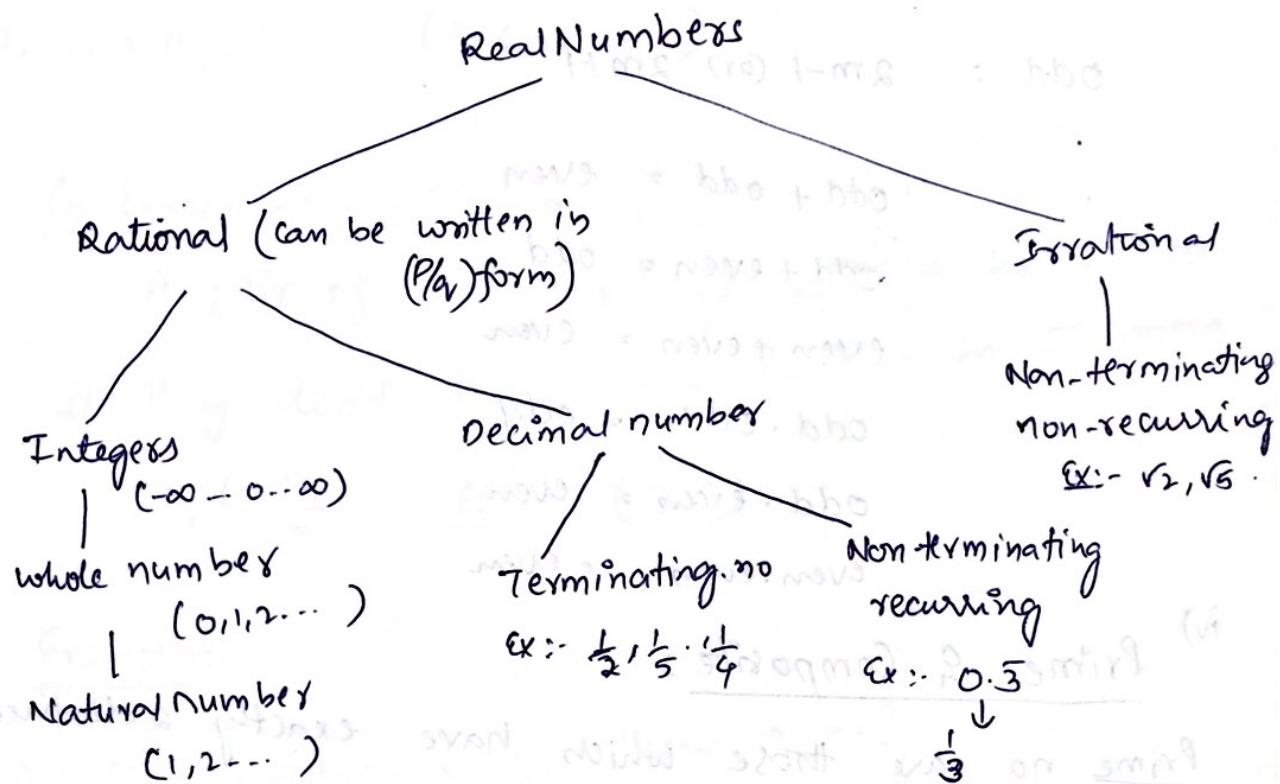


I N D E X

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12 B IT 0077

Number Systems

I. i) Classification



ii) Conversion from Decimal to $(\frac{P}{Q})$ form :-

$$a) y = 0.\overline{333} \dots$$

$$10y = 3.\overline{333} \dots$$

$$\underline{- \quad -}$$

$$-9y = -3$$

$$y = \frac{3}{9} = \frac{1}{3}$$

$$b) y = 0.\overline{1666} \dots$$

$$10y = 1.\overline{666} \dots$$

$$100y = 16.\overline{666} \dots$$

$$\underline{- \quad -}$$

$$90y = 15$$

$$y = \frac{15}{90} = \frac{1}{6}$$

General form :-

$$a) 0.\overline{abcabc} \dots$$

$$0.\overline{abc}$$

$$= \overbrace{\overbrace{\overbrace{\overbrace{abc - a} \overline{990}} \overline{990}} \overline{990}}$$

$$b) 0.\overline{abcccc} \dots$$

$$0.\overline{ab\overline{c}}$$

$$\frac{abc - ab}{990}$$

iii) Odd AND EVEN

General form
Even : $2m$

Odd : $2m-1$ (or) $2m+1$

Odd + Odd = Even

Odd + Even = Odd

Even + Even = Even

Odd . Odd = Odd

Odd . Even = Even

Even . Even = Even

iv) Prime & Composite

Prime no are those which have exactly 2 distinct factors

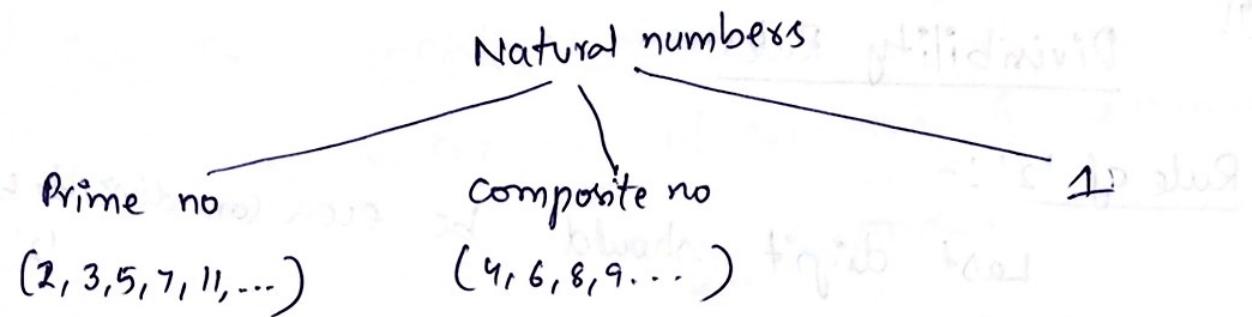
(1 - 100)

25 - Prime number

* → Any prime no. greater than '3' can be written in the form $\underline{(6n-1)}$ (or) $\underline{(6n+1)}$

$$\text{Ex: } 59 \rightarrow 6(10)-1$$

Composite Any natural number having more than 2 distinct factors



Co-Primes :-

A pair of numbers are said to be co-primes if they don't have any factors in common except '1' ex:- (2, 3)

Examples:-

1) if $0.\overline{abab\dots} = \frac{5}{11}$, then $a, b = ?$

$$\text{Sol: } 0.\overline{ab} = \frac{5}{11} \Rightarrow \frac{ab}{99} = \frac{5}{11} \Rightarrow ab = 45$$

$$\Rightarrow a=4; b=5$$

2. If 'p' is a prime no. greater than '3' then what could be remainder if 'p' is divided by 6?

$$\text{Sol: } \frac{6n+1}{6}; \frac{6n-1}{6}$$

$$\Rightarrow \frac{1}{6} = 1 \quad \frac{-1}{6} = 5$$

(1, 5) \rightarrow are remainders

II

Divisibility Rules:

Rule of '2':-

Last digit should be even (or) divisible by 2.

Rule of '4':-

Last two digits should be divisible by 4.

Rule of '8':-

Last three digits should be divisible by 8.

Similarly for 16, 32 - -

Rule of '3':-

Sum of digits should be divisible by 3.

Rule of '9':-

Sum of digits should be divisible by 9.

Rule of '7':-

Double the last digit, subtract from rest of number, the obtained number should be divisible by 7.

'7':

$$\text{Ex: } 672 \Rightarrow 67 - 4 = 63/7 = 0$$

Rule by '11':-

(sum of unit's place - sum of ten's place)

should be divisible by 11.

$$\text{Ex: } \begin{array}{r} 567896 \\ 707070 \end{array}$$

$$(6+8+6) - (9+7+5) =$$

Rule for composite Numbers :-

Find the co-primes of numbers, & the number should be divisible by co-primes

Ex:-



III

Cyclicity of Unit's Place :-

- 1) a; b, c, d, a,b, c, d, a, b, c, d, a, b, ...

14th term = ? cycle of '4'

$$\frac{14}{4} = \frac{\text{remainder}}{2} \Rightarrow \text{2nd term} \Rightarrow \text{'b'}$$

2)

$$12\textcircled{4} \times 17\textcircled{7}$$

unit's digit depends only on last digits

$$\Rightarrow 4 \times 7 = 2\textcircled{8} \rightarrow \underline{\text{units place of product}}$$

$$\text{Ex:- } 786\textcircled{7} \times 45\textcircled{2} \times 32\textcircled{1}$$

$$7 \times 2 \times 1 = \underline{1\textcircled{4}}$$

3)

$$\text{units place of } 2^{61}, 5^{32} \dots ?$$

$2^1 = 2$	$3^1 = 3$	$4^1 = 4$ even	$5^1 = 5$
$2^2 = 4$	$3^2 = 9$	$4^2 = 6$	$5^2 = 5$
$2^3 = 8$	$3^3 = 7$	$4^3 = 4$	$5^3 = 5$
$2^4 = 6$	$3^4 = 1$	$4^4 = 6$	$5^4 = 5$
$2^5 = 2$	$3^5 = 3$	$4^5 = 4$	$5^5 = 5$
$2^6 = 4$	$3^6 = 9$	$4^6 = 6$	$5^6 = 5$
$2^7 = 8$	$3^7 = 1$	$4^7 = 4$	$5^7 = 5$
$2^8 = 6$	$3^8 = 3$	$4^8 = 6$	$5^8 = 5$
\vdots	\vdots	\vdots	\vdots
\downarrow cycle of '4'	\downarrow cycle of '4'	\downarrow cycle of '2'	\downarrow cycle of '1'
cycle of '4' - $2^n, 3^n, 8^n, 7^n$			
cycle of '2' - $4^n, 9^n$			
cycle of '1' - $1^n, 5^n, 6^n$			

Ex :-

$$\therefore \text{units place of } 1) \quad 2^{78} \quad \frac{78}{4} = \frac{\text{Remainder}}{2} = 2 \Rightarrow 4$$

12⁵⁰
only last digit

Addition of numbers in powers

$$\begin{array}{c}
 & + \\
 2^1 = 2 & + 8 = 8^1 \\
 2^2 = 4 & + 4 = 8^2 \\
 2^3 = 8 & + 2 8^3 \\
 2^4 = 6 & + 6 = 8^4
 \end{array}$$

$$2^n + 8^n = \underset{\text{if } n \text{ is odd}}{=} 0 \text{ unit place}$$

$$3^n + 7^n = 0$$

$$4^n + 6^n = 0$$

$$1^n + 9^n = 0$$

Ex:- $872^{915} \times 733^{700} \times 147^{140}$

$$\begin{array}{c|c}
 2^3 \times 3^4 \times 7^4 & 8+1+1 \\
 8 \cdot 1 \cdot 1 & = 0 \\
 & \text{addition}
 \end{array}$$

multiplication

Ex:-

$$122^{201} - 78^{86}$$

$$2 - 4$$

$$- - 2$$

$$- 8 \rightarrow \text{unit place}$$

Ex:

$$\text{units place of } 17^{23}$$

$$= 1^{23} = 1$$

$$17^1 = 7$$

Ex:

$$19^{18} 19$$

Q

Ten's place of $(5^1 \times 2^{20})$

$$(5^1 \times 2^1) \times (2^{19})$$

\downarrow \leftarrow
 • 0 \rightarrow unit's place

If '5' & '2' \rightarrow unit's place $\Rightarrow 0$

$$(5^1 \times 2^1) \times 2^{19}$$

units place will be ten's place

$$\underline{-80}$$

$$\text{i) } (5^2 \times 2^5)$$

thousands place

$$(5^2 \times 2^2) \times 2^3$$

$$\underline{800}$$

IV

Factorisation

a) factorise a number to prime powers

Ex:- $864 = 8 \times 108$

$$= 2^3 \times 4 \times 27$$

$$= 2^3 \times 2^2 \times 3^3$$

$$\Rightarrow 2^5 \times 3^3$$

Uses of Factorisation:-

1. Even or odd (If '2' is multiple)

2. Divisibility

3. Perfect square, Cube, fourth etc.
(powers = even) (multiples of 3)

4. Units place & trailing zeroes

Ex: By which least number should 1440 be divided
so that quotient is an odd number?

Sol:- $1440 = 12 \times 12 \times 10$

$$= 3 \times 2^2 \times 3 \times 2^2 \times 5 \times 2$$

$$= 2^5 \times 3^2 \times 5$$

$$\Rightarrow \frac{2^5 \times 3^2 \times 5}{2^5} \Rightarrow 3^2 \times 5 \Rightarrow \text{odd}$$

$\Rightarrow "2^5"$ should be divided

- Prime no Power Prime no Power
- \times prime no
- 1) if powers are multiples of '2' then perfect squares
 - 2) if powers are multiples of '3' then perfect cubes
 - 3) if powers are both multiples of '2' & '3'
then p. squares as well as p. cubes

Ex:- By which least natural no. should 720 be multiplied so that resultant is a cube as well as a square?

Sol:

$$720 = 2^4 \times 3^2 \times 5$$

$$= 2^2 \times 3 \times 2 \times 3 \times 5 \times 2$$

$$= 2^5 \times 3^2 \times 5$$

$$\Rightarrow 2^4 \times 3^5 \times 5^5 \Rightarrow \text{should be multiplied}$$

QX:- no. of trailing zeroes $((280)^5 \times 12^3 \times 18^2)$

$$\Rightarrow (5^5 \times 2^13) = 10^{13}$$

V

Factorials

$$0! = 1 ; 1! = 1 ; 2! = 2 ; 3! = 6$$

$$\begin{aligned}4! &= 24 \\&= 4 \times 6 \\&= 2^2 \times 3 \times 2 \\&= 2^3 \times 2\end{aligned}$$

$$\begin{aligned}90 &= 9 \times 10 \\&= 3^2 \times 5 \times 2 \\&= 2 \times 3^2 \times 5\end{aligned}$$

$$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$$

$$= 2^? \times 3^? \times 5^? \times 7^? \times 11^?$$

$$15! = 2^? \times 3^? \times 5^? \times 7^? \times 11^? \times 13^?$$

i) ~~Ques.~~ What is the highest power of '2' in $10!$?

Sol:- multiples of '2' = $\frac{10}{2} = 5$

$$\frac{5}{2} = 2$$

$$\frac{2}{2} = 1$$

$$\frac{1}{8}$$

'2' is the highest power of '2' in $10!$

ii) highest power of '3' in $30!$

$$\frac{30}{3} = 10$$

$$\frac{10}{3} = 3$$

$$\frac{3}{3} = 1$$

'3' is the highest power

Factorise

10!

~~10! = 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1~~
2, 3, 5, 7 → are prime no. under '10'

$$\begin{array}{rcl}
 \frac{10}{2} = 5 & \frac{10}{3} = 3 & \frac{10}{5} = 2 \\
 \frac{5}{2} = 2 & \frac{3}{3} = 1 & \underline{\quad} \\
 \frac{2}{2} = 1 & & \underline{\quad} \\
 & \underline{4} & \\
 & \underline{8} &
 \end{array}$$

$$10! = 2^8 \times 3^4 \times 5^2 \times 7^1$$

Ex:- highest power of '4' in 10!

Sol:-

$$\begin{array}{r}
 2^8 \\
 \diagdown \quad \diagup \\
 2 \quad 2
 \end{array}$$

$(2^2)^4 = 4$ is the highest power of '4'

Ex:- highest power of 6! in 50!

Sol:-

$$\begin{array}{rcl}
 6! = \frac{6}{2} = 3 & \frac{6}{3} = 2 & \frac{6}{5} = 1 \quad 6! = 2^4 \times 3^2 \times 5^1 \\
 \frac{3}{2} = 1 & \underline{\quad} & \underline{\quad} \\
 & \underline{4} &
 \end{array}$$

50!

$$\begin{array}{rcl}
 \frac{50}{2} = 25 & \frac{50}{3} = 16 & \frac{50}{5} = 10 \\
 \frac{25}{2} = 12 & \frac{16}{3} = 5 & \frac{10}{3} = 2 \\
 \frac{12}{2} = 6 & \frac{5}{3} = 1 & \underline{\quad} \\
 \frac{6}{2} = 3 & & \underline{12} \\
 \frac{3}{2} = 1 & & \\
 & \underline{22} & \\
 & \underline{47} &
 \end{array}$$

$50! = 2^{47} \times 3^{22} \times 5^{12}$
 \Rightarrow

$$= 2^{\frac{47}{4}} \times 3^{\frac{22}{2}} \times 5^{12}$$

$$= \cancel{2}^1 \times \cancel{3}^1 \times 2^4 \times 3^2 \times 2^3 \times 5^{12}$$

$$(2^4 \times 3^2 \times 5^1)^{11} \times 2^3 \times 5^1$$

$\Rightarrow (6!)^{11} \Rightarrow 11$ is the highest power 6! in

so!

Ex:- what is the highest power of 5 that will divide
the product of first 50 multiples of 5?

$$\text{Sol:- } \frac{(5 \times 1) \times (5 \times 2) \times (5 \times 3) \times (5 \times 4) \dots}{5^?}$$

$$= \frac{5^{50} \times 50!}{5^?}$$

$$\frac{50}{5} = 10$$

$$\frac{10}{5} = 2$$

$$\underline{12}$$

$$= \frac{5^{50} \times 5^{12}}{5^{?}} = \frac{5^{62}}{5^x} \Rightarrow x = 62$$

Ex:- what will be the remainder when

$$1! + 2! + 3! + 4! + 5! + \dots + 100! \quad \text{is divided by 4}$$

$$1+2+6$$

$$= \frac{9}{4} = 1 \text{ is the remainder}$$

Ex: Find the least value of n such that $n!$ is

divisible by 910

Sol:-

$$\frac{n!}{9!0!} = \frac{n!}{7 \times 10 \times 13!} \quad \text{because '13!' should be there in numerator.}$$

Ex:- Find the unit's place of

$$2^{1!+2!+3!+\dots+10!} + 3^{1!+2!+3!+\dots+10!}$$

= 2" → cycle of "4"

$$\frac{1! + 2! + 3! + 4! + \dots + 10!}{4} = \frac{(1+2+6)}{4} = (1+6) = 7$$

$$2' + 3' + 4 \xrightarrow{\text{P}^{\text{odd}}} 5 + 6 + 7 + 8 + \dots$$

$$\rightarrow \partial = -\frac{q}{2} \times$$

VI

Number of factors

factors are those which completely divide a no.

factors of 6 - 1, 2, 3, 6

" " 10 - 1, 2, 5, 10

" " 24 - 1, 2, 3, 4, 6, 8, 12, 24

factors of 24 - $2^3 \times 3^1$
 $2^0, 2^1, 2^2, 2^3$ $3^0, 3^1$

$$4 \times 2 = 8 \text{ factors}$$

~~so~~

$$= (3+1)(1+1)$$

$$= 4 \times 2 =$$

$$N = P_1^a \times P_2^b \times P_3^c \times P_4^d \times \dots$$

$$\text{no. of factors} = (a+1)(b+1)(c+1)(d+1) \dots$$

Ex:- if $N = 2^5 \times 3^4 \times 5^2$ then

Q How many factors of 'N' are

a) Prime - 3 (2, 3, 5)

$$b) \text{Composite} - 90 - 3 = (87 - 1) = 86$$

$$c) \text{odd} - (4+1)(2+1) = 15$$

$$d) \text{even} - 90 - 15 = 75$$

$$\text{Sol: Total no. of factors} = (5+1)(4+1)(2+1)$$

$$= 6 \times 5 \times 3 = 90$$

Ex:- If $N = 2^5 \times 3^4 \times 5^2$ then

i) How many factors of N are $\nmid 3$

1) Perfect cubes

2) Perfect Squares

3) will end up with atleast one zero

1) Perfect cubes,

$$\begin{array}{c} 2^5 \\ 2^0, 2^3 \\ \times \end{array} \quad \begin{array}{c} 3^4 \\ 3^0, 3^3 \\ \times \end{array} \quad \begin{array}{c} 5^2 \\ 5^0 \\ \times \end{array}$$

$$2 \times 2 \times 1 = 2^4$$

2) Perfect squares:

$$2^0, 2^2, 2^4, 3^0, 3^2, 3^4, 5^0, 5^2$$

$$3 \times 3 \times 2 = 18$$

3) atleast one-zero,

$$\begin{array}{c} 2^5 \\ 2^0, 2^2, 2^3, 2^4, 2^5 \\ \times \end{array} \quad \begin{array}{c} 3^4 \\ (3^0, 3^3) \\ \times \end{array} \quad \begin{array}{c} 5^2 \\ 5^0, 5^2 \\ \times \end{array}$$

$$5 \times 5 \times 2 = 50$$

Ex: How many factors of 540, 360 & 900 are common to each other?

Ex:- How many factors of $(2^6 \times 3^4 \times 5^3)$ are multiples of 4 as well as perfect cubes?

Ex:- How many factors of $(12^2 \times 15^3 \times 20^2 \times 36)$ are perfect cubes but not squares

Application of no. of factors:

factors of $12 : \{1, 2, 3, 4, 6, 12\}$ } even no. of factors

factors of $35 : \{1, 5, 7, 35\}$

factors of $36 : \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ } perfect square odd no. of factors

factors of $45 : \{1, 3, 5, 9, 15, 45\}$ } even no. of factors

Ex: If there are 8 factors between 1 and \sqrt{N} ,

then find the total no. of factors of N .

Sol:-



$8+8+2 = 18$ factors if 'N' is not a perfect square

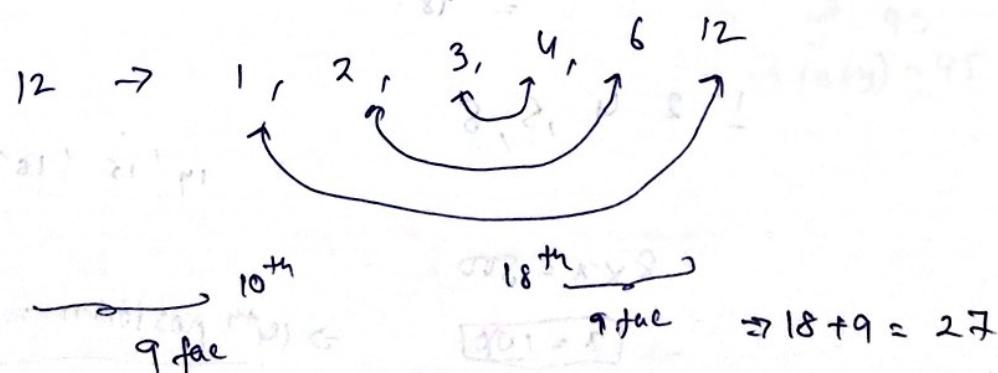
$18+1 = 19$ factors if 'N' is a perfect square

Ex: If the product of factors which are at 10^{th} position

and 18^{th} position result in the number, N itself

find how many factors of N are there?

Sol:-



10^{th}

9 fac

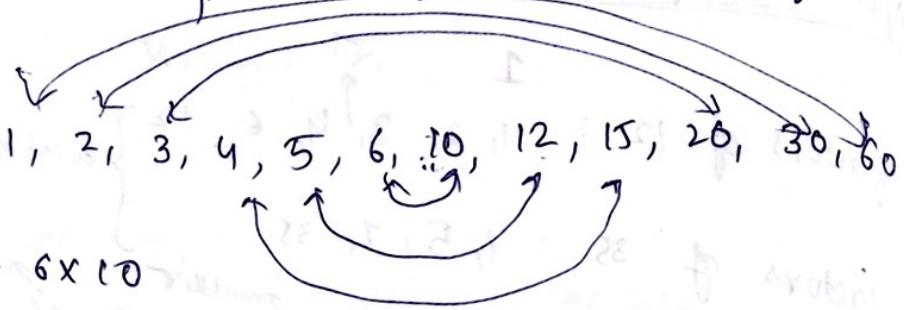
18^{th}

9 fac

$$\Rightarrow 18 + 9 = 27$$

Ex: what is the product of all factors of 60

Sol:-



$$60 = 6 \times 10$$

$$= 2 \times 3 \times 5 \times 2$$

$$= 2^2 \times 3 \times 5$$

$$\text{no. of factors} = 3 \times 2 \times 2$$

$$= 12$$

$$\Rightarrow \frac{\text{no. of factors}}{2}$$

if 'N' is perfect square, then

$$\text{ex:- } \frac{\text{no. of factors}}{(N)^{\frac{1}{2}}} \cdot \sqrt{N}$$

which factor of 800 will occupy 14th position if all the factors of 800 are written in ascending order?

Sol:-

$$800 \rightarrow 8 \times 10 \times 10$$

$$= 2^3 \times 2 \times 5 \times 2 \times 5$$

$$= 2^5 \times 5^2$$

$$\text{no. of factors} = (5+1) \times (2+1)$$

$$= 18.$$

$$1, 2, 4, 5, 8$$

$$8 \times n^{th} = 800$$

$$\Rightarrow n = 100$$

14th is 18th

14th position = 100

Ex: In how many ways can 240 can be written as product of two integers? [may be distinct]

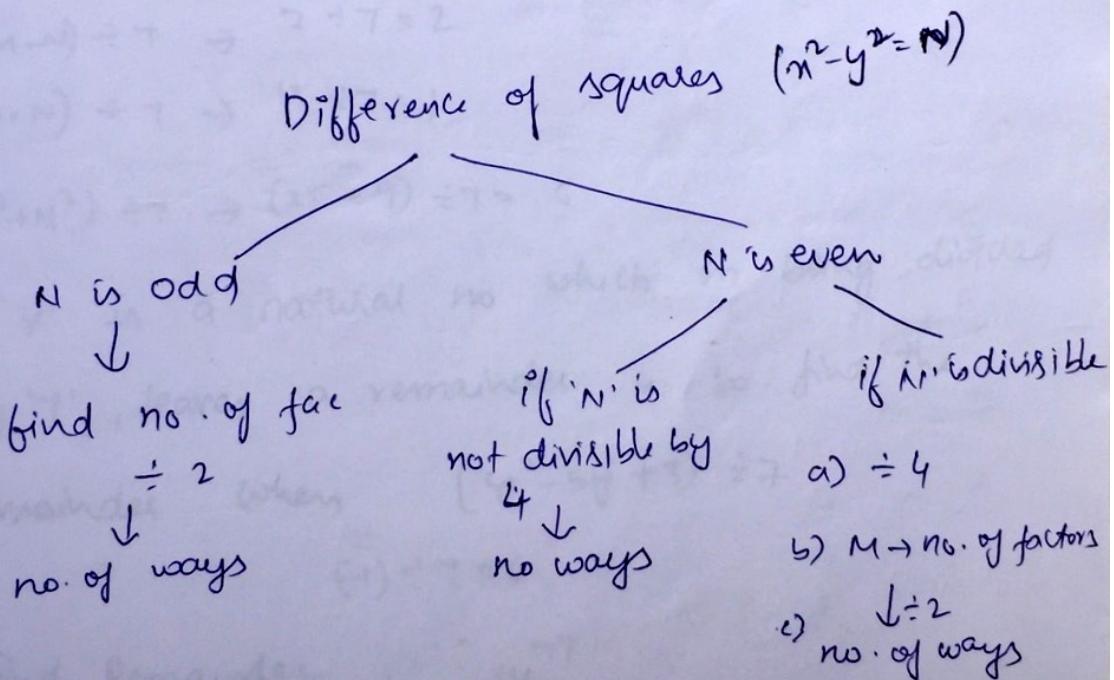
Sol:- $240 = 12 \times 2 \times 10$
 $= 3 \times 4 \times 2 \times 5 \times 2$
 $= 2^4 \times 3 \times 5 \Rightarrow (5) \times 2 \times 2 = 20 \text{ factors}$

$\frac{20}{2} = 10$ ways of natural no.

20 ways of integers (integral)

if P. square
 $10 + 1 \rightarrow$ nat. ways
 $20 + 2 \rightarrow$ integral ways

Q



Ex: In how many ways can 45 be written as difference of squares of two natural numbers?

