

**KEY SHEET****MATHEMATICS**

1)	1	2)	3	3)	2	4)	3	5)	3
6)	1	7)	1	8)	3	9)	2	10)	3
11)	2	12)	1	13)	2	14)	1	15)	2
16)	1	17)	1	18)	2	19)	2	20)	4
21)	3	22)	1	23)	1	24)	2	25)	1

PHYSICS

26)	3	27)	2	28)	4	29)	3	30)	3
31)	4	32)	4	33)	2	34)	1	35)	1
36)	1	37)	3	38)	2	39)	1	40)	2
41)	1	42)	2	43)	4	44)	3	45)	3
46)	20	47)	20	48)	13	49)	200	50)	2

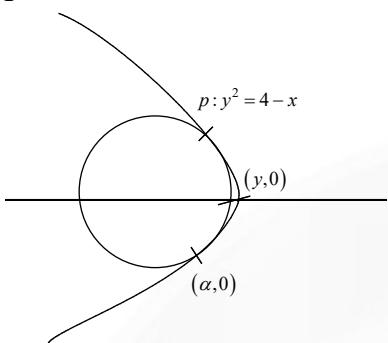
CHEMISTRY

51)	2	52)	2	53)	2	54)	1	55)	2
56)	2	57)	3	58)	4	59)	4	60)	2
61)	3	62)	3	63)	4	64)	3	65)	1
66)	2	67)	3	68)	3	69)	3	70)	2
71)	24	72)	8	73)	1	74)	136	75)	6

SOLUTIONS

MATHEMATICS

1.



$$\text{Circle: } (x - \lambda)^2 + (y - 0)^2 = \lambda^2$$

$$x^2 + y^2 - 2\lambda x = 0$$

$$\text{Intersect: } x^2 + (4 - x) - 2\lambda x = 0$$

$$D = 0$$

$$\lambda = \frac{3}{2}, -\frac{5}{2} \Rightarrow \lambda = \frac{3}{2}$$

$$p: y^2 = 4 - x$$

$$x^2 + y - x - 3x = 0$$

$$c: x^2 + y^2 - 3x = 0$$

$$x = 2$$

$$r = \frac{3}{2}, c \equiv \left(\frac{3}{2}, 0 \right)$$

$$\alpha = 2$$

2.

$$AM \geq GM$$

$$\frac{16^{x^2+y} + 16^{y^2+x}}{2} \geq \left(16^{x^2+y} 16^{y^2+x} \right)^{\frac{1}{2}}$$

$$\frac{4x^2 + 4x + 4y^2 + 4y}{2}$$

$$\frac{1}{2} \geq 2$$

$$1 \geq 2^{-2}$$

$$RHS \geq 1$$

$$\therefore 2x + 1 = 0, 2y + 1 = 0$$

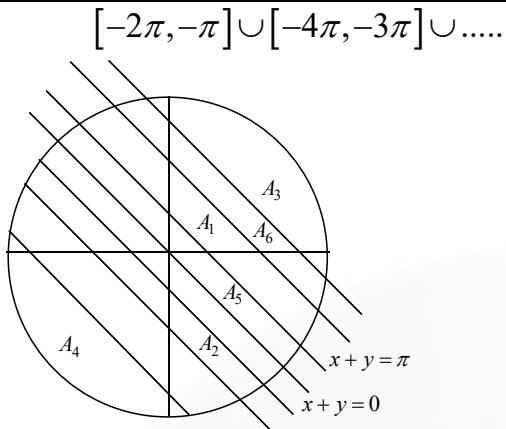
$$x = -\frac{1}{2}, y = -\frac{1}{2}$$

$$\therefore 32(x^4 + y^4) = 2^{32} \left(\frac{1}{16} + \frac{1}{16} \right) = 4$$

3.

$$\sin(x + y) \geq 0$$

$$\Rightarrow x + y \in [0, \pi] \cup [2\pi, 3\pi] \cup [4\pi, 5\pi] \cup$$



By observation

$$A_3 = A_4, A_1 = A_2$$

$$\begin{aligned}\text{Required area} &= A_2 + A_4 + A_5 + A_6 \\ &= A_5 + A_1 + A_6 + A_3 \\ &= \frac{1}{2}(\text{Area of circle}) \\ &= 50\pi\end{aligned}$$

4. 3

$$f(n) = \frac{4n + \sqrt{(2n+1)(2n-1)}}{\sqrt{2n+1} + \sqrt{2n-1}}$$

Rationalize

$$f(n) = \frac{1}{2} \left[(2n+1)^{3/2} - (2n-1)^{3/2} \right]$$

$$\sum f(n) = \frac{1}{2} \left[\frac{3}{81^2 - 1} \right] = 364$$

5. 3

by observation

$$\sqrt{2 + \sqrt{2}} < \sqrt{3 + \sqrt{3}} < \sqrt{4 + \sqrt{4}} < \dots$$

and

$$\begin{aligned}\sqrt{3 + \sqrt{3}} &< \sqrt{3 + \sqrt{3 + \sqrt{3}}}, & < \sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3}}}} &< \dots \\ &< \dots < \sqrt{3 + \sqrt{3 + \dots \infty}}\end{aligned}$$

\therefore solve

$$\sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}} = 8$$

$$\sqrt{x+8} = 8 \Rightarrow x = 56$$

$x = 56$ will be included as this converges to 8 & will be less than 8.

$$x = 0, 1, 2, \dots, 56$$

$$\text{No. of integers} = 57$$

6. 1



$$x_{n+1} = \frac{x_n + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}x_n}, \quad x_1 = 1$$

put $x_n = \tan \theta_n$

$$x_{n+1} = \tan\left(\theta_x + \frac{\pi}{6}\right)$$

$\theta_1 = 45$	$x_1 = 1$
$\theta_2 = 75$	$x_2 = 2 + \sqrt{3}$
$\theta_3 = 105$	$x_3 = -2 - \sqrt{3}$
$\theta_4 = 135$	$x_4 = -1$
$\theta_5 = 165$	$x_5 = -2 + \sqrt{3}$
$\theta_6 = 195$	$x_6 = 2 - \sqrt{3}$
$\theta_7 = 225 = 180 + 45$	$x_7 = x_1$
$\theta_8 = 225 = 180 + 75$	$x_8 = x_2$
⋮	⋮
⋮	⋮

$$\therefore x_1 + x_2 + \dots + x_6 = x_7 + \dots + x_{12} =$$

$$x_{13} + \dots + x_{18} = x_{19} + \dots + x_{24} = 0$$

$$\sum_{R=1}^{2008} x_R = 0 + 0 + 0 + \dots + 0 + (x_1 + x_2 + x_3 + x_4) \\ = 0$$

7.

