

Sol. $|x - 2|^2 + 2|x - 2| - |x - 2| - 2 = 0$

$$\Rightarrow (|x - 2| + 2)(|x - 2| - 1) = 0$$

$$\Rightarrow |x - 2| = 1$$

$$\Rightarrow x = 2 \pm 1 = 3, 1$$

$$\Rightarrow \text{sum of square of roots} = 9 + 1 = 10$$

$$x^2 - 2|x - 3| - 5 = 0$$

Case-I $x - 3 \geq 0$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x - 1)^2 = 0$$

$$\Rightarrow x = 1$$

But $x \geq 3$

$$\Rightarrow x \in \emptyset$$

Case-II $x - 3 < 0$

$$x^2 + 2x - 11 = 0, D > 0 \Rightarrow \text{Real & distinct roots}$$

$$f(x) = x^2 + 2x - 11$$

$$f(3) > 0, \frac{-p}{2a} = -1 < 3$$

\Rightarrow both roots < 3 , both roots acceptable

$$\text{Sum of square of roots} = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= 4 + 22 = 26$$

$$\Rightarrow \text{Final sum} = 10 + 26 = 36$$

7. Let a be the length of a side of a square $OABC$ with O being the origin. Its side OA makes an acute angle α with the positive x -axis and the equations of its diagonals are $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 0$ and $(\sqrt{3}-1)x - (\sqrt{3}+1)y + 8\sqrt{3} = 0$. Then a^2 is equal to

(1) 48

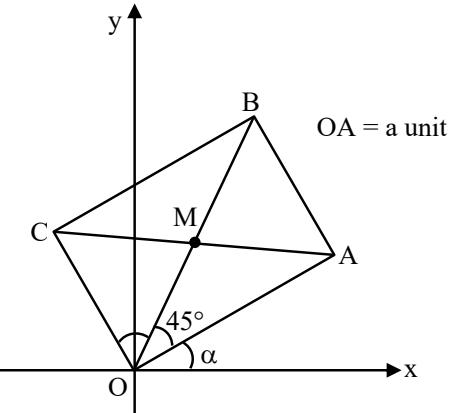
(2) 32

(3) 16

(4) 24

Ans. (1)

Sol.



$$\text{Slope of diagonal } OB = \frac{\sqrt{3}+1}{1-\sqrt{3}} = \tan 105^\circ$$

$$\therefore \alpha = 60^\circ$$

$$\therefore A(\cos 60^\circ, \sin 60^\circ)$$

$$\therefore A\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$$

A lies on other diagonal

$$\therefore \left(\frac{\sqrt{3}-1}{2}\right)a - \left(\frac{\sqrt{3}+1}{2}\right). \sqrt{3}a + 8\sqrt{3} = 0$$

$$a\left[\frac{\sqrt{3}-1-3-\sqrt{3}}{2}\right] = -8\sqrt{3}$$

$$a = 4\sqrt{3}$$

$$\therefore a^2 = 48$$

8. Let $f(x)$ be a positive function and

$$I_1 = \int_{-\frac{1}{2}}^1 2xf(2x(1-2x)) dx \text{ and } I_2 = \int_{-1}^2 f(x(1-x)) dx.$$

Then the value of $\frac{I_2}{I_1}$ is equal to _____

(1) 9

(2) 6

(3) 12

(4) 4

Ans. (4)



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Sol. Let, $I = \pi^2 \int_{-1}^{3/2} |x \sin \pi x| dx$

$$= \pi^2 \left\{ \int_{-1}^1 x \sin \pi x dx - \int_1^{3/2} x \sin \pi x dx \right\}$$

$$= \pi^2 \left\{ 2 \int_0^1 x \sin \pi x dx - \int_{-1}^{3/2} x \sin \pi x dx \right\}$$

