



Sec: Sr.Super60_STERLING BT

JEE-ADV-2023_P1

Date: 24-08-2025

Time: 09.00Am to 12.00Pm

RPTA-03

Max. Marks: 180

KEY SHEET

MATHEMATICS

1	AC	2	BC	3	CD	4	C	5	A	6	C
7	C	8	1	9	16	10	7	11	5	12	9
13	12	14	A	15	B	16	D	17	B		

PHYSICS

18	ABD	19	AC	20	AD	21	A	22	A	23	C
24	C	25	13	26	30	27	20	28	7	29	4
30	3	31	B	32	A	33	C	34	C		

CHEMISTRY

35	ABCD	36	BCD	37	ABD	38	A	39	D	40	C
41	D	42	4	43	4	44	2	45	3	46	4
47	3	48	D	49	A	50	A	51	A		

SOLUTIONS MATHEMATICS

1. $x^2 + x + 1$ has no factors

$$p(x) = (x^2 + x + 1) Q(x) \text{ when } Q(x) \text{ polynomial is } x$$

$$p(x-1) = ((x-1)^2 + x - 1 + 1) Q(x-1)$$

$$= (x^2 - x + 1) Q(x-1)$$

$$(x^2 + x + 1) p(x-1) = (x^2 - x + 1) p(x)$$

$$\Rightarrow Q(x-1) = Q(x)$$

$Q(x)$ be constant

$$p(x) = k(x^2 + x + 1) \quad k \in R$$

$$P(1) = 3 \quad k=1$$

$$P(x) = x^2 + x + 1$$

$$P(x) > 0 \quad \forall x \in R$$

$$|p(x)| = p(x)$$

$$|p(x)| - 1 = |x^2 + x|$$

One local maxima two local minima

$$\int_0^1 \frac{1}{p(x)+x} dx = \int_0^1 \frac{1}{x^2+2x+1} dx = \int_0^1 \frac{1}{(1+x)^2} dx = \frac{1}{2}$$

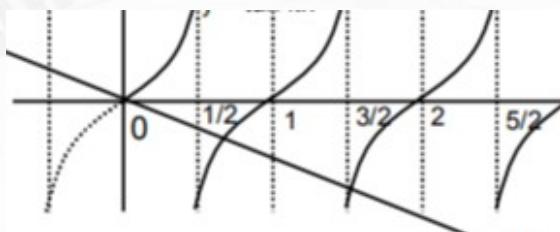
2.

$$f(x) = x \sin \pi x$$

$$f'(x) = \pi x \cos \pi x + \sin \pi x$$

$$f'(x) = 0$$

$$\tan \pi x = -\pi x$$



intersecting in $\left(\frac{1}{2}, 1\right) \cup \left(\frac{3}{2}, 2\right)$

3.

$$f'(x) > 3f(x) \quad f(0) = 1$$

$$f'(x) - 3f(x) > 0$$

$$\frac{d}{dx}(e^{-3x} f(x)) > 0$$

$$\text{let } g(x) = e^{-3x} f(x)$$

$$g(0) = 1$$

$$\therefore g(x) > 1 \quad \forall x > 0$$

$$f(x) > e^{3x} \quad \forall x > 0$$

$$f(x) > 0$$

$$f'(x) > 0 \quad f(x) \text{ increases in } (0, \infty)$$



4. $f'(x) = x(e^x - 1)(x-1)(x-2)^3(x-3)^5$

+ +

- 0 - 1 2 - 3

5. Conceptual

6. Conceptual

7.

$$\text{Let } f(x) = \begin{cases} xe^{ax} \\ x + ax^2 - x^3 \end{cases}$$

$$f(x) = \begin{cases} xe^{ax} \\ x + ax^2 - x^3 \end{cases} \quad x > 0 \quad [f(x) \text{ is continuous} = 0] \Rightarrow f(x) = \begin{cases} (ax+1)e^x & ; x \leq 0 \\ 1 + 2ax - 3x^2 & ; x > 0 \end{cases}$$

