or equal to k.If both the sides are equal to k for same value of θ,then solution exist and if they are equal for different value of θ,then solution does not exist.

Solution of Inequations:

If $a<b$, then

(i) $(x-a)(x-b)<0 \Rightarrow a<x<b$
(ii) $(x-a)(x-b)\le 0 \Rightarrow a\le x\le b$
(iii) $(x-a)(x-b)>0 \Rightarrow x<a$ or $x>b$
(iv) $(x-a)(x-b)\ge 0 \Rightarrow x\le a$ or $x\ge b$.

EXERCISE-I

CRTQ & SPQ LEVEL-I

PRINCIPAL SOLUTION

C.R.T.Q Class Room Teaching Questions

1. If $4Cos^2\theta=3$ then $\theta=$----
 1) $\frac{\pi}{6},\frac{5\pi}{6}$ 2) $\frac{\pi}{4},\frac{3\pi}{4}$
 3) $\frac{\pi}{3},\frac{2\pi}{3}$ 4) $\pm\frac{\pi}{2}$

2. If $Tan\theta+Sec\theta=\sqrt{3}$, then the principal value of $\left(\theta+\frac{\pi}{6}\right)$ is
 1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{2\pi}{3}$ 4) $\frac{\pi}{2}$

3. If $Tan^2\theta=\sqrt{3}+(\sqrt{3}-1)Tan\theta$ and θ lies in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ then $\theta=$
 1) $\left\{\frac{-\pi}{3},\frac{\pi}{4}\right\}$ 2) $\left\{\frac{\pi}{3},\frac{-\pi}{4}\right\}$
 3) $\left\{\frac{\pi}{6},\frac{-\pi}{3}\right\}$ 4) $\left\{\frac{-\pi}{12},\frac{\pi}{12}\right\}$

4. If $Sin\left(\frac{\pi Cot\theta}{4}\right)=Cos\left(\frac{\pi Tan\theta}{4}\right)$ and θ is in the first quadrant then $\theta=$----
 1) $\frac{\pi}{3}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

5. If $3Tan^4\alpha-10Tan^2\alpha+3=0$ then principal values of 'α' are
 1) $\pm45^0,\pm36^0$ 2) $\pm30^0,\pm60^0$
 3) $\pm75^0,\pm36^0$ 4) $\pm60^0,\pm15^0$

6. If $Sin(x+28^0)=Cos(3x-78^0)$ then $x=$
 1) $37^0, 8^0$ 2) 39^0 3) $35^0, 8^0$ 4) 47^0

7. If $\sin 2\theta-\cos 3\theta=0$ and θ is acute then $\theta=$
 1) 36^0 2) 54^0 3) 18^0 4) 30^0

8. The smallest value of θ satisfying the equation $\sqrt{3}(Cot\theta+Tan\theta)=4$ is
 1) $\frac{2\pi}{3}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{12}$

9. If $3Tan\theta=Cot\theta$ then $\theta=$-----
 1) $\pm30^0$ 2) $\pm60^0$ 3) $\pm45^0$ 4) $\pm15^0$

10. The principal value of $\left(\theta+\frac{\pi}{4}\right)$ where $\sin\theta+\cos\theta=1$ is
 1) 0^0 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

11. If $\cos 2\theta=2\sin^2\theta$ then $\theta=$
 1) $\pm30^0$ 2) $\pm60^0$ 3) $\pm45^0$ 4) $\pm90^0$

12. If $Tan(\pi Cos\theta)=Cot(\pi Sin\theta)$ then the value(s) of $Cos\left(\theta-\frac{\pi}{4}\right)$ is (are)
 1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\pm\frac{1}{2\sqrt{2}}$ 4) $\frac{1}{3\sqrt{2}}$

13. If A and B are acute angles such that $SinA=Sin^2B$ and $2Cos^2A=3Cos^2B$ then A=
 1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

14. If $\sqrt{3}\cos\theta-\sin\theta=1$ then $\theta=$
 1) π 2) $\frac{\pi}{2}$ 3) 1 4) $\frac{\pi}{6}$

15. If $\cot\theta - \tan\theta = 2$ then principal value of θ

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{8}$ 4) $\frac{3\pi}{4}$

S.P.Q. Student Practice Questions

16. If SinA = SinB, CosA = CosB then A =

1) $n\pi + B, \forall n \in Z$ 2) $2n\pi + B, \forall n \in Z$
3) $2n\pi - B, \forall n \in Z$ 4) $2n\pi \pm B, \forall n \in Z$

17. The general solution of $\frac{\tan 5x - \tan 4x}{1 + \tan 5x \tan 4x} = 1$ is

1) $n\pi + \frac{\pi}{4}; \forall n \in Z$ 2) $n\pi \pm \frac{\pi}{4}, \forall n \in Z$
3) ϕ 4) $n\pi + \frac{\pi}{6}, \forall n \in Z$

18. Solution of $Cot^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)Cot\theta + 1 = 0$ is

1) $n\pi - \frac{\pi}{6}, n\pi - \frac{\pi}{3}, \forall n \in Z$
2) $n\pi + \frac{\pi}{6}, n\pi + \frac{\pi}{3}, \forall n \in Z$
3) $n\pi + \frac{\pi}{12}, \forall n \in Z$
4) $n\pi + \frac{\pi}{4}, \forall n \in Z$

19. 3Sinx+4Cosx-6=0 then the general solution of x=

1) $n\pi + (-1)^n \frac{\pi}{6}, \forall n \in Z$
2) $n\pi + (-1)^n \frac{\pi}{4}, \forall n \in Z$
3) $n\pi + (-1)^n \frac{\pi}{3}, \forall n \in Z$
4) Empty Set

20. $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) - \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = 2$ then the general solution of x =

1) $n\pi \pm \frac{\pi}{4}, \forall n \in Z$ 2) $n\pi + \frac{\pi}{4}, \forall n \in Z$
3) $n\pi - \frac{\pi}{4}, \forall n \in Z$ 4) $n\pi, \forall n \in Z$

21. If TanA+Tan2A+$\sqrt{3}$ TanATan2A=$\sqrt{3}$ then the general solution of $\frac{A}{2}$=

1) $\frac{n\pi}{3} + \frac{\pi}{9}, \forall n \in Z$ 2) $n\pi + \frac{\pi}{9}, \forall n \in Z$
3) $\frac{n\pi}{6} + \frac{\pi}{18}, \forall n \in Z$ 4) $\frac{n\pi}{3} + \frac{\pi}{4}, \forall n \in Z$

22. Solution of Tanx+Tan(120°+x)+Tan(x-120°) = 0 is

1) $\frac{n\pi}{3}, \forall n \in Z$ 2) $\frac{n\pi}{6}, \forall n \in Z$
3) $\frac{n\pi}{4}, \forall n \in Z$ 4) $\frac{n\pi}{2}, \forall n \in Z$

23. If $Tanp\theta = Cotq\theta$ then solutions of θ are in

1) A.P 2) G.P 3) H.P 4) A.G.P

24. If Sinx + Cosx = $\sqrt{2}Cos\alpha \Rightarrow$ x = ___

1) $2n\pi - \frac{\pi}{4} \pm \alpha$ 2) $n\pi - \frac{\pi}{4} + \alpha$
3) $2n\pi + \frac{\pi}{4} \pm \alpha$ 4) $n\pi - \frac{\pi}{4} \pm \alpha$

25. The solution of $(\sin x - 2)^2 + (\cos x - 3/2)^2 = 0$ is

1) $x = \pi/2$ 2) $x = \pi$
3) $x = \pi/4$ 4) No solution

26. The values of θ satisfying $Sin5\theta = Sin3\theta - Sin\theta$ and $0 < \theta < \frac{\pi}{2}$ are

1) $\frac{\pi}{6}, \frac{\pi}{3}$ 2) $\frac{\pi}{6}, \frac{\pi}{4}$ 3) $\frac{\pi}{4}, \frac{\pi}{3}$ 4) $\frac{\pi}{4}, \frac{\pi}{2}$

27. If $5Cos2\theta + 2Cos^2(\theta/2) + 1 = 0, \ 0 < \theta < \pi \Rightarrow \theta =$

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{3}, Cos^{-1}\left(\frac{3}{5}\right)$
3) $Cos^{-1}\left(\frac{3}{5}\right)$ 4) $\frac{\pi}{3}, \pi - Cos^{-1}\left(\frac{3}{5}\right)$

28. The number of solutions of the equation $Sin3\alpha = 4Sin\alpha.Sin(x+\alpha).Sin(x-\alpha)$ where $0<\alpha<\pi$ for x in the interval $[0,\pi]$ is

1) 1 2) 2 3) 4 4) 5

29. General solution of $2^{\sin x}+2^{\cos x}=2^{1-\frac{1}{\sqrt{2}}}$ is

1) $n\pi-\frac{\pi}{4}$ 2) $2n\pi+\frac{5\pi}{4}$

3) $n\pi+(-1)^n\frac{\pi}{4}$ 4) $2n\pi\pm\frac{\pi}{4}$

GENERAL SOLUTION

C.R.T.Q Class Room Teaching Questions

30. The most general value of θ satisfying the equations $Sin\theta=\frac{1}{\sqrt{2}}$, $Cos\theta=-\frac{1}{\sqrt{2}}$ is

1) $2n\pi+\frac{\pi}{4},\forall n\in Z$ 2) $2n\pi+\frac{3\pi}{4},\forall n\in Z$

3) $n\pi+\frac{\pi}{6},\forall n\in Z$ 4) $n\pi+\frac{\pi}{3},\forall n\in Z$

31. The general solution of $\frac{Tan2x-Tanx}{1+Tanx.Tan2x}=1$ is

1) $n\pi+\frac{\pi}{4},\forall n\in Z$ 2) $n\pi\pm\frac{\pi}{4},\forall n\in Z$

3) ϕ 4) $n\pi+\frac{\pi}{6},\forall n\in Z$

32. If $11Sin^2x+7Cos^2x=8$ then x =-----

1) $n\pi\pm\frac{\pi}{6},\forall n\in Z$ 2) $n\pi\pm\frac{\pi}{4},\forall n\in Z$

3) $n\pi\pm\frac{\pi}{3},\forall n\in Z$ 4) $n\pi\pm\frac{\pi}{2},\forall n\in Z$

33. The general solution of Sin2x=4Cosx is

1) $(2n+1)\frac{\pi}{2},\forall n\in Z$ 2) $n\pi,\forall n\in Z$

3) Empty Set 4) $2n\pi,\forall n\in Z$

34. $Tan\left(\frac{\pi}{4}+\frac{A}{2}\right)+Tan\left(\frac{\pi}{4}-\frac{A}{2}\right)=\frac{4}{\sqrt{3}}$; then the general solution of A =

1) $2n\pi\pm\frac{\pi}{6},\forall n\in Z$ 2) $n\pi+\frac{\pi}{4},\forall n\in Z$

3) $2n\pi\pm\frac{\pi}{4},\forall n\in Z$ 4) $n\pi,\forall n\in Z$

S.P.Q. Student Practice Questions

35. If Tanx+2Tan2x+4Tan4x+8Cot8x= $\sqrt{3}$ then the general solution of x =

1) $n\pi+\frac{\pi}{3},\forall n\in Z$ 2) $n\pi+\frac{\pi}{6},\forall n\in Z$

3) $n\pi+\frac{\pi}{4},\forall n\in Z$ 4) $n\pi,\forall n\in Z$

36. The general value of θ satisfies the equation $Tan\theta Tan(120^0+\theta)Tan(120^0-\theta)=\frac{1}{\sqrt{3}}$ is

1) $(6n+1)\frac{\pi}{18},\forall n\in Z$

2) $(3n+1)\frac{\pi}{3},\forall n\in Z$

3) $(6n+1)\frac{\pi}{6},\forall n\in Z$

4) $(3n+1)\frac{\pi}{6},\forall n\in Z$

37. If $Tanm\theta=Cotn\theta$ then the G.S of θ=

1) $\frac{(k+1)\pi}{2(m+n)},\forall k\in Z$ 2) $\frac{(2k+1)\pi}{2(m+n)},\forall k\in Z$

3) $\frac{(2k+1)\pi}{m+n},\forall k\in Z$ 4) $\frac{(2k+1)\pi}{m-n},\forall k\in Z$

38. If Cosx+Cosy=1 and CosxCosy=1/4 then the G.S are

1) $x=2n\pi\pm\frac{\pi}{4},n\in Z,y=2m\pi\pm\frac{\pi}{4},m\in Z$

2) $x=2n\pi\pm\frac{\pi}{3},n\in Z,y=2m\pi\pm\frac{\pi}{3},m\in Z$

3) $x=2n\pi\pm\frac{\pi}{6},n\in Z,y=2m\pi\pm\frac{\pi}{6},m\in Z$

4) $x=2n\pi\pm\frac{\pi}{5},n\in Z,y=2m\pi\pm\frac{\pi}{5},m\in Z$

39. If $\sin x \sin\left(60^\circ+x\right)\sin\left(60^\circ-x\right)=\frac{1}{8}$, then $x=$

1) $n\pi+(-1)^n\frac{\pi}{6}$ 2) $\frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$

3) $n\pi+(-1)^n\frac{\pi}{3}$ 4) $\frac{n\pi}{3}+(-1)^n\frac{\pi}{9}$

SOLUTIONS IN THE GIVEN INTERVAL

C.R.T.Q Class Room Teaching Questions

40. The value of θ satisfying $\sin 7\theta=\sin 4\theta-\sin\theta$ in $0<\theta<\frac{\pi}{2}$ are

1) $\frac{\pi}{9},\frac{\pi}{4}$ 2) $\frac{\pi}{3},\frac{\pi}{9}$ 3) $\frac{\pi}{6},\frac{\pi}{4}$ 4) 0,1

41. If $1+\sin^2\theta=3\sin\theta\cos\theta$ then the solution set in $\left(0,\frac{\pi}{2}\right)$ is

1) $\left\{\frac{\pi}{4},\cos^{-1}\frac{1}{3}\right\}$ 2) $\left\{\frac{\pi}{4},\tan^{-1}\frac{1}{2}\right\}$

3) $\left\{\frac{\pi}{3},\tan^{-1}\frac{1}{3}\right\}$ 4) $\left\{\frac{\pi}{6},\sin^{-1}\frac{1}{3}\right\}$

S.P.Q Student Practice Questions

42. The equation $\sqrt{3}\sin x+\cos x=4$ has

1) Only one solution
2) Two solutions
3) Infinitely many solutions 4) No solution

43. If $y+\cos\theta=\sin\theta$ has a real solution then

1) $-\sqrt{2}\le y\le\sqrt{2}$ 2) $y>\sqrt{2}$

3) $y\le-\sqrt{2}$ 4) $y\le 1$

KEY

01) 1	02) 1	03) 2	04) 3	05) 2
06) 3	07) 3	08) 3	09) 1	10) 3
11) 1	12) 3	13) 2	14) 4	
16) 2	17) 1	18) 1	19) 4	
21) 3	22) 1	23) 1	24) 3	
26) 1	27) 4	28) 2	29) 2	
31) 3	32) 1	33) 1	34) 1	
36) 1	37) 2	38) 2	39) 2	
41) 2	42) 4	43) 1		

Hints & Solutions

1. $4\cos^2\theta=3$

$\Rightarrow\cos^2\theta=\left(\frac{\sqrt{3}}{2}\right)^2\Rightarrow\theta=n\pi\pm\frac{\pi}{6}$

2. $\tan\theta+\sec\theta=\sqrt{3}\Rightarrow\sin\theta+1=\sqrt{3}\cos\theta$

$\Rightarrow\sqrt{3}\cos\theta-\sin\theta=1\Rightarrow\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta=\frac{1}{2}$

$\Rightarrow\cos\theta\cos\frac{\pi}{6}-\sin\theta\sin\frac{\pi}{6}=\frac{1}{2}\Rightarrow\cos\left(\theta+\frac{\pi}{6}\right)=\frac{1}{2}$

$\therefore$ P.S. of $\theta+\frac{\pi}{6}=\frac{\pi}{3}$

3. $\tan^2\theta=\sqrt{3}+\left(\sqrt{3}-1\right)\tan\theta$ $\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

on verification $\left\{\frac{\pi}{3},-\frac{\pi}{4}\right\}$ satisfies

4. $\sin\left(\frac{\pi\cot\theta}{4}\right)=\cos\left(\frac{\pi\tan\theta}{4}\right)$

on verification $\theta=\frac{\pi}{4}$ satisfies

5. $3\tan^4\alpha-10\tan^2\alpha+3=0$

$\Rightarrow 3\tan^4\alpha-9\tan^2\alpha-\tan^2\alpha+3=0$

$\Rightarrow\left(3\tan^2\alpha-1\right)\left(\tan^2\alpha-3\right)=0$

$\Rightarrow \tan^2\alpha = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$; $\tan^2\alpha = 3 = \left(\sqrt{3}\right)^2$

$\therefore \alpha = \pm 30^0, \pm 60^0$

6. $3x - 78^0 = 90^0 - \left(x + 28^0\right)$ or $90 + x + 28^0$

$4x = 90^0 - 28^0 + 78^0 = 140^0 \Rightarrow x = \frac{140^0}{4} = 35^0$

$x = 8^0$

7. $\sin 2\theta = \cos 3\theta$

$3\theta = \frac{\pi}{2} - 2\theta \Rightarrow 5\theta = 90^0 \Rightarrow \theta = 18^0$

8. $2\operatorname{cosec}2\theta = \frac{4}{\sqrt{3}}$

$\sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow 2\theta = \frac{\pi}{3}$; $\theta = \frac{\pi}{6}$

9. $3\tan\theta = \cot\theta \Rightarrow \tan^2\theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$ according to options $\pm 30^0$

10. $\sin\theta + \cos\theta = 1$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$,

$\therefore$ P.S of $\theta + \frac{\pi}{4} = \frac{\pi}{4}$

11. $1 - 2\sin^2\theta = 2\sin^2\theta$, $\quad 4\sin^2\theta = 1$

$\sin\theta = \pm\frac{1}{2}$, $\theta = \pm 30^0$

12. $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$\pi\cos\theta + \pi\sin\theta = \frac{\pi}{2} \Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

13. $\sin A = \sin^2 B, 2\cos^2 A = 3\cos^2 B$

$\Rightarrow 2\cos^2 A = 3(1 - \sin A)$

$\Rightarrow 2 - 2\sin^2 A - 3 + 3\sin A = 0$

$\Rightarrow 2\sin^2 A - 3\sin A + 1 = 0$

$\Rightarrow 2\sin^2 A - 2\sin A - \sin A + 1 = 0$

$\Rightarrow (2\sin A - 1)(\sin A - 1) = 0$

$\Rightarrow \sin A = \frac{1}{2}$, $\therefore A = \frac{\pi}{6}$ ($\because A$ is acute)

14. $\theta = \frac{\pi}{6}$ satisfies

15. $\cot\theta - \tan\theta = 2$ on verification $\theta = \frac{\pi}{8}$

satisfies $\left(\cot\frac{\pi}{8} = \sqrt{2} + 1; \tan\frac{\pi}{8} = \sqrt{2} - 1\right)$

16. $\sin A - \sin B = 0; \cos A - \cos B = 0$

$2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) = 0;$

$-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) = 0$

$\therefore \sin\left(\frac{A-B}{2}\right) = 0 \Rightarrow \frac{A-B}{2} = n\pi \Rightarrow A = 2n\pi + B$

17. $\tan(5x - 4x) = 1$

$\Rightarrow \tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}, n \in Z$

18. $\cot^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta + 1 = 0$

$\Rightarrow \cot^2\theta + \sqrt{3}\cot\theta + \frac{1}{\sqrt{3}}\cot\theta + 1 = 0$

$\Rightarrow \cot\theta\left(\cot\theta + \sqrt{3}\right) + \frac{1}{\sqrt{3}}\left(\cot\theta + \sqrt{3}\right) = 0$

$\Rightarrow \left(\cot\theta + \sqrt{3}\right)\left(\cot\theta + \frac{1}{\sqrt{3}}\right) = 0$

$\Rightarrow \cot\theta = -\sqrt{3}; \cot\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}; \tan\theta = -\sqrt{3}$

$\theta = n\pi - \frac{\pi}{6}; n\pi - \frac{\pi}{3}$

19. $3\sin x+4\cos x=6$, Here $|c|>\sqrt{a^2+b^2}$
$\therefore$ There exists no solution

20. $\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)-\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=2\tan x$
$\therefore 2\tan x=2\Rightarrow \tan x=1$
$\Rightarrow x=n\pi+\frac{\pi}{4}, n\in Z$

21. $\tan A+\tan 2A=\sqrt{3}(1-\tan A\tan 2A)$
$\Rightarrow \frac{\tan 2A+\tan A}{1-\tan 2A\tan A}=\sqrt{3}\Rightarrow \tan 3A=\sqrt{3}$
$\Rightarrow 3A=n\pi+\frac{\pi}{3}\Rightarrow A=\frac{n\pi}{3}+\frac{\pi}{9}\Rightarrow \frac{A}{2}=\frac{n\pi}{6}+\frac{\pi}{18}$

22. $\tan x+\tan(120^0+x)+\tan(x-120^0)=0$
$\Rightarrow 3\tan 3x=0\Rightarrow \tan 3x=0$
$\Rightarrow 3x=n\pi\Rightarrow x=\frac{n\pi}{3}$

23. $\tan p\theta=\tan\left(\frac{\pi}{2}-q\theta\right)$, The solutions are in A.P

24. $\cos\left(x-\frac{\pi}{4}\right)=\cos\alpha$
$\Rightarrow x-\frac{\pi}{4}=2n\pi\pm\alpha\Rightarrow x=2n\pi+\frac{\pi}{4}\pm\alpha$

25. sinx-2=0, cosx-3/2=0
but $\sin x\neq 2$, $\cos x\neq\frac{3}{2}$

26. $\sin 5\theta+\sin\theta=\sin 3\theta$
$\Rightarrow 2\sin 3\theta.\cos 2\theta=\sin 3\theta$
$\Rightarrow \sin 3\theta=0$ or $\cos 2\theta=\frac{1}{2}, \frac{\pi}{6}, \frac{\pi}{3}$ satisfies

27. $5(2\cos^2\theta-1)+2\left(\frac{1+\cos\theta}{2}\right)+1=0$
$10\cos^2\theta-5\cos\theta+6\cos\theta-3=0$
$(5\cos\theta+3)(2\cos\theta-1)=0$
$\cos\theta=-\frac{3}{5}$, $\cos\theta=\frac{1}{2}$,
$\theta=\frac{\pi}{3}, \pi-\cos^{-1}\left(\frac{3}{5}\right)$

28. $3\sin\alpha-4\sin^3\alpha=4\sin\alpha(\sin^2x-\sin$
$\Rightarrow 3\sin\alpha-4\sin^3\alpha=4\sin\alpha\sin^2x-4\sin$
$\Rightarrow 4\sin\alpha\sin^2x=3\sin\alpha\Rightarrow \sin^2x=\frac{3}{4}=\Big($
$\Rightarrow x=n\pi\pm\frac{\pi}{6}, n\in z$ $\quad x=\left\{\frac{\pi}{6},\frac{5\pi}{6}\right\}$

29. The given equality holds for minmum of $\sin x+\cos x$ i.e., $-\sqrt{2}$
$\therefore x=\frac{5\pi}{4}$ G.S is $2n\pi+\frac{5\pi}{4}$

30. $\sin\theta=\frac{1}{\sqrt{2}};\cos\theta=-\frac{1}{\sqrt{2}}$
$\therefore \theta\in$ II quadrant
$\therefore$ P.S. of $\theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$
G.S. of $\theta=2n\pi+\frac{3\pi}{4}, \forall n\in Z$

31. $\tan(3x-2x)=1\Rightarrow \tan x=1\Rightarrow x=\frac{\pi}{4}$
But for $x=\frac{\pi}{4}, 2x=\frac{\pi}{2}\Rightarrow \tan 2x$ is undefined $\therefore \exists$ no solutions

32. $11\sin^2x+7\cos^2x=8\Rightarrow 4\sin^2x+7=8$
$\Rightarrow 4\sin^2x=1\Rightarrow \sin^2x=\frac{1}{4}=\left(\frac{1}{2}\right)^2$
$\therefore x=n\pi\pm\frac{\pi}{6}$

33. $\sin 2x=4\cos x\Rightarrow 2\sin x\cos x=4\cos x$
$\Rightarrow \cos x=0$ or $\sin x=2$ (which is not possible), $\therefore x=(2n+1)\frac{\pi}{2}, n\in Z$

34. $\tan\left(\frac{\pi}{4}+\frac{A}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{A}{2}\right)=\frac{4}{\sqrt{3}}$
$\Rightarrow \tan\left(\frac{\pi}{4}+\frac{A}{2}\right)+\cot\left(\frac{\pi}{4}+\frac{A}{2}\right)=\frac{4}{\sqrt{3}}$

$\Rightarrow 2\operatorname{cosec}\left(2\left(\frac{\pi}{4}+\frac{A}{2}\right)\right)=\frac{4}{\sqrt{3}}(\because \tan\theta+\cot\theta=2\operatorname{cosec}2\theta)$

$\Rightarrow 2\operatorname{cosec}\left(\frac{\pi}{2}+A\right)=\frac{4}{\sqrt{3}}\Rightarrow \sec A=\frac{2}{\sqrt{3}}\Rightarrow \cos A=\frac{\sqrt{3}}{2}$

$\Rightarrow A=2n\pi\pm\frac{\pi}{6},\forall n\in Z$

35. $\tan x+2\tan 2x+4\tan 4x+8\cot 8x=\sqrt{3}$

$\cot x-8\cot 8x+8\cot 8x=\sqrt{3}\Rightarrow \cot x=\sqrt{3}$

$\Rightarrow x=n\pi+\frac{\pi}{6},n\in Z$

36.

$\tan\theta\tan\left(120^0+\theta\right)\tan\left(120^0-\theta\right)=\frac{1}{\sqrt{3}}$

$\Rightarrow \tan 3\theta=\frac{1}{\sqrt{3}}\Rightarrow 3\theta=n\pi+\frac{\pi}{6}\Rightarrow \theta=\frac{n\pi}{3}+\frac{\pi}{18}$

$=\left(n+\frac{1}{6}\right)\frac{\pi}{3}=(6n+1)\frac{\pi}{18}$

37. $\tan m\theta=\cot n\theta=\tan\left(\frac{\pi}{2}-n\theta\right)$

$m\theta=K\pi+\frac{\pi}{2}-n\theta\Rightarrow\theta=\frac{K\pi+\frac{\pi}{2}}{m+n}=\frac{(2K+1)\pi}{2(m+n)}$

38. $\cos x+\cos y=1;\cos x\cos y=\frac{1}{4}$

on verification $x=y=\frac{\pi}{3}$ satisfies

39. $\frac{1}{4}\sin 3x=\frac{1}{8}\Rightarrow 3x=n\pi+(-1)^n\frac{\pi}{6}$

40. $\sin 7\theta+\sin\theta=\sin 4\theta$

$2\sin 4\theta.\cos 3\theta=\sin 4\theta$

$\sin 4\theta=0,\qquad \cos 3\theta=\frac{1}{2}$

$\theta=\frac{\pi}{4}\qquad 3\theta=\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{9}$

41. Divide both sides with $\cos^2\theta$

$\sec^2\theta+\tan^2\theta=3\tan\theta$

$\Rightarrow 2\tan^2\theta-3\tan\theta+1=0$

$\Rightarrow(2\tan\theta-1)(\tan\theta-1)=0$

$\Rightarrow \tan\theta=1$ or $\tan\theta=\frac{1}{2}$

$\Rightarrow\theta=\frac{\pi}{4}$ or $\theta=\tan^{-1}\left(\frac{1}{2}\right)$

42. $\sqrt{3}\sin x+\cos x=4$ (no solution)

[$a\cos x+b\sin x=c$ has a solution

if $-\sqrt{a^2+b^2}\le c\le\sqrt{a^2+b^2}$]

43. $y=\sin\theta-\cos\theta$ It has solutions if

$|c|\le\sqrt{a^2+b^2}\Rightarrow-\sqrt{2}\le y\le\sqrt{2}$

EXERCISE-II

CRTQ & SPQ LEVEL-II

PRINCIPAL SOLUTION

C.R.T.Q Class Room Teaching Questions

1. If $\cos 2\theta.\cos 3\theta.\cos\theta=\frac{1}{4}$ for $0<\theta<\pi$, then $\theta=$

1) $\frac{\pi}{7},\frac{5\pi}{7},\pi$ 2) $\frac{\pi}{2},\frac{\pi}{4},\frac{3\pi}{4},\frac{\pi}{6},\frac{5\pi}{6}$

3) $\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8},\frac{\pi}{3},\frac{2\pi}{3}$ 4) $\frac{\pi}{9},\frac{4\pi}{9},\frac{5\pi}{9}$

2. If α and β are two different solutions lying between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ of the equation $2\tan\theta+\sec\theta=2$ then $\tan\alpha+\tan\beta$ is

1) 0 2) 1 3) 4/3 4) 8/3

3. If $\gamma\sin\theta=3,\gamma=4(1+\sin\theta),0\le\theta\le 2\pi$ then $\theta=$ ----

1) $\frac{\pi}{6},\frac{5\pi}{6}$ 2) $\frac{\pi}{3},\frac{2\pi}{3}$

3) $\frac{\pi}{4},\frac{5\pi}{4}$ 4) $\frac{\pi}{2},\pi$

4. If $32Tan^8\theta = 2Cos^2\alpha - 3Cos\alpha$ and $3Cos2\theta = 1$ then the general value of 'α' is

1) $n\pi \pm \frac{\pi}{3}$ 2) $2n\pi \pm \frac{2\pi}{3}$

3) $n\pi \pm \pi$ 4) $2n\pi \pm \frac{\pi}{2}$

5. If $Cos20^0 = k$ and $Cosx = 2k^2-1$, then the possible values of x between 0^0 and 360^0 are

1) 140^0 2) 40^0 and 140^0

3) 40^0 and 320^0 4) 50^0 and 130^0

6. The equation $(Cosp - 1)x^2 + (Cosp)x + Sinp = 0$, where x is a variable with real roots. then the interval of p may be any one of the following.

