sound signals are non electric signals.

→ Signals are represented mathematically as function of one or more independent variable.

Ex: (1) $S(t) = t$

Here 't' is an independent variable & $S(t)$ is dependent variable.

(2) $S(x,y) = x^2 + xy$

When the function depends on single variable the signal is said to be one-dimensional signal.

Ex: A speech signal is one dimensional signal whose amplitude varies with time when The function depends on two or more variables The signal is said to be multidimensional signal.

Ex: An Images The horizontal & vertical co-ordinates of The Image Represents The two dimension.

System: "A system is formally defined as an Entity That manipulates one or More signals to accomplish [carryout] a function, Thereby yielding New signals."

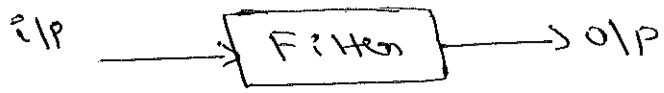
" A system is defined as a Physical device That performs an operation on a signal".

Ex: Communication system, Control system, Remote sensing system Any audio amplifier, TV set, ... Etc.

Block diagram Representation of a System

Examples

1)



A filter is used to reduce a noise & interference Corrupting a desired information varying signals. In This case The filter performs some operations on The signal which has The Effect of reducing The noise & interference from The desired info varying signals.

Ex: MUX, Demux, modulator Etc..

2) Communication System

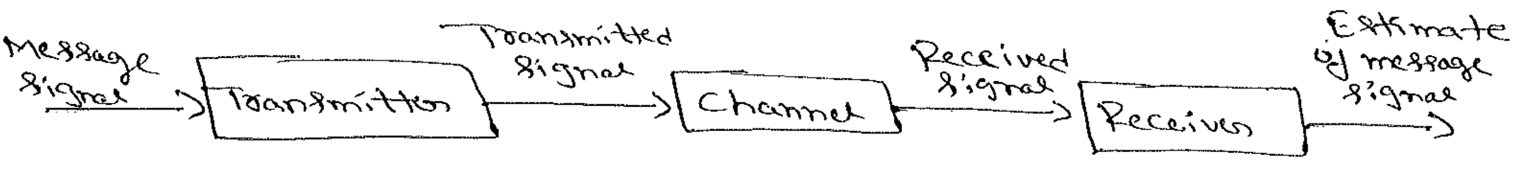


Fig: Elements of Communication System

The Three basic elements of communication systems are

i) Transmitter ii) Channel iii) Receiver

Function of The transmitter is to convert The message signal into a form suitable for transmission over The channel. The message signal could be a speech signal, a TV signal or computer data. The channel may be an optical fiber, a coaxial cable, free space etc...

As The transmitted signals propagates over The channel, it is distorted due to The physical characteristics of The channel such as Noise & interfering signals.

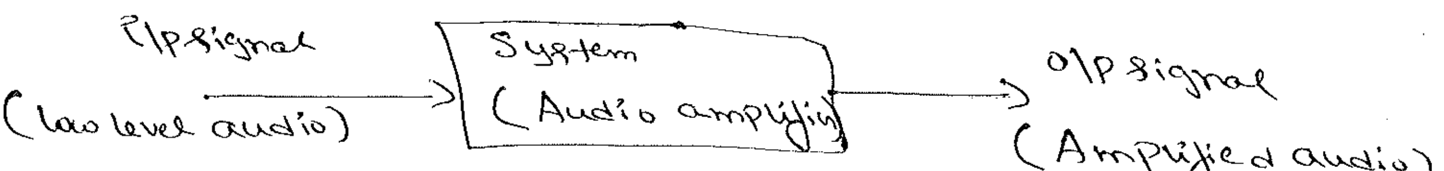
The function of The Receiver is to operates on The receiver signal so as to reconstruct a recognizable form of The original message signals.

The communication systems can be of analog or digital type. The design of an analog commn is easier.

Signals and Systems Relationship

* Every system has one or more inputs its called excitation
* Every system has one or more o/ps it is called Response.
* The i/p & o/ps of The systems are always signals.

Ex:
* The below figure shows an audio amplifier it is a system.



→ i/p to The audio amplifier is low level audio

→ o/p of The audio amplifier is amplified audio

Control Systems

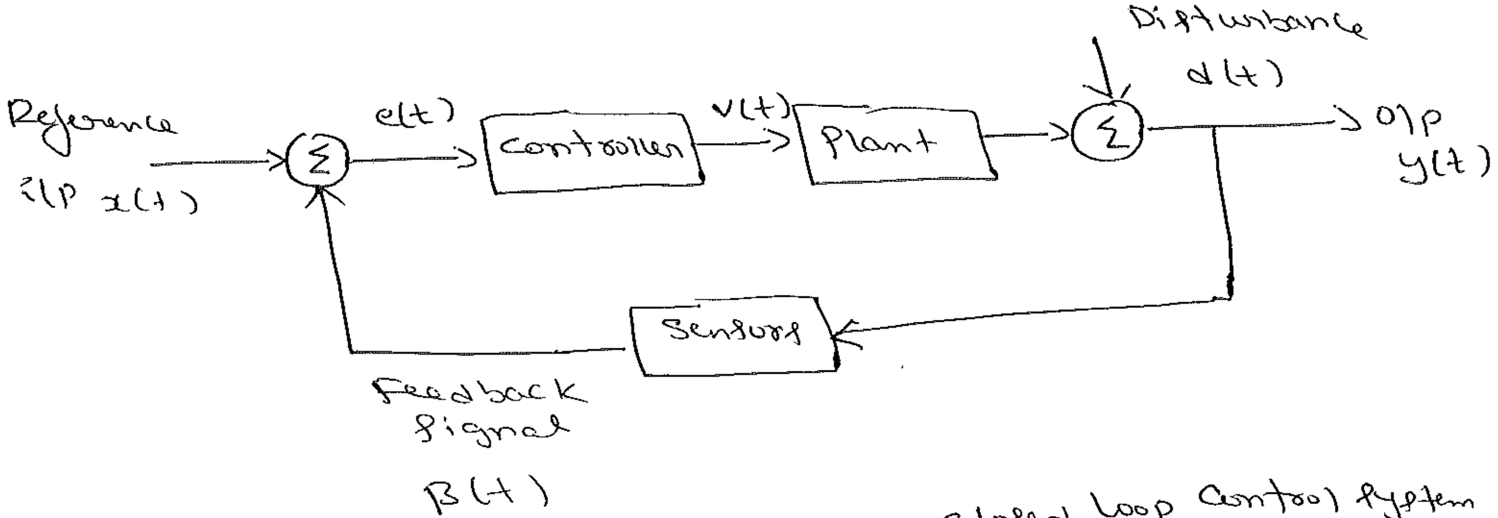


Fig: Block diagram of closed loop control system

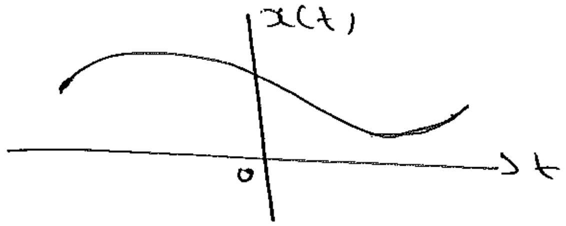
In any control system, The plant is represented by ~~op~~ mathematical operations That generated The o/p y(t) in response to The plant input v(t) & The external disturbance d(t). The sensors existing in The feedback loop measure The plant o/p y(t) and converts it into another form B(t) known as feed back signal. it is compared against The Reference input x(t) to produce an error signal e(t). This error signal is applied to a controller which in turn produces The actuating signal v(t) That performs The controlling action of The plant.

EX: In an ~~air~~ aircraft landing system, The plant is represented by The aircraft body & actuator. The sensors are used by The pilot to determine The lateral position of The aircraft & The controller is a digital computer.

Sampling of analog signals

D.T sequences are usually obtained from a continuous time signal by sampling it at a uniform rate.

$$x(n) = x(t)\big|_{t=nT} = x(nT)$$



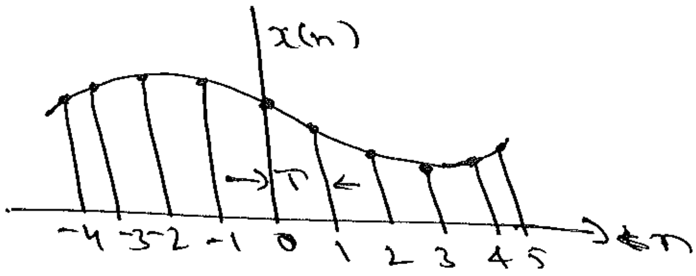


Fig: Discrete-time signal $x(n)$ obtained from a C.T signal $x(t)$ with sampling period T.

Consider T is the sampling period and n denote an integer $(-\infty < n < \infty)$. Sampling a CT signal $x(t)$ at time $t = nT$ gives a sample value $x(nT)$.

$$x(n) = x(nT) \quad ; n = 0, \pm 1, \pm 2 \ldots$$

D.T signal is represented by a sequence of numbers $x(-3), x(-2), x(-1), x(0), x(1) \ldots$

Classification of Signals

* Continuous time (CT) Signals.
* Discrete time (DT) Signals.
* Even and odd Signals
* Periodic and Non periodic Signals
* Energy and Power Signals
* Random and Deterministic Signals.

* Continuous time (CT) & Discrete time Signals (DT)

"A signal $x(t)$ is said to be a continuous-time signal if it has value of Amplitude for all time 't'."

A CT signal is defined continuously with respect to time t.

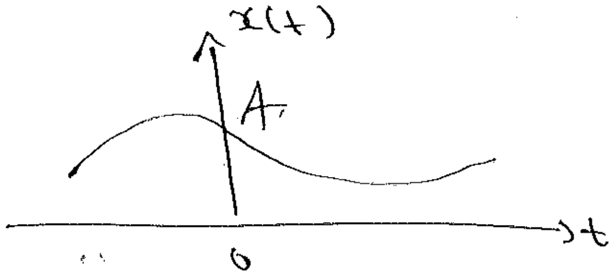


Fig: Continuous time Signal

Continuous time signals arise Naturally when a Physical Phenomenon (Ex: Heart beat) is converted into an Electrical signal using appropriate transducer.

Discrete time Signal

"A discrete time signal is defined only at discrete instants of a time"

"A Discrete time signal is defined only at specific or Regular time instants".

* A Discrete time signals are Represented mathematically as sequence of number.

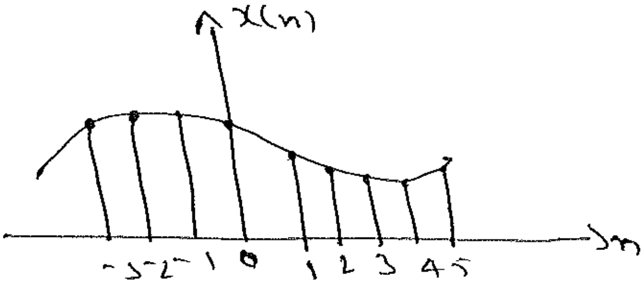
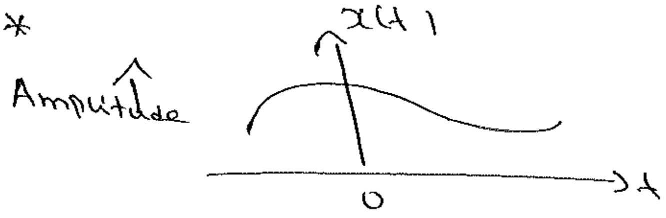
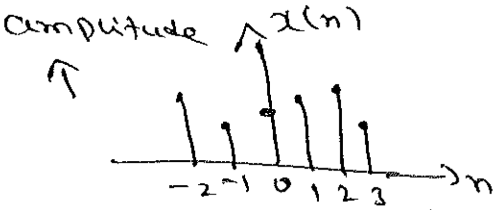


Fig: DT signals

Difference b/w Continuous time & Discrete time signals.

Continuous time signal	Discrete time signals.
* it's Amplitude is continuous with respect to time.	* it is defined only at specific or Regular time intervals.
* There are signals in Analog Circuits	* There are signals in discrete circuits.
* 	

(2) Even Signal & odd Signals

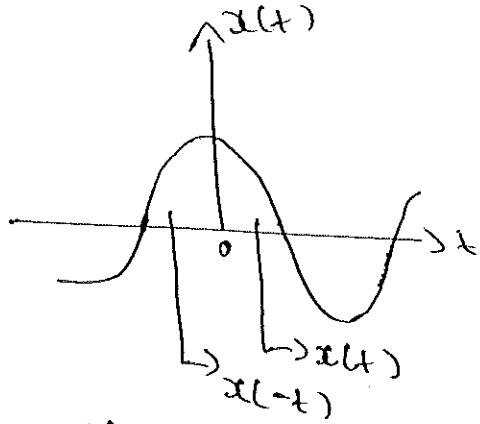
" A signal is said to be even signal if inversion of time axis does not change the amplitude".

" A continuous time signal $x(t)$ is said to an even if $x(t) = x(-t)$ for all t.

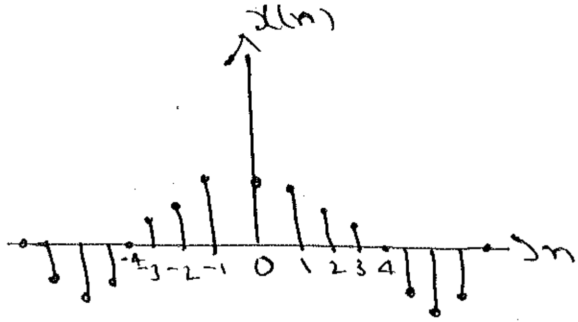
$|||^{ly}$ Discrete time $\Rightarrow x(n) = x(-n)$ for all n.

* Even signals are also called symmetric signals.

Ex: Cos signal



$x(t) = x(-t)$



$x(n) = x(-n)$

Odd signal: "A signal is said to be odd signal if inversion of time axis also inverts amplitude of the signal".

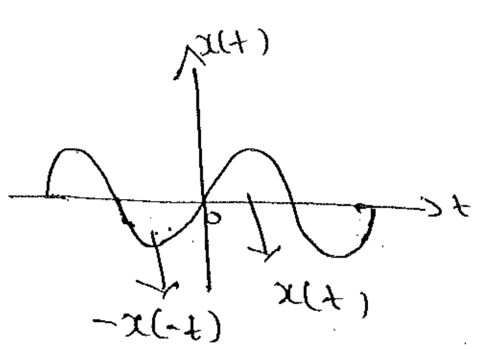
" The signal x(t) is said to an odd signal if

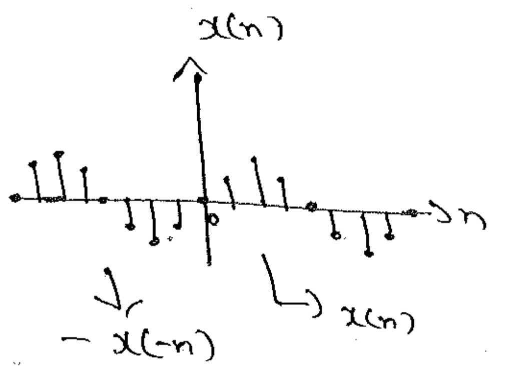
$x(-t) = -x(t)$ for all t. $x(n) = -x(-n)$ for all n,

or $x(t) = -x(-t)$ for all t.

* Odd signals are also called Anti-symmetric signals.

Ex: Sine signal





* Even or odd symmetry property is used in filter design

~~Even si~~

Representation of signals in Even and odd parts

Let The signal be Represented into its Even & odd Parts as,

$$x(t) = x_e(t) + x_o(t) \quad -(1)$$

Here $x_e(t)$ is even Part of $x(t)$

$x_o(t)$ is odd Part of $x(t)$

Substitute $-t$ for t in eqn (1)

$$x(-t) = x_e(-t) + x_o(-t) \quad -(2)$$

Now by definition of even signal

$$x_e(t) = x_e(-t)$$

& odd signal $\quad x_o(t) = -x_o(-t)$ or $-x_o(t) = x_o(-t)$

Add eqn (1) & 2

$$x(t) + x(-t) = x_e(t) + x_o(t) + x_e(-t) + x_o(-t)$$

$$x(t) + x(-t) = x_e(t) + x_o(t) + x_e(t) - x_o(t)$$

$$x_e(t) = \tfrac{1}{2}\left[x(t) + x(-t)\right] \quad \text{even formula}$$

Sub eqn 1 & 2

$$x(t) - x(-t) = x_e(t) + x_o(t) - x_e(-t) - x_o(-t)$$

$$x(t) - x(-t) = x_e(t) + x_o(t) - x_e(t) + x_o(t)$$

$$x(t) - x(-t) = 2x_o(t)$$

$$x_o(t) = \tfrac{1}{2}\left[x(t) - x(-t)\right] \quad \text{odd formla.}$$

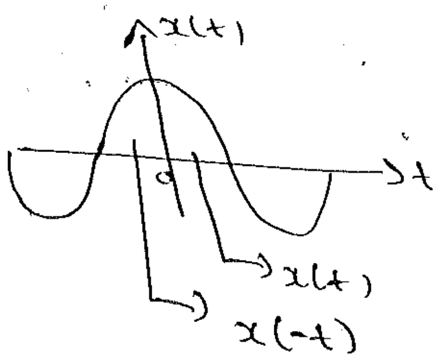
Even part $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

Odd part $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

Difference b/w Even & odd signals

Even signal

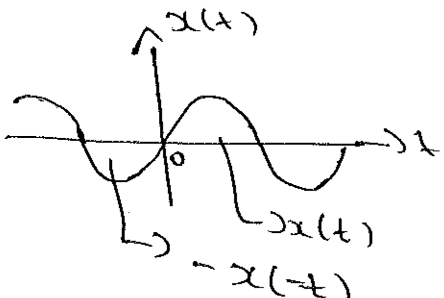
* Inversion of time axis doesnot change the amplitude
* if $x(t) = x(-t)$ for all t is even signal
* it is also called symmetric signal
* Ex: Cos signal



$x(t) = x(-t)$

Odd signal

* Inversion of time axis also inverts the amplitude.
* if $x(t) = -x(-t)$ for all t is odd signal
* it is also called Anti symmetric
* Ex: Sine signal



$x(t) = -x(-t)$

Periodic & non Periodic signal

Periodic signal

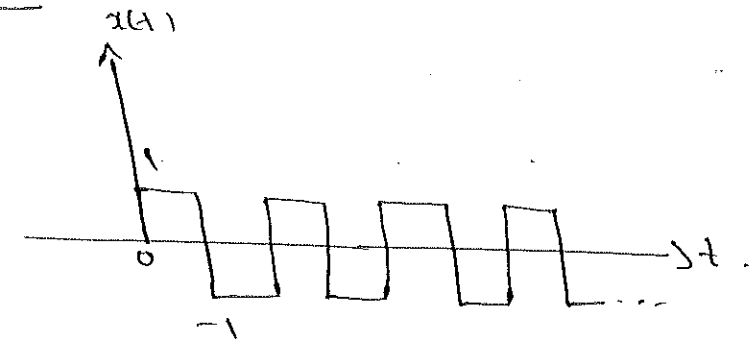
"A signal is said to be periodic if it repeats at regular intervals".

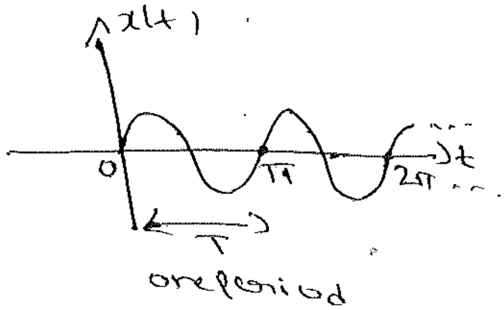
" A periodic signal $x(t)$ is a function of time that satisfies the condition $x(t) = x(t+T)$ for all T. for CT

$x(n) = x(n+N)$ for all N for D.T.

" The Ratio of Rational number is called periodic signal"

Example





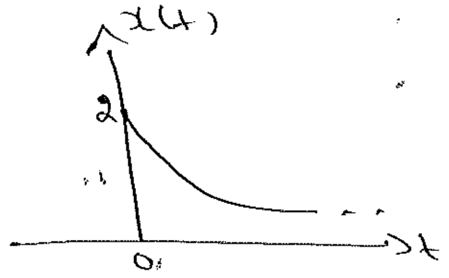
Non periodic (Aperiodic signal)

"Non periodic signal donot repeat at Regular intervals of

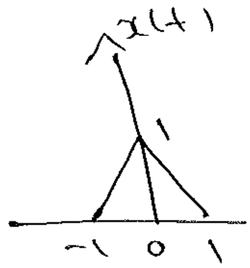
"The Ratio of irrational number is also called nonperiodic"

" it does not Satisfy The condition $x(t) \neq x(t+T)$

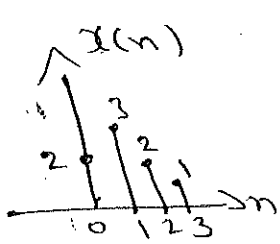
Ex :



(a)



(b)



The Reciprocal of The fundamental period T is called The fundamental frequency of The periodic signal x(t)

$$f = \frac{1}{T} \text{ Hz \underline{or} cycles/second}$$

The angular frequency

$$\omega = 2\pi f \text{ rad/sec}$$

$$\omega = 2\pi \times \frac{1}{T} = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

The fundamental angular frequency of D.T ($x(n)$) is

$$\Omega = \frac{2\pi}{N} \text{ radius} \quad \underline{\text{or}} \quad \Omega = \frac{2\pi}{N} \cdot m$$

m is an integer.

$$\boxed{N = \frac{2\pi}{\Omega} \cdot m}$$

Energy & Power Signal

Energy Signal: "A signal is referred to as an energy signal if and only if the total energy of the signals satisfies the condition $0 < E < \infty$"

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \quad \text{for C.T signal}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \text{for D.T signal.}$$

~~Power Si~~ Power Signal: "A signal is refferred to as a power signal if and only if the average power of the signal satisfies the condition $0 < P < \infty$".

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \quad \text{for C.T signal}$$

$$P = \frac{1}{T} \int_{0}^{T} |x(t)|^2 \, dt \quad \text{if } x(t) \text{ is periodic}$$

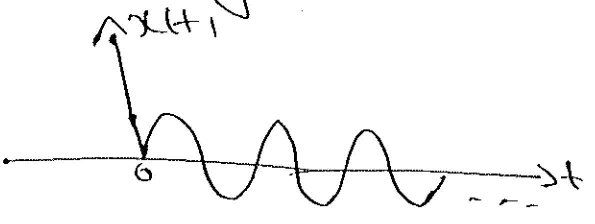
4 $$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \quad \text{for D.T signal}$$

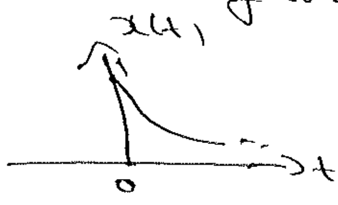
$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 \quad \text{if } x(n) \text{ is periodic}$$

Difference b/w i) Energy & Power signal
ii) Periodic & non periodic signal.

Energy signal	Power signal
* For Energy signal, total Energy is finite & non zero $0 < E < \infty$	* For Power signal, total power is finite & non zero ($0 < P < \infty$)
* Most of non periodic signals are Energy signal	* Most of periodic signals are Power signal
* Power of the Energy signal is zero	* Energy of the power signal is finite
* $E = \int_{-\infty}^{\infty} \lvert x(t) \rvert^2 dt$ for C.T $E = \sum_{n=-\infty}^{\infty} \lvert x(n) \rvert^2$ for D.T	* $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \lvert x(t) \rvert^2 dt$ for C.T $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x(n) \rvert^2$ for D.T

Periodic Signal	Non periodic Signal
* ~~It Repeats~~ A signal is said to be Periodic if it repeats at regular intervals.	* A signal is said to be non Periodic if it does not repeats ~~at~~ at regular intervals.
* $x(t) = x(t+T)$ for all t for C.T $x(n) = x(n+N)$ for all n for D.T	* $x(n) \neq x(n+N)$ $x(t) \neq x(t+T)$
* Most of the Power signals are Periodic	* Most of Energy signals are non periodic.
* The Ratio of rational number	* The ratios of irrational number.





Deterministic and Random Signal

" Signal where Amplitude can be predicted before its actual occurance it is called Deterministic signal"

Ex: ECG signal from good heart

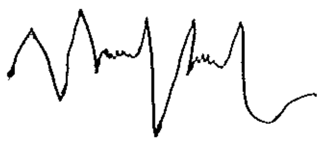
" Signal can be completely Represented by mathematical equation at any time Ex: $x(t) = \cos \omega t$

$x(n) = 2\pi f n$

Random Signal

Signal where Amplitude cannot predicted before its actual occurance is called Random signal.

Ex: Failure ECG



* A signal can't be represented by any mathematical equation is called Random signal.

Basic operations on signals

① Dependent Variable ② Independent Variable

Dependent Variables corresponding to the amplitude or value of the signal.

Independent Variable corresponding to the time term for CT & DT Respectively.

Operation Performed on dependent Variables

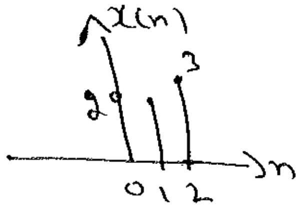
(a) **Amplitude Scaling** : let $x(t)$ denote a Continuous time Signal. Then The Signal $y(t)$ resulting from amplitude scaling applied to $x(t)$ is defined by

$$y(t) = C\,x(t)$$

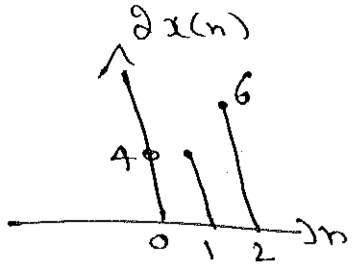
where 'C' is Scaling factor

IIIly for D.T $\quad y(n) = C\,x(n)$

Ex:



find $2x(n)$



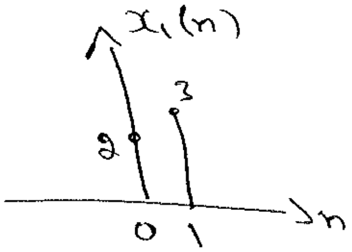
(b) **Addition** :

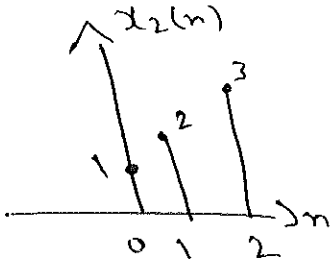
$$y(t) = x_1(t) + x_2(t) \quad \text{for C.T}$$

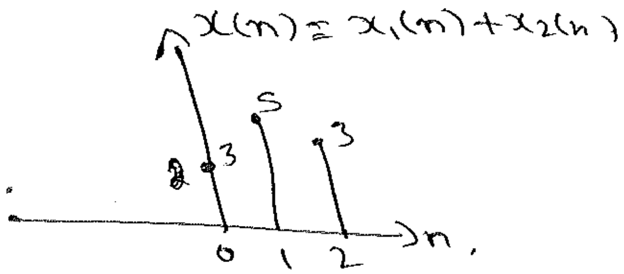
$$y(n) = x_1(n) + x_2(n) \quad \text{for D.T}$$

let $x_1(t)$ & $x_2(t)$ denote a pair of C.T signals. The Signal $y(t)$ obtained by The addition of $x_1(t) + x_2(t)$

Ex:







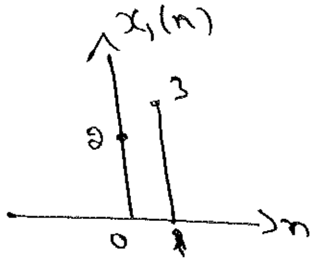
Multiplication :

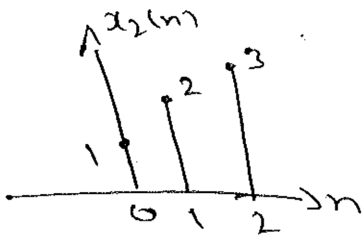
$$y(t) = x_1(t) \cdot x_2(t) \quad \text{for C.T}$$

$$y(n) = x_1(n) \cdot x_2(n) \quad \text{for D.T}$$

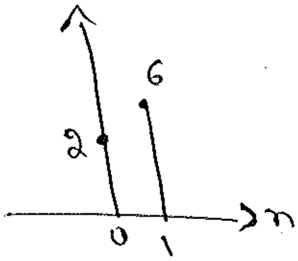
Let $x_1(t)$ & $x_2(t)$ denote a pair of C.T signal. Then the signal $y(t)$ resulting from the multiplication of $x_1(t)$ & $x_2(t)$

EX :





$x_1(n) \cdot x_2(n) = y(n)$

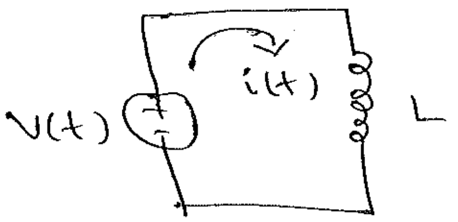


Differentiation :

$$y(t) = \frac{d}{dt} x(t)$$

Let $x(t)$ denote a continuous time signal then the derivative of $x(t)$ with respect to time is $y(t) = \frac{d}{dt} x(t)$

Ex :



Voltage ~~across~~ across the L is

$$V(t) = L \frac{d}{dt} i(t)$$

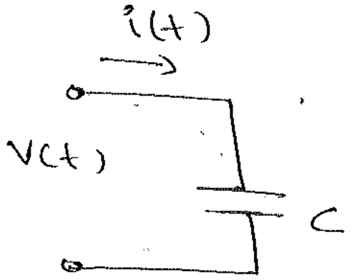
An inductor performs differentiation. Let $i(t)$ denote the current flowing through an inductor of inductance L. is in above figure

Integration

Let $x(t)$ denote a C.T signal Then The integral of $x(t)$ with respect to time 't' is defined by

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$ τ is The integration variable.

Ex:



The voltage across The capacitor is

$$V(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau)\, d\tau$$

Operation performed on The independent variable

ⓐ Time Scaling: $y(t) = x(at)$

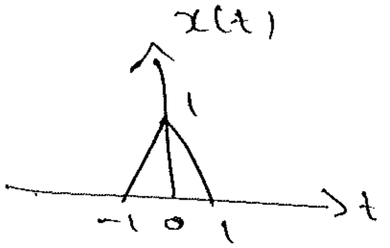
Let $x(t)$ denote a CT signal Then The signal $y(t)$ obtained by scaling The independent variable, time t by a factor a.

Conditions:

1) if $a > 1$ The signal $y(t)$ is compressed version of $x(t)$

② if $a < 1$ The signal $y(t)$ is expanded version of $x(t)$

Ex:



ⓐ find $y(t) = x(2t)$

ⓑ $y(t) = x(\frac{1}{2}t)$.

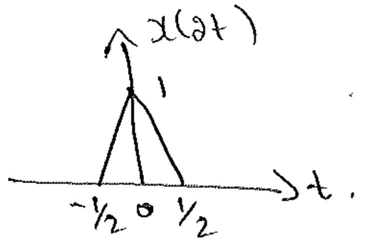
Soln

$y(t) = x(2t)$

$-1 < t < 1$

$-1 < 2t < 1$

$-\frac{1}{2} < t < \frac{1}{2}$

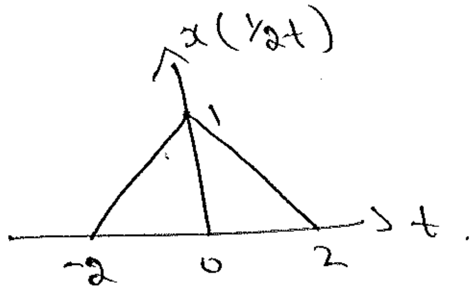


(b) $y(t) = x(\frac{1}{2}t)$

$-1 < t < 1$

$-1 < \frac{1}{2}t < 1$

$-2 < t < 2$

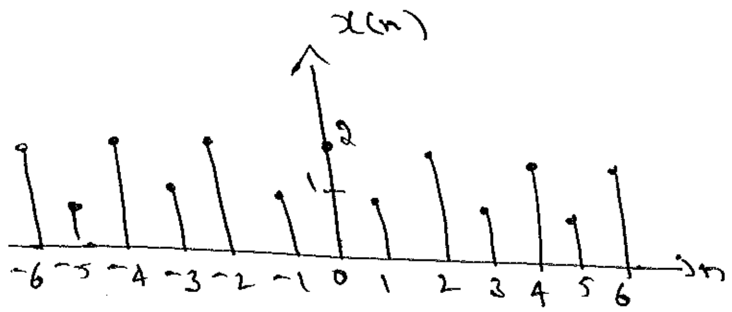


In discrete time

$y(n) = x(kn) \qquad k > 0$

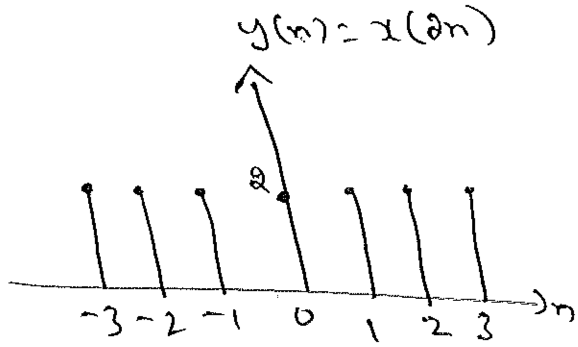
if $k > 1$, Then some values of The discrete time signal $y(n)$ are lost.

Ex:



find $y(n) = x(2n)$

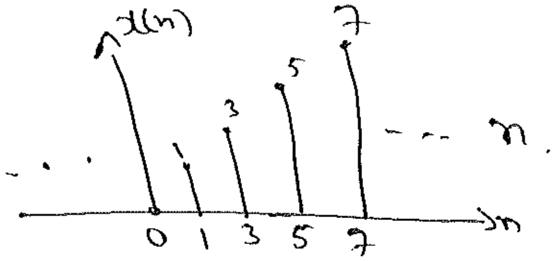
$k = 2 \qquad k > 1$



(2) let $x(n) = \begin{cases} n & \text{for } n = \text{odd} \\ 0 & \text{otherwise} \end{cases}$

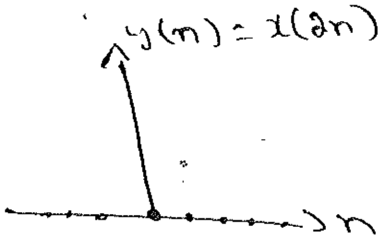
Determine $y(n) = x(2n)$

Soln



$y(n) = x(2n)$

$K = 2.$



$y(n) = 0$ for all n values

(2) Reflection: Let $x(t)$ denote a continuous time signal. The Reflected version of $x(t)$ is obtained by Replacing t with $-t$.

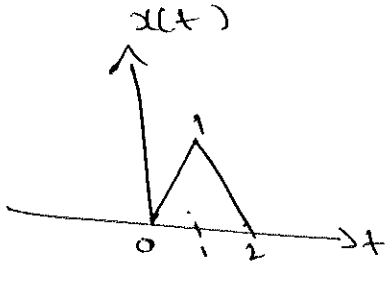
$$y(t) = x(-t)$$

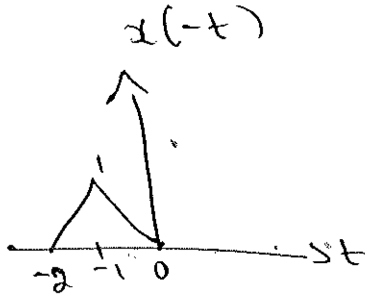
The signal $y(t)$ Represents a Reflected version of $x(t)$ about the vertical axis.

|||ly for D.T

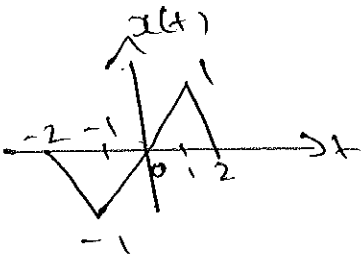
$$y(n) = x(-n).$$

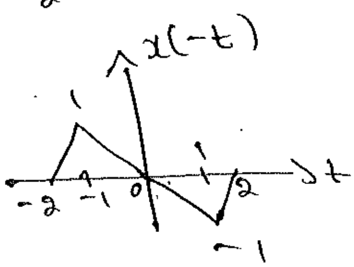
Ex: (1)





(2)

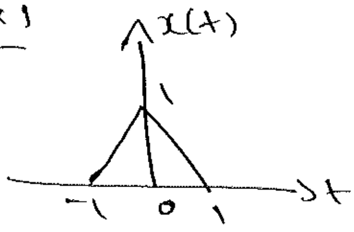


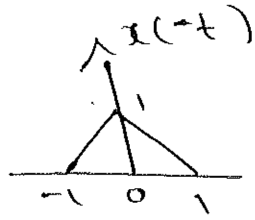


Note:

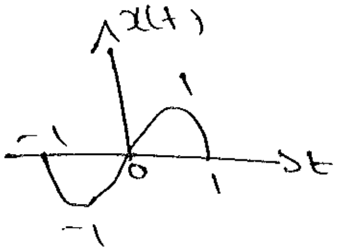
* For an Even signal $x(t)$, The signal & its reflected version $x(-t)$ are identical.
* For odd signal $x(t)$, The signal & its Reflected version $x(-t)$ are negative $[x(-t) = -x(t)]$ of each other.

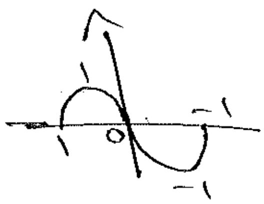
Ex 1





Ex 2





③ **Time Shifting**

Let $x(t)$ denotes a Continuous time signal Then The time shifted version of $x(t)$ is defined by

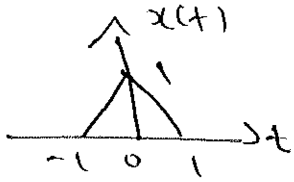
$$y(t) = x(t - t_0) \quad t_0 = \text{time shift}.$$

* if $t_0 > 0$ The waveform of $y(t)$ is obtained by shifting $x(t)$ towards right relative to the time axis
* if $t_0 < 0$ $x(t)$ is shifted to The left.

|||^{ly} for D.T signal

$$y(n) = x(n - n_0)$$

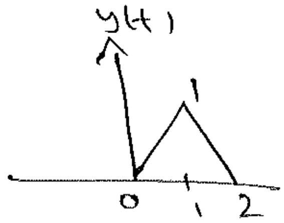
Ex: if $x(t)$ is



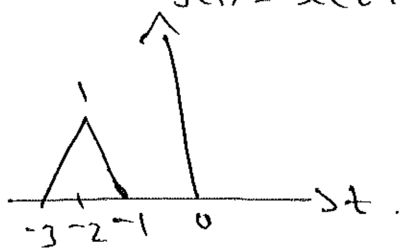
find i) $y(t) = x(t-1)$

ii) $y(t) = x(t+2)$

(i) $y(t) = x(t-1)$



ii) $y(t) = x(t+2)$



Problems

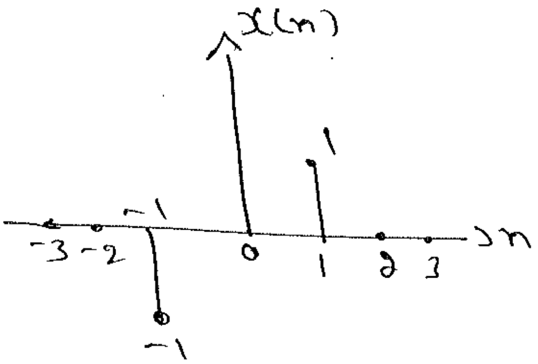
1) The discrete time signal

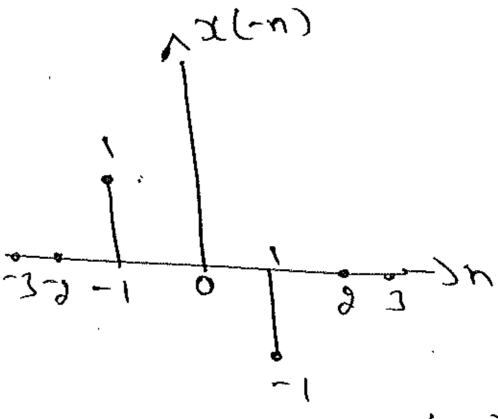
$$x(n) = \begin{cases} 1, & n=1 \\ -1, & n=-1 \\ 0, & n=0 \ \& \ |n|>1 \end{cases}$$

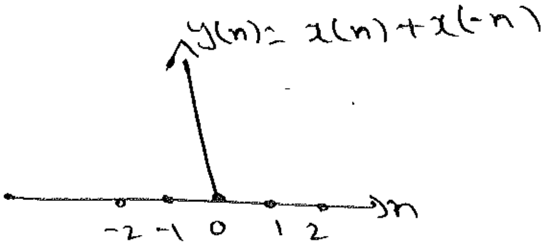
Find the composite signal

$$y(n) = x(n) + x(-n)$$

Soln







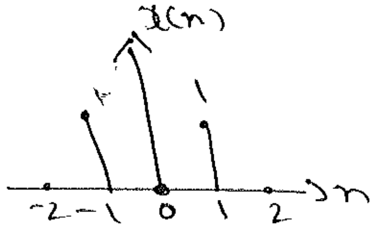
$y(n) = 0$ for all integer values of n

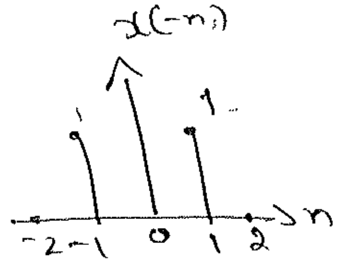
(2) The discrete time signal

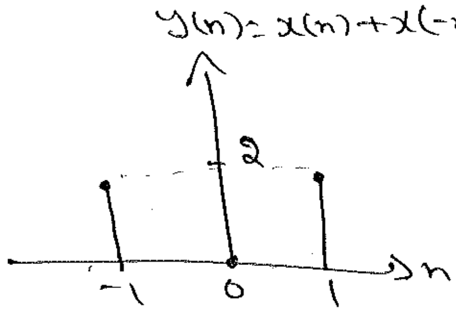
$$x(n) = \begin{cases} 1 & , n=-1 \ \& \ n=1 \\ 0 & , n=0 \ \& \ |n|>1 \end{cases}$$

find $y(n) = x(n) + x(-n)$

Soln







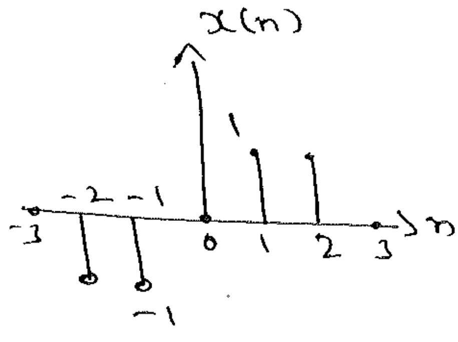
$$y(n) = \begin{matrix} 2 & , n=1 \ \& \ n=-1 \\ 0 & n=0 \ \& \ |n|>1 \end{matrix}$$

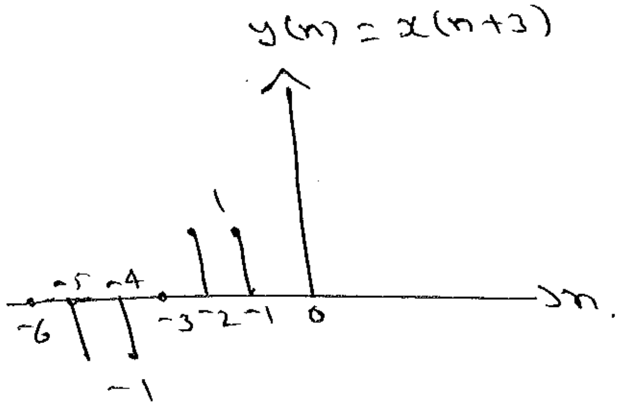
(3) The discrete time signal

$$x(n) = \begin{cases} 1 & , n=1,2 \\ -1 & , n=-1,-2 \\ 0 & , n=0 \ \& \ |n|>2 \end{cases}$$

Find The time-shifted signal $y(n) = x(n+3)$

Soln

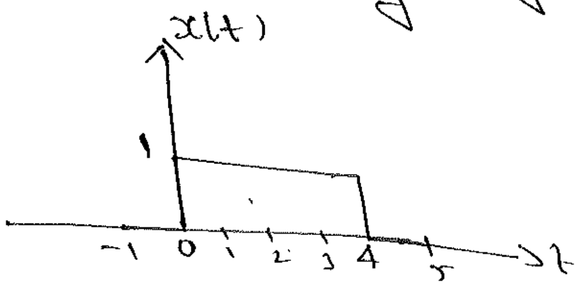




$$y(n) = \begin{cases} 1 & , n=-1,-2 \\ -1 & , n=-4,-5 \\ 0 & , n=-3 \ \& \ n<-5 \ \& \ n>-1 \end{cases}$$

//

④ A continuous time signal $x(t)$ is shown in below figure. Sketch & label each of the following signal.



ⓐ $x(t-2)$

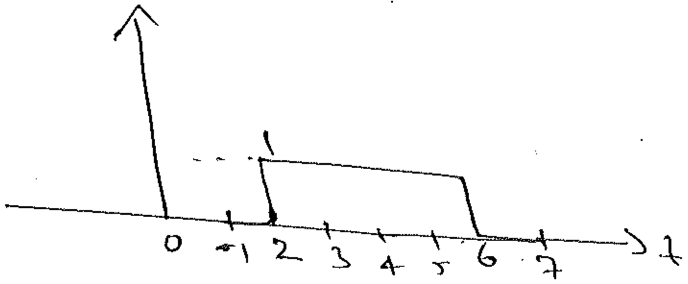
ⓑ $x(2t)$

ⓒ $x(t/2)$

ⓓ $x(-t)$

Soln

ⓐ $x(t-2)$

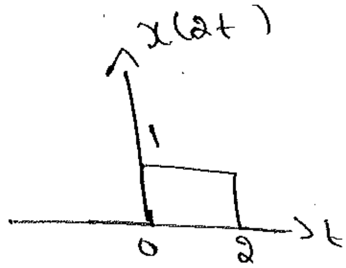


ⓑ $x(2t)$

$0 < t < 4$

$0 < 2t < 4$

$0 < t < 2$

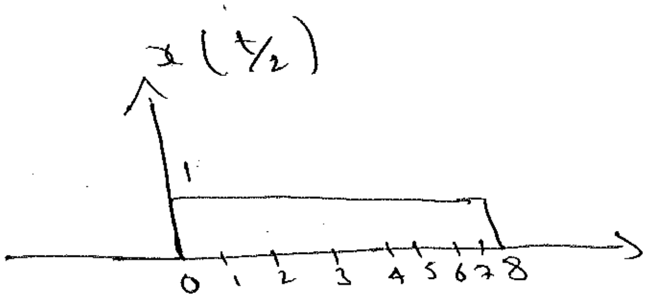


ⓒ $x(t/2)$

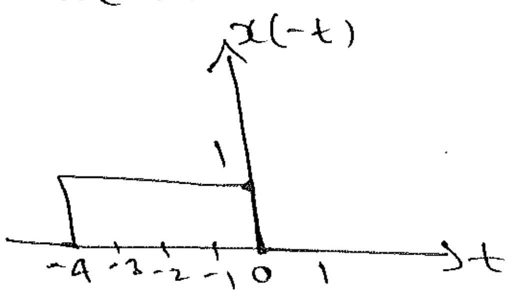
$0 < t < 4$

$0 < \frac{1}{2}t < 4$

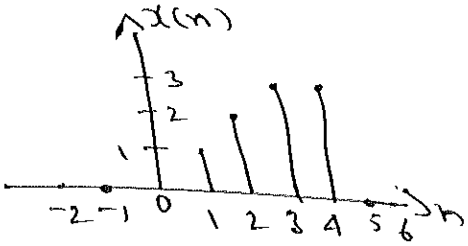
$0 < t < 8$



ⓓ $x(-t)$



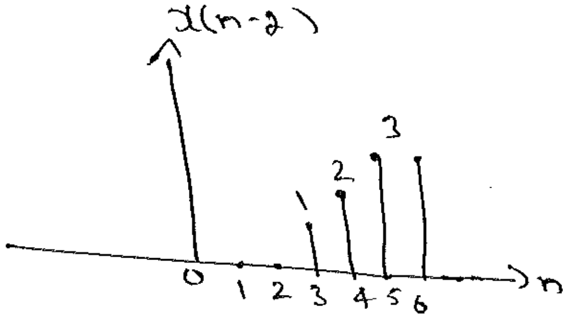
(5) A Discrete time signal $x(n)$ is shown in below figure Sketch & label each of The following signals.



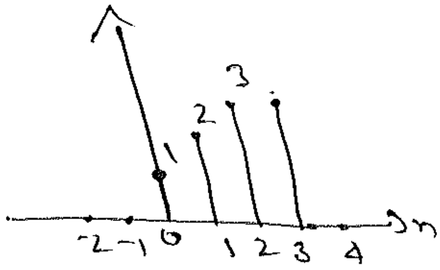
(a) $x(n-2)$
(b) $x(2n)$
(c) $x(-n)$
(d) $x(n+1)$

Soln

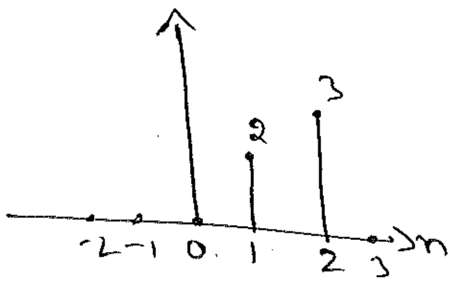
(a) $x(n-2)$



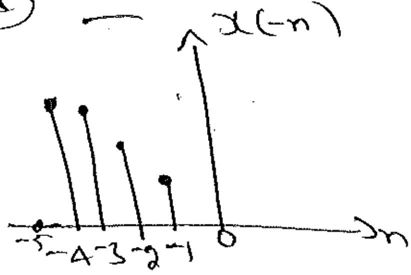
(d) $x(n+1)$



(b) $x(2n)$



(d) $x(-n)$



Precedence Rule for time shifting and time scaling

Let $y(t)$ denote a continuous-time signal That is derived from another continuous-time signal $x(t)$ Through a Combination of time shifting & time scaling

i.e $\quad y(t) = x(at - b)$

This Reation b/w $y(t)$ & $x(t)$ satisfies The conditions

$$y(0) = x(-b)$$

& $\quad y(b/a) = x(0)$

which Provide useful checks on $y(t)$ interms of Corresponding Value of $x(t)$.

To obtain $y(t)$ from $x(t)$, The time shifting & time scaling operations must be formed in the correct order.

Step1 : The time shifting operation is performed first on $x(t)$.

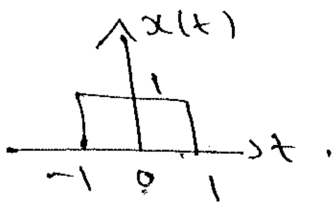
$$v(t) = x(t-b)$$

Step2 : time scaling operation is performed on $v(t)$

$$y(t) = v(at)$$
$$= x(at-b)$$

Problems

1) Consider The Reactangular pulse $x(t)$ of unit amplitude & a duration of 2 time units shown in below figure find $y(t) = x(2t+3)$

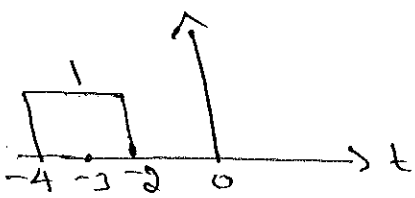


Soln

$$y(t) = x(2t+3)$$

Step1:

$$v(t) = x(t+3)$$



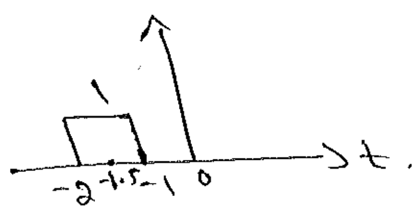
Step2

$$y(t) = v(2t)$$

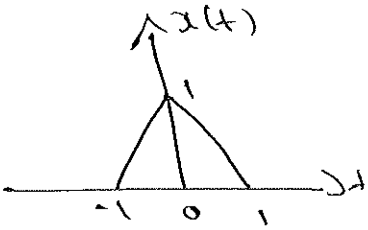
$-4 < t < -2$

$-4 < 2t < -2$

$-2 < t < -1$



(2) A triangular pulse signal $x(t)$ is depicted in below figure. Sketch each of the following signals derived from $x(t)$

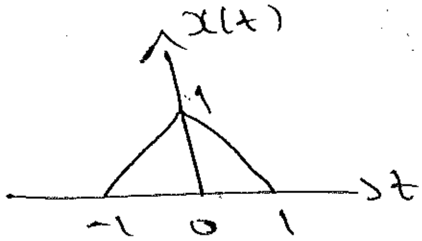


(a) $x(3t)$ (d) $x(2(t+2))$

(b) $x(3t+2)$ (e) $x(2(t-2))$

(c) $x(-2t-1)$ (f) $x(3t)+x(3t+2)$

Soln:

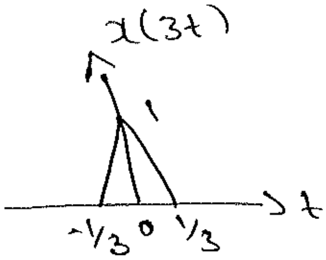


(a) $x(3t)$

$-1 < t < 1$

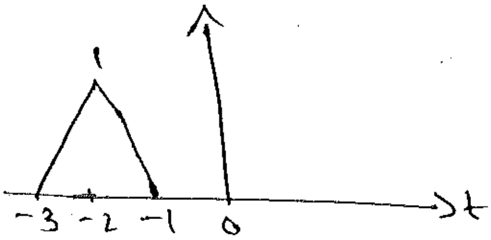
$-1 < 3t < 1$

$-1/3 < t < 1/3$



(b) $x(3t+2)$

Step 1: $v(t) = x(t+2)$

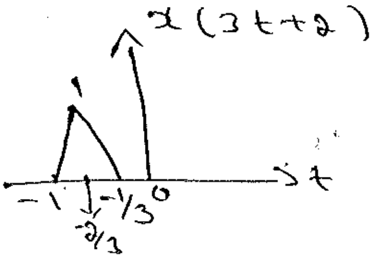


Step 2: $y(t) = v(3t)$

$-3 < t < -1$

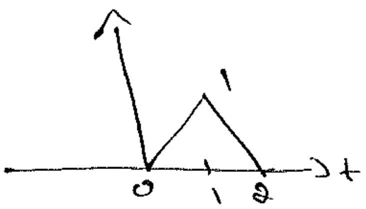
$-3 < 3t < -1$

$-1 < t < -1/3$



(c) $x(-2t-1)$

$v(t) = x(t-1)$

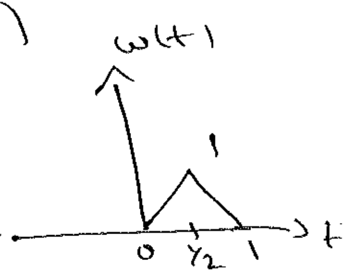


$w(t) = v(2t)$

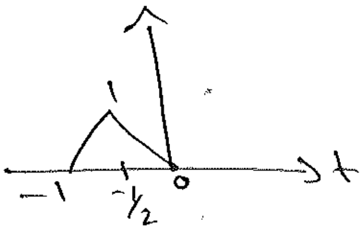
$0 < t < 2$

$0 < 2t < 2$

$0 < t < 1$



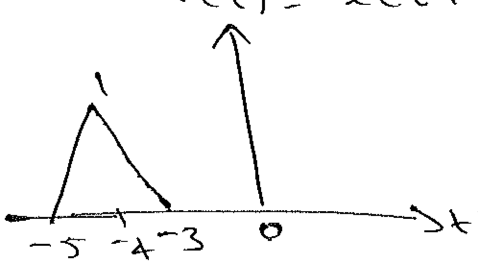
$y(t) = w(-t)$



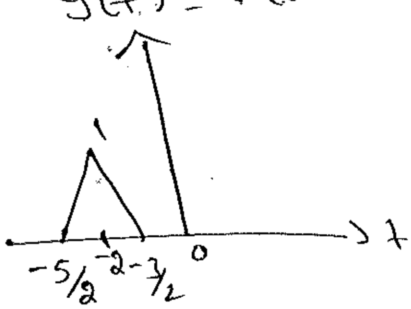
(b) $x(2(t+2))$

$x(2t+4)$

$v(t) = x(t+4)$



$y(t) = v(2t)$



$-5 < t < -3$

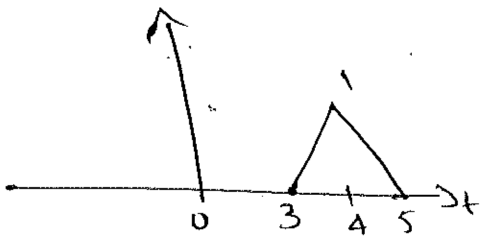
$-5 < 2t < -3$

$-5/2 < t < -3/2$

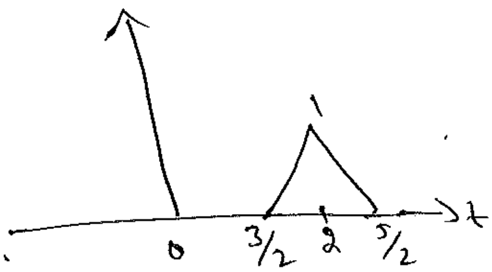
(c) $x(2(t-2))$

$x(2t-4)$

$v(t) = x(t-4)$



$y(t) = v(2t)$



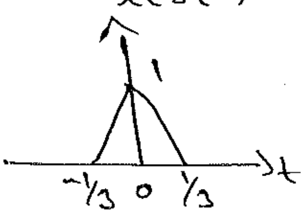
$3 < t < 5$

$3 < 2t < 5$

$3/2 < t < 5/2$

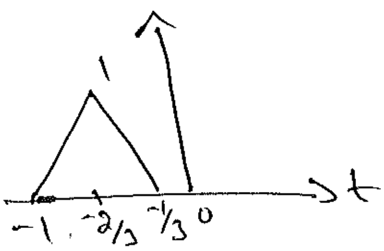
(d) $x(3t) + x(3t+2)$

$x(3t)$

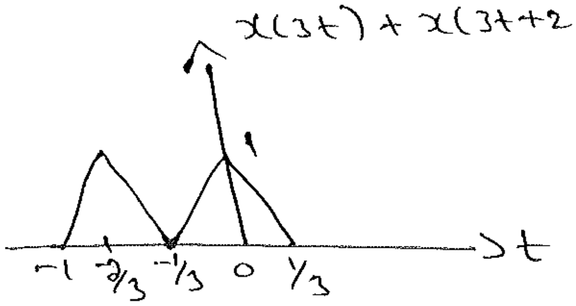


$+$

$x(3t+2)$



$x(3t) + x(3t+2)$

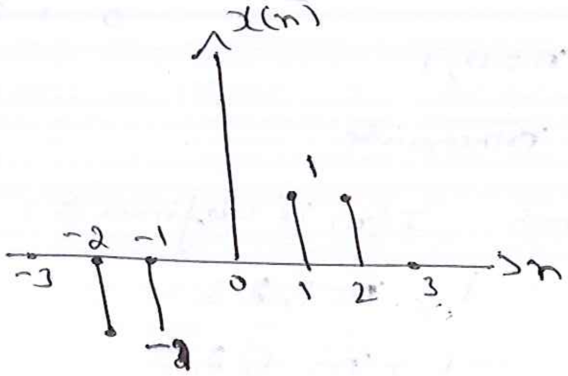


(3) A discrete time signal is defined by

$$x(n) = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = 0, \text{ and } |n| > 2 \end{cases}$$

Find $y(n) = x(2n+3)$

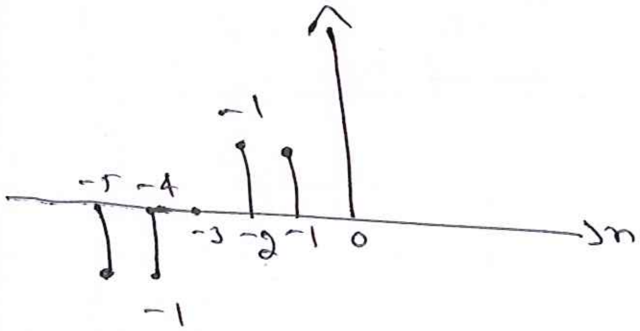
<u>Soln</u>



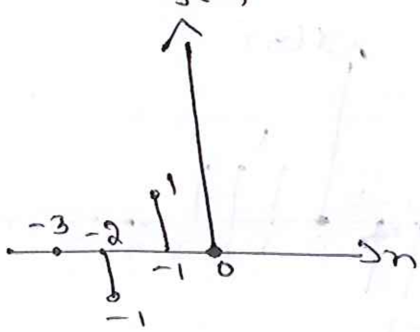
K > 1 signal loss

$y(n) = x(2n+3)$

$v(n) = x(n+3)$



$y(n) = v(2n)$



$y(0) = v(0)$

$y(-1) = v(-2)$

$y(-2) = v(-4)$

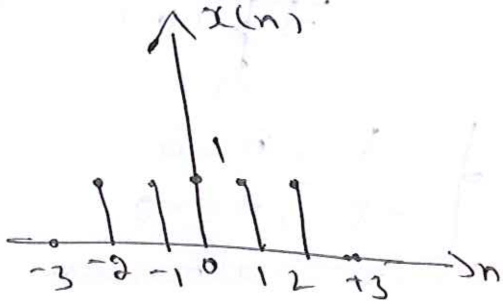
$y(-3) = v(-6)$

(4) Consider a discrete-time signal

$$x(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & |n| > 2 \end{cases}$$

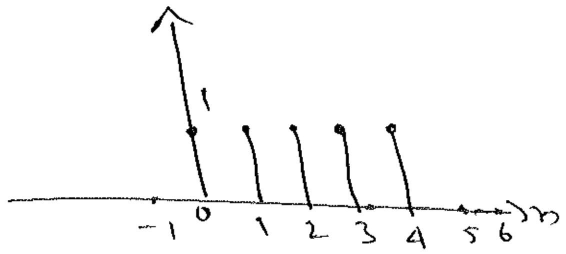
Find $y(n) = x(3n-2)$

<u>Soln</u>



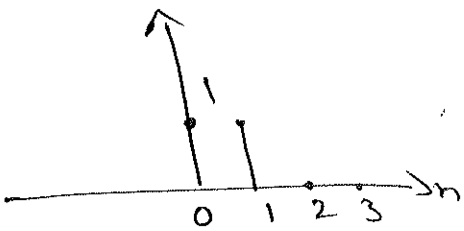
$y(n) = x(3n-2)$

$v(n) = x(n-2)$ 　　　　 $y(n) = v(3n)$



- $k > 1$ & $v(n)$ signal will be loss

$y(n)$



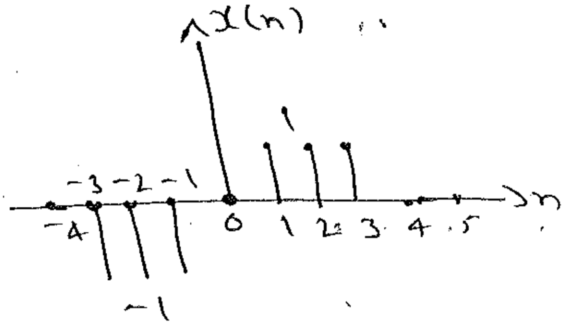
$$y(n) = \begin{cases} 1 & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

(5) A discrete time signal $x(n)$ is defined by

$$x(n) = \begin{cases} 1, & n = 1, 2, 3 \\ -1, & n = -1, -2, -3 \\ 0, & n = 0 \text{ \& } |n| > 3 \end{cases}$$

Find $y(n) = x(2n+2)$

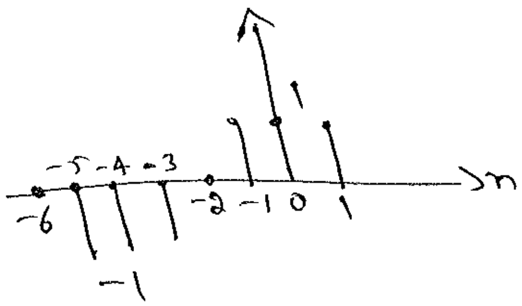
<u>Soln</u>

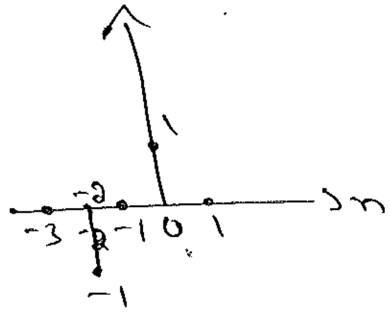


$y(n) = x(2n+2)$

$v(n) = x(n+2)$ 　　　　 $y(n) = v(2n)$ 　 $k > 1$

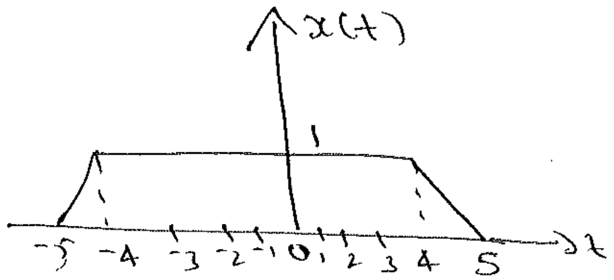
$y(n) = x(2n+2)$





$$y(n) = \begin{cases} 1, & n = 0 \\ -1, & n = -2 \\ 0 & \text{otherwise} \end{cases}$$

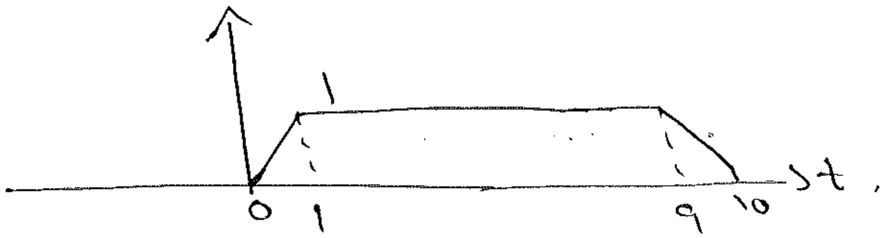
6 For The Signal $x(t)$ shown in below figure sketch
$y_1(t) = x(10t-5)$ & $y_2(t) = x(2t)$



Soln:

(a) $y_1(t) = x(10t-5)$

$V(t) = x(t-5)$ $\quad t_0 = 5$



$y_1(t) = V(10t)$

$0 < t < 10$

$0 < 10t < 10$

$0 < t < 1$

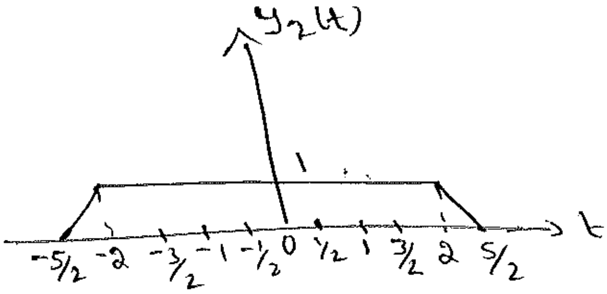
(b) $y_2(t) = x(2t)$

$x(t)$ time period is

$-5 < t < 5$

$-5 < 2t < 5$

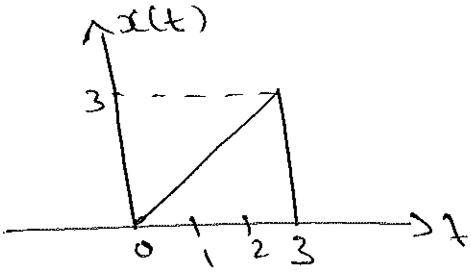
$-5/2 < t < 5/2$



(7) A signal $x(t)$ is shown in below figure. Sketch & label each of the following signals.

i) $x(-t-1)$ ii) $x(2(t+1))$

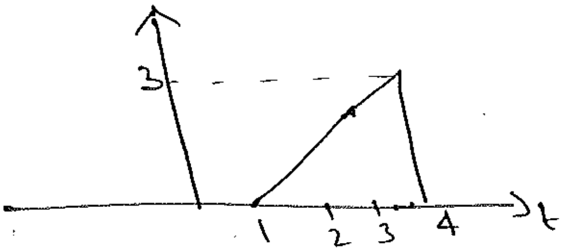
iii) $x(t/2-2)$



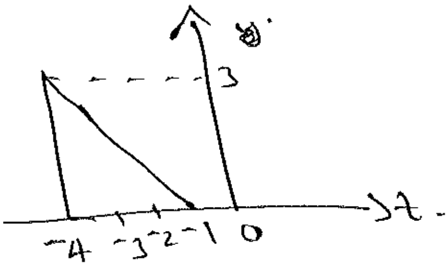
Soln

(i) $x(-t-1)$.