1

$$i^k + i^{k+1} + i^{k+2} + i^{k+3} = 0$$

Sum of 4 consecutive powers
of $i = 0$

8.

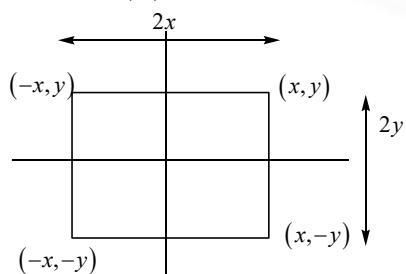
3

Let $z = x + iy$

$$\bar{z} = x - iy$$

$$\bar{z} - 2\operatorname{Re}(\bar{z}) = -x - iy$$

$$z - 2\operatorname{Re}(z) = -x + iy$$





9. 2
 $2x = 2y = 4 \text{ (it is a square)}$

$$\begin{aligned} z &= -2e^{i\left(-\frac{\pi}{3}\right)} \\ &= 2(-1)e^{i\left(-\frac{\pi}{3}\right)} \\ &= 2e^{i\pi}e^{i\left(-\frac{\pi}{3}\right)} \\ &= 2e^{i\frac{2\pi}{3}} \end{aligned}$$

$$|z| = 2, \quad \arg(z) = \frac{2\pi}{3}$$

10. 3
 $z = x + iy$

$$z^2 = (x^2 - y^2) + i(2xy)$$

$$|z| = \sqrt{x^2 + y^2}$$

ATQ

$$x^2 - y^2 + \sqrt{x^2 + y^2} = 0$$

$$2xy = 0 \quad \dots \dots \dots (1)$$

using equation (1)

case I $\rightarrow x = 0 \Rightarrow y = \pm 1, 0$

case II $\rightarrow y = 0 \Rightarrow x = 0$

$$\therefore (0,0)(0,1)(0,-1)$$

$$z = 0, i, -1$$

Number of solution = 3

11. 2
 $z_1 = x_1 + iy_1$
 $z_2 = x_2 + iy_2$

ATQ

$$x_1 = |(x_1 - 1) + iy_1|$$

$$y_1^2 = 2x_1 - 1 \quad (1) \quad \& \quad y_2^2 = 2x_2 - 1 \quad (2)$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$\operatorname{Arg}(z_1 - z_2) = \frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$$

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{\pi}{\sqrt{3}}$$

Equation (1) – (2)

$$y_1^2 - y_2^2 = 2(x_1 - x_2)$$

$$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right)$$

$$= 2\sqrt{3}$$

12. 1

$$z_1 = r_1 e^{i\theta_1} \quad z_3 = r_2 e^{i\theta_2}$$

$$z_2 = r_1 e^{-i\theta_1} \quad z_4 = r_2 e^{-i\theta_2}$$

$$\frac{z_1}{z_4} = \frac{r_1}{r_2} e^{i(\theta_1 + \theta_2)}$$

$$\frac{z_2}{z_3} = \frac{r_1}{r_2} e^{-i(\theta_1 + \theta_2)}$$

AT θ

$$(\theta_1 + \theta_2) + -(\theta_1 + \theta_2) \\ = 0$$

13. 2

$$z = (e^{-i3\theta})^5 \frac{\left(\frac{i\sin 5\theta - \cos 5\theta}{i}\right)^4}{(e^{-i4\theta})^6}$$

$$= e^{-i15\theta} \frac{\frac{(-1)^4}{(i)^4} (\cos 5\theta - i\sin 5\theta)^4}{e^{-i24\theta}}$$

$$z = e^{-i11\theta}$$

$$z = \cos 11\theta - i\sin 11\theta$$

$$= \cos \frac{\pi}{6} + i\sin \frac{\pi}{6}$$

$$= e^{\frac{\pi}{6}}$$

14. 1

$$z = \frac{(1+i)^2}{a-i}$$

Squaring

$$|z| = \left| \frac{(1+i)^2}{a-i} \right|$$

$$\frac{2}{5} = \frac{4}{a^2 + 1} \Rightarrow a^2 = 9 \Rightarrow a = 3$$

$$z = \frac{(1+i)^2}{3-i}$$

$$z = -\frac{1}{5} + \frac{3i}{5}$$

$$\bar{z} = -\frac{1}{5} - \frac{3i}{5}$$

15. 2

$$\text{Let } \sqrt{a+ib} = x+iy$$

square

$$a+ib = (x^2 - y^2) + i(2xy)$$

$$a = x^2 - y^2 \quad \& \quad b = 2xy$$

Compare & solve

16. 3

Using inequality let $|z| = r$

$$\left| |z| - \frac{2}{|z|} \right| \leq \left| z - \frac{2}{z} \right| \leq |z| + \left| \frac{2}{z} \right|$$

$$\left| r - \frac{2}{r} \right| \quad (I) \quad \leq 1 \leq r + \frac{2}{r} \quad (II)$$

Case. I

$$\left| r - \frac{2}{r} \right| \leq 1$$

$$-1 \leq r - \frac{2}{r} \leq 1$$

$$-1 \leq r - \frac{2}{r} \Rightarrow r \in [1, \infty)$$

$$r - \frac{2}{r} \leq 1 \Rightarrow r \in (0, 2]$$

Combining

$$r \in [1, 2]$$

Case II

$$1 \leq r + \frac{2}{r}$$

$$\Rightarrow r^2 - r + 2 \geq 0$$

its discriminant is negative.