Sol:- $45 \times = 5 \times 9$
 $= 3^2 \times 5$

$$x^2 - y^2 = 45$$

$$(x-y)(x+y) = 45$$

no. of fac = $3 \times 2 = \frac{6}{2} = 3$ ways

Q. find integral solutions to $\frac{1}{a} + \frac{1}{b} = \frac{1}{8}$

soli:-

$$(a-8) \cdot (b-8) = 64$$

1

7 factors

↓

G factors

↓

3 nat no way

1

6 nat. no 80/13

十一

$\pm \rightarrow (P, S)$

$$\begin{array}{l} \checkmark \quad (\text{longer, } 12 \text{ integral soln's}) \\ (8 \times 8) \\ \cancel{x(8 \times 8)} \\ \hline \end{array}$$

+
 $\boxed{1} \rightarrow (p.s)$
 per sq

12 Integral sol'n's
 $\int \rightarrow (P.S)$
 P.E. - S.Q.

$$\mathbb{F} \rightarrow (\mathbf{p}, \mathbf{s})$$

(11)

Remainder

$$\frac{1}{6} = 1 \quad ; \quad \frac{5}{6} = 5 \text{ (Q) } (1) \text{ (R)}$$

$\begin{array}{r} \text{div}) N \text{ (quo} \\ \hline \text{Rem} \end{array}$

$$\begin{array}{l} \text{N} = (\text{divisor} \times \text{quotient}) + (\text{remainder}) \\ \text{dividend} \end{array}$$

$$M \div 7 \rightarrow \overline{5}^{\text{rem}}$$

$$N \div 7 \rightarrow 3$$

$$(M+N) \div 7 \rightarrow 8 \div 7 = 1$$

$$(M-N) \div 7 \rightarrow 2 \div 7 = 2$$

$$(M \cdot N) \div 7 \rightarrow 15 \div 7 = 1$$

$$(M^2+N^2) \div 7 \rightarrow (25+9) \div 7 = 6$$

Ex:- 'y' is a natural no which on being divided by '7', leaves a remainder 4, so. find the remainder when $(y^2 - 5y + 3) \div 7$

$$(1) \div 7 = 0$$

Ex:- find Remainder , $\frac{14^{77}}{11}$

$$= \underbrace{14 \times 14 \times 14 \cdots}_{11} \times 14_{17 \text{ times}}$$

$$= \underbrace{3 \times 3 \times \cdots}_{11}^{74 \text{ times}} \times 3^{77} \div 11$$

form cycle,

$$3^1 \div 11 = 3$$

$$3^2 = 9 \div 11 = 2$$

$$3^3 = 6 \div 11 = 6$$

$$3^4 = 18 \div 11 = 4$$

$$3^5 = 12 \div 11 = 5$$

$$3^6 = 15 \div 11 = 1$$

$$3^7 = 8 \div 11 = 3$$

cycle of '6'

$$3^{77} = \overbrace{3}^{3 \text{ rem}} \overbrace{22}^{5} = \overbrace{3}^{3 \text{ rem}}$$

$$3^1 \div 11 = 3 \div 11 = 3$$

$$3^2 \div 11 = 9 \div 11 = 2$$

$$3^3 \div 11 = 27 \div 11 = 5$$

$$3^4 \div 11 = 15 \div 11 = 4$$

$$3^5 \div 11 = 12 \div 11 = 1$$

∴ cycle of '5'

$$77 \div 5 = 2$$

$$3 = \overbrace{1}^{1 \text{ rem}} \overbrace{1}^{1 \text{ rem}}$$

∴ ~~Remainder~~ $\Rightarrow 3^2 \Rightarrow 9$ is the remainder

$$3^{80} = \overbrace{3}^{3 \text{ rem}} \overbrace{27}^{9} = \overbrace{3}^{3 \text{ rem}}$$

if '1' comes, then it is
stop of the cycle

$$3^{80} = \overbrace{3}^{3 \text{ rem}} \overbrace{27}^{9} = \overbrace{3}^{3 \text{ rem}}$$

$$= 80 \div 5 = 0$$

5th term $\Rightarrow 1$
 $\Rightarrow 1$ is the remainder

if '1' comes, then the
cycle length is double

Ex... if '1' at 3, then
cycle length = '6'

Objective : Frame such a cycle which gives a remainder of $\equiv 1 \pmod{7}$

Q.

$$19^{200} \pmod{7}$$

$$\Rightarrow 5^{200} \pmod{7}$$

a

$$5^1 = 5 \pmod{7} = 5$$

$$5^2 = 25 \pmod{7} = 4$$

$$5^3 = 20 \pmod{7} = 6 \quad (\text{double the cycle length})$$

here, we will get 1

$$5^4 = 30 \pmod{7} = 2$$

$$5^5 = 10 \pmod{7} = 3$$

$$5^6 = 15 \pmod{7} = 1$$

$$\Rightarrow \frac{200}{6} = 200/6 = 2$$

second term of cycle remainder

$\Rightarrow 4$

Q A Number, n when divided by 437, leaves a remainder of 150. Find the remainder when same no. is divided by 23.

$$n \div 437 = 150$$

$$n = 437 \times a + 150 \quad \text{'o' remainder}$$

$$n \div 23 \Rightarrow \frac{n}{23} = \frac{437 \times a}{23} + \frac{150}{23} \Rightarrow \frac{n}{23} = \frac{150}{23} = 12$$

$$\therefore \text{Remainder} = 12$$

$$\text{when, } \frac{15^{23} + 23^{23}}{19}$$

Q find the remainder

sol:- grouping

$$\frac{15^{23}}{19} \quad \frac{(-4)^{23}}{19}$$

$$\frac{(23)^{23}}{19} \quad \frac{(4)^{23}}{19}$$

$$\Rightarrow \frac{-4^{23} + 4^{23}}{19} = 0$$

Negative Remainder

$$\frac{19^{93}}{5} = \frac{(-1)^{93}}{5} = -1 \equiv 4$$

$$\frac{20^{23}}{5} = 0$$

$$\text{Ex, } \frac{21^{23}}{5} \equiv 1$$

$$\frac{2^{100}}{9} \text{ bec } \frac{(2^3)^{33}}{9} \text{ fo } \frac{8^{23}}{9} \cdot 2 \equiv \frac{(-1)^{23} \cdot 2}{9}$$

$$\Rightarrow \frac{-2}{9} \equiv 7$$



$$\text{Q} \quad \frac{2^{100}}{17} = \frac{(2^4)^{25}}{17} = \frac{(16)^{25}}{17} = \frac{(-1)^{25}}{17} = \frac{-1}{17} = 16$$

\Rightarrow 'N' is a natural number, when divided by a divisor leaves a remainder of 24. When $2N$ is divided by same divisor, remainder is 11. Find the value of divisor.

Sol:-

$$\begin{aligned} N &= x.a + 24 \rightarrow \textcircled{1} \\ 2N &= x.b + 11 \rightarrow \textcircled{2} \end{aligned}$$

$$\textcircled{1} \times 2 \Rightarrow 2N = 2x.a + 48$$

$$\Rightarrow x.b + 11 = 2x.a + 48$$

$$x.(b - 2a) = 37$$

Product of two natural numbers

\Rightarrow factors concept

$$\Rightarrow (1 \times 37) \text{ or } (37 \times 1)$$

$$\Rightarrow x = 37$$

$$\frac{37}{17} = \frac{4}{11} \quad \frac{4}{11} = \frac{1}{11}$$

$$23^{21} \stackrel{17}{\not{\equiv}} \left(23^{21}\right)^{17}$$

* while finding unit's digit, we come from top to bottom

Ex:

$$23^{21} \stackrel{17}{\not{\equiv}} \Rightarrow 23^1 \Rightarrow 3^1 = 3$$

* but while finding remainder we go from bottom to top

Ex:

$$\begin{array}{r} 23^{21} \\ \hline 11 \end{array} \stackrel{17}{\not{\equiv}} \frac{14^{21}}{11} \stackrel{39}{\not{\equiv}} \frac{3^{21}}{11} \stackrel{39}{\not{\equiv}}$$

$$\cancel{23^{21}} \rightarrow 3^1 \div 11 = 3 \text{ R } 0$$

$$3^2 \div 11 = 9 \div 11 = 9$$

$$3^3 \div 11 = 27 \div 11 = 5$$

$$3^4 \div 11 = 15 \div 11 = 4$$

$$(1+55) \rightarrow (1+2) \div 11 = 1$$

cycle of 5

$$\Rightarrow \frac{3^{21}}{11} \stackrel{39}{\not{\equiv}} \frac{3^N}{11} = \frac{N}{5} = \frac{21}{5}$$

$$\Rightarrow \frac{1}{5} = \frac{1}{5} \stackrel{\text{remainder}}{\not{\equiv}} \frac{3^1}{11}, 3$$

⇒ remainder $\underline{\underline{= 3}}$

$$\frac{40}{12} \Rightarrow \text{rem } 4 \quad (\text{we are asked to make group of '12'})$$

↓

$$\frac{10}{3} \Rightarrow 1 \quad (\text{we are making group of '3'})$$

Ex:

$$\frac{17^{100}}{51}$$

$$17 \div 51 = 17$$

$$17^2 \div 51 = 289 \div 51 = ?$$

tedious

$$\frac{17 \times 17^{99}}{17 \times 3}$$

$$\frac{17^{99}}{3}$$

↓
remainder $\times 17$

the number
which we have
cancelled should
be multiplied

$$\Rightarrow \frac{17^{100}}{51} = \text{remainder of } \left(\frac{17^{99}}{3}\right) \times 17$$

Finding last two Digits

i) Divide by '100'

$$\frac{15^{40}}{100}$$

$$= \frac{15^3 \times 15^{38}}{5 \times 5 \times 4}$$

$$= \frac{9 \times 15^{38}}{4}$$

$$\frac{9 \times (-1)^{38}}{4}$$

rem

$$1 \times$$

$$\downarrow$$

$$1 \times 5 \times 5$$

multiply what you
have & cancel

$$= 25,$$

Using

Binomial Theorem

$$(a+b)^n = n_{c_0} a^n b^0 + n_{c_1} a^{n-1} b^1 + n_{c_2} a^{n-2} b^2 + \dots$$

$$n_{c_{n-1}} a^{n-1} b^{n-1} + n_{c_n} a^{(n-n)} b^n$$

→ observe patterns

$$n_{c_0} = 1 \quad ; \quad n_{c_n} = 1$$

$$n_{c_1} = n \quad - n_{c_{n-1}} = n$$

$$n_{c_n} = \frac{n!}{(n-n)! n!}$$

Q Find last two digits of 41^{47}

Sol:

$$(1+40)^{47} = {}^{47}C_0 1^{47} \cdot 40^0 + {}^{47}C_1 1^{46} \cdot 40^1 +$$

$$(80+1)^{47}$$

$$(1+40)^{47}$$

$${}^{47}C_0 1^{47} \cdot 40^0 + {}^{47}C_1 1^{46} \cdot 40^1 +$$

$${}^{47}C_2 1^{45} \cdot 40^2 + \dots$$

$$\boxed{(10+50)+100+1000+10000+\dots}$$

→ last '2' digits depends on because we are getting ~~two~~ '2' zeroes afterwards

Only for above prob, 40^2 (we get 2 zeroes)

and last digits does not depend

$$\Rightarrow (1+40)^{47} = (1) + (47 \times 40)$$

$$\begin{array}{r} 47 \\ \times 40 \\ \hline 1880 \\ +40 \\ \hline 1880 \end{array}$$

$\Rightarrow 1881$
→ last two digits

$$\underline{\underline{69}}^{50} \Rightarrow (-1+70)^{50}$$

$$= {}^{50}C_0 (-1)^{50} (70)^0 + {}^{50}C_1 (-1)^{49} (70)^1 +$$

$$= 1 + 50 (-1) \times 70$$

$$\Rightarrow 71 - 3500$$

$$\Leftarrow \cancel{3500} - 3499$$

$$\begin{array}{r} 3500 \\ - 3499 \\ \hline 1 \end{array}$$

$$\Rightarrow 10000$$

$$\begin{array}{r} 3499 \\ \hline 6499 \end{array}$$

$$\begin{array}{r} 10000 \\ 3499 \\ \hline 6501 \end{array}$$

\Rightarrow ~~99~~ \rightarrow last two digits

\Rightarrow '01' last two digits

Q

$$(37)^{64}$$

\hookrightarrow last digits $\neq 1, 9$

$$(37^2)^{32} \cdot 37 = (1369)^{32} \cdot 37$$

\rightarrow last two digit of a number depend only on last two digits

$$\Rightarrow (69)^{32}$$

$$= (1+70)^{32}$$

Q

$$(43)^{37} = (43^2)^{18} \cdot 43^1$$

$$= (49)^{18} \times 43^1$$

last two digits

to calculate the

last digit

$\div 100$

unit's place

2, 4, 6, 8, 5

1, 9, 3, 7

Binomial theorem

$$\frac{44^{70}}{100}$$

(01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99)

(01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99)

(201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1839, 1840, 1841, 1842, 1843, 1844, 1845, 1846, 1847, 1848, 1849, 1849, 1850, 1851, 1852, 1853, 1854, 1855, 1856, 1857, 1858, 1859, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1879, 1880, 1881, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1889, 1890, 1891, 1892,

LCM of a set of numbers will be the least number which is divided by all the numbers in that given set.

GCD/HCF
(Highest common factor)

HCF of (24, 36 and 108)

24 - 1, 2, 3, 4, 6, 8, 12, 24

36 - 1, 2, 3, 4, 6, 9, 12, 18, 36

108 - 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108

~~KC = C~~

$$\text{HCF}(24, 36, 108) = 12$$

In general,

1) Factorise given numbers

$$24 = (2^3) \times (3^1)$$

$$36 = (2^2) \times (3^2)$$

$$108 = (2^2) \times (3^3)$$

Take the common factors, which are present in all the numbers \Rightarrow $\underline{\underline{2^2 \times 3^1}} = 4 \times 3 = 12$

Q if HCF of two natural numbers is 12. product of these two natural numbers is 1728. How many pairs of number are possible?

Sol:

$$(12a) \times (12b) = 1728$$

$$ab = \frac{1728}{144} = 12$$

$$ab = 12 \quad [\because a, b \text{ are co-primes}]$$

$$\curvearrowleft 1 \times 12$$

$$\curvearrowleft 3 \times 4$$

$$\times 2 \times 6$$

$$(1, 12) \Rightarrow (12, 144)$$

$$\Rightarrow (3, 4) \Rightarrow (36, 48)$$

Q find the greatest number that on dividing 150, 194 & 245 leaves remainder of 6, 2 & 5 respectively

$$\text{Sol: } 150 = N \times a + 6 \Rightarrow Na = 144$$

$$N \times b = 192$$

$$194 = N \times b + 2$$

$$N \times c = 240$$

$$245 = N \times c + 5$$

N is the H.C.F of 144, 192, 240

$$144 = 12 \times 2^2 \times 3$$

$$192 = 12 \times 2^4$$

$$240 = 12 \times 2^2 \times 5$$

$$\therefore \underline{\text{H.C.F}} = 48$$

$$\Rightarrow \underline{N = 48}$$

Q Find the greatest number that on dividing 480, 600 & 800 leaves the same remainder in each case.

Sol:

$$480 = N \times a + r \rightarrow ①$$

$$600 = N \times b + r \rightarrow ②$$

$$800 = N \times c + r \rightarrow ③$$

eliminate 'r'

$$③ - ② \quad 200 = N(c-b)$$

$$③ - ① \quad 120 = N(b-a)$$

$$③ - ① \quad 320 = N(c-a)$$

H.C.F of (200, 120, 320)

$$120 = 10 \times 12 = 10 \times 2^2 \times 3$$

$$200 = 10 \times 20 = 10 \times 2^2 \times 5$$

$$320 = 10 \times 32 = 10 \times 2^5$$

Q 48 rose plants, 72 marigold plants & 108 lotus plants have to be planted in rows such that each row has equal no of plants & each row has plants of a particular variety only.

What is the least no of rows required?

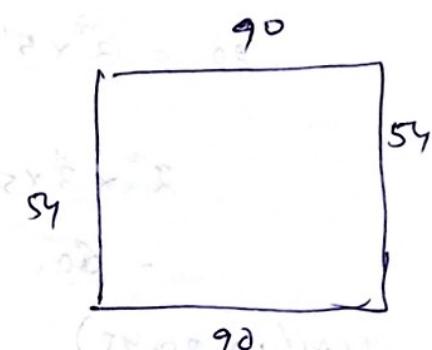
Sol:

$$\text{H.C.F}(48, 72, 108)$$

Q A rectangular cloth of (54×90) units is to be cut into equal squares. Find how many min. no of squares pieces are required such that no cloth is wasted.

Sol: HCF $(54, 90)$

= $\boxed{18 \times 18}$ \rightarrow dimension



Q. If the HCF of $120 + N$ is 8, how many two digit values could 'N' assume.

Sol:

$$120 = 8 \times 2$$

$$\boxed{x = 15}$$

$$N = 8 \times y$$

$$y = 1, 2, 4, 7, 8, 11$$

\Rightarrow ~~8~~ \Rightarrow 5 possible 2 digit no.

In Proper Fraction:-

A proper fraction is a fraction, whose numerator is less than denominator.

Ex:- $\frac{5}{11}, \frac{7}{9}$

Improper Fraction:-

A fraction, in which numerator is equal or greater than denominator is called Improper Fraction.

Ex:- $\frac{56}{18}$

Improper fractions can be expressed as mixed fractions
Ex: $\frac{56}{18} = 3\frac{2}{18}$

L.C.M

L.C.M (6, 12, 20)

$$6 = 2^1 \times 3^1$$

$$12 = 2^2 \times 3^1$$

$$20 = 2^2 \times 5^1$$

$2^2 \times 3^1 \times 5^1$ (Take the highest power, by
= 60.

L.C.M (12, 30, 45)

$$12 = 2^2 \times 3^1$$

$$30 = 2 \times 3 \times 5$$

$$45 = 3^2 \times 5$$

$$= 2^2 \times 3^2 \times 5^1 = 180$$

} another method,
take multiples of
highest no and
start checking

Ex:-

traffic signals, green light - after every 3 min

red light - after 5 min
when together will glow?

Sol:-

$$\text{L.C.M} = 3 \times 5 = 15 \text{ min}$$

after every 15 min, 30 min, 45 min ... cycle

Ex:-



A = 4 min to complete round

B = 5 min to complete round

after 20 min, they will meet,

20, 40, 60, 80 ...

Case I:
Q Find the greatest 3 digit number, that when divided by (10, 12, 15) leaves a remainder of 5 in each case.

$$\text{Sol: } N = 10a + 5 \Rightarrow (N-5) = 10a$$

$$N = 12b + 5 \Rightarrow (N-5) = 12b$$

$$N = 15c + 5 \Rightarrow (N-5) = 15c$$

$$\Rightarrow (N-5) \text{ L.C.M. of } 10, 12, 15 = 60$$

$$(N-5) = 60y \quad [\because y = 1, 2, 3, 4, \dots]$$

$$N = 60y + 5$$

$$y=16 \Rightarrow N = 965, \text{ greatest}$$

Three digit no:

$$N = (L.C.M.) \times y + \text{remainder}$$

Case II:

Find the least three-digit no. that when divided by 10, 15, 20 leaves remainders of 8, 13 & 18 respectively.

Sol:

$$N = 10a + 8$$

$$N = 15b + 13$$

$$N = 20c + 18$$

$$= N+2 = (10a+8)+2 = 10(a+1)$$

$$N+2 = (15b+13)+2 = 15(b+1)$$

$$N+2 = (20c+18)+2 = 20(c+1)$$

$$\Rightarrow N = L.C.M.(10, 15, 20) - 2$$

$$N = \frac{60xy}{2} \quad [y=1, 2, 3, \dots]$$

$$N = 60x2 \frac{1}{2}$$

$$\underline{N = 120}$$

Case-II:

Find the least 3-digit no., that on divided

by 8 & 11 leaves remainders of 6 & 8 resp.

Sol:-

$$\cancel{N = 11x + 8} \quad N = 11x + 8$$

$$N = 8x + 6$$

$$11x + 8 = 8, 19, \textcircled{30}, 41, 52, 63, 74, 85, 96, 107,$$

$$8x + 6 = 6, 14, \textcircled{30}, 38, 46, 54, 62, 70, 78, 86,$$

$$94, 102, 110, \textcircled{118}$$

Least 3-digit no. = 30

next greatest no. = 118

at a gap of L.C.M(8, 11) = 88

$$\Rightarrow \boxed{N = 88x + 30}$$

$$N = 88 \times 1 + 30 = 118$$

$$= 88 \times 10 + 30 = 910$$

$$= 88 \times 11 + 30 = \textcircled{998}$$

greatest three digit.

Three firms

1st Case:

$N \div (9, 10, 12)$, remainder was same in each case

general form $\rightarrow N = L.C.M (9, 10, 12) \times a + r$

2nd Case:

$N \div (9, 10, 12)$, remainder was diff case but common

diff b/w divisors & respective remainder
was constant

gen. form of $N = (L.C.M (9, 10, 12) \times a) + (\text{Common diff})$

3rd Case:-

$N \div (9, 10)$, remainder was diff each case, but
diff b/w divisors & respective remainder
was not constant

general form of $N = L.C.M (9, 10) \times a + (\text{last no})$

Q Find the remainder, when $2^{100} \div 99$

Sol:-

$$\frac{2^{100}}{99} \Rightarrow 99 = 11 \times 9 \Rightarrow \frac{2^{100}}{11 \times 9}$$

$$2^{100} \div 11$$

$$2^1 \div 11 = 2 \text{ Rem } 2$$

$$2^2 \div 11 = 4$$

$$2^3 \div 11 = 2 \text{ Rem } 8$$

$$2^4 \div 11 = 5 \text{ Rem } 1$$

$$2^5 \div 11 = -1$$

$$2^6 \div 11 = 7$$

$$\frac{100}{10} \div 9 \text{ Rem } 0$$

$$\Rightarrow 2^{10} = 1$$

$$11 \left(\frac{2^{100}}{1} \right)$$

$$2^{100} \div 9$$

$$2^1 \div 9 = 2 \text{ Rem } 2$$

$$2^2 \div 9 = 4$$

$$2^3 \div 9 = -1$$

$$2^4 \div 9 = 1$$

$$6) 100 \overline{) 4}$$

$$\Rightarrow 2^4 = 2^3 \times 2 = (-1) \times 2$$

$$\Rightarrow \underline{\text{remainder } 2}$$

$$(1) 2^{100} \quad (2) 2^{100} \quad (3) 2^{100}$$

$$N = 11x + 1$$

$$N = 9x + b + 7$$

3rd type

$$\text{general form} \rightarrow \underline{99x + 34}$$

Q If eggs are removed from a basket two, three, four, five & six at a time, there remain resp.

one, two, three, four, & five eggs. But if eggs are removed seven at a time, no eggs remain what is the least no of eggs, that could have been in basket?

Ques. Let 'n' be the least no. of eggs

$$n \div 2 = 1 \quad \text{rem} \quad n = (2 \times a) + 1$$

$$n \div 3 = 2 \quad n = (3 \times b) + 2$$

$$n \div 4 = 3 \quad n = (4 \times c) + 3$$

$$n \div 5 = 4 \quad n = (5 \times d) + 4$$

$$n \div 6 = 5 \quad n = (6 \times e) + 5$$

$$n \div 7 = 6$$

$$\text{gen. form of } = \text{l.c.m. } (2, 3, 4, 5, 6) - 1$$

$$N = 60 \times y - 1$$

$$y = 1 \Rightarrow N = 59 \times$$

$$y = 2 \Rightarrow 119 \checkmark$$

∴ 119 eggs

Profit loss and Discount

$$\begin{array}{c} \text{(Cost Price)} \quad \text{(C.P)} \\ \text{₹ 400} \end{array} \xrightarrow{\text{Profit = SP - C.P}} \begin{array}{c} \text{Selling price (S.P)} \\ \text{(500)} \end{array}$$

$\Rightarrow \text{Profit} = 500 - 400 = 100$

$$\text{Profit \%} = \frac{P}{C.P} \times 100 = \frac{100}{400} \times 100 = 25\%$$

$$\frac{S.P}{C.P} = \frac{500}{400} \Rightarrow S.P = \frac{5}{4} (C.P)$$

$$S.P = 25\% \text{ more of C.P}$$

$$\begin{array}{l} \text{C.P} \\ \text{800} \end{array} \rightarrow 12.5\% \text{ profit} \Rightarrow S.P = ?$$

$$800 \times \frac{9}{8} = 900$$

Q Profit \% = 16.66\% ; S.P = 700 ; C.P = ?

Sol:

$$C.P = 600 \quad S.P = C.P + \frac{C.P}{6}$$

$$S.P = C.P \left(1 + \frac{1}{6}\right)$$

$$\Rightarrow C.P = \frac{6}{7} S.P$$

$$= \frac{6}{7} \times 700$$

$$C.P = 600$$

Base fractions :

$$\frac{1}{2} \rightarrow 50\%$$

$$Y_{11} = 9.09\%$$

$$\frac{1}{3} \rightarrow 33.\overline{3}\%$$

$$Y_{13} = 7.69\%$$

$$\frac{1}{5} = 20\%$$

$$Y_1 = 5.88\%$$

$$\frac{1}{7} = 14.28\%$$

$$Y_9 = 5.26\%$$

* Profit or loss % are always in terms of C.P

$$\begin{array}{c}
 \text{Q} \quad \frac{C-P}{S-P} \\
 \text{at } 300 \rightarrow 200 \quad 33.\overline{3}\% \text{ loss} \\
 \text{at } 300 \text{ selling price} = S.P = C.P - \frac{C.P}{3} \\
 \text{at } 300 \text{ selling price} = S.P \left(\frac{2}{3} \right) = 300 \times \frac{2}{3} \\
 \text{at } 300 \text{ selling price} = 200
 \end{array}$$

Margin % :-

The relationship between profit & selling price is Margin %.

$$\begin{array}{c}
 50 + 9.2 = 59.2 \\
 400 \rightarrow 500 \\
 P = 100
 \end{array}$$

$$\frac{P}{S.P} = \frac{100}{500} = \frac{1}{5} = 20\% \text{ - Margin.}$$

Profit is 20% of S.P \Rightarrow Margin %.

Q:

$$P = 10\% \quad ; \text{margin} \%$$

$$\frac{P}{C.P} = \frac{P}{S.P} = \frac{10}{10+10} = \frac{P}{S.P} = \frac{1}{(P+C.P)} = \frac{1}{11}$$

$$\Rightarrow \text{Margin \%} = \frac{1}{11} = 9.09\%$$

Ex: Profit \% \rightarrow Margin %.

$$\text{Margin} = \frac{P}{(C.P + P)} \Rightarrow \frac{25}{25+5} = \frac{25}{30} = \frac{5}{6} = 83\frac{1}{3}\%$$

Q:

An article sold at a certain price resulted in a loss of 7\%, whereas when it was sold at ₹ 100 more, it resulted in a 13\% profit. Find the cost price of the article.

R:

$$20\% = 100$$

$$100\% = 500 \Rightarrow C.P$$

(or)

$$S.P = \frac{93}{100} C.P \quad ; S.P' = S.P + 100$$

$$S.P' = \frac{113}{100} C.P$$

$$25 \Rightarrow S.P + 100 = \frac{113}{100} C.P \Rightarrow \frac{93}{100} C.P + 100 = \frac{113}{100} C.P$$

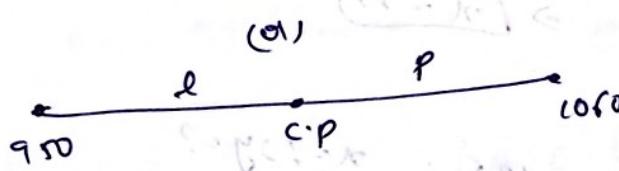
$$\Rightarrow C.P = 700$$

Q. Profit earned by selling an article at ₹ 1060 is 20% more than the loss incurred by the selling the article for ₹ 950. Find the C.P of the article.

Sol:

$$1060 - C.P = \frac{6}{5} (C.P - 950)$$

$$\Rightarrow C.P = 1000$$



$$P = \frac{6}{5}l$$

$$P + l = 110 \Rightarrow \frac{6}{5}l + l = 110$$

$$\Rightarrow l = 50$$

$$\Rightarrow C.P = 950 + 50 = 1000$$

Important note: If two variables change for opposite effect, then consider them with their signs first. If one variable changes with respect to another, then consider the change in the second variable first.

Example: If x and y are two variables such that x increases by 20% and y decreases by 20%, then the effect on the product xy will be:

$\frac{2000}{x} \times \frac{800}{y} = \frac{1600}{xy}$

$\therefore xy$

Ratios

Ratio is a relationship & relationship between numbers \Rightarrow quantitative relationship

\rightarrow Fractions

Ex:- Boys Girls $\frac{B}{G} = \frac{30}{20} = \frac{3}{2} = \frac{3:2}{B:G}$

$$B : G$$

$$3x : 2x$$

$$75 = 50$$

$$\text{if } 3x = 75$$

$$\Rightarrow x = 25$$

$\therefore x:y$ & $xy = 135$; $x=?$, $y=?$

$$\frac{3z}{5z} \Rightarrow (3z)(5z) = 135 \Rightarrow 15z^2 = 135$$

$$z^2 = 9$$

$$z = \pm 3$$

$$\Rightarrow 3z = 9; 5z = 15$$

Q The ratio of marks obtained by Ram & Shyam

is 5:7. If five times the marks obtained by

Ram is 12 more than thrice the marks obtained

by Shyam. Find their marks.

