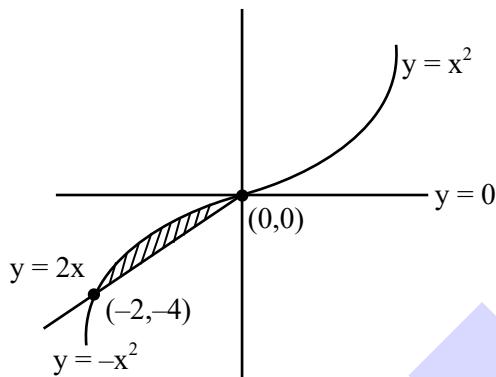


4. The area enclosed between the curves $y = x|x|$ and $y = x - |x|$ is :

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$
(3) 1 (4) $\frac{4}{3}$

Ans. (4)

Sol.



$$A = \int_{-2}^0 -x^2 - 2x = \frac{4}{3}$$

5. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50th word is :

(1) OBBHJ (2) HBBJO
(3) OBBJH (4) JBBOH

Ans. (3)

Sol. B B H J O

$$\boxed{B} \quad 4! = 24$$

$$\boxed{H} \quad \frac{4!}{2!} = 12$$

$$\boxed{J} \quad \frac{4!}{2!} = 12$$

O B B H J

O B B J H \rightarrow 50th rank

6. Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$

and \vec{c} be three vectors such that

$$(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i}) = \vec{a} \times (\vec{c} + \hat{i}) \quad \vec{a} \cdot \vec{c} = -29,$$

then $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$ is equal to :

- (1) 10 (2) 5
(3) 15 (4) 12

Ans. (2)

Sol. Let's assume $\vec{v} = \vec{a} + \vec{b} + \hat{i}$

$$= 5\hat{i} + 3\hat{j} + \hat{k}$$

and $\vec{c} + \hat{i} = \vec{p}$

So,

$$\vec{p} \times \vec{v} = \vec{a} \times \vec{p}$$

$$\vec{p} \times \vec{v} + \vec{p} \times \vec{a} = \vec{0}$$

$$\vec{p} \times (\vec{v} + \vec{a}) = \vec{0}$$

$$\Rightarrow \vec{p} = \lambda(\vec{v} + \vec{a})$$

$$\vec{c} + \hat{i} = \lambda(7\hat{i} + 8\hat{j})$$

$$\vec{a} \cdot \vec{c} + \vec{a} \cdot \hat{i} = \lambda \vec{a} \cdot (7\hat{i} + 8\hat{j})$$

$$-29 + 2 = \lambda(14 + 40)$$

$$\lambda = -\frac{1}{2}$$

$$\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) + \hat{i} \cdot (-2\hat{i} + \hat{j} + \hat{k}) = \lambda(7\hat{i} + 8\hat{j}) \cdot (-2\hat{i} + \hat{j} + \hat{k})$$

$$= -\frac{1}{2}(-14 + 8) + 2 = 5$$

7. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2, |\vec{b}| = 3$

and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between

the vectors \vec{b} and \vec{c} , then the minimum value of

$$27|\vec{c} - \vec{a}|^2$$

is equal to :

- (1) 110 (2) 105
(3) 124 (4) 121

Ans. (3)



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11. Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be defined as : $f(x) = |x - 1|$ and $g(x) = \begin{cases} e^x, & x \geq 0 \\ x + 1, & x \leq 0 \end{cases}$. Then the function $f(g(x))$ is

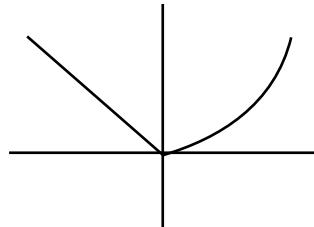
- neither one-one nor onto.
- one-one but not onto.
- both one-one and onto.
- onto but not one-one.