Consider

$$\int x \sin \pi x dx$$

$$= -x \cdot \frac{1}{\pi} \cos \pi x + \int 1 \cdot \frac{1}{\pi} \cos \pi x dx$$

$$= -\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2}$$

$$I = \pi^2 \left\{ 2 \left(-\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2} \right) \Big|_0^1 - \left(-\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2} \right) \Big|_1^{3/2} \right\}$$

$$= \pi^2 \left\{ \frac{2}{\pi} - \left(-\frac{1}{\pi^2} - \frac{1}{\pi} \right) \right\}$$

$$= \pi^2 \left\{ \frac{3}{\pi} + \frac{1}{\pi^2} \right\}$$

$$= 3\pi + 1$$

12. A line passing through the point $P(a, 0)$ makes an acute angle α with the positive x-axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clock-wise direction. If in the new position, the slope of the line is $2 - \sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2 \tan^2 \alpha - 2\sqrt{3}$ is

(1) 4

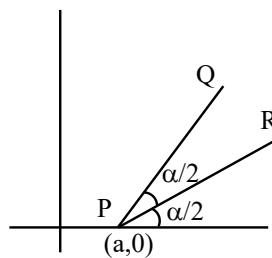
(2) 6

(3) 5

(4) 8

Ans. (1)

Sol.



$$m_{PR} = 2 - \sqrt{3} = \tan 15^\circ$$

$$\therefore \frac{\alpha}{2} = 15^\circ \Rightarrow \alpha = 30^\circ$$

equation of PR :

$$y = \tan 15^\circ (x - a)$$

$$y = (2 - \sqrt{3})(x - a)$$

$$\perp \text{distance from origin} = \frac{1}{\sqrt{2}}$$

$$\left| \frac{\sqrt{3}a - 2a}{\sqrt{4 + 3 - 4\sqrt{3} + 1}} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{|a|(2 - \sqrt{3})}{2\sqrt{(2 - \sqrt{3})}} = \frac{1}{\sqrt{2}}$$

$$|a| = \frac{\sqrt{2}}{\sqrt{2 - \sqrt{3}}} = \sqrt{2}(\sqrt{2 + \sqrt{3}})$$

$$a^2 = 2(2 + \sqrt{3})$$

$$3a^2 \tan^2 \alpha - 2\sqrt{3}$$

$$3 \times (4 + 2\sqrt{3}) \cdot \frac{1}{3} - 2\sqrt{3} = 4$$

13. There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is

(1) 230

(2) 220

(3) 200

(4) 210

Ans. (4)

Sol. ${}^{12}C_3 - {}^5C_3 = 210$



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14. Let $A = \left\{ \theta \in [0, 2\pi] : 1 + 10 \operatorname{Re} \left(\frac{2 \cos \theta + i \sin \theta}{\cos \theta - 3i \sin \theta} \right) = 0 \right\}$.

Then $\sum_{\theta \in A} \theta^2$ is equal to

- (1) $\frac{21}{4}\pi^2$ (2) $8\pi^2$
 (3) $\frac{27}{4}\pi^2$ (4) $6\pi^2$

Ans. (1)

Sol. $1 + 10 \operatorname{Re} \left(\frac{2 \cos \theta + i \sin \theta}{\cos \theta - 3i \sin \theta} \right) = 0$

$$\begin{aligned} \therefore z + \bar{z} &= 2 \operatorname{Re}(z) \\ \frac{2 \cos \theta + i \sin \theta}{\cos \theta - 3i \sin \theta} + \frac{2 \cos \theta - i \sin \theta}{\cos \theta + 3i \sin \theta} &= 2 \times \left(\frac{-1}{10} \right) \\ \frac{(2 \cos^2 \theta - 3 \sin^2 \theta) + (2 \cos^2 \theta) - (3 \sin^2 \theta)}{\cos^2 \theta + 9 \sin^2 \theta} &= \frac{-2}{10} \\ \Rightarrow \frac{2 \cos^2 \theta - 3 \sin^2 \theta}{\cos^2 \theta + 9 \sin^2 \theta} &= \frac{-1}{10} \\ \Rightarrow 20 \cos^2 \theta - 30 \sin^2 \theta &= -\cos^2 \theta - 9 \sin^2 \theta \\ 21 \cos^2 \theta - 21 \sin^2 \theta &= 0 \\ \Rightarrow \cos 2\theta &= 0 \end{aligned}$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\Rightarrow \sum \theta^2 = \frac{\pi^2}{16} + \frac{9\pi^2}{16} + \frac{25\pi^2}{16} + \frac{49\pi^2}{16} = \frac{84\pi^2}{16} = \frac{21\pi^2}{4}$$