Sol:-

$$R : S$$

$$5:7$$

$$5x : 7x$$

$$5 \times 8x = 3 \times 7x + 12$$

$$40x = 21x + 12$$

$$\Rightarrow x = 3$$

$$\frac{R = 5}{15 = 21}$$

Q If the ages of a husband, wife + child in the ratio $13:11:3$ & avg age of family = 36 ages?

sol:

$$h : w : c$$

$$13 : 11 : 3$$

$$13x : 11x : 3x$$

$$\frac{13x + 11x + 3x}{3} = 36 \Rightarrow \frac{27x}{3} = 3 \times 36$$

$$x = 4$$

$$\Rightarrow h : w : c$$

$$52 : 44 : 12$$

Ans: 52, 44, 12

$$Q A : B : C = ? : ? : ? \quad | A+B+C = 200000 \text{ rs}$$

$$2x = 3x = 5x$$

$$\text{A's share} = \frac{2x}{10x} \times 200000$$

$$Q A : B : C = ? : ? : ? \quad | \left(\frac{1}{4}, \frac{1}{3}, \frac{1}{3} \right)^{\times 60}$$

$$A : B$$

$$B : C$$

$$5 : 3$$

using L.C.M.

$$A : B = (3 : 4) \times 5$$

$$\frac{B : C}{(5 : 3) \times 4} = 15 : 20 : 12$$

$$Q A : B : C = ? : ? : ? \quad | 5x = x_1 + x_2$$

$$2A = 3B = 5C$$

$$\frac{2A}{B} = \frac{3}{2}$$

$$\frac{B}{C} = \frac{5}{3}$$

$$A : B : C = 15 : 10 : 6$$

(Q1) ~~Sum of 3rd & 4th standard is 10 copys with 11~~

$$\frac{A}{4} : \frac{B}{3} : \frac{C}{5} = 1 : 2 : 3 \quad \text{Ans} + 8 = 11 : 21 \quad \text{after 3y}$$

$$A = 4K, B = 3K, C = 5K$$

$$4K, 3K : 5K$$

$$\Rightarrow 4 : 3 : 5 \quad \text{Ans} = \underline{\underline{K+K+K+K}}$$

Q The present ages of a father

and son is $8:1$. Eight years hence, the ratio would be $10:3$. Present age of father?

$$\text{Soln} - f : s \quad \text{after 8 yrs,}$$

$$8 : 1 \quad \frac{8x+8}{x+8} = \frac{10}{3}$$

$$8x : x$$

$$(8x+8) : (x+8) \quad 24x+24 = 10x+80$$

$$24x = 56$$

$$x = 24$$

$$\Rightarrow \text{Father's age} = 32; \quad \text{son's age} = 4 \text{ at present}$$

Ex-1

Milk : water

$$3 : 2$$

$$P \times \frac{3}{(3+2)} \quad \text{Total} \rightarrow 10 \text{ lit}$$

$$3x + 2x = 10$$

$$M : T : 8$$

$$8 : 2$$

$$2 : 1$$

$$W : T$$

$$2 : 3$$

$$3 : 5$$

$$M : T : 8 : 5$$

$$\frac{M}{T} = \frac{3x}{5x}$$

$$3 : 5$$

$$\Rightarrow \frac{M}{T} = \frac{3x}{5x}$$

$$\Rightarrow \frac{M}{50} = \frac{3x}{5x} \Rightarrow M = \frac{3x}{5} \times 50$$

$$\underline{M = 30 \text{ lit}}$$

Ques Milk: Water
8 : 2

18 lit water.

$$\frac{W_a}{T_0} = \frac{2x}{5x} \Rightarrow \frac{28}{T_0} = \frac{2}{5} \Rightarrow T_0 = \frac{28 \times 5}{2}$$

$$\underline{\text{Total} = 70 \text{ lit}}$$

$$\underline{\text{Milk} = 42 \text{ lit}}$$

Concept

AOPR BOPR C Total = 2,00,000

2 : 3 : 5

/ figure out new relations. like

A+B+C; A+Total, B+Total, C+Total, etc

Q The number of 1 Re, 50 paise & 2 paisa coins in a bag is in the ratio 3:4:5. If the bag has 2300 coins, find the no of 50 paise coins:

Ans: $\frac{2300 \times 4}{3+4+5} = \frac{2300 \times 4}{12} = 766.66$

$\frac{2300 \times 4}{3+4+5} = \frac{2300 \times 4}{12} = 766.66$

801:-

$$\text{Req} : R_{0.50} \quad R_{0.25} = \frac{M}{T}$$

no. of coins

value :-

$$8x = M \quad 2x = \frac{M}{0.25}$$

$$3x : 4x : 5x$$

$$\text{Total value} = M$$

$$(3x \text{ Re}) : (4x \text{ 50 p}) : (5x \text{ 25 p}) \\ \times 1 \text{ Re/coin} : 0.5 \text{ Re/coin} : 0.25 \text{ Re/coin}$$

$$3x : 2x : 5x$$

$$3x + 2x + \frac{5x}{4} = 300$$

$$5x + \frac{5x}{4} = 300 \rightarrow \frac{25x}{4} = 300$$

$$\Rightarrow x = 48$$

$$\text{no. of } R_{1 \text{ coin}} = 3 \times 48 = 144 ; \text{ no. of } 0.50 \text{ p coins} = 192$$

$$\text{no. of } 0.25 \text{ p coins} = 240$$

(Or)

add the sum of rupees

$$\text{Initial sum} = ₹ 3 + ₹ 2 + ₹ 1.5 = ₹ 6.25$$

$$6.25 \times 4 \rightarrow 4 \text{ (0.50 p coins) in est?}$$

$$6.25 \times 4 \rightarrow ?$$

$$\text{no. of coins} = \frac{300 \times 4}{6.25} = \frac{1200 \times 4}{6.25 \times 25} = 240 \text{ coins}$$

Variations

Direct Variation:

Two variables, x and y are said to vary directly if when one of them increases (or decreases), the other also increases (or decreases).

increases (or decreases) proportionally.

The word "proportionally" is important here. If one of them doubles, the other should also double. If one of them becomes one-third, the other should also become one-third.

concept

$$x \propto y$$

(proportionally means in the same ratio)

$$\frac{x}{y} = k$$

Mathematically, if the ratio is constant

for all instances of (x, y) , then x and y are vary directly, and it is represented

$$\text{as } x \propto y$$

$$x \propto y^2$$

$$\frac{x}{y^2} = k$$

The value of the diamond varies directly as the square of its weight. If a diamond weighing 1.5 gms has a value of ₹ 9000, what will be the value of a diamond weighing 2.5 gms?

$$\text{Sol: } V \propto W^2$$

$$\frac{V_1}{V_2} = \frac{W_1^2}{W_2^2} = \frac{9000}{(2.5)^2}$$

$$V_2 = \frac{25}{9} \times 9000 = \underline{\underline{25,000}}$$

The volume of a sphere is directly proportional to the cube of its radius. If the volumes of two spheres are in the ratio 8:21, find the ratio of their radii.

$$\frac{V_1}{V_2} = \frac{r_1^3}{r_2^3}$$

$$\text{Sol: } \frac{V_1}{V_2} = \frac{r_1^3}{r_2^3} \Rightarrow r_1^3 = 8 \cdot r_2^3$$

$$\therefore \sqrt[3]{\frac{r_1}{r_2}} = \frac{r_1}{r_2}$$

$$\therefore 2:1$$

$$\Rightarrow \frac{r_1}{r_2} = 2:1$$

Inverse Variation :-

Two variables x & y , are said to vary inversely if when one of them increases (or decreases), the other decreases (or increases) proportionally.

Thus, if one of them doubles, the other should halve; if one of them becomes one-third, the other should become thrice.

$$x \propto \frac{1}{y}$$

$$\frac{x_1}{x_2} = \frac{y_2}{y_1}$$

The cost per kg rice varies inversely with the square of the quantity of rice produced.

in a year - when 1 million tons of rice was produced, its cost was ₹ 36/kg. How much was the production in the year when the cost of rice was ₹ 46/kg?

Sol:-

$$C \propto \frac{1}{P_2^2}$$

$$\text{Given } C_1 \propto \frac{1}{P_2^2} \Rightarrow \frac{C_1}{C_2} = \frac{P_2^2}{P_1^2} \Rightarrow \frac{36}{49} = \frac{P_2^2}{7^2}$$

Q) Given $C_1 \propto \frac{1}{P_2^2}$ nett \therefore given

$$\text{Given } C_1 \propto \frac{1}{P_2^2} \Rightarrow \frac{36}{49} \times \frac{49}{7^2} = \frac{36}{49} \text{ tonnes}$$

$$P_2 = 6 \text{ mi tons}$$

$$\Rightarrow P_2 = 5 \cancel{14}$$

eff. cost per tonne for one item

constant cost per one item; total cost per one item

Joint Variation:-

There would be many situations in which more than two variables would be

involved whereas the direct and inverse

relation can be established only between two variables (with others being

constant) for example movie ex:

per person cost \propto (no. of persons) (proportionally)

so if per cost is constant $C \propto np$

Q:- 'x' varies directly with 'y' & 'z' when $y=3$, $z=5$.

if $y=9$, $z=15$, the value of x ? when $y=9$, $z=15$.

$$x \propto y z \Rightarrow \frac{x}{yz} = k$$

Sol:-

$$\frac{x_1}{y_1 z_1} = \frac{x_2}{y_2 z_2}$$

$$\text{constant} \propto \frac{2}{3x8} = \frac{x}{3x8} = \frac{x=6}{\text{constant}}$$

Q The value of a bar weighing 400 gm, varies inversely with the square root of the fraction of impurities of a bar containing 25 gm of impurities as £ 45,000, how many gms of pure gold is there in a bar costing £ 90,000?

Q1. $V \propto \frac{1}{\sqrt{\frac{I_1}{T_1}}}$ (or) $V^2 \propto \frac{1}{\frac{I_1}{T_1}}$ → unchanged

$\therefore \frac{V_1}{V_2} = \frac{\sqrt{\frac{I_2}{T_2}}}{\sqrt{\frac{I_1}{T_1}}} \Rightarrow \frac{V_1^2}{V_2^2} = \frac{I_2}{I_1}$

given we have $\frac{(45000)^2}{(60000)^2} \times 25 = I_2$ required

$\frac{25}{4} = I_2 \Rightarrow I_2 = 6.25$

Q The value of a silver coin varies directly as the square of its diameter, when thickness is constant and varies directly as its thickness when diameter remains constant two silver coins have the diameters in the

ratio 4:3. Find the ratio of the thickness

If the value of the first coin is

four times the value of second coin,

sol:

$$V \propto d^2 \quad ; \quad d = k$$

$$V \propto t \quad ; \quad d = k$$

$$V \propto d^2 t$$

$$\frac{V_1}{d_1^2 t_1} = K \Rightarrow \frac{V_1}{d_1^2 t_1} = \frac{V_2}{d_2^2 t_2}$$

→ Problems on joint variations → problems
on chain rule.

Q. 6 labourers can build a wall in 10 days.

How many labourers of double the
efficiency are needed to build 3 such walls
in 5 days?

MVT & W

$$\frac{M_1 T_1}{W_1} = \frac{M_2 T_2}{W_2}$$

$$6 \times 10^2 = 2 \times 18$$

therefore answer

∴ no. of men, $x = 18$

Q Observe the variation b/w Variable

A garrison has enough food to sustain its 40 men for 60 months. If each man consumes 900 gm/day. Reinforcement of 20 men joins the garrison. For how long will the food last now if each man consumes only 600 gm/day.

Sol:- Men $\propto \frac{1}{\text{ration}}$; time $\propto k$

Men $\propto \frac{1}{\text{time}}$; ration $\propto k$

$$M_1 R_1 T_1 = M_2 R_2 T_2$$

$$40 \times 900 \times 60 = 60 \times 600 \times t_2$$

$$\Rightarrow t_2 = \frac{G \times \frac{2}{3} \times \frac{3}{2}}{R}$$

$$\underline{t_2 = 6 \text{ months}}$$

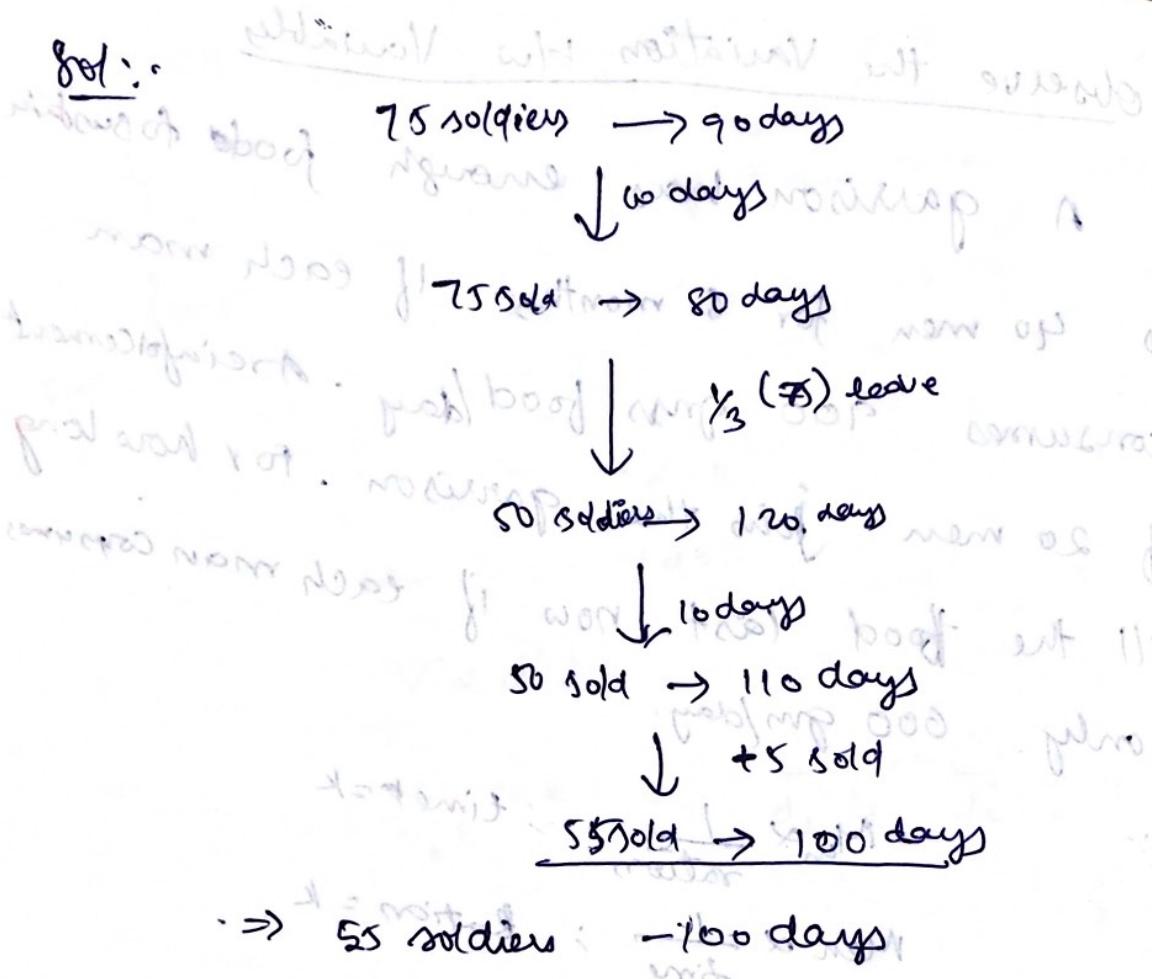
~~If~~ A garrison has sufficient food for 75 soldiers

for a period of 90 days. After 10 days, ~~1/3~~ of the soldiers leave. After another 10 days,

5 soldiers return. From this day on, how many

days will the food last?

Sol:



Q Four inlet pipes with circular cross section, take 9 hrs to fill a cistern. How many pipes of half the radius are reqd. to fill the cistern in 6 hrs, if the speed of water now is three times the speed of water in previous case?

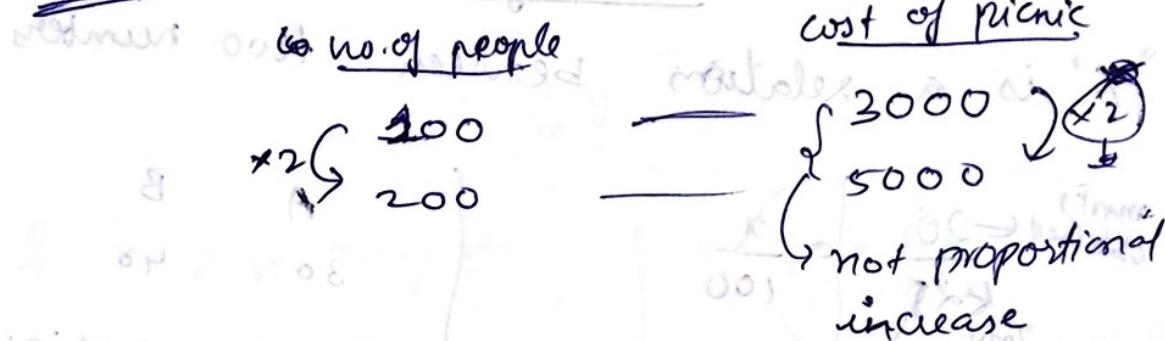
Ans:

$$\frac{4 \times \pi \times r^2}{1 \text{ hr}} = \frac{\pi \times 6 \times 3 \times \frac{1}{4} \text{ hrs}}{1 \text{ hr}} \left(\begin{array}{l} \text{Area} \propto r^2 \\ \text{Speed} \propto r^2 \end{array} \right)$$

$$\therefore 2 = 8 \text{ pipes}$$

Inverse & Direct Relation analysis

Concept



Total Cost = fixed cost + variable cost

$$y = K_1 + K_2 x$$

direct relation, if fixed cost is involved

$$y = \frac{K_1}{x} + K_2$$

inverse relation, if fixed cost is involved
sharing of fixed cost

Ex: Bus, Picnic, Food example

Picnic cost = bus cost + food

bus cost = fixed cost
P.C. = bus cost + (food) × no. of ppl

Indirect:

$$P.C. = \frac{\text{bus. cost}}{\text{no. of ppl}} + \frac{(\text{food}) \times \text{no. of ppl}}{\text{no. of ppl}}$$

Ex:

no. of ppl	cost per head	share: fixed cost + lunch
20	80	20
30	60	20
50	?	20
5:3		5:3
		24

Percentages

"%" is a relation between two numbers

$$\frac{\text{mark obtained}}{\text{Total}} \times 100 = \frac{x}{100}$$

both have same relationships

$$A = \frac{30}{40} \times 100 = 75\%$$

$$B = \frac{40}{30} \times 100 = 133.33\%$$

→ 'A' is what % of 'B'?

→ 'B' is what % of 'A'?

Q 'A' is what % of 'B'?

$$A = \frac{30}{40} \times 100 = 75\%$$

Ans

Q P is increased by 10%.

$$P' = P + \frac{10P}{100} = 1.1P$$

Q 'Z' is decreased by 20%.

$$Z' = Z - \frac{20}{100}Z = 0.8Z$$

Q

x is decreased by 16.66%.

$$x' = x - \frac{16.66}{100}x = \frac{5}{6}x$$

Q A is 75% of B; B is what % of A?

$$A = \frac{3}{4}B$$

$$B = \frac{4}{3}A$$

B is 133.33 % of A

~~Q~~ x is 10% more of y . $10\% \times y$ is 10% of y

~~Q~~ Let no. of eggs be x . x is 10% more than y .

$$x = \frac{11}{10}y$$

~~Q~~ P is \uparrow by 25%. $A = \frac{8}{7}B$

$$P' = P + \frac{5}{4}$$

~~Q~~ A is 14.28% of B

~~Q~~ A is 14.28% more than B

~~Q~~ A is 114.28% of B .

~~Q~~ A is 114.28% more than B .

~~Q~~ ~~Rectangular~~, $l \uparrow 25\%$; $b \uparrow 25\%$.

~~Q~~ γ : change in area

$$\text{Sol: } \frac{\text{New area}}{\text{old area}} = \frac{l \times b}{l \times b} = \frac{25}{16}$$

$$\cancel{\text{So, } A'} = A + \frac{9}{16}A$$

~~Q~~ Total $\Rightarrow A' \uparrow 0\%$ by 55% .

~~Q~~ The price of chocolates dropped by 15%.

~~Q~~ The price of chocolates was $\$1.35$ and thus Rashi was able to save $\$1.35$

~~Q~~ and thus Rashi was able to save $\$1.35$ and thus Rashi was able to save $\$1.35$ on each chocolate. Find the new price of one chocolate?

Sol:

$$15\% = \$0.35$$

$$0.15 = 1.35 \Rightarrow \frac{1.35}{0.15} = 9,$$

$$\text{new price} = 9 - 1.35 = \underline{\underline{\$7.65}}$$

Q A man mixes milk & water in the ratio 5:3. Find the percentages of milk in the resulting mixture?

Sol:-

$$\frac{5}{8} \times 100 = 62.5\%$$

Q To pass an exam of 250 marks, a candidate should obtain 40% marks. Raj scores 10% more than the passing marks. Find the % of marks scored by Raj.

Sol:-

$$\frac{25}{25} \times 40 = (0.4) \times 250 = 100$$

$$10\% \text{ more than passing} = (0.1)100 + 100$$

$$= 110$$

% marks obtained = 110

Rei per beggar γ -of marks = $\frac{110}{250} \times 44.\bar{4}\% \text{ of total}$

Q A's salary is 80% of B's salary, whereas B's salary is 110% of 'C' salary. What % of A's sal is C's salary?

Sol:-

$$A = \frac{4}{5} B$$

$$B = \frac{9}{5} C$$

$$A = \frac{4}{5} \left(\frac{9}{5} C \right) = \frac{16}{25} C$$

$$\cancel{A = 68\% \text{ off}}$$

$$C = \frac{25}{16} A$$

$$\text{Forward formula: } C = A + \frac{q}{100} A$$

Successive Percentages formula:

$$1) +10\% \quad 2) +20\% \quad 3) -16.66\%$$

$$\Rightarrow \left(\left(P \times \frac{11}{10} \right) \times \frac{6}{5} \right) \times \frac{5}{6}$$

App. of successive % A
⇒ (1.1) P.

$$= \left(\frac{11}{10} \right)^2 P \times \frac{11}{10} = \frac{1}{2} \left(\frac{2+21}{2} \right)$$

$$2) \quad \begin{array}{c} +25\% \\ A \end{array} \quad \begin{array}{c} +20\% \\ B \end{array} \quad \begin{array}{c} -20\% \\ C \end{array} \quad \begin{array}{c} -12.5\% \\ D \end{array} \quad \begin{array}{c} +14.28\% \\ E \end{array}$$

$$P \quad P \times \frac{5}{4} \quad \left(P \times \frac{6}{5} \right) \times \left(P \times \frac{7}{8} \right) = P \times \frac{63}{32}$$

$P = \frac{8}{7} P$
A, E prices are same
discounts (25%, 20%, 12.5%)

Successive Increments formula:

if $a\%$, $b\%$, ... are successive increments,
then final change in % will be \rightarrow only applicable to 2 nos

$$a + b + \frac{ab}{100} \rightarrow \text{resultant}$$

$$\text{Ex: } 250 ; a = 10\% ; b = 20\%$$

$$10 + 20 + \frac{200}{100} = \underline{\underline{32\%}} \rightarrow \begin{array}{l} \text{final price} \\ \text{change} \\ = 32\% \end{array}$$

$$25 \times \frac{11}{10} \times \frac{6}{5} = 33$$

Q

successive changes
 $l = 20\% \uparrow$; $b = 20\% \downarrow$ change in area:

~~$A \propto l b$~~ ~~proportional increase~~
 formula

$$= a + b + \frac{ab}{100}$$

$$= 20 - 20 - \frac{20}{100}$$

$$= -4\%$$

\therefore Area decreasing by 4%.

$$\Rightarrow \left(1b \cdot \frac{6}{5}\right) \frac{4}{5} = \frac{24}{25} l b = \left(1 - \frac{1}{25}\right) l b$$

- decrease in 4%.

Q Travelling from home to office, if I increase my speed by 37.5% of my usual speed, what percent of time would I save?

Ans:-

B. $S \propto D$

Increase in distance $S_2 T_1 = S_1 T_2$

at same time $(1+37.5)$

~~$S_1 T_1 = S_1 T_2$~~

$$\text{therefore } T_2 = \frac{11}{8} T_1$$

$$T_2 = \left(1 - \frac{3}{11}\right) T_1$$

$$\Rightarrow 9.09 \times 3$$

- 27.27% decrease in time

while preparing the price tag, instead of increasing the price by 12.5%, a shopkeeper absent mindedly decreased the prices by 12.5%. find the % diff in the price on the tag compared to what should have been the price of?

sol:

$$\frac{\text{what should be}}{\text{what is there}} = \frac{9}{88}$$

$$\frac{7}{8} = (wSB) - wST$$

$$1 - (0.0) \cdot \frac{7}{9} = (0.0) (p.0)$$

Since the price of mangoes increased by 6.66%, the whole seller was able to purchase 18 mangoes less in his budget of ₹ 864. what is the increased price of each mangoes?

sol:

$$xp = 864$$

$$(x-18) \cdot \frac{16}{15} p = 864$$

$$\frac{x}{(x-18) \frac{16}{15}} = 1 \Rightarrow 15x = (x-18) 16$$

$$15x = 16x - 16 \times 18$$

$$x = 16 \times 18$$

$$\Rightarrow 270$$

$$P' = \frac{\text{old price}}{\left(\frac{864}{270}\right) \times \frac{16}{15}}$$

A large watermelon 20 kg with 96% of its weight being water. If it is allowed to stand in the sun for a while and some of the water evaporates so that now only 95% of its weight remains.

Its reduced weight will be?

sol:

concentrate on pulp	95% post sun
96% of water \Rightarrow 4% of pulp	95% of water 5% of pulp

$$\rightarrow (0.04)(20) = (0.05)y$$

$$0.8 = \frac{8x}{5y} \Rightarrow \frac{8x}{5} = 0.8y$$

$$y = 16 \text{ kg}$$

new weight = 16 kg

$$16 = 9x$$

$$16 = 9 \frac{21}{21} (21-x)$$

$$21(81-x) = 16x$$

$$81x - 21x = 16x$$

$$1 = \frac{x}{21(81-x)}$$

$$(21 \times 21 - x) =$$

Profit & Loss

→ Mark price (list price)

$$\begin{array}{ccc}
 \underline{C.P} & \xrightarrow{\text{S.P}} & \underline{M.P / L.P} \\
 800 & \xrightarrow{R=100} 900 & \xrightarrow{\text{Discount} = 100} 1000 \\
 & \curvearrowleft \text{markup amt} = 200 &
 \end{array}$$

$$P\% = \frac{100}{800} = 12.5 \quad ; \quad \text{discount \%} = \frac{100}{1000} = 10\%$$

connects discount & M.P

$$S.P = \frac{9}{8} C.P \quad ; \quad S.P = \frac{9}{10} M.P$$

Markup % connects C.P & M.P

$$\text{Markup \%} = \frac{200}{800} = \frac{1}{4} = 25\% \quad \xrightarrow{\text{cost price}}$$

$$M.P = \frac{5}{4} C.P$$

markup % works on cost price

Ex:- M.P % = 20% ; d = 10% ; P = ?

$$\begin{aligned}
 & \left(\left(C.P \times \frac{6}{5} \right) \times \frac{9}{10} \right) - \frac{9}{10} C.P = \frac{54}{50} C.P \\
 & \downarrow M.P = \frac{9}{10} S.P \\
 & S.P = \left(1 + \frac{4}{5} \right) C.P \\
 & = 87. P%
 \end{aligned}$$

Q:-

$$C.P, M.P = 300 ; C.P = ?$$

$$d.l. = 20\% ; P = 20\%$$

Sol:-

$$S.P = \frac{4}{5} M.P ; S.P = \frac{6}{5} C.P$$

$$\Rightarrow \frac{4}{5} \cdot M.P = \frac{6}{5} \cdot C.P$$

$$C.P = 200$$

Q

$$M.P = 250 ; P = 10\% ; d\% = ?$$

~~C.P < M.P~~

$$M.P = \frac{5}{4} C.P$$

$$S.P = \frac{11}{10} C.P$$

$$\frac{M}{5} M.P = \frac{10}{11} S.P \Rightarrow S.P = \frac{44}{50} M.P$$

$$\Rightarrow S.P = \left(1 - \frac{6}{50}\right) M.P$$

$$\Rightarrow 12\% \text{ of } d\%.$$



Concept:- Buy 5 get 1 free

5 items \rightarrow 1 item free

M.P. = $6x$ (for 6 shirts)

= ~~6x~~ $= 5x$ (for 5 shirts)

Q. 1 shirt free

$$\text{M.P.} = 6x ; \text{discount} = x$$

$$\Rightarrow \text{d.r.s.} = \frac{x}{6x} = \frac{1}{6} = 16.66\%$$

Q.

Buy 7 get 2 free } which one to prefer
 $\text{d.r.s.} = 20\%$

$$\frac{2x}{9x} = \frac{66.66}{92.33} = \frac{2}{3} \quad 22.22\%$$

$\Rightarrow d = 20\%$ is not preferable

$$92.33 \times 1.1 = 92.33 + 9.23 = 92.33 + 9.2$$

Concept:

20% extra



15

Buy 5 → get 1 free



$(\frac{1}{6}) \Rightarrow 16.66\% \text{ discount}$

$\Rightarrow 20\% \text{ extra} \Rightarrow 16.66\% \text{ discount}$

Ex:-

14. 28% extra
 $\Rightarrow \frac{1}{7}$

Buy 7 → get 1

$\frac{1}{8} \Rightarrow 12.5\% \text{ discount}$

A trader
 $M.P = 50\%$. $\Rightarrow 25\% \text{ extra} \text{ is } P.O. ?$

$M.P = \frac{3}{2} C.P$ $\Rightarrow M.P = 1.5 C.P$

$25\% \text{ extra} \Rightarrow \frac{1}{4} \Rightarrow \text{Buy } \frac{4}{5} \Rightarrow \text{get } \frac{1}{2}$

$= \frac{1}{5} = 20\% \text{ discount}$

$S.P = \frac{4}{5} M.P \Rightarrow M.P = \frac{5}{4} S.P$

$\frac{3}{2} \cdot C.P = \frac{5}{4} S.P$

$S.P = \frac{12}{10} C.P \Rightarrow S.P = \left(1 + \frac{1}{5}\right) C.P$

$\Rightarrow 20\% \text{ profit}$

Cheating and Error and Dilution

Application

Cheating

Actual weight - 800 gms \Rightarrow markup $\rightarrow C.P = 100$
 shown wt $\Rightarrow 1000 \text{ gms}$ $\rightarrow M.P = 100$

\downarrow

$$M.P \% = \frac{200}{800} = 25\%$$

If act wt - 1000
 shown wt - 900 \Rightarrow 10% discount

Q) ~~actual wt 800 gms~~ \rightarrow discount ; P% = ?
 showing wt 1000 gms

~~(or)~~ $M.P \% = 25\%$.

