

FINAL JEE-MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 06th April, 2023)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

SECTION-A

1. Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$. Then $18 \int_1^2 f(x) dx$

is equal to:

- (1) $10 \log_e 2 - 6$
(2) $10 \log_e 2 + 6$
(3) $5 \log_e 2 + 3$
(4) $5 \log_e 2 - 3$

Official Ans. by NTA (1)
Allen Ans. (1)

Sol. $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3 \dots\dots\dots(1)$

replace $x \rightarrow \frac{1}{x}$

$$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3 \dots\dots\dots(2)$$

Eq. (1) $\times 5$ – eq. (2) $\times 4$

$$f(x) = \frac{1}{9} \left(\frac{5}{x} - 4x + 3 \right)$$

$$I = 18 \int_1^2 \frac{1}{9} \left(\frac{5}{x} - 4x + 3 \right) dx = 10 \log_e 2 - 6$$

2. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to

- (1) 82
(2) 123
(3) 164
(4) 75

Official Ans. by NTA (2)
Allen Ans. (2)

Sol. Probability of success $= \frac{1}{9} = p$

Probability of failure $q = \frac{8}{9}$

$P(\text{at least 4 success}) = P(4 \text{ success}) + P(5 \text{ success})$

$$= {}^5C_4 p^4 q + {}^5C_5 p^5 = \frac{41}{3^{10}} = \frac{123}{3^{11}}$$

$k = 123$

TEST PAPER WITH SOLUTION

3. If ${}^{2n}C_3 : {}^nC_3 = 10:1$, then the ratio

$$(n^2 + 3n) : (n^2 - 3n + 4) \text{ is}$$

- (1) 35: 16
(2) 65:37
(3) 27:11
(4) 2:1

Official Ans. by NTA (4)
Allen Ans. (4)

Sol. $\frac{{}^{2n}C_3}{{}^nC_3} = 10 \Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 10$

$$n = 8$$

$$\text{So } (n^2 + 3n) : (n^2 - 3n + 4) = 2$$

4. If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of

$$\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}} \right)^n \text{ is } \sqrt{6} : 1, \text{ then the third term from the}$$

beginning is:

- (1) $60\sqrt{2}$
(2) $60\sqrt{3}$
(3) $30\sqrt{2}$
(4) $30\sqrt{3}$

Official Ans. by NTA (2)
Allen Ans. (2)

Sol. $\frac{{}^nC_4 2^{\frac{n-4}{4}} \cdot \left(\frac{-1}{3^{\frac{1}{4}}}\right)^4}{{}^nC_4 3^{-\left(\frac{n-4}{4}\right)} \cdot \left(\frac{1}{2^{\frac{1}{4}}}\right)^4} = \frac{\sqrt{6}}{1}$

$$\Rightarrow n = 10$$

$$\text{So } T_3 = {}^{10}C_2 2^{\frac{1}{4} \cdot 8} \cdot 3^{-\frac{1}{4} \cdot 2} = \frac{45.4}{\sqrt{3}} = 60\sqrt{3}$$

5. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} - 2\hat{k}$ and

$$\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}. \text{ If } \vec{d} \text{ is a vector perpendicular to}$$

both \vec{b} and \vec{c} and $\vec{a} \cdot \vec{d} = 18$, Then $|\vec{a} \times \vec{d}|^2$ is equal to

- (1) 640
(2) 760
(3) 680
(4) 720

Official Ans. by NTA (4)
Allen Ans. (4)

Sol. $\vec{a} = \lambda(\vec{b} \times \vec{c})$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{d} = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{a} \cdot \vec{d} = 18$$

$$\lambda = 2$$

$$\text{So } \vec{d} = 2(2\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{d} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 4 \\ 2 & 3 & 4 \end{vmatrix} = -20\hat{i} - 8\hat{j} + 16\hat{k}$$

$$|\vec{d} \times \vec{a}|^2 = 720$$

6. The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line $L: 9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2 , then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on