\therefore always true

Case I \cap Case II

$$[1, 2] \cup R$$

$$\Rightarrow [1, 2]$$

$$a = 1, \quad b = 2$$

$$a + b = 3$$

17. 1

$$C_3 \rightarrow C_3 - (C_1 + C_2)$$



$$\begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1) & 0 \end{vmatrix} = 0$$

$$\Rightarrow f(100) = 0$$

18. 2

Let $C = AB - BA$

$$\begin{aligned} C' &= (AB - BA)' \\ &= (AB)' - (BA)' \\ &= BA - AB \\ &\therefore \text{skew-symmetric} \end{aligned}$$

19. 2

Given expression

$$\begin{aligned} &2(a^2 + b^2 + c^2) - 2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) \\ &= 6 - 2 \left[\frac{(\bar{a} + \bar{b} + \bar{c})^2 - (a^2 + b^2 + c^2)}{2} \right] \\ &= 6 - \left[(\bar{a} + \bar{b} + \bar{c})^2 - 3 \right] \\ &= 9 - (\bar{a} + \bar{b} + \bar{c})^2 \end{aligned}$$

Max is 9

Where $\bar{a} + \bar{b} + \bar{c} = 0$

20. 4

$$\text{Volume} = \frac{1}{6} [\bar{a} \bar{b} \bar{c}] = \frac{3}{4}$$

$$[\bar{a} \bar{b} \bar{c}] = \frac{9}{2}$$

For new tetrahedron

$$\begin{aligned} \text{Volume} &= \frac{1}{6} [3(\bar{a} \times \bar{b}) \cdot 4(\bar{b} \times \bar{c}) \cdot 5(\bar{c} \times \bar{a})] \\ &= \frac{1}{6} \times 3 \times 4 \times 5 [\bar{a} \times \bar{b} \cdot \bar{b} \times \bar{c} \cdot \bar{c} \times \bar{a}] \\ &= 10 [\bar{a} \bar{b} \bar{c}]^2 \\ &= 10 \times \frac{81}{4} = \frac{810}{4} = 202.5 \text{ cubic units} \end{aligned}$$

Statement II is incorrect

21. 3

$$xdy = y(dx + ydy)$$

$$\frac{xdy - ydx}{y^2} = dy$$



$$-d\left(\frac{x}{y}\right) = dy$$

Integration both sides

$$-\frac{x}{y} + c = y$$

$$\frac{x}{y} + y = c$$

$$x + y^2 = cy$$

Put $x = 1, y = 1$

$$\Rightarrow c = 2$$

Curve is $\frac{x}{y} + y = 2$

Put $x = -3, y = -1 \text{ or } 3$

then, $y > 0, y = 3$

22.

1

Apply King's
Rule & Add

$$f(t) = \int_0^\pi \frac{\pi dx}{1 - \cos^2 x + \sin^2 x}$$

$$f(t) = \int_0^\pi \frac{\pi \sec^2 x - dx}{\sec^2 x - \cos^2 t \cdot \tan^2 x}$$

$$f(t) = 2\pi \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \cdot \tan^2 x}$$

Put $\tan x = m$

$$\sec^2 x dx = dm$$

$$f(t) = 2\pi \int_0^\infty \frac{dm}{1 + m^2 - m^2(1 - \sin^2 t)}$$

$$= 2\pi \int_0^\infty \frac{dm}{1 + (m \sin t)^2}$$

$$= 2\pi \times \frac{\pi}{2} \times \frac{1}{\sin t} - 0$$

$$f(t) = \frac{\pi^2}{\sin t}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\pi^2 dt}{f(t)} = \int_0^{\frac{\pi}{2}} \sin t dt = 1$$

23. 1



$$\frac{3}{2 + \cos \theta + i \sin \theta} = a + ib$$

Rationalize and comparing

$$a = \frac{6 + 3 \sin \theta}{5 + 4 \cos \theta}, \quad b = \frac{-3 \sin \theta}{5 + 4 \cos \theta}$$

$$(a - 2)^2 + b^2 = 1 \text{ after simplification}$$

24. 2

$$w = \frac{5 + 3z}{5 - 5z}$$

$$\Rightarrow z = \frac{5(w-1)}{3+5w}$$

$$|z| < 1$$

$$5|w-1| < |3+5w|$$

$$25(w\bar{w} - w - \bar{w} + 1) < 9 + 25w\bar{w} + 15w + 15\bar{w}$$

$$\Rightarrow 16 < 40w + 40\bar{w}$$

$$\Rightarrow w + \bar{w} > \frac{16}{40}$$

$$\Rightarrow 2 \operatorname{Re}(w) > \frac{16}{40}$$

$$\Rightarrow 5 \operatorname{Re}(w) > 1$$

Min integral value is 2.

25. 1

$$z = r_1 e^{i\theta} \quad w = r_2 e^{i\phi}$$

$$|zw| = 1$$

$$\left| r_1 r_2 e^{i(\theta+\phi)} \right| = 1$$

$$\Rightarrow r_1 r_2 = 1$$

$$\arg(z) - \arg(w) = \frac{\pi}{2}$$

$$\theta - \phi = \frac{\pi}{2}$$

$$\therefore \bar{z} w = r_1 r_2 e^{-i(\theta-\phi)}$$

$$= 1 \cdot e^{-i \frac{\pi}{2}}$$

$$= -i$$

$$|\bar{z}w| = 1$$



PHYSICS

26. Initial and final states for both the process are same.

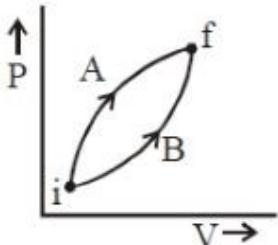
$$\therefore \Delta U_A = \Delta U_B$$

Work done during process A is greater than in process B.