1) $(0,2\pi)$ 2) $(-\pi,0)$

3) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ 4) $(0,\pi]$

7. In a right angled triangle, the hypotenuse is $2\sqrt{2}$ times the length of the perpendicular drawn from the opposite vertex to the hypotenuse. Then the other two angles are

1) $\frac{\pi}{3},\frac{\pi}{6}$ 2) $\frac{\pi}{4},\frac{\pi}{4}$ 3) $\frac{\pi}{8},\frac{3\pi}{8}$ 4) $\frac{\pi}{12},\frac{5\pi}{12}$

8. Number of solutions of the equation Tan x+Secx = 2 Cos x in the interval $[0,2\pi]$ is

1) 0 2) 1 3) 2 4) 3

9. The number of integral values of k for which the equation 7 cosx+ 5 sinx=2k +1 has a solution is

1) 4 2) 8 3) 10 4) 12

10. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

1) 0 2) 2 3) 1 4) 3

11. If the solutions for θ $Cosp\theta + Cosq\theta = 0$, $p > q > 0$ are in ... then numerically smallest com... difference of A.P is

1) $\frac{\pi}{p+q}$ 2) $\frac{2\pi}{p+q}$

3) $\frac{\pi}{2(p+q)}$ 4) $\frac{1}{p+q}$

S.P.Q. Student Practice Questions

12. The solution set of Tan(4k+... −Tan(4k+1)x− Tan(4k+2)xTan(4k+1)... 1, $k \in Z$ is

1) ϕ 2) $\frac{\pi}{4}$

3) $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$ 4) $\left\{2n\pi + \frac{\pi}{4} : n \in Z\right.$

13. The general solution of the equation $Sin^2\theta Sec\theta + \sqrt{3}Tan\theta = 0$ is

1) $\theta = n\pi + (-1)^n \frac{\pi}{3}, \forall n \in Z$

2) $\theta = n\pi, \forall n \in Z$

3) $\theta = n\pi + (-1)^n \frac{\pi}{4}, \forall n \in Z$

4) $\theta = \frac{n\pi}{6}, \forall n \in Z$

14. $Cos^3\alpha + Cos^3(120^0 + \alpha) + Cos^3(120^0 - \alpha) = \frac{3\sqrt{3}}{4}$ then the general solution of α is

1) ϕ 2) $2n\pi \pm \frac{\pi}{3}, \forall n \in Z$

3) $(2n+1)\frac{\pi}{2}, \forall n \in Z$ 4) $n\pi, \forall n \in Z$

15. The general solution of $\sin^{100}x - \cos^{100}x = 1$ is

1) $2n\pi + \frac{\pi}{3}, n \in I$ 2) $n\pi + \frac{\pi}{4}, n \in I$

3) $n\pi + \frac{\pi}{2}, n \in I$ 4) $2n\pi - \frac{\pi}{3}, n \in I$

16. The equation $4Sin^2x + 4Sinx + a^2 - 3 = 0$ has a solution if
1) $-2 \le a \le 2$ 2) $-1 \le a \le 1$
3) $-3 \le a \le 3$ 4) $-4 \le a \le 4$

17. If the complex numbers $Sinx + i\,Cos2x$ and $Cosx - i\,Sin2x$ are conjugate to each other, then the set of values of x =
1) $n\pi$ 2) $(2n+1)\frac{\pi}{2}$ 3) $\{0\}$ 4) ϕ

18. Given that tanA, tanB are the roots of the equation $x^2 - bx + c = 0$, the value of $\sin^2(A+B)$ is
1) $\frac{b}{(b+c)^2}$ 2) $\frac{b^2}{b^2+c^2}$
3) $\frac{b^2}{c^2+(1-b^2)}$ 4) $\frac{b^2}{b^2+(1-c)^2}$

19. The smallest positive x satisfying $\log_{Cosx} Sinx + \log_{Sinx} Cosx = 2$ is
1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

20. The number of solutions of the equation $2^{\cos x} = |\sin x|$ in $[-2\pi, 2\pi]$
1) 1 2) 4 3) 3 4) 8

21. If the equation $k\cos x - 3\sin x = k+1$ has a solution for 'x' then
1) $K \le 4$ 2) $K \ge 4$
3) $1 \le k \le 6$ 4) $0 \le k \le 8$

SOLUTIONS IN THE GIVEN INTERVAL

C.R.T.Q Class Room Teaching Questions

22. If $Cos6\theta + Cos4\theta + Cos2\theta + 1 = 0$ for $0 \le \theta \le \pi$, then θ =
1) $\frac{\pi}{7}, \frac{5\pi}{7}, \pi$ 2) $\frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6}$
3) $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{\pi}{3}, \frac{2\pi}{3}$ 4) $\frac{\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}$

23. If $\alpha, \beta, \gamma, \delta$ are the four solutions of the equation $Tan\left(\theta + \frac{\pi}{4}\right) = 3Tan\,3\theta$. No two of which have equal tangents, then the value of $Tan\alpha + Tan\beta + Tan\gamma + Tan\delta$ =
1) 1 2) 0 3) -1 4) 4

24. If $0 < x, y < \frac{\pi}{2}$ then the system of equations Sinx.Siny = 3/4 and Tanx.Tany = 3 has a solution at
1) $x = \frac{\pi}{6}, y = \frac{\pi}{6}$ 2) $x = \frac{\pi}{3}, y = \frac{\pi}{3}$
3) $x = \frac{\pi}{12}, y = \frac{\pi}{12}$ 4) $x = \frac{\pi}{4}, y = \frac{\pi}{4}$

25. The solution set of $Sin\left(x + \frac{\pi}{4}\right) = Sin2x$
1) $\frac{n\pi - \left(\frac{\pi}{4}\right)}{1+(-1)^n 2}$ 2) $\frac{n\pi + \left(\frac{\pi}{4}\right)}{1+(-1)^n 2}$
3) $\frac{n\pi + \left(\frac{\pi}{4}\right)}{1-(-1)^n 2}$ 4) $\frac{n\pi - \left(\frac{\pi}{4}\right)}{1-(-1)^n 2}$

26. In a ΔABC, the angle A is greater than angle B. If the values of angles A and B satisfy the equation $3Sinx - 4Sin^3x - k = 0, 0 < k < 1$, then the measure of angle C =
1) $\frac{\pi}{3}$ 2) $\frac{\pi}{2}$ 3) $\frac{2\pi}{3}$ 4) $\frac{5\pi}{6}$

27. The equation 8Cosx.Cos2x.Cos4x = $\frac{Sin6x}{Sinx}$ has a solution if
1) sinx = 0 2) Cos7x = 0
3) sin7x = 0 4) sin8x = 0

28. If $x \in (-\pi, \pi)$ such that $y = 1 + |\cos x| + |\cos^2 x| + |\cos^3 x| + ...$ and $8^y = 64$, then no.of values of x is
1) 1 2) 2 3) 3 4) 4

29. If a is any real number, the number of roots of Cotx - Tanx = a in the first quardrant is
1) 2 2) 0 3) 1 4) 4

30. The number of roots of the equation $x^3+x^2+2x+\sin x=0$ in $(-2\pi,2\pi)$
1) 4 2) 3 3) 2 4) 1

31. If $\begin{vmatrix} \cos(A+B) & -\sin(A+B) & \cos 2B \\ \sin A & \cos A & \sin B \\ -\cos A & \sin A & \cos B \end{vmatrix}=0$ then B =
1) $(2n+1)\frac{\pi}{2}$ 2) $n\pi$
3) $(2n+1)\pi$ 4) $2n\pi$

32. If 1/6 sinx, cosx, tan x are in G.P. then x =
1) $n\pi\pm\frac{\pi}{3}, n\in z$ 2) $2n\pi\pm\frac{\pi}{3}, n\in z$
3) $n\pi\pm(-1)^n\frac{\pi}{3}, n\in z$ 4) $n\pi\pm\frac{\pi}{6}, n\in z$

S.P.Q. Student Practice Questions

33. The sum of the solutions of the equaiton $\tan x.\tan 4x=1$ for $0<x<\pi$ is
1) 10π 2) $\frac{3\pi}{2}$ 3) $\frac{5\pi}{2}$ 4) 2π

34. If $Tan^2 2\theta = Cot^2\alpha$ then the general solution is
1) $\theta=\left(\frac{1}{4}\right)\left\{n\pi\pm\left(\frac{\pi}{2}-\alpha\right)\right\}, \forall n\in Z$
2) $\theta=\left(\frac{1}{2}\right)\left\{n\pi\pm\left(\frac{\pi}{4}-\alpha\right)\right\}, \forall n\in Z$
3) $\theta=\left(\frac{1}{2}\right)\left\{n\pi\pm\left(\frac{\pi}{2}-\alpha\right)\right\}, \forall n\in Z$
4) $\theta=\left(\frac{1}{4}\right)\left\{n\pi\pm\left(\frac{\pi}{2}+\alpha\right)\right\}, \forall n\in Z$

35. If Sinx + Cosx = 1 + Sinx.Cosx then x = ---
1) $n\pi+\frac{\pi}{3}$ 2) $n\pi+(-1)^n\frac{\pi}{6}$
3) $n\pi+(-1)^n\frac{\pi}{2}\cup 2n\pi$ 4) $n\pi$

36. The inequation $3^{\sin^2\theta}+3^{\cos^2\theta}\geq 2\sqrt{3}$ h...
1) for all real values of θ
2) some real values of θ
3) for imaginary values of θ
4) for no value of θ

37. The set of values of a for which equa-tion $Sin^4x + Cos^4x = a$ ha... solution is
1) $\left[\frac{-1}{2},\frac{1}{2}\right]$ 2) $\left[\frac{1}{4},1\right]$
3) $\left[\frac{1}{2},1\right]$ 4) $\left[\frac{1}{4},\frac{1}{2}\right]$

38. If Sin2x.Cos2x.Cos4x = γ has a solu... then γ lies in the interval
1) $\left[-\frac{1}{2},\frac{1}{2}\right]$ 2) $\left[-\frac{1}{4},\frac{1}{4}\right]$
3) $\left[-\frac{1}{3},\frac{1}{3}\right]$ 4) $[-1,1]$

39. If α and β satisfying the equation $Sin\alpha+Sin\beta=\sqrt{3}(Cos\alpha-Cos\beta)$, then
1) $Sin3\alpha+Sin3\beta=1$ 2) $Sin3\alpha+Sin3\beta=$...
3) $Sin3\alpha-Sin3\beta=0$ 4) $Sin3\alpha-Sin3\beta=$...

40. If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number and $0<\theta<2\pi$, then θ =
1) π 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/6$

41. The number of solutions of the equatio... $Sinx = x$ is
1) 3 2) 1 3) 0 4) 2

42. The smallest positive root of the equaiton $\tan x-x=0$ is in
1) $\left(0,\frac{\pi}{2}\right)$ 2) $\left(\pi,\frac{3\pi}{2}\right)$
3) $\left(\frac{\pi}{2},\pi\right)$ 4) $\left(\frac{3\pi}{2},2\pi\right)$

KEY

01) 3	02) 4	03) 1	04) 2	05) 3
06) 4	07) 3	08) 3	09) 2	10) 3
11) 2	12) 1	13) 2	14) 1	15) 3
16) 1	17) 4	18) 4	19) 3	20) 4
21) 1	22) 2	23) 2	24) 2	25) 4
26) 3	27) 2	28) 4	29) 3	30) 4
31) 1	32) 2	33) 4	34) 3	35) 3
36) 1	37) 3	38) 2	39) 2	40) 1
41) 2	42) 2			

Hints & Solutions

1. $4\cos 2\theta \cos 3\theta \cos\theta = 1 \quad 0<\theta<\pi$

$2\cos 2\theta(\cos 4\theta + \cos 2\theta) - 1 = 0$

$\Rightarrow 2\cos 4\theta \cos 2\theta + 2\cos^2 2\theta - 1 = 0$

$\Rightarrow 2\cos 4\theta \cos 2\theta + \cos 4\theta = 0$

$\Rightarrow \cos 4\theta = 0$ or $\cos 2\theta = -\frac{1}{2}$

$4\theta = (2n+1)\frac{\pi}{2}, 2\theta = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{8}, \theta = n\pi \pm \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}; \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

2. $\sec\theta = 2(1-\tan\theta)$

$1+\tan^2\theta = 4(1-2\tan\theta+\tan^2\theta)$

$\Rightarrow 3\tan^2\theta - 8\tan\theta + 3 = 0 \Rightarrow \tan\alpha + \tan\beta = \frac{8}{3}$

3. $\gamma\sin\theta = 3, \gamma = 4(1+\sin\theta), 0\le\theta\le 2\pi$

on verification with the options $\frac{\pi}{6}, \frac{5\pi}{6}$ satisfies

4. $3\cos 2\theta = 1 \Rightarrow \frac{3(1-\tan^2\theta)}{1+\tan^2\theta} = 1 \Rightarrow \tan^2\theta = \frac{1}{2}$

$32\left(\frac{1}{2}\right)^4 = 2\cos^2\alpha - 3\cos\alpha$

$2\cos^2\alpha - 3\cos\alpha - 2 = 0$

$\Rightarrow (2\cos\alpha+1)(\cos\alpha-2) = 0 \Rightarrow \cos\alpha = \frac{-1}{2}$

$\alpha = 2n\pi \pm \frac{2\pi}{3}$

5. $\cos 20^0 = K; \cos x = 2K^2 - 1$

$= 2\cos^2 20^0 - 1 = \cos 40^0$

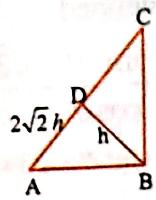
on verification $x = 40^0$ & 320^0 satisfies

6. $\Delta \ge 0 \Rightarrow \cos^2 p - 4\sin p(\cos p - 1) \ge 0$

$\Rightarrow \cos^2 p + 4\sin p(1-\cos p) \ge 0$

$\cos^2 p \ge 0$ and $1-\cos p \ge 0$ for all values of p and $\sin p \ge 0$ for $p \in (0,\pi]$

7. In $\Delta ABD, \sin A = \frac{h}{AB}$



In $\Delta ABC, \cos A = \frac{AB}{2\sqrt{2}h}$

$\frac{h}{\sin A} = 2\sqrt{2}h\cos A \Rightarrow \sin 2A = \frac{1}{\sqrt{2}}$

$\Rightarrow A = \frac{\pi}{8}, B = \frac{3\pi}{8}$

8. $\frac{\sin x + 1}{\cos x} = 2\cos x \qquad (\cos x \ne 0)$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0,$

$\Rightarrow \sin x = \frac{1}{2}$

but $\sin x + 1 \ne 0 \because \cos x \ne 0)$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$ in $[0, 2\pi]$

9. $-\sqrt{49+25} \le 2k+1 \le \sqrt{49+25}$

$-1-\sqrt{74} \le 2k \le -1+\sqrt{74}$

$$\frac{-1-\sqrt{74}}{2} \le k \le \frac{-1+\sqrt{74}}{2}$$

k=-4,-3,-2,-1,0,1,2,3

10. $(\cos x-\sin x)(2\cos^2 x-\sin x\cos x-\sin^2 x)=0$

$\sin x=\cos x$

11. $2\cos\left(\frac{p+q}{2}\right)\theta.\cos\left(\frac{p-q}{2}\right)\theta=0$

$\left(\frac{p+q}{2}\right)\theta=(2n+1)\frac{\pi}{2}$

$\theta=(2n+1)\frac{\pi}{p+q}$, $\quad c.d.=\frac{2\pi}{p+q}$

12. $\frac{\tan(4k+2)x-\tan(4k+1)x}{1+\tan(4k+2)x\tan(4k+1)x}=1$

$\Rightarrow \tan x=1 \Rightarrow x=\frac{\pi}{4}, \Rightarrow x=\frac{\pi}{4}$

But for $x=\frac{\pi}{4}, \tan(4K+2)x$ is not defined

13. $\frac{\sin^2\theta}{\cos\theta}+\frac{\sqrt{3}\sin\theta}{\cos\theta}=0 \ (\cos\theta\neq 0)$

$\Rightarrow \sin^2\theta+\sqrt{3}\sin\theta=0 \Rightarrow \sin\theta(\sin\theta+\sqrt{3})=0$

$\sin\theta=0 \Rightarrow \therefore \theta=n\pi$

$\sin\theta=-\sqrt{3}$ which is not possible

14. $\frac{3}{4}\cos 3\alpha=\frac{3\sqrt{3}}{4} \Rightarrow \cos 3\alpha=\sqrt{3}(>1)$

$\therefore$ There exists no solution

15. $\sin^{100}x=1+\cos^{100}x$

As $\sin^{100}x$ cannot be >1

$\Rightarrow \cos x=0 \Rightarrow x=(2n+1)\frac{\pi}{2}=n\pi+\frac{\pi}{2}, n\in Z$

16. $\sin x=\frac{-4\pm\sqrt{16-16(a^2-3)}}{8}$

$-1\le\frac{-1\pm\sqrt{4-a^2}}{2}\le 1$

$-2\le -1\pm\sqrt{4-a^2}\le 2$

$0\le\sqrt{4-a^2}\le 3$

$4-a^2\ge 0 \Rightarrow a\in[-2,2]$

17. $\sin x-i\cos 2x=\cos x-i\sin 2x$

$\Rightarrow \sin x=\cos x$ *and* $\cos 2x=\sin 2x$

No 'x' satisfies both the above equations simultaneously

18. $\tan A+\tan B=b; \tan A\tan B=c$

$\tan(A+B)=\frac{b}{1-c} \Rightarrow \sin^2(A+B)=\left(\frac{b}{\sqrt{b^2+(1-c)^2}}\right)^2$

19. $\sin x=\cos x$

20. Draw the graphs of,

$y=2^{\cos x}$ and $y=|\sin x|$

No. of points of intersection is 8.

21. $\frac{|k+1|}{\sqrt{k^2+9}}\le 1$

22. $\cos 6\theta+\cos 4\theta+\cos 2\theta+1=0$

$0\le\theta\le\pi$

$\Rightarrow(\cos 6\theta+\cos 2\theta)+(1+\cos 4\theta)=0$

$\Rightarrow 2\cos 4\theta\cos 2\theta+2\cos^2 2\theta=0$

$\Rightarrow 2\cos 2\theta(\cos 4\theta+\cos 2\theta)=0$

$\Rightarrow 4\cos 2\theta\cos 3\theta\cos\theta=0$

$\theta=(2n+1)\frac{\pi}{2}$ or $2\theta=(2n+1)\frac{\pi}{2}$ or $3\theta=(2n+1)\frac{\pi}{2}$

$\theta=\frac{\pi}{2}; \theta=\frac{\pi}{4},\frac{3\pi}{4}; \theta=\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6}$

23. $\frac{1+\tan\theta}{1-\tan\theta}=3\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

$\Rightarrow 3\tan^4\theta-6\tan^2\theta+8\tan\theta-1=0$

$\tan\alpha+\tan\beta+\tan\gamma+\tan\delta=0$

24. $\sin x\sin y=3\cos x\cos y$

$\cos x.\cos y=\frac{1}{4}$; $\sin x.\sin y=\frac{3}{4}$

$\cos(x+y)=\frac{1}{4}-\frac{3}{4}=-\frac{1}{2}$

$\cos(x-y)=\frac{4}{4}=1$

$x-y=0, x+y=\frac{2\pi}{3}, \quad x=y=\frac{\pi}{3}$

25. $\left(x+\frac{\pi}{4}\right)=n\pi+(-1)^n 2x$

$x\left(1-2(-1)^n\right)=n\pi-\frac{\pi}{4}, x=\frac{n\pi-\frac{\pi}{4}}{1-2(-1)^n}$

26. $\sin 3x=K \quad 0<K<1$

$\sin 3A=\sin(\pi-3B) \quad (\because A>B)$

$3A+3B=\pi$

$A+B=\frac{\pi}{3} \qquad C=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

27. $\sin 8x=\sin 6x, \sin x\neq 0$

$2\cos 7x.\sin x=0, \Rightarrow \cos 7x=0$

28. $y=\frac{1}{1-|\cos x|}, \quad 8^{\frac{1}{1-|\cos x|}}=64=8^2$

$1-|\cos x|=\frac{1}{2}, |\cos x|=\frac{1}{2}\Rightarrow \cos x=\pm\frac{1}{2}$

29. $2\cot 2x=a \Rightarrow \cot 2x=\frac{a}{2}$

From graph, $y=\frac{a}{2}$ intersection at only one point with $y=\cot 2x$

30. $x^3+x^2+2x+\sin x=0$
only x = 0 satisfies the given equation

31. $\cos(A+B)[\cos(A+B)]+\sin(A+B)$

$[\sin(A+B)]+\cos 2B(1)=0$

$\Rightarrow 1+\cos 2B=0, \; B=(2n+1)\frac{\pi}{2}$

32. $6\cos^2 x=\frac{\sin^2 x}{\cos x}$

$6\cos^3 x+\cos^2 x-1=0$

$\cos x=\frac{1}{2}, \; x=2n\pi\pm\frac{\pi}{3}$

33. $\sin x\sin 4x-\cos x\cos 4x=0$

$\Rightarrow \cos(x+4x)=0\Rightarrow \cos 5x=0$

$\Rightarrow 5x=(2n+1)\frac{\pi}{2}$

$\Rightarrow x=\frac{\pi}{10},\frac{3\pi}{10},\frac{7\pi}{10},\frac{9\pi}{10}$ but $x\neq\frac{5\pi}{10}$

$\therefore$ sum of solutions 2π

34. $\tan^2 2\theta=\cot^2\alpha=\tan^2\left(\frac{\pi}{2}-\alpha\right)$

$\therefore 2\theta=n\pi\pm\left\{\frac{\pi}{2}-\alpha\right\}\Rightarrow \theta=\frac{1}{2}\left\{n\pi\pm\left(\frac{\pi}{2}-\alpha\right)\right\}$

35. $\sin x+\cos x-\sin x\cos x-1=0$

$(\sin x-1)(1-\cos x)=0$

$\sin x=1$ or $\cos x=1$

$x=n\pi+(-1)^n\frac{\pi}{2}$ or $x=2n\pi$

36. $3^{\sin^2\theta}=a, \quad A.M\geq G.M$

$\frac{a+\frac{3}{a}}{2}\geq\sqrt{a\times\frac{3}{a}}, \; a+\frac{3}{a}\geq 2\sqrt{3}$

37. $\sin^4 x+\cos^4 x=a$

$\Rightarrow 1-2\sin^2 x\cos^2 x=a$

$\Rightarrow 1-\frac{1}{2}\sin^2 2x=a$

$0\leq\sin^2 2x\leq 1\Rightarrow 0\leq\frac{1}{2}\sin^2 2x\leq\frac{1}{2}$

$\frac{1}{2}\leq 1-\frac{1}{2}\sin^2 2x\leq 1, \quad \therefore a\in\left[\frac{1}{2},1\right]$

38. $\sin 4x\cos 4x=2\gamma$

$\sin 8x=4\gamma \Rightarrow -1\leq 4\gamma\leq 1$

$\Rightarrow -\frac{1}{4}\leq\gamma\leq\frac{1}{4}$

39. $\frac{\sin\alpha+\sin\beta}{\cos\alpha-\cos\beta}=\sqrt{3}\Rightarrow \cot\left(\frac{\alpha-\beta}{2}\right)=-\sqrt{3}$

$\Rightarrow\frac{\alpha-\beta}{2}=\frac{5\pi}{6}\Rightarrow 3\alpha-3\beta=5\pi$

$\sin 3\alpha=\sin(5\pi+3\beta)$

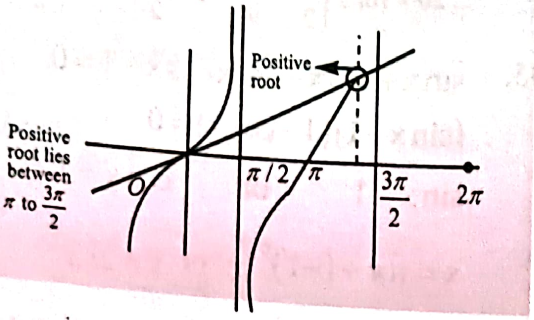
40. $\frac{(3+2i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$ is real

Imaginary part=0, $6\sin\theta+2\sin\theta=0$

$\sin\theta=0\Rightarrow\theta=\pi$

41. y=x, y=sinx
From the graph, the only one point of intersection

42. Draw the graph of $y=\tan x$ and $y=x$



EXERCISE-III
CRTQ & SPQ LEVEL-III

SOLUTIONS IN THE GIVEN INTERVAL

C.R.T.Q Class Room Teaching Questions

1. The number of points of intersection of the two curves $y=2\sin x$ and $y=5x^2+2x+3$ is
   1) 0 2) 1 3) 2 4) ∞

2. A solution (x,y) of $x^2+2x\sin(xy)+1=0$ is
   1) $(1,0)$ 2) $(1,7\pi/2)$
   3) $(-2,7\pi/2)$ 4) $(-1,0)$

3. The solution set of equation $\cos^5 x=1+\sin^4 x$ is
   1) $\{n\pi, n\in I\}$ 2) $\{2n\pi, n\in I\}$
   3) $\{4n\pi, n\in I\}$ 4) $\{n\pi/2, n\in I\}$

4. Number of solution (s) of the equation $\sin x=[x]$ [where$[\bullet]$ denotes greatest integer function is
   1) 1 2) 2
   3) 0 4) infinite

5. The roots of the equation $\cos^7 x+\sin^4 x=1$ in the interval $(-\pi,\pi)$ are
   1) $\left\{-\frac{\pi}{2},0,\frac{\pi}{2}\right\}$ 2) $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$
   3) $\left\{\frac{\pi}{2}\right\}$ 4) $\{\pi\}$

6. Let n be a positive integer such that $\sin\frac{\pi}{2n}+\cos\frac{\pi}{2n}=\frac{\sqrt{n}}{2}$. Then
   1) $n=6$ 2) $n=1,2,3,...$
   3) $n=5$ 4) $n=4$

7. If $x=X\cos\theta-Y\sin\theta$, $y=X\sin\theta+Y\cos\theta$ and $x^2+4xy+y^2=AX^2+BY^2$, $0\le\theta\le\frac{\pi}{2}, n\in Z$, then
   1) $\theta=\frac{\pi}{6}$ 2) $\theta=\frac{\pi}{4}$
   3) $A=3$ 4) Both 2 and 3

8. $\sin\theta=\frac{1}{2}\left(\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}\right)$ necessarily implies
   1) $x>y$
   2) $x<y$
   3) $x=y$
   4) both x and y are purely imaginary

9. The equation $2\cos^2\left(\frac{x}{2}\right)\sin^2 x=x^2+\frac{1}{x^2}, 0\le x\le\frac{\pi}{2}$ has
   1) one real solution
   2) no solution
   3) more than one real solution
   4) Two solutions

10. Let $2\sin^2 x+3\sin x-2>0$ and $x^2-x-2<0$ (x is measured in radians). Then x lies in the interval
   1) $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$ 2) $\left(-1,\frac{5\pi}{6}\right)$
   3) $(-1, 2)$ 4) $\left(\frac{\pi}{6},2\right)$

11. If $\sin^4 x+\cos^4 y+2=4\sin x\cos y$ and $0\le x,y\le\frac{\pi}{2}$ then $\sin x+\cos y$ is equal to

1) -2 2) 0 3) 2 4) 5

12. If $0\le x\le 2\pi$ and $|\cos x|\le\sin x$, then

1) $x\in\left[0,\frac{\pi}{4}\right]$ 2) $x\in\left[\frac{\pi}{4},2\pi\right]$

3) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$ 4) $[0,\pi]$

13. Number of ordered pairs (a,x) satisfying the equation $\sec^2(a+2)x+a^2-1=0$; $-\pi<x<\pi$ is

1) 2 2) 1 3) 3 4) infinite

14. The number of the solutions of the equation $\cos\left(\pi\sqrt{x-4}\right)\cos\left(\pi\sqrt{x}\right)=1$ is

1) >2 2) 2 3) 1 4) 0

S.P.Q. Student Practice Questions

15. The number of values of x in $[0,2\pi]$ satisfying the equation $|\cos x-\sin x|\ge\sqrt{2}$, is

1) 0 2) 1 3) 2 4) 3

16. In $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, $\log_{\sin\theta}(\cos 2\theta)=2$ has

1) no solution 2) a unique solution
3) two solutions
4) infinetly many solutions

17. Consider the solutions of the equation $\sqrt{2}\tan^2 x-\sqrt{10}\tan x+\sqrt{2}=0$ in the range $0\le x\le\pi/2$. Then only one of the following is true

1) no solutions exist for x in the given range
2) two solutions x_1 and x_2 exist with $x_1+x_2=\pi/4$
3) two solutions x_1 and x_2 exist with $x_1+x_2=\pi/6$
4) two solutions x_1 and x_2 exist with $x_1+x_2=\pi/2$

18. The number of solutions of the equation $|\cos x|=2[x]$ is (where $[\bullet]$ denotes the reatest integer function)

1) 0 2) 1 3) 2 4) 3

19. If n be the number of solutions of the equation $|\cot x|=\cot x+\frac{1}{\sin x}(0<x<2\pi)$, then n =

1) 1 2) 2 3) 3 4) 4

20. If $\theta\in[0,5\pi]$ and $r\in R$ such that $2\sin\theta=r^4-2r^2+3$ then the maximum number of values of the pair (r,θ) is

1) 8 2) 10 3) 6 4) 4

21. If $\frac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\frac{\cos\theta}{\sqrt{1+\cot^2\theta}}-2\tan\theta\cot\theta=-1,\theta\in[0,2\pi]$ then

1) $\theta\in\left(0,\frac{\pi}{2}\right)-\left(\frac{\pi}{4}\right)$

2) $\theta\in\left(\frac{\pi}{2},\pi\right)-\left\{\frac{3\pi}{4}\right\}$

3) $\theta\in\left(\pi,\frac{3\pi}{2}\right)-\left\{\frac{5\pi}{4}\right\}$

4) $\theta\in(0,\pi)-\left\{\frac{\pi}{4},\frac{\pi}{2}\right\}$

22. The least difference between the roots, in the first quadrant $\left(0\le x\le\frac{\pi}{2}\right)$, of the equation $4\cos x\,(2-3\sin^2 x)+(\cos 2x+1)=0$, is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

23. The set of all x in $(-\pi,\pi)$ satisfying $|4\sin x-1|<\sqrt{5}$ is given by

1) $\left(-\frac{3\pi}{10},\frac{3\pi}{10}\right)$ 2) $\left(-\frac{\pi}{10},\pi\right)$

3) $(-\pi,\pi)$ 4) $\left(-\pi,\frac{3\pi}{10}\right)$

24. The value of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x}=1$ are in AP with common difference

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{8}$ 3) $\frac{3\pi}{8}$ 4) $\frac{5\pi}{8}$

25. The no. of values of x in the interval $[0,3\pi]$ satisfying the equation $2\sin^2 x+5\sin x-3=0$ is

1) 1 2) 4 3) 6 4) 2

26. If $2\tan^2 x-5\sec x=1$ for exactly 7 distinct values of $x\in\left[0,\frac{n\pi}{2}\right], n\in N$ then the greatest value of n is

1) 13 2) 17 3) 19 4) 15

27. The number of solution of $\sum_{r=1}^{5}\cos rx=5$ in the interval $[0,2\pi]$ is

1) 0 2) 1 3) 5 4) 2

28. The number of solution of $x\in[0,2\pi]$ for which $[\sin x+\cos x]=3+[-\sin x]+[-\cos x]$ (where [.] denotes the greatest integer function) is

1) 0 2) 4 3) infinite 4) 1

GENERAL SOLUTION

C.R.T.Q Class Room Teaching Questions

29. The most general values of θ for which $\sin\theta-\cos\theta=\min_{a\in R}(1,a^2-6a+11)$ are given by

1) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}, n\in I$

2) $n\pi+(-1)^n\frac{\pi}{4}+\frac{\pi}{4}, n\in I$

3) $2n\pi+\frac{\pi}{4}, n\in I$

4) $n\pi+\frac{\pi}{2}, n\in I$

30. Let $[x]$ = the greatest integer less than equal to x and let $f(x)=\sin x+$ ... Then the most general solution of $f(x)=\left[f\left(\frac{\pi}{10}\right)\right]$ are

1) $2n\pi\pm\frac{\pi}{2}, n\in Z$ 2) $n\pi, n\in Z$

3) $2n\pi-\frac{\pi}{2}, n\in Z$ 4) $2n\pi$ or $2n\pi$ ...

31. The equation $1+\sin^2 ax=\cos x$ has unique solution then a is

1) rational 2) irrational

3) integer 4) whole numb...

S.P.Q. Student Practice Questions

32. The general solution of the equation $\frac{1-\sin x+....+(-1)^n\sin^n x+.....}{1+\sin x+....+\sin^n x+.......}=\frac{1-\cos 2x}{1+\cos 2x}$

1) $(-1)^n\left(\frac{\pi}{3}\right)+n\pi, \forall n\in I$

2) $(-1)^n\left(\frac{\pi}{6}\right)+n\pi, \forall n\in I$

3) $(-1)^{n+1}\left(\frac{\pi}{6}\right)+n\pi, \forall n\in I$

4) $(-1)^{n-1}\left(\frac{\pi}{3}\right)+n\pi, \forall n\in I$

33. The solution set satisfying tan x > 1 is

1) $\left(n\pi+\frac{\pi}{4}, n\pi+\frac{\pi}{2}\right)$ 2) $\left(n\pi+\frac{\pi}{4}, n\pi\right)$

3) $\left(n\pi+\frac{\pi}{4}, \infty\right)$ 4) ϕ

PRINCIPAL SOLUTION

C.R.T.Q Class Room Teaching Questions

34. If α,β are solutions of $\sin^2 x + a\sin x + b = 0$ and $\cos^2 x + c\cos x + d = 0$ then $\sin(\alpha+\beta)$ equals

1) $\frac{2ac}{a^2+c^2}$ 2) $\frac{a^2+c^2}{2ac}$

3) $\frac{2bd}{b^2+d^2}$ 4) $\frac{b^2+d^2}{2bd}$

35. If $\sqrt{3}\sin\pi x + \cos\pi x = x^2 - \frac{2}{3}x + \frac{19}{9}$, then x is equal to

1) $-\frac{1}{3}$ 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{4}{3}$

36. If $\frac{1-\tan x}{1+\tan x} = \tan y$ and $x - y = \frac{\pi}{6}$, then x,y are respectively

1) $\frac{5\pi}{24},\frac{\pi}{24}$ 2) $-\frac{7\pi}{24},-\frac{11\pi}{24}$

3) $-\frac{115\pi}{24},-\frac{119\pi}{24}$ 4) All the above

37. $0\le a\le 3, 0\le b\le 3$ and the equation, $x^2+4+3\cos(ax+b)=2x$ has atleast one solution then the value of a + b

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) π

S.P.Q. Student Practice Questions

38. If the equation $\cot^4 x - 2\operatorname{cosec}^2 x + a^2 = 0$ has at least one solution, then the sum of all possible integral values of a is equal to

1) 4 2) 3 3) 2 4) 0

39. The equation $\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$ is solvable for

1) $-\frac{5}{2}\le\alpha\le\frac{1}{2}$ 2) $-3\le\alpha\le 1$

3) $-\frac{3}{2}\le\alpha\le\frac{1}{2}$ 4) $-1\le\alpha\le 1$

40. The equation $\cos^8 x + b\cos^4 x + 1 = 0$ will have a solution if b belongs to

1) $(-\infty,2]$ 2) $[2,\infty)$

3) $(-\infty,-2]$ 4) $[1,\infty)$

KEY

01) 1	02) 2	03) 2	04) 2	05) 1	06) 1
07) 4	08) 3	09) 2	10) 4	11) 3	12) 3
13) 3	14) 3	15) 3	16) 2	17) 4	18) 1
19) 1	20) 3	21) 4	22) 1	23) 1	24) 1
25) 2	26) 4	27) 4	28) 3	29) 2	30) 4
31) 2	32) 2	33) 1	34) 1	35) 2	36) 4
37) 4	38) 4	39) 3	40) 3		

Hints & Solutions

1. We have,

$$y=5x^2+2x+3=5\left[x^2+\frac{2}{5}x+\frac{3}{5}\right]$$

$$=5\left[\left(x+\frac{1}{5}\right)^2+\frac{3}{5}-\frac{1}{25}\right]=5\left(x+\frac{1}{5}\right)^2+\frac{14}{5}>2$$

since $y=2\sin x\le 2$, so there cannot be any point of intersection.

