JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 29TH JANUARY 2025) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

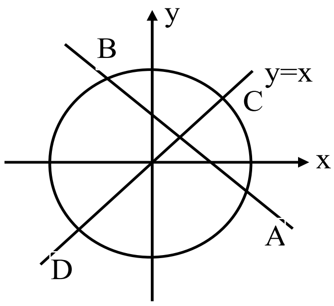
1. Let the line $x + y = 1$ meet the circle $x^2 + y^2 = 4$ at the points A and B. If the line perpendicular to AB and passing through the mid point of the chord AB intersects is the circle at C and D, then the area of the quadrilateral ADBC is equal to

(1) $3\sqrt{7}$ (2) $2\sqrt{14}$

(3) $5\sqrt{7}$ (4) $\sqrt{14}$

Ans. (2)

Sol.



By solving $x = y$ with circle

We get

$C(\sqrt{2},\sqrt{2})$

$D(-\sqrt{2},-\sqrt{2})$

By solving $x + y = 1$ with circle $x^2 + y^2 = 4$

we set

$A\left(\frac{1+\sqrt{7}}{2},\frac{1-\sqrt{7}}{2}\right)$

& $B\left(\frac{1-\sqrt{7}}{2},\frac{1+\sqrt{7}}{2}\right)$

$\therefore$ Area of Quadrilateral ACBD

$= 2 \times$ Area of ΔBCD

$$= 2\times\frac{1}{2}\begin{vmatrix}\sqrt{2} & \sqrt{2} & 1\\ \frac{1-\sqrt{7}}{2} & \frac{1+\sqrt{7}}{2} & 1\\ -\sqrt{2} & -\sqrt{2} & 1\end{vmatrix}$$

$= 2\sqrt{14}$

2. Let M and m respectively be the maximum and the minimum values of

$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}, x\in R$$

Then $M^4 - m^4$ is equal to :

(1) 1280 (2) 1295

(3) 1040 (4) 1215

Ans. (1)

Sol. $$\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}, x\in R$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 \to R_1$

$$f(x)\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ -1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}$$

Expand about R_1 , use get

$f(x) = 2 + 4\sin 4x$

$\therefore$ M = max value of $f(x) = 6$

M = min value of $f(x) = -2$

$\therefore M^4 - M^4 = 1280$

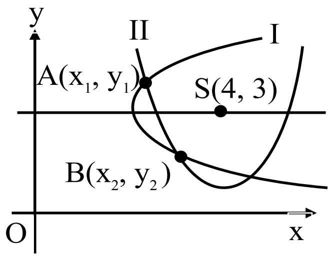
3. Two parabolas have the same focus (4,3) and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then $(AB)^2$ is equal to

(1) 192 (2) 384

(3) 96 (4) 392

Ans. (1)

Sol.



Let intersection points of these two parabolas are $A(x_1, y_1)$ & $B(x_2,y_2)$

$\because$ equation of parabola I and II are given below

$\therefore (x-4)^2 + (y-3)^2 = x^2$ .....(1)

& $(x-4)^2 + (y-3)^2 = y^2$ .....(2)

Here $A(x_1, y_1)$ & $B(x_2,y_2)$ will satisfy with equation

Also from equations (1) & (2), we get = x = y ..(3)

Put x = y in equation (1)

We get $x^2 - 14x + 25 = 0$

$x_1 + x_2 = 14$

$x_1x_2 = 25$

$\therefore AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= 2(x_1 - x_2)^2$

$= 2[(x_1+x_2)^2 - 4x_1x_2]$

$= 192$

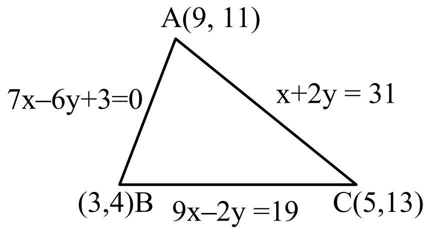
4. Let ABC be a triangle formed by the lines $7x - 6y + 3 = 0$, $x + 2y - 31 = 0$ and $9x - 2y - 19 = 0$, Let the point (h,k) be the image of the centroid of ΔABC in the line $3x + 6y - 53 = 0$. Then $h^2 + k^2 + hk$ is equal to

(1) 37 (2) 47

(3) 40 (4) 36

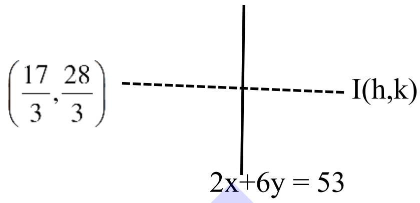
Ans. (1)

Sol.



