INDEX

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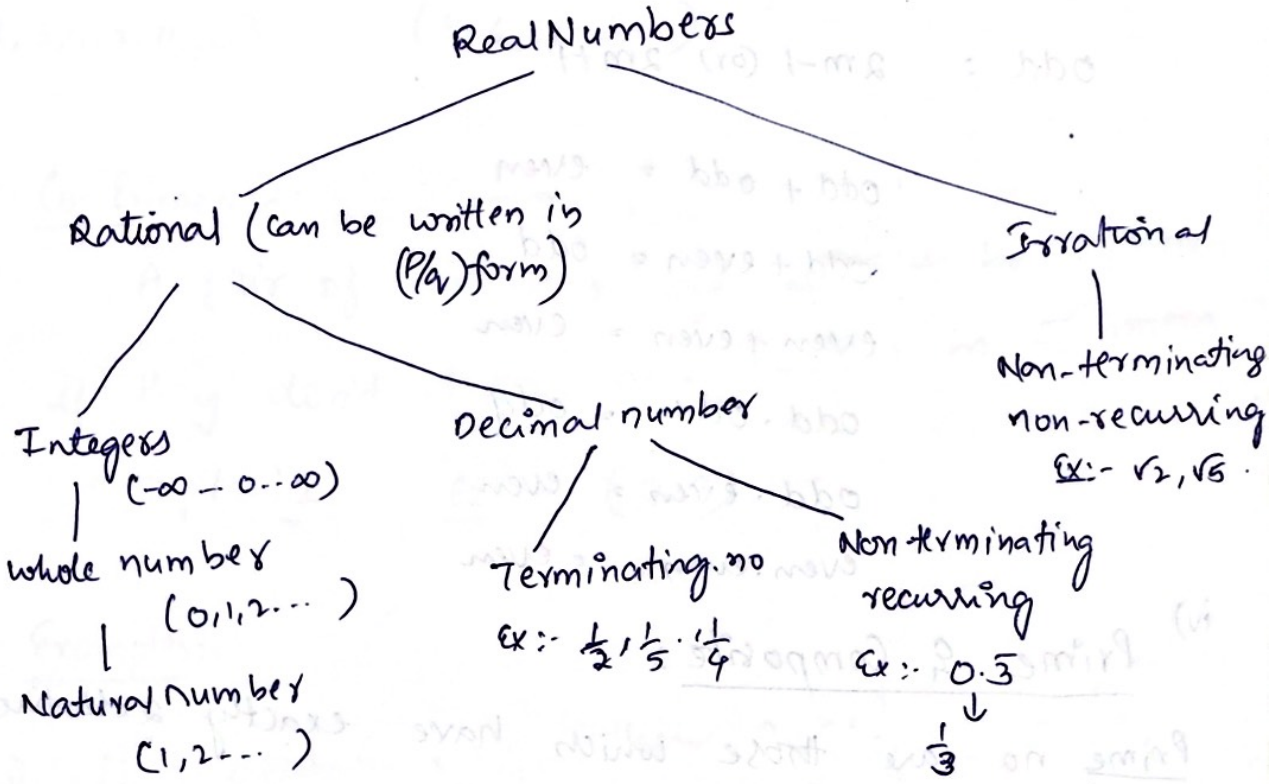
S. No.	Date	Title	Page No.	Teacher's Sign / Remarks
		Numbers Systems		
01.		classification - Number Systems		
02.		Divisibility		
03.		Cyclicity		
04.		Factorisation		
05.		Factorials		
06.		Number of Trailing Zeroes		
07.		Number of Factors		
08.		Remainders		
09		Last two digits of number		
10.		HCF		
11.		LCM		

Number Systems

I. ;) classification



ii) Conversion from Decimal to (P/q) form:-

a) $y = 0.333\ldots$

$10y = 3.333\ldots$

$-9y = -3$

$y = 3/9 = 1/3$

b) $y = 0.1666\ldots$

$10y = 1.666\ldots$

$100y = 16.666\ldots$

$90y = 15$

$y = \frac{15}{90} = \frac{1}{6}$

General form:-

a) $0.abcbcbc\ldots$

$0.a\overline{bc}$

$= \frac{abc - a}{990}$

b) $0.abcccc\ldots$

$0.ab\overline{c}$

$= \frac{abc - ab}{900}$

iii) Odd AND EVEN

General form

Even : $2m$

Odd : $2m-1$ (or) $2m+1$

odd + odd = even

odd + even = odd

even + even = even

odd . odd = odd

odd . even = even

even . even = even

iv) Prime & Composite :

Prime no are those which have exactly 2 distinct factors

(1 - 100)

↓

25 - prime number

*) → Any prime no. greater than '3' can be written in the form $(6n-1)$ (or) $(6n+1)$

Ex :- 59 → $6(10)-1$

Composite :- Any natural number having more than 2 distinct factors

Natural numbers

Prime no (2, 3, 5, 7, 11, ...)

composite no (4, 6, 8, 9 ...)

1

Co-Primes :-

A pair of numbers are said to be co-primes if they don't have any factors in common except '1' Ex:- (2,3)

Examples :-

1) if $0.abab\ldots = 5/11$; then $a, b = ?$

Sol:- $0.\overline{ab} = 5/11 \Rightarrow \frac{ab}{99} = \frac{5}{11} = ab = 45$

$\Rightarrow a = 4 ; b = 5$

2. If 'p' is a prime no. greater than '3' then the what could be remainder . if 'p' is divided by 6 ?

Sol:- $\frac{6n+1}{6}$; $\frac{6n-1}{6}$

$= \frac{1}{6} = 1$; $\frac{-1}{6} = 5$

(1, 5) → are remainders

II Divisibility Rules:-

Rule of '2':-

Last digit should be even (or) divisible by '2'

Rule of '4':-

Last two digits should be divisible by '4'

Rule of '8':-

Last three digits should be divisible by '8'

Similarly for. 16, 32 ---

Rule of '3':-

Sum of digits should be divisible by '3'.

Rule of '9':-

Sum of digits should be divisible by '9'.

Rule of '7':-

Double the last digit, subtract from rest of number, the obtained number should be divisible by '7'.

Ex:. 672 ⇒ 67-4 = 63/7 = 0

Rule by '11':-

(Sum of unit's place - Sum of ten's place) = 0, 11, 22...

should be divisible by '11'.

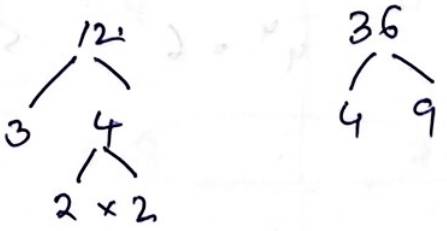
Ex:- 5 6 7 8 9 6
T U T U T U

(6+8+6) - (9+7+5) =

Rule for Composite Numbers:-

Find the co-primes of numbers, & the number should be divisible by co-primes

Ex:-



III

Cyclicity of Unit's Place:-

1) a, b, c, d, a, b, c, d, a, b, c, d, a, b, ...

14th term = ? cycle of '4'.

$\frac{14}{4} = \overset{\text{remainder}}{2} \Rightarrow 2^{nd}$ term $\Rightarrow$ 'b'.

2) $12\textcircled{4} \times 17\textcircled{7}$

unit's digit depends only on last digits

$\Rightarrow 4 \times 7 = 2\textcircled{8}$ → units place of product

Ex:- $78\textcircled{7} \times 45\textcircled{2} \times 32\textcircled{1}$

$7 \times 2 \times 1 = 1\textcircled{4}$

3) units place of 2^{61}, 5^{32} ... ?

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 6$
$2^5 = 2$
$2^6 = 4$
$2^7 = 8$
$2^8 = 6$

$3^1 = 3$
$3^2 = 9$
$3^3 = 7$
$3^4 = 1$
$3^5 = 3$
$3^6 = 9$

$4^1 = 4$ → odd
$4^2 = 6$ → even
$4^3 = 4$
$4^4 = 6$

$5^1 = 5$
$5^2 = 5$
$5^3 = 5$
$5^4 = 5$

↓ cycle of '4'

↓ cycle of '4'

↓ cycle of '2'

↓ cycle of '1'

cycle of '4' - $2^n, 3^n, 8^n, 7^n$,

cycle of '2' - $4^n, 9^n$

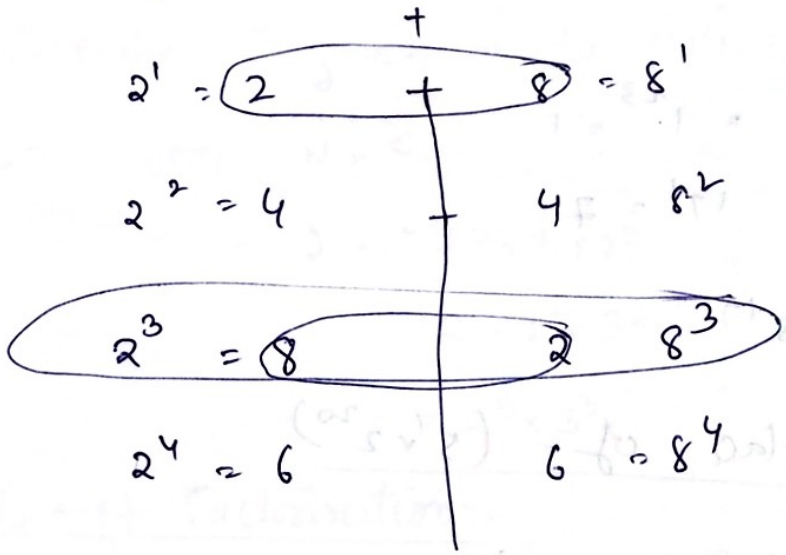
cycle of '1' = $1^n, 5^n, 6^n$

Ex:-

units place of 1) 2^{78} $\frac{78}{4} = \frac{\text{Remainder}}{2} = 2^2 = 4$

$= 50\,\%\,4 = 2 = 2^2 = 4$

Addition of numbers in power



$2^n + 8^n = \underset{\text{unitsplace}}{0}$ → (if 'n' is odd)

$3^n + 7^n = 0$

$4^n + 6^n = 0$

$1^n + 9^n = 0$

Ex:- $872^{915} \times 733^{700} \times 147^{140}$

$2^3 \times 3^4 \times 7^4$

$8 \cdot 1 \cdot 1$

$= 8$

multiplication

$8 + 1 + 1$

$= 0$

addition

Ex:-

$122^{201} - 78^{86}$

$2 - 4$

$= -2$

$= 8 \rightarrow$ unitsplace

Ex:

Units place of $17^{21^{23}}$

$= 1^{23} = 1$

$17^1 = 7$

Ex:

$19^{18^{19}}$

Q Ten's place of $(5^1 \times 2^{20})$

$(5^1 \times 2^1) \times (2^{19})$

↓

$\underline{0}$ → units place

If '5' & '2' → units place ⇒ '0'

$(5^1 \times 2^1) \times 2^{19}$

↓

units place will be ten's place

$\underline{80}$

ii) $(5^2 \times 2^5)$

thousands place

$(5^2 \times 2^2) \times 2^3$

$\underline{800}$

IV Factorisation

a) factorise a number to prime powers

Ex:- 864 — 8×108

$2^3 \times 4 \times 27$

$2^3 \times 2^2 \times 3^3$

$= 2^5 \times 3^3$

Uses of Factorisation:-

1. Even (or) odd (If '2' is multiple)
2. Divisibility
3. Perfect Square, Cube, fourth etc.. (powers = even) (multiples of 3)
4. Units place & trailing zeroes

Ex: By which least number should 1440 be divided so that quotient is an odd number?

Sol:- $1440 = 12 \times 12 \times 10$

$3 \times 2^2 \times 3 \times 2^2 \times 5 \times 2$

$= 2^5 \times 3^2 \times 5$

$\Rightarrow \frac{2^5 \times 3^2 \times 5}{2^5} \Rightarrow 3^2 \times 5 \Rightarrow$ odd

$\Rightarrow$ "2^5" should be divided

$\text{prime no}^{\text{Power}} \times \text{prime no}^{\text{Power}}$

1) if powers are multiples of '2' then perfect square
2) if powers are " '3' then perfect cube
3) if powers are both multiples of '2' & '3'

then p. squares as well as p. cubes

Ex:- By which least natural no. should 720 be multiplied so that resultant is a cube as well as a square?

Sol:

$$720 = 24 \times 30$$
$$= 2^2 \times 3 \times 2 \times 3 \times 5 \times 2$$
$$= 2^5 \times 3^2 \times 5$$

$\Rightarrow 2 \times 3^4 \times 5^5 \Rightarrow$ should be multiplied

Ex:- no. of trailing zeroes $((280)^5 \times 12^3 \times 18^2)$

$$= \left(5^{15} \times 2^{13}\right) = 10^{13}$$

V

Factorials

$0! = 1$; $1! = 1$; $2! = 2$; $3! = 6$

$4! = 24$

$= 4 \times 6$

$= 2^2 \times 3 \times 2$

$= 2^3 \times 2$

$90 = 9 \times 10$

$= 3^2 \times 5 \times 2$

$= 2 \times 3^2 \times 5$

$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$= 2^? \times 3^? \times 5^? \times 7^?$

$15! = 2^? \times 3^? \times 5^? \times 7^? \times 11^? \times 13^?$

1) what is the highest power of '2' in 10! ?

Sol:- multiples of '2' $= \frac{10}{2} = 5$

$\frac{5}{2} = 2$

$\frac{2}{2} = 1$

8

'8' is the highest power of '2' in 10!

2) highest power of '3' in 30!

$\frac{30}{3} = 10$

$\frac{10}{3} = 3$

$\frac{3}{3} = 1$

14

'14' is the highest power

Factorise 10!

2, 3, 5, 7 → are prime no. under '10'

$\frac{10}{2} = 5$
$\frac{5}{2} = 2$
$\frac{2}{2} = 1$
8

$\frac{10}{3} = 3$
$\frac{3}{3} = 1$
4

$\frac{10}{5} = 2$
2

$\frac{10}{7} = 1$
1

$10! = 2^8 \times 3^4 \times 5^2 \times 7^1$

Ex:- highest power of '4' in 10!

Sol:-

$(2^2)^4$ = '4' is the highest power of '4'

Ex:- highest power of 6! in 50!

Sol:-

$6! = 2^4 \times 3^2 \times 5^1$

$\frac{6}{2} = 3$
$\frac{3}{2} = 1$
4

$\frac{6}{3} = 2$
2

$\frac{6}{5} = 1$
1

50!

$\frac{50}{2} = 25$
$\frac{25}{2} = 12$
$\frac{12}{2} = 6$
$\frac{6}{2} = 3$
$\frac{3}{2} = 1$
47

$\frac{50}{3} = 16$
$\frac{16}{3} = 5$
$\frac{5}{3} = 1$
22

$\frac{50}{5} = 10$
$\frac{10}{5} = 2$
12

$50! = 2^{47} \times 3^{22} \times 5^{12} \ldots$
⇒

$= 2^{47/4} \times 3^{22/2} \times 5^{12}$

$= 2^{44} \times 3^{22} \times 2^{3} \times 5^{12}$

$= (2^{4} \times 3^{2} \times 5^{1})^{11} \times 2^{3} \times 5^{1}$

$\Rightarrow (6!)^{11}$. $\Rightarrow$ '11' is highest power 6! in 50!

Ex):- what is the highest power of '5' that will divide the product of first 50 multiples of 5?

Sol:-

$$\frac{(5\times1)\times(5\times2)\times(5\times3)\times(5\times4)\cdots(5\times50)}{5^{?}}$$

$$= \frac{5^{50} \times 50!}{5^{?}}$$

$\frac{50}{5} = 10$

$\frac{10}{5} = 2$

12

$$= \frac{5^{50} \times 5^{12}}{5^{x=?}} = \frac{5^{62}}{5^{x}} \Rightarrow x = 62$$

Ex:- what will be the remainder when

$$\frac{1!+2!+3!+\underbrace{4!+5!+\cdots+100!}_{\text{has } 4!\text{ is d'ble by } 4}}{4}$$

$= 1+2+6$

$= \frac{9}{4} =$ '1' is the remainder

Ex:- Find the least value of 'n' such that $n!$ is divisible by 910

Sol:-

$$\frac{n!}{910} = \frac{n!}{7\times10\times13} = \frac{13!}{7\times10\times13}$$

↳ because '13' should be there in numerator.

Ex:- Find the unit's place of

$$2^{1!+2!+3!+\cdots+10!} + 3^{1!+2!+3!+\cdots+10!} + \cdots + 9^{1!+2!+3!+\cdots}$$

$= 9^n \Rightarrow$ cycle of "4"

$$\frac{1!+2!+3!+4!+\cdots 10!}{4} = \frac{1+2+6}{4} = 1$$

$$2^1 + 3^1 + 4^{\text{odd}} + 6^{\text{odd}} + \cdots$$

$$= 2+3+4+5+6+7+8+9$$

$$= 4$$

VII

Number of factors

Factors are those which completely divide a no.

factors of 6 — 1, 2, 3, 6

" " 10 — 1, 2, 5, 10

" " 24 — 1, 2, 3, 4, 6, 8, 12, 24

factors of 24 — $2^3 \times 3^1$

$2^0, 2^1, 2^2, 2^3$ $\quad$ $3^0, 3^1$

$4 \times 2 = 8$ factors

$= (3+1)(1+1)$

$= 4 \times 2 =$

$N = P_1^a \times P_2^b \times P_3^c \times P_4^d \times \cdots$

no. of factors $= (a+1)(b+1)(c+1)(d+1) \cdots$

Ex:- if $N = 2^5 \times 3^4 \times 5^2$ then

How many factors of N are

a) Prime — 3 (2, 3, 5)

b) Composite — $90 - 3 = (87 - 1) = 86$ (1 ↑)

c) odd — $(4+1)(2+1) = 15$

d) even — $90 - 15 = 75$

Sol:- Total no. of factors $= (5+1)(4+1)(2+1)$

$= 6 \times 5 \times 3 = 90$

Ex:- If $N = 2^5 \times 3^4 \times 5^2$ then

i) How many factors of N are

1) Perfect cubes
2) Perfect squares
3) will end up with atleast one zero

1) Perfect cubes :-

2^5 $\quad 3^4 \quad 5$

$2^0, 2^3 \quad 3^0, 3^3 \quad 5^0$

$2 \times 2 \times 1 = 4$

2) Perfect squares:

$2^0, 2^2, 2^4 \quad , \quad 3^0, 3^2, 3^4 \quad , \quad 5^0, 5^2$

$3 \times 3 \times 2 = 18$

3) atleast one-zero :-

2^5

$2^1, 2^2, 2^3, 2^4, 2^5 \quad (3^4) \quad 5^1, 5^2$

$5 \times 5 \times 2 = 50$

Ex: How many factors of 540, 360 & 900 are common to each other?

Ex:- How many factors of $(2^6 \times 3^4 \times 5^3)$ are multiples of 4 as well as perfect cubes?

Ex:- How many factors of $(12^2 \times 15^3 \times 20^2 \times 36)$ are perfect cubes but not squares

Application of no. of factors :

factors of 12 : 1, 2, 3, 4, 6, 12 } even no. of factors

factors of 35 : 1, 5, 7, 35

factors of 36 : 1, 2, 3, 4, (6), 9, 12, 18, 36 → odd no. of factors (perfect square)

factors of 45 : 1, 3, 5, 9, 15, 45 } even no of factors

Ex:- If there are 8 factors between 1 and $\sqrt{N}$, then find the total no of factors of N.

Sol:-

1 —8— $\sqrt{N}$ —8— N

8 + 8 + 2 = 18 factors if 'N' is not a perfect square

18 + 1 = 19 factors if 'N' is a perfect square

Ex:- If the product of factors which are at 10th position and 18th position result in the number, N itself find how many factors of N are there ?

Sol:-

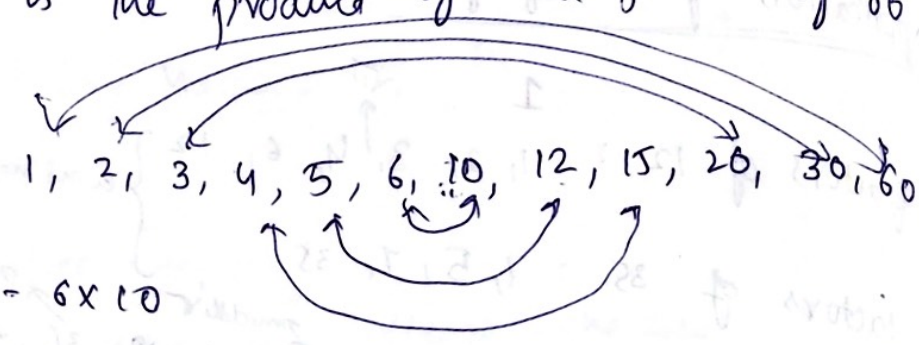
12 → 1, 2, 3, 4, 6, 12

10th — 9 fac — 18th — 9 fac

⇒ 18 + 9 = 27

Ex:- what is the product of all factors of 60

Sol:-



$60 = 6 \times 10$

$= 2 \times 3 \times 5 \times 2$

$= 2^2 \times 3 \times 5$

no of factors $= 3 \times 2 \times 2$

$= 12$

$\Rightarrow 60 = \frac{\text{no. of factors}}{2}$

if 'N' is perfect-square, then

$= (N)^{\frac{\text{no. of factors}}{2}} \cdot \sqrt{N}$

Ex:- which factor of 800 will occupy 14th position if all the factors of 800 are written in ascending order?

Sol:-

$800 \rightarrow 8 \times 10 \times 10$

$= 2^3 \times 2 \times 5 \times 2 \times 5$

$= 2^5 \times 5^2$

no. of fac $= (5+1) \times (2+1)$

$= 18.$

1 2 4 5 8

14 is 18-17 800

$8 \times x = 800$

$\Rightarrow \boxed{x = 100}$ $\Rightarrow$ 14th position $= 100$

Ex:- In how many ways can 240 can be written as product of two integers? [may be distinct]

Sol:- $240 = 12 \times 2 \times 10$

$= 3 \times 4 \times 2 \times 5 \times 2$

$= 2^4 \times 3^1 \times 5^1 \Rightarrow (5) \times 2 \times 2 = 20$ factors

$\frac{20}{2} = 10$ ways of natural no.