2. Verify from the options

3. $\cos^5 x = 1+\sin^4 x$ Minimum value of R.H.S. is 1 and maximum value of L.H.S. is 1. So equality holds only when both sides are simultaneously 1.

i.e., cos x =1 and sin x =0

cos x= 1 $\Rightarrow x=2n\pi$ and

sin x=0$\Rightarrow$ $x = n\pi$

So, solution set x= $2n\pi$.

4. $y=\sin x=[x]$ from graph there are two solutions. x = 0 and $x=\frac{\pi}{2}$

5. $\cos^7 x+\sin^4 x=1$ in the interval $(-\pi,\pi)$.
In L.H.S. one term has to be one and one term is 0, otherwise it is not possible.

$\cos^7 x=0,\ \sin^4 x=1\Rightarrow x=\pm\frac{\pi}{2}$

or $\cos^7 x=1,\ \sin x=0\Rightarrow x=0$

6. Verification by n =6.

7. Let $E=x^2+4xy+y^2$
Substituting respective values , we have

$E=(1+4\sin\theta\cos\theta)X^2$

$+4(\cos^2\theta-\sin^2\theta)XY$

$+(1-4\sin\theta\cos\theta)Y^2$

$\Rightarrow(1+2\sin 2\theta)X^2+4\cos(2\theta)XY+(1-2\sin 2\theta)Y^2$

$=AX^2+BY^2$

$\Rightarrow\theta=\frac{\pi}{4}$, and $A=1+2=3,\ B=1-2=-1$

8. $\sin\theta\le 1$

9. Since $x^2+x^{-2}=(x-x^{-1})^2+2\ge 2$ and

$2\cos^2\frac{x}{2}\sin^2 x\le 2.$

$\therefore$ The given equation is valid only if

$2\cos^2\frac{x}{2}\sin^2 x=2$

$\Rightarrow\cos\frac{x}{2}=\sin x=1$, which cannot be true

10. we have $2\sin^2 x+3\sin x-2>0$

$\Rightarrow(2\sin x-1)(\sin x+2)>0$

$\Rightarrow 2\sin x-1>0[\because\sin x+2>0\forall x\in R]$

$\Rightarrow\sin x>1/2\Rightarrow x\in(\pi/6,5\pi/6)$ Also \

$x^2-x-2<0$

$\Rightarrow(x-2)(x+1)<0\Rightarrow -1<x<2.$

As $2<\frac{5\pi}{6}$, we obtain that x must lie in $\left(\frac{\pi}{6},2\right)$

11. We have $\sin^4 x+\cos^4 y+2=4\sin x\cos y$

$\Rightarrow\sin^4 x+\cos^4 y+2-4\sin x\cos y=0$

$\Rightarrow(\sin^2 x-1)^2+(\cos^2 y-1)^2+2\sin^2 x$

$+2\cos^2 y-4\sin x\cos y=0$

$\Rightarrow(\sin^2 x-1)^2+(\cos^2 y-1)^2+2(\sin x-\cos y)^2$

Which is true if $\sin^2 x=1,\cos^2 y=1$

sin x = cos y. As, $0\le x,\ y\le\pi/2$, we g

sin x = cos y = 1 $\Rightarrow\sin x+\cos y=2$

12. We have,

$|\cos x|\le\sin x\Rightarrow\sin x\ge 0(\because|\cos x|\ge 0)$

$\Rightarrow x\notin(\pi,2\pi)$ If $x=2\pi,|\cos 2\pi|\le\sin 2$

which is not possible

$\therefore x\in\left[0,\frac{\pi}{2}\right]$, then

$|\cos x|\le\sin x\Rightarrow x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

If $x\in\left(\frac{\pi}{2},\pi\right)$, then $|\cos x|\le\sin x$

$\Rightarrow-\cos x\le\sin x\ (\because\cos x<0)$

$\Rightarrow\tan x\le-1\Rightarrow x\in\left(\frac{\pi}{2},\frac{3\pi}{4}\right]$

$\Rightarrow x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right]$

13. Given equation is

$\sec^2(a+2)x+a^2-1=0$

$\Rightarrow\tan^2(a+2)x+a^2=0$

$\Rightarrow\tan^2(a+2)x=0$ and $a=0$

$\Rightarrow\tan^2 2x=0\Rightarrow x=0,\frac{\pi}{2},-\frac{\pi}{2}$

$\therefore(0,0),(0,\pi/2),(0,-\pi/2)$ are ordered pairs satisfying the equation.

14. Clearly, $x \ge 4\left(Since\ \sqrt{x-4}\ is\ real\right)$ so that $\sqrt{x}$ is also real. Again,

if $\cos\left(\pi\sqrt{x}\right)<1$ then

$\cos\left(\pi\sqrt{x-4}\right)>1$ (since their product =1)

But both of these are not possible (since $\cos\theta$ cannot be greater than 1)

$\therefore \cos\left(\pi\sqrt{x-4}\right)=1$ and $\cos\left(\pi\sqrt{x}\right)=1$

$\therefore x-4=0$ and $x=0$

But x =0 is not possible $\therefore x=4$ is the only solution

15. Given equation is $|\cos x-\sin x|\ge\sqrt{2}$

Since $|\cos x-\sin x|\le\sqrt{1+1}=\sqrt{2}$

$\therefore$ we must have $|\cos x-\sin x|=\sqrt{2}$

$$\Rightarrow\left|\cos\left(x+\frac{\pi}{4}\right)\right|=1\Rightarrow\cos\left(x+\frac{\pi}{4}\right)=1,-1$$

$$\therefore x=\frac{3\pi}{4},\frac{7\pi}{4}$$

16. We have, $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$

$\therefore -1\le\sin\theta\le1$, here $0<\sin\theta<1$. Now,

$\log_{\sin\theta}\cos 2\theta=2\Rightarrow 1-2\sin^2\theta=\sin^2\theta$

$$\Rightarrow\sin\theta=\frac{1}{\sqrt{3}}$$

The given equation has a unique solution

17. $\tan x_1+\tan x_2=\sqrt{5}, \tan x_1\tan x_2=1$

18. $|\cos x|=2[x]=y,\ y=|\cos x|$ and $y=2[x]$

Graph of $|\cos x|$ and 2 [x] don't cut each other for any real value of x.Hence number of solutions is zero.

19. $|\cot x|=\begin{cases}-\cot x, & \cot x<0\\ \cot x, & \cot x\ge 0\end{cases}$

CASE I : $\cot x\ge 0$

$$\cot x=\cot x+\frac{1}{\sin x}\Rightarrow\frac{1}{\sin x}=0$$

$\Rightarrow\sin x=\infty$, This is not possible.

CASE II : $\cot x<0$

$$\Rightarrow -\cot x=\cot x+\frac{1}{\sin x}$$

$$\therefore\cos x=-\frac{1}{2}=-\cos\frac{\pi}{3}$$

$$\Rightarrow\cos x=\cos\left(\pi+\frac{\pi}{3}\right)\ or\ \cos\left(\pi-\frac{\pi}{3}\right)$$

$$\therefore x=\frac{2\pi}{3},\frac{4\pi}{3}$$

But, $x=\frac{4\pi}{3}\Rightarrow\cot x>0$, $\therefore n=1$

20. But max value of $2\sin\theta$ is 2 and it is attained at r =1, -1 $\Rightarrow\theta=n\pi+(-1)^n\frac{\pi}{2}$

$$\therefore\theta=\frac{\pi}{2},\frac{5\pi}{2},\frac{9\pi}{2}\{\because\theta\in[0,5\pi]\}$$

$\therefore$ The number of values of the pair $(r,\theta)=2\times3=6$

21. $\sin\theta-\cos\theta\ne0$, $\therefore\theta\ne\frac{\pi}{4},\frac{5\pi}{4}$

Now $1+\cos\theta(\sin\theta-|\sin\theta|)-2=-1$

$\cos\theta(\sin\theta-|\sin\theta|)=0$

$$\therefore\theta\in(0,\pi)-\left\{\frac{\pi}{4},\frac{\pi}{2}\right\}$$

22. we have

$4\cos x\left(3\cos^2x-1\right)+2\cos^2x=0$

$2\cos x(3\cos x+2)(2\cos x-1)=0$

either $\cos x=0$ which gives

$x=\pi/2$ or $\cos x=-2/3$

Which gives no values of x for whic

$0\le x\le\pi/2$ or $\cos x=1/2$

which gives $x=\pi/3$

So, the required difference $=\frac{\pi}{6}$

23. We have , $|4\sin x-1|<\sqrt{5}$

$\Rightarrow -\left(\frac{\sqrt{5}+1}{4}\right)<\sin x<\frac{\sqrt{5}+1}{4}$

$\Rightarrow \sin\left(\frac{-3\pi}{10}\right)<\sin x<\sin\left(\frac{3\pi}{10}\right)$

24. $\sin 2x=\pm\frac{1}{\sqrt{2}}$

25. Given equation is

$2\sin^2 x+5\sin x-3=0$

$\Rightarrow(\sin x+3)(2\sin x-1)=0$

$\Rightarrow \sin x\neq -3$ or $\sin x=\frac{1}{2}$

given $0^0\leq x\leq 540^0$, $\therefore$ no. of values are 4. i.e., 30^0, 150^0, 390^0, 510^0

26. $\sec x=3\Rightarrow \cos x=\frac{1}{3}$

Which gives two values of x in each of $[0,2\pi],(2\pi,4\pi],(4\pi,6\pi]$ and one value in $6\pi+\frac{3\pi}{2}=15\frac{\pi}{2}$

$\therefore$ Greatest value of n = 15

27. $\sum_{r=1}^{5}\cos rx=5\Rightarrow \cos x+\cos 2x+\cos 3x$ $+\cos 4x+\cos 5x=5$

Which is possible only when

$\cos x=\cos 2x=\cos 3x$ $=\cos 4x=\cos 5x=1$ and is satisfied by x=0 and $x=2\pi$.

28. $[\sin x+\cos x]=3+[-\sin x]+[-\cos x]$

Maximum value of left - hand side is 1 and minimum of right hand side is also 1

$\Rightarrow[\sin x+\cos x]=3+[\sin x]+[-\cos x]=1$

$\Rightarrow[\sin x+\cos x]=1,$

$[-\sin x]=-1,[-\cos x]=-1$

29. We have

$\sin\theta-\cos\theta=\min_{a\in R}\{1,a^2-6a+11\}$

Since $a^2-6a+11=(a-3)^2+2$ for all a $\therefore \sin\theta-\cos\theta=1$

$\Rightarrow \sin\left(\theta-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\sin\frac{\pi}{4}$

$\Rightarrow \theta-\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{4}$

$\therefore \theta=n\pi+(-1)^n\frac{\pi}{4}+\frac{\pi}{4}$ where $n\in Z$

30. Here $f\left(\frac{\pi}{10}\right)=\sin 18^0+\cos 18^0$

$=\sqrt{2}\sin(45^0+18^0)=\sqrt{2}\sin 63^0$

Since $\sin 63^0>\sin 45^0=\frac{1}{\sqrt{2}}$ and $\sqrt{2}\sin 63^0$

$\therefore 1<f\left(\frac{\pi}{10}\right)<\sqrt{2}\Rightarrow\left[f\left(\frac{\pi}{10}\right)\right]=1$

$\therefore$ *The equation is* $\sin x+\cos x=1$

$\Rightarrow \cos\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\cos\frac{\pi}{4}$

$\Rightarrow x-\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}$

31. $1+\sin^2 ax=\cos x$

Minimum value of L.H.S is 1 and maxim value of R.H.S is 1, equality exist on when both sides are1

$\Rightarrow \sin^2 ax=0$ and $\cos x=1$

$\Rightarrow ax=n\pi$ and $x=2n\pi$

If a is rational, there are infinite or common solution but when a is irratio only one common solution i.e., x =0

32. The equation

$\frac{1-\sin x+....+(-1)^n\sin^n x+....}{1+\sin x+......+\sin^n x+.....}=\frac{1-\cos 2x}{1+\cos 2x}$

$\Rightarrow \frac{1}{1+\sin x}.\frac{1-\sin x}{1}=\frac{2\sin^2 x}{2\cos^2 x}$

$\Rightarrow 2\sin^2 x+\sin x-1=0$

$\Rightarrow \sin x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$

$\Rightarrow \sin x = -1$ or $\sin x = \frac{1}{2}$

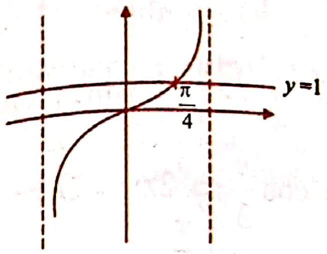
Since, $\sin x \neq -1$, we have

$\sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$,

$\therefore x = n\pi + (-1)^n \frac{\pi}{6}$

33. Graph of y= tanx is as shown

$\therefore$ For $\tan x > 1$



$n\pi + \frac{\pi}{4} < x < n\pi + \frac{\pi}{2}$

34. $\sin\alpha + \sin\beta = -a$, $\cos\alpha + \cos\beta = -c$

Apply transformations and on dividing

$\tan\frac{\alpha+\beta}{2} = \frac{a}{c}$, $\sin(\alpha+\beta) = \frac{2ac}{a^2+c^2}$

35. Verify the options.

36. $\tan\left(\frac{\pi}{4} - x\right) = \tan y = \tan\left(x - \frac{\pi}{6}\right)$

$\left\{\because x - y = \frac{\pi}{6}\right\}$

$\therefore \frac{\pi}{4} - x = n\pi + x - \frac{\pi}{6} \Rightarrow \left(\frac{5\pi}{12} - n\pi\right) = 2x$

$\Rightarrow x = (5-12n)\frac{\pi}{24}$ and $y = x - \frac{\pi}{6}$

Options (1)(2),(3) correspond to n =0,1, 10 respectively.

37. $x^2 - 2x + 4 = -3\cos(a\alpha + b)$

$\Rightarrow (x-1)^2 + 3 = -3\cos(ax+b)$

As $-1 \le \cos(ax+b) \le 1$ and $(x-1)^2 \ge 0$

$\therefore$ equation (i) is only possible if,

$\cos(ax+b) = -1$ and $(x-1) = 0$ so

$a + b = \pi$, (where $a + b \le 6$)

38. $\cot^4 x - 2\cot^2 x - 2 + \alpha = 0$

$(\cot^2 x - 1)^2 = 3 - \alpha, 3 - \alpha \ge 0$,

$\alpha = -1, 0, 1$

39. $\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$

Let $\sin 2x = y$. then the given equatiion becomes $y^2 - 2y - 2(1+\alpha) = 0$ where $-1 \le y \le 1$ $(\because -1 \le \sin 2x \le 1)$, for real, discriminant ≥ 0

40. $\cos^8 x + b\cos^4 x + 1 = 0$

$\Rightarrow b = -\left(\cos^4 x + \frac{1}{\cos^4 x}\right) \le -2, \forall x \in R$

$\Rightarrow b \in (-\infty, -2]$

EXERCISE-IV

LEVEL-IV

1. **Assertion (A): 3 sinx+ 4 cos x =7 has no solution.**

Reason (R): a cos x+ b sinx = c has no solution if |c| > $\sqrt{a^2+b^2}$

1) Both A and R are true and R is correct explanation of A
2) Both A and R are true and R is not the correct explanation of A
3) A is true but R is false
4) A is false but R is true

2. **Statement I: Principal value of $\cos\theta = -1$ is π**

Statement II: Principal value of $\sin\theta = 0$ is π

Which of the above statement is correct?

1) Only I 2) Only II
3) Both I and II 4) Neither I or II

3. **Statement I: The set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is**

$\left\{n\pi + \frac{\pi}{4}, n \in I\right\}$

Statement II: The expression $\left(1+\tan x+\tan^2 x\right)\left(1-\cot x+\cot^2 x\right)$ is positive for all defined values of x

III) $e^{\sin x}-e^{-\sin x}\neq 4$ for any real values of x.

Which of the following is correct?

1) only I, II
2) only II, III
3) only I, III
4) I,II,III

4. Statement I: If cot $\left(\frac{\pi}{3}\cos 2\pi x\right)=\sqrt{3}$, then the general solution of the equation is $x=n\pm\frac{1}{6}, n\in I$.

Statement II: If tan P θ = tan Q θ, then the values of θ from an A.P with common difference $\frac{\pi}{P-Q}$.

Which of the above statements are correct?

1) only I
2) only II
3) Both I and II
4) neither I or II

5. Statement I: If x lies in the 1st Quadrant and cos x+ cos 3x=cos 2x then x = 30° or 45°

Statement II: x $\in$ (0, 2π) and cosec x + 2=0 then $x=\frac{7\pi}{6},\frac{11\pi}{6}$

Statement III: x $\in$ [0,2π] and (2 cos x-1) (3+2 cos x) =0 then x = $\frac{\pi}{3},\frac{5\pi}{3}$

Which of the above statements are correct?

1) only I, II
2) only II, III
3) only I, III
4) I,II,III

KEY

01) 1 02) 1 03) 2 04) 3 05) 2

Hints & Solutions

1. Assertion is true since $|c|>\sqrt{a^2+b^2}$

2. By range of principal solutions

3. I. for $x=\frac{\pi}{4}$, $\tan 2x$ is undefined

II. G.E. $=1+\cot^2 x+\tan^2 x>0$

III. If $e^{\sin x}-e^{-\sin x}=4$ then

$\sin x=\log_e\left(2+\sqrt{5}\right)>1$ which is not tr

4. I. $\cos 2\pi x=\cos\frac{\pi}{3}\Rightarrow 2\pi x=2n\pi\pm\frac{\pi}{3}$

$\Rightarrow x=n\pm\frac{1}{6}, n\in z$

II. $P\theta=n\pi+Q\theta \Rightarrow \theta=\frac{n\pi}{P-Q}$

$\therefore$ common difference $=\frac{\pi}{P-Q}$

5. I. $\cos x+\cos 3x=\cos 2x$

$\Rightarrow 2\cos 2x\cos x=\cos 2x$

$\Rightarrow \cos 2x=0$ or $2\cos x=1$

$\therefore x=45^0$ or $x=60^0$

II. $\operatorname{cosec} x=-2$

$\therefore x=\frac{7\pi}{6},\frac{11\pi}{6}$ satisfies

III. $\left(2\cos x-1\right)\left(3+2\cos x\right)=0$

$\Rightarrow \cos x=\frac{1}{2}$ or $\cos x=\frac{-3}{2}$

(which is not possible)

$\therefore x=\frac{\pi}{3},\frac{5\pi}{3}$

INTEGER & NUMERICAL
ANSWER TYPE QUESTIONS

1. Number of values of P for which equation $\sin^3 x+1+P^3-3P\sin x=0 \, (P>0)$ has a root is______

2. Number of roots of the equation $|\sin x+\cos x|+\sqrt{2+\tan^2 x+\cot^2 x}=\sqrt{3}, x\in[0,4\pi]$, are__

3. Number of roots of the equation $(3+\cos x)^2=4-2\sin^8 x, x\in[0,5\pi]$, are______

4. Number of solution(s) of the equation $\frac{\sin y}{\cos 3x}+\frac{\sin 3x}{\cos 9x}+\frac{\sin 9x}{\cos x27x}=0$ in the interval $\left(0,\frac{\pi}{4}\right)$ is______

5. Number of solutions of the equation $\left(\sqrt{3}+1\right)^{2x}+\left(\sqrt{3}-1\right)^{2x}=2^{3x}$ is______

6. Number of integral value(s) of m for which the equation $\sin x-\sqrt{3}\cos x=\frac{4m-6}{4-m}$ has solutions, $x\in[0,2\pi]$, is______

7. The number of solutions of the equation $\cos^2\left(x+\frac{\pi}{6}\right)+\cos^2 x-2\cos\left(x+\frac{\pi}{6}\right)$ $+\cos^2 x-2\cos\left(x+\frac{\pi}{6}\right).\cos\frac{\pi}{6}=\sin^2\frac{\pi}{6}$ in interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is______

8. If $\cos 4x=a_0+a_1\cos^2 x+a_2\cos^4 x$ is true for all values of $x\in K$, then the value of $5a_0+a_1+a_2$ is______

9. Number of integral values of a for which the equation $\cos^2 x-\sin x+a=0$ has roots when $x\in(0,\pi/2)$ is______

10. Number of roots of the equation $2^{\tan\left(x-\frac{\pi}{4}\right)}-2(0.25)^{\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x}}+1=0$ is______

11. Number of solutions of the equation $\sin^4 x-\cos^2 x, \sin x+2\sin^2 x+\sin x=0$ in $0\le x\le 3\pi$ is______

12. Let K be sum of all x in the interval $[0,2\pi]$ such that $3\cot^2 x+8\cot x+3=0$, then the value of K/π is______

13. If $\theta\in[0,5\pi]$ and $r\in R$ such that $2\sin\theta=r^4-2r^2+3$ then the maximum number of values of the pair (r,θ) is______

14. If $2\tan^2 x-5\sec x=1$ is stisified by exactly seven distinct values of $x\in\left[0,(2x+1)\frac{\pi}{2}\right]$, $n\in N$, then the greatest value of n is______

15. If $\sin x+\sin y\ge\cos\alpha\cos x \, \forall\in R$, then $\sin y+\cos\alpha$ is equal to______

16. If $\sin(\sin x+\cos x)=\cos(\cos x-\sin x)$, and largest possible value of $\sin x$ is $\frac{\pi}{K}$, then the value of K is______

17. The number of solution of the equation $1+\cos x+\cos x2x+\sin x+\sin 2x+\sin 3x=0$, which satisfy the condition $\frac{\pi}{2}<\left|3x-\frac{\pi}{2}\right|\le\pi$ is______

18. Number of solutions of the equations $\tan x+\sec x=2\cos x$ lying in the interval $[0,2\pi]$ is______

19. The number of values of x in the interval $[0,3\pi]$ satisfying the equation $2\sin^2 x+5\sin x-3=0$ is______

20. If a is any real number, the number of roots of $\cot x-\tan x=a$ in the first quadrant is______

21. If $0\le x<2\pi$, then the number of real values of x, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$, is______

22. If $\sin^4 x+\cos^4 y+2=4\sin x\cos y, 0\le x,y<\pi/2$ then $\sin x+\cos y=$______

23. If $x\in(-\pi,\pi)$ such that $y=1+|\cos x|+|\cos^2 x|+|\cos^3 x|+......$ and $8^y=64$, then $y=$______

24. The number of solutions of the equation $\sin^5 x-\cos^5 x=\frac{1}{\cos x}-\frac{1}{\sin x},(\sin x\ne\cos x)$ is______

25. If $a_1+a_2\sin x+a_3\cos x+a_4\sin 2x+a_5\cos 2x=0$ holds for all x then the number of all possible 5-tuples is______

26. The number of real solutions of the equation $\cos^5 x+\sin^3 x=1$ in the interval $[0,2\pi]$ is______

27. The solution of the equation $\cos^2 x-2\cos x=4\sin x-\sin 2x$, $(0\le x\le\pi)$ i is $\pi+\tan^{-1}\left(-\frac{1}{K}\right)$. Find K.

28. If $\left(\frac{\sin\theta}{\sin\phi}\right)^2=\frac{\tan\theta}{\tan\phi}=3$, then the value of θ and φ are $\theta=n\pi\pm\frac{\pi}{A},\phi=n\pi\pm\frac{\pi}{B}$ Find A+B.

29. The number of solutions of the equation $\tan x+\sec x=2\cos x$ lying in the interval $(0,2\pi)$ is

30. The number of integeral values of k, for which the equation $7\cos x+5\sin x=2k+1$ has a solution, is

31. Let $f(x)=\cos\sqrt{x}$, then $f(x)$ is periodic with period $A\pi$. Find A. (if no such value of A exist mark 0)

32. In a right angled triangel the hupotenuse is $2\sqrt{2}$ times the length of perpendicular drawn from the opposite vertex on the hypotenuse, then the other two angles are $\frac{A\pi}{8},\frac{B\pi}{8}$. Find B–A.

33. In a triangle ABC, $\angle B=\frac{\pi}{3}$ and $\angle C=$ and D divides BC internally in the ratio 1: Then $\frac{\sin\angle BAD}{\sin\angle CAD}$ is equal to $\frac{1}{\sqrt{A}}$. Find A

34. If p_1,p_2,p_3 are altitudes of a triangle AB form the vertices A, B, C and Δ, the ar of the triangle, then $p_1^{-1}+p_2^{-1}-p_3^{-1}$ is equ to $\frac{s-xa-yb-zc}{\Delta}$. Find x+y+z.

35. A spherical balloon of radius r subtends angle α at the eye of an observer. If th angle of elevation of the center of the ball be β. The height of the center of the ball is $r\,\mathrm{cosec}\left(\frac{\alpha}{A}\right)\sin\left(\frac{\beta}{B}\right)$. Find A+B.

36. A tower AB leans towards west making a angle α with the vertical. The angula elevation of β, the top most poin of th tower is β as observed from a point C du east of A at a distance d from A. If th angular elevation of B from a point D du east of C at a distane 2d from C is γ, then tan α can be given as A cot β – B cot γ Find A+B.

37. The number of solution of the given equation $a\sin x+b\cos x=c$, where $|c|>\sqrt{a^2+b^2}$, is

38. The number of pairs (x,y) satisfying the equations $\sin x+\sin y=\sin(x+y)$ and $|x|+|y|=1$ is

39. If the two angle on the base of a triangle are $\left(22\frac{1}{2}\right)^0$ and $\left(112\frac{1}{2}\right)^0$, then the ratio

of the height of the triangle to the length of the base is 1 : K. Find K.

40. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to___

41. The number of distinct solutions of the equation $\log_{\frac{1}{2}}|\sin x| = 2-\log_{\frac{1}{2}}|\cos x|$ in the interval $[0,2\pi]$, is_____.

42. The minimum value of α for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=\alpha$ has at least one solution in $\left(0,\frac{\pi}{2}\right)$ is_____.

43. The number of integral values of 'k' for which the equation 3sinx + 4 cosx = k + 1 has a solution, $k\in R$ is

44. The number of solutions of the equation $\log_4(x-1)=\log_2(x-3)$ is

45. If $\sqrt{3}(\cos^2 x)=(\sqrt{3}-1)\cos x+1$, the number of solutions of the given equation when $x\in\left[0,\frac{\pi}{2}\right]$ is

46. The number of solutions of the equation $|\cot x| = \cot x+\frac{1}{\sin x}$ in the interval $[0,2\pi]$ is

47. If the sum of solutions of the system of equations $2\sin^2\theta-\cos 2\theta=0$ and $2\cos^2\theta+3\sin\theta=0$ in the interval $[0,2\pi]$ is $k\pi$, then k is equal to_____.

48. Let $S=\left[-\pi,\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right\}$. Then the number of elements in the set $A=\{\theta\in S:\tan\theta(1+\sqrt{5}\tan(2\theta))=\sqrt{5}-\tan(2\theta)\}$ is _____

49. Let $S=\{\theta\in(0,2\pi):7\cos^2\theta-3\sin^2\theta-2\cos^2 2\theta=2\}$ Then, the sum of roots of all the equations $x^2-2(\tan^2\theta+\cot^2\theta)x+6\sin^2\theta=0$ $\theta\in S$, is

50. The number of solutions of the equation $\sin x=\cos^2 x$ in the interval (0,10) is_____.

51. The number of values of x in the interval $\left(\frac{\pi}{4},\frac{7\pi}{4}\right)$ for which $14\operatorname{cosec}^2 x-2\sin^2 x=21-4\cos^2 x$ holds, is_______

52. The number of elements in the set $S=\{\theta\in[-4\pi,4\pi] : 3\cos^2 2\theta+6\cos 2\theta-10\cos^2\theta+5=0\}$ is__________.

53. The number of solutions of the equation $2\theta-\cos^2\theta+\sqrt{2}=0$ is R is equal to_____.

Answers

1) 1	2) 0	3) 3	4) 6	5) 1
6) 4	7) 2	8) 5	9) 1	10) 0
11) 4	12) 5	13) 6	14) 7	15) 1
16) 4	17) 2	18) 2	19) 4	20) 1
21) 7	22) 2	23) 2	24) 0	25) 1
26) 3	27) 2	28) 9	29) 2	30) 8
31) 0	32) 2	33) 6	34) 1	35) 3
36) 4	37) 0	38) 6	39) 2	40) 1
41) 8	42) 9	43) 11	44) 1	45) 1
46) 1	47) 3	48) 5	49) 16	50) 4
51) 4	52) 32	53) 1		

Hints & Solutions

1. $\sin^3 x + P^3 + 1 = 3P\sin x$

$\Rightarrow (\sin x + P + 1)(\sin^2 x + 1 + P^2 - \sin x - P - P\sin x) = 0$

Therefore, either $\sin x + P + 1 = 0$

$\Rightarrow P = -1(1+\sin x)$ or $\sin x = 1 = P$.

Hence, only one vaoue of $P(P>0)$ is possible which is given by $P=1$.

2. $|\sin x\cos x| + |\tan x + \cot x| = \sqrt{3}$

$\Rightarrow |\sin x\cos x| + \frac{1}{|\sin x\cos x|} = \sqrt{3}$

$|\sin x\cos x| + \frac{1}{|\sin x\cos x|} \geq 2$

Hence, there is no solution.

3. $4 \leq L.H.S \leq 16; 2 \leq R.H.S \leq 4$

Hence, equality can occur only when both sides are 4, which is possible if $x = \pi, 3\pi, 5\pi$.

That is, there are three solutions.

4. Conceptual

5. $(\sqrt{3}+1)^{2x} + (\sqrt{3}-1)^{2x} = 2^{3x} = (2\sqrt{2})^{2x}$

$= \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^{2x} + \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^{2x} = 1$

$= (\sin 75^0)^{2x} + (\cos 75^0)^{2x} = 1$

$\Rightarrow x = 1$

6. Since $-2 \leq \sin x - \sqrt{3}\cos x \leq 2$, we have

$-2 \leq \frac{4m-6}{4-m} \leq 2$ or $-1 \leq \frac{2m-3}{4-m} \leq 1$

If $\frac{2m-3}{4-m} \leq 1$, we have

$\frac{(2m-3)-(4-m)}{4-m} \leq 0$

or $\frac{3m-7}{m-4} \geq 0$ .....(i)

Also, $-1 \leq \frac{2m-3}{4-m}$

$\Rightarrow \frac{m+1}{m-4} \leq 0$ .....(ii)

From Eqs. (i) and (ii), we get $m \in \left[-1, \frac{7}{3}\right]$

Therefore, the possible integers are $-1,0,1,2$

7. We have,

$\left(\cos\left(x+\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{6}\right)\right)^2 = \sin^2 x$

$\Rightarrow 4\sin^2\left(\frac{x}{2}\right).\sin^2\left(\frac{x}{2}+\frac{\pi}{6}\right) = 4\sin^2\left(\frac{x}{2}\right).\cos^2\left(\frac{x}{2}\right)$

$\therefore \sin\frac{1}{2} = 0$ or $\sin^2\left(\frac{x}{2}+\frac{\pi}{6}\right) = \cos^2\frac{x}{2}$

$\Rightarrow x = 2m\pi, m \in Z$

or $\frac{\pi}{2} - \left(\frac{x}{2}+\frac{\pi}{6}\right) = n\pi \pm \frac{x}{2}, n \in I$

$\therefore x = 0$ or $x = \frac{\pi}{3}$.

8. $\cos 4x = 2\cos^2 2x - 1$

$= 2(2\cos^2 x - 1)^2 - 1 = 2(4\cos^4 x + 1 - 4\cos^2 x) - 1$

$= 8\cos^4 x - 8\cos^2 x + 1$

$\therefore a_0 = 1, a_1 = -8, a_2 = 8$

$\therefore 500 + a_1 + a_2 = 5$

9. $1 - \sin^2 x - \sin x + a = 0$

$\Rightarrow \sin^2 x + \sin x = (a+1)$

For $x \in (0, \pi/2)$, the range of $\sin^2 x + \sin x$ is (0,2).

$\Rightarrow 0 < (a+1) < 2 \Rightarrow a \in (-1,1)$

10. $$\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x}=\frac{\frac{1}{2}(\sin x-\cos x)^2}{\cos^2 x-\sin^2 x}$$

$$\frac{\frac{1}{2}(\sin x-\cos x)}{\cos x+\sin x}=-\frac{1}{2}\tan\left(x-\frac{\pi}{4}\right)$$

Given equation reduces to

$$2^{\tan\left(x-\frac{\pi}{4}\right)}-2^{(0.25)^{-\frac{1}{2}\tan\left(x-\frac{\pi}{4}\right)}}+1=0$$

$$=2^{\tan\left(x-\frac{\pi}{4}\right)}=1$$

$\Rightarrow x=\pi/4$, which is not possible as $\cos 2x=0$ for this value of x_1 which is not defining the original equation.

11. $\sin^4 x-\cos^2 x\sin x+2\sin^2 x+\sin x=0$

or $\sin x[\sin^3 x-\cos^2 x+2\sin x+1]=0$

or $\sin x[\sin^3 x+\sin^2 x+2\sin x]=0$

or $\sin x[\sin^2 x+\sin x+2]=0$

or $\sin x=0$, where $x=0,\pi,2\pi,3\pi$.