15. Let $A = \{0, 1, 2, 3, 4, 5\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $\max\{x, y\} \in \{3, 4\}$. Then among the statements

(S₁) : The number of elements in R is 18, and
 (S₂) : The relation R is symmetric but neither reflexive nor transitive

- (1) both are true (2) both are false
 (3) only (S₂) is true (4) only (S₁) is true

Ans. (3)

Sol. $A = \{0, 1, 2, 3, 4, 5\}$

$$R = \{(0, 3), (3, 0), (0, 4), (4, 0), (1, 3), (3, 1), (1, 4), (4, 1), (2, 3), (3, 2), (2, 4), (4, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$$

Total 16 elements

Not reflexive as $(0, 0), \dots, (2, 2) \notin R$

Symmetric $\because \forall$ all a, b

$(a, b) \& (b, a) \in R$

Not transitive $\because (0, 3), (3, 1) \in R$

but $(0, 1) \notin R$

\Rightarrow Only S₂ correct

16. The number of integral terms in the expansion of

$$\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}} \right)^{1016}$$

- is
 (1) 127 (2) 130
 (3) 129 (4) 128

Ans. (4)

Sol. $T_r = {}^{1016}C_r (5^{\frac{1016-r}{2}})^{\frac{r}{8}}$
 $\Rightarrow r = 0, 8, 16, 24, \dots, 1016$
 $1016 = 0 + (n-1)8$

$$\Rightarrow n-1 = \frac{1016}{8} = 127$$

So, $n = 128$.

17. Let $f(x) = x - 1$ and $g(x) = e^x$ for $x \in \mathbb{R}$. If

$$\frac{dy}{dx} = \left(e^{-2\sqrt{x}} g(f(f(x))) - \frac{y}{\sqrt{x}} \right), y(0) = 0, \text{ then } y(1)$$

is :-

- (1) $\frac{1-e^2}{e^4}$ (2) $\frac{2e-1}{e^3}$
 (3) $\frac{e-1}{e^4}$ (4) $\frac{1-e^3}{e^4}$

Ans. (3)

Sol. $f(x) = x - 1$
 $f(f(x)) = f(x) - 1 = x - 1 - 1 = x - 2$
 $g(f(f(x))) = e^{x-2}$

$$\therefore \frac{dy}{dx} = e^{-2\sqrt{x}} \times e^{x-2} - \frac{1}{\sqrt{x}} y$$

$$\frac{dy}{dx} + \frac{1}{\sqrt{x}} y = e^{x-2\sqrt{x}-2} \text{ which is L.D.E}$$

$$\text{I.F.} = e^{\int \frac{dy}{\sqrt{x}}} = e^{2\sqrt{x}}$$

Its solution is

$$y \times e^{2\sqrt{x}} = \int e^{2\sqrt{x}} \times e^{x-2\sqrt{x}-2} dx + c$$

$$y \times e^{2\sqrt{x}} = \int e^{x-2} dx + c$$

$$y \times e^{2\sqrt{x}} = e^{x-2} + c$$

$$\text{Given } x = 0, y = 0 \Rightarrow 0 = e^{-2} + c ; c = -e^{-2}$$

$$\therefore y \times e^{2\sqrt{x}} = e^{x-2} - e^{-2}$$

$$\text{when } x = 1, y \times e^2 = e^{-1} - e^{-2}$$

$$y = \frac{e^{-1} - e^{-2}}{e^2} = \frac{\frac{1}{e} - \frac{1}{e^2}}{e^2} = \frac{e^2 - e}{e^5} = \frac{e-1}{e^4}$$

Option (1) is correct



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18. The value of $\cot^{-1} \left(\frac{\sqrt{1+\tan^2(2)} - 1}{\tan(2)} \right) - \cot^{-1}$

$$\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)} + 1}{\tan\left(\frac{1}{2}\right)} \right)$$

- (1) $\pi - \frac{5}{4}$ (2) $\pi - \frac{3}{2}$
 (3) $\pi + \frac{3}{2}$ (4) $\pi + \frac{5}{2}$

