Mathematics

61. If the area (in sq. units of the region

$\left\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\right\}$

is $a\sqrt{2}$ +b, then a – b is equal to :

A. $\frac{10}{3}$ B. $\frac{8}{3}$

C. $-\frac{2}{3}$ D. 6

Ans: D

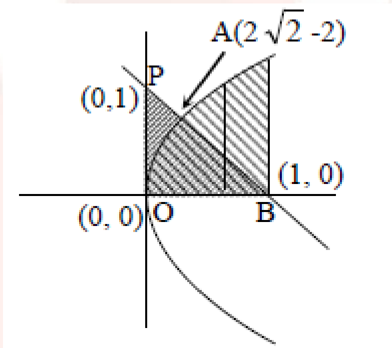
It is given that,

$C_1 : y^2 \le 4x$

$C_2 : x + y \le 1$

$x \ge 0$

$y \ge 0$



So, now,

$y^2 = 4x;$

$y^2 = 4(1–y)$ (since, $C_2 : x + y \le 1$)

$y^2 + 4y + 4 = 0$

So, solving the above equation, we get,

$y = 2\sqrt{2} - 2, \; -2\sqrt{2} - 2$

Now, plotting the points on graph, we get,

The required area is shaded region of curve OAB,

Thus,

A = Area of Δ_{OBP} – Area of region OAP

$$\Delta_{OBP} = \frac{1}{2}\times 1\times 1 = \frac{1}{2}$$

$$\text{Area of OAP} = \int_0^{2\sqrt{2}-2} \frac{y^2}{4}dy + \int_{2\sqrt{2}-2}^{1} (1-y)$$

$$= \frac{1}{12}\left[y^3\right]_0^{2\sqrt{2}-2} + \left[y - \frac{y^2}{2}\right]_{2\sqrt{2}-2}^{1}$$

$$= \frac{1}{12}\left[\left(2\sqrt{2}-2\right)^3\right] + \left[\left(1-\frac{1}{2}\right) - \left\{\left(2\sqrt{2}-2\right) - \frac{\left(2\sqrt{2}-2\right)^2}{2}\right\}\right]$$

$$= \frac{23}{6} - \frac{8}{3}\sqrt{2}$$

$$A = \frac{1}{2} - \frac{23}{6} + \frac{8\sqrt{2}}{3}$$

$$a = \frac{8}{3}, b = -\frac{20}{6}$$

$$\therefore a - b = 6$$

62. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4,4) then PQ is equal to :

A. $\frac{\sqrt{221}}{2}$ B. $\frac{5\sqrt{5}}{2}$

C. $\frac{\sqrt{157}}{2}$ D. $\frac{\sqrt{61}}{2}$

Ans: B

Equation of given ellipse is $3x^2 + 4y^2 = 12$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1 \qquad \ldots(1)$$

Now, let point P($2\cos\theta$, $\sqrt{3}\sin\theta$), so equation of tangent to ellipse 1 at point P is

$$\frac{x\cos\theta}{2}+\frac{y\sin\theta}{\sqrt{3}}=1 \qquad \ldots(ii)$$

$$2x\sec\theta-\sqrt{3}\operatorname{cosec}\theta=1$$

Slope of normal = $\frac{-2\sec\theta}{-\sqrt{3}\operatorname{cosec}\theta}=\frac{2}{\sqrt{3}}\frac{\sin\theta}{\cos\theta}$

Since, normal parallel to $2x + y = 4$

$\therefore$ slope of normal = slope of line

then $\frac{2}{\sqrt{3}}\tan\theta=-2;\ \tan\theta=-\sqrt{3}$

$$\theta=\frac{2\pi}{3}$$

$$(\sin\theta,\cos\theta)=\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)$$

Now, point $P\left(-1,\frac{3}{2}\right), Q(4,4)$

$$PQ=\sqrt{(4+1)^2+\left(4-\frac{3}{2}\right)^2}=\sqrt{\frac{125}{4}}=\frac{5\sqrt{5}}{2}$$

63. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :

A. $\frac{3}{20}$ B. $\frac{1}{5}$

C. $\frac{1}{10}$ D. $\frac{3}{10}$

Ans: C

Since, there is a regular hexagon, then the number of ways choosing three vertices is $^6C_3$

Number of total triangle = $^6C_3$

Equilateral triangle = 2

Required $\text{probability} = \frac{2}{{}^6C_3} = \frac{2}{\frac{6!}{3!3!}} = \frac{1}{10}$

64. If the volume of parallelepiped formed by the vectors $\hat{i} + \lambda\hat{j} + k, \hat{j} + \lambda k$ and $\lambda\hat{i} + k$ is minimum, then λ is equal to:

A. $-\sqrt{3}$

B. $\frac{1}{\sqrt{3}}$

C. $\sqrt{3}$

D. $-\frac{1}{\sqrt{3}}$

Ans: B

Given, vectors are $\hat{i} + \lambda\hat{j} + \hat{k}, \hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ which forms a parallelepiped.

$\therefore$ Volume of parallelepiped

$$\begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix} = 1[1-0] - \lambda[-\lambda^2] + 1[-\lambda]$$

$V = |1 + \lambda^3 - \lambda|$

on differentiating w.r.t., λ, we get

$\frac{dV}{d\lambda} = 3\lambda^2 - 1 \Rightarrow \text{for maxima or minima } \frac{dV}{d\lambda} = 0 \Rightarrow \lambda = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

$\frac{d^2V}{d\lambda^2} = 6\lambda$

$\frac{d^2V}{d\lambda^2} > 0 \text{ at } \lambda = \frac{1}{\sqrt{3}}$

So, volume minimum for $\lambda = \frac{1}{\sqrt{3}}$

65. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2k$ and $\vec{b} = \hat{i} + 2\hat{j} - 2k$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is:

A. $4\left(2\hat{i}+2\hat{j}-k\right)$ B. $4\left(-2\hat{i}-2\hat{j}+k\right)$

C. $4\left(2\hat{i}-2\hat{j}-k\right)$ D. $4\left(2\hat{i}+2\hat{j}+k\right)$

Ans: C

Given vectors are

$\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$

and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$

Now, vectors $\vec{a}+\vec{b}=4\hat{i}+4\hat{j}$

and $\vec{a}-\vec{b}=2\hat{i}+4\hat{k}$

Let a vector $\vec{r}$ perpendicular to $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ and magnitude is 12, then

$\vec{r}=12.\hat{n}$

$$\therefore\ \hat{n}=\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}])|} \qquad ...(1)$$

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 4 & 0\\ 2 & 0 & 4\end{vmatrix}$$

$$=\hat{i}(16)-\hat{j}(16)+\hat{k}(-8)=8[2\hat{i}-2\hat{j}-\hat{k}]$$

$$|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|=8\times\sqrt{4+4+1}=8\times3$$

$$\hat{n}=\frac{(2\hat{i}-2\hat{j}-\hat{k})}{3}$$

Putting values in eq (1) we get

$\vec{r}=4.(2\hat{i}-2\hat{j}-\hat{k})$

66. If α and β are the roots of the equation $375x^2-25x-2=0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r+\lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to:

A. $\frac{29}{358}$ B. $\frac{7}{116}$

C. $\frac{1}{12}$ D. $\frac{21}{346}$

Ans: C

Given, α and β are roots quadratic equation

$375x^2 - 25x - 2 = 0$

The sum of roots is $\alpha + \beta = \frac{25}{375} = \frac{1}{15}$

The product of roots is $\alpha\beta = \frac{-2}{375}$

And, $\alpha \,\&\, \beta \in (-1, 1)$, then,

$$\lim_{n\to\infty} \sum_{r=1}^{n} (\alpha^r + \beta^r)$$

$$\Rightarrow (\alpha + \alpha^2 \text{.......upto infinite term}) + (\beta + \beta^2 \text{......upto infinite term})$$

$$\Rightarrow \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} \qquad \left[\because S_\infty = \frac{\alpha}{1-r} \text{ for GP}\right]$$

$$\Rightarrow \frac{\alpha(1-\beta) + \beta(1-\alpha)}{(1-\alpha)(1-\beta)} = \frac{(\alpha+\beta) - 2\alpha\beta}{1 - (\alpha+\beta) + \alpha\beta}$$

$$\Rightarrow \frac{\frac{1}{15} + \frac{4}{375}}{1 - \frac{1}{15} - \frac{2}{375}} = \frac{1}{12}$$