20 ways of integers (integral)

if P. square $10 + 1 \rightarrow$ nat. ways

$20 + 2 \rightarrow$ integral ways

Q

Difference of squares $(x^2 - y^2 = N)$

N is odd → find no. of fac $\div 2$ → no. of ways

N is even

if 'N' is not divisible by 4 → no ways

if 'N' is divisible

a) $\div 4$

b) M → no. of factors $\downarrow \div 2$

c) no. of ways

Ex:- In how many ways can '45' be written as difference of squares of two natural numbers?

$x^2 - y^2 = 45$

$(x-y)(x+y) = 45$

Sol:- $45 = 5 \times 9$

$= 3^2 \times 5$

no. of fac $= 3 \times 2 = \frac{6}{2} = 3$ ways

Q. find integral solutions to $\frac{1}{a} + \frac{1}{b} = \frac{1}{8}$

Soln:-

$$(a-8)\cdot(b-8) = 64$$

↓

7 factors

↓

6 factors

↓

3 nat no ways

↓

6 nat. no soln's

+

1 → (p, s)

12 integral soln's

+

1 → (p, s)

per. sq

(8 × 8)

~~(8 × 8)~~

(11)

Remainders

$\frac{1}{6} = 1^{rem}$; $\frac{5}{6} = 5\ (Q) + (1)$

(div)N (quo)

Rem

N = (divisor × quotient) + (Remainder)

↙ Dividend

$M \div 7 \rightarrow 5^{rem}$

$N \div 7 \rightarrow 3$

$(M+N) \div 7 \rightarrow 8 \div 7 = 1$

$(M-N) \div 7 \rightarrow 2 \div 7 = 2$

$(M \cdot N) \div 7 \rightarrow 15 \div 7 = 1$

$(M^2+N^2) \div 7 \rightarrow (25+9) \div 7 = 6$

Ex:- 'y' is a natural no which on being divided by '7', leaves a remainder 4, so. find the remainder when $(y^2-5y+3) \div 7$

$(4) \div 7 = 6$

Ex:- find Remainder, $\frac{14^{77}}{11}$

$= \frac{14 \times 14 \times 14 \ldots\ldots \times 14_{77\ times}}{11}$

$= \frac{3 \times 3 \times \ldots\ldots 77\ times}{11} = \frac{3^{77}}{11}$

form cycle,

$3^1 \div 11 = 3 \div 11 = 3$

$3^2 = 9 \div 11 = 2$

$3^3 = 6 \div 7 = 6$

$3^4 = 18 \div 7 = 4$

$3^5 = 12 \div 7 = 5$

$3^6 = 15 \div 7 = 1$ — cycle of '6'

$3^7 = 3 \div 7 = 3$

$3^{77/6} = 3^{\text{rem } 5} = 3^5$ ← remainder

$3^1 \div 11 = 3 \div 11 = 3$

$3^2 \div 11 = 9 \div 11 = 9$

$3^3 \div 11 = 27 \div 11 = 5$

$3^4 \div 11 = 15 \div 11 = 4$

$3^5 \div 11 = 12 \div 11 = 1$

∴ cycle of '5'

$77/5 = 2$

~~$3^2 = 9 \div 11 = 2$~~

~~Remainder = '2'~~

$2 \Rightarrow 3^2 \Rightarrow$ 9 is the remainder

If '1' comes, then it is stop of the cycle.

3^{80}

$80/5 = 0$

5th term

$\Rightarrow$ '1' is the remainde

→

If '-1' comes, then the cycle length is double

Ex:..

if '-1' at 3, then

cycle length = '6'

Objective : Frame such a cycle which gives a remainder of '1' (or) '-1'

Q.

$19^{200} \div 7$

$\Rightarrow 5^{200} \div 7$

$5^1 = 5 \div 7 = 5$

$5^2 = 25 \div 7 = 4$

$5^3 = 20 \div 7 = 6$ (or) -1 → double the cycle length here, we will get 1

$5^4 = 30 \div 7 = 2$

$5^5 = 10 \div 7 = 3$

$5^6 = 15 \div 7 = 1$

$\Rightarrow \frac{200}{6}$ $200 \% 6 = 2$

$\Rightarrow$ second term of cycle Remainder

$\Rightarrow$ '4'

Q A Number, n when divided by 437, leaves a remainder of 150. Find the remainder when same no. is divided by '23'.

Sol: $N \div 437 = 150$

$N = 437 \times a + 150$ → '0' remainder

$N \div 23 \Rightarrow \frac{N}{23} = \frac{437 \times a}{23} + \frac{150}{23}$

$\Rightarrow \frac{N}{23} = \frac{150}{23} = 12$

$\therefore$ Remainder = 12

Q Find the remainder when, $\frac{15^{23} + 23^{23}}{19}$

Sol:- grouping

$$\frac{15^{23}}{19} = \frac{(-4)^{23}}{19}$$

$$\frac{(23)^{23}}{19} = \frac{(4)^{23}}{19}$$

$$\Rightarrow \frac{-4^{23} + 4^{23}}{19} = 0$$

Negative Remainder

$$\frac{19^{93}}{5} = \frac{(-1)^{93}}{5} = -1/5 = 4$$

$$\frac{20^{23}}{5} = 0$$

$$\frac{21^{23}}{5} = 1$$

$$\frac{2^{100}}{9} = \frac{(2^3)^{33} \cdot 2}{9} = \frac{8^{23} \cdot 2}{9} = \frac{(-1)^{23} \cdot 2}{9}$$

$$= \frac{-2}{9} = 7$$

Q $\frac{2^{100}}{17} \cdot \frac{(2^4)^{25}}{17} = \frac{(16)^{25}}{17} = \frac{(-1)^{25}}{17} = \frac{-1}{17} = 16$

Q 'N' is a natual number, when divided by a divisor leaves a remainder of 24. when 2N is divided by same divisor, remainder is 11. find the value of divisor.

Sol:-

$N = x.a + 24 \rightarrow (1)$

$2N = x.b + 11 \rightarrow (2)$

$(1) \times 2 \Rightarrow 2N = 2x.a + 48$

$\Rightarrow xb + 11 = 2x.a + 48$

$\boxed{x.(b - 2a) = 37}$

Product of two natural numbers

$\Rightarrow$ factors concept

$\Rightarrow (1 \times 37)$ (or) (37×1)

$\Rightarrow \boxed{x = 37}$

$$23^{21^{17}} \neq (23^{21})^{17}$$

* while finding unit's digit, we come from top to bottom

ex:-

$$23^{21^{17}} \Rightarrow 23^{1} \Rightarrow 3^1 = 3$$

* but while finding remainder we go from bottom to top

Ex:-

$$\frac{14^{21^{39}}}{11} \Rightarrow \frac{3^{21^{39}}}{11}$$

~~23 21 17~~

$3^1 \div 11 = 3$

$3^2 \div 11 = 9 \div 11 = 9$

$3^3 \div 11 = 27 \div 11 = 5$

$3^4 \div 11 = 15 \div 11 = 4$

$3^5 \div 11 = 12 \div 11 = 1$

} cycle of '5'

$$\Rightarrow \frac{3^{21^{39}}}{11} \Rightarrow \frac{3^N}{11} = \frac{N}{5} = \frac{21^{39}}{5}$$

$$= \frac{1^{39}}{5} = \overset{\text{remainder}}{1} \Rightarrow \frac{3^1}{11} = 3$$

$\Rightarrow$ remainder = 3

Q

$\frac{40}{12} \Rightarrow$ rem 4 ✓ (we are asked to make group of '12')

↓ but

$\frac{10}{3} \Rightarrow 1$ ✗ (we are making group of '3')

Ex: $\frac{17^{100}}{51}$

$17 \div 51 = 17$

$17^2 \div 51 = 289 \div 51 = ?$

tedious → $\frac{17 \times 17^{99}}{17 \times 3}$

$\Rightarrow \frac{17^{99}}{3}$

↓

(remainder) × (17)

↓

the number which we have cancelled should be multiplied

$\Rightarrow \frac{17^{100}}{51}$ = remainder of $\left(\frac{17^{99}}{3}\right) \times 17$

Finding last two Digits

i) Divide by '100'

$$\frac{15^{40}}{100} = \frac{\overset{3}{15}\times\overset{3}{15}\times 15^{38}}{5\times 5\times 4} = \frac{9\times 15^{38}}{4}$$

$$= \frac{9\times(-1)^{38}}{4} = \text{rem}$$

$1 \times$ ~~5~~

$\downarrow$

$1 \times (5\times 5)$ → multiply what you have cancelled

$= 25$

Using

Binomial Theorem

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \cdots$$

$$\cdots {}^nC_{n-1}\, a^1 b^{n-1} + {}^nC_n\, a^{(n-n)} b^n$$

→ observe patterns

$${}^nC_0 = 1 \quad ; \quad {}^nC_n = 1$$

$${}^nC_1 = n \quad - \quad {}^nC_{n-1} = n$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

Q Find last two digits of 41^{47}

Sol:-

~~$(40+1)^{47}$~~ $(1+40)^{47}$

$(1+40)^{47} = {}^{47}C_0\, 1^{47} \cdot 40^0 + {}^{47}C_1\, 1^{46}\, 40^1 +$

${}^{47}C_2\, 1^{45}\, 40^2 + \cdots$

$10 + 50 + 100 + 1000 + 10000 + \cdots$

↳ last '2' digits depends on because we are getting '2' zeroes afterwards

11'ly for, above pblm, 40^2 (we get 2 zeroes) 3rd term.

and last digits does not depend

$\Rightarrow (1+40)^{47} = (1) + (47 \times 40)$

$47 \times 40 = 1880$

$= 1881$

↳ last two digits

Q2 69^{50}

$\Rightarrow (-1+70)^{50}$

$= {}^{50}C_0 (-1)^{50} (70)^0 + {}^{50}C_1 (-1)^{49} (70)^1 + \cdots$

$= +1 + 50(-1) \times 70$

$= 1 - 3500$

$= -3499$

$\Rightarrow$ 10000
3499
6501

$\Rightarrow$ ~~last two digits~~

$\Rightarrow$ '01' last two digits

Q $(37)^{64}$

↳ last digits $\neq 1, 9$

$(37^2)^{32} \cdot 37 = (1369)^{32} \cdot 37$

→ last two digit of a number depend only on last two digits

$\Rightarrow (69)^{32}$

$= (-1+70)^{32}$

Q $(43)^{37} = (43^2)^{18} \cdot 43^1$

$= (49)^{18} \times 43^1$

$=$

last two digits

÷100

unit's place

2, 4, 6, 8, 5

$\frac{44^{70}}{100}$

1, 9, 3, 7

Binomial theorem

LCM

LCM of a set of numbers will be the least number which is divided by all the numbers in that given set.

GCD/HCF

(Highest common factor)

HCF of (24, 36 and 108)

24 - 1, 2, 3, 4, 6, 8, (12), 24

36 - 1, 2, 3, 4, 6, 9, (12), 18, 36

108 - 1, 2, 3, 4, 6, 9, (12), 18, 27, 36, 54, 108

HCF(24, 36, 108) = 12

In general,

1) Factorise given numbers

$24 = (2^3) \times (3^1)$

$36 = (2^2) \times 3^2$

$108 = (2^2) \times 3^3$

Take the common factors, which are present in all the numbers $\Rightarrow 2^2 \times 3^1 = 4 \times 3 = 12$

Q if HCF of two natural numbers is 12. product of these two natural numbers is 1728. How many pairs of number are possible?

Sol:

$(12a) \times (12b) = 1728$

$ab = \frac{1728}{144} = 12$

$a \times b = 12$ [$\because$ a, b are co-primes]

- 1×12
- 3×4
- 2×6

$(1, 12) \Rightarrow (12, 144)$

$\Rightarrow (3, 4) = (36, 48)$

Q Find the greatest number that on dividing 150, 194 & 245 leaves remainder of 6, 2 & 5 respectively

Sol:

$150 = N \times a + 6 \Rightarrow Na = 144$

$194 = N \times b + 2 \quad Nb = 192$

$245 = N \times c + 5 \quad Nc = 240$

'N' is the H.C.F of 144, 192, 240

$144 = 12 \times 2^2 \times 3$

$192 = 12 \times 2^4$

$240 = 12 \times 2^2 \times 5$

H.C.F = 48

$\Rightarrow N = 48$

Q Find the greatest number that on dividing 480, 600 & 800 leaves the same remainder in each case

Sol:

$$480 = N \times a + r \rightarrow (1)$$

$$600 = N \times b + r \rightarrow (2)$$

$$800 = N \times c + r \rightarrow (3)$$

eliminate 'r'

(3) − (2) $\quad 200 = N(c-b)$

(2) − (1) $\quad 120 = N(b-a)$

(3) − (1) $\quad 320 = N(c-a)$

H.C.F of (200, 120, 320)

$$120 = 10 \times 12 = 10 \times 2^2 \times 3$$

$$200 = 10 \times 20 = 10 \times 2^2 \times 5$$

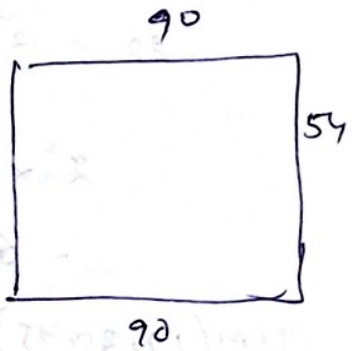
$$320 = 10 \times 32 = 10 \times 2^5$$

$$= 10 \times 2^2 = 40$$

Q 48 rose plants, 72 marigold plants & 108 lotus plants have to be planted in rows such that each row has equal no of plants & each row has plants of a particular variety only. what is the least no of rows required

Sol: HCF (48, 72, 108)

Q A rectangular cloth of (54 x 90) units is to be cut into equal squares. Find how many min. no of squares pieces are required such that no cloth is wasted.



Sol:- HCF (54, 90)

= 18 x 18 → dimension

Q If the HCF of 120 & N is 8., how many two digit values could 'N' assume.

Sol:-

$120 = 8 \times x$

$x = 15$

$N = 8 \times y$

$y = \overset{\times}{1}, 2, 4, 7, 8, 11.$

⇒ ~~6~~ ⇒ '5' possible '2' digit no.

For Proper Fraction:-

A proper fraction is a fraction, whose numerator is less than denominator.

Ex:- 5/11, 7/9

Improper Fraction:-

A fraction, in which numerator is equal (or) greater than denominator is called Improper Fraction

Ex:- $\frac{56}{18}$

Improper fractions can be expressed as Mixed Fractions Ex: $\frac{56}{18} = 3\frac{2}{18}$

LCM

Lcm (6, 12, 20)

$6 = 2^1 \times 3^1$

$12 = 2^2 \times 3$

$20 = 2^2 \times 5^1$

$2^2 \times 3^1 \times 5^1$ (Take the highest power, because the no should be divisible by all these)

$= 60$.

LCM (12, 30, 45)

$12 = 2^2 \times 3^1$

$30 = 2 \times 3 \times 5$

$45 = 3^2 \times 5$

$= 2^2 \times 3^2 \times 5^1 = 180$

another method, take multiples of highest no and start checking

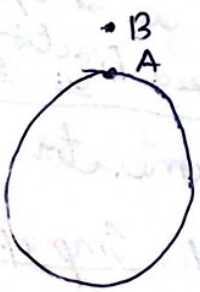
Ex:- traffic signals, green light - after every 3 min
red light - " " 5 min
when together will glow?

Sol:-

L.C.M = $3 \times 5 = 15$ min

after every 15 min, 30 min, 45 min ..... } cycle

Ex:-



A = 4 min to complete round

B = 5 min to complete round

after 20 min, they will meet,

20, 40, 60, 80 ....

Case I:

Q. Find the greatest 3 digit number, that when divided by (10, 12, 15) leaves a remainder of 5 in each case

Sol:

$N = 10 \times a + 5 \Rightarrow (N-5) = 10a$

$N = 12 \times b + 5 \quad (N-5) = 12b$

$N = 15 \times c + 5 \quad (N-5) = 15c$

$\Rightarrow (N-5)$ l.c.m of $10, 12, 15 = 60$

$\Rightarrow (N-5) = 60 * y \quad [\because y = 1, 2, 3, 4 \ldots]$

$$N = 60y + 5$$

$y = 16 \Rightarrow N = 965$, greatest three digit no.

$$N = (\text{LCM}) \times y + \text{remainder}$$

Case II:

Find the least three-digit no. that when divided by 10, 15, 20 leaves remainder of 8, 13 & 18 respectively

Sol:

$N = (10 \times a + 8)$

$N = (15 \times a + 13)$

$N = (20 \times a + 18)$

$N + 2 = (10a + 8) + 2 = 10(a+1)$

$N + 2 = (15b + 13) + 2 = 15(b+1)$

$N + 2 = (20c + 18) + 2 = 20(c+1)$

$$N = \text{L.CM}(10, 15, 20) - 2$$

$N = 60xy + 2$ $[y = 1, 2, 3 \ldots]$

$N = 60 \times 2 + 2$

$N = 122$

Case - II:

Find the least 3-digit no, that on divided by 8 & 11 leaves remainders of 6 & 8 resp

Sol:-

$N = 11xa + 8$

$N = 8xb + 6$

$11xa + 8 = 8, 19, 30, 41, 52, 63, 74, 85, 96, 107, 118, \ldots$

$8xb + 6 = 6, 14, 22, 30, 38, 46, 54, 62, 70, 78, 86, 94, 102, 110, 118 \ldots$

Least 3 digit no = 30

next greatest no = 118

at a gap of L.C.M(8, 11) = 88

$\Rightarrow N = 88xa + 30$

$N = 88 \times 1 + 30 = 118$

$= 88 \times 10 + 30 = 910$

$= 88 \times 11 + 30 = 998$

greatest three digit.

Three forms

1st Case:

N ÷ (9, 10, 12), remainder was same in each case

general form → N = L.C.M (9, 10, 12) * a + r

2nd Case:

N ÷ (9, 10, 12), remainder was diff case but common diff b/w divisors & respective remainder was constant

gen. form of N = (L.C.M (9, 10, 12) * a) − (Common diff)

3rd Case:-

N ÷ (9, 10), remainder was diff each case, but diff. b/w divisors & respective remainder was not constant

general form of N = L.C.M (9, 10) * a + (least no)

Q Find the remainder, when $2^{100} \div 99$

Sol:-

$$\frac{2^{100}}{99} \Rightarrow 99 \Rightarrow 11 \times 9 \Rightarrow \frac{2^{100}}{11,9}$$

$2^{100} \div 11$

	Rem
$2^1 \div 11$	2
$2^2 \div 11$	4
$2^3 \div 11$	8
$2^4 \div 11$	5
$2^5 \div 11$	-1
$2^{10} \div 11$	1

$\frac{100}{10}$ Rem 0

$\Rightarrow 2^{10} = 1$

$11)2^{100}($

$\frac{}{1}$

$2^{100} \div 9$

	Rem
$2^1 \div 9$	2
$2^2 \div 9$	4
$2^3 \div 9$	-1
$2^6 \div 9$	1

$6)100($

$\frac{4}{}$

$\Rightarrow 2^4 = 2^3 \times 2 = \frac{(-1)\times 2}{9}$

$= -\frac{2}{9}$

$11)2^{100}($

$\frac{1}{}$

$9)2^{100}($

$\frac{7}{}$

$\Rightarrow$ remainder = 7

~~NOTE~~ $N = 11 \times a + 1$

$N = 9 \times b + 7$

3rd type

general form = $99 \times c + 34$

Q If eggs are removed from a basket two, three, four, five & six at a time, there remain resp one, two, three, four, & five eggs. But if eggs are removed seven at a time, no eggs remain what is the least no of eggs, that could have been in basket?

sol:-

Let 'n' be the least no. of eggs

$n \div 2 = 1$ (rem)	:	$n = (2 \times a) + 1$
$n \div 3 = 2$		$n = (3 \times b) + 2$
$n \div 4 = 3$		$n = (4 \times c) + 3$
$n \div 5 = 4$		$n = (5 \times d) + 4$
$n \div 6 = 5$		$n = (6 \times e) + 5$
$n \div 7 = 0$		

gen. form of = L.C.M (2, 3, 4, 5, 6) − 1

$N = 60 \times y - 1$

$y = 1 \Rightarrow N = 59$ ✗

$y = 2 \Rightarrow 119$ ✓

$\Rightarrow$ '119' eggs

Profit loss and Discount

(Cost Price) (C.P) → Selling price (S.P)

₹ 400 → (500)

(buying price)

Profit (P) = SP − CP = 100

$$\text{Profit \%} = \frac{P}{C.P} \times 100 = \frac{100}{400} \times 100 = 25\%$$

$$\frac{SP}{C.P} = \frac{500}{400} \Rightarrow S.P = \frac{5}{4}(C.P)$$

SP = 25% more of C.P

C.P

800 → 12.5% profit ⇒ S.P = ?

$$800 \times \frac{9}{8} = 900$$

Q

Profit % = 16.66% ; SP = 700 ; C.P = ?

Sol:-

$$S.P = C.P + \frac{C.P}{6}$$

$$S.P = C.P\left(1 + \frac{1}{6}\right)$$

$$\Rightarrow C.P = \frac{6}{7} S.P$$

$$= \frac{6}{7} \times 700^{100}$$

C.P = 600

Base fractions :

$\frac{1}{2}$ = 50%.

$\frac{1}{3}$ = 33.33%.

$\frac{1}{5}$ = 20%.

$\frac{1}{7}$ = 14.28%.

$\frac{1}{11}$ = 9.09%.

$\frac{1}{13}$ = 7.69%.

$\frac{1}{17}$ = 5.88%.

$\frac{1}{19}$ = 5.26%.

→

* Profit or loss % are always in terms of C.P

Q

C.P → S.P

300 → 200 (Loss) 33.33% loss

~~SP = CP(1-i)~~

$= SP = CP - \frac{CP}{3}$

$= CP\left(\frac{2}{3}\right)$

$= 300 \times \frac{2}{3}$

SP = 200

Margin % :-

The relationship between profit & selling price is **Margin %**.

400 → 500

P = 100

$\frac{P}{S.P} = \frac{100}{500} = \frac{1}{5} = 20\%$ - Margin %.

Profit is 20% of S.P ⇒ Margin %.

Q)

P = 10% ; Margin%.