$v(t) = x(t-1)$



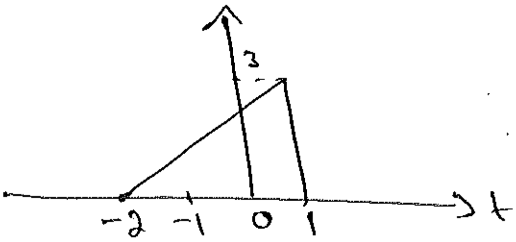
$y(t) = v(-t)$



ii) $x(2(t+1))$

$x(2t+2)$

$v(t) = x(t+2)$



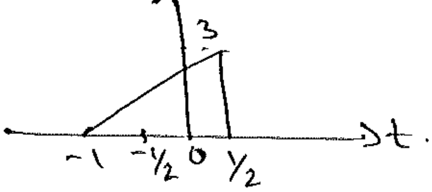
$y(t) = v(2t)$

$-2 < t < 1$

$-2 < 2t < 1$

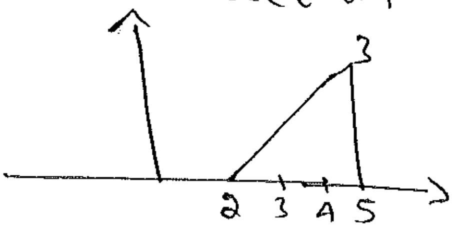
$-1 < t < 1/2$

$y(t) = x(2t+2)$



iii) $x(t/2-2)$

$v(t) = x(t-2)$

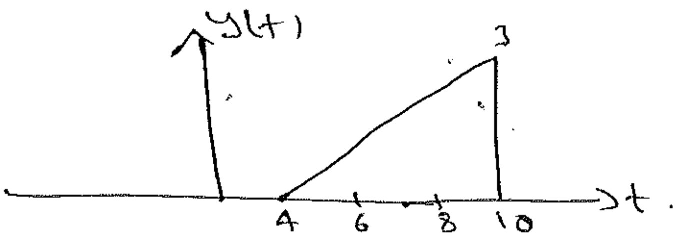


$y(t) = v(1/2 t)$

$2 < t < 5$

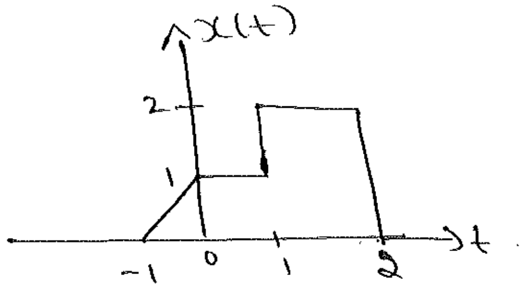
$2 < 1/2 t < 5$

$4 < t < 10$



7. For given $x(t)$ in below figure. Sketch the following signals

(i) $x(4-t)$ (ii) $x(\frac{2}{3}t-1)$ (iii) $x(-t+1)$

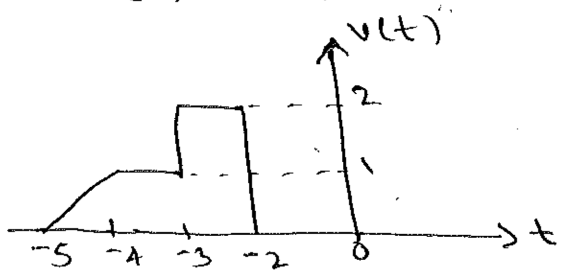


Soln

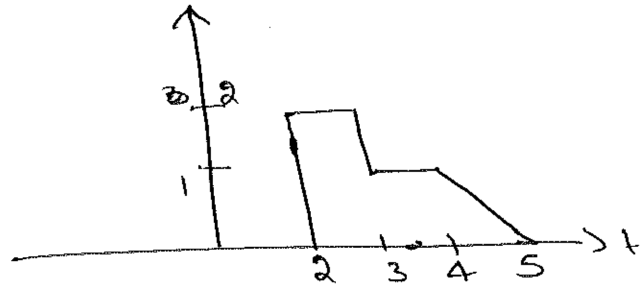
(a) $x(4-t)$

$x(-t+4)$

$v(t) = x(t+4)$

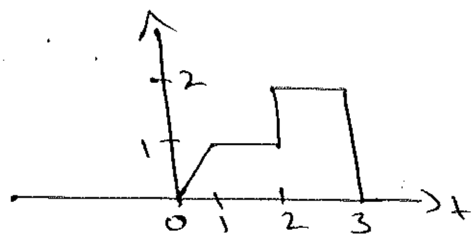


$y(t) = v(-t)$



(b) $x(\frac{2}{3}t-1)$

$v(t) = x(t-1)$

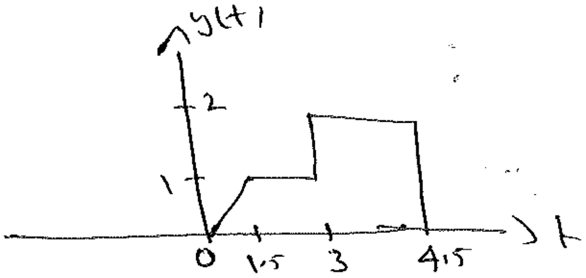


$y(t) = v(\frac{2}{3}t)$

$0 < t < 3$

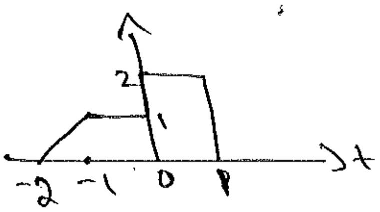
$0 < \frac{2}{3}t < 3$

$0 < t < 4.5$

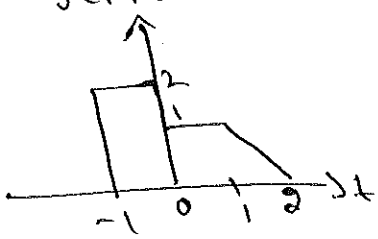


(c) $x(-t+1)$

$v(t) = x(t+1)$

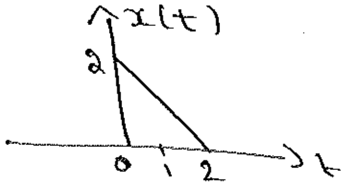


$y(t) = v(-t)$



(8) For The signal $x(t)$ shown in below figure sketch

(i) $x(2(t-2))$ ii) $x(-2t-1)$ iii) $x(t/2+2)$ iv) $x(-t)$

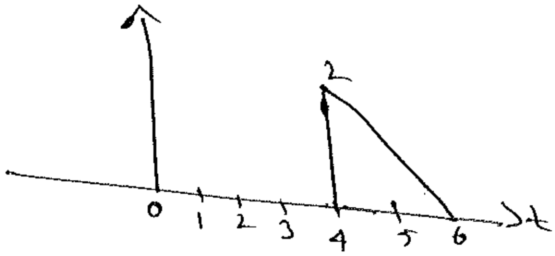


Soln

$x(2(t-2))$

$x(2t-4)$

$v(t) = x(t-4)$

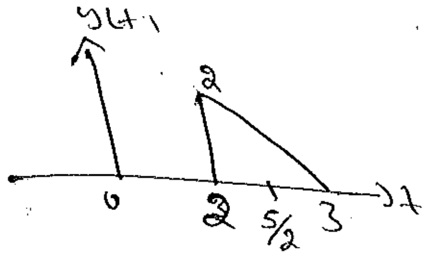


$y(t) = v(2t)$

$4 < t < 6$

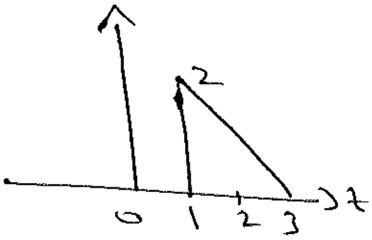
$4 < 2t < 6$

$2 < t < 3$



ii) $x(-2t-1)$

$v(t) = x(t-1)$

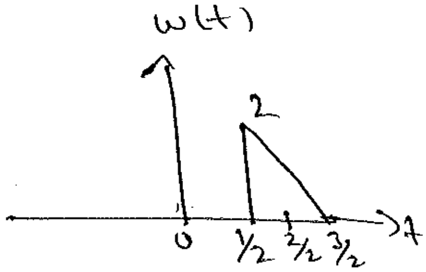


$w(t) = v(2t)$

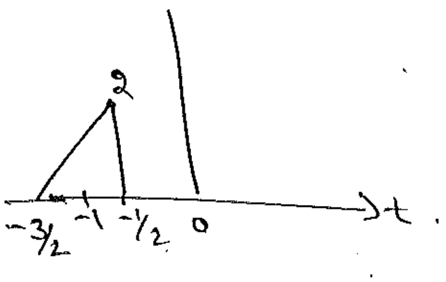
$1 < t < 3$

$1 < 2t < 3$

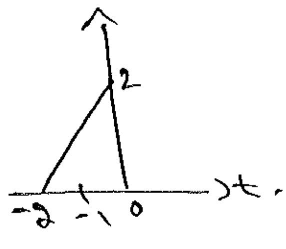
$1/2 < t < 3/2$



$y(t) = w(-t)$

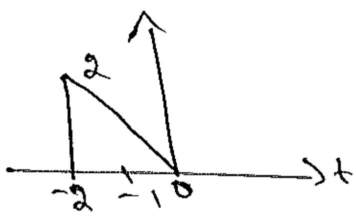


iv) $x(-t)$



iii) $x(t/2+2)$

$v(t)\ x(t+2)$

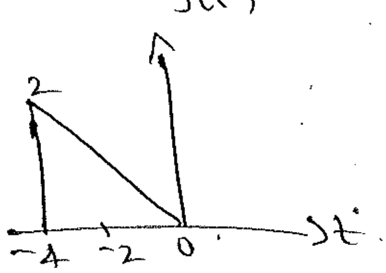


$y(t) = v(1/2 t)$

$-2 < t < 0$

$-2 < 1/2 t < 0$

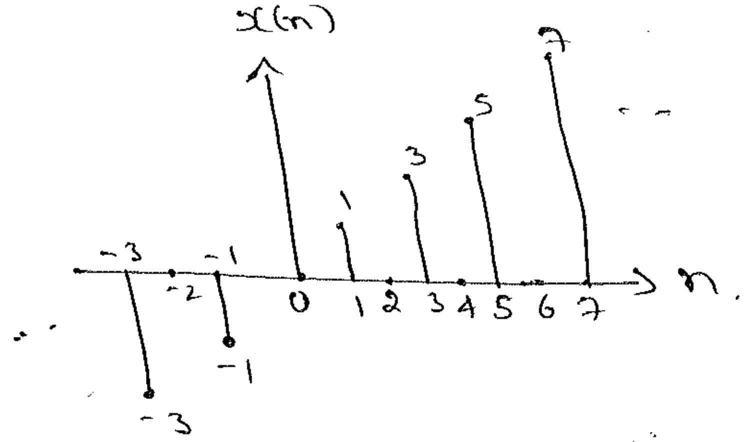
$-4 < t < 0$



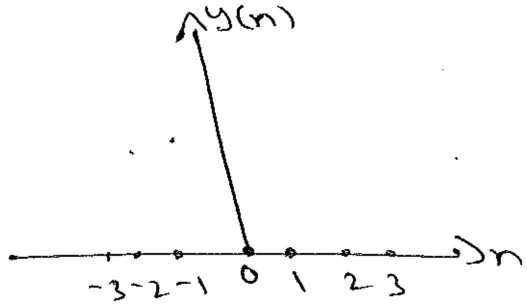
(9) $x(n) = \begin{cases} n & \text{for } n \text{ odd} \\ 0 & \text{otherwise} \end{cases}$

find $y(n) = x(2n)$

Soln



$y(n) = x(2n)$. $K > 1$ Signal will be loss



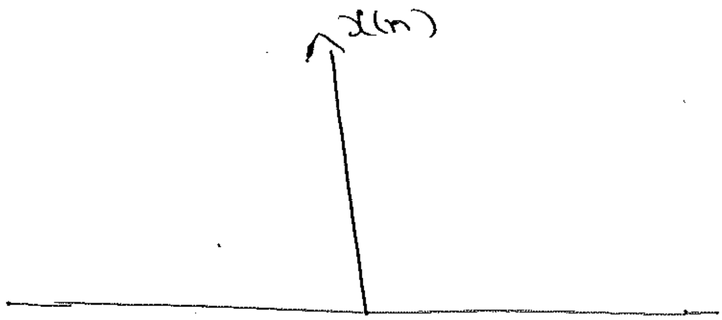
$y(n) = 0$ for all values of n

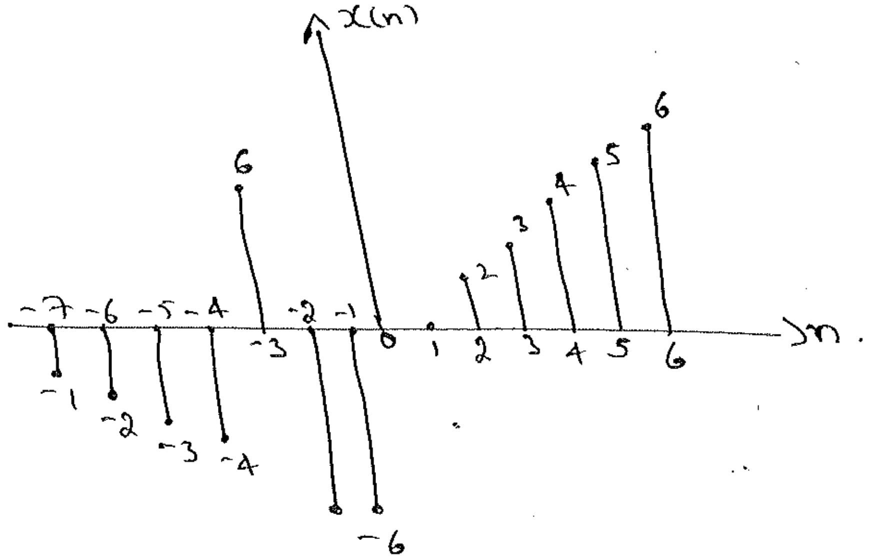
(10) A function $x(n)$ is defined by

$$x(n) = \begin{cases} -(n+8) & \text{for } -8 < n < -3 \\ 6 & \text{for } n = -3 \\ -6 & \text{for } -3 < n < 0 \\ n & \text{for } 1 < n < 7 \\ 0 & \text{otherwise} \end{cases}$$

Sketch $y(n) = 3 \cdot x\left(\frac{n}{2}+1\right)$

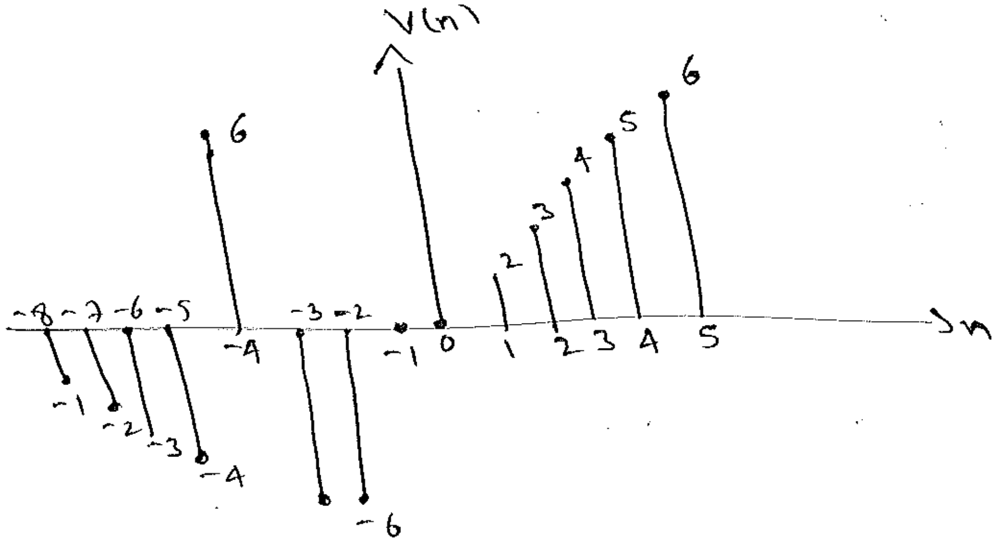
Soln



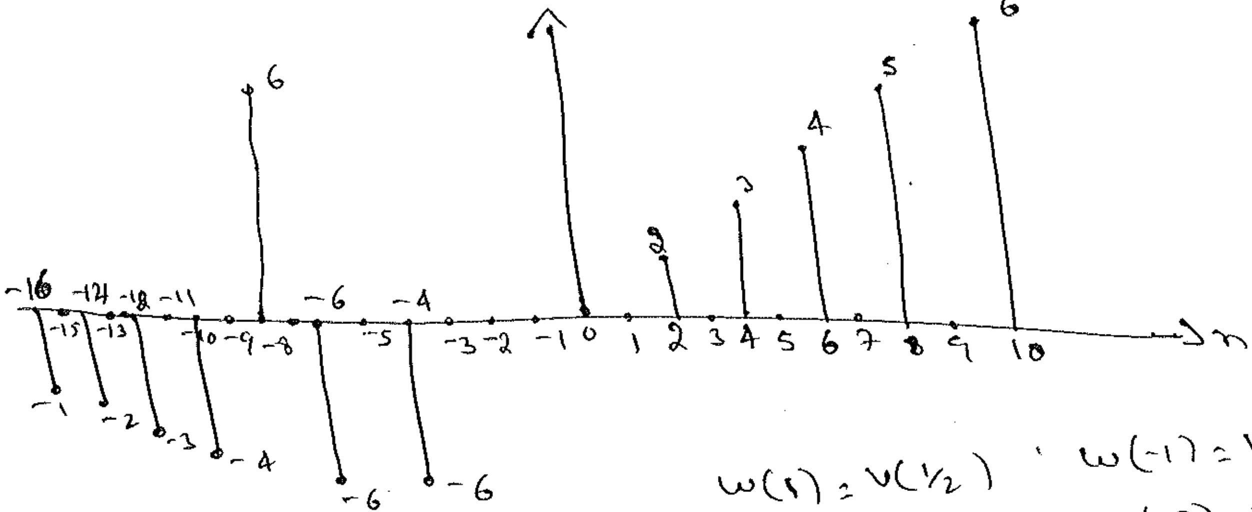


$y(n) = 3 \cdot x\left(\frac{n}{2} + 1\right)$

$v(n) = x(n+1)$



$w(n) = v\left(\frac{n}{2}\right)$



$w(1) = v(1/2)$, $w(-1) = v(-1/2)$

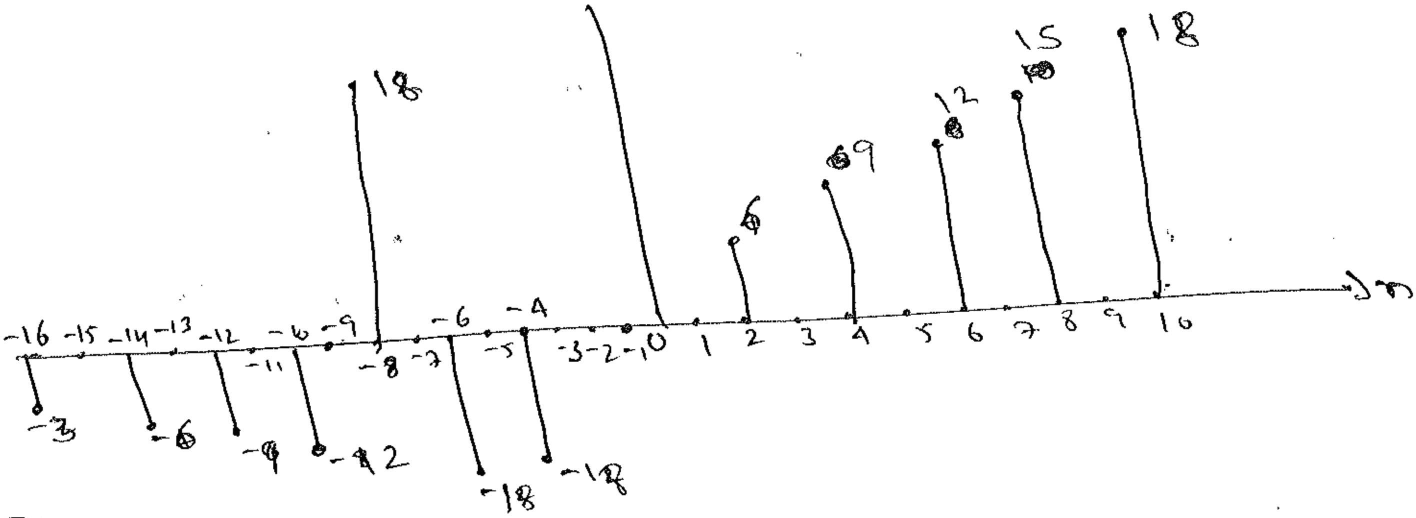
$w(3) = v(3/2)$ $w(-3) = v(-3/2)$

$w(5) = v(5/2)$ $w(-5) = v(-5/2)$

$w(7) = v(7/2)$ $w(-7) = v(-7/2)$

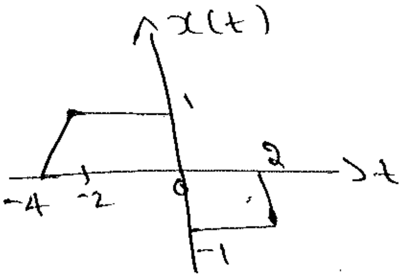
$w(9) = v(9/2)$ all are undefined signals.

$y(n) = 3 \cdot w(n)$



(10) The signal $x(t)$ as shown in below figure

(i) $x(0.5t)$ (ii) $x(2t+1)$

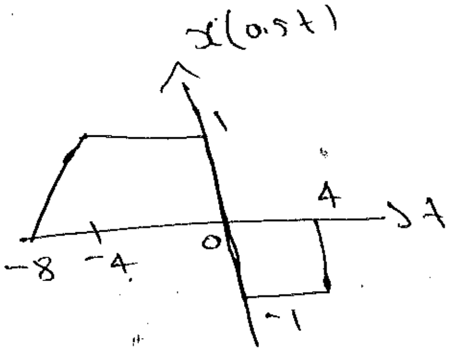


Soln (a) $x(0.5t)$

$-4 < t < 2$

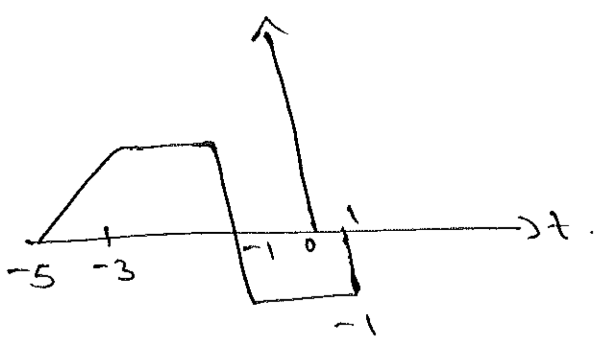
$-4 < \frac{1}{2}t < 2$

$-8 < t < 4.$



(b) $x(2t+1)$

$v(t) = x(t+1)$

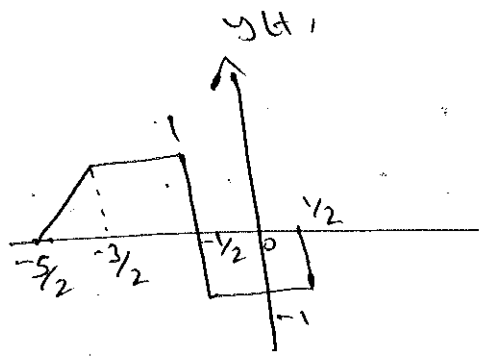


$y(t) = v(2t)$

$-5 < t < 1$

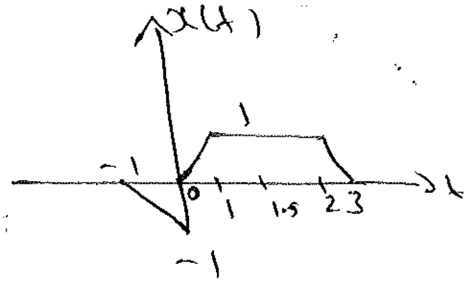
$-5 < 2t < 1$

$-5/2 < t < 1/2$

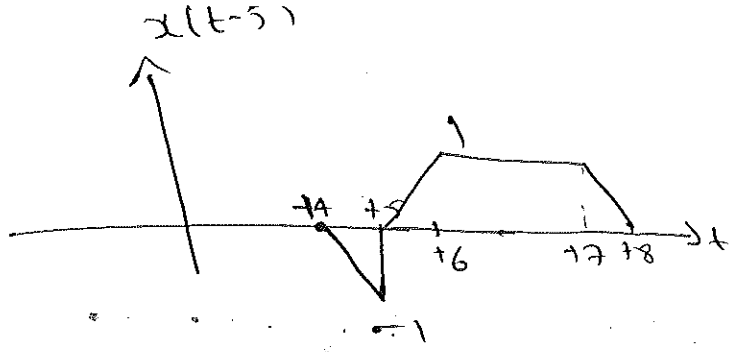


(11) Sketch & label The signal

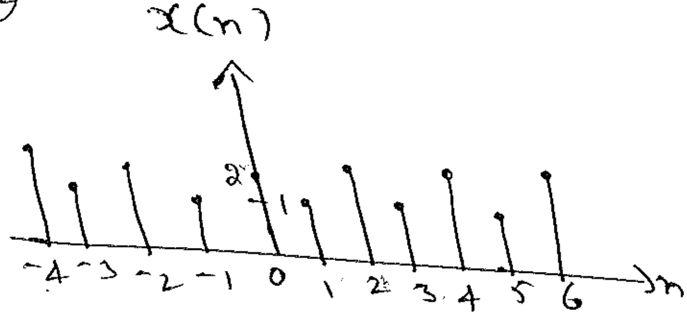
(i) $x(t-5)$



Soln

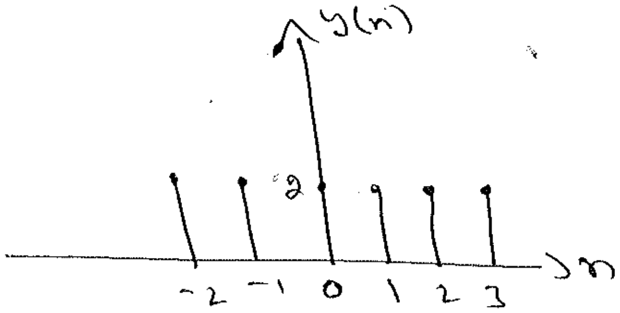


(12)

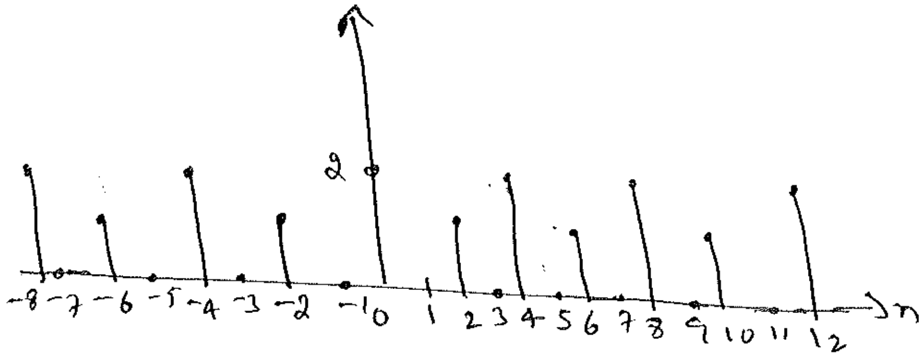


find $y(n) = x(2n)$ & $y(n) = x(\frac{1}{2}n)$

$K > 1$ $\quad k = 2$

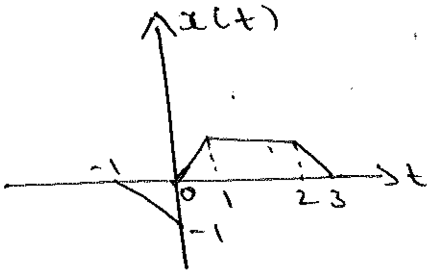


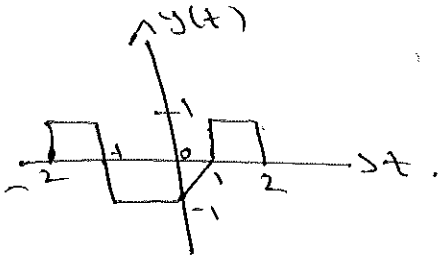
(b) $y(n) = x(\frac{1}{2}n)$



$y(-1), y(-3), y(-5), y(-7)$
$y(1), y(3), y(5), y(7), y(9), y(11)$
are undefined signals.

1) Let $x(t)$ & $y(t)$ be given in below figure a & b. Sketch the following signals





(a)

(a) $x(t)\ y(t-1)$

(b) $x(t-1)\ y(-t)$

(c) $x(t+1)\ y(t-2)$

(d) $x(t)\ y(-1-t)$

(e) $x(t)\ y(2-t)$

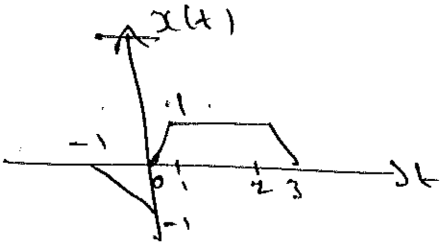
(f) $x(2t)\ y(\frac{1}{2}t+1)$

(g) $x(4-t)\ y(t)$

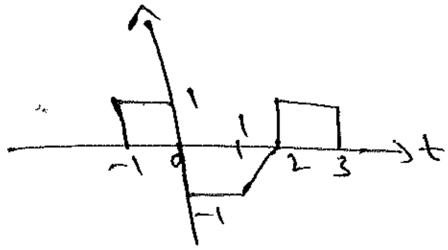
(h) $x(t) \cdot y(2-t)$

(I) ~~x(t)·y(2t)~~ $x(2t) \cdot y(2t+1)$

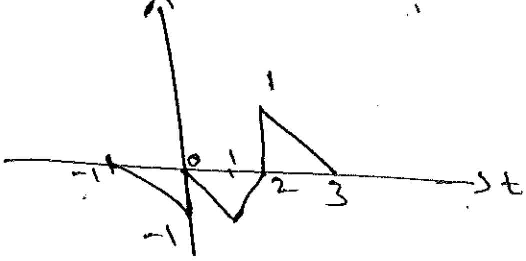
Soln (a) $x(t)\ y(t-1)$



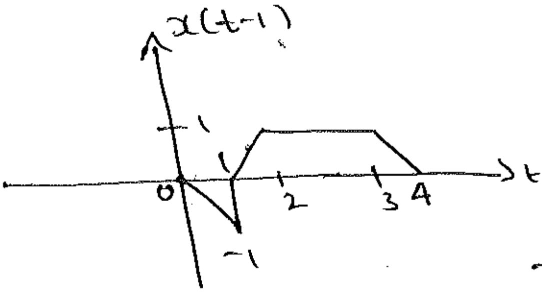
$y(t-1)$



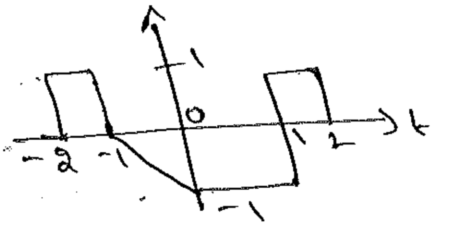
$x(t) \cdot y(t-1)$



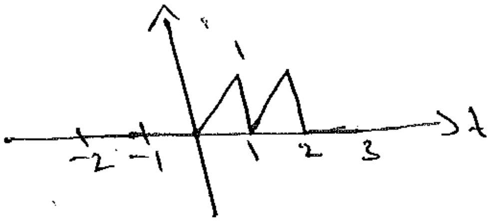
(b) $x(t-1)\ y(-t)$



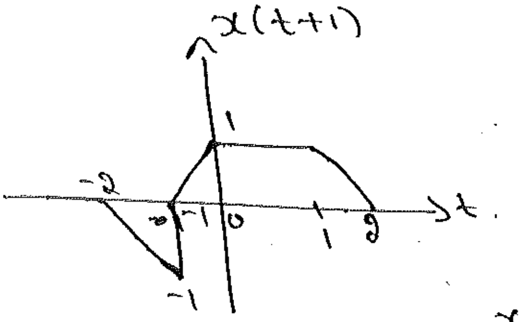
$y(-t)$

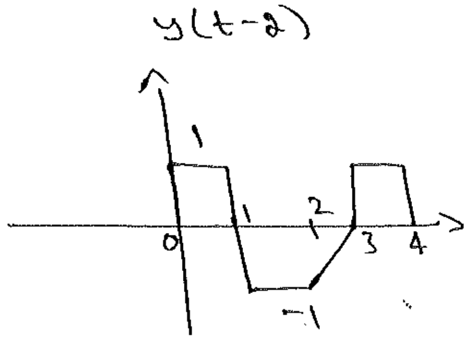


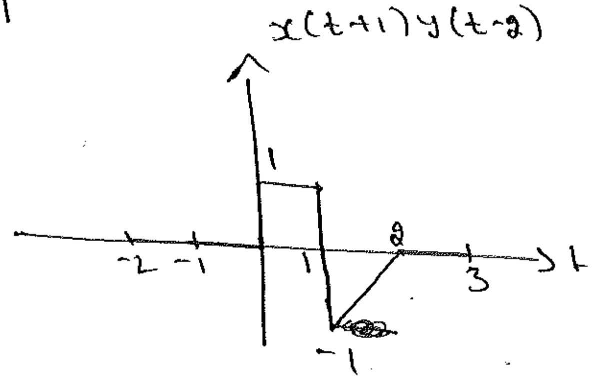
$x(t-1)\ y(-t)$



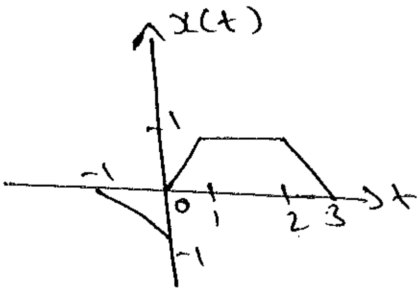
(c) $x(t+1)\, y(t-2)$

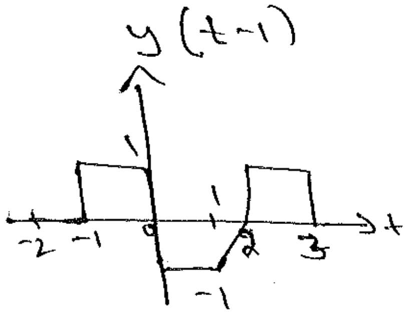


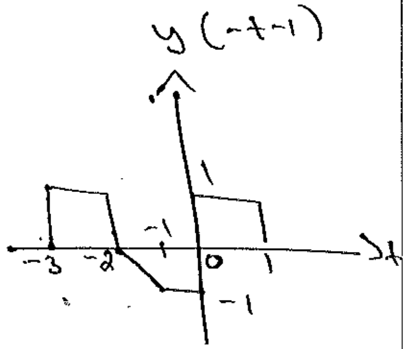


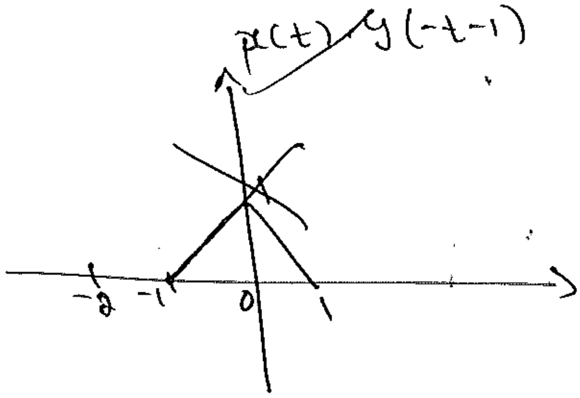


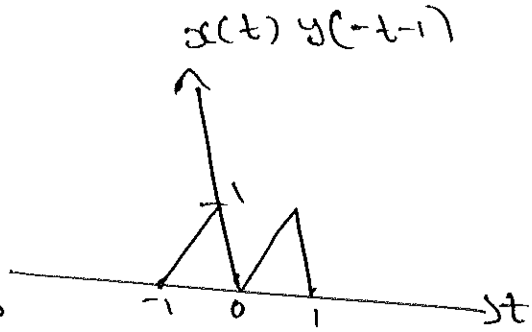
(d) $x(t)\, y(-1-t)$



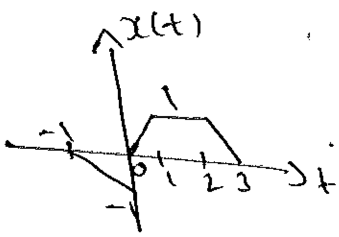


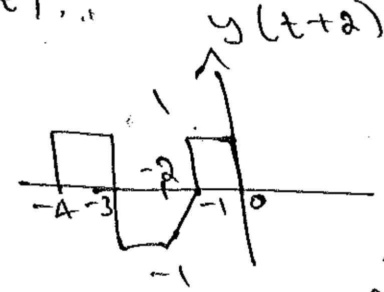


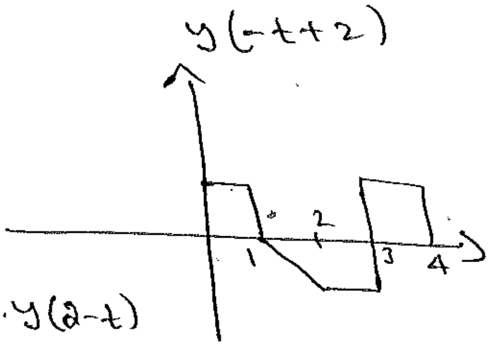


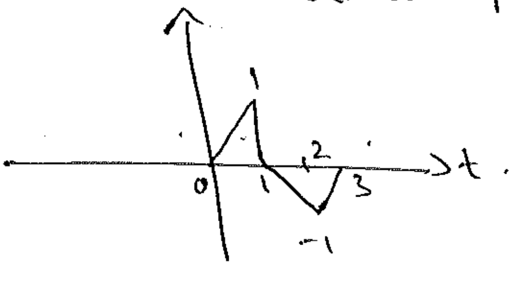


(e) $x(t) \cdot y(2-t)$..

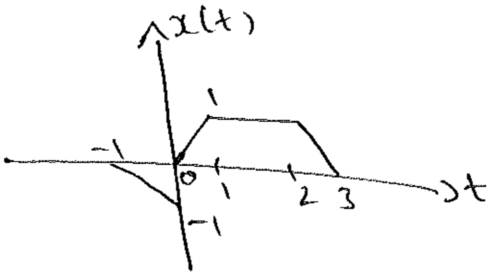


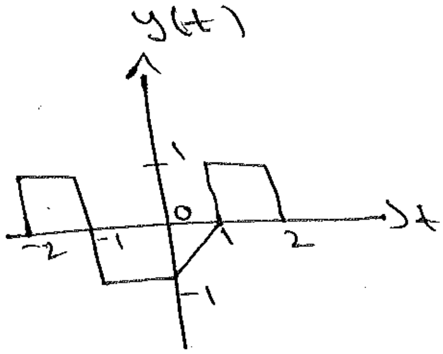






(d) $x(2t)\ y(\tfrac{1}{2}t+1)$

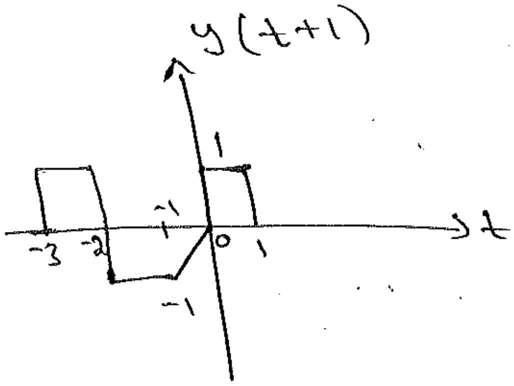


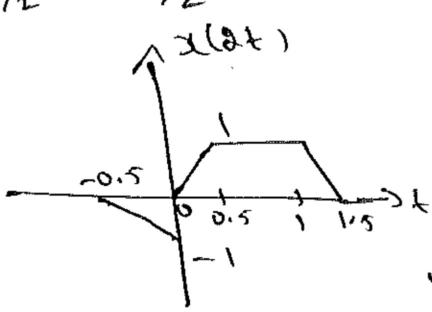


$-1 < t < 3$

$-1 < 2t < 3$

$-\tfrac{1}{2} < t < \tfrac{3}{2}$

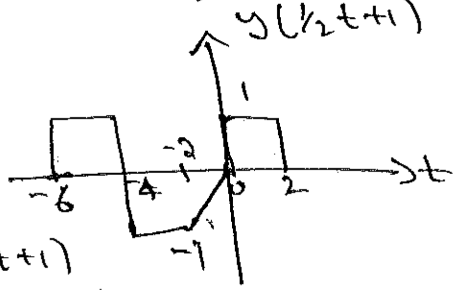


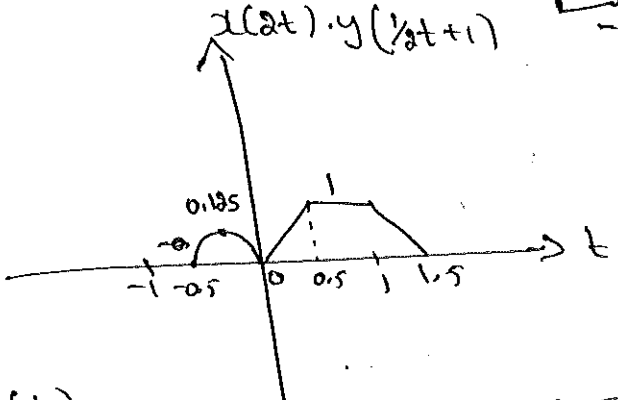


$-3 < t < 1$

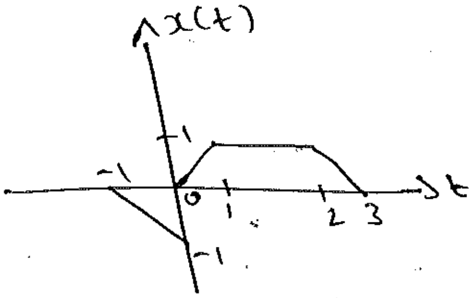
$-3 < \tfrac{1}{2}t < 1$ $\quad -6 < t < 2$

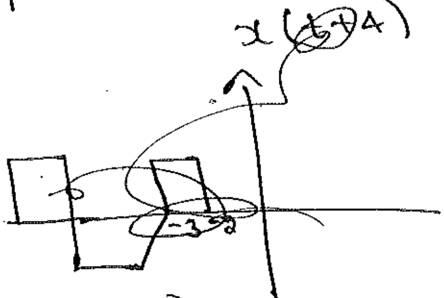
(*)

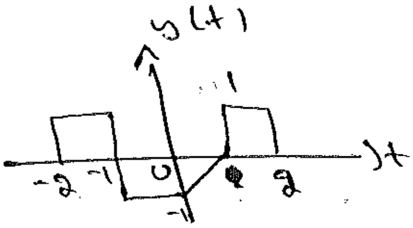


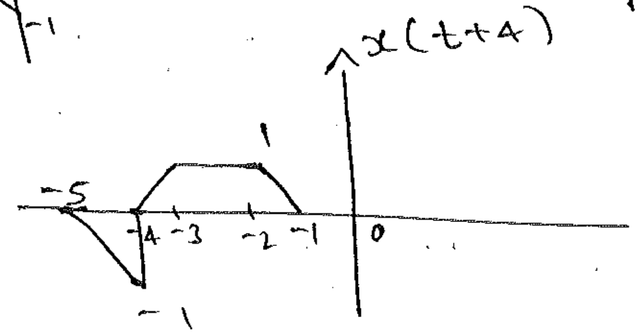


(g) $x(4-t)\ y(t)$

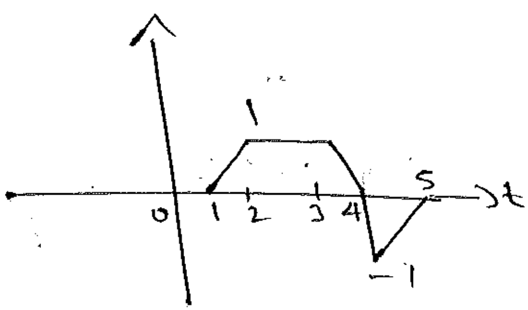


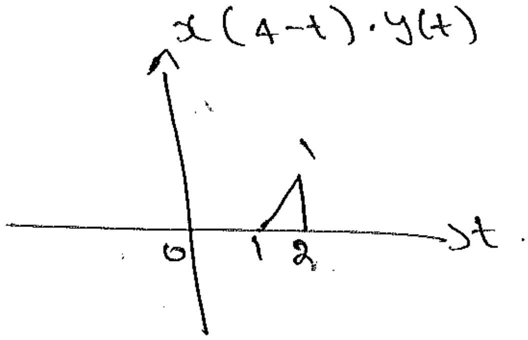




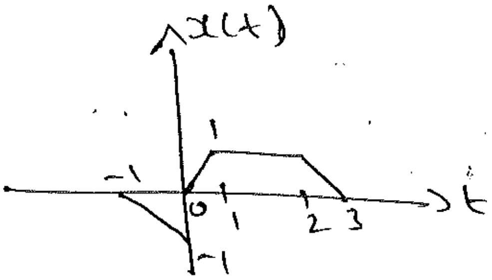


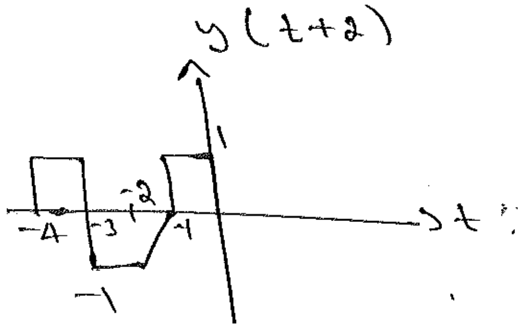
$x(-t+4)$



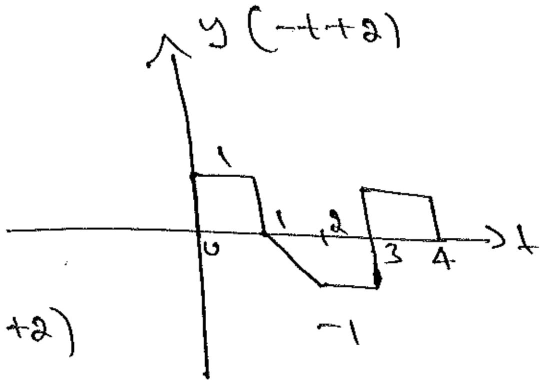


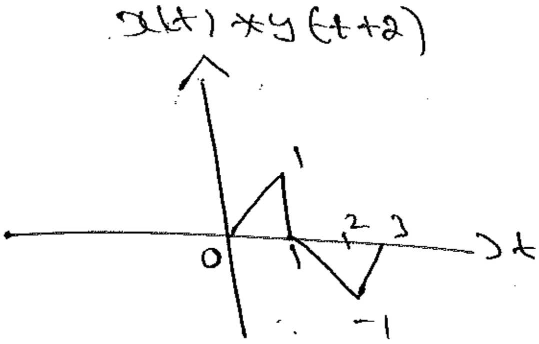
(h) $x(t)\, y(2-t)$



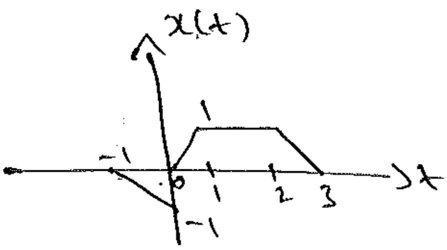


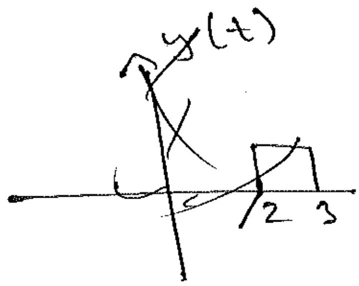
$*$

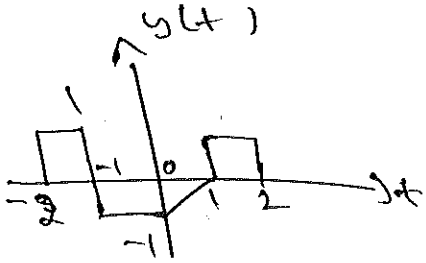


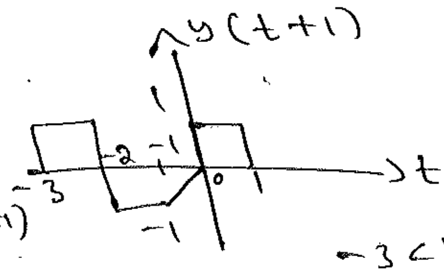


(I) $x(2t)\cdot y(2t+1)$



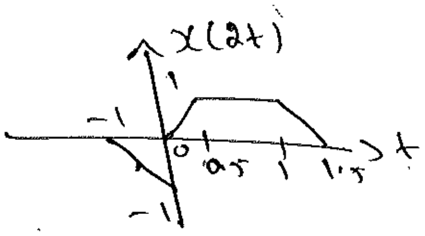


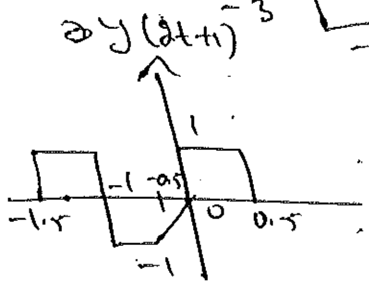




$-1 < t < 3$

$-1 < 2t < 3 = -\tfrac{1}{2} < t < \tfrac{3}{2}$

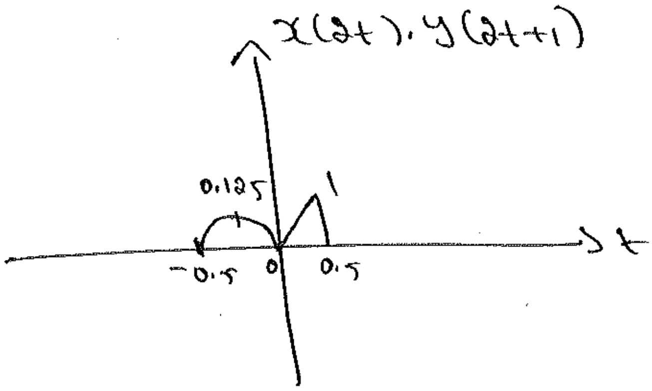




$-3 < t < 1$

$-3 < 2t < 1$

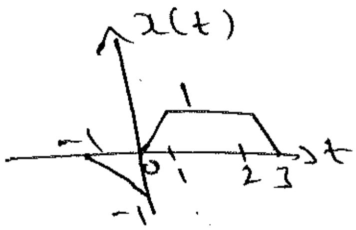
$-1.5 < t < \tfrac{1}{2}$

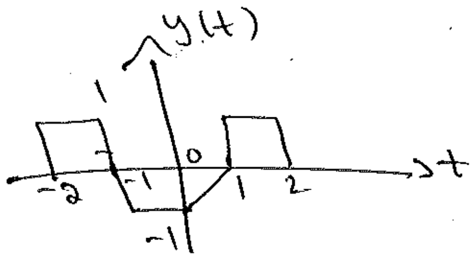


(2) Two continuous time signals $x(t)$ & $y(t)$ are given in below figure a & b Draw

$$z(t) = x(2t) \cdot y(2t+1)$$

S

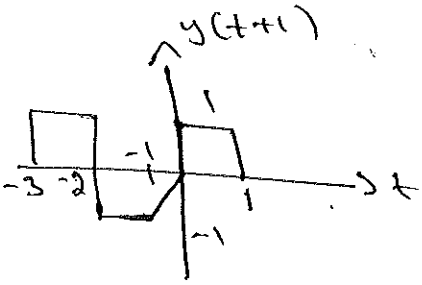


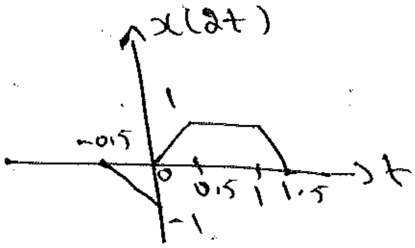


$-1 < t < 3$

$-1 < 2t < 3$

$-0.5 < t < 1.5$



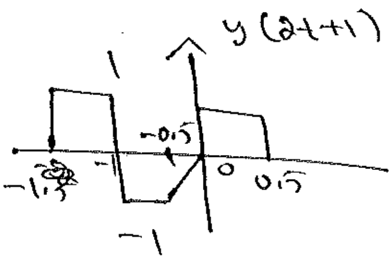


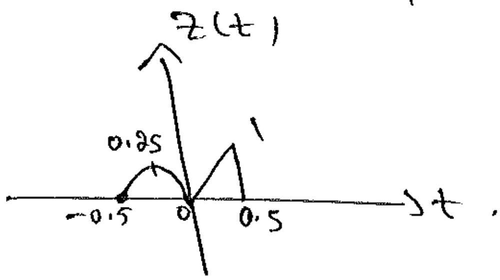
$-3 < t < 1$

$-3 < 2t < 1$

$-1.5 < t < 0.5$

*





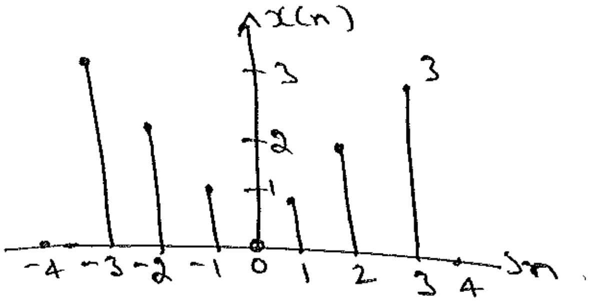
③ Let $x(n)$ & $y(n)$ be given in below figure a & b. Sketch the following signals

ⓐ $x(2n)$ ⓑ $x(3n-1)$ ⓒ $y(1-n)$ ⓓ $y(2-2n)$

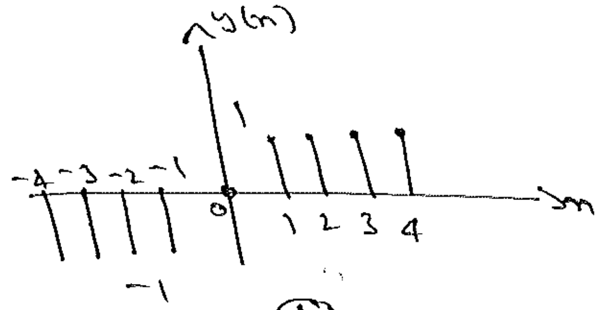
ⓔ $x(n-2)+y(n+2)$ ⓕ $x(2n)+y(n-4)$ ⓖ $x(n+2)\,y(n-2)$

ⓗ $x(3-n)\,y(n)$ ⓘ $x(-n)\,y(-n)$ ⓙ $x(n)\,y(-2-n)$

ⓚ $x(n+2)\,y(6-n)$.