Ans. (1)

Sol. $f(g(x)) = |g(x) - 1|$

$$f \circ g = \begin{cases} |e^x - 1| & x \geq 0 \\ |x+1-1| & x \leq 0 \end{cases}$$

$$f \circ g = \begin{cases} e^x - 1 & x \geq 0 \\ -x & x \leq 0 \end{cases}$$



12. Let the circle $C_1 : x^2 + y^2 - 2(x + y) + 1 = 0$ and C_2 be a circle having centre at $(-1, 0)$ and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :
- 2
 - 1
 - 6
 - 4

Ans. (1)

Sol. $S_1 : x^2 + y^2 - 2x - 2y + 1 = 0$

$$S_2 : x^2 + y^2 + 2x - 3 = 0$$

$$\text{Common chord} = S_1 - S_2 = 0$$

$$-4x - 2y + 4 = 0$$

$$2x + y = 2 \Rightarrow P(0, 2)$$

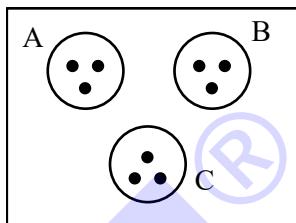
$$d_{(c,p)}^2 = (1-0)^2 + (2-1)^2 = 2$$

13. Let the set $S = \{2, 4, 8, 16, \dots, 512\}$ be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \emptyset$. The maximum number of such possible partitions of S is equal to :

- 1680
- 1520
- 1710
- 1640

Ans. (1)

Sol.



$$\frac{9!}{(3!3!3!)^3} \times 3!$$

14. The values of m, n, for which the system of equations

$$\begin{aligned} x + y + z &= 4, \\ 2x + 5y + 5z &= 17, \\ x + 2y + mz &= n \end{aligned}$$

has infinitely many solutions, satisfy the equation :

- $m^2 + n^2 - m - n = 46$
- $m^2 + n^2 + m + n = 64$
- $m^2 + n^2 + mn = 68$
- $m^2 + n^2 - mn = 39$

Ans. (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 5 \\ 1 & 2 & m \end{vmatrix} = 0 \Rightarrow m = 2$

$$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & 17 \\ 1 & 2 & n \end{vmatrix} = 0 \Rightarrow n = 7$$

15. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set $\{1, 2, 3, 4, 5, 6\}$. If the probability of this equation having one real root bigger than the other is p, then $216p$ equals :

- 57
- 38
- 19
- 76

Ans. (2)



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19. If $y(\theta) = \frac{2\cos\theta + \cos 2\theta}{\cos 3\theta + 4\cos 2\theta + 5\cos\theta + 2}$,

then at $\theta = \frac{\pi}{2}$, $y'' + y' + y$ is equal to:

(1) $\frac{3}{2}$

(2) 1

(3) $\frac{1}{2}$

(4) 2

Ans. (4)

$$\text{Sol. } y = \frac{2\cos\theta + 2\cos^2\theta - 1}{4\cos^3\theta - 3\cos\theta + 8\cos^2\theta - 4 + 5\cos\theta + 2}$$

$$y = \frac{(2\cos^2\theta + 2\cos\theta - 1)}{(2\cos^2\theta + 2\cos\theta - 1)(2\cos\theta + 2)}$$

$$y = \frac{1}{2} \left(\frac{1}{1 + \cos\theta} \right)$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = \frac{1}{2}$$

$$y' = \frac{1}{2} \left(\frac{-1}{(1 + \cos\theta)^2} \times (-\sin\theta) \right)$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = \frac{1}{2}$$

$$y'' = \frac{1}{2} \left[\frac{\cos\theta(1 + \cos\theta)^2 - \sin\theta(2)(1 + \cos\theta)(-\sin\theta)}{(1 + \cos\theta)^4} \right]$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = 1$$

20. For $x \geq 0$, the least value of K, for which $4^{1+x} + 4^{1-x}$, $\frac{K}{2}$, $16^x + 16^{-x}$ are three consecutive terms of an A.P. is equal to :

(1) 10

(2) 4

(3) 8

(4) 16

Ans. (1)

$$\text{Sol. } k = 4 \left(4^x + \frac{1}{4^x} \right) + \left(4^{2x} + \frac{1}{4^{2x}} \right) \geq 2 \geq 2$$

$$k \geq 10$$

- SECTION-B**
21. Let the mean and the standard deviation of the probability distribution

X	α	1	0	-3
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ , respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to ____.