~~(or)~~ $M.P = \frac{5}{4} C.P$

$S.P = \frac{9}{10} M.P$

$\frac{5}{4} C.P = \frac{10}{9} \cdot S.P$

$S.P = \frac{45}{40} C.P$

$= (1 + 1/8) C.P$

$= \underline{\underline{12.5\% \text{ profit}}}$

(or)

$C.P \times \frac{5}{4} \times \frac{9}{10} = S.P$

Q

showing - 150 gms

$d.r. = 20\% ; P = 20\%$

$S.P = \frac{6}{5} C.P ; \cancel{S.P = \frac{4}{3} M.P}$

$\frac{6}{8} C.P = \frac{4}{3} M.P \Rightarrow M.P = \frac{30}{24} C.P$

$M.P \% = 50\%$

$\Rightarrow \text{actual wt} = 150 = \frac{3}{2} C.P$

$\Rightarrow C.P = \frac{2}{3} \times 150 = 100 \text{ gms}$

S-19: Mixing Impurities

5 lit milk + 1 lit water \Rightarrow 6
 Milk up

Another Concept:-

Impurities = 20% of Total

$$\text{imp} = \frac{1}{5} * T$$

$$\frac{1}{5} * 100 = 20$$

$$\text{Milk} + \text{Imp} = T$$

Ex:-

Ques. Milk % = 10% of total; d = 10%; P = ?

$$P + I = T$$

$$9 + 1 = 10$$

$$\Rightarrow \text{Markup} = \frac{10}{9} - 1 = \frac{1}{9}$$

$$9.2 \xrightarrow{d=10\%} 9.2 \times \frac{10}{9} \times \frac{1}{10} = 1.2$$

\Rightarrow no loss or profit

$9.2 \times \frac{1}{9} = 1.02 \approx 10\%$

$$\text{Profit} = 0.1 \times \frac{1}{9} = 1.1$$

Another Key Word:-

20% extra \Rightarrow 20% of pure form is added as impurity to pure form

Ex:-

$$5 + 1 \xrightarrow{20\% \text{ extra}} 6$$

~~Markup~~ $\frac{1}{5} = 20\%$

20% of extra \neq 20% of total

Q A trader gives 20% extra. & $d.l. = 10\%$... it makes P.I. = 35%.
what is wt shown when 400gms is kept on machine.

Sol:-

20% extra \rightarrow one markup

(weighting machine) \rightarrow second markup

$$\cancel{C.P \times \frac{6}{5} \times \frac{a}{b} \times \frac{9}{10} = \frac{27}{20} S.P}$$

$$\frac{a}{b} = \frac{5}{4}$$

\therefore markup provided by wt machine = $5/4$
 $\Rightarrow 12.5\%$.

$$= 12.5\%$$

$$400 \times \frac{5}{4} = 500 \text{ gms}$$

(Meter Scale)

Shrinkage & Expansions

→ Meter Scale → expands in summer

shinks in winter

mark up provided

$$\text{Actual length} = 100 \text{ cm} + \xrightarrow{\text{mark up } 10\%} \text{mark up} = 10 \Rightarrow \text{discount}$$

$$(\text{summer}) \rightarrow 110 \text{ cm}$$

$$(\text{winter}) = 90 \text{ cm}$$

Ex:-

20% exp ⇒ Markup

$$100 \rightarrow 120$$

$$120 \Rightarrow (1 - \frac{1}{6})$$

$$100 \Rightarrow 5\% \text{ discount}$$

16.66%

winter

$$80 \times \frac{20}{80} = 1 + \frac{1}{4} = 5\% \text{ markups}$$

$$\frac{2}{2} = \frac{P}{2}$$

P = winter for

2000

Ans -

$$2000 = 2 \times 1000$$

Averages & Weighted Averages

$$\text{Avg} = \frac{\text{sum of samples}}{\text{no. of samples}}$$

Ex: Avg^{wt} of a class of '9' students = 80 kg
 (new person of 90 kg enters \rightarrow new avg?)

$$80 \times 9 = 720$$

$$A' = \frac{720 + 90}{10} = \frac{810}{10} = 81.$$

Divide Equally Approach:

$$\begin{array}{rcl} 90 & & 10 \\ \downarrow & & \downarrow \\ 80 & & 90 \end{array}$$

$$\text{avg} - 80 - 10 \text{ extra}$$

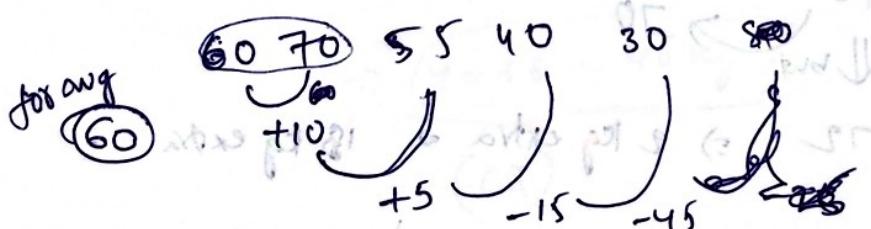
divide 10 \rightarrow equally to all 10 persons

$$\Rightarrow \text{new avg} = 81$$



60, 70, 55, 40, 30 \leftarrow ~~80~~

Divide equally approach



50 - deficit = -45 \Rightarrow divide it equally for all

$$\frac{-45}{5} = -9$$

$$\Rightarrow 60 - 9 = 51$$

Ans 51

Ques. 55. $66, 70, 55, 40, 30$

(55) $\sum_{i=1}^5 x_i = 20$

$$\frac{-20}{5} = -4 \Rightarrow 55 - 4 = 51$$

~~Q~~ Ques. 56. $n=8$, avg $= 56$ kg, wt $= 32$ kg, new person comes at avg $= 56$

~~Q~~ Ques. 57. $n=9$, avg $= 56$ kg, wt $= 24$ kg, new person comes at avg $= 56$

~~Q~~ Ques. 58. $n=10$, avg $= 70$ kg, new person comes at avg $= 72$ kg

$n=9$, avg $= 56$ kg, wt $= 24$ kg, new person comes at avg $= 56$ kg

$70 - 56 = 14$ kg extra

$72 - 56 = 16$ kg extra

No. of players $= 9$ (avg $= 56$) \Rightarrow new person. $\Rightarrow 70 - 18 = 52$ kg

$$P = \frac{24}{9}$$

$n = 11$ \downarrow 96 kg
 Avg = ?
 \downarrow
 $\text{Avg}' = \text{Avg} + 3$ $\Rightarrow 12 \times 3 = 36 \text{ kg added}$
 $\Rightarrow \text{Avg}' = \frac{(96 - 36) + 3}{12} = 60 + 3 = 63$

$n_1 = 4$ $n_2 = 6$ $\Rightarrow \text{Avg}_{\text{new}}$?
 $A_1 = 40 \text{ kg}$ $\text{Avg}_2 = 50$
 $\text{Avg}_{\text{new}} = \frac{A_1 n_1 + A_2 n_2}{n_1 + n_2} = \frac{40 \times 4 + 50 \times 6}{10} = 46$

Divide equally approach
 $n_1 = 4$ $n_2 = 6$
 $A_1 = 40$ $A_2 = 50$
Ex: consider '40' as Avg and bring '50' as
 '40' and 6 people each carrying 10 kg
 extra things $\frac{60 \text{ kg}}{10} = 6$. and divide
 60 kg equally to '10'.

$$\Rightarrow 40 + 6 = 46 \text{ kg}$$

(or)

$$\begin{aligned}
 A &= 50 \\
 40 &\quad 50 \\
 &\quad \swarrow \uparrow \\
 -10 \times 4 &= -\frac{40}{10} = -4 \\
 \Rightarrow 50 - 4 &= \underline{\underline{46 \text{ kg}}}
 \end{aligned}$$

$$\text{Average} = \frac{\text{sum of all samples}}{\text{no. of samples}}$$

~~12 kgs of rice costing ₹10/kg is mixed with 48 kgs of another variety of rice costing ₹20/kg. What is the average cost/kg of mixture.~~

Sol:

$$\text{Avg cost} = \frac{\text{Total cost}}{\text{Total quantity}} = \frac{12 \times 10 + 48 \times 20}{60}$$

Or ~~average price about ₹18~~

$$\begin{array}{rcl} 12 & & 48 \\ 10 & & 20 \\ \hline -10 \times 12 & = & -120 \\ & & \hline 60 & & \end{array}$$

~~20 - 2 = ₹18~~

$$\begin{array}{rcl} 10 \text{ kg} & & 20 \text{ kg} \\ | & & \downarrow \\ A = ₹20/\text{kg} & + & \text{original price} \\ & & 20 \text{ kg} \\ & & \downarrow \\ & & ₹35/\text{kg} \\ \hline -15 \times 10 & = & -150 \\ & & \hline 30 & & \end{array}$$

$\Rightarrow 35 - 5 = ₹30/\text{kg}$

$$P = \frac{OP}{OQ} = \frac{4 \times OI}{OI}$$

Rice example

concept:

$$\text{Average} = \frac{\overset{\text{cost}}{R_1 w_1} + \overset{\text{wt}}{R_2 w_2}}{w_1 + w_2}$$

$$A = \frac{R_1 w_1 + R_2 w_2}{w_1 + w_2}$$

$$\Rightarrow \frac{w_1}{w_2} = \frac{R_2 - A}{A - R_1}$$

Weighted Averages:

$$n_1 = 5 \quad n_2 = 6$$

$$A_1 = 40$$

$$A_2 = 56$$

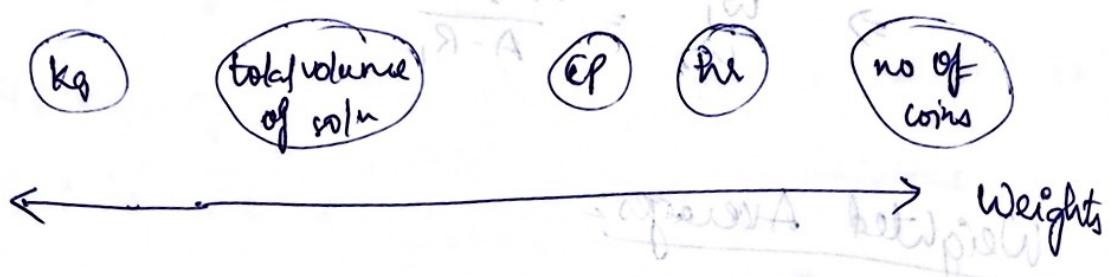
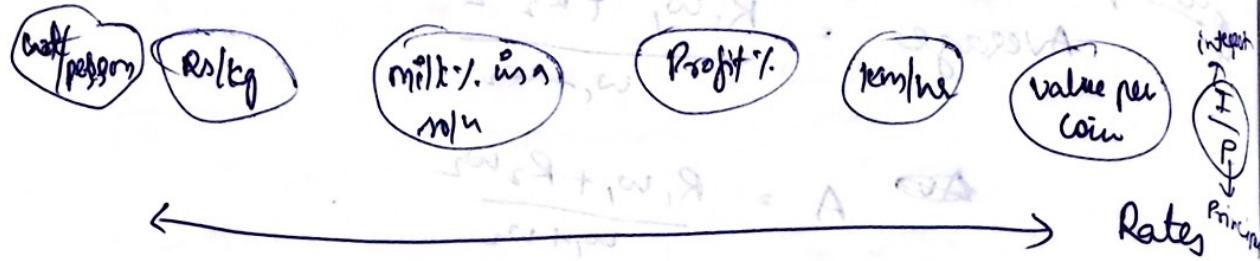
$$A_m = \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2}$$

$$n_1 A_m + n_2 A_m = n_1 A_1 + n_2 A_2$$

$$\Rightarrow n_1 (A_m - A_1) = n_2 (A_2 - A_m)$$

$$\frac{n_1}{n_2} = \frac{(A_2 - A_m)}{(A_m - A_1)}$$

10Rs/kg	8kg	2kg	2kg	1kg	4kg
A (E)	15	18	15	18	12
20Rs/kg	8kg	8kg	2kg	4kg	1kg



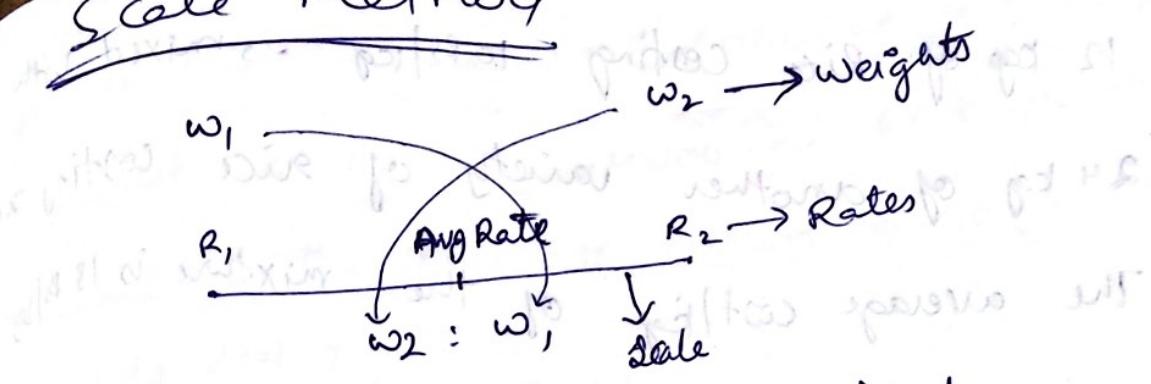
$$A_m = \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2}$$

$$R_m = \frac{w_1 R_1 + w_2 R_2}{w_1 + w_2}$$

$$\frac{\omega_1}{\omega_2} = \frac{R_2 - R_m}{R_m - R_1}$$

Rate \rightarrow numerator
denominator \rightarrow weight

Scale Method

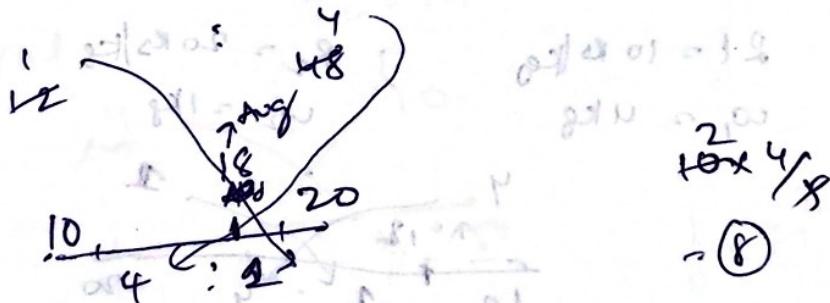


1. Write the rates on the line [scale]
2. Write the weights corresponding to each rate above the rates
3. Find the ratios off col. $w_2 : w_1$ reverse the ratio write it under the line
4. Divide the scale into ratios written under the line
5. The division point on the scale is the average rate
(or) resultant rate.

$$\frac{w_1}{w_2} = \frac{R_2 - \text{Avg}}{\text{Avg} - R_1}$$

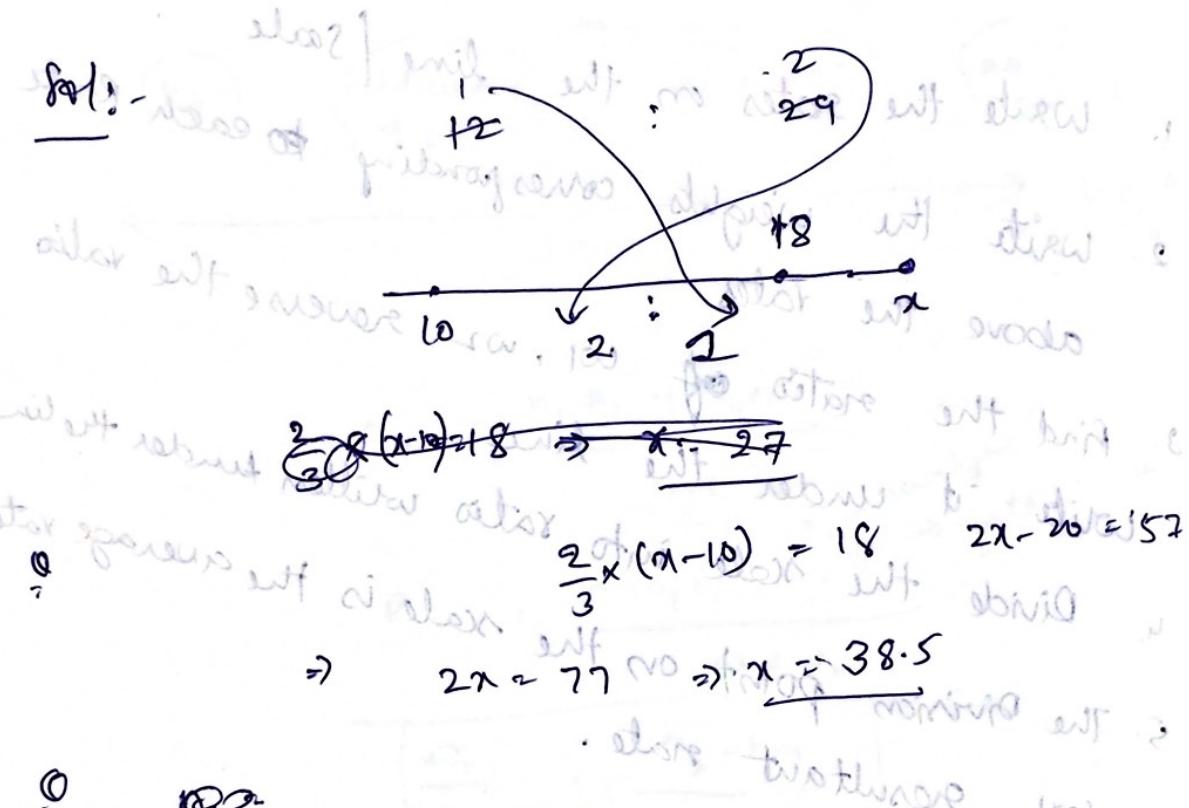
Q. 12 kg of Rice costing 10 Rs/kg is mixed with 48 kg of another variety of rice costing 20 Rs/kg what is the average cost per kg of the mixture

Sol:



$$\frac{10 \times 48}{12 + 48} = 16$$

Q. 12 kg of rice costing 10 Rs/kg is mixed with 24 kg of another variety of rice costing x . The average cost/kg of the mixture is 18 Rs/kg.



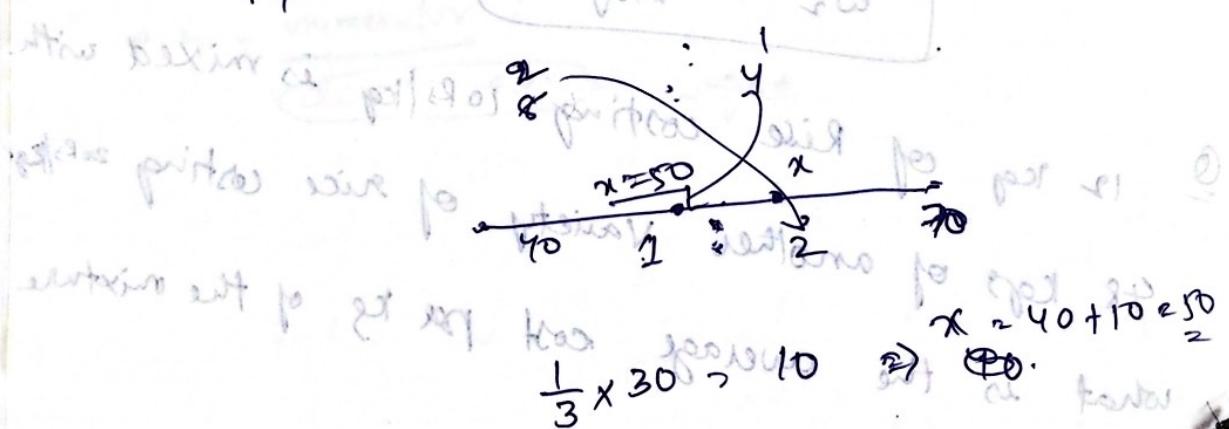
Q. $\omega_1 = 8$
 $R_1 = 10$

$$\frac{2x + 10 \times 12}{36} = 18$$

$$2x + 120 = 648$$

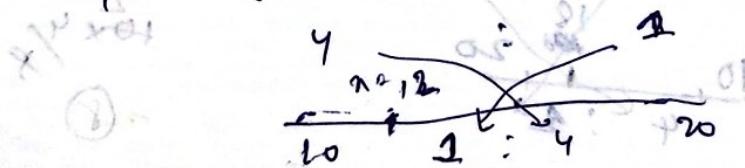
$$2x = 528$$

$$x = 264/2 = 27$$



Q. $R_1 = 10$ Rs/kg
 $\omega_1 = 4$ kg

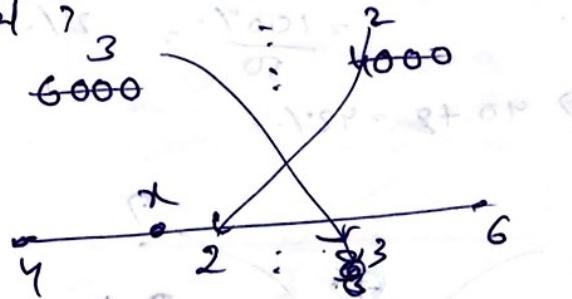
$R_2 = 20$ Rs/kg
 $\omega_2 = 1$ kg



$$\frac{1}{5} \times 10 = 2$$

$$\Rightarrow \underline{\underline{Ave}} = 10 + 2 = 12$$

A man borrows ₹ 6000 at 9% simple interest & another ₹ 4000 at 6% simple interest what is the overall interest ratio for the whole deal?



$$\frac{2}{5} \times (2) = \frac{4}{5} = 0.8$$

$$\Rightarrow x = 4 + 0.8 = 4.8$$

Final & for new approach,

Divide equally approach, (no matter which part matters)

split into 2 parts: 4000 (no matter which part matters)

1 part: 3000 (no matter which part matters)

entry out for 4%.

$$\Rightarrow \frac{4}{5} = 0.8\%$$

divide '4' among'st

$$\Rightarrow \underline{4.8\%}$$

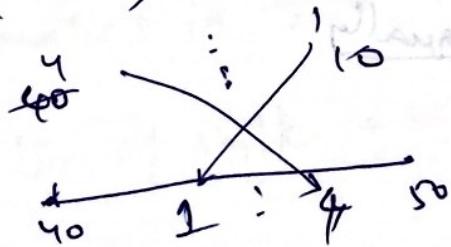


$$S_1 = 40\%$$

$$T_{total} = 40\%$$

$$S_2 = 50\%$$

$$T_2 = 10\%$$



$$\frac{1}{5} \times 10^2 = 2$$

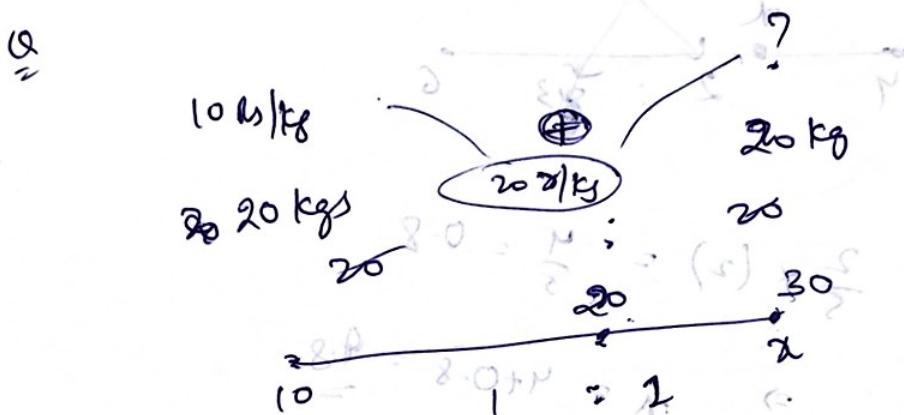
$$\Rightarrow A \% = 40 + 2 = \underline{42\%} \text{ milk}$$

~~Divide equally approach~~

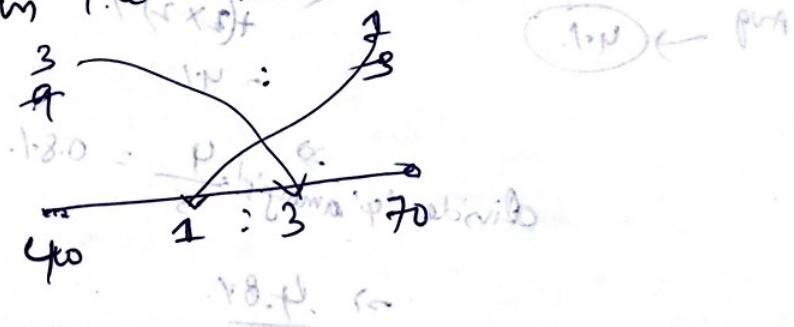
Divide equally approach
 $\text{Total water} = 40 \text{ l}$ $\text{Milk} = 10 \text{ l}$
 $\text{Water} = 40 \text{ l}$ $\text{Milk} = 10 \text{ l}$
 $80\% \rightarrow 10\%$
 $40\% \rightarrow 10\%$
 $(10 \times 10) \rightarrow 100\%$

$$\frac{100\%}{50} = 2\% \text{ less salt}$$

$$\Rightarrow 40 + 2 = 42\%$$



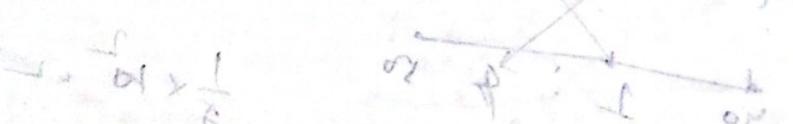
Q litres of milk and water sol having
 (either water or) $\frac{3}{4}$ litres of milk and $\frac{1}{4}$ litres of water
 40% milk is mixed with $\frac{3}{4}$ litres of milk
 and water sol having 70% milk. Find %
 of milk in the mixture of two sols.



$$\frac{1}{4} \times 30 = 7.5 \Rightarrow A = 40 + 7.5$$

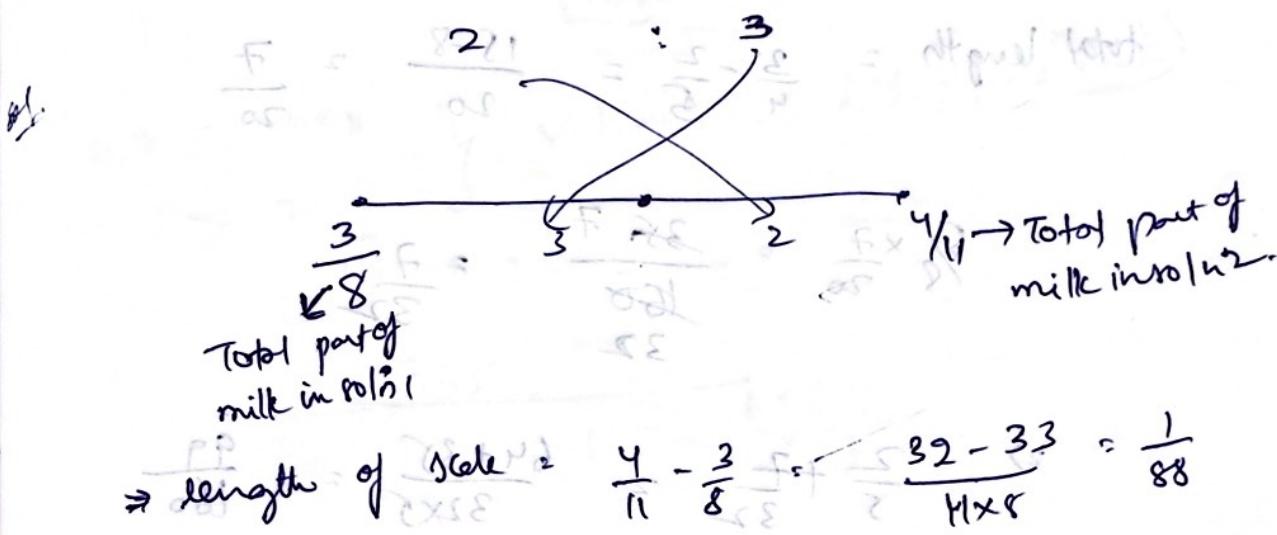
$$A = 47.5$$

Divide equally:



Divide equally: $\frac{1}{4} \times 30 = 7.5$

Two solns having milk + water in the ratios $3:5$ and $4:7$ are mixed in the ratio $2:3$. Find the ratio of milk and water in the resultant soln.