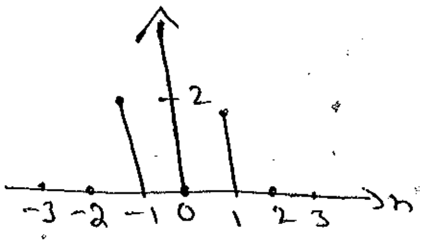
ⓐ



ⓑ

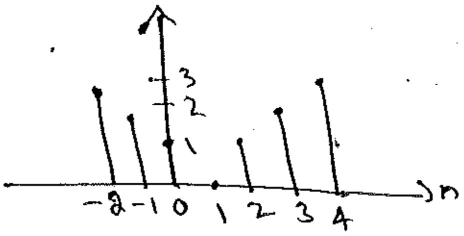
Soln

ⓐ $x(2n)$

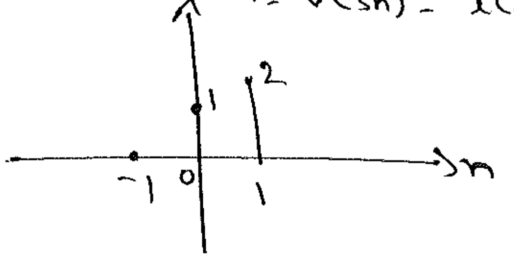


ⓑ $x(3n-1)$

$v(n) = x(n-1)$

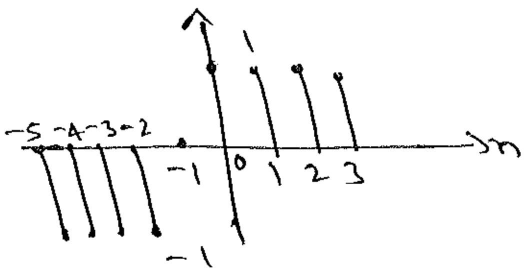


$y(n) = v(3n) = x(3n-1)$

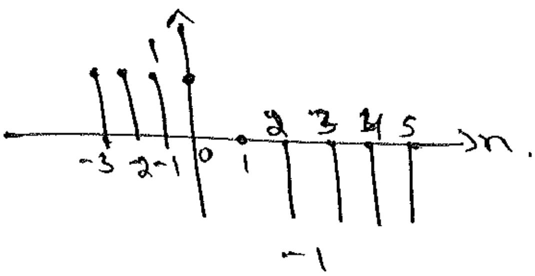


ⓒ $y(1-n)$

$v(n) = y(n+1)$



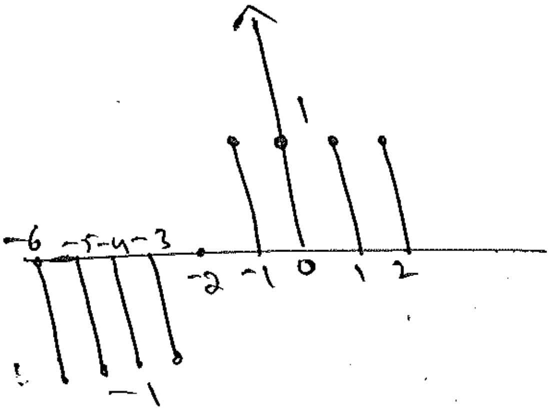
$w(n) = v(-n)$



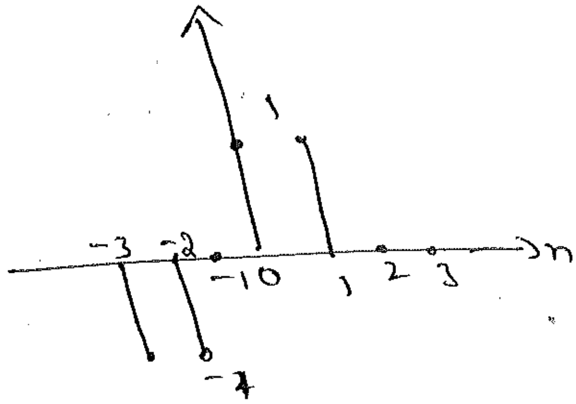
(d) $y(2-2n)$

$y(-2n+2)$

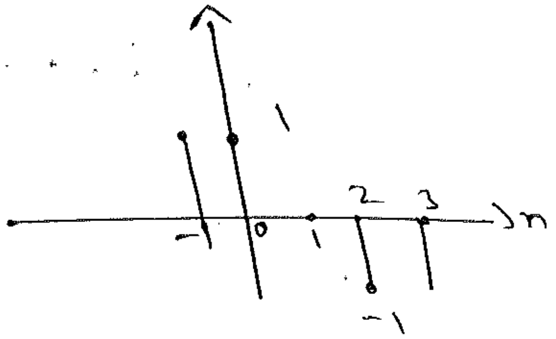
$v(n) = y(n+2)$



$w(n) = v(2n)$

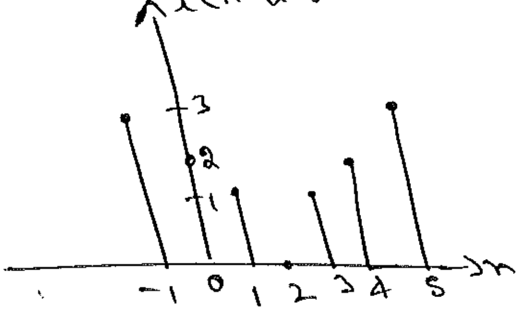


$z(n) = w(-n)$



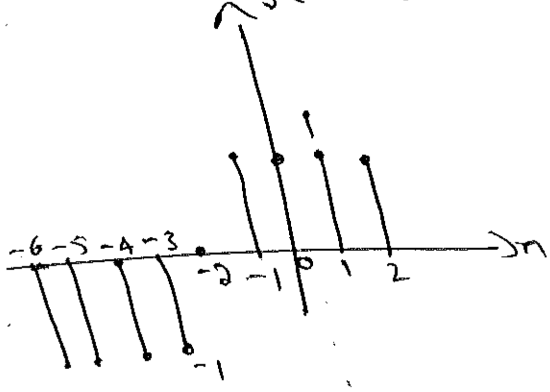
(e) $x(n-2) + y(n+2)$

$x(n-2)$

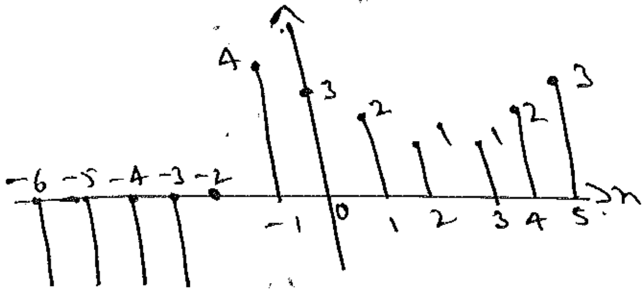


+

$y(n+2)$

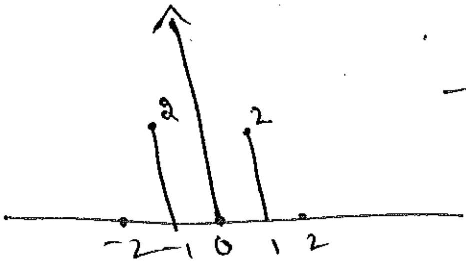


$x(n-2)+y(n+2)$



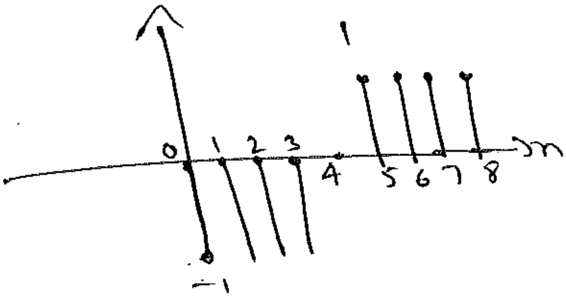
(f) $x(2n) + y(n-4)$

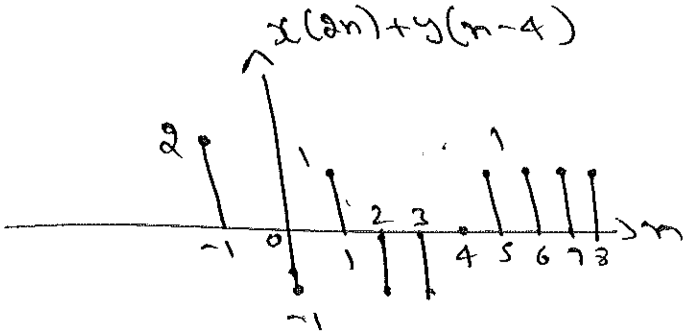
$x(2n)$



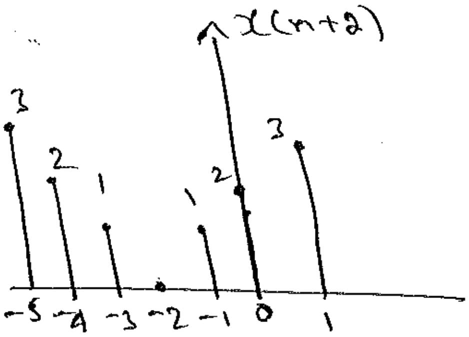
+

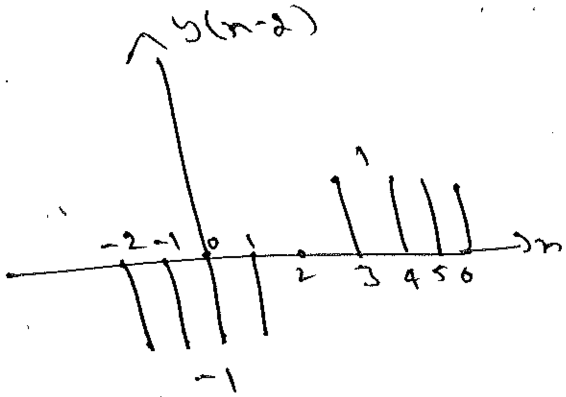
$y(n-4)$

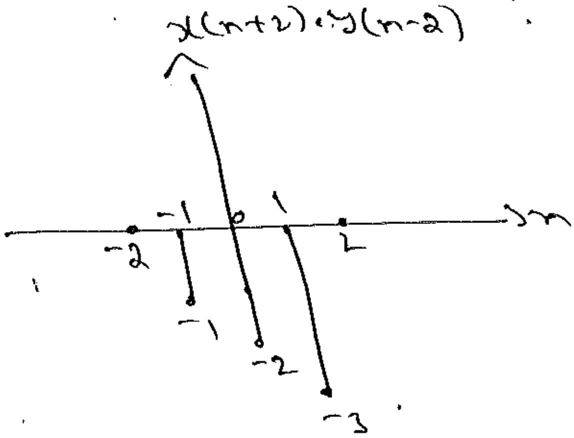




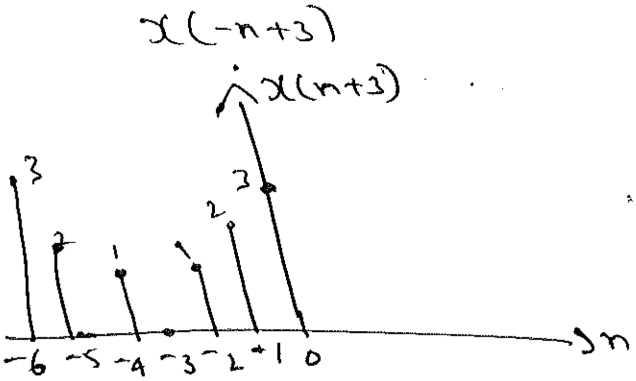
(g) $x(n+2) \cdot y(n-2)$

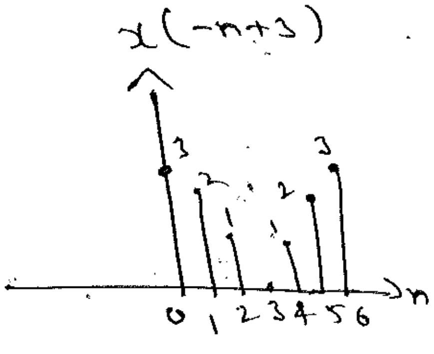


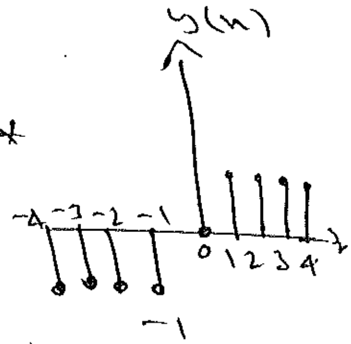


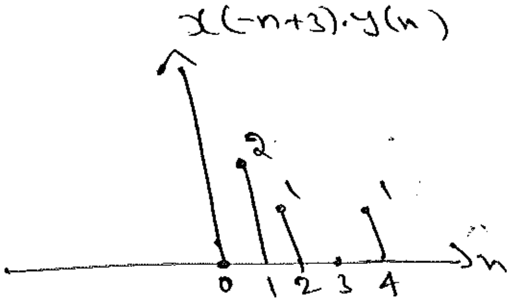


(h) $x(3-n)\ y(n)$

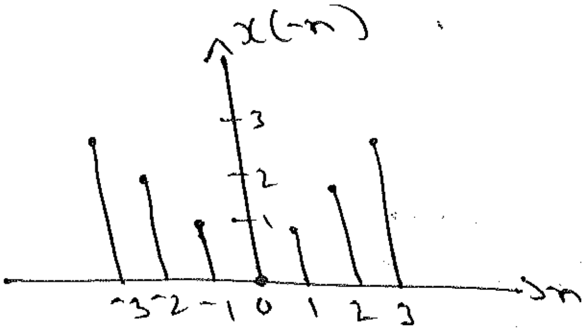








(I) $x(-n) \cdot y(-n)$



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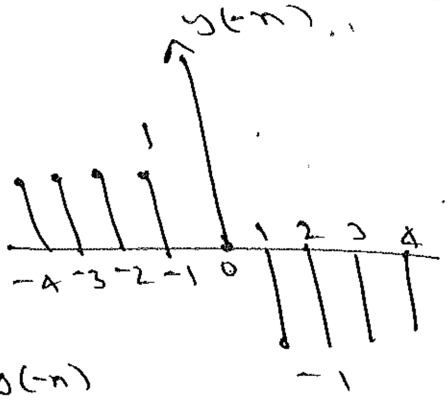
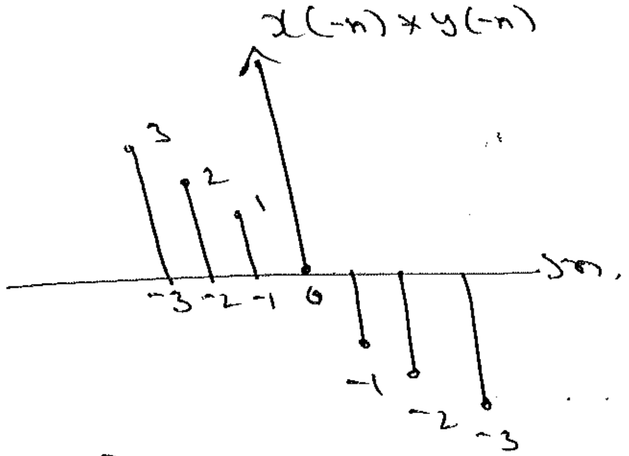
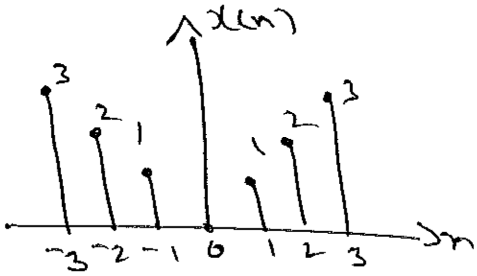
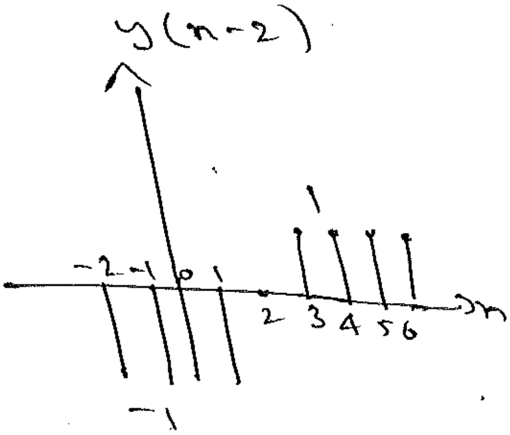




(J) $x(n)\ y(-2-n)$





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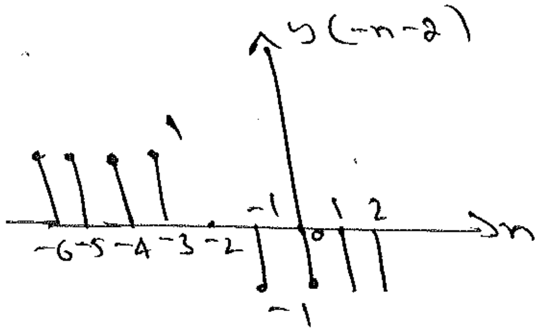
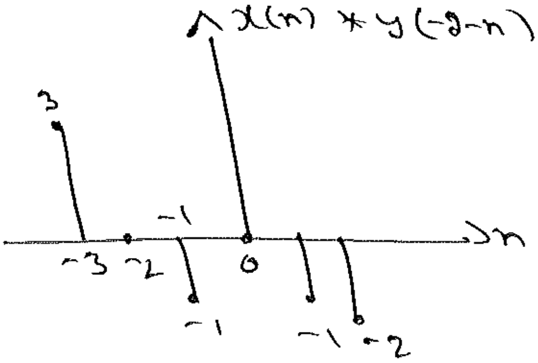
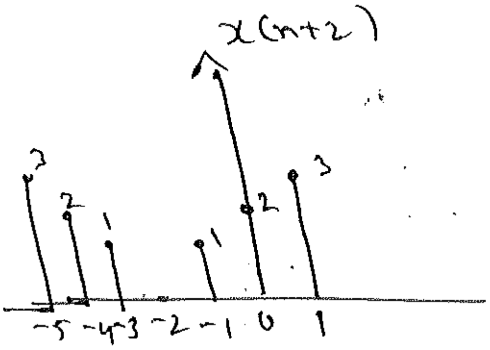
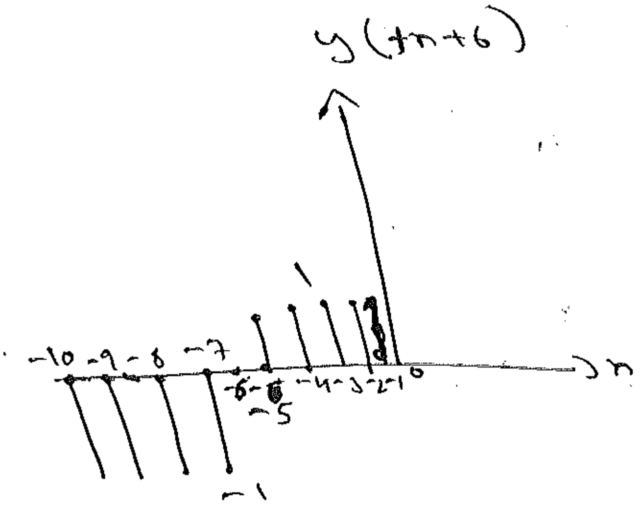
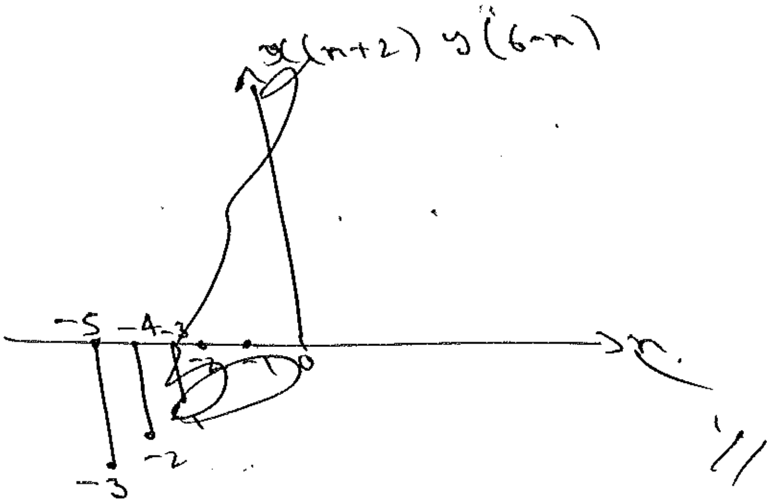


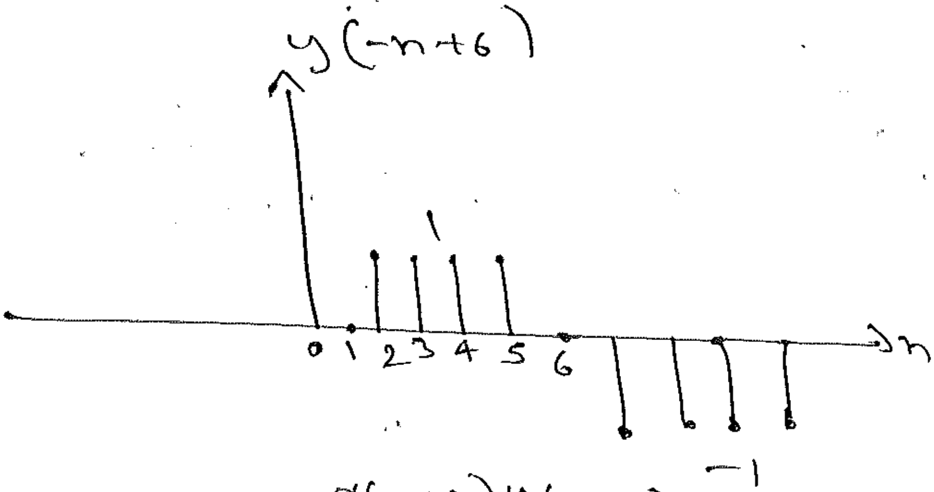


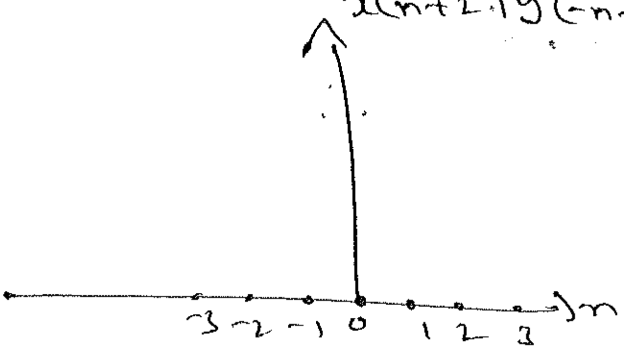
(k) $x(n+2)\,y(6-n)$







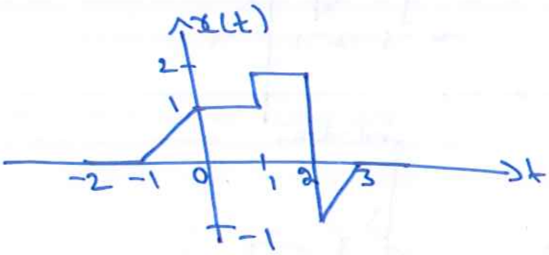




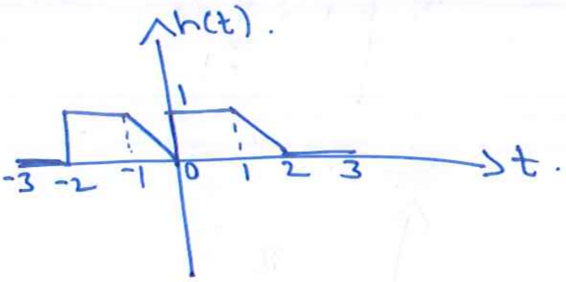
Consider The signals $x(t)$ & $h(t)$ shown in fig (a) & (b) respectively Sketch & label The following signals.

(a) $x(t)\,h(t+1)$ (b) $x(t)\cdot h(-t)$

(c) $x(t-1)\,h(1-t)$ and (d) $x(1-t)\,h(t-1)$.



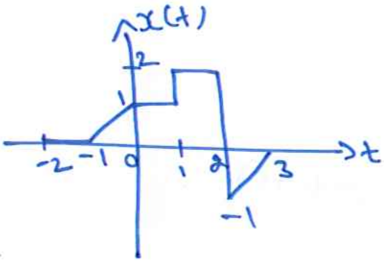
(a).

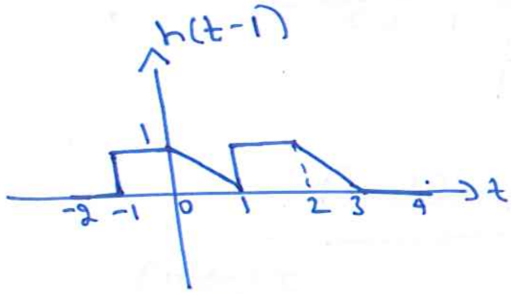


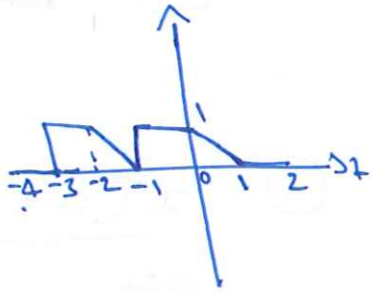
(b)

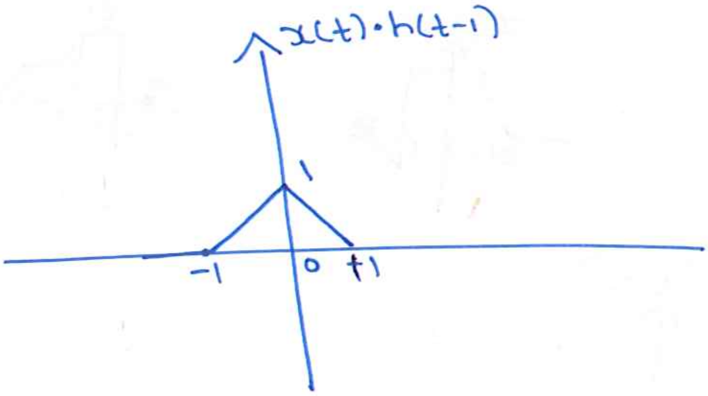
Soln :

(a) $x(t)\cdot h(t-1)$

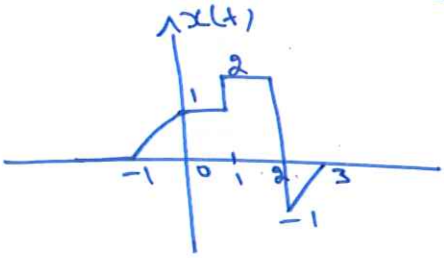








(b) $x(t)\cdot h(-t)$



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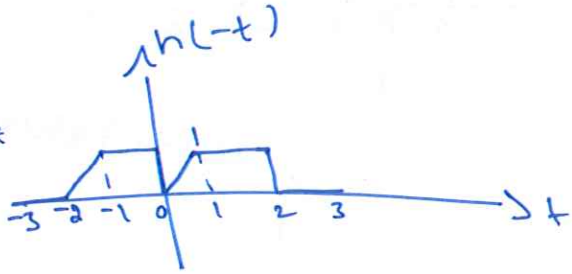
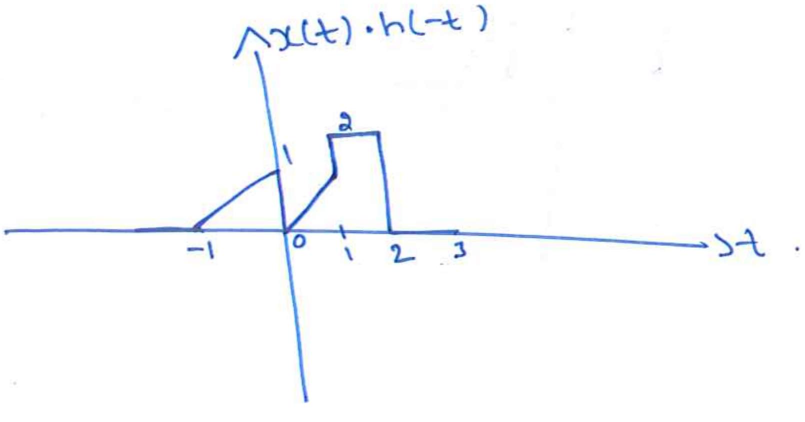
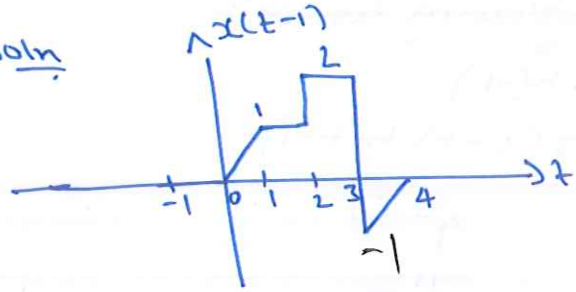


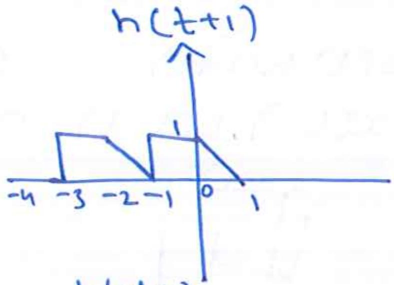


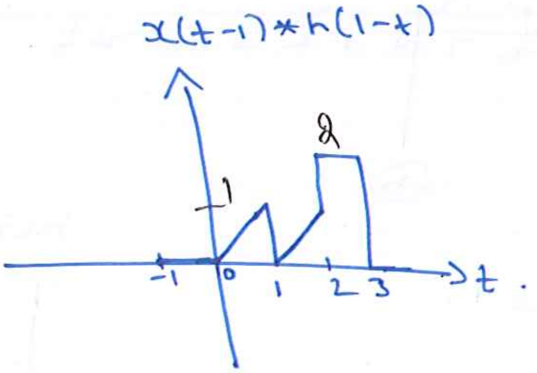
(c) $x(t-1)\, h(1-t)$

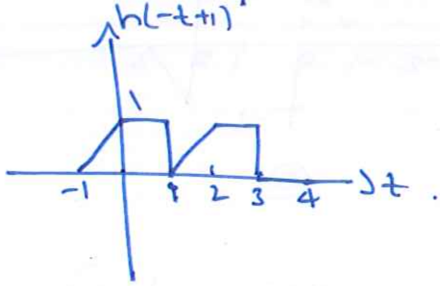
Soln



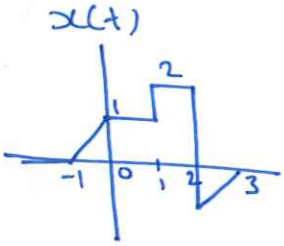
$h(-t+1) =$

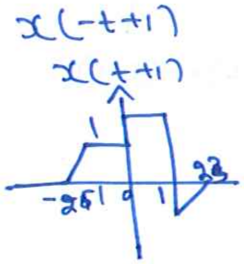


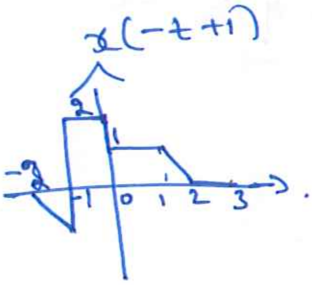


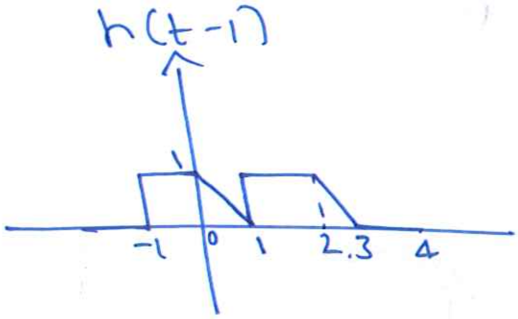


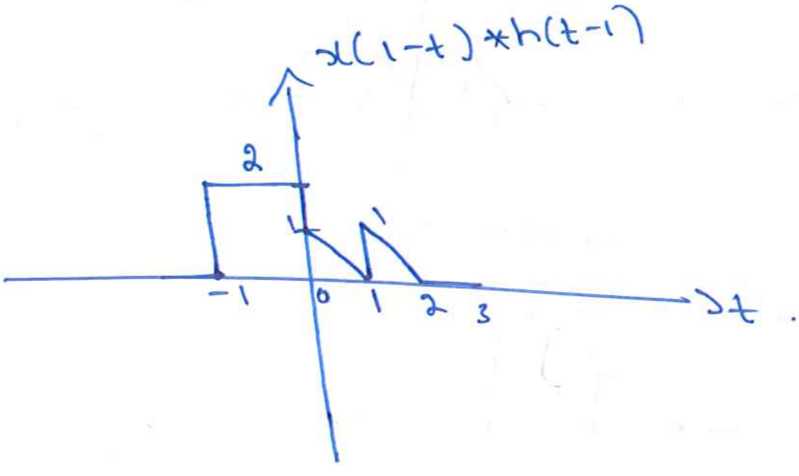
(d) $x(1-t)\, h(t-1)$.





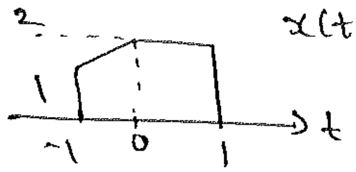


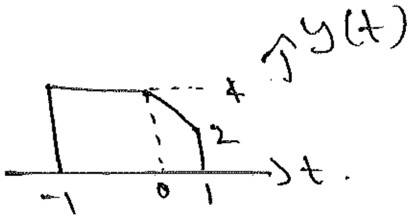




(4) i) $x(t)$ & $y(t)$ are shown in below figure

Sketch $x(1-t)\cdot y(t/2)$





Soln

$x(1-t)\cdot y(t/2)$

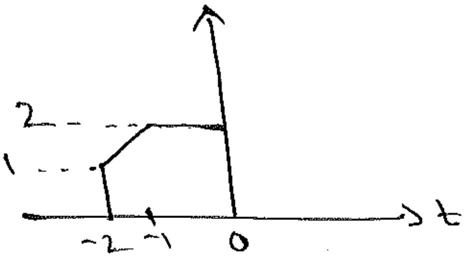
$x(-t+1)$

$-1 < t < 1$

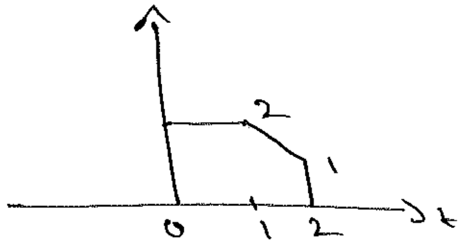
$-1 < \frac{1}{2}t < 1$

$-2 < t < 2$

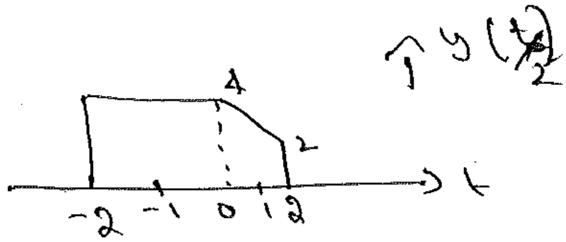
$x(t+1)$



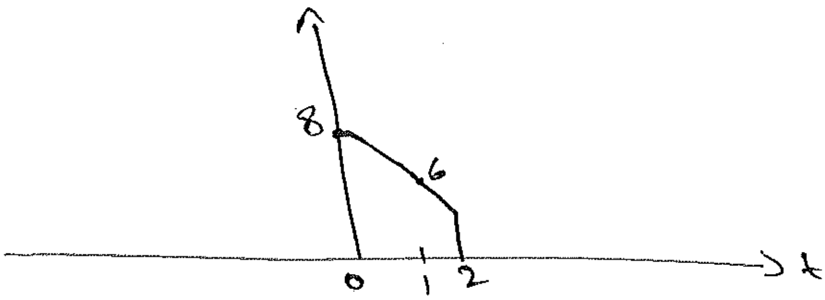
$x(-t+1)$



$\times$



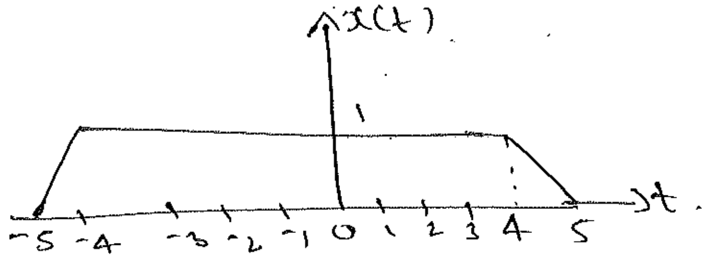
$x(-t+1)\cdot y(t/2)$



(5) The trapezoidal pulse $x(t)$ as ~~shown~~ shown in below figure is time scaled, producing the equation

$$y(t) = x(at)$$

Sketch $y(t)$ for (a) $a = 5$ & (b) $a = 0.2$.



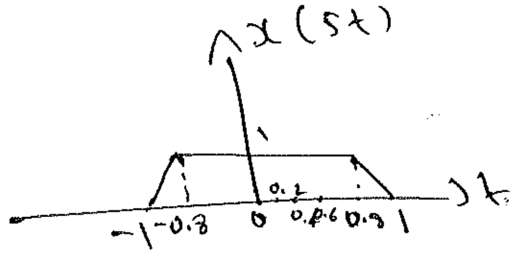
Soln (a) $y(t) = x(at)$ $a = 5$

$y(t) = x(5t)$.

$-5 < t < 5$

$-5 < 5t < 5$

$-1 < t < 1$



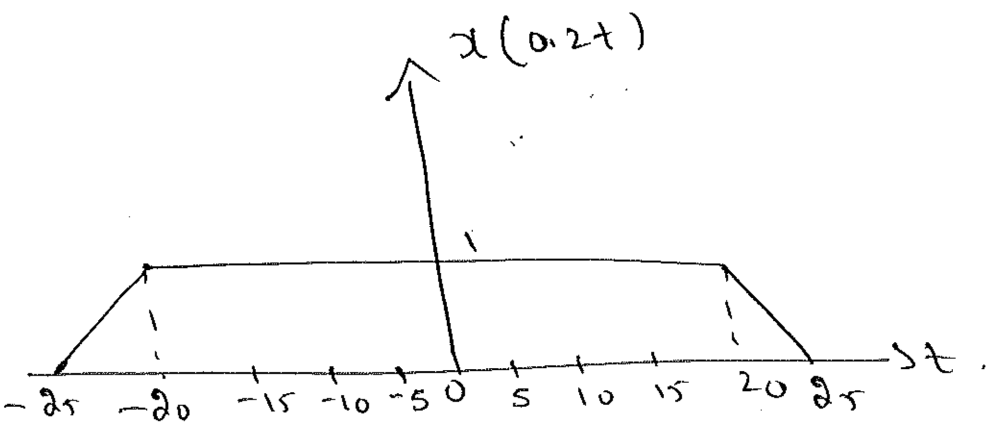
(b) $y(t) = x(at)$ $a = 0.2$

$y(t) = x(0.2t)$

$-5 < t < 5$

$-5 < 0.2t < 5$

$-25 < t < 25$



Problems on Even and ~~S~~ Odd Signals :-

1) Consider The Signal

$$x(t) = \begin{cases} \sin\left(\frac{\pi t}{T}\right) & -T < t \leq T \\ 0 & \text{Otherwise} \end{cases}$$

is The signal $x(t)$ an even or ~~and~~ odd function of time t ?

Soln :

$$x_e(t) = \tfrac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = \tfrac{1}{2}\left[x(t) - x(-t)\right].$$

Replace t with $-t$.

$$x(-t) = \sin\left(\frac{-\pi t}{T}\right) \qquad \sin(-\theta) = -\sin\theta$$

$$x(-t) = -\sin\left(\frac{\pi t}{T}\right)$$

$$x(-t) = -x(t) \quad \text{for all } t.$$

Hence $x(t)$ is an odd function.

or

$$x_e(t) = \tfrac{1}{2}\left[\sin\left(\frac{\pi t}{T}\right) - \sin\left(\frac{\pi t}{T}\right)\right]$$

$$x_e(t) = 0.$$

$$x_o(t) = \tfrac{1}{2}\left[\sin\left(\frac{\pi t}{T}\right) - \left(-\sin\frac{\pi t}{T}\right)\right]$$

$$x_o(t) = \sin\left(\frac{\pi t}{T}\right)$$

(2) Find The even & odd ~~signal~~ component of The signal

$$x(t) = e^{-2t}\cos t$$

Soln: Replacing t with $-t$.

$$x(-t) = e^{2t}\cos(-t) \qquad \cos(-\theta) = \cos\theta$$

$$x(-t) = e^{2t}\cos t$$

WKT $x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

$= \frac{1}{2}\left[e^{-2t}\cos t + e^{2t}\cos t\right]$

$x_e(t) = \frac{1}{2}\cos t\left[e^{2t} + e^{-2t}\right]$

$x_e(t) = \cos t \cosh(2t)$

$\frac{1}{2}\left[e^{t} + e^{-t}\right] = \cosh t$

$\frac{1}{2}\left[e^{t} - e^{-t}\right] = \sinh t.$

$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$

$= \frac{1}{2}\left[e^{-2t}\cos t - e^{2t}\cos t\right]$

$= \frac{1}{2}\cos t\left[e^{-2t} - e^{2t}\right]$

$x_o(t) = -\sinh(2t)\cos t.$

3) Find The even & Odd Components of The Signal.

$x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$

Soln

Replace t with $-t$

$x(-t) = \cos(-t) + \sin(-t) + \sin(-t)\cos(-t)$

$x(-t) = \cos t - \sin t - \sin t \cos t.$

$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right].$

$x(-t) = \cos(t) - \sin(t) - \sin(t)\cos t$. — ③

Sub eqn 2 & 3 in ① we get

$x_e(t) = \frac{1}{2}\left[\{\cos t + \cancel{\sin t} + \cancel{\sin t \cos t}\} + \cos t - \cancel{\sin t} - \cancel{\sin t \cos t}\right]$

$= \frac{1}{2}\{2\cos t\} = \cos t$.

$x_e(t) = \cos(t)$

III^ly

$x_o(t) = \frac{1}{2}\left[\cancel{\cos t} + \sin t + \sin t \cos t - \cancel{\cos t} + \sin t + \sin t \cos t\right]$

$= \frac{1}{2}\left[2\sin t + \sin t \cos t\right]$

$x_o(t) = \sin t\,[1 + \cos t]$

4) Find the even and odd part of the signal
$x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$.

Soln : we have $x_e(t) = \frac{1}{2}\{x(t) + x(-t)]$
$x_o(t) = \frac{1}{2}\{x(t) - x(-t)]$ — ①.

given $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$ — ②.

Replace t by $-t$ in eqn ②

$x(-t) = 1 - t + 3t^2 - 5t^3 + 9t^4$. — ③.

Sub eqn 2 & 3 in ①

$x_e(t) = \frac{1}{2}\left[1 + \cancel{t} + 3t^2 + \cancel{5t^3} + 9t^4 + 1 - \cancel{t} + 3t^2 - \cancel{5t^3} + 9t^4\right]$

$= \frac{1}{2}\left[2(1 + 3t^2 + 9t^4)\right]$

$x_e(t) = 1 + 3t^2 + 9t^4$

III^ly

$$x_o(t) = \frac{1}{2}\left[\cancel{1}+t+\cancel{3t^2}+5t^3+\cancel{9t^4}-\cancel{1}+t-\cancel{3t^2}+5t^3-\cancel{9t^4}\right]$$

$$= \frac{1}{2}\left[2\ (t+5t^3)\right])$$

$$x_o(t) = t+5t^3$$

(3) Find The even and odd Components of The Signal

$x(t) = (1+t^3)\cos^3(10t)$. July 07 - 2m.

Soln : we have $x_e(t) = \frac{1}{2}\left[x(t)+x(-t)\right]$. - (1).

$$x_o(t) = \frac{1}{2}\left[x(t)-x(-t)\right]$$

given $x(t) = (1+t^3)\cos^3(10t)$. (2)

Replace t by $-t$ in eqn (2).

$$x(-t) = (1+(-t)^3]\cos^3(-10t)$$

$$x(-t) = (1-t^3)\cos^3(10t) \quad -(3).$$

Sub eqn 2&3 in (1).

$$x_e(t) = \frac{1}{2}\left[(1+t^3)\cos^3(10t)+(1-t^3)\cos^3(10t)\right]$$

$$x_e(t) = \frac{1}{2}\left[\cos^3(10t)+\cancel{t^3\cos^3(10t)}+\cos^3(10t)-\cancel{t^3\cos^3(10t)}\right]$$

$$= \frac{1}{2}\left[2\cos^3(10t)\right]$$

$$x_e(t) = \cos^3(10t)$$

III^ly

$$x_o(t) = \frac{1}{2}\left[\cancel{\cos^3(10t)}+t^3\cos^3(10t)-\cancel{\cos^3(10t)}+t^3\cos^3(10t)\right]$$

$$x_o(t) = t^3\cos^3(10t)$$

6) Find The even & odd signal component

$$x(t) = 1 + t\cos(t) + t^2 \sin(t) + t^3 \sin(t).\cos t \quad \text{--- (1)}$$

<u>Soln</u>

W.K.T

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Replace t by -t in eqn (1)

$$x(-t) = 1 + (-t)\cos(-t) + (-t)^2 \sin(-t) + (-t)^3 \sin(-t).\cos(-t)$$

W.K.T

$$\cos(-\theta) = \cos\theta \quad \sin(-\theta) = -\sin\theta$$

$$x(-t) = 1 - t\cos t - t^2 \sin t + t^3 \sin t.\cos t \quad \text{--- (2)}$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_e(t) = \frac{1}{2}\left[1 + t\cos t + t^2 \sin t + t^3 \sin t.\cos t + 1 - t\cos t - t^2 \sin t + t^3 \sin t.\cos t\right]$$

$$x_e(t) = \frac{1}{2}\left[2 + 2t^3 \sin t.\cos t\right]$$

$$x_e(t) = 1 + t^3 \sin t.\cos t$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$x_o(t) = \frac{1}{2}[1 + t\cos t + t^2 \sin t + t^3 \sin t.\cos t - 1 + t\cos t + t^2 \sin(t) - t^3 \sin t.\cos t]$$

$$= \frac{1}{2}[2t\cos t + 2t^2 \sin t]$$

$$x_o(t) = t\cos t + t^2 \sin t$$

<u>Note</u>:

The above definitions of even & odd signals holds good for only real valued signals. But complex valued signals are characterized by conjugate symmetry.

A complex valued signal $x(t)$ is said to be conjugate symmetric if

$$x(-t) = x^*(t) \quad -(1)$$

where the asterisk denotes complex conjugation.

let $x(t) = a(t) + j b(t)$

where $a(t)$ is the real part, $b(t)$ is the imaginary part & $j = \sqrt{-1}$

Then complex conjugate of $x(t)$ is.

$$x^*(t) = a(t) - j b(t) \quad -(2)$$

<u>Substituting</u> $x(t)$ & $x^*(t)$ in eqn (1).

$$a(-t) + j b(-t) = a(t) - j b(t)$$

By equating the real part on the left with that on the right & similarly ~~by equating~~ for the imaginary part. we find that $a(-t) = a(t)$ & $b(-t) = -b(t)$.

it follows that a complex valued signal $x(t)$ is conjugate symmetric if its real part is even & its imaginary part is odd.

7) Find The even & odd Component of $x(t) = e^{jt}$

Soln

even part

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)].$$

$$x(-t) = e^{-jt}.$$

$$x_e(t) = \frac{1}{2}[e^{jt} + e^{-jt}] = \cos t$$

W.K.T. $\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta.$ $\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta$

$$x_o(t) = \frac{1}{2}[e^{jt} - e^{-jt}] = j\sin\theta = j\sin t$$

8) Show that The product of two even signals or two odd signals is an even signal while The product of an even & odd signal is an odd signal.

Soln

let $y(t) = y_1(t) \cdot y_2(t)$

if $y_1(t)$ & $y_2(t)$ are both even Then

$$y(-t) = y_1(-t) \cdot y_2(-t)$$

$$y(-t) = y_1(t) \cdot y_2(t) = y(t).$$

& Thus $y(t)$ is even. if $y_1(t)$ & $y_2(t)$ are both odd.

Then $y(-t) = y_1(-t) \cdot y_2(-t)$

$$= -y_1(+t) \cdot [-y_2(+t)]$$

$$y(-t) = y_1(+t) \cdot y_2(+t) = y(t).$$

& Thus $y(t)$ is even. if $y_1(t)$ is even & $y_2(t)$ is odd, Then

$$y(-t) = y_1(-t) \cdot y_2(-t)$$

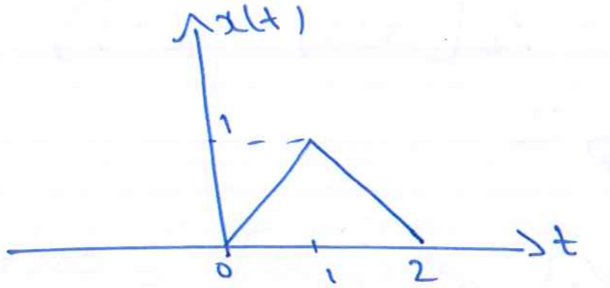
$$= y_1(t)[-y_2(t)]$$

$$y(-t) = -y_1(t)\, y_2(t)$$

$$y(-t) = -y(t) \qquad \therefore y(t) = -y(-t)$$

Thus $y(t)$ is odd

1) Determine & sketch the even & odd parts of the signal shown in below figure.

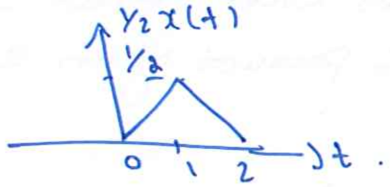


Soln

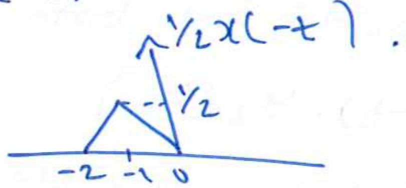
$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t).$$

first find $\frac{1}{2}x(t)$. $x(t)$ is given

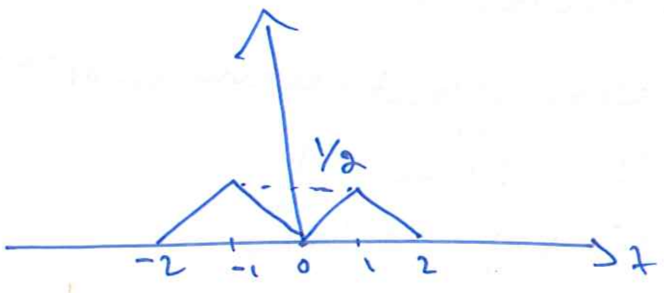


find $\frac{1}{2}x(-t)$

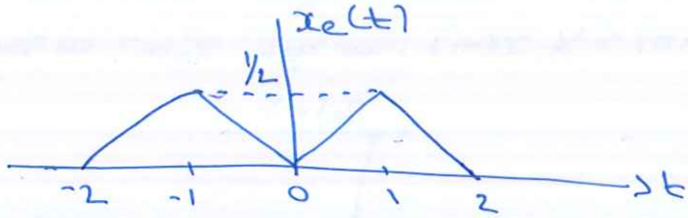


Then add $\frac{1}{2}x(t) + \frac{1}{2}x(-t)$ we get $x_e(t)$

$$x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$$



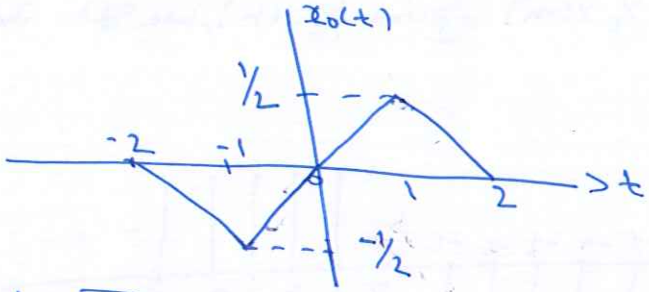
Adding $\frac{1}{2}x(t)$ & $\frac{1}{2}x(-t)$, we get $x_e(t)$ as shown in below figure.



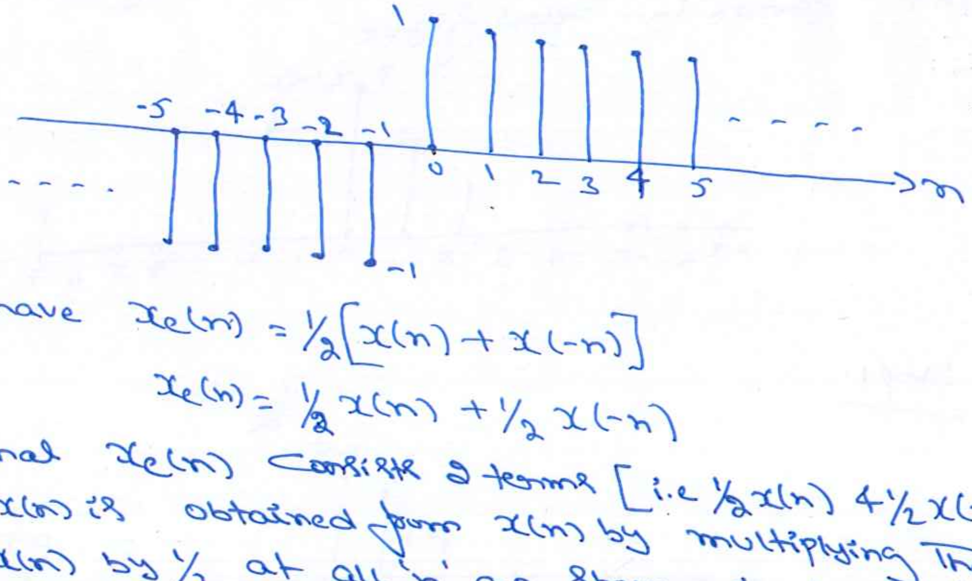
$|||^{ly}$, we have $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$$x_o(t) = \frac{1}{2}x(t) \left(-\frac{1}{2}x(-t)\right)$$

Subtracting $\frac{1}{2}x(-t)$ from $\frac{1}{2}x(t)$, we get $x_o(t)$ as shown below figure



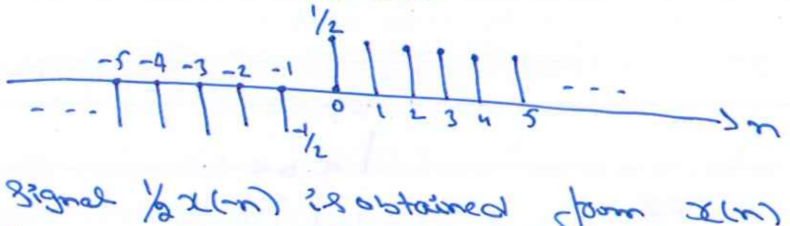
2) Determine & sketch the even & odd components of the discrete-time signal $x(n)$ shown in below figure



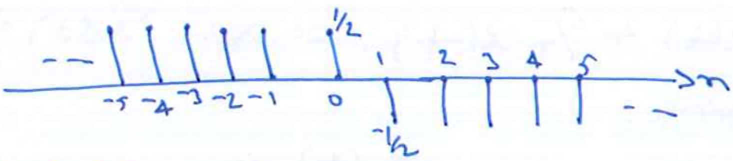
Soln :- we have $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$$x_e(n) = \frac{1}{2}x(n) + \frac{1}{2}x(-n)$$

→ The signal $x_e(n)$ consists 2 terms [i.e $\frac{1}{2}x(n)$ & $\frac{1}{2}x(-n)$]. The signal $\frac{1}{2}x(n)$ is obtained from $x(n)$ by multiplying the sample values of $x(n)$ by $\frac{1}{2}$ at all 'n' as shown in below figure

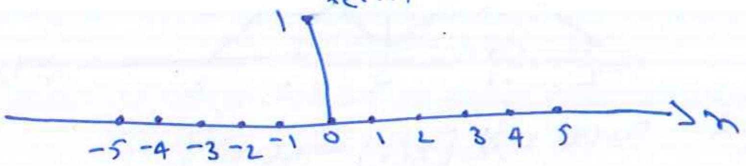


→ Similarly the signal $\frac{1}{2}x(-n)$ is obtained from $x(n)$ by taking the mirror image of $x(n)$ to get $x(-n)$, then multiplying its sample values by $\frac{1}{2}$ as shown below.



→ Adding $\frac{1}{2}x(n)$ & $\frac{1}{2}x(-n)$, we get $x_e(n)$ as shown below.

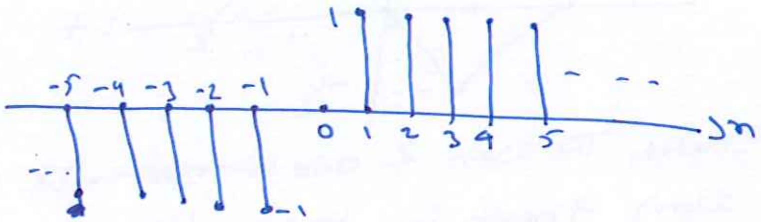
$x_e(n)$



Ill^{ly}, we have $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

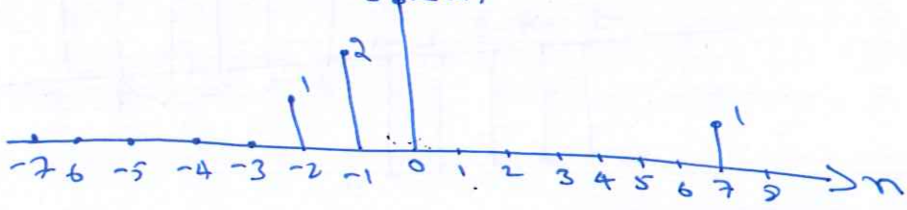
$$x_o(n) = \frac{1}{2}x(n) - \frac{1}{2}x(-n)$$

Subtracting $\frac{1}{2}x(-n)$ from $\frac{1}{2}x(n)$, we get $x_o(n)$



3) Determine & draw the even & odd parts of the discrete-time signal $x(n)$ shown in below figure.

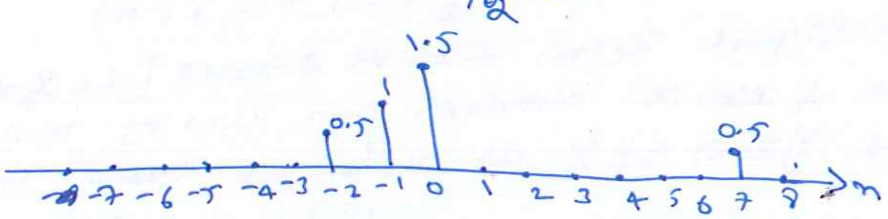
$x(n)$



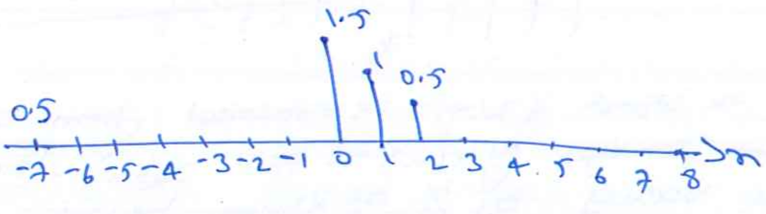
Soln:

Step 1:

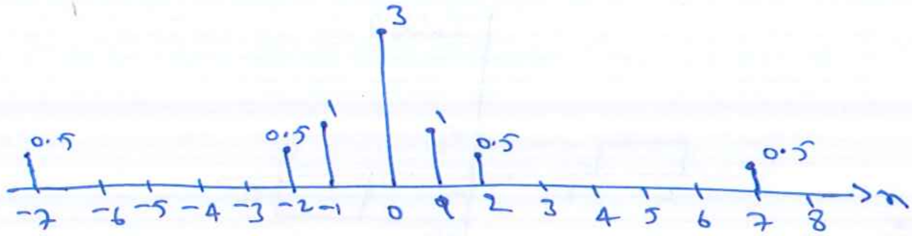
$\frac{1}{2}x(n)$



Step 2:

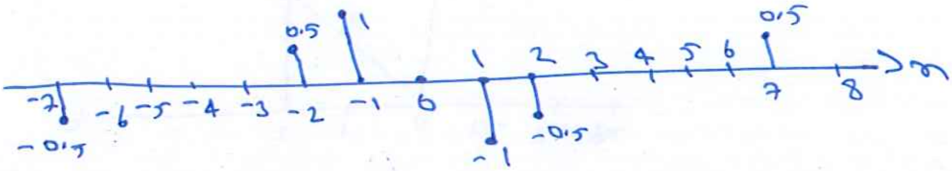


adding $\frac{1}{2}x(n)$ & $\frac{1}{2}x(-n)$, we get $x_e(n)$ as shown below.

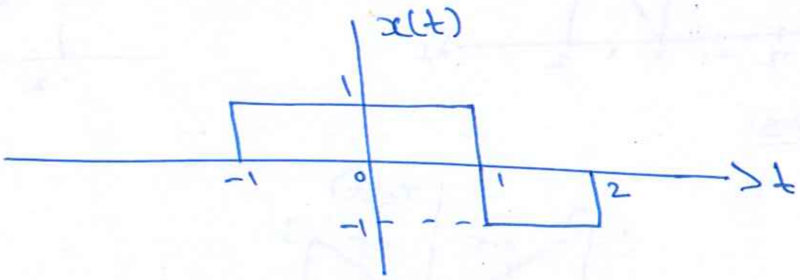


|||ly

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right].$$

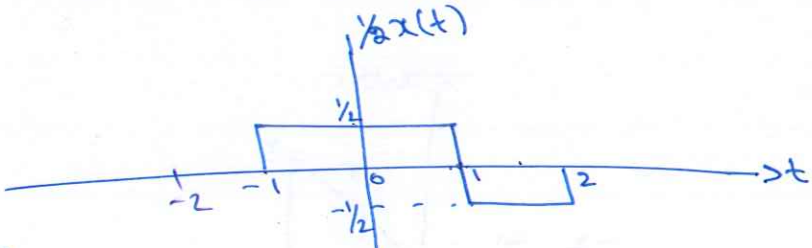


(4) Find The odd & even part of the signal $x(t)$ shown in below figure.

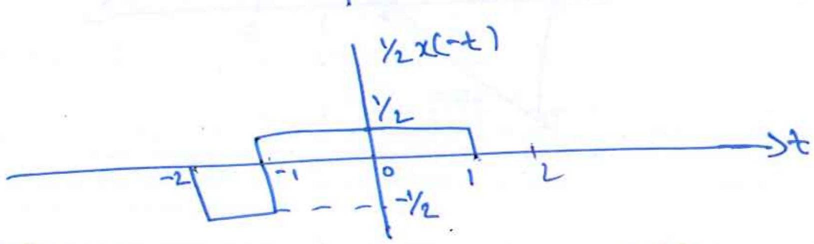


Soln

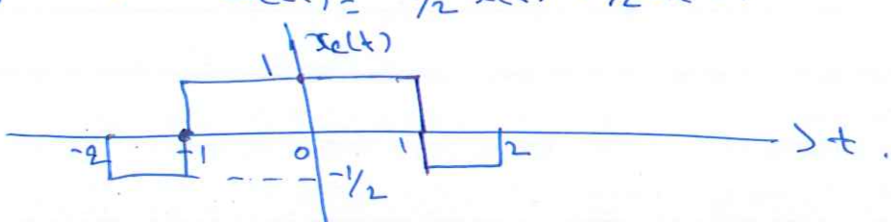
Step1: $\frac{1}{2}x(t)$



Step2: $\frac{1}{2}x(-t)$

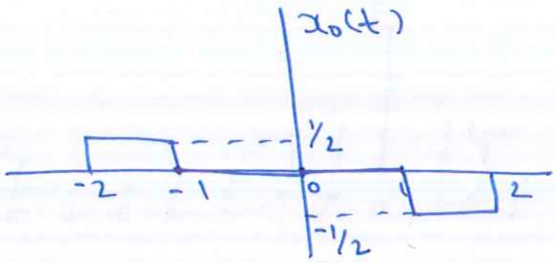


Step3: adding 2 signal $x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$



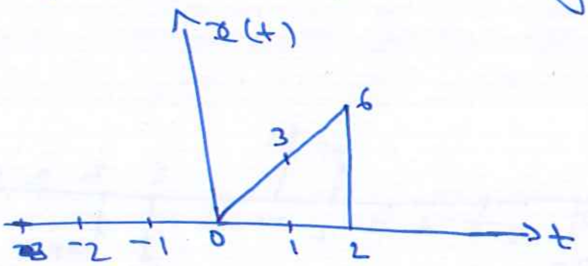
Step 4: $x_o(n) = \frac{1}{2}x(n) - \frac{1}{2}x(-n)$.