Ans. (1)

$$\text{Sol. } \cot^{-1} \left(\frac{|\sec 2| - 1}{\tan 2} \right) - \cot^{-1} \left(\frac{\left| \sec \frac{1}{2} \right| + 1}{\tan \frac{1}{2}} \right)$$

$$= \cot^{-1} \left(\frac{-1 - \cos 2}{\sin 2} \right) - \cot^{-1} \left(\frac{1 + \cos \frac{1}{2}}{\sin \frac{1}{2}} \right)$$

$$= \pi - \cot^{-1} (\cot 1) - \cot^{-1} \left(\cot \frac{1}{4} \right)$$

$$= \pi - 1 - \frac{1}{4} = \pi - \frac{5}{4}$$

19. Let $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$.

If $\det(\text{adj}(\text{adj}(3A))) = 2^m \cdot 3^n$, $m, n \in \mathbb{N}$, then

- m + n is equal to
 (1) 22 (2) 24
 (3) 26 (4) 20

Ans. (2)

$$\text{Sol. } |A| = \begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$$

$$C_3 \rightarrow C_3 - C_2 - C_1 \times \frac{q}{2}$$

$$\text{Then } C_3 \rightarrow C_2 - C_1 \times \left(1 + \frac{p}{2} \right)$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & 0 & 0 \\ 4 & 2 & 2+p \\ 6 & 6 & 8+3p \end{vmatrix}$$

$$\Rightarrow |A| = 2(16 + 6p - 12 - 6p) = 8 = 2^3$$

$$|\text{adj}(\text{adj}(3A))| = |3A|^{(3-1)^2} = |3A|^4$$

$$= (3^3 |A|)^4 = (3^3 \times 2^3)^4 = 2^{12} \times 3^{12}$$

$$\Rightarrow m + n = 24$$

20. Given below are two statements :

Statement I :

$$\lim_{x \rightarrow 0} \left(\frac{\tan^{-1} x + \log_e \sqrt{\frac{1+x}{1-x}} - 2x}{x^5} \right) = \frac{2}{5}$$

$$\text{Statement II : } \lim_{x \rightarrow 1} \left(x^{\frac{2}{1-x}} \right) = \frac{1}{e^2}$$

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Statement I is false but Statement II is true
 (2) Statement I is true but Statement II is false
 (3) Both Statement I and Statement II are false
 (4) Both Statement I and Statement II are true

Ans. (4)



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Sol.

$$\lim_{x \rightarrow 0} \frac{\tan^{-1} x + \frac{1}{2} [\ln(1+x) - \ln(1-x)] - 2x}{x^5}$$

$$= \lim_{x \rightarrow 0} \left(x - \frac{x^3}{3} + \frac{x^5}{5} \dots \right) + \frac{1}{2} \left[x - \frac{x^2}{2} + \frac{x^3}{3} \dots - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} \dots \right) \right] - 2x$$

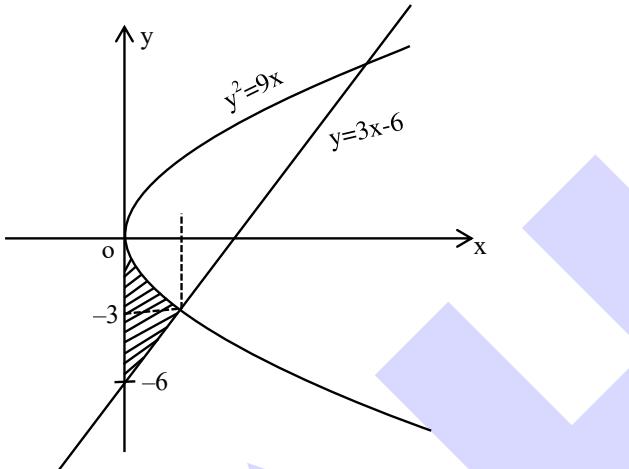
$$= \lim_{x \rightarrow 0} \frac{2x + \frac{2x^5}{5} \dots - 2x}{x^5} = \frac{2}{5}$$

$$\lim_{x \rightarrow 1} x^{\frac{2}{(1-x)}} = e^{\lim_{x \rightarrow 1} \left(\frac{2}{1-x} \right)(x-1)} = e^{-2}$$