$[f^{11}(x) \text{ is continuous} = 0]$

Now $f^{11}(x) > 0 \Rightarrow 2a - 6x > 0 \text{ or } 2ae^{ax} + a^2 \times e^{ex} > 0 \text{ if } x \leq 0$

$\therefore f^{11}(x) \text{ increases at } x \in \left(-\frac{2}{a}, \frac{a}{3}\right)$

8.

$$f(x) = \int_0^x e^{x-t} dt + \int_x^1 e^{t-x} dt$$

$$f(x) = e^x \int_0^x e^{-t} dt + e^{-x} \int_x^1 e^t dt$$

$$f(x) = e^x (1 - e^{-x}) + e^{-x} (e - e^x)$$

$$f(x) = e^x - 1 + e e^{-x} - 1$$

$$f'(x) = e^x - e e^{-x} = 0$$

$$x = \frac{1}{2}$$

$$f''(x) = e^x + e e^{-x} > 0$$

$$f(0) = e - 1$$

$$f(1) = e - 1$$

$$[e - 1] = 1$$

9. Conceptual

10.

$$\text{by LMVT, } f(x) = \frac{f(2) - f(0)}{2 - 0} = \frac{f(2) + 3}{2}$$

$$f'(x) \leq 5$$

$$\frac{f(2) + 3}{2} \leq 5$$

$$\Rightarrow f(2) \leq 7$$



$$\text{Area} = (x_2 - x_1)y = \left(\frac{30-y}{2} - \frac{y}{3}\right)y \quad A'(y) = 0, y = 9$$

11. $g(0)=1+2^{1/3}, g(1)=2^{1/3}+1 \quad A''(y)=-10<0$

$$A_{\max} = \frac{810-405}{6} = \frac{405}{6} = \frac{132}{2}$$

12. $f'(x) = a - 2\sin 2x + \cos x - \sin x$

$$\text{Let } g(x) = -2\sin 2x + \cos x - \sin x = -2\{(\cos x - \sin x)^2 - 1\} + \cos x - \sin x$$

$$\text{where } \cos x - \sin x = t - 2t^2 + t + 2 \forall t \in [-\sqrt{2}, \sqrt{2}]$$

$$f''(c) = 0 \text{ for } c \in (0,1)$$

13. we have $f(3+x) = f(3-x)$

on replacing x by $x-3$, we get

$$f(x) = f(6-x)$$

On differentiating (1) w.r.t. x , we get

$$f'(x) = -f'(6-x)$$

$$\text{Putting } x = \frac{3}{2}, 2, 3, 5 \text{ in (2), We get}$$

$$f'\left(\frac{3}{2}\right) = 0 = f'(2) = f'(3) = f'(5) = f'\left(\frac{9}{2}\right) = f'(4)$$

$$= f'(1)$$

$\therefore f'(x) = 0$ has minimum 7 roots in $[1, 6]$

Now, consider a function $y = f'(x)$

As $f'(x)$ satisfy Rolle's Theorem in intervals $\left[1, \frac{3}{2}\right], \left[\frac{3}{2}, 3\right], [2, 3], [3, 4], \left[4, \frac{9}{2}\right]$ and $\left[\frac{9}{2}, 5\right]$ respectively.

so, by Rolle's Theorem, the equation $f''(x) = 0$ has minimum 6 roots

$$\text{Now, } g(x) = (f''(x))^2 + f'(x)f'''(x) = \frac{d}{dx}(f'(x)f'(x))$$

$$= h'(x), \text{ where } h(x) = f'(x)f''(x)$$

Hence, again by Rolle's theorem, $g(x) = h'(x)$ has minimum 12 zeroes in $[1, 6]$

Hence, the correct answer is (12).