$\therefore$ centroid of $\Delta ABC = \left(\frac{9+3+5}{3}, \frac{11+4+13}{3}\right)$

$= \left(\frac{17}{3}, \frac{28}{3}\right)$



Let image of centroid with respect to line mirror is (h,k)

$$\therefore \left(\frac{k - \frac{28}{3}}{h - \frac{17}{3}}\right)\left(-\frac{1}{2}\right) = -1$$

$$\& \ 3\left(\frac{h + \frac{17}{3}}{2}\right) + 6.\left(\frac{k + \frac{28}{3}}{2}\right) = 53$$

Solving (1) & (2) we get h = 3, k = 4

$\therefore h^2 + k^2 + hk = 37$

5. Let $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$ and $(\vec{a} + \vec{c}).(\vec{b} + \vec{c}) = 168$. Then the maximum value of $|\vec{c}|^2$ is :

(1) 77 (2) 462

(3) 308 (4) 154

Ans. (3)

Sol. $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - 5\hat{j} + 3\hat{k}$

$\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$

$\vec{a} \times \vec{c} + \vec{b} \times \vec{c} = 0$

$(\vec{a}+\vec{b})\times\vec{c}=0$

$\Rightarrow \vec{c}=\lambda(\vec{a}+\vec{b})$

$\vec{c}=\lambda(5\hat{i}-6\hat{j}+4\hat{k})$ .....(1)

$|\vec{c}|^2=\lambda^2(25+36+16)$

$|\vec{c}|^2=77\lambda^2$

$(\vec{a}+\vec{c}).(\vec{b}+\vec{c})=168$

$\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{c}.\vec{b}+|\vec{c}|^2=168$

$14+\vec{c}.(\vec{a}+\vec{b})+77\lambda^2=168$

using equation (1)

$\lambda\left|5\hat{i}-6\hat{j}+4\hat{k}\right|^2+77\lambda^2=154$

$77\lambda+77\lambda^2-154=0$

$\lambda^2+\lambda-2=0$

$\lambda=-2, 1$

$\therefore$ Maximum value of $|\vec{c}|^2$ occurs when $\lambda=-2$

$|\vec{c}|^2=77\lambda^2$

$=77\times4$

$=308$

6. Let P be the set of seven digit numbers with sum of their digits equal to 11. If the numbers in P are formed by using the digits 1,2 and 3 only, then the number of elements in the set P is :

(1) 158 (2) 173

(3) 164 (4) 161

Ans. (4)

Sol. (i) number of numbers created using

$1111133=\dfrac{7!}{5!2!}\Rightarrow 21$

(ii) number of numbers created using

$1111223=\dfrac{7!}{4!2!}\Rightarrow 105$

(iii) number of numbers created using

$1112222=\dfrac{7!}{4!3!}\Rightarrow 35$

Total = 161

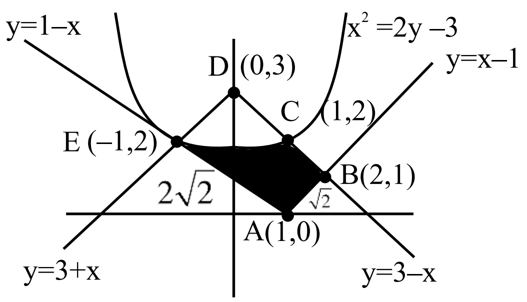
7. Let the area of the region $\{(x,y):2y\le x^2+3,\ y+|x|\le 3,\ y\ge|x-1|\}$ be A. Then 6A is equal to:

(1) 16 (2) 12

(3) 18 (4) 14

Ans. (4)

Sol.