(1) $6x + y = 10$

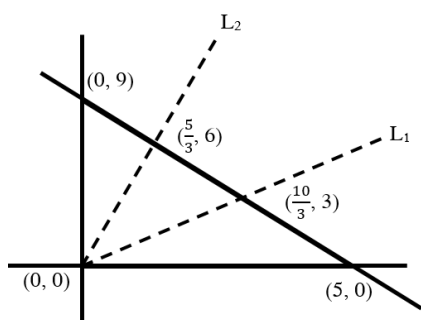
(2) $6x - y = 15$

(3) $y - x = 5$

(4) $y - 2x = 5$

Official Ans. by NTA (3)

Allen Ans. (3)



Sol.

$$m_{l_1} = \frac{3.3}{10} = \frac{9}{10}$$

$$m_{l_2} = \frac{6.3}{5} = \frac{18}{5}$$

$$y = (m_1 + m_2)x$$

$$y = \frac{9}{2}x$$

$$\text{Point of intersection with } L \text{ is } \left(\frac{10}{7}, \frac{45}{7}\right)$$

7. From the top A of a vertical wall AB of height 30 m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively. B and Q are on the same horizontal level. If C is a point on AB such that $CB = PQ$, then the area (in m^2) of the quadrilateral BCPQ is equal to

(1) $600(\sqrt{3} - 1)$

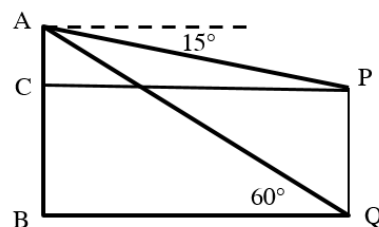
(2) $300(\sqrt{3} + 1)$

(3) $200(3 - \sqrt{3})$

(4) $300(\sqrt{3} - 1)$

Official Ans. by NTA (1)

Allen Ans. (1)



Sol.

$$\tan 60^\circ = \sqrt{3} = \frac{30}{BQ}$$

$$BQ = 10\sqrt{3}\text{m} = CP$$

$$\tan 15^\circ = 2 - \sqrt{3} = \frac{AC}{CP}$$

$$AC = 10\sqrt{3}(2 - \sqrt{3})$$

$$\text{Area} = 10\sqrt{3}(60 - 20\sqrt{3}) = 600(\sqrt{3} - 1)$$

8. The sum of the first 20 terms of the series $5 + 11 + 19 + 29 + 41 + \dots$ is

(1) 3450

(2) 3250

(3) 3420

(4) 3520

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $S_{20} = 5 + 11 + 19 + 29 + \dots$

$$\text{Let } T_r = ar^2 + br + c$$

$$T_1 = a + b + c = 5$$

$$T_2 = 4a + 2b + c = 11$$

$$T_3 = 9a + 3b + c = 19$$

$$a = 1, b = 3, c = 1$$

$$\text{Hence } S_{20} = \sum_{r=1}^{20} r^2 + 3 \sum_{r=1}^{20} r + \sum_{r=1}^{20} 1 = 3520$$

9. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to

- (1) 9
(2) 12
(3) 11
(4) 10

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Combine var. $= \frac{n_1\sigma^2 + n_2\sigma^2}{n_1 + n_2} + \frac{n_1n_2(m_1 - m_2)^2}{(n_1 + n_2)^2}$

$$13 = \frac{15.14 + 15.\sigma^2}{30} + \frac{15.15(12-14)^2}{30 \times 30}$$

$$13 = \frac{14 + \sigma^2}{2} + \frac{4}{4}$$

$$\sigma^2 = 10$$

10. Let $A = [a_{ij}]_{2 \times 2}$ where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$, then $3a^2 + 4b^2$ is equal to

- (1) 7
(2) 14
(3) 3
(4) 4

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$$A^2 = \begin{bmatrix} p^2 + qr & pq + qs \\ pr + rs & qs + s^2 \end{bmatrix}$$

$$\Rightarrow p^2 + qr = 1 \quad (1) \quad pq + qs = 0 \Rightarrow q(p+s) = 0 \quad (3)$$

$$\Rightarrow s^2 + qr = 1 \quad (2) \quad pr + rs = 0 \Rightarrow r(p+s) = 0 \quad (4)$$