⇒ Both statements correct

SECTION-B

- 21.** Let the area of the bounded region $\{(x, y) : 0 \leq 9x \leq y^2, y \geq 3x - 6\}$ be A. Then $6A$ is equal to _____
- Ans. (15)**
- Sol.** $0 \leq 9x \leq y^2$ & $y \geq 3x - 6$



$$A = \text{Required Area} = \left[\int_0^1 (-3\sqrt{x}) dx - \int_0^1 (3x - 6) dx \right]$$

$$A = -3 \left[\frac{x^{3/2}}{3/2} \right]_0^1 - \left[\frac{3x^2}{2} - 6x \right]_0^1$$

$$A = -2[1-0] \left[\frac{3}{2} - 6 \right]$$

$$A = -2 - \frac{3}{2} + 6 = \frac{5}{2} \text{ Sq. unit}$$

$$\therefore 6A = 6 \times \frac{5}{2} = 15$$

- 22.** Let the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha, \beta]$ and the domain of $g(x) = \log_2(2 - 6\log_{27}(2x+5))$ be (γ, δ) . Then $|7(\alpha + \beta) + 4(\gamma + \delta)|$ is equal to _____

Ans. (96)

Sol. $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$

$$\Rightarrow -1 \leq \frac{4x+5}{3x-7} \leq 1$$

$$\left(\frac{4x+5}{3x-7} \right) \geq -1$$

$$\frac{4x+5+3x-7}{3x-7} \geq 0$$

$$\Rightarrow \frac{7x-2}{3x-7} \geq 0$$

$$\begin{array}{c} \leftarrow \bullet \\ \frac{2}{7} \\ \hline \end{array} \quad \begin{array}{c} \circ \rightarrow \\ \frac{7}{3} \\ \hline \end{array}$$

$$x \in \left(-\infty, \frac{2}{7}\right] \cup \left(\frac{7}{3}, \infty\right)$$

$$\& \frac{4x+5}{3x-7} \leq 1 \Rightarrow \frac{x+12}{3x-7} \leq 0$$

$$\begin{array}{c} \bullet \circ \\ -12 \quad \frac{7}{3} \\ \hline \end{array}$$

∴ Domain of $f(x)$ is

$$\left[-12, \frac{2}{7}\right] \quad \boxed{\alpha = -12, \beta = \frac{2}{7}}$$

$g(x) = \log_2(2 - 6 \log_{27}(2x+5))$

Domain

$$2 - 6 \log_{27}(2x+5) > 0$$

$$6 \log_{27}(2x+5) < 2$$

$$\log_{27}(2x+5) < \frac{1}{3}$$

$$2x+5 < 3$$

$$x < -1$$

$$\& 2x+5 > 0 \Rightarrow x > -\frac{5}{2}$$

$$\text{Domain is } x \in \left(-\frac{5}{2}, -1\right)$$

$$\boxed{\gamma = -\frac{5}{2}, \delta = -1}$$

$$|7(\alpha + \beta) + 4(\gamma + \delta)| = |7(-12 + \frac{2}{7}) + 4(-\frac{5}{2} - 1)|$$

$$|-82 - 14| = 96$$



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23. Let the area of the triangle formed by the lines

$$x+2=y-1=z, \frac{x-3}{5}=\frac{y}{-1}=\frac{z-1}{1}$$

and $\frac{x}{-3}=\frac{y-3}{3}=\frac{z-2}{1}$ be A. Then A^2 is equal to _____

Ans. (56)

Sol. $L_1 : x + 2 = y - 1 = z = \ell$

$$L_2 : \frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1} = m$$

$$L_3 : \frac{x}{-3} = \frac{y-3}{5} = \frac{z-2}{1} = n$$

Point of intersection of L_1 and L_2

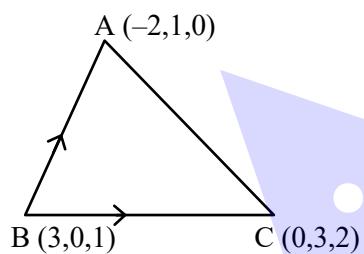
$$\begin{cases} \ell - 2 = 5m + 3 \\ \ell + 1 = -m \\ \ell = m + 1 \end{cases} \left\{ \begin{array}{l} \ell = 0, m = -1 \\ A(-2, 1, 0) \end{array} \right.$$