Ans. (5)

$$\text{Sol. } \frac{1}{3} + k + \frac{1}{6} + \frac{1}{4} = 1 \Rightarrow k = \frac{1}{4}$$

$$\mu = \frac{\alpha}{3} + \frac{1}{4} - \frac{3}{4}$$

$$\mu = \frac{\alpha - 1}{3} - \frac{1}{2}$$

$$\sigma = \sqrt{\left(\alpha^2 \frac{1}{3} + \frac{1}{4} + 9 \frac{1}{4} \right) - \left(\frac{\alpha - 1}{3} - \frac{1}{2} \right)^2}$$

$$\sigma = \sqrt{\frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4}}$$

$$\sigma = \mu + 2$$

$$\sigma^2 = (\mu + 2)^2 \Rightarrow \frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4} = \frac{\alpha^2}{9} + \frac{9}{4} + \alpha$$

$$\frac{\alpha^2}{9} - \frac{2\alpha}{3} = 0$$

$$\alpha = 0, (\text{reject}) \text{ or } \alpha = 6$$

($\because x = 0$ is already given)

$$\Rightarrow \sigma + \mu = 2\mu + 2$$

$$= 5$$

22. Let $y = y(x)$ be the solution of the differential

$$\text{equation } \frac{dy}{dx} + \frac{2x}{(1+x^2)^2} y = xe^{\frac{1}{(1+x^2)}}; y(0) = 0.$$

Then the area enclosed by the curve

$$f(x) = y(x)e^{-\frac{1}{(1+x^2)}} \text{ and the line } y - x = 4 \text{ is } ____.$$

Ans. (18)



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Sol. $IF = e^{\int \frac{2x}{(1+x^2)^2} dx} = e^{\frac{-1}{1+x^2}}$

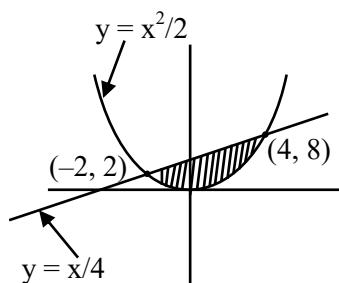
$$y \cdot e^{\frac{-1}{1+x^2}} = \int x \cdot e^{\frac{-1}{1+x^2}} \cdot e^{\frac{-1}{1+x^2} dx}$$

$$y \cdot e^{\frac{-1}{1+x^2}} = \frac{x^2}{2} + C$$

$$(0, 0) \Rightarrow C = 0$$

$$y(x) = \frac{x^2}{2} e^{\frac{-1}{1+x^2}}$$

$$f(x) = \frac{x^2}{2}$$



$$A = \int_{-2}^4 (x+4) - \frac{x^2}{2} dx = 18$$

- 23.** The number of solutions of $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x-1)^2 = 0$, where $-\pi \leq x \leq \pi$, is

Ans. (2)

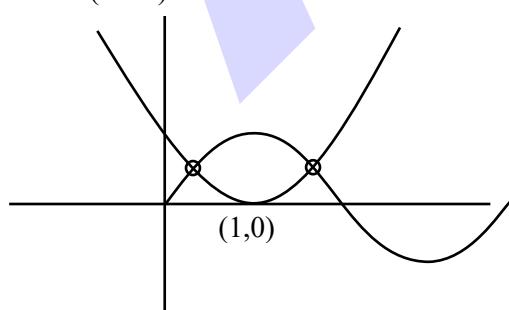
Sol. $\sin^2 x - (x^2 - 2x - 2)\sin x - 3(x-1)^2 = 0$
 $\sin^2 x - (x-1)^2 \sin x - 3(x-1)^2 = 0$

roots :

$$-3 \quad (x-1)^2$$

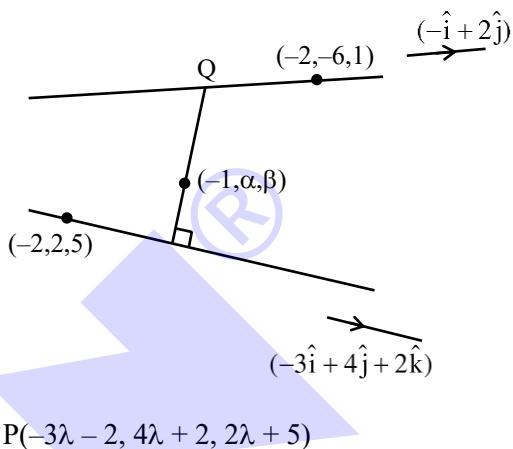
$$\sin x = -3 \text{ (reject)} \text{ or } (x-1)^2$$

$$\sin x = (x-1)^2$$



- 24.** Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$. Then $(\alpha - \beta)^2$ is equal to _____.
Ans. (25)

Sol.