$$\frac{3}{5} \times \frac{1}{88} = \frac{3}{440}$$

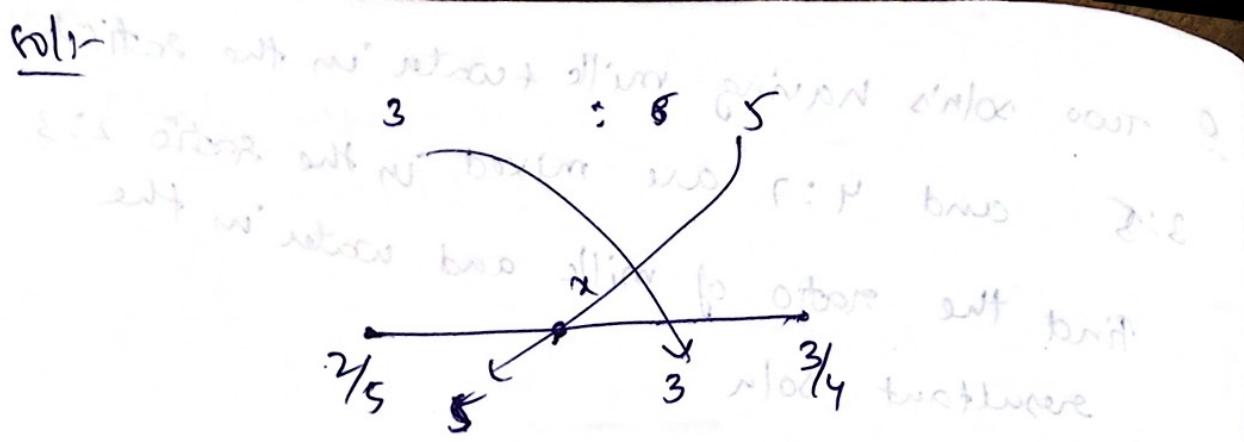
18:100 = 3 stages over

$$\Rightarrow \frac{3}{8} + \frac{3}{440} = \frac{134}{440} = \frac{21}{58}$$

$$1:2:3 \quad 21:34$$

Two vessels having volumes in the ratio $3:5$ are filled with water and milk solns. The ratio of milk and water in the two vessels is $2:3$ and $3:1$ respectively. If the contents of both the vessels are emptied into a larger vessel, find the ratio of milk + water in the larger vessel.

$$\text{Vessel} = \frac{1}{2} + \frac{1}{2} = \text{equal}$$



$$\text{Total length} = \frac{3}{4} - \frac{2}{5} = \frac{15-8}{20} = \frac{7}{20}$$

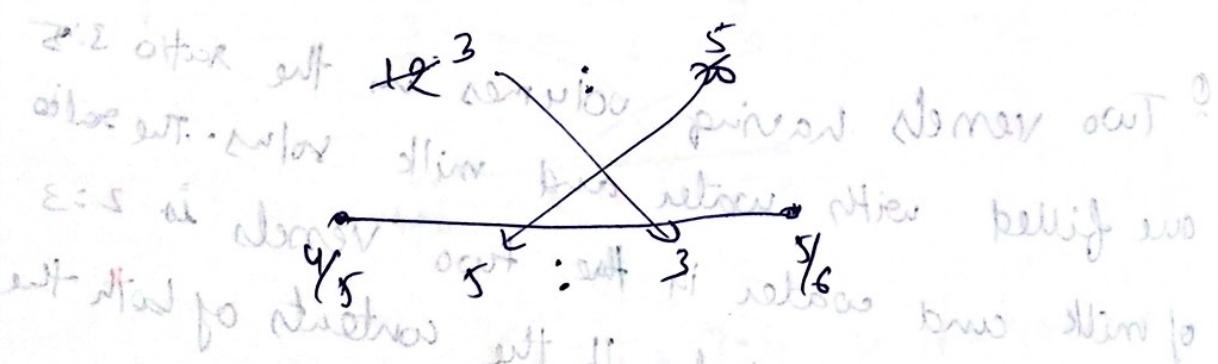
Probability of getting milk
in 1st glass

$$\frac{5}{8} \times \frac{7}{20} = \frac{35}{160} = \frac{7}{32}$$

Probability of getting milk
in 2nd glass

$$\frac{1}{8} = \frac{2}{16} \quad \frac{2}{5} + \frac{7}{32} = \frac{16+35}{32 \times 5} = \frac{99}{160}$$

$$\Rightarrow \text{new ratio} = 99 : 61$$



$$\text{Total length} = \frac{5}{6} - \frac{4}{5} = \frac{25-24}{30} = \frac{1}{30}$$

Probability of getting milk
in 1st glass

$$\frac{5}{8} \times \frac{1}{30} = \frac{5}{240} = \frac{1}{48}$$

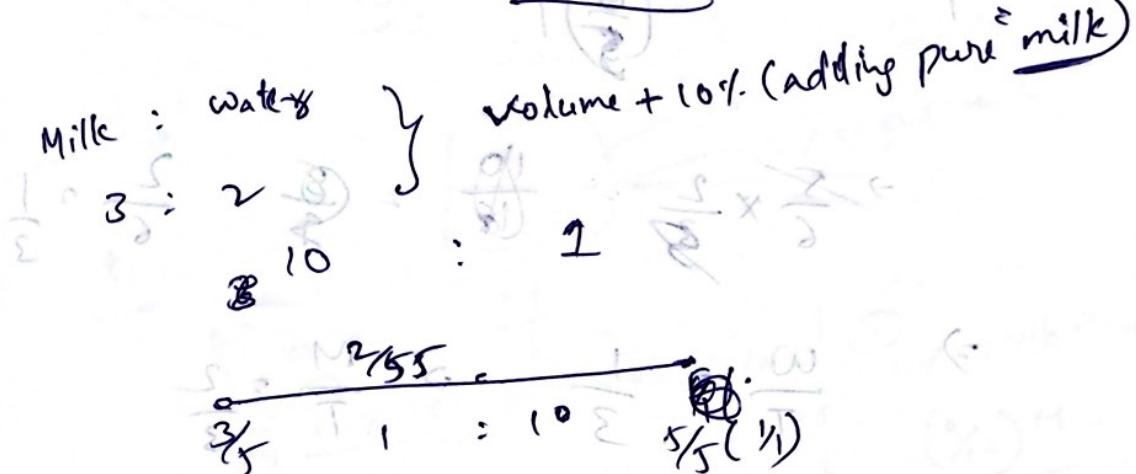
$$\text{Average} = \frac{4}{5} * \frac{1}{48} = \frac{192+5}{48 \times 5} = \frac{197}{240}$$

⇒

197:43

Successive Removal + Replacement

Problems

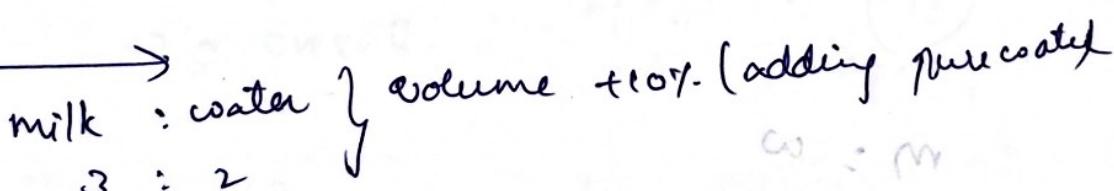


Total length of scale $= \frac{5}{5} - \frac{3}{5} = \frac{2}{5}$

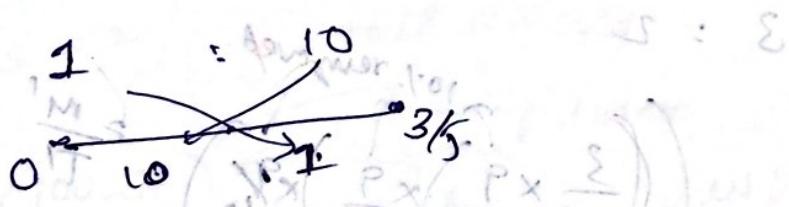
$\frac{1}{11} \times \frac{2}{5} = \frac{2}{55}$

⇒ $\frac{3}{5} + \frac{2}{55} = \frac{165 + 10}{55 \times 5} = \frac{175}{275} = \frac{7}{11}$

New ratio = 7:4



$w : m$



$\Rightarrow 0 + \frac{6}{11} = \frac{6}{11} \Rightarrow \underline{\underline{6:5}}$

$$M : \omega + 20\% \text{ milk}$$

3 : 2

~~before & after mixing~~

$$\frac{\omega}{T} \rightarrow \frac{\omega}{(\frac{3}{5})T} \rightarrow \frac{\frac{5}{6} \times \omega}{T}$$

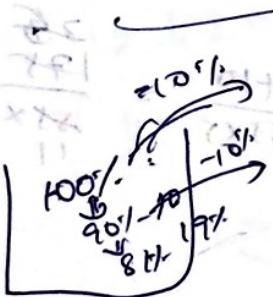
~~(After mixing with water)~~ ~~for 1st iteration~~

$$\Rightarrow \frac{5}{6} \times \frac{2}{3} = \frac{10}{18} = \frac{5}{9} = \frac{2}{6} = \frac{1}{3}$$

$$\Rightarrow \frac{\omega}{T} = \frac{1}{3} \Rightarrow \frac{M}{T} = \frac{2}{3}$$

$$\frac{\omega}{T} = \frac{\omega - 2}{T} \Rightarrow M : \omega = 2 : 1$$

Removal & Replacement



Remove 10% of milk +
10% of water & do it
recursively.

$$m : \omega$$

$$3 : 2$$

$$\left(\left(\left(\frac{3}{5} \times \frac{9}{10} \right) \times \frac{9}{10} \right) \times \frac{9}{10} \right)$$

~~10% removed~~

$$= \frac{81}{100} = \frac{3}{8} \times \frac{9}{11}$$

$m : w$ 85 : 17
 3 : 2
 3) 10% removed
 2) 20% removed
 1) 25% removed

$$\left(\left(\frac{3}{5} \times \frac{9}{10} \right) \times \frac{4}{5} \right) \times \frac{3}{4} = \frac{M}{T}$$

10% M removed
 10% w put back.

① 10% removed from total
 ② Milk removed

$$\frac{M}{(9/10)T} = \frac{(9/10)M}{(9/10)T}$$

③ 10% again put back
 so, total again changes

$$\frac{(9/10)M}{T} \Rightarrow \text{milk changed}$$

$$\frac{(9/10)M}{T} \times 10 \text{ lit removed} \Rightarrow \text{Total unchanged}$$

⇒ overall, multiple factor $\left(\frac{9}{10}\right)$

Q

$$\frac{M}{T} = \left(\frac{9}{10} \right)^n \frac{M}{T} \Rightarrow \frac{\frac{1}{10}M}{\left(\frac{9}{10}\right)^n T} = \frac{1}{10} \left(\frac{M}{T} \right)$$

100 lit coln
 10% removed
 water put back
 10% removed
 10% put back

$$\frac{9}{11} \times \frac{3}{5} = \frac{27}{55} \Rightarrow 27:28$$

Again

$$\left(\frac{9}{11} \left(\frac{M}{T} \right) \right) \cdot \frac{4}{5}$$

~~Now "20" removed out of "110"~~

$$+30\% \rightarrow \left(\frac{9}{11} \left(\frac{M}{T} \right) \right) \frac{11}{10}$$

$$\left(\frac{9}{11} \left(\frac{M}{T} \right) \right) \frac{8}{11}$$

$$\frac{20}{110} \rightarrow \frac{2}{11}$$

$$\frac{9}{11} \left(\frac{M}{T} \right) \rightarrow \frac{9}{11} \left(\frac{M}{T} \right) + \frac{2}{11}$$

$$\rightarrow \left(\frac{9}{11} \left(\frac{M}{T} \right) \right) \frac{9}{11} + \frac{2}{11} \rightarrow \frac{120}{110} - \frac{12}{11}$$

Q A man purchases a cow and a calf for ₹ 1300.

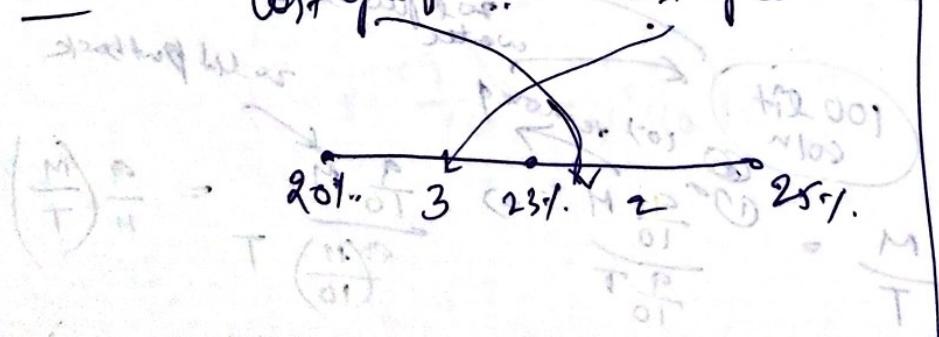
He sold the calf at a profit of 20% and the cow at a profit of 25%.

His total profit was 231.50.

The selling price of the cow

Sol:- Cost of calf : Cost of cow

Here
 $C.P.$ = weight
 $P.L.$ = rate



$$\Rightarrow 3a + 2a = 1300$$

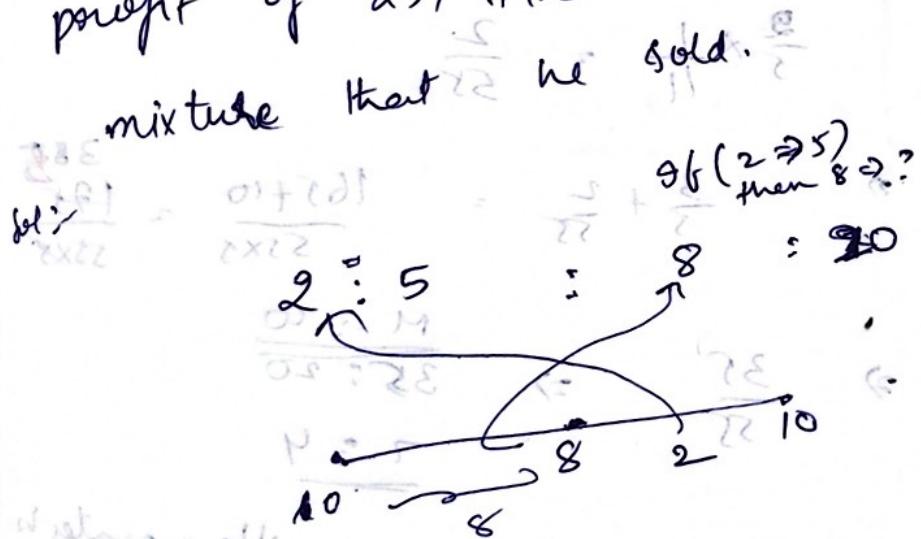
$$\Rightarrow a = 260$$

$$\Rightarrow \text{cost of calf} = 2a = 520; \text{cost of cow} = 3a = 780$$

cow sold at 25/-

$$\Rightarrow \frac{\$}{4} \times 780 = 195 \Rightarrow \text{cow sold at S.P.} = 975$$

Q A milkman purchased milk at Rs 10/liter mixed 5 liters of water in it. By selling the mixture at the rate of Rs 10/liter he earns a profit of 25%. Find the total amount of mixture that he sold.



$$S.P. = \frac{5}{7} C.P.$$

$$C.P. = \cancel{2} \times 10$$

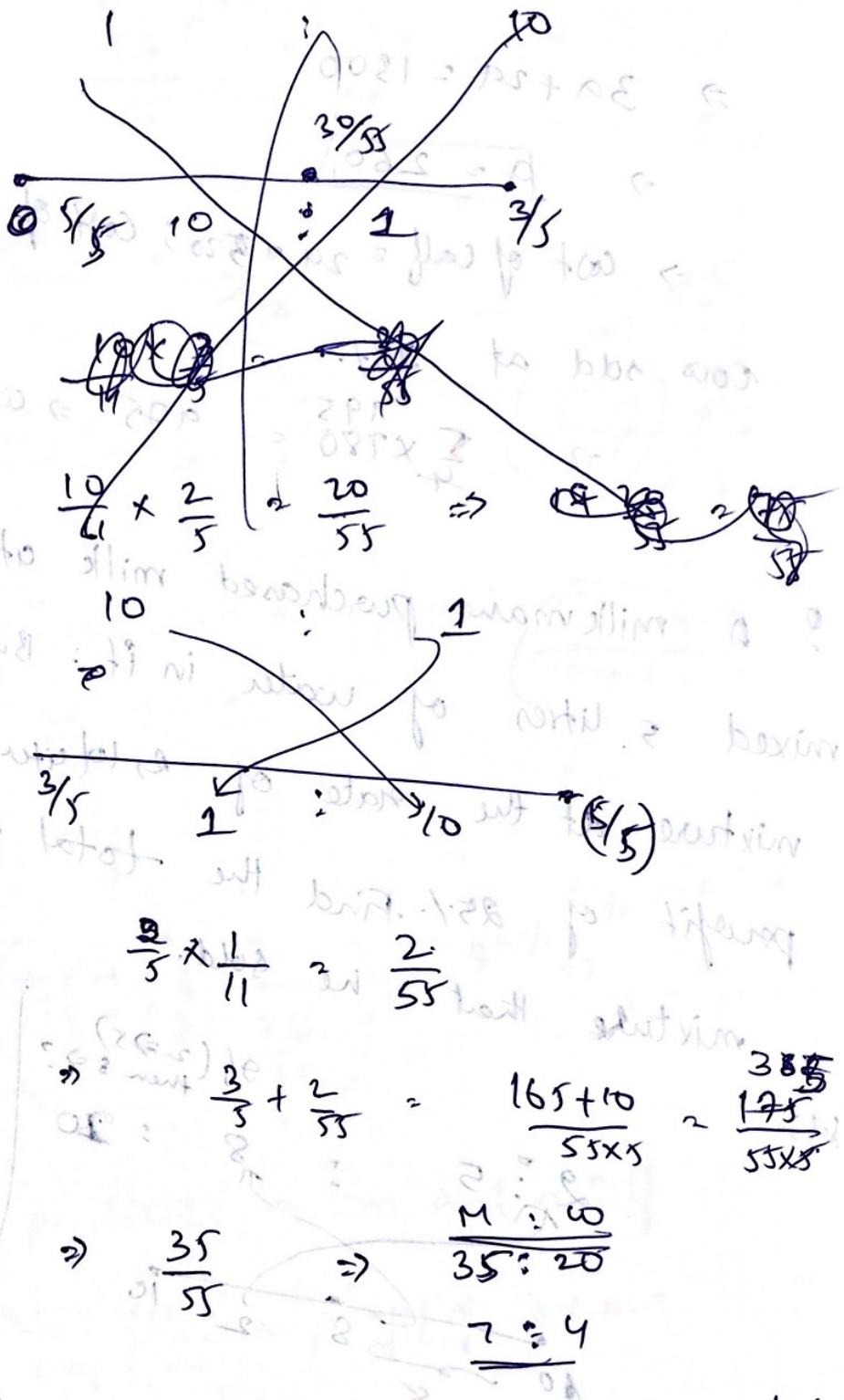
$$C.P. = \frac{4}{5} \times 10$$

$$C.P. = 8$$

What is the ratio after adding 10%? \Rightarrow total mixture = $20 + 5 = 25$ liters.

Q Ratio of milk & water in a mixture is increased by 10% by adding pure milk. What is the ratio of milk & water in the resulting mixture?

Sol:



Q When 20% of a soln of milk + water is removed & replaced with water, the ratio of milk and water in the resulting soln become 2 : 3. Find the ratio of milk + water in the original soln.

$\frac{4}{5}$ of soln + water ($\frac{1}{5}$ of soln)

4 : : 1

water (parts of A) = 4

initial $\frac{4}{5}$: 1 $\frac{2}{5}$ 4 0

→ what 'A' parts remain after removing

just removed

100 $\xrightarrow{-20}$ 20 } 100 : 1
S : 1

-20

100 : 1

removed + replaced

100 $\xrightarrow{-20}$ 80
(80) $\xleftarrow{+20}$ 100

100 $\xrightarrow{-25}$ 75
75 $\xrightarrow{+25}$ 100
3 : 1

stability of 4 equal parts

now parts left $\frac{5}{3} -$ parts 8 - A
bottom below open. $\frac{8}{3} -$ parts 8 - $\frac{1}{3}$ 8

P = 81.55 Pa and 81.55 R

$\frac{3}{8} \times 8 = 3$ drops without loss of parts

stable

no loss of parts

parts 8 - 8 parts 8 - 8 parts 8 - 8 parts 8

parts 8 - 8 parts 8 - 8 parts 8 - 8 parts 8

Time & Work

A - 12 days worked

B - 16 days

B - works 4 days & leaves;

How many days A take to finish work

$$\text{M.R.} \quad 1 : p \quad A - 12 - \frac{4}{3} \text{ days} \quad L.C.M = 48 \\ B - 16 - 3 \frac{3}{4} \text{ days}$$

$$4 + 3 = 7 \frac{1}{4} \text{ days}$$

$$= 4 \text{ days } \leftarrow 4 \times 72 = 288, 120 \text{ remain}$$

$$\frac{20}{4} = 5 \text{ days}$$

\Rightarrow A take 5 days to complete

$$\text{Q.2} \quad A - 8 \text{ days} - \frac{72}{8} = 9 \quad \text{last 2 days A worked} \\ B - 9 \text{ days} - \frac{72}{9} = 8 \quad \text{days worked together?}$$

$$9 \times 2 = 18 \rightarrow 72 - 18 = 54$$

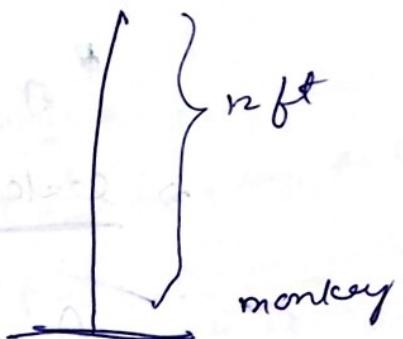
$$\text{days worked together} = \frac{54}{17} = 3 \frac{3}{17}$$

Negative Work:

① first hr = +3 ft

second hr = -2 ft

when $12 \text{ ft} = ?$



$$9 \text{ ft} + 3 \text{ ft} = 12 \text{ ft}$$

$$\downarrow$$

18 hrs + 1 hr $\Rightarrow 19 \text{ hours}$

②

A 8 days

how many days
wall built?

- B (destroys)
wall 9 days

72 units

$$\begin{array}{l} A - 9 \text{ u/d} \\ B - 8 \text{ u/d} \end{array} \quad \left. \begin{array}{l} 2 \text{ days} \\ \hline \end{array} \right\} = +1 \text{ unit}$$

$$63 \text{ u} + 9 \text{ u}$$

$$12 \frac{1}{2} \text{ d} + 1 \text{ d} \Rightarrow 13 \frac{1}{2} \text{ days}$$

③

P_A

$$\begin{array}{l} 12 \text{ hrs} \rightarrow 4 \text{ u/hr} \\ 16 \text{ hrs} \rightarrow 3 \text{ u/hr} \end{array}$$

$$\begin{array}{l} 4 \times 3 \\ 4 \times 4 \end{array} \Rightarrow 4 \times 3 \times 4 \Rightarrow 48$$

(leak) P_B

simultaneously

$$4 \text{ u} + 3 \text{ u} = 7 \text{ u}$$

$$\text{every hr. } 4 - 3 = 1 \text{ litre coming}$$

$$\frac{1}{2} \text{ hr} \Rightarrow 48 \text{ lit} \Rightarrow 48 \text{ liters}$$

$$A = 8 \text{ hrs (actually)}$$

$$\cancel{A = 12 \text{ hrs (due to leak)}}$$

\Rightarrow leak ?

$$A = 8 = 3 \text{ u/hr}$$

$$A_2 = 12 = 2 \text{ u/hr}$$

$$\Rightarrow \text{leak} = 3 - 2 = 1 \text{ u/hr}$$

\Rightarrow leak takes 24 hrs to empty tank

Q

$$A + B = 10 \text{ days} = 6 \text{ u/d}$$

$$B + C = 12 \text{ days} = 5 \text{ u/d}$$

$$C + A = 15 \text{ days} = 4 \text{ u/d}$$

$$(A + B + B + C + C + A) = 15 \text{ u/d}$$

$$2(A + B + C) = 15 \text{ u/d}$$

$$\underbrace{A + B + C}_{\text{L.C.M}} = 7.5 \text{ u/d}$$

L.C.M

$$5 \times 2$$

$$6 \times 2$$

$$5 \times 3$$

$$C = 7.5 - 6$$

$$\boxed{C = 1.5 \text{ u/d}}$$

$$\frac{5 \times 2 \times 6}{15}$$

$$\therefore C = \frac{600}{15} = 40 \text{ days}$$

$$A + B + C = 7.5$$

$$A + B + C = 7.5$$

$$B = 3.5$$

$$\boxed{A = 2.5}$$

$$\frac{600}{24} = 25 \text{ days}$$

$$\frac{600}{35} = 17 \frac{5}{7}$$

A, B

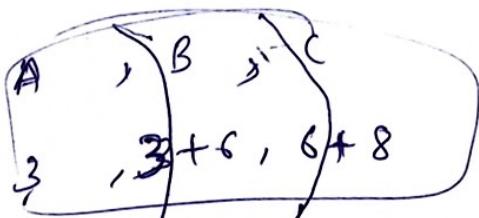
10d

Rate
6

B, C

12d

5



A+B
3 days
 $6 \times 3 = 18$

B+C
6 days
 $6 \times 5 = 30$

12 units left

8 days

$$\Rightarrow \frac{12}{8} = 1.5 \text{ units/day}$$

LCM
60

A works for 3 days then left
B works for 9 days then left
& C finishes remaining work
in 14 days

A, B, C
3, 9, 14

$$\Rightarrow \text{C. rate} = 10.5 \text{ u/d}$$

II

A, B

Rate
 $\frac{1}{2.5+3.5}$

B, C

$3.5 + 1.5$

\Rightarrow how long for 'A' + 'C'

$$2.5 + 1.5 = 4 \text{ u/d}$$

$\frac{60}{4} = 15 \text{ days}$

Time, Speed & Distance

- 1) Time × Speed = Distance
 - 2) Basic Variation
 - 3) Relative speed
 4. Average Speed
 5. Boats & Streams
 6. Circular motion (Clocks)
- Q A person travels three distances in the ratio 1:2:3 at speeds of 20 m/s, 30 m/s & 40 m/s. If the total time taken is 23 min, find the total distance covered.

Given: $d_1 : d_2 : d_3$
 $t_1 = 2x : 3x : 4x$; $d_1 + d_2 + d_3 = 23 \text{ min}$

Now, $\frac{d_1}{20} : \frac{d_2}{30} : \frac{d_3}{40} = 1 : 2 : 3$

$$t_1 = \frac{x}{20}, t_2 = \frac{2x}{30}, t_3 = \frac{3x}{40}$$

$$\Rightarrow \left(\frac{x}{20} + \frac{2x}{30} + \frac{3x}{40} \right) = 23 \times 60$$

$$x = \frac{120}{120} = 23 \times 60 \Rightarrow x = 7200 \text{ m.}$$

Q I travel for three stretches of time in the ratio 1:2:3 at speeds of 20 kmph, 30 kmph, 40 kmph. If I cover a total distance of 100 km

Find the total time for which I'm travelling

Sol:

$$d_1 = d_2 = d_3 \text{ (speed & dist)}$$

$$1x : 2x : 3x$$

$$\text{speed: } 20 \text{ kmph} : 30 \text{ kmph} : 40 \text{ kmph}$$

$$d = 100 \text{ km}$$

$$\cancel{d_1 + d_2 + d_3} \quad D = S \times T$$

$$d_1 = 20x ; d_2 = 30x ; d_3 = 40x$$

$$d_1 + d_2 + d_3 = d \Rightarrow 20x + 30x + 40x = 100$$

$$x = 0.5$$

$$t_1 : t_2 : t_3$$

$$0.5 : 1.5 : 2.5$$

$$30 \text{ min} : 60 \text{ min} : 90 \text{ min}$$

Q I cover a total dist of 300km in 8 hours, partly at a speed of 30 kmph and partly at a speed of 50 kmph. For what time was I travelling at a speed of 30 kmph.

Sol:

$$d_1 + d_2 = 300$$

$$\Rightarrow 30t_1 + 50t_2 = 300$$

$$t_1 + t_2 = 8$$

Q Find the ratio of the time taken to cover two distance in the ratio 4:5 at speeds in the ratio 4:3 respectively.