A $\Rightarrow$ Rectangle ABDE – Area of region EDC

$$A\Rightarrow 4-2\int_0^1(3-x)-\left(\frac{x^2+3}{2}\right)dx$$

$$A\Rightarrow 4-2\left\{3x-\frac{x^2}{2}-\frac{x^3}{6}-\frac{3}{2}x\right\}_0^1$$

$$A\Rightarrow 4-2\left\{3-\frac{1}{2}-\frac{1}{6}-\frac{3}{2}\right\}=\frac{7}{3}$$

So 6A = 14

8. The least value of n for which the number of integral terms in the Binomial expansion of $\left(\sqrt[3]{7}+\sqrt[12]{11}\right)^n$ is 183, is :

(1) 2184 (2) 2148

(3) 2172 (4) 2196

Ans. (1)

Sol. General term $= {}^nC_r\{7^{1/3}\}^{n-r}\left(11^{1/12}\right)^r$

$$={}^nC_r\{7\}^{\frac{n-r}{3}}(11)^{r/12}$$

For integral terms, r must be multiple of 12

$\therefore$ r = 12k, k $\in$ W

Total values of r = 183

Hence max r = 12(182)

= 2184

Min value of n = 2184

9. The number of solutions of the equation

$$\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0 \text{ is :}$$

(1) 2 (2) 4

(3) 1 (4) 3

Ans. **(2)**

Sol. Consider $\frac{1}{\sqrt{x}}=\alpha$ $\boxed{x>0}$

$\{9\alpha^2-9\alpha+2\}\{2\alpha^2-7\alpha+3\}=0$

$(3\alpha-2)(3\alpha-1)(\alpha-3)(2\alpha-1)=0$

$\alpha=\frac{1}{3},\frac{1}{2},\frac{2}{3},3$

$x=9,4,\frac{9}{4},\frac{1}{9}$

So, no. of solutions = 4

10. Let y = y(x) be the solution of the differential equation

$\cos x(\log_e(\cos x))^2 dy+(\sin x-3y\sin x\log_e(\cos x))dx=0$,

$x\in\left(0,\frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{4}\right)=\frac{-1}{\log_e 2}$, then $y\left(\frac{\pi}{6}\right)$ is :

(1) $\frac{2}{\log_e(3)-\log_e(4)}$ (2) $\frac{1}{\log_e(4)-\log_e(3)}$

(3) $-\frac{1}{\log_e(4)}$ (4) $\frac{1}{\log_e(3)-\log_e(4)}$

Ans. **(4)**

Sol.

$$\cos x(\ln(\cos x))^2 dy+(\sin x-3y(\sin x)\ln(\cos x))dx=0$$

$$\cos x(\ln(\cos x))^2\frac{dy}{dx}-3\sin x.\ln(\cos x)y=-\sin x$$

$$\frac{dy}{dx}-\frac{3\tan x}{\ln(\cos x)}y=\frac{-\tan x}{(\ln(\cos x))^2}$$

$$\frac{dy}{dx}+\frac{3\tan x}{\ln(\sec x)}y=\frac{-\tan x}{(\ln(\sec x))^2}$$

$$\text{I.F.}=e^{\int\frac{3\tan x}{\ln(\sec x)}dx}=(\ln(\sec x))^3$$

$$y\times(\ln(\sec x))^3=-\int\frac{\tan x}{(\ln(\sec x))^2}(\ln(\sec x))^3dx+C$$

$$y\times(\ln(\sec x))^3=-\frac{1}{2}(\ln(\sec x))^2+C$$

Given : $x=\frac{\pi}{4},\ y=-\frac{1}{\ln 2}$

$$\frac{-1}{\ln 2}\times(\ln\sqrt{2})^3=-\frac{1}{2}\times(\ln\sqrt{2})^2+C$$

$$\Rightarrow\frac{-1}{8\ln 2}\times(\ln 2)^3=\frac{-1}{2}\times\frac{1}{4}(\ln 2)^2+C$$

$$-\frac{1}{8}(\ln 2)^2=\frac{-1}{8}(\ln 2)^2+C$$

$\Rightarrow C=0$

$$\therefore y(\ln(\sec x))^3=\frac{-1}{2}(\ln(\sec x))^2+0$$

$$y=\frac{-1}{2\ln(\sec x)}$$

$$y=\frac{1}{2\ln(\cos x)}$$

$$\therefore y\left(\frac{\pi}{6}\right)=\frac{1}{2\ln\left(\cos\frac{\pi}{6}\right)}$$

$$=\frac{1}{2\ln\left(\frac{\sqrt{3}}{2}\right)}$$

$$=\frac{1}{2\left(\frac{1}{2}\ln 3-\ln 2\right)}$$

$$=\frac{1}{\ln 3-\ln 4}$$

Option (4)

11. Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by x R y if and only if $\sec^2 x - \tan^2 y = 1$. Then R is :

(1) an equivalence relation

(2) both reflexive and transitive but not symmetric

(3) both reflexive and symmetric but not transitive

(4) reflexive but neither symmetric not transitive

Ans. **(1)**

Sol. $\sec^2 x - \tan^2 x = 1$ (on replacing y with x)

$\Rightarrow$ Reflexive

$\sec^2 x - \tan^2 y = 1$

$\Rightarrow 1 + \tan^2 x + 1 - \sec^2 y = 1$

$\Rightarrow \sec^2 y - \tan^2 x = 1$

$\Rightarrow$ symmetric

$\sec^2 x - \tan^2 y = 1$,

$\sec^2 y - \tan^2 z = 1$

Adding both

$\Rightarrow \sec^2 x - \tan^2 y + \sec^2 y - \tan^2 z = 1 + 1$

$\sec^2 x + 1 - \tan^2 z = 2$

$\sec^2 x - \tan^2 z = 1$

$\Rightarrow$ Transitive

hence equivalence releation

Option (1)

12. Let the ellipse, $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, A < B have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A,B,C and D, then the area of the quadrilateral ABCD equals:

(1) $6\sqrt{6}$

(2) $\frac{18\sqrt{6}}{5}$

(3) $\frac{12\sqrt{6}}{5}$

(4) $\frac{24\sqrt{6}}{5}$

Ans. **(4)**

Sol. $2ae = 4$

$2a\left(\frac{1}{\sqrt{3}}\right) = 4$

$\Rightarrow a = 2\sqrt{3}$

$\Rightarrow 1 - \frac{b^2}{12} = \frac{1}{3} \Rightarrow b^2 = 8$

Now $\frac{2b^2}{a} \cdot \frac{2A^2}{B} = \frac{32}{\sqrt{3}} \Rightarrow 2\left(\frac{8}{2\sqrt{3}}\right)\frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow A^2 = 2B$

$1 - \frac{A^2}{B^2} = \frac{1}{3} \Rightarrow 1 - \frac{2B}{B^2} = \frac{1}{3} \Rightarrow B = 3$

$\Rightarrow A^2 = 6$

$\frac{x^2}{12} + \frac{y^2}{8} = 1$ .....(1)

$\frac{x^2}{6} + \frac{y^2}{9} = 1$ .....(2)

On solving (1) & (2) we get

$(x, y) \equiv \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right)$

The four points are vertices of rectangle and its area = $\frac{24\sqrt{6}}{5}$

13. Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is :

(1) 84

(2) 122

(3) 90

(4) 108

Ans. **(3)**

Sol. $S_3 = 3a + 3d = 54$

$\Rightarrow a + d = 18$

$S_{20} = 10(2a + 19d)$

$\Rightarrow 10(36 + 17d)$

$\Rightarrow 1600 < 10(36 + 17d) < 1800$

$\Rightarrow 160 < 36 + 17d < 180$

$\Rightarrow 124 < 17d < 144$

$\Rightarrow 7\frac{5}{17} < d < 8\frac{8}{17}$

Common difference will be natural number

$\Rightarrow d = 8 \Rightarrow a = 10$

$\Rightarrow a_{11} = 10 + 10 \times 8 = 90$

14. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 7\hat{j} + 3\hat{k}$. Let

$L_1 : \vec{r} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\vec{a}, \lambda \in R$ and

$L_2 : \vec{r} = \left(\hat{j} + \hat{k}\right) + \mu\vec{b}, \mu \in R$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_2, and is parallel to $\vec{a} + \vec{b}$, then L_3 passes through the point:

(1) (8, 26, 12) (2) (2,8, 5)

(3) (–1, –1, 1) (4) (5, 17, 4)

Ans. (1)

Sol. $L_1 : \vec{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow \vec{r} = (\lambda - 1)\hat{i} + 2(\lambda + 1)\hat{j} + (\lambda + 1)\hat{k}$

$L_2 : \vec{r} = (\hat{j} + \hat{k}) + \mu(2\hat{i} + 7\hat{j} + 3\hat{k})$

$\Rightarrow \vec{r} = 2\mu\hat{i} + (1 + 7\mu)\hat{j} + (1 + 3\mu)\hat{k}$

For point of intersection equating respective components

$\Rightarrow \lambda - 1 = 2\mu$ ...(1)