P% = $\frac{P}{CP} = \frac{1x}{10x} =$ Margin% $= \frac{P}{SP} = \frac{1}{(P+CP)} = \frac{1}{11}$

$= 9.09\%$

$\Rightarrow$ Margin% $= \frac{1}{11} = 9.09\%$

Ex:- Profit% $\longrightarrow$ Margin%.

Margin $= \frac{P}{(CP+P)} \Rightarrow$ P% $\longrightarrow$ Margin%

25% $\longrightarrow$ 20%

4 $\longrightarrow$ 5

$= \frac{4}{5}$

Q)

An article sold at a certain price resulted in a loss of 7%, whereas when it was sold at ₹100 more, it resulted in a 13% profit find the cost price of the article.

Sol:-

20% = 100

100% = 500 $\Rightarrow$ C.P

(or)

$S.P = \frac{93}{100} CP$; $SP' = S.P + 100$

$S.P' = \frac{113}{100} CP$

$\Rightarrow S.P + 100 = \frac{113}{100} C.P \Rightarrow \frac{93}{100} CP + 100 = \frac{113}{100} C.P$

$\Rightarrow$ C.P = 500

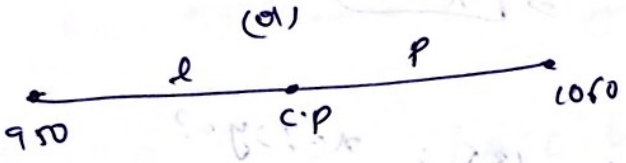
Q. Profit earned by selling an article at 1060 is 20% more than the loss incurred by the selling the article for Rs 950. Find the C.P of the article.

Sol:

$$1060 - CP = \frac{6}{5}(CP - 950)$$

$\Rightarrow$ C.P = 1000

(or)



$$P = \frac{6}{5}l$$

$\Rightarrow P + l = 110 \quad \Rightarrow \quad \frac{6}{5}l + l = 110$

$\Rightarrow l = 50 \quad \Rightarrow$ C.P = 950 + 50 = 1000

$./x \times 100 = ?$

Ratios

Ratio is a relationship & relationship between numbers ⇒ quantitative Relationship

→ Fractions

Ex:- Boys 30, Girls x

$\frac{B}{G} = \frac{30}{20} = \frac{3}{2} = \frac{3:2}{B:G}$

B : G

3x : 2x

75 : 50

if $3x = 75$

$\Rightarrow \boxed{x = 25}$

Q $x : y$

3 : 5 & $xy = 135$; $x = ? ; y = ?$

$\frac{3z}{5z} \Rightarrow (3z)(5z) = 135 \Rightarrow 15z^2 = 135$

$z^2 = 9$

$z = \pm 3$

$\Rightarrow 3z = 9 ; 5z = 15$

Q The ratio of marks obtained by Ram & Shyam is 5:7. If five times the marks obtained by Ram is 12 more than thrice the marks obtained by Shyam. Find their marks.

Sol:-

R : S

5 : 7

5x : 7x

$5 \times 5x = 3 \times 7x + 12$

$25x = 21x + 12 \Rightarrow \boxed{x = 3}$

R : S

15 : 21

Q If the ages of a husband, wife + child in the ratio 13 : 11 : 3 & avg age of family = 36 ages?

Sol:- $h : w : c$

$13 : 11 : 3$

$13x : 11x : 3x$

$$\frac{13x + 11x + 3x}{3} = 36 \Rightarrow 27x = 3 \times 36$$

$$\boxed{x = 4}$$

$\Rightarrow h : w : c$

$52 : 44 : 12$

Q $A : B : C$ | $A + B + C = 200000$

$2x : 3x : 5x$

A's share : $\frac{2x}{10x} \times 20000$

Q $A : B : C$

$\frac{1}{4} : \frac{1}{3} : \frac{1}{5}$ | $\left(\frac{1}{4} : \frac{1}{3} : \frac{1}{5}\right) \times 60$

$= 15 : 20 : 12$

$A : B$ | $B : C$

$3 : 4$ | $5 : 3$

$A : B$

$(3 : 4) \times 5$

$B : C$

$(5 : 3) \times 4$

$15 : 20 : 12$

Q $A : B : C$

$2A = 3B = 5C$

$2A = 3B$

$\frac{A}{B} = \frac{3}{2}$; $\frac{B}{C} = \frac{5}{3}$

$A : B$

$3 : 2$

$B : C$

$5 : 5$

$15 : 10 : 6$

(or)

$\frac{A}{4} : \frac{B}{3} : \frac{C}{5} = K$

$A = 4K \; ; \; B = 3K \; ; \; C = 5K$

$4K : 3K : 5K$

$\Rightarrow 4 : 3 : 5$

Q. The ratio of the present ages of a father and son is 8:1. Eight years hence, the ratio would be 10:3. present age of father?

Sol:- f : Son — after 8 yes,

8 : 1 — $\frac{8x+8}{x+8} = \frac{10}{3}$

8x : x

$\Rightarrow 24x + 24 = 10x + 80$

$14x = 56$

$x = 4$

$\Rightarrow$ Father's age = 32 ; son's age = 4 at present

Ex:-

Milk : water — Total : 50 lit

3 : 2

$3x + 2x = 50$

M : T ; W : T | M : T

$\frac{M}{T} = \frac{3x}{5x}$

$\Rightarrow \frac{M}{T} = \frac{3x}{5x}$

$\Rightarrow \frac{M}{50} = \frac{3x}{5x}$ $\Rightarrow M = \frac{3x}{5x} \times 50^{10}$

$M = 30$ lit

20

Milk : Water

3 : 2 ; 28 lit water.

$\frac{Wa}{To} = \frac{2x}{5x} \Rightarrow \frac{28}{To} = \frac{2}{5}$ $\therefore To = \frac{28 \times 5}{2}^{14}$

Total = 70 lit

Milk = 42 lit

Concept

A : B : C Total = 2,00,000

2 : 3 : 5

figure out new relations. like

A+B & C ; A & Total, B & Total, C & total, etc

8 The number of 1Re, 50 paise & 25 paise coins in a bag is in the ratio 3:4:5. If the bag has ₹ 300, find the no of 50 paise coins?

Sol:-

Re 1 : R 0.50 : R 0.25

8 : 4 : 1

no. of coins 3x : 4x : 5x

value :- (3x coins × 1 Re/coin) : (4x coins × 0.5 Re/coin) : (5x coins × 0.25 Re/coin)

3x : 2x : $\frac{5x}{4}$

$3x + 2x + \frac{5x}{4} = 300$

$5x + \frac{5x}{4} = 300 \Rightarrow \frac{25x}{4} = 300^{12}$

$\Rightarrow \boxed{x = 48}$

no of Re 1 coins = 3 × 48 = 144 ; no of 0.50 p coins = 192

no of 0.25 p coins = 240

(OR)

add the sum of rupees

$\Rightarrow$ ₹ 3 + ₹ 2 + ₹ 1.5 = ₹ 6.25

If 6.25 → 4 (0.5 p coins)

300 → ?

no. of coins = $\frac{300 \times 4}{6.25} = \frac{1200 \times 4}{625 \times 25} = 240$ coins

Variations

Direct Variation:-

Two variables, x and y are said to vary directly if when one of them increases (or decreases), the other also increases (or decreases) proportionally

The word "proportionally" is important here. If one of them doubles, the other should also double, if one of them becomes one-third, the other should also become one-third.

= Concept

$x \propto y$

(proportionally means in the same ratio)

$$\frac{x}{y} = k$$

Mathematically, if the ratio is constant for all instances of (x, y), then 'x' and 'y' are vary directly, and it is represented as $x \propto y$

$x \propto y^2$

$\Rightarrow \frac{x}{y^2} = k$

Q The value of the diamond varies directly as the square of its weight. If a diamond weighing 1.5 gms has a value of ₹ 9000, what will be the value of a diamond weighing 2.5 gms?

Sol: $V \propto W^2$

$\frac{V_1}{V_2} = \frac{W_1^2}{W_2^2} = \frac{9000}{V_2} = \left(\frac{1.5}{2.5}\right)^2$

$V_2 = \frac{25}{9} \times 9000 = 25,000$

Q The volume of a sphere is directly proportional to the cube of the its radius. If the volume of two sphere are in the ratio 8:1, find the ratio of the radii of the spheres?

$V \propto r^3$

Sol: $\frac{V_1}{V_2} = \frac{r_1^3}{r_2^3}$

$= \sqrt[3]{\frac{8}{1}} = \frac{r_1}{r_2}$

$= 2:1 \Rightarrow r_1 : r_2 = 2:1$

Inverse Variation :-

Two variables x & y, are said to vary inversely if when of them increases (or decreases), the other decreases (or increases) proportionally.

Thus, if one of them doubles, the other should halve; if one of them becomes one-third, the other should become thrice.

$$x \propto \frac{1}{y}$$

$$x y = k$$

$$x_1 y_1 = x_2 y_2$$

$$\frac{x_1}{x_2} = \frac{y_2}{y_1}$$

Q. The cost per kg rice varies inversely with the square of the quantity of rice produced in a year. When 7 million tons of rice was produced, its cost was Rs 36/kg. How much was the production in the year when the cost of rice was ₹ 46/kg?

Sol:- $C \propto \frac{1}{P^2}$

$\frac{C_1}{C_2} = \frac{P_2^2}{P_1^2}$ $\Rightarrow$ $\frac{36}{49} = \frac{P_2^2}{7^2}$

$P_2^2 = \frac{36}{49} \times 49 = 36$ tonnes

$P_2 = 6$ m tons

~~$\Rightarrow P_2 = 5$ tons~~

Joint Variation:-

There would be many situations in which more than two variables would be involved whereas the direct and inverse relations can be established only between pairs of variables (with others being constant).

more ex:

Cost $\propto$ (no. of persons) (price per ticket)

$C \propto np$

Q: 'x' varies directly with 'y' & 'z'; when $x=2$, $y=3$, $z=5$, the value of $x=?$ when $y=9$, $z=15$.

Sol:- $x \propto yz \Rightarrow \frac{x}{yz} = k$

$\frac{x_1}{y_1 z_1} = \frac{x_2}{y_2 z_2}$

$$\Rightarrow \frac{2}{3\times 8} = \frac{x}{\frac{9}{3}\times 4} \Rightarrow x = 6$$

Q The value of a bar weighing 400 gm, varies inversely with the square root of the fraction of impurities $\left(\frac{\text{Impurities}}{\text{Total weight}}\right)$ in it. If the value of a bar containing 25 gm of impurities is ₹ 45,000, how many gms of pure gold is there in a bar costing ₹ 90,000?

Sol:

$$V \propto \frac{1}{\sqrt{\frac{I}{T}}} \quad (\text{or}) \quad V^2 \propto \frac{1}{\frac{I}{T}}$$

T → unchanged

$$\Rightarrow \frac{V_1}{V_2} = \frac{\sqrt{\frac{I_2}{T_2}}}{\sqrt{\frac{I_1}{T_1}}} \Rightarrow \frac{V_1^2}{V_2^2} = \frac{I_2}{I_1}$$

$$\frac{(45000)^2}{(90,000)^2} \times 25 = I_2$$

$$\frac{25}{4} = I_2 \Rightarrow I_2 = 6.25$$

Q The value of a silver coin varies directly as the square of its diameter, when thickness is constant and varies directly as its thickness when diameter remains constant. Two silver coins have the diameters in the

ratio 4:3. Find the ratio of the thickness if the value of the first coin is four times the value of second coin.

Sol:

$V \propto d^2 \quad ; d = k$

$V \propto t \quad ; d = k$

$V \propto d^2 t$

$\frac{V}{d^2 t} = k \Rightarrow \frac{V_1}{d_1^2 t_1} = \frac{V_2}{d_2^2 t_2}$

→ Problems on joint variations ⇒ problems on chain rule.

Q 6 labourers can build a wall in 10 days. How many labourers of double the efficiency are needed to build 3 such walls in 5 days?

Sol:

$M T \propto W$

$\frac{M_1 T_1}{W_1} = \frac{M_2 T_2}{W_2} \Rightarrow \frac{6 \times 10}{1} = \frac{x \times 5}{3}$

$\Rightarrow$ No. of men, $x = 18$

Q. Observe the Variation b/w Variably

A garrison has enough foode to sustain its 40 men for 6 months. If each man consumes 900 gms food/day. A reinforcement of 20 men joins the garrison. For how long will the food last now if each man consumes only 600 gm/day.

Sol:- $Men \propto \frac{1}{ration}$; time = k

$Men \propto \frac{1}{time}$; Ration = k

$\rightarrow M_1 R_1 T_1 = M_2 R_2 T_2$

$\rightarrow 40 \times 900 \times 6 = 60 \times 600 \times t_2$

$\rightarrow t_2 = 6 \times \frac{2}{3} \times \frac{3}{2}$

$t_2 = 6$ months

Q. A garrison has sufficient food for 75 soldiers for a period of 90 days. After 10 days, 1/3rd of the soldiers leave. After another 10 days 5 sold. return. From this days on, how many days will the food last?

Sol:-

75 soldiers → 90 days

↓ 60 days

75 sold → 80 days

↓ 1/3 (25) leave

50 soldiers → 120 days

↓ 10 days

50 sold → 110 days

↓ +5 sold

55 sold → 100 days

⇒ 55 soldiers – 100 days

Q- Four inlet pipes with circular cross section, take 9 hrs to fill a cistern. How many pipes of half the radius are req. to fill the cistern in 6 hrs, if the speed of water now is three times the speed of water in previous case?

Sol:-

$$\frac{4 \times 9}{1} = \frac{x \times 6^2 \times 3}{1} \times \frac{1}{4}$$

$\left[\because \text{Area} = \pi r^2 \right.$
$\left. \pi \left(\frac{r}{2}\right)^2 = \pi \frac{r^2}{4} \right]$

$\therefore x = 8$ pipes

Inverse & Direct Relation analysis

Concept

no. of people		cost of picnic
100	——	3000
×2 → 200	——	5000 (×2)

→ not proportional increase

Total Cost : fixed cost + variable cost

$$y = K_1 + K_2 x$$

↓ direct relation, if fixed cost is involved

$$y = \frac{K_1}{x} + K_2$$

↓ Inverse Relation, if fixed cost is involved

↓ sharing of fixed cost

Ex: Bus, Picnic, food example

picnic cost = bus cost + food

Direct

p.c = bus. cost (fixed cost) + (food) × no. of ppl

Indirect:

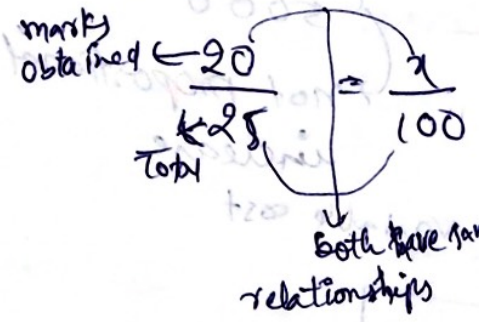
$$p.c = \frac{\text{bus. cost}}{\text{no. of ppl}} + \frac{(\text{food}) \times \text{no. of ppl}}{\text{no. of ppl}}$$

Ex:

no. of ppl	cost per head	share : fixed cost 60	+ lunch
20	80	40	20
30 (3:5)	60	24 (5:3)	20
50	?		20

Percentages

"%" is a relation between two numbers



A	B
30	40

→ 'A' is what % of 'B'?
$= \frac{30}{40} \times 100 = 75\%$

→ 'B' is what % of 'A'
$\frac{40}{30} \times 100 = 133.32\%$

Q '40' is what % of 80

Q P is increased by 10%. Q 'Z' is decreased by 20%

$P = P + \frac{10P}{100}$

$= 1.1P$

$Z = Z - \frac{20}{100} Z$

$= 0.8Z$

Q x is decreased by 16.66%.

$x - \frac{x}{6} = \frac{5}{6}x$

Q A is 75% of B; B is what % of A

$A = \frac{3}{4}B$

$B = \frac{4}{3}A$

→ B is 134.64 % of A

Q "x" is 10% more of y (or) x is 110% of y

$$x = \frac{11}{10} y$$

Q P is ↑ by 25%

$P' = P * \frac{5}{4}$

Q $A = \frac{8}{7} B$

A is B + 14.28% of B

(or)

A is 14.28% more than B

(or)

A is 114.28% of B

Q Rectangle; l = ↑ 25%; b ↑ 25%

% change in area

Sol:- % change = $\frac{\text{New area}}{\text{old area}} = \frac{\frac{5}{4} l \times \frac{5}{4} b}{lb} = \frac{25}{16}$

$A' = A + \frac{9}{16} A$

⇒ A' ↑ by 56.25%

Q The prices of chocolates dropped by 15%. and thus Rashi was able to save ₹1.35 on each chocolate. Find the new price of one chocolate?

Sol:-

15% = ₹1.35

0.15 = 1.35

1 = x

$\Rightarrow \frac{1.35}{0.15} = 9$

new price = 9 - 1.35 = ₹7.65

Q A man mixes milk & water in the ratio 5:3. Find the percentages of milk in the resulting mixture?

Sol:-

$= 5/8 \times 100$

Q To pass an exam of 250 marks, a candidate should obtain 40% marks. Raj scores 10% more than the passing marks. Find the % of marks scored by Raj?

Sol:-

40% of $(0.4) \times 250 = 100$

10% more than passing $= (0.1)100 + 100$

$= 110$

Raj marks obtained $= 110$

% of marks $= \frac{110}{250} =$ 44% of total

Q A's salary is 80% of B's salary, whereas B's sal is 80% 'C' salary. what % of A's sal is C's salary?

Sol:-

$A = \frac{4}{5}B$

$B = \frac{4}{5}C$

$A = \frac{4}{5}\left(\frac{4}{5}C\right) = \frac{16}{25}C$

~~A = 68% of C~~ $\quad C = \frac{25}{16} A$

$C = A + \frac{9}{16} A$

Successive Percentages formula :-

1) +10% 2) +20% 3) -16.66%

$= \left(\left(P \times \frac{11}{10}\right) \times \frac{6}{5}\right) \times \frac{5}{6}$

$= (1.1) P$

$P \times \frac{11}{10}$

2) +25% −20% −12.5% +14.28%

A → B → C → D → E

$P \quad P \times \frac{5}{4} \quad \left(P \times \frac{5}{4}\right) \times \frac{4}{5} \quad \left(P \times \frac{7}{8}\right) = P$

$P = \frac{8}{7} P$

A, E prices are same

Successive Increments formula

if a%, b%, are successive increments, then final change in % will be

$$a + b + \frac{ab}{100} \text{ - resultant}$$

→ only applicable to '2' no's

for discounts $\left(x + y - \frac{xy}{100}\right)$

Ex:- 250 ; a = 10% ; b = 20%

$10 + 20 + \frac{200}{100} = 32\%$ ⇒ ∴ final price change = 32%

$\frac{5}{250} \times \frac{11}{10} \times \frac{6}{5}$ $= 5 \times 66 = 330$

Q. successive changes
$l = 20\% \uparrow$; $b = 20\% \downarrow$; change in area?

$A \propto l\,b$

formula

$= a + b + \frac{ab}{100}$

$= 20 - 20 - \frac{20}{100}$

$= -4\%$.

∴ Area decreasing by 4%.

$\Rightarrow \left(16 \cdot \frac{6}{5}\right)\frac{4}{5} = \frac{24}{25}\,16 = \left(1 - \frac{1}{25}\right)16$

∴ decrease in 4%.

Q. Travelling from home to office, if I increase my speed by 37.5% of my usual speed, what percent of time would I save?

Sol: $S\,T \propto D$

$S_1 T_1 = S_2 T_2$

$S_1 T_1 = \left(1 + \frac{3}{8}\right) S_1 T_2$

$T_1 = \frac{11}{8} T_2$

$T_2 = \frac{8}{11} T_1 = T_2 = \left(1 - \frac{3}{11}\right) T_1$

$\Rightarrow \quad 9.09 \times 3$

$= 27.27\%$ decrease in time

Q. While preparing the price tag, instead of increasing the price by 12.5%, a shopkeeper absent mindedly decreased the prices by 12.5%. Find the % diff in the price on the tag compared to what should have been the price tag?

sol:

$$\frac{\text{what should be}}{\text{what is there}} = \frac{9}{8} \Big/ \frac{7}{7}$$

$$= \frac{7}{9}(WSB) = WIST$$

$$\Rightarrow \frac{7}{9} \times 100 = 22.22\%$$

Q. Since the price of mangoes increased by 6.66%, the wholeseller was able to purchase 18 mangoes less in his budget of ₹864. what is the increased price of each mangoes?

sol:

$$xp = 864$$

$$(x-18)\frac{16}{15}p = 864$$

$$\frac{x}{(x-18)\frac{16}{15}} = 1 \Rightarrow 15x = (x-18)16$$

$$\Rightarrow 15x = 16x - 16 \times 18$$

$$\Rightarrow x = 16 \times 18$$

$$= 288$$

$$p' = \left(\frac{864}{270}\right) \times \frac{16}{15}$$ (old price)

Q. A large watermelon 20 kg with 96% of its weight being water. It is allowed to stand in the sun for a while and some of the water evaporates so that now only 95% of its water

It's reduced weight will be?

Sol:

concentrate on pulp	post sun
96% of water ⇒ 4% of pulp	95% of water 5% of pulp

$$\rightarrow (0.04)(20) = (0.05)y$$

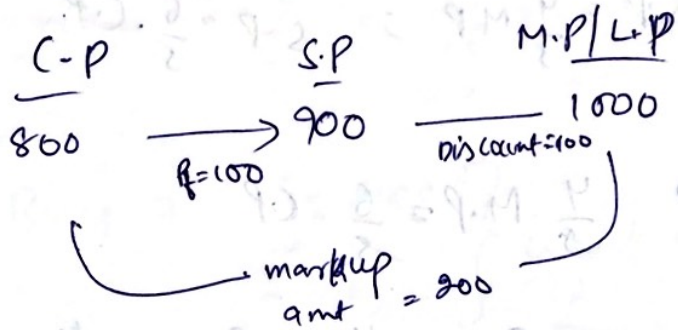
$$\frac{0.8}{5\times10^{-2}} = \frac{8\times10^{-1}}{5\times10^{-2}} = \frac{80}{5} = y$$

$$\boxed{y = 16\ kg}$$

new weight = 16kg

Profit & Loss

→ Mark price (List price)



P.% = $\frac{100}{800}$ = 12.5 ; discount % = $\frac{100}{1000}$ = 10%

↓ connects discount & M.P

S.P = $\frac{9}{8}$ C.P ; S.P = $\frac{9}{10}$ MP

Markup %:- connects C.P & M.P

Markup % = $\frac{200}{800}$ = $\frac{1}{4}$ = 25%

→ cost price

∴ $M.P = \frac{5}{4} C.P$

markup % works as cost price

Ex:- Mup % = 20% ; d = 10% ; P.% = ?