Hence, there are four solutions.

12. $3\cot^2 x+8\cot x+3=0$

$$\therefore \cot x=\frac{-4\pm\sqrt{7}}{3}$$

Both roots are negative.

$$\therefore \frac{\pi}{2}<x_1,x_2<\pi$$

$$\therefore \pi<x_1+x_2<2\pi$$

product of roots $=\cot x_1\cot x_2=1$

$$\Rightarrow \cot x_1\cot\left(\frac{3\pi}{2}-x_1\right)=1$$

$$\Rightarrow x_1+x_2=\frac{3\pi}{2}$$

similarly, $x_1+x_2=\frac{7\pi}{2}$

13. $2\sin\theta=r^4-2r^2+3$

$$\Rightarrow 2\sin\theta=\left(r^2-1\right)^2+2$$

This is possible only when $\sin\theta=1$ and $r^2=1$ or $r=\pm1$ so, $\theta=\pi/2,\ 5\pi/2,\ 9\pi/2$

$\therefore$ Number of values of the pair $(r,\theta)=6$

14. Here $(2\sec x+1)(\sec x-3)=0$ which gives two values of θ in each of $[0,2\pi]$, $[2\pi,4\pi]$, $[4\pi,6\pi]$ and one value in $\left[6\pi,6\pi+\frac{3\pi}{2}\right]$.

15. $\cos\alpha\cos x-\sin x\le\sin y\,\forall x\in R$

$\therefore \sin y\ge$ greatest value of $(\cos\alpha\cos x-\sin x)$

$\therefore \sin y\ge\sqrt{\cos^2\alpha+1}$

16. $\sin(\sin x+\cos x)=\cos(\cos x-\sin x)$

or

$$\cos(\cos x-\sin x)=\cos\left(\frac{\pi}{2}-(\sin x+\cos x)\right)$$

or

$$\cos x-\sin x=2x\pi\pm\left(\frac{\pi}{2}-\sin x-\cos x\right)$$

Taking +ve sign, we get

$$\cos x-\sin x=2x\pi+\frac{\pi}{2}-\sin x-\cos x$$

$$\Rightarrow \cos x=n\pi+\frac{\pi}{4}$$

For $n=0,\cos x=\frac{\pi}{4}$, which is the only possible value

$$\therefore \sin x=\frac{\left(\sqrt{16-\pi^2}\right)}{4}$$

Taking -ve sign, we get $\sin x=\frac{\pi}{4}$

From (i) and (ii), we get $\frac{\pi}{4}$ as the larges value. Hence, $K=4$.

17. Given $\frac{\pi}{2}<\left|3x-\frac{\pi}{2}\right|\le\pi$

$\Rightarrow\frac{\pi}{2}<\left(3x-\frac{\pi}{2}\right)\le\pi$ or

$-\pi\le\left(3x-\frac{\pi}{2}\right)<-\frac{\pi}{2}$

$\Rightarrow\pi<3x\le\frac{3\pi}{2}$ or $-\frac{\pi}{6}\le x<0$

$\therefore x\in\left(\frac{-\pi}{6},0\right)\cup\left(\frac{\pi}{2},\frac{\pi}{2}\right)$

Now, $1+\cos x+\cos 2x+\sin x$

$+\sin 2x+\sin 3x=0$

$\Rightarrow 2\cos^2 x+\cos x+\sin 2x+2\sin 2x\cos x=0$

$\Rightarrow \cos x(2\cos x+1)+\sin 2x(2\cos x+1)=0$

$\Rightarrow(\cos x+2\sin x)(2\cos x+1)=0$

$\Rightarrow\cos x(1+2\sin x)(2\cos x+1)=0$

$\Rightarrow\cos x=0$ or $\sin x=\frac{-1}{2}$ (as for given interval, $\cos x>0$)

$\Rightarrow x=\frac{\pi}{2}$ or $x=-\frac{\pi}{6}$.

Hence, there are 2 solutions.

18. The given equation can be written as

$\frac{\sin x+1}{\cos x}=2\cos x$

$\Rightarrow\sin x+1=2\cos^2 x(\cos x\ne 0)$

$\Rightarrow 2\sin^2 x+\sin x-1=0$

$\Rightarrow 2(\sin x-1)(\sin x+1)=0$

$\Rightarrow\sin x=\frac{1}{2}(\because \sin x+1\ne 0$ as $\cos x\ne 0)$

$\Rightarrow x=\pi/6, 5\pi/6$ in $[0,2\pi]$.

The equired number of solutions is 2.

19. $2\sin^2 x+5\sin x-3=0$

$\Rightarrow(\sin x-3)(2\sin x-1)=0$

$\sin x=1/2$

$\therefore x$ has 4 values in $[0,3\pi]$

20. $\cot x-\tan x=a\Rightarrow\frac{1-\tan^2 x}{2\tan x}=\frac{a}{2}$

$\Rightarrow\cot 2x=\frac{a}{2}$

21. $\cos x+\cos 2x+\cos 3x+\cos 4x=0$

$\Rightarrow(\cos x+\cos 4x)+(\cos 2x+\cos 3x)=0$

$\Rightarrow 2\cos\frac{5x}{2}\cos\frac{3x}{2}+2\cos\frac{5x}{2}\cos\frac{x}{2}=0$

$\Rightarrow 2\cos\frac{5x}{2}\left[\cos\frac{3x}{2}+\cos\frac{x}{2}\right]=0$

$\Rightarrow 4\cos\frac{5x}{2}\cos x\cos\frac{x}{2}=0$

$\Rightarrow\cos\frac{x}{2}=0$ or $\cos x=0$ or $\cos\frac{5x}{2}=0.$

$\Rightarrow x=\pi$ or $x=\frac{\pi}{2},\frac{3\pi}{2}$

or $x=\frac{\pi}{5},\frac{3\pi}{2},\frac{5\pi}{5},\frac{7\pi}{5},\frac{9\pi}{5}$.

The number of real values of x is 7.

22. The given equation can be written as

$\sin^4 x+\cos^4 y+2-4\sin x\cos y=0$

$\Rightarrow\left(\sin^2 x-1\right)^2+\left(\cos^2 y-1\right)^2+2\sin^2 x$

$+2\cos^2 y-4\sin x\cos y=0$

$\Rightarrow\left(\sin^2 x-1\right)^2+\left(\cos^2 y-1\right)^2$

$+2(\sin x-\cos y)^2=0$

which is true if $\sin^2 x=1$, $\cos^2 y=1$ and $\sin x=\cos y$, So $\sin x+\cos y=2$ as $0\le x,\ y\le\pi/2$.

23. $8^y=64\Rightarrow y=2$

24. The given equation can be written as

$\sin^5 x-\cos^5 x=\frac{\sin x-\cos x}{\sin x\cos x}$

$\Rightarrow \sin x\cos x\left[\dfrac{\sin^5 x-\cos^5 x}{-\sin x-\cos x}\right]=1$

$\Rightarrow (1/2)\sin 2x[\sin^4 x+\sin^3 x\cos x$

$+\sin^2 x\cos^2 x+\sin x\cos^3 x\cos^4 x]=1$

$\Rightarrow \sin 2x[(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x$

$+\sin x\cos x(\sin^2 x+\cos x)+\sin^2 x.\cos^2 x]=2$

$\Rightarrow \sin 2x\left[1-\sin^2 x\cos^2 x+\sin x\cos x\right]=2$

$\Rightarrow \sin 2x\left[1-\frac{1}{4}\sin^2 2x+\frac{1}{2}\sin 2x\right]=2$

$\Rightarrow \sin^3 2x-2\sin^2 2x-4\sin 2x+8=0$

$\Rightarrow (\sin 2x-2)^2(\sin 2x+2)=0$

$\Rightarrow \sin 2x=\pm 2$, which is not possible for any x.

25. Given $a_1+a_2\sin x+a_3\cos x+a_4$

$\sin 2x+a_5\cos 2x=0$

put $x=0^0, 180^0$ we get

$a_1+a_3+a_5=0\rightarrow(1)$

$a_1-a_3+a_5=0\rightarrow(2)$

$(1)-(2)\Rightarrow a_3=0\Rightarrow a_1+a_5=0\rightarrow(3)$

Again put $x=90^0$ and 270^0 we get

$a_1+a_2-a_5=0\rightarrow(4)$

$a_1-a_2-a_5=0\rightarrow(5)$

$(4)-(5)\Rightarrow a_2=0\Rightarrow a_1=a_5=0$

put $a_1=a_2=a_3=a_5=0$ in the given equation we get $a_4\sin 2x=0$

$\because a_4=0$

There exists only one 5-tuples consisting of zeroes

26. $\cos^5 x\le\cos^2 x$ and $\sin^3 x\le\sin^2 x$

So, $\cos^5 x+\sin^3 x=1$ is possible only when $\cos^5 x=\cos^2 x$ and $\sin^3 x=\sin^2 x$

which is possible only at $x=0,\pi/2$ and $2\pi\forall x\in[0,2\pi]$.

27. Given equationis $\cos^2 x-2\cos x=4\sin x-\sin 2x,$

$\Rightarrow \cos^2 x-2\cos x=4\sin x-2\sin x\cos x$

$\Rightarrow \cos x(\cos x-2)=2\sin x(2-\cos x)$

$\Rightarrow (\cos x-2)(\cos x+2\sin x)=0$

$\Rightarrow \cos x+2\sin x=0\ \ (\because \cos x\ne 2)$

$\Rightarrow \tan x=-\frac{1}{2}\Rightarrow x=n\pi+\tan^{-1}(-1/2), n\in I$

As $0\le x\le\pi$, therfore,

$x=\pi+\tan^{-1}(-1/2)$.

28. $\left(\dfrac{\sin\theta}{\sin\phi}\right)^2=\dfrac{\tan\theta}{\tan\phi}$

$\Rightarrow \sin\theta.\cos\theta=\sin\phi\cos\phi$

$\Rightarrow \sin 2\theta=\sin 2\phi$

$2\theta=\pi-2\phi\Rightarrow\theta=\frac{\pi}{2}-\phi$

But $\dfrac{\tan\theta}{\tan\phi}=3\Rightarrow\dfrac{\tan\theta}{\tan\phi}=3\tan^2\theta=3$

$\Rightarrow\theta=n\pi\pm\frac{\pi}{3}$, so that $\Rightarrow\phi=n\pi\pm\frac{\pi}{6}$.

A = 3, B = 6

29. Given, $\tan x+\sec x=2\cos x$ .....(i)

$\Rightarrow (\sin x+1)=2\cos^2 x$

$\Rightarrow (\sin x+1)=2(1-\sin x)(1+\sin x)$

$\Rightarrow (1+\sin x)[2(1-\sin x)-1]=0\Rightarrow 2(1-\sin x)-1=0$

$[\because \sin x\ne -1$ *otherwise* $\cos x=0$ *and* $\tan x, \sec x$ *will be undefined*$]$

$\Rightarrow \sin x=\frac{1}{2}\Rightarrow x=\frac{\pi}{6},\frac{5\pi}{6}$ in $(0,2\pi)$.

30. $-\sqrt{7^2+5^2}\le(7\cos x+5\sin x)\le\sqrt{7^2+5^2}$

So, for solution $-\sqrt{74}\le(2k+1)\le\sqrt{74}$

or $-8.6\le(2k+1)\le 8.6$ or

$-9.6\le 2k\le 7.6$

or $-4.8 \le k \le 3.8$ So, integral values of k are $-4, -3, -2, -1, 0, 1, 2, 3$ (eight values).

31. If $f(x) = \cos\sqrt{x}$, then $f(x)$ is not a periodic function.

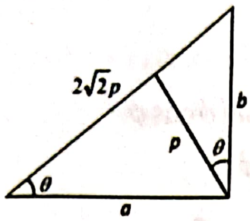
32. Here $a^2 + b^2 = 8p^2$ ......(i)

where $p = a\sin\theta = b\cos\theta$

By(i), $p^2\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right) = 8p^2$

$\Rightarrow 2\sin^2 2\theta = 1$

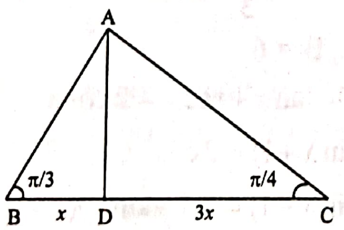
$\Rightarrow \sin 2\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \quad \theta = \frac{\pi}{8}.$



Here the other angle is $\frac{3\pi}{8}$.

B = 3, A = 1

33. Let $\angle BAD = \alpha, \angle CAD = \beta$



In ΔADB, applying sine formula we get

$\frac{x}{\sin\alpha} = \frac{AD}{\sin(\pi/3)}$ ....(i)

In ΔADC, applying sine formula we get

$\frac{3x}{\sin\beta} = \frac{AD}{\sin(\pi/4)}$ .....(ii)

Dividing (i) by (ii), we get

$\frac{x}{\sin\alpha} \times \frac{\sin\beta}{3x} = \frac{AD}{\sin(\pi/3)} \times \frac{\sin(\pi/4)}{AD}$

$\frac{\sin\beta}{3\sin\alpha} \frac{1/\sqrt{2}}{\sqrt{3}/2} = \frac{\sqrt{2}}{\sqrt{3}}$

$\frac{\sin\beta}{\sin\alpha} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{6}$

$\therefore \frac{\sin\angle BAD}{\sin\angle CAD} = \frac{\sin\alpha}{\sin\beta} = \frac{1}{\sqrt{6}}$

34. We have, $\frac{1}{2}ap_1 = \Delta, \frac{1}{2}bp_2 = \Delta, \frac{1}{2}cp_3 = \Delta$

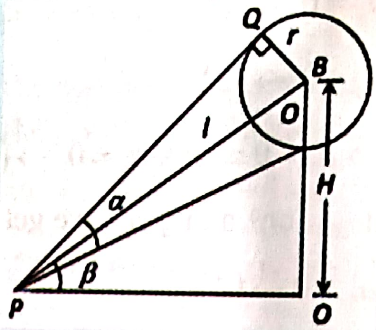
$p_1 = \frac{2\Delta}{a} p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$

$\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{a}{2\Delta} + \frac{b}{2\Delta} = \frac{a+b-c}{2\Delta} = \frac{2(s-c)}{2\Delta} = \frac{s-c}{\Delta}.$

x = y = 0, z = 1

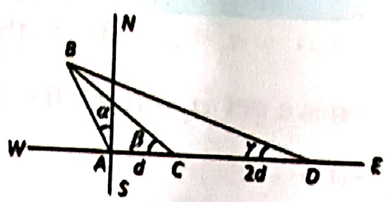
35. In $\Delta PQB, I = \frac{r}{\sin\frac{\alpha}{2}}$ and in ΔPOB

$H = l\sin\beta \Rightarrow H = r\sin\beta \operatorname{cosec}\frac{\alpha}{2}.$



36. By m – n theorem at C

$(d + 2d)\cot\beta = d\cot\phi - 2d\cot(90^0 + \alpha)$



$3d\cot\beta = d\cot\gamma + 2d\tan\alpha$

$\Rightarrow 3\cot\beta = \cot\gamma + 2\tan\alpha$

$\therefore 2\tan\alpha = 3\cot\beta - \cot\gamma.$

37. $\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x = \frac{c}{\sqrt{a^2+b^2}}$

$\sin(x+\alpha)=\frac{c}{\sqrt{a^2+b^2}}>1$ (as given)

Hence there is no solution.

The first equation can be written as

$$2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)=2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x+y)$$

$\therefore$ Either $\sin\frac{1}{2}(x+y)=0$ or $\sin\frac{1}{2}x=0$

or $\sin\frac{1}{2}y=0$

Thus $x+y=-1, x-y=-1$.

When $x+y=0$, we have to reject $x+y=1$ and check with the options or $x+y=-1$ and solve it with $x-y=1$ or $x-y=-1$ which gives $\left(\frac{1}{2},-\frac{1}{2}\right)$ or $\left(-\frac{1}{2},-\frac{1}{2}\right)$ as the possible solution. Again solving with $x=0$, we get $(0,\pm1)$ and solving with $y=0$, we get $(\pm1,0)$ as the other solution. Thus, we have six pairs of solution for x and y.

In $\Delta ACD, \frac{h}{\sin 67.5^0}=\frac{AC}{\sin 90^0}$

$\Rightarrow\frac{h}{AC}=\sin 67.5^0$ ...(i)

In $\Delta ABC, \frac{AC}{\sin 22.5^0}=\frac{x}{\sin 45^0}$

$\Rightarrow\frac{AC}{x}=\sqrt{2}\sin 22.5^0$ .....(ii)

From (i) and (ii), $\frac{h}{x}=\frac{1}{2}$.

$\Rightarrow K=2$

$\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha}=\frac{1}{7}\Rightarrow\tan\alpha=\frac{1}{7}$

$\sin\beta=\frac{1}{\sqrt{10}}\Rightarrow\tan\beta=\frac{1}{3}\Rightarrow\tan 2\beta=\frac{3}{4}$

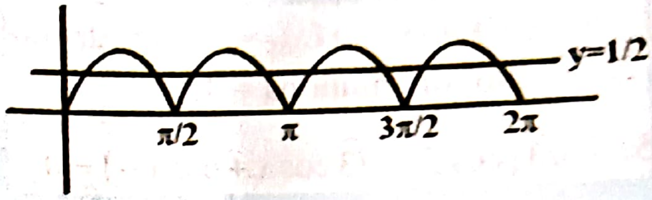
$\tan(\alpha+2\beta)=\frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}=1$

41. $\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x|; x\in[0,2\pi]$

$\Rightarrow\log_{1/2}|\sin x|+\log_{1/2}|\cos x|=2$

$\Rightarrow\log_{1/2}(|\sin x\cos x|)=2$

$\Rightarrow|\sin x\cos x|=\frac{1}{4}\Rightarrow|\sin 2x|=\frac{1}{2}$



$\Rightarrow$ 8 solutions

42. Let $f(x)=\frac{4}{\sin x}+\frac{1}{1-\sin x}$

$\Rightarrow f'(x)=0\Rightarrow\sin x=2/3$

$\therefore f(x)_{\min}=\frac{4}{2/3}+\frac{1}{1-2/3}=9$

$f(x)\max\to\infty$

$f(x)$ continuous function

$\therefore\alpha_{\min}=9$

43. $3\sin x+4\cos x=k+1$

$\Rightarrow k+1\in\left[-\sqrt{3^2+4^2},\sqrt{3^2+4^2}\right]$

$\Rightarrow k+1\in[-5,5];\quad\Rightarrow k\in[-6,4]$

No. of integral values of k = 11

44. $\log_4(x-1)=\log_2(x-3)$

$\Rightarrow\frac{1}{2}\log_2(x-1)=\log_2(x-3)$

$\Rightarrow\log_2(x-1)^{1/2}=\log_2(x-3)$

E-9 : $Cos^{-1}(\cos 10)$ *is equal to....*

Sol: We know that $Cos^{-1}(\cos\theta)=\theta$ if $0\le\theta\le\pi$

Here $\theta=10\,rad$, clearly, it does not lie between 0 and π. But $4\pi-10$ lies between 0 and π

$\therefore Cos^{-1}(\cos 10)=Cos^{-1}(\cos(4\pi-10))=4\pi-10$

E-10: $Tan^{-1}(Tan(-6))$ *is equal to.....*

Sol: We know that $Tan^{-1}(Tan\theta)=\theta$ if $\frac{-\pi}{2}<\theta<\frac{\pi}{2}$

Here $\theta=-6\,rad$, does not lie between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$

But $2\pi-6$ lies between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$

Now $Tan(2\pi-6)=-Tan6=Tan(-6)$

$\therefore Tan^{-1}(Tan(-6))$

$=Tan^{-1}(Tan(2\pi-6))=2\pi-6$

Concept Based Questions

1. $Tan(Cos^{-1}x)=$

1) $\frac{\sqrt{1-x^2}}{x}$ 2) $\frac{x}{1-x^2}$
3) $\frac{\sqrt{1+x^2}}{x}$ 4) $\sqrt{1-x^2}$

2. The value of $\sin\left(Cot^{-1}\left(\cos\left(Tan^{-1}x\right)\right)\right)$ is

1) $\sqrt{\frac{x^2+2}{x^2+1}}$ 2) $\sqrt{\frac{x^2+1}{x^2+2}}$
3) $\frac{x}{\sqrt{x^2+2}}$ 4) $\frac{1}{\sqrt{x^2+2}}$

3. $\tan\left[2Tan^{-1}(\cos x)\right]=$

1) 2tanx cosx 2) 2tanx cosecx
3) 2cotx cosecx 4) 2secx

4. $\cos^2\left[Tan^{-1}\left(\sin(Cot^{-1}x)\right)\right]=$

1) $\frac{x^2-1}{x^2+2}$ 2) $\frac{x^2+1}{x^2-2}$ 3) $\frac{x^2+1}{x^2+2}$ 4) $\frac{x^2-1}{x^2+1}$

5. If a> b> c>0 , then

$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)$

1) 0 2) $\pi/2$
3) π 4) $3\pi/2$

6. If $\sec^{-1}\sqrt{1+x^2}+\operatorname{cosec}^{-1}\frac{\sqrt{1+y^2}}{y}+\cot^{-1}\frac{1}{z}=\pi$, then

1) $x+y+z=0$ 2) $x+y+z=1$
3) $x+y+z=xyz$ 4) $x+y+z=-xyz$

7. $\tan\left[\frac{1}{2}Sin^{-1}\left(\frac{2x}{1+x^2}\right)-\frac{1}{2}Cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]=$

1) 0 2) 1 3) $\frac{x-y}{1+xy}$ 4) $\frac{2x}{1-x^2}$

8. $Cos[Sin^{-1}(2\cos^2\theta-1)+Cos^{-1}(1-2\sin^2\theta)]=$

1) 0 2) 1 3) -1 4) $\frac{\pi}{2}$

9. $Cos^{-1}(x/a)+Cos^{-1}(y/b)=\theta$ then

$\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\theta+\frac{y^2}{b^2}=$

1) $\sin^2\theta$ 2) $\cos^2\theta$
3) $\tan^2\theta$ 4) $\cot^2\theta$

10. $2Sin^{-1}x=Sin^{-1}\left(2x\sqrt{1-x^2}\right)$ holds good for

1) $x\in[0,1]$ 2) $x\in[-1,1]$
3) $x\in\left[-\frac{1}{\sqrt2},\frac{1}{\sqrt2}\right]$ 4) $x\in[-1,0]$

11. $\operatorname{cosec}^{-1}(\cos x)$ is defined if

1) $x \in [-1, 1]$ 2) $x \in R$

3) $x-(2n\pi+1)\frac{\pi}{2}$, $n \in z$

4) $x = n\pi$, $n \in z$

12. If $0<x<1$ then $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$ is equal to

1) $\frac{1}{2}\cos^{-1}x$ 2) $\cos^{-1}\sqrt{\frac{1+x}{2}}$

3) $\sin^{-1}\sqrt{\frac{1-x}{2}}$ 4) All the above

KEY

01) 1 02) 2 03) 3 04) 3 05) 3
06) 3 07) 3 08) 1 09) 1 10) 3
11) 4 12) 4

Hints & Solutions

1. $\cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

2. $\tan^{-1}x = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

3. Apply $2\tan^{-1}x$ formula

4. $\operatorname{Cot}^{-1}x=\operatorname{Sin}^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$ &

$\operatorname{Tan}^{-1}x=\operatorname{Cos}^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

5. $\operatorname{Cot}^{-1}(-x)=\pi-\operatorname{Cot}^{-1}x$

6. Put x = tanA, y = tanB, z = tanC

7. Put x = tanA ; y = tanB

8. $2\cos^2\theta-1=1-2\sin^2\theta=\cos 2\theta$

9. Apply $\cos^{-1}x+\cos^{-1}y$ formula

10. Standard $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

11. $\operatorname{Cosec}^{-1}(\cos x)$ is defined only if $\cos x = \pm 1$

$\Rightarrow x = n\pi;\ n \in z$

12. Put $x=\frac{1}{2}$, G.E $=\frac{\pi}{6}$

EXERCISE-I

CRTQ & SPQ LEVEL-I

DOMAIN, RANGE PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

C.R.T.Q Class Room Teaching Questions

1. The domain of $\operatorname{Sin}^{-1}x+\operatorname{Cos}^{-1}x$ is

1) $(-\pi,\pi)$ 2) $[-1,1]$

3) $(0,2\pi)$ 4) $(-\infty,\infty)$

2. The domain of $\log_e \sin^{-1}(x)$ is

1) (0, 1] 2) (0, 2]

3) $(0, \infty)$ 4) $(-\infty, 0]$

3. The range of $\operatorname{Tan}^{-1}x$ is

1) R 2) $(0,\pi)$

3) $[0,\pi]$ 4) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

4. The principal value of $\operatorname{Cos}^{-1}\left(\cos\frac{7\pi}{6}\right)$

1) $\frac{7\pi}{6}$ 2) $\frac{5\pi}{3}$ 3) $\frac{5\pi}{6}$ 4) $\frac{13\pi}{6}$

5. $\operatorname{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)-\operatorname{Tan}^{-1}(-\sqrt{3})$ is

1) $\frac{-\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{6}$ 4) 0

$\cot^{-1}\left(2+\sqrt{3}\right)=$

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{15}$ 3) $\frac{\pi}{5}$ 4) $\frac{3\pi}{10}$

$\cot\left[\sin^{-1}\sqrt{\frac{13}{17}}\right]-\sin\left[\tan^{-1}\frac{2}{3}\right]=$

1) $-\frac{2}{\sqrt{13}}$ 2) 0 3) $\frac{2}{\sqrt{13}}$ 4) $\frac{2}{3\sqrt{13}}$

$\sec^2(\cot^{-1}\frac{1}{2})+\operatorname{cosec}^2(\tan^{-1}1/3)=$

1) 5 2) 10 3) 15 4) 50

Find the value of $\sin\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{7}{25}\right)$

1) $\frac{119}{125}$ 2) $\frac{117}{125}$ 3) $\frac{118}{125}$ 4) $\frac{113}{125}$

$\cos(2\cos^{-1}(7/25))=$

1) $\frac{527}{625}$ 2) $-\frac{527}{625}$ 3) $\pi-\frac{527}{625}$ 4) $\frac{24}{25}$

If $x+\frac{1}{x}=2$, the principle value of $\sin^{-1}x$ is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) π 4) $\frac{3\pi}{2}$

The numerical value of $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$ is

1) 1 2) 0 3) $\frac{7}{17}$ 4) $-\frac{7}{17}$

$2\tan^{-1}(1/2)+\sin^{-1}(3/5)=$

1) $\tan^{-1}(\frac{12}{25})$ 2) $\frac{\pi}{4}$

3) $\frac{\pi}{2}$ 4) $\tan^{-1}(\frac{25}{12})$

$\tan^{-1}(2)+\tan^{-1}(3)=$

1) $-\frac{\pi}{4}$ 2) $\frac{\pi}{4}$ 3) $\frac{3\pi}{4}$ 4) $\frac{5\pi}{4}$

$\tan^{-1}(\frac{m}{n})-\tan^{-1}(\frac{m-n}{m+n})=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{4}$

16. $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=$

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{4}$

17. $\sec[\tan^{-1}5+\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{3}{4}]=$

1) 3/5 2) 5/3 3) 4/5 4) $\sqrt{2}$

18. $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=$

1) π 2) $\pi/2$ 3) $\pi/4$ 4) $3\pi/4$

19. $\tan^{-1}\left(\frac{5}{12}\right)+\sin^{-1}\left(\frac{24}{25}\right)=\cos^{-1}(x)\Rightarrow x=$

1) $\frac{-31}{325}$ 2) $\frac{-33}{325}$ 3) $\frac{-36}{325}$ 4) $\frac{-39}{325}$

20. If $\sin^{-1}(\frac{12}{13})+\sec^{-1}(\frac{13}{x})=\frac{\pi}{2}$ then $x=$

1) 12 2) 13 3) 11 4) 5

21. $\tan^{-1}(\frac{a}{x})+\tan^{-1}(\frac{b}{x})=\frac{\pi}{2}$ then $x=$

1) ab 2) $\sqrt{ab}$

3) $\sqrt{a^2+b^2}$ 4) a^2+b^2

22. $\tan^{-1}\left(x+\sqrt{1+x^2}\right)=$

1) $\frac{\pi}{4}-\frac{1}{2}\tan^{-1}x$ 2) $\frac{1}{2}\tan^{-1}x$

3) $\frac{\pi}{2}-\frac{1}{2}\tan^{-1}x$ 4) $\frac{\pi}{4}+\frac{1}{2}\tan^{-1}x$

23. If $\tan^{-1}(\frac{3a^2x-x^3}{a^3-3ax^2})=k\tan^{-1}\left(\frac{x}{a}\right)$ then $k=$

1) 2 2) 3 3) -2 4) 4

24. If $\tan^{-1}(\frac{2x}{x^2-1})+\cos^{-1}\frac{x^2-1}{x^2+1}=\frac{2\pi}{3}$ then $x=$

1) $2-\sqrt{3}$ 2) $\sqrt{3}-\sqrt{2}$ 3) $2+\sqrt{3}$ 4) $+\sqrt{2}$

25. If $Tan^{-1}\left(\frac{x-1}{x-2}\right)+Tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$ then x=

1) $\frac{1}{\sqrt{2}}$ 2) $\pm\frac{1}{\sqrt{2}}$ 3) $\pm\frac{1}{\sqrt{3}}$ 4) $\frac{1}{\sqrt{3}}$

26. If $Cos^{-1}x = Cot^{-1}(4/3)+Tan^{-1}(1/7)$ then x =

1) ½ 2) $\frac{\sqrt{3}}{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{3}{5}$

27. If $n \in N$, $\sum_{k=1}^{n} Sin^{-1}(x_k)=\frac{n\pi}{2}$ then $\sum_{k=1}^{n} x_k =$

1) n 2) k 3) $\frac{k(k+1)}{2}$ 4) $\frac{n(n+1)}{2}$

28. If $Sin^{-1}(x-\frac{x^2}{2}+\frac{x^3}{4}-\ldots)+Cos^{-1}(x^2-\frac{x^4}{2}+\frac{x^6}{4}-\ldots)$ $=\frac{\pi}{2}$ for $0<|x|<\sqrt{2}$ then x =

1) 1/2 2) 1 3) - 1/2 4) -1

29. The value of $Sin^{-1}(\sin 10)$ is

1) 10 2) $10-3\pi$ 3) $3\pi-10$ 4) -10

30. The greatest of $Tan^{-1}1, Sin^{-1}1, \sin 1, \cos 1$ is

1) $\sin 1$ 2) $\cos 1$
3) $\tan^{-1}1$ 4) $\sin^{-1}1$

31. If $Tan^{-1}x, Tan^{-1}y, Tan^{-1}z$ are in A.P. then $\frac{2y}{1-y^2}=$

1) $\frac{x-z}{1+xz}$ 2) $\frac{x+z}{1-xz}$ 3) x + z 4) xz

S.P.Q. Student Practice Questions

32. The domain of $Sin^{-1}\frac{2x+1}{3}$ is

1. (-2, 1] 2. [-2, 1] 3. R 4. [-1, 1]

3. The domain of $Cos^{-1}\sqrt{2x}$ is

1) [-1, 1] 2) [-1/2, 1/2]
3) [0, 1/2] 4) (1, 1/2)

4. The range of $Cot^{-1}x$ is

1) $(0,\pi)$ 2) $[0,\pi]$
3) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ 4) R

35. The principal value of $Sin^{-1}\left(Sin\left(\frac{2\pi}{3}\right)\right)$

1) $\frac{2\pi}{3}$ 2) $\frac{\pi}{3}$ 3) $\frac{-\pi}{3}$ 4) $\frac{-2\pi}{3}$

36. The principal value of $Sin^{-1}(Tan(\frac{-5\pi}{4}))$

1) $\frac{\pi}{4}$ 2) $\frac{-\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{-\pi}{2}$

37. $Sec^{-1}(\frac{2\sqrt{2}}{1+\sqrt{3}})$

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{3}$ 3) $\frac{3\pi}{4}$ 4) $\frac{\pi}{6}$

38. Thevalue of $Sin(2Tan^{-1}\frac{1}{3})+Cos(Tan^{-1}2\sqrt{2})$ is

1) $\frac{12}{13}$ 2) $\frac{13}{14}$ 3) $\frac{14}{15}$ 4) $\frac{16}{15}$

39. $Sec^2(Tan^{-1}(2))+Cosec^2(Cot^{-1}(2))=$

1) 5 2) 10 3) 15 4) 50

40. $Cos^{-1}(\frac{4}{5})+Cos^{-1}(\frac{63}{65})=$

1) $Cos^{-1}(\frac{204}{325})$ 2) $Cos^{-1}(\frac{300}{325})$
3) $Cos^{-1}(\frac{201}{300})$ 4) $Sin^{-1}(\frac{204}{325})$