14. $A \rightarrow p; B \rightarrow r; C \rightarrow s; D \rightarrow q$

$$y - y_1 = f'(x_1, y_1)(x - x_1)$$

$$A) xy^2 = 1$$

$$y - y_1 = f'(x_1, y_1)(x - x_1)$$

$$A) xy^2 = 1$$

$$y = \frac{1}{\sqrt{x}} \Rightarrow \frac{dy}{dx} = \frac{1}{2x^{3/2}}$$

$$\text{Tangent } y - \frac{1}{\sqrt{x}} = -y = -\frac{1}{2x^{3/2}}(x - x_1)$$

$$y = 0, x = x_1 + 2x_1 = 3x_1$$

$$x_1, x_2, x_3, \dots, \text{G.I}$$



B) $y = e^{-3x}$ then $\frac{dy}{dx} = 3e^{-3x}$

Tangent $y - e^{-3x_1} - 3e^{-3x_1}(x - x_1)$

$y = 0$

$x_{1+1} = \frac{1}{3} + x_1$

x_1, x_2, x_3, \dots increasing A.P.

C) $y = -\cot^{-1} x = -\frac{\pi}{2} + \tan^{-1} x$ then $\frac{dy}{dx} = \frac{1}{1+x^2}$

$y = 0, x_{1+1} = x_1 + (1+x^2)\cot^{-1} x_1$

x_1, x_2, x_3, \dots increasing

D) $y^2 = x$ $y = \sqrt{x}$

$y - \sqrt{x_1} = \frac{1}{2\sqrt{x_1}}(x - x_1)$

$y = 0$

$x_{1+1} = x_1 - 2x_1 = -x_1$

$x_{1+2} = -x_{1+1} = x_1$

periodic ($T = 2$)

Hence, the correct answer is (B).

15

$A \rightarrow r; B \rightarrow q; C \rightarrow t; D \rightarrow s$

A) $\frac{dy}{dx} = \frac{4t}{3}$

Tangent is $y - dt^4 = \frac{4t}{3}(x - at^3)$;

The point of intersection of tangent with the axes are

$\left(\frac{at^3}{4}, 0\right)$ and $\left(0, \frac{-at^4}{4}\right)$

$\therefore \frac{m}{n} = \frac{3}{4}$

$m = 3, n = 4$

$m+n=7$

B) $\frac{dy}{dx} = e^{\sin y} \cos y$

slope of normal = -1

equation of normal is $x+y=1$

Area = $\frac{1}{2}$

C) $y = \frac{1}{x^2}; \frac{dy}{dx} = \frac{2}{x^3}$

slope of tangent = -2

$y = e^{2-2x}; \frac{dy}{dx} = x^{2-2x}(-2)$

$\therefore \tan \theta = 0$

D) $\frac{dy}{dx} = \frac{be^{x/3}}{3}$

Length of subtangent = $\left| \frac{y}{y'} \right| = \left| \frac{be^{x/3}}{b \frac{1}{3} e^{x/3}} \right| = 3$

16. $f(x) = 4\alpha x^2 + \frac{1}{x}$, $f'(x) = 8\alpha x - \frac{1}{x^2}$



$$f'(x) = 0 \Rightarrow x = \frac{1}{2\alpha^{1/3}}$$

17.

let $OA, OB = y$ $AB = R$

$$AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos 120^\circ$$

$$R^2 = x^2 + y^2 - 2xy \left(\frac{-1}{2} \right)$$

$$R^2 = x^2 + y^2 + xy$$

$$2R \frac{dR}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} + \left(x \frac{dy}{dt} + y \frac{dx}{dt} \right)$$

$$B) g'(x) = f'(x) - f'(1-x) \& f''(x) > 0$$

$$\Rightarrow x = 1 - x \Rightarrow x = 1/2$$

$$g''(x) = f''(x) + f''(1-x) > 0, g \text{ is increasing}, \alpha = \frac{1}{2}$$

$$\text{sub } \alpha = \frac{1}{2}$$

C) Let $BP = x$, from similar triangle property,

$$\frac{AO}{l_1} = \frac{l_2}{x} \Rightarrow \left(AO = \frac{l_1 l_2}{x} \right)$$

$$\text{when } x = \frac{l_1}{2}, \frac{d(AO)}{dt} = \frac{-2l_2}{5} \text{ m/s}$$