$\left(\left(C.P \times \frac{6}{5}\right) \times \frac{9}{10}\right)$

↓ M.P

S.P

$= \frac{54}{50} C.P$

$= \left(1 + \frac{4^2}{50}\right) C.P$

$= 8\%$ P%

Q:- C.P, M.P = 300 ; C.P = ?

d.% = 20% ; P = 20%.

Sol:- $S.P = \frac{4}{5} MP$; $S.P = \frac{6}{5} . C.P$

$\Rightarrow \frac{4}{5} . M.P = \frac{6}{5} . C.P$

$\Rightarrow C.P = \frac{4}{6} M.P = \frac{4}{6} \times 300^{50}$

$\boxed{C.P = 200}$

Q

$m_{up}\% = 25\%$; $P = 10\%$; $d\% = ?$

$M.P = \frac{5}{4} C.P$

$S.P = \frac{11}{10} C.P$

$\frac{4}{5} M.P = \frac{10}{11} SP \Rightarrow S.P = \frac{44}{50} M.P$

$\Rightarrow S.P = \left(1 - \frac{6}{50}\right) M.P$

$\Rightarrow$ 12% of d%.

Concept:- Buy 5 get 1 Free

5 items → 1 item free

M.P = 6x (for 6 shirts)

= 5x (for 5 shirts)

* 1 shirt free

- M.P = 6x ; discount = x

$\Rightarrow$ d% = $\frac{x}{6x} = \frac{1}{6} = 16.66\%$

Q

Buy 7 · 2 free
(or)
d = 20%.

} which one to prefer

$\frac{2x}{9x} = \frac{66.66}{3}$ = 22.22%

$\Rightarrow$ d = 20% is not preferable

Concept:

20% extra

↓

1/5

↓

Buy 5 ⟶ get 1 free

↓

$\left(\frac{1}{6}\right) \Rightarrow$ 16.66% discount

$\Rightarrow$ 20% extra $\Rightarrow$ 16.66% discount

Ex:-

14.28% extra

= 1/7

Buy 7 ⟶ get 1

1/8 $\Rightarrow$ 12.5% discount

Q. A trader Mup = 50%. ⇒ 25% extra ; P% = ?

M.P = $\frac{3}{2}$ C.P

25% extra ⇒ $\frac{1}{4}$ ⇒ Buy 4 → get 1

= $\frac{1}{5}$ = 20% discount

S.P = $\frac{4}{5}$ M.P ⇒ M.P ⇒ $\frac{5}{4}$ S.P

= $\frac{3}{2}$ · C.P = $\frac{5}{4}$ S.P

$\boxed{S.P = \frac{12}{10} C.P}$ ⇒ S.P = $\left(1+\frac{1}{5}\right)^{\frac{6}{5}}$ C.P

⇒ 20% profit

Cheating and Error and Dilution

Application

Cheating

Actual weight - 800 gms
shown wt = 1000 gms
} mark up — CP = 800, M.P = 1000

↓

Mup% = $\frac{200}{800}$ = 25%

If act wt - 1000
shown wt - 900
} 10% discount

Q act. wt 800 gms → 10% ⇒ discount ∴ P% = ?
shown wt 1000 gms

Mup% = 25%

M.P = 5/4 C.P

S.P = 9/10 M.P

5/4 C.P = 10/9 · S.P

S.P = 45/40 C.P

= (1 + 1/8) CP

= 12.5% profit

(or)

C.P × 5/4 × 9/10 = S.P

Q showing = 150 gms

d.r. = 20% ; P% = 20%

S.P = 6/5 C.P ; S.P = 4/5 MP

6/5 C.P = 4/5 MP ⇒ M.P = 30/20 C.P

Mup% = 50%

⇒ actual wt = 150 = 3/2 C.P

⇒ C.P = 2/3 × 150 = 100 gms

Mixing Impurities

5 lit milk + 1 lit water ⇒ 6

mark up

$\frac{1}{5} = 20\%$

Another Concept :-

impurities = 20% of Total

$imp = \frac{1}{5} * T$

Pure + Imp = T

4 + 1 = 5

Mup % of total = 5/4

Ex :-

Mup % = 10 of total ; d = 10% ; P % = ?

B P + I = T

9 + 1 = 10

(9) (1) (1)

⇒ Markup = $\frac{10}{9}$

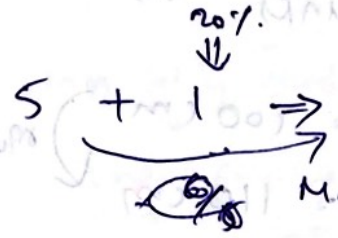
⇒ CP × $\frac{10}{9}$ × $\frac{9}{10}$ = SP

⇒ no loss or profit

Another keyword:-

20% extra ⇒ 20% of pure form is added as impurity to pure form

Ex:-



20% of extra ≠ 20% of total

Q A trader gives 20% extra & d% = 10%... & makes P% = 35%. what is CP of 400 gms is kept on machine?

Sol:-

20% extra → one markup

(weighing machine) → second markup

$$C.P \times \frac{5}{5} \times \frac{a}{b} \times \frac{9}{10} = \frac{27}{20} SP$$

$$\frac{a}{b} = \frac{5}{4}$$

∴ markup provided by wt machine = 5/4

⇒ 25%.

= 12.5%.

$$400 \times \frac{5}{4} = 500 \text{ gms}$$

(Meter Scale)

Shrinkage & Expansion

→ Meter Scale → expands in Summer
shrinks in winter

Actual length = 100 cm
(summer) = 110 cm
(winter) = 90 cm

markup provided ⇒ Markup = 10 ⇒ discount

Ex :-

20% exp ⇒ Markup

100 → 120

120 ⇒ $(1-\frac{1}{6})$

⇒ $\frac{5}{6}$%

$\frac{5}{6}$% discount ⇒ 16.66%

winter

80 → 100

$x \Rightarrow \frac{20}{80} = 17\frac{1}{4} = \frac{5}{4}$% markup

Averages & Weighted Averages

$$\text{Avg} = \frac{\text{sum of samples}}{\text{no. of samples}}$$

Ex: Avg wt of a class of '9' students = 80 kg
(new person of 90 kg enters → new wt.?

$80 \times 9 = 720$

$$A' = \frac{720+90}{10} = \frac{810}{10} = 81$$

Divide Equally Approach:

9P → 80
1P → 90

avg (9P) - 80 — 10 extra
↓
divide 10 ⇒ equally to all 10 persons

⇒ new avg = 81

Q
60, 70, 55, 40, 30 ~~80~~

Divide equally approach

for avg (60)

60 70 55 40 30 ~~80~~
+10 +5 −15 −45

so - deficit = −45 ⇒ divide it equally for all

$$\frac{-45}{5} = -9$$

⇒ 60 − 9 = 51
↓
Avg wt

(55) 60, 70, 55, 40, 30

+5 +20 +5 −20

$\frac{-20}{5} = -4 \Rightarrow 55 - 4 = 51$

Q

n = 8 | new person

avg = 56 kg | wt = 32 kg

56 kg

$\Rightarrow$ 32 + 24

8

Consider new person comes at avg = 56

$\Rightarrow$ 56 (32, −24, 8)

$8 \quad -\frac{24}{9}$

$= 56 - \frac{24}{9}$

Q.

n = 10 ; 1 person left

Avg = 70 ; Avg' = 72

n = 9 70 → new person 70

⇓ but

72 $\Rightarrow$ 2 kg extra $\Rightarrow$ 18 kg extra

$\Rightarrow$ new person. $\frac{1}{2}$ 70 − 18 = 52 kg

Q $n = 11$ ↙ 96 kg

Avg = ?

↓

Avg' = Avg + 3 $\Rightarrow 12 \times 3 = 36$ kg added

$\Rightarrow$ Avg' = $(96 - 36) + 3$

$= 60 + 3 = 63$

Q $n_1 = 4$ ⊕ $n_2 = 6$ $\Rightarrow$ Anew = ?

$A_1 = 40$ kg $Avg_2 = 50$

$$Anew = \frac{A_1 \times n_1 + A_2 \times n_2}{n_1 + n_2} = \frac{40 \times 4 + 50 \times 6}{10}$$

$= 46$

Q Divide equally approach (extra)

$n_1 = 4$ ⊕ $n_2 = 6$

$A_1 = 40$ $A_2 = 50$

A = 40 ← 50

Sol: Consider '40' as Avg and bring '50' as '40' and 6 people each carrying 10 kg extra brings 60 kg extra. and divide 60 kg equally to '10'

$\Rightarrow \frac{60}{10} = 6$

$\Rightarrow$ 40 + 6 = 46 kg

(OR)

A = 50 40 → 50

$-10 \times 4 = \frac{-40}{10} = -4$

$\Rightarrow 50 - 4 = 46$ kg

$$\text{Average} = \frac{\text{Sum of all samples}}{\text{no. of samples}}$$

Q. 12 kgs of rice costing ₹10/kg is mixed with 48 kgs of another variety of rice costing ₹20/kg what is the average cost/kg of mixture.

Sol:

$$\text{Avg cost} = \frac{\text{Total cost}}{\text{Total quantity}} = \frac{12\times10+48\times20}{60}$$

$$= 18$$

(or)

12 — 48

10 — 20

$-10\times12 = -\frac{120}{60} = -2$

$\Rightarrow 20-2 = ₹18$

Q.

10 kg — 20 kg

A — ₹20/kg — (+) — ₹35/kg

$-15\times10 = -\frac{150}{30} = -5$

$\Rightarrow 35-5 = ₹30/kg$

Rice example

Concept:-

$$\text{Average} = \frac{R_1 w_1 + R_2 w_2}{w_1 + w_2}$$

(R_1: cost, w_1: wt)

$$A = \frac{R_1 w_1 + R_2 w_2}{w_1 + w_2}$$

$$\Rightarrow \frac{w_1}{w_2} = \frac{R_2 - A}{A - R_1}$$

Weighted Averages:-

$n_1 = 5$ $\quad n_2 = 6$

$A_1 = 40$ $\quad A_2 = 56$

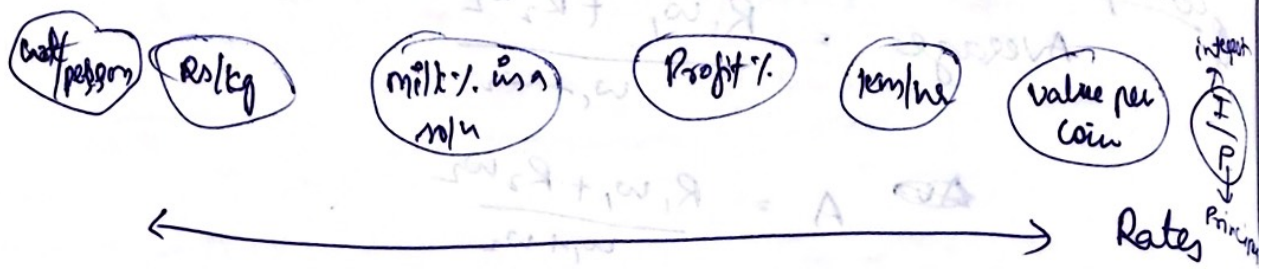
$$A_m = \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2}$$

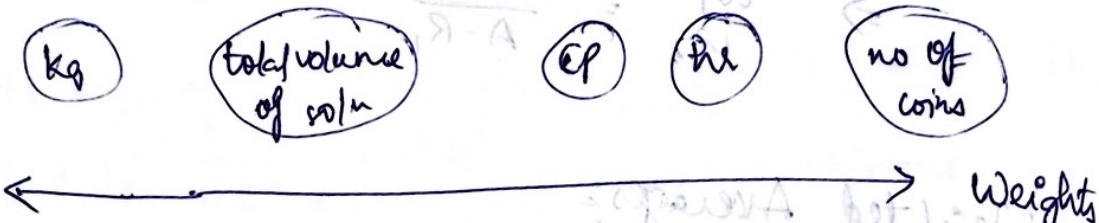
$$n_1 A_m + n_2 A_m = n_1 A_1 + n_2 A_2$$

$$\Rightarrow n_1 (A_m - A_1) = n_2 (A_2 - A_m)$$

$$\frac{n_1}{n_2} = \frac{(A_2 - A_m)}{(A_m - A_1)}$$

				1 kg	4 kg
10 Rs/kg	8 kg	2 kg	2 kg	18	12
A (₹)	15	18	15		
			2 kg	4 kg	1 kg
20 Rs/kg	8 kg	8 kg			





$$A_m = \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2}$$

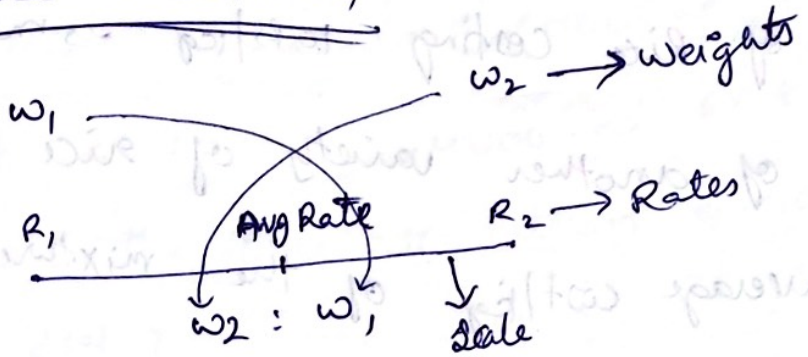
$$R_m = \frac{w_1 R_1 + w_2 R_2}{w_1 + w_2}$$

(weight, Rate)

$$\frac{w_1}{w_2} = \frac{R_2 - R_m}{R_m - R_1}$$

Rate = numerator / denominator → weight

Scale Method

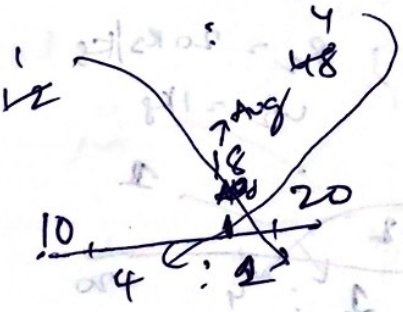


1. Write the rates on the line / scale
2. Write the weights corresponding to each Rate above the rates
3. Find the ratio of w_1, w_2 reverse the ratio write it under the line
4. Divide the scale into ratio written under the line
5. The Division point on the scale is the average rate (or) resultant rate.

$$\frac{w_1}{w_2} = \frac{R_2 - Avg}{Avg - R_1}$$

Q. 12 kg of Rice costing 10 Rs/kg is mixed with 48 kgs of another variety of rice costing 20 Rs/kg what is the average cost per kg of the mixture

Sol:-

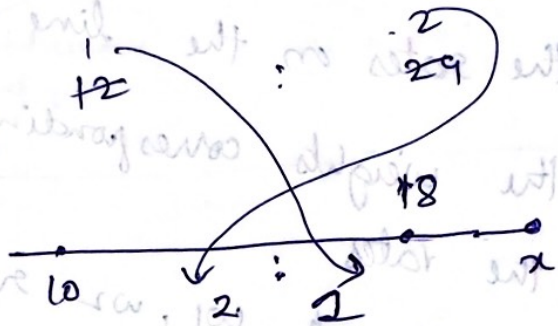


$10 \times \frac{2}{5} + 4/5$

$= 8$

Q. 12 kg of Rice costing 10 Rs/kg is mixed with 24 kg of another variety of rice costing x Rs/kg. The average cost/kg of the mixture is 18 Rs/kg.

Sol:-



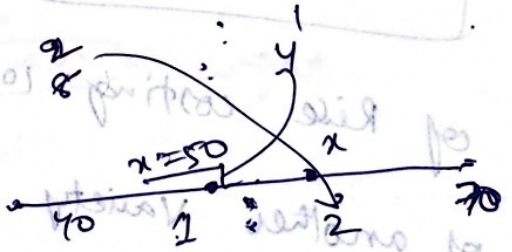
$\frac{2}{3} \times (x-10) = 18 \Rightarrow x = 27$

Q. $\frac{2}{3} \times (x-10) = 18 \quad 2x - 20 = 57$

$\Rightarrow 2x = 77 \Rightarrow x = 38.5$

Q. $w_1 = 8 \quad ; w_2 = 4$

$R_1 = 40 \quad ; R_2 = 70$

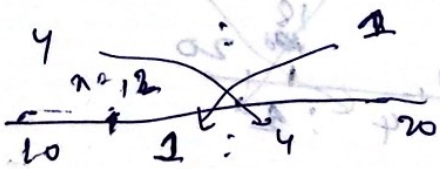


$x = 40 + 10 = 50$

$\frac{1}{3} \times 30 = 10 \Rightarrow$

Q. $R_1 = 10$ Rs/kg $\quad ; R_2 = 20$ Rs/kg

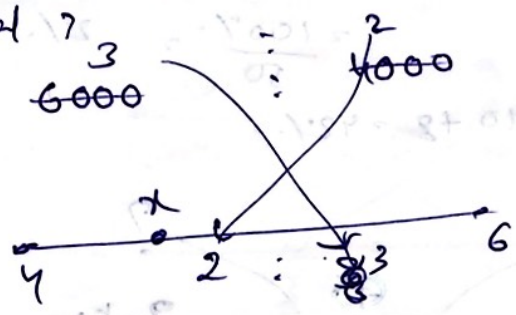
$w_1 = 4$ kg $\quad w_2 = 1$ kg



$\frac{1}{5} \times 10 = 2$

$\Rightarrow$ Ans $= 10 + 2 = 12$

Q A man borrows ₹ 6000 at 4% simple interest & another 4000 at 6% simple interest what is the over all interest ratio for the whole deal?



$$\frac{2}{5} \times (2) = \frac{4}{5} = 0.8$$

$$\Rightarrow x = 4 + 0.8 = 4.8$$

Divide equally approach,

$$\frac{6000}{4000} = \frac{3}{2}$$ (no doesn't matter ratio matters)

⇒ 3 ppl & 2 ppl

4% 6%

+(2×2)%

= 4%

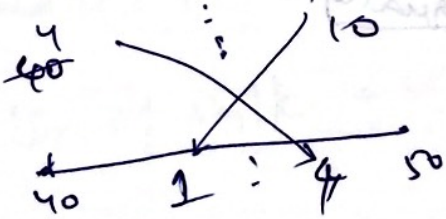
Avg → (4%)

$$\Rightarrow \text{divide '4' among's} = \frac{4}{5} = 0.8\%$$

$$\Rightarrow \underline{4.8\%}$$

$S_1 = 40\%$; $S_2 = 50\%$

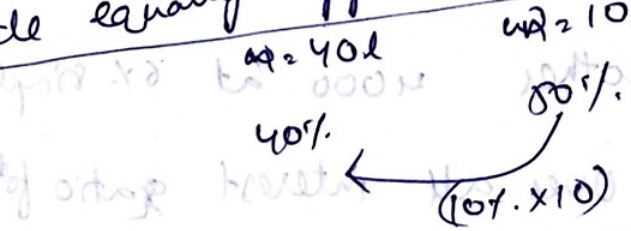
$T_1 = 40$; $T_2 = 10$



$$\frac{1}{5} \times 10^2 = 2$$

$$\Rightarrow A\% = 40 + 2 = \underline{42\% \text{ milk}}$$

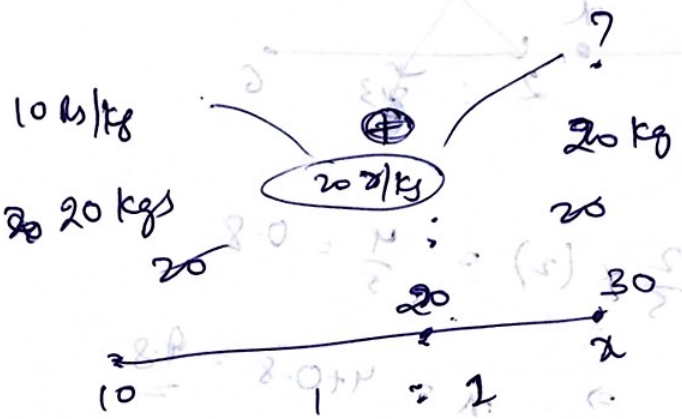
Divide equally approach



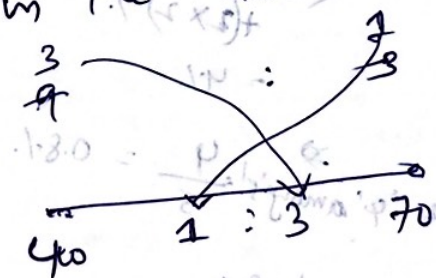
$= \frac{100\%}{50} = 2\%$

$\Rightarrow 40 + 2 = 42\%$

Q



Q 9 litres of milk and water sol^n having 40% milk is mixed with 3 litres of milk and water sol^n having 70% milk. Find % of milk in the mixture of two sol^{ns}



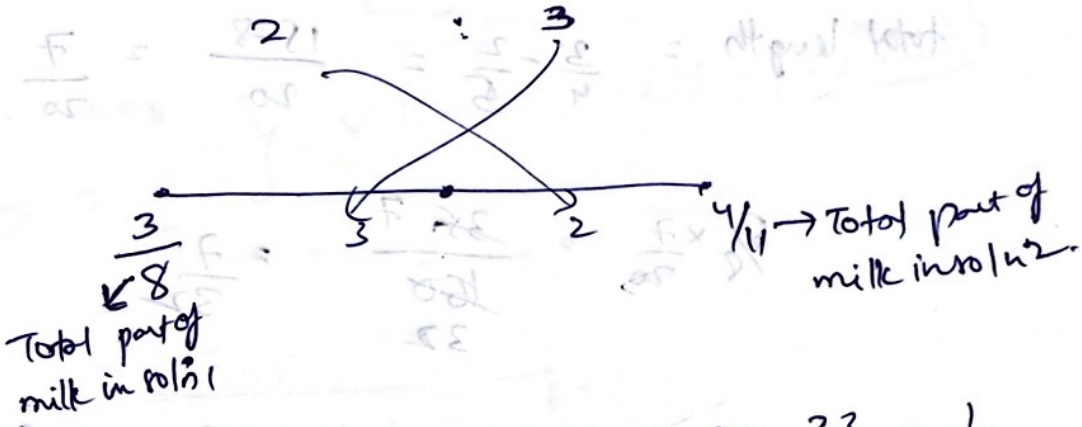
$\frac{1}{4} \times 30 = 7.5 \Rightarrow A = 40 + 7.5$

$= 47.5$

Divide equally:

Q Two soln's having milk & water in the ratios 3:5 and 4:7 are mixed in the ratio 2:3 find the ratio of milk and water in the resultant soln

Sol:



$\Rightarrow$ length of scale $= \frac{4}{11} - \frac{3}{8} = \frac{32-33}{11\times 8} = \frac{1}{88}$

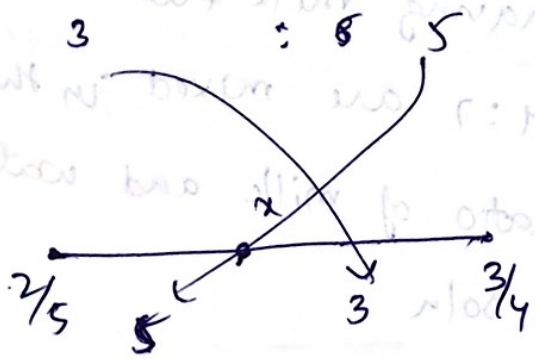
$\frac{3}{5} \times \frac{1}{88} = \frac{3}{440}$

$\Rightarrow \frac{3}{8} + \frac{3}{440} = \frac{1344}{440\times 8} = \frac{21}{55}$

21 : 34

Q Two vessels having volumes in the ratio 3:5 are filled with water and milk solns. The ratio of milk and water in the two vessels is 2:3 and 3:1 respectively. If the contents of both the vessels are emptied in to a larger vessel, find the ratio of milk & water in the larger vessel.