Equation (1) – equation (2)

$$p^2 = s^2 \Rightarrow p+s=0$$

$$\text{Now } 3a^2 + 4b^2$$

$$= 3(p+s)^2 + 4(ps-qr)^2$$

$$= 3.0 + 4(-p^2 - qr)^2 = 4(p^2 + qr)^2 = 4$$

11. Let $I(x) = \int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$. If $I(0) = 0$ the I

$\left(\frac{\pi}{4}\right)$ is equal to

(1) $\log_e \frac{(x+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$

(2) $\log_e \frac{(x+4)^2}{16} + \frac{\pi^2}{4(\pi+4)}$

(3) $\log_e \frac{(x+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$

(4) $\log_e \frac{(x+4)^2}{32} + \frac{\pi^2}{4(\pi+4)}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $I(x) = \int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$

$$\text{Let } x \tan x + 1 = t$$

$$I = x^2 \left(\frac{-1}{x \tan x + 1} \right) + \int \frac{2x}{x \tan x + 1} dx$$

$$I = x^2 \left(\frac{-1}{x \tan x + 1} \right) + 2 \int \frac{x \cos x}{x \sin x + \cos x} dx$$

$$I = x^2 \left(\frac{-1}{x \tan x + 1} \right) + 2 \ln |x \sin x + \cos x| + C$$

$$\text{As } I(0) = 0 \Rightarrow C = 0$$

$$I\left(\frac{\pi}{4}\right) = \ln \left(\frac{(\pi+4)^2}{32} \right) - \frac{\pi^2}{4(\pi+4)}$$

12. If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$. then a

$+ b + c$ is equal to

- (1) 14
(2) 12
(3) 13
(4) 15

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Equation of family of plane

$$(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$$

$$x(2 + 4\lambda) - y(1 + 3\lambda) + z(1 + 5\lambda) - 3 + 9\lambda = 0$$

Parallel to the line

$$-2(2 + 4\lambda) - (1 + 3\lambda)4 + (1 + 5\lambda)5 = 0$$

$$5\lambda = 3$$

$$\lambda = \frac{3}{5}$$

equation of plane

$$11x - 7y + 10z + 6 = 0$$

$$a + b + c = 14$$

13. Statement $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is logically equivalent to

(1) $(P \vee R) \Rightarrow Q$

(2) $(P \Rightarrow R) \wedge (Q \Rightarrow R)$

(3) $(P \Rightarrow R) \vee (Q \Rightarrow R)$

(4) $(P \wedge R) \Rightarrow Q$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$

We known that $P \Rightarrow Q \equiv \sim P \vee Q$

$$\Rightarrow (\sim P \vee Q) \wedge (\sim R \vee Q)$$

$$\Rightarrow (\sim P \wedge \sim R) \vee Q$$

$$\Rightarrow \sim(P \vee R) \vee Q$$

$$\Rightarrow (P \vee R) \Rightarrow Q$$

14. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is:

(1) $9 + \sqrt{3}$

(2) $11 + \sqrt{3}$

(3) $9 - \sqrt{3}$

(4) $11 - \sqrt{3}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. For $x \leq 3$ or $x \geq 5$

$$x^2 - 8x + 15 - 2x + 7 = 0$$

$$x = 5 + \sqrt{3}$$

$$\text{For } 3 < x < 5, \quad x^2 - 8x + 15 + 2x - 7 = 0$$

$$x = 4$$

$$\text{Hence sum} = 9 + \sqrt{3}$$

15. Let $a_1, a_2, a_3, \dots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}}$$

(1) 1

(2) \sqrt{d}

(3) $\frac{1}{\sqrt{d}}$

(4) 0

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}}$

On rationalising each term

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{d} \left(\sqrt{a_n} - \sqrt{a_1} \right)}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{d} \left(\frac{(n-1)d}{(\sqrt{a_n} + \sqrt{a_1})} \right)} = 1$$