PL

$$\frac{s_1 T_1}{D_1} = \frac{s_2 T_2}{D_2} = \frac{4 \times t_1}{4t} = \frac{3 \times t_2}{3t}$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{1}{3} \Rightarrow 3 : 1$$

estimating \rightarrow work of legs

$$D = S \times T$$

$$S = \frac{D}{T}$$

$$S \propto \frac{D}{T}$$

$$D \propto S T$$

$$D \propto S \quad (t = \text{constant})$$

$$D \propto T \quad (S = \text{constant})$$

A person travels from home to office

if

10 km/hr

arrive 10 min late

15 km/hr

10 min early

① find actual time, it is ② distance b/w A-B

③ speed to be travelled to reach office at correct time

sol :-

10 km/hr

15 km/hr

+ 10 min

- 10 min

\Rightarrow Dist \Rightarrow Constant

$$\Rightarrow s_1 T_1 = s_2 T_2 \Rightarrow 10 \left(t + \frac{1}{6} \right) = 15 \left(t - \frac{1}{6} \right)$$

$$\Rightarrow 10 \left(\frac{6t+1}{6} \right) = 15 \left(\frac{6t-1}{6} \right) \Rightarrow 60t + 10 = 90t - 15$$

$$30t = 25$$

$$t = \frac{5}{6}$$

$$\Rightarrow \underline{\underline{50 \text{ min}}}$$

i) distance = $10 \times 1 = 10 \text{ km}$

ii) Speed travelled = $\frac{10^2}{\left(\frac{5}{6}\right)} = 12 \text{ km/h}$

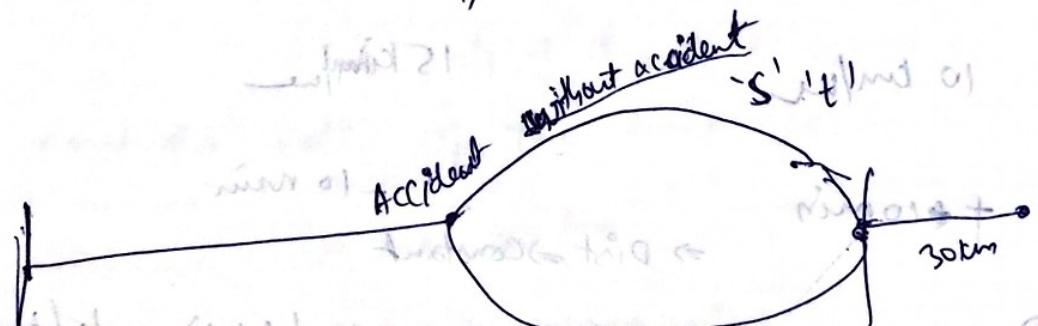
Q A train meets with an accident and travels at $\frac{4}{7}$ of its regular speed hereafter and hence it reaches its destination 36 min late. Had the accident occurred 30 kms further, the train would have been late by only 21 min . Find the regular speed of train.

Sol:

$$S_1 = S \Rightarrow t_1 = t; d = d$$

$$S_2 = \frac{4}{7} S; d_2 = t + 36; d = d$$

$$S_3 = \frac{4}{7} S; t_2 = t + 21; d = (d - 30)$$

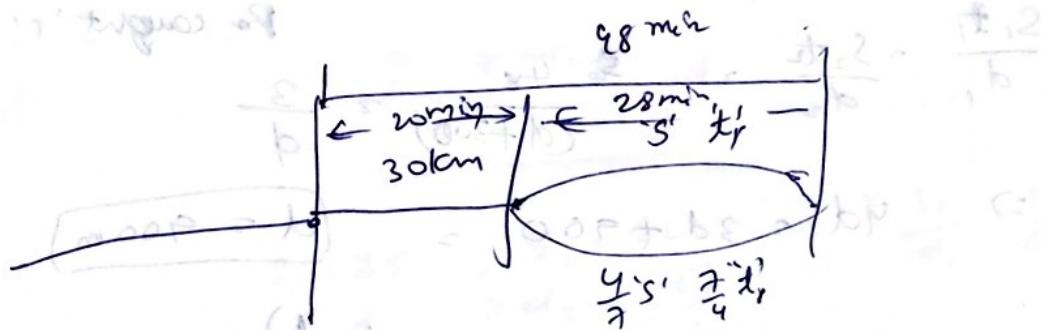


$$\left(\frac{4}{7}t\right) + (t+36) = \left(\frac{4}{7}t\right) + \left(t + \frac{21}{7}\right)$$

I past:
diff in time : $\frac{3}{4} t = 36 \Rightarrow \underline{\underline{t = 48}}$

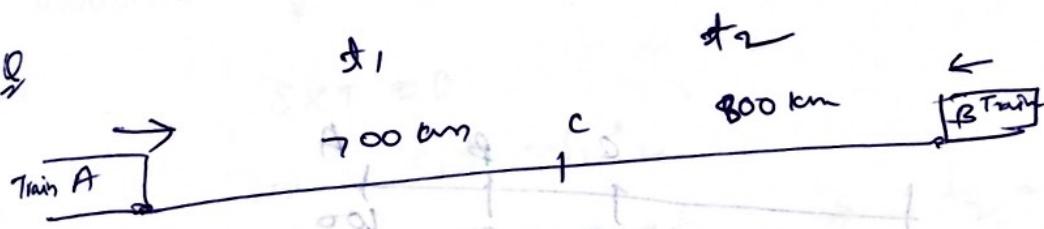
~~diff~~ in time : $\frac{3}{4} t = 36 \Rightarrow \underline{\underline{t = 48}}$

$$\underline{t = 48 \text{ min}}$$



$$\frac{3}{4} x_1 = 21 \Rightarrow x_1 = 28 \text{ min}$$

$$d = \frac{t}{\text{km/h}} \times 30 \text{ km} = \frac{30 \text{ km}}{\frac{20}{3} \text{ h}} = 30 \times 3 = 90 \text{ km/h}$$



→ A, B moving towards each other & meet at 'C'.

$$\text{What is } \frac{d_1}{s_1} = \frac{d_2}{s_2} \Rightarrow \frac{d_1}{s_1} = \frac{700}{50} = \frac{800}{50}$$

$$\Rightarrow \frac{s_1}{s_2} = \frac{7}{8} \Rightarrow \underline{s_1 : s_2 = 7:8}$$

Police chasing thief

When thief caught
'P' ran 300m more
than thief
→ at what distance
P caught 'T'?

$$S \times T = D$$

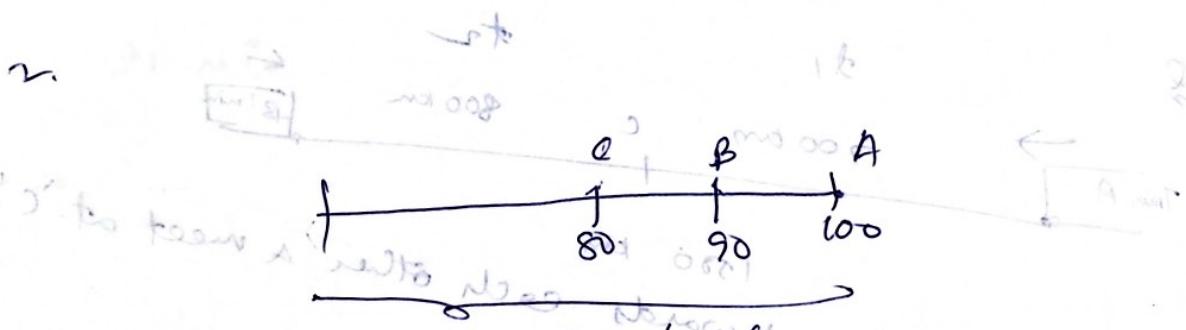
$$\frac{S_1 \cdot T}{d_1} = \frac{S_2 \cdot T}{d_2} \Rightarrow \frac{4}{(d+300)} = \frac{3}{d}$$

$$\Rightarrow 4d = 3d + 900 \Rightarrow d = 900 \text{ m}$$

②

Races (Application of Variation)

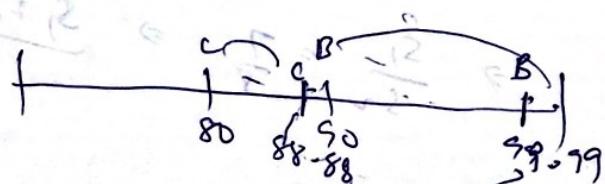
1. A beats B by 10 mt in a 100 mt race
by how much the beat in a 1000 mt race.



In 200 mt race, by how much B beats C.

~~$\frac{200}{80} = \frac{200}{12}$~~

- Q In 100 mt race, how much B beats C?

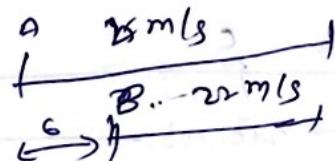


$$\Rightarrow \text{In 200 mt} = 11.11 \text{ mt}$$

$$200 \text{ mt} \Rightarrow 22.22 \text{ mt}$$

what should be the length of the race so that two runners A & B running at speeds of 25 m/s & 22 m/s resp. reach the end pt. together even though 'B' was given a head of 6 mts.

sol:



$$s_1 = 25 ; s_2 = 22 \quad \Rightarrow \frac{s_1}{d_1} = \frac{s_2}{d_2}$$

$$\frac{25}{d} = \frac{22}{d-6}$$

$$\therefore d = 50 \text{ m}$$

(Q) Over a certain distance, the speed is increased by 10%. what would be % change in time?

Ans:

$$SXT \Rightarrow$$

$$\Rightarrow S_1 t_1 = S_2 t_2$$

$$S t_1 = \frac{11}{10} S t_2$$

$$t_2 = \frac{10}{11} t_1$$

$$\therefore \cancel{t_2 = (1.1) t_1}$$

$$\Rightarrow \frac{9.1 t_1}{10 t_1} \downarrow$$

Over a certain dist, the speed was increased by 4 kmph because which the time for the journey reduced from 8 hrs to 7.5 hrs what was speed?

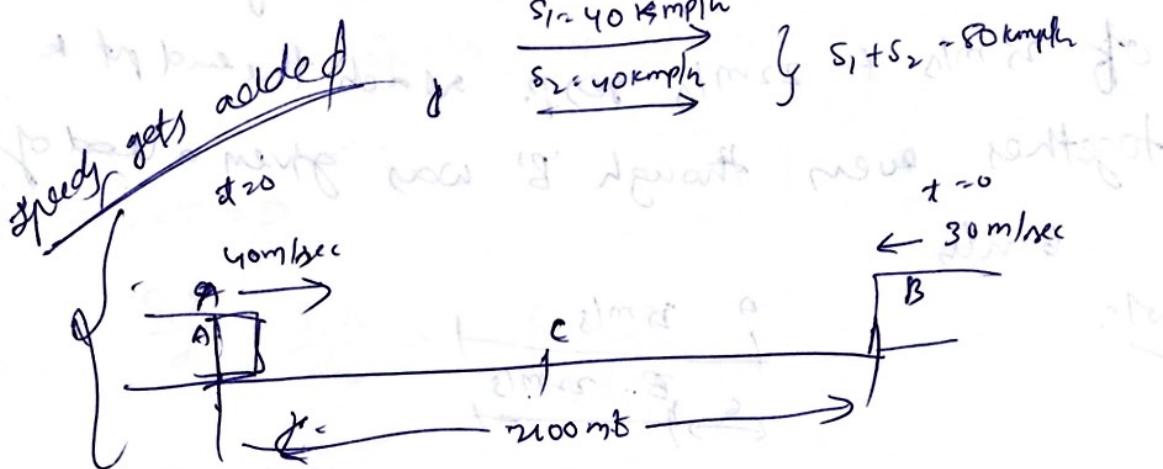
what was speed? what is speed.

60 kmph; 64 kmph

Sol:

Relative Speed & Application

Beeps to police & a runner out back.



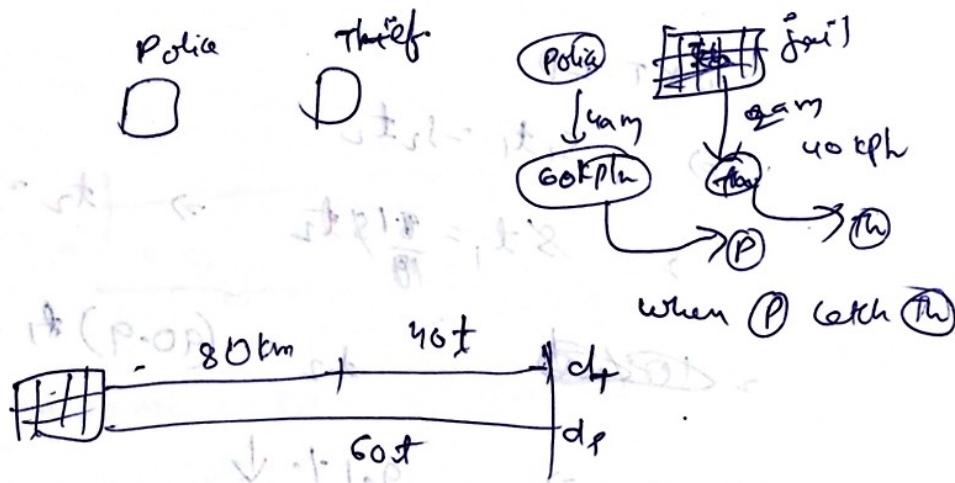
c = ? , when & where

$$\text{d' - time}$$

at $t = 0$ $d_1 = 40t$; $d_2 = 30t$

$$40t + 30t = 200$$

in beep left, probably with speeds & all blow back. Now get known
Subtracted



Position w.r.t. ~~time~~ \rightarrow $d_1 = 60t$, $d_2 = \text{constant}$

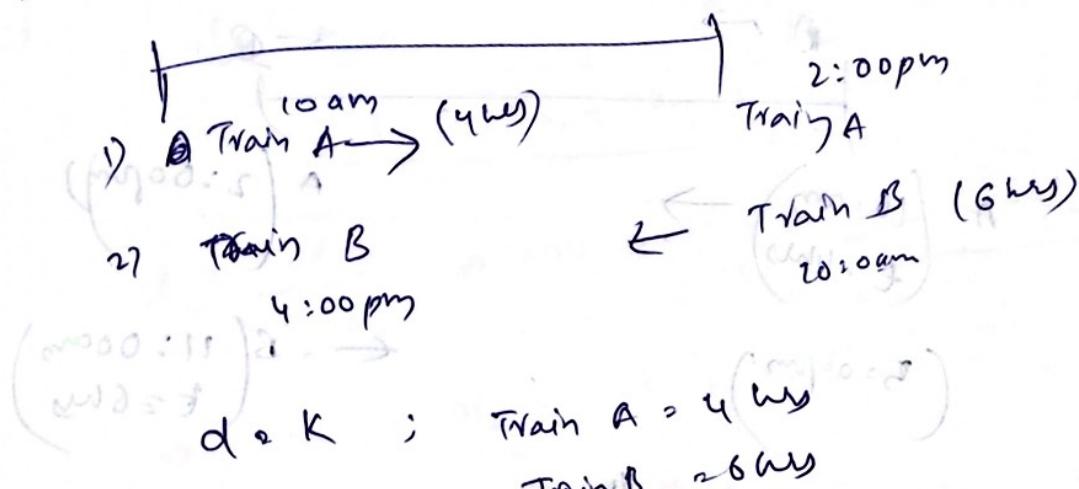
$$40t + 80 = 60t$$

$$20t = 80 \Rightarrow t = 4 \text{ hours}$$

$$t = \frac{80}{20} \approx 4 \text{ hours}$$

$$40t + 80 \Rightarrow 240 \text{ km}$$

Leaving from two points at same time



when they will meet?

to solve it we will assume

$$(S_A 4) \Rightarrow S_B 6 \Rightarrow S_A : S_B = 2 : 3$$

$$\frac{S_A}{S_B} = \frac{2}{3} \Rightarrow S_A = \frac{2}{3} S_B$$

$$t = \frac{d}{S_1 + S_2}$$

$$t = \frac{d}{\frac{2}{3} S_B + S_B} = \frac{d}{\frac{5}{3} S_B}$$

$$t = \frac{d}{\frac{5}{3} S_B} = \frac{d}{\frac{5}{3} \times 6} = \frac{d}{10}$$

$$t = \frac{12}{10} = 1.2 \text{ hours}$$

(Q1)

take L.C.M of S_1, S_2

$$t = \frac{12}{S_1 + S_2}$$

$$t = \frac{12}{6 + 2} = \frac{12}{8} = 1.5 \text{ hours}$$

$$t = 1.5 \text{ hours} = 1 hour 30 minutes$$

$\approx 2.4 \text{ hr}$

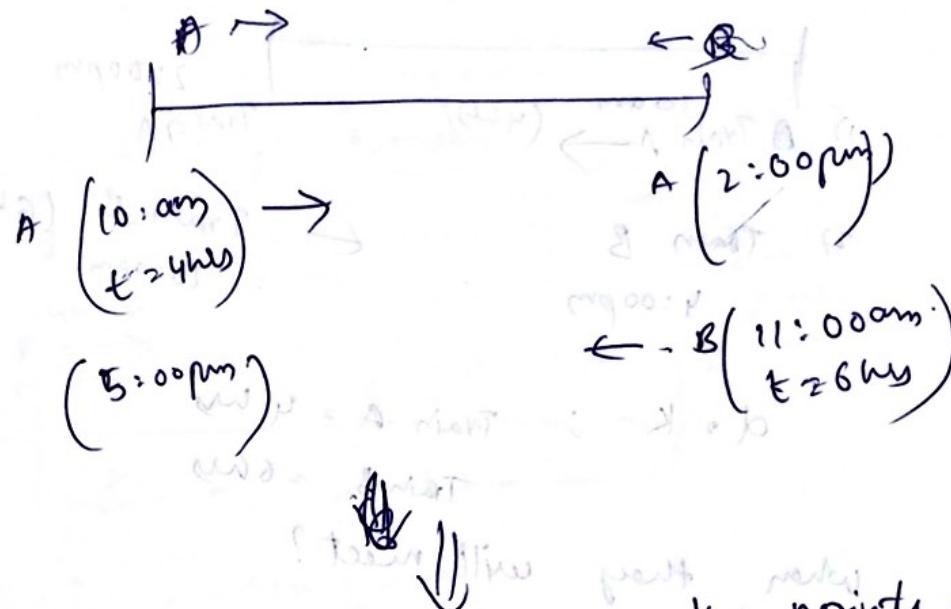
now we have to do about '2' if make it

then what is the result in meter (m)

$$5 \times 8^2 \times 8^2$$

Leaving from two pts at diff times

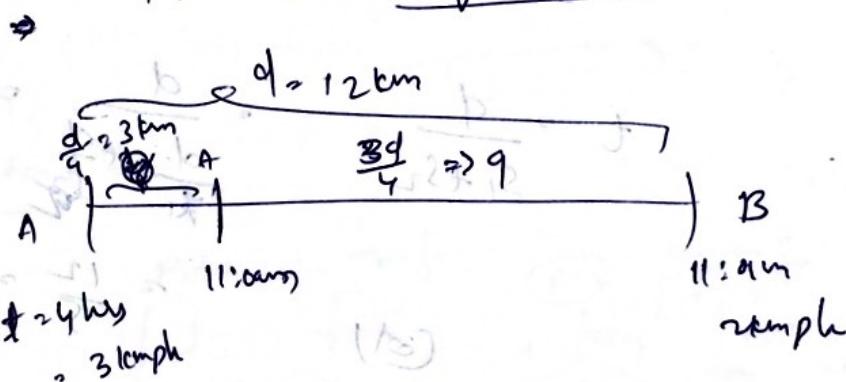
Q



make them to leave the points at

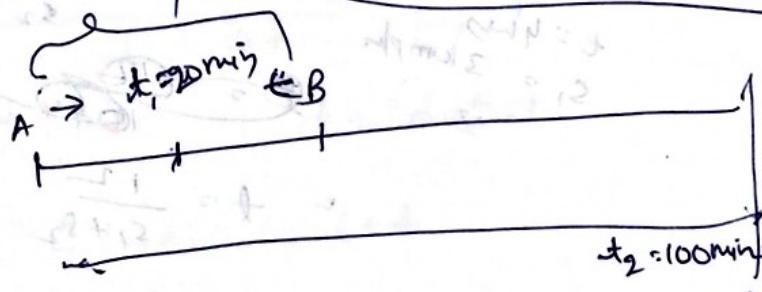
same time

⇒ move 'A' say 1 hour i.e. at 11:00 am



Q

(towards & departing from each other)



- i) when 'A' & 'B' towards each other met in 20 min
- ii) when in opp directions → after 100 min

$$S_A = ? ; S_B = ?$$

$$t_1 = \frac{d}{s_1 + s_2} \Rightarrow 20 = \frac{d}{s_1 + s_2}$$

$$\Rightarrow d = 20s_1 + 20s_2$$

$$t_2 = \frac{d}{s_1 - s_2} \Rightarrow 100 = \frac{d}{s_1 - s_2} \Rightarrow 100s_1 - 100s_2 = d$$

$$20s_1 + 20s_2 = 100s_1 - 100s_2$$

$$120s_2 = 80s_1 \Rightarrow 3s_2 = 2s_1$$

$$\Rightarrow \frac{s_1}{s_2} = \frac{3}{2}$$

B) Superman flying \downarrow w/ train

(SM) \rightarrow 100 km/hr

and 08 \rightarrow 80 km/hr \rightarrow 180 km/hr \rightarrow 100 km/hr

TB \rightarrow 40 km/hr

3600 km \rightarrow $\frac{3600}{120} = 30$ hrs until they met

\rightarrow Superman flies \downarrow w/ train with a accident \rightarrow distance travelled by SM?

and 08 \rightarrow 100 \times 30 hrs \rightarrow 3000 km

Dog, Thief, Police

Thief - 2:00 \rightarrow 40 km/h 405-80 km
 Police - 5:00 \rightarrow 60 km/h 240 km

- Q Dog moves to the thief & comes back to police
 & again moves to thief.
 what is the total distance covered by dog in forward direction?

Given: $d = 360 \text{ km}$
 Chase $\Rightarrow \frac{360}{60} = 6 \text{ hrs}$

dog moves $= 80 \times 6 = 480 \text{ km}$

displacement $= 360 \text{ km}$

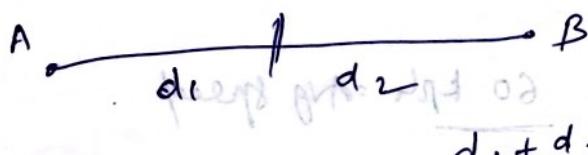
$480 - 360 = 120 \text{ km}$

$\frac{120}{2} = 60 \text{ (forward dist)}$

$480 - 60 = 420 \text{ km}$ Dog moves

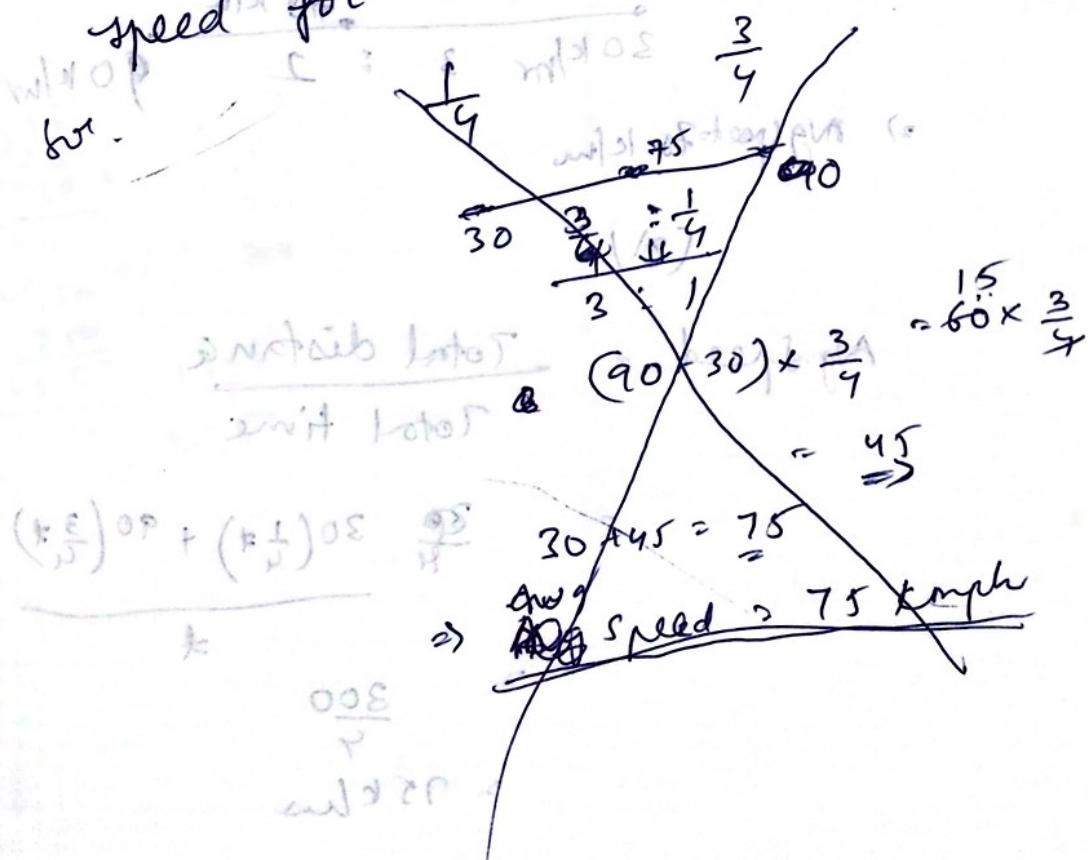
Average Speed

$$A = \frac{\text{Total distance}}{\text{Total time}}$$



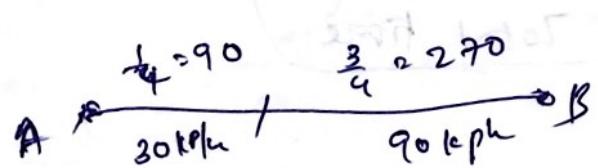
$$\text{Avg speed} = \frac{d_1 + d_2}{\frac{d_1}{40} + \frac{d_2}{60}}$$

Ques. A man walked $\frac{1}{4}$ th of the dist at a speed of 30 kmph and the remaining dist at a speed of 90 kmph. What is my avg speed for the journey?



SQ: take L.C.M (speeds)

→ 360 (or) 180 whichever is convenient



$$\frac{90}{30} = 3 \text{ hrs}$$

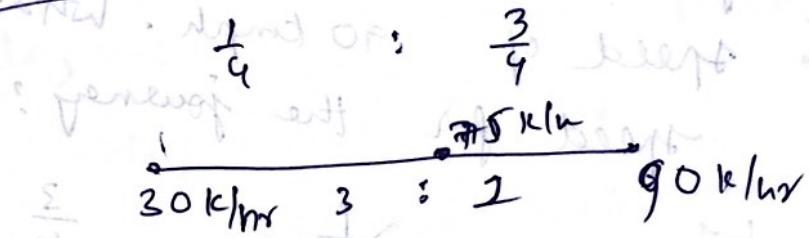
$$\frac{270}{90} = 3 \text{ hrs}$$

$$\Rightarrow \frac{360}{6} = \underline{\underline{60 \text{ kph}}} = \text{Avg speed}$$

Now

(Q) ii) $\frac{1}{4}$ hr of time travelled with 30 kph
& remaining at a speed of 90 kph

Now scale method can be used



⇒ Avg speed 75 kph

(Q)

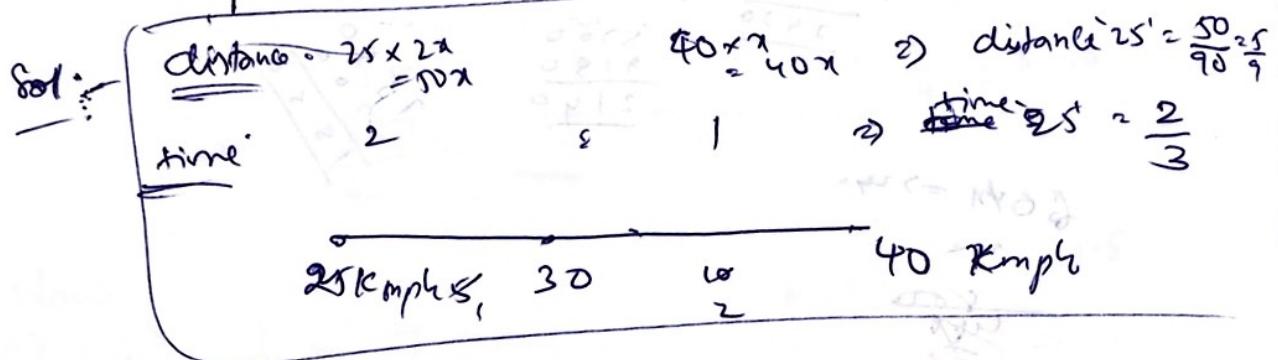
Avg speed

$$\frac{\text{Total distance}}{\text{Total time}}$$

$$\frac{30 \left(\frac{1}{4} \right) + 90 \left(\frac{3}{4} \right)}{4}$$

$$\frac{300}{4} \\ = 75 \text{ kph}$$

Q Two stretches of a journey were done at speeds of 25 kmph & 40 kmph and the avg speed for the journey was 30 kmph. what fraction of distance / time were completed at 25 kmph?



Q $\frac{1}{3}$ rd of the distance for a journey was done at a speed of 45 kmph, half the distance was done at a speed of 60 kmph and the remaining distance at a speed of 36 kmph. what is the avg speed for the journey?