Point of intersection of L_2 and L_3

$$\begin{cases} 5m + 3 = -3n \\ -m = 3n + 3 \\ m + 1 = n + 2 \end{cases} \left\{ \begin{array}{l} m = 0, n = -1, B(3, 0, 1) \\ m + 1 = n + 2 \end{array} \right.$$

Point of intersection L_3 and L_4

$$\begin{cases} -3n = \ell - 2 \\ 3n + 3 = \ell + 1 \\ n + 2 = \ell \end{cases} \left\{ \begin{array}{l} \ell = 2, n = 0, C(0, 3, 2) \\ n + 2 = \ell \end{array} \right.$$



$$Ar(\Delta ABC) = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -5 & 1 \\ -3 & 3 & 1 \end{vmatrix}$$

$$A = \frac{1}{2} | \hat{i}(4) - \hat{j}(-8) + \hat{k}(-12) |$$

$$A = \frac{1}{2} \sqrt{16 + 64 + 144} = \sqrt{56}$$

$$A^2 = 56$$

24. The product of the last two digits of $(1919)^{1919}$ is

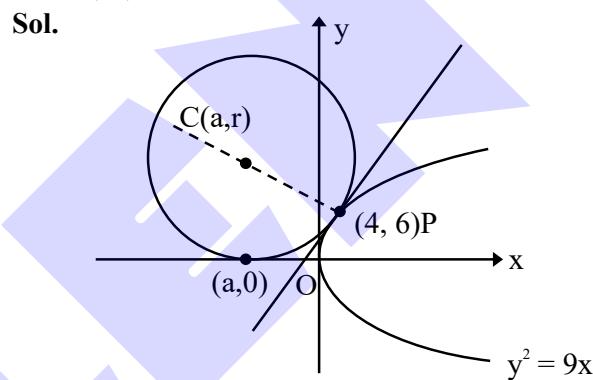
Ans. (63)

$$\begin{aligned} (1919)^{1919} &= (1920 - 1)^{1919} \\ &= {}^{1919}C_0(1920)^{1919} - {}^{1919}C_1(1920)^{1918} + \dots \\ &\quad + {}^{1919}C_{1918}(1920)^1 - {}^{1919}C_{1919} \\ &= 100\lambda + 1919 \times 1920 - 1 \\ &= 100\lambda + 3684480 - 1 \\ &= 100\lambda + \dots \text{last two digit} \\ \Rightarrow \text{Number having last two digit } 79 \\ \therefore \text{Product of last two digit } 63 \end{aligned}$$

25. Let r be the radius of the circle, which touches x-axis at point $(a, 0)$, $a < 0$ and the parabola $y^2 = 9x$ at the point $(4, 6)$. Then r is equal to _____

Ans. (30)

Sol.



$$(x - a)^2 + (y - r)^2 = r^2$$

$$(4 - a)^2 + (6 - r)^2 = r^2$$

$$16 + a^2 - 8a + 36 + r^2 - 12r = r^2$$

$$a^2 - 8a - 12r + 52 = 0$$

Tangent to parabola at $(4, 6)$ is

$$6.4 = 9 \cdot \left(\frac{x+4}{2} \right) \text{ i.e. } 3x - 4y + 12 = 0$$

This is also tangent to the circle

$$\therefore CP = r$$

$$\frac{3a - 4r + 12}{5} = \pm r$$

$$3a + 12 = 4r \pm 5r \begin{cases} ar \\ -r \end{cases} \dots \dots (1)$$

equation of circle is

$$(x - a)^2 + (y - r)^2 = r^2$$

$$\text{satisfy } P(4, 6) \Rightarrow a^2 - 8a - 12r + 52 = 0 \dots \dots (2)$$

From equation (1)

If $a + 4 = 3r$ then $a = +6$ (rejected)

If $3a + 12 = -r$ then $a = -14$ and $r = 30$



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