D) Consider $g(x) = (f(x))^3$ which is continuous in $[2, 7]$ differentiable in $(2, 7)$ by Lmvt, $c \in [2, 7]$ such that

$$g'(c) = \frac{g(7) - g(2)}{7 - 2}$$

$$5f^2(c)f'(c) = (f(7) - f(2)) \frac{(f(7)^2 + f(2)^2 + f(7)f(2))}{3}$$

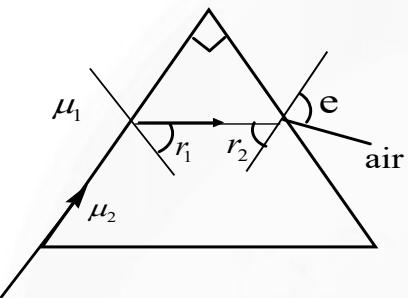
PHYSICS

18. CONCEPTUAL

19. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{20} + \frac{1}{-60} = \frac{3-1}{60} = \frac{1}{30} \Rightarrow F=30$

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{w} \Rightarrow \frac{1}{30} = \frac{1}{v} - \frac{1}{-30} \Rightarrow v=\infty(\text{infinity})$$

20.



a) $\sin e = \frac{1}{\mu_2} = \frac{1}{\sqrt{3}}$

$$r_1 = 90^\circ - r_2 = 90^\circ - e$$

$$\sin r_1 = \sin(90^\circ - e)$$

$$= \cos e$$

$$= \sqrt{1 - \sin^2 e}$$

$$\sin r_1 = \sqrt{\frac{2}{3}}$$

b) from snell's law

$$\frac{\mu_2}{\mu_1} = \frac{\sin 90^\circ}{\sin r_1}$$

$$\sin r_1 = \frac{\mu_2}{\mu_1} \rightarrow 1$$

$$\frac{\sin e}{\sin r_2} = \mu_2$$

$$\sin r_2 = \frac{\sin e}{\mu_2}$$

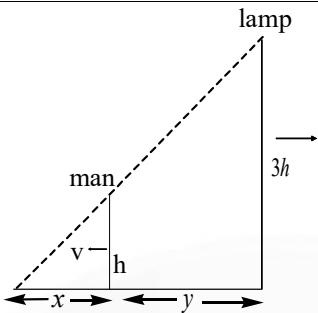
$$\sin(90^\circ - r) = \frac{\sin e}{\mu_2}$$

$$\cos r_1 = \frac{\sin e}{\mu_2}$$

$$\sqrt{1 - \sin^2 r_1} = \frac{\sin e}{\mu_2} \rightarrow 2$$

$$\text{from 1 \& 2, } \mu_1^2 + \sin^2 e = \mu_2^2$$

21. Let 'x' is the length of shadow, when 'y' is the distance from the man to the lamppost.
From similar triangle



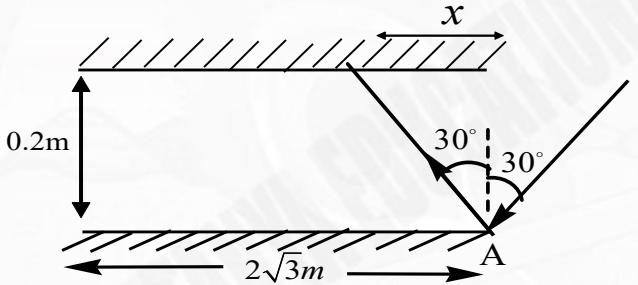
$$\frac{h}{3h} = \frac{x}{x+y}$$

$$x+y = 3x$$

$$y = 2x \Rightarrow x = \frac{y}{2} \Rightarrow \frac{dx}{dt} = \frac{1}{2} \frac{dy}{dt}$$