Sol:

choose the L.C.M of (45, 60, 36)

such that it gives clear cut 'hours'

up to speed up to speeds and A

Up to speed up to speeds and A

$d \rightarrow$ distance
 $\frac{1}{3} d \rightarrow 45 \text{ kmph}; \frac{1}{2} d \rightarrow 60 \text{ kmph}; \frac{1}{3} d \rightarrow 36 \text{ kmph}$

$$\text{Avg. Speed} = \frac{d}{t_1 + t_2 + t_3}$$

$$= \frac{d}{\frac{(d)}{45} + \frac{(d)}{60} + \frac{(d)}{36}}$$

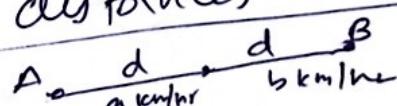
$$\text{clock} = \frac{360 \times 6}{100 \times 6}$$

$$= \frac{\frac{360 \times 6}{(100 \times 6)}}{\frac{3}{3 \times 45} + \frac{3}{180 \times 6} + \frac{5}{6 \times 36}}$$

$$= \frac{\frac{360 \times 6}{100}}{\frac{6}{22}}$$

Two equal distances at diff speeds

Ex:



Avg. speed

$$\text{Avg. Speed} = \frac{\text{total distance}}{\text{total speed}}$$

$$= \frac{\frac{d+a}{d+b}}{\frac{1}{a} + \frac{1}{b}} = \frac{2}{\frac{1}{a} + \frac{1}{b}}$$

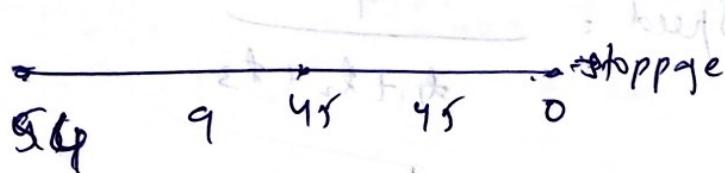
$$\boxed{\text{Avg Speed} = \frac{2ab}{a+b}}$$

Q A bus travels at a speed of 54 kmph but because of stoppages its overall speed is reduced to 45 kmph. what is the stoppage time per hour?

Sol:-

$$\frac{5}{45} : \frac{1}{45} \Rightarrow \text{stoppage} = \frac{1}{6} \times 60$$

$$= 10 \text{ min}$$



$$\Rightarrow \frac{(P)}{S} + \frac{(P)}{S} + \frac{(P)}{S}$$

"Excess" \Rightarrow stoppage

$$\left(\frac{2 \times 0.05}{54 \times 2} \right) + \left(\frac{2 \times 0.05}{54 \times 2} \right) + \left(\frac{2 \times 0.05}{54 \times 2} \right)$$

$$\frac{2 \times 0.05}{54 \times 2} = \frac{2 \times 0.05}{E + P + S}$$

Stoppage to overall hours lost

$$\frac{2}{54} = \frac{2 \times 0.05}{E + P + S}$$

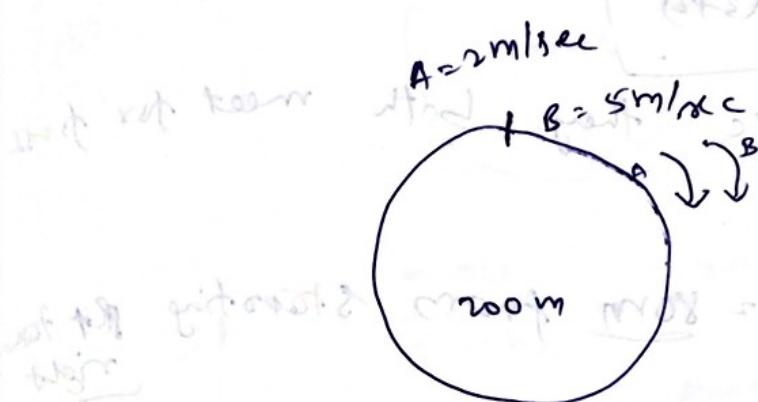
total lost

Stoppage

$$\frac{2}{54} = \frac{2 \times 0.05}{E + P + S}$$

$$\boxed{\frac{2}{54} = \frac{2 \times 0.05}{E + P + S}}$$

Circular Tracks



i) when both 'A' & 'B' will meet at "starting pt."

to calculate time to complete one rotation

$$A = \frac{200}{2} = 100 \text{ sec}$$

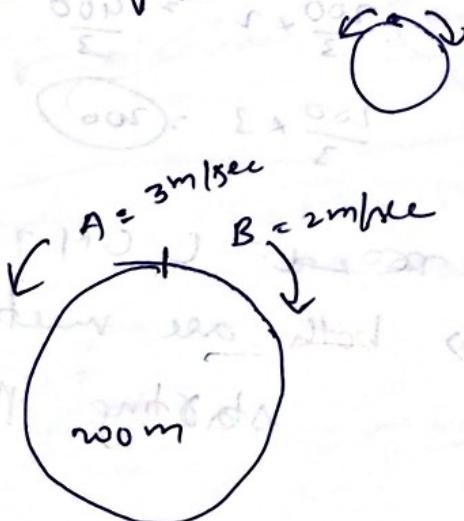
$$B = \frac{200}{5} = 40 \text{ sec}$$

L.C.M ($100, 40$) ~~by ratio~~ \Rightarrow first time they meet at starting pt

$$\Rightarrow L.C.M (100, 40) = 200$$

at 200 sec they ~~will~~ meet for first time

No change in direction changes also



! when they both meet for first time and where?

sol:

adding speeds

$$t = \frac{200}{(3+2)} = 40 \text{ sec}$$

at "40" sec they both meet for first time &

$40 \times 2 = 80 \text{ m}$ from starting pt forward right

& 120 m from starting pt towards left

and they will meet at intervals of

40 sec. i.e. $0, 40, 80, 120, 160$ at same pt

ii)

at starting pt.

$$\text{L.C.M} \left(\frac{200}{3}, \frac{200}{2} \right)$$

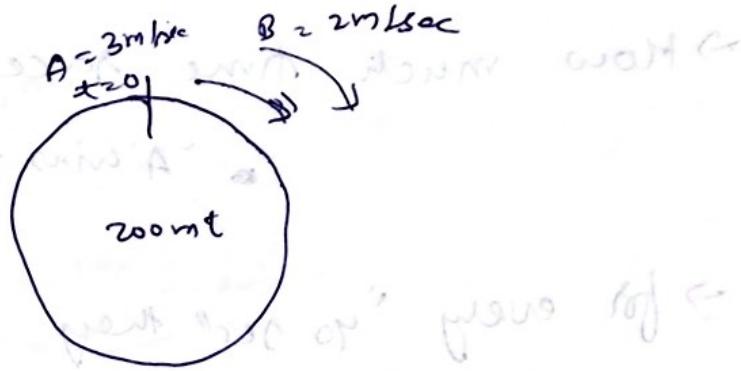
finding L.C.M

$$\begin{aligned} \text{After dividing } \frac{200}{3} \times 1 &= \frac{200}{3} & \text{so } 100 \times 1 = 100 \\ \frac{200}{3} \times 2 &= \frac{400}{3} & 100 \times 2 = 200 \\ \frac{200}{3} \times 3 &= 200 & \end{aligned}$$

$$\Rightarrow \text{L.C.M} = 200$$

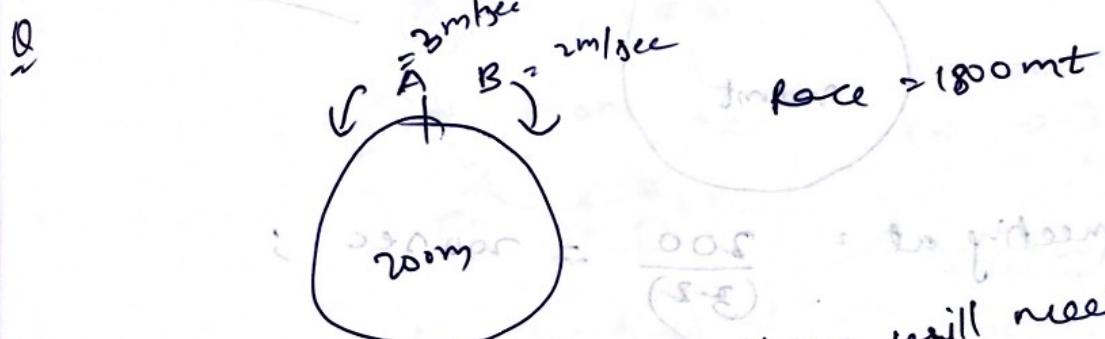
\Rightarrow both are meeting at 200 sec at starting pt.

~~Speeds
gets
subtracted~~



at $\frac{200}{(3-2)} = 200$ sec they first meet at
at 200 sec they first meet at
meet at intervals
200, 400, 600...

part two



Q) How many times they will meet?

sol: first meeting = $\frac{200}{5} = 40$ sec

~~no of times~~ ~~800~~ ~~80~~

∴ 'B' = $40 \times 2 = 80$ mt from starting towards right

(0²)

~~first~~
~~2nd~~
~~3rd~~
~~4th~~
~~5th~~
~~6th~~
~~7th~~
~~8th~~
~~9th~~
~~10th~~
~~11th~~
~~12th~~
~~13th~~
~~14th~~
~~15th~~
⇒ 15 times

$\frac{15}{120} = 15$ times

→ How much time race lasts,

• A wins \Rightarrow

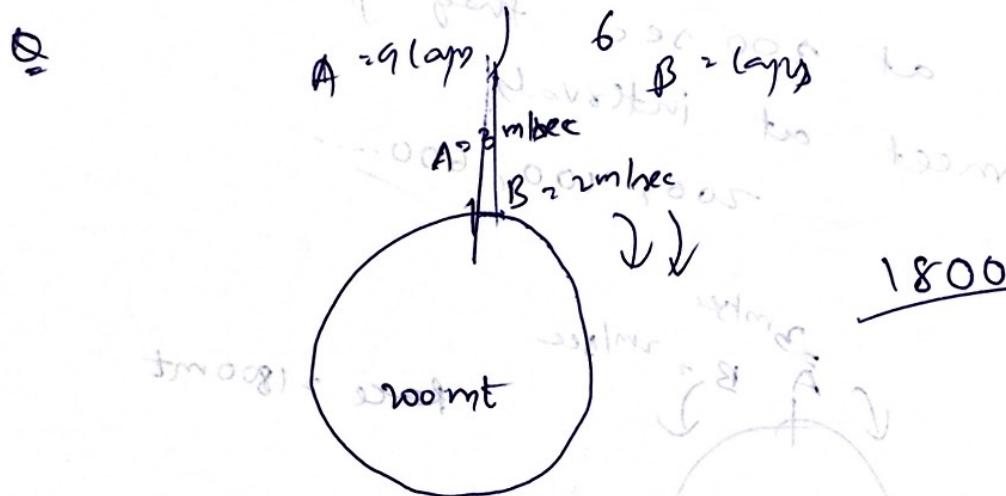
process

$$\frac{1800}{3}$$

$$= 600 \text{ sec}$$

→ for every "40 sec" they meet

$$\frac{600}{40} = 15 \text{ times they will meet}$$



1800m race

$$\text{meeting at } \frac{200}{(3-2)} = 200 \text{ sec};$$

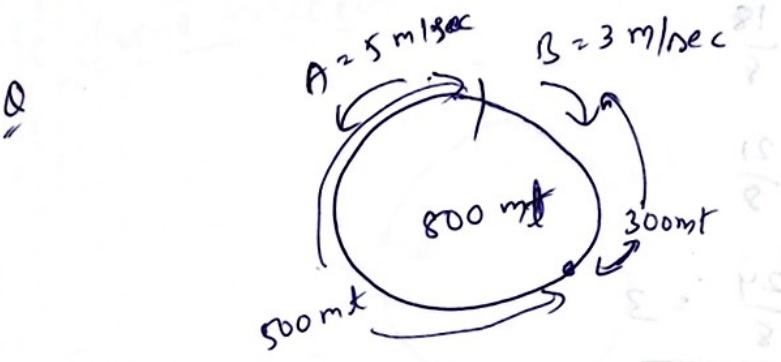
and race lasts $\approx 600 \text{ sec}$

so \Rightarrow 3 meetings

if given to \Rightarrow whenever faster one completes the lap they'll meet slower one

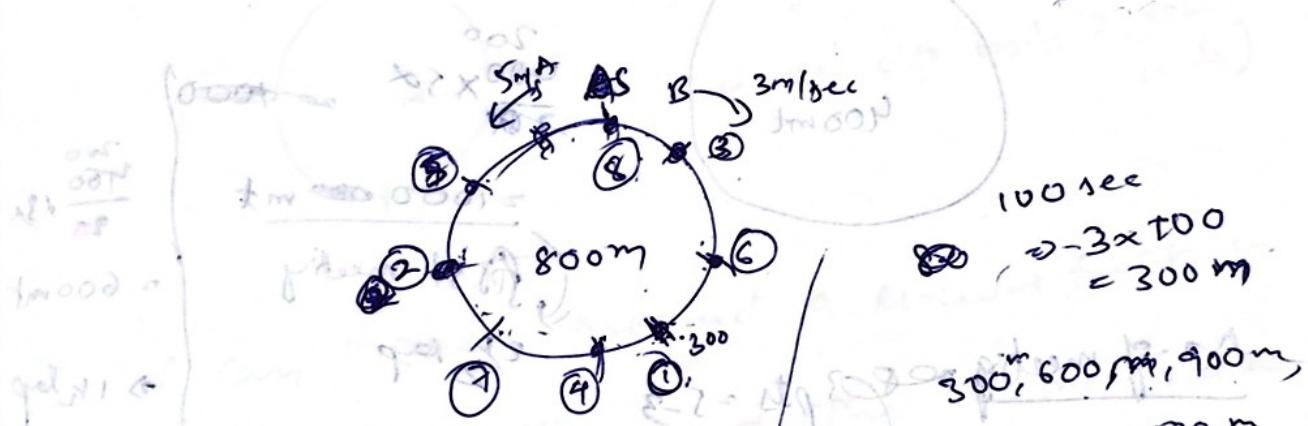
so \Rightarrow 9 + 6 = 15 times whenever they completed a lap they'll meet

they completed a lap they'll meet

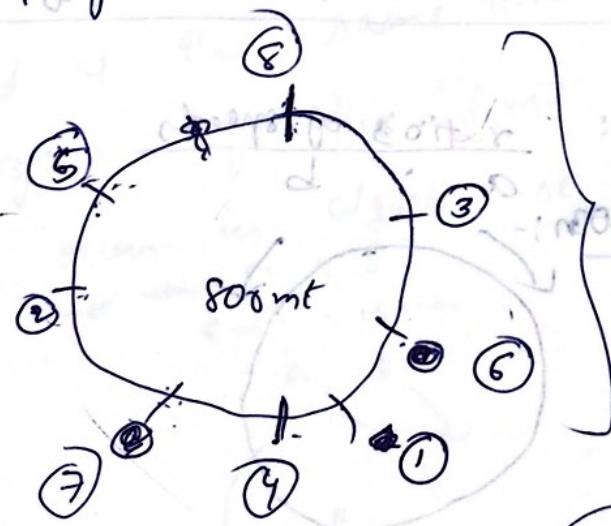


first meet $= \frac{800}{(5+3)}$ 100 sec

100, 200, 300, 400



→ circle, meetings at
a regular interval spots



no. of meeting pts $= (5+3)$
 $= 8$

$\frac{5}{8}$	$\frac{3}{8}$	$\frac{20}{8}$
$\frac{10}{8}$	$\frac{6}{8}$	$\frac{25}{8}$
$\frac{15}{8}$	$\frac{9}{8}$	$\frac{30}{8}$
$\frac{18}{8}$	$\frac{12}{8}$	$\frac{35}{8}$
$\frac{19}{8}$	$\frac{13}{8}$	$\frac{38}{8}$

$$\frac{30}{8}$$

$$\frac{18}{8}$$

$$\frac{30}{8}$$

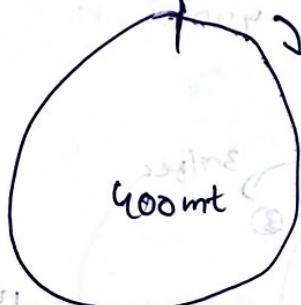
$$\frac{21}{8}$$

$$\underline{\frac{40}{8} \Rightarrow 5}$$

$$\underline{\frac{24}{8} = 3}$$

$\frac{200}{(2+3)} = 40$ laps

$$A : B = 2 : 3$$



$$200$$

$$\frac{400 \times 5x}{2x}$$

$$= 1000 \text{ mt}$$

first meeting

$$2 \times \Sigma \text{ lap}$$

$$\frac{200}{2x} \times 3x$$

$\approx 600 \text{ mt}$

$\Rightarrow 1 \text{ lap}$

no. of meeting pts = $5-3$

$200 \times 3 = 600$

$200 \times 2 = 400$

$200 \times 1 = 200$

$200 \times 0 = 0$

$200 \times -1 = -200$

$200 \times -2 = -400$

$200 \times -3 = -600$

$200 \times -4 = -800$

$200 \times -5 = -1000$

$200 \times -6 = -1200$

$200 \times -7 = -1400$

$200 \times -8 = -1600$

$200 \times -9 = -1800$

$200 \times -10 = -2000$

$200 \times -11 = -2200$

$200 \times -12 = -2400$

$200 \times -13 = -2600$

$200 \times -14 = -2800$

$200 \times -15 = -3000$

$200 \times -16 = -3200$

$200 \times -17 = -3400$

$200 \times -18 = -3600$

$200 \times -19 = -3800$

$200 \times -20 = -4000$

$200 \times -21 = -4200$

$200 \times -22 = -4400$

$200 \times -23 = -4600$

$200 \times -24 = -4800$

$200 \times -25 = -5000$

$200 \times -26 = -5200$

$200 \times -27 = -5400$

$200 \times -28 = -5600$

$200 \times -29 = -5800$

$200 \times -30 = -6000$

$200 \times -31 = -6200$

$200 \times -32 = -6400$

$200 \times -33 = -6600$

$200 \times -34 = -6800$

$200 \times -35 = -7000$

$200 \times -36 = -7200$

$200 \times -37 = -7400$

$200 \times -38 = -7600$

$200 \times -39 = -7800$

$200 \times -40 = -8000$

$200 \times -41 = -8200$

$200 \times -42 = -8400$

$200 \times -43 = -8600$

$200 \times -44 = -8800$

$200 \times -45 = -9000$

$200 \times -46 = -9200$

$200 \times -47 = -9400$

$200 \times -48 = -9600$

$200 \times -49 = -9800$

$200 \times -50 = -10000$

$200 \times -51 = -10200$

$200 \times -52 = -10400$

$200 \times -53 = -10600$

$200 \times -54 = -10800$

$200 \times -55 = -11000$

$200 \times -56 = -11200$

$200 \times -57 = -11400$

$200 \times -58 = -11600$

$200 \times -59 = -11800$

$200 \times -60 = -12000$

$200 \times -61 = -12200$

$200 \times -62 = -12400$

$200 \times -63 = -12600$

$200 \times -64 = -12800$

$200 \times -65 = -13000$

$200 \times -66 = -13200$

$200 \times -67 = -13400$

$200 \times -68 = -13600$

$200 \times -69 = -13800$

$200 \times -70 = -14000$

$200 \times -71 = -14200$

$200 \times -72 = -14400$

$200 \times -73 = -14600$

$200 \times -74 = -14800$

$200 \times -75 = -15000$

$200 \times -76 = -15200$

$200 \times -77 = -15400$

$200 \times -78 = -15600$

$200 \times -79 = -15800$

$200 \times -80 = -16000$

$200 \times -81 = -16200$

$200 \times -82 = -16400$

$200 \times -83 = -16600$

$200 \times -84 = -16800$

$200 \times -85 = -17000$

$200 \times -86 = -17200$

$200 \times -87 = -17400$

$200 \times -88 = -17600$

$200 \times -89 = -17800$

$200 \times -90 = -18000$

$200 \times -91 = -18200$

$200 \times -92 = -18400$

$200 \times -93 = -18600$

$200 \times -94 = -18800$

$200 \times -95 = -19000$

$200 \times -96 = -19200$

$200 \times -97 = -19400$

$200 \times -98 = -19600$

$200 \times -99 = -19800$

$200 \times -100 = -20000$

$200 \times -101 = -20200$

$200 \times -102 = -20400$

$200 \times -103 = -20600$

$200 \times -104 = -20800$

$200 \times -105 = -21000$

$200 \times -106 = -21200$

$200 \times -107 = -21400$

$200 \times -108 = -21600$

$200 \times -109 = -21800$

$200 \times -110 = -22000$

$200 \times -111 = -22200$

$200 \times -112 = -22400$

$200 \times -113 = -22600$

$200 \times -114 = -22800$

$200 \times -115 = -23000$

$200 \times -116 = -23200$

$200 \times -117 = -23400$

$200 \times -118 = -23600$

$200 \times -119 = -23800$

$200 \times -120 = -24000$

$200 \times -121 = -24200$

$200 \times -122 = -24400$

$200 \times -123 = -24600$

$200 \times -124 = -24800$

$200 \times -125 = -25000$

$200 \times -126 = -25200$

$200 \times -127 = -25400$

$200 \times -128 = -25600$

$200 \times -129 = -25800$

$200 \times -130 = -26000$

$200 \times -131 = -26200$

$200 \times -132 = -26400$

$200 \times -133 = -26600$

$200 \times -134 = -26800$

$200 \times -135 = -27000$

$200 \times -136 = -27200$

$200 \times -137 = -27400$

$200 \times -138 = -27600$

$200 \times -139 = -27800$

$200 \times -140 = -28000$

$200 \times -141 = -28200$

$200 \times -142 = -28400$

$200 \times -143 = -28600$

$200 \times -144 = -28800$

$200 \times -145 = -29000$

$200 \times -146 = -29200$

$200 \times -147 = -29400$

$200 \times -148 = -29600$

$200 \times -149 = -29800$

$200 \times -150 = -30000$

$200 \times -151 = -30200$

$200 \times -152 = -30400$

$200 \times -153 = -30600$

$200 \times -154 = -30800$

$200 \times -155 = -31000$

$200 \times -156 = -31200$

$200 \times -157 = -31400$

$200 \times -158 = -31600$

$200 \times -159 = -31800$

$200 \times -160 = -32000$

$200 \times -161 = -32200$

$200 \times -162 = -32400$

$200 \times -163 = -32600$

$200 \times -164 = -32800$

$200 \times -165 = -33000$

$200 \times -166 = -33200$

$200 \times -167 = -33400$

$200 \times -168 = -33600$

$200 \times -169 = -33800$

$200 \times -170 = -34000$

$200 \times -171 = -34200$

$200 \times -172 = -34400$

$200 \times -173 = -34600$

$200 \times -174 = -34800$

$200 \times -175 = -35000$

$200 \times -176 = -35200$

$200 \times -177 = -35400$

$200 \times -178 = -35600$

$200 \times -179 = -35800$

$200 \times -180 = -36000$

$200 \times -181 = -36200$

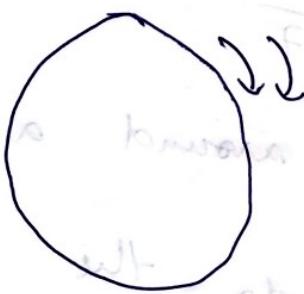
$200 \times -182 = -36400$

$200 \times -183 = -36600$

$200 \times -184 = -3680$

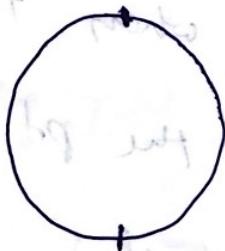
ii) same direction

a : b



$$\text{no. of int pt} = a - b$$

Ex: 5 : 3

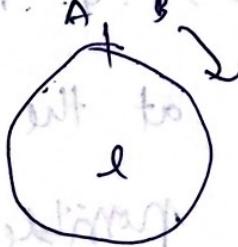


$$\text{no. of int pt} = 5 - 3 = 2$$

(divide circle into 2 equal parts)

Q A car runs around a circular track in 8 min. B can run around the same track in 15 min. If they start from the same point and at the same time, when they meet again for the first time? Both run in clock wise direction?

of them run in speed ratio is 8 : 15



$$\Rightarrow t = \frac{l}{s_1 - s_2}$$

$$\Rightarrow \frac{t}{15} - \frac{t}{8} = \frac{1}{15} - \frac{1}{8}$$

\Rightarrow Take L.C.M (15, 8)

$$t = \frac{120}{15.8} - \frac{120}{7}$$

Q A can run around a circular track in 4 min, B can do the same in 7 min. If they start running together from the same starting pt, when they'll meet for the first time at the pt diametrically opposite to the starting pt?

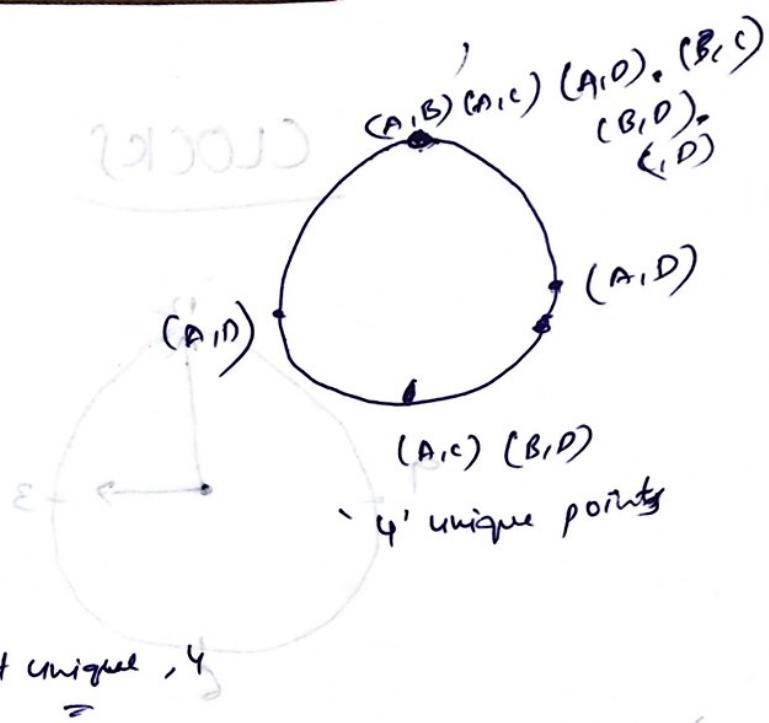
Sol: They'll never meet in diametrically opposite positions

** meeting pt

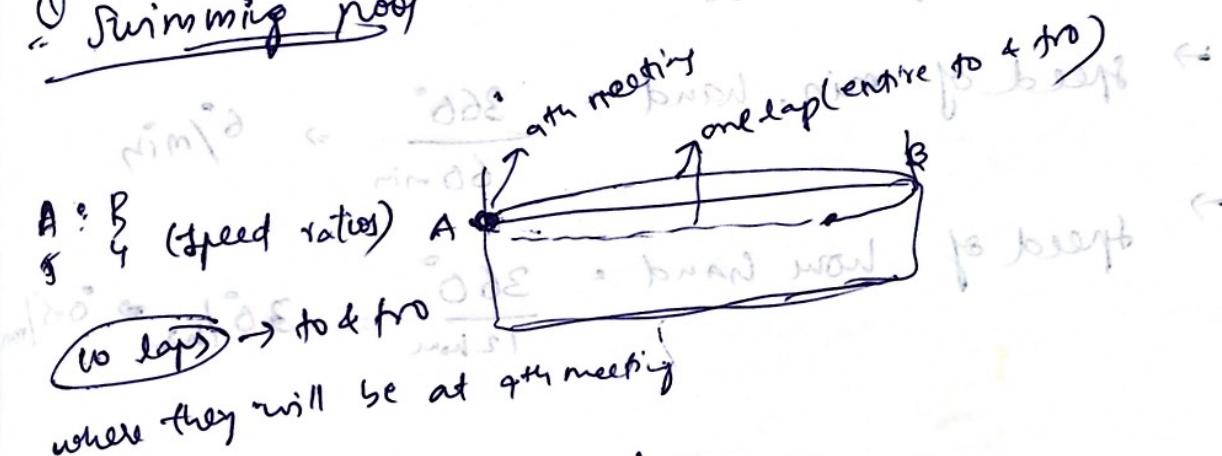
Q Four runners A, B, C & D are running around a circular track with speeds in the ratio 1:2:3:4 respectively. They start together and at the same pt. what is the no of unique possible locations of meeting?

meeting pts

A, B -	1
A, C -	2
A, D -	3
B, C -	1
B, D -	2
C, D -	1
	<u>10</u>



Swimming pool

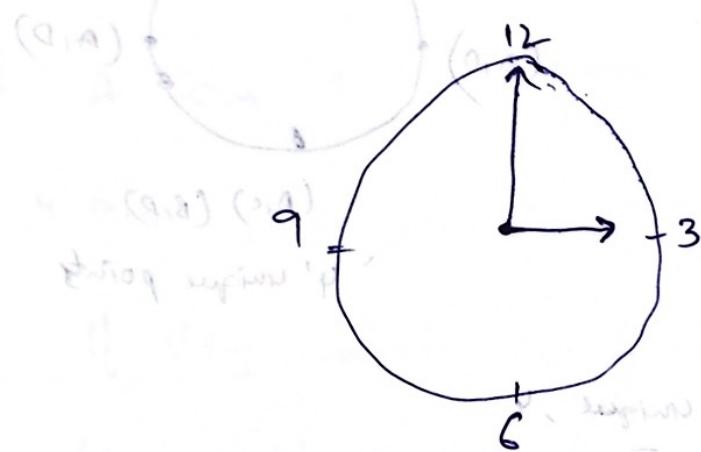


1st meeting = 1 lap

9th meeting = 9 laps

ratio = 5 : 4 \Rightarrow at starting pt
 \Rightarrow $\frac{5}{11}$ laps \Rightarrow B 4 laps
 A 5 laps

CLOCKS



- track length = 360°
- speed of min. hand = $\frac{360^\circ}{60 \text{ min}} \rightarrow 6^\circ/\text{min}$
- speed of hour hand = $\frac{360^\circ}{12 \text{ hour}} \rightarrow 30^\circ/\text{hr} \rightarrow 0.5^\circ/\text{min}$

Arithmetic And GeometricProgressions1. Sequence:

A sequence is a set of numbers written in a particular order. We sometimes write u_1 for the first term of the sequence, u_2 for the second term and so on. We write the n^{th} term as u_n .

