Game Theory and its Applications (CS9071)

Instructor

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Elective allocation mechanism

- Mechanism M1:

  - Students are ranked (1, 2, …, n) according to CGPA/Gate Score
  - Students present their preferences by providing an ordering of all the subjects offered under the pool
    - For example, if there are three subjects S1, S2, S3 in a particular pool and a student prefers S2 over S1 over S3, her preference may be represented as S2 > S1 > S3

      [Note that the student must put all the subjects of the pool in her preference list]
  - For i = 1, 2, …, n:

    Student ranked i is assigned to her favourite choice among the options still available

- Is M1 a good mechanism?
  - How to judge?
  - Which criteria/properties should be considered?
- Is there any better alternative?

One of the primary goals of this course is to make you familiar with the tools and techniques that allows you to reason about these types of questions

Few quality metrics

Pareto Optimality

- The property of an outcome that you can't make anyone better off without making someone else worse off

- **Claim 1:** a mechanism that fails to produce a Pareto optimal outcome is not that great
  - If you can make some people better off without hurting anyone else, why didn't you do that?

Pareto Optimality

- The property of an outcome that you can't make anyone better off without making someone else worse off

- **Claim 2:** Pareto optimality is necessary for a mechanism to be good but may not be sufficient
    - ➢Pareto optimality says nothing about equity or fairness
    - A simple example is the distribution of a pie among three people
        - The assignment of, say, a half section to each of two individuals and none to the third is Pareto-optimal despite not being fair, because none of the recipients could be made better off without decreasing someone else's share

✓Pareto optimality may be considered as an initial sanity check for a mechanism to be a reasonable one

Nash Equilibrium

- The most commonly-used solution concept for non-cooperative games
- A Nash equilibrium is a situation where no player could gain by changing their own strategy (holding all other players' strategies fixed)Pareto optimality may be considered as an initial sanity check for a mechanism to be a reasonable one
- Example:
  - If two players Alice and Bob choose strategies A and B, (A, B) is a Nash equilibrium:
    - if Alice has no other strategy available that does better than A at maximizing her payoff in response to Bob choosing B

      and
    - Bob has no other strategy available that does better than B at maximizing his payoff in response to Alice choosing A
  - In a game in which Carol and Dan are also players, (A, B, C, D) is a Nash equilibrium if A is Alice's best response to (B, C, D), B is Bob's best response to (A, C, D), and so forth…..

Strategyproofness (SP)

- The property of a mechanism that motivates participants to be honest (truthful)
  - ✓It is not possible to obtain a better result by lying about preferences
  - ✓It advocates to adopt “honesty is always the best policy”
  - ✓Prevents participants to be over-smart (offers no incentive to “game the system”)
  - ✓Requires no effort to consider what other participants are doing or may do
  - ✓No regret about the submitted preferences

- Presenting the actual preference appears to be the dominant strategy for each and every participant
- SP is also called truthful or dominant-strategy-incentive-compatible (DSIC)

Individual Rationality (IR)

- A truthtelling bidder must not regret participating in
  - i.e., truthful bidding should guarantee non-negative utility

Social-welfare maximization

- An allocation is said to be maximizing social-welfare if the item is given to the bidder who values it the most

Mechanism M1 and Pareto Optimality

- **Proposition 1: Outcome of mechanism M1 is Pareto optimal**
  - ➢ Every assignment different from the outcome of M1 makes someone worse off (quite intuitive)
  - ▪ **Proof** (by induction):
    - ✓ Assumption: First i students are assigned identically in the two assignments (M1 and the new one)
    - ✓ Base case (with $i = 0$):
      - Trivial, since two empty assignments coincide
    - ✓ Inductive step (for a value of $i \geq 1$):
      - By the inductive hypothesis, the first i - 1 students are assigned identically in the two assignments.
      - Thus, the remaining options for student i in the new assignment are precisely the options remaining when i is considered in the allocation.
      - M1 gives student i her favourite option among those remaining
      - If the new assignment does anything different, then student i is worse off
        .

Mechanism M1 and Strategyproofness

- **Proposition 2: Mechanism M1 is strategyproof**
  - **Proof**:
    - ✓A student can't affect the rank i that she gets
      - Ranks are computed independently of the ordered lists of choices that students submit
    - ✓She can't affect the choices made by the i - 1 students before her
      - Their choices are independent of the ordered list of choices that she submits
    - ✓Thus, the available options at the time she is considered by the mechanism M1 does not depend on her submitted list
    - ✓Since M1 assigns her to her highest-preferred available option, it pays to be honest
    - ✓Any lie in her ranked list could only cause her to instead receive a remaining option that is not her favourite

Elective allocation mechanism

- Exercise:
  - Consider the following mechanism (M2):

    - Students present their preferences by providing an ordering of all the subjects offered under the pool
      - For example, if there are three subjects S1, S2, S3 in a particular pool and a student prefers S2 over S1 over S3, her preference may be represented as S2 > S1 > S3

        [Note that the student must put all the subjects of the pool in her preference list]
    - Each student is assigned a number in $\{1, 2, \ldots , n\}$
    - For i = 1, 2, …, n:

      Student i is assigned to her favourite choice among the options still available

    Note: Students do not know their rank in advance [i.e., while presenting their preferences]

  - Is M2 strategyproof?

Elective allocation mechanism

Famous Draw (process that assigns Stanford students to rooms in dorms and houses)

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  - Is M2 strategyproof?
    - Yes [Proposition 1.2, Roughgarden's lecture note l1, Page 3]

Elective allocation mechanism

- Mechanism M3: [Applicable when the total number of elective offered (m) is very large]

  - Each student submits a ranked list of at most c (some constant) subjects, out of total m electives
  - Each student is assigned a number in the range (1, 2, …, n)
    [Either by ranking according to CGPA/Gate Score or arbitrarily but independently of the submitted lists]
  - For i = 1, 2, …, n:
    Student ranked i is assigned to her favourite choice among the options still available
    If none of her c options are still available, the least popular subject is assigned to the student

- Is M3 strategyproof?

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- Is M3 strategyproof?
  - No
    - Getting least popular subject may be a pretty bad outcome
    - This motivates putting a "safety" in the m-th slot, a subject unpopular enough that a student very likely to get that, but still at least a little better than the least popular one

Few related problems

- Hostel room allocation problem
- Course assignment problem
- …