41. $\sin(1/2\ Cot^{-1}(-3/4))=$

1) $1/\sqrt{5}$ 2) $2/\sqrt{5}$ 3) $-2/\sqrt{5}$ 4) $-1/\sqrt{5}$

42. If 'x' is a negative real number then the value of $Cos^{-1}x + Sec^{-1}x$ is

1) rational number 2) irrational number
3) integer 4) imaginary

43. $\tan(2\ Tan^{-1}(\frac{\sqrt{5}-1}{2}))=$

1) 4 2) 3 3) ½ 4) 2

44. $Tan^{-1}(\frac{3}{2}) - Tan^{-1}(\frac{1}{5}) =$

1) $\frac{-\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{4}$

45. $4\ Tan^{-1}\frac{1}{5} - Tan^{-1}\frac{1}{239} =$

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{4}$

46. $Cot(\cos ec^{-1}\frac{5}{3} + Tan^{-1}\frac{2}{3}) =$

1) $\frac{6}{17}$ 2) $\frac{3}{17}$ 3) $\frac{4}{17}$ 4) $\frac{5}{17}$

47. $2Tan^{-1}\frac{1}{5} + Tan^{-1}\frac{1}{7} + 2Tan^{-1}\frac{1}{8} =$

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{4}$

48. $Cos^{-1}(\sqrt{3}x) + Cos^{-1}x = \frac{\pi}{2}$ then x =

1) 0 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{1}{2}$ 4) 1

49. $Sin^{-1}(\frac{5}{x}) + Sin^{-1}(\frac{12}{x}) = \frac{\pi}{2}$, $(x > 0)$ then x =

1) 12 2) 13 3) 14 4) 15

50. $Tan^{-1}(\frac{1+\sin x}{\cos x}) =$

1) $\frac{\pi}{4} - \frac{x}{2}$ 2) $\frac{\pi}{4} - x$ 3) $\frac{\pi}{4} + x$ 4) $\frac{\pi}{4} + \frac{x}{2}$

51. A value of $Tan^{-1}\left\{\sin\left(Cos^{-1}\sqrt{\frac{2}{3}}\right)\right\}$ is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

52. $Tan[\frac{1}{2}Sin^{-1}(\frac{2a}{1+a^2}) + \frac{1}{2}Cos^{-1}(\frac{1-a^2}{1+a^2})] =$

1) $\frac{2a}{1+a^2}$ 2) $\frac{2a}{1-a^2}$ 3) $\frac{a}{1+a^2}$ 4) $\frac{a}{1-a^2}$

53. If $3Tan^{-1}(2-\sqrt{3}) - Tan^{-1}(x) = Tan^{-1}(1/3)$ then x =

1) 1/2 2) 2 3) 3 4) 1/3

54. If $Cos^{-1}(\frac{8}{17}) - Cos^{-1}(\frac{5}{13}) = Cos^{-1}x$ then x =

1) $\frac{140}{221}$ 2) $\frac{220}{221}$

3) $\frac{7}{11}$ 4) $\frac{221}{220}$

55. The equation $2Cos^{-1}x = Cos^{-1}(2x^2-1)$ is satisfied by

1) $-1 \le x \le 1$ 2) $0 \le x \le 1$

3) $x \ge 1$ 4) $x \le 1$

56. If $\sum_{r=1}^{n} Cos^{-1}x_r = 0$, then $\sum_{r=1}^{n} x_r$ equals to

1) 0 2) n 3) $\frac{n(n+1)}{2}$ 4) $\frac{n}{2}$

57. $Sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right] =$

1) $\sin^{-1}x + \sin^{-1}\sqrt{x}$ 2) $\sin^{-1}x - \sin^{-1}\sqrt{x}$

3) $\sin^{-1}\sqrt{x} - \sin^{-1}x$ 4) 0

58. If $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ then $Sin^{-1}(\sin\theta) =$

1) θ 2) $\pi - \theta$ 3) $2\pi - \theta$ 4) $-\pi + \theta$

59. $Cos^{-1}(\frac{a-b}{a+b}) =$

1) $2\ Tan^{-1}b$ 2) $2\ Tan^{-1}\sqrt{\frac{b}{a}}$

3) $2\ Tan^{-1}\sqrt{\frac{a}{b}}$ 4) $2\ Tan^{-1}(\frac{ab}{a-b})$

60. If $\theta = Tan^{-1}a, \phi = Tan^{-1}b$ and ab=-1, then $\theta - \phi$ is equal to :

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) π

50. $\frac{1+Sinx}{Cosx} = Tan(\frac{\pi}{4}+\frac{x}{2})$

51. $Tan^{-1}[Sin(Sin^{-1}\sqrt{\frac{1}{3}})] = Tan^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$

52. Put $a = Tan\theta$

53. $2-\sqrt{3} = Tan15^0$ then $Tan^{-1}x = \frac{\pi}{4} - Tan^{-1}\frac{1}{3}$

54. Apply $Cos^{-1}x - Cos^{-1}y = Cos^{-1}(xy+\sqrt{1-x^2}\sqrt{1-y^2})$

55. Given equation satisfied for $0 \le x \le 1$

56. $Cos^{-1}x_1 = Cos^{-1}x_2 = ... = Cos^{-1}x_n = 0$

$\Rightarrow x_1 = x_2 = ... = x_n = +1$

$\therefore \sum_{r=1}^{n} x_r = n$

57. $Sin^{-1}[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}] = Sin^{-1}[x\sqrt{1-(\sqrt{x})^2} - \sqrt{x}\sqrt{1-x^2}]$

$= Sin^{-1}x - Sin^{-1}\sqrt{x}$

58. $Sin^{-1}(Sin\theta) = Sin^{-1}(Sin(\pi-\theta)) = \pi-\theta$

59. $Cos^{-1}\left(\frac{1-\frac{b}{a}}{1+\frac{b}{a}}\right) = Cos^{-1}\left(\frac{1-\left(\sqrt{\frac{b}{a}}\right)^2}{1+\left(\sqrt{\frac{b}{a}}\right)^2}\right) = 2Tan^{-1}\sqrt{\frac{b}{a}}$

60. $Tan(\theta-\Phi) = \frac{Tan\theta - Tan\Phi}{1+Tan\theta Tan\Phi} = \frac{a-b}{1-1} = \infty$

$\theta - \Phi = \frac{\pi}{2}$

EXERCISE-II

CRTQ & SPQ LEVEL-II

DOMAIN, RANGE PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

C.R.T.Q Class Room Teaching Questions

1. Range of $Sin^{-1}x + Cos^{-1}x + Tan^{-1}x$ is

1) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ 2) $(0, \pi]$ 3) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ 4) $[0, \pi]$

2. Range of $f(x) = Sin^{-1}x + Tan^{-1}x + ...$

1) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ 2) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

3) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ 4) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

3. The domain of $Sin^{-1}[\log_2(x^2/2)]$ is

1) $[-2, -1]$ 2) $[1, 2]$

3) $[-2, -1] \cup [1, 2]$ 4) $[-2, 0]$

4. Arrange the following functions in increasing order of their domains

$f(x) = Sin^{-1}x$; $g(x) = Cos^{-1}\sqrt{2x}$;

$h(x) = Sin^{-1}2x$; $k(x) = Tan^{-1}(5x)$

1) $f(x), g(x), h(x), k(x)$

2) $g(x), h(x), k(x), f(x)$

3) $g(x), h(x), f(x), k(x)$

4) $g(x), f(x), h(x), k(x)$

5. The value of $\cos(2Cos^{-1} 0.8)$ is

1) 0.48 2) 0.96 3) 0.6 4) ...

6. The value of the expression

$Sin^{-1}\left(Sin\frac{22\pi}{7}\right) + Cos^{-1}\left(Cos\frac{5\pi}{3}\right) + Tan^{-1}\left(Tan\frac{5\pi}{7}\right) + Sin^{-1}(Cos2)$ is

1) $\frac{17\pi}{42} - 2$ 2) -2

3) $-\frac{\pi}{21} - 2$ 4) $\frac{\pi}{21}$

7. $\cos\left[2\,Sin^{-1}\sqrt{\frac{1-x}{2}}\right] =$

1) x 2) $\frac{1}{x}$ 3) 2x 4) 3...

8. If $Sin^{-1}(6x) + Sin^{-1}(6\sqrt{3}x) = -\frac{\pi}{2}$ then value of x is

1) $\frac{1}{12}$ 2) $-\frac{1}{12}$ 3) $-\frac{1}{4\sqrt{3}}$ 4) ...

If $\mathrm{Tan}^{-1}(2x)+\mathrm{Tan}^{-1}(3x)=\frac{\pi}{4}$ then x =

1) 1/2 2) 1/4 3) 1/6 4) 6

If $\mathrm{Cot}^{-1}\left(\frac{1-x^2}{2x}\right)+\mathrm{Cos}^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\frac{2\pi}{3}$; $x>0$; $x\neq 1$ then x =

1) $\frac{1}{\sqrt{2}}$ 2) $\pm\frac{1}{\sqrt{2}}$ 3) $\pm\frac{1}{\sqrt{3}}$ 4) $\frac{1}{\sqrt{3}}$

If $Tan^{-1}\frac{1}{1+2x}+Tan^{-1}\frac{1}{4x+1}=Tan^{-1}\frac{2}{x^2}$ then x =

1) 1 2) 0 3) -3 4) $\frac{2}{3}$

$\mathrm{Tan}\left[\mathrm{Tan}^{-1}\left(\frac{1}{a+b}\right)+\mathrm{Tan}^{-1}\left(\frac{b}{a^2+ab+1}\right)\right]=$

1) a 2) 1/a 3) b 4) 1/b

In a ΔABC, If C is a right angle then

$Tan^{-1}\left(\frac{a}{b+c}\right)+Tan^{-1}\left(\frac{b}{c+a}\right)=$

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{5\pi}{2}$

. If $\mathrm{Tan}^{-1}\left(\frac{x+1}{x-1}\right)+\mathrm{Tan}^{-1}\left(\frac{x-1}{x}\right)$ $=\pi+\mathrm{Tan}^{-1}(-7)$ then x =

1) 2 2) -2 3) 1 4) no solution

. The number of real solutions of

$\mathrm{Tan}^{-1}\left(\sqrt{x(x+1)}\right)+\mathrm{Sin}^{-1}\sqrt{(x^2+x+1)}=\frac{\pi}{2}$ is

1) 0 2) 1 3) 2 4) ∞

6. If $x=\mathrm{Tan}^{-1}(1)+\mathrm{Cos}^{-1}(-\tfrac{1}{2})+\mathrm{Sin}^{-1}(-\tfrac{1}{2})$ and $y=\cos\left[\tfrac{1}{2}\mathrm{Cos}^{-1}(1/8)\right]$ then

1) $x=2\pi y$ 2) $y=3\pi x$

3) $x=\pi y$ 4) $y=\pi x$

7. Let a,b,c be a positive real numbers

$\theta=\mathrm{Tan}^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\mathrm{Tan}^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+$ $\mathrm{Tan}^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ then $\tan\theta=$

1) 0 2) 3π 3) 1 4) 4π

18. If $\left[\sin^{-1}\left(\cos^{-1}\left(\sin^{-1}\left(\tan^{-1}x\right)\right)\right)\right]=1$, where $[\bullet]$ denotes the greatest integer function, then $x\in$

1) $[\tan\sin\cos 1,\ \tan\sin\cos\sin 1]$

2) $(\tan\sin\cos 1,\ \tan\sin\cos\sin 1)$

3) $[-1,\ 1]$

4) $[\sin\cos\tan 1,\ \sin\cos\sin\tan 1]$

19. If $x(3-x)\geq 2$ then $\mathrm{Sin}^{-1}(x)+\mathrm{Sin}^{-1}(x^2)+\ldots\ldots+\mathrm{Sin}^{-1}(x^{10})=$

1) $\frac{\pi}{2}$ 2) 2π 3) 5π 4) 10π

20. If $\mathrm{Cos}^{-1}x+\mathrm{Cos}^{-1}y=\frac{\pi}{2}$ and $\mathrm{Tan}^{-1}x-\mathrm{Tan}^{-1}y=0$ then $x^2+xy+y^2=$

1) 0 2) $-\frac{1}{2}$ 3) $\frac{1}{2}$ 4) $\frac{3}{2}$

21. If $0<x<1$, then $\sqrt{1+x^2}\left[\left\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\right\}^2-1\right]=$

1) $\frac{x}{\sqrt{1+x^2}}$ 2) x 3) $x\sqrt{1+x^2}$ 4) $\sqrt{1+x^2}$

22. The value of 'a' for which

$ax^2+\mathrm{Sin}^{-1}(x^2-2x+2)$ $+\mathrm{Cos}^{-1}(x^2-2x+2)=0$

has a real solution, is

1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$ 3) $\frac{2}{\pi}$ 4) $-\frac{2}{\pi}$

23. If minimum value of $\left(\mathrm{Sin}^{-1}x\right)^2+\left(\mathrm{Cos}^{-1}x\right)^2$ is $\frac{\pi^2}{k}$, then value of k is

1) 4 2) 6 3) 8 4) 10

24. If $6\mathrm{Sin}^{-1}(x^2-6x+12)=2\pi$, then the value of x, is

1) 1 2) 2 3) 3 4) does not exists

25. If $\cos^{-1}x > \sin^{-1}x$, then x belongs to the interval
1) $(-\infty, 0)$ 2) $(-1, 0)$
3) $[0, \frac{1}{\sqrt{2}})$ 4) $[-1, \frac{1}{\sqrt{2}})$

26. The least integral value of x for which $Tan^{-1}x > Cot^{-1}x$ is
1) 1 2) 2 3) 3 4) 4

27. The value of
$\sin^{-1}(\sin 12)+\sin^{-1}(\cos 12)=$
1) 0 2) $24-2\pi$
3) $4\pi-24$ 4) 8π

28. If $\log_2^x \geq 0$ then
$\log_{\frac{1}{\pi}}\{\sin^{-1}\frac{2x}{1+x^2}+2Tan^{-1}x\}=$
1) $\log_{\frac{1}{\pi}}(4\tan^{-1}x)$ 2) 0 3) -1 4) -2

29. If $Sin^{-1}x+Sin^{-1}y+Sin^{-1}z=\frac{3\pi}{2}$ then
$\sum(\frac{x^{201}+y^{201}}{x^{603}+y^{603}})(\frac{x^{402}+y^{402}}{x^{804}+y^{804}})=$
1) 0 2) 1 3) 2 4) 3

30. The ascending order of minimum values of the function $P: Sin^{-1}x-Cos^{-1}x$
$Q: Tan^{-1}x+Cot^{-1}x$,
$R: \sec^{-1}x-Cosec^{-1}x$
1) P, Q, R 2) P, R, Q
3) Q, P, R 4) Q, R, P

31. $\tan^{-1}\left(\frac{1}{1+(1)(2)}\right)+\tan^{-1}\left(\frac{1}{1+(2)(3)}\right)+$ ....$+\tan^{-1}\left(\frac{1}{1+(n-1)(n)}\right)=$
1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\tan^{-1}\left(\frac{n-1}{n+1}\right)$

S.P.Q. Student Practice Questions

32. Range of $Sin^{-1}x-Cos^{-1}x$ is
1) $[\frac{-3\pi}{2},\frac{\pi}{2}]$ 2) $[\frac{-5\pi}{3},\frac{\pi}{3}]$
3) $[\frac{-3\pi}{2},\pi]$ 4) $[0,\pi]$

33. The ascending order of $A=\sin^{-1}(\log$ ...
$B=\cos^{-1}\left(\log_3\left(\frac{1}{2}\right)\right)$, $C=\tan^{-1}(\log$ ...
1) C,B,A 2) B,A,C
3) C, A,B 4) B,C,A

34. The domain of $Tan^{-1}(5x)$
1) R 2) $(0,\pi)$ 3) $[0,\pi]$ 4) $\left(\frac{-\pi}{2},\right.$ ...

35. The decreasing order of ,
$A=\left(\sin^{-1}\left(\frac{1-\sqrt{3}}{2\sqrt{2}}\right)\right)$ $B=\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$,
$C=\tan^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$ is
1) B,A,C 2) B,C,A 3) C,A,B 4) C ...

36. The value of $Sin^{-1}(Cos(\frac{33\pi}{5}))$ is
1) $\frac{3\pi}{5}$ 2) $\frac{7\pi}{5}$ 3) $\frac{\pi}{10}$ 4) $\frac{-\pi}{10}$

37. $\cos^{-1}\left(\cos\frac{10\pi}{7}\right)+\sin^{-1}\left(\sin\frac{35\pi}{11}\right)+$
$\tan^{-1}\left(\tan\frac{24\pi}{13}\right)+Cot^{-1}\left(\cot\frac{26\pi}{5}\right)$ is
1) $\frac{4\pi}{7}+\frac{\pi}{55}+\frac{2\pi}{13}$ 2) $\frac{4\pi}{7}-\frac{\pi}{55}$ ...
3) $\frac{4\pi}{7}-\frac{\pi}{55}+\frac{2\pi}{13}$ 4) $-\frac{4\pi}{7}+\frac{\pi}{55}$ ...

38. The value of $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$ is
1) $\frac{3+\sqrt{5}}{2}$ 2) $3+\sqrt{5}$
3) $\frac{1}{2}(3-\sqrt{5})$ 4) $2-\sqrt{3}$

39. If $Tan^{-1}\frac{5}{12}+Tan^{-1}\frac{24}{25}=Cos^{-1}x$ then x =

1) $\frac{-31}{325}$ 2) $\frac{-33}{325}$ 3) $\frac{-36}{325}$ 4) $\frac{-39}{325}$

40. $6Sin^{-1}(x^2-6x+8.5)=\pi$, then x =

1) 1 2) 2 3) 3 4) 0

41. If $Tan^{-1}(x+1)+Tan^{-1}(x-1)=Tan^{-1}\frac{8}{31}$ then x =

1) 1 2) 1/2 3) -1/2 4) 1/4

42. If $Cot^{-1}\left(\frac{1}{x+1}\right)+Cot^{-1}\left(\frac{1}{x-1}\right)=$ $Tan^{-1}3x-Tan^{-1}x$ then $x=$

1) $\pm1/2$ 2) $-1, \pm 1/3$ 3) $2, \pm1$ 4) $-1, \pm1/2$

43. $Tan^{-1}\frac{5}{6}+\frac{1}{2}Tan^{-1}\frac{11}{60}=$

1) π 2) $\pi/2$ 3) $\pi/4$ 4) $3\pi/4$

44. $Tan\left(\frac{\pi}{4}+\frac{1}{2}Cos^{-1}\frac{a}{b}\right)+Tan\left(\frac{\pi}{4}-\frac{1}{2}Cos^{-1}\frac{a}{b}\right)$

1) b/a 2) a/b 3) 2a/b 4) 2b/a

45. $Cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(Cos\frac{9\pi}{10}-Sin\frac{9\pi}{10}\right)\right\}=$

1) $\frac{3\pi}{20}$ 2) $\frac{7\pi}{10}$ 3) $\frac{7\pi}{20}$ 4) $\frac{17\pi}{20}$

46. If $Tan^{-1}\left(\frac{x-1}{x-2}\right)+Tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$ then $x=$

1) $\frac{1}{\sqrt{2}}$ 2) $\pm\frac{1}{\sqrt{2}}$ 3) $\pm\frac{1}{\sqrt{3}}$ 4) $\frac{1}{\sqrt{3}}$

47. The number of solutions of the equation $2\left(Sin^{-1}x\right)^2-5Sin^{-1}x+2=0$ is

1) 0 2) 1 3) 2 4) 3

48. If $\frac{1}{\sqrt{2}}<x<1$ then

$Cos^{-1}x+Cos^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)=$

1) $2Cos^{-1}x-\frac{\pi}{4}$ 2) $2Cos^{-1}x$

3) $\frac{\pi}{4}$ 4) 0

49. Value of $Tan^{-1}2+Tan^{-1}3+Tan^{-1}4=$

1) $\pi-Tan^{-1}\frac{3}{5}$ 2) $\pi+Tan^{-1}\frac{5}{3}$

3) $\pi-Tan^{-1}\frac{5}{3}$ 4) $\pi+Tan^{-1}\frac{3}{5}$

50. The function $f(x)=Tan^{-1}(\sin x+\cos x)$ is an increasing function in

1) $(\frac{\pi}{4},\frac{\pi}{2})$ 2) $(-\frac{\pi}{2},\frac{\pi}{4})$

3) $(0,\frac{\pi}{2})$ 4) $(-\frac{\pi}{2},\frac{\pi}{2})$

51. If $\left[Cot^{-1}x\right]+\left[Cos^{-1}x\right]=0$ where x is non-negative real number and $[\bullet]$ denotes the greatest integer function, then complete set of values of x is

1) $(\cos1,\ 1]$ 2) $(\cos1,\ \cot1)$

3) $(\cot1\ \ 1]$ 4) $(0,\ \cos1)$

52. For the equation $Cos^{-1}x+Cos^{-1}2x+\pi=0$ the number of real solution is

1) 0 2) 1 3) 2 4) 3

53. If $\alpha=2Tan^{-1}\left(\sqrt{2}-1\right)$,

$\beta=3Sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+Sin^{-1}(-\frac{1}{2})$ and

$\gamma=Cos^{-1}\frac{1}{3}$ then

1) $\alpha<\beta<\gamma$ 2) $\alpha<\gamma<\beta$

3) $\beta<\gamma<\alpha$ 4) $\gamma<\beta<\alpha$

54. The value of

$$\sin^{-1}\left(\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right)$$ is

1) 0 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) π

55. The equation $2\cos^{-1}x+\sin^{-1}x=\frac{11\pi}{6}$ has

1) No solution 2) One solution
3) Two solutions 3) Three solutions

56. The smallest and the largest values of $\tan^{-1}\left(\frac{1-x}{1+x}\right), 0\le x\le 1$ are

1) $0,\pi$ 2) $0,\frac{\pi}{4}$
3) $-\frac{\pi}{4},\frac{\pi}{4}$ 4) $\frac{\pi}{4},\frac{\pi}{2}$

57. If α,β,γ are the roots of the equation $x^3+mx^2+3x+m=0$, then the general value of $\tan^{-1}\alpha+\tan^{-1}\beta+\tan^{-1}\gamma$ is

1) $(2n+1)\frac{\pi}{2}$ 2) $n\pi$ 3) $\frac{n\pi}{2}$
4) dependent upon the value of m

58. For $0\le \cos^{-1}x\le\pi$ and $-\frac{\pi}{2}\le \sin^{-1}x\le\frac{\pi}{2}$ then value of $\cos\left(\sin^{-1}x+2\cos^{-1}x\right)$ at $x=\frac{1}{5}$ is

1) $\frac{-2\sqrt{6}}{5}$ 2) $\frac{-\sqrt{6}}{5}$ 3) $\frac{2\sqrt{6}}{5}$ 4) $\frac{\sqrt{6}}{5}$

59. If 'a' is twice the tangent of the arithmetic mean of $\sin^{-1}x$ and $\cos^{-1}x$, 'b' is the geometric mean of $\tan x$ and $\cot x$. Then $x^2-ax+b=0\Rightarrow x=$

1) 2 2) 3 3) 1 4) 0

60. If x>1 then $2\tan^{-1}x$ cannot be equal to

1) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ 2) $\pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right)$
3) $\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$
4) $\pi-\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

61. $\cos^{-1}(\cos 12)-\sin^{-1}(\sin 12)=$

1) 0 2) π
3) $8\pi+24$ 4) $8\pi-24$

62. If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\frac{5\pi}{12}$ and $\sin^{-1}\frac{x}{a}-\sin^{-1}\frac{y}{b}=\frac{\pi}{12}$ then $\frac{x^2}{a^2}+\frac{y^2}{b^2}=$

1) 1 2) $\frac{1}{4}$ 3) $\frac{3}{4}$ 4) $\frac{5}{4}$

63. $2\cos^{-1}x=\sin^{-1}(2x\sqrt{1-x^2})$ is valid for

1) $-1\le x\le 1$ 2) $0\le x\le 1$
3) $0\le x\le\frac{1}{\sqrt{2}}$ 4) $\frac{1}{\sqrt{2}}\le x\le 1$

64. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$ and $(x-y)^2+(y-z)^2+(z-x)^2=0$ then $x^2+y^2+z^2=$

1) 0 2) 4 3) 1 4) 2

65. The ascending order of

$A=\cos(\sin^{-1}\frac{12}{13})$

$B=\sin(2\tan^{-1}\frac{3}{4})$ $C=\tan(\frac{1}{2}\cos^{-1}\frac{1}{3})$ is

1) A, B, C 2) A, C, B
3) B, C, A 4) B, A, C

66. $\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)=$

1. $\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$ 2. $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$

2. If x takes negative permissible value, then $\sin^{-1}x=$

1) $\cos^{-1}\sqrt{1-x^2}$ 2) $-\cos^{-1}\sqrt{1-x^2}$
3) $\cos^{-1}\sqrt{x^2-1}$ 4) $\pi-\cos^{-1}\sqrt{1-x^2}$

3. $\cos^{-1}\sqrt{\dfrac{a-x}{a-b}}=\sin^{-1}\sqrt{\dfrac{x-b}{a-b}}$ $(a\neq b)$ is possible if

1) $a\geq x\geq b$ or $a\leq x\leq b$ 2) $a=x=b$
3) $a>b$ and x takes any value
4) $a<b$ and x takes any value

4. The value of

$\sin^{-1}\left(\cos\left\{\cos^{-1}(\cos x)+\sin^{-1}(\sin x)\right\}\right)$

where $x\in\left(\dfrac{\pi}{2},\pi\right)$ is

1) $\dfrac{\pi}{2}$ 2) $-\pi$
3) π 4) $-\dfrac{\pi}{2}$

5. If $x=\sin\left(2\operatorname{Tan}^{-1}2\right)$ and $y=\sin\left[\dfrac{1}{2}\operatorname{Tan}^{-1}\dfrac{4}{3}\right]$ then

1) $x=\dfrac{y}{2}$ 2) $x>y$
3) $\dfrac{}{\sqrt{5}}$ 4) $x<y$

6. $\sin^{-1}(\sin 3)+\cos^{-1}(\cos 7)-\tan^{-1}(\tan 5)=$

1) $\pi-1$ 2) π 3) $3\pi-1$ 4) $2\pi-1$

7. If $y=(\sin^{-1}x)^3+(\cos^{-1}x)^3$ then

1) min $y=\dfrac{\pi^3}{8}$ 2) min $y=\dfrac{\pi^3}{32}$
3) max $y=\dfrac{\pi^3}{8}$ 4) max $y=\dfrac{7\pi^3}{32}$

8. $\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)+\sin^{-1}\left(\dfrac{\sqrt{2}-1}{\sqrt{6}}\right)+\ldots+\sin^{-1}\left(\dfrac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)+\ldots\infty=$

1) π 2) $\dfrac{\pi}{2}$ 3) $\dfrac{\pi}{4}$ 4) $\dfrac{3\pi}{2}$

9. $\cos^{-1}\left\{\dfrac{1}{2}x^2+\sqrt{1-x^2}\sqrt{1-\dfrac{x^2}{4}}\right\}=\cos^{-1}\dfrac{x}{2}-\cos^{-1}x$ holds for :

1) $|x|\leq 1$ 2) $x\in R$
3) $0\leq x\leq 1$ 4) $-1\leq x\leq 0$

10. The set of values of x from which the identity $\cos^{-1}x+\cos^{-1}\left(\dfrac{x}{2}+\dfrac{1}{2}\sqrt{3-3x^2}\right)=\dfrac{\pi}{3}$ holds good is

1) $[0,1]$ 2) $[0,\frac{1}{2}]$ 3) $[\frac{1}{2},1]$ 4) $\{-1,0,1\}$

S.P.Q. Student Practice Questions

11. If $A=\operatorname{Tan}^{-1}\left(\dfrac{x\sqrt{3}}{2k-x}\right)$ and $B=\operatorname{Tan}^{-1}\left(\dfrac{2x-k}{k\sqrt{3}}\right)$ then A-B =

1) 0^0 2) $\pi/6$ 3) $\pi/4$ 4) $\pi/3$

12. If a_1, a_2, a_3 .... a_n are in A.P. with common difference 'd', then

$$\tan\left[\operatorname{Tan}^{-1}\left(\frac{d}{1+a_1a_2}\right)+\operatorname{Tan}^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots\ldots+\operatorname{Tan}^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]=$$

1) $\dfrac{(n-1)d}{a_1+a_n}$ 2) $\dfrac{(n-1)d}{1+a_1a_n}$
3) $\dfrac{nd}{1+a_1a_n}$ 4) $\dfrac{a_n-a_1}{a_n+a_1}$

13. $\operatorname{Tan}^{-1}\left(\dfrac{c_1x-y}{c_1y+x}\right)+\operatorname{Tan}^{-1}\left(\dfrac{c_2-c_1}{1+c_2c_1}\right)+\operatorname{Tan}^{-1}\left(\dfrac{c_3-c_2}{1+c_3c_2}\right)+\ldots+\operatorname{Tan}^{-1}\left(\dfrac{1}{c_n}\right)=$

1) $\operatorname{Tan}^{-1}\left(\dfrac{2x}{y}\right)$ 2) $\operatorname{Tan}^{-1}(xy)$
3) $\operatorname{Tan}^{-1}\left(\dfrac{x}{y}\right)$ 4) $\operatorname{Tan}^{-1}\left(\dfrac{y}{x}\right)$

14. $\operatorname{Tan}^{-1}\left(\dfrac{1}{3}\right)+\operatorname{Tan}^{-1}\left(\dfrac{2}{9}\right)+\operatorname{Tan}^{-1}\left(\dfrac{4}{33}\right)+\ldots\ldots\infty$

1) $\dfrac{\pi}{4}$ 2) $\dfrac{\pi}{2}$ 3) π 4) 2π

15. Value of $\mathrm{Sec}^{-1}\left(\frac{1}{1-2x^2}\right)+4\mathrm{Cos}^{-1}\sqrt{\frac{1+x}{2}}=$

1) $2\mathrm{Tan}^{-1}x$ 2) $\mathrm{Tan}^{-1}\left(\frac{\sqrt{1-x^2}}{1-x}\right)$

3) $Cot^{-1}\left(\frac{\sqrt{1-x^2}}{1-x}\right)$ 4) constant for all x

16. If x_1,x_2,x_3,x_4 are roots of the equation $x^4-x^3\sin 2\beta+x^2\cos 2\beta-x\cos\beta-\sin\beta=0$ then

$\mathrm{Tan}^{-1}x_1+\mathrm{Tan}^{-1}x_2+\mathrm{Tan}^{-1}x_3+\mathrm{Tan}^{-1}x_4=$

1) β 2) $\frac{\pi}{2}-\beta$

3) $\pi-\beta$ 4) $-\beta$

17. If $Cot^{-1}\frac{n}{\pi}>\frac{\pi}{6}, n\in N$ then the maximum value of 'n' is

1) 6 2) 7 3) 5 4) 10

18. If α is the only real root of the equation $x^3+bx^2+cx+1=0 (b<c)$ then the value of $\mathrm{Tan}^{-1}\alpha+\mathrm{Tan}^{-1}\left(\frac{1}{\alpha}\right)=$

1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$ 3) 0 4) π

19. $\frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\mathrm{Tan}^{-1}\frac{\alpha}{\beta}\right)+\frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\mathrm{Tan}^{-1}\left(\frac{\beta}{\alpha}\right)\right)=$

1) $(\alpha-\beta)(\alpha^2+\beta^2)$

2) $(\alpha+\beta)(\alpha^2-\beta^2)$

3) $(\alpha+\beta)(\alpha^2+\beta^2)$ 4) 0

NO. OF SOLUTIONS

C.R.T.Q Class Room Teaching Questions

20. The number of solutions of $\mathrm{Sin}^{-1}(1+b+b^2+....\infty)+\mathrm{Cos}^{-1}\left(a-\frac{a^2}{3}+\frac{a^3}{9}+..\infty\right)=\frac{\pi}{2}$

1) 1 2) 2 3) 3 4) ∞

21. The number of solutions of the equation $\mathrm{Tan}^{-1}(x-1)+\mathrm{Tan}^{-1}(x)+\mathrm{Tan}^{-1}(x+1)=\mathrm{Tan}^{-1}(3x)$ is

1) 1 2) 2 3) 3 4) 4

22. The solution set of the equation $\mathrm{Tan}^{-1}x-Cot^{-1}x=\mathrm{Cos}^{-1}(2-x)$ is :

1) $[0,1]$ 2) $[-1,1]$ 3) $[1,3)$ 4) $(1,3)$

23. The number of solutions of the equation

$$3\mathrm{Cos}^{-1}x-\pi x-\frac{\pi}{2}=0$$

1) 0 2) 1 3) 2 4) infinitely many

24. Number of real solutions of the equation $\sqrt{1+\cos 2x}=\sqrt{2}\,\mathrm{Sin}^{-1}(\sin x)$ where $-\pi\le x\le\pi$

1) 0 2) 1 3) 2 4) 4

25. If the equation

$$\mathrm{Sin}^{-1}\left(x^2+x+1\right)+\mathrm{Cos}^{-1}(\lambda x+1)=\frac{\pi}{2}$$

has exactly two solutions, then λ can have the integral value.

1) -1 2) 0 3) 1 4) 2

S.P.Q. Student Practice Questions

26. If $\left(Cot^{-1}x\right)^2-7\left(Cot^{-1}x\right)+10>0$, then x lies in the interval

1) $(\cot 5,\cot 2)$

2) $(-\infty,\cot 5)\cup(\cot 2,\infty)$

3) $(-\infty,\cot 5)$

4) $(\cot 2,\infty)$

Which is possible if x lies in the second quadrand only and so n must be odd integer.