By first law of thermodynamics

$$\Delta Q = \Delta U + W$$

$$\Rightarrow \Delta Q_A > \Delta Q_B$$



27. The peak current in a series RC circuit is given by

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

Given that $E_{rms} = 110 \text{ V}$, so peak voltage is given by

$$E_0 = \sqrt{2} E_{rms} = 1.414 \times 110 = 155.5 \text{ V}$$

$$\Rightarrow I_0 = \frac{155.5}{\sqrt{(40)^2 + \left(\frac{1}{376.8 \times 10^{-4}}\right)^2}}$$

$$\Rightarrow I_0 = 3.24 \text{ A}$$

28. In time t heat produced by a DC current i_{dc} is given by

$$H_1 = i_{dc}^2 R t$$

$$\Rightarrow H_1 = (2)^2 (2)t = 8t$$

RMS value of AC is given by

$$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ A}$$

Heat produced by the rms current in time t is given by

$$H_2 = i_{rms}^2 R t$$

$$\Rightarrow H_2 = (\sqrt{2})^2 (1)t = 2t$$

$$\Rightarrow \frac{H_1}{H_2} = \frac{8t}{2t} = 4$$

29. Given, phase difference, $\phi = \frac{\pi}{4}$

As we know, for R-L or R-C circuit,



$$\tan \phi = \frac{\text{Capacitive reactance}(X_C) \text{ or inductive reactance}(X_L)}{\text{Resistance}(R)}$$

$$\Rightarrow \tan \frac{\pi}{4} = \frac{X_C \text{ or } X_L}{R}$$

$$I = \frac{X_C \text{ or } X_L}{R} \Rightarrow R = X_C \text{ or } X_L$$

Also, given $e = e_0 \sin(100t)$

Comparing the above equation with general equation of emf, i.e. $e = e_0 \sin \omega t$, we get

$$\omega = 100 \text{ rad/s} = 10^2 \text{ rad/s}$$

Now, checking option wise,

For R-C circuit, with

$$R = 1k\Omega = 10^3 \Omega \text{ and } C = 1\mu F = 10^{-6} F$$

$$\text{So, } X_C = \frac{1}{\omega C} = \frac{1}{10^2 \times 10^{-6}} = 10^4 \Omega \Rightarrow R \neq X_C$$

- 30. Statement – I is true, statement – II is false
- 31. Statement – I is false, statement – II is true
- 32. In the series L-C-R circuit V_R

(voltage across resistance) is in the phase with current (i), while V_C (voltage across inductance) leads

i by 90° , while V_C lags behind

i by 90°

Resultant emf E is

$$E^2 = V_R^2 + (V_L - V_C)^2$$

$$\text{Since, } V_L = V_C = 300V$$

$$\therefore E = V_R = 220V$$

Reading of ammeter,

$$i = \frac{E}{R} = \frac{220}{100} = 2.2A$$

- 33. Statement – II is true, statement – I is true; statement – II is not correct explanation for statement – I
- 34. The equation of a semicircular wave is

$$\text{or } x^2 + y^2 = I_0^2$$

$$y^2 = I_0^2 - x^2$$

$$\therefore i_{rms}^2 = \frac{1}{2I_0} \int_{-I_0}^{I_0} y^2 dx$$

$$\begin{aligned}
 &= \frac{1}{2I_0} \int_{-I_0}^{I_0} \left(I_0^2 - x^2 \right) dx \\
 &= \frac{1}{2I_0} \left| I_0^2 x - \frac{x^3}{3} \right|_{-I_0}^{I_0} \\
 &= \frac{1}{2I_0} \left[\left(I_0^3 - \frac{I_0^3}{3} \right) - \left(-I_0^3 + \frac{I_0^3}{3} \right) \right] \\
 &= \frac{2I_0^2}{3}
 \end{aligned}$$

or $I_{rms} = \sqrt{\frac{2I_0^2}{3}} = \sqrt{\frac{2}{3}} I_0$

35. As $M \propto N_1 N_2$, therefore, M remains the same.

36. Capacitance of wire

$$C = 0.014 \times 10^4 \times 200$$

$$= 2.8 \times 10^6 F = 2.84 \mu F$$

For impedance of the circuit to be minimum

$$X_L = X_C$$

$$\Rightarrow 2\pi v L = \frac{1}{2\pi v C}$$

$$L = \frac{1}{4\pi^2 v^2 C} = \frac{1}{4(3.14)^2 \times (5 \times 10^3)^2 \times 2.8 \times 10^{-6}} = 0.35 \times 10^{-3} H = 0.35 mH$$

37.

$$\vec{p} \propto S^a I^b h^c$$

$$\left[M^1 L^1 T^{-1} \right] = k \left[M^1 L^0 T^{-2} \right]^a \left[M^1 L^2 T^0 \right]^b \left[M^1 L^2 T^{-1} \right]^c$$

$$1 = a + b + c \quad \text{(1)}$$

$$1 = 2b + 2c \quad \text{(2)}$$

$$-1 = -2a - 2c \quad \text{(3)}$$

solving 1,2 & 3

$$a = \frac{1}{2}, b = \frac{5}{2}, c = -2$$

$$\therefore \vec{p} = \left[S^{1/2} I^{5/2} h^{-2} \right]$$



38. Given, initial position of particle

$$\mathbf{r}_0 = (2\hat{i} + 4\hat{j})m,$$

Initial velocity of particle, $\mathbf{u} = (5\hat{i} + 4\hat{j})m/s$

Acceleration of particle, $\mathbf{a} = (4\hat{i} + 5\hat{j})m/s^2$

According to second equation of motion,

Position of particle at time t is, $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