Ex:- Fibonacci sequence

$$1 \ 1 \ 2 \ 3 \ 5 \ 8 \ 13 \ \dots$$

$$\Rightarrow u_n = (-1)^{n+1}/\sqrt{5}$$

2. Series:

A series is something we obtain from a sequence by adding all the terms together.

sequence by adding all the terms

Ex:- sequence : $u_1, u_2, u_3, u_4, \dots, u_n$

series : $u_1 + u_2 + u_3 + u_4 + \dots + u_n$

$S_2 \rightarrow$ sum up to '2' terms

$S_n \rightarrow$ sum up to 'n' terms

$$\Rightarrow S_n = u_1 + u_2 + u_3 + \dots + u_n$$

3. Arithmetic Progressions:
 An arithmetic progression, or AP is a sequence where each new term after the first is obtained by adding, a constant(d) called the common difference, to the preceding term. If the first term of the sequence is a , then

$$a, a+d, a+2d, a+3d, \dots, a+(n-1)d$$

$\boxed{+ n^{\text{th}} \text{ term}, l = a + (n-1)d}$

4. The sum of an Arithmetic Series:

$$S_n = a + (a+d) + (a+2d) + \dots + (l-2d) + (l-d) + l$$

$$S_n = l + (l-d) + (l-2d) + \dots + (a+2d) + (a+d) + a$$

$$+ 2S_n = (a+l) + (a+l) + (a+l) + \dots + (a+l) + (a+l) + (a+l)$$

$$S_n = \frac{n(a+l)}{2}$$

$$= \frac{n}{2} [a + a + (n-1)d]$$

$$\cdot \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow S_n = \frac{n}{2} (a+l)$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

2 Geometric Progressions (GP):

A geometric progression, or GP, is a sequence where each new term after the first is obtained by multiplying the preceding term by a constant, r , called the common ratio. If the first term of sequence is a , then the GP is a, ar, ar^2, ar^3, \dots

$\underset{m\text{th term}}{ar^{n-1}}$

Sum of a Geometric Series:

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

$$S_n = a + ar + ar^2 + \dots$$

$$= r \cdot S_n$$

$$S_n - rS_n = a - ar^n$$

$$\Rightarrow S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$\Rightarrow S_n = \frac{a(1-r^n)}{(1-r)}$$

7. Convergence of Geometric Series

The sum to infinity of a G.P with starting value a & common ratio r is given by,

$$S_{\infty} = \frac{a}{1-r}$$

where

$$-1 < r < 1$$

Some Important Series

$$1) 1 + 2 + 3 + 4 + 5 + \dots + n = \frac{n(n+1)}{2}$$

$$2) 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$3) 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

$$0.0001_{\text{pol}} = 8 \quad < 0001_{\text{pol}}$$

Summation for Integrals

$$r_p \text{ pol} + r_q \text{ pol} = (r_p + r_q) \text{ pol}$$

$$r_p \text{ pol} - r_q \text{ pol} = (r_p - r_q) \text{ pol}$$

$$1 \rightarrow r_p \text{ pol}$$

$$0 \rightarrow r_q \text{ pol}$$

$$(r_p \text{ pol}) q = (q r_p) \text{ pol}$$

$$\frac{1}{r_p \text{ pol}} \rightarrow r_q \text{ pol}$$

$$r_p \text{ pol} \rightarrow r_q \text{ pol}$$

logarithms

i) logarithms:-

If 'a' is a +ve real no, other than 1, and $a^m = x$, then we write $m = \log_a x$. and we say that the value of $\log x$ to the base a is

$$\begin{array}{c} (\text{Hence}) \\ \frac{x}{a} = a^{m-1} \\ m = \log_a x \end{array}$$

Ex:-

$$10^3 = 1000 \Rightarrow 3 = \log_{10} 1000$$

ii) Properties of logarithms:-

i)

$$\log_a(xy) = \log_a x + \log_a y$$

ii)

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

iii)

$$\log_a 1 = 0$$

iv)

$$\log_a a = 1$$

v)

$$\log_a(x^p) = p(\log_a x)$$

vi)

$$\log_a x = \frac{1}{\log_x a}$$

vii)

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Statistics:-

Avg., Weighted Avg., Median

* If numbers are in A.P., then Mean = median (the middle term in the sequence)

* If 'n' is even, then median =

$$\frac{n}{2} (a + a + (n-1)d)$$

$$= \frac{n}{2} (2a + (n-1)d)$$

$$\Rightarrow \frac{200}{5} = 40$$

$$\frac{x}{3} = 50$$

$$x = 150$$

$$\frac{-4 + 4 - 4}{3}$$

$$37, 1, 1, 1$$

$$(7) \quad 24$$

$$23, 20, 21, 19$$

$$\begin{matrix} 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \end{matrix}$$

$$m + 2(S.D) = 86$$

$$m + S.D = 63$$

$$(63 - m)$$

$$m + 2(63 - m) = 86$$

$$m + 2(63 - m) = 86$$

$$m = 126 + 86$$

2

$$2, 3, 4, 5, 6, 7, 8, 9, 10$$

$$S = 2, 9 \\ T = 2, 8$$

$$T = 1, 1$$

$$\begin{matrix} 2.5 & (5, 0) \\ 2.5 & +3 \\ 2.5 & -3 \\ 2.5 & \\ 20.0 & \end{matrix}$$

$$\begin{matrix} 10 - 4 \\ 20 - 3 \\ 30 - 3 \\ 40 - 1 \\ 50 - 2 \\ 60 - 3 \\ 70 - 4 \end{matrix}$$

$$\begin{aligned} 10 + 40 + 90 + 160 + 150 + 140 + 70 \\ = 300 + 410 \\ 16 + K + M = 224 \cdot \frac{1730}{14} \\ K + M = 8 \end{aligned}$$

$$17 \\ 26 \\ 35 \\ 44$$

$$\begin{aligned} 40 + 60 + 60 + 40 + 100 + 100 + 200 \\ = 500 \\ 650 \\ 260 \\ 120 \\ 320 \\ 210 \\ 130 \\ 110 \\ 20 \\ 30 \end{aligned}$$

To start off with the notes:

1. अंतर्विद्या के बारे में:

Quant

5 types of qustns

i) MCQ

ii) MAQ

iii) DI → LL ✓

iv) N.E.B.V → ... ?

v) Quantitative Comp.

↳ Exponents

→ Numbers

→ I&M (Inequalities & Modulus)

10 classes :: 13 topics

1) GRE 2 : Revision - 15 days → 1 topic/day

2. Tests

10 tests

section wise

GRE 1 - 5

GRE 2 - 14
27 tests

FLTs

8 FLTs

2 FLTs ETS prep

books

off guide

Repetetive tests

12 "

2 per day

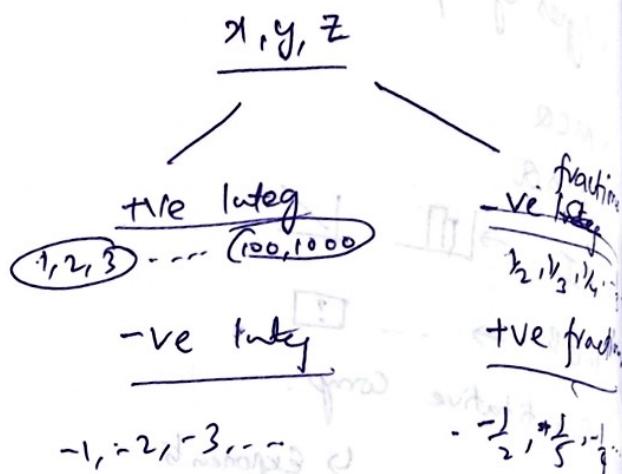
→ cover all the test materials
↓ Note down & check

Q & Ans answers strategies -

Ineq & Modules

$$\text{range of } \begin{cases} x < 2 \\ y < 3 \end{cases}$$

$$\frac{QA}{xy} \quad \frac{QB}{6}$$



②

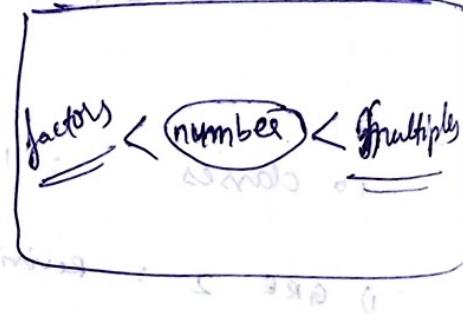
$$x > 0$$

(excluding 0 with respect to range)

+ve Integ
1, 2, ..., 1000

+ve fractions

$$\frac{1}{2}, \frac{1}{16}, \frac{1}{64}, \dots$$



* ③ substitute Special Numbers... 0, 1, 2, ...

about

"0" - neither +ve nor -ve

- is an even integer

- divisible by all numbers except itself

- non-negative means +ve & 0

about 1:

(A: no is prime, if it has 2 factors, '1' & itself)
(A no is composite, if it has more than '2' factors)

① "1" is neither prime nor composite

② different "1" and itself

about 2:

- "2" only even prime number

about $0 < x < 1$:

(arithmetic) $x^n < x^2 < x^3 < x^4 < x^5 \dots$ } $x > 1$
 $x > x^2 > x^3 > x^4 > x^5 \dots$ } $0 < x < 1$

④ factors of st^4 = t. factors except t
Factors can be -ve
-ve nos are also factors

For ex: st^4 factors are $\pm 1, \pm 2, \pm 4$

⑤ Prime Numbers are Positive Integers

NO NEGATIVE

⑥ $-\frac{1}{4}$ $-\frac{1}{64}$ } which is greater fraction

(graphical method we can try for question A)

and result more real life situations in a)

⑦ $\frac{-2^4}{2} = -16$
Integers and earning neither

$$(-2)^4 = 16 \downarrow$$

⑧ $2^{2^3} = 2^8$
float has "L" parallel

: 8 books

Q3. In $(2^2)^3 = 2^6$ now what is?

⑨

$$\sqrt{N} \leftarrow N \quad \text{X}$$

$$\begin{cases} \sqrt{N} = N & (1 \text{ and } 0) \\ \sqrt{N} < N & \text{two ways} \end{cases}$$

$1 \leq x \leq N \rightarrow \sqrt{x} \leq \sqrt{N}$ (fractions)

$1 < x \leq N \rightarrow \sqrt{x} < \sqrt{N}$

⑩

i) $\sqrt{4} = +2 \checkmark$

ii) square root of 4 = ± 2 ~~root of~~ \checkmark

$\sqrt{N} = \begin{cases} \text{+ve root of } N \\ \text{-ve root of } N \end{cases}$

" $\sqrt{}$ "; symbol denotes only for +ve root

Summary

Never forget -ve no's and fractions

without steps in always

$$\left\{ \begin{array}{l} \frac{1}{p} \\ \frac{1}{q} \end{array} \right.$$

Exponents Rules :-

$$1) a^m \times a^n = a^{m+n}$$

$$2) \frac{a^m}{a^n} = a^{m-n}$$

$$3) a^{-m} = \frac{1}{a^m}$$

$$4) (a^{mn}) = a^{mn}$$

$$5) (a \times b)^m = a^m \cdot b^m$$



$$a^{-3} + b^{-3} = 0 \text{ if } i) a=0 \text{ and } b=0$$

$$\underline{a=-b}$$



$x, \sqrt{x} \rightarrow \text{Relationships}$

$$\underline{x \geq \sqrt{x}}$$

→ Rule Except 1, 2

$$\text{(Smaller Number)}^{\text{(Larger No)}} > (L)^{\text{(S)}}$$

→ Unknown common factors should not be cancelled

$$\cancel{x^2 = 2x} \quad \Rightarrow$$

$$x^2 - 2x = 0 \\ x(x-2) = 0 \\ x=0; \underline{x=2}$$

$$N + \frac{1}{N} > 2 ; \quad N \neq 0 ; \quad N \neq 1 ;$$

selected answer

1. $x + \frac{1}{x} = 2$ (for $x=1$) ; If ($x > 2$; then $x + \frac{1}{x} > 2$)

2. We can add or subtract a variable on both sides of an inequality : BUT we cannot multiply or divide a variable on both sides of an inequality

Ex:- $\frac{x}{y} > 1$; then we think $x > y$, multiplying by y on R.S.

Consider this, $\frac{-4}{-3} > 1$

Is $x > y$? No

$\frac{x}{y} > 1$

So we cannot multiply a variable on R.S. of inequality
divide
Unless & Until we know sign of the variable

if $x > 0$ then $x^2 > 0$

(2) (i) $x^2 < 0$ (disagrees)

followed by two lines of notes

$$0 = x^2 - x^2 \rightarrow 0 = x^2 - x^2$$

new & application between 2 numbers & a
relation of these two numbers & a
new one 'P', there is a need of x^4 found at the last

$$S_n$$

$$254$$

$$(x+2)(x+1) \approx 254$$

$$\begin{aligned} & (x+1) \approx \frac{(x+2)(x+1) + x^2 + 2}{2} \\ & \therefore 2S_6 + 4 = 316 \end{aligned}$$

$$\begin{aligned} & (x+1)x^2 + (x+1)^2 \\ & = \end{aligned}$$

② What is the 60th term in the sequence?

$$\begin{aligned} & S_n = a r^{n-1} \\ & (1, 2, 4, 7, 11, 16, 22) \text{ etc.} \\ & \therefore n=60 \quad a=1 \quad r=2 \end{aligned}$$

$$1) \cancel{1675} = (x+1)^2 \Rightarrow \text{⑤} \quad \therefore a r^{n-1}$$

$$2) \cancel{1760}$$

$$3) \cancel{1761} = (x+1)^2 = 1(2)^{n-1} = 1(2)^{60-1}$$

$$\cancel{1771} \quad \therefore 1(2)^{60-1} = 2^{60}-1$$

$$10787 \quad \therefore 1(2)^{60-1} = 2^{60}-1$$

③ The sum of the squares of first 15 integers

$$1^2 + 2^2 + 3^2 + \dots + 15^2 = 1240$$

$$= 16^2 + 17^2 + \dots + 30^2 =$$

$$16^2 - 86 \quad 9 \times 900 \\ 900$$

$$(15+1)^2 + (15+2)^2 + \dots + 30^2 = 1240$$

9 A certain club has 5 members initially. x new men will be brought to each member of the club in next n weeks. In n weeks, y new men.

10 Ques

$$\begin{aligned}
 & 5 + 5 + 5(1+x) + 5(1+x)^2 + 5(1+x)^3 \\
 & = 5(1+x) + 5x(1+x) + 5x^2(1+x) + 5x^3(1+x) \\
 & = 5(1+x)(1+x + x + x^2) \\
 & = 5(1+x)(1+x)^2 \\
 & = 5(1+x)^3 \\
 & = 5(1+x)^3 = y
 \end{aligned}$$

→

$$\sqrt{2+}$$

negative or positive

except at power zero

$$0 \neq 1$$

$$= 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$$

$$0 \neq 1$$

$$0 \neq 1$$

$$0 \neq 1$$

Sequence & Series

Orgairam

omniumentatus regalis nesha

<u>Arithmetic</u>	<u>Geometric</u>
<u>Sequence</u>	<u>Sequence</u>
$1, 4, 7, 10, 13, \dots$	$1, 3, 9, 27, 81, 243, \dots$
$1+4+7+10+13, \dots$	$1+3+9+27+81+\dots$
$4-1=3$	$\frac{3}{1}=3$
$7-4=3$	$\frac{9}{3}=3$
$10-7=3$	$\frac{27}{9}=3$
⋮	
$a_{k+1} - a_k = d$ (common diff)	$\frac{a_{k+1}}{a_k} = r$ (common ratio)

ARITHMETIC SEQUENCE

$$1, 7, 13, 19, 25, 31, 37 \dots$$

FIND THE COMMON DIFFERENCE,

$$\frac{d = a_{n+1} - a_n}{d = 31 - 25 = 6}$$

Find the n^{th} number

$$x_n = a_1 + (n-1)d$$

$$n=25; \quad d_{25} = 1 + (24)^6 \\ = 247145$$

ARITHMETIC SERIES

$$1+7+13+19+25+31+37+\dots$$

FIND THE SUM OF THE FIRST 'n' NUMBERS

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_{10} = \frac{10(1+55)}{2} = \frac{10(56)}{2} = 10(28) = 280$$

GEOMETRIC SEQUENCE

1, 4, 16, 64, 256, 1024, 4096, ...

$$\frac{a_{n+1}}{a_n} = R \text{ (common ratio)}$$

$$\frac{4}{1} = 4 ; \frac{16}{4} = 4 ; \frac{64}{16} = 4 ; \dots$$

FIND nth number in series

$$a_n = a_1 (R)^{n-1}$$

$$a_5 = 1 (4)^{5-1}$$

$$= 4^4 = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots$$

$$a_5 = 256$$

GEOMETRIC SERIES

1 + 4 + 16 + 64 + 256 + 1024 + 4096 + ...

FIND THE COMMON RATIO

$$R = \frac{a_{n+1}}{a_n} = \frac{256}{64} = 4 ; \frac{1024}{256} = 4$$

FIND THE nth NUMBER,

$$a_n = a_1 (R)^{n-1}$$

FIND THE SUM OF THE FIRST "n" NUMBERS,

$$S_n = \frac{a_1 (R)^n - a_1}{R-1}$$

Sum

S_{3-7} (Sum from 3rd term to 7th term)

$$\Rightarrow S_{3-7} = S_7 - S_3$$

$$= \frac{(1)(4)^7 - 1}{4 - 1} - \frac{(1)(4)^3 - 1}{4 - 1}$$

$$= \frac{16384 - 1}{3} - \frac{64 - 1}{3}$$

$$= \frac{16383}{3} = 63 \quad 5461 - 21 \\ = 5440.$$

FINITE GEOMETRIC SERIES

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{128}$$

$$R = \frac{a_{n+1}}{a_n} = \frac{\frac{1}{16}}{\frac{1}{8}} = \frac{1}{16} \cdot \frac{8}{1} = \frac{1}{2}$$

$$S_7 = \frac{a_1(R)^n - 1}{R - 1} \Rightarrow S_7 = \frac{(1)(\frac{1}{2})^7 - 1}{\frac{1}{2} - 1} = \frac{\frac{1}{128} - 1}{-\frac{1}{2}}$$

$$S_\infty = \frac{1(\frac{1}{2})^\infty - 1}{\frac{1}{2} - 1} = \frac{\left(\frac{1}{2}\right)^\infty - 1}{-\frac{1}{2}} = (-1)\left(\frac{-2}{1}\right)^\infty = 2$$

Find 'n' if $a_n = \frac{1}{128}$; given a_1

Functions & Sequences

Function

$$h(x) = \frac{\sqrt{x+10}}{100x^2}$$

$$\underline{h(5)} ?.$$

$$\frac{\sqrt{5+10}}{100 \times 5 \times 5}$$

$$\underline{h(x)} = \frac{\sqrt{x+10}}{100x^2}$$

$$\underline{h(a+b)} = \frac{1}{\sqrt{(a+b)^2}} \cdot \underline{a+b}$$

$$\frac{\sqrt{(a+b)+10}}{100(a+b)^2} = 1. \quad \underline{a+b} =$$

Sequences:

$$1, 5, 7, 9, 10, \dots$$

Rep / May Repa

Consecutive / May not be consecutive.

Arithmetic Sequence:

"COMMON DIFFERENCE".

$$2, 4, 6, 8, 10, 12.$$

Geometric Sequence:

$$\begin{aligned} \text{Diff} &= 2. & 3, 6, 9, \dots \\ &4, 8, 12, \dots \end{aligned}$$

Evenly spaced set:

$$10, 20, 30, 40, \dots$$

$$n^{\text{th}} \text{ term} = a + (n-1)d.$$

$$d = 10$$

a : 1st term.

Median = Ascending order.

n : n^{th} term.

odd no of γ : Median = Pnt.
Int-terms

d : difference.

$$1, \underline{3}, \underline{5}, \underline{6}, 7, 9, 10$$

odd even no of γ : Median, terms $\frac{5}{2} \times \frac{6}{2} = 3$.

$$\frac{2+4}{2} = 3.$$

$$\text{Mean} = \frac{\text{Sum (no.)}}{\text{No. of terms}}$$

$$\text{Mean} = \text{Median}$$

$$\text{Mean} = \frac{\text{First term} + \text{Last term}}{2}$$

Sum of such sequences:

$$\text{Sum} = (\text{No. of terms}) \times (\text{Mean})$$

$$= 10 \times []$$

$$= []$$

$$\text{No. of terms} = \frac{\text{Last term} - \text{First term}}{1} + 1$$

$$\begin{array}{r} 2 \\ 4, 6, 8, 10, 12 \\ \hline 14, 16, 18 \end{array}$$

$$\frac{4+16}{2} = \frac{20}{2}$$

Product of n consecutive int,

= divisible by $n!$

$$16-4 = \frac{12}{2} = 6+1.$$

$$\begin{array}{r} 100 \times 101 \times 102 \\ \hline 3 \times 2 \times 1 \end{array}$$

$$\begin{array}{r} 98 \times 99 \\ \hline 2 \times 1 \end{array}$$

$$\begin{array}{r} 6 \\ 7 \\ \hline 49 \end{array}$$

$$50$$

sum of ' n ' consecutive ~~int~~,

"ODD" sum will always DIVISIBLE

"EVEN" sum may / may not be.

$$8-1$$

~~$\frac{n}{2}$~~

Geometric Sequence:

"COMMON RATIO".

$$\begin{array}{r} 2, 2^2, 2^3, 2^4, \dots \\ \hline \frac{2^2}{2} = 2 \\ \frac{2^3}{2^2} = 2 \\ \frac{2^4}{2^3} = 2 \end{array}$$

$$n^{\text{th}} = a(r^n - 1)$$

$$\text{sum} = \frac{a(r^n - 1)}{r - 1}$$

1. Trifling - trivial, inconsequential
2. glean - to extract, elicit
3. quashed - Abrogate, cancel, rescind, annul
4. repertoire - repository
5. exactitude - exact
6. Copious - abundant
7. Impassive - emotionless, expressionless
8. reprehend - Admonish, Rebuke, Censure, berate
9. Rambunctious - uncontrollably exuberant, boisterous
10. Nebulous - unclear, Amorphous, cloudy
11. Antiques - old, museum piece
12. Antics - foolish, silly, buffoonery
13. Rosters - schedules of assigned work
14. Anecdotes - A short amusing or interesting story about a real incident or person
- tales
15. Sabotage - WRECK, Vandalize, destroy
16. Profanity - blasphemy (or) obscene language
(dirty words)
17. Expunge - Efface, erase, remove, Purge

18. Motley - Disparate, ~~Cold~~ Eclectic, Diverse
19. languorous - Lazy, Lethargy, Lassitude (Languor)
20. Abnegation - Renunciation, Abdication, Austerity.
21. ^{Slight} Verisimilar - Appearing to be true; probable; likely
truth
22. Explicate - Explain, Exound
23. Crooner - A singer who sings sentimental song in a soft, low voice
24. Contravene - Infringe, Deny, Transgress, PLOW
25. peruse - Examine in a thorough and careful manner
26. blandishment - flattery, fawning, Coquetry, GROVEL
27. Injunction - an authoritative warning or order
dictum, command, exhortation, decree
28. Parochial - Narrow minded, Provincial, Myopic
29. haughty - Arrogantly superior, Disdainful
Egotist, Conceit, Condescending
30. Impertinent - Rude, insolent, impolite, uncivilized
disconcerting.

31. Cosset - Care for and protect in an over pampered manner, pamper, spoon-feed, coddle
32. Dandle - Cradle, cuddle, cosset, Pamper
Move up & down in a playful manner.
33. Edify - educate, instruct, indoctrinate
instruct (or) improve (someone) morally
(or) intellectually
34. Despise - Detest, loathe, abhor, execrate,
disdain, spurn
35. chasten - subdue, humiliate, put down
(= subdue - chaste)
(= "chake - chane") Mortify
36. Wizened - shrivelled or wrinkled with age
"a wizened, weather beaten old man."
37. Revile - Re + Vile
Repeat Vile
gets criticized, condemned, attack, denounce, lambaste
38. benign - kind hearted, gentle, warm, altruist
39. Cohort - group, category, batch
division

40. Sundry - Several, various, associated, multifarious
* Some rituals are practised by
sundry organisations

41. Implore - To plead, beg, exhort (someon)
to do something, beseech, petition

42. Conjure - concoct, beg someone to do
something (implore)

43. Gothicism → horror → Hotel Transylvania

44. subterfuge - deceit used to achieve one's goal.
intrigue, chicanery, pretence
arouse interest → secret plan to do
something wrong.
(collude)

45. Forlorn - Recluse, unhappy & lonely

46. exoneration - Acquittal

47. Stifle - kill by cutting off supply of air.
choke / suffocate

48. Sombre - dark (or) dull.

49. dearth - Scarcity, Insufficiency
(Dearth low fuel)

50. Sonorous
Euphony

Pleasing sound

51. Rostrum - podium, raised platform for someone to give speech
52. Abominate - abhor, hate, dislike
53. Germane - very appropriate
54. vociferous - express opinions in a strong and confident way, outspoken
54. languor - a pleasant & lazy feeling
54. meek - quite
55. eccentric - Maverick, iconoclast
56. fiasco - embarrassment
57. Dawdle - procrastinate
" → coddle, pamper
58. forbade - proscribe, ban, preclude, Detar
59. Bereft - Deprived
60. Obsequies - funeral rites, bural
Obsequious - fawning
61. Tantrums - outburst of anger, blowout
62. Relics - Antiques, remains, artefact, fossil.
63. Dogma - rules, principle, tenet, maxim
64. LAX - slipshod, negligent, heedless

65. exonerate - exptd exculcate, acquit, clear
66. vindicate (win + dit case)
when you win the case in court
you are free from accusations.
66. acquiesce - accept something reluctantly but without protest.
to accede, ascent, Quiet
Act QUIE~~E~~ sce, → who is quiet will accept
without any protest.
67. impunity - exonerate, immunity, Amnesty,
Exemption
68. vivacious - bubbly, ebullient, effervescent
scintillating VS LUGRUBIOUS
69. imperative - important, crucial, mandatory
bossy
70. Veil - cover, blanket, canopy
71. fitful - intermittent, sporadic, spasmodic
72. Jaded - bored (or) lacking enthusiasm,
cloyed, satiated.

73. lackadaisical - lax, uninteresting, careless
languid.
74. rebut - refute, deny, disprove
confute
75. subtle - sharp, incisive, razor-like, astute
perspicacious, trenchant
76. - ~~the~~ fine
- delicately complex & understated
77. degradation - attack (or) plunder, robbery
despoliation.
78. blaze - fire, burn fiercely
79. tantamount - equivalent to
80. Iniquity - Reprobate, vs Probity
81. Cornucopia - abundant
82. hiatus - break, pause, intermission
(Hi + atus) → after some gap, if you meet someone, you say "Hi"
83. fictions - fictitious, artificially developed

84. ignominy - ignoble, public shame, obliquity
opprobrium, humiliation, stigma

85. filial - love b/w father & children

avuncular - " " Uncle & niece/nephew

sororal - " " sisters

uxorious - husband & wife

86. perjury - mendacious, forewear

87. insidious - stealthy, surreptitious, speaking
cunning

88. unprecedented - never done before

89. dissimulation - wrong simulation (pretence, duplicity)
- subterfuge, deceit, double stand

90. callousness - heartless, uncaring, cold hearted, hard
biting

91. chuffing - make soft sound, or move slowly
(Duck) (Suean)

92. dotage - old age, declining years, superannuity
decrepitude

93. cortege - procession, parade, caravan
"victory Cortege"

94. mendicant - beggar

95. prepossessing - winsome
 ↳ attractive, winning
 alluring, capitivating
96. Adage - proverb, cliché, platitude.
 97. confess - admit, acknowledge, disclose, reveal
98. blighted - infected, diseased, marred, grieved, ruined, wrecked
99. blithe - care-free, heedless, nonchalant, happy
 (flexible) lithe - agile, graceful, loose-limbed.
100. Construe - interpret, decode, understand
101. contort - twist, quirk.
102. Amble - walk or move in a relaxed pace.
103. marauding - raid and plunder, thieving, foray
 savagery.
104. rapacious - greedy, avaricious, acquisitive,
 voracious
105. serenade - music played at night below GFs
 window.
106. Peril - hazard, danger, menace, Pitfall
107. felicity - happiness, joy, (Ability to find appropriate
 expression for one's thought).
108. Despotic - dogmatic, authoritarian, autocratic
109. protégé - student, trainee, apprentice

root word
 mendes
 ↓
 fault