EXERCISE-IV
LEVEL-IV

1. **Assertion (A) : If** $0 < x < \frac{\pi}{2}$ **then**

$\sin^{-1}(\cos x)+\cos^{-1}(\sin x)=\pi-2x$

Reason (R) : $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$ $\forall\ x \in [0,1]$

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not correct explanation of A
3) A is true but Ris false
4) A is false but R is true

2. **Statement -1: Number of roots of the equation**

$\cot^{-1}x+\cos^{-1}2x+\pi=0$ **is zero.**

Statement - 2: Ragne of $\cot^{-1}x$ **and** $\cos^{-1}x$ **is** $(0,\pi)$ **and** $[0,\pi]$ **respectively**

1) Both the statements are true and statement 2 is the correct explanation of statement 1
2) Both the statements are true but statement 2 is not the correct explanation of statement 1
3) statement 1 is true and statement 2 is false
4) statement 1 is false and statement 2 is true

3. **Let** (x,y) **be such that**

$\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\pi/2$

a) If $a=1,b=0,then$ (x,y) lies on   p) $x^2+y^2=1$

b) If $a=1,b=1,then$ q) $(x^2-1)(y^2-1)=0$

c) $a=1,b=2,then$ r) $y=x$

d) $a=2,b=2then$ s) $(4x^2-1)(y^2-1)$

1) $a\to p,b\to q,c\to p,d\to s$
2) $a\to p,b\to q,c\to s,d\to r$
3) $a\to r,b\to q,c\to s,d\to r$
4) $a\to p,b\to q,c\to r,d\to s$

4. **Which of the following function(s) is/are identical**

(i) $f(x)=\cot(\cot^{-1}x)$; $g(x)=x$

(ii) $f(x)=\sin^{-1}(3x-4x^3)$; $g(x)=3\sin^{-1}x$

(iii) $f(x)=\sec^{-1}x+\operatorname{cosec}^{-1}x$; $g(x)=\frac{\pi}{2}$

(iv) $f(x)=\tan(\cot^{-1}x)$; $g(x)=\cot(\tan^{-1}x)$

(v) $f(x)=e^{\ln \cot^{-1}x}$; $g(x)=\cot^{-1}x$

(vi) $f(x)=e^{\ln \sec^{-1}x}$; $g(x)=\sec^{-1}x$

1) only i,iv,v   2) i, iii,iv only
3) ii,v,vi only   4) i,iv only

5. **Which of the following function(s) is/are periodic?**

(A) $f(x)=\frac{2^x}{2^{[x]}}$ **where [ ] denotes greatest integer function**

(B) $g(x)=\operatorname{sgn}\{x\}$ **where** $\{x\}$ **denotes the fractional part function**

3. $\cosh(0)=$
1) 0 2) 1
3) e 4) not defined

4. $\sinh(3)-\cosh(3)=$
1) e^{-3} 2) $-e^{-3}$ 3) e^{3} 4) $-e^{3}$

5. $(\cosh 2+\sinh 2)^{n}=$
1) $\cosh(2n)+\sinh(2n)$
2) $\cosh(2n)-\sinh(2n)$
3) $\cosh(2n)$ 4) $\sinh(2n)$

6. If $\cosh x=\sec\alpha$ then $\operatorname{cosech} x=$
1) $\sin\alpha$ 2) $\cos\alpha$
3) $\tan\alpha$ 4) $\cot\alpha$

7. If $\sinh x=\frac{3}{4}$ then $\sinh(2x)=$
1) $\frac{5}{8}$ 2) $\frac{15}{8}$ 3) $\frac{7}{8}$ 4) $\frac{17}{8}$

8. If $\tanh x=\frac{3}{5}$ then $\sinh(2x)=$
1) $\frac{15}{8}$ 2) $\frac{15}{17}$ 3) $\frac{18}{17}$ 4) $\frac{17}{8}$

9. $\cosh^{4}(x)-\sinh^{4}(x)=$
1) $\cosh x$ 2) $\cosh 2x$
3) $\sinh x$ 4) $\sinh 2x$

10. If $\cosh x=\frac{5}{4}$ then $\cosh(3x)=$
1) $\frac{61}{16}$ 2) $\frac{63}{16}$ 3) $\frac{61}{63}$ 4) $\frac{65}{16}$

11. $\sinh^{-1}\left(2^{\frac{3}{2}}\right)=$
1) $\log_e\left(2+\sqrt{18}\right)$ 2) $\log_e\left(3+\sqrt{8}\right)$
3) $\log_e\left(3-\sqrt{8}\right)$ 4) $\log_e\left(\sqrt{8}+\sqrt{27}\right)$

12. $\cosh^{-1}(2)=$
1) $\log_e\left(2+\sqrt{3}\right)$ 2) $\log_e\left(2+\sqrt{5}\right)$
3) $\log_e\left(2-\sqrt{5}\right)$ 4) $\log_e\left(2+\sqrt{2}\right)$

13. $\operatorname{Tanh}^{-1}\left(\frac{1}{3}\right)=$
1) $\frac{1}{2}\log_e(2)$ 2) $2\log_e(2)$
3) $-\frac{1}{2}\log_e(2)$ 4) $-2\log_e(2)$

14. $2\operatorname{Tanh}^{-1}\left(\frac{1}{2}\right)=$
1) 0 2) $\log_e 2$ 3) $\log_e 3$ 4) $\log_e 4$

15. $\operatorname{cosech}^{-1}(-4)=$
1) $\log_e\left(\frac{\sqrt{17}-1}{4}\right)$ 2) $\log_e\left(\sqrt{17}-1\right)$
3) $\log_e\left(\sqrt{17}+1\right)$ 4) $\log_e\left(\frac{\sqrt{17}+1}{4}\right)$

16. If $\sinh^{-1}(x)=\log_e\left(5+\sqrt{26}\right)$ then $x=$
1) 1 2) 2 3) 3 4) 5

17. If $\cosh^{-1}(k)=\log_e\left(3+2\sqrt{2}\right)$ then $k=$
1) 1 2) 2 3) 3 4) 4

18. $e^{\sinh^{-1}(\cot\theta)}=$
1) $\cot\theta+\operatorname{cosec}\theta$ 2) $-\cot\theta+\operatorname{cosec}\theta$
3) $\sec\theta-\tan\theta$ 4) $\sec\theta+\tan\theta$

19. $\operatorname{sech}^{-1}(\sin\theta)=$
1) $\log_e\left(\cos\frac{\theta}{2}\right)$ 2) $\log_e\left(\sin\frac{\theta}{2}\right)$
3) $\log_e(\cos\theta)$ 4) $\log_e\left(\cot\frac{\theta}{2}\right)$

20. If $x=\log_e\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$ then $\cosh x=$
1) $\sec\theta$ 2) $\operatorname{cosec}\theta$
3) $\sin\theta$ 4) $\cos\theta$

21. If $x=\log_e\left[\cot\left(\frac{\pi}{4}+\theta\right)\right]$ then $\sinh x=$
1) $\tan 2\theta$ 2) $\cot 2\theta$
3) $-\tan 2\theta$ 4) $-\cot 2\theta$

2. If $\cosh 2x = 99$ then $\tanh x =$

1) $\frac{5}{7\sqrt{2}}$ 2) $\frac{7}{5\sqrt{2}}$ 3) $\frac{5\sqrt{7}}{2}$ 4) $\frac{7\sqrt{5}}{2}$

3. $\sinh(ix) =$

1) isinx 2) $\sin(ix)$
3) -isinx 4) isin(ix)

S.P.Q. Student Practice Questions

4. The domain of $\coth x$ is
1) $(-\infty,\infty)$ 2) $(0,\infty)$ 3) $R-\{0\}$ 4) $(-\infty,0)$

5. The range of $\tanh x$ is
1) $[-1,1]$ 2) $(-1,0)$ 3) $(0,1)$ 4) $(-1,1)$

6. $\sinh(0) =$
1) 0 2) 1 3) e 4) not defined

7. $\cosh(1)-\sinh(1)$
1) e^2 2) e 3) $\frac{1}{e}$ 4) $\frac{1}{e^2}$

8. $(\cosh x - \sinh x)^n =$
1) $\cosh nx - \sinh nx$
2) $2\cosh nx$
3) $2\sinh nx$
4) $\cosh nx + \sinh nx$

9. If $\cosh x = \frac{5}{3}$ then $\sinh x =$
1) $\frac{3}{4}$ 2) $\frac{4}{3}$ 3) $\frac{4}{5}$ 4) $\frac{3}{5}$

10. If $\cosh x = \frac{5}{2}$ then $\cosh(2x) =$
1) $\frac{5\sqrt{21}}{2}$ 2) $\frac{23}{5\sqrt{21}}$ 3) $\frac{23}{2}$ 4) $\frac{7}{2}$

11. If $\operatorname{sech} x = \frac{3}{5}$ then $\tanh(2x) =$
1) $\frac{40}{41}$ 2) $\frac{41}{40}$ 3) $\frac{21}{41}$ 4) $\frac{16}{41}$

32. $\cosh^{-1}(2) - \sinh^{-1}(2) =$
1) $\cosh(2)$ 2) $\cosh(3)$
3) $\cosh(4)$ 4) $\cosh(8)$

33. If $\sinh x = \frac{3}{4}$ then $\sinh(3x) =$
1) $\frac{61}{16}$ 2) $\frac{63}{16}$ 3) $\frac{61}{63}$ 4) $\frac{65}{16}$

34. $\sinh^{-1}(2) =$
1) $\log_e(2+\sqrt{5})$ 2) $\log_e(2+\sqrt{7})$
3) $\log_e(3+\sqrt{10})$ 4) $\log_e(5+\sqrt{26})$

35. $\cosh^{-1}(1) =$
1) 2 2) 3 3) $\log_e(1+\sqrt{2})$ 4) 0

36. $\tanh^{-1}\left(\frac{1}{2}\right) =$
1) $\frac{1}{2}\log_e 3$ 2) $\frac{1}{2}\log_e 2$
3) $\log_e 3$ 4) $\log_e 5$

37. $\tanh^{-1}\left(\frac{1}{4}\right) + \coth^{-1}(4) =$
1) $\frac{1}{2}\log_e\left(\frac{5}{3}\right)$ 2) $\log_e\left(\frac{5}{3}\right)$
3) $\log_e(5)$ 4) $\log_e(3)$

38. $\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{2}}\right) =$
1) $\log_e(\sqrt{2}-1)$ 2) $\log_e(\sqrt{2}+1)$
3) $\log_e(\sqrt{3}+1)$ 4) $\log_e(\sqrt{3}-1)$

39. If $\cosh^{-1}(x) = \log_e(2+\sqrt{3})$ then $x =$
1) 2 2) 3 3) 1 4) 5

40. If $x = \log_e\left(y+\sqrt{y^2-1}\right)$ then $y =$
1) $\tanh x$ 2) cothx 3) sinhx 4) coshx

EXERCISE-II

CRTQ & SPQ LEVEL-II

DOMAIN, RANGE, PROPERTIES OF HYPERBOLIC FUNCTION & INVERSE HYPERBOLIC FUNCTION

C.R.T.Q Class Room Teaching Questions

1. The domain of $\cosh^{-1}(2x)$ *is*

1) $(-\infty,\infty)$ 2) $[1,\infty)$ 3) $[2,\infty)$ 4) $\left[\frac{1}{2},\infty\right)$

2. The range of $\cosh\left(\frac{x}{2}\right)$ *is*

1) R 2) $(0,\infty)$ 3) $(0,1)$ 4) $[2,\infty)$

3. The value of $\frac{\cosh 2\theta-1}{\sinh 2\theta}$ is equal to

1) $\cosh\theta$ 2) $\tanh\theta$ 3) $\operatorname{cosech}\theta$ 4) $\operatorname{sech}\theta$

4. If $\tanh^2 x=\cos\theta$ *then* $\cosh 2x=$

1) $\tan^2\left(\frac{\theta}{2}\right)$ 2) $\cot^2\left(\frac{\theta}{2}\right)$

3) $\sec^2\left(\frac{\theta}{2}\right)$ 4) $\sin^2\left(\frac{\theta}{2}\right)$

5. If $\cosh x=\sec\theta$ *then* $\tanh^2\left(\frac{x}{2}\right)=$

1) $\tan^2\frac{\theta}{2}$ 2) $\cot^2\frac{\theta}{2}$

3) $-\tan^2\frac{\theta}{2}$ 4) $-\cot^2\frac{\theta}{2}$

6. $\frac{\tanh x}{\operatorname{sech} x-1}+\frac{\tanh x}{\operatorname{sech} x+1}=$

1) cosechx 2) 2cosechx
3) -cosechx 4) -2cosechx

7. If *sinx coshy* $=\cos\theta, \cos x\sinh y=\sin\theta$ then $\sinh^2 y=$

1) $\cosh^2 x$ 2) $\cos^2 x$
3) $\cosh^3 x$ 4) $\sinh^2 x$

8. If x is an acute angle and $y=\log_e\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$ then cosx. coshy =

1) 0 2) -1 3) 1 4) 2

9. $\sinh\left(\cosh^{-1}x\right)=$

1) $\sqrt{x^2+1}$ 2) $\frac{1}{\sqrt{x^2+1}}$

3) $\sqrt{x^2-1}$ 4) $\frac{1}{\sqrt{x^2-1}}$

10. $\log_e\left[(x-1)+\sqrt{x^2-2x}\right], x\geq 2$ is equal to

1) $\sinh^{-1}(x-1)$ 2) $\cosh^{-1}(x-1)$
3) $\sinh^{-1}(x+1)$ 4) $\cosh^{-1}(x+1)$

S.P.Q. Student Practice Questions

11. The domain of $\operatorname{sech}^{-1}\left(\frac{x}{2}\right)$ is

1) $\left(0,\frac{1}{2}\right)$ 2) $\left[0,\frac{1}{2}\right]$ 3) $\left(0,\frac{1}{2}\right]$ 4) (0,2]

12. $\frac{\sinh(2x)}{1+\cosh(2x)}=$

1) $\tanh(x)$ 2) $\tanh^2(x)$
3) $\coth(x)$ 4) $\coth^2(x)$

13. *cosh2x+sinh2x=*

1) $\frac{1+\tanh(x)}{1-\tanh(x)}$ 2) $\frac{1-\tanh x}{1+\tanh x}$

3) $\frac{\tanh x-1}{\tanh x+1}$ 4) $\frac{1-\tanh x}{\tanh x-1}$

14. $\tanh^2(x)=\tan^2\theta$ then $\cosh 2x=$

1) $\cos 2\theta$ 2) $\sec 2\theta$ 3) $\sin 2\theta$ 4) $\cos\theta$

15. $\frac{\cosh(x)}{1-\tanh x}+\frac{\sinh(x)}{1-\coth(x)}=$

1) $\sinh(x)+\coth(x)$ 2) $\tanh(x)+\coth(x)$
3) $\tanh(x)-\coth x$ 4) $\sinh(x)+\cosh(x)$

(iv) If $\cos^2 A+\cos^2 B+\cos^2 C=1$ then the triangle is right angled.

(v) If $\cos A=\dfrac{\sin B}{2\sin C}$ then the triangle is isosceles.

(vi) If $\dfrac{a}{\cos A}=\dfrac{b}{\cos B}=\dfrac{c}{\cos C}$ then the triangle is equilateral .

(vii) If $\cos A+\cos B+\cos C=3/2$ then the triangle is equilateral.

(viii) If $\sin A+\sin B+\sin C=\dfrac{3\sqrt{3}}{2}$ then the triangle is equilateral.

(ix) If $\tan A+\tan B+\tan C=3\sqrt{3}$ then the triangle is equilateral .

(x) If $\cot A+\cot B+\cot C=\sqrt{3}$ then the triangle is equilateral .

Concept Based Questions

1. If the cosines of angles of a triangle are proportional to opposite sides, then the triangle is
   1) isosceles 2) right angled
   3) equilateral 4) isosceles or right angled
2. If the circumcentre of a triangle lies inside the triangle then the triangle is
   1) right angled 2) acute angled
   3) obtuse angled 4) scalene
3. If twice the square of the radius of a cirucm circle is equal to half the sum of the squares of the sides of inscribed triangle ABC, then $\sin^2A + \sin^2B + \sin^2C =$
   1) 1 2) 2 3) 4 4) 8
4. The radius of the excircle opposite to any vertex of an equilateral triangle of side'a' is
   1) a 2) $\dfrac{a}{2}$ 3) $\sqrt{3}\dfrac{a}{2}$ 4) $\sqrt{3}a$

JEE - MAIN

5. If the line joining the circumcentre 'O' and the incentre I is parallel to BC, then cosB + cosC =
   1) 3/2 2) 1 3) 3/4 4) 2
6. Circum radius of excentral triangle is
   1) R 2) 3R 3) 2R 4) 4R
7. For a ΔABC it is given that $\text{CosA+CosB+CosC}=\dfrac{3}{2}$ then the triangle is
   1) isosceles 2) equilateral
   3) right angled 4) scalene
8. In a ΔABC,
   1) $R\geq 2r$ 2) $R\leq 2r$
   3) $R>2r$ 4) $R<2r$
9. If the length of tangents from vertices to Incircle are in H.P then r_1,r_2,r_3 are in
   1) A.P 2) G.P
   3) H.P 4) A.G.P
10. Match the following.

Column-I	Column-II
A) If the angles A, B, C of a triangle ABC are in A.P and the sides a, b and c opposite to these angles are in G.P. Then a^2,b^2,c^2 are in	p) G.P
B) If r_1,r_2,r_3 in a triangle be in H.P., Then the sides are in	q) H.P
C) If the cotangents of half the angles of a traingle are in A.P. Then its sides are in	r) A.P
D) In a traingle ΔABC, $2\sin\dfrac{A}{2}\sin\dfrac{C}{2}=\sin\dfrac{B}{2}$ if a,b,c are in	s) A.G.P

1) A-r, B-r, C-r, D-r 2) A-r, B-r, C-p, D-q
3) A-p, B-p, C-p, D-q 4)A-p, B-p, C-p, D-p

EXERCISE-I

CRTQ & SPQ LEVEL-I

SINE RULE

C.R.T.Q Class Room Teaching Questions

1. In $\Delta ABC, \Sigma \frac{\sin(B-C)}{bc} =$

1) Δ 2) s 3) 0 4) s^2

2. In $\Delta ABC, \Sigma \frac{(b^2-c^2)}{a}\cos A =$

1) s 2) 0 3) Δ 4) 2Δ

3. In $\Delta ABC, \Sigma a^3\cos(B-C) =$

1) abc 2) 2abc 3) 3abc 4) 0

4. In $\Delta ABC, b^2\sin 2C + c^2\sin 2B =$

1) Δ 2) 2Δ 3) 4Δ 4) $\Delta/2$

5. In ΔABC, if $a\cos A + b\cos B + c\cos C = \frac{2\Delta}{k}$ then $k =$

1) r 2) R 3) s 4) R^2

6. A triangle ABC is such that aCosA = bCosB, then the triangle is

1) right angled 2) isosceles

3) right angled or isosceles 4) equilateral

7. If $a = 5$, $b = 6$, $\sin A = 5/6$, then $B =$

1) π 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/4$

8. If $a^2+b^2+c^2 = 8R^2$, then the triangle is

1) right angled 2) isosceles

3) equilateral 4) Scalene

In ΔABC, if $A=30^0, b=8, a=6, B=\sin^{-1}x$, then $x=$

1) 1/2 2) 1/3 3) 2/3 4) 1

If the angles of a triangle ABC are in A.P., $a=2, c=4$, then $b=$

1) $2\sqrt{3}$ 2) $\sqrt{21}$ 3) 8 4) 14

The angles of a triangle are in the ratio 3:5:10. Then the ratio of the smallest side to the greatest side is

1) $1:\sin 10^0$ 2) $1:2\sin 10^0$

3) $1:\cos 10^0$ 4) $1:2\cos 10^0$

The sides of a right angled triangle are in A.P., then they are in the ratio

1) 2:3:1 2) 2:3:5 3) 3:4:5 4) 3:1:2

In a ΔABC, If a is the arithmetic mean and $b, c (b \neq c)$ are the geometric means then $\frac{\sin^3 B + \sin^3 C}{\sin A \sin B \sin C} =$

1) 0 2) 1 3) 2 4) 4

S.P.Q. Student Practice Questions

In ΔABC, $\sum a^3 \sin(B-C) =$

1) $a^3b^3c^3$ 2) 3abc

3) $3a^2b^2c^2$ 4) 0

In ΔABC, $\sum \frac{b^2-c^2}{a^2}\sin 2A =$

1) 0 2) a^2+b^2 3) b^2+c^2 4) c^2+a^2

In ΔABC, $\frac{1}{2}a^2\frac{\sin B \sin C}{\sin A} =$

1) Δ 2) 2Δ 3) 3Δ 4) 4Δ

In ΔABC, $\Sigma a \cos A =$

1) 4R cos A cos B cos C

2) 4R sin A sin B sin C

3) 2R sin A sin B sin C

4) 2R cos A cos B cos C

18. In ΔABC, if bCosA=aCosB then the triangle is

1) right angled 2) isosceles

3) equilateral 4) scalene

19. If $a=\sqrt{3}+1$, $b=2$, $B=45^\circ$, then $A=$

1) 30° 2) 60° 3) 75° 4) 45°

20. In a triangle ABC if tan A + tan B + tan C $=3\sqrt{3}$, then the triangle is

1) isosceles 2) right angled

3) equilateral 4) scalene

21. If the angles of a ΔABC are $30^0, 45^0$ and the included side is $\sqrt{3}+1$, then the remaining sides are

1) $2, \sqrt{2}$ 2) $2, 2\sqrt{3}$

3) $\sqrt{2}, 4$ 4) $2, 4\sqrt{3}$

22. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. The sides of the triangle are

1) 2,3,4 2) 3,4,5

3) 4,5,6 4) 5,6,

23. In ΔABC, If $(\sin A+\sin B+\sin C)(\sin A+\sin B-\sin C) = 3\sin A \sin B$ then

1) $A=60^0$ 2) $B=60^0$ 3) $C=60^0$ 4) $C=90^0$

COSINE RULE

C.R.T.Q Class Room Teaching Questions

24. In ΔABC, If $\frac{b}{c+a}+\frac{c}{a+b}=1$, then

1) $A=60^0$ 2) $B=60^0$

3) $C=60^0$ 4) $A=90^\circ$

25. In ΔABC, If $\frac{b}{c^2-a^2}+\frac{a}{c^2-b^2}=0$, then

1) $A=60^0$ 2) $B=60^0$

3) $C=60^0$ 4) $C=90^\circ$

26. If the sides of a triangle are 4 cm, 5 cm, 6 cm then ratio of the cosine of the least and the greatest angles is

1) 6 : 1 2) 2 : 1 3) 3 : 4 4) 5 : 6

27. If the sides of a triangle are in the ratio $x:y:\sqrt{x^2+xy+y^2}$, then the greatest angle is

1) 90° 2) 120°

3) $\cos^{-1}\left(\frac{x+y}{x-y}\right)$ 4) 30°

28. If two sides of a triangle are the roots of the equation $4x^2-(2\sqrt{6})x+1=0$ and the included angle is 60°, then the third side is

1) $\sqrt{3}$ 2) $\sqrt{3}/2$ 3) $1/\sqrt{3}$ 4) $2\sqrt{3}$

29. The sides of a triangle are $\sin\alpha, \cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $\alpha, 0<\alpha<\frac{\pi}{2}$. Then the greatest angle of the triangle is

1) 60^0 2) 150^0 3) 120^0 4) 90^0

30. Two sides of a triangle are given by the roots of the equation $x^2-5x+6=0$ and the angle between the sides is $\pi/3$. Then the perimeter of the triangle is

1) $5+\sqrt{2}$ 2) $5+\sqrt{3}$ 3) $5+\sqrt{5}$ 4) $5+\sqrt{7}$

31. In a ΔABC, if D is the mid point of BC and AD is perpendicular to AC, then $\cos B =$

1) $\frac{2b}{a}$ 2) $\frac{-b}{c}$

3) $\frac{(b^2+c^2)}{ca}$ 4) $\frac{(c^2+a^2)}{ca}$

S.P.Q. Student Practice Questions

32. In ΔABC, If $A=60^\circ$ then $\frac{b}{c+a}+\frac{c}{a+b}=$

1) 1 2) 2 3) 3 4) 4

33. In ΔABC, $\frac{a}{b^2-c^2}+\frac{c}{b^2-a^2}=0$ then B =

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{2\pi}{3}$ 4) $\frac{\pi}{3}$

34. If $7, 4\sqrt{3}, \sqrt{13}$ are the sides of ΔABC then least angle is

1) 45^0 2) 60^0 3) 90^0 4) 30^0

35. In ΔABC, c(acosB - b cos A) =

1) $a^2 - b^2$ 2) $a^2 + b^2$

3) $b^2 - c^2$ 4) $b^2 + c^2$

36. If the angles of the ΔABC are in A.P., then $a^2+c^2-ac=$

1) bc 2) b^2c 3) abc 4) b^2

37. If $x^2+x+1, 2x+1, x^2-1$ are the sides of a triangle, then its largest angle is

1) $\pi/3$ 2) $\pi/4$ 3) $\pi/6$ 4) $2\pi/3$

38. Two straight roads intersect at an angle of 60^0. A bus on one road is 2 km. away from the intersection and a car on the other road is 3 km away from the intersection. Then the distance between the two vehicles is

1) 1 km 2) $\sqrt{2}$ km 3) 4 km 4) $\sqrt{7}$ km

39. If $a=26, b=30, \cos C=\frac{63}{65}$ then c=

1) 8 2) 25 3) 24 4) 6

40. In ΔABC, If $\sin A$ and $\sin B$ are the roots of the equation

$c^2x^2-c(a+b)x+ab=0$ then $\sin C=$

1) 0 2) 1/2 3) $1/\sqrt{2}$ 4) 1

41. In ΔABC, If $\tan A, \tan B, \tan C$ are in H.P., then a^2, b^2, c^2 are in

1) H.P 2) G.P 3) A.P 4) A.G.P

42. If $\frac{CosA}{bc}+\frac{CosB}{ac}+\frac{CosC}{ab}=\frac{a^2+b^2+c^2}{3k}$ then k =

1) a 2) bc 3) $a+b+c$ 4) abc

TANGENT RULE & MOLLIWEIDE RULE

C.R.T.Q Class Room Teaching Questions

43. In ΔABC, If $x=\tan\left(\frac{B-C}{2}\right)\tan\frac{A}{2}$, $y=\tan\left(\frac{C-A}{2}\right)\tan\frac{B}{2}$, $z=\tan\left(\frac{A-B}{2}\right)\tan\frac{C}{2}$, then $x+y+z$ (in terms of x, y, z only) is

1) xyz 2) 2xyz 3) -xyz 4) $\frac{1}{2}$xyz

44. In a ΔABC, $a=2b$ and $|A-B|=\frac{\pi}{3}$, then $\angle C$ is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3}$

45. In ΔABC if the angle A, B, C are in A.P. then $\frac{a+c}{\sqrt{a^2-ac+c^2}}=$

1) $Cos\left(\frac{A-C}{2}\right)$ 2) $2Cos\left(\frac{A-C}{2}\right)$

3) $2Sin\left(\frac{A-C}{2}\right)$ 4) $2Cos\left(\frac{A+C}{2}\right)$

46. In a ΔABC

$\sum(b+c)\tan\frac{A}{2}\tan\left(\frac{B-C}{2}\right)=$

1) a 2) b 3) c 4) 0

S.P.Q. Student Practice Questions

47. If in a triangle ABC, in the usual notation, $2a\cos\left(\frac{B-C}{2}\right)=b+c$ and $B\neq C$, then the measure of the angle A is

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

48. In ΔABC, If C = A - B = 60°, then the value of $\frac{a-b}{c}=$

1) $\frac{1}{\sqrt{3}}$ 2) 2 3) 3 4) 4

49. $\frac{b-c}{b+c}\cot\frac{A}{2}+\frac{b+c}{b-c}\tan\frac{A}{2}=$

1) $2\sin(B-C)$ 2) $2\operatorname{cosec}(B-C)$

3) $2\tan(B-C)$ 4) $2\cot(B-C)$

PROJECTION RULE

C.R.T.Q Class Room Teaching Questions

50. In ΔABC, $\frac{\cos C+\cos A}{c+a}+\frac{\cos B}{b}=$

1) $\frac{1}{a}$ 2) $\frac{1}{b}$ 3) $\frac{1}{c}$ 4) $\frac{c+a}{b}$

51. In a ΔABC, $b\cos(C+\theta)+c\cos(B-\theta)$

1) $a\sin\theta$ 2) $a\cos\theta$ 3) $a\tan\theta$ 4) $a\cot\theta$

S.P.Q. Student Practice Questions

52. In ΔABC, if a (b cos C + c cos B) = 2ka², then k =

1) 0 2) 1 3) $\frac{1}{2}$ 4) 2

53. In ΔABC, Σ (b+c) cos A =

1) s 2) 2s 3) 2(s-a) 4) 4 s

HALF ANGLES

C.R.T.Q Class Room Teaching Questions

54. In ΔABC, $(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}$ =

1) c 2) c/2 3) 2c 4) c²

55. In ΔABC, $(a+b+c)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)=$

1) $2c\cot\frac{A}{2}$ 2) $2c\cot\frac{B}{2}$
3) $2c\cot\frac{C}{2}$ 4) $2c\tan\frac{C}{2}$

56. In ΔABC, If a, b, c are in A.P., then $\text{Cot}\frac{A}{2}$, $\text{Cot}\frac{B}{2}$, $\text{Cot}\frac{C}{2}$ are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P

57. In ΔABC, If a, b, c are in A.P. then $\text{Cot}\frac{A}{2}\text{Cot}\frac{C}{2}=$

1) 1 2) 2 3) 3 4) 4

58. In ΔABC, $4R\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=$

1) Δ 2) s 3) r 4) 2s

59. In ΔABC, If a = 13 cm, b = 12 cm, c = 5 cm, then $\sin\frac{A}{2}=$

1) $1/\sqrt{5}$ 2) $1/\sqrt{2}$ 3) $\sqrt{32/35}$ 4) 2/3

60. In a ΔABC, If $\cot\frac{A}{2}:\cot\frac{B}{2}:\cot\frac{C}{2}=1:4:15$, then the greatest angle is

1) $\pi/3$ 2) $\pi/4$ 3) $\pi/6$ 4) $2\pi/3$

61. In a ΔABC, If $\tan\frac{A}{2},\tan\frac{B}{2},\tan\frac{C}{2}$ are in H.P., then a ,b,c are in

1) H.P. 2) G .P 3) A.P 4) A.G.P.