At $t = 2s$, position of particle is,

$$\mathbf{r} = (2\hat{i} + 4\hat{j}) + (5\hat{i} + 4\hat{j}) \times 2 + \frac{1}{2}(4\hat{i} + 5\hat{j}) \times 4$$

$$\text{or } \mathbf{r} = (2+10+8)\hat{i} + (4+8+8)\hat{j}$$

$$\Rightarrow \mathbf{r} = 20\hat{i} + 20\hat{j}$$

\therefore Distance of particle from origin is,

$$|\mathbf{r}| = 20\sqrt{2}m$$

39. Substituting, $m_1 = 5kg$ and $m_2 = 10kg$

$$\text{We get, } \mu(10+m)g \geq 5g \Rightarrow 10+m \geq \frac{5}{0.15}$$

$$m \geq 23.33kg$$

40. Given, Force, $F = -kv^2$

$$\therefore \text{Acceleration, } a = -\frac{k}{m}v^2 \quad \text{or} \quad \frac{dv}{dt} = -\frac{k}{m}v^2 \Rightarrow \frac{dv}{v^2} = -\frac{k}{m}dt$$

Now, with limits, we have

$$\int_{10}^v \frac{dv}{v^2} = \frac{k}{m} \int_0^1 dt$$

$$\Rightarrow \left(-\frac{1}{v}\right)_{10}^v = -\frac{k}{m}t \Rightarrow \frac{1}{v} = 0.1 + \frac{kt}{m} \Rightarrow v = \frac{1}{0.1 + \frac{kt}{m}} = \frac{1}{0.1 + 1000k}$$

$$\Rightarrow \frac{1}{2} \times m \times v^2 = \frac{1}{8} \times v_0^2 \Rightarrow v = \frac{v_0}{2} = 5 \Rightarrow \frac{1}{0.1 + 1000k} = 5$$

$$\Rightarrow 1 = 0.5 + 5000k \Rightarrow K = \frac{0.5}{5000} \Rightarrow k = 10^{-4} kg/m$$

41. Let density of cone = ρ

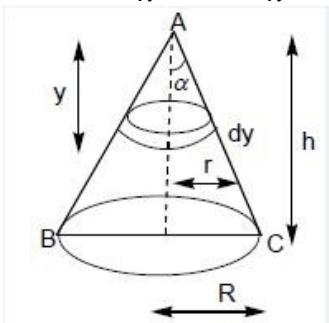
$$\text{Center of mass, } y_{cm} = \frac{\int ydm}{\int dm} = \frac{\int_0^h y\pi r^2 dy \rho}{\frac{1}{3}\pi R^2 h \rho} = \frac{\int_0^h r^2 y dy}{\frac{1}{3}R^2 h} \quad \dots(i)$$

For a cone, we know that

$$\frac{r}{R} = \frac{y}{h} \therefore r = \frac{y}{h}R$$



$$y_{cm} = \frac{\int_0^h 3y^3 dy}{h^3} = \frac{3 \frac{y^4}{4}}{h^3} = \frac{3}{4} h$$



42. Maximum possible volume of cube will occur when

$$\sqrt{3}a = 2R \quad (a = \text{side of cube})$$

$$\therefore a = \frac{2}{\sqrt{3}} R$$

Now, density of sphere, $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

$$\text{Mass of cube, } m = (\text{volume of cube})(\rho) = (a^3)(\rho) = \left[\frac{2}{\sqrt{3}} R \right]^3 \left[\frac{m}{\frac{4}{3}\pi R^3} \right] = \left(\frac{2}{\sqrt{3}\pi} \right) M$$

Now, moment of inertia of the cube about the said axis is

$$I = \frac{ma^2}{6} = \frac{\left(\frac{2}{\sqrt{3}\pi} \right) M \left(\frac{2}{\sqrt{3}} R \right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$$

43. $F \propto \frac{1}{R^n}$

$$\frac{mv^2}{R} \propto \frac{1}{R^n} \quad \text{or} \quad v \propto \frac{I}{R^{\frac{n-1}{2}}}$$

$$T = \frac{2\pi R}{v} \Rightarrow T \propto \frac{R}{v} \Rightarrow T \propto R^{\frac{n-1}{2}} \quad \text{or} \quad T \propto R^{\frac{n+1}{2}}$$

44. Kinetic energy is minimum at $\frac{T}{4}$

45. Thermal stress $\sigma = Y\alpha \Delta\theta$

Given, $\sigma_1 = \sigma_2$

$$\therefore Y_1 \alpha_1 \Delta\theta = Y_2 \alpha_2 \Delta\theta \quad \text{or} \quad \frac{Y_1}{Y_2} = \frac{\alpha_1}{\alpha_2} = \frac{3}{2}$$

46. Comparing $E = 200\sqrt{2} \sin(100t)$ with



$$E = E_0 \sin wt; E_0 = 200\sqrt{2}V \text{ and } w = 100(\text{rad/s})$$

$$X_C = \frac{1}{wC} = \frac{1}{100 \times 10^{-6}} = 10^4 \Omega$$

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_0}{\sqrt{2}X_c} = \frac{200\sqrt{2}}{\sqrt{2} \times 10^4} = 20mA$$

47. Wheatstone bridge is balanced so that $R = 4\Omega$.

$$e = iR = Blv$$

$$v = \frac{iR}{Bl} = 2 \times 10^{-2} ms^{-1}$$

48. For a series LR circuit, impedance is given by

$$Z = \sqrt{R^2 + X_L^2}$$

Inductive reactance X_L is given by

$$X_L = L\omega = 2\pi fL \Rightarrow X_L = 2\left(\frac{22}{7}\right)(50)\left(\frac{175}{11} \times 10^{-3}\right) \Omega \Rightarrow X_L = 5000 \times 10^{-3} \Omega = 5\Omega$$

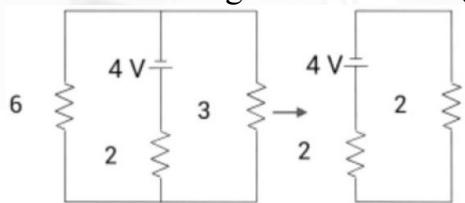
$$\Rightarrow Z = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13\Omega$$

$$49. E_{eq} = 6 + \frac{12 \times 4}{12 + 4} = 9\Omega \therefore i_{main} = \frac{72}{9} = 8amp$$

$$\therefore i_{10\Omega} = \frac{4}{12 + 4} \times 8 = 2amp$$

$$\therefore Q = CV = 10 \times 10^{-6} \times (10 \times 2) = 200\mu C$$

50. Current through the connector(i)



$$\frac{E}{R_p + r} = \frac{4}{2 + 2} = 1A$$

Magnetic force on the connector

$$= Bil = (1)(1) = 2N$$

Therefore, to keep the connector moving with a constant velocity, a force of 2N has to be applied to the right side.