$\therefore x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$



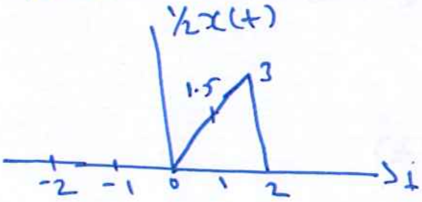
(5) Find even & odd Component for The given signal.

a)

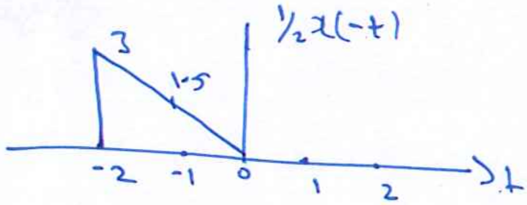


Soln:

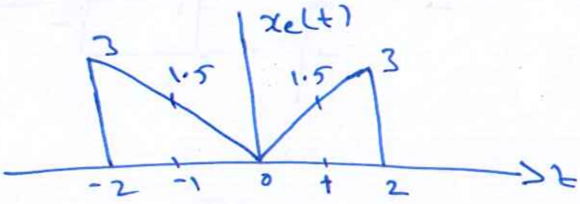
Step 1: $\frac{1}{2}x(t)$



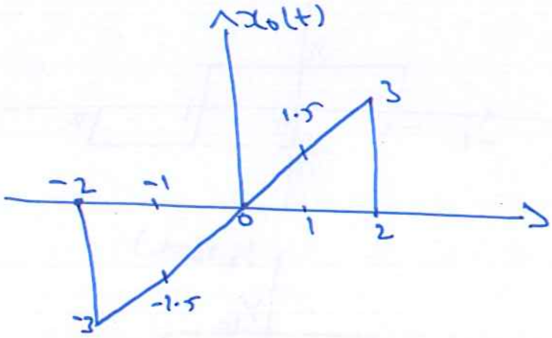
Step 2: $\frac{1}{2}x(-t)$



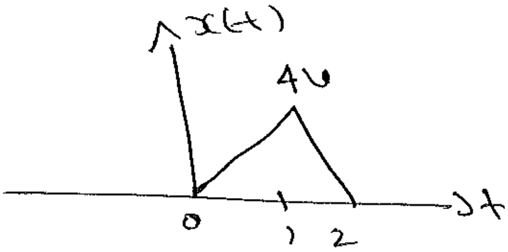
adding



Sub

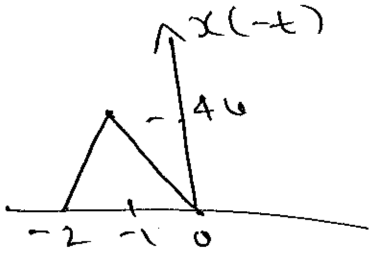


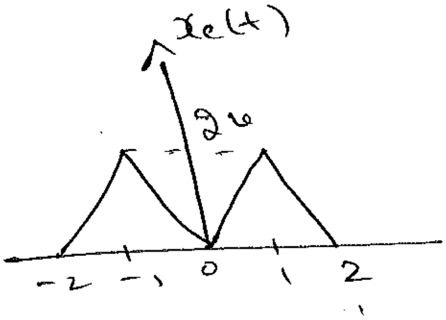
(b)



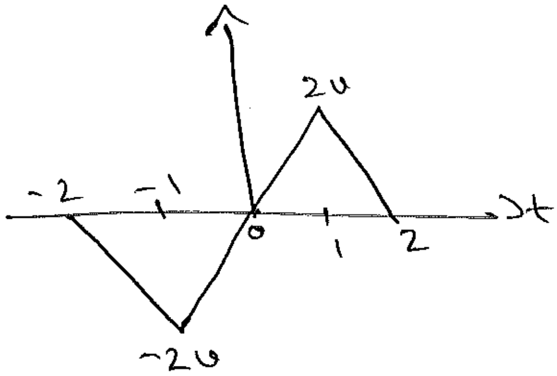
Soln

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

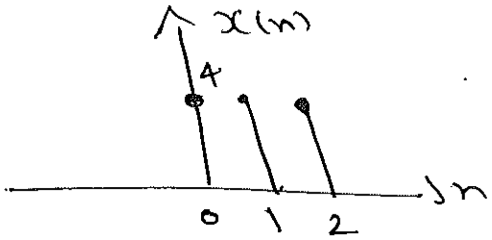




$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

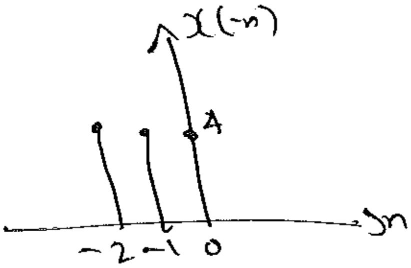


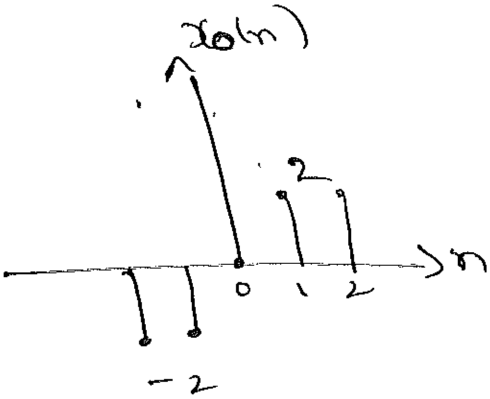
(c)

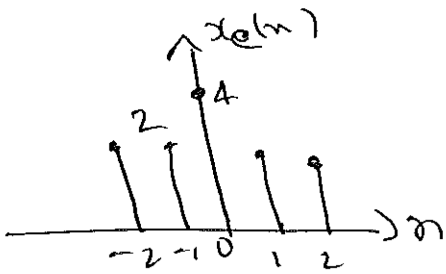


Soln

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right] \qquad x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$







(e) $x(n) = e^{-(n/4)} u(n)$

or

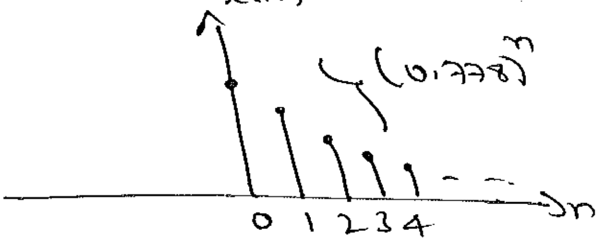
$$x(n) = \begin{cases} e^{-(n/4)} & 0 \le n \le \infty \\ 0 & \text{otherwise} \end{cases}$$

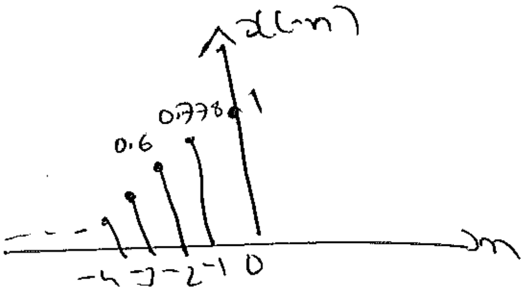
Soln

$$u(n) = \begin{cases} 1 & n \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

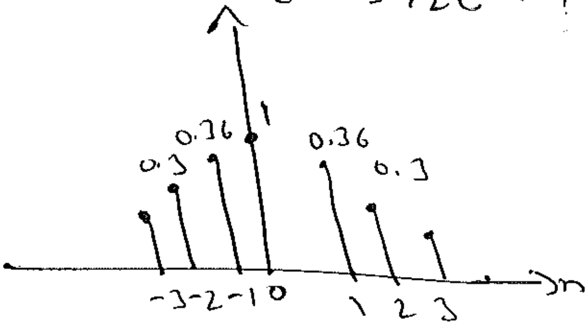
$x(n) = e^{-n(1/4)}$

$x(n) = (0.778)^n$

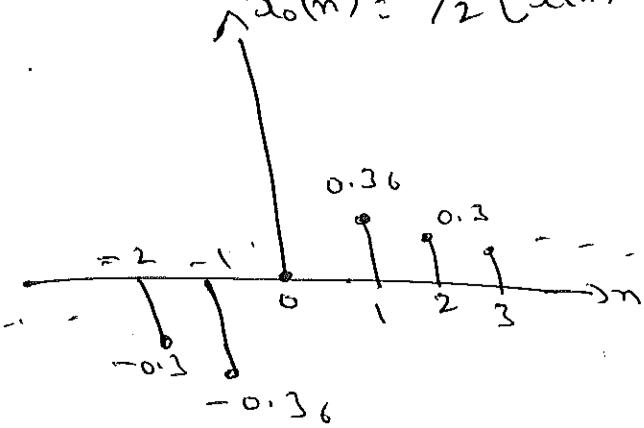




$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$



$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$



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ii) $x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \end{cases}$

Soln

$x(t) = t \quad t = 0, 1$

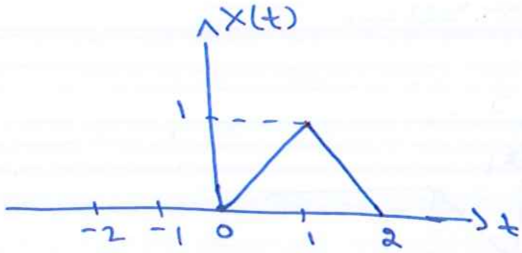
$x(0) = 0$

$x(1) = 1$

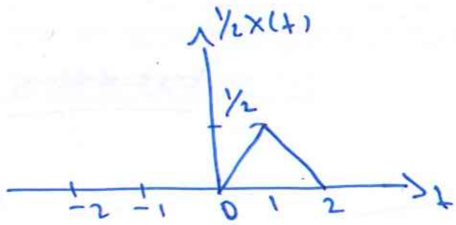
$x(t) = 2 - t$

$x(1) = 2 - 1 = 1 \quad t = 1, 2$

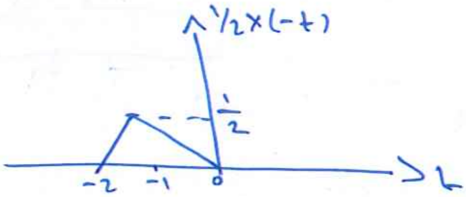
$x(2) = 2 - 2 = 0$



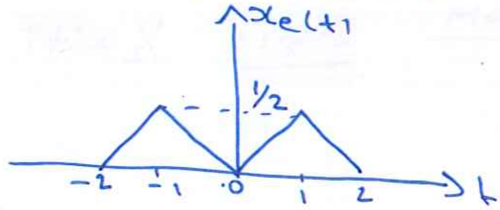
Step 1:



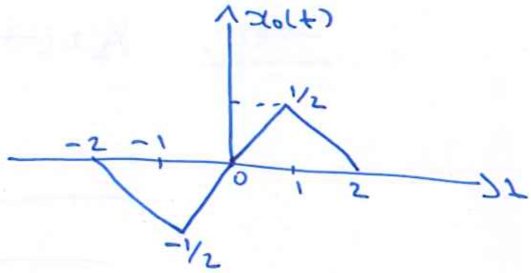
Step 2:



Step 3: $x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$



Step 4: $x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$



iii) $x(t) = \begin{cases} t & 0 \le t \le 2 \\ 4-t & 2 \le t \le 4 \end{cases}$

$x(t) = t \quad t = 0, 1, 2$

$x(0) = 0$

$x(1) = 1$

$x(2) = 2$

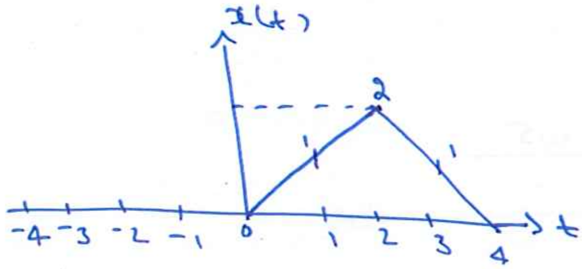
$x(t) = 4 - t$

$t = 2, 3, 4$

$x(2) = 2$

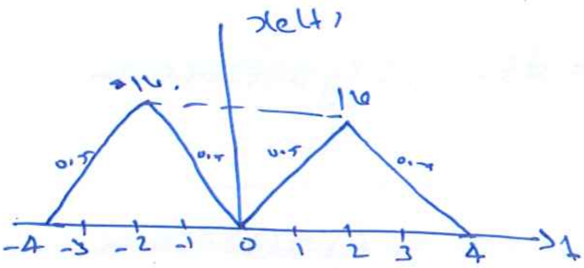
$x(3) = 4 - 3 = 1$

$x(4) = 4 - 4 = 0$

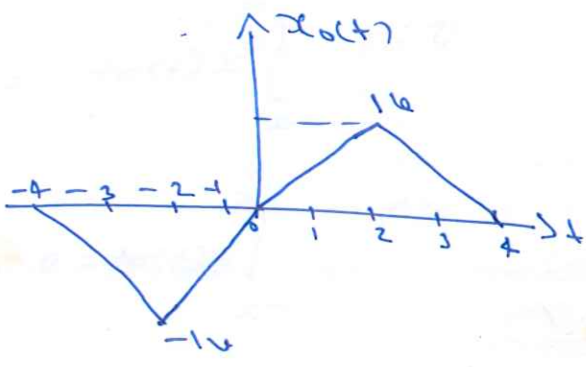


Step 1:

$x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$



$x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x(-t).$



iv) $x(n) = \begin{cases} 2^n & 0 \le n \le \infty \\ 0 & \text{otherwise.} \end{cases}$

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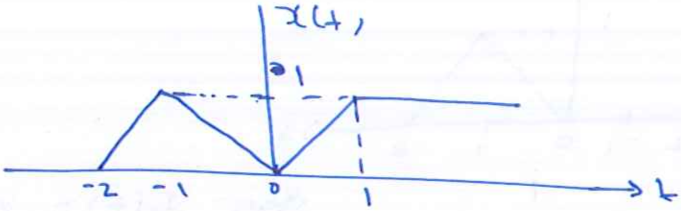
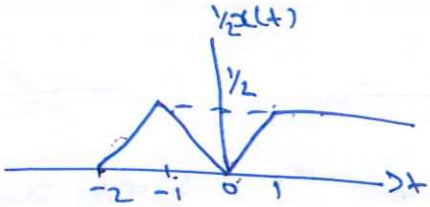
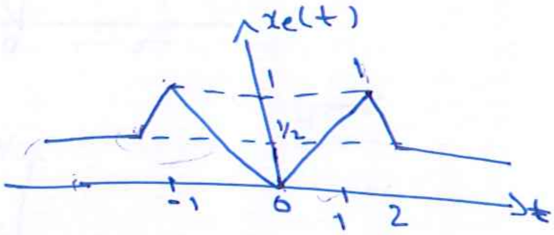
v) Determine & sketch the even & odd components of the signal $x(t)$ shown in below.



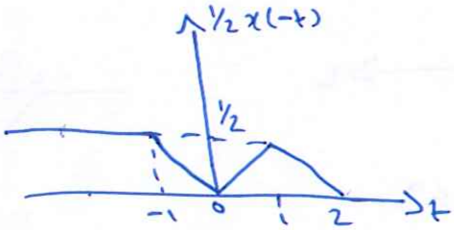
Soln. Step 1: $\frac{1}{2}x(t)$



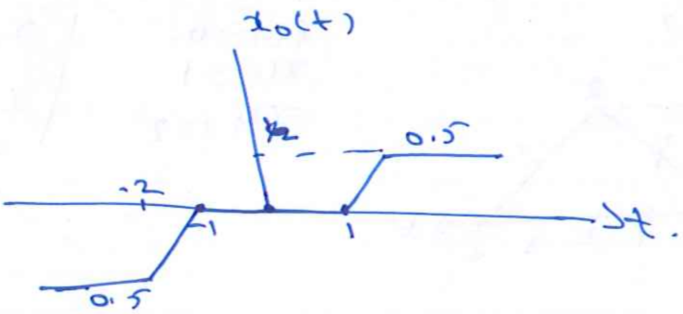
by adding:



Step 2: $\frac{1}{2}x(-t)$



iii) Subtracting:



vi) Prove that

(i) $\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt.$; if $x(t)$ is even

ii) $\int_{-a}^{a} x(t)\,dt = 0$; if $x(t)$ is odd.

Soln (i) P.T $\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$; if $x(t)$ is even

$$LHS = \int_{-a}^{a} x(t)\,dt$$

$$= \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$

$$= -\int_{0}^{-a} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$

Put $t = -t$ in first term

$$= -\int_{-t=0}^{-t=-a} x(-t)(-dt) + \int_{0}^{a} x(t)\,dt$$

$x(t) =$ even

$x(-t) = x(t)$

$$= \int_{0}^{a} x(t)\,dt + \int_{0}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$$

LHS = RHS

(ii) To prove $\int_{-a}^{a} x(t)\,dt = 0$; if $x(t)$ is odd

Soln

$$LHS = \int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$

$$= -\int_{0}^{-a} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$

Put $t = -t$ in first term

$$= -\int_{0}^{a} x(-t)(-dt) + \int_{0}^{a} x(t)\,dt$$

$$= \int_{0}^{a} x(-t)\,dt + \int_{0}^{a} x(t)\,dt$$

if $x(t)$ is odd

$x(-t) = -x(t)$

$$= -\int_{0}^{a} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$

$$= 0$$

① S.T if $x(n)$ is an odd signal then $\sum_{n=-\infty}^{\infty} x(n) = 0$

Soln

$$LHS = \sum_{n=-\infty}^{\infty} x(n)$$

$$= \sum_{n=-\infty}^{-1} x(n) + x(0) + \sum_{n=1}^{\infty} x(n)$$

$$= \sum_{n=-1}^{-\infty} x(n) + x(0) + \sum_{n=1}^{\infty} x(n)$$

Put $n=-n$ in first term

$$= \sum_{n=1}^{\infty} x(-n) + \cancel{x(0)}^{0} + \sum_{n=1}^{\infty} x(n)$$

if $x(n)$ is odd $\quad x(-n) = -x(n)$

$$= -\sum_{n=1}^{\infty} x(n) + \sum_{n=1}^{\infty} x(n) = 0$$

② $x(t) = [\sin(\pi t) + \cos(\pi t)]^2$ find Even & odd signals

Soln $\quad x_e(t) = \frac{1}{2}[x(t) + x(-t)] \qquad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

Replace t by $-t$

$$x(-t) = [\sin(-\pi t) + \cos(-\pi t)]^2$$

$$x(-t) = [\sin(-\pi t)]^2 + [\cos(-\pi t)]^2 + 2\sin(-\pi t)\cos(-\pi t)$$

$$= \sin^2(\pi t) + \cos^2 \pi t - 2\sin(\pi t)\cdot\cos(\pi t)$$

$$x(t) = \sin^2(\pi t) + \cos^2(\pi t) + 2\sin\pi t\cdot\cos\pi t.$$

$$x_e(t) = \frac{1}{2}[\sin^2\pi t + \cos^2\pi t + \sin^2\pi t + \cos^2\pi t + 2\sin\pi t\cdot\cos\pi t - 2\sin\pi t\cdot\cos\pi t]$$

$$x_e(t) = 1$$

$$x_o(t) = \frac{1}{2}[\sin^2\pi t + \cos^2\pi t + 2\sin\pi t\cdot\cos\pi t - \sin^2\pi t - \cos^2\pi t + 2\sin\pi t\cdot\cos\pi t]$$

$$x_o(t) = 2\sin\pi t\cos\pi t = \sin(2\pi t)$$

③ Show that if $x_1(n)$ is an odd signal & $x_2(n)$ is an even signal Then $x_1(n) \cdot x_2(n)$ is an odd signal.

Soln

Consider $y(n) = x_1(n) \cdot x_2(n)$ — ①

Given $x_1(n)$ is an odd signal

$x_1(-n) = -x_1(n)$

$x_2(n)$ is an even signal

$x_2(-n) = x_2(n)$

Also

Put $n = -n$ in eqn ①

$y(-n) = x_1(-n) \cdot x_2(-n)$

$y(-n) = -x_1(n) \cdot x_2(n)$

$y(-n) = -y(n)$

$\therefore$ $y(n) = x_1(n) \cdot x_2(n)$ is an odd signal //

④ let $x(n)$ be an arbitrary signal with even & odd parts denoted by $x_e(n)$ & $x_o(n)$ Respectively. Show that

$$\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n)$$

Soln

$$LHS = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$= \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$$

$$= \sum_{n=-\infty}^{\infty} [x_e^2(n) + x_o^2(n) + 2x_e(n) \cdot x_o(n)]$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \cancel{2\sum_{n=-\infty}^{\infty} x_e(n) * x_o(n)}^{\,0}$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n)$$

//

(5) if $x(n)$ is an even signal Then show that

$$\sum_{n=-\infty}^{\infty} x(n) = x(0) + 2\sum_{n=1}^{\infty} x(n)$$

Soln

$$LHS = \sum_{n=-\infty}^{\infty} x(n)$$

$$= \sum_{n=-\infty}^{-1} x(n) + x(0) + \sum_{n=1}^{\infty} x(n)$$

$$= \sum_{n=-1}^{-\infty} x(n) + x(0) + \sum_{n=1}^{\infty} x(n)$$

Replace n by $-n$ in first term

$$= \sum_{n=1}^{\infty} x(-n) + x(0) + \sum_{n=1}^{\infty} x(n)$$

$x(n)$ is even Then

$$x(-n) = x(n)$$

$$= \sum_{n=1}^{\infty} x(n) + x(0) + \sum_{n=1}^{\infty} x(n)$$

$$= x(0) + 2\sum_{n=1}^{\infty} x(n) \quad //$$

LHS = RHS //

1) Find The Even & odd Parts of The Signals.

(a) $x(t) = e^{t} + e^{-t}$

Soln $\quad x_e(t) = \frac{1}{2}[x(t) + x(-t)] \qquad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$x(-t) = e^{-t} + e^{t}$

$x_e(t) = \frac{1}{2}[\cancel{x}\, e^{t} + e^{-t} + e^{-t} + e^{t}]$

$x_e(t) = e^{t} + e^{-t} = 2\cosh t$

$x_o(t) = \frac{1}{2}[e^{t} + e^{-t} - e^{-t} - e^{t}]$

$x_o(t) = 0$

(b) $x(t) = \sin(\omega_0 t + \pi/4)$

$x(-t) = \sin(-\omega_0 t + \pi/4) = \sin(-(\omega_0 t - \pi/4))$

$x(-t) = -\sin(\omega_0 t - \pi/4)$

$x_e(t) = \frac{1}{2}[\sin(\omega_0 t + \pi/4) - \sin(\omega_0 t - \pi/4)]$

$\left[\because \begin{array}{l} \sin(A+B) = \sin A \cos B + \cos A \sin B \\ \sin(A-B) = \sin A \cos B - \cos A \sin B \end{array}\right]$

$x_e(t) = \frac{1}{2}\left[[\sin\omega_0 t \cdot \cos\pi/4 + \cos\omega_0 t \sin\pi/4] - [\sin\omega_0 t \cdot \cos\pi/4 - \cos\pi/4 \sin\omega_0 t]\right]$

$= \frac{1}{2}\{\cancel{\sin\omega_0 t \cos\pi/4} + \cos\omega_0 t \sin\pi/4 - \cancel{\sin\omega_0 t \cdot \cos\pi/4} + \cos\pi/4 \sin\omega_0 t\}$

$= \cos\omega_0 t \sin\pi/4 = \frac{1}{\sqrt{2}} \cos\omega_0 t.$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$x_o(t) = \frac{1}{2}\left[[\sin\omega_0 t \cos\pi/4 + \cos\omega_0 t \sin\pi/4 + \sin\omega_0 t \cos\pi/4 - \cos\pi/4 \sin\omega_0 t]\right]$

$= \frac{1}{2}[2\sin\omega_0 t \cos\pi/4)$

$= \sin\omega_0 t \cos\pi/4 = \frac{1}{\sqrt{2}} \sin\omega_0 t$

(2) Sketch The following signals

(a) $x_1(n) = \text{rect}(n/6)$ $\quad$ (b) $x_2(n) = \text{rect}\left(\frac{n-2}{4}\right)$

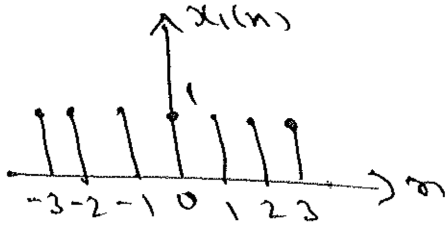
(c) $x_3(n) = 6\,\text{tri}\left(\frac{n-4}{3}\right)$

Soln (a) $x_1(n) = \text{rect}(n/6)$.

$x_1(n)$ is a 7 sample rectangular pulse from $n = -3$ to 3

$\text{rect}(n) = 1 \quad |n| \le N_0$

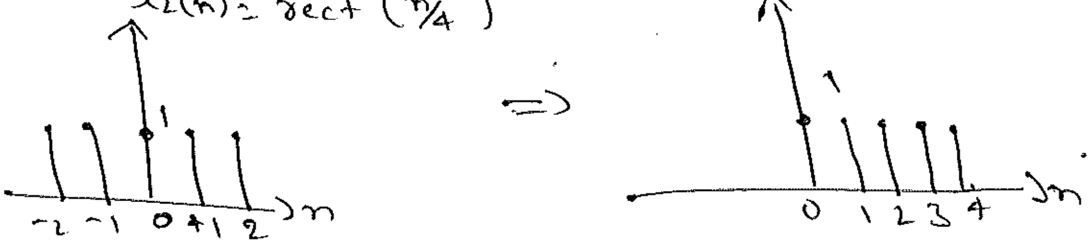
$\quad\quad 0 \quad |n| > N_0$



(b) $x_2(n) = \text{rect}\left(\frac{n-2}{4}\right)$

$x_2(n)$ is a 5 sample rectangular pulse centered at $n = 2$

$x_2(n) = \text{rect}(n/4) \quad \Rightarrow \quad x_2(n) = \text{rect}\left(\frac{n-2}{4}\right)$



(c) $x_3(n) = 6\,\text{tri}\left(\frac{n-4}{3}\right)$

$x_3(n)$ is a 7 samples centered at $n = 4$ with end values being zero.

$6\,\text{tri}(n/3)$

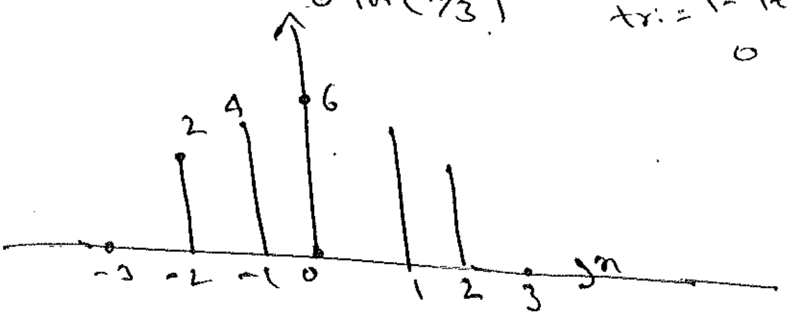
$\text{tri} = 1 - |t| \quad |t| \le 1$

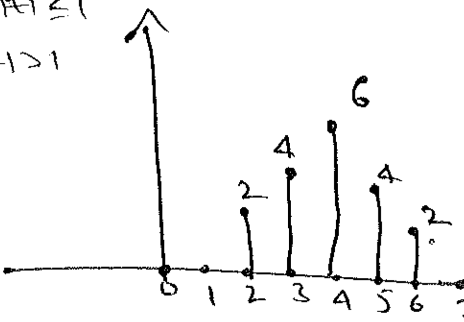
$\quad\quad 0 \quad |t| > 1$

$x_3(n) = 6\,\text{tri}\left(\frac{n-4}{3}\right)$

$1 - \frac{|n|}{N_0}$

$|n| \le N_0$



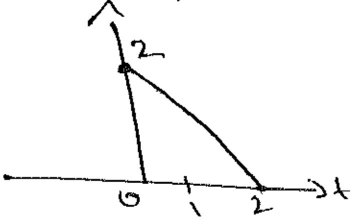


(3) Find the even & odd parts of the signal

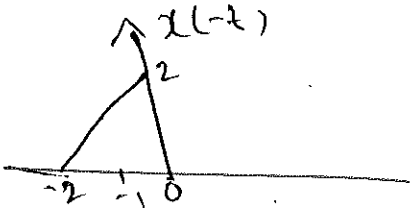
$x(t) = -t + 2 \quad ; \; 2 \ge t \ge 0$

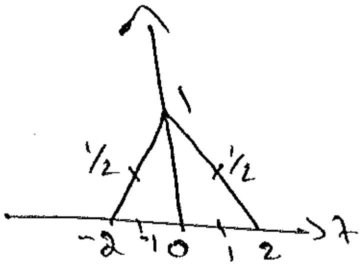
$\quad\quad 0 \quad ; \;$ otherwise

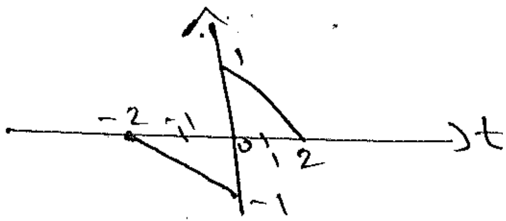
Soln



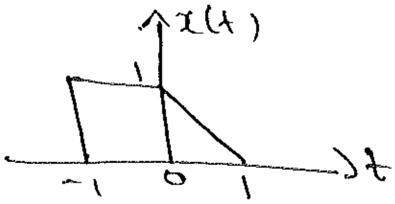
$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \quad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$



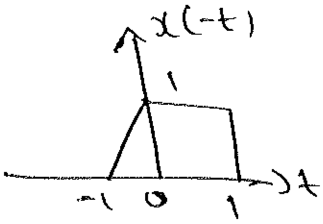




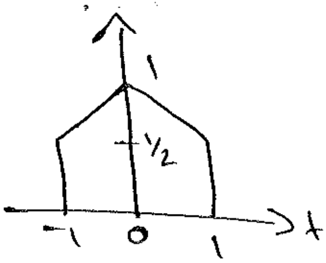
(b)



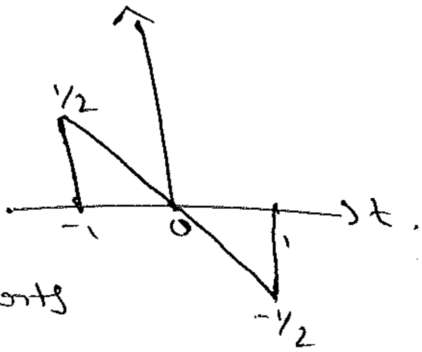
Soln



$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$



$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$



(4)

$x(n) = n^2 (\frac{1}{2})^{n-2}$ find Even & odd parts

Soln $x(n) = n^2 (\frac{1}{2})^n \cdot 2^2 = 4n^2 (\frac{1}{2})^n$

$x(-n) = 4n^2 (\frac{1}{2})^{-n}$

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$x_e(n) = \frac{1}{2}[4n^2 (\frac{1}{2})^n + 4n^2 (\frac{1}{2})^{-n}]$

$x_e(n) = 2n^2 [(\frac{1}{2})^n + (\frac{1}{2})^{-n}]$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

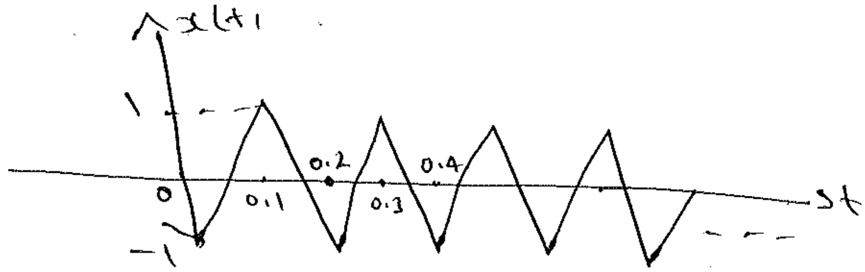
$x_o(n) = \frac{1}{2}[4n^2 (\frac{1}{2})^n - 4n^2 (\frac{1}{2})^{-n}]$

$x_o(n) = 2n^2 [(\frac{1}{2})^n - (\frac{1}{2})^{-n}]$

Problems on Periodic & non Periodic signals

1) For The triangular wave what is The fundamental frequency of This wave? Explain The fundamental frequency in units of Hz & rad/sec

Soln

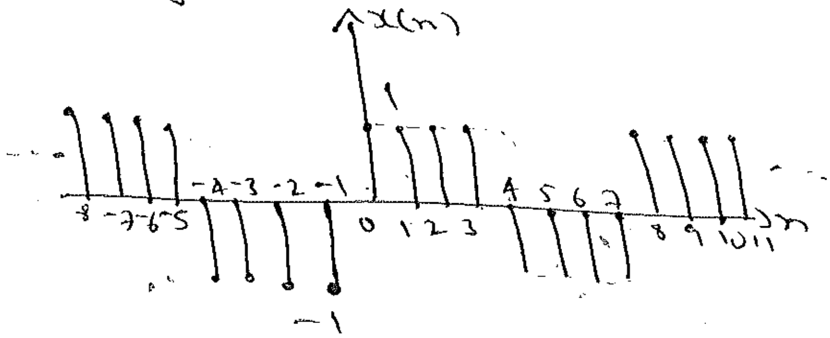


$T = T_1 - T_0 = 0.2 - 0 = 0.2$

fundamental frequency $f = \frac{1}{T} = \frac{1}{0.2} = 5\ Hz.$

$\omega = 2\pi f = 2\pi \times 5 = 10\pi$ rad/sec

(2) Determine the fundamental frequency of the discrete time signal square wave shown in figure.



Soln

$N = N_1 - N_0 = 8 - 0 = 8$ or $x(n) = x(n+N)$ $N=8$)

$x(0) = x(0+8)$

$\Omega = \frac{2\pi}{N} = \frac{2\pi}{8} = \pi/4$ radians

$x(0) = x(8)$

$N = \frac{2\pi}{\Omega} =$

(3) For each of the following signals, determine whether it is periodic and if it is, find the fundamental period.

(a) $x(t) = \cos^2(2\pi t)$

Soln

w.k.T $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$x(t) = \frac{1+\cos 4\pi t}{2}$

$x(t) = \frac{1}{2} + \frac{1}{2}\cos 4\pi t$

DC.

$x(t) = \frac{1}{2}\cos 4\pi t$ $x(t) = A\cos(\omega t + \phi)$.

$\omega = 4\pi$ $\phi = 0$

w.k.T $\omega = \frac{2\pi}{T}$

$T = \frac{2\cancel{\pi}}{4\cancel{\pi}} = 1/2$ sec.

$\therefore$ it is periodic with fundamental period is $1/2$ sec.

26d

(b) $x(t) = \sin^3(2t)$

Soln

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$$

$$x(t) = \frac{3}{4}\sin(2t) - \frac{1}{4}\sin(6t).$$

$\omega_1 = 2$ $\qquad\qquad$ $\omega_2 = 6.$

$\omega_1 = \frac{2\pi}{T_1}$ $\quad T_1 = \frac{2\pi}{2}.$ $\qquad\qquad$ $\omega_2 = \frac{2\pi}{T_2}$ $\quad T_2 = \frac{2\pi}{6}$

$T_1 = \pi$ $\qquad\qquad$ $T_2 = \pi/3$

$$T = \frac{T_1}{T_2} = \frac{\pi}{\pi/3} = 3$$

$$T = \frac{T_1}{T_2} = \frac{3}{1}$$

$$T = T_1 \times 1 = T_2 \times 3$$

$$T = \pi \quad = \pi/3 \times 3 = \pi$$

$T = \pi$ sec. it is Periodic.

c) $x(t) = e^{-2t}\cos(2\pi t)$

Soln

e^{-2t} is an exponential ~~[illegible]~~

it is non-periodic

$\therefore x(t)$ is also non Periodic.

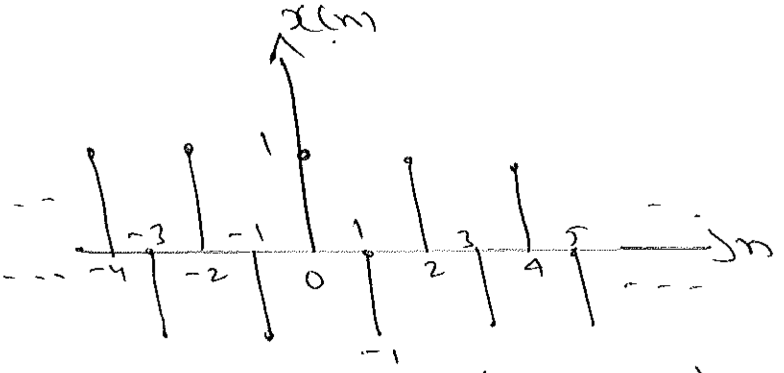
(d) $x(n) = (-1)^n.$

Soln

$x(n) = (-1)^n$

i.e $\quad x(n) = 1$ for n = even

$\qquad\qquad = -1$ for n = odd.



it is Periodic with fundamental Period is $\underline{N=2}$ samples $x(n) = x(n+N)$.

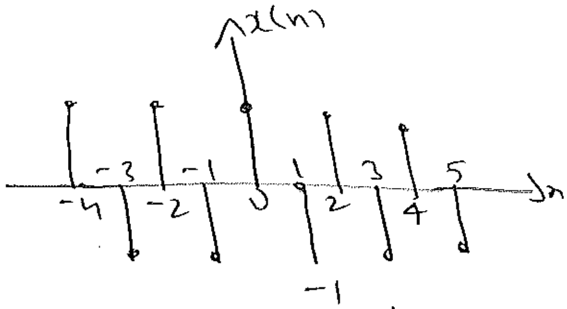
$x(0) = x(0+2) = x(2)$

(e.) $x(n) = (-1)^{n^2}$

Soln

$x(n) = 1$ for n even

-1 $n =$ odd.



Periodic with fundamental period of $\underline{2 \text{ samples}}$

(f.) $x(n) = \cos(2n)$.

Soln

$\Omega = \frac{2\pi}{N} \cdot m$

$\cos(\Omega n + \phi)$.

$\Omega = 2.$

$N = \frac{2\pi}{2} = \pi$

it is non periodic.

(9) $x(n) = \cos(2\pi n)$

Soln

$\Omega = 2\pi$

$\Omega = \frac{2\pi}{N} \cdot m$

$N = \frac{2\pi}{\Omega} = \frac{2\pi}{2\pi} = 1.$

$\underline{\underline{N = 1 \text{ samples}}}$

It is periodic with a fundamental period of 1 samples.

1) $\cos(0.01\pi n)$

Soln

$\Omega = 0.01\pi$

$\Omega = \frac{2\pi}{N}$

$N = \frac{2\pi}{0.01\pi}$

$\underline{\underline{N = 200}}$

It is periodic with fundamental period 200 samples

2) $\sin 3n$

Soln

$\Omega = 3$

$\Omega = \frac{2\pi}{N}$

$N = \frac{2\pi}{3}$

It is non periodic.

3) $\cos \frac{2\pi}{5} n + \cos \frac{2\pi}{7} n.$

Soln

$\Omega_1 = \frac{2\pi}{5}$ $\Omega_2 = \frac{2\pi}{7}.$

$N_1 = \frac{2\pi}{2\pi/5}$ $N_2 = \frac{2\pi}{2\pi/7}$

$N_1 = 5$ $N_2 = 7.$

$\underline{\underline{N = 35}}$ LCM of N_1 & $N_2 = 35.$

or $N = \frac{N_1}{N_2} = \frac{5}{7}$ $= N_1 \times 7 = N_2 \times 5$

$= 5 \times 7 = 7 \times 5 = 35.$

It is periodic with fundamental period 35 samples

(4) $x(n) = \cos(n/8) \cdot \cos(n\pi/8)$.

Soln

$\Omega_1 = 1/8$ $\qquad$ $\Omega_2 = \pi/8$.

$N_1 = \dfrac{2\pi}{1/8}$ $\qquad$ $N_2 = \dfrac{2\pi}{\pi/8}$

$N_1 = 16\pi$ $\qquad$ $N_2 = 16$

$N = 16\pi$ $\qquad$ LCM of 16π & 16.

it is nonperiodic.

(5) $x(n) = \sin(\pi + 0.2n)$.

Soln $\quad \Omega = 0.2$

$N = \dfrac{2\pi}{0.2} = 10\pi$ $\qquad$ It is non periodic

(6) $x(n) = e^{\left(j\frac{\pi}{4}\right)n}$.

Soln. $\quad x(n) = \cos \pi/4\, n + j \sin \pi/4\, n$. $\qquad e^{j\theta} = \cos\theta + j\sin\theta$

$\Omega_1 = \pi/4$ $\qquad$ $\Omega_2 = \pi/4$ $\qquad$ $e^{j\Omega n}$

$N_1 = \dfrac{2\pi}{\pi/4}$ $\qquad$ $N_2 = \dfrac{2\pi}{\pi/4}$ $\qquad$ $\Omega = \pi/4$

$\Omega = \dfrac{2\pi}{N}$

$N_1 = 8$ $\qquad$ $N_2 = 8$. $\qquad$ $N = \dfrac{2\pi}{\pi/4}$

$\underline{\underline{N = 8}}$ $\qquad$ LCM of N_1 & N_2. $\qquad$ $N = 8$.

it is periodic with fundamental period 8 samples

(7) $x(t) = \cos(t + \pi/4)$.

Soln $\quad \omega = 1$.

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{1}$

$x(t)$ is periodic with fundamental period $T = 2\pi$ seconds

(8) $x(t) = \sin\left(\frac{2\pi}{3}t\right)$.

Soln

$\omega = 2\pi/3$

$\omega = \frac{2\pi}{T}$ $\quad T = \frac{2\pi}{2\pi/3} = 3$ seconds

it is Periodic with fundamental Period 3 seconds.

(9) $x(t) = \cos \pi/3\, t + \sin \pi/4\, t$.

Soln

$\omega_1 = \pi/3$ $\quad \omega_2 = \pi/4$

$T_1 = \frac{2\pi}{\pi/3}$ $\quad T_2 = \frac{2\pi}{\pi/4} = 8$

$T_1 = 6$

$$\begin{array}{r|l} 2 & 6, 8 \\ \hline & 3, 4 \end{array}$$

$T = 24$ seconds. LCM of 6 & 8

it is periodic

$T = \frac{T_1}{T_2} = \frac{6}{8} = 3/4$

$T = T_1 \times 4 = T_2 \times 3$

$T = 24 = 24 = 24$ seconds.

(10) $x(t) = \cos t + \sin\sqrt{2}t$.

$x(t) = x_1(t) + x_2(t)$

Soln

$\omega_1 = 1$ $\quad \omega_2 = \sqrt{2}$

$T_1 = \frac{2\pi}{1}$ $\quad T_2 = \frac{2\pi}{\sqrt{2}}$.

$T_1 = 2\pi$ $\quad T_2 = \frac{2\pi}{\sqrt{2}}$

$T = \frac{T_1}{T_2} = \frac{2\pi}{2\pi/\sqrt{2}} = \sqrt{2}$. it is irrational number

$\therefore$ it is non periodic

(11) $x(t) = \sin^2 t$.

Soln $\quad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$= \frac{1 - \cos 2t}{2} = 1/2 - 1/2 \cos 2t$.

$x(t) = -\frac{1}{2}\cos 2t$

$A = -\frac{1}{2}$ $\omega = 2$

$\omega = \frac{2\pi}{T}$ $T = \frac{2\pi}{2}$

$T = \pi$ seconds it is periodic with fundamental period π seconds.

(12) $x(t) = e^{j(\frac{\pi}{2}t - 1)}$

Soln

$x(t) = e^{j\frac{\pi}{2}t} \cdot e^{-j}$

$\omega = \frac{\pi}{2}$ $e^{j\omega t}$

$T = \frac{2\pi}{\pi/2} = 4$ seconds it is period with fundamental period 4 seconds.

(13) $x(n) = 5\cos(0.2\pi n)$

$\Omega = 0.2\pi$

$N = \frac{2\pi}{0.2\pi} = 10$ samples.

it is periodic with fundamental period 10 samples

(14) $x(t) = 2\cos(3t + \frac{\pi}{4})$.

$\omega = 3$.

$T = \frac{2\pi}{3}$

it is periodic with fundamental period. $2\pi/3$.

(15) $x(t) = e^{-j2t}$

Soln $e^{j\omega t}$

$\omega = 2$.

$T = \frac{2\pi}{2}$

$T = \pi$

periodic with FP π sec

(16) $x(t) = \{e^{j(\pi t - 1)}\}$

$x(t) = e^{j(\pi t - 1)}$

$= e^{j\pi t} \cdot e^{-1}$

$\omega = \pi$

$T = \frac{2\pi}{\pi} = 2$ seconds.

periodic, Period $= 2$.

(269) (a)

(17) $x(t) = [\sin(t - \pi/6)]^2$.

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$.

Soln $x(t) = \frac{1 - \cos 2(t - \pi/6)}{2}$.

$x(t) = \underbrace{1/2}_{D.C} - 1/2 \cos(2t - \pi/3)$

$x(t) = -1/2 \cos(2t - \pi/3)$

$x(t) = A \cos(\omega t + \phi)$.

$A = 1/2 \quad \omega = 2$.

$\omega = 2$

$\omega = \frac{2\pi}{T} \quad T = \frac{2\pi}{2} = \pi$ seconds.

Periodic, period $= \pi$ seconds

(18) $x(t) = Re(e^{j2t}) + Img\{e^{j3t}\}$.

$x(t) = \cos 2t + j\sin 3t$.

$\omega_1 = 2 \qquad \omega_2 = 3$

$T_1 = \frac{2\pi}{2} = \pi \qquad T_2 = \frac{2\pi}{3}$

$T = \frac{T_1}{T_2} = \frac{\pi}{2\pi/3} = 3/2$

LCM of T_1 & T_2

π, 2π is $\underline{\underline{2\pi}}$

$T = T_1 \times 2 = T_2 \times 3$

$= 2\pi = \frac{2\pi}{3} \times 3$

$\underline{\underline{T = 2\pi \text{ seconds}}}$.

Periodic, period $= 2\pi$ sec

(19) $x(n) = \cos\left(\frac{8\pi}{7} n + 2\right)$

$\Omega = \frac{8\pi}{7}$

$m = 4$.

$N = \frac{2\pi}{8\pi/7} = 7/4$.

$\frac{N}{m} = 7$. periodic, period $= 7$.

(20) $x(n) = \cos\left(\frac{1}{3}\pi n\right) \sin(2n)$.

Soln

it is non periodic

$\Omega_1 = \frac{1}{3}\pi$ $\qquad$ $\Omega_2 = 2$

$N_1 = \frac{2\pi}{\frac{1}{3}\pi} = 6$ $\qquad$ $N_2 = \frac{2\pi}{\pi} = \pi$

$N = 6\pi$ it is non periodic.

$N = \frac{N_1}{N_2} = \frac{6}{\pi} = N_1 \times \pi = N_2 \times 6$

$= 6\pi = \pi \times 6$

$= 6\pi$

(21) Determine whether the D.T signal $z(n)$ is periodic where $z(n) = z_1(n) + z_2(n)$ where $z_1(n)$ & $z_2(n)$ are periodic with period of 90 & 54 respectively.

Soln

$z(n) = z_1(n) + z_2(n)$.

Period of $z_1(n) = N_1 = 90$

— " — $z_2(n) = N_2 = 54$.

Step 1: $\frac{N_1}{N_2} = \frac{90}{54} = 5/3$ = rational

$\therefore z(n)$ is periodic.

$N = \frac{N_1}{N_2} = 5/3$

$N = N_1 \times 3 = N_2 \times 5$

$N = 90 \times 3 = 54 \times 5$

$N = 270$ samples

LCM of N_1 & N_2

2	90, 54
3	45, 27
3	15, 9
	5, 3

1) Let $x_1(t)$ and $x_2(t)$ be two periodic signals with fundamental periods T_1 & T_2 respectively. under what conditions, the sum $x(t) = x_1(t) + x_2(t)$ is periodic and what is the fundamental period of $x(t)$, if it is periodic?

Soln:

Since $x_1(t)$ and $x_2(t)$ are periodic with fundamental periods T_1 and T_2 respectively, we have

$$x_1(t) = x_1(t+T_1) = x_1(t+mT_1), \; m = \text{a positive integer}.$$

$$x_2(t) = x_2(t+T_2) = x_2(t+kT_2) \quad k = \text{a positive integer}.$$

Thus

$$x(t) = x_1(t+mT_1) + x_2(t+kT_2)$$

in order for $x(t)$ to be periodic with period T, it is required that.

$$x(t+T) = x_1(t+T) + x_2(t+T).$$

$$= x_1(t+mT_1) + x_2(t+kT_2)$$

Thus we must have

$$mT_1 = kT_2 = T$$

$$\frac{T_1}{T_2} = \frac{k}{m} = \text{a rational number}.$$

$\therefore$ The sum of two periodic signals is periodic, only if the ratio of their respective periods can be expressed as a rational number. Then, the fundamental period is the LCM of T_1 & T_2.

1) For each of The following signals, determine whether it is Periodic, and if it is, find The fundamental Period.

1) $\cos[3n]$ 2) $x(n) = \cos(2\pi n)$.

Soln

1) $\cos[3n]$

$\Omega = \frac{2\pi}{N} \cdot m$

$\Omega = 3.$

$\frac{N}{m} = \frac{2\pi}{\Omega}$

$\frac{N}{m} = \frac{2\pi}{3}$

$N = 2\pi$

nonperiodic.

2) $x(n) = \cos(2\pi n)$

$\Omega = 2\pi$

$\Omega = \frac{2\pi}{N} \cdot m$

$\frac{N}{m} = \frac{2\pi}{\Omega}$

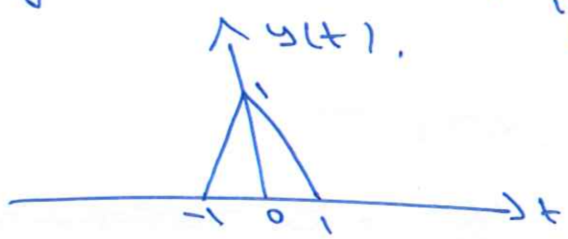
$\frac{N}{m} = \frac{2\pi}{2\pi}$

$\frac{N}{m} = \frac{1}{1}$

$N = 1$

Periodic, with a fundamental period of 1 sample.

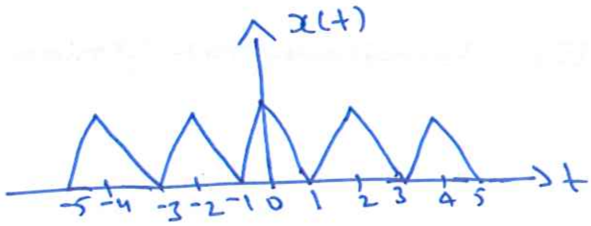
2) Determine whether The signal $x(t) = \sum_{k=-2}^{2} y(t-2k)$ for $y(t)$ shown in below fig. is periodic or not. if Periodic, find its fundamental Period.



Soln

$x(t) = \sum_{k=-2}^{2} y(t-2k)$

$= y(t+4) + y(t+2) + y(t) + y(t-2) + y(t-4).$



non-periodic

Here The cycle repeats only between $t=-5$ & $t=5$. For The signal to be periodic, The cycle must repeat between $t=-\infty$ & $t=\infty$
The given $x(t)$ is non periodic.

1) Check for periodicity of the signal $x(t) = e^{j\pi t}$.

Soln $x(t) = e^{j\pi t}$

$$\omega = \frac{2\pi}{T}$$

$$x(t) = \cancel{\cos\omega t + j\sin\omega t} \cos\pi t + j\sin\pi t.$$

$\cancel{\omega}$ $\omega = \pi$

$$T = \frac{2\pi}{\pi}$$

$$\underline{\underline{T = 2 \text{ sec}}}.$$

2) Determine whether the continuous-time signal ~~x(t)~~ $x(t) = x_1(t) + x_2(t) + x_3(t)$ is periodic, where $x_1(t)$, $x_2(t)$ & $x_3(t)$ have periods of $8/3$, 1.26 & $\sqrt{2}$ sec respectively.

Soln

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

Period of $x_1(t)$: $T_1 = 8/3$ sec

Period of $x_2(t)$: $T_2 = 1.26$ sec

Period of $x_3(t)$: $T_3 = \sqrt{2}$ sec

Step 1: $\frac{T_1}{T_2} = \frac{8/3}{1.26} = \frac{8}{3.78} = \frac{800}{378} = \frac{400}{189}$ = rational.

$\frac{T_1}{T_3} = \frac{8/3}{\sqrt{2}} = \frac{8}{3\sqrt{2}}$ = not rational.

T_1/T_3 cannot be brought to the form of ratio of integers.

Step 2: $\therefore x(t) = x_1(t) + x_2(t) + x_3(t)$ is not periodic

Note:

The following steps can be used to determine the period of the summation of N periodic signals $x_1(t), x_2(t) \ldots x_m(t)$

1. Obtain the ratio T_1/T_i & convert it into a ratio of integer (rational), where T_1 is the fundamental period of $x_1(t)$ & T_i is the fundamental period of $x_i(t)$ where $2 \le i \le m$.
2. if this conversion is not possible, then the sum signal is not periodic
3. if possible, then find greatest common divisor (G.C.D) from the numerator & denominator of each individual ratio.
4. Then find least common multiple (L.C.M) of the denominators of the resulting ratios, say this is 'λ'.
5. Then the period of the sum signal is given by $T = T_1 \cdot \lambda$

2) Determine whether the CT signal $y(t) = y_1(t) + y_2(t) + y_3(t)$ is periodic, where $y_1(t)$, $y_2(t)$ & $y_3(t)$ have periods of 1.08, 3.6 & 2.025 sec respectively.

Soln:

Given $y(t) = y_1(t) + y_2(t) + y_3(t)$

Periods of $y_1(t)$: $T_1 = 1.08\,sec$

—"— $y_2(t)$: $T_2 = 3.6\,sec$

—"— $y_3(t)$: $T_3 = 2.025\,sec$

Step1: $\dfrac{T_1}{T_2} = \dfrac{1.08}{3.6} = \dfrac{108}{360} = \dfrac{3}{10}$ = rational.

$\dfrac{T_1}{T_3} = \dfrac{1.08}{2.025} = \dfrac{1080}{2025} = 8/15$ = rational

Step2: $\dfrac{T_1}{T_2} = \dfrac{3}{10}$ & $\dfrac{T_1}{T_3} = \dfrac{8}{15}$

g.c.d of $(3, 8) = 1$

g.c.d of $(10, 15) = 5$

$$\therefore \frac{T_1}{T_2} = \frac{3(1)}{2(5)} \ \& \ \frac{T_1}{T_3} = \frac{8(1)}{3(5)}$$

Step 4: LCM of the denominators

i.e LCM of $(2), (3), (5) = 30 = L$.

Step 5:

Period of the sum signal $y(t)$ is

$$T = T_1 \cdot L = 1.08(30) = 32.4 \text{sec}.$$

Determine whether the following DT & CT signals are periodic or not? if periodic determine fundamental period.

a) $2\cos 100\pi t + 5\sin 50t$.

Soln

$\omega_1 = 100\pi$ $\qquad$ $\omega_2 = 50$

$T_1 = \frac{2\pi}{100\pi}$ $\qquad$ $T_2 = \frac{2\pi}{50}$

$T_1 = \frac{1}{50}$ $\qquad$ $T_2 = \frac{\pi}{25}$.

$$T = \frac{T_1}{T_2} = \frac{25}{50\pi} = \frac{1}{2\pi}$$

$T = \frac{1}{2\pi}$ which is not rational

$\therefore$ signal is non periodic

b) $x(t) = \cos 100\pi t + \sin 50\pi t$

Soln

$\omega_1 = 100\pi$ $\qquad$ $\omega_2 = 50\pi$ $\qquad$ $\omega = \frac{2\pi}{T}$.

$T_1 = \frac{2\pi}{100\pi}$ $\qquad$ $T_2 = \frac{2\pi}{50\pi}$

$T_1 = \frac{1}{50}$ $\qquad$ $T_2 = \frac{1}{25}$

$$T = \frac{T_1}{T_2} = \frac{25}{50} = \frac{1}{2}.$$

$$T = T_1 \times 2 = T_2 \times 1$$

$T = \frac{2}{50} = \frac{1}{25}$ $\qquad$ $T = 1/25$ $\qquad$ signal is periodic

c) $x(t) = 2\cos t + 3\cos t/3$.

$\omega = \frac{2\pi}{T}$ $\quad \omega_1 = 1$ $\quad \omega_2 = 1/3$

$T_1 = 2\pi$ $\quad T_2 = \frac{2\pi}{1/3}$

$T_2 = 6\pi$

$T = \frac{T_1}{T_2} = \frac{2\pi}{6\pi} = \frac{1}{3}$

$T = T_1 \times 3 = T_2 \times 1$

$= 2\pi \times 3 = 6\pi \times 1$

$T = 6\pi$ sec $\quad$ periodic

D) $x(n) = \cos\left(\frac{1}{5}\pi n\right) \cdot \sin\left(\frac{1}{3}\pi n\right)$.

Soln $x(n) = \cos\left(\frac{1}{5}\pi n\right) \cdot \sin\left(\frac{1}{3}\pi n\right)$

$\sin A \cdot \cos B = 1/2\left[\sin(A+B) + \sin(A-B)\right]$.

$x(n) = \frac{1}{2}\left\{\sin\left[\frac{1}{3}\pi n + \frac{1}{5}\pi n\right] + \sin\left[\frac{1}{3}\pi n - \frac{1}{5}\pi n\right]\right\}$

$x(n) = \frac{1}{2}\left\{\sin\left(\frac{8\pi n}{15}\right) + \sin\left[\frac{2\pi n}{15}\right]\right\}$.

~~cos~~: $\Omega_1 = \frac{8\pi}{15}$ $\quad \Omega_2 = \frac{2\pi}{15}$

$N_1 = \frac{\cancel{2\pi}}{\frac{\cancel{48\pi}}{15}}$ $\quad N_2 = \frac{\cancel{2\pi}}{\frac{\cancel{2\pi}}{15}}$

$N_1 = \frac{15}{4}$ $\quad N_2 = 15$.

$N = \frac{N_1}{N_2} = \frac{\frac{15}{4}}{15} = \frac{1}{4}$

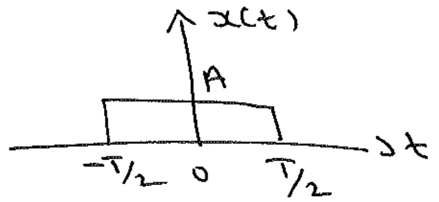
$N = N_1 \times 4 = N_2 \times 1$

$= \frac{15}{4} \times 4 = 15 \times 1$

$N = 15$ sec $\quad$ Periodic

Problems on Energy & Power Signals

1) What is the total Energy of the Rectangular pulse shown in figure



Or

$$x(t) = \begin{cases} A & ; |t| \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

Soln The given signal is nonperiodic

∴ it is Energy signal

$$x(t) = \begin{cases} A & ; -T/2 < t < T/2 \\ 0 & ; \text{otherwise} \end{cases}$$

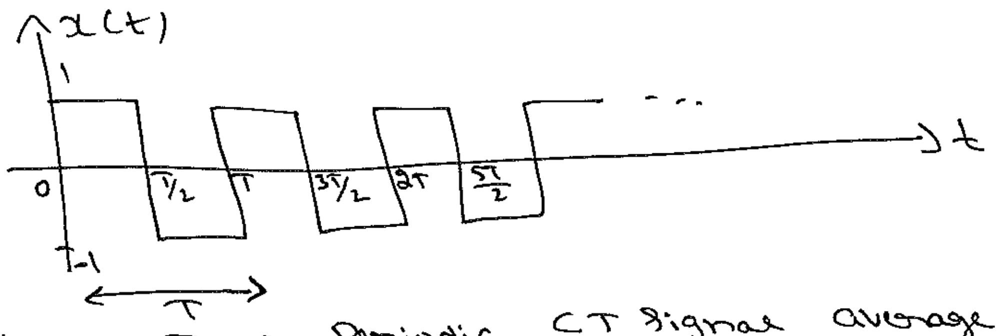
$$E = \int_{-\infty}^{\infty} (x(t))^2 \, dt$$

$$E = \int_{-T/2}^{T/2} A^2 \, dt = A^2 t \Big|_{-T/2}^{T/2}$$

$$= A^2 \left[T/2 - (-T/2) \right]$$

$$E = A^2 T \; J$$

2) What is the average power of the square wave shown in below figure.



Soln Note: In a Periodic CT signal average power is defined for one cycle.

$$T = T - 0 = T$$

$$x(t) = \begin{cases} 1 & \text{for } 0 \leq t < T/2 \\ -1 & \text{for } T/2 < t < T \end{cases}$$

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

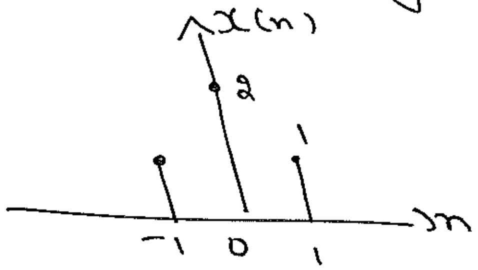
$$= \frac{1}{T}\left[\int_0^{T/2} 1^2\,dt + \int_{T/2}^{T} (-1)^2\,dt\right]$$

$$= \frac{1}{T}\left[t\Big|_0^{T/2} + t\Big|_{T/2}^{T}\right]$$

$$= \frac{1}{T}\left[[T/2 - 0] + T - T/2\right] = \frac{1}{T}[T]$$

$$P = \underline{\underline{1}} \text{ watt}$$

(3) what is The total Energy of The D.T signal $x(n)$ shown in below figure



Soln: it is non Periodic

$\therefore$ it is Energy signal

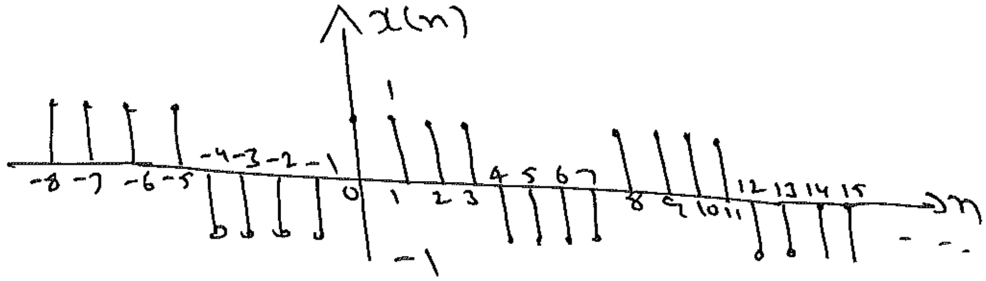
$$E = \sum_{n=-\infty}^{\infty} (x(n))^2$$

$$E = \sum_{n=-1}^{1} (x(n))^2 = (x(-1))^2 + (x(0))^2 + (x(1))^2$$

$$E = 1 + 4 + 1 = 6\,J$$

4) Check The following signals are Energy or Power if it is find corresponding signal

(a)



Soln

it is periodic signal

$\therefore$ it is Power signal

$$P = \frac{1}{T}\int_0^T (x(t))^2\,dt \quad \text{for C.T}$$

$$P = \frac{1}{N}\sum_{n=0}^{N-1} (x(n))^2 \quad \text{for D.T}$$

N = 8 samples

$$P = \frac{1}{8}\sum_{n=0}^{7} (x(n))^2$$

$$P = \frac{1}{8}\left[(x(0))^2 + (x(1))^2 + (x(2))^2 + (x(3))^2 + \cdots + (x(7))^2\right]$$

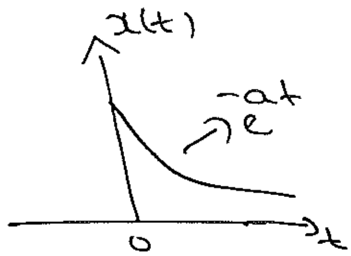
$$P = \frac{1}{8}\left[1+1+1+1+1+1+1+1\right]$$

$$P = \frac{1}{8}\,[8]$$

P = 1 watt

(b)

$$x(t) = e^{-at} \;;\; 0 \le t \le \infty$$
$$0 \;;\; \text{otherwise}$$



Soln

it is nonperiodic signal

$\therefore$ it is Energy signal

$e^{\infty} = \infty$

$e^{-\infty} = 0$

$e^{0} = 1$

$\frac{1}{\infty} = 0$

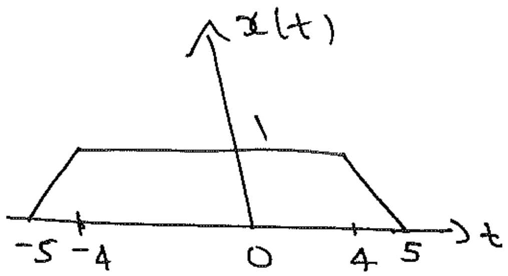
$\frac{1}{e^{\infty}} = 0$

$a^{mn} = a^{m\,n} = a^{mn}$

$$E = \int_{-\infty}^{\infty} (x(t))^2\,dt = \int_0^{\infty} (e^{-at})^2\,dt = \int_0^{\infty} e^{-2at}\,dt$$

$$E = \left.\frac{e^{-2at}}{-2a}\right|_0^{\infty} = \frac{e^{-\infty} - e^{0}}{-2a} = \frac{-1}{-2a} = 1/2a \text{ J}$$

(c) For The trapezoidal pulse $x(t)$ shown in below figure

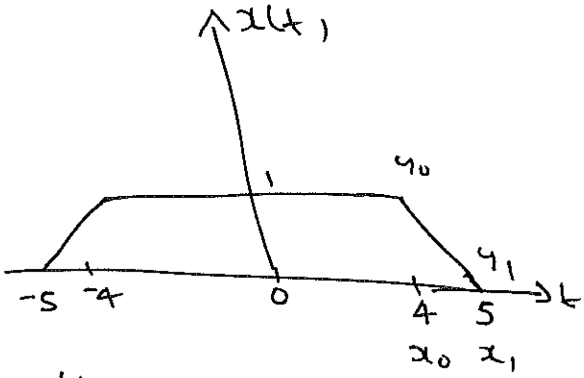


Soln :

it is nonperiodic signal

$\therefore$ it is Energy signal

$$x(t) = \begin{cases} 5-t & 4 \le t \le 5 \\ 1 & -4 \le t \le 4 \\ t+5 & -5 \le t \le -4 \\ 0 & \text{otherwise} \end{cases}$$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

$$E = \int_4^5 (5-t)^2 dt + \int_{-4}^{4} (1)^2 dt + \int_{-5}^{-4} (t+5)^2 dt$$

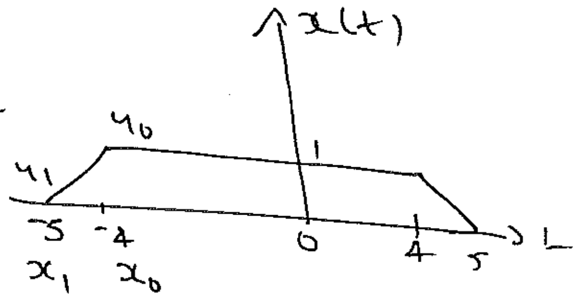
$$\frac{y-0}{x-5} = \frac{0-1}{5-4} = \frac{-1}{1}$$

$$E = \int_4^5 (25+t^2-10t)dt + \int_{-4}^{4} dt + \int_{-5}^{-4} (t^2+25+10t)dt$$

$$y = 5-x$$

$$y = 5-t$$

$$E = 25t\Big|_4^5 + \frac{t^3}{3}\Big|_4^5 - \frac{10}{2}t^2\Big|_4^5 + t\Big|_{-4}^{4} + \frac{t^3}{3}\Big|_{-5}^{-4} + 25t\Big|_{-5}^{-4} + \frac{10t^2}{2}\Big|_{-5}^{-4}$$



$E = 8.66 J$ //

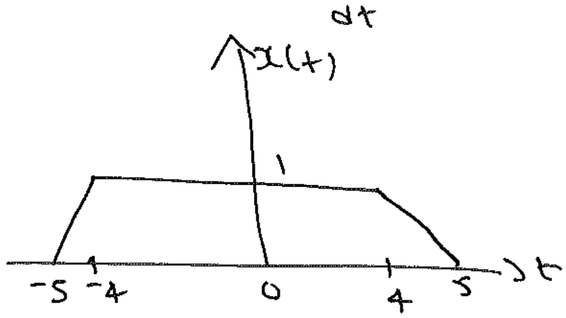
$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

$$\frac{y-0}{x+5} = \frac{0-1}{-5+4} = 1$$

$$y = 5+t$$ //

(d) The trapezoidal pulse $x(t)$ shown in below figure is applied to a differentiator defined by

$$y(t) = \frac{d}{dt} x(t)$$



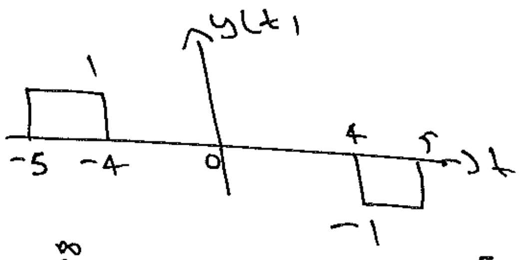
(i) Find the resulting o/p $y(t)$ of the differentiator.