Thus, if α and β are the roots of the equation

$375\,x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \alpha^r + \lim_{n\to\infty} \sum_{r=1}^{n} \beta^r$ is equal to $\frac{1}{12}$

67. The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to:

A. $\pi - \sin^{-1}\left(\frac{63}{65}\right)$ B. $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

C. $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$ D. $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

Ans: C

It is given that

$$\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$$

Now,

$\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$

$=\sin^{-1}\left[\frac{12}{13}\sqrt{1-\frac{9}{25}}-\frac{3}{5}\sqrt{1-\frac{144}{169}}\right]$

$\left[\because \sin^{-1}x-\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)\right]$

$=\sin^{-1}\left(\frac{33}{65}\right)$

$=\cos^{-1}\left(\frac{56}{65}\right)$

$=\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$

Thus,

The value of $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ is equal to $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$.

68. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to:

A. $2\sqrt{14}$ B. $\sqrt{14}$

C. 14 D. $2\sqrt{7}$

Ans: A

Equation of given line is

$\frac{x-2}{3}=\frac{y+1}{g}=\frac{z-1}{-1}=\lambda$

Now, coordinates of a general point line is

$(3\lambda+2, 2\lambda-1, -\lambda+1)$

Since, p is the point of intersection and the plane

$2x+3y-z+13=0$ then

$2(3\lambda+2)+3(2\lambda-1)-(-\lambda+1)+13=0$

$\lambda=-1$

So, point P$(-1,-3,2)$

Line intersect plane : $3x + y + 4z = 16$ at Q then

$3(3\lambda + 2) + 2\lambda - 1 + 4(-\lambda + 1) = 16 \Rightarrow \lambda = 1$

So, point Q(5, 1, 0) then PQ = $\sqrt{(5+1)^2 + (1+3)^2 + (0-2)^2}$

$PQ = 2\sqrt{14}$

69. If A is a symmetric matrix and B is a skew-symmetrix matrix such that

$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

A. $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ B. $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

C. $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$ D. $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Ans: B

Given, matrix A is symmetric and matrix B is a skew-symmetric

A is symmetric $A^T = A$

B is skew symmetry $B^T = -B$

Since, $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ ...(i)

On taking transpose both sides, we get

$(A + B)^T - \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}^T$

Transpose

$A^T + B^T = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$ ...(ii)

From (i) + (ii)

$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$

From (i) - (ii)

$B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

So, $AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

70. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x, x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is:

A. 4 B. 3

C. 5 D. 7

Ans: C

The Given equation is

$1 + \sin^4 x = \cos^2 3x;\ x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$

$\Rightarrow$ L.H.S. ≥ 1

R.H.S. ≤ 1

Both satisfy when

L.H.S. = R.H.S. = 1

Since, $x \in \left[\frac{-5\pi}{2}, \frac{5\pi}{2}\right]$

$\sin^4 x = 0;\ \cos^2 3x = 1$

Hence, the value of x is

$x = -2\pi, -\pi, 0, \pi, 2\pi$

Thus, there are five different values of x is possible.