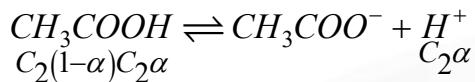


CHEMISTRY



$$C_1(1-\alpha)C_1\alpha \quad C_1\alpha$$

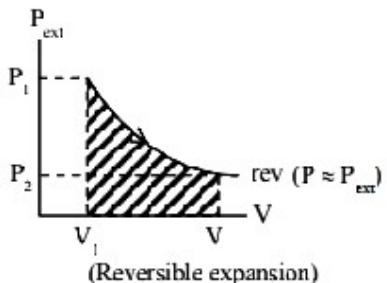
$$\left[H^+ \right]_L = C_1\alpha = \sqrt{(Ka)_1 \times C_1}$$



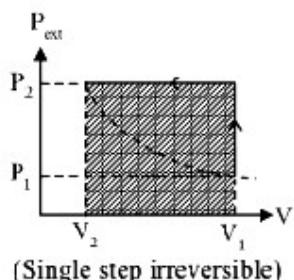
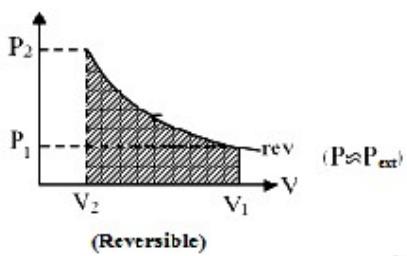
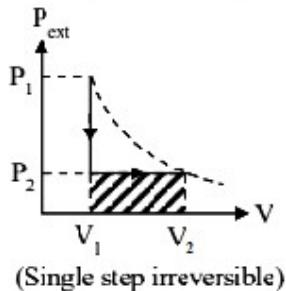
$$\therefore \left[H^+ \right]_R = \sqrt{(Ka)_2 \times C_2}$$

$$E = E^0 - \frac{0.0591}{1} \log \frac{\left[H^+ \right]_L}{\left[H^+ \right]_R}$$

52. Conceptual



53.

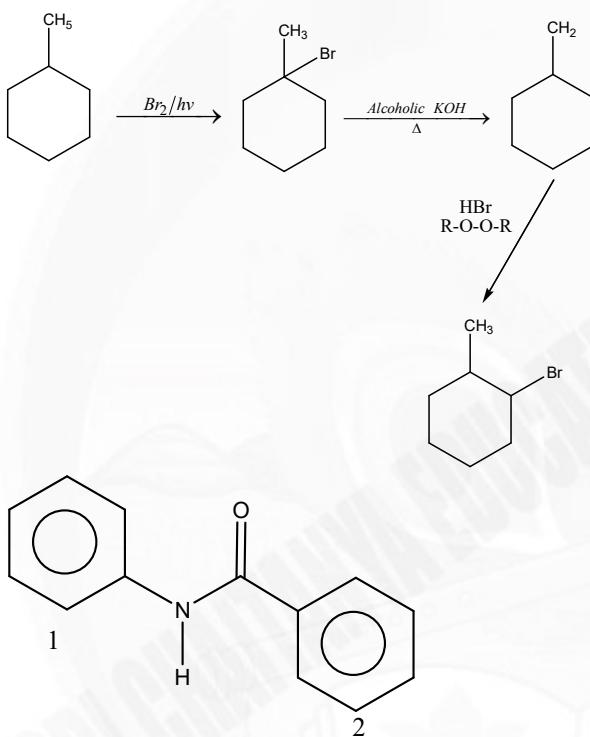
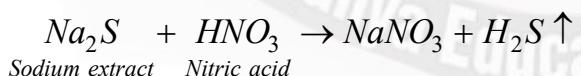
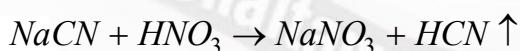




54.

	B	Al	Ga	In	Tl
Atomic radius (pm)	88	143	135	167	170
Electronegativity	2	1.5	1.6	1.7	1.8
Density (g/cm^3)	2.35	2.7	5.9	7.31	11.85
Ionisation energy (kJ/mol)	801	577	579	558	589

55.

56. Ring 1 is activated by $-NH$ group57. Nitric acid is added to sodium extract before addition of silver nitrate for testing halogens. Because it decomposes NaCN and Na_2S or else they interfere in the test. The reactions are as follows :

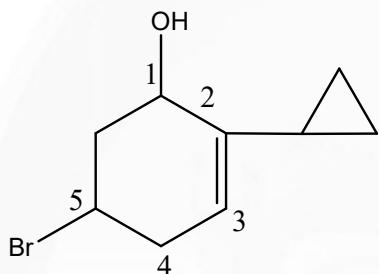
$$58. \lambda = \frac{C}{v} = \frac{3 \times 10}{5.16 \times 10^{14}} = 581.39 \text{ nm}$$

59.