Sol:-



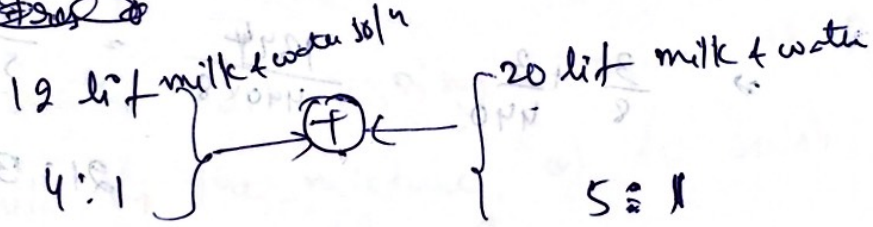
total length = $\frac{3}{4} - \frac{2}{5} = \frac{15-8}{20} = \frac{7}{20}$

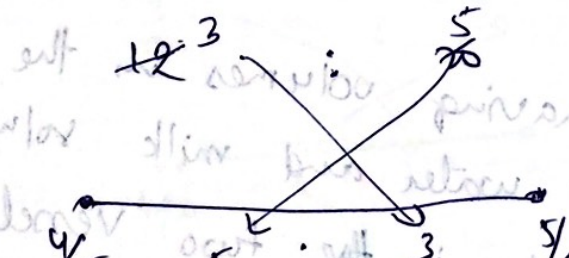
$\frac{5}{8} \times \frac{7}{20} = \frac{\cancel{35}\ 7}{\cancel{160}\ 32} = \frac{7}{32}$

2) $\frac{2}{5} + \frac{7}{32} = \frac{64+35}{32\times5} = \frac{99}{160}$

3) new ratio = 99 : 61

Q





Total length = $\frac{5}{6} - \frac{4}{5} = \frac{25-24}{30} = 1/30$

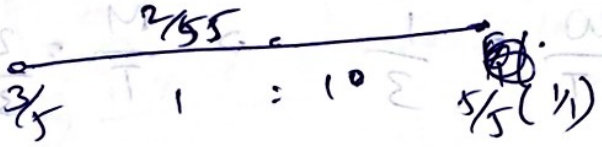
$\frac{5}{8} \times \frac{1}{30} = \frac{5}{240} = \frac{1}{48}$

Average = $\frac{4}{5} * \frac{1}{48} = \frac{192+5}{48\times5} = \frac{197}{240}$

⇒ 197 : 43

Successive Removal & Replacement Problems

Milk : water } volume + 10% (adding pure milk)
3 : 2
10 : 1



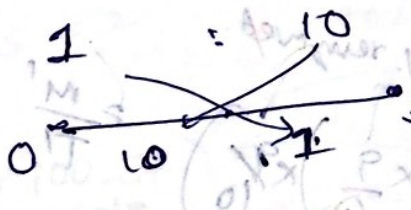
Total length of scale = $\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$

$\frac{1}{11} \times \frac{2}{5} = \frac{2}{55}$

$\Rightarrow \frac{3}{5} + \frac{2}{55} = \frac{165+10}{55\times5} = \frac{175}{55\times5} = \frac{7}{11}$

new ratio = 7 : 4

→ milk : coater } volume +10% (adding pure coater)
3 : 2



$\frac{10}{11} \times \frac{3}{5} = \frac{30}{55} = \frac{6}{11}$

$\Rightarrow 0 + \frac{6}{11} = \frac{6}{11} \Rightarrow$ 6 : 5

Q. M : W + 20% milk

3 : 2

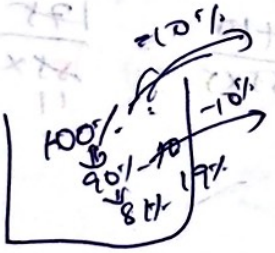
$\frac{W}{T} \rightarrow \frac{W}{\left(\frac{5}{6}\right)T} \Rightarrow \frac{5}{6} \times \frac{W}{T}$

$\Rightarrow \frac{5}{6} \times \frac{2}{5} = \frac{10}{15} = \frac{2}{3}$

$\Rightarrow \frac{W}{T} = \frac{1}{3} \Rightarrow \frac{M}{T} = \frac{2}{3}$

$\Rightarrow$ M : W = 2 : 1

Removal + Replacement



Remove 10% of milk + 10% of water + do it recursively.

M : W

3 : 2

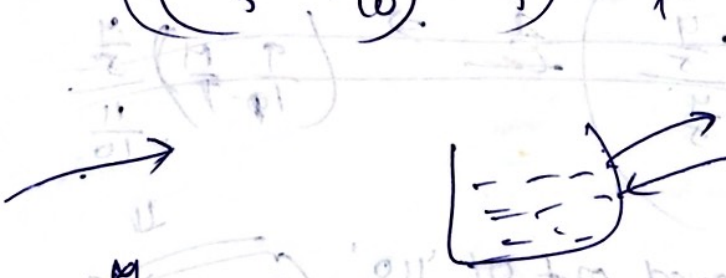
$\left(\left(\frac{3}{5} \times \frac{9}{10}\right) \times \frac{9}{10}\right) \times \frac{9}{10} = \frac{M'}{T'}$

(10% removed)

M : W
3 : 2

1) 10% → removed
2) 20% → removed
3) 25% → removed

$$\left(\left(\frac{3}{5} \times \frac{9}{10}\right) \times \frac{4}{5}\right) \times \frac{3}{4} = \frac{M'}{T'}$$



M/T

M → q → ① 10% removed from total → M/((9/10)T)

⇒

② 10% Milk removed → ((9/10)M)/((9/10)T)

③ 10% again putback
So, total again changes

(9/10)·M ⇒ milk changed
T ⇒ Total unchanged

⇒ (9/10)(M/T)

⇒ So overall, multiple factor (9/10)

Q

M : W
3 : 2

100 lit Soln → 10 lit removed → 10 l putback water

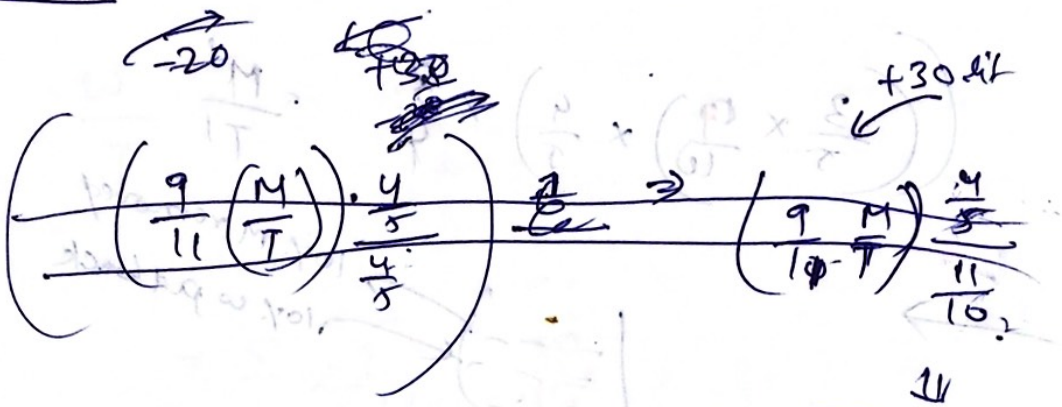
① 10% removed

$$\frac{M}{T} = \frac{\frac{9}{10}M}{\frac{9}{10}T} \Rightarrow \frac{\frac{9}{10}M}{\frac{11}{10}T} = \frac{9}{11}\left(\frac{M}{T}\right)$$

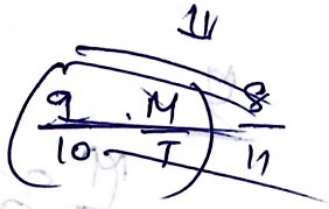
10 lit putback

$\frac{9}{11} \times \frac{3}{5} = \frac{27}{55}$ $\Rightarrow 27 : 28$

Again

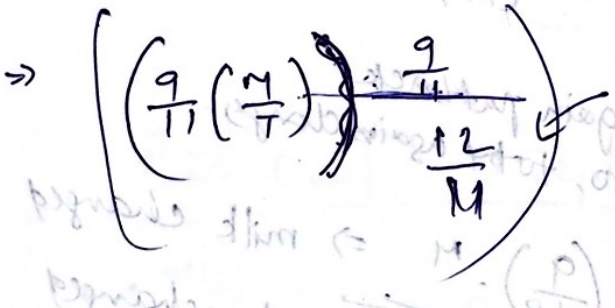


Now "20" removed out of "110"



$\frac{20}{110} = \frac{2}{11}$

$1 - \frac{2}{11} = \frac{9}{11}$

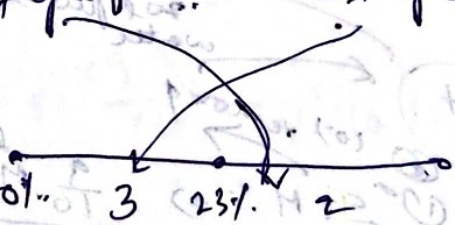


30% adding to 110 $\Rightarrow \frac{120}{110} = \frac{12}{11}$

Q. A man purchases a cow and a calf for ₹ 1300. He sold the calf at a profit of 20% and the cow at a profit of 25%. His total profit was 23%. Find the selling price of the cow.

Here CP = weight
P/- = rate

Sol:- Cost of calf : Cost of cow



$\Rightarrow 3a + 2a = 1300$

$\Rightarrow \boxed{a = 260}$

$\Rightarrow$ cost of calf $= 2a = 520$; Cost of cow $= 3a = 780$

cow sold at 25%.

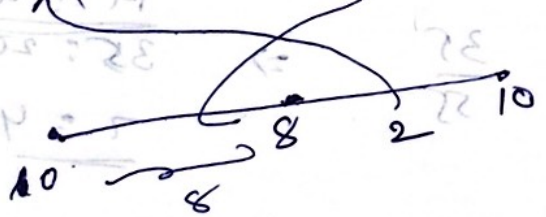
$\Rightarrow \frac{5}{4} \times 780 = 975 \Rightarrow$ cow sold at SP = 975

Q A milkman purchased milk at Rs 10/liter mixed 5 litres of water in it. By selling the mixture at the rate of Rs 10/liter he earns a profit of 25%. Find the total amount of mixture that he sold.

Sol:-

If $(2 \Rightarrow 5)$ then $8 \Rightarrow ?$

$2 : 5 : 8 : 20$



$SP = \frac{5}{4} C.P$

$\Rightarrow$ [illegible]

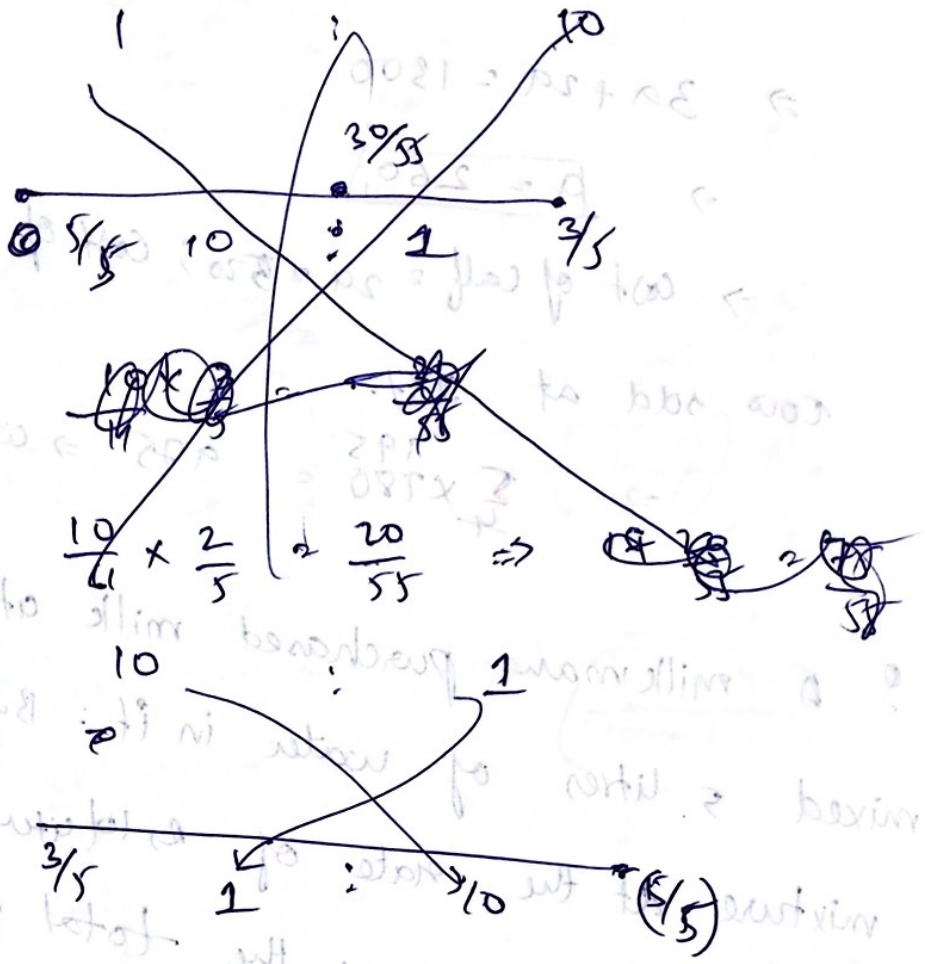
$C.P = \frac{4}{5} \times 10$

$C.P = 8$

$\Rightarrow$ total mixture $= 20 + 5 = 25$ lit

Q Volume of a sol^n which has milk + water in the ratio 3:2 is increased by 10% by adding pure milk. What is the ratio of milk & water in the resulting sol^n

Sol:



$$\frac{2}{5} \times \frac{1}{11} = \frac{2}{55}$$

$$\Rightarrow \frac{3}{5} + \frac{2}{55} = \frac{165+10}{55\times5} = \frac{175}{55\times5}$$

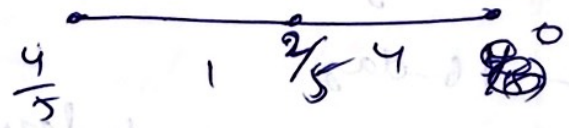
$$\Rightarrow \frac{35}{55} \Rightarrow \frac{M:W}{35:20}$$

$$7:4$$

Q When 20% of a sol^n of milk & water is removed & replaced with water, the ratio of milk and water in the resulting sol^n become 2:3. Find the ratio of milk & water in the original sol^n.

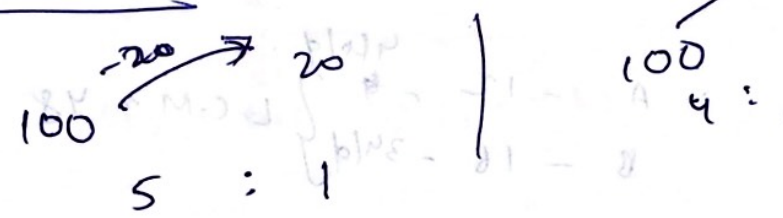
$\frac{4}{5}$ of soln + water ($\frac{1}{5}$ of soln) — volume of water

4 : : 1

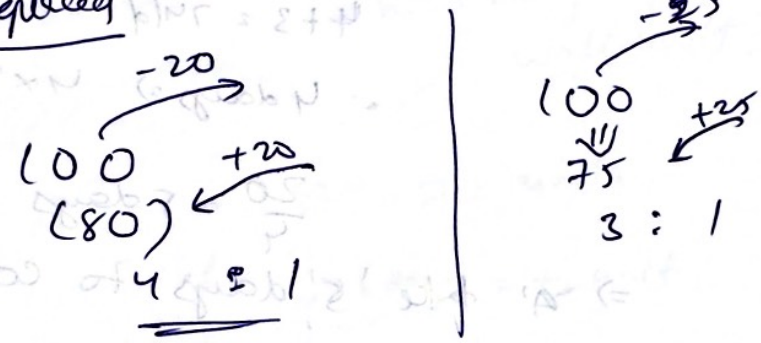


→

just removed



removed + replaced



Time & Work

A - 12 days work.

B - 16 days

B works '4' days & leaves ;

How many days 'A' take to finish work

Sol :- A - 12 - 4 u/d
B - 16 - 3 u/d
} L.C.M = 48

4 + 3 = 7 u/d

∴ 4 days ⇒ 4 × 7 = 28 u, 20 u remaining

$\frac{20}{4} = 5$ days

⇒ 'A' take '5' days to complete

Q2

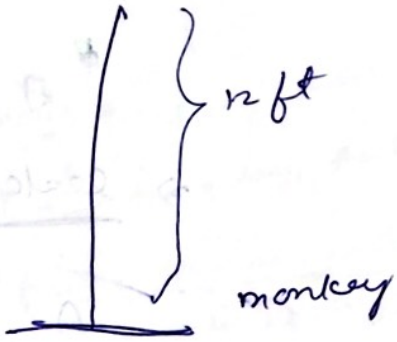
A - 8 days - $\frac{72}{8} = 9$
B - 4 days - $\frac{72}{4} = 8$
} last '2' days 'A' worked
days worked together ?

9 × 2 = 18 u ⇒ 72 - 18 = 54

days worked together $= \frac{54}{17} = 3\frac{3}{17}$

Negative work:

① first hr = +3 ft
second hr = −2 ft
when 12 ft = ?



9 ft + 3 ft = 12 ft
⇓ ↓
(8 hrs + 1 hour ⇒ 19 hours

② A 8 days — how many days wall built?
− B (destroys wall) 9 days

~~12~~ ⇒ 12 units

A − 9 u/d
B − 8 u/d
} 2 days = +1 unit

63 u + 9 u
12 d + 1 d ⇒ ~~12~~ 13 days

③ P_A = 12 hrs ⇒ 4 u/hr
(leak) P_B = 16 hrs ⇒ 3 u/hr

4 × 3 ⇒ 4 + 3 × 4
4 × 4 = 48

simultaneously

~~48 units~~
~~11~~ 8 + 1 = ~~9 hrs~~

every hr ⇒ 4 − 3 = 1 litre coming in
⇒ 48 lit ⇒ 48 hrs

A = 8 hrs (actually)

A = 12 hrs (due to leak)

⇒ leak = ?

4×2
4×3

A → 8 = 3 l/hr (24)

A → 12 = 2 l/hr

⇒ leak = 3 − 2 = 1 l/hr

⇒ leak takes 24 hrs to empty tank

Q

A, B − 10 days = 6 l/d
B, C − 12 days = 5 l/d
C, A − 15 days = 4 l/d

60

LCM

5×2
6×2
5×3

(A+B + B+C + C+A) = 15 l/d

2(A+B+C) = 15 l/d

A+B+C = 7.5 l/d (12)

C = 7.5 − 6

5×2×6×5

C = 1.5 l/d

⇒ C = 60/15 = 40 days

A+B+C = 7.5

B = 3.5

⇒ 60/35 = 17 5/35 ⇒ 17 1/7

A+B+C = 7.5

A = 2.5

24/60/25 = 24 days

		Rate
A, B	10d	6
B, C	12d	5

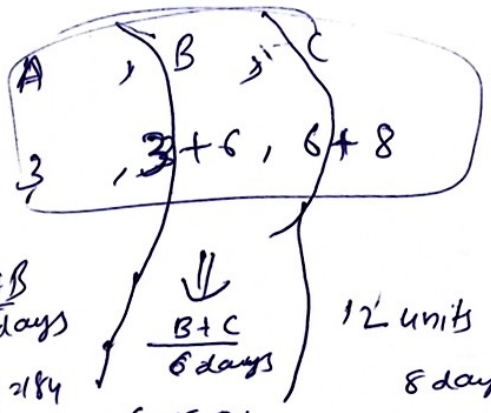
L.C.M (60)

A works for 3 days then left the B works for 9 days then left & 'C' finishes remaining work in 14 days

⇓

A, B, C

3, 9, 14



$\Rightarrow \frac{12}{8} = 1.5$ u/day $\Rightarrow$ ~~C = (1.5×1) = 2 u/d~~

$\Rightarrow$ C. rate = 1.5 u/d

⇓

	Rate
A, B	2.5+3.5
B, C	3.5+1.5

$\Rightarrow$ now long for 'A' & 'C'

$\rightarrow$ 2.5+1.5 = 4 u/d

$\rightarrow \frac{60}{4} = 15$ days

Time, Speed & Distance

1) Time × Speed = Distance
2) Basic Variation
3) Relative speed
4) Average speed
5) Boats & Streams
6) Circular motion (clocks)

Q A person travels three distances in the ratio 1 : 2 : 3 at speeds of 20 m/s, 30 m/s & 40 m/s. If the total time taken is 23 min, find the total dist covered

Sol:-

$d_1 : d_2 : d_3$

$1x : 2x : 3x$; $t_1 + t_2 + t_3 = 23$ min

20 m/s : 30 m/s : 40 m/s

$t_1 = \frac{x}{20}$; $t_2 = \frac{2x}{30}$; $t_3 = \frac{3x}{40}$

$\Rightarrow \left(\frac{x}{20} + \frac{x}{15} + \frac{3x}{40}\right) = 23 \times 60$

$= \frac{23x}{120} = 23 \times 60 \Rightarrow x = 7200$ m.