$2(\lambda + 1) = 1 + 7\mu$ ....(2)

$\lambda + 1 = 1 + 3\mu$ .....(3)

We get

$\Rightarrow \lambda = 3$ and $\mu = 1$

$\Rightarrow \vec{a} + \vec{b} = 3\hat{i} + 9\hat{j} + 4\hat{k}$

$L_3 : \vec{r} = 2\hat{i} + 8\hat{j} + 4\hat{k} + \alpha(3\hat{i} + 9\hat{j} + 4\hat{k})$

For $\alpha = 2$, $\vec{r} = 8\hat{i} + 26\hat{j} + 12\hat{k}$

15. The value of $\lim\limits_{n\to\infty}\left(\sum\limits_{K=1}^{n}\frac{k^3 + 6k^2 + 11k + 5}{(k+3)!}\right)$ is :

(1) $\frac{4}{3}$ (2) 2

(3) $\frac{7}{3}$ (4) $\frac{5}{3}$

Ans. (4)

Sol. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{k^3 + 6k^2 + 11k + 5}{(k+3)!}$

$= \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{k^3 + 6k^2 + 11k + 6 - 1}{(k+3)!}$

$= \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{(k+1)(k+2)(k+3) - 1}{(k+3)!}$

$= \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{(k+1)(k+2)(k+3)}{(k+3)!} - \frac{1}{(k+3)!}$

$= \lim\limits_{k=1}\sum\limits_{k=1}^{n}\left(\frac{1}{k!} - \frac{1}{(k+3)!}\right)$

$= \lim\limits_{k=1}\left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} .... + \frac{1}{n!} - \frac{1}{4!} - \frac{1}{5!} - \frac{1}{6!} .... - \frac{1}{(n+3)!}\right)$

$= \frac{1}{1} + \frac{1}{2} + \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$

16. The integral $80\int\limits_0^{\frac{\pi}{4}}\left(\frac{\sin\theta + \cos\theta}{9 + 16\sin 2\theta}\right)d\theta$ is equal to :

(1) $3\log_e 4$ (2) $6\log_e 4$

(3) $4\log_e 3$ (4) $2\log_e 3$

Ans. (3)

Sol. $I = 80\int\limits_0^{\frac{\pi}{4}}\left(\frac{\sin\theta + \cos\theta}{9 + 16(2\sin\theta.\cos\theta)}\right)d\theta$

$= 80\int\limits_0^{\frac{\pi}{4}}\frac{\sin\theta + \cos\theta}{9 - 16(1 - 2\sin\theta.\cos\theta - 1)}d\theta$

$= = 80\int\limits_0^{\frac{\pi}{4}}\frac{\sin\theta + \cos\theta}{9 + 16 - 16(\sin\theta - \cos\theta)^2}d\theta$

Let $\sin\theta - \cos\theta = t$

$(\cos\theta + \sin\theta)d\theta = dt$

$= 80\int\limits_{-1}^{0}\frac{dt}{25 - 16t^2}$

$= \frac{80}{16}\int\limits_{-1}^{0}\frac{dt}{\left(\frac{5}{4}\right)^2 - t^2}$

$= \frac{5}{2\left(\frac{5}{4}\right)}\ln\left[\left(\frac{\frac{5}{4} + t}{\frac{5}{4} - t}\right)\right]_{-1}^{0}$

$= 2\ln(1) + 4\ln 3$

$= 4\ln 3$

17. Let $L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$ and

$L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α,β,γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha - 11\beta - 8\gamma|$ equals :

(1) 18 (2) 16

(3) 25 (4) 20

Ans. (3)

Sol. DR's of $L_3 = \vec{m} \times \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -1 & 2 & 1 \end{vmatrix}$