(ii) Find the total Energy of $y(t)$.

Soln:

$$x(t) = \begin{cases} 5-t & 4 \le t \le 5 \\ 1 & -4 \le t \le 4 \\ 5+t & -5 \le t \le -4 \end{cases}$$

$$y(t) = \frac{d}{dt} x(t) = \begin{cases} -1 & 4 \le t \le 5 \\ 0 & -4 \le t \le 4 \\ 1 & -5 \le t \le -4 \end{cases}$$



$$E = \int_{-\infty}^{\infty} (x(t))^2 dt = \int_4^5 (-1)^2 dt + \int_{-5}^{-4} (1)^2 dt$$

$$= t\Big|_4^5 + t\Big|_{-5}^{-4}$$

$$= (5-4) + [-4-(-5)]$$

$$E = 2 \text{ J}$$

(c) $x(n) = \begin{cases} \cos \pi n & -4 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$

Soln

it is nonperiodic

$\therefore$ it is Energy signal

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 = \sum_{n=-4}^{4} (\cos \pi n)^2$$

$$= (\cos(-4\pi))^2 + (\cos(-3\pi))^2 + (\cos(-2\pi))^2 + (\cos(-\pi))^2 + (\cos(0))^2 + (\cos \pi)^2 + (\cos 2\pi)^2 + (\cos 3\pi)^2 + (\cos 4\pi)^2$$

$$E = 1+1+1+1+1+1+1+1+1$$

$$E = 9 \text{ J}$$

(d) $x(t) = \begin{cases} 5 \cos \pi t & -0.5 \le t \le 0.5 \\ 0 & \text{otherwise} \end{cases}$

Soln

it is non periodic

$\therefore$ it is Energy signal

$$E = \int_{-\infty}^{\infty} (x(t))^2 \, dt = \int_{-0.5}^{0.5} (5 \cos \pi t)^2 \, dt$$

$$E = 25 \int_{-0.5}^{0.5} \cos^2 \pi t \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= 25 \int_{-0.5}^{0.5} \left(\frac{1+\cos 2\pi t}{2}\right) dt$$

$$= \frac{25}{2} \left[\int_{-0.5}^{0.5} 1 \, dt + \int_{-0.5}^{0.5} \cos 2\pi t \, dt \right]$$

$$E = 12.5 \left[t \Big|_{-0.5}^{0.5} + \frac{\sin 2\pi t}{2\pi} \Big|_{-0.5}^{0.5} \right] = 12.5 \text{ J}$$

(g) $x(n) = \begin{cases} n & 0 \le n \le 5 \\ 10-n & 5 < n \le 10 \\ 0 & \text{otherwise} \end{cases}$

Soln: it is non periodic signal

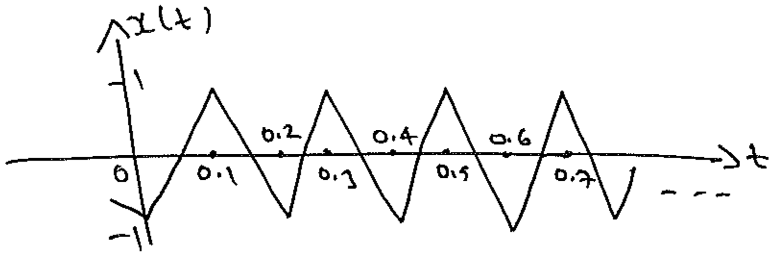
$\therefore$ it is Energy signal

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 = \sum_{n=0}^{5} n^2 + \sum_{n=6}^{10} (10-n)^2$$

$$E = 0+1+4+9+16+25+16+9+4+1+0$$

$$E = 85\,J$$

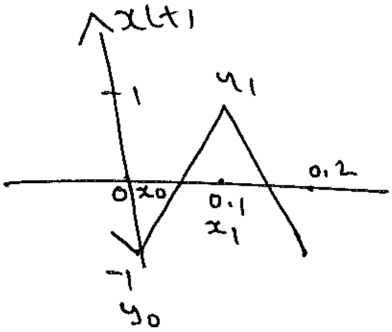
(h)



Soln: it is periodic signal

$\therefore$ it is periodic $T = 0.2$ sec.

$$P = \frac{1}{T} \int_0^T (x(t))^2\, dt$$

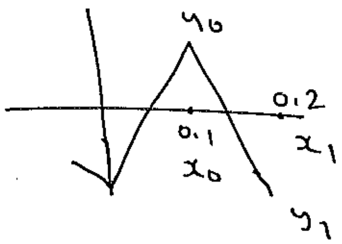


$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

$$\frac{y-1}{x-0.1} = \frac{1+1}{0.1-0} = \frac{2}{0.1} = 20$$

$$y-1 = 20x-2$$

$$y = 20t-1$$



$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

$$\frac{y+1}{x-0.2} = \frac{-1-1}{0.2-0.1} = \frac{-2}{0.1} = -20$$

$$y+1=-20x+4$$

$$y=-20t+3$$

$$x(t)=\begin{cases} 20t-1 & 0\le t\le 0.1 \\ -20t+3 & 0.1\le t\le 0.2 \end{cases}$$

$$P=\frac{1}{0.2}\left[\int_0^{0.1}(20t-1)^2\,dt+\int_{0.1}^{0.2}(-20t+3)^2\,dt\right]$$

$$P=\frac{1}{0.2}\left[\int_0^{0.1}400t^2\,dt+\int_0^{0.1}dt-\int_0^{0.1}40t+\int_{0.1}^{0.2}400t^2\,dt+\int_{0.1}^{0.2}9\,dt-\int_{0.1}^{0.2}120t\,dt\right]$$

$P=1/3$ watts.

$$P=\frac{1}{0.2}\left[\frac{400}{3}t^3\Big|_0^{0.1}+t\Big|_0^{0.1}-\frac{40}{2}t^2\Big|_0^{0.1}+\frac{400}{3}t^3\Big|_{0.1}^{0.2}+9t\Big|_{0.1}^{0.2}-\frac{120}{2}t^2\Big|_{0.1}^{0.2}\right]$$

$P=1/3$ watts

2) Categorize each of The following signals as an Energy or Power signal & find The Energy or time averaged Power of The signals.

(a)
$$x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Soln. it is nonperiodic signal

$\therefore$ it is Energy signal

$$E = \int_{-\infty}^{\infty} (x(t))^2 dt$$

$$E = \int_0^1 t^2 dt + \int_1^2 (2-t)^2 dt$$

$$E = \frac{t^3}{3}\Big|_0^1 + \int_1^2 (4+t^2-4t)dt$$

$$E = \frac{1}{3}[1-0] + 4t\Big|_1^2 + \frac{t^3}{3}\Big|_1^2 - \frac{4}{2}t^2\Big|_1^2$$

$$E = 1/3 + 4[2-1] + \frac{1}{3}[8-1] - 2[4-1] = 2/3$$

$E = 0.666$ J//

(b)
$$x(n) = \begin{cases} n & 0 \le n < 5 \\ 10-n & 5 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$$

Soln → it is nonperiodic signal

$\therefore$ it is Energy signal

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2$$

$$E = \sum_{n=0}^{4} n^2 + \sum_{n=5}^{10} (10-n)^2$$

$$E = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + (10-5)^2 + (10-6)^2 + (10-7)^2 + (10-8)^2 + (10-9)^2 + 0^2$$

$$E = 1 + 4 + 9 + 16 + 25 + 16 + 9 + 4 + 1$$

$$E = 85 \text{ J}$$

(c) $x(t) = 5\cos\pi t + \sin 5\pi t \quad ; -\infty < t < \infty.$

Soln

$$x(t) = 5\cos\pi t + \sin 5\pi t$$

$$\omega_1 = \pi \qquad \omega_2 = \pi$$

$$T_1 = \frac{2\pi}{\pi} = 2 \quad T_2 = \frac{2\pi}{5\pi} = 2/5$$

$$T = \text{LCM}(T_1, T_2) = 2 \text{ sec}$$

it is periodic with fundamental period is = 2 sec.

$$P = \frac{1}{T} \int_{-T/2}^{T/2} (x(t))^2 \, dt$$

$$P = \frac{1}{2} \int_{-T/2}^{T/2} \left[5\cos\pi t + \sin 5\pi t\right]^2 dt$$

$$P = \frac{1}{2}\left[\int_{-T/2}^{T/2} \left(25\cos^2\pi t + \sin^2 5\pi t + 10\cos\pi t \sin 5\pi t\right) dt\right.$$

$$P = \frac{1}{2}\left[\int_{-T/2}^{T/2} \left[25\left(\frac{1+\cos 2\pi t}{2}\right) + \left(\frac{1-\cos 10\pi t}{2}\right) + 10\cos\pi t \sin 5\pi t\right] dt\right.$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2}\left[25\left[\frac{1}{2}+\frac{\cos 2\pi t}{2}\right) + \left(\frac{1}{2}-\frac{1}{2}\cos 10\pi t\right) + 10\times\frac{1}{2}\left[\sin 6\pi t+\sin 4\pi t\right)\right] dt$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right)$$

$$P = \frac{1}{T}\left[25\left[\frac{1}{2}t+\frac{\sin 2\pi t}{4\pi}\right]_{-T/2}^{T/2} + \left[\frac{1}{2}t-\frac{\sin 10\pi t}{20\pi}\right]_{-T/2}^{T/2} + 5\left[-\cancel{\frac{\cos 6\pi t}{6\pi}}^{0} - \cancel{\frac{\cos 4\pi t}{4\pi}}^{0}\right]_{-T/2}^{T/2}\right.$$

$$P = \frac{1}{T}\left[25\left[\frac{1}{2}\left(T/2+T/2\right)+0\right] + \frac{1}{2}\left[T/2+T/2\right]+0\right.$$

$$P = \frac{1}{T}\left[25\times\frac{T}{2}+T/2\right]$$

$$P = \frac{T}{T}\left[12.5+\frac{1}{2}\right] \qquad P = 13\,\omega //.$$

(d) Consider The sinusoidal signal given by $x(t) = A\cos\omega_0 t$ find The averaged power.

<u>Soln</u>

$$P = \frac{1}{T_0}\int_0^{T}(x(t))^2\,dt = \frac{1}{T_0}\int_0^{T_0}(A\cos\omega_0 t)^2\,dt$$

$$P = \frac{1}{T_0}\int_0^{T_0}A^2\cos^2\omega_0 t\,dt = \frac{A^2}{T_0}\int_0^{T_0}\left(\frac{1+\cos 2\omega_0 t}{2}\right)dt$$

$$P = \frac{A^2}{2T_0}\left[\int_0^{T_0} 1\, dt + \int_0^{T_0} \cos 2\omega_0 t \, dt\right]$$

$$P = \frac{A^2}{2T_0}\left[t \Big|_0^{T_0} + \frac{\sin 2\omega_0 t}{2\omega_0}\Big|_0^{T_0}\right]$$

$$P = \frac{A^2}{2T_0}\left[(T_0) + \frac{1}{2\omega_0}\left[\sin 2\omega_0 T_0 - \sin 0\right]\right] \qquad \sin 0 = 0$$

$$P = \frac{A^2}{2T_0}\left[T_0 + \frac{1}{2\omega_0}\left[\cancel{\sin 2 \times \frac{2\pi}{\cancel{T_0}} \cancel{T_0}}\right]^{\to 0.}\right] \qquad \omega_0 = 2\pi f, \quad \omega_0 = \frac{2\pi}{T_0}$$

$$P = \frac{A^2 \cancel{T_0}}{2\cancel{T_0}} = \frac{A^2}{2} \text{ watts}$$

(c)

$$x(t) = \begin{cases} 1 & ; -\frac{\epsilon}{2} \le t \le \frac{\epsilon}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

Soln: it is non periodic signal

$\therefore$ it is Energy signal

$$E = \int_{-\infty}^{\infty} (x(t))^2 dt = \int_{-\epsilon/2}^{\epsilon/2} 1^2 dt = t \Big|_{-\epsilon/2}^{\epsilon/2}$$

$$E = \left[\epsilon/2 + \epsilon/2\right]$$

$$E = \epsilon \text{ J}$$

(d)

$$x(n) = \begin{cases} \sin(\pi n) & ; -4 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Soln: it is non periodic signal
∴ it is Energy signal

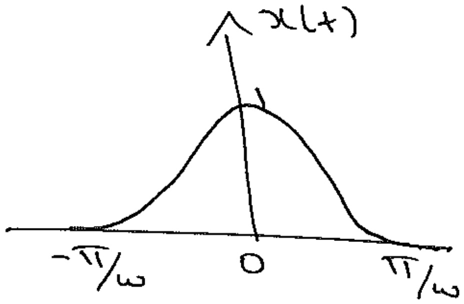
$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 = \sum_{n=-4}^{4} \sin^2(\pi n)$$

$E = 0\ J$ // or Zero signal.

(9) The raised-cosine pulse $x(t)$ shown in below fig is given by

$$x(t) = \begin{cases} \frac{1}{2}[\cos(\omega t) + 1] & ; -\pi/\omega \le t \le \pi/\omega \\ 0 & ; \text{otherwise} \end{cases}$$

Find the total Energy of $x(t)$



Soln:

$$E = \int_{-\infty}^{\infty} (x(t))^2\, dt$$

$$E = \int_{-\pi/\omega}^{\pi/\omega} \left(\tfrac{1}{2}[\cos\omega t + 1]\right)^2 dt$$

$$E = \frac{1}{4}\int_{-\pi/\omega}^{\pi/\omega} (\cos^2\omega t + 1 + 2\cos\omega t)\, dt$$

$$E = \frac{1}{4}\left[\int_{-\pi/\omega}^{\pi/\omega} \left(\frac{1+\cos 2\omega t}{2}\right) + 1 + 2\cos\omega t\right) dt\right]$$

$$E = \frac{1}{4}\left[\int_{-\pi/\omega}^{\pi/\omega} \frac{1}{2}\,dt + \int_{-\pi/\omega}^{\pi/\omega} \frac{\cos 2\omega t}{2}\,dt + \int_{-\pi/\omega}^{\pi/\omega} 1\,dt + \int_{-\pi/\omega}^{\pi/\omega} 2\cos\omega t\,dt\right]$$

$$E = \frac{1}{4}\left[\frac{1}{2}t\Big|_{-\pi/\omega}^{\pi/\omega} + \frac{1}{4\omega}\sin 2\omega t\Big|_{-\pi/\omega}^{\pi/\omega} + t\Big|_{-\pi/\omega}^{\pi/\omega}\right.$$

$$\left. - \frac{2\sin\omega t}{\omega}\Big|_{-\pi/\omega}^{\pi/\omega}\right]$$

$$E = \frac{3\pi}{4\omega}\ \text{J}$$

(h)

$$x(t) = \begin{cases} 5\cos\pi t & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Soln

it is non periodic signal

$\therefore$ it is Energy signal

$$E = \int_{-\infty}^{\infty} (x(t))^2\,dt$$

$$E = \int_{-1}^{1} (5\cos\pi t)^2\,dt = \int_{-1}^{1} 25\cos^2\pi t\,dt$$

$$E = 25\int_{-1}^{1} \left(\frac{1+\cos 2\pi t}{2}\right) dt$$

$$E = \frac{25}{2}\left[\int_{-1}^{1} (1+\cos 2\pi t)\,dt\right]$$

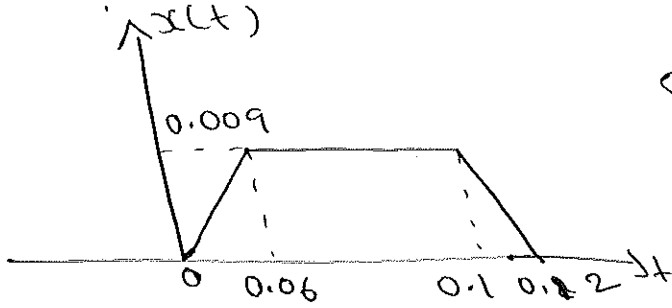
$$E = \frac{25}{2}\left[\int_{-1}^{1} 1\,dt + \int_{-1}^{1} \cos 2\pi t\,dt\right]$$

$$E = \frac{25}{2}\left[t\Big|_{-1}^{1} + \frac{\sin 2\pi t}{2\pi}\Big|_{-1}^{1} \right]$$

$$E = \frac{25}{2}\left[1-(-1) + \tfrac{1}{2\pi}\left[\sin(2\pi) - \sin(-2\pi) \right] \right].$$

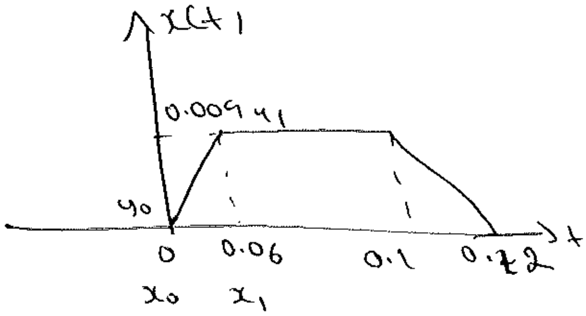
$$E = \underline{\underline{25\ J}}$$

1)



Find energy or Power & also find Corresponding values.

Soln



$$x(t) = \begin{cases} 0.009 & 0.06 < t < 0.1 \\ 0.15t & 0 < t < 0.06 \\ 0.45t - 0.054 & 0.1 < t < 0.12 \end{cases}$$

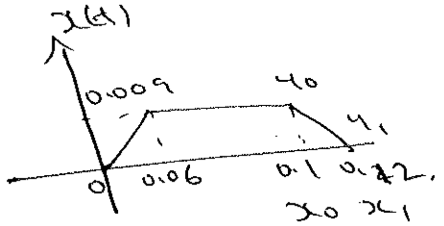
$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\frac{y - 0.009}{x - 0.06} = \frac{0.009}{0.06} = 0.15$$

$$y - 0.009 = 0.15(x - 0.06)$$

$$y - 0.009 = 0.15t - 0.009$$

$$y = \underline{0.15t}$$



$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\frac{y - 0}{x - 0.12} = \frac{0 - 0.009}{0.12 - 0.1} = -0.45$$

$$y = 0.45(x - 0.12)$$

$$\cancel{y = 0.09t - 0.018}$$

$$y = -0.45t + 0.054.$$

$$x(t) = \begin{cases} 0.009 & 0.06 < t < 0.1 \\ 0.15t & 0 < t < 0.06 \\ -0.45t + 0.054 & 0.1 < t < 0.12. \end{cases}$$

$$E = \int_{0.06}^{0.1} (0.009)^2\, dt + \int_{0}^{0.06} (0.15t)^2\, dt + \int_{0.1}^{0.12} (-0.45t + 0.054)^2\, dt$$

$$E = \underline{\qquad}\ J$$

(2) Check whether The following are energy or power signals? also find The Corresponding value.

(i) $x_1(n) = \cos(\pi n)$; $-4 \le n \le 4$
$\qquad 0$; otherwise

ii) $x_2(n) = \cos(\pi n)$; $n \ge 0$
$\qquad 0$; otherwise

Soln (i) $x_1(n) = \cos(\pi n)$; $-4 \le n \le 4$
$\qquad 0$; otherwise

The given signal is non periodic & exits b/w $-4 \le n \le 4$.
$\therefore$ it is energy signal

$$E = \sum_{n=-\infty}^{\infty} |x_1(n)|^2$$

$$E = \sum_{n=-4}^{4} (\cos \pi n)^2$$

$$E = |\cos(\pi \times -4)|^2 + |\cos(-3\pi)|^2 + |\cos(-2\pi)|^2 + |\cos(-\pi)|^2 + |\cos(0)|^2 + |\cos(\pi)|^2 + |\cos(2\pi)|^2 + |\cos(3\pi)|^2 + |\cos(4\pi)|^2.$$

$$E = 1+1+1+1+1+1+1+1+1$$

$$E = \underline{\underline{9 J}}$$

Or

$$E = \sum_{n=-4}^{4} |\cos(n\pi)|^2$$

$$\cos n\pi = (-1)^n$$

$$E = \sum_{n=-4}^{4} (-1)^{2n}.$$

Put $m = n+4$

$n = m-4$

$$E = \sum_{m-4=-4}^{m-4=4} (-1)^{2(m-4)} = E \sum_{m=0}^{8} (-1)^{2m} \cdot (-1)^{-8}$$

$$E = (-1)^{-8} \sum_{m=0}^{8} (-1)^{2m} = 1 \sum_{m=0}^{8} 1$$

W.K.T $\sum_{n=0}^{N-1} 1 = N$ $\qquad (-1)^{2m} = 1.$

$$E = \sum_{m=0}^{9-1} 1 = \underline{9\,J}.$$

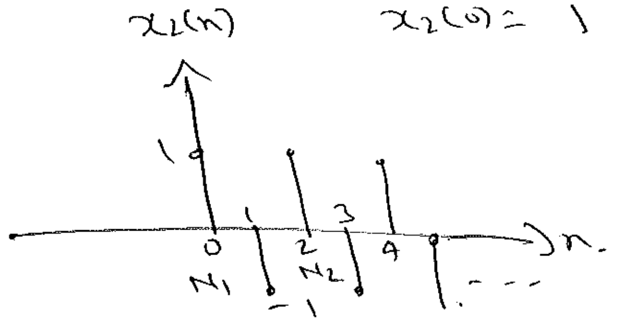
ii) Given $x_2(n) = \cos(\pi n) \quad n \geq 0$

$\qquad\qquad 0 \quad$ otherwise.

Soln

$x_2(n) = \cos(\pi n) \qquad n = 0, 1 \ldots$

$x_2(0) = 1$



$x_2(1) = \cos\pi = -1$

$x_2(2) = \cos(2\pi) = 1$

$x_2(3) = \cos(3\pi) = -1$

$x_2(4) = \cos(4\pi) = 1$

The given signal is periodic $\therefore$ it is power signal

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 \qquad \begin{aligned} N &= N_2 - N_1 \\ &= 2 - 0 \\ &= 2. \end{aligned}$$

$$P = \frac{1}{2} \sum_{n=0}^{1} |x(n)|^2 = \frac{1}{2} \sum_{n=0}^{1} |\cos(\pi n)|^2$$

$$P = \frac{1}{2}\left[(\cos(0))^2 + (\cos(\pi))^2 \right]$$

$$P = \frac{1}{2}\left[1 + 1 \right] = 1\omega$$

or

$$P = \frac{1}{2} \sum_{n=0}^{1} (\cos n\pi)^2$$

$$P = \frac{1}{2} \sum_{n=0}^{1} (-1)^{n \times 2} \qquad \cos n\pi = -1$$

$$P = \frac{1}{2} \sum_{n=0}^{1} (-1)^{2n} = \frac{1}{2} \sum_{n=0}^{1} 1$$

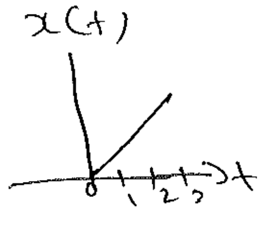
w.k.T $\sum_{n=0}^{N-1} 1 = N$.

$$P = \frac{1}{2} \sum_{n=0}^{2-1} 1$$

$$P = \frac{1}{2} \times 2$$

$$P = 1 \text{ watt.}$$

(3) $x(t) = \begin{cases} t & 0 \le t \le \infty \\ 0 & \text{otherwise} \end{cases}$



Soln The given signal is nonperiodic
∴ it is Energy signal.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{0}^{\infty} t^2 dt.$$

$$E = \left. \frac{t^3}{3} \right|_0^{\infty} = \frac{1}{3}\left[\infty^3 - 0^3 \right] = \frac{1}{3}[\infty - 0] = \infty/3$$

$E = \infty$ Energy is '∞' Hence given signal is neither energy nor power.

1) The angular frequency Ω of the sinusoidal signal $x(n) = A\cos(\Omega n + \phi)$ satisfies the condition of periodicity. Determine the average power of $x(n)$.

Soln:

The given signal $x(n)$ is periodic with fundamental period $N = \frac{2\pi}{\Omega}$. The av power is given by

$$P = \frac{1}{N}\sum_{n=0}^{N-1}(x(n))^2 = \frac{1}{N}\sum_{n=0}^{N-1} A^2\cos^2(\Omega n + \phi).$$

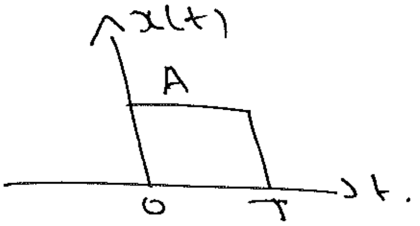
$$P = \frac{A^2}{N}\sum_{n=0}^{N-1} A\cos^2\left(\frac{2\pi}{N}n + \phi\right) //$$

(2) A rectangular pulse $x(t)$ is given by

$$x(t) = \begin{cases} A & ; 0 \le t \le T \\ 0 & \text{Otherwise} \end{cases}$$

The pulse $x(t)$ is applied to an integrator defined by $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$. Find the total energy of the o/p $y(t)$.

Soln:



$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau = \int_0^t A\,d\tau = \int_0^t A\,d\tau$$

$$y(t) = A\tau\Big|_0^t = At.$$

$$\text{Energy} = \int_{-\infty}^{\infty}(y(t))^2\,dt = \int_0^T A^2 t^2\,dt$$

$$= A^2\frac{t^3}{3}\Big|_0^T = \frac{A^2 T^3}{3} //$$

③ Compute The Energy of The length N sequence.

$$x(n) = \cos\left(\frac{2\pi}{N} kn\right) \quad 0 \le n \le N-1$$

Soln

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 = \sum_{n=0}^{N-1} \cos^2\left(\frac{2\pi}{N} kn\right).$$

$$E = \sum_{n=0}^{N-1} \frac{1 + \cos \frac{4\pi}{N} kn}{2}$$

$$E = \frac{1}{2} \sum_{n=0}^{N-1} 1 + \sum_{n=0}^{N-1} \cos \frac{4\pi}{N} kn. \rightarrow 0.$$

$$E = 1/2 \cdot N$$

$$E = \frac{N}{2}$$

$$\sum_{n=0}^{N-1} 1 = N.$$

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha} \quad \alpha \ne 1$$

$$= N \quad \alpha = 1.$$

④ $x(n) = (0.5)^n \quad 0 \le n \le \infty$

0 otherwise

Soln it is nonperiodic signal ∴ it is Energy signal

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 = \sum_{n=0}^{\infty} ((0.5)^n)^2 = \sum_{n=0}^{\infty} (0.25)^n$$

W.K.T $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad \alpha < 1$

$$E = \frac{1}{1-0.25} = 4/3 \, J$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad \alpha < 1$$

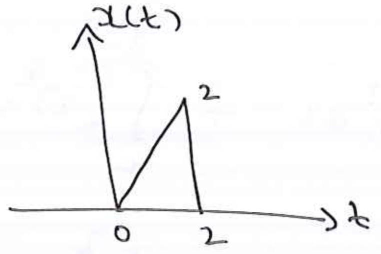
$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha} \quad \alpha \ne 1$$

$$= N \quad \alpha = 1$$

(5) $x(t) = t \quad 0 \leq t \leq 2$ Check Energy or Power.

$0 \quad$ otherwise find Corresponding Values.

Soln:



it is non periodic signal

∴ it is Energy signal

$$E = \int_{-\infty}^{\infty} (x(t))^2 dt = \int_0^2 t^2 dt = \frac{t^3}{3}\Big|_0^2$$

$$E = \frac{1}{3}[8-0] = 8/3 \text{ J}$$

(6) $x(t) = t^2 \quad -1 \leq t \leq 1$ Check Energy or Power

$0 \quad$ otherwise find Corresponding Values.

Soln:

it is non periodic signal

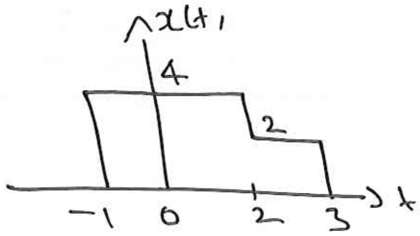
∴ it is Energy signal

$$E = \int_{-\infty}^{\infty} (x(t))^2 dt = \int_{-1}^{1} (t^2)^2 dt$$

$$E = \int_{-1}^{1} t^4 dt = \frac{t^5}{5}\Big|_{-1}^{1}$$

$$E = \frac{1}{5}\left[(1)^5 - (-1)^5\right] = 2/5 \text{ J}$$

(7)



Soln

it is non periodic signal

∴ it is Energy signal

$$x(t) = \begin{cases} 4 & -1 \text{ to } 2 \\ 2 & 2 \text{ to } 3 \end{cases}$$

$$E = \int_{-\infty}^{\infty} (x(t))^2 dt$$

$$E = \int_{-1}^{2} (4)^2 dt + \int_{2}^{3} (2)^2 dt$$

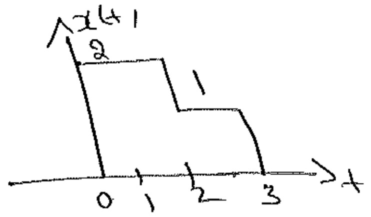
$$E = 16t\Big|_{-1}^{2} + 4t\Big|_{2}^{3}$$

$$E = 16[2-(-1)] + 4[3-2]$$

$$E = 48 + 4$$

$$E = 52 J$$

8

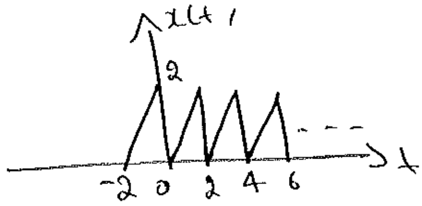


Soln it is energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^2 (2)^2 dt + \int_2^3 1^2 dt$$

$$E = 4t\Big|_0^2 + t\Big|_2^3 = 9J$$

9



Soln The given signal is power signal

Period $= 2 = T$

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{2}\int_0^2 t^2 dt = \frac{1}{2}\,\frac{t^3}{3}\Big|_0^2$$

$P = 4/3$ watts.

10 Compute the energy of the length-N sequence:

$$x(n) = \cos\left(\frac{2\pi k n}{N}\right) \quad 0 \le n \le N-1$$

Soln

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2.$$

$$E = \sum_{n=0}^{N-1} \left|\cos\left(\frac{2\pi k n}{N}\right)\right|^2$$

$$E = \sum_{n=0}^{N-1} \frac{1 + \cos\frac{4\pi k n}{N}}{2} = \frac{1}{2}\sum_{n=0}^{N-1} 1 + \cos\left(\frac{4\pi k n}{N}\right)$$

$$E = \frac{1}{2}\left[\sum_{n=0}^{N-1} 1 + \sum_{n=0}^{N-1} \cos\frac{4\pi k n}{N}\right]$$

$$E = \frac{1}{2}\left[N + \sum_{n=0}^{N-1} \cos\frac{4\pi k n}{N}\right]$$

let $A = \sum_{n=0}^{N-1} \cos\left(\frac{4\pi kn}{N}\right)$, & $B = \sum_{n=0}^{N-1} \sin\left(\frac{4\pi kn}{N}\right)$

$$A = \sum_{n=0}^{N-1} \frac{e^{j\frac{4\pi kn}{N}} + e^{-j\frac{4\pi kn}{N}}}{2} \qquad B = \sum_{n=0}^{N-1} \frac{e^{j\frac{4\pi kn}{N}} - e^{-j\frac{4\pi kn}{N}}}{2j}$$

$$\therefore \quad A + jB = \sum_{n=0}^{N-1} e^{j\frac{4\pi kn}{N}}.$$

$$= \frac{1 - e^{j\frac{4\pi k}{N}N}}{1 - e^{j\frac{4\pi k}{N}}} \qquad \sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha} \quad \alpha \neq 1$$

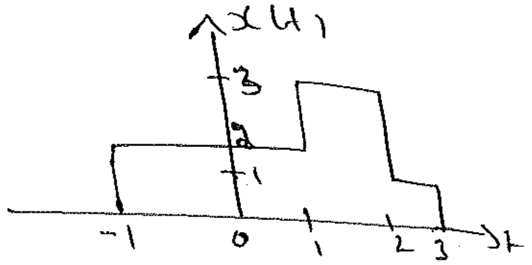
$$N. \quad \alpha = 1$$

$$A + jB = 0 \qquad (\because e^{j4\pi k} = 1).$$

$$\therefore \quad A = 0 \;\&\; B = 0$$

$$E = N/2$$

(11)



Soln

it is non periodic $\therefore$

it is Energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

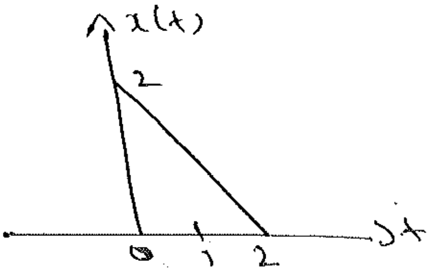
$$E = \int_{-1}^{1} (2)^2 dt + \int_{1}^{2} |3|^2 dt + \int_{2}^{3} 1^2 dt$$

$$= 4t\Big|_{-1}^{1} + 9t\Big|_{1}^{2} + t\Big|_{2}^{3}$$

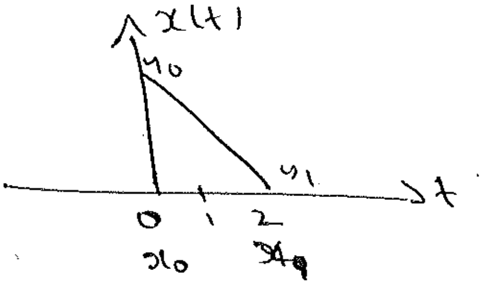
$$= 4[1-(-1)] + 9[2-1] + [3-2]$$

$$= 8 + 9 + 1 = 18\ J$$

(12)



Soln



$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

$$\frac{y-0}{x-2} = \frac{0-2}{2-0} = \frac{-2}{2}$$

$$\frac{y}{x-2} = -1$$

$y = 2-x$ or $2-t$.

it is non periodic signal

$\therefore$ it is Energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E = \int_0^2 (2-t)^2 \, dt$$

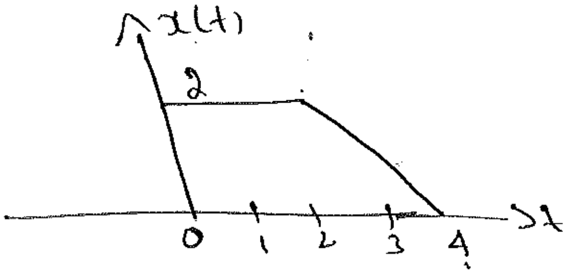
$$= \int_0^2 (4+t^2-4t)\,dt = \int_0^2 4\,dt + \int_0^2 t^2\,dt - \int_0^2 4t\,dt$$

$$= 4t\Big|_0^2 + \frac{1}{3}t^3\Big|_0^2 - 4\frac{t^2}{2}\Big|_0^2$$

$$= 4(2-0) + 1/3\,[8-0] - 4/2\,[4-0]$$

$$= 8 + 8/3 - 8 = 2.66\,J$$

(13)



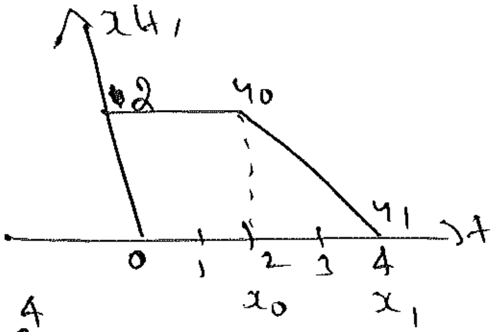
Check Energy or power ?

Soln

It is non periodic signal

$\therefore$ it is Energy signal.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$



$$E = \int_0^2 |2|^2 dt + \int_2^4 (4-t)^2 dt$$

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

$$\frac{y-0}{x-4} = \frac{0-2}{4-2} = -\cancel{2}/_{\cancel{2}}$$

$$\frac{y}{x-4} = -1$$

$$y = 4-x$$

$$y = (4-t)$$

$$E = 4t\Big|_0^2 + \int_2^4 (16+t^2-8t)\,dt$$

$$E = 8 + \int_2^4 16\,dt + \int_2^4 t^2 dt - \int_2^4 8t\,dt$$

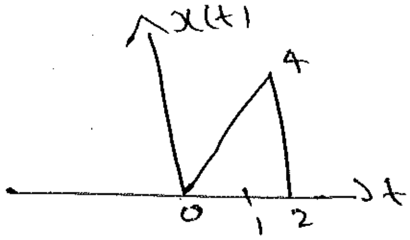
$$E = 8 + 16t\Big|_2^4 + \frac{t^3}{3}\Big|_2^4 - \frac{8t^2}{2}\Big|_2^4$$

$$E = 8 + 16(2) + 1/3\,[64-8] - 4\,[16-4]$$

$$E = 8+32+18.66-48$$

$$E = 10.66\ J //$$

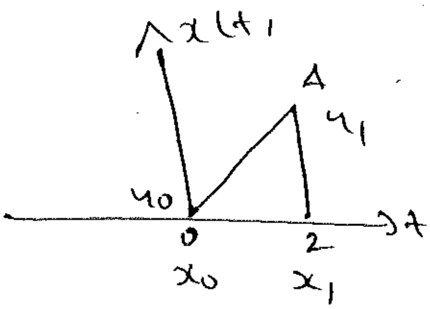
(14)



Check E or P

Soln The given signal is non periodic

∴ it is Energy signal



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \int_0^2 (2t)^2 dt$$

$$= \int_0^2 4t^2\,dt = \frac{4t^3}{3}\Big|_0^2$$

$$= 4/3\,[8-0] = \frac{32}{3} =$$

$$E = 10.66\ J //$$

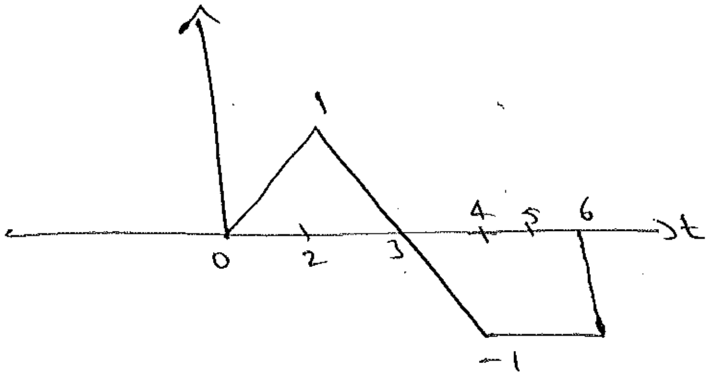
$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

$$\frac{y-4}{x-2} = \frac{4-0}{2-0} = 2$$

$$y-4 = 2x-4$$

$$y = 2t //$$

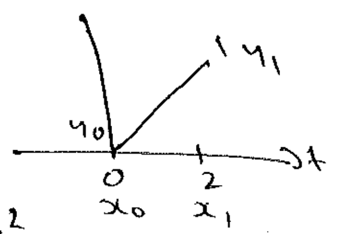
(1) Find Energy or power signals



Soln: it is non periodic signal

∴ it is Energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$



$$E = \int_0^2 \left(\tfrac{1}{2}t\right)^2 dt + \int_2^4 (3-t)^2 dt + \int_4^6 (1)^2 dt$$

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

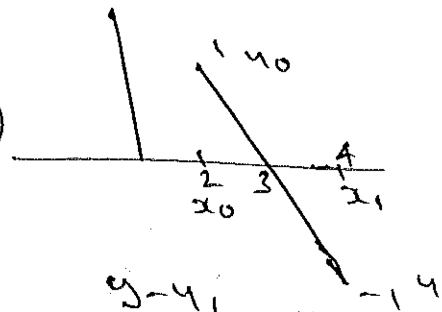
$$E = \cancel{\frac{1}{2}\frac{t^2}{2}}\int_0^2$$

$$\frac{y-1}{x-2} = \frac{1-0}{2-0} = \frac{1}{2}$$

$$E = \frac{1}{2} \cdot \frac{t^2}{2}\Big|_0^2 + 9t\Big|_2^4 + \frac{t^3}{3}\Big|_2^4 - \frac{6t^2}{2}\Big|_2^4 + t\Big|_4^6$$

$$y - 1 = \tfrac{1}{2}x - 1$$

$$y = \tfrac{1}{2}t$$



$$E = \frac{1}{4}(2) + 9(2) + \frac{1}{3}(64-8) - 3(16-4) + (6-4)$$

$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$$

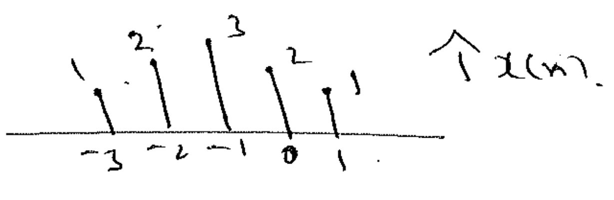
$$E = 3.32\,J$$

$$\frac{y+1}{x-4} = \frac{-1-1}{4-2} = \frac{-2}{2} = -1$$

$$y + 1 = -x + 4$$

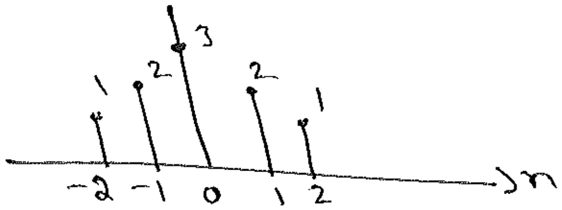
$$y = 3 - t$$

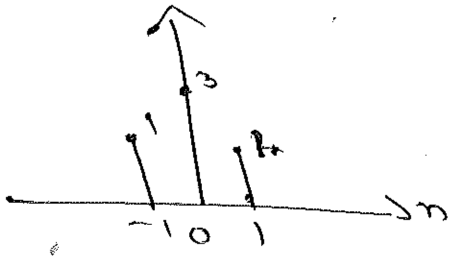
(2) if $x(n)$ is as shown in below figure, find the Energy of the signal $x(2n-1)$.



Soln $x(2n-1)$

$v(n) = x(n-1)$ $\qquad$ $y(n) = v(2n)$





$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=-1}^{1} |x(n)|^2$$

$$E = |x(-1)|^2 + |x(0)|^2 + |x(1)|^2$$

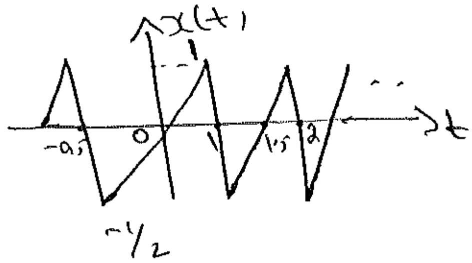
$$E = 1^2 + 3^2 + 1^2$$

$$= 1 + 9 + 1$$

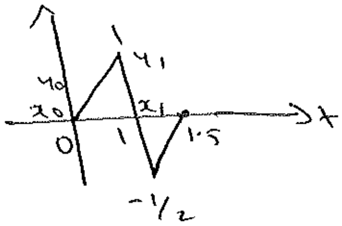
$$E = 11\ J$$

(34)

1) Check energy or Power find Corresponding Value.



Soln



The given signal is Periodic ∴ It is Power signal.

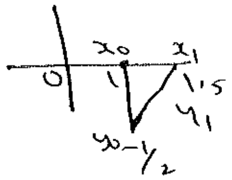
$$x(t) = \begin{cases} t & 0 \le t \le 1 \\ t - 1.5 & 1 \le t \le 1.5 \end{cases}$$

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\frac{y - 1}{x - 1} = \frac{1 - 0}{1 - 0}$$

$$y - 1 = x - 1$$

$$y = t$$



$$\frac{y - 0}{x - 1.5} = \frac{0 - (-1/2)}{1.5 - 1}$$

$$y = x - 1.5$$

$$y = t - 1.5$$

$$T = T_2 - T_1 = 1.5 - 0 = 1.5$$

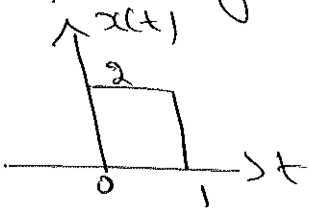
$$P = \frac{1}{T}\left[\int_0^T |x(t)|^2 dt\right] = P = \frac{1}{1.5}\left[\int_0^1 t^2 dt + \int_1^{1.5} (t - 1.5)^2 dt\right]$$

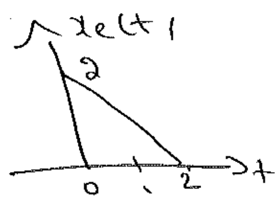
$$P = \frac{1}{1.5}\left[\frac{t^3}{3}\Big|_0^1 + \frac{t^3}{3}\Big|_1^{1.5} + 2.25t\Big|_1^{1.5} - \frac{3t^2}{2}\Big|_1^{1.5}\right]$$

$$P = 0.25\,\text{w}$$

(2) Figure a & b shows part of the signal x(t) & its even part [$x_e(t)$ respectively] for $t \ge 0$ only. And even part ~~x(t)~~ for $t < 0$ is not shown. Complete the Plot of x(t) & $x_e(t)$. Also draw the odd part of x(t) [i.e $x_o(t)$].

July 05 - 8M



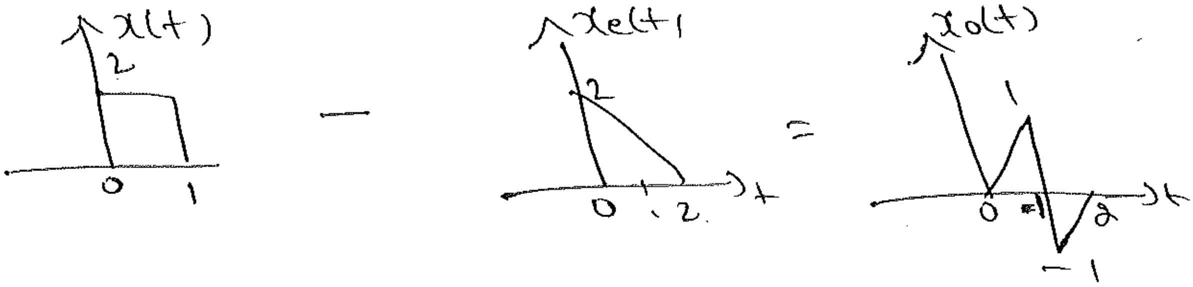


Soln

W.K.T $x(t) = [x_e(t) + x_o(t)]$

$$x_o(t) = x(t) - x_e(t).$$

Step 1:

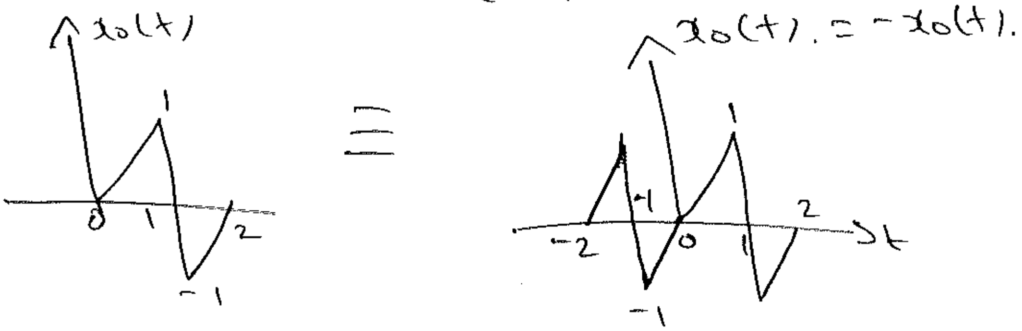


Step2 : w.k.T even part of any signal is symmetric

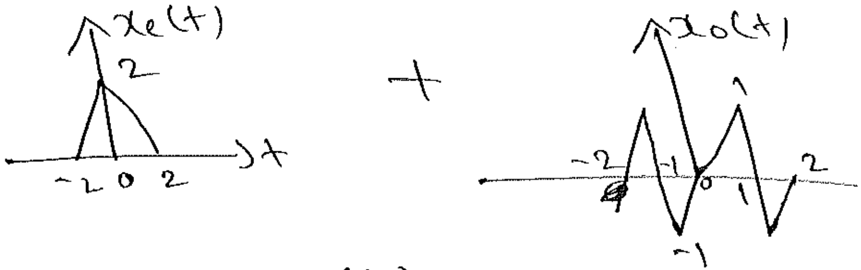
$x_e(t) = x_e(-t)$

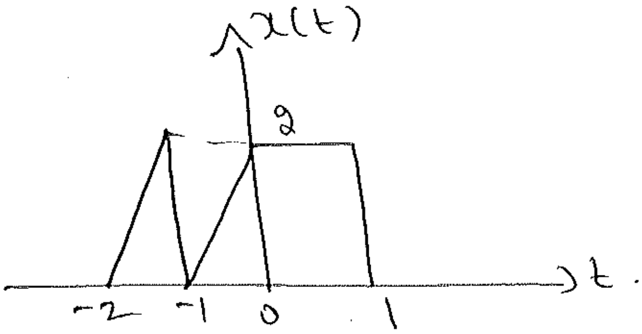
lly odd part of any signal is antisymmetric

$x_o(t) = -x_o(-t)$



finally $x(t) = x_e(t) + x_o(t)$





Elementary Signals

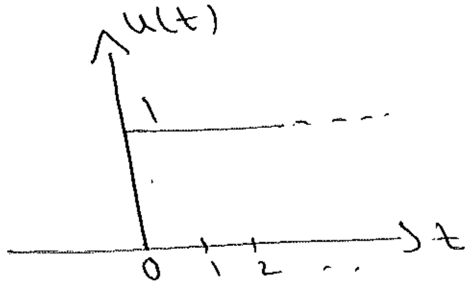
Elementary signals are fundamental building blocks of any complex signal it can classified as.

1) unit step function or step function.
2) unit impulse function or Impulse function.
3) unit Ramp function or Ramp function.
4) Exponential signals.
5) Sinusoidal signals.
6) Exponentially damped sinusoidal signals.

1) Unit Step function or Step function [$u(t)$]

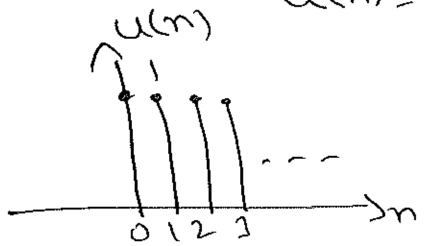
The continuous time (CT) unit step function $u(t)$ is defined by

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0 & \text{otherwise } [t<0] \end{cases}$$



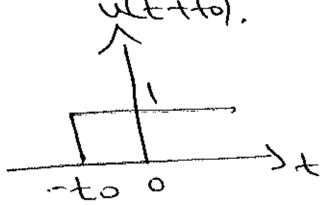
The discrete time (D.T) unit step function $u(n)$ is defined by

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0 & n < 0 \end{cases}$$



NOTE:

$$u(t+t_0) = \begin{cases} 1 & \text{for } t+t_0 > 0 \text{ or } t > -t_0 \\ 0, & t+t_0 < 0 \text{ or } t < -t_0. \end{cases}$$



Significance: 1) D.T unit step signal is sampled version of C.T unit step.

2) unit step signal generated when a DC supply is applied to the circuit

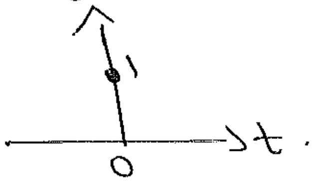
3) unit step is generate when a switch is closed at $t=0$.

2) Unit Impulse or Impulse function

The continuous time unit impulse function $\delta(t)$ defined by

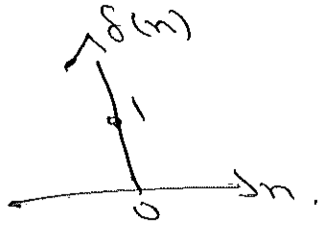
$$\delta(t)=0 \quad \text{for} \quad t\neq 0 \qquad \& \quad \int_{-\infty}^{\infty}\delta(t)\,dt = 1.$$

$$\delta(t)=1 \quad \text{for} \quad t=0.$$



The discrete-time version of the impulse function $\delta(n)$ defined as

$$\delta(n)=\begin{cases}1, & n=0\\ 0, & n\neq 0.\end{cases}$$



Note: * Impulse as '0' width & infinite length.

* it's total area is unity

* $\int_{-\infty}^{\infty}\delta(t)\,dt=1$

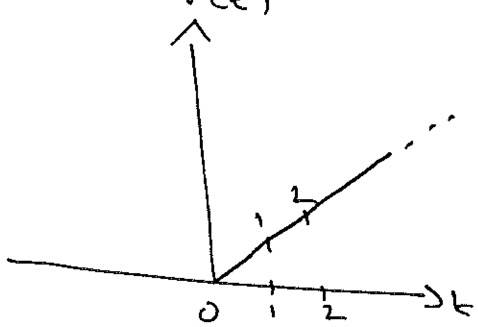
* $\int_{-\infty}^{\infty}K\,\delta(t)\,dt=K$.

③ Unit Ramp function or Ramp function

* it is linearly growing function for positive values of Independent variables.

* A Ramp function is defined by $r(t)$ is given by

$$r(t) = \begin{cases} t & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$



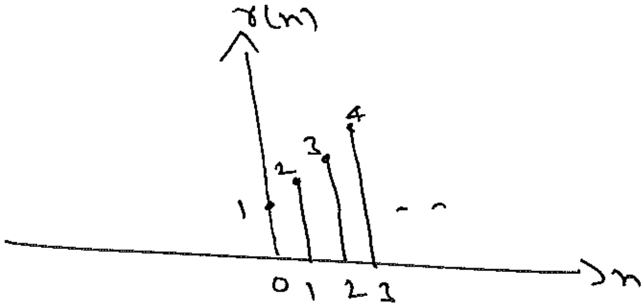
We can write $r(t) = t.u(t)$

Since $u(t) = 1$ for $t \geq 0$ & 0 for $t < 0$

* The discrete time version of The Ramp function is

$$r(n) = \begin{cases} n & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

$r(n) = n.u(n)$



Significance :

* Ramp function indicates Linear relationship
* it indicates constant current charging of the capacitor

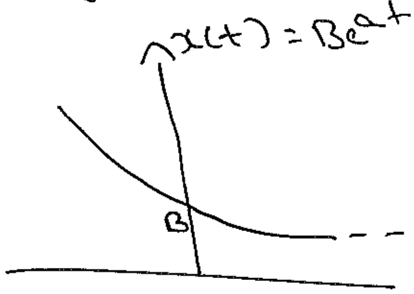
(4) Exponential Signals

* it is exponentially growing or decaying signal.
* A real exponential signal for C.T is given by

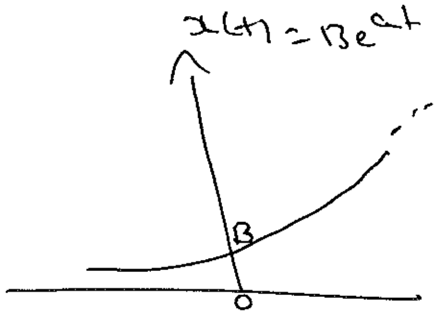
$$x(t) = Be^{at}$$

B = amplitude of the exponential signal.

* if $a < 0$ $x(t)$ said to be decaying exponential
* if $a > 0$ $x(t)$ —"— growing exponential



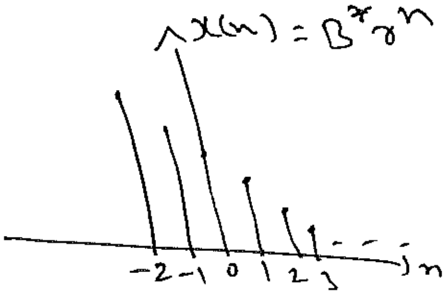
Decaying $a < 0$



growing $a > 0$

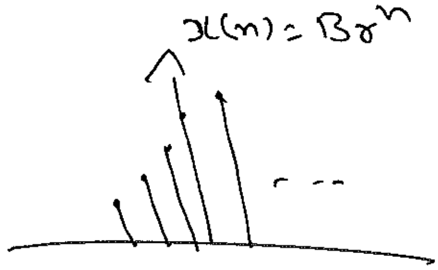
* |||ly for D.T $x(n) = B\gamma^n$

→ if $\gamma < 1$ $x(n)$ is called decaying exponential signal
→ if $\gamma > 1$ —"— growing —"—



$\gamma < 1$

decaying



$\gamma > 1$

growing

Significance of C.T exponential signal.

1) Charging and discharging of a capacitor.
2) Current flowthrough an inductor.
3) Radioactive decay.

Significance of D.T exponential signal

1) Population growth as a function of generation.
2) Return on investment as a function of day, month or quater.

Complex exponential signals

Definition: when exponent is purely imaginary, then the signal is said to be complex exponential signal.
it is given as,

CT : $x(t) = e^{j\omega_0 t} \Rightarrow \cos\omega_0 t + j\sin\omega_0 t$

DT : $x(n) = e^{j\omega_0 n}$. $\cos\omega_0 n + j\sin\omega_0 n$

by Eulers identity $e^{j\theta} = \cos\theta + j\sin\theta$

5) Sinusoidal signals.

→ General expression for continuous time sinusoidal signal is

$$x(t) = A\cos(\omega t + \phi) \quad -(1).$$

where $A \rightarrow$ Amplitude.
$\omega \rightarrow$ angular frequency in rad/sec.
$\phi \rightarrow$ Phase angle in ~~radian~~ radians.

→ The period of a sinusoidal signal is defined by $T = \frac{2\pi}{\omega}$.

→ A continuous-time sinusoid is always periodic with a Period $= T$.

this can be proved as follows.

Consider

$$x(t+T) = A\cos(\omega(t+T) + \phi)$$
$$= A\cos(\omega t + \omega T + \phi) \qquad \because T = \frac{2\pi}{\omega}$$
$$= A\cos(\omega t + 2\pi + \phi) \qquad \because \omega T = 2\pi$$
$$= A\cos(\omega t + \phi)$$

$$x(t+T) = x(t)$$

∴ a C.T sinusoidal signal always periodic.

→ The discrete-time version of a sinusoidal signal is written as,

$$x(n) = A\cos[\Omega n + \phi].$$

→ The period of a discrete-time sinusoid is measured in samples.

Let the period of $x(n)$ be N. Then, $[\because x(n) = x(n+N)]$.

$$x(n+N) = A\cos(\Omega n + \Omega N + \phi] \quad -①.$$

eqn ① satisfies the condition of periodic if $\Omega N = 2\pi m$ radians

or $\quad \Omega = \dfrac{2\pi}{N} m$ radians/cycle,

m & N are integer.

Ex:

① $x(n) = \sin 0.3\pi n.$

Soln $\quad \Omega = 0.3\pi$

$$\Omega = \frac{2\pi m}{N} \Rightarrow N = \frac{2\pi m}{\Omega} = \frac{2\cancel{\pi}}{0.3\cancel{\pi}} m$$

$$N = \frac{2m}{0.3} = \frac{20m}{3} \Rightarrow \frac{N}{m} = \frac{20}{3}$$

for $m = 3, 6, 9 \ldots$

$N = 20, 40, 60 \ldots$

The smallest of $N = 20$ & this is the fundamental period & given signal is periodic

→ Alternatively a discrete-time sinusoid, $x(n)$ is periodic if and only if $\dfrac{\Omega}{2\pi}$ is a rational function

i.e $\quad \dfrac{\Omega}{2\pi} = \dfrac{p}{q}$ where p & q are integers.

if $\dfrac{\Omega}{2\pi}$ is rational, then fundamental period, $N = q$ samples

on the other hand, if $\dfrac{\Omega}{2\pi}$ is not rational, then $x(n)$ is not periodic

Ex: $x(n) = \sin(0.3n)$

Soln

$$\Omega = 0.3$$

$$\Omega = \frac{2\pi}{N} \cdot m$$

$$\frac{N}{m} = \frac{2\pi}{\Omega} = \frac{2\pi}{0.3}$$

$$N = 2\pi$$

$\therefore x(n)$ is non periodic

3) A pair of sinusoidal signals with a common angular frequency is defined by

$$x_1(n) = \sin(5\pi n)$$

&

$$x_2(n) = \sqrt{3}\cos(5\pi n)$$

a) Both $x_1(n)$ and $x_2(n)$ are periodic. Find their common fundamental period.

b) express the composite sinusoidal signal

$$y[n] = x_1(n) + x_2(n) \text{ in the form}$$

$y(n) = A\cos(\Omega n + \phi]$ & evaluate the amplitude A & ϕ.