Q I travel for three stretches of time in the ratio 1 : 2 : 3 at speeds of 20 kmph, 30 kmph, 40 kmph. If I cover a total distance of 100 km

Find the total time for which I'm travelling for!:

$$t_1 : t_2 : t_3$$

$$1x : 2x : 3x$$

speed: 20 kmph : 30 kmph : 40 kmph

$$d = 100 \text{ km}$$

~~$d_1 = S$~~ $\quad D = S \times T$

$$d_1 = 20x \; ; \; d_2 = 60x \; ; \; d_3 = 120x$$

$$d_1 + d_2 + d_3 = d \Rightarrow \quad 200x = 100$$

$$\boxed{x = 0.5}$$

$$\Rightarrow \quad t_1 : t_2 : t_3$$

$$0.5 : 1 : 1.5$$

30 min : 60 min : 90 min

Q: I cover a total dist of 300 km in 8 hours, partly at a speed of 30 kmph and partly at a speed of 50 kmph. For what time was I travelling at a speed of 30 kmph.

Sol:

$$d_1 + d_2 = 300$$

$$\Rightarrow 30t_1 + 50t_2 = 300$$

$$t_1 + t_2 = 8$$

Q: Find the ratio of the time taken to cover two distance in the ratio 4:5 at speeds in the ratio 4:3 respectively.

$$\frac{S_1 T_1}{D_1} = \frac{S_2 T_2}{D_2} = \frac{4 \times t_1}{4} = \frac{3 \times t_2}{5}$$

$$\frac{t_1}{t_2} = \frac{3}{5} \Rightarrow 3:5$$

$\Rightarrow$ 1:1

$\longrightarrow$

$D = S \times T$

$S = \frac{D}{T}$

$D \propto S T$

$D \propto S$ (t = constant)

$D \propto T$ (S = constant)

$S \propto \frac{D}{T}$

Q A person travels from home to office

if

10 km/hr

10 min late

15 km/hr

10 min early

Find actual time. ① distance b/w h-o ② distance b/w h-o

③ Speed to be travelled to reach office at correct time

Sol:-

10 km/hr

t 10 min

15 km/hr

$t-10$ min

$\Rightarrow$ Dist $\Rightarrow$ constant

$\Rightarrow$ $S_1 T_1 = S_2 T_2 \Rightarrow 10\left(t + \frac{1}{6}\right) = 15\left(t - \frac{1}{6}\right)$

$10\left(\frac{6t+1}{6}\right) = 15\left(\frac{6t-1}{6}\right) \Rightarrow 60t + 10 = 90t - 15$

$30t = 25$

$\boxed{t = \frac{5}{6}} \Rightarrow 50 \text{ min}$

i) distance $= 10 \times 1 = 10$ km

ii) Speed to travelled $= \frac{10}{\left(\frac{5}{6}\right)} = 12$ km/hr

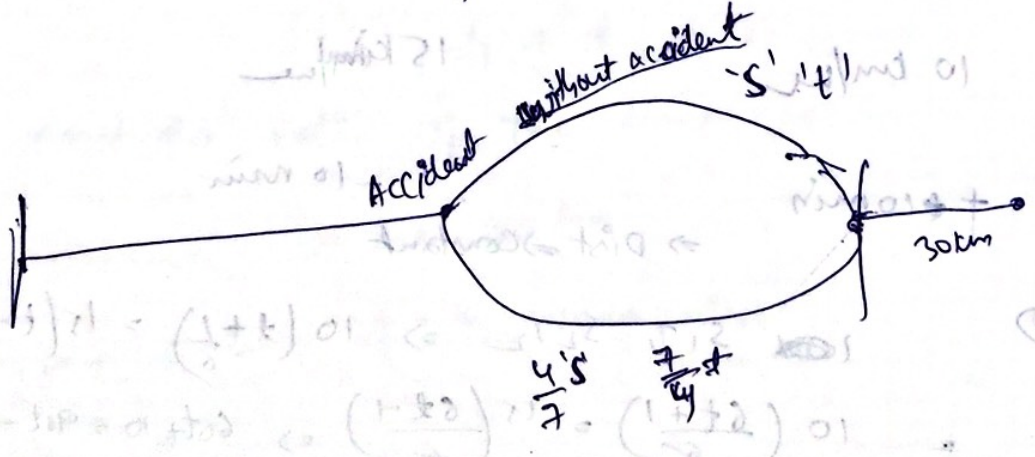
Q A train meds with an accident and travels at 4/7 of its regular speed hereafter and hence it reaches its destination 36 min late. Had the accident occured 30 kms further, the train would have been late by only 21 min. Find the regular speed of train

Sol:-

$S_1 = S$; $t_1 = t$; $d = d$

$S_2 = \frac{4}{7}S$; $t_2 = t + 36$; $d = d$

$S_3 = \frac{4}{7}S$; $t_2 = t + 21$; $d = (d - 30)$



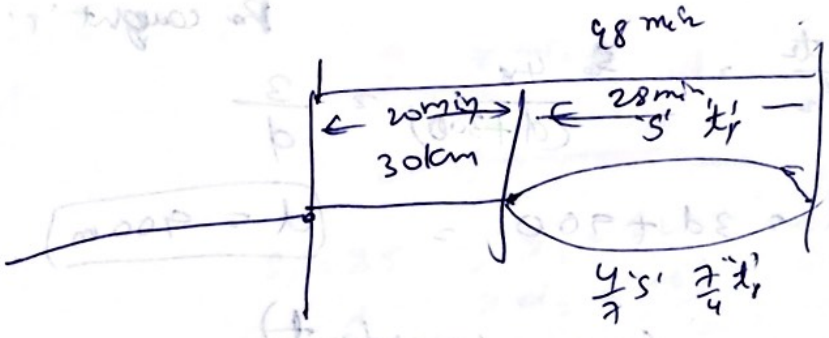
$1 - \frac{3}{4}$

$\frac{1}{4}$

If late:

diff in time : $\frac{3}{4}t = 36 \Rightarrow \underline{t = 48}$

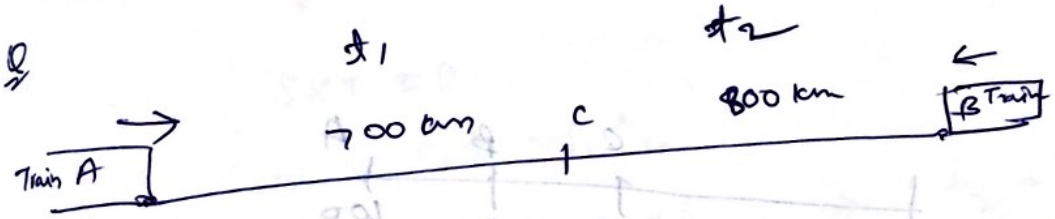
$\underline{t = 48 \text{ min}}$



$\frac{3}{4}t_1 = 21 \Rightarrow \underline{t_1 = 28 \text{ min}}$

$$s = \frac{d}{t} = \frac{30 \text{ km}}{\frac{28}{60} \text{ h}} = 30 \times 3 = 90 \text{ km/h}$$

Q



1500 km

→ A, B moving towards each other & meet at 'C'.

what is their ratio of speeds.

Sol: $t_1 = t_2 \Rightarrow \frac{d_1}{s_1} = \frac{d_2}{s_2} = \frac{700}{s_1} = \frac{800}{s_2}$

$\Rightarrow \frac{s_1}{s_2} = \frac{7}{8} \Rightarrow \underline{s_1 : s_2 = 7 : 8}$

Police chasing thief

	P	T
Speeds :	4	3

When thief caught 'P' ran 300m more than thief

→ at what distance P caught 'T'?

$S \times T_1 = D$

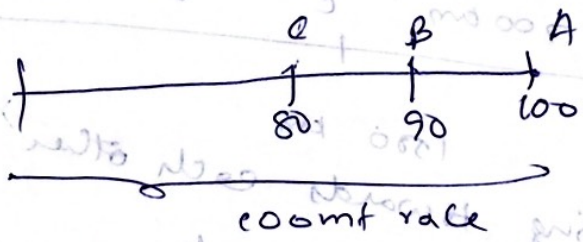
$\frac{S_1 t_1}{d_1} = \frac{S_2 t_2}{d_2} \Rightarrow \frac{4x}{(d+300)} = \frac{3}{d}$

$\Rightarrow 4d = 3d + 900 \Rightarrow \boxed{d = 900\,m}$

Races (Application of Variation) → (Time constant)

1. A beats B by 10 mt in a 100 mt race by how much he beat in a 1000 mt race.

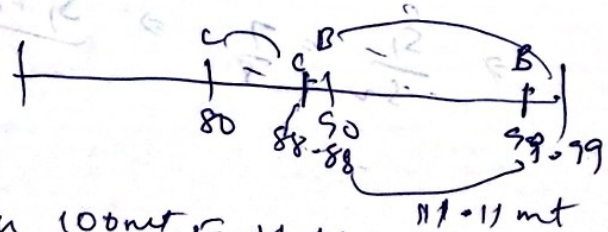
2.



In 200 mt race, by how much 'B' beat's.

Sol:-

Q In 100 mt race, how much 'B' beat's

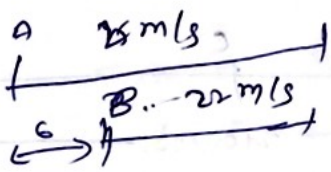


⇒ In 100mt = 11.11

200mt = 22.22 mt

what should be the length of the race so that two runners A & B running at speeds of 25 m/s & 22 m/s resp. reach the end pt together even though 'B' was given a lead of 6 mts

Sol:



$S_1 = 25$; $S_2 = 22$ ⇒ $\frac{S_1}{d_1} = \frac{S_2}{d_2}$

$d_1 = d$; $d_2 = (d-6)$

$= \frac{25}{d} = \frac{22}{d-6}$ ⇒ $d = 50$ mts

Q) Over a certain distance, the speed is increased by 10%. what would be % change in time

Sol: $S \times T = D$

⇒ $S_1 t_1 = S_2 t_2$

⇒ $S t_1 = \frac{110}{100} S t_2$ ⇒ $t_2 = \frac{100}{110} t_1$

$= (90.9)\, t_1$

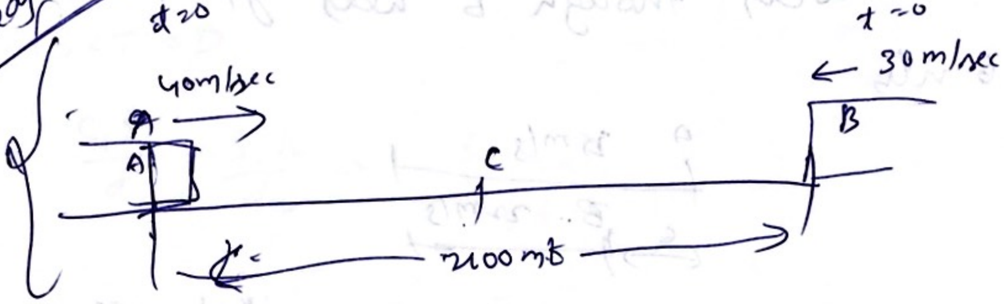
$t_2 = 9.1\%$ ↓

Q) Over a certain dist, the speed was increased by 4 kmph because which the time for the journey reduced from 8 hrs to 7.5 hrs what was speed? what is speed.

Sol: 60 kmph ; 64 kmph

Relative Speed & Applns

Speeds gets added ; $\xrightarrow{S_1 = 40\ kmph}$ $\xrightarrow{S_2 = 40\ kmph}$ } $S_1 + S_2 = 80\ kmph$



$t = \frac{d}{s}$ C = ? ; when & where

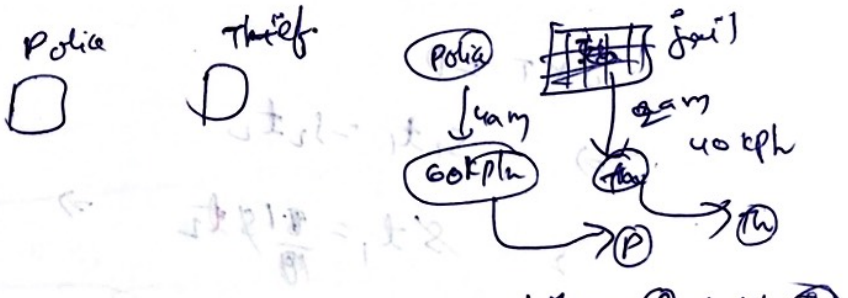
't' - time

$d_1 = 40t$; $d_2 = 30t$

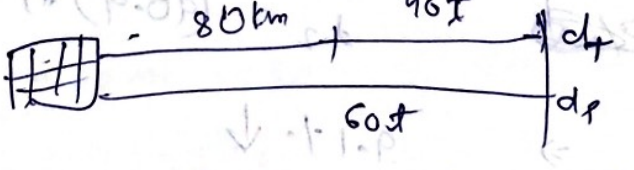
$40t + 30t = 2100$

$t = 30\ sec$

Subtracted



when P catch Th



~~t = const~~ t = constant

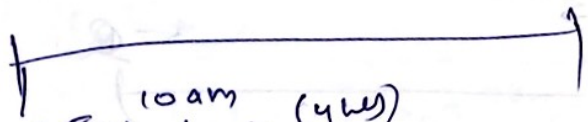
$40t + 80 = 60t$

$\Rightarrow 20t = 80$

$t = \frac{80}{20} = 4\ hrs$

$\Rightarrow 40t + 80 \Rightarrow 240\ km$

Leaving from two points at same time



1) Train A → (4 hrs) 10 am — 2:00 pm Train A

2) Train B ← Train B (6 hrs) 20:0 am — 4:00 pm

d.K ; Train A = 4 hrs

Train B = 6 hrs

when they will meet?

Sol:

$(S_A\ 4) = (S_B\ 6)$

$\Rightarrow \frac{S_A}{S_B} = \frac{3}{2}$ $\Rightarrow S_A : S_B = 3:2$

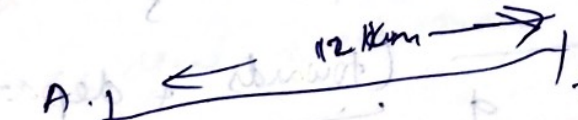
$t = \frac{d}{S_1 + S_2} = \frac{d}{\frac{d}{4} + \frac{d}{6}} = \frac{1}{\frac{1}{4} + \frac{1}{6}}$

$= \frac{12}{5} = 2.4$

(or)

take L.C.M of S_1 & S_2

$\Rightarrow 12$



A ← 12 km → B

$t = 4$ hrs, $S_1 = 3$ kmph

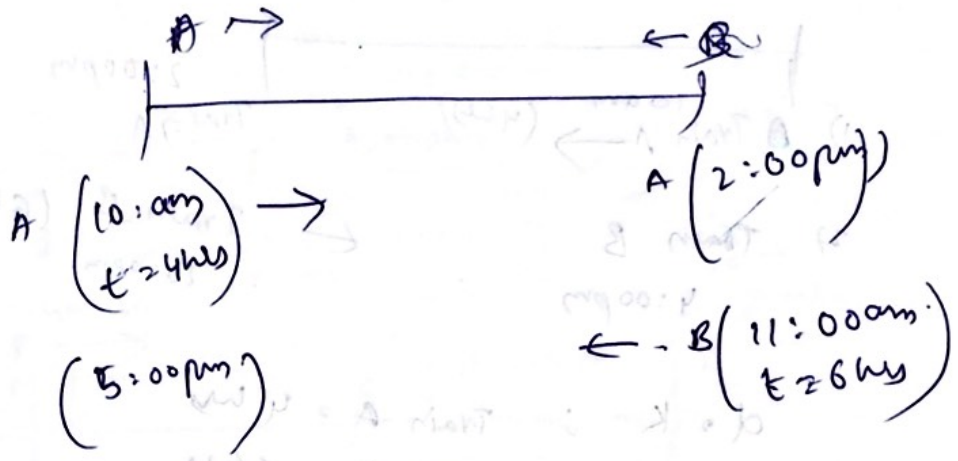
$t = 6$ hrs, $S_2 = 2$ kmph

$t = \frac{12}{S_1 + S_2}$

$= \frac{12}{5} = 2.4$ hr

Leaving from two pts at diff times

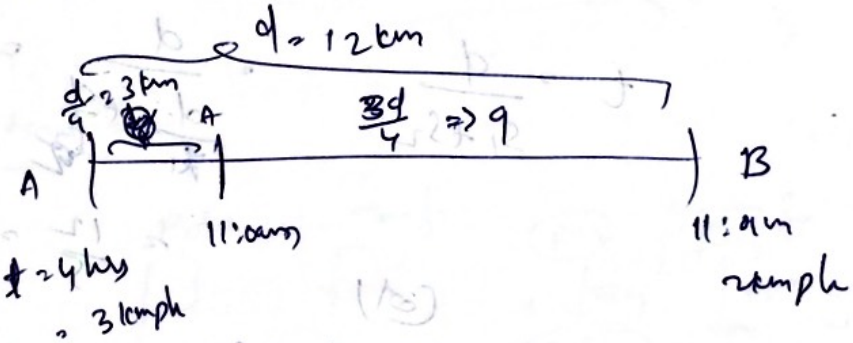
Q



make them to leave the points at same time.

⇒ move 'A' by 1 hour i.e at 11:00 am

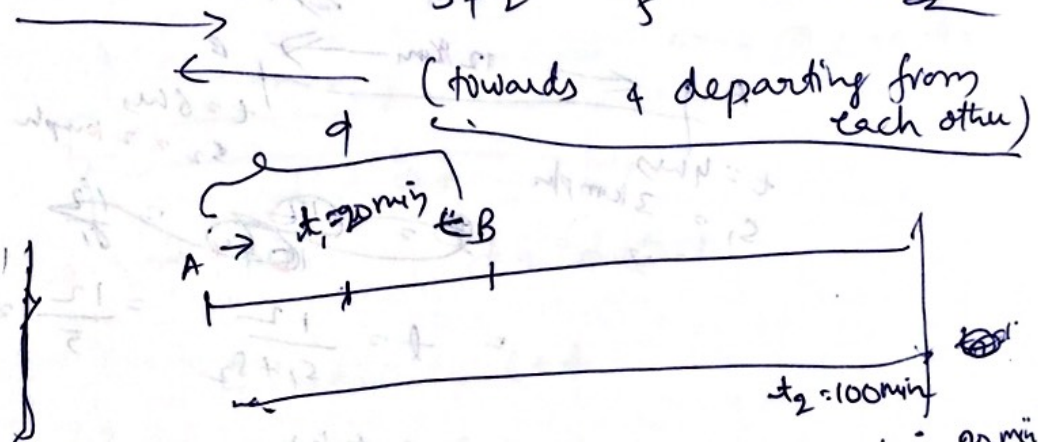
⇒



⇒ $\frac{9}{3+2} = \frac{9}{5} = 1.8$ hrs

(3)

Q



i) when 'A' & 'B' towards each other met in 20 min

ii) when in opp directions ⇒ after 100 min

$S_A = ?$; $S_B = ?$

$t_1 = \frac{d}{s_1+s_2} \Rightarrow 20 = \frac{d}{s_1+s_2}$

$\Rightarrow$

$\Rightarrow \boxed{d = 20s_1 + 20s_2}$

$t_2 = \frac{d}{s_1-s_2} \Rightarrow 100 = \frac{d}{s_1-s_2} = 100s_1 - 100s_2 = d$

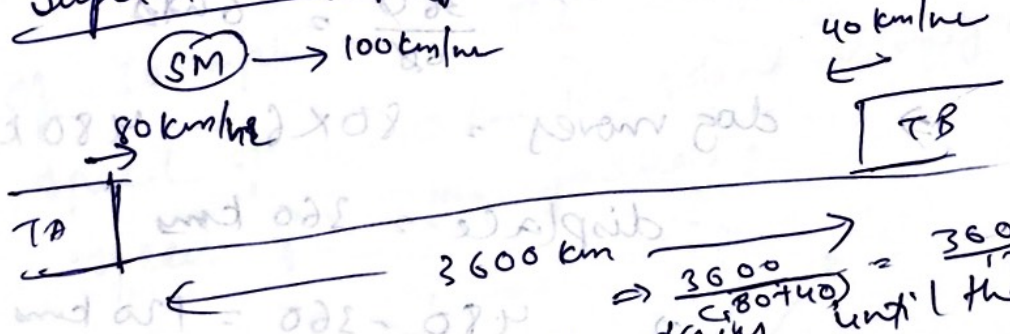
$\Rightarrow 20s_1 + 20s_2 = 100s_1 - 100s_2$

$\Rightarrow 120s_2 = 80s_1$

$\Rightarrow 3s_2 = 2s_1$

$\therefore \boxed{\frac{s_1}{s_2} = \frac{3}{2}}$

Q. Superman flying b/w trains



$\Rightarrow \frac{3600}{(80+40)} = \frac{3600}{120} = 30$ hrs

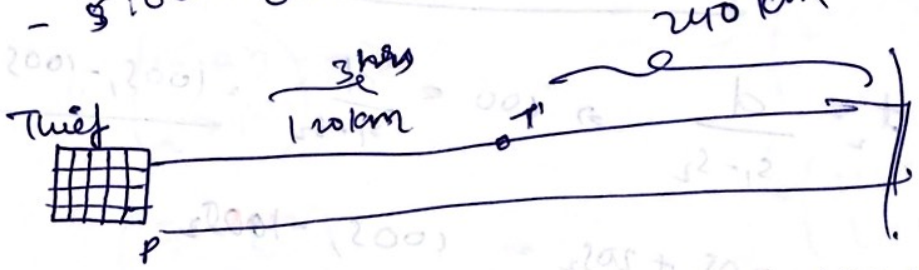
→ Superman flies b/w trains until they met with accident → distance travelled by SM=?