71. The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to: (Here C is a constant of integration)

A. $\frac{1}{2}\log_e \frac{\left(x^3+1\right)^2}{\left|x^3\right|} + C$

B. $\log_e \frac{\left|x^3+1\right|}{x^2} + C$

C. $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2}+C$

D. $\log_e\left|\frac{x^3+1}{x}\right|+C$

Ans: C

Let

$I=\int\frac{2x^3-1}{x^4+x}dx$

$\Rightarrow I=\int\frac{(4x^3+1)-(2x^3+2)}{x^4+x}dx$

$\Rightarrow I=\int\frac{4x^3+1}{x^4+x}dx-2\int\frac{1}{x}dx$

Now, putting $x^4+x=t$ and differentiating, we get,

$\Rightarrow (4x^3+1)dx=dt$

Now,

$I=\int\frac{dt}{t}-2\int\frac{1}{x}dx$

$\Rightarrow I=\ell n\,|\,t\,|-2\ell nx+c$

$\Rightarrow I=\ell n\left|\frac{x^4+x}{x^2}\right|+C \Rightarrow I=\ell n\left|\frac{x^3+1}{x}\right|+C$

72. If $\int_0^{\frac{\pi}{2}}\frac{\cot x}{\cot x+\operatorname{cosec} x}dx=m(\pi+n)$, then m n is equal to :

A. 1　　B. $\frac{1}{2}$

C. $-\frac{1}{2}$　　D. -1

Ans: D

$$I = \int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx$$

$$I = \int_0^{\pi/2} \frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{1}{\sin x}} dx$$

$$I = \int_0^{\pi/2} \frac{\cos x}{1 + \cos x} = \int_0^{\pi/2} \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}}$$

$$I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2 x / 2\right) dx$$

$$I = \left[x - \frac{2}{2}\tan(x/2)\right]_0^{\pi/2}$$

$$\Rightarrow \left(\frac{\pi}{2} - 1\right) = \frac{(\pi - 2)}{2} = \frac{1}{2}(\pi - 2)$$

Since, $I = m(\pi - n)$

on comparing both sides, we get

$m = \frac{1}{2}$, $n = -2$

Now, $m \cdot n = -1$

73. The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents :

A. A circle of radius $\frac{1}{2}$.

B. The line through the origin with slope -1.

C. A circle of radius 1.

D. The line through the origin with slope 1.

Ans: D

$|z - i| = |z - 1|$ (Given)

Let $z = x + iy$

$|x + i(y - 1)| = |(x - 1) + iy|$

$\Rightarrow x^2 + (y - 1)^2 = (x - 1)^2 + y^2$

$1 - 2x = 1 - 2y$

$\Rightarrow$ y = x
x – y = 0
This is equation of straight line with slope 1.

74. For x ϵ R, let [x] denote the greatest integer ≤ x, then the sum of the series

$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+.....+\left[-\frac{1}{3}-\frac{99}{100}\right]$ is :

A. -135 B. -153
C. -133 D. -131

Ans (C)

75. Consider the differential equation, $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If value of y is 1 when x =1. Then the value of x for which y =2 is :

A. $\frac{3}{2}-\frac{1}{\sqrt{e}}$ B. $\frac{3}{2}-\sqrt{e}$
C. $\frac{5}{2}+\frac{1}{\sqrt{e}}$ D. $\frac{1}{2}+\frac{1}{\sqrt{e}}$

Ans B)
Explanation:

76. Let a random variable X have a binomial distribution with mean 8 and variance 4.

If P(X ≤ 2) = , then k is equal to :

A. 121 B. 17
C. 137 D. 1

Ans D)
Explanation:

77. If the data $x_1, x_2, \ldots.. x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

A. $\sqrt{2}$ B. 2

C. 4 D. $2\sqrt{2}$

Ans B)

78. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to :

A. $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ B. $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

C. $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ D. $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$

Ans: D

$e^y + xy = e$ …(i)

Put x = 0 in (i)

$\Rightarrow e^y = e \Rightarrow y = 1$

Differentiate (i) wr. to x

$$e^y \frac{dy}{dx} + x\frac{dy}{dx} + y = 0 \qquad \ldots\text{(ii)}$$

Put y = 1 in (ii)

$$e\frac{dy}{dx} + 0 + 1 = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{e}$$

Differentiate (ii) w. r, to x

$$e^y \frac{d^2y}{dx^2} + \frac{dy}{dx}.e^y \frac{dy}{dx} + x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0 \qquad \ldots\text{(iii)}$$

Put $y=1,\ x=0,\ \dfrac{dy}{dx}=-\dfrac{1}{e}$

$e\dfrac{d^2y}{dx^2}+\dfrac{1}{e}-\dfrac{2}{e}=0 \Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{1}{e^2}$

$\Rightarrow \left(\dfrac{dy}{dx},\dfrac{d^2y}{dx^2}\right)=\left(-\dfrac{1}{e},\ \dfrac{1}{e^2}\right)$

79. If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0.3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M is equal to.