$$= \frac{v}{2}$$

22. Let 'x' is distance occupied by the ray in one reflection,



$$x = 0.2 \tan 30^\circ = \frac{0.2}{\sqrt{3}} m$$

$$\therefore \text{no.of reflections} = \frac{\text{length of mirror}}{x}$$

$$= \frac{2\sqrt{3}\sqrt{3}}{0.2} = 30$$

23. Let angle of incidence is i , the angle of refraction first face of the prism will be 30° .
Now by Snell's law

$$\mu = \frac{\sin i}{\sin r} \Rightarrow 45^\circ$$

24. Use lens makers formula

$$\frac{1}{7} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$25. f = \frac{D^2 - x^2}{4D} = \frac{(60)^2 - (20)^2}{4 \times 60} \Rightarrow \frac{40}{3} m$$

26. Image formed by the first lens



$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \Rightarrow v_1 = 15\text{cm}$$

It acts like object for second lens, then image formed by second lens

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_2} \Rightarrow \frac{1}{v_2} - \frac{1}{10} = \frac{1}{-10}$$

So, image due to second lens forms at infinity and it acts like object for third lens

$$\frac{1}{v_3} - \frac{1}{u_3} = \frac{1}{f_3}$$

$$\frac{1}{v_3} - \frac{1}{\infty} = \frac{1}{30}$$

$$\therefore v_3 = 30\text{cm}$$

So final image is formed 30m to the right of the third lens.

27. Image produced by plane mirror is 5 cm inside the mirror.

Hence , distance image formed by the convex mirror

$$V=25-5=20\text{ cm}$$

\therefore By lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{20} - \frac{1}{u} = \frac{1}{10} \Rightarrow \frac{1}{u} = \frac{1}{20} - \frac{1}{10}$$

$$\therefore u = 20\text{cm}$$

28. Total apparent depth of the coin = $\frac{d_1}{\mu_1} + \frac{d_2}{\mu_2}$

$$= \frac{6}{9} \times 4 + \frac{6}{4} \times 3$$

$$= \frac{24}{9} + \frac{18}{4} \Rightarrow 2.66 + 4.5$$

$$= 7.16\text{cm}$$

29. lateral shift

$$\alpha = \frac{t \sin(i-r)}{\cos r} = 3.64\text{cm}$$

30. Total deviation = S+S¹

$$= \left(\frac{\mu_V + \mu_R}{2} - 1 \right) A + \left(\frac{\mu_V^1 + \mu_R^1}{2} - 1 \right) A^1$$

$$= 3$$

31. P) Parallel beam can be obtain from concave mirror and convex lens when point object is at focus.

Q) Real image for a real object is for concave mirror and convex lens

R) Virtual and diminished images are obtained for convex mirror and concave lens

S) Real and magnified image is obtained for concave mirror and convex lens

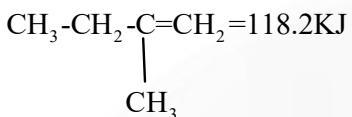
T) The direction of motion of image is in the same direction as motion of object in lens and opposite in mirror



32. When a ray of light enters from rarer to denser medium it bends towards the normal and its opposite when ray of light travels from denser to rarer , it bends away from normal
- $\mu_1 < \mu_2$, the ray bends towards normal
 - $\mu_1 > \mu_2$ the ray bends away from normal
 - $\mu_2 = \mu_3$ the ray comes out without deviation
 - $\mu_2 > \mu_3$ the ray deviates away from the normal
33. A) Object for diverging lens will be virtual . From $v = \frac{uf}{u+f}$ v may be less than, equal to or more than f.
- B) Object for diverging lens will be a real object (diverging rays fall on diverging lens). Hence, $v < f$
- C) For converging lens, object distance will be between f and 2f . So , image formed by the converging lens will be beyond 2f hence $L_2 > f$
- D) For converging lens, object distance will be greater than 2f. So, image formed by it will be between F and 2F. Hence, $L_2 > f$.
34. A) Applying Snell's law at PQ,
- $$1 \times \sin 45^\circ = \sqrt{2} \times \sin r$$
- $$\sin r = \frac{1}{\sqrt{2}}$$
- $$r = 30^\circ$$
-
- Critical angle for QR = $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$*
- Since, angle of incidence is greater than the critical angle, TIR takes place at surface QR.