Soln: The angular frequency of both $x_1(n)$ & $x_2(n)$ is

$$\Omega = 5\pi$$

$$N = \frac{2\pi}{\Omega} m \qquad N = \frac{2}{5} m$$

$$= \frac{2\pi}{5\pi} m$$

For $x_1(n)$ & $x_2(n)$ to be periodic, N must be an integer. This can be so only for $m = 5, 10, 15 \ldots$

which results in $N = 2, 4, 6 \ldots$

b) $y(n) = A\cos(\Omega n + \phi)$

$y(n) = A\cos(\Omega n)\cos\phi - A\sin\Omega n \cdot \sin\phi$

letting $\Omega = 5\pi$

$y(n) = A\cos(5\pi n)\cdot\cos\phi - A\sin(5\pi n)\sin\phi$

$x_1(n) + x_2(n) = A\cos(5\pi n)\cdot\cos\phi - A\sin(5\pi n)\cdot\sin\phi)$

$\sin(5\pi n) + \sqrt{3}\cos(5\pi n) = A\cos(5\pi n)\cdot\cos\phi - A\sin(5\pi n)\cdot\sin\phi$.

On Comparision

$-A\sin\phi = 1 \qquad A\cos\phi = \sqrt{3}$

$\tan\phi = \dfrac{\sin\phi}{\cos\phi} = \dfrac{\text{amplitude of } x_1(n)}{\text{amplitude of } x_2(n)}$

$\tan\phi = -1/3 \qquad \left[\dfrac{-A\sin\phi}{A\cos\phi} = 1/\sqrt{3}\right)$

$\phi = -\pi/3$

$-A\sin\phi = 1$

$-A\sin(-\pi/3) = 1$

$A = -1/\sin(-\pi/3)$

$A = \underline{2}$

$\therefore$

$y(n) = 2\cos(5\pi n - \pi/3)$

4) Determine The fundamental Period of The Sinusoidal Signal

$$x(n) = 10\cos\left(\frac{4\pi}{31}n + \frac{\pi}{5}\right).$$

Soln:

$$x(n) = 10\cos\left(\frac{4\pi}{31}n + \pi/5\right).$$

Comparing with

$$x(n) = A\cos(\Omega n + \phi).$$

$$\Omega = \frac{2\pi m}{N}$$

$$\frac{\cancel{2}\pi m}{N} = \frac{\cancel{4}\pi^{2}}{31}$$

$$N = \frac{31}{2}m \qquad \frac{N}{m} = \frac{31}{2}.$$

Since N is the smallest integer hence m=2

So that $N = \frac{31}{2} \times 2 = \underline{\underline{31}}$

N = 31 Samples.

5) Consider The following Sinusoidal Signals.

(a) $x(n) = 5\sin[2n]$

$$\Omega = 2$$

$$\Omega = \frac{2\pi}{N}m$$

$$N = \frac{2\pi}{2}m$$

$$N = \pi m$$

$$\frac{N}{m} = \pi \qquad \frac{m}{N} = \frac{1}{\pi} \Rightarrow \text{irrational So non-periodic.}$$

b) $x(n) = 5\cos[0.2\pi n]$

$$\Omega = 0.2\pi$$

$$\Omega = \frac{2\pi}{N}m \qquad \frac{m}{N} = \frac{1}{10}$$

$$N = \frac{2\cancel{\pi}}{0.2\cancel{\pi}}m$$

$$N = 10m$$

$\underline{\underline{N = 10}}$

Periodic, fundamental Period = 10.

c) $x(n) = 5\cos[6\pi n]$.

$\cancel{x(n)}$ $\Omega = 6\pi$

$$\Omega = \frac{2\pi}{N} m \Rightarrow N = \frac{\cancel{2}^{1}\cancel{\pi}}{\cancel{6}_{3}\cancel{\pi}} m$$

$$\frac{N}{\cancel{m}} = \frac{1}{3} m$$

$$\frac{N}{m} = \frac{1}{3} \quad \text{or} \quad \frac{m}{N} = \frac{3}{1}$$

Periodic, fundamental period $= 1$.

d) $x(n) = 5\sin[6\pi n/35]$.

$$N = \frac{2\pi}{\Omega} m \qquad \Omega = \frac{6\pi}{35}$$

$$N = \frac{\cancel{2}^{1}\cancel{\pi}}{\cancel{6}_{3}\cancel{\pi}/35}$$

$$N = \frac{35}{3} m$$

$$\frac{N}{m} = \frac{35}{3}$$

$$N = \underline{\underline{35}}.$$

e) $x(n) = \cos 2\pi n$

$$\Omega = 2\pi$$

$$N = \frac{\cancel{2\pi}}{\cancel{2\pi}} m$$

$$N = 1m$$

$$\frac{N}{m} = \frac{1}{1}$$

$$N = 1$$

Periodic, fundamental period $\underline{\underline{= 1}}$

f) $x(n) = \cos 2n$.

$$\Omega = 2$$

$$N = \frac{\cancel{2}\pi}{\cancel{2}} m$$

$$\frac{N}{m} = \pi$$

or

$$\frac{m}{N} = \frac{1}{\pi} \rightarrow \text{irrational}$$

6) Find the smallest angular frequencies for which discrete-time sinusoidal signals with the following fundamental period ~~periods~~ periods would be periodic.

a) $N=8$ b) $N=32$ c) $N=64$ d) $N=128$.

Soln

a) $N=8$

$$\Omega = \frac{2\pi}{N}$$

$$\Omega = \frac{\cancel{2}\pi}{\cancel{8}4}$$

$$\Omega = \pi/4$$

b) $N=32$

$$\Omega = \frac{2\pi}{N}$$

$$\Omega = \frac{\cancel{2}\pi}{\cancel{32}16}$$

$$\Omega = \pi/16.$$

c) $N=64$

$$\Omega = \frac{2\pi}{16}$$

$$\Omega = \pi/8.$$

d) $N=128$

$$\Omega = \frac{2\pi}{128}$$

$$\Omega = \pi/64$$

Relation between Sinusoidal & Complex exponential signals.

Consider the complex exponential $e^{j\theta}$. Using Euler's identity, ~~we may expont~~ we can write.

$$e^{j\theta} = \cos\theta + j\sin\theta$$

this result indicates that we may express the continuous-time sinusoidal signal of eqn. $x(t) = A\cos(\omega t+\phi)$ as the real part of the complex exponential signal $Be^{j\omega t}$, where

$$B = Ae^{j\phi}$$

ie
$$A\cos(\omega t+\phi) = Re\{Be^{j\omega t}\}$$

$$A\sin(\omega t+\phi) = Im\{Be^{j\omega t}\}.$$

$|||^{ly}$ discrete-time case

$$A\cos(\Omega n+\phi) = Re\{Be^{j\Omega n}\}$$

&

$$A\sin(\Omega n+\phi) = Im\{Be^{j\Omega n}\}.$$

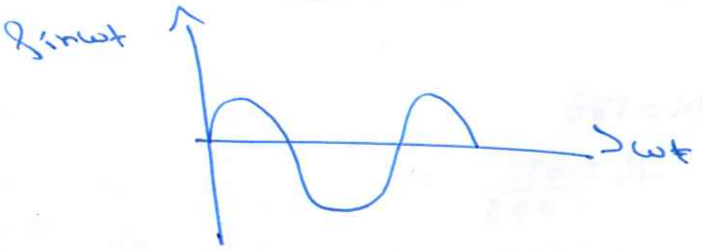
6) Exponentially damped sinusoidal signals

The multiplication of a sinusoidal signal, $A\sin(\omega t+\phi)$ by a real valued decaying exponential signal, $e^{-\alpha t}$ results in a new signal referred to as an exponentially damped sinusoidal signal.

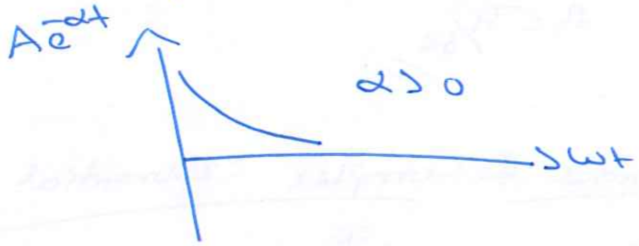
$$x(t) = Ae^{-\alpha t}\sin(\omega t+\phi), \quad \alpha > 0$$

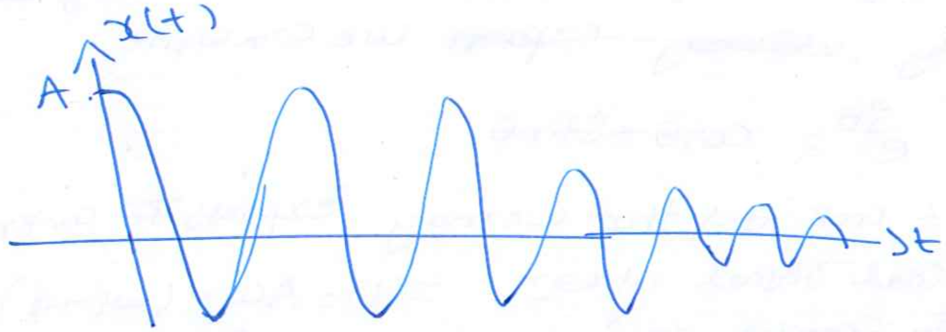
$^{\text{lly}}$ discrete-time version

$$x(n) = Br^{n}\sin(\Omega n+\phi) \qquad \underline{0<|r|<1}.$$



$|e|\ \phi = 0$.





Exponentially damped sinusoidal signal shown for $\underline{t \geq 0}$.

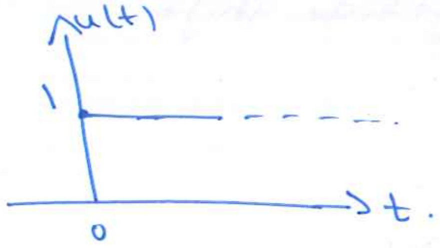
Problems on ~~exponential~~ elementary signal

1) Sketch the following signals.

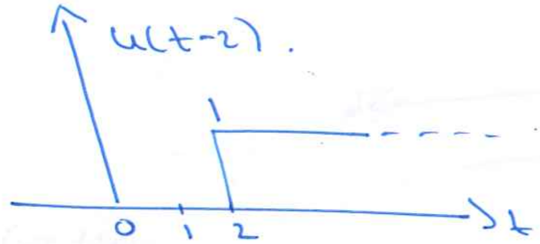
$$x(t) = u(t) - u(t-2).$$

Soln: Given $x(t) = u(t) - u(t-2)$.

$$\boxed{\text{w.k.T} \quad u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}}$$



The signal $u(t-2)$ is obtained by shifting $u(t)$ to the right by 2 units.



i.e $u(t-2) = 1 \quad t \geq 2$

$\qquad\qquad\quad 0 \; ; t > 2$

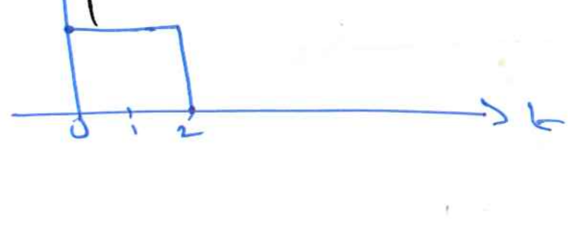
Now $x(t) = u(t) - u(t-2)$ is obtained as follows.

for $t < 0$; $u(t) = u(t-2) = 0$ $\quad \therefore x(t) = 0 - 0 = 0$

for $0 < t < 2$; $u(t) = 1$ & $u(t-2) = 0$ $\quad \therefore x(t) = 1 - 0 = 1$

for $t > 2$; $u(t) = 1$ & $u(t-2) = 1$ $\quad \therefore x(t) = 1 - 1 = 0$.

$x(t) = u(t) - u(t-2)$.

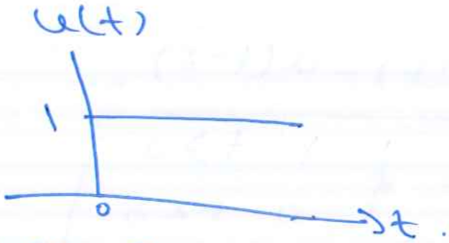


	$u(t)$	$u(t-2)$	$x(t)$
$t<0$	0	0	$0-0=0$
$0<t<2$	1	0	$1+0=1$
$t>2$	1	1	$1-1=0$

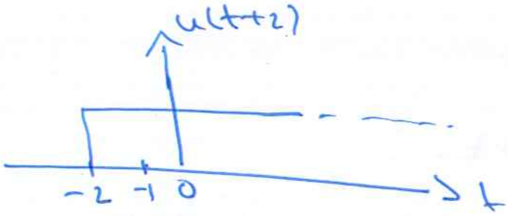
2) Sketch

$$x(t) = -u(t+2) + u(t-1).$$

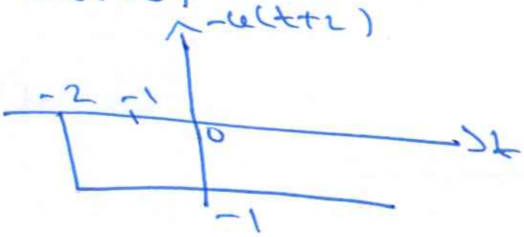
Soln. w.k.t.



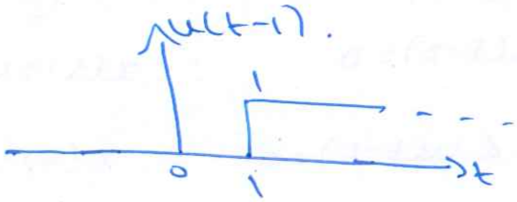
Step: $u(t+2)$ left side shift



Step2: $-u(t+2)$

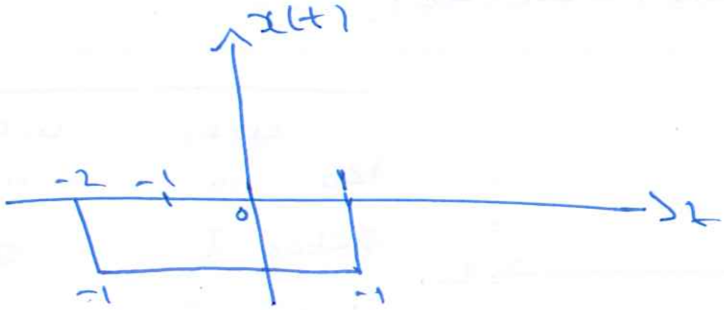


Step3: $u(t-1)$ right side shift



	$-u(t+2)$	$u(t-1)$	$x(t)$
$t<-2$	0	0	0
$-2<t<1$	1	0	1
$t>1$	1	1	0

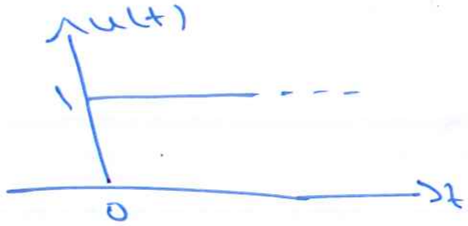
Step4: $x(t) = -u(t+2) + u(t-1)$



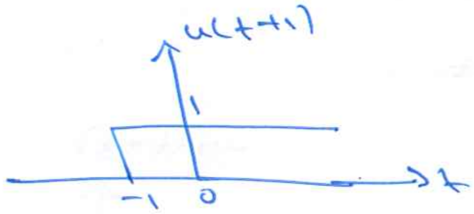
3) Sketch

$x(t) = u(t+1) - 2u(t) + u(t-2)$.

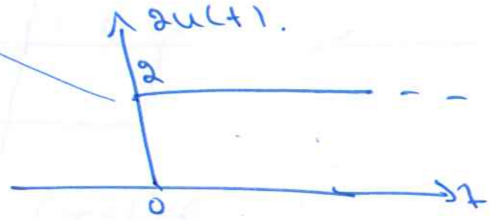
Soln: w.k.T



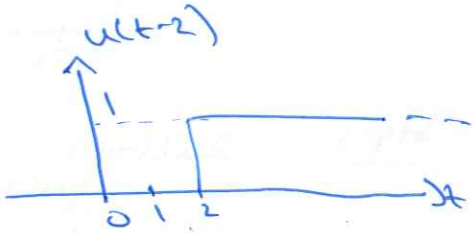
Step 1: $u(t+1)$ left side shift



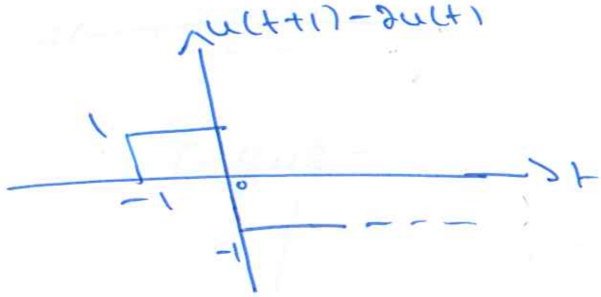
Step 2: $2u(t)$



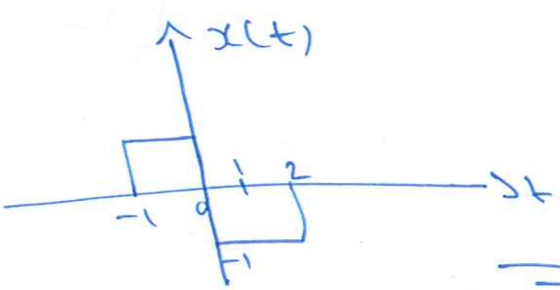
Step 3: $u(t-2)$ right side shift



Step 4: $u(t+1) - 2u(t) = y(t)$



Step 5 $= y(t) + u(t-2). = x(t)$



	$u(t+1)$	$-2u(t)$	$+u(t-2)$	
$t < -1$	0	0	0	0
$-1 < t < 0$	1	0	0	1
~~0<t~~ $0 < t < 2$	1	2	0	-1
$t > 2$	1	2	1	0

(4) $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$.

Soln:

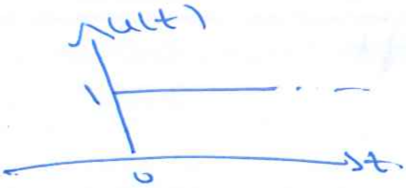
$x_1(t) = -u(t+3)$

$x_2(t) = 2u(t+1)$

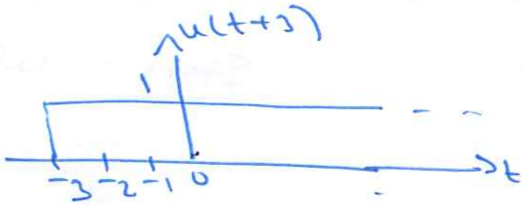
$x_3(t) = -2u(t-1)$

$x_4(t) = u(t-3)$.

Step 1:



Step 2: $u(t+3)$



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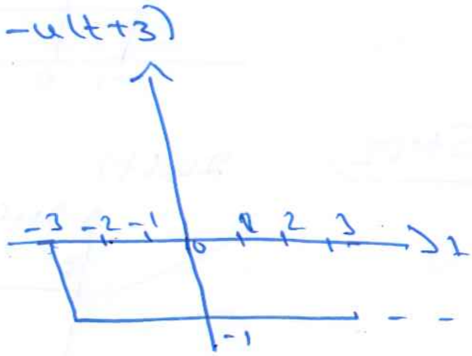
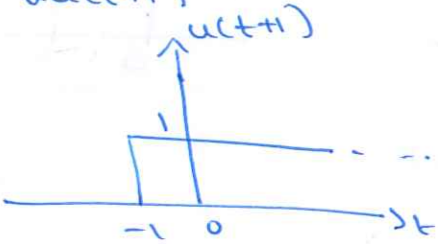


Step 3 $2u(t+1)$



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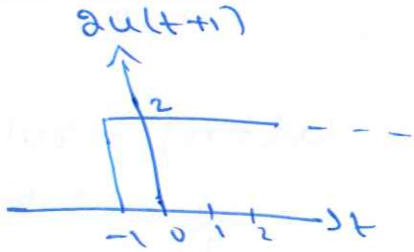
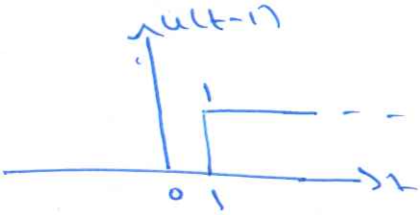


Step 4

$-2u(t-1)$



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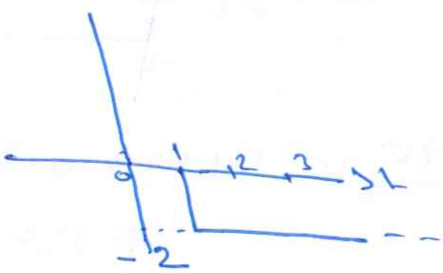
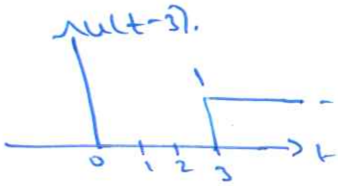


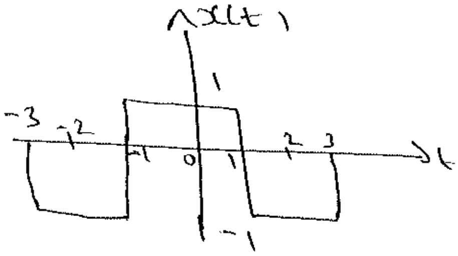
Step 5 $u(t-3)$



Now $x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t)$

	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x(t)$
for $t < -3$	0	0	0	0	$0+0+0+0=0$
for $-3 < t < -1$	-1	0	0	0	-1
for $-1 < t < 1$	-1	2	0	0	1

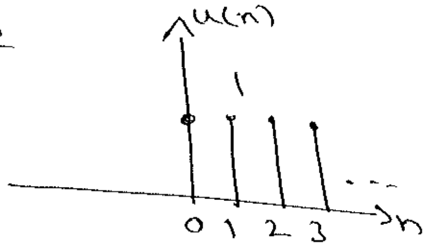
	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x(t)$
for $1<t<3$	-1	2	-2	0	-1
for $t>3$	-1	2	-2	1	0



Problems Step function [D.T] Sketch The following signals

(a) $x(n) = u(n) - u(n-4)$

Soln



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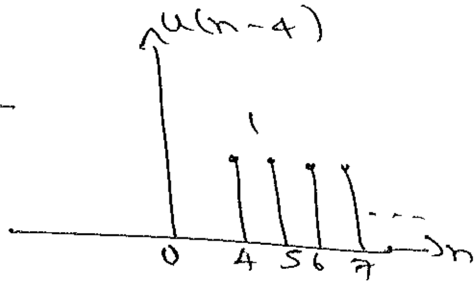
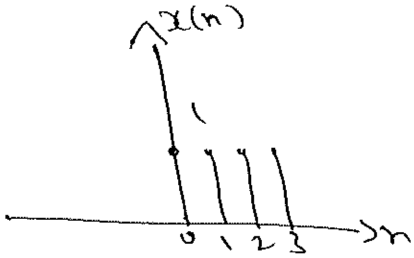
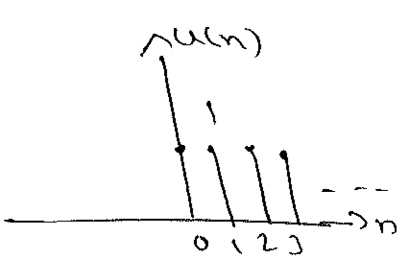
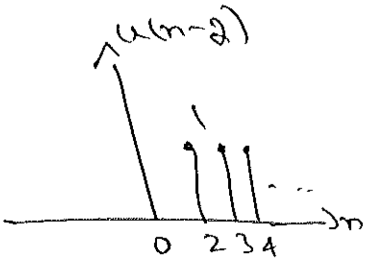




(b) $x(n) = 2\left[u(n-2) - u(n-6)\right]$

Soln





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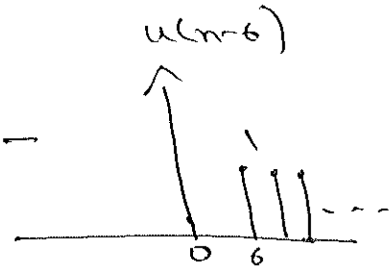
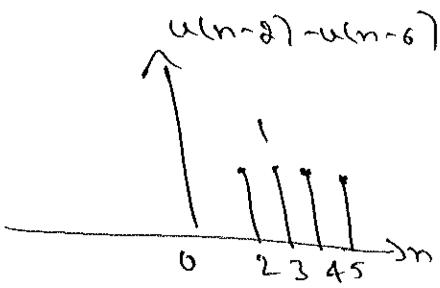
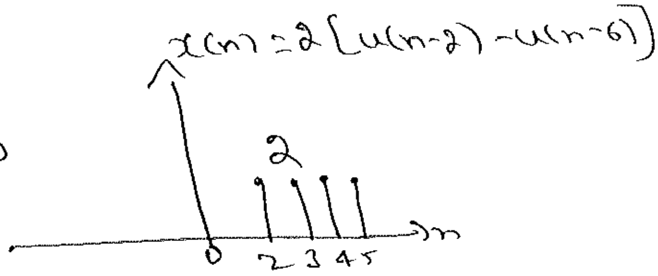




$\Rightarrow$



Problems on Impulse function

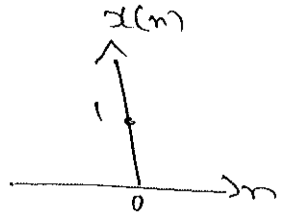
(1) Sketch The following signals

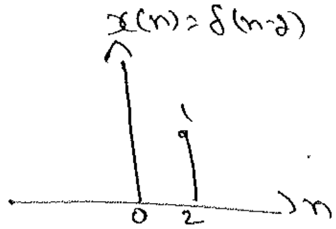
(a) $x(n) = \delta(n)$

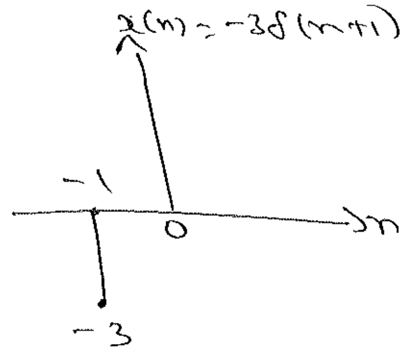
(b) $\delta(n-2)$

(c) $-3\,\delta(n+1)$

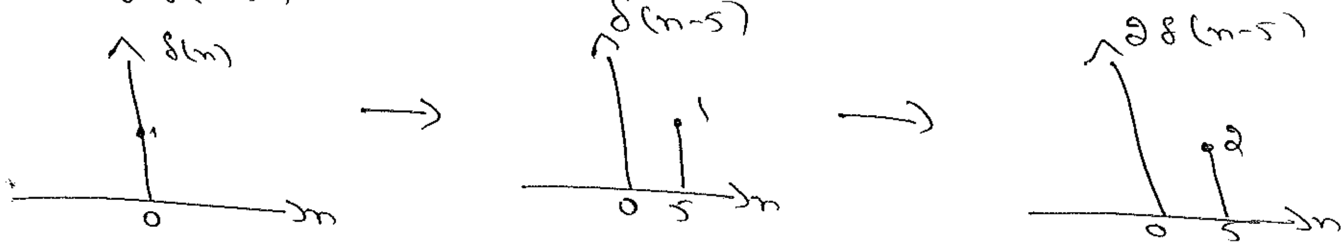
Soln





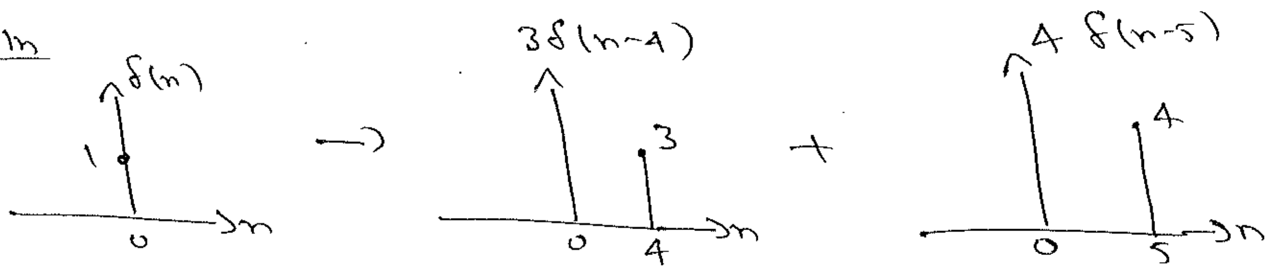


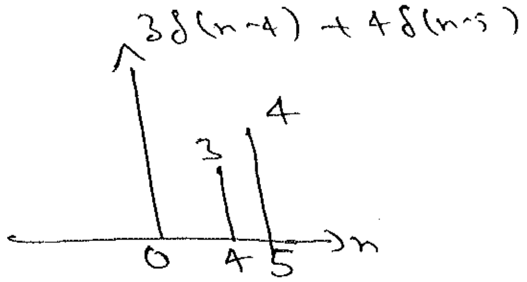
(d) $2\,\delta(n-5)$



(e) $3\delta(n-4) + 4\,\delta(n-5)$

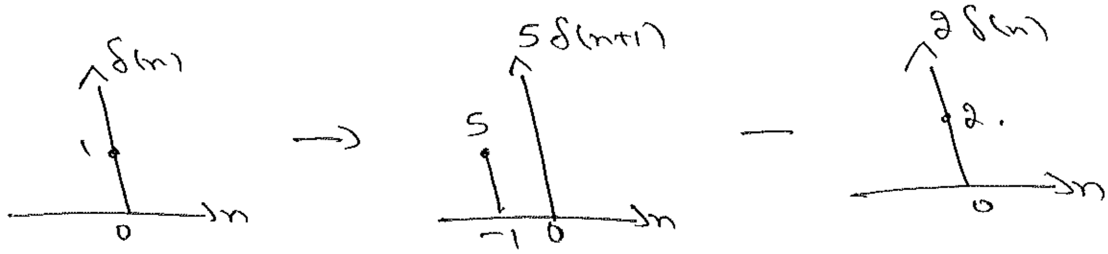
Soln

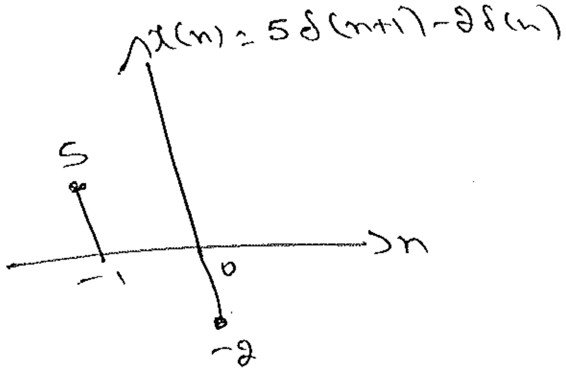




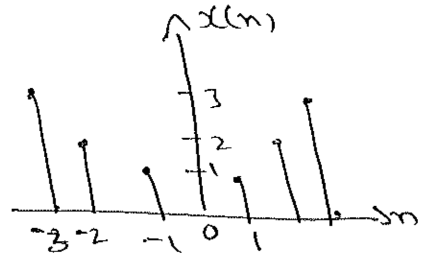
(d) $5\,\delta(n+1) - 2\,\delta(n)$

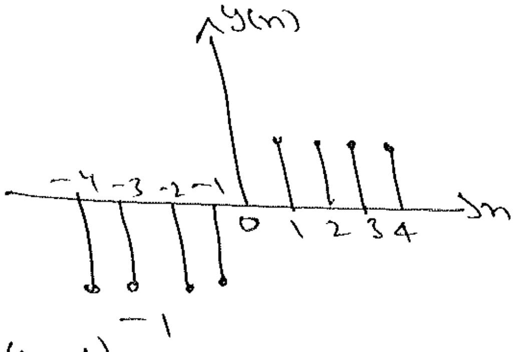
Soln





(2) The discrete time signals $x(n)$ & $y(n)$ as shown in below figure a & b respectively. Sketch the signal.





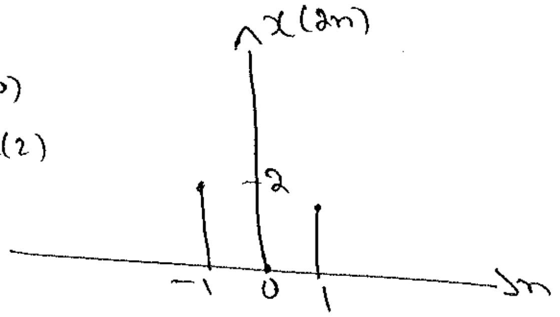
(a) $z(n) = x(2n) \cdot y(n-4)$

Soln

$x(2\times0) = x(0)$

$x(2\times1) = x(2)$

$x(-2)$



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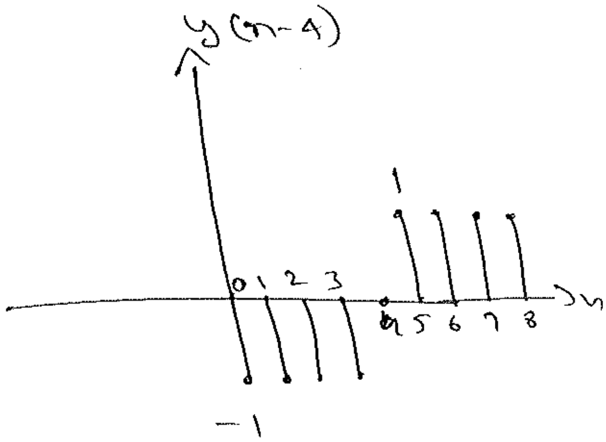
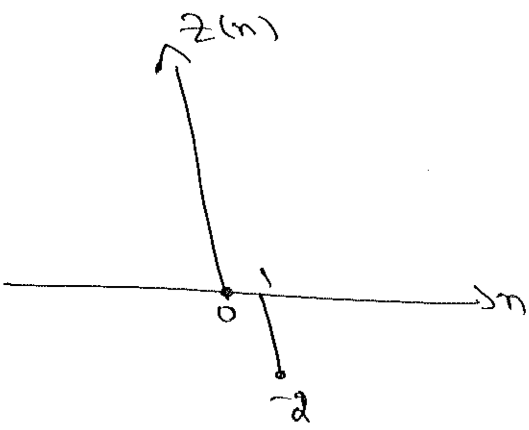
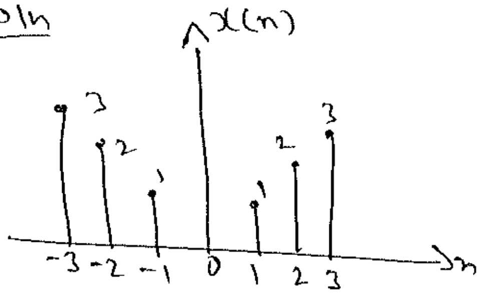
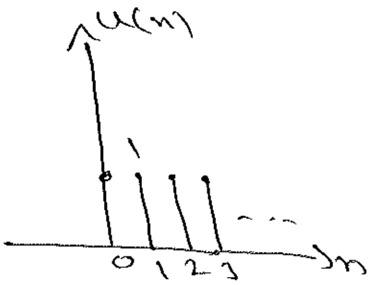


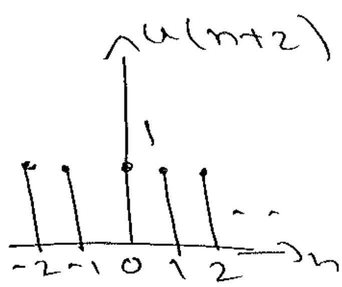


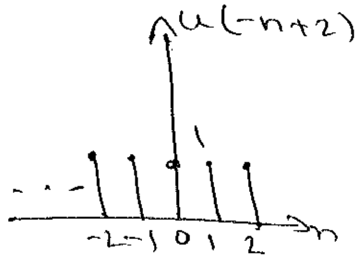
(b) $z(n) = x(n) \cdot u(2-n)$ $\qquad$ $u(2-n) = u(-n+2)$

Soln









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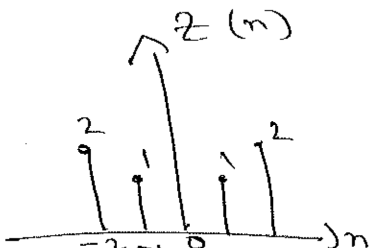
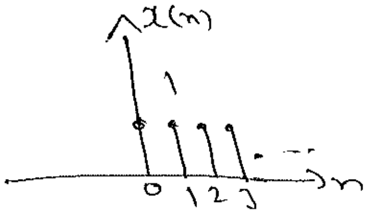


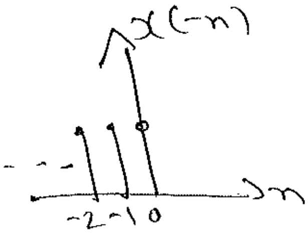
(3) Find the even & odd signals

(a) $x(n) = u(n)$ (b) $x(n) = \delta(n)$

Soln $x(n) = u(n)$

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$





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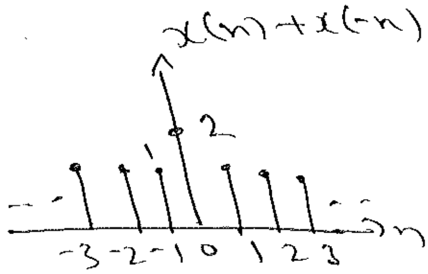
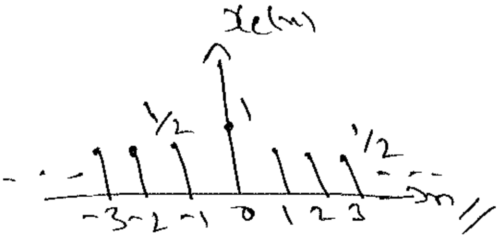
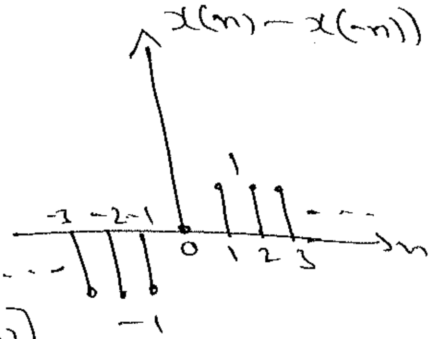
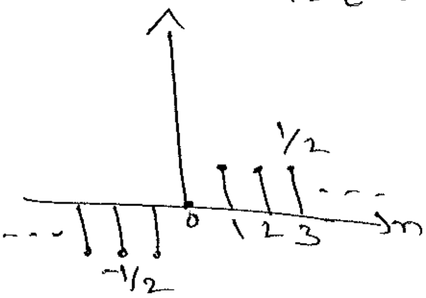


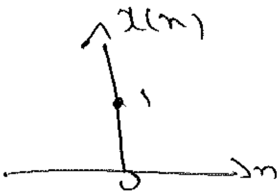


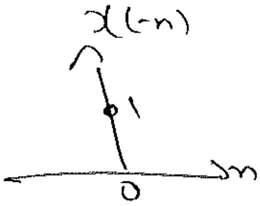


$$x_o(n) = \frac{1}{2}\left[x(n) \ast x(-n)\right]$$

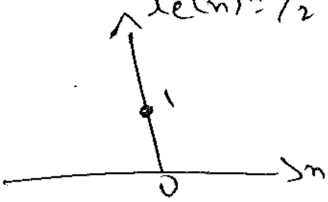


(b) $x(n) = \delta(n)$

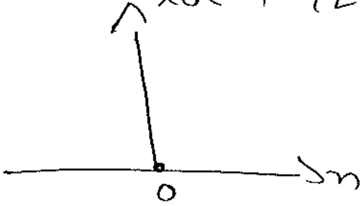




$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$



$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$



1) Sketch each of the following signals & find its energy or Power appropriate.

a) $x(n) = (\overset{\downarrow}{2}, 4, 1, 1)$.

b) $x(n) = (-3, -2, \overset{\downarrow}{-1}, 1, 0)$

c) $x(n) = \cos(n\pi/2)$

d) $x(n) = 2(0.5)^n u(n)$

Soln:

a) $x(n) = (\overset{\downarrow}{2}, 4, 1, 1)$

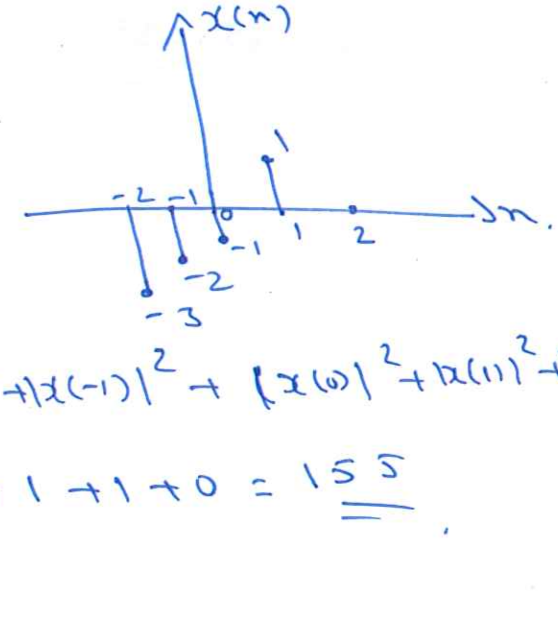
The given signal is energy

$$x(n) = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$x(n) = \sum_{n=0}^{3} |x(n)|^2 = |x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2$$

$$x(n) = 4 + 16 + 1 + 1 = \underline{\underline{22\,J}}$$

b) $x(n) = (-3, -2, \overset{\downarrow}{-1}, 1, 0)$



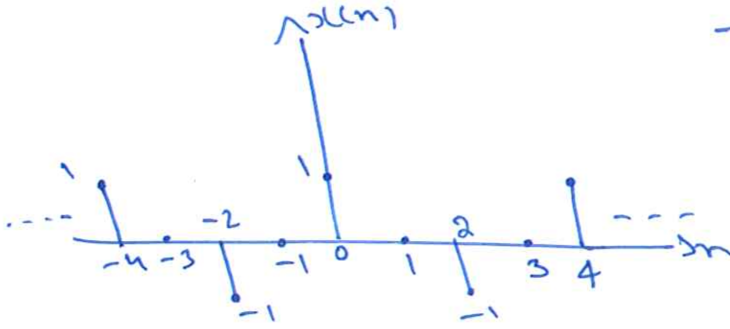
The given signal is energy.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=-2}^{2} |x(n)|^2 = |x(-2)|^2 + |x(-1)|^2 + |x(0)|^2 + |x(1)|^2 + |x(2)|^2$$

$$= 9 + 4 + 1 + 1 + 0 = \underline{\underline{15\,J}}$$

c) $x(n) = \cos(n\pi/2)$.



The given signal is Power.

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

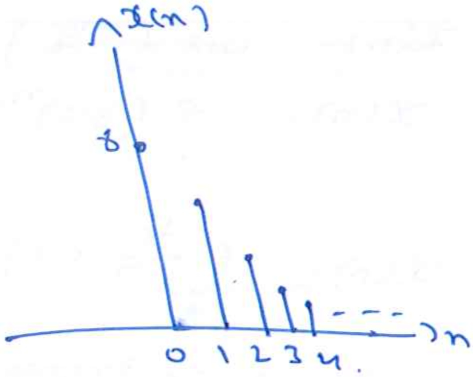
$\underline{\underline{N = 4}}$ or $\Omega = \pi/2$

$N = \frac{2\pi}{\pi/2} = 4$

$$P = \frac{1}{4} \sum_{n=0}^{3} |x(n)|^2 = \frac{1}{4}\left[|x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2\right]$$

$$P = 1/4\,[1+0+1+0] = 1/2 = 0.5\,W$$

d) $x(n) = 8(0.5)^n u(n)$.



The given signal is Energy

$$E = \sum_{n=0}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} |8(0.5)^n|^2.$$

$$= 64 \sum_{n=0}^{\infty} (0.25)^n$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}.$$

$$= 64 \cdot \frac{1}{1-0.25}$$

$$= 85.335$$

Classify the following signals as energy signals, power signals or neither.

a) $x(n) = \cos(n\pi)$

(b) $x(n) = e^{jn\pi}$

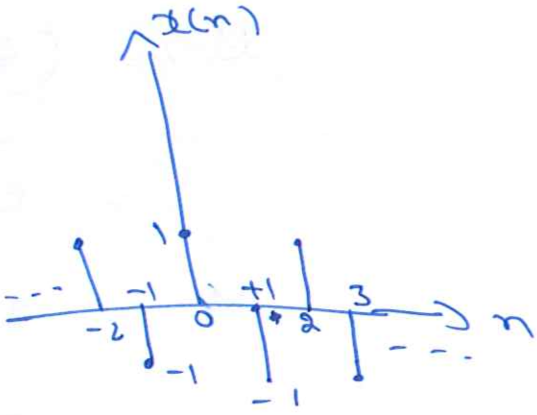
(c) $x(n) = (j)^n + (j)^{-n}$

Soln:

a) $x(n) = \cos(n\pi)$.

$\Omega = \pi \quad N = \frac{2\pi}{\pi}$ or

$N = 2$



$N = 2$.

The given signal is power signal

$N = 2$

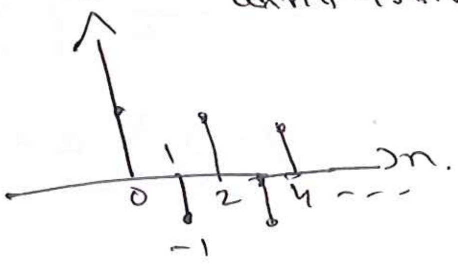
$$P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

$$P = \frac{1}{2}\sum_{n=0}^{1} |x(n)|^2 = \frac{1}{2}\left[|x(0)|^2 + |x(1)|^2\right]$$

$$P = \tfrac{1}{2}[1+1] = \underline{1\omega}$$

b) $x(n) = e^{jn\pi}$.

$e^{jn\pi} = \cancel{\cos n\pi + j\sin n\pi} - \cos n\pi + j\sin n\pi$



$\Omega = \pi \quad N = \frac{2\pi}{\pi} \quad N = 2.$

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

$$P = \frac{1}{2}\sum_{n=0}^{1} |x(n)|^2 = \tfrac{1}{2}\left[|x(0)|^2 + |x(1)|^2\right]$$

$$P = \tfrac{1}{2}[1+1] = \underline{1\omega}$$

$\cos\pi + j\sin\pi$

c) $x(n) = (j)^n + (j)^{-n}$

w.k.T $j = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$

$= e^{j\pi/2}$

$$x(n) = \left(e^{j\pi/2}\right)^n + \left(e^{j\pi/2}\right)^{-n}$$

$$= e^{jn\pi/2} + e^{-jn\pi/2}.$$

$$= 2\cos\left(n\pi/2\right).$$

$$= \cos n\pi/2 + j\sin n\pi/2 + \cos n\pi/2 - j\sin n\pi/2$$

$$= 2\cos\left(n\pi/2\right).$$

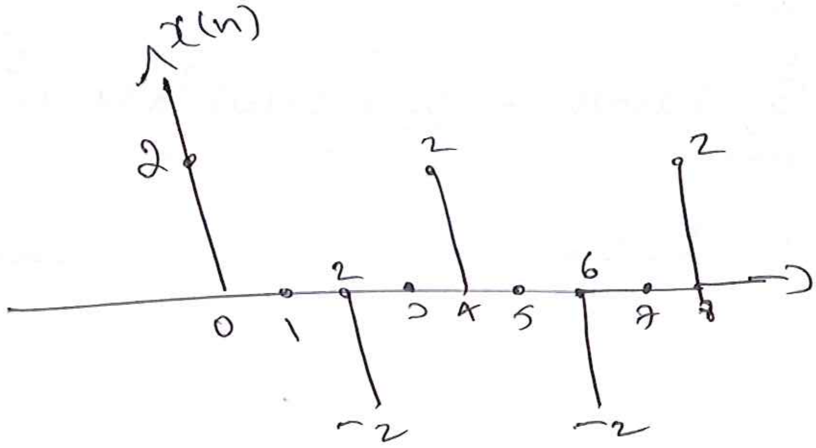
$\Omega = \pi/2 \quad N = \frac{2\pi}{\Omega} = 2\pi/\pi/2 = \underline{\underline{4}}$

$x(n) = \{2, 0, -2, 0\}.$

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

$$P = \frac{1}{4}\sum_{n=0}^{3} |x(n)|^2 = \frac{1}{4}\left[|x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2\right]$$

$$P = \frac{1}{4}\left[4+0+4+0\right] = \underline{2w}$$



$x(n) = \{2, 0, -2, 0\}$

$N = N_1 - N_0 = 4 - 0 = 4_{//}$

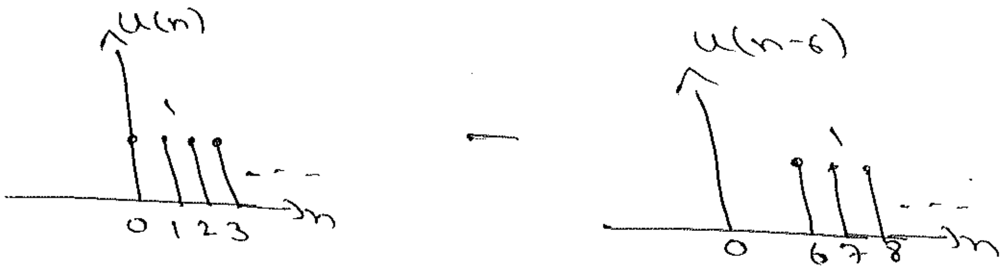
(4) Given a sequence $x(n) = (6-n)[u(n)-u(n-6)]$ make a sketch of

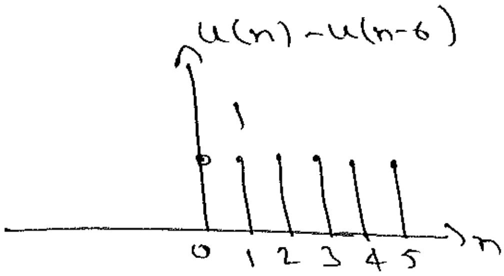
(a) $y_1(n) = x(4-n)$

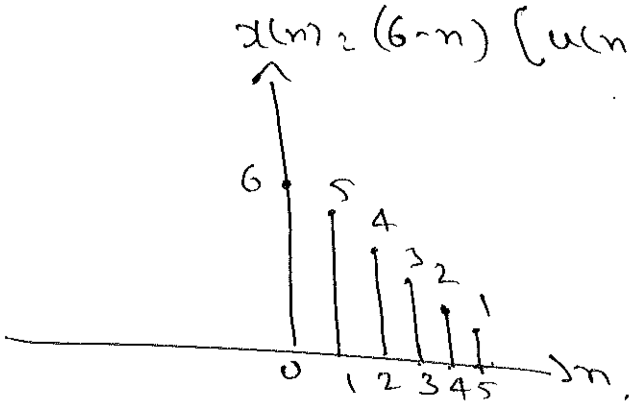
(b) $y_2(n) = x(2n-3)$

(c) $y_3(n) = x(n+2)$

Soln

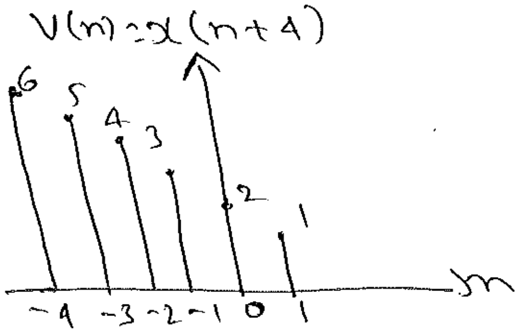


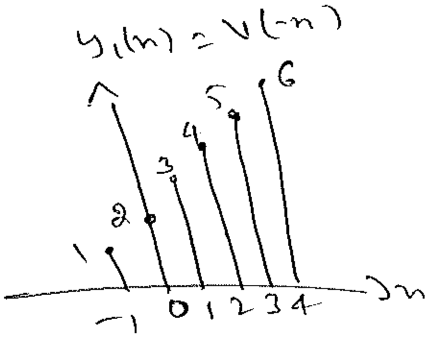




(a) $y_1(n) = x(4-n)$

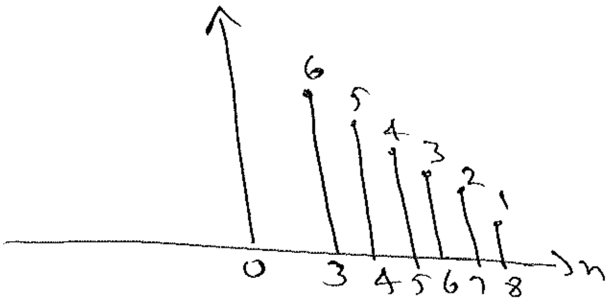
$y_1(n) = x(-n+4)$



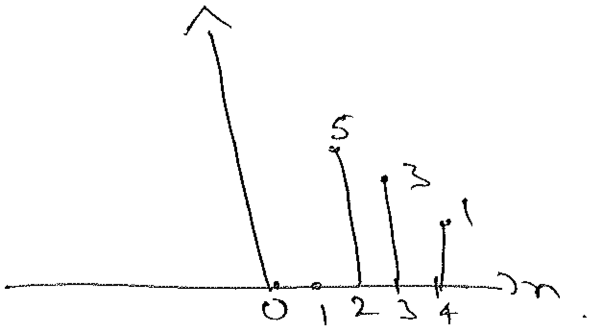


(b) $y_2(n) = x(2n-3)$

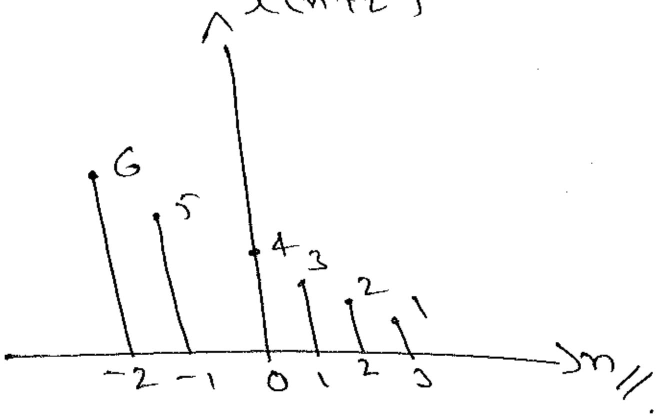
$v(n) = x(n-3)$



$y_2(n) = v(2n)$



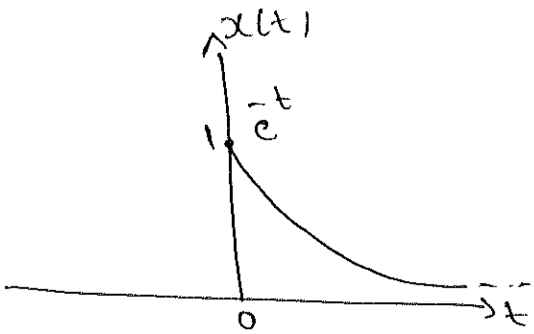
(c) $y_3(n) = x(n+2)$

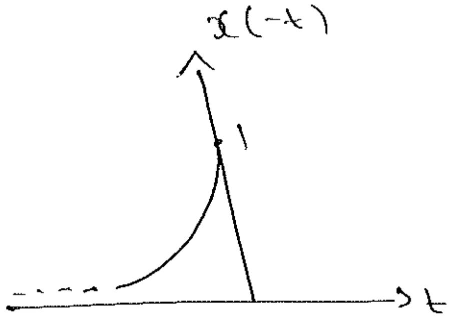


(2) Find The even & odd part

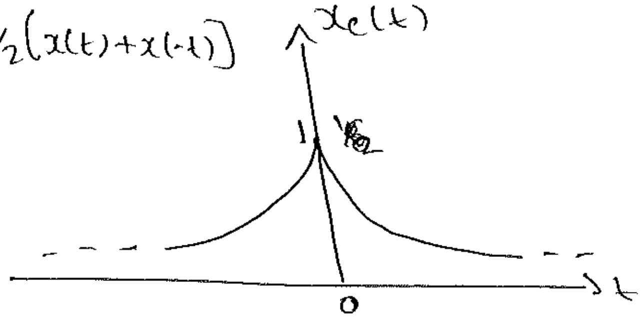
$$x(t) = e^{-t}\,u(t)$$

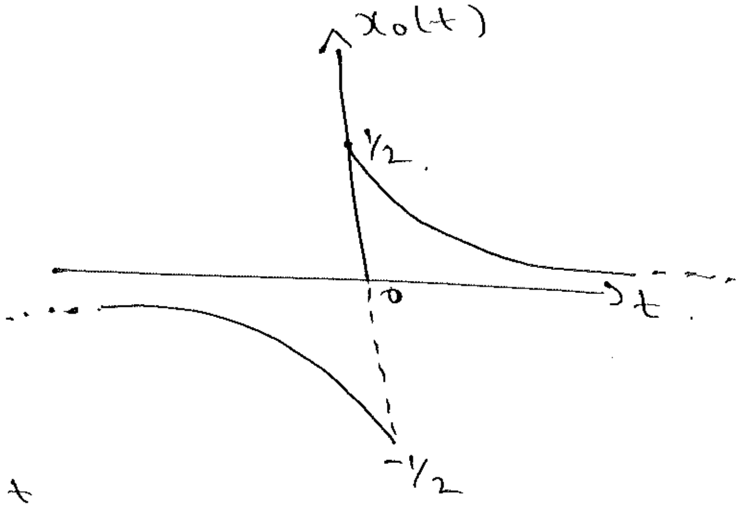
Soln $\quad x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] \qquad x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$





$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$



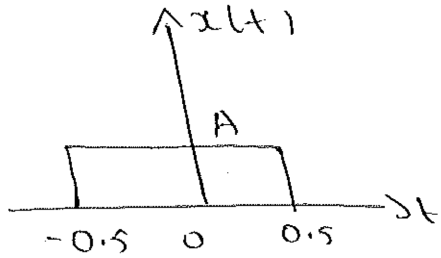


$$x(-t) = e^{t}\,u(-t)$$

$$x_e(t) = \frac{1}{2}\left[e^{-t}u(t) + e^{+t}u(-t)\right]$$

$$x_o(t) = \frac{1}{2}\left\{e^{-t}u(t) - e^{t}u(-t)\right]$$ //.

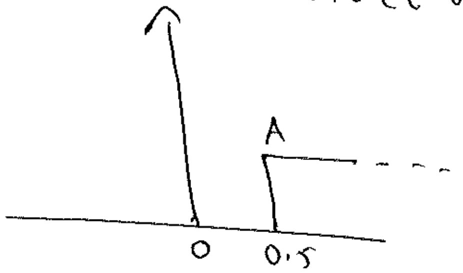
(3) Consider The Reactangular pulse $x(t)$ shown in figure express $x(t)$ as a weighted sum of two step functions.



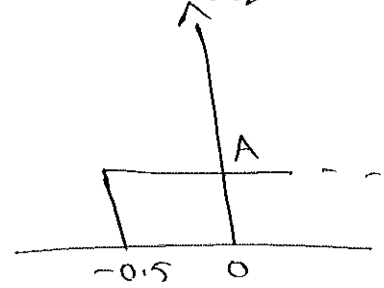
Soln The Reactangular pulse $x(t)$ may be written as

$$x(t) = \begin{cases} A & -0.5 < t < 0.5 \\ 0 & |t| > 0.5 \end{cases}$$

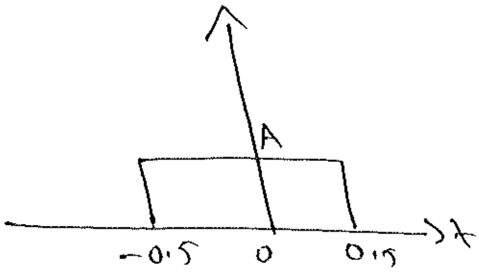
$x_1(t) = Au(t-0.5)$



$x_2(t) = Au(t+0.5)$



$x(t) = -x_1(t) + x_2(t)$



$x(t) = -Au(t-0.5) + Au(t+0.5)$ //

or

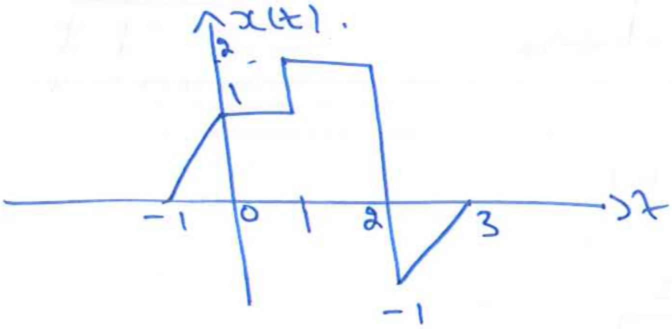
$x(t) = Au(t+0.5) - Au(t-0.5)$ //

$u(t)$ is step function

Additional Problems

1) A Continuous-time signal $x(t)$ shown in below figure. Draw the signal

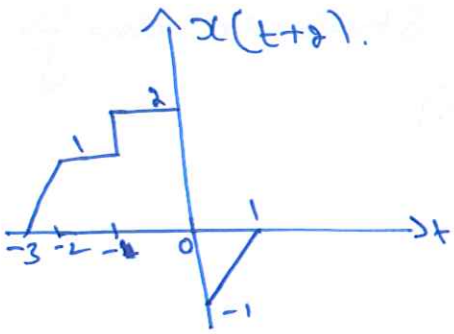
$$y(t) = \{x(t) + x(2-t)\}\, u(1-t).$$



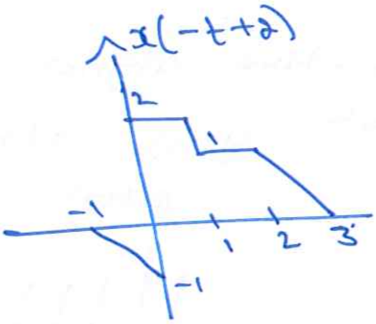
Soln

$x(t)$ is given, find $x(2-t)$

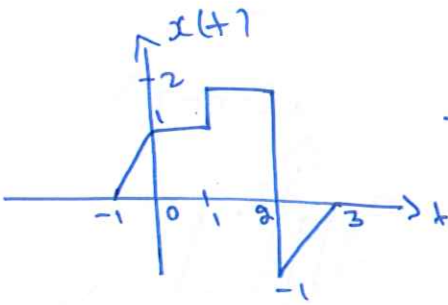
Step 1: $x(2-t) = x(-t+2)$.



