DESIGN OPTIMIZATION

ASSIGNMENT - 2

1]

$$f = 2x_1^2 - 4x_1 x_2 + 1.5x_2^2 + x_2$$

$$\frac{\partial f}{\partial x_1} = 4x_1 - 4x_2 + 0 + 0 = 0 \Rightarrow 4x_1 - 4x_2 = 0$$

$$\frac{\partial f}{\partial x_2} = 0 - 4x_1 + 3x_2 + 1 = 0 \Rightarrow -4x_1 + 3x_2 + 1 = 0$$

Solving the abv. eq.

$$\begin{array}{r} 4x_1 - 4x_2 + 0 = 0 \\ -4x_1 + 3x_2 + 1 = 0 \\ \hline -x_2 = -1 \end{array}$$

$$x_2 = 1$$

$$\therefore x_1 = 1$$

$$x^* \rightarrow [x_1 \; x_2] = [1, 1]^T$$

$$g = \begin{bmatrix} \partial f/\partial x_1 \\ \partial f/\partial x_2 \end{bmatrix} = \begin{bmatrix} 4x_1 - 4x_2 \\ -4x_1 + 3x_2 + 1 \end{bmatrix}$$

$$g_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ at stationary point

Hessian ;

$$H = \begin{bmatrix} \partial^2 f / \partial x_1^2 & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 f}{\partial x_1 \partial x_2} & \dfrac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial x_1^2} = \frac{\partial}{\partial x}\left(4x_1 - 4x_2 \right) = 4$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial f}{\partial x_1}\left(\frac{\partial f}{\partial x_2}\right) = \frac{\partial f}{\partial x_2}(4x_1 - 4x_2) = -4$$

$$\frac{\partial^2 f}{\partial x_1 x_2} = -4$$

$$\frac{\partial^2 f}{\partial x_2^2} = \frac{\partial f}{\partial x}\left(-4x_1 + 3x_2 + 1\right) : 3$$

$$H = \begin{bmatrix} 4 & -4 \\ -4 & 3 \end{bmatrix} \Rightarrow \det \begin{vmatrix} 4 & -4 \\ -4 & 3 \end{vmatrix} = -4$$

as one eigen value is +ve and other is -ve. ~~stati~~ ∴ stationary point is saddle point.

Taylor's Expansion $f(x) \rightarrow e^{\infty}$

$$f(x) = f(x_0) + \frac{\partial f}{\partial x}(x - x_0) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(x - x_0) \cdot\cdot$$

Taylor's Expansion matrix form:

$$f'(x) = f(x_0) + g^T (x - x_0) + \frac{1}{2} (x - x_0)^T H (x - x_0)$$

$$f(x_0) = 2(1)^2 - 4(1)(1) + 1.5(1) + 1 = 2 - 4 + 1 = 0.5$$

$[x_1, x_2] \to [1, 1]$

$$f'(x) = 0.5 + \cancelto{0}{g^T (x - x_0)} + \frac{1}{2} (x - x_0)^T H (x - x_0)$$

$$f'(x) = 0.5 + \frac{1}{2} \underbrace{\begin{bmatrix} x_1 - \cancel{x}1 & x_2 - 1 \end{bmatrix} \begin{bmatrix} 4 & -4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_2 - 1 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} x_1 - 1 & x_2 - 1 \end{bmatrix} \begin{bmatrix} 4(x_1 - 1) - 4(x_2 - 1) \\ -4(x_1 - 1) + 3(x_2 - 1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 - 1 & x_2 - 1 \end{bmatrix} \begin{bmatrix} 4x_1 - \cancel{4} - 4x_2 + \cancel{4} \\ -4x_1 + 4 + 3x_2 - 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 - 1 & x_2 - 1 \end{bmatrix} \begin{bmatrix} 4x_1 - 4x_2 \\ -4x_1 + 3x_2 + 1 \end{bmatrix}$$

$$\Rightarrow (x_1 - 1)(4x_1 - 4x_2) + (x_2 - 1)(-4x_1 + 3x_2 + 1)$$

$$\Rightarrow 4x_1^2 - 4x_1 x_2 - 4x_1 + 4x_2 + (-4x_1 x_2) + 3x_2^2$$
$$+ x_2 + 4x_1 - 3x_2 - 1$$

$\Rightarrow \quad 4x_1^2 - 4x_1 - 4x_1x_2 + 4x_2 - 4x_1x_2 + 4x_1 + 3x_2^2 - 3x + x_2 - 1$

$\Rightarrow \quad 4x_1^2 + 3x_2^2 - 8x_1x_2 - 2x_2 - 1$

$f(x) = \cancel{\tfrac{1}{2}x} + \frac{1}{2}(4x_1^2 + 3x_2^2 - 8x_1x_2 - 2x_2 - 1)$

For, $f'(x) < 0$

$$\frac{1}{2}([2x_1 - x_2][2x_1 - 3x_2])$$

The abv. is in the form of $(ax+by)(cx+dy) < 0$

$(ax+by) < 0$ and $(cx+dy) > 0$ or vice versa

i.e $(2x_1 - x_2) < 0$ and $(2x_1 - 3x_2) > 0$

also can be verified from fig: 1 below