CHEMISTRY

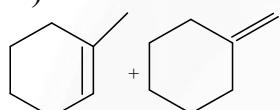
35. a) Syn 1,2 elimination
 b) Cope elimination
 c) Hofmann elimination
 d) Hofmann elimination
36. A) Isobutylene $(CH_3)_2C=CH_2 = 117.8\text{KJ}$



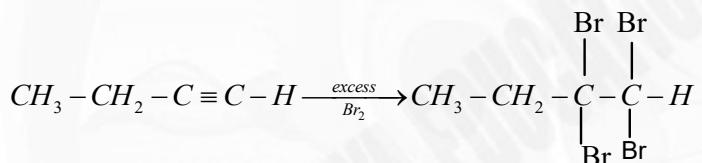
B) bredict rule

C) In E2 elimination reactivity of the substance $3^0 > 2^0 > 1^0$ (alkyl halide)

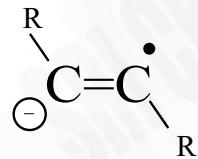
D)



37. LiAlH₄ and NaBH₄ differ in primary and tertiary

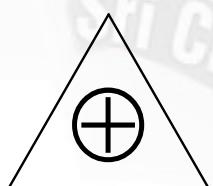


38. A

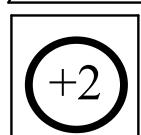


40. C

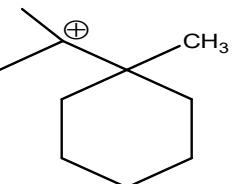
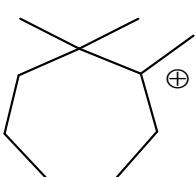
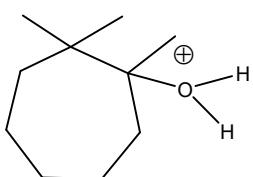
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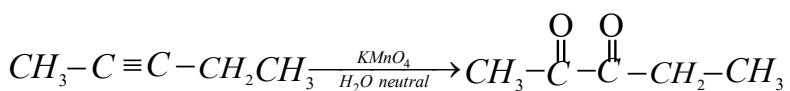
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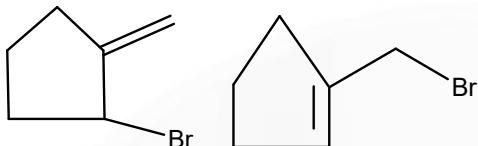
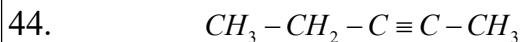
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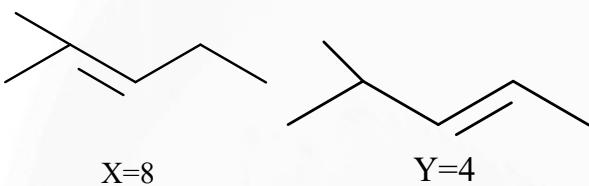


43. 4



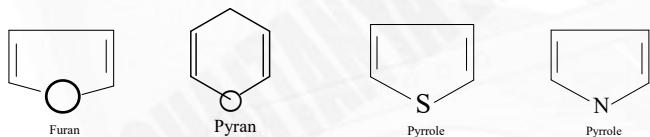
45. 3

46.



47. 3

48.



49.

- 1) Markonikoves
- 2) Markonikoves
- 3) Anti Markonikoves
- 4) Markonikoves

50.

- 1) Syn Addition
- 2) Anti Product
- 3) Dehydration is takes place after that dehydrogenation
- 4) Hydroboration

51.

- 1) Cyclo octa tetraene
- 2) pyrrole is produced
- 3) vinyl cyanide is produced
- 4) benzene is produced