omsairam om srivenkatesayanamaha om mahaganadhipatayanamaha om namaha shivayaha.

Greedy Algorithms

→ Locally optimal choice

→ Myophic (blind) choice

Coin Changing Problem:-

1, 5, 10, 25, 50 → cent coin's

92-cents = ? # smallest no. of coins possible

= $1 \times 50 + 1 \times 25 + 1 \times 10 + 1 \times 5 + 2 \times 1 \Rightarrow 92¢$

Ex 2: If you have denominations like

1, 14, 25

28¢ = ?

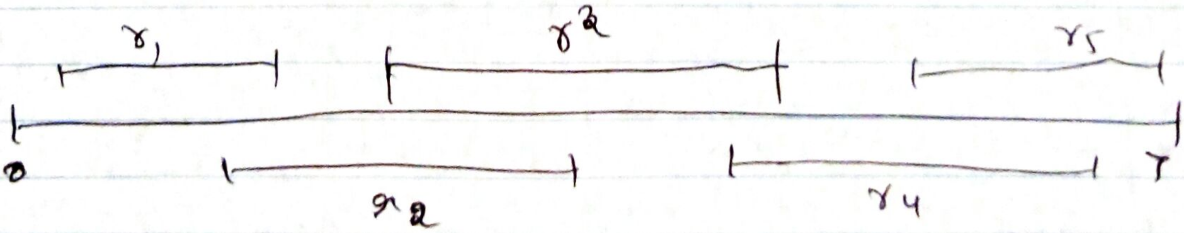
Greedy algorithm will choose,

$25¢ + 3 \times 1¢ = 4$ coins,

but we can do it 2 coins

greedy algo fails,

Room Scheduling Algorithm



We have a meeting room, and there are requests to the meeting room, but the ti[me] slots overlap.

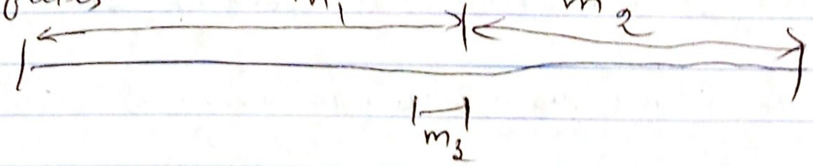
But we can accomodate only one request at a time

→ We should make sure that, maximum # of meetings take place.

Choices:

① Shortest meeting First :-

fails



→ In the above case, you can schedule only one meeting. but we could have done "2" meetings.

2) S.M.F is sub-optimal.

② Earliest Ending meeting First:-

Greedy Algorithm - Continuation:-

1) Huffman Code :-

* To compress the information and use less memory, we encode characters.

* There are different types of encoding patterns (or) standards

Ex:- ASCII.

* In ASCII (American Standard for Information Interchange), every character is given a hexadecimal - number.

Ex:- a - 0x40 ; a - z , A - Z

b - 0x41 } 8 bits

So, if we have 10,000 characters, we use 80000 bits.

* To compress the data, We can use "small length" codes for most repeating chars and "large length" codes for less frequent chars,

So that we can use less memory

→ For example, a, e → repeats more in english characters, so, assign bits of small length to a, e, and for less repeating characters assign bigger length bits.

e ::= 1

a ::= 01

z ::= 0000000 11111111

But, Decoding = ?

→ Coding & Decoding made easy in Prefix Codes:

Prefix Codes:-

Ex:- a, b

Code(a) cannot be a proper prefix of codeword(b)

(or) any character

(or) any character

Proper Prefix = ?

b_1 : 0001

b_2 : 000

} " b_2 " is proper prefix of " b_1 "

→ ASCII codes are prefix codes

Ex:-

x? y?

x : $\boxed{0000}$01

y : 0000

→ We can arrange code-words in the form of a tree in "Pre-fix" Codes

(a, b, c, d, e)