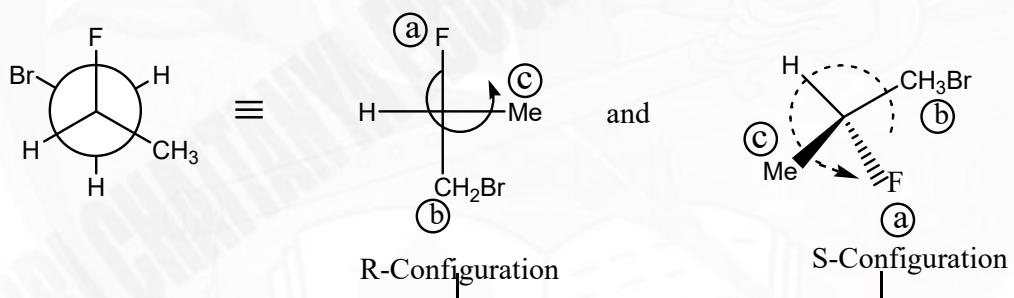
Cations	Group No	Group reaction



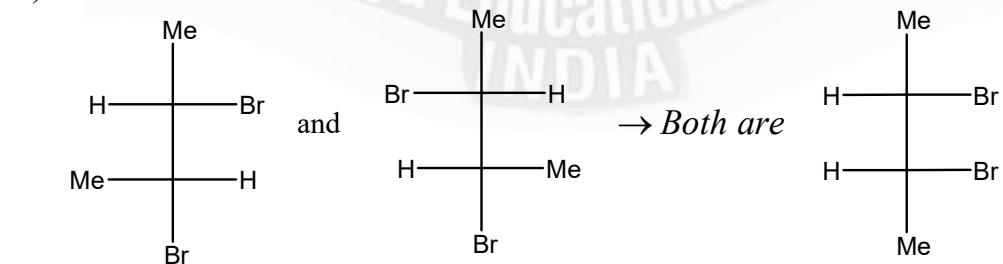
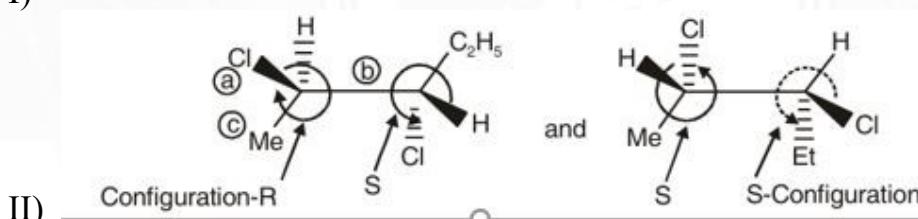
Pb^{2+}, Cu^{2+}	II	i) H_2S gas in presence of dilute HCl
Al^{3+}, Fe^{3+}	III	ii) NH_4OH in presence of NH_4Cl
Co^{2+}, Ni^{2+}	IV	iii) H_2S in presence of NH_4OH
Ba^{2+}, Ca^{2+}	V	iv) $(NH_4)_2CO_3$ in presence of NH_4OH



60. Priority sequence is alcohol > alkene > halide



61. I)

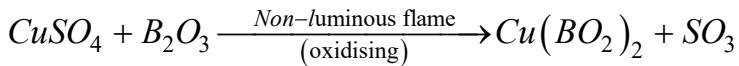


III) Identical ← Meso



62. Both Statement I and Statement II are false

On treatment with metal salt, boric anhydride forms metaborate of the metal which gives different colours in oxidizing and reducing flame. For example, in the case of copper sulphate, following reactions occur.

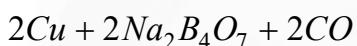


Two reactions may take place in reducing flame (Luminous flame)

(i) The blue-green $Cu(BO_2)_2$ is reduced to colourless cuprous metaborate as:

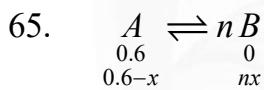


(ii) Cupric metaborate may be reduced to metallic copper and bead appears red opaque.



63. Conceptual

64. SO_2 produce turbidity in Baryta water and turns acidified dichromate solution green.



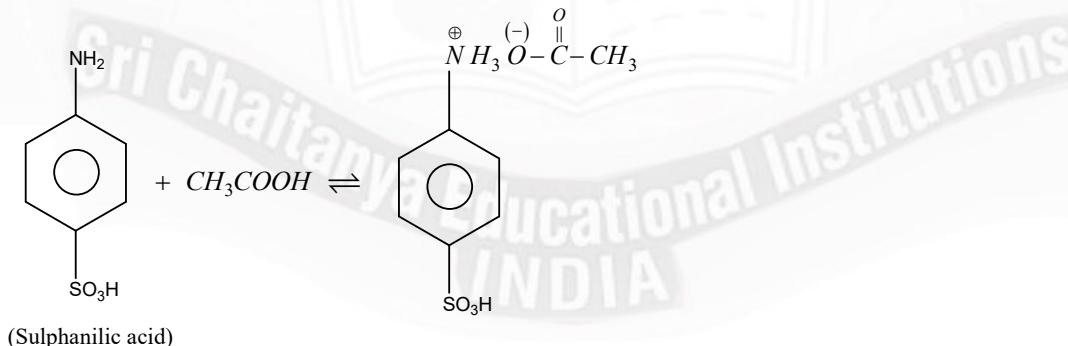
$$(B) = mx = 0.6 \quad \text{and} \quad [A] = 0.6 - x = 0.3$$