16. If the system of equations

$$x + y + az = b$$

$$2x + 5y + 2z = 6$$

$$x + 2y + 3z = 3$$

has infinitely many solutions, then $2a + 3b$ is equal to

(1) 23

(2) 28

(3) 25

(4) 20

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11 - 4 - a = 0$

$$a = 7$$

$$\Delta_1 = \begin{vmatrix} b & 1 & a \\ 6 & 5 & 2 \\ 3 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11b - 12 - 21 = 0$$

$$b = 3$$

$$2a + 3b = 23$$

17. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at $(2, 2)$ is equal to

(1) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$ (2) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

(3) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$ (4) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $2x^y + 3y^x = 20$

$$2x^y \left[\frac{y}{x} + (\ln x) y' \right] + 3y^x \left[\frac{xy'}{y} + \ln y \right] = 0$$

$$y' = \frac{-(12 \ln 2 + 8)}{12 + 8 \ln 2} = -\left(\frac{2 + \log_e 8}{3 + \log_e 4}\right)$$

18. One vertex of a rectangular parallelopiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex $(3, 4, 5)$. Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is:

(1) $\frac{12}{\sqrt{5}}$ (2) $\frac{12}{5\sqrt{5}}$

(3) $12\sqrt{5}$ (4) $\frac{12}{5}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol Equation of OP is $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

$$a_1 = (0, 0, 0) \quad a_2 = (3, 0, 5)$$

$$b_1 = (3, 4, 5) \quad b_2 = (0, 0, 1)$$

Equation of edge parallel to z axis

$$\frac{x-3}{0} = \frac{y-0}{0} = \frac{z-5}{1}$$

$$S.D = \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\begin{vmatrix} 3 & 0 & 5 \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix} = \frac{3(4)}{4\hat{i} - 3\hat{j}} = \frac{12}{5}$$

19. Let the position vectors of the points A, B, C and D be $5\hat{i} + 5\hat{j} + 2\lambda\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $-2\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 6\hat{k}$. Let the set $S = \{\lambda \in \mathbb{R} : \text{The points A, B, C and D are coplanar}\}$. Then $\sum_{\lambda \in S} (\lambda + 2)^2$ is equal to

(1) 41 (2) 25

(3) 13 (4) $\frac{37}{2}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Since A, B, C, D are coplanar

$$\text{Hence } [\vec{BA} \quad \vec{CA} \quad \vec{DA}] = 0$$

$$\begin{vmatrix} 4 & 3 & 2\lambda - 3 \\ 7 & 5 - \lambda & 2\lambda - 4 \\ 6 & 0 & 2\lambda - 6 \end{vmatrix} = 0$$

$$\lambda = 2, 3 \text{ Hence } \sum_{\lambda \in S} (\lambda + 2)^2 = 41$$

20. Let $A = \{x \in \mathbb{R} : [x+3] + [x+4] \leq 3\}$,

$$B = \left\{x \in \mathbb{R} : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r} \right)^{x-3} < 3^{-3x}\right\}, \text{ where } [t]$$

denotes greatest integer function. Then,

(1) $A \cap B = \phi$

(2) $A = B$

(3) $B \subset C, A \neq B$

(4) $A \subset B, A \neq B$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $[x] + 3 + [x] + 4 \leq 3$

$$2[x] \leq -4$$

$$[x] \leq -2 \Rightarrow x \in (-\infty, -1) \dots\dots\dots(A)$$

$$3^x \left(\frac{3 \cdot \frac{1}{10}}{1 - \frac{1}{10}} \right)^{x-3} < 3^{-3x}$$

$$27 < 3^{-3x}$$

$$-3x > +3$$

$$x < -1 \dots\dots\dots(B)$$

$$A = B$$

SECTION-B

21. Let $a \in \mathbb{Z}$ and $[t]$ be the greatest integer $\leq t$. Then the number of points, where the function $f(x) = [a + 13 \sin x]$, $x \in (0, \pi)$ is not differentiable, is _____

Official Ans. by NTA (25)