Ex:- a : 01

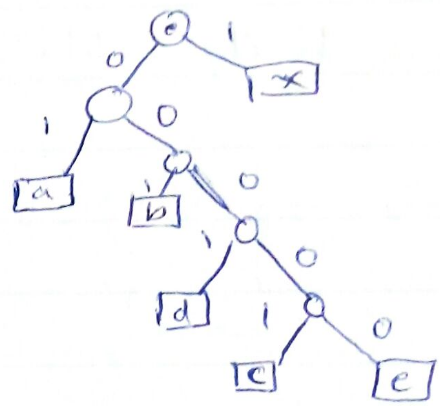
b : 001

c : 00001

d : 0001

e : 00000

decision tree



Ex:-

$\underbrace{00001}_{c}\ \underbrace{01}_{a}\ \underbrace{01}_{a}\ \underbrace{1}_{x}\ \underbrace{1}_{x}\ \underbrace{01}_{a}\ \underbrace{1}_{x}\ \underbrace{01}_{a}\ \underbrace{1}_{x}\ \underbrace{01}_{a}\ \underbrace{1}_{x}\ \underbrace{01}_{a}$

→ ASCII is a 8 bit prefix code

↓ can be stored in "Hash Table"

Optimal Pre-fix Code :-

If we have document containing, 10^6 chars containing

a, b, c, d, e
0.3, 0.2, 0.1, 0.05, 0.35 → fractions of occurence

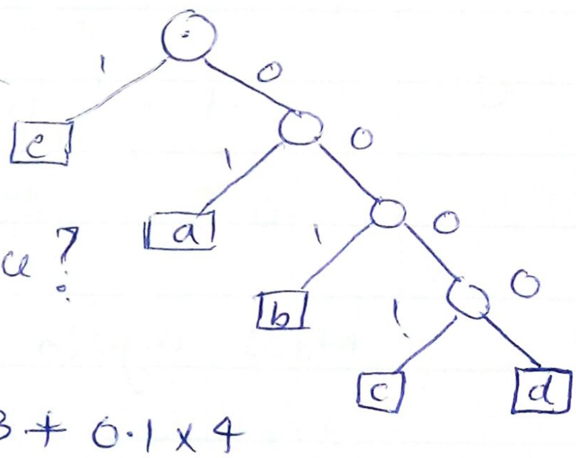
a : 000
b : 001
c : 010
d : 011
e : 100

} If we use '3' bits, we need 3×10^6 bits of space.

↳ considering above occurences,

using this code

10^6 bits document size requires how much space?



$= 0.35 + 0.3 \times 2 + 0.2 \times 3 + 0.1 \times 4 + 0.05 \times 4$

$= 2.15$ bits per character.

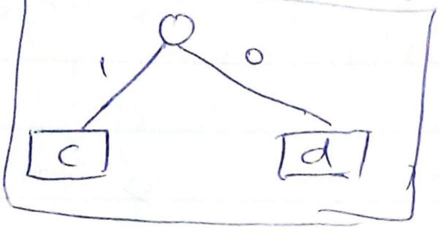
H.W : Design probabilities of chars, that gives even worse compression

Huffman Code:- (Going Bottom up)
= ↳ Optimal

a	b	c	d	e
0.3	0.2	0.1	0.05	0.35

① Take the two least frequent chars & make a sub-tree;
(Combine two chars & make a composite character & assign frequency by adding freq. of (these) two composite chars)

Tree -



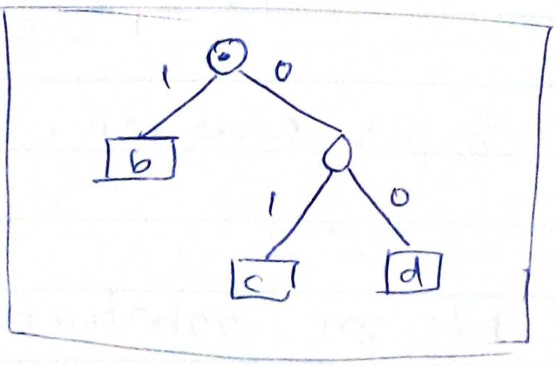
→ composite char

"cd"
⇓
0.1 + 0.05 = 0.15

②

a	b	"cd"	e
0.3	0.2	0.15	0.35

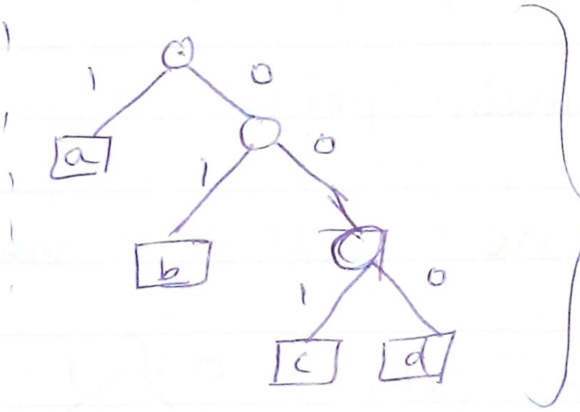
Now again you take, two least frequently occuring char