A. $\left(3,3\sqrt{3}\right)$ B. $\left(4,3\sqrt{3}\right)$

C. $\left(4,3\sqrt{2}\right)$ D. $\left(5,3\sqrt{6}\right)$

Ans: B

$f(x)=x\sqrt{kx-x^2}=\sqrt{kx^3-x^4}$

$f'(x)=\dfrac{\left(3kx^2-4x^3\right)}{2\sqrt{kx^3-x^4}}\geq 0 \text{ for } x\in[0,\ 3]$

$\Rightarrow 3k-4x\geq 0$

$3k\geq 4x$

$3k\geq 4x \text{ for } x\in[0,\ 3]$

Hence, $k\geq 4$

i.e., m = 4

For k = 4,

$\Rightarrow f(x)=x\sqrt{4x-x^2}$

For max. value, f’(x) = 0

⇒ x = 3

i.e., y = 3√3

Hence, M = 3√3

80. The equation y = sinx sin(x + 2) – $\sin^2$ (x + 1) represents a straight line laying in :

A. First, second and fourth quadrants

B. First , third and fourth quadrants

C. Third and fourth quadrants only

D. Second and third quadrants only

Ans: C

It is given that

y = sin x sin(x + 2) – $\sin^2$(x + 1)

Multiply and dividing the above equation by 2, we get,

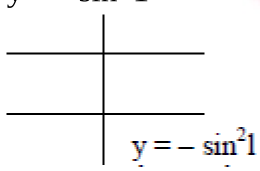
$$y = \frac{1}{2}[2\sin x \sin(x+2)] - \frac{1}{2}[2\sin^2(x+1)]$$

$$y = \frac{1}{2}[\cos(2) - \cos(2x+2)] - \frac{1}{2}[1 - \cos 2(x+1)]$$

$$y = \frac{1}{2}[\cos 2 - 1]$$

$$y = (-)\frac{1}{2}\sin^2 1$$

$$y = -\sin^2 1$$



Thus, the graph of y lies in IIIrd & IVth quadrant.

81. If the truth value of the statement $p \rightarrow (\sim q \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively:

A. T, F, T B. F, T, T

C. T, F, F D. T, T, F

Ans: D

It is given that

$p \rightarrow (\sim q \vee r)$ is false

It is true when

$p \rightarrow T$ & $(\sim q \vee r) =$ false

It will true : ~ q false & r false

$\sim q \rightarrow F \mid r \rightarrow F$

$\Rightarrow q \rightarrow T$

Truth value of p, q, r $\Rightarrow$ T, T, F

82. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is:

A. 2^{21} B. 2^{20}

C. $2^{20} + 1$ D. $2^{20} - 1$

Ans: B

Given, that, out of 31 objects 10 are identical and remaining 21 are distinct, so in following ways, we can choose 10 objects.

0 identical + 10 distincts, number of ways

$= 1 \times {}^{21}C_{10}$

1 identical + 9 distincts, number of ways

$= 1 \times {}^{21}C_{9}$

2 identical + 8 distincts, number of ways

$= 1 \times {}^{21}C_{8}$

10 identicals + 0 distinct, number of ways

$= 1 \times {}^{21}C_{0}$

So, total number of ways in which we can choose 10 objects is

$^{21}C_{10} + {}^{21}C_9 + {}^{21}C_8 + \ldots. + {}^{21}C_0 = x$ (let) …. (i)