$\Rightarrow$ 100×30 hrs $= 3000$ km

Dog, Thief, Police :=

Thief - 2:00 ⇒ 40 kmph dog - 80 kmph

police - 3:00 → 60 kmph



@ Dog moves to the thief & comes back to police & again moves to thief.

what is the total distance covered by dog in forward direction?

Sol: d = 360 kms

Chase ⇒ $\frac{360}{60} = 6$ hrs

⇒ dog moves = 80 × 6 = 480 kms

- displace = 360 kms

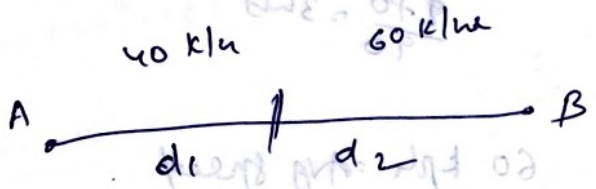
⇒ 480 - 360 = 120 kms

⇒ $\frac{120}{2}$ = 60 (forward dist)

⇒ 480 - 60 = 420 kms Dog moves

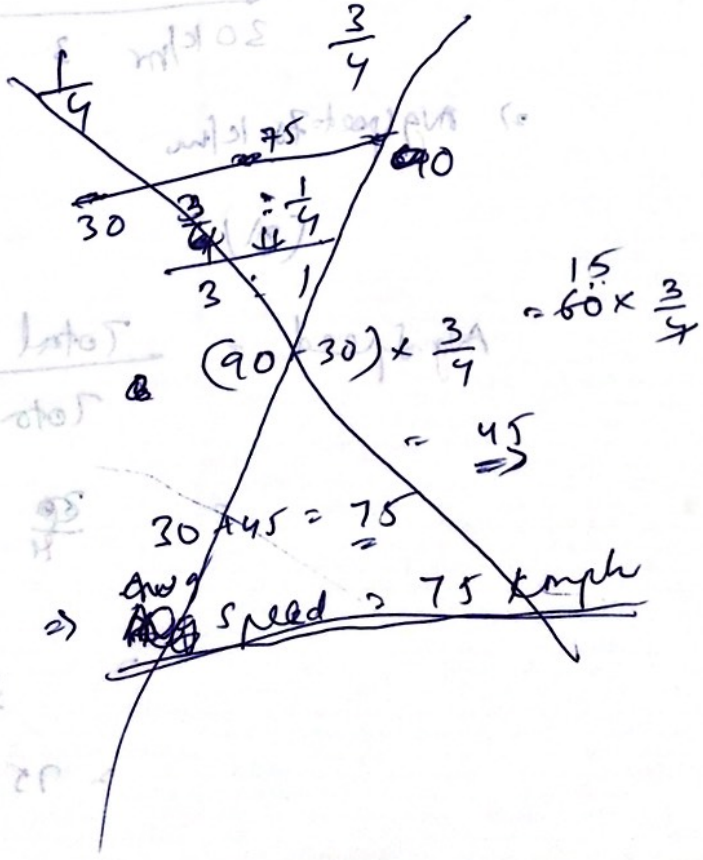
Average Speed

$$\text{Avg speed} = \frac{\text{Total distance}}{\text{Total time}}$$



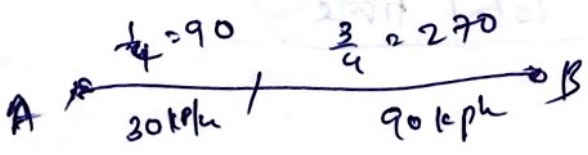
$$\text{Avg speed} = \frac{d_1 + d_2}{\frac{d_1}{40} + \frac{d_2}{60}}$$

Q A travelled 1/4th of the dist at a speed of 30 kmph and the remaining dist at a speed of 90 kmph. What is my avg speed for the journey?



Sol:- take L.C.M (Speeds)

$= 360$ (or) 180 whichever is convinient



$\frac{90}{30} = 3$ hrs $\qquad \frac{270}{90} = 3$ hrs

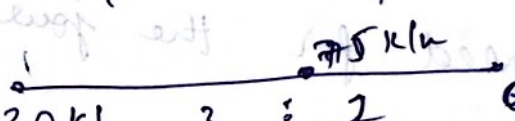
$\Rightarrow \frac{360}{6} = \underline{60 \text{ kph}}$ = Avg speed

Now

Q) ii) $\frac{1}{4}$th of time travelled with 30 kph & remaining at a speed of 90 kph

Sol:- Now Scale method can be used

$\frac{1}{4} : \frac{3}{4}$



$\Rightarrow$ Avg speed = 75 k/hr

(or)

Avg speed = $\frac{\text{Total distance}}{\text{Total time}}$

$= \frac{30\left(\frac{1}{4}t\right) + 90\left(\frac{3}{4}t\right)}{t}$

$= \frac{300}{4}$

$= 75$ k/hr

Q Two stretches of a journey were done at speeds of 25 kmph & 40 kmph and the avg speed for the journey was 30 kmph. what fraction of distance / time were completed at 25 kmph?

Sol:- distance: $25x$, $40x$, $50x$

time: 2 : 3 : 1

$\Rightarrow$ distance '25' $= \frac{50}{90} = \frac{5}{9}$

$\Rightarrow$ time '25' $= \frac{2}{3}$

25 kmph — 30 — 40 kmph

Q $\frac{1}{3}$rd of the distance for a journey was done at a speed of 45 kmph, half the distance was done at a speed of 60 kmph and the remaining distance at a speed of 36 kmph. what is the avg speed for the journey?

Sol:- Choose the L.C.M of (45, 60, 36) such that it gives clear cut 'hours'

d → distance

$\frac{1}{3}d$ → 45 kmph ; $\frac{1}{2}d$ → 60 kmph ; remaining $(1/3)$ → 36 kmph

Avg. Speed = $\frac{d}{t_1 + t_2 + t_3}$

$= \frac{d}{\frac{(d/3)}{45} + \frac{(d/2)}{60} + \frac{(d/6)}{36}}$

choose $= \frac{360 \times 6}{\frac{360 \times 6}{3 \times 45} + \frac{360 \times 6}{2 \times 60} + \frac{360 \times 6}{6 \times 36}}$

$= \frac{360 \times 6}{8 + 9 + 5} = \frac{360 \times 6}{22}$

Ex:- Two equal distances at diff speeds

A —(d, a km/hr)— d —(b km/hr)— B

~~Avg. speed~~ :

Avg speed = $\frac{\text{total distance}}{\text{total speed}}$

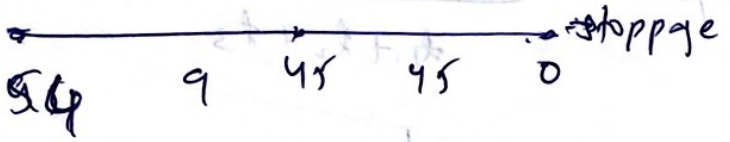
$= \frac{d+d}{\frac{d}{a} + \frac{d}{b}} = \frac{2}{\frac{1}{a} + \frac{1}{b}}$

Avg speed = $\frac{2ab}{a+b}$

Q: A bus travels at a speed of 54 kmph but because of stoppages its overall speed is reduced to 45 kmph. What is the stoppage time per hour?

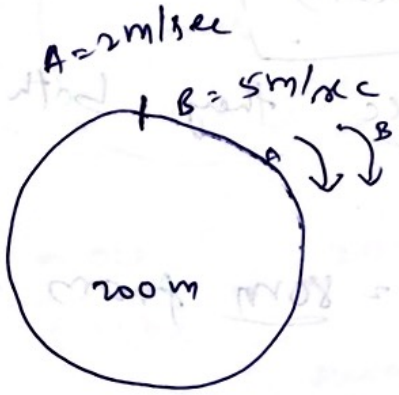
Sol:-

$$\frac{5}{45} : \frac{1}{9} \Rightarrow \text{stoppage} = \frac{1}{6} \times 60 = 10 \text{ min}$$



$\Rightarrow$

Circular Tracks



Q i) when both 'A' & 'B' will meet at starting pt?

$A = \frac{200}{2} = 100$ sec (time to complete one rotation)

$B = \frac{200}{5} = 40$ sec (time to complete one rotation)

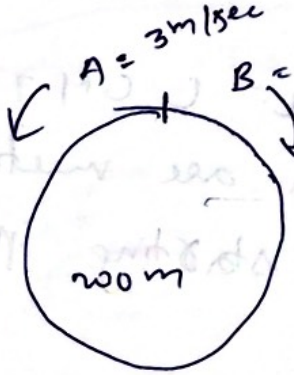
L.C.M (100, 40) ⇒ first time they meet at starting pt

⇒ L.C.M (100, 40) = 200

⇒ at 200 sec they meet for first time

No change if direction changes also

2Q



! when they both meet for first time and where ?

Sol:- adding speeds

$$t = \frac{200}{(3+2)} = 40 \text{ sec}$$

at "40" sec they both meet for first time &

$40 \times 2 = 80$ m from starting pt towards right

& 120 m from starting pt towards left

and they meet at intervals of 40. like 40, 80, 120, 160 at same pt

ii) at starting pt:

$$L.C.M\left(\frac{200}{3}, \frac{200}{2}\right)$$

Finding L.C.M

$\frac{200}{3} \times 1 = \frac{200}{3}$ | 100 × 1 = 100

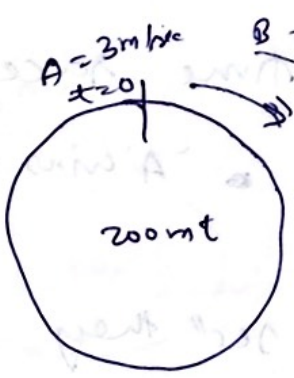
$\frac{200}{3} \times 2 = \frac{400}{3}$ | 100 × 2 = 200

$\frac{200}{3} \times 3 = 200$

⇒ L.C.M = 200

⇒ both are meeting at 200 sec at starting pt.

300 gets
speeds ↑ subtracted



$$t = \frac{200}{(3-2)} = 200 \text{ sec}$$

at 200 sec they first meet at
meet at intervals
200, 400, 600....

Q

Race = 1800 mt

(i) How many times they will meet?

Sol: first meeting = $\frac{200}{5}$ = 40 sec

(i) ~~No of times = 1800/40 = 45 times~~

⇒ 'B' = 40 × 2 = 80 mt from starting towards right

(OR)

⇒ ~~1800/120 = 15 times~~ ⇒ ~~2 times~~

→ How much time race lasts,

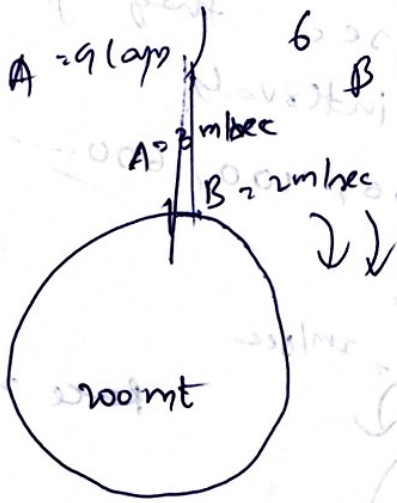
• 'A' wins ⇒ $\frac{1800}{3}$

= 600 sec

→ for every "40 sec" they meet

⇒ $\frac{600}{40}$ = 15 times they will meet

Q.



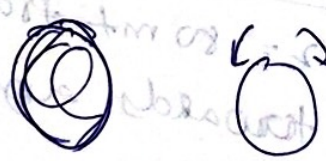
1800 mt race

meeting at = $\frac{200}{(3-2)}$ = 200 sec ;

and race lasts for = 600 sec

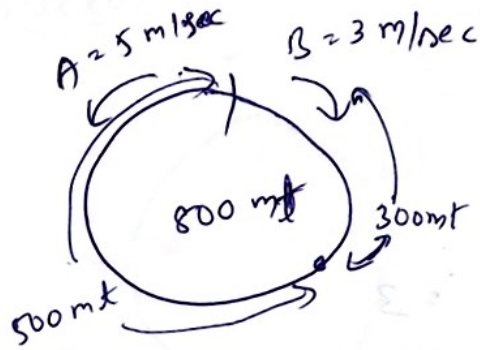
⇒ 3 meetings

= 9 − 6 = 3 times } whenever faster one completes the lap, they'll meet compared to slower one



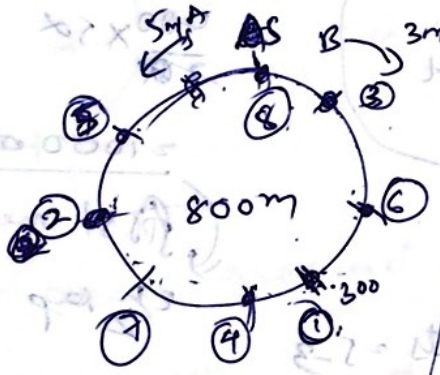
= 9 + 6 = 15 times } whenever they completed a lap they'll meet

Q



first meet = $\frac{800}{(5+3)}$ = 100 sec

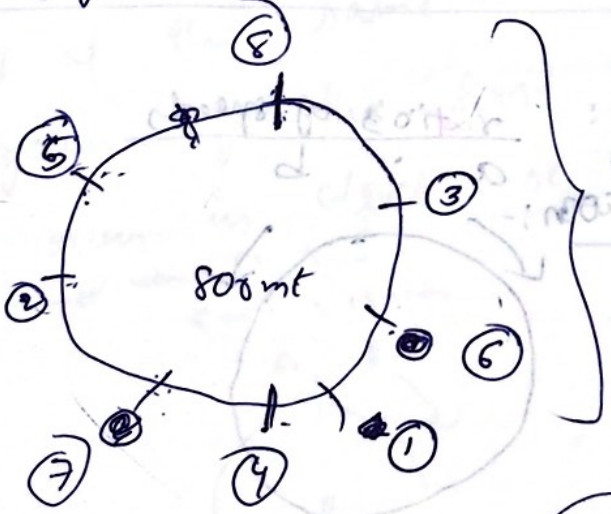
100, 200, 300, 400 ...



100 sec
⇒ 3 × 100
= 300 m

⇒ circle, meetings at a regular interval spots

300m, 600m, 900m, 1200m, 1500m, 1800m.



⇒ no. of meeting pts = (5+3) = 8

3/8
5/8
6/8
10/8
9/8
15/8

$\frac{20}{8}$ $\frac{12}{8}$
$\frac{25}{8}$ $\frac{15}{8}$

$\frac{30}{8}$ $\frac{18}{8}$

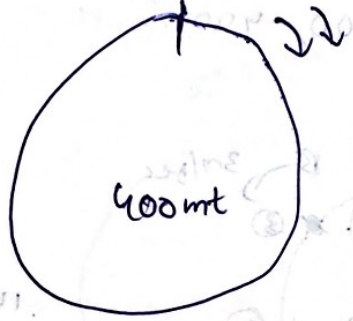
$\frac{35}{8}$ $\frac{21}{8}$

$\frac{40}{8} \Rightarrow 5$ $\frac{24}{8} : 3$

speeds (ratios)

A : B

5 : 3



200

400 × 5

= 1000 mt

1st meeting

2½ lap

$\frac{200}{400} \times 3$

= 600 mt

⇒ 1½ lap

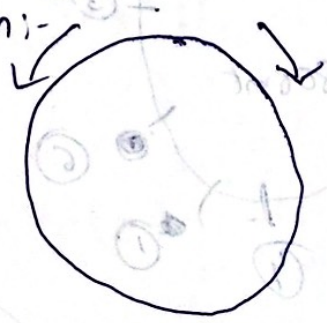
no. of meeting pts = 5 − 3

= 2

no. of meeting pts = diff in ratios of speeds

Summary: ratio's of speeds a : b

1) Opposite Direction:-



no. of meeting pts = a + b

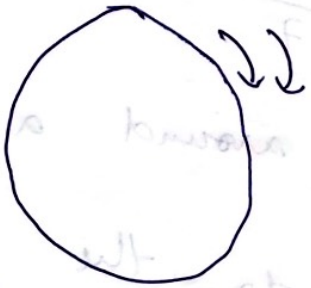
Ex:-

⇒ no. of mt pts = 5 + 3 = 8

⇒ 8 equal parts

ii) Same Directions

a : b



no. of int pts = a-b

Ex:

5 : 3

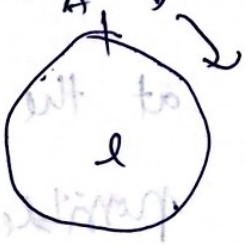
; no. of int pts = 5-3
= 2
(divide circle into 2 equal parts)

Q A car runs around a circular track in 8 min., B car runs around the same track in 15 min. If they start from the same pt and at the same time, when they meet again for the first time? Both of them runs in clockwise direction?

Sol:

speeds ratios 15 : 8

8 min : 15 min



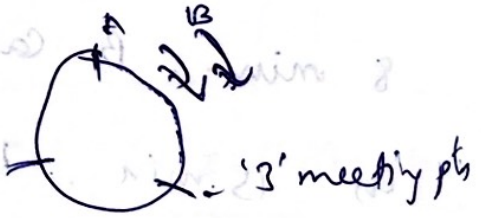
$\Rightarrow t = \frac{l}{S_1 - S_2} = \frac{l}{\frac{l}{15} - \frac{l}{8}} = \frac{1}{\frac{1}{15} - \frac{1}{8}}$

$\Rightarrow$ Take L.C.M (15, 8)

$$t = \frac{120}{15-8} = \frac{120}{7}$$

Q: A can run around a circular track in 4 min, B can do the same in 7 min. If they start running together from the same starting pt, when they'll meet for the first time at the pt diametrically opposite to the starting pt?

Sol:- They'll never meet in diametrically opposite positions

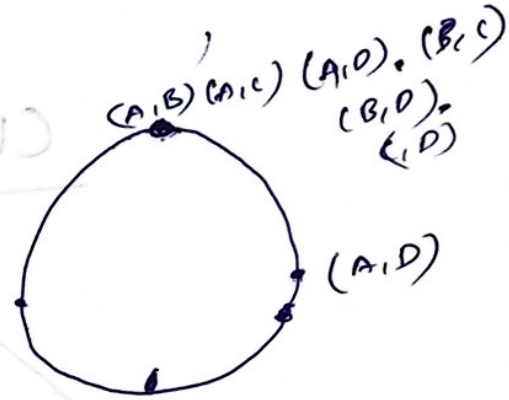


** Q: Four runners A, B, C & D are running around a circular track with speeds in the ratio 1:2:3:4 respectively. They start together and at the same pt. what is the no of unique possible locations of meeting?

meeting pts

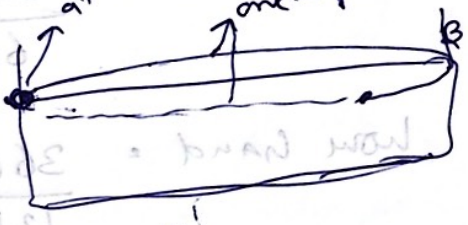
A, B - 1
A, C - 2
A, D - 3
B, C - 1
B, D - 2
C, D - 1

10 but unique, 4



① Swimming pool

A : B (speed ratios)
5 4



(10 laps) → to & fro

where they will be at qth meeting

1st meeting = 1 lap

qth meeting = q laps

ratio = 5 : 4

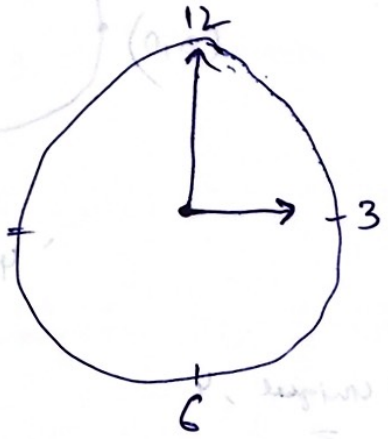
⇒ ⇒ at starting pt

⇒ 5 : 4

⇓ ↓ B 4 laps

A 5 laps

CLOCKS



→ track length = 360°

→ speed of min. hand = $\frac{360^\circ}{60\ \text{min}}$ ⇒ 6°/min

→ speed of hour hand = $\frac{360^\circ}{12\ \text{hour}}$ = 30°/hr ⇒ 0.5°/min

Arithmetic And Geometric Progressions

1. Sequence:-

A sequence is a set of numbers written in a particular order. we sometimes write u_1 for the first term of the sequence, u_2 for the second term and soon. we write the n^{th} term as u_n.

Ex:- Fibonocci sequence

1 1 2 3 5 8 13 ...

$\Rightarrow u_n = (-1)^{n+1}/n$.