$$DRS \text{ of } PQ = (3\lambda - \mu, 2\mu - 4\lambda - 8, -2\lambda - 4)$$

$$DRS \text{ of } PQ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$$

$$= (4\hat{i} + 2\hat{j} + 2\hat{k})$$

OR

$$(2, 1, 1)$$

$$\frac{3\lambda - \mu}{2} = \frac{2\mu - 4\lambda - 8}{1} = \frac{-2\lambda - 4}{1}$$

$$\Rightarrow \mu = \lambda + 2 \text{ & } 7\lambda = \mu - 8$$

$$\lambda = -1 \quad \mu = 1$$

$$Q : (-3, -4, 1)$$

$$L_{PQ} = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$$

$$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha+4}{1} = \frac{\beta-1}{1}$$

$$\Rightarrow \alpha = -3, \beta = 2$$

$$(\alpha - \beta)^2 = 25$$



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25. If

$$1 + \frac{\sqrt{3} - \sqrt{2}}{2\sqrt{3}} + \frac{5 - 2\sqrt{6}}{18} + \frac{9\sqrt{3} - 11\sqrt{2}}{36\sqrt{3}} + \frac{49 - 20\sqrt{6}}{180} + \dots$$

$$\text{upto } \infty = 2 \left(\sqrt{\frac{b}{a}} + 1 \right) \log_e \left(\frac{a}{b} \right), \text{ where } a \text{ and } b \text{ are}$$

integers with $\gcd(a, b) = 1$, then $11a + 18b$ is equal to _____.

Ans. (76)

Sol. $S = 1 + \frac{x}{2\sqrt{3}} + \frac{x^2}{18} + \frac{x^3}{36\sqrt{3}} + \frac{x^4}{180} + \dots \infty$

Put $\frac{x}{\sqrt{3}} = t$, where $x = \sqrt{3} - \sqrt{2}$

$$S = 1 + \frac{t}{2} + \frac{t^2}{6} + \frac{t^3}{12} + \frac{t^4}{20} + \dots$$

$$S = 1 + t \left(1 - \frac{1}{2} \right) + t^2 \left(\frac{1}{2} - \frac{1}{3} \right) + t^3 \left(\frac{1}{3} - \frac{1}{4} \right) + t^4 \left(\frac{1}{4} - \frac{1}{5} \right)$$

$$S = \left(1 + t + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{4} + \dots \right) - \left(\frac{t}{2} + \frac{t^2}{3} + \frac{t^3}{4} + \frac{t^4}{5} + \dots \right)$$

$$S = \left(t + \frac{t^2}{2} + \dots \right) - \frac{1}{t} \left(t + \frac{t^2}{2} + \frac{t^3}{3} + \dots \right) + 2$$

$$S = 2 + \left(1 - \frac{1}{t} \right) (-\log(1-t)) = \left(\frac{1}{t} - 1 \right) \log(1-t) + 2$$

$$S = 2 + \left(\frac{\sqrt{3}}{\sqrt{3} - \sqrt{2}} - 1 \right) \log \left(1 - \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3}} \right)$$

$$S = 2 + \left(\frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} \right) \log e \frac{\sqrt{2}}{\sqrt{3}}$$

$$S = 2 + \frac{(\sqrt{6} + 2)}{2} \log e \frac{2}{3} = 2 + \left(\sqrt{\frac{3}{2}} + 1 \right) \log e \frac{2}{3}$$

$$a = 2, b = 3$$

$$11a + 18b = 11 \times 2 + 18 \times 3 = 76$$

26. Let $a > 0$ be a root of the equation $2x^2 + x - 2 = 0$.

$$\text{If } \lim_{x \rightarrow \frac{1}{a}} \frac{16(1 - \cos(2 + x - 2x^2))}{(1 - ax^2)} = \alpha + \beta\sqrt{17}, \text{ where}$$

$\alpha, \beta \in \mathbb{Z}$ then $\alpha + \beta$ is equal to _____.