Allen Ans. (25)

Sol. $f(x) = [a + 13 \sin x]$, $x \in (0, \pi)$

For $[n \sin x]$; Total number of non differentiable points are $= 2n - 1$ for $x \in (0, \pi)$

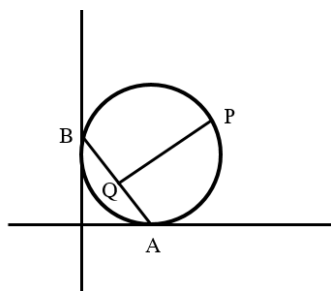
So number of non differentiable points for $[13 \sin x] \Rightarrow 25$ Points

22. A circle passing through the point $P(\alpha, \beta)$ in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then the value of $\alpha\beta$ is _____

Official Ans. by NTA (121)

Allen Ans. (121)

Sol.



Let equation of circle is $(x - a)^2 + (y - a)^2 = a^2$

which is passing through $P(\alpha, \beta)$

then $(\alpha - a)^2 + (\beta - a)^2 = a^2$

$$\alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 = 0$$

Here equation of AB is $x + y = a$

Let $Q(\alpha', \beta')$ be foot of perpendicular of P on AB

$$\frac{\alpha' - \alpha}{1} = \frac{\beta' - \beta}{1} = \frac{-(\alpha + \beta - a)}{2}$$

$$PQ^2 = (\alpha' - \alpha)^2 + (\beta' - \beta)^2 = \frac{1}{4}(\alpha + \beta - a)^2 + \frac{1}{4}(\alpha + \beta - a)^2$$

$$121 = \frac{1}{2}(\alpha + \beta - a)^2$$

$$242 = \alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 + 2\alpha\beta$$

$$242 = 2\alpha\beta$$

$$\Rightarrow \alpha\beta = 121$$

23. The number of ways of giving 20 distinct oranges to 3 children such that each child gets atleast one orange is _____

Official Ans. by NTA (171)

Allen Ans. (Bonus)

- Sol.** 20 distinct oranges distributed among 3 children so that each child gets at least one orange

$$= 3^{20} - {}^3C_1 2^{20} + {}^3C_2 1^{20}$$

Bonus

24. If the area of the region

$S = \{(x, y) : 2y - y^2 \leq x^2 \leq 2y, x \geq y\}$ is equal to

$\frac{n+2}{n+1} - \frac{\pi}{n-1}$, then the natural number n is equal to _____

Official Ans. by NTA (5)

Allen Ans. (5)

- Sol.** $x^2 + y^2 - 2y \geq 0$ & $x^2 - 2y \leq 0$, $x \geq y$

$$\text{Hence required area} = \frac{1}{2} \times 2 \times 2 - \int_0^2 \frac{x^2}{2} dx - \left(\frac{\pi}{4} - \frac{1}{2} \right)$$

$$= \frac{7}{6} - \frac{\pi}{4} \Rightarrow n = 5$$

25. Let the point $(p, p + 1)$ lie inside the region

$E = \{(x, y) : 3 - x \leq y \leq \sqrt{9 - x^2}, 0 \leq x \leq 3\}$ If the set of all values of p is the interval (a, b) . then $b^2 + b - a^2$ is equal to _____

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $3 - x \leq y \leq \sqrt{9 - x^2}$

Points $(p, p + 1)$ lies on $y = x + 1$

So point of intersection between

$$y = x + 1 \text{ \& } y = 3 - x \text{ is } x = 1, y = 2$$

and point of intersection between

$$x + 1 = \sqrt{9 - x^2} \text{ is } x = \frac{-1 + \sqrt{17}}{2}$$

$$\text{Hence } p \in \left(1, \frac{-1 + \sqrt{17}}{2}\right)$$

$$\text{Hence } b^2 + b - a^2 = 3$$

26. Let $y = y(x)$ be a solution of the differential equation $(x \cos x)dy + (x y \sin x + y \cos x - 1)dx = 0$,

$$0 < x < \frac{\pi}{2}. \text{ If } \frac{\pi}{3} y\left(\frac{\pi}{3}\right) = \sqrt{3}, \text{ then}$$