62. In a ΔABC, If $A=60^0$, then the value of $\left(1+\frac{a}{c}+\frac{b}{c}\right)\left(1+\frac{c}{b}-\frac{a}{b}\right)=$

1) 3 2) 2 3) 1 4) 0

S.P.Q. Student Practice Questions

63. In ΔPQR, $\underline{|R}=\frac{\pi}{4}$, $Tan\frac{P}{3}, Tan\frac{Q}{3}$ are the roots of the equation $ax^2+bx+c=0$ then

1) $a+b=c$ 2) $b+c=0$
3) $a+c=0$ 4) $b=c$

64. In ΔABC if $\text{Tan}\frac{A}{2}=\frac{5}{6}$ and $\text{Tan}\frac{B}{2}=\frac{20}{37}$ then the sides are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

65. In ΔABC, 4R s $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=$

1) Δ 2) 2Δ 3) r 4) 2r

66. In a ΔABC, If a,b,c are in A.P., then $\cos\left(\frac{A-C}{2}\right)\operatorname{cosec}\left(\frac{B}{2}\right)=$

1) 1/2 2) 2 3) 1/3 4) 3

67. In a ΔABC, If $Co\sec^2\frac{A}{2}, Co\sec^2\frac{B}{2}, Co\sec^2\frac{C}{2}$ are in A.P., then a,b,c are in

1) A.P 2) G.P 3) H.P 4) A.G.P

68. In a ΔABC, $\frac{s-a}{11}=\frac{s-b}{12}=\frac{s-c}{13}$, then $\tan^2\left(\frac{A}{2}\right)=$

1) 143/432 2) 13/33
3) 11/39 4) 12/37

AREA OF TRIANGLE

C.R.T.Q Class Room Teaching Questions

69. If two sides of a triangle are 3 feet and 12 feet and included angle is 150° then the area of the triangle in square feet is

1) 36 2) 24 3) 72 4) 9

70. If the sides of a triangle are 5,7,8, then its area is

1) $10\sqrt{3}$ 2) $3\sqrt{10}$ 3) $3/\sqrt{10}$ 4) $10/\sqrt{3}$

71. In ΔABC, If $c^2 = a^2 + b^2, 2s = a+b+c$, then $4s(s-a)(s-b)(s-c)$

1) s^4 2) b^2c^2 3) c^2a^2 4) a^2b^2

72. The perimeter of a triangle is 16 cm. One of the sides is of length 6 cm. If the area of the triangle is 12 sq. cm, then the triangle is

1) Right angled 2) Isosceles
3) Equilateral 4) Scalene

73. In $\Delta ABC, \frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$; If b=2, then the area of the triangle is

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) 2 4) 3

74. In ΔABC,
$R^2(\sin 2A + \sin 2B + \sin 2C) =$

1) Δ 2) 3Δ 3) 2Δ 4) 4Δ

S.P.Q. Student Practice Questions

75. In ΔABC, If a =2, B = 120°, C = 30°, then the area of the triangle is

1) $2\sqrt{3}$ 2) $\sqrt{3}$ 3) $\frac{\sqrt{3}}{2}$ 4) $4\sqrt{3}$

76. If the length of each side of an equilateral triangle is 10 cm, then its area is

1) 75 2) $\frac{\sqrt{3}}{25}$ 3) $\frac{25}{\sqrt{3}}$ 4) $25\sqrt{3}$

77. In ΔABC if a=30, b=24, c=18 then $\Delta =$

1) 16 2) 216 3) $\sqrt{216}$ 4) 17

78. A triangular park is enclosed on two sides by a fence and the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is

1) $\sqrt{\frac{x^3}{8}}$ 2) $\frac{1}{2}x^2$ 3) πx^2 4) $\frac{3}{2}x^2$

79. If $\Delta = a^2 - (b-c)^2$, is the area of the triangle ABC, then $\tan A =$

1) 1/16 2) 8/15 3) 3/4 4) 4/3

80. $\sum a\cos^2\frac{A}{2} =$

1) Δ 2) $\frac{\Delta}{R}$ 3) $s+\frac{\Delta}{R}$ 4) 2Δ

81. If α, β, γ are the lengths of the altitudes of ΔABC, then

$\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}-\frac{2ab}{(a+b+c)\Delta}\cos^2\frac{C}{2} =$

1) 0 2) 1 3) 2s 4) Δ

CIRCLES CONNECTED TO TRIANGLE :

C.R.T.Q Class Room Teaching Questions

82. In triangle ABC, if $rr_1 = r_2r_3$, then the triangle is

1) equilateral 2) isosceles
3) right angled 4) scalene

83. If p_1, p_2, p_3 are respectively the perpendiculars from the vertices of ΔABC to the opposite sides, then

(I) $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3} =$

1) $\frac{1}{r}$ 2) $\frac{1}{r_1}$ 3) $\frac{1}{r_2}$ 4) $\frac{1}{r_3}$

(II) $\frac{1}{p_1}+\frac{1}{p_2}-\frac{1}{p_3} =$

1) $\frac{1}{r}$ 2) $\frac{1}{r_1}$ 3) $\frac{1}{r_2}$ 4) $\frac{1}{r_3}$

(III) $\frac{\cos A}{p_1}+\frac{\cos B}{p_2}+\frac{\cos C}{p_3} =$

1) $\frac{1}{r}$ 2) $\frac{1}{R}$ 3) $\frac{1}{r_1}$ 4) $\frac{1}{r_3}$

(IV) $p_1p_2p_3 =$

1) $\frac{abc}{8R^2}$ 2) $\frac{a^2b^2c^2}{8R^2}$
3) $\frac{a^2b^2c^2}{8R^3}$ 4) $\frac{a^3b^3c^3}{8R^2}$

84. In a triangle ABC, if $(r_1+r_2)(r_2+r_3)(r_3+r_1) = 4RK$ then K=

1) s 2) Δ 3) Δ^2 4) s^2

85. If in a triangle PQR; sin P, sin Q, sin R are in A.P; then

1) the altitudes are in A.P
2) the altitudes are in H.P
3) the altitudes are in G.P
4) the medians are in A.P

86. If the radius of the incircle of a triangle with its sides 5k, 6k and 5k is 6, then k is equal to

1) 3 2) 4 3) 5 4) 6

87. If d_1, d_2, d_3 are the diameters of three excircles of a triangle then $d_1d_2+d_2d_3+d_3d_1 =$

1) Δ^2 2) $4s^2$ 3) $4\Delta^2$ 4) $ab+bc+ca$

88. If ΔABC is right angled at A then $r_2+r_3 =$

1) $r_1 - r$ 2) $r_1 + r$ 3) $r - r_1$ 4) R

89. In an equilateral triangle $r : R : r_1$ is

1) 1 : 1 : 1 2) $1:\sqrt{2}:3$
3) 1 : 2 : 3 4) $2:\sqrt{3}:\sqrt{3}$

90. If, in ΔABC, $r_3 = r_1 + r_2 + r$, then $\angle A+\angle B=$

1) 120^0 2) 100^0 3) 90^0 4) 80^0

91. If $r : R : r_1 = 2 : 5 : 12$, then A =

1) 45° 2) 60° 3) 30° 4) 90°

92. In a triangle ABC, if $\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+\frac{1}{r^2} =$

1) $\frac{a+b+c}{\Delta}$ 2) $\frac{a^2+b^2+c^2}{\Delta^2}$
3) $\frac{a^2+b^2+c^2}{\Delta}$ 4) $\frac{a+b+c}{\Delta^2}$

93. In a triangle ABC, if $a\cot A+b\cot B+c\cot C=$

1) r + R 2) 2 (r + R)
3) r - R 4) 2 (r - R)

94. In a triangle ABC, if $r^2\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

1) Δ 2) 2Δ 3) 4Δ 4) $\frac{\Delta}{2}$

95. The diameter of the circumcircle of the triangle whose sides 61,60,11 is

1) 61 2) 60 3) 11 4) 50

96. In a triangle ABC, if $4Rr\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} =$

1) $(r_2+r_3)\sec^2\frac{A}{2}$ 2) $r_1r_2r_3$
3) Δ 4) s

97. If the area of the triangle ABC is $a^2-(b-c)^2$, then its circumradius R =

1) $(a/6)\sin^2(A/2)$
2) $(a/6)\operatorname{cosec}^2(A/2)$
3) $(b/16)\sin^2(B/2)$
4) $(c/16)\sin^2(C/2)$

98. In triangle ABC, if a = 13, b = 14, c = 15, then $r_1 =$

1) 21/2 2) 14 3) 65/8 4) 4

99. In triangle ABC, if $r_1 = 3, r_2 = 10, r_3 = 15$, then c =

1) 5 2) 12 3) 13 4) 13/2

100. If the area of triangle ABC is 96 sq. cm. Its perimeter is 16 cm, then inradius is

1) 10 cm 2) 11 cm 3) 12 cm 4) 13 cm.

101. The perimeter of a triangle ABC right angled at C is 70, and the inradius is 6 then $|a-b|=$

1) 1 2) 2 3) 8 4) 9

102. In triangle ABC,

$\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)=$

1) $\frac{abc}{\Delta^3}$ 2) 0 3) $4Rr^2$ 4) $1/r$

S.P.Q. Student Practice Questions

103. If $\frac{r}{r_2}=\frac{r_1}{r_3}$ then

1) $A=90^0$ 2) $B=90^0$
3) $C=90^0$ 4) $C=60^0$

104. If α,β,γ are the lengths of the altitudes of ΔABC, then

$\frac{\cos A}{\alpha}+\frac{\cos B}{\beta}+\frac{\cos C}{\gamma}=$

1) Δ 2) $1/\Delta$ 3) R 4) $1/R$

105. $(r_1-r)(r_2-r)(r_3-r)=$

1) 4Rr 2) 4Rs 3) $4R\Delta$ 4) $4Rr^2$

106. If the sides of a triangle are 13,14,15, then the radius of the incircle is

1) $\frac{65}{8}$ 2) $\frac{65}{4}$ 3) 4 4) 24

107. In ΔABC if $r_1-r=r_2+r_3$, then

1) $A=90^0$ 2) $B=90^0$
3) $C=90^0$ 4) $A=45^0$

108. If a,b,c are in A.P., then r_1, r_2, r_3 are in
1) A.P. 2) G.P. 3) H.P. 4) A.G.P

109. In a triangle ABC, if

$\frac{s-a}{\Delta}=\frac{1}{8},\frac{s-b}{\Delta}=\frac{1}{12},\frac{s-c}{\Delta}=\frac{1}{24}$ then b=

1) 16 2) 20 3) 24 4) 28

110. $\frac{\Delta^2}{a^2+b^2+c^2}\left\{\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+\frac{1}{r^2}\right\}=$

1) 1 2) 0 3) $4Rr^2$ 4) $1/r$

111. $\frac{a\cos A+b\cos B+c\cos C}{a+b+c}=$

1) $\frac{r}{2R}$ 2) $\frac{r}{R}$ 3) $\frac{2r}{R}$ 4) $\frac{R}{r}$

112. $r\left(\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\right)=$

1) Δ 2) s 3) 2s 4) 2Δ

113. If the sides of a triangle ABC are 6,8,10 units, then the radius of its circumcircle is

1) 4 2) 3 3) 6 4) 5

114. In triangle ABC, if $r_1=3$, $r_2=10$, $r_3=15$, then R =

1) 5 2) 12 3) 13 4) 13/2

115. If in a triangle ABC, $r_1=2$, $r_2=3$ and $r_3=6$ then b =

1) 1 2) 2 3) 3 4) 4

116. In ΔABC, if a = 26, b = 30,

$\cos C=63/65$, then $r_1:r_2:r_3=$

1) 4:12:1 2) 3:4:12
3) 1:4:12 4) 4:12:3

NATURE OF A TRIANGLE

C.R.T.Q Class Room Teaching Questions

117. In ΔABC, if $\sin^2 A+\sin^2 B=\sin^2 C$, then the triangle is

1) equilateral 2) right angled
3) isosceles 4) scalene.

118. In ΔABC, if

$\tan B\tan C+\tan C\tan A+\tan A\tan B=\sqrt{3}\tan A\tan B\tan C$ then the triangle is

1) isosceles
2) equilateral
3) right angled
4) right angled isosceles

2. If the perimeter of a triangle is six times the arithmetic mean of the sine angles and the side 'a' is unity, then A =

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

3. In ΔABC, if $C = 90^0$ then $\frac{a^2+b^2}{a^2-b^2}$ $\sin(A-B) =$

1) 1 2) 2 3) 3 4) 4

4. In triangle ABC, $\frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B} =$

1) $\frac{a^2+b^2}{b^2+c^2}$ 2) $\frac{a^2+b^2}{a^2+c^2}$

3) $\frac{a^2-b^2}{b^2-c^2}$ 4) $\frac{a^2-b^2}{a^2-c^2}$

5. If in a triangle ABC, 3 sin A = 6 sin B $= 2\sqrt{3}$ sin C, then the angle A is

1) 30^0 2) 45^0 3) 60^0 4) 90^0

6. In a triangle ABC, $\angle B = \frac{\pi}{3}, \angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1:3. Then $\frac{\sin\angle BAD}{\sin\angle CAD}$ is equal to

1) $\frac{1}{\sqrt{6}}$ 2) $\frac{1}{3}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{\frac{2}{3}}$

7. If the sides a,b,c of a triangle are in G.P, and largest angle exceeds the smallest by 60^0, then $\cos B =$

1) 1 2) $\frac{\sqrt{13}-1}{4}$ 3) $\frac{1}{2}$ 4) $\frac{1-\sqrt{13}}{4}$

8. If in a triangle ABC sines of angles A and B satisfy the equation

$4x^2 - 2\sqrt{6}x + 1 = 0$, then $\cos(A-B) =$

1) 0 2) 1/2 3) $1/\sqrt{2}$ 4) $\sqrt{3}/2$

9. In triangle ABC, if $\frac{\sin 3B}{\sin B} = \left(\frac{a^2-c^2}{2ac}\right)^2$ then a^2, b^2, c^2 are in

1) A.P 2) G.P 3) H.P 4) A.G.P

10. In ΔABC, if $\angle A = \frac{\pi}{4}, \angle B = \frac{5\pi}{12}$ then $a + \sqrt{2}c =$

1) b 2) 2b 3) b/2 4) $\sqrt{3}b$

11. In triangle ABC, $(a^2-b^2-c^2)\tan A + (a^2-b^2+c^2)\tan B =$

1) $(a^2+b^2-c^2)\tan C$

2) $(a^2+b^2+c^2)\tan C$

3) $(b^2+c^2-a^2)\tan C$ 4) 0

12. In triangle ABC, $C = 90^0$, Then $\frac{a^2-b^2}{a^2+b^2} =$

1) $\sin(A+B)$ 2) $\sin(A-B)$

3) $\cos(A+B)$ 4) $\cos(A-B)$

13. If the angles A,B and C of a triangle are in an arithmetic progression and if a,b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is

1) $\frac{1}{2}$ 2) $\frac{\sqrt{3}}{2}$ 3) 1 4) $\sqrt{3}$

14. If in a triangle ABC, $4\sin A = 4\sin B = 3\sin C$, then $\cos C =$

1) 1/3 2) 1/9 3) 1/27 4) 1/18

15. In a triangle ABC, if $(\sqrt{3}-1)a = 2b, A = 3B$, then $\angle C$ is

1) 60^0 2) 120^0 3) 30^0 4) 45^0

S.P.Q. Student Practice Questions

16. In ΔABC, If $\dfrac{SinA}{SinC}=\dfrac{Sin(A-B)}{Sin(B-C)}$ then a^2, b^2, c^2 are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

17. The radius of the circumcircle of an isosceles triangle PQR is equal to PQ (=PR), then the angle P is

1) $\pi/6$ 2) $\pi/3$ 3) $\pi/2$ 4) $2\pi/3$

18. In ΔABC, If a:b:c = 3:4:5, then sin A : sin B : sin C =

1) 3: 4 : 5 2) 9 : 16 : 25
3) 9 : 8 : 7 4) 7 : 9 : 8

19. If the angles of a right angled triangle are in A.P., then ratio of its sides is

1) $1:\sqrt{3}:2$ 2) $1:1:\sqrt{2}$
3) $2:3:\sqrt{13}$ 4) 1 : 2 :3

20. The sides of a triangle are in ratio $1:\sqrt{3}:2$, then angles of the triangle are in the ratio

1) 1:2:3 2) 1:3:5 3) 2:3:4 4) 3:2:1

21. In a triangle ABC, if $a=100, c=100\sqrt{2}$ and $A=30^0$ then the angle $B=$

1) 105^0 or 15^0 2) 45^0 or 135^0
3) 60^0 or 120^0 4) 30^0 or 60^0

22. $a^2-b^2=bc \Rightarrow A:B=$

1) 2 : 1 2) 3 : 1
3) 4 : 1 4) 5 : 1

23. The sides of a triangle are in A.P and the greatest angle exceeds the least by 90^0. The sides are in the ratio

1) $1:2:\sqrt{2}$ 2) $1:\sqrt{3}:2$
3) $\sqrt{7}+1:\sqrt{7}:\sqrt{7}-1$ 4) 1:2:3

24. If the angles A, B,C of a triangle are in A.P and sides a,b,c are in G.P, then a^2,b^2,c^2 are in

1) A.P 2) H.P 3) G.P 4) A.G.P

25. In triangle ABC, if A,B,C are in A.P and $b:c=\sqrt{3}:\sqrt{2}$, then

1) $\sin(2C-A)=\sin(B/4)$
2) $\sin(A-C)=\sin(B/4)$
3) $\sin(A+C)=\cos B$
4) $\cos(A-C)=\sin(B/2)$

26. In a triangle ABC, if $\dfrac{a^2+b^2}{a^2-b^2}\sin(A-B)=1$ and the triangle is not right angled, then $\cos(A-B)=$

1) $\sin\left(\dfrac{C}{2}-\dfrac{\pi}{4}\right)$ 2) $\cos\left(\dfrac{C}{2}-\dfrac{\pi}{4}\right)$
3) $\tan\left(\dfrac{C}{2}-\dfrac{\pi}{4}\right)$ 4) $\cot\left(\dfrac{C}{2}-\dfrac{\pi}{4}\right)$

27. In a triangle ABC $\sum a^2(\cos^2 B-\cos^2 C)=$

1) 0 2) 1 3) 2 4) 3

28. If angles A, B and C are in A.P, then $\dfrac{a+c}{b}$ is equal to

1) $2\sin\dfrac{A-C}{2}$ 2) $2\cos\dfrac{A-C}{2}$
3) $\cos\dfrac{A-C}{2}$ 4) $\sin\dfrac{A-C}{2}$

29. In ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix}=0$, then $\sin^2 A+\sin^2 B+\sin^2 C$ is equal to

1) $\dfrac{4}{9}$ 2) $\dfrac{9}{4}$ 3) $3\sqrt{3}$ 4) 1

30. If a, b and A are given and c_1, c_2 are two values of the side c in the ambiguous case, then $c_1^2+c_2^2-2c_1c_2.\cos 2A=$

1) $4a^2\cos^2 A$ 2) $4a^2\sin^2 A$
3) $4a^2$ 4) $4a^2\cos 2A$

COSINE RULE

C.R.T.Q Class Room Teaching Questions

31. If in a triangle ABC, $a^2+b^2+c^2=ca+ab\sqrt{3}$, then the triangle is
1) equilateral
2) right angled and isosceles
3) right angled with one of the acute angles measuring $\frac{\pi}{3}$
4) obtuse angled

32. In ΔABC, the lengths of the two larger sides are 10 and 9 units respectively. If the angles are in A.P. then the length of the third side can be
1) $5\pm\sqrt{6}$ 2) $3\sqrt{3}$
3) 5 4) $\pm\sqrt{6}$

33. In ΔABC, if $c^4-2(a^2+b^2)c^2+a^4+a^2b^2+b^4=0$ then $\underline{|C}=$
1) 30^0 2) 45^0 3) 60^0 4) 75^0

34. In a triangle ABC, the sides a,b,c are such that they are the roots of then $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=$
1) 1 2) $\frac{3}{4}$ 3) $\frac{9}{16}$ 4) $\frac{1}{2}$

35. If $b+c:c+a:a+b=11:12:13$, then $\cos A:\cos B:\cos C=$
1) $7:9:11$ 2) $14:11:6$
3) $7:19:25$ 4) $8:6:5$

36. The roots of $x^2-2\sqrt{3}x+2=0$ represent two sides of a triangle. If the angle between them is $\frac{\pi}{3}$, then the perimeter of the triangle is
1) $2\sqrt{3}+6$ 2) $2\sqrt{3}+\sqrt{6}$
3) $3\sqrt{2}+6$ 4) $3\sqrt{2}+\sqrt{6}$

37. The base of a triangle is 80 and one of the base angles is 60^0. If the sum of the lenghts of the other two sides is 90, then the shortest side is
1) 15 2) 17 3) 19 4) 21

S.P.Q. Student Practice Questions

38. If 6, 10, 14 are the sides of a triangle then its obtuse angle is
1) 110^0 2) 120^0 3) 135^0 4) 115^0

39. In ΔABC, if $a^4+b^4+c^4=2c^2(a^2+b^2)$ then $\underline{|C}=$
1) $\frac{\pi}{8}$ or $\frac{3\pi}{8}$ 2) $\frac{\pi}{12}$ or $\frac{5\pi}{12}$
3) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ 4) $\frac{\pi}{2}$

40. If one angle of a triangle is 30^0 and the lengths of the sides adjacent to it are 40 and $40\sqrt{3}$, then the triangle is
1) equilateral 2) right angled
3) isosceles 4) scalene

41. If in ΔABC, $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ca}$, then the value of the angle A, in degrees is
1) 30 2) 60 3) 90 4) 120

42. In ΔABC, If $\frac{b+c-a}{4}=\frac{c+a-b}{3}=\frac{a+b-c}{2}$, then Cos A=
1) $\frac{5}{7}$ 2) $\frac{3}{7}$ 3) $\frac{2}{7}$ 4) $\frac{1}{7}$

43. In triangle ABC, $\frac{bc\cos A+ca\cos B+ab\cos C}{\cot A+\cot B+\cot C}=$
1) Δ 2) 2Δ 3) $\frac{1}{2}\Delta$ 4) Δ^2

TANGENT RULE & PROJECTION RULE

C.R.T.Q Class Room Teaching Questions

4. If one side of a triangle is double that of an other and the angles opposite to these sides differ by 60^0, then the triangle is
1) isosceles
2) right angled
3) equilateral
4) right angled isosceles

15. If in ΔABC, A=3C, then $\frac{a-c}{2c}=$
1) $\sin\frac{B}{2}$ 2) $\sin\frac{C}{2}$
3) $\cos\frac{B}{2}$ 4) $\cos\frac{C}{2}$.

16. In triangle ABC, $a(b^2+c^2)\cos A+b(c^2+a^2)\cos B+c(a^2+b^2)\cos C=$
1) $3abc^2$ 2) $2a^2bc$
3) $3abc$ 4) $3ab^2c$

17. In triangle ABC, a = 5, b = 4 and $\cos(A+B)=\frac{31}{32}$. In this triangle, c =
1) $\sqrt{6}$ 2) 36 3) 6 4) $\frac{\sqrt{319}}{2}$

S.P.Q. Student Practice Questions

48. If in ΔABC, A=B-C=60^0, then $\frac{b-c}{b+c}=$
1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

49. In a triangle ABC, $a=3b$ and $|A-B|=60^0$ then $\tan\left(\frac{C}{2}\right)=$
1) $\frac{2}{\sqrt{3}}$ 2) $\frac{\sqrt{3}}{2}$ 3) $\sqrt{3}$ 4) $2\sqrt{3}$

HALF - ANGLES

C.R.T.Q Class Room Teaching Questions

50. In a triangle ABC, $(a+b+c)(b+c-a)=\lambda bc$ if
1) $\lambda<0$ 2) $\lambda>0$ 3) $0<\lambda<4$ 4) $\lambda>4$

51. In ΔABC, If (a-b)(s-c)=(b-c)(s-a) then r_1, r_2, r_3 are in
1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

52. In ΔABC if a=(b-c)Secθ then $\frac{2\sqrt{bc}}{b-c}\text{Sin}\frac{A}{2}=$
1) Cosθ 2) Secθ 3) Tanθ 4) Cotθ

53. In a triangle ABC, B=90^0, then the value of $\tan\frac{A}{2}=$
1) $\sqrt{\frac{b+c}{b-c}}$ 2) $\sqrt{\frac{b-c}{b+c}}$
3) $\sqrt{\frac{a+c}{a-c}}$ 4) $\sqrt{\frac{a-c}{a+c}}$

54. In a triangle ABC if $\cot\frac{A}{2}.\cot\frac{B}{2}=c$, $\cot\frac{B}{2}.\cot\frac{C}{2}=a$ and $\cot\frac{C}{2}.\cot\frac{A}{2}=b$ then $\frac{1}{s-a}+\frac{1}{s-b}+\frac{1}{s-c}=$
1) -1 2) 0 3) 1 4) 2

55. The area Δ of a triangle ABC is given by $\Delta=a^2-(b-c)^2$ then $Tan\frac{A}{2}=$
1) -1 2) 0 3) 1/4 4) 1/2

56. In ΔABC, If $\cot\frac{A}{2}:\cot\frac{B}{2}:\cot\frac{C}{2}=3:7:9$, then a:b:c =
1) 7 : 9 : 11 2) 14 : 11 : 6
3) 7 : 19 : 25 4) 8 : 6 : 5

57. If AD, BE, CF are internal bisectors of the angles of ΔABC, then $\frac{\cos A/2}{AD}+\frac{\cos B/2}{BE}+\frac{\cos C/2}{CF}=$

1) $\frac{abc}{2s}$ 2) $\frac{2s}{abc}$

3) $\frac{ab+bc+ca}{abc}$ 4) $\frac{a+b+c}{abc}$

58. In a triangle ABC, $\frac{\cot(A/2)+\cot(B/2)+\cot(C/2)}{\cot A+\cot B+\cot C}=$

1) $\frac{(a+b+c)^2}{a^2+b^2+c^2}$ 2) $\frac{a^2+b^2+c^2}{(a+b+c)^2}$

3) s 4) Δ

59. In ΔABC, the value of $\frac{\cot\frac{A}{2}\cot\frac{B}{2}-1}{\cot\frac{A}{2}\cot\frac{B}{2}}$ is

1) $\frac{a}{a+b+c}$ 2) $\frac{2c}{a+b+c}$

3) $\frac{2a}{a+b+c}$ 4) $\frac{2b}{a+b+c}$

S.P.Q. Student Practice Questions

60. In ΔABC, If (a + b + c) (b + c - a) = 3 bc, then

1) $A=60^0$ 2) $B=60^0$

3) $C=60^0$ 4) $A=90^0$

61. In ΔABC, If a, b, c are in H.P., then $\text{Sin}^2\frac{A}{2}$, $\text{Sin}^2\frac{B}{2}$, $\text{Sin}^2\frac{C}{2}$ are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

62. If the sides of a triangle are in A.P and difference between the greatest and least angles is $\frac{\pi}{2}$, then $\text{Sin}\left(\frac{B}{2}\right)=$

1) $\frac{1}{2}$ 2) $\frac{1}{2\sqrt{2}}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{\sqrt{3}}{2}$

63. In ΔABC, If $3Tan\frac{A}{2}.Tan\frac{C}{2}=1$, then a,b,c are in

1) A.P 2) G.P 3) H.P 4) A.G.P.

64. If a:b:c=3:4:5, then $\cot\frac{A}{2}:\cot\frac{B}{2}:\cot\frac{C}{2}=$

1) 5:6:7 2) 1:2:3 3) 3:2:1 4) 4:5:6

65. In ΔABC if a=13, b=14, c=15 then $\text{Tan}\frac{B}{2}=$

1) $\frac{3}{7}$ 2) $\frac{16}{49}$ 3) $\frac{4}{7}$ 4) $\frac{2}{\sqrt{7}}$

66. In any triangle ABC, $1-\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)=$

1) $\frac{2a}{a+b+c}$ 2) $\frac{2b}{a+b+c}$

3) $\frac{2c}{a+b+c}$ 4) $a+b$

67. In ΔABC, If a, b, c are in A.P. then $\frac{2\,Sin\frac{A}{2}\,Sin\frac{C}{2}}{Sin\frac{B}{2}}=$

1) 1 2) 2 3) 3 4) 4

68. In a triangle ABC, if the sides of a = 3, b = 5 and c=4, then $\sin\frac{B}{2}+\cos\frac{B}{2}$ is equal to

1) $\sqrt{2}$ 2) $\frac{\sqrt{3}+1}{2}$ 3) $\frac{\sqrt{3}-1}{2}$ 4) 1

AREA OF TRIANGLE

C.R.T.Q Class Room Teaching Questions

69. In ΔABC, area of incircle : area of ΔABC =

1) π : $\text{Tan}\frac{A}{2}\ \text{Tan}\frac{B}{2}\ \text{Tan}\frac{C}{2}$

2) π : $\text{Cot}\frac{A}{2}\ \text{Cot}\frac{B}{2}\ \text{Cot}\frac{C}{2}$

3) 1 : 1

4) π : r

70. If $\frac{2}{5}\frac{y}{z}+\frac{2}{5}\frac{z}{x}, \frac{2}{5}\frac{z}{x}+\frac{2}{5}\frac{x}{y}, \frac{2}{5}\frac{x}{y}+\frac{2}{5}\frac{y}{z}$ are the sides of a triangle then its area is

1) $\frac{4}{25}\sqrt{\frac{x}{y}+\frac{y}{z}+\frac{z}{x}}$ 2) xyz

3) 1 4) x + y + z

71. If p_1, p_2, p_3 are the lengths of the altitudes of a tringle from the vertices A, B, C, then $\frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=$

1) $\frac{2ab\cos^2 C/2}{\Delta(a+b+c)}$ 2) $\frac{1}{R}$

3) $\frac{\cot A+\cot B+\cot C}{\Delta}$ 4) $2R$

S.P.Q. Student Practice Questions

72. If Δ is the area of triangle whose sides are a,b,c then the area of the triangle with 2a,2b,2c as the sides is

1) 2Δ 2) 4Δ 3) 8Δ 4) 16Δ

73. Which of the following statements is true

I : In a ΔABC if 4s(s - a) (s - b) (s - c) = a^2b^2 then it is right angled triangle

II : In a ΔABC the expression sinA + sinB + sinC is maximum then the triangle is equilateral

1) only I 2) only II

3) both I and II 4) neither I nor II

74. In a triangle ABC, two sides are a = 6, b = 3 and $\cos(A-B)=\frac{4}{5}$, then the area of the triangle is

1) 9 Sq. units 2) 3 Sq. units

3) 18 Sq. units 4) $\frac{9}{2}$Sq.units

CIRCLES CONNECTED TO TRIANGLE

C.R.T.Q Class Room Teaching Questions

75. In ΔABC, if $C=90^0$, then $R+r=$

1) a+ b 2) $\frac{a+b}{2}$

3) b + c 4) $\frac{b+c}{2}$

76. If the diameter of any excircle of a triangle is equal to its perimeter, then the triangle is

1) equilateral 2) isosceles

3) right angled 4) scalene

77. In ΔABC, if $\sqrt{3}r+a=\sqrt{3}r+b=s$ then the triangle is

1) right angled 2) isosceles

3) equilateral 4) scalene

78. If the area of a triangle is 96 and the radii of the escribed circles are 8, 12, 24, then the greatest side of the triangle is

1) 18 2) 20 3) 16 4) 30

79. In ΔABC, r = 1, R=4, Δ = 8 then the value of ab + bc + ca =

1) 18 2) 81 3) 72 4) 27

80. In triangle ABC, $r=\frac{R}{6}$ and $r_1=7r$. Then the measure of angle A=

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

81. The equation whose roots are radii of escribed circles is

1) $x^3-2x^2(r+R)+s^2x-s^2r=0$

2) $x^3-x^2(r+4R)+s^2x-s^2r=0$

3) $x^3-x^2(r+4R)+s^2x-\Delta^2s=0$

4) $x^3-4Rrx^2+s^2x-sr=0$

82. $\frac{\sqrt{rr_1r_2r_3}}{2Rr(\sin A+\sin B+\sin C)}=$

1) 1 2) 1 / 3 3) 1 / 4 4) 1 / 2

83. If 3, 10 and 15 are the altitudes of a triangle then area of incircle is

1) 2π 2) 9π 3) 4π 4) 16π

84. In ΔABC, right angled at A, $\cos^{-1}\left(\frac{R}{r_2+r_3}\right)$ is

1) 30^0 2) 60^0 3) 90^0 4) 45^0

85. If r_1, r_2, r_3 are the radii of the escribed circles of a ΔABC and if r is the radius of its incircle then $r_1 r_2 r_3 - r(r_1 r_2 + r_2 r_3 + r_3 r_1) =$

1) 0 2) 1 3) 2 4) 3

86. In ΔABC, If $C = 90^0$, $1 + \sin A - \sin B =$

1) $\frac{r}{2R}$ 2) $\frac{2r}{R}$ 3) $\frac{2r_1}{R}$ 4) $\frac{r_1}{R}$

87. In a ΔABC, if $r_1 = 2r_2 = 3r_3$, then $a:b=$

1) $\frac{5}{4}$ 2) $\frac{4}{5}$ 3) $\frac{7}{4}$ 4) $\frac{4}{7}$

88. $(r+r_1)\tan\frac{B-C}{2}+(r+r_2)\tan\frac{C-A}{2}+(r+r_3)\tan\frac{A-B}{2}=$

1) 2s 2) s 3) Δ 4) 0

89. In an ambiguous case of solving a triangle when $a=\sqrt{5}, b=2, \angle A=\frac{\pi}{6}$ and the two possible values of third side are c_1 and c_2 then

1) $|c_1-c_2|=2\sqrt{6}$ 2) $|c_1-c_2|=4\sqrt{6}$

3) $|c_1-c_2|=4$ 4) $|c_1-c_2|=6$

90. In a triangle ABC, $\frac{r_1-r}{a}+\frac{r_2-r}{b}+\frac{r_3-r}{c}=$

1) $\frac{r_1+r_2+r_3}{3}$ 2) $r_1+r_2+r_3$

3) $\frac{r_1+r_2+r_3}{2}$ 4) $\frac{r_1+r_2+r_3}{s}$

91. If A, A_1, A_2, A_3 are areas of excircles and incircle of a triangle, then $\frac{1}{\sqrt{A_1}}+\frac{1}{\sqrt{A_2}}+\frac{1}{\sqrt{A_3}}=$

1) $\frac{2}{\sqrt{A}}$ 2) $\frac{3}{\sqrt{A}}$ 3) $\frac{1}{\sqrt{A}}$ 4) $\frac{4}{\sqrt{A}}$

92. In triangle $ABC, (r_3+r_1)\sqrt{\left[\frac{rr_2}{r_3 r_1}\right]}=$

1) a 2) b 3) c 4) 0

93. The sum of the raddi of inscribed and circumscribed circles of an n sided regular polygon of side a, is

1) $a\cot\left(\frac{\pi}{n}\right)$ 2) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$

3) $a\cot\left(\frac{\pi}{2n}\right)$ 4) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

S.P.Q. Student Practice Questions

94. If $2R + r = r_1$, then

1) $A=90^0$ 2) $B=90^0$

3) $C=90^0$ 4) $A=60^0$

95. If a circle is inscribed in a right angled triangle having right angle at B, then diameter of the circle is

1) AB+BC-AC 2) BC+CA-AB

3) AC+AB-BC 4) AB+BC+CA

96. In a triangle ABC if $2a=\sqrt{3}b+c$ then

1) $c^2=a^2+b^2-ab$ 2) $a^2=b^2+c^2$

3) $b^2=a^2+c^2-ac\sqrt{3}$ 4) $a=b=c$

97. If $r_1=8, r_2=12, r_3=24$, then C=

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

98. The ratio between the circumradius and inradius of a right angled isosceles triangle is

1) 2:1 2) $(\sqrt{2}+1):1$

3) $1:(\sqrt{2}+1)$ 4) $(\sqrt{2}+1):\sqrt{2}$

1) $\frac{a^2+b^2}{2}$ 2) a^2+b^2
3) $2(a^2+b^2)$ 4) $4(a^2+b^2)$

5. **If a flagstaff subtends the same angle at the points A, B, C, D on the horizontal plane through the foot of the flag staff then A, B, C, D are the vertices of a**
1) square 2) cyclic quadrilateral
3) rectangle 4) parallelogram