Ans. (170)

Sol. $2x^2 + x - 2 = 0$

$$2x^2 - x - 2 = 0$$

$$\lim_{x \rightarrow \frac{1}{a}} 16 \cdot \frac{\left(1 - \cos 2\left(x - \frac{1}{a}\right)\left(x - \frac{1}{b}\right)\right)}{4\left(x - \frac{1}{b}\right)^2} \times \frac{4\left(x - \frac{1}{b}\right)^2}{a^2\left(x - \frac{1}{a}\right)^2}$$

$$= 16 \times \frac{2}{a^2} \left(\frac{1}{a} - \frac{1}{b} \right)^2$$

$$= \frac{32}{a^2} \left(\frac{17}{4} \right) = \frac{17.8}{a^2} = \frac{17 \times 8 \times 16}{(-1 + \sqrt{117})^2}$$

$$= \frac{136.16}{18.2\sqrt{7}} \times \frac{18 + 2\sqrt{7}}{18 + 2\sqrt{7}}$$

$$= \frac{136}{256} (18 + 2\sqrt{7}) \cdot 16$$

$$= 153 + 17\sqrt{17} = \alpha + \beta\sqrt{17}$$

$$\alpha + \beta = 153 + 17 = 170$$

27. If $f(t) = \int_0^\pi \frac{2x dx}{1 - \cos^2 t \sin^2 x}$, $0 < t < \pi$, then the value

of $\int_0^{\frac{\pi}{2}} \frac{\pi^2 dt}{f(t)}$ equals _____.

Ans. (1)

Sol. $f(t) = \int_0^\pi \frac{2x}{1 - \cos^2 t \sin^2 x} dx \quad \dots(1)$



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$$= 2 \int_0^{\pi} \frac{(\pi - x) dx}{1 - \cos^2 t \sin^2 x} \quad \dots(2)$$

$$2f(t) = 2 \int_0^{\pi} \frac{\pi}{1 - \cos^2 t \sin^2 x} dx$$

$$f(t) = \int_0^{\pi} \frac{\pi}{1 - \cos^2 t \sin^2 x} dx$$

divide & by $\cos^2 x$

$$f(t) = \pi \int_0^{\pi} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \tan^2 x}$$

$$f(t) = 2\pi \int_0^{\pi/2} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \tan^2 x}$$

$\tan x = z$

$$\sec^2 x dx = dz$$

$$f(t) = 2\pi \int_0^{\infty} \frac{dz}{1 + \sin^2 t \cdot z^2}$$

$$= \frac{\pi^2}{\sin t}$$

$$\text{Then } \int_0^{\pi/2} \frac{\pi^2}{f(t)} dt$$

$$= \int_0^{\pi/2} \sin t dt$$

$$= 1$$

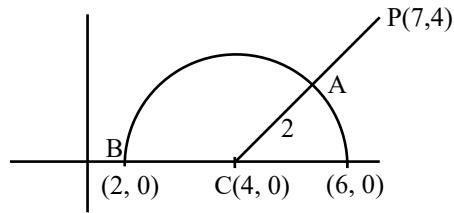
28. Let the maximum and minimum values of $(\sqrt{8x - x^2 - 12} - 4)^2 + (x - 7)^2$, $x \in \mathbb{R}$ be M and m respectively. Then $M^2 - m^2$ is equal to _____.
Ans. (1600)

Sol. $(x - 7)^2 + (y - 4)^2$

$$y = \sqrt{8x - x^2 - 12}$$

$$y^2 = -(x - 4)^2 + 16 - 12$$

$$(x - 4)^2 + y^2 = 4$$



$$m = 9$$

$$M = 41$$

$$M^2 - m^2 = 41^2 - 9^2 = 1600$$

29. Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x - 9)$ at the point P. The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is _____.
Ans. (10)

Sol. $y^2 = 4(x - 9)$

slope of tangent = $\frac{-1}{2}$

Point of contact P $\left(9 + \frac{1}{(-\frac{1}{2})^2}, \frac{2 \times 1}{-\frac{1}{2}}\right)$

P(13, -4)

center of circle C(7, 4)

$$\text{distance CP} = \sqrt{(13 - 7)^2 + (-4 - 4)^2} \\ = 10$$

30. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is _____.
Ans. (3)

30. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is _____.
Allen Ans. (3)

- Sol.** Case I : $x \geq -5$

$$x^2 + 5x + 2x + 12 = 0$$

$$x^2 + 7x + 12 = 0$$

$$x = -3, -4$$

- Case II : $-7 < x < -5$

$$-x^2 - 5x + 2x + 14 - 2 = 0$$



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$$-x^2 - 3x + 12 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 48}}{2}$$

