5)

a) Given $n \rightarrow$ lamps

$m \rightarrow$ mirrors

$I_k \rightarrow$ Intensity at

power output :

$P = [P_1, \cdots P_n]^T$

actual reflection $a_k^T P$

$$\min \sum_{k=1}^{m} (a_k^T P - I)^2 \quad \forall \; 0 \le P_i \le P_{max}$$

gradient $\Rightarrow 2(a_k^T P - I)\, a_k$

$$= 2 a_k^T P \cdot a_k - 2 - 2I a_k$$

$$= 2 a_k \cdot a_k^T P - 2I a_k$$

Hessian $\Rightarrow$ $2 a_k \cdot a_k^T$

$\therefore H = 2 a_k \cdot a_k^T$

b)

For $d^T H d \ge 0 \;\forall\; d \neq 0$ (d: non zero vector) then Hessian is p.s.d

$$d^T H d = 2 d^T [a] [a]^T d$$

$$= 2 \cdot (d^T a_k)^2$$

$\therefore$ it will be +ve

c) Unique solution $\rightarrow$ convex.

c) ~~from~~ Hessian $= \sum 2 \cdot a_k a_k^T$

$$\sum d^T a_k a_k^T d$$

Hessian is p.d

$$\therefore \sum_k 2 d^T a_k a_k^T d = \sum 2 (a_k^T)^2 d^2 > 0$$

for vector $d \neq 0$

$\therefore$ mirror number > lamp number

For the abv. condition to apply there is only a unique solution

d) from c; mirror no $> \frac{1}{2}$ lamp no. , Solution is unique

$$H = \sum a_k^{T^2} d^2$$

$\downarrow$ +ve and $d \neq 0$