$$\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right| \text{ is equal to } \underline{\hspace{2cm}}$$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $(x \cos x)dy + (x y \sin x + y \cos x - 1)dx = 0, 0 < x < \frac{\pi}{2}$

$$\frac{dy}{dx} + \left(\frac{x \sin x + \cos x}{x \cos x}\right)y = \frac{1}{x \cos x}$$

$$\text{IF} = x \sec x$$

$$y \cdot x \sec x = \int \frac{x \sec x}{x \cos x} dx = \tan x + c$$

$$\text{Since } y\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{\pi}$$

$$\text{Hence } c = \sqrt{3}$$

$$\text{Hence } \left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + y'\left(\frac{\pi}{6}\right)\right| = |-2| = 2$$

27. The coefficient of x^{18} in the expansion of

$$\left(x^4 - \frac{1}{x^3}\right)^{15} \text{ is } \underline{\hspace{2cm}}$$

Official Ans. by NTA (5005)

Allen Ans. (5005)

Sol. $\left(x^4 - \frac{1}{x^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(\frac{-1}{x^3}\right)^r$$

$$60 - 7r = 18$$

$$r = 6$$

$$\text{Hence coeff. of } x^{18} = {}^{15}C_6 = 5005$$

28. Let $A = \{1, 2, 3, 4, \dots, 10\}$ and $B = \{0, 1, 2, 3, 4\}$.

The number of elements in the relation $R = \{(a, b)$

$$\in A \times A: 2(a - b)^2 + 3(a - b) \in B\} \text{ is } \underline{\hspace{2cm}}$$

Official Ans. by NTA (18)

Allen Ans. (18)

Sol. $A = \{1, 2, 3, \dots, 10\}$

$$B = \{0, 1, 2, 3, 4\}$$

$$R = \{(a, b) \in A \times A: 2(a - b)^2 + 3(a - b) \in B\}$$

$$\text{Now } 2(a - b)^2 + 3(a - b) = (a - b)(2(a - b) + 3)$$

$$\Rightarrow a = b \text{ or } a - b = -2$$

$$\text{When } a = b \Rightarrow 10 \text{ order pairs}$$

$$\text{When } a - b = -2 \Rightarrow 8 \text{ order pairs}$$

$$\text{Total} = 18$$

29. Let the image of the point $P(1, 2, 3)$ in the plane $2x - y + z = 9$ be Q . If the coordinates of the point R are $(6, 10, 7)$, then the square of the area of the triangle PQR is $\underline{\hspace{2cm}}$

Official Ans. by NTA (594)

Allen Ans. (594)

Sol. Let $Q(\alpha, \beta, \gamma)$ be the image of P , about the plane

$$2x - y + z = 9$$

$$\frac{\alpha - 1}{2} = \frac{\beta - 2}{-1} = \frac{\gamma - 3}{1} = 2$$

$$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$$

$$\text{Then area of triangle } PQR = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}|$$

$$= |-12\hat{i} - 3\hat{j} + 21\hat{k}| = \sqrt{144 + 9 + 441} = \sqrt{594}$$

$$\text{Square of area} = 594$$

30. Let the tangent to the curve $x^2 + 2x - 4y + 9 = 0$ at the point $P(1, 3)$ on it meet the y -axis at A . Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B . If B lies on the line $2x - 3y = 8$. then $(AB)^2$ is equal to

Official Ans. by NTA (292)

Allen Ans. (292)

Sol. Equation of tangent at $P(1, 3)$ to the curve

$$x^2 + 2x - 4y + 9 = 0 \text{ is } y - x = 2$$

Then the point A is $(0, 2)$

Equation of line passing through P and parallel to the line $x - 3y = 6$.

The possible coordinate of B are $(4, 4)$ or $(16, 8)$

But $(4, 4)$ does not satisfy $2x - 3y = 8$

Thus the point B is $(16, 8)$

Then $(AB)^2 = 292$