$$= \frac{-3 \pm \sqrt{57}}{2}$$

$$x = \frac{-3 - \sqrt{57}}{2}, \frac{-3 + \sqrt{57}}{2} \text{ (rejected)}$$

Case III : $x \leq -7$

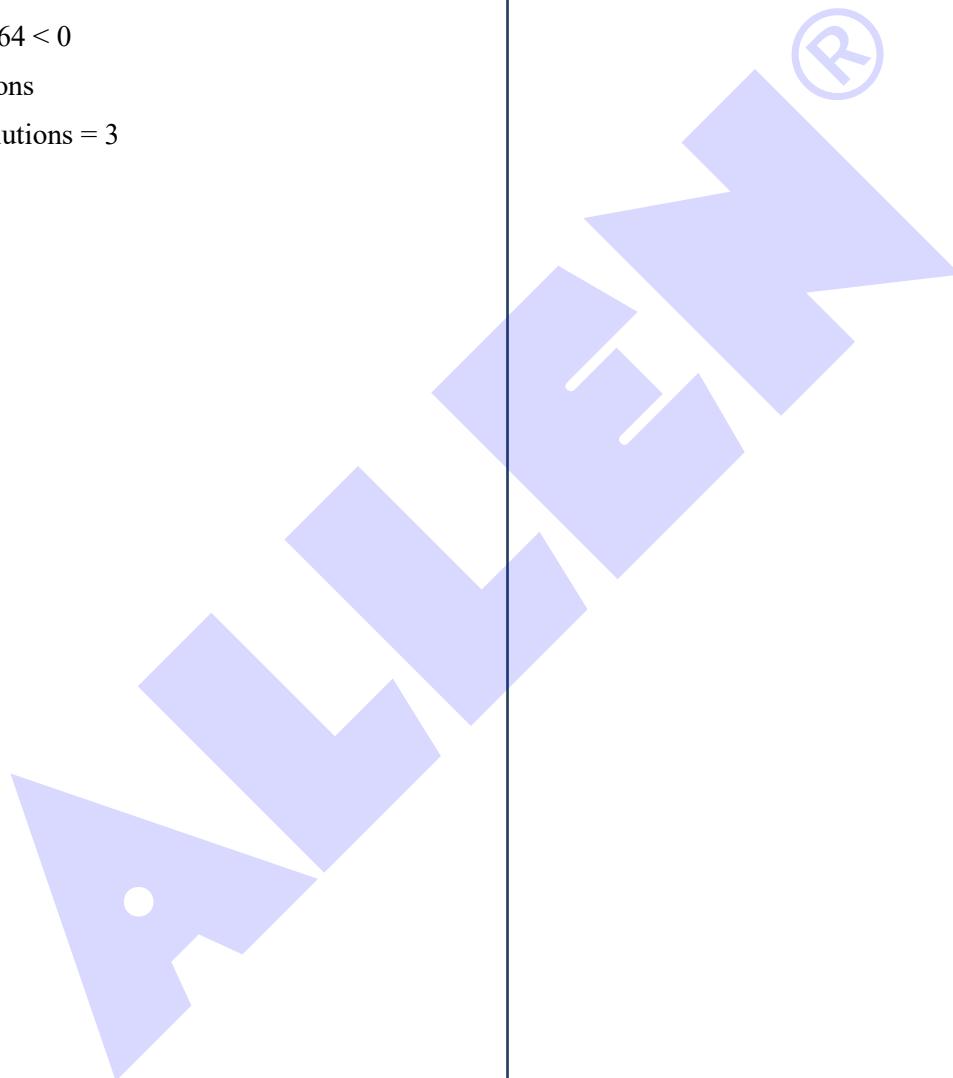
$$-x^2 - 5x - 2x - 14 - 2 = 0$$

$$x^2 + 7x + 16 = 0$$

$$D = 49 - 64 < 0$$

No solutions

No. of solutions = 3



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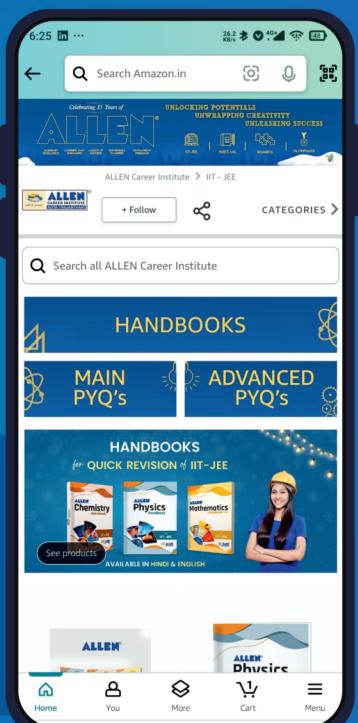
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