6. **The angle of elevation of the top of the tower observed from each of the three points A, B, C on the ground, forming a triangle is the same angle α. If R is the circum radius of the ΔABC, then the height of the tower is**
1) $R\sin\alpha$ 2) $R\cos\alpha$
3) $R\cot\alpha$ 4) $R\tan\alpha$

7. **A pole stands vertically, inside a triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then ΔABC, the foot of the pole is at the**
1) centriod 2) circumcentre
3) incentre 4) orthocentre

8. **A tower subtends an angle α at a point A on the same level as the foot of the tower B is a point vertically above A and AB=h metres. The angle of depression of the foot of the tower from B is β. The height of the tower is**
1) $h\tan\alpha\cot\beta$ 2) $h\tan\alpha\tan\beta$
3) $h\cot\alpha\cot\beta$ 4) $h\cot\alpha\tan\beta$

9. **At a point on a level plane, a tower subtends an angle α, and a flag-staff of height 'a' metres standing on its top subtends an angle β. The height of the tower in metres is**
1) $\frac{a\sin\alpha\sin(\alpha+\beta)}{\sin\beta}$
2) $\frac{a\cos\alpha\sin(\alpha+\beta)}{\sin\beta}$
3) $\frac{a\sin\alpha\cos(\alpha-\beta)}{\sin\beta}$
4) $\frac{a\sin\alpha\cos(\alpha+\beta)}{\sin\beta}$

10. **A sperical balloon of radius r subtends an angle α at the eye of an observer, while the angle of elevation of its centre is β. The height of the centre of the balloon is**
1) $r\cos\left(\frac{\beta}{2}\right)\sin\alpha$ 2) $r\operatorname{cosec}\beta\sin\left(\frac{\alpha}{2}\right)$
3) $r\operatorname{cosec}\left(\frac{\alpha}{2}\right)\sin\beta$ 4) $r\operatorname{cosec}\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)$

11. **PQ is a vertical tower. A, B, C are three points in a horizontal line through Q, the foot of the tower. If the angles of elevation of the top of the tower from A,B,C are α, β, γ respectively. Then $BC\cot\alpha - CA\cot\beta + AB\cot\gamma =$**
1) 3 2) 1 3) 2 4) 0

KEY

01) 1	02) 1	03) 4	04) 3	05) 2
06) 4	07) 2	08) 1	09) 4	10) 3
11) 4				

Hints & Solutions

1. $\tan\beta = \frac{h}{AC} \Rightarrow AC = \frac{h}{\tan\beta}$

$\tan\alpha = \frac{h-x}{AC} \Rightarrow AC = \frac{h-x}{\tan\alpha}$

$\frac{h}{\tan\beta} = \frac{h-x}{\tan\alpha} \Rightarrow x = \frac{h(\tan\beta-\tan\alpha)}{\tan\beta}$

In right angled triangle ABD

$\Rightarrow \sin\beta = \dfrac{h}{r\operatorname{cosec}\alpha/2}$

$\therefore h = r\operatorname{cosec}\dfrac{\alpha}{2}\sin\beta$

11. $CQ = X,\ BC = Y,\ AB = Z$

$\operatorname{Cot}\alpha = \dfrac{x+y+z}{h}$,

$\operatorname{COT}\ \beta = \dfrac{x+y}{h}$, $\cot\gamma = \dfrac{x}{h}$

Sub. $\cot\alpha, \cot\beta, \cot\gamma$ values in

$BC\cot\alpha - CA\cot\beta + AB\cot\gamma$

EXERCISE–I

CRTQ & SPQ LEVEL-I

ANGLE OF ELEVATION, ANGLE OF DEPRESSION, LAW OF COSINES, SINES

C.R.T.Q Class Room Teaching Questions

. The angle of elevation of the top of a flag - staff when observed from a point, distance 60 metres from its foot is 30^0 the height of the flag-staff in metres is

1) $20\sqrt{3}$ 2) $10\sqrt{3}$

3) $60\sqrt{3}$ 4) $30\sqrt{3}$

From the top of a building of height h metres, the angle of depression of an object on the ground is 'α'. The distance (in metres) of the object from the foot of the building is --

1) $h\tan\alpha$ 2) $h\cos a$

3) $h\cot\alpha$ 4) $h\sin\alpha$

The tops of two poles of heights 18 metres and 12 metres are connected by a rope. If the rope makes an angle 30^0 with the hori-zontal, the length of the rope in metres is

1) 12 2) 18 3) 24 4) 30

4. The angle of elevation of an electric pole from a point A on the ground is 60^0 and from a Point B towards the pole on the line joining the foot of the pole to the point A, is 75^0. If the distance AB = a, then the height of the pole is

1) $\dfrac{a(3+2\sqrt{3})}{2}$ 2) $a(4+2\sqrt{3})$

3) $\dfrac{a(2+\sqrt{3})}{2}$ 4) $\dfrac{a(2\sqrt{3}-3)}{2}$

5. A man standing on a level plane observes the anlge of elevation of top of pole to be 'α'. He walks a distance equal to double the height of pole towards it and finds that the elevation is 2α then $\alpha =$

1) $\dfrac{\pi}{12}$ 2) $\dfrac{\pi}{4}$ 3) $\dfrac{\pi}{6}$ 4) $\dfrac{\pi}{3}$

6. The shadow of a tower standing on a level plane is found to be 60 mt longer when the sun's altitude is 30^0 than when it is 45^0. Then the height of the tower in metres.

1) 30m 2) $60\sqrt{3}\,m$

3) $30(\sqrt{3}+1)m$ 4) 60 m

7. There are two stations P,Q due north, due south of a tower of height 15 metres. The angle of depression of P and Q as seen from top a tower are $\cot^{-1}\dfrac{12}{5}, Sin^{-1}\dfrac{3}{5}$. The distance between P and Q is -----

1) 48 2) 56 3) 65 4) 25

8. The angles of elevation of a cliff at a point A on the ground and at a point B 100 mt vertically above A are α are β respectively.

The height of the cliff in metres is

1) $\dfrac{100\tan\beta}{\operatorname{Cot}\beta - \operatorname{Cot}\alpha}$ 2) $\dfrac{100\operatorname{Cot}\beta}{\operatorname{Cot}\beta - \operatorname{Cot}\alpha}$

3) $\dfrac{100\tan\beta}{\operatorname{Cot}\alpha + \operatorname{Cot}\beta}$ 4) $\dfrac{100\cot\beta}{\operatorname{Cot}\alpha + \operatorname{Cot}\beta}$

9. If from the top of a tower of 60 metre heigh, the angles of depression of the top and floor of a house are α and β respectively and if the height of the house is $\frac{60\sin(\beta-\alpha)}{x}$, then $x=$

1) $\sin\alpha\sin\beta$ 2) $\cos\alpha\cos\beta$
3) $\sin\alpha\cos\beta$ 4) $\cos\alpha\sin\beta$

10. Two towers are standing on a level ground. From a point on the ground mid-way between them, the angles of elevation of their tops are 60^0 and 30^0 respectively. If the height of the first tower is 100 metres, the height of the second tower is

1) 5/3 2) 100/3 3) 80/3 4) 135/3

11. The upper 3/4th portion of a vertical pole subtends an angle $\tan^{-1}(3/5)$ at a point in the horizontal plane through its foot and at a distance 40m from the foot. A possible height of the vertical pole is

1) 40 m 2) 60 m 3) 80 m 4) 20 m

12. A flag staff stands vertically on a pillar. The height of the flag staff being double the height of the pillar. A man on the ground at a distance finds both pillar and the flag staff subtends equal angle 'θ' at his eye then $\theta=$

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{8}$ 3) $\frac{\pi}{10}$ 4) $\frac{\pi}{6}$

13. A flag staff of height 10 metres is placed on the top of a tower of height 30 metres. At the top of a tower of height 40 metres, the flag staff and the tower subtend equal angles then the distance between the two towers in metres is

1) $10\sqrt{2}$ 2) $20\sqrt{2}$ 3) $30\sqrt{2}$ 4) $40\sqrt{2}$

14. From a point at a height of 27 metres above a lake the angle of elevation of the top of a tree on opposite side is 30^0 and the angle of depression of the image is 45^0. The height of the tree from water level is -- mts

1) $10(2+\sqrt{3})$ 2) $10(2-\sqrt{3})$
3) $27(2-\sqrt{3})$ 4) $27(2+\sqrt{3})$

15. The angle of elevation of the top of the tower is 45^0 on walking up a slope inclined at an angle of 30^0 to the horizontal a distance 20 metres, the angle of elevation of top of tower is observed to be 60^0. The height of the tower is

1) $10(\sqrt{3}+1)m$ 2) $20(\sqrt{3}+1)m$
3) $100\sqrt{3}\,m$ 4) $50(3+\sqrt{3})m$

16. A tower substends angles $\alpha, 2\alpha, 3\alpha$ at A,B, C all lying on the horizontal line through the foot of the tower then $\frac{AB}{BC}=$

1) $\frac{\text{Sin}3\alpha}{\text{Sin}2\alpha}$ 2) $1+2\text{Cos}2\alpha$
3) $2+2\text{Cos}2\alpha$ 4) $\frac{\text{Sin}2\alpha}{\text{Sin}\alpha}$

17. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60^0. After 10 seconds the elevation is observed to be 30^0 then the uniform speed per hour of the aeroplane is

1) $235\sqrt{5}\,km$ 2) $235\sqrt{3}\,km$
3) $240\sqrt{3}\,km$ 4) $240\sqrt{2}\,km$

18. From the top of a chiff 30 metres high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is

1) 30 mts 2) $15\sqrt{2}\,mts$
3) 60 mts 4) $30\sqrt{2}\,mts$

19. Two pillars are 120 ft a part and the height of one is double that of the other. From the middle point of the line joining their feet, an observer finds that the angular elevations of their tops are complementary. The height of the longer tower is ------ feet.

1) $35\sqrt{2}$ 2) $60\sqrt{2}$ 3) $50\sqrt{2}$ 4) $40\sqrt{2}$

S.P.Q. Student Practice Questions

20. The angle of elevation of the top of a tower from a point on the same level as foot of tower is 15^0 if the point is at a distance of $6(2+\sqrt{3})$ metres from the foot of tower then height of tower is ---

1) 6 mts
2) $6(2-\sqrt{3})mts$
3) $12(2-\sqrt{3})mts$
4) $10(2-\sqrt{3})mts$

21. From the top of a tree, a man observes the angle of depression of a point which is at a distance of 40 metres from the foot is 75^0. The height of the tree is

1) $40\sqrt{3}\,mts$
2) $21\sqrt{3}\,mts$
3) $40(2+\sqrt{3})mts$
4) $3\sqrt{21}\,mts$

22. The tops of two poles of heights 24 mt and 20 mt are connected by wire. If the wire makes an angle 45^0 with the horizontal, then the length of wire is

1) $8\sqrt{3}\,mt$
2) $8\sqrt{2}\,mt$
3) $8\sqrt{5}\,mt$
4) $4\sqrt{2}\,mts$

23. From a point on the level ground, the angle of elevation of the top of a pole is 30^0 on moving 20 metres nearer, the angle of elevation is 45^0. Height of the pole in metres is ----

1) $10(\sqrt{3}+1)$
2) $10(\sqrt{3}-1)$
3) 20
4) 15

24. A preson standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60^0. When he retires 40 metres from the bank, he finds the angle of elevation as 30^0 breadth of the river is

1) 25 m 2) 20 m 3) $20\sqrt{3}$ 4) 30 m

25. The angles of elevation of a tower from two points which are at distance h_1 and h_2 from the foot of the tower on the same side are complementary. The height of the tower is

1) h_1h_2
2) $\frac{h_1}{h_2}$
3) $\sqrt{\frac{h_1}{h_2}}$
4) $\sqrt{h_1h_2}$

26. From the top of a light house the angle of depression of boats on opposite sides of the light house observed to be 30^0 and 45^0 if the distance between the boats is 20 metres then the height of the light house is

1) $10(\sqrt{3}-1)m$
2) $20(\sqrt{3}-1)m$
3) $20(\sqrt{3}+1)m$
4) $10(\sqrt{3}+1)m$

27. If the angle of elevation of the top of a tower from a point is 60^0 and 40 metres vertically above this point the angle of elevation is 45^0. The height of the tower in metres is

1) 64.64
2) 94.64
3) 54.64
4) 74.64

28. The horizontal distance between two towers is 60 m and angular depression of the top of the first as seen from the second, which is 150 m height is 30^0. The height of the first tower is

1) $(150+20\sqrt{3})$ mt
2) $(150+15\sqrt{3})$ mt
3) $(150-20\sqrt{5})$ mt
4) $(150-20\sqrt{3})$ mt

29. Two pillars of equal height stand at a distance of 100 metres. At a point between them the elevation of their tops are found to be 30^0 and 60^0. Then height of the each pillar in metres is

1) $25\sqrt{3}$
2) $20\sqrt{3}$
3) $50\sqrt{3}$
4) $35\sqrt{3}$

30. The angle of elevation of the top of an unfinished tower from a point at a distance of 120 metres from its base is 45^0. The height of the tower that must be raised so that the angle of elevation at the same point may be 60^0 is

1) $120(\sqrt{3}-1)$m
2) $120(\sqrt{3}+1)$m
3) $120\sqrt{3}$ m
4) $140\sqrt{3}$ m

31. A Ladder of 20mt long reaches a point 20mt below the top of a flag staff. If the angle of elevation of the top of the flag staff at the foot of the ladder is 60^0 then the height of the flagstaff is

1) $300(\sqrt{3}+1)m$ 2) $120(\sqrt{3}-1)m$
3) $30m$ 4) $10(\sqrt{3}-1)m$

32. A pole 50 m high stands on a building 250 m high. To an observer at a height of 300m, the building and the pole subtend equal angles. The horizontal distance of the observer from the pole is

1) 25 m 2) 50 m
3) $25\sqrt{6}\ m$ 4) $25\sqrt{3}\ m$

33. The angle of elevation of stationary cloud from a point 25 meters above the lake is 15^0 and the angle of depression of reflection in the lake is 45^0. The height of the cloud above the lake level is

1) $25\sqrt{3}mt$ 2) $\frac{25}{\sqrt{3}}mts$
3) $20\sqrt{3}$ 4) $\frac{50}{\sqrt{3}}mt$

34. The angle of elevation of the top point P of a vertical tower PQ of height 'h' from a Point 'A' is 30^0 and from a B is 45^0, where B is the point at a distance 40 metersfrom the point 'A' measured along the line AB which makes an angle 15^0 with AQ. Height of the tower is

1) $10(\sqrt{6}-\sqrt{2})mts$ 2) $10(\sqrt{6}+\sqrt{2})mts$
3) $10\sqrt{6}mts$ 4) $20(\sqrt{6}+\sqrt{2})mts$

35. A balloon is observed simultaneously from the 3 points A,B,C on a straight road directly beneath it. The angles elevation at B is twice that at A and the angular eleva-tion on at C is thrice that at A. If AB = 20, BC = 40 then the height of the balloon is

1) $3\sqrt{15}$ 2) $5\sqrt{15}$
3) $8\sqrt{15}$ 4) $15\sqrt{5}$

36. The height of a hill is 3300 mts. From the point P on the ground the angle of elevation of the top of the hill is 60^0. A balloon is moving with constant speed vertically upwards from P. After 5 minutes of its movement a person sitting in it observes the angle of elevation of the top of hill as 30^0. The speed of balloon is

1) 2.64 km/hr 2) 26.4 km/hr
3) 22.4 km/hr 4) 2.24 km/hr

37. From the top of a cliff $x\,mts$ heigh, the angle of depression of the foot of a tower is found to be double the angle of elevation of the tower. If the height of the tower is h, the angle of elevation is

1) $Sin^{-1}\sqrt{\frac{x}{2-h}}$ 2) $Tan^{-1}\sqrt{3-\frac{2h}{x}}$
3) $Sin^{-1}\sqrt{\frac{2h}{x}}$ 4) $Cos^{-1}\sqrt{\frac{2h}{x}}$

38. Two towers of heights h_1 and h_2 subtend angles 60^0 and 30^0 respectively at the midpoint of the line joining their feet then $h_1 : h_2 =$

1) 1 : 2 2) 1 : 3 3) 2 : 1 4) 3 : 1

KEY

01) 1	02) 3	03) 1	04) 1	05) 1
06) 3	07) 2	08) 2	09) 4	10) 2
11) 1	12) 4	13) 1	14) 4	15) 1
16) 2	17) 3	18) 3	19) 2	20)1
21)3	22)4	23)1	24)2	25)4
26)1	27)2	28)4	29)1	301)1
31)3	32) 3	33)1	34)2	35)2
36)2	37)2	38)4		

Hints & Solutions

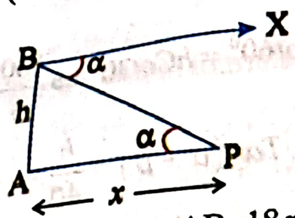
Height of tower AB=h mts $\tan 30^0 = \frac{h}{60}$

$\Rightarrow h = 60 \times \tan 30$

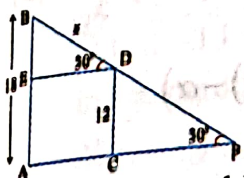
Let distance of object 'p' form foot of the tower be x mts

$\lfloor XBP = \lfloor BPA = \alpha$

(Alternative angles) $Cot\alpha = \frac{x}{h}$



Height of pole AB=18 mts,
Height of pole CD=12 mts
Length of rope BD=x mts



DE is horizontal line drawn through 'D'

BE=18-12=6 mts

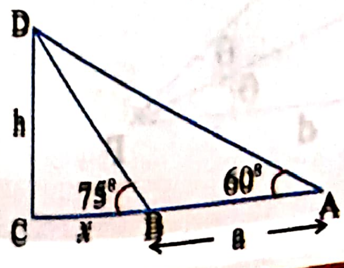
From ΔBDE, $Sin30^0 = \frac{6}{x}$

Height of the pole CD= h mts, AB=a mts

From ΔACD, $Cot60^0 = \frac{x+a}{h}$

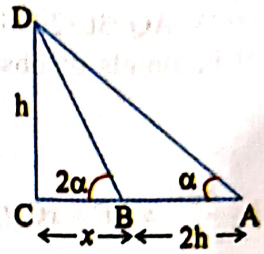
From ΔBCD, $Cot75^0 = \frac{x}{h}$

$\Rightarrow h(Cot60^0 - Cot75^0) = a$

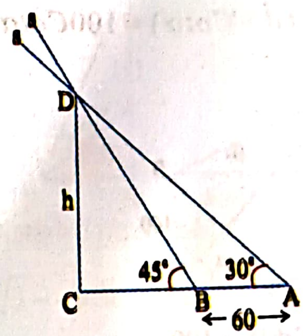


5. Height of pole CD = h mts, $\alpha = \alpha, \beta = 2\alpha$
d = AB = 2h mts

height of pole $h = \frac{d \sin\alpha \sin\beta}{\sin(\beta - \alpha)}$



6. Let S be position of sun length shadow BC=x mts, when sun altitude is 45^0. Length of shadow CA=(x+60) mts, when sun altitude is 30^0. d=AB=60 mts



$\Rightarrow h = \frac{d}{Cot\alpha - Cot\beta} = \frac{60}{\sqrt{3}-1} = 30(\sqrt{3}+1)mts$

7. If height of tower AT=15 mts. Angles of depression of P and Q from T are respectively α and β.

$\lfloor XTP = \alpha = \lfloor TPA$

$\lfloor YTQ = \beta = \lfloor PQT$

$\alpha = Cot^{-1}\frac{12}{15} \Rightarrow Cot\alpha = \frac{12}{5}$,

$\beta = Sin^{-1}\frac{3}{5} \Rightarrow Sin\beta = \frac{3}{5}$

$Cot\alpha = \frac{PA}{15} \Rightarrow PA = 36\,mts$

$Sin\beta = \frac{3}{5} \Rightarrow Cot\beta = \frac{4}{3} = \frac{AQ}{15} \Rightarrow AQ = 20\,mts$

EXERCISE–II

CRTQ & SPQ LEVEL-II

ANGLE OF ELEVATION, ANGLE OF DEPRESSION, LAW OF COSINES, SINES

C.R.T.Q Class Room Teaching Questions

1. At a certain point the angle of elevation of a tower is found to be $Cot^{-1}\left(\frac{3}{5}\right)$. On walking 32 metres directly towards the tower its angle of elevation is $Cot^{-1}\left(\frac{2}{5}\right)$. The height of the tower in metres is

   1) 32 2) 160 3) 320 4) 340

2. At a particular instant the height of the tower is equl to the length of its shadow after some time the length of the shadow is $\sqrt{3}$ times of the height of the tower, then the time lapsed between the two observations in hours is

   1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) 24

3. A tower of height 50 metres stands on a level ground. A flag-staff standing on the tower subtends an angle of $Tan^{-1}\left(\frac{1}{3}\right)$ at a point 100 metres away from the tower on the ground. The length of the flag-staff in metres is

   1) 50 2) 75 3) 100 4) 125

4. A pole of height h stands at one corner of a park in the shape of an equilateral triangle. If α is the angle which the pole subtends at the midpoint of the opposite side, the length of each side of the park is

   1) $\left(\frac{\sqrt{3}}{2}\right)h\cot\alpha$ 2) $\left(\frac{2}{\sqrt{3}}\right)h\cot\alpha$

   3) $\left(\frac{\sqrt{3}}{2}\right)h\tan\alpha$ 4) $\left(\frac{2}{\sqrt{3}}\right)h\tan\alpha$

5. Three vertical poles of heights h_1, h_2 and h_3 at the vertices A, B and C of a ΔABC subtend angles α,β and γ respectively at the cicumcentre of triangle. If $\cot\alpha, \cot\beta$ and $\cot\gamma$ are in A.P. then h_1, h_2, h_3 are in

   1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

6. A tower stands at the top of a hill whose height is three times the height of the tower. The tower is found to subtend an angle of $Tan^{-1}\left(\frac{1}{7}\right)$ at a point 2 km away on the horizontal throught the foot of the hill. Then the height of the tower is

   1) $\frac{1}{2}km$ or $\frac{1}{3}km$ 2) $\frac{1}{3}km$ or $\frac{2}{3}km$

   3) $\frac{2}{3}km$ or $\frac{1}{2}km$ 4) $\frac{3}{4}km$ or $\frac{1}{2}km$

7. A sphere of radius 'a' subtends an angle 60^0 at a point P. Then the distance of P from the centre of the sphere is

   1) $\frac{a}{\sqrt{3}}$ 2) 2a 3) $\frac{a\sqrt{3}}{2}$ 4) $\frac{2a}{\sqrt{3}}$

8. A person in a balloon, who has ascended vertically from flat land at the sea level, observes the angle of depression of a ship at anchor to be 30^0. After descending vertically 600 metres, he finds the angle of depression to be 15^0. The horizontal distance of the ship from the foot of ascent in metres is

   1) $300(3+\sqrt{3})$ 2) $300(3+2\sqrt{3})$

   3) $150(3+2\sqrt{3})$ 4) $150(3-2\sqrt{3})$

9. C is the midpoint of the line joining two points A,B on the ground. A tower at C slightly leans towards B. If the angles of elevation of the top of the tower from A and B are $30^0, 60^0$ respectvely, the angle made by the tower with the horizontal is

   1) 45^0 2) 60^0 3) 75^0 4) 30^0

10. The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at the point in the horizontal plane through its foot. The tangent of the angle subtended by the pole at the same point is
1) 1 or 2 2) 2 or 3
3) 3 or 4 4) 4 or 1

11. A straight pole A subtends a right angle at a point B of another pole at a distance of 30 metres from A, the top of A being 60^0 above the horizontal line joining the point B to the pole A. The length of the pole A is, in metres.
1) $20\sqrt{3}$ 2) $40\sqrt{3}$ 3) $60\sqrt{3}$ 4) $\frac{40}{\sqrt{3}}$

12. A vertical tower stands on a declivity which is inclined at 15^0 to the horizon. From the foot of the tower a man ascends the declivity from 80feet and then finds that the tower subtends an angle of 30^0. The height of the tower is
1) $40\left(\sqrt{6}+\sqrt{2}\right)$ 2) $20\left(\sqrt{6}-\sqrt{2}\right)$
3) $40\left(\sqrt{6}-\sqrt{2}\right)$ 4) $10\left(\sqrt{6}-\sqrt{2}\right)$

S.P.Q. Student Practice Questions

13. At a point, the angle of elevation of top of a tower is found to be $\tan^{-1}\left(\frac{5}{12}\right)$ on walking 240 metres nearer the tower, the elevation is found to be $\tan^{-1}\left(\frac{3}{4}\right)$. The height of the tower in metres is
1) 175 2) 225 3) 275 4) 300

14. A tower of height 30 metres casts a shadow of length 40 meters at a certain instant. When the sun's elevation increases by $Tan^{-1}\frac{1}{7}$, the length of the shadow cast by the tower, in meters is
1) 20 2) 25 3) 30 4) 45

15. A flag-staff 20 metres long standing on a wall 10 metres high subtends an angle whose tangent is 0.5 at a point on the ground. If θ is the angle subtended by the wall at that point then $\tan\theta=$
1) 1 only 2) 1/3 only
3) 1 or 1/3 4) 2

16. ABC is a triangular park with AB=AC=100cm A clock tower is situated at the midpoint of BC. The angles of elevation of the top of the tower at A and B are $Cot^{-1}3.2$ and $Cosec^{-1}2.6$. The height of the tower is
1) 25 mt 2) 50 mt
3) 100 mt 4) $50\sqrt{2}mt$

17. A person observes that the top of three poles standing infront of him are in a line. If the height of the poles are in A.P. Then the horizontal distances of the poles from the person are in
1) A.P 2) G.P 3) H.P 4) AGP

18. A vertical pole more than 100 ft high consists of two portions, the lower being 1/3 of the whole. If the upper portion subtends an angle $\tan^{-1}\left(\frac{1}{2}\right)$ at a point distant 40 ft. From the foot of the pole. The height of the pole is
1) 105 2) 120
3) 135 4) 150

19. A spherical balloon of radius 5 subtends an angle 60^0 at a point on the horizontal. If the angle of elevation of its centre is 30^0, the height of the centre of the balloon is
1) $5\sqrt{2}$ units 2) 10 units
3) 15 units 4) 20 units

6. The angle of elevation of the top of a tower standing on a horizontal plane from the two points lying on a line passing through the foot of the tower at distances a and b res-pectively are complementary angles. if the line joining the two points sutends an angle θ at the top of the tower, then $\sin\theta$ is

1) $\frac{a+b}{a-b}$ 2) $\frac{a-b}{a+b}$ 3) $\frac{b-a}{a+b}$ 4) $\frac{a}{b}$

S.P.Q. Student Practice Questions

7. Two vertical poles 20m and 80m heigh stand apart on a horizontal plane. The height of the point of intersection of the lines joining the top of each pole to the foot of the other is

1) 13m 2) 14 m 3) 15 m 4) 16m

8. On one side of a road of width 'd' meters there is a point of observation P at a height 'h' meters from ground. If a tree on the other side of the road, makes a right angle at P, height of the tree in meters is

1) $\frac{h^2-d^2}{h}$ 2) $\frac{h^2+d^2}{h}$

3) $\frac{d^2-h^2}{h}$ 4) $\frac{2d^2+h^2}{h}$

9. An obsercver finds that the angular elevation of a tower is θ. On advancing a metres towards the tower the elevation is 45^0 and an advancing 'b' metres nearer the elevation is $90^0-\theta$ then the height of the tower in metres is

1) $\frac{ab}{a+b}$ 2) $\frac{ab}{a-b}$ 3) $\frac{2ab}{a+b}$ 4) $\frac{2ab}{a-b}$

10. From a point E on the horizontal due to east of a tree , the top of the tree makes an angle of elevation 60^0. From a point S on the horizontal due west of the tree the same top makes an angle of elevation 30^0. The angle of elevation made by the top of the tree at the mid point of ES is

1) 30^0 2) 60^0 3) 90^0 4) 40^0

11. The length of the shadow of vertical pole of height h, thrown by the sun's rays at three different movements are h,2h,3h. The sum of the angles of elevation of rays at these three moments is equal to

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

12. A pole is slightly inclined towords the east. At two points due west of it at distance 'a' and b, the angles of elevation of the top of the pole are α and β respectively. The inclination of the pole to the horizon is

1) $Tan^{-1}\left[\frac{a+b}{b\cot\alpha-a\cot\beta}\right]$

2) $Tan^{-1}\left[\frac{b-a}{b\cot\alpha-a\cot\beta}\right]$

3) $\cos^{-1}\left[\frac{a-b}{b\cot\alpha-a\cot\beta}\right]$

4) $\sin^{-1}\left[\frac{a-b}{b\cot\alpha-a\cot\beta}\right]$

ANGLE OF DEPRESSION

C.R.T.Q Class Room Teaching Questions

13. A flag staff of the height (a-b) stands on the top of a tower subtends the same angle at the point on the horizontal plane through the foot of the tower which are at distant a and b from the tower. The height of the tower is

1) b 2) a+b 3) a 4) a-b

14. A tower 51 m height has a mark at height of 25 m from the ground. At what distance the two parts subtend equal angle to an eye at the height of 5 m from the ground.

1) 20 m 2) 30 m 3) 15 m 4) 160 m

15. A vertical pole subtends an angle $\tan^{-1}\frac{1}{2}$ at a point P on the ground. The angle subtended by the upper half the pole at p is

1) $\tan^{-1}\frac{1}{4}$ 2) $\tan^{-1}\frac{1}{8}$

3) $\tan^{-1}\frac{2}{3}$ 4) $\tan^{-1}\frac{2}{9}$

S.P.Q. Student Practice Questions

16. A tower MPQ surmounted by a spiral QR stands on a horizontal plane. At the extremity 'A' of a horizontal line AM it is found that MP and QR subtend equal angles. if MP = 3 m, PQ = 28 m and QR = 5 m then MA =

1) $\sqrt{36\times93}$ 2) $\sqrt{18\times93}$

3) $\sqrt{34\times36}$ 4) $\sqrt{34\times93}$

17. A tower ABCD stands on a level ground with foot A. At a point P on the ground the portion AB,AC and AD subtends angles α,β,γ respectively. If AB=a, AC = b, AD= c, AP = x and $\alpha+\beta+\gamma=180^0$ then (a+b+c) x^2 =

1) abc 2) a + b + c

3) a + b − c 4) a - b - c

3D-MODELS

C.R.T.Q Class Room Teaching Questions

18. If each side of length 'a' of an equilateral triangle subtends an angle of 60^0 at the top of a tower h metrs heigh situated at the centre of the triangle ,then

1) $3a^2 = 2h^2$ 2) $2a^2 = 3h^2$

3) $a^2 = 3h^2$ 4) $3a^2 = h^2$

19. A vertical tower stands on a triangular field. Angle of elevation of the top of the tower from each of the vertex is θ. If the length of the sides of the field are 30 m, 50 m and 70 m. The height of the tower is

1) $70\sqrt{3}\tan\theta$ 2) $\frac{70}{\sqrt{3}}\tan\theta$

3) $\frac{50}{\sqrt{3}}\tan\theta$ 4) $75\sqrt{3}\tan\theta$

20. The angle of elevation of a top of a tower from a point A due south of it is $\tan^{-1}$ (6) and that from B due to west of it is $\tan^{-1}(7.5)$. If h is the height of the tower and AB = λ h then λ^2 =

1) $\frac{21}{700}$ 2) $\frac{42}{1300}$ 3) $\frac{41}{900}$ 4) $\frac{52}{1100}$

21. The angular elevation of a tower OP at a point A due south of it is 60^0 and at a point B due to west of A, the evaluation is 30^0. If AB = 3 m, the height of the tower is

1) $2\sqrt{3}m$ 2) $2\sqrt{6}m$

3) $\frac{3\sqrt{3}}{2}m$ 4) $\frac{3\sqrt{6}}{4}m$

S.P.Q. Student Practice Questions

22. A lamp post is sittuated at the middle point M of the side AC of a triangular plot ABC with BC = 7 m CA = 8 m and AB = 9 m. Lamp post subtends an angle 15^0 at the point B. The height of the lamp post is

1) $7(2+\sqrt{3})$ m 2) $7(2-\sqrt{3})$ m

3) $14(2-\sqrt{3})$ m 4) $14(2+\sqrt{3})$ m

23. Two ships leave a port at the same time. One goes 24 Km per hour in the direction N 45^0 E and other travels 32 Km per hour in the direction S 75^0 E. The distance between the ships at the end of 3 hours is ______ Km

1) 86.4 2) 96.4 3) 66.8 4) 98.4