"bcd"
⇓
0.2 + 0.15
= 0.35

(3)

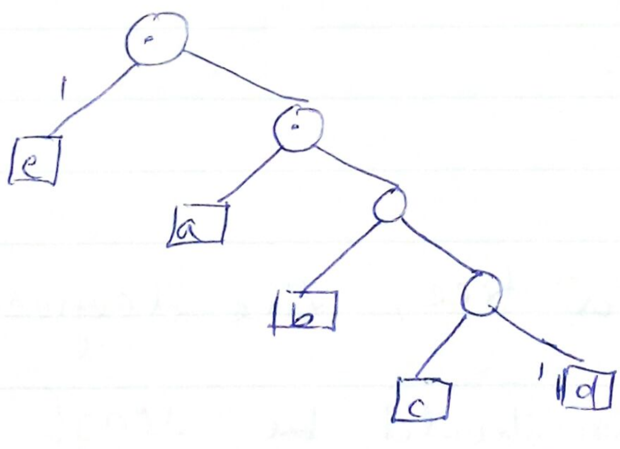
a	bcd	e
0.3	0.35	0.35



"abcd"
⇓
0.3 + 0.35
= 0.65

4)

abcd	e
0.65	0.35



→

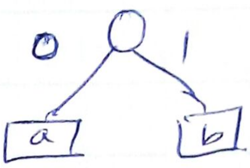
If you go from "Top to Down" (Shannon's Algo). It'll be sub-optimal.

Why Huffman-Code Optimal?

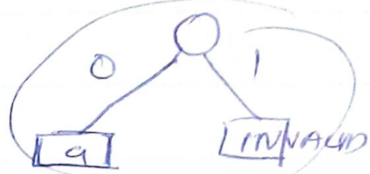
Proof of Optimality:-

Three main points

1) Deepest node always has a sibling. It should be like

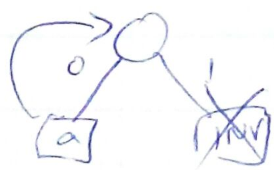


but cannot be

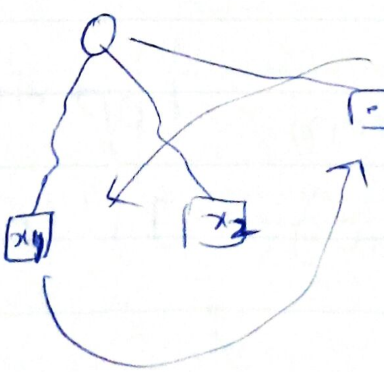


→ Huffman-code will not give invalid nodes

2)



2) In a tree, the deepest nodes, x_1 & x_2 are (or) should be least frequent characters

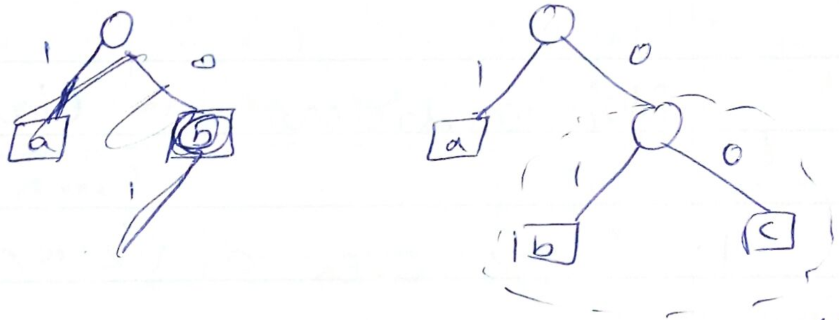


(or) If that is not the case, then exchange with the least frequent characters, node, which is up in the tree.

③ Optimal - Substructure

Transform the input-problem, ~~to~~ by taking, ~~to~~ '2' Least frequently occuring characters & ~~more~~ combining to 'one' character by adding probabilities (or) frequencies.

then, the remaining tree must be optimal for the other problem



Sub-problem