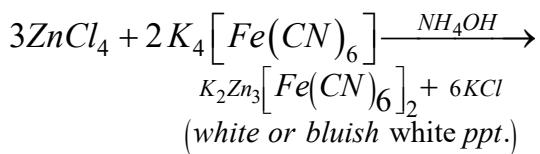
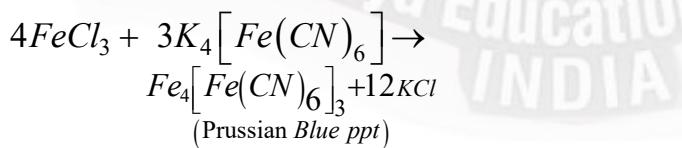
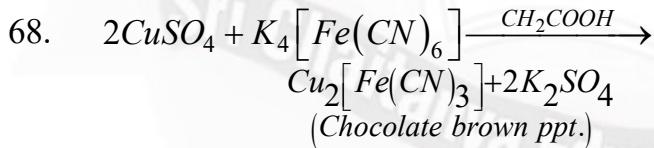
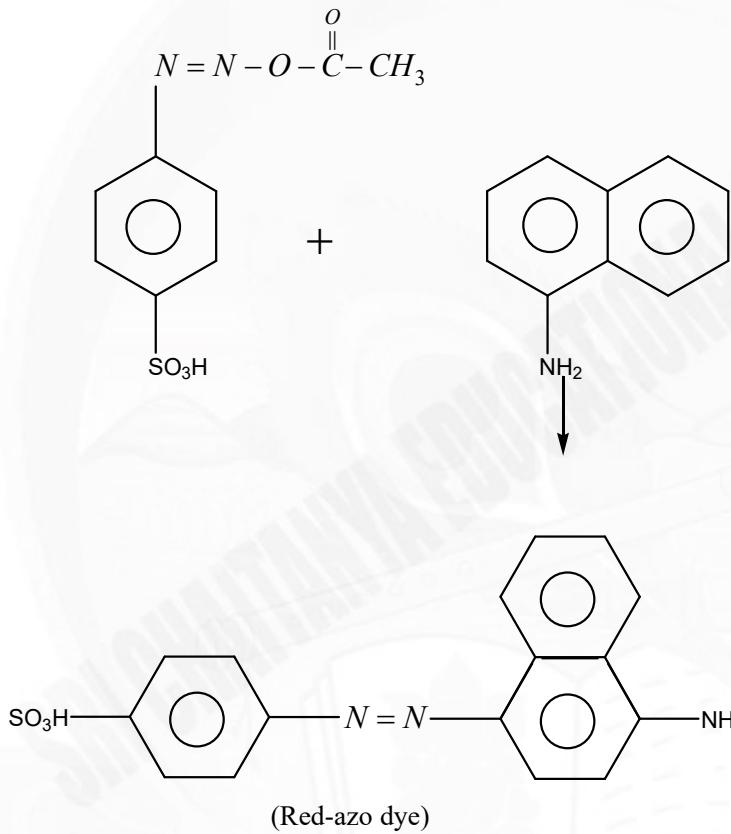
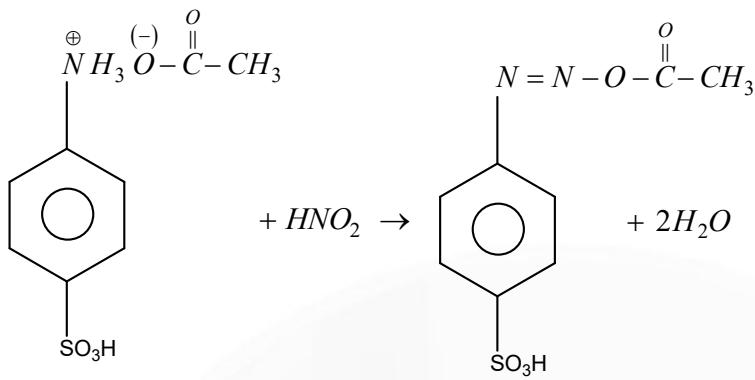
$$\text{So, } n = 2 \quad \text{and} \quad x = 0.3$$

$$K_C = \frac{[B]^2}{[A]} = \frac{0.6 \times 0.6}{0.3} = 1.2$$

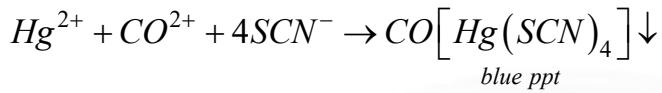
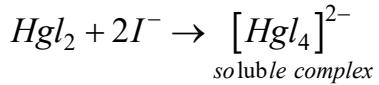
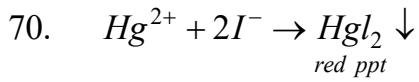
66. The ions of group II of salt analysis are precipitated by HCl and H_2S .

67. 0.1 mole of red-azo dye (Molar Mass = 327 gm/mol) will have 32.7 gm mass. Nearly 33 gm.





69. Conceptual



71. Rate at ($t = 10$ min) = $0.04 \text{ mol L}^{-1} = kC_{10} = R_{10}$

Rate at ($t = 20$ min) = $0.03 \text{ mol L}^{-1} \text{ sec}^{-1} = kC_{20} = R_{20}$

$$\text{So, } \frac{C_{10}}{C_{20}} = \frac{R_{10}}{R_{20}} = \frac{4}{3}$$

$$K = \frac{2.303}{(20-10)} \log \frac{C_{10}}{C_{20}} \text{ min}^{-1} = \frac{2.303}{10} \log \frac{4}{3} \text{ min}^{-1}$$

$$t_{1/2} = \frac{0.693}{K} = \frac{0.693 \times 10}{2.303 \left(\log \frac{4}{3} \right)} = \frac{3}{\log \frac{4}{3}} = \frac{3}{0.124} \text{ min} = 24.19 \text{ min}$$

72. So, bond angles are

- 1) $\angle N^I Co O^I$
- 2) $\angle N^I Co O^{II}$
- 3) $\angle N^I Co O^{III}$
- 4) $\angle N^I Co O^{IV}$
- 5) $\angle N^{II} Co O^I$
- 6) $\angle N^{II} Co O^{II}$
- 7) $\angle N^{III} Co O^{III}$
- 8) $\angle N^{II} Co O^{IV}$

So, total asked bond angles are 8.

73. $K_{sp} = [Zn^{2+}][S^{2-}] = [S^{2-}] = \frac{10^{-21}}{10^{-2}} = 10^{-19}$

$$K_{a_1} \times K_{a_2} = \frac{[H^+]^2 [S^{2-}]}{[H_2S]}$$

$$10^{-20} \frac{[H^+]^2 \times 10^{-19}}{0.1}$$

$$[H^+] = 0.1 \text{ pH} = 1$$

74. $CaSO_4$ does not evolve any gas with concentrated H_2SO_4 .

$NaBr \rightarrow \text{evolve } Br_2$

$NaNO_3 \rightarrow \text{evolve } NO_2$

$KI \rightarrow \text{evolve } I_2$

75. 6