2 Series:-

A series is something we obtain from a sequence by adding all the terms together

Ex:- sequence : $u_1, u_2, u_3, u_4 \ldots u_n$

series : $u_1 + u_2 + u_3 + u_4 \ldots + u_n$

$S_2 \rightarrow$ sum upto '2' terms

$S_n \rightarrow$ sum upto 'n' terms

$\Rightarrow \boxed{S_n = u_1 + u_2 + u_3 + \ldots u_n}$

3. Arithmetic Progressions:-

An arithmetic progression, or AP is a sequence where each new term after the first is obtained by adding, a constant(d) called the common difference, to the preceding term. If the first term of the sequence is 'a', then the AP is

$$a, a+d, a+2d, a+3d, \ldots a+(n-1)d$$

& n^{th} term, $l = a+(n-1)d$

4. The sum of an Arithmetic Series:-

$$S_n = a + (a+d) + (a+2d) + \ldots + (l-2d) + (l-d) + l$$

$$+ \quad S_n = l + (l-d) + (l-2d) + \ldots (a+2d) + (a+d) + a$$

$$2S_n = (a+l) + (a+l) + (a+l) + \ldots (a+l) + (a+l) + (a+l)$$

$$S_n = \frac{n(a+l)}{2}$$

$$= \frac{n}{2}\left[a + a+(n-1)d\right] \quad \cdot \frac{n}{2}\left[2a+(n-1)d\right]$$

$$\Rightarrow \boxed{S_n = \frac{n}{2}(a+l)}$$

$$\boxed{S_n = \frac{n}{2}\left[2a+(n-1)d\right]}$$

Geometric Progressions:- (GP):

A geometric progression, or GP, is a sequence where each new term after the first is obtained by multiplying the preceding term by a constant, r, called the common ratio. If the first term of sequence is a, then the GP is

$$a, ar, ar^2, ar^3 \cdots$$ then

n^{th} term $\boxed{ar^{n-1}}$

Sum of a Geometric Series:

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

$$r \cdot S_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n$$

$$S_n - rS_n = a - ar^n$$

$$\Rightarrow \underline{S_n - rS_n = a - ar^n}$$

$$S_n(1-r) = a(1-r^n)$$

$$\Rightarrow \boxed{S_n = \frac{a(1-r^n)}{(1-r)}}$$

7. Convergence of Geometric Series :

The sum to infinity of a G.P with starting value 'a' & common ratio, r is given by,

$$\boxed{S_\infty = \frac{a}{1-r}}$$

where $\underline{-1 < r < 1}$

Some Important Series

1) $1+2+3+4+5+\cdots+n = \frac{n(n+1)}{2}$

2) $1^2+2^2+3^2+4^2+5^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

3) $1^3+2^3+3^3+4^3+5^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Logarithms

1) **Logarithm:-**

If 'a' is a +ve real no, other than 1, and $a^m = x$, then we write $m = \log_a x$. and we say that the value of $\log x$ to the base a is m

$$\Rightarrow a^m = x$$

$$\underline{\underline{m = \log_a x}}$$

Ex:-

$$10^3 = 1000 \Rightarrow 3 = \log_{10} 1000.$$

II) **Properties of logarithms:-**

i) $\log_a(xy) = \log_a x + \log_a y$

ii) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

iii) $\log_x x = 1$

iv) $\log_a 1 = 0$

v) $\log_a(x^p) = p\left(\log_a x\right)$

vi) $\log_a x = \frac{1}{\log_x a}$

vii) $\log_a x = \frac{\log_b x}{\log_a x}$

Statistics:-

Avg, Weighted Avg, Median

* If numbers are in A.P, then Mean = median (the middle term in the sequence)

* if 'n' is even, then median =

$$\frac{n}{2}\left(a + a + (n-1)d\right)$$

$$= \frac{n}{2}\left(2a + (n-1)d\right)$$

$$\Rightarrow \frac{200}{5} = 40$$

$$\frac{x}{3} = 50$$

$$x = 150$$

$$\frac{-4 + 4 - 4}{3}$$

$$\frac{10 + 40 + 90 + 160 + 150 + 140 + 70}{14}$$

$$= \frac{300 + 410}{14}$$

$$16 + k + m = 24 \cdot 30$$

$$k + m = 8$$

$$m + 2(S.D) = 86$$

$$m + S.D = 63$$

$$(63 - m)$$

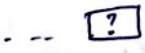
$$m + 2(63 - m) = 86$$

$$m + 126 - 2m = 86$$

$$m = 126 + 86$$

Quants

5 types of qstns

i) MCQ
ii) MAQ
iii) DI → 
iv) N.E.B → ... 
v) Quantitative Comp.
    ↳ Exponents
    → Number. S
    → IEM (Inequalities & Modulus)

→ 10 classes : 13 topics

1) GRE 2 : Revision .. 15 days → 1 topic/day.

2. Tests

Section wise
GRE 1 - 5
GRE 2 - 14
27 tests
(2) per day

FLTs
8 FLTs
2 FLTs ETS prep
2 FLTs - off guide
12 "

→ 1) Note taking & check

Quantitative Comp

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Ineq & Modules

① vague { $x < 2$, $y < 3$

QA: xy QB: 6

x, y, z

- +ve integ: 1, 2, 3 ... 100, 1000
- -ve integ: $-1, -2, -3, \ldots$
- -ve fractions: $-\frac{1}{2}, -\frac{1}{5}, -\frac{1}{9} \ldots$
- +ve fractions: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots$

② $x > 0$

- +ve integ: $1, 2, \ldots 1000$
- +ve fractions: $\frac{1}{2}, \frac{1}{16}, \frac{1}{10} \ldots$

factors < number < multiples

* ③ Substitute Special Numbers .. 0, 1, 2, $0 < x < 1$

about "0"

- neither +ve nor -ve
- is an even integer | 2)0(0=x
- divisible by all numbers except itself
- non-negative means +ve & 0

about 1: (A: no is prime, if it has 2 +ve factors, '1' & itself)
(A no is composite, if it has more than '2' +ve factors)

1) "1" is neither prime nor composite

2) ↓ div by "1" and itself

about 2:

- "2" only even prime number

about $0 < x < 1$:-

$x < x^2 < x^3 < x^4 < x^5 \cdots$ } $x > 1$

$x > x^2 > x^3 > x^4 > x^5 \cdots$ } $0 < x < 1$

4) factors of "4"

→ 1, 2, 4
→ -1, -2, -4
} Factors can be -ve
-ve no's are also factors

5) Prime Numbers are Positive Integers

NO NEGATIVE

6) $-\frac{1}{4}$ $-\frac{1}{64}$ } which is greater fraction

(7) $-2^4 = -16$

$(-2)^4 = 16$

(8) $2^{2^3} = 2^8$

$(2^2)^3 = 2^6$

(9) 

$\sqrt{N} = N$ (1 and 0)

$\sqrt{N} < N$

$\sqrt{N} > N$ (fractions)

(10)

(i) $\sqrt{4} = +2$ ✓

(ii) square root of 4 = +2 (or) −2

$\sqrt{N}$ = +ve Root of N

$= \sqrt{}$ "; symbol denotes only for +ve root

Summary:-

Never forget -ve no's and fractions

Exponents Rules :-

1) $a^m \times a^n = a^{m+n}$

2) $\frac{a^m}{a^n} = a^{m-n}$

3) $a^{-m} = \frac{1}{a^m}$

4) $(a^{mn}) = a^{mn}$

5) $(a \times b)^m = a^m \cdot b^m$

→

$a^{-3}+b^{-3} = 0$ if i) $a=0$ & $b=0$

(or)

$a = -b$

→ x , $\sqrt{x}$ → Relationships

$x \geqq \sqrt{x}$

→ Rule Except 1, 2

(Smaller Number)$^{(\text{Larger No})} > (1)^{(\text{Same})}$

→ Unknown common factors should not be cancelled

$x^2 = 2x$ ⇒ $x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 ; x = 2$

(X)

$N + \frac{1}{N} > 2$; $N > 0$; $N \neq 1$;

1. $x + \frac{1}{x} = 2$ (for $x = 1$) ; If ($x > 2$; then $x + \frac{1}{x} > 2$) always

2. We can add (or) subtract a variable on both sides of an inequality. BUT we cannot multiply (or) divide a variable on both sides of an inequality

Ex:- $\frac{x}{y} > 1$; then we think $x > y$, multiplying 'y' on B.S

Why? $\frac{x}{y} \times (y) > (y)$

Consider this, $\frac{-4}{-3} > 1$

$x > y$ ✗

Is $x > y$; (NO)

So we cannot multiply/divide a variable on B.S of inequality Unless & Until we know sign of the variable

S_n :

$2S_4 + 4 =$

$2S_6 + 4 = 316$

② What is the 60th term in the sequence?

(1, 2, 4, 7, 11, 16, 22)

1) ~~167~~

2) ~~1760~~

3) ~~176~~

4) ~~1771~~

$a_n = 60$

$= ar^{n-1}$

$= 1(2)^{60-1}$

$= 2^{60} - 1$

③ The sum of the squares of first 15 integers

$1^2 + 2^2 + 3^2 + \cdots 15^2 = 1240$

$16^2 + 17^2 + \cdots 30^2 =$

$(15+1)^2 + (15+2)^2 + \cdots$

1) A certain club has 5 members initially. 'x' new mem's will be brought to each member of the club. in next following week. 12th week, 'y' new mem.

$5 + 5x + (5+5x)$

$(1+x)(5+5x)$

$5(1+x)^2$

$5(1+x)$ $5(1+x) + 5x(1+x)$

$5(1+x)^2 + 5x(1+x)^2$

$= 5(1+x)^2(1+x)$

$= 5(1+x)^3$

$5 + 5x + (5+5x) + (5+5x)^x$

$$\boxed{5(1+x)^{11} = y}$$

$(1+x) = \left(\frac{y}{5}\right)^{1/11}$

$\rightarrow$

$\sqrt{2+\cdot}$

Sequence & Series

Arithmetic

Sequence

1, 4, 7, 10, 13, ..

4 − 1 = 3
7 − 4 = 3
10 − 7 = 3
⋮

$a_{k+1} - a_k = d$ (common diff)

Series

1 + 4 + 7 + 10 + 13 ...

Geometric

Sequence

1, 3, 9, 27, 81, 243 ..

$\frac{3}{1} = 3$
$\frac{9}{3} = 3$
$\frac{27}{9} = 3$

$\frac{a_{k+1}}{a_k} = r$ (common ratio)

Series

1 + 3 + 9 + 27 + 81 + 243 + ...

ARITHMETIC SEQUENCE

1, 7, 13, 19, 25, 31, 37 ..

FIND THE COMMON DIFFERENCE,

$$d = a_{n+1} - a_n$$

$$d = 31 - 25 = 6$$

find the n^{th} number

$$t_n = a_1 + (n-1)d$$

$n = 25$;

$$t_{25} = 1 + (24)\,6$$

$$= 145$$

ARITHMETIC SERIES :

1 + 7 + 13 + 19 + 25 + 31 + 37 + ...

FIND THE SUM OF THE FIRST 'n' NUMBERS

$$S_n = \frac{n(a_1 + a_n)}{2}$$ → average of the series

$$S_{10} = \frac{10(1+55)}{2} = \frac{10(56)}{2} = 10(28) = 280$$

GEOMETRIC SEQUENCE

1, 4, 16, 64, 256, 1024, 4096, ...

$$\frac{a_{n+1}}{a_n} = R \text{ (common ratio)}$$

$$\frac{4}{1} = 4 \; ; \; \frac{16}{4} = 4 \; ; \; \frac{64}{16} = 4$$

FIND n^{th} number in series

$$a_n = a_1 (R)^{n-1}$$

$$a_5 = 1(4)^{5-1}$$

$$= 4^4$$

$$a_5 = 256$$

GEOMETRIC SERIES

$$1+4+16+64+256+1024+4096+\ldots$$

FIND THE COMMON RATIO

$$R = \frac{a_{n+1}}{a_n} = \frac{256}{64} = 4 \; ; \; \frac{1024}{256} = 4$$

FIND THE n^{th} NUMBER,

$$\boxed{a_n = a_1 (R)^{n-1}}$$

FIND THE SUM OF THE FIRST 'n' NUMBERS.

$$S_n = \frac{a_1 (R)^n - a_1}{R-1}$$

Soln S_{3-7} (Sum from 3rd term to 7th term)

$$\Rightarrow S_{3-7} = S_7 - S_3$$

$$= \frac{(1)(4)^7 - 1}{4-1} - \frac{(1)(4)^3 - 1}{4-1}$$

$$= \frac{16384 - 1}{3} - \frac{64-1}{3}$$

$$= \frac{16383 - 63}{3} = 5461 - 21$$

$$= 5440.$$

FINITE GEOMETRIC SERIES

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{128}$$

$$R = \frac{a_{n+1}}{a_n} = \frac{\frac{1}{16}}{\frac{1}{8}} = \frac{1}{16} \cdot \frac{8}{1} = \frac{1}{2}$$

$$S_n = \frac{a_1(R)^n - a_1}{R-1} \Rightarrow S_7 = \frac{(1)\left(\frac{1}{2}\right)^7 - 1}{\frac{1}{2} - 1} = \frac{\frac{1}{128} - 1}{-1/2}$$

$$= \frac{127}{64}$$

$$S_\infty = \frac{1\left(\frac{1}{2}\right)^\infty - 1}{\frac{1}{2} - 1} = \frac{\left(\frac{1}{\infty}\right)^{\to 0} - 1}{-1/2} = (-1)\left(\frac{-2}{1}\right) = 2$$

Find 'n' if $a_n = \frac{1}{128}$; given a_1

Functions & Sequences

Functio $h(x) \rightarrow$

$h(x) = \frac{\sqrt{x+10}}{100x^2}$ $\quad h(5)$?.

$\frac{\sqrt{5+10}}{100 \times 5 \times 5}$

$h(x) = \frac{\sqrt{x+10}}{100x^2}$ $\quad h(a+b) = 1$? $\quad a+b$

$\sqrt{(a+b)+10}$ $\quad x = a+b$

$\frac{\sqrt{(a+b)+10}}{100[a+b]^2} = 1.$ $\quad a+b = .$

Sequences:

1, 5, 7, 9, 10 ....

Rep / May Repe

Consecutive / May not be consecutive.

Arithmetic Sequence:

"COMMON DIFFERENCE".

2, 4, 6, 8, 10, 12.

Diff = 2. 3, 6, 9...

4, 8, 12....

Geometric Sequence:

Evenly spaced set:

Sum of Terms $S = \frac{n}{2}(2a + (n-1)d)$.

10, 20, 30, 40 ...

n^{th} term $= a + (n-1)d$.

$d = 10$

a: 1st term.

Median = Ascending order.

n: n^{th} term.

odd no of S: Median = Int. - Int terms

d: difference.

1, 3, 5, 6, 7, 9, 10

even no of S: Median, 1, 2, 4, 5. 1, 2, 4, 5

terms $\frac{5}{2} \times \frac{6}{2} = 3.$

Mean = $\frac{\text{Sum (nos)}}{\text{No of terms}}$

Mean = Median ⟹ Evenly spaced

Mean = $\frac{\text{First} + \text{last term}}{2}$

Sum of such sequences:

Sum = (No of terms) × (Mean)

= 10 × [ ]

= [ ]

No of terms = (Last term − First term) [Inclusive of End points] + 1

2, 4, 6, 8, 10, 12, 14, 16, 18, ...

$\frac{4+16}{2} = \frac{20}{2}$

2 . . 18, ...

4 . . 16.

16 − 4 = 12 = 6 + 1.

6 . . 50.

7 . . 49

49 − 7 + 1

Product of n consecutive int,

= divisible by n!

n=3, $\frac{100 \times 101 \times 102}{3 \times 2 \times 1}$

n=2, $\frac{98 \times 99}{2 \times 1}$

Sum of 'n' consecutive int,

= odd

"ODD" sum will always DIVISIBLE

"EVEN" sum may / may not be.

Geometric Sequence:

"COMMON RATIO".

$2, 2^2, 2^3, 2^4, \ldots$

$\frac{2^2}{2} = 2 \quad \frac{2^3}{2^2} = 2 \quad \frac{2^4}{2^3} = 2$

$n^{th} = a(r^{n-1})$

$n = \text{sum} = \frac{a(r^n - 1)}{r - 1}$

$r^n - 1$

r^{n-1}

1. Trifling - trivial, inconsequential
2. glean - to extract, elicit
3. quashed - Abrogate, cancel, rescind, annul
4. repertoire - repository
5. exactitude - exact
6. copious - Abundant
7. Impassive - emotionless, expressionless
8. reprehend - Admonish. Rebuke, Censure, berate
9. Rambunctious - Uncontrollably exuberant, boisterous
10. Nebulous - Unclear, Amorphous. Cloudy
11. Antiques - old, museum piece
12. Antics - foolish, silly, buffoonery.
13. Rosters - schedules of assigned work
14. Anecdotes - A short amusing or interesting story about a real incident or person
    - tales
15. Sabotage - WRECK, Vandalize, destroy
16. Profanity - blasphemy (or) Obscene language (Dirty words)
17. Expunge - Efface, erase, remove, Purge

18. Motley - Disparate, ~~Eclatt~~ Eclectic, Diverse

19. languorous - Lazy, Lethargy, lassitude (Languish)

20. Abnegation - Renunciation, Abdication, austerity.

21. Verisimilar - Appearing to be true; probable; likely

22. Explicate - Explain, Expound

23. Crooner - A singer who sings sentimental songs in a soft, low voice

24. Contravene - Infringe, Deny, Trangress, FLOUT

25. Peruse - Examine in a thorough and careful manner

26. blandishment - flattery, fawning, Cajolery, GROVEL

27. Injunction - an authoritative warning (or order) dictum, command, exhortation, decree

28. Parochial - Narrow minded, Provincial, Myopic

29. haughty - Arrogantly superior, Disdainful Egotist, Conceit, Condescending ~~Proud~~ Imperious.

30. Impertinent - rude, insolent, impolite, uncivilised discourteous.

31. Cosset - Care for and protect in an overpampered manner,

pamper, spoon-feed, coddle

32. Dandle - Cradle, cuddle, cosset, Pamper

Move up & down in a playful manner.

33. Edify - educate, instruct, indoctrinate

instruct or improve (someone) morally or intellectually

34. Despise - Detest, loathe, abhor, execrate, disdain, spurn

35. chasten - subdue, humiliate, put down Mortify

(= Subdue = "chalu-chalu") = inka apu"

36. Wizened - shrivelled or wrinkled with age

"a wizened, weather beaten old man".

37. Revile - Re + Vile

Re → Repeat vile → gets critized,

Vile → extremely unpleasant, horrid, dreadful, repugnant

condemn, attack, denounce, lambaste

38. benign - Kind hearted, gentle, warm, altruist

39. Cohort - groups, category, batch division

40. Sundry - Several, various, assorted, multifarious
* Some rituals are practised by sundry organisations.

41. Implore - To plead, beg, exhort (someone) to do something, beseech, petition

42. Conjure - Concot, beg someone to do something (implore)

43. Gothicism → horror → Ex: Hotel Transylvania

44. subterfuge - deceit used to achieve one's goal
intrigue, chicanery, pretence
arouse interest / secret plan to do something wrong.
collude

45. Forlorn - Recluse, Unhappy & lonely

46. exoneration - Acquittal

47. Stifle - kill by cutting off supply of air.
choke / suffocate

48. Sombre - dark (or) dull.

49. dearth - Scarcity, Insufficiency
(Dearth low fuel)

50. Euphony (sonorous) - Pleasing sound

51. Rostrum - podium, or raised platform for someone to give speech

52. Abominate - Abhor, hate, Dislike

53 Germane - very appropriate

54. vociferous - express opinions in a strong and confident way, outspoken

54. languor - a pleasant & lazy feeling

54. meek - quite

55. eccentric - Maverick, Iconoclast

56. fiasco - Embarrasment

57. Dawdle - procrastinate
↳ cuddle, pamper

58 forbade - proscribe, ban, preclude, Debar

59 Bereft - Deprived

60. Obsequies - funeral rites, bureal

Obsequious - fawning

61 Tantrums - Outburst of anger, blowout

62. Relics - Antiques, remains, artefact, fossil.

63 Dogma - rules, principle, tenet, maxim

64. LAX - slipshod, negligent, heedless

65. exonerate - ~~expd~~ exculpate, acquit, clear

~~64.~~

~~66.~~ vindicate
(vin + dit case)
↓
when you win the case in court

you are free from accusations.

66. acquiesce - accept something reluctantly but without protest.
to accede, ascent, Quiet

Act QUIEsce, → who is quiet will accept without any protest.

67. Impunity - exonerate, immunity, Amnesty, Exemption

68. VIVAcious - bubbly, ebullient, effervescent scintillating VS LUGRUBIOUS

69. Imperative - important, crucial, mandatory Bossy

70. Veil - cover, blanket, canopy

71. Fitful - intermittent, sporadic, spasmodic

72 Jaded - bored (or) lacking enthusiasm, cloyed, satiated.

73. lackadaisical - lax, uninteresting, careless
languid.

74. rebut - refute, deny, disprove
confute

75. subtle - shap, incisive, razor-like, astute
perspicacious, trenchant

76. - ~~fine~~ fine
- delicately complex & understated

77. degradation - attack (or) plunder, robbery
despoliation.

78. blaze - fire, burn fiercely

79. tantamount - equivalent to

80. Iniquity - Reprobate, vs Probity

81. Cornucopia - ~~abandon~~ abundant

82. hiatus - break, pause, intermission
(Hi+atus) → ~~when~~ after some
gap. if you meet someone, you
say "Hi"

83. factitious - fictitious, artificially developed

84. ignominy - ignoble, public shame, obliquy, opprobrium, humiliation, stigma

85. filial - love blw father & children

avuncular - " " uncle & niece/nephew

sororal - " " sisters

uxorious - " " husband & wife

86. perjury - mendacious, forswear

87. Insidious - stealthy, surreptitious, sneaking, cunning

88. unprecedented - never done before

89. Dissimulation - wrong simulation (pretence, duplicity)

- subterfuge, deceit, double standard

90. callously - heartless, uncaring, cold hearted, hard-bitten

91. shuffling - make soft sound, or move slowly

(~~Duck~~) (Swan)

92. dotage / decrepit - old age, declining years, superannuation

decrepitude

93. cortege - procession, parade, caravan

"victory cortege"

94. mendicant - begger

95. prepossessing - winsome
↳ attractive, winning
allwing, capitivating

root word
mendus
↓
fault

96. Adage - proverb, cliche, platitude.

97. confess - admit, acknowledge, disclose, reveal

98. blighted - infected, diseased, mar, quell, ruin, wreck

99. blithe - care-free, heedless, non-chalant, happy
(flexible) lithe - agile, graceful, loose-limbed.

100. Construe - interpret, decode, understand

101. contort - twist, quirk.

102. Amble - walk or move in a relaxed pace.

103 marauding - ride and plunder, thieving, foray
ravage.

104. rapacious - greedy, avaricious, acquisitive,
voracious

105. serenade - music played at night below GFs
window.

106. Peril - hazard, danger, menace, pitfall

107. felicity - happiness, joy, (Ability to find appropriate
expression for one's thought).

108. Despotic - dogmatic, authoritarian, autocratic

109. protège - student, trainee, apprentice