$= -5\hat{i} - 3\hat{j} + \hat{k}$

$L_3 : \frac{x-\alpha}{-5} = \frac{y-\beta}{-3} = \frac{z-\gamma}{1} = \lambda$

$A(\alpha - 5\lambda, \beta - 3\lambda, \gamma + \lambda)$

$L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2} = k$

$B(k + 1, -k + 2, 2k + 1)$

Now

$\alpha - 5\lambda = k+1 \Rightarrow \alpha = 5\lambda + k + 1$

$\beta - 3\lambda = -k+2 \Rightarrow \beta = 3\lambda - k + 2$

$\gamma + \lambda = 2k-1 \Rightarrow \gamma = -\lambda + 2k + 1$

$|5\alpha - 11\beta - 8\gamma| = |-25|$

$= 25$

18. Let $x_1, x_2, \ldots\ldots x_{10}$ be ten observations such that $\sum_{i=1}^{10}(x_i - 2) = 30$, $\sum_{i=1}^{10}(x_i - \beta)^2 = 98$, $\beta > 2$ and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1-1) + 4\beta, 2(x_2-1) + 4\beta, \ldots\ldots, 2(x_{10}-1) + 4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to :

(1) 100 (2) 110

(3) 120 (4) 90

Ans. (1)

Sol. $\frac{4}{5} = \frac{\Sigma x_i^2}{10} - \left(\frac{\Sigma x_i}{10}\right)^2$

$\frac{4}{5} = \frac{\Sigma x_i^2}{10} - 25$

$\Rightarrow \Sigma x_i^2 = 258$

Now $\sum_{i=1}^{10}(x_i - \beta)^2 = 98$

$\sum_{i=1}^{10}\left(x_i^2 - 2\beta.x_i + \beta^2\right) = 98$

$258 - 2\beta(50) + 10\beta^2 = 98$

$(\beta - 8)(\beta - 2) = 0$

$\beta =$ or $\beta = 2$ (as $\beta > 2$)

$\therefore \beta = 8$

Now,

$= 2(x_1-1) + 4\beta, 2(x_2-1) + 4\beta, \ldots. 2(x_{10}-1) + 4\beta$

$= 2x_1 + 30, 2x_2 + 30, \ldots\ldots 2x_{10} + 30$

$\mu = 2(5) + 30 = 40$

$\sigma^2 = 2^2\left(\frac{4}{5}\right) = \frac{16}{5}$

$\therefore \frac{B\mu}{\sigma^2} = \frac{8 \times 40}{16/5} = 100$

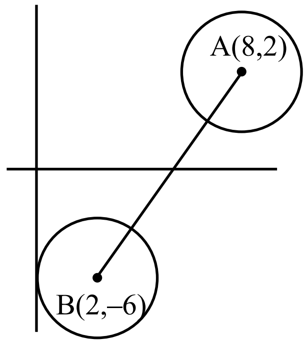
19. Let $|z_1 - 8 - 2i| \le 1$ and $|z_2 - 2 + 6i| \le 2$, $z_1, z_2 \in C$. Then the minimum value of $|z_1 - z_2|$ is :

(1) 3 (2) 7

(3) 13 (4) 10

Ans. (2)

Sol.



$\because AB = \sqrt{100} = 10$

$\therefore |Z_1 - Z_2|_{min} = 10 - 2 - 1 = 7$

20. Let $A = [a_{ij}] = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$.

If A_{ij} is the cofactor of a_{ij}, $C_{ij} = \sum_{k=1}^{2} a_{ik} A_{jk}$, $1 \le i, j \le 2$, and $C = [C_{ij}]$, then $8|C|$ is equal to :

(1) 262 (2) 288

(3) 242 (4) 222

Ans. (3)

$|A| = \frac{11}{2}$

$C_{11} = \sum_{k=1}^{2} a_{1k}.A_{1k} = a_{11}A_{11} + a_{12}A_{12} = |A| = \frac{11}{2}$

$C_{12} = \sum_{k=1}^{2} a_{1k}.A_{2k} = 0$

$C_{21} = \sum_{k=1}^{2} a_{2k}.A_{1k} = 0$

$C_{22} = \sum_{k=1}^{2} a_{2k}.A_{2k} = |A| = \frac{11}{2}$

$C = \begin{bmatrix} 11/2 & 0 \\ 0 & 11/2 \end{bmatrix}$

$|C| = \frac{121}{4}$

$8|C| = 242$

SECTION-B

21. Let $f : (0,\infty) \to R$ be a twice differentiable function. If for some $a \neq 0$, $\int_0^1 f(\lambda x)d\lambda = af(x)$, $f(1) = 1$ and $f(16) = \frac{1}{8}$, then $16 - f'\left(\frac{1}{16}\right)$ is equal to ______,

Ans. (112)