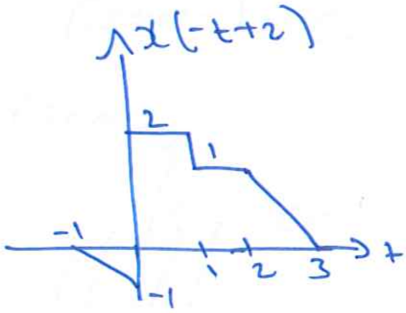
$\Rightarrow$



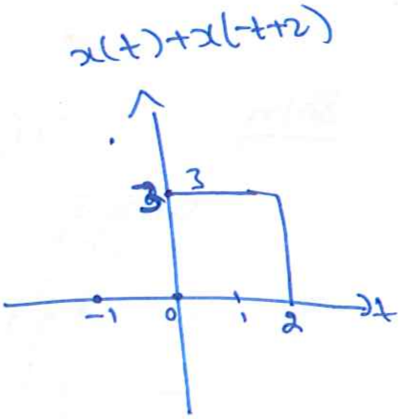
$x(t) + x(-t+2)$



$+$

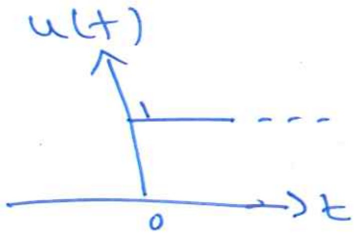


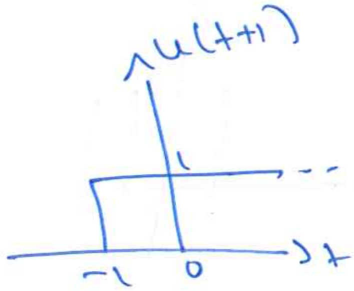
$=$



Step 2: $u(1-t) \Rightarrow u(-t+1)$

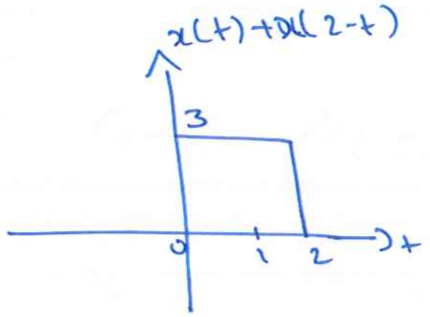
W.K.T





$=$

$$y(t) = \{x(t) + x(2-t)\}\, u(1-t).$$



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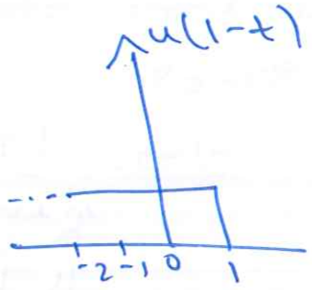
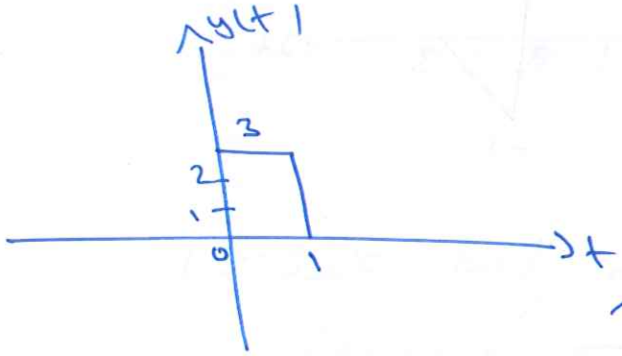
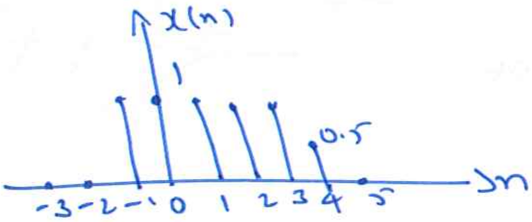




//.

2) A discrete-time signal $x(n)$ is shown in below fig. Sketch the signal

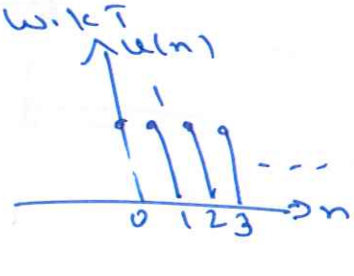
$$y(n) = x(n) \cdot u(2-n).$$



<u>Soln</u>

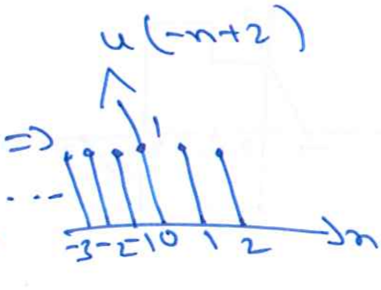
$$u(2-n) \Rightarrow u(-n+2)$$

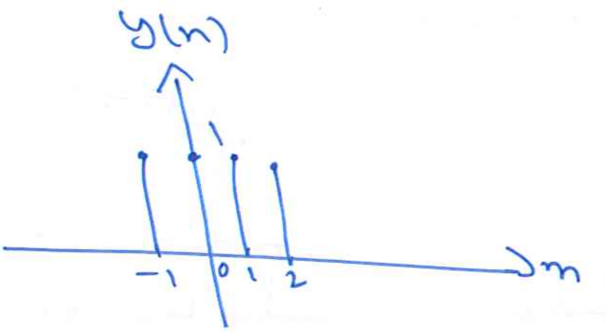
w.k.T



$\Rightarrow$

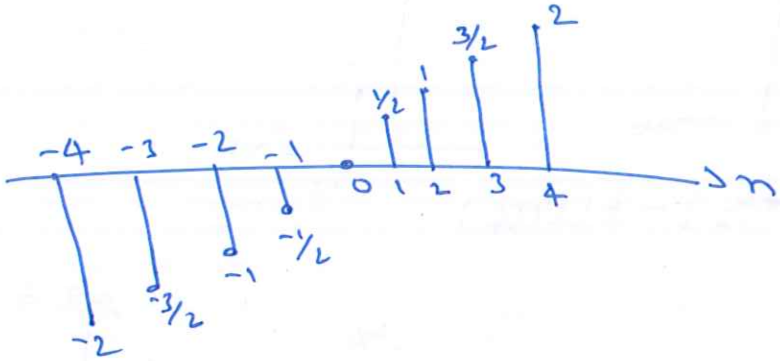
$\Rightarrow$





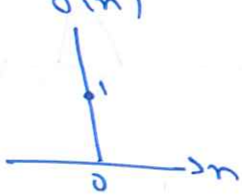
3) A discrete-time sequence $h(n)$ is shown in below figure. Sketch the signal

$x(n) = h(3n) \cdot \delta(n-1)$.

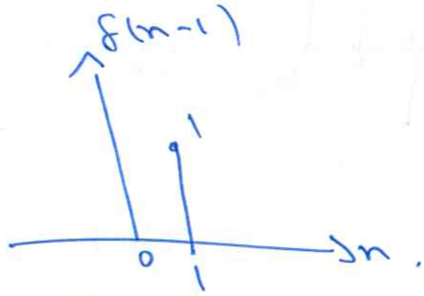


Soln

W.K.T

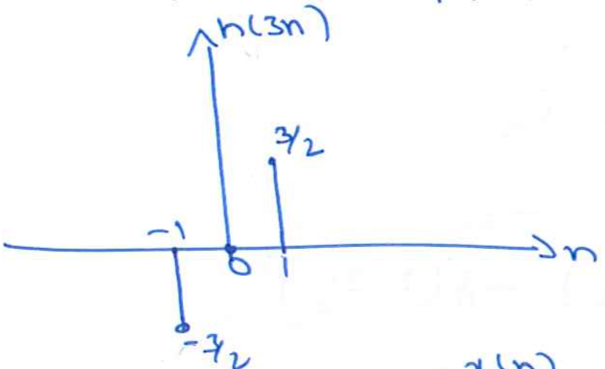


$\Rightarrow$



$h(n)$ is given find $h(3n)$.

$K = 3$ $\quad$ $K > 1$ some signal loss in $h(n)$.



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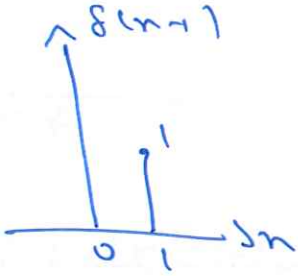
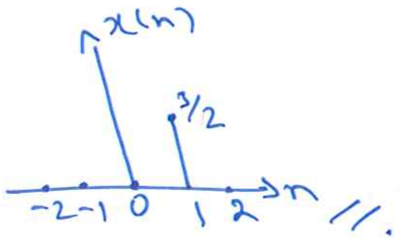
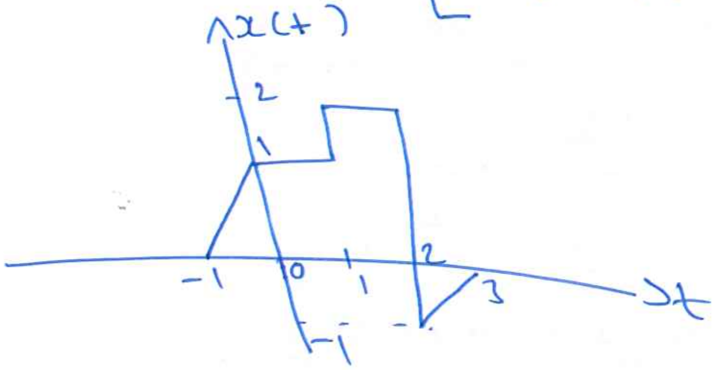




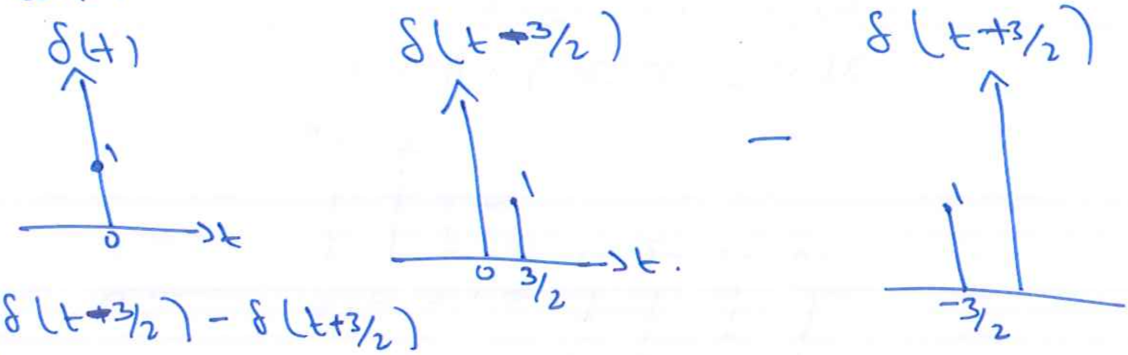
//.

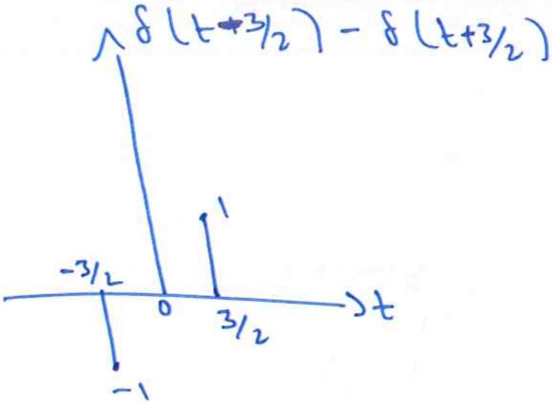
4) A continuous time signal is shown in below figure find $y(t) = x(t)\left[\delta(t-3/2) - \delta(t+3/2)\right]$.



Soln.

w.k.T





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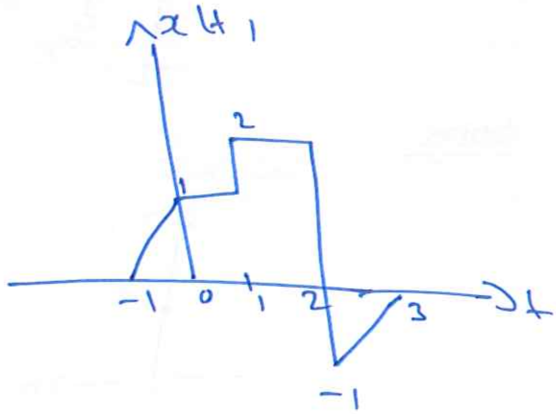
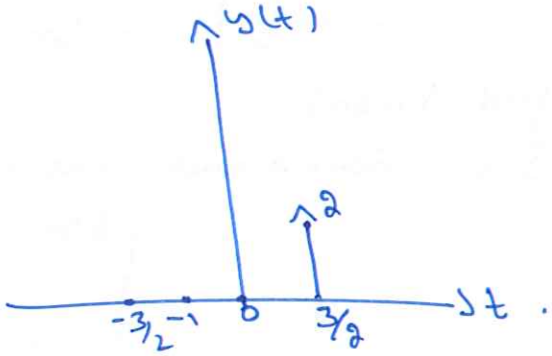
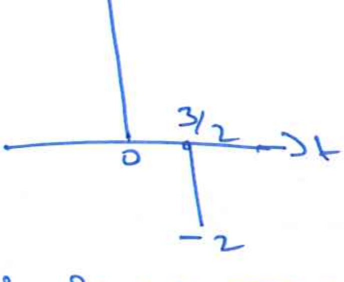




Assign

Find $y(t) = x(t)\left[\delta(t+3/2) - \delta(t-3/2)\right]$

Ans



5) Find & sketch the even & odd components of the signal

$$x(t) = e^{-(t/4)} \cdot u(t).$$

Soln

$$x(t) = e^{(-t/4)} \cdot u(t)$$

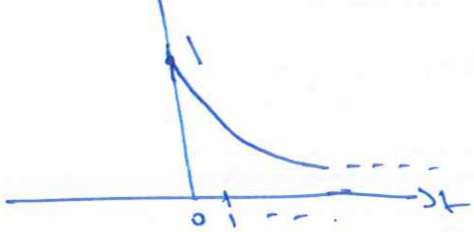
$$x(t) = (0.779)^t \cdot u(t)$$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

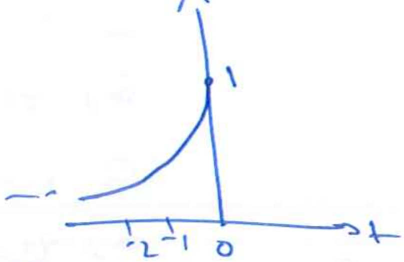
45C

$x(t) = (0.779)^t u(t)$

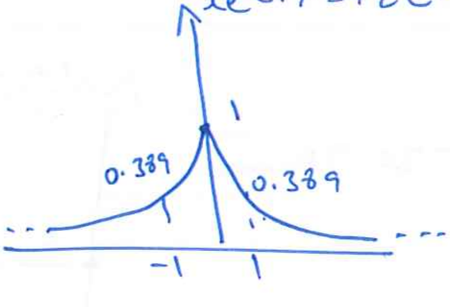


+

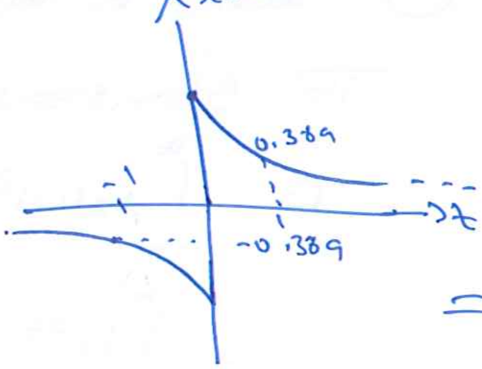
$x(-t)$



$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$



$x_o(t)$



=

Sketch the signal & find its energy or power as appropriate.

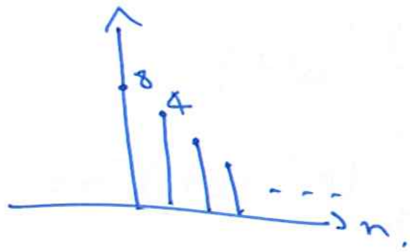
$$x(n) = 8(0.5)^n u(n)$$

Soln

w.k.T

$u(n)$

$x(n)$



$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} |8(0.5)^n|^2 \qquad \alpha < 1$$

$$= 64 \sum_{n=0}^{\infty} (0.25)^n \qquad \text{w.k.T} \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$$

$$= \frac{64}{1-0.25} = \underline{85.33\ J}$$

Compute the energy for each of the following signals

ⓐ $x(t) = e^{2t} u(-t)$

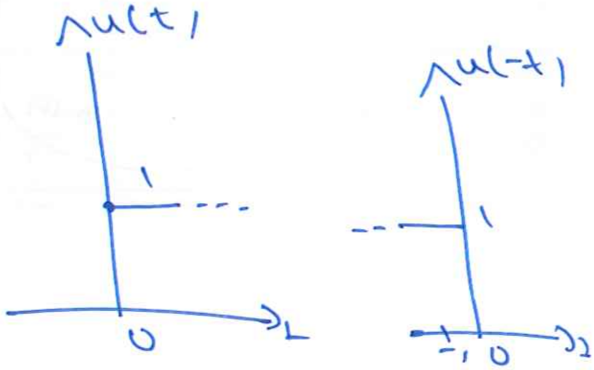
ⓑ $x(t) = e^{t-1} u(-t)$

ⓒ $x(t) = e^{1+2t} u(1-t)$.

d) $x(n) = 2^n u(-n)$

Soln

ⓐ $x(t) = e^{2t} u(-t)$



The given signal is Energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E = \int_{-\infty}^{\infty} |e^{2t} u(-t)|^2 \, dt.$$

$$= \int_{-\infty}^{0} |e^{4t}| \, dt = \frac{e^{4t}}{4} \Big|_{-\infty}^{0} = \frac{1-0}{4} = 0.25 \text{ J}$$

ⓑ $x(t) = e^{t-1} u(-t)$

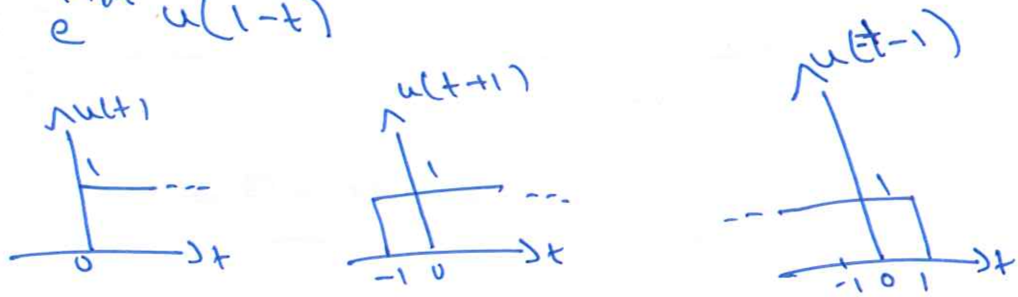
$$E = \int_{-\infty}^{\infty} (e^{t-1} u(-t))^2 \, dt = \int_{-\infty}^{0} e^{2t} \cdot e^{-2} \, dt$$

$$E = e^{-2} \int_{-\infty}^{0} e^{2t} \, dt = e^{-2} \left[\frac{e^{2t}}{2} \Big|_{-\infty}^{0} \right] = \frac{e^{-2}}{2} [1]$$

$E = 0.068$ J

c) $x(t) = e^{1+2t} u(1-t)$

Soln



$$u(1-t) = \begin{cases} 1 & t < 1 \text{ or } 1-t > 0 \\ 0 & t > 1 \end{cases}$$

$$E = \int_{-\infty}^{\infty} \left| e^{2t+1} \cdot u(1-t) \right|^2 dt$$

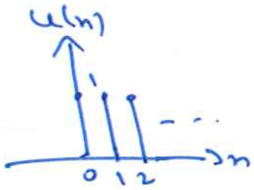
$$E = \int_{-\infty}^{1} \left| e^{2t+1} \right|^2 dt = \int_{-\infty}^{1} e^{4t} \cdot e^{2}\, dt.$$

$$E = e^{2} \left[\frac{e^{4t}}{4} \Big|_{-\infty}^{1} \right]$$

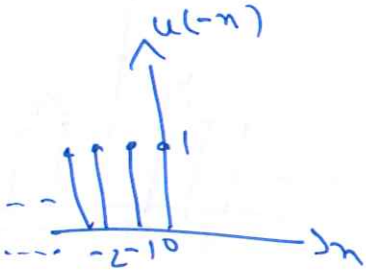
$$E = e^{6} \times 0.25 = \underline{100.865}.$$

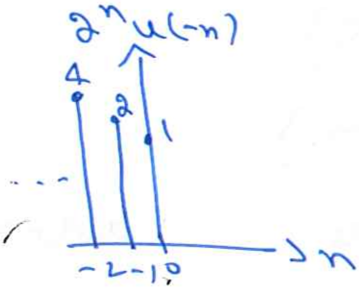
d) $x(n) = 2^n u(-n)$

Soln



$\Rightarrow$





it is energy signal

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |2^n u(-n)|^2$$

$$= \sum_{n=-\infty}^{0} |2^n|^2$$

$$E = \sum_{n=-\infty}^{0} 4^n = \sum_{n=0}^{-\infty} 4^n \quad \text{Replace } n \text{ by } -n$$

$$= \sum_{n=0}^{\infty} 4^{-n} = \sum_{n=0}^{\infty} \left(\tfrac{1}{4}\right)^n$$

$$= \frac{1}{1-\frac{1}{4}} = 4/3\ J \;//.$$

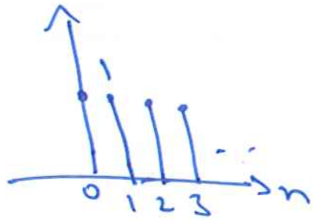
Find the even & odd parts of the following signals.

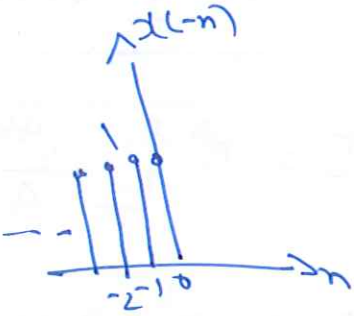
a) $x(n) = u(n)$ (b) $y(n) = \alpha^n u(n)$

(c) $x(t) = e^{-t} u(t)$ d) $x(n) = 8(0.5)^n u(n)$

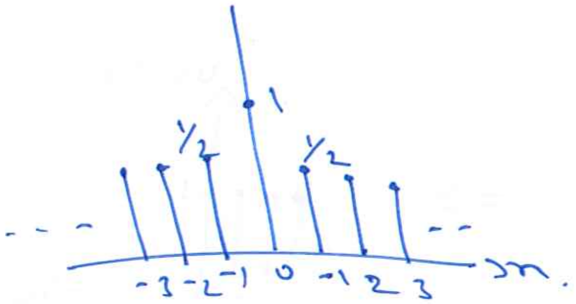
Soln:

(a) $x(n) = u(n)$

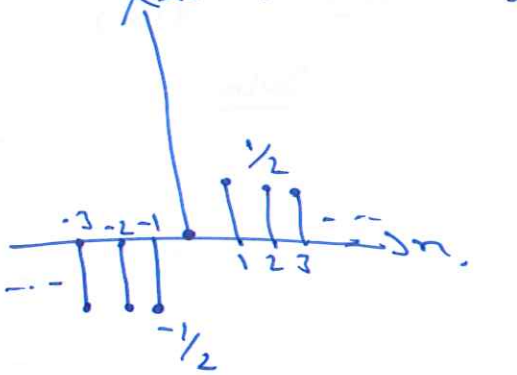




$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$



$x_o(n) = \frac{1}{2}(x(n) - x(-n))$



$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$u_e(n) = \frac{1}{2}[u(n) + u(-n)]$

$$u_e(n) = \begin{cases} 1 & n = 0 \\ \frac{1}{2} & n \neq 0 \end{cases}$$

$$u_o(n) = \begin{cases} \frac{1}{2} & n > 0 \\ -\frac{1}{2} & n < 0 \\ 0 & n = 0 \end{cases}$$

u_e ~~(n)~~ =

(b) $y(n) = \alpha^n u(n)$

$y_e(n) = \frac{1}{2}[y(n) + y(-n)]$

$= \frac{1}{2}[\alpha^n u(n) + \alpha^{-n} u(-n)]$

$$u_e(n) = \begin{cases} \frac{1}{2}\alpha^n & n > 0 \\ 1 & n = 0 \\ \frac{1}{2}\alpha^{-n} & n < 0 \end{cases}$$

$$y_o(n) = \frac{1}{2}\left[y(n) - y(-n)\right]$$
$$= \frac{1}{2}\left[\alpha^n u(n) - \alpha^{-n} u(-n)\right]$$
$$= \frac{1}{2}\alpha^{|n|}\, \mathrm{sgn}(n)$$

$$y_o(n) = \begin{cases} \frac{1}{2}\alpha^n & n > 0 \\ 0 & n = 0 \\ \frac{1}{2}\alpha^{-n} & n < 0 \end{cases}$$

c) $x(t) = e^{-t}u(t)$

Soln

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$
$$x_e(t) = \frac{1}{2}\left[e^{-t}u(t) + e^{t}u(-t)\right]$$
$$x_o(t) = \frac{1}{2}\left[e^{-t}u(t) - e^{t}u(-t)\right]$$

(d) $x(t) = \left[\sin(\pi t) + \cos \pi t\right]^2$.

Soln $x(t) = \sin^2\pi(t) + \cos^2\pi t + 2\sin\pi t \cdot \cos\pi t$.

$x(t) = 1 + 2\sin\pi t \cdot \cos\pi t$

$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$ $\qquad x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$

$x(-t) = 1 - 2\sin\pi t \cdot \cos\pi t$

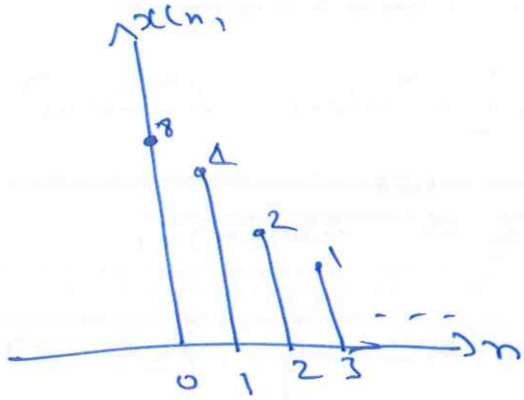
$x_e(t) = \frac{1}{2}\left[1 + 2\sin\pi t \cdot \cos\pi t + 1 - 2\sin\pi t \cdot \cos\pi t\right] = 1$

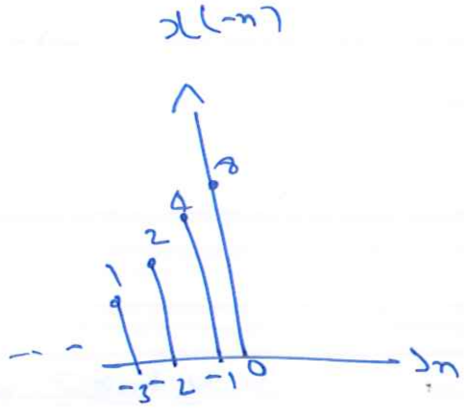
$x_o(t) = \frac{1}{2}\left[1 + 2\sin\pi t \cdot \cos\pi t - 1 + 2\sin\pi t \cdot \cos\pi t\right)$

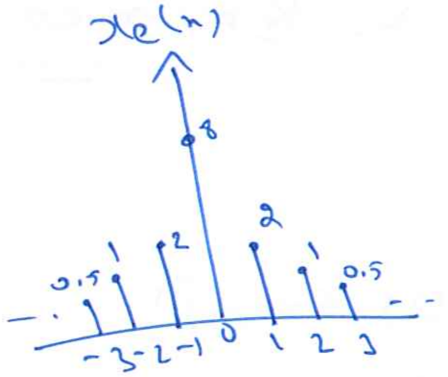
$x_o(t) = 2\sin\pi t \cdot \cos\pi t = \sin(2\pi t)$

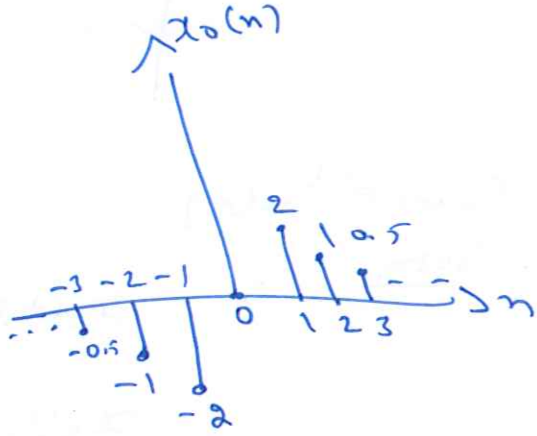
d) $x(n) = 8(0.5)^n u(n)$

Soln









Determine the average power & the energy of the following sequences.

(i) $x_1(n) = u(n)$ iii) $x_3(n) = A_0 e^{j\Omega_0 n}$

ii) $x_2(n) = n u(n)$

Soln

(i) $x_1(n) = u(n)$

Energy: $E = \sum_{n=-\infty}^{\infty} |x_1(n)|^2 = \sum_{n=0}^{\infty} 1^2 = \underline{\underline{\infty}}$

Average power:

$$P = \mathrm{Lt}_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_1(n)|^2$$

$$= \mathrm{Lt}_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1^2 \qquad \sum_{n=0}^{N-1} 1 = N$$

$$= \mathrm{Lt}_{N\to\infty} \frac{1}{2N+1} (N+1) = 1/2 \qquad \sum_{n=0}^{N} 1 = N+1$$

45d

$$P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_1(n)|^2$$

$$P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} 1^2$$

$$P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} (N+1)$$

$$\sum_{n=0}^{N-1} 1 = N$$

$$\sum_{n=0}^{N} 1 = N+1$$

$$P = \underset{N \to \infty}{Lt} \frac{\cancel{N}(1 + 1/N)}{\cancel{N}(2 + 1/N)}$$

$$P = \underset{N \to \infty}{Lt} \frac{1 + 1/N}{2 + 1/N}$$

$$P = \frac{1 + 1/\infty}{2 + 1/\infty} \qquad 1/\infty = 0$$

$$P = \frac{1}{2} \text{ watts}$$

ii) $x_2(n) = n\,u(n)$.

$$E = \sum_{n=-\infty}^{\infty} |x_2(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} |n\,u(n)|^2$$

$$= \sum_{n=0}^{\infty} n^2$$

$$E = \infty \text{ J}$$

Average power:

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_2(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |n^2 u(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} n^2$$

$$= \underline{\underline{\infty}}$$

iii) $x_3(n) = A_0 e^{j\Omega_0 n}$.

Energy: $$E = \sum_{n=-\infty}^{\infty} |x_3(n)|^2 = \sum_{n=-\infty}^{\infty} |A_0 e^{j\Omega_0 n}|^2.$$

$$E = A_0^2 \sum_{n=-\infty}^{\infty} 1 = \infty \qquad |e^{j\Omega_0 n}| = 1.$$

Av power

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_3(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |A_0 e^{j\Omega_0 n}|^2 \qquad |e^{j\Omega_0 n}| = 1$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} A_0^2.$$

$$= \underset{N\to\infty}{Lt} \frac{A_0^2}{2N+1} \sum_{n=-N}^{N} 1 \qquad \underline{\sum_{n=-N}^{N} 1 = 2N+1}$$

Replace n by $n-N$

$$\sum_{n=0}^{N+N} 1 = N+1+N = \underline{\underline{2N+1}}$$

$$= \underset{N\to\infty}{Lt} \frac{2N+1}{2N+1} \cdot A_0^2 = A_0^2$$

Determine each of The following

ⓐ $x(t) = e^{-3(t-1)}\delta(t)$

Soln

$$\delta(t) = 1 \qquad t = 0$$

$$e^{-3(t-1)}\delta(t)\Big|_{t=0}$$

$$x(t) = e^{3} = \underline{\underline{20.08}}$$

ⓑ $x(t) = \int_{-1}^{1} \delta(t^2-4)\,dt$

Soln

$$\delta(x^2-a^2) = \frac{1}{2|a|}\left[\delta(x+a) + \delta(x-a)\right]$$

$$\delta(t^2-2^2) = \frac{1}{2|2|}\left[\delta(t+2) + \delta(t-2)\right]$$

$$x(t) = \frac{1}{4}\int_{-1}^{1}\left[\delta(t+2) + \delta(t-2)\right]dt$$

$$x(t) = \frac{1}{4}\left[\int_{-1}^{1}\cancel{\delta(t+2)}^{\,0}\,dt + \int_{-1}^{1}\cancel{\delta(t-2)}^{\,0}\,dt\right]$$

$$x(t) = 0 \,//$$

(c) $x(t) = \int_{-\infty}^{\infty} \delta(t^2-4)\,dt$

<u>Soln</u>

$$\delta(x^2-a^2) = \frac{1}{2|a|}\left[\delta(x+a) + \delta(x-a)\right]$$

$$\delta(t^2-2^2) = \frac{1}{2|2|}\left[\delta(t+2) + \delta(t-2)\right]$$

$$x(t) = \frac{1}{4}\left[\int_{-\infty}^{\infty}(\delta(t+2) + \delta(t-2))\,dt\right]$$

$$x(t) = 1/4\left[\int_{-\infty}^{\infty}\delta(t+2)\,dt + \int_{-\infty}^{\infty}\delta(t-2)\,dt\right]$$

$$x(t) = 1/4\left[1+1\right]$$

$$x(t) = 1/2$$ //

Properties of Impulse function

CT	D.T
* $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$	* $x(n) \cdot \delta(n) = x(0) \cdot \delta(n)$
* $\int_{-\infty}^{\infty} K\,\delta(t) = K$	* $x(n)\,\delta(n-n_0) = x(n_0) \cdot \delta(n-n_0)$
* $x(t) \cdot \delta(t) = x(0)\,\delta(t)$ [Product property]	* $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n) = x(0)$
* $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$	* $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$
* $\delta(at) = \frac{1}{a}\,\delta(t)\ ;\ a > 0$ timescaling property	
* $\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0)\,dt = x(t_0)$ Shifting property.	
* $x(t) \cdot \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)$	

Sampling property

* $$\delta(t-t_0) = \frac{d}{dt}\, u(t-t_0)$$

* $$u(t-t_0) = \int_{-\infty}^{t} \delta(\tau - t_0)\, d\tau$$

$$= \begin{cases} 1, & t > t_0 \\ 0 & t < t_0 \end{cases}$$

* $$\int_{-\infty}^{\infty} \delta(at - t_0)\,dt = \frac{1}{|a|} \int_{-\infty}^{\infty} \delta\left(t - \frac{t_0}{a}\right) dt.$$

Evaluate The following integrals

1) $\int_{-1}^{1} (3t^2+1)\,\delta(t)\,dt.$

Soln $\int_{-1}^{1} (3t^2+1)\,\delta(t)\,dt$ exits only when $t=0$

$= (3t^2+1)\Big|_{t=0}$

$= 1$ //

2) $\int_{1}^{2} (3t^2+1)\,\delta(t)\,dt$

Soln $\delta(t)$ does not exits (appear) in the range of integration

$\therefore \int_{1}^{2} (3t^2+1)\,\delta(t)\,dt = 0$

3) $\int_{-\infty}^{\infty} (t^2+\cos\pi t)\,\delta(t-1)\,dt.$

Soln $\delta(t-1)$ exits only $t=1$.

$(t^2+\cos\pi t)\Big|_{t=1}$

$= 1+(-1) = 0$ //

$\cos\pi = -1$ //

(4) $$\int_{-\infty}^{\infty} e^{-t} \cdot \delta(2t-2)\,dt$$

<u>Soln</u> $$\int_{-\infty}^{\infty} e^{-t}\, \delta(2\cdot(t-1))\,dt.$$

W.K.T $\delta(at) = \frac{1}{a}\delta(t)$

$$\int_{-\infty}^{\infty} e^{-t}\, \frac{1}{2}\, \delta(t-1)\,dt$$

$$= \frac{1}{2} e^{-t} \Big|_{t=1}$$

$$= \frac{1}{2} \cdot e^{-1} = \frac{1}{2e}$$ //.

(5) $$\int_{-3}^{-1} e^{-t}\, \delta(t)\,dt$$

<u>Soln</u> $$\int_{-3}^{-1} e^{-t}\, \delta(t)\,dt = 0$$

$\because$ ~~integral not in range~~

[$\delta(t)$ does not appear in The range of integration]

(6) $$\int_{-\infty}^{\infty} (\sin\pi t + \cos^2\pi t)\ \delta(t-1)\,dt$$

<u>Soln</u> $$(\sin\pi t + \cos^2\pi t) \Big|_{t=1}$$

$= \sin\pi + \cos^2\pi$

$= +1$

(7) $\int_{-1}^{1} \left(\frac{t^2}{2}+5\right) \delta(t)\, dt$

Soln

$\left.\frac{t^2}{2}+5\right|_{t=0}$

$= 5$

(8) $\int_{-\infty}^{\infty} e^{-2t}\, \delta(2t-1)\, dt$

Soln $\int_{-\infty}^{\infty} e^{-2t}\, \delta\left(2\left(t-\tfrac{1}{2}\right)\right) dt$

$\because \quad \delta(at) = \frac{1}{a}\delta(t)$

$= \int_{-\infty}^{\infty} e^{-2t}\, \frac{1}{2} \cdot \delta\left(t-\tfrac{1}{2}\right) dt$

$\left.\tfrac{1}{2} e^{-2t}\right|_{t=1/2}$

$= \tfrac{1}{2} e^{-1}$

$= \frac{1}{2e}$

(9) $\int_{-2}^{1} (t+t^2)\, \delta(t-3)\, dt$

Soln $\quad \delta(t-3)\Big|$ exists at $t=3$

$= 0$

(10) $\int_{-2}^{4} (t+t^2)\, \delta(t-3)\, dt$

Soln. $\delta(t-3)$ exists at $t=3$

$(t+t^2)\Big|_{t=3} = 3+9 = 12$

(11) $\int_{-\infty}^{\infty} \cos t\, u(t-1)\, \delta(t)\, dt$.

Soln

$u(t-1) = 1 \quad t \geq 1$

$\qquad\quad\;\; 0 \quad t < 1$

$\delta(t) = 1 \quad t = 0$

$\qquad\;\; 0 \quad t \neq 0$

$\therefore \; = 0$.

(12) $\int_{0}^{3} e^{t-2}\, \delta(2t-4)\, dt$

Soln

w.k.t $\quad \delta(at) = \tfrac{1}{a}\delta(t)$

$\int_{0}^{3} e^{t-2}\, \delta(2(t-2))\, dt$

$\int_{0}^{3} e^{t-2}\, \frac{1}{2}\, \delta(t-2)\, dt$

$\delta(t-2)\Big|_{t=2} = 1$

$\therefore \; e^{0} \cdot \tfrac{1}{2}$

$= \tfrac{1}{2}$

**

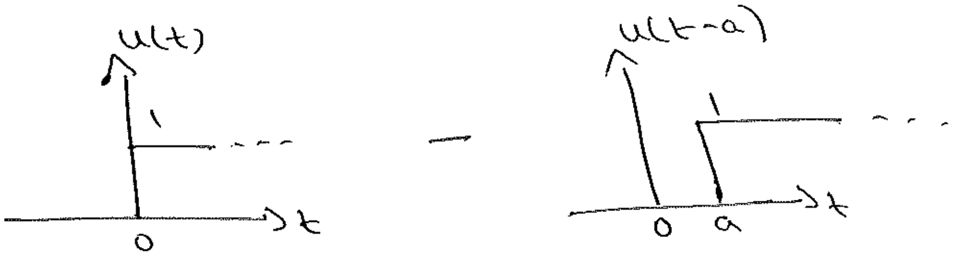
1) Find and Sketch The first derivatives of The following Signals

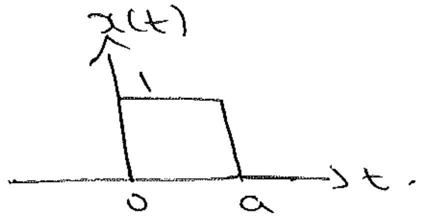
(a) $x(t) = u(t) - u(t-a)$, $a > 0$.

(b) $y(t) = t[u(t) - u(t-a)]$, $a > 0$

Soln

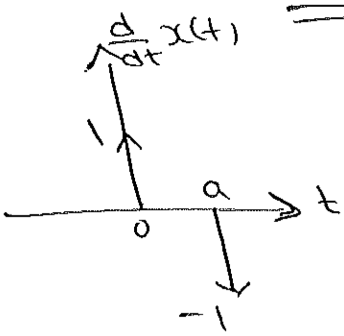
(a) $x(t) = u(t) - u(t-a)$, $a > 0$.





$x(t) = u(t) - u(t-a)$

$\frac{d}{dt} x(t) = \delta(t) - \delta(t-a)$



(b) $y(t) = t[u(t) - u(t-a)]$

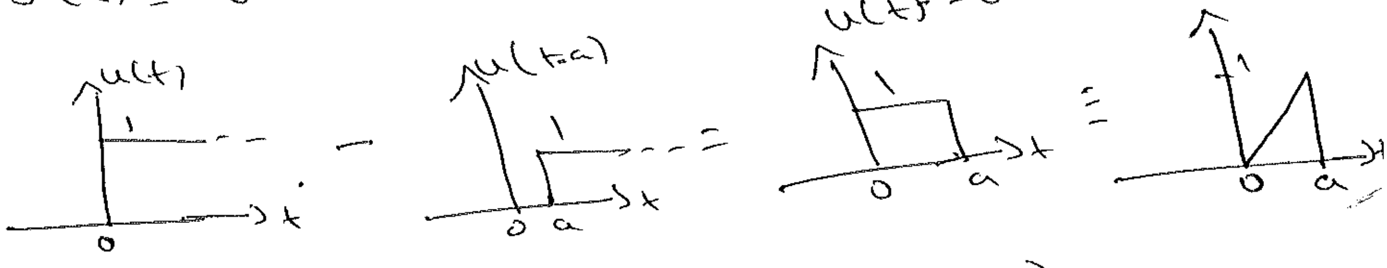
$y'(t) = \frac{d}{dt} y(t) =$

$y'(t) = \frac{d}{dt} y(t) = t[\delta(t) - \delta(t-a)] + 1 \times [u(t) - u(t-a)]$.

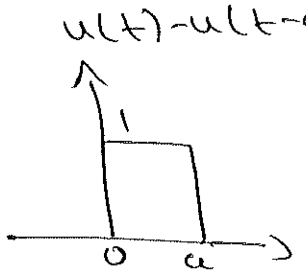
$y'(t) = t\,\delta(t) - t\,\delta(t-a) + u(t) - u(t-a)$

$y'(t) = t\big|_{t=0}\delta(t) - t\big|_{t=a}\delta(t-a) + u(t) - u(t-a)$.

$y'(t) = 0 - a\delta(t-a) + [u(t) - u(t-a)]$.



$y'(t) = -a\delta(t-a) + u(t) - u(t-a)$



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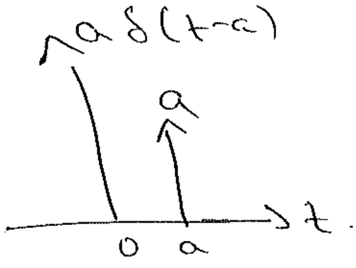
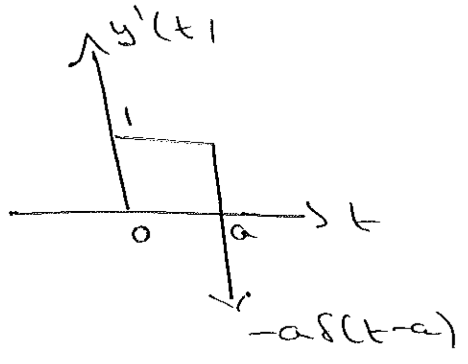
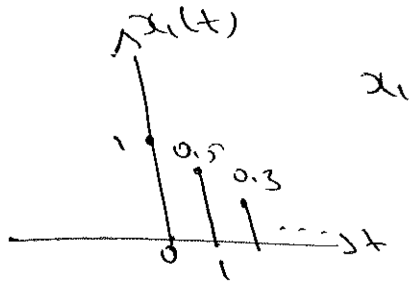




2) Classify each signal as a power signal, energy signal or neither & compute the signal power or energy where appropriate.

(a) $x_1(t) = \dfrac{1}{1+|t|}$

Soln



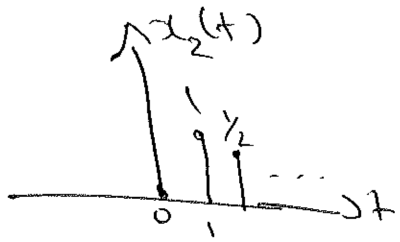
$x_1(t)$ it is energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{0}^{\infty} \left|\frac{1}{1+|t|}\right|^2 dt$$

$$E = 2\int_0^\infty \frac{1}{(1+t)^2} = \frac{-2}{1+t}\Bigg|_0^\infty$$

$E = \underline{+2J}$

(b) $x_2(t) = \dfrac{1}{t} \quad t \geq 1$

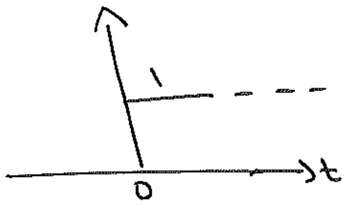


it is energy signal

$$E = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_1^\infty \frac{1}{t^2} dt .$$

$$E = \int_1^\infty t^{-2} dt = \frac{t^{-1}}{-1}\Bigg|_1^\infty = -1\left[t^{-\infty} - 1^{-1}\right] = 1J$$

(c) $x_3(t) = u(t)$



it is power signal

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T/2} 1^2 \, dt$$

$$P = \lim_{T \to \infty} \frac{1}{T} \left[t \Big|_0^{T/2} \right]$$

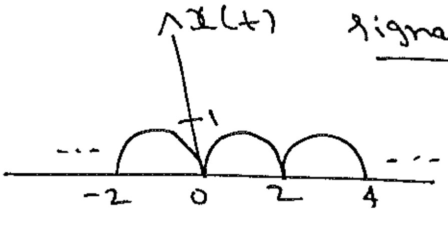
$$P = \lim_{T \to \infty} \frac{1}{2}$$

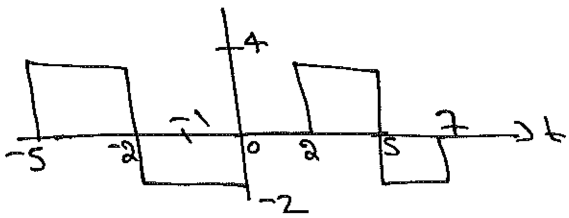
$P = 1/2$ watts

(d) $x_4(t) = e^{|1-t|}$

Soln $x_4(t) = e^{|1-t|}$ is a two sided growing exponential So, neither power signal nor a Energy signal.

2) For Each of The periodic signal shown in below figure find The Energy E in one period, The signal power p. & Rms Value





Soln :

Signal 1

Energy in one period $E = \int_0^2 (\sin \pi/2\, t)^2 \, dt$

$$E = \int_0^2 \frac{1 - \cos 2\pi/2\, t}{2} \, dt$$

$$E = 1/2 \left[\int_0^2 dt - \int_0^2 \cos \pi t \, dt \right]$$

$$E = 1/2 \left[t \Big|_0^2 - \sin \pi t \Big|_0^2 dt \right]$$

$$E = \frac{2}{2} = 1 \text{ J}$$

Signal power $P = \dfrac{\text{Energy in one period}}{T}$

$$P = \frac{1}{2} \text{ W} \qquad T = 2.$$

$$Rms = \sqrt{P} = \sqrt{1/2} = 0.707$$

Signal 2 :

Energy in one period $E = \int_0^T |x(t)|^2 dt$

$$E = \int_2^5 4^2 dt + \int_5^7 2^2 dt$$

$$E = 16t \Big|_2^5 + 4t \Big|_5^7$$

$$E = 56 \text{ J}$$

Signal power $P = \dfrac{\text{Energy in one period}}{T} = \dfrac{56}{7} = 8$

$$Rms = \sqrt{8} = 2.8284$$

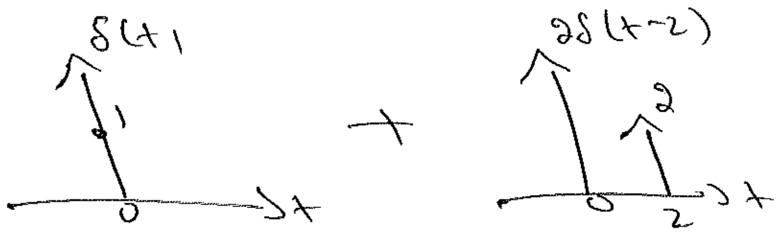
1) Simplify $x(t) = 2\delta(2t) + 6\delta(3(t-2))$.

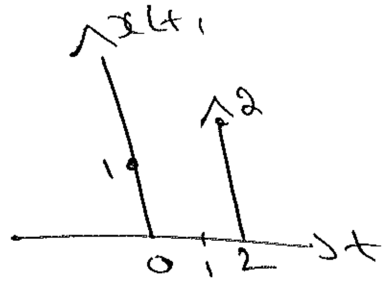
Soln

w.k.T $\delta(at) = \frac{1}{a}\delta(t)$

$$x(t) = \frac{2}{2}\delta(t) + \frac{6}{3}\delta(t-2)$$

$$x(t) = \delta(t) + 2\delta(t-2)$$





//

(2) $\int_{-1}^{\infty}[\delta(t+3) - 2\delta(4t)]\,dt$ find the numerical value.

Soln

$$\int_{-1}^{\infty}\delta(t+3)\,dt - 2\int_{-1}^{\infty}\delta(4t)\,dt$$

w.k.T $\delta(at) = \frac{1}{a}\delta(t)$

$$\int_{-1}^{\infty}\delta(t+3)\,dt - \frac{2}{4}\int_{-1}^{\infty}\delta(t)\,dt$$

$$= \cancel{\delta(t+3)}^{0}\Big|_{t=-3} - \frac{1}{2}\,\delta(t)\Big|_{t=0}$$

$$= 0 - 1/2 = \underline{-0.5}$$

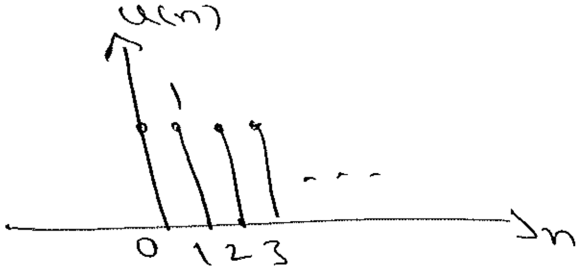
~~Problems on~~

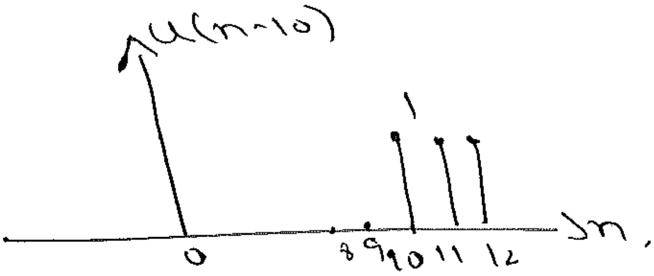
1) A discrete-time signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{otherwise} \end{cases}$$

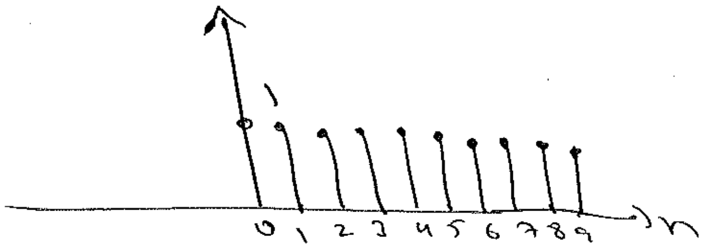
Using $u(n)$ describe $x(n)$ as the superposition of two step functions.

Soln





$x(n) = u(n) - u(n-10)$

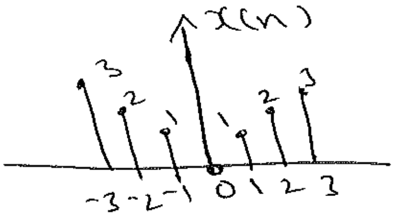


(2) Let $x(n)$ & $y(n)$ be given in fig a & b respectively. Carefully sketch the following signals.

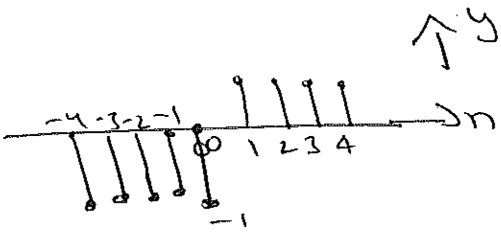
(a) $x(2n)$ (b) $x(3n-1)$ (c) $y(1-n)$ (d) $y(2-2n)$

(e) $x(n-2)+y(n-2)$ (f) $x(2n)+y(n-4)$ (g) $x(n+2)\,y(n-2)$

(h) $x(-n)\,y(-n)$



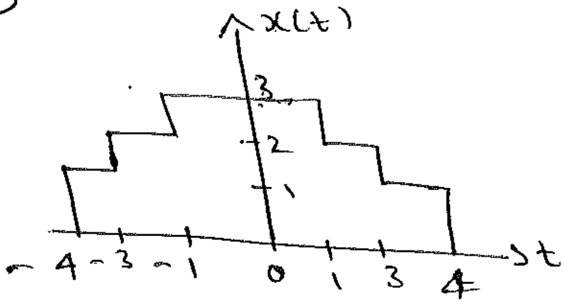
(a)



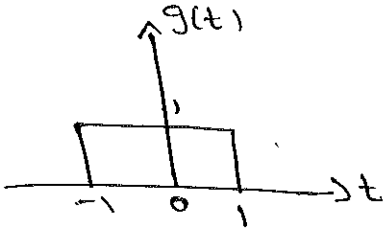
(b).

1) A C.T Signal $x(t)$ & $g(t)$ as shown in below figure a & b Respectively. express $x(t)$ interms of $g(t)$

1)

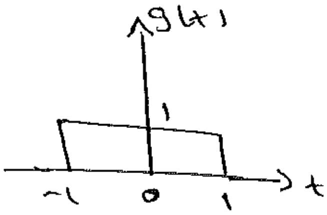


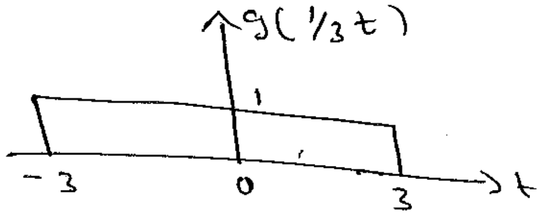
ⓐ

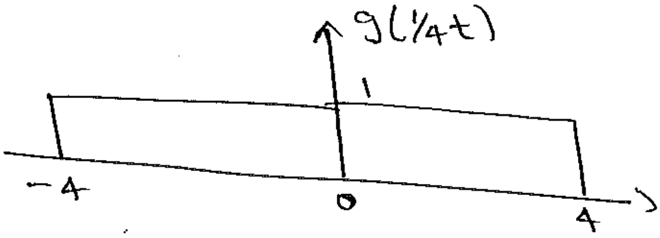


ⓑ

Soln







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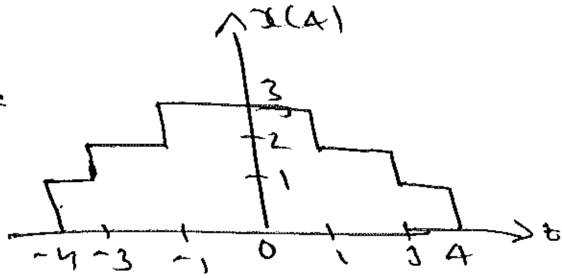
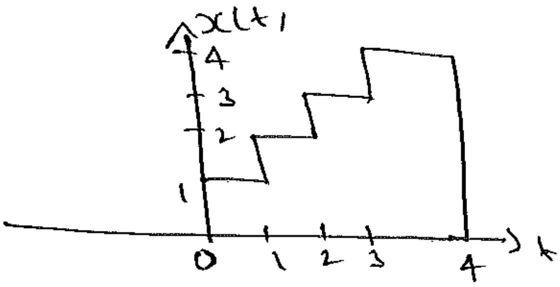
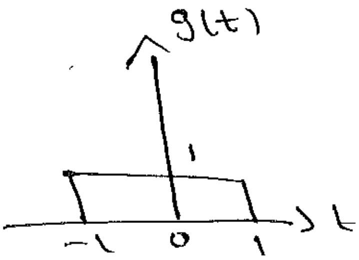


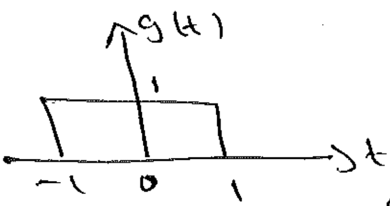
$\therefore x(t) = g(t) + g(\tfrac{1}{3}t) + g(\tfrac{1}{4}t)$ //

2)

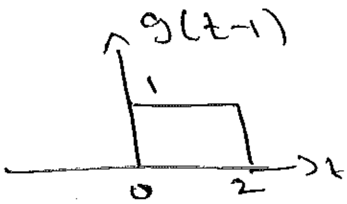




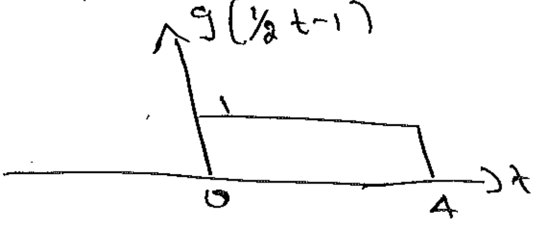
Soln



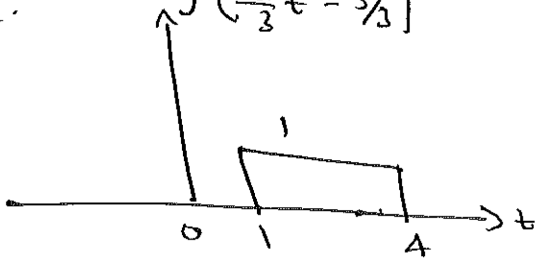
$\longrightarrow$



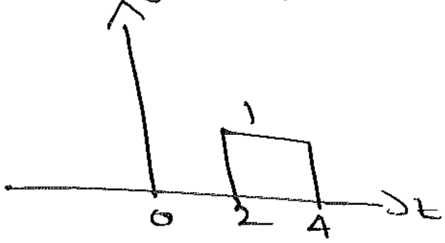
Step 1:



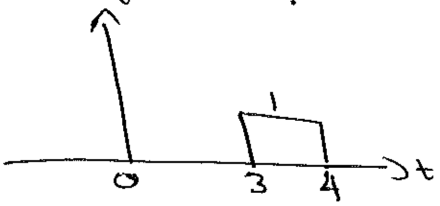
Step 2. $g\left(\frac{2}{3}t - 5/3\right)$



Step 3 : $g(t-3)$

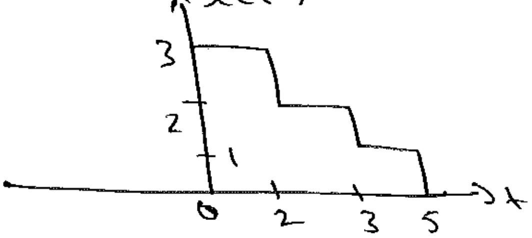


Step 4 $g(2t-7)$

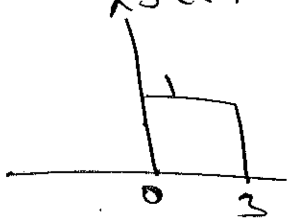


$$x(t) = g\left(\tfrac{1}{2}t-1\right) + g\left(\tfrac{2}{3}t - 5/3\right) + g(t-3) + g(2t-7)$$ //

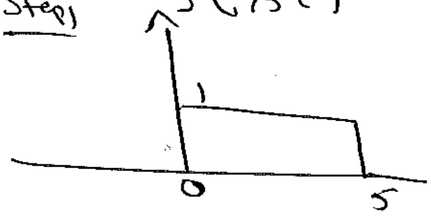
(3) $x(t)$



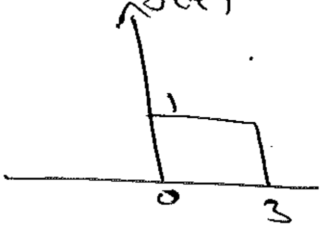
$g(t)$



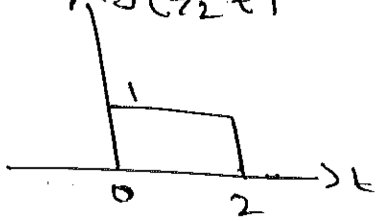
Soln. Step 1 $g(3/5\, t)$



Step 2 $g(t)$

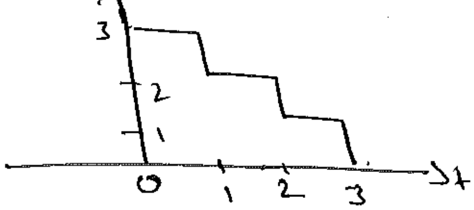


Step 3 $g(3/2\, t)$

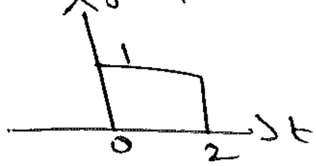


$$x(t) = g(3/5\, t) + g(t) + g(3/2\, t)$$ //.

