Game Theory and its Applications (CS9071)

Instructor

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Algorithmic Mechanism Design

- Conventional Algorithm:
  - ✓ No matter how complex the algorithmic task at hand, it is normally assumed that the input x does not depend on algorithm A
- **Mechanism:**
  - ✓ Algorithms where the inputs are provided by self-interested (and potentially strategic) agents
  - ✓ Inputs x to an algorithm A may depend on the algorithm
    - For example, underlying an auction is typically a simple optimization algorithm (with a single item for sale, the algorithm is just the max function)
    - However, inputs come from bidders who might (and will) optimize their bids to get an improved outcome (for example, get the item for as little money as possible)
  - ➢ Incentive issues:
    - May encourage agents to employ certain strategy (game the system) while presenting the input to get some reward

Algorithmic Mechanism Design

- Incentives:
  - Not always money (favourable outcome such as winning the game)

- Abstractly, mechanism design addresses the problem of aggregating preferences of multiple parties into an outcome
  - ➢ Manages user incentives and misbehaviours as strategic entities
    - Solicits private preferences from users in the form of private values: i.e., how much each agent prefers each outcome and then chooses an outcome based on these reported values
    - For incentive compatibility it is required that although users could misreport their preferences, it would not be in any user's best interest to do so
- Examples:
  - ➢ housing market, voting, college admissions etc.

  [The housing market explicitly involve money, while voting and college admissions do not.]

Matching Problem in One-sided Market

- **Motivational Problem:**

- Elective allocation:
  - Given the preferences (choices) of individual students, objective is to allocate the elective subjects of a particular pool to the students such that the number of students allocated to a subject does not exceed the maximum class size.

➢ A one-sided market, since only one side of the market (the students) has preferences

Motivational Problem

- Elective allocation problem:
  - Given the preferences (choices) of individual students, objective is to allocate the elective subjects of a particular pool to the students such that the number of students allocated to a subject does not exceed the maximum class size.

  - **Solution Recipe:**
    - Students are ranked (1, 2, …, n) according to CGPA/Gate Score
    - Students present their preferences by providing an ordering of all the subjects offered under the pool
      - For example, if there are three subjects S1, S2, S3 in a particular pool and a student prefers S2 over S1 over S3, her preference may be represented as S2 > S1 > S3

      [Note that the student must put all the subjects of the pool in her preference list]
    - For i = 1, 2, …, n:

      Student ranked i is assigned to her favourite choice among the options still available

Mechanism

- A procedure for making a decision or taking an action, as a function of what people want (i.e., of participants' preferences)
  - May be thought of as a synonym of algorithm in Game theoretic terminology

- **Serial dictatorship:**
  - A mechanism that orders the participants, and in this order allows each player to dictate their favourite feasible option (given the choices made by previous players)

- **Mechanism design without money:**
  - Where there are significant incentive issues but where money is infeasible or illegal

Elective allocation mechanism

- Mechanism M1:

  - Students are ranked (1, 2, …, n) according to CGPA/Gate Score
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- Is M1 a good mechanism?
  - How to judge?
  - Which criteria/properties should be considered?
- Is there any better alternative?

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One of the primary goals of this course is to make you familiar with the tools and techniques that allows you to reason about these types of questions

Elective allocation mechanism

- Mechanism M3: [Applicable when the total number of elective offered (m) is very large]

  - Each student submits a ranked list of at most c (some constant) subjects, out of total m electives
  - Each student is assigned a number in the range (1, 2, …, n)
    [Either by ranking according to CGPA/Gate Score or arbitrarily but independently of the submitted lists]
  - For i = 1, 2, …, n:
    Student ranked i is assigned to her favourite choice among the options still available
    If none of her c options are still available, the least popular subject is assigned to the student

- Is M3 strategyproof?

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    - Student ranked i is assigned to her favourite choice among the options still available
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- Is M3 strategyproof?
  - No
    - Getting least popular subject may be a pretty bad outcome
    - This motivates putting a “safety” in the m-th slot, a subject unpopular enough that a student very likely to get that, but still at least a little better than the least popular one