Sol. $\int_0^1 f(\lambda x)d\lambda = af(x)$

$\lambda x = t$

$d\lambda = \frac{1}{x}dt$

$\frac{1}{x}\int_0^x f(t)dt = af(x)$

$\int_0^x f(t)dt = axf(x)$

$f(x) = a(x f'(x) + f(x))$

$(1-a)f(x) = a.x f'(x)$

$\frac{f'(x)}{f(x)} = \frac{(1-a)}{a}\frac{1}{x}$

$\ell nf(x) = \frac{1-a}{a}\ell nx + c$

$x = 1, f(1) = 1 \Rightarrow c = 0$

$x = 16, f(16) = \frac{1}{8}$

$\frac{1}{8} = (16)^{\frac{1-a}{a}} \Rightarrow -3 = \frac{4-4a}{a} \Rightarrow a = 4$

$f(x) = x^{-\frac{3}{4}}$

$f'(x) = -\frac{3}{4}x^{-\frac{7}{4}}$

$\therefore\ 16 - f'\left(\frac{1}{16}\right)$

$= 16 - \left(-\frac{3}{4}(2^{-4})^{-7/4}\right)$

$= 16 + 96 = 112$

22. Let $S = \left\{m \in Z : A^{m^2} + A^m = 3I - A^{-6}\right\}$, where $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$. Then n(S) is equal to ____.

Ans. (2)

Sol. $A=\begin{bmatrix}2&-1\\1&0\end{bmatrix}$

$A^2=\begin{bmatrix}3&-2\\2&-1\end{bmatrix}, A^3=\begin{bmatrix}4&-3\\3&-2\end{bmatrix}, A^4=\begin{bmatrix}5&-4\\4&-3\end{bmatrix}$

and so on

$A^6=\begin{bmatrix}7&-6\\6&-5\end{bmatrix}$

$A^m=\begin{bmatrix}m+1&-m\\m&-m-1\end{bmatrix}$,

$A^{m^2}=\begin{bmatrix}m^2+1&-m^2\\m^2&-(m^2-1)\end{bmatrix}$

$A^{m^2}+A^m=3I-A^{-6}$

$\begin{bmatrix}m+1&-m^2\\m^2&-(m^2-1)\end{bmatrix}+\begin{bmatrix}m+1&-m\\m&-(m-1)\end{bmatrix}$

$=3\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}-5&6\\-6&7\end{bmatrix}$

$=\begin{bmatrix}8&-6\\6&-4\end{bmatrix}$

$=m^2+1+m+1=8$

$=m^2+m-6=0\Rightarrow m=-3, 2$

$n(s)=2$

23. Let [t] be the greatest integer less than or equal to t. Then the least value of $p\in N$ for which

$$\lim_{x\to0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+....+\left[\frac{9^2}{x^2}\right]\right)\right)\ge1$$

is equal to ____.

Ans. **(24)**

Sol. $\lim_{x\to0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\left[\frac{9^2}{x^2}\right]\right)\right)\ge1$

$(1+2+......+p)-(1^2+2^2+....9^2)\ge1$

$\frac{p(p+1)}{2}-\frac{9.10.19}{6}\ge1$

$p(p+1)\ge572$

Least natural value of p is 24

24. The number of 6-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is 4 ____.

Ans. **(1405)**

Sol. (i) Single letter is used , then no. of words = 5

(ii) Two distinct letters are used, then no. of words

$${}^5C_2\times\left(\frac{6!}{2!4!}\times2+\frac{6!}{3!3!}\right)=10(30+20)=500$$

(iii) Three distinct letters are used, then no. of words

$${}^5C_3\times\frac{6!}{2!2!2!}=900$$

Total no. of words = 1405

25. Let $S=\left\{x:\cos^{-1}x=\pi+\sin^{-1}x+\sin^{-1}(2x+1)\right\}$. Then $\sum_{x\in S}(2x-1)^2$ is equal to ______.

Ans. **(5)**

Sol. $\cos^{-1}x=\pi+\sin^{-1}x+\sin^{-1}(2x+1)$

$2\cos^{-1}x-\sin^{-1}(2x+1)=\frac{3\pi}{2}$

$2\alpha-\beta=\frac{3\pi}{2}$ where $\cos^{-1}x=\alpha$, $\sin^{-1}(2x+1)=\beta$