$\Rightarrow {}^{21}C_{11} + {}^{21}C_{12} + {}^{21}C_{13} + \ldots. + {}^{21}C_0 = x$ ….(ii)

On adding both equation (i) and (ii) we get

$2x = {}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \ldots. + {}^{21}C_{10} + {}^{21}C_{11} + {}^{21}C_{12} + \ldots + {}^{21}C_{21}$

$\Rightarrow 2x = 2^{21}$

$\Rightarrow x = 2^{20}$

83. For $x \epsilon$ (0, 3/2) let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to :

A. $\tan\frac{11\pi}{12}$

B. $\tan\frac{5\pi}{12}$

C. $\tan\frac{\pi}{12}$

D. $\tan\frac{7\pi}{12}$

Ans: A

Given that

For x ϵ (0, 3/2), functions

$$f(x) = \sqrt{x}$$
$$g(x) = \tan x$$
$$\text{and } h(x) = \frac{1-x^2}{1+x^2}$$

Now,

$$\phi(x) = ((hof)og)(x) = (hof)(g(x))$$
$$= h(f(g(x))) = h(f(\tan x))$$
$$= h\left(\sqrt{\tan x}\right) = \frac{1-\left(\sqrt{\tan x}\right)^2}{1+\left(\sqrt{\tan x}\right)^2}$$
$$= \frac{1-\tan x}{1+\tan x}$$

84. If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3 x 3 matrix A, then the sum of all value of α for which det (A) + 1 = 0, is :

A. 2
B. 1
C. -1
D. 0

Ans: B

Given, matrix B is the inverse matrix of 3 × 3 matrix A.

where $B = \begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & 1 \end{vmatrix}$

If B is inverse of A then

AB = I

det(AB) = det(I)

det(A).det(B) = 1

Given det (A) = –1

then det (B) = –1

$$\begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = -1$$

$5(-2-3) - 2\alpha[0-\alpha] + 1\,[-2\alpha] = -1$

$2\alpha^2 - 2\alpha - 25 = -1$

$2\alpha^2 - 2\alpha - 24 = 0$

$(\alpha - 4)(\alpha + 3) = 0$; $\alpha = 4, -3$

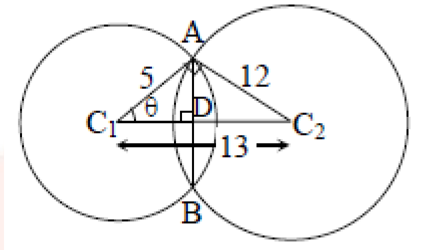
Sum of value of $\alpha = 4 - 3 = 1$

85. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is:

A. $\frac{13}{5}$
B. $\frac{60}{13}$

C. $\frac{13}{2}$ D. $\frac{120}{13}$

Ans: D



In ΔAC_1C_2

$\tan\theta = \frac{12}{5} \Rightarrow \sin\theta = \frac{12}{13}$ ...(i)

In ΔACD:

$\sin\theta = \frac{AB/2}{5}$ ...(ii)

from (i) & (ii)

$\Rightarrow \frac{AB}{2.5} = \frac{12}{13}$

$AB = \frac{120}{13}$

Length of common chord $(AB) = \frac{120}{13}$

86. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec., then the rate (in cm/sec) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

A. $\frac{25}{3}$ B. 25

C. $25\sqrt{3}$ D. $\frac{25}{\sqrt{3}}$

Ans: D

Given, $\frac{dy}{dt} = -25$ at $y = 1$

$x^2 + y^2 = 4$ …(1)

On differentiating both sides of Equation (1) w.r.t. t.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = \frac{xdy}{xdt}$$

At $y = 1;\ x = \sqrt{3}$

$\left(\because\ x^2 + y^2 = 4 \Rightarrow x^2 + 1 = 4 \Rightarrow x = \sqrt{3}\right)$

then

$$\frac{dx}{dt} = \frac{25}{\sqrt{3}}\text{cm / sec}$$

87. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. IF S and S' denote the foci of the hyperbola where S lies on the positive x – axis then P divides SS' in a ratio:

A. 14: 13 B. 5: 4

C. 2: 1 D. 13: 11

Ans: B

Equation of given parabola $y^2 = 12x$ …(i)

and hyperbola $8x^2 – y^2 = 8$ …(ii)

Now, equation of tangent to parabola $y^2 = 12x$

having slope 'm' is $y = mx + 3/m$ …(iii)

and equation of tangent to hyperbola

$\frac{x^2}{1} - \frac{y^2}{8} = 1$ having slope 'm' is

$y = mx \pm \sqrt{1^2 m^2 - 8}$ ...(iv)

Since, tangents (iii) and (iv) represent the same line

$$\therefore\ m^2 - 8 = \left(\frac{3}{m}\right)^2$$

$\Rightarrow m^4 – 8m^2 – 9 = 0$

⇒ $(m^2 - 9)(m^2 + 1) = 0$

⇒ $m = \pm 3$

Now, equation of common tangents to the parabola (i) and hyperbola (ii) are

$y = 3x + 1$ and $y = -3x - 1$

∵ Point 'P' is point of intersection of above common tangents,

∴ P(-1/3, 0)

and focus of hyperbola S(3, 0) and S'(-3, 0).

Thus, the required ratio

$$= \frac{PS}{PS'} = \frac{3+1/3}{3-1/3} = \frac{10}{8} = \frac{5}{4}$$

88. Let S_n denote the sum of the first n terms of an A.P..
If $S_4 = 16$ and $S_6 = -48$, the S_{10} is equal to :

A. – 380 B. – 320

C. – 260 D. – 410

Ans: B

S_n = Sum of n terms of an A.P.

Given, $S_4 = 16 = a + 3d$ …..(i)

and $S_6 = -48 = a + 5d$ …..(ii)

On soving equation (i) & (ii), we get,

d = – 32 a = 112

$$S_{10} = \frac{10}{2}[2.(112) + (10 - 1)(-32)] = 5[-64]$$

$S_{10} = -320$

Thus, the S_{10} is equal to -320.

89. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}\left(1+x+x^2\right)^9$ is:

A. 126 B. -126

C. 84 D. – 84

Ans: C

Given expression is

$(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

$= (1 + x)(1 - x)[(1 - x)(1 + x + x^2)]^9$

$= (1 - x^2)(1 - x^3)^9$

Now, coefficient of x^{18} in the product

$(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

= coefficient of x^{18} in the product

$(1 - x^2)(1 - x^3)^9$

= coefficient of x^{18} in $(1 - x^3)^9$ – coefficient of x^{16} in $(1 - x^3)^9$

Since, (r + 1)th term in the expansion of

$(1 - x^3)^9$ is ${}^9C_r(-x^3)^r = {}^9C_r(-1)^r x^{3r}$

Now, for x^{18}, $3r = 18 \Rightarrow r = 6$

and for x^{16}, $3r = 16$

$\Rightarrow r = \frac{16}{3} \notin N.$

$\therefore$ required coefficient is ${}^9C_6 = \frac{9!}{3!3!} = \frac{9 \times 8 \times 7}{3 \times 2} = 84$

90. Let $f: R \to R$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2) = \frac{1}{48}$. IF $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim_{x \to 2} g(x)$ is equal to:

A. 18 B. 36

C. 12 D. 24

Ans: A

Given that

$$\int_{6}^{f(x)} 4t^3 \cdot dt = (x-2)g(x); f(2) = 6; f'(2) = \frac{1}{48}$$

$$g(x) = \frac{\int_{6}^{f(x)} 4t^3 dt}{(x-2)}$$

$$\lim_{x\to 2} \frac{\int_{6}^{f(x)} 4t^3 dt}{(x-2)}$$

$$\lim_{x\to 2} \frac{4[f(x)]^3.f'(x)}{1}$$

At x = 2,

$$\lim_{x\to 2} g((x)) = 4[f(2)]^3.f'(2)$$

$$\Rightarrow 4(6)^3\left(\frac{1}{48}\right) = 18$$