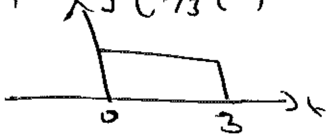
(4) $x(t)$



$g(t)$

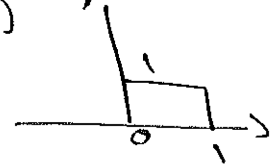


Soln. (i) $g(2/3\, t)$



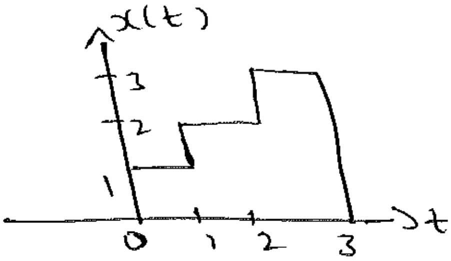
ii) $g(t)$

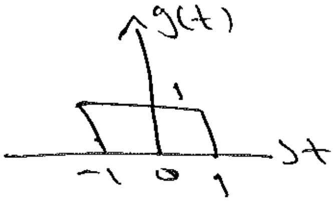
iii) $g(2t)$



$$x(t) = g(2/3\, t) + g(t) + g(2t)$$

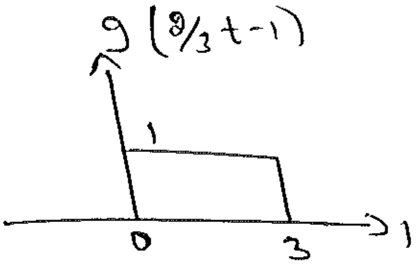
(5)

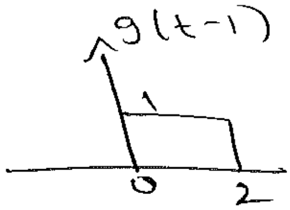




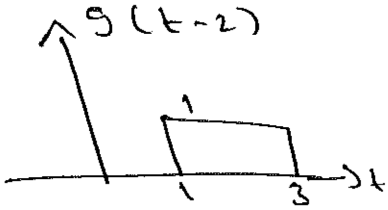
Soln

(i)

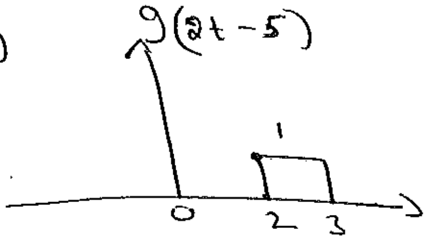




ii)

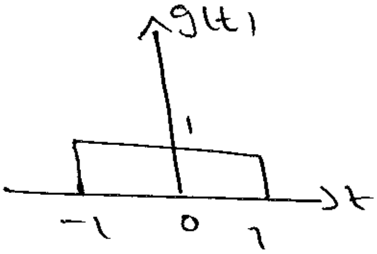


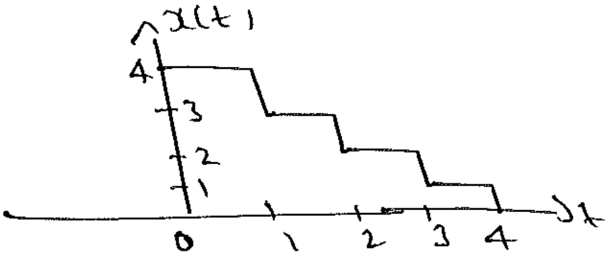
iii)



$$x(t) = g\left(\tfrac{2}{3}t-1\right) + g(t-2) + g(2t-5)$$

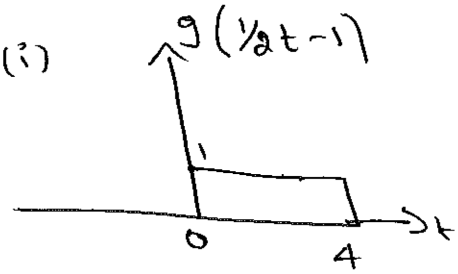
(6)

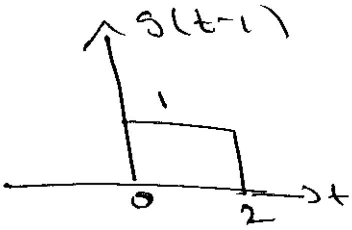




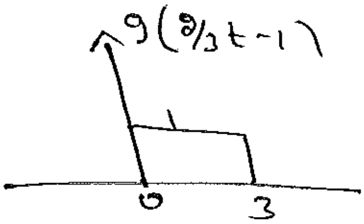
Soln

(i)

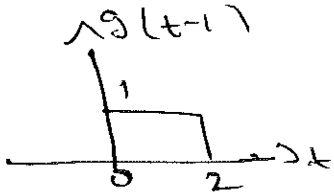




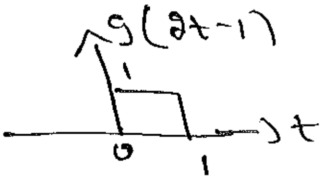
ii)



iii)



iv)



$$x(t) = g\left(\tfrac{1}{2}t-1\right) + g\left(\tfrac{2}{3}t-1\right) + g(t-1) + g(2t-1)$$

Determine The Energy of The sequence

$$x(n) = \begin{cases} (1/2)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Soln

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=0}^{\infty} ((1/2)^n)^2 = \sum_{n=0}^{\infty} (1/4)^n$$

$$= \frac{1}{1-1/4} = 4/3 \text{ J}$$

Determine whether C.T signal x(t) even part of $[\cos(2\pi t)]u(t)$ is periodic. if periodic find its fundamental period

Soln

$$\omega = 2\pi$$

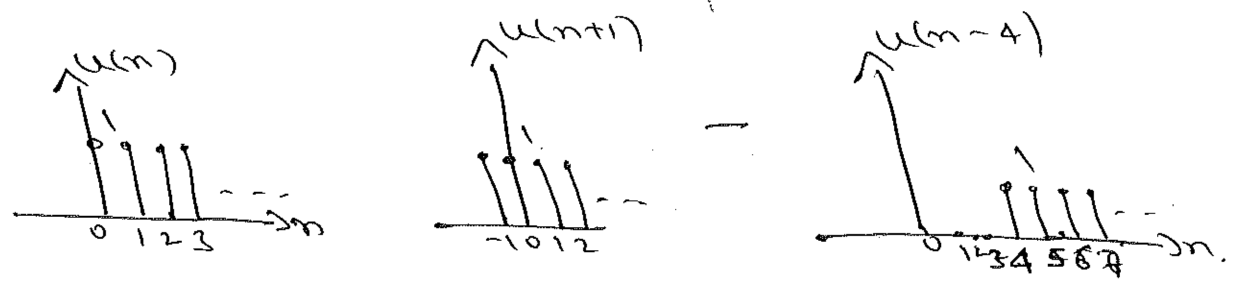
$$T = \frac{2\pi}{2\pi} = 1 \text{ sec} \quad \text{periodic}$$

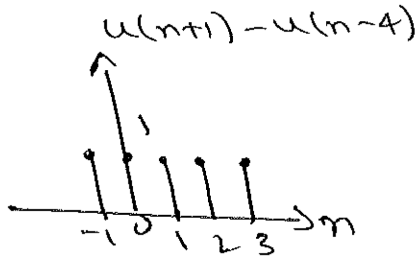
1) let $x(n) = 2^n [u(n+1) - u(n-4)]$. Sketch the following signal

(a) $y_1(n) = x(n-3)$ (b) $y_2(n) = x(n+1)$ (c) $y_3(n) = x(-n+4)$

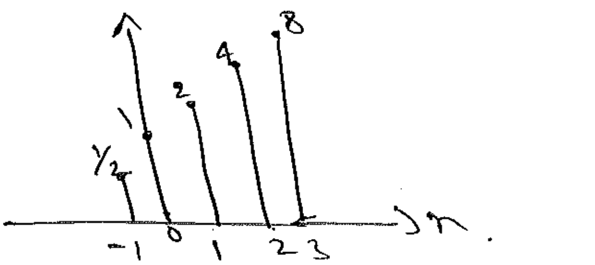
(d) $y_4(n) = x(-n+2)$.

Soln

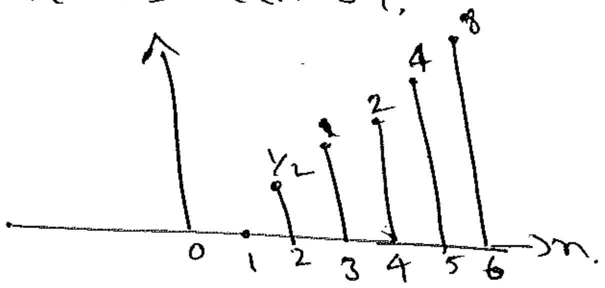




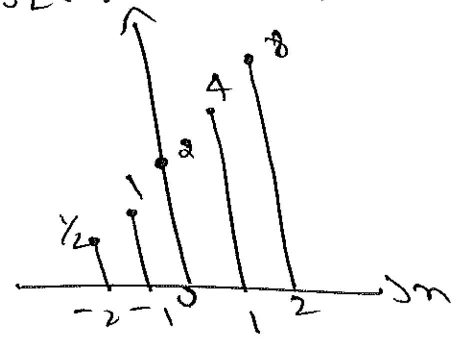
⇒



(a) $y_1(n) = x(n-3)$.

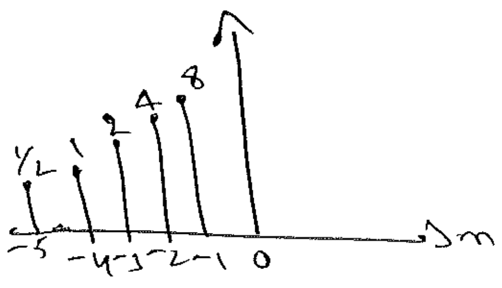


(b) $y_2(n) = x(n+1)$

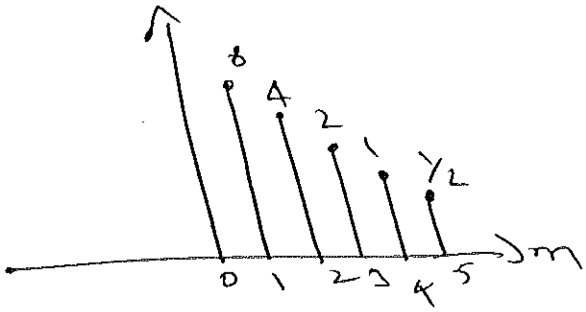


(c) $y_3(n) = x(-n+4)$

$v(n) = x(n+4)$

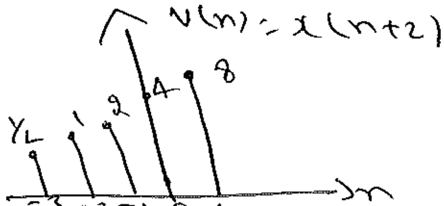


$y_3(n) = v(-n) = x(-n+4)$



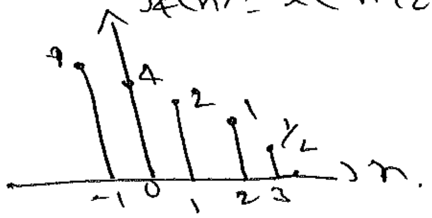
(d) $y_4(n) = x(-n+2)$

$v(n) = x(n+2)$

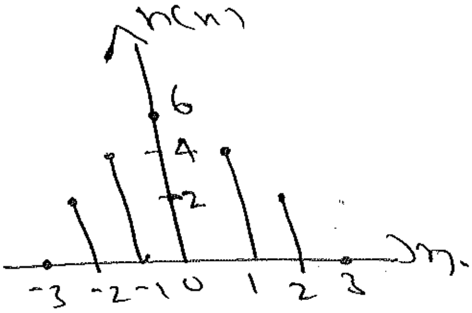


⇒

$y_4(n) = x(-n+2)$



③ Mathematically describe The signal in below fig. in atleast Three different ways.



Soln :

ⓐ A numeric sequence $x(n) = \{0, 2, 4, \underset{\uparrow}{6}, 4, 2, 0\}$

ⓑ A sum of impulse

$x(n) = 2\delta(n+2) + 4\delta(n+1) + 6\delta(n) + 4\delta(n-1) + 2\delta(n-2)$

ⓒ A pulse function

$x(n) = 6\ \text{tri}\left(\frac{n}{3}\right)$.

④ Evaluate The following

ⓐ $\int_{-1}^{1} (3t^2+1)\,\delta(t)\,dt$

Soln $(3t^2+1)\,\delta(t)\Big|_{t=0}$

1 //

ⓑ $\int_{-\infty}^{\infty} (t^2 + \cos \pi t)\,\delta(t-1)\,dt$

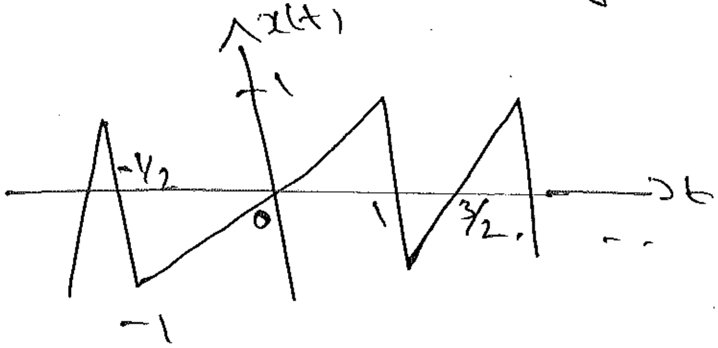
Soln $(t^2 + \cos \pi t)\,\delta(t-1)\Big|_{t=1}$

$1 + \cos \pi$

$= 1 - 1 = 0$ //

(47d)

Determine The power of signals



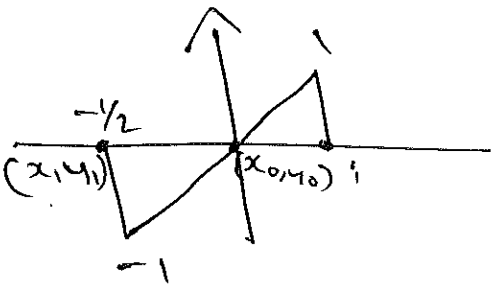
Soln

$P = \frac{1}{T}\int_0^T |x(t)|^2 d$.

$T = T_2 - T_1 = 1-(-1/2)$

$= 1 + 1/2$

$T = 3/2$

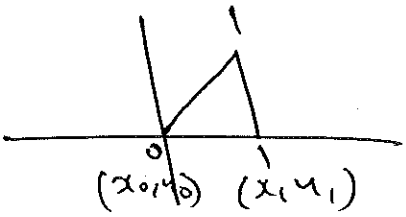


$\frac{y-y_1}{x-x_1} = \frac{y_1-y_0}{x_1-x_0}$

$\frac{y+1}{x+1/2} = \frac{-1-0}{-1/2-0} = 2$

$y+1 = 2x+1$

$y = 2x+1-1 = 2t$ $\quad -1/2 < t < 0$



$\frac{y-1}{x-1} = \frac{1-0}{1-0} = 1$

$y-1 = x-1$

$y = x = t$ $\quad 0 \leq t < 1$.

$P = \frac{1}{3/2}\left[\int_{-1/2}^{0}(2t)^2 dt + \int_0^1 (t)^2 dt\right]$.

$P = 2/3 \left[4\frac{t^3}{3}\Big|_{-1/2}^{0} + \frac{t^3}{3}\Big|_0^1\right]$

$$P = 2/3 \left[\frac{4}{3} \left[0 - (-1/2)^3 \right] + 1/3 \left[1^3 - 0^3 \right] \right].$$

$$P = 2/3 \left[4/3 \left[1/8 \right] + 1/3 \right].$$

$$P = 2/3 \left[1/6 + 1/3 \right].$$

$$P = 2/3 \left[1/2 \right] = 2/6 = 1/3 \text{ watts.}$$

Check whether signal x(t) is power or Energy signal

$$x(t) = A \cdot e^{-j2\pi t}$$

Soln

$$\omega = 2\pi \quad T = \frac{2\pi}{2\pi} = 1 \text{ sec}$$

$\therefore$ it is periodic

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} (x(t))^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 (e^{-j2\pi t})^2 dt$$

$$P = \frac{A^2}{T} \int_{-T/2}^{T/2} e^{-j4\pi t} dt. \qquad e^{-j4\pi t} = \cos 4\pi t - j \sin 4\pi t = 1$$

$$P = \frac{A^2}{T} \int_{-T/2}^{T/2} 1 \, dt$$

$$P = \frac{A^2}{T} t \Big|_{-T/2}^{T/2}$$

$$P = \frac{A^2}{T} \left[T/2 - (-T/2) \right]$$

$$P = \frac{A^2}{T} \cdot T = \underline{A^2} \text{ watts.}$$

(6) Find The derivatives of The signals

$x(t) = \cancel{Seg}\ Sgn(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$

Soln Sgn(t) fn changes its value from -1 to 1 at $t = 0$

Hence

$$\frac{d}{dt} x(t) = \begin{cases} \frac{1-(-1)}{0} = \infty & \text{at } t = 0 \\ 0 & \text{for } t > 0 \text{ \& } t < 0 \end{cases}$$

RHS of above equation is impulse fn.

$$\frac{d}{dt} x(t) = \delta(t)$$

(7) Prove The following relationship b/w fns

(a) $u(n) = \sum_{k=-\infty}^{\infty} \delta(k)$

Soln

$$\sum_{k=-\infty}^{n} \delta(k) = \begin{cases} 0 & \text{for } k < 0 \\ 1 & \text{for } k \geq 0. \end{cases}$$

LHS = RHS

(b) $u(n) = \sum_{k=0}^{\infty} \delta(n-k)$

Soln

$$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0 & \text{for } k < 0 \\ 1 & \text{for } k \geq 0 \end{cases}$$

(c) $\delta(n) = u(n) - u(n-1)$.

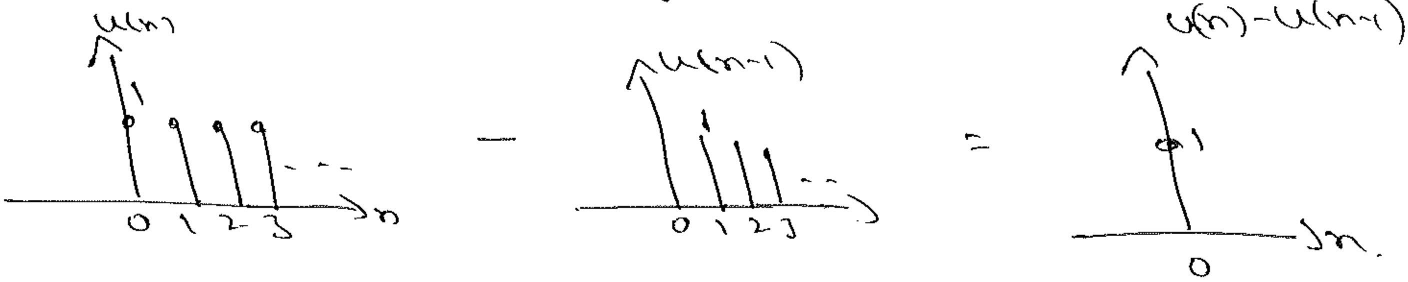
Soln

We know that

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

&

$$u(n-1) = \begin{cases} 1 & \text{for } n \geq 1 \\ 0 & \text{for } n < 1 \end{cases}$$



$$u(n) - u(n-1) = \delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0. \end{cases}$$

(7) Determine ~~The~~ whether the following signals are energy or power signals & calculate their energy or power.

(a) $x(n) = (1/2)^n u(n)$

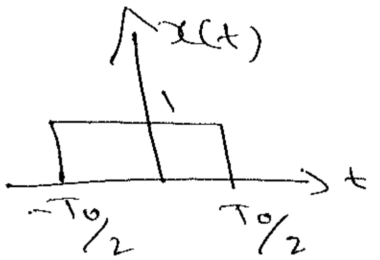
Soln it is non periodic $\therefore$ it is Energy signal

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |(1/2)^n u(n)|^2$$

$$E = \sum_{n=0}^{\infty} (1/4)^n = \frac{1}{1 - 1/4} = 4/3$$

(b) $x(t) = \text{rect}\left(\frac{t}{T_0}\right)$

Soln The ~~rect~~ rect (t/T_0) fn is given as



$$\text{rect}(t/T_0) = \begin{cases} 1 & \text{for } \frac{-T_0}{2} \leq t \leq \frac{T_0}{2} \\ 0 & \text{elsewhere} \end{cases}$$

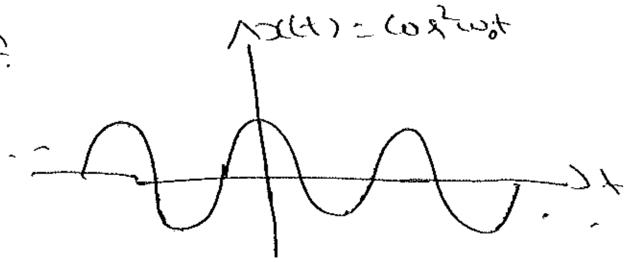
it is non periodic. Hence it is energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\frac{T_0}{2}}^{T_0/2} (1)^2 dt = t \Big|_{-T_0/2}^{T_0/2} = \frac{T_0}{2} + \frac{T_0}{2} = T_0 //.$$

(c) $x(t) = \cos^2 \omega_0 t$.

Sol



it is periodic signals ∴ it is power signal.

$$P = \lim_{T \to \infty} \frac{1}{T_0} \int_{-T/2}^{T_0/2} |x(t)|^2 dt$$

$$P = \lim_{T \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (\cos^2 \omega_0 t)^2 dt =$$

$\cos^4 \omega_0 t$ can be expanded by standard trignometeric relation as follow.

$$P = \lim_{T \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \left(\frac{1 + \cos 2\omega_0 t}{2} \right)^2 dt$$

$$P = \lim_{T \to \infty} \frac{1}{T_0} \left[\frac{1}{4} \int_{-T_0/2}^{T_0/2} (1 + \cos 2\omega_0 t)^2 dt \right].$$

$$P = \lim_{T \to \infty} \frac{1}{T_0} \left[\frac{1}{4} \int_{-T_0/2}^{T_0/2} (1 + \cos^2 2\omega_0 t + 2\cos 2\omega_0 t) dt \right].$$

$$P = \lim_{T \to \infty} \frac{1}{T_0} \left[\frac{1}{4} \int_{-T_0/2}^{T_0/2} \left(1 + \frac{1 + \cos 4\omega_0 t}{2} + 2\cos 2\omega_0 t\right) dt \right].$$

$$P = \lim_{T \to \infty} \frac{1}{T_0} \left[1/8 \int_{-T_0/2}^{T_0/2} (3 + \cos 4\omega_0 t + 4\cos 2\omega_0 t) dt \right.$$

$$P = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{T_0/2} \frac{3}{8}\, dt + \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} 4 \cos 2\omega_0 t \, dt +$$

$$\lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \cos 4\omega_0 t \, dt$$

$$P = \lim_{T_0 \to \infty} \frac{1}{T_0} \cdot \frac{3}{8} t \Big|_{-T_0/2}^{T_0/2} + 0 + 0$$

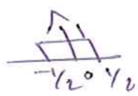
$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \; 3/8 \left[T_0/2 + \frac{T_0}{2} \right]$$

$$P = 3/8$$

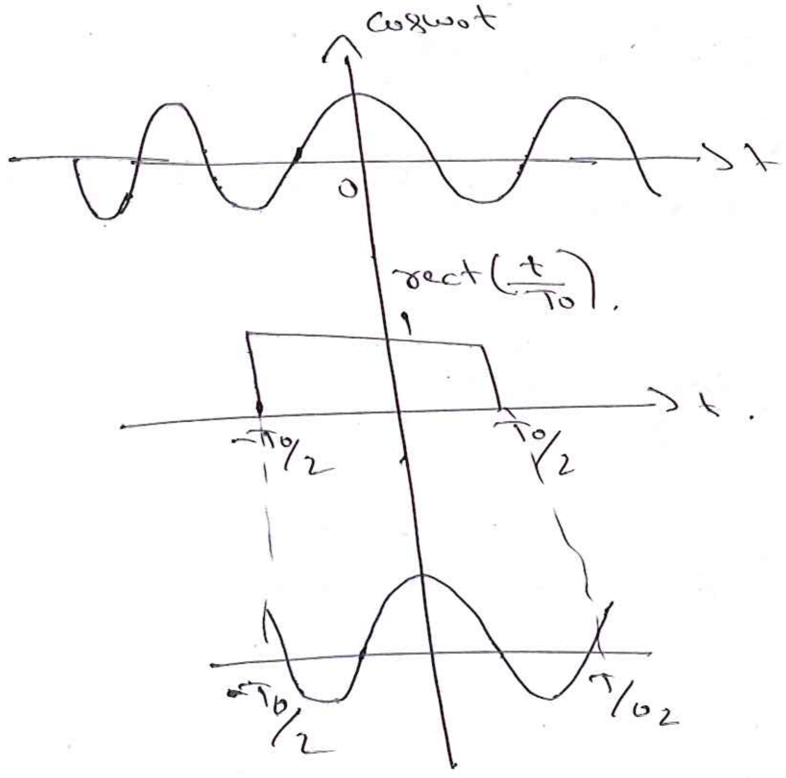
(d) $x(t) = \text{rect}\left(\frac{t}{T_0}\right) \cos \omega_0 t.$

$\text{rect}(t) = 1 \quad |t| \leq 1/2$

$\phantom{\text{rect}(t) =} 0 \quad |t| > 1/2$



Soln



it is Energy signal.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-T_0/2}^{T_0/2} [\cos \omega_0 t]^2 \, dt.$$

$$E = \int_{-T_0/2}^{T_0/2} \left(\frac{1+\cos 2\omega_0 t}{2}\right) dt$$

$$E = \int_{-T_0/2}^{T_0/2} \frac{1}{2}\, dt + \int_{-T_0/2}^{T_0/2} \frac{\cos 2\omega_0 t}{2}\, dt$$

$$= \frac{1}{2}\left[t\Big|_{-T_0/2}^{T_0/2}\right] + \cancel{\frac{\sin 2\omega_0 t}{2\times 2\omega_0}}^{\,0} \Bigg|_{-T_0/2}^{T_0/2}$$

$$= \frac{1}{2}\left[\frac{T_0}{2} - \left(-\frac{T_0}{2}\right)\right]$$

$$E = \frac{T_0}{2}$$

(e) $x(t) = A e^{-\alpha t} u(t)$ $\quad \alpha < 0$

Soln

it is nonperiodic $\therefore$ it is Energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |A e^{-\alpha t} u(t)|^2 dt$$

$$E = \int_{0}^{\infty} A^2 e^{-2\alpha t}\, dt$$

$$E = A^2 \cdot \frac{e^{-2\alpha t}}{-2\alpha}\Bigg|_0^{\infty}$$

$$E = \frac{A^2}{-2\alpha} \cdot \left[e^{-2\alpha \times \infty} - e^0\right] = \frac{A^2}{2\alpha} \quad [0-1]$$

$$= \frac{A^2}{2\alpha}$$

Problems on Ramp function

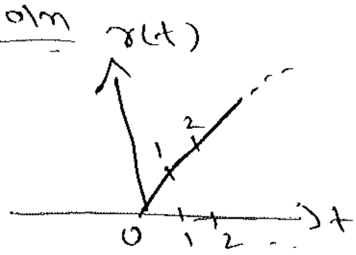
(i) Sketch The waveform of The following signals.

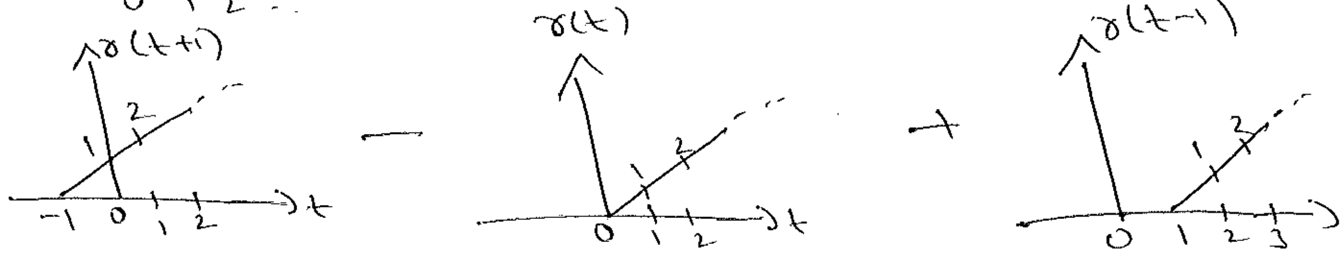
(a) $x(t) = r(t+1) - r(t) + r(t-1)$.

Soln

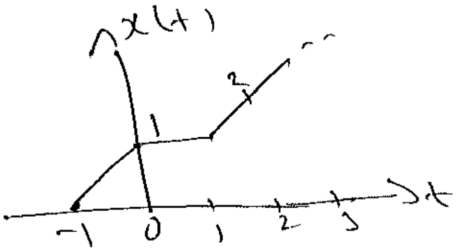
$r(t) = t \quad t \geq 0$

$\quad\;\; 0 \quad t < 0$



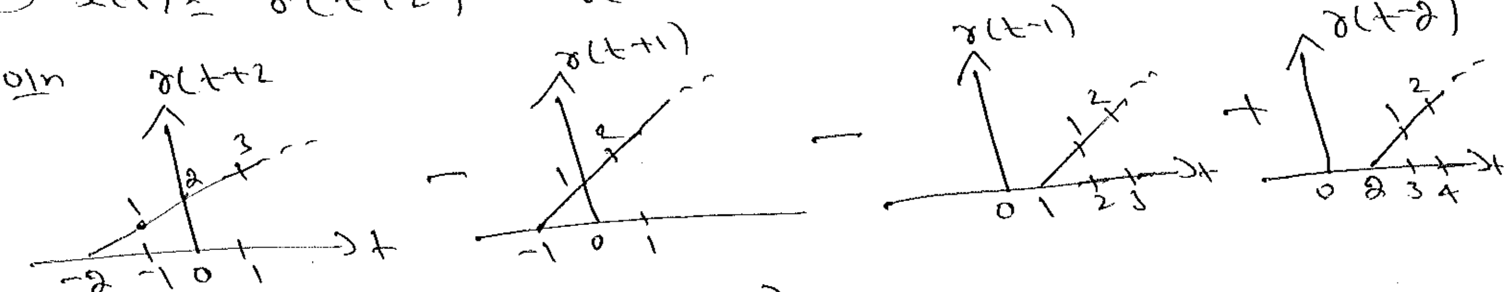


time	$r(t+1)$	$-r(t)$	$r(t-1)$	$x(t)$
$t=-1$	0	0	0	0.
$t=0$	1	0	0	1
$t=1$	2	1	0	1
$t=2$	3	2	1	2
$t=3$	4	3	2	3.

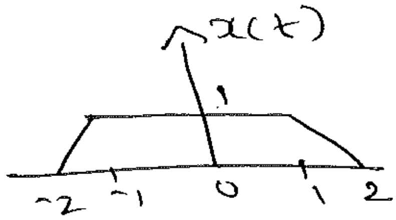


(b) $x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$.

Soln



Soln



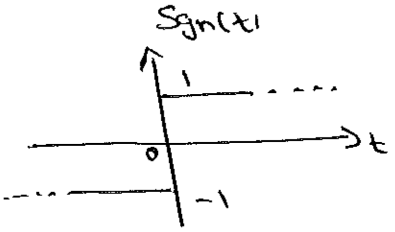
Other elementary signals

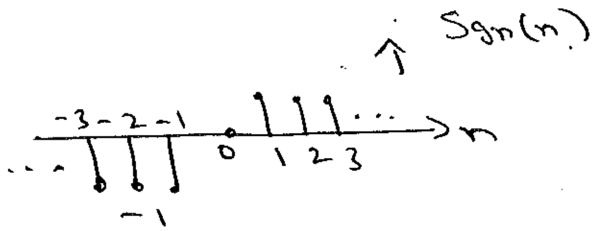
1) Signum function

The signum function is defined as

$$\mathrm{Sgn}(t) = \begin{cases} -1 & ; t < 0 \\ 0 & ; t = 0 \\ 1 & ; t > 0 \end{cases} \quad \text{for C.T}$$

$$\mathrm{Sgn}(n) = \begin{cases} 1 & ; n > 0 \\ 0 & ; n = 0 \\ -1 & ; n < 0 \end{cases} \quad \text{for D.T}$$



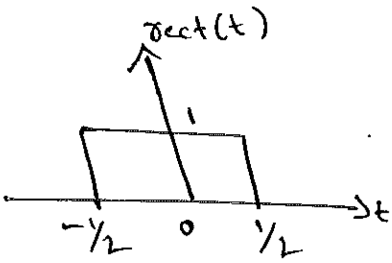


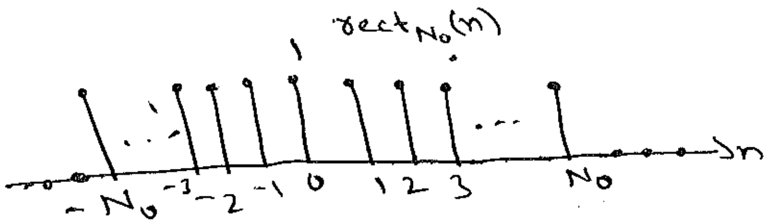
2) Rectangular function

Rectangular function is defined as

$$\mathrm{rect}(t) = \begin{cases} 1 & ; |t| \le 1/2 \\ 0 & ; |t| > 1/2 \end{cases} \quad \text{for C.T}$$

$$\mathrm{rect}_{N_0}(n) = \begin{cases} 1 & ; |n| \le N_0 \\ 0 & ; |n| > N_0 \end{cases} \quad \text{for D.T}$$

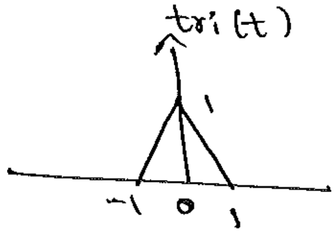




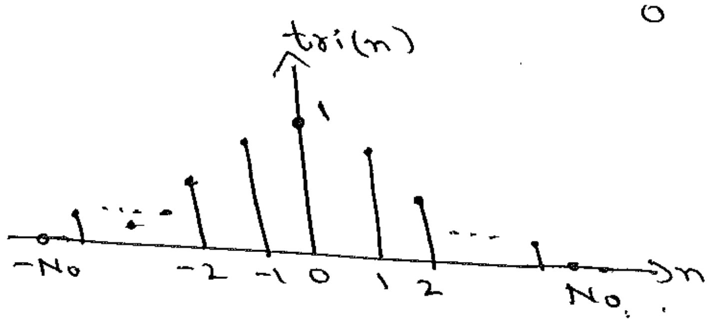
③ **Triangular function**

The triangular function is defined as

$$tri(t) = \begin{cases} 1 - |t| & ; |t| \leq 1 \\ 0 & ; |t| > 1 \end{cases} \quad \text{for C.T}$$



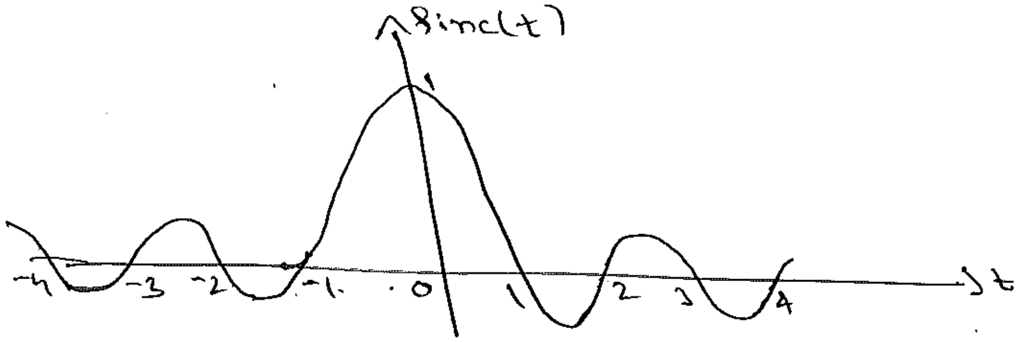
$$tri(n) = \begin{cases} 1 - \frac{|n|}{N_0} & ; |n| \leq N_0 \\ 0 & ; |n| > N_0 \end{cases} \quad \text{for D.T}$$



4) **Sinc function**

The Sinc function is defined as

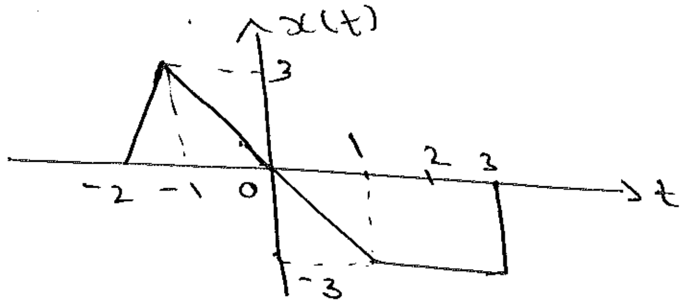
$$Sinc(t) = \frac{Sin(\pi t)}{\pi t}$$



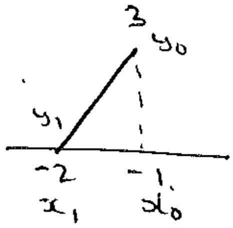
At $t=0$; $Sinc(t)\Big|_{t=0} = 1$ (using L-Hospital's Rule).

Additional Problems

1) Obtain the functional expression for the signal shown in below figure



Soln



$$x(t) = \begin{cases} 0 & ; \quad t < -2 \\ 3(t+2) & ; \quad -2 < t < 1 \\ -3t & ; \quad -1 < t < 1 \\ -3 & ; \quad 1 < t < 3 \\ 0 & ; \quad t > 3 \end{cases}$$

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_0}{x_1 - x_0}$$

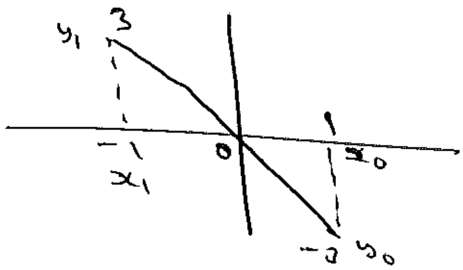
$$\frac{y-0}{x+2} = \frac{0-3}{-2+1} = 3$$

$$y = 3(t+2)$$

$$\frac{y-3}{x+1} = \frac{3+3}{-1-1} = -3$$

$$y - 3 = -3t - 3$$

$$y = -3t$$



2) Find even & odd component of the following signals

(a) $x(t) = \text{Sinc}(t)$

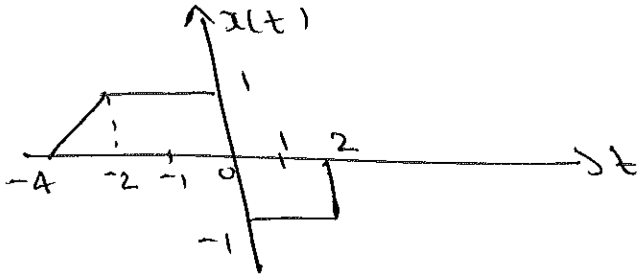
Soln The fn Sinc(t) is $\text{Sinc}(t) = \frac{\sin \pi t}{\pi t}$

$$x(t) = \frac{\sin \pi t}{\pi t} \qquad x(-t) = \frac{-\sin \pi t}{-\pi t} = \frac{\sin \pi t}{\pi t} = \text{Sinc}(t)$$

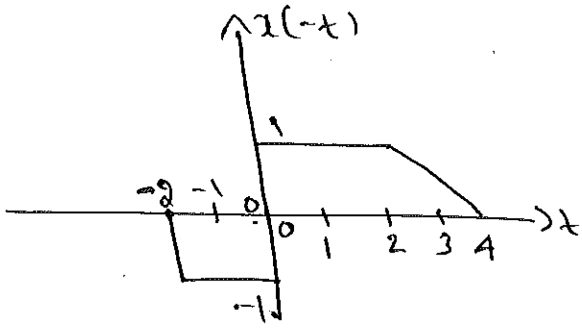
$$x_e(t) = \tfrac{1}{2}[x(t) + x(-t)] = \tfrac{1}{2}[\text{Sinc}(t) + \text{Sinc}(t)] = \text{Sinc}(t)$$

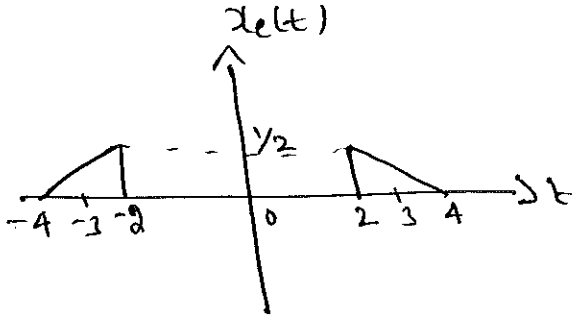
$$x_o(t) = \tfrac{1}{2}[x(t) - x(-t)] = \tfrac{1}{2}[\text{Sinc}(t) - \text{Sinc}(t)] = 0$$

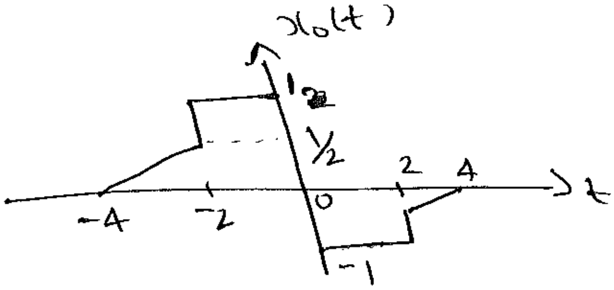
(b)



Soln $x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$ $\quad x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$







Problems on periodic

1) $x(t) = 10\cos(\pi t) \cdot \sin(4\pi t)$

Soln $\sin a \cdot \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$

$\sin 4\pi t \cdot \cos \pi t = \frac{1}{2}\left[\sin 5\pi t + \sin 3\pi t\right]$

$\therefore x(t) = 5\left[\sin 5\pi t + \sin 3\pi t\right]$

$\omega_1 = 5\pi$ $\qquad \omega_2 = 3\pi$

$T_1 = \frac{2\pi}{5\pi} = 2/5$ $\qquad T_2 = \frac{2\pi}{3\pi} = 2/3$

$T = \text{Lcm}(T_1, T_2) = 2$ sec

$T = \frac{T_1}{T_2} = \frac{2/5}{2/3} = 3/5$

$T = T_1 \times 5 = T_2 \times 3$

$= \frac{2}{\cancel{5}} \times \cancel{5} = \frac{2}{\cancel{3}} \times \cancel{3}$

$T = 2$ sec

Periodic.

3) $x(t) = e^{-j10t} + e^{j15t}$

<u>Soln</u> W.K.T $e^{j\omega\theta}$

$\omega_1 = 10 \qquad \omega_2 = 15$

$T_1 = \frac{2\pi}{10} = \frac{\pi}{5} \qquad T_2 = \frac{2\pi}{15}.$

$T = \frac{T_1}{T_2} = \frac{\pi/5}{2\pi/15} = \frac{3}{2}$ The Ratio of Rational no

$\therefore$ it is periodic.

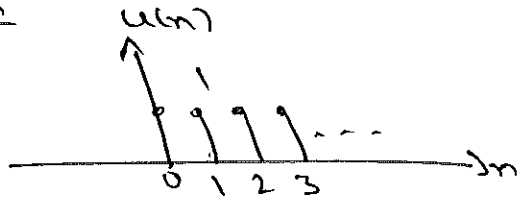
$T = T_1 \times 2 = T_2 \times 3$

$T = \frac{\pi}{5} \times 2 = \frac{2\pi}{15} \times 3$

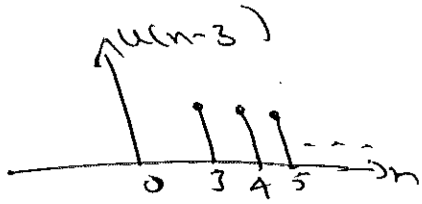
$T = \frac{2\pi}{5}$ sec

(4) $x(n) = u(n-3)$

<u>Soln</u>



$\Rightarrow$



$\therefore$ Non periodic step fn delayed by 3 units.

(5) $x(n) = e^{an}$

<u>Soln</u> W.K.T $x(n+N) = x(n)$

$\therefore x(n+N) = e^{a(n+N)}$

$= e^{an} \cdot e^{aN}$; if $e^{aN} = 1$

$= x(n)$

But $e^{aN} = 1$ only if $N = 0$

$\therefore x(n)$ is non periodic

(6) $x(n) = n$ modulo 6

<u>Soln</u> $\therefore x(n+N) = (n+N)$ modulo 6

$= x(n)$; only if $N = 6m$; minimum value of $m = 1$

$\therefore x(n)$ is periodic

fundamental period $N = 6$

(7) $x(n) = \cos\left(\frac{\pi}{8} n^2\right)$

Soln

For periodic $x(n+N) = x(n)$

$$\cos\left(\frac{\pi}{8}(n+N)^2\right) = \cos\left(\frac{\pi}{8}n^2\right) = \cos\left(\frac{\pi}{8}n^2 + 2k\pi\right)$$

$$\frac{\pi}{8}(n+N)^2 = \frac{\pi}{8}n^2 + 2k\pi \qquad k = \text{integer}$$

$$\frac{\pi}{8}(N^2 + 2nN) = 2k\pi$$

$$\therefore N^2 + 2nN = 16k \quad -(1)$$

eqn (1) will be satisfied if $N = 8, 16, 32 \ldots$

$\therefore x(n)$ is periodic with period $N = 8$

Problems on Energy & power signal

(1) Check whether the following signals are energy or power signals

(a) $x(t) = A$

Soln

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad \text{or} \quad E = \underset{T \to \infty}{Lt} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$E = \int_{-\infty}^{\infty} A^2 dt \qquad E = \underset{T \to \infty}{Lt} \int_{-T/2}^{T/2} A^2 dt$$

$$= A^2 t \Big|_{-\infty}^{\infty} \qquad = \underset{T \to \infty}{Lt} A^2 \, t \Big|_{-T/2}^{T/2}$$

$$= A^2 \cdot \infty \qquad = \underset{T \to \infty}{Lt} A^2 \cdot T = A^2 \cdot \infty$$

$$E = \infty$$

$\therefore x(t) = A$ is not an energy signal $= \infty$.

Power $P = \underset{T \to \infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \underset{T \to \infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} A^2 dt$

$$= \underset{T \to \infty}{Lt} \frac{1}{T} A^2 \, t \Big|_{-T/2}^{T/2}$$

$$= \underset{T \to \infty}{Lt} \frac{1}{\cancel{T}} A^2 \cancel{T}$$

$$P = A^2 < \infty \qquad \therefore \text{it is power signal}$$

(47K)

(b) $x(t) = u(t)$

Soln

$$E = \underset{T\to\infty}{Lt} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$E = \underset{T\to\infty}{Lt} \int_{0}^{T/2} 1 \, dt$$

$$= \underset{T\to\infty}{Lt} \; t \Big|_0^{T/2}$$

$$= \underset{T\to\infty}{Lt} \; \frac{T}{2}$$

$$E = \infty$$

$\therefore$ $x(t) = u(t)$ is not an Energy signal

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{0}^{T/2} 1 \, dt$$

$$\Rightarrow P = \underset{T\to\infty}{Lt} \frac{1}{T} \; t \Big|_0^{T/2}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{T} \left[T/2 \right]$$

$$P = 1/2 \quad < \infty$$

$\therefore$ $x(t)$ is a power signal.

(c) $x(t) = e^{t} u(t)$

Soln

$$E = \underset{T\to\infty}{Lt} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \int_{0}^{T/2} (e^{t})^2 dt$$

$$= \underset{T\to\infty}{Lt} \int_{0}^{T/2} e^{2t} dt$$

$$= \underset{T\to\infty}{Lt} \frac{e^{2t}}{2} \Big|_0^{T/2}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2} \left[e^{T} - 1 \right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2} \left[e^{\infty} - 1 \right]$$

$$E = \infty$$

not an energy signal

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{0}^{T/2} (e^{t})^2 u(t)$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \left[\frac{e^{2t}}{2} \Big|_0^{T/2} \right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \left[e^{T} - 1 \right]$$

$$P = \infty$$

$\therefore$ $x(t) = e^{t} u(t)$ is neither an energy nor a power signal

(d) $x(t) = e^{j(2t + \pi/4)}$

Soln

$|x(t)|^2 = 1$

$\because |e^{j\theta}| = 1]$

$$E = \underset{T \to \infty}{Lt} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$E = \underset{T \to \infty}{Lt} \int_{-T/2}^{T/2} 1 \, dt$$

$$= \underset{T \to \infty}{Lt} \, t \Big|_{-T/2}^{T/2} = T$$

$$= \infty //$$

not an Energy signal

$$\text{Power } P = \underset{T \to \infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \underset{T \to \infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} 1 \, dt$$

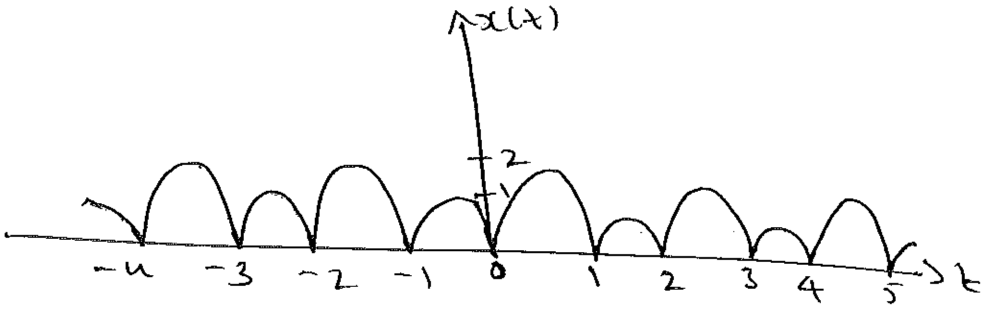
$$= \underset{T \to \infty}{Lt} \frac{1}{T} \left[t \Big|_{-T/2}^{T/2} \right]$$

$$= \underset{T \to \infty}{Lt} \frac{1}{\cancel{T}} [\cancel{T}]$$

$$P = 1 \text{ W} \qquad 1 < \infty //$$

$\therefore$ it is an power signal.

1) Determine the Power of the periodic signal $x(t)$ shown in below fig



Soln The period of the signal

$T = 2$

$$x(t) = \begin{cases} 2\sin(\pi t) & 0 \le t \le 1 \\ \sin(\pi(t-1)) & 1 \le t \le 2 \end{cases}$$

$$P = \frac{1}{T} \int_0^T |x(t)|^2 dt$$

$$P = \frac{1}{2} \left[4 \int_0^1 \sin^2 \pi t + \int_1^2 \sin^2 (\pi(t-1)) dt \right]$$

$P = 5/4$ watt.

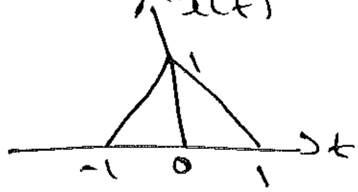
(2) Find the Energy of the signal

$$x(t) = 3\,\text{tri}(t/4)$$

Soln

w.k.t $\quad \text{tri}(t) = \begin{cases} 1 - |t| & ; |t| \le 1 \\ 0 & ; |t| > 1 \end{cases}$

The sketch of $x(t) = \text{tri}(t)$ is



Energy $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |3\,\text{tri}(t/4)|^2 dt$

$$E = 9 \int_{-\infty}^{\infty} |\text{tri}(t/4)|^2 dt$$

$$\mathrm{tri}(t/4) = 1 - \left|\tfrac{t}{4}\right| \le 1 \quad ; \left|\tfrac{t}{4}\right| \le 1 \Rightarrow |t| \le 4$$

$$0 \quad ; \left|\tfrac{t}{4}\right| > 1 \Rightarrow |t| > 4$$

$$\therefore \quad E = 9 \int_{-4}^{4} (1 - |t/4|)^2 \, dt$$

Since $\mathrm{tri}(t/4) = 1 - |t/4|$ is an even fn $\left[\because \int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt\right]$.

$$E = 9 \times 2 \int_0^4 (1 - t/4)^2 \, dt$$

$$= 18 \int_0^4 \left(1 - t/2 + \frac{t^2}{16}\right) dt$$

$$E = 24 //$$

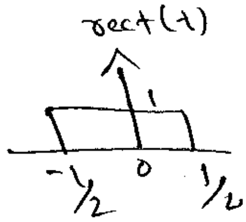
(3) Find the Energy of the Signal

(a) $x(t) = 2\,\mathrm{rect}(t)$

Soln

$$\mathrm{rect}(t) = 1 \quad ; |t| \le 1/2$$

$$0 \quad ; |t| > 1/2$$



$$\therefore \quad E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |2\,\mathrm{rect}(t)|^2 \, dt$$

$$= 4 \int_{-1/2}^{1/2} 1\,dt = 4\, t \Big|_{-1/2}^{1/2}$$

$$= 4 //$$

(b) $x(t) = \mathrm{rect}(t) \cdot \cos 4\pi t$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |\mathrm{rect}(t) \cdot \cos 4\pi t|^2 \, dt$$

$$E = \int_{-1/2}^{1/2} \cos^2 4\pi t \, dt = \frac{1}{2} \int_{-1/2}^{1/2} \frac{1 + \cos 8\pi t}{\cancel{2}} \, dt$$

(47M)

$$E = \frac{1}{2}\left[\int_{-1/2}^{1/2} 1\, dt + \int_{-1/2}^{1/2} \cos 8\pi t\, dt\right]$$

$$E = 1/2$$

(c) $x(t) = \text{rect}(t) \cdot \sin 2\pi t$

Soln

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |\text{rect}(t) \cdot \sin 2\pi t|^2 dt$$

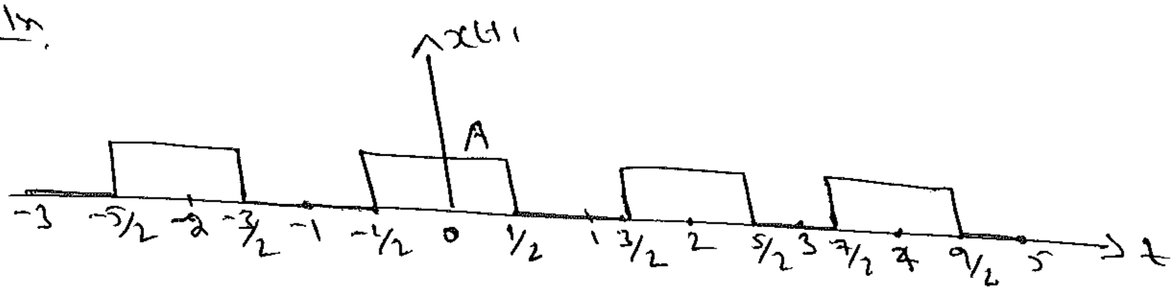
$$E = \int_{-1/2}^{1/2} \sin^2 2\pi t\, dt = \frac{1}{2}\int_{-1/2}^{1/2} [1 - \cos 4\pi t]\, dt$$

$$E = 1/2 \;//$$

(4) Sketch & find the power of the signal.

(a) $x(t) = A \sum_{k=-\infty}^{\infty} \text{rect}(t - 2k)$.

Soln

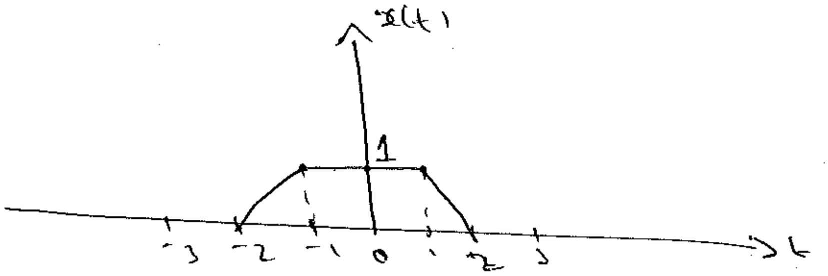


fundamental period $T = T_1 - T_0 = 3/2 - (-1/2) = 2$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2} \int_{-1/2}^{1/2} A^2 dt$$

$$P = \frac{1}{2} A^2 \;//$$

t	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x(t)$
$t=-2$	0	0	0	0	0
$t=-1$	1	0	0	0	1
$t=0$	2	-1	0	0	1
$t=1$	3	-2	0	0	1
$t=2$	4	-3	-1	0	0
$t=3$	5	-4	-2	1	0



~~Systems~~ :

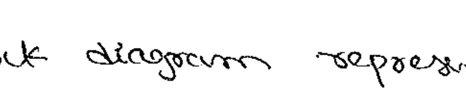
System Viewed as Interconnection of operations

→ A system is an interconnection of operations that transforms an input signal into an output signal. The properties of these output signals are entirely different from that of the input signal.

→ Consider a continuous-time system represented by an operator 'H'. An input signal $x(t)$ applied to this system results in an output signal $y(t)$ is described as.

$$y(t) = H\{x(t)\} \quad -①.$$

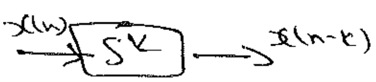
The block diagram representation of eqn ① is.



$|||^{ly}$ The block diagram representation for a discrete-time system described by

$$y(n) = H\{x(n)\}$$

is shown in fig ②



②.

Example

1) Find the overall operator of a system whose output signal $y(n)$ is given by,

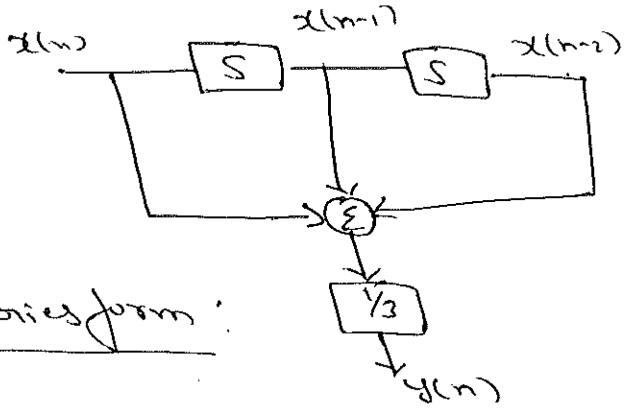
$$y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right]$$

Also draw the block diagram representation.

Soln:

The block diagram representation is shown below fig.

Series form:



S → The operator which delays the signal by one unit.

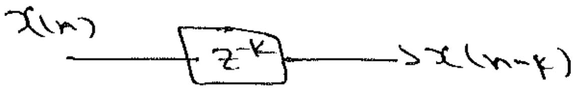
where H = S is the shift operator

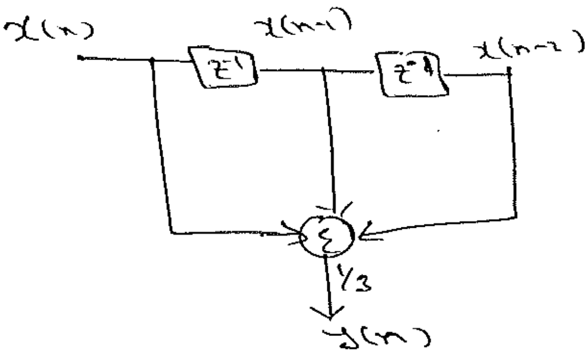
$$H = \frac{1}{3}\left[\cancel{\delta}\, 1 + S + S^2\right]$$

Or

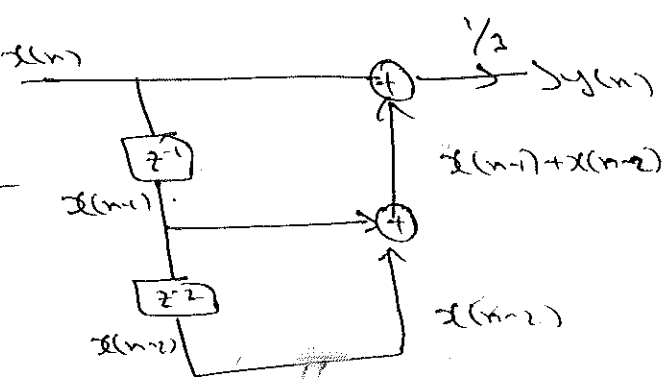
Using Z-transform.

z^{-k} is the shift operator



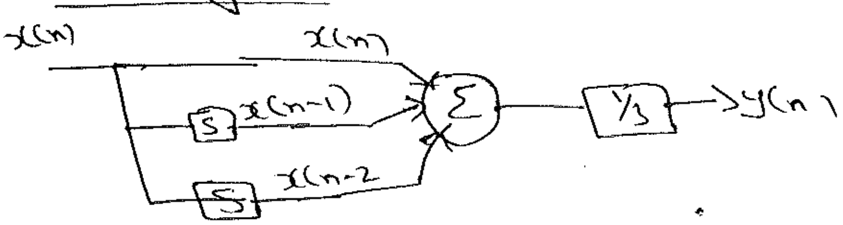


or



or

Parallel form



(2) A discrete time system is represented by the following input-o/p relation

$y(n) = 2x(n) + 3x(n-1) + 4x(n-2) + 5x(n-3)$

Draw two different Representation for the slm operator H.

Soln

$$y(n) = 2 + 3S^{1} + 4S^{2} + 5S^{3}$$

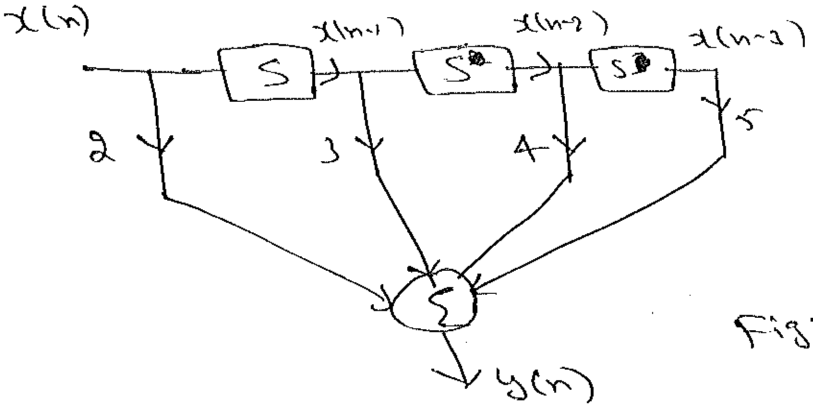


Fig: Cascade implementation

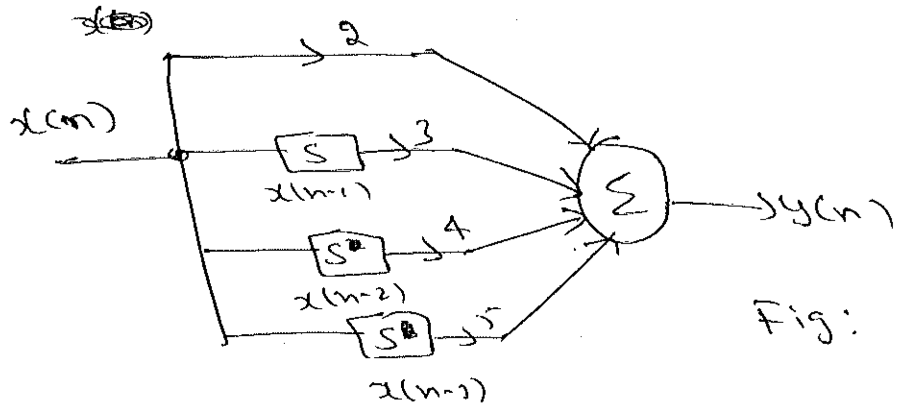
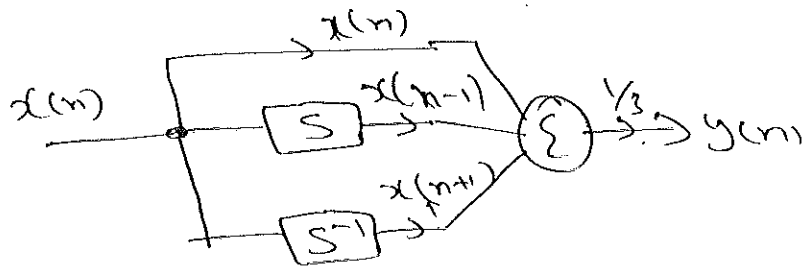


Fig: Parallel implementation

(3) Find the overall operator of a slm whose o/p signal $y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$. Also draw the block diagram representation.

Soln

$$y(n) = \frac{1}{3}\left[S^{-1} + 1 + S\right].$$



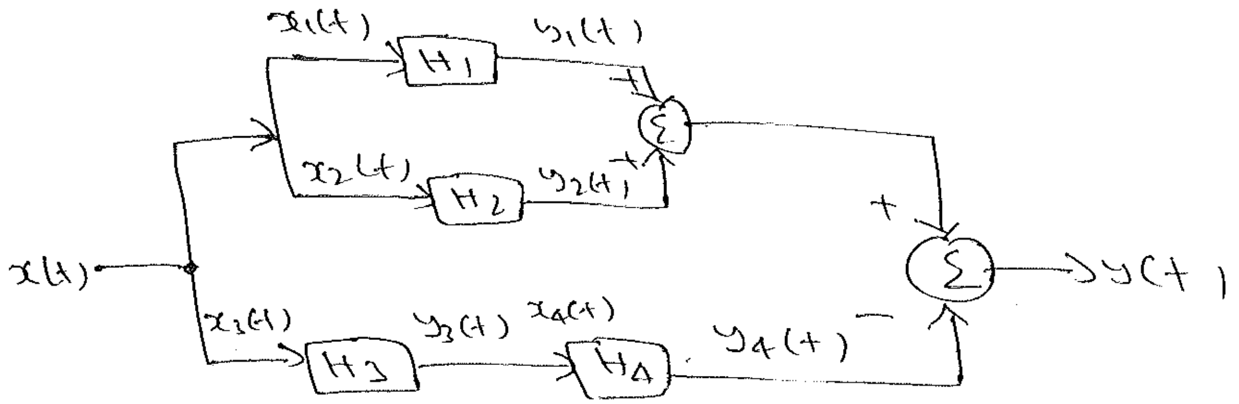
(4) A system consists of several subsystems connected as shown in below fig. Find the operator H relating $x(t)$ to $y(t)$ for the subsystem operators given by

$H_1 : y_1(t) = x_1(t) \cdot x_1(t-1)$

$H_2 : y_2(t) = |x_2(t)|$

$H_3 : y_3(t) = 1 + 2x_3(t)$

$H_4 : y_4(t) = \cos(x_4(t))$.



Soln

$$y(t) = \{y_1(t) + y_2(t)\} - y_4(t).$$

$$y(t) = \{x_1(t) \cdot x_1(t-1) + |x_2(t)|\} - \cos\{x_4(t)\}$$

$$y(t) = \{x_1(t) \cdot x_1(t-1) + |x_2(t)|\} - \cos\{y_3(t)\}.$$

$$y(t) = \{x_1(t) \cdot x_1(t-1) + |x_2(t)|\} - \cos\{1 + 2x_3(t)\}.$$

we have $x_1(t) = x_2(t) = x_3(t) = x(t)$

$$y(t) = \{x(t) \cdot x(t-1) + |x(t)|\} - \cos\{1 + 2x(t)\}$$