$2\alpha=\frac{3\pi}{2}+\beta$

$\cos2\alpha=\sin\beta$

$2\cos^2\alpha-1=\sin\beta$

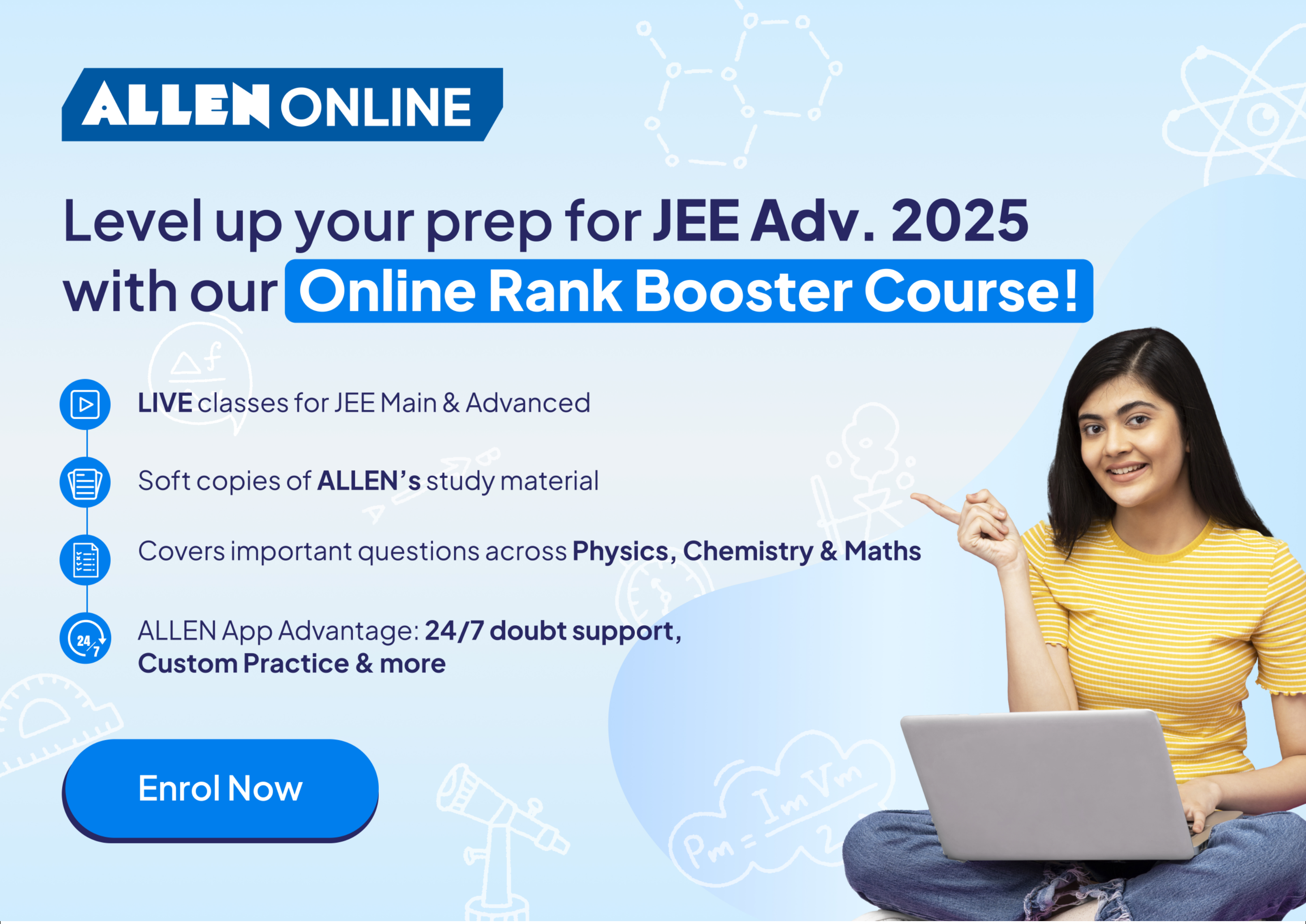
$2x^2-1=2x+1$

$x^2-x-1=0$

$$\Rightarrow n=\frac{1\pm\sqrt5}{2}=\begin{cases}n=\frac{1+\sqrt5}{2}\ \text{rejedcted}\\ n=\frac{1-\sqrt5}{2}\end{cases}$$

$\therefore 4x^2-4x=4$

$(2x-1)^2=5$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