Sri Chaitanya IIT Academy.,India.
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A right Choice for the Real Aspirant
ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_STERLING BT	RPTA-05	Date: 07-09-2025
Time: 09:00AM to 12:00PM	JEE-ADV_(2021-P1)	Max. Marks: 180

KEY SHEET

PHYSICS

1	**D**	2	**A**	3	**B**	4	**B**	5	**5**	6	**17**
7	**64**	8	**8**	9	**2.5**	10	**1.5**	11	**AC**	12	**ABCD**
13	**ABC**	14	**ABCD**	15	**ABC**	16	**CD**	17	**9**	18	**4**
19	**6**										

CHEMISTRY

20	**C**	21	**D**	22	**B**	23	**C**	24	**2**	25	**67.5**
26	**4**	27	**4**	28	**3**	29	**6**	30	**ABC**	31	**BD**
32	**AC**	33	**AB**	34	**AB**	35	**AC**	36	**12**	37	**44**
38	**12**										

MATHEMATICS

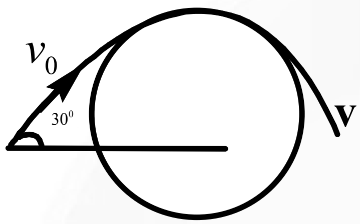
39	**A**	40	**C**	41	**B**	42	**D**	43	**6**	44	**4**
45	**100**	46	**3.2**	47	**6**	48	**1**	49	**BC**	50	**AB**
51	**ABD**	52	**ABCD**	53	**CD**	54	**ABC**	55	**4**	56	**2**
57	**6**										

SOLUTIONS
PHYSICS

1. $mv_0 \sin 30 = mvR.....(1)$

Energy conservation

$$\frac{1}{2}mv_0^2 - \frac{GM}{5R} = \frac{1}{2}mv^2 - \frac{GMm}{R}.....(2) \Rightarrow v_0 = \sqrt{\frac{32GM}{105R}} \ \& \ v = \sqrt{\frac{40GM}{21R}}$$



2. $F = \frac{GMm}{r^{5/2}} = \frac{mV_0^2}{r}$

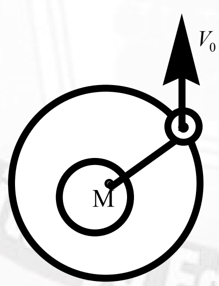
$V_0^2 = GMr^{-3/2}$

$2LnV_0 = LnGM - \frac{3}{2}Lnr$

$LnV_0 = LnGM - \frac{3}{4}Lnr$

$y = C + mx$

$m = \left|\frac{3}{4}\right| = 0.75$



3. We consider on angular element as shown in figure. Force on element is

$$dF = \lambda(R.d\theta).E_0$$

Perpendicular distance between two equal and opposite force pairs of dF will be

$$r = 2R\sin\theta$$

Torque on ring is

$$d\tau = dF.r = 2\lambda R^2 E_0 \sin\theta.d\theta$$

$$\Rightarrow \tau = \int_d^\tau d\tau = 2\lambda R^2 E_0$$

These pair of forces will not provide net force but due to rotation tendency force of frication on ring is f in forward direction as shown

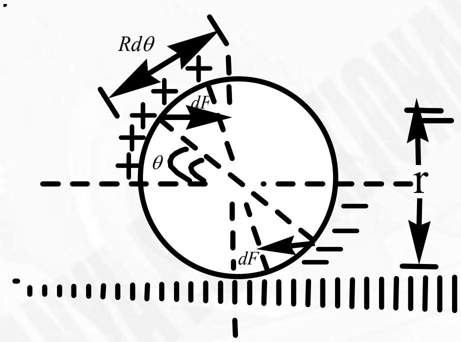
For pure rolling to take place, we use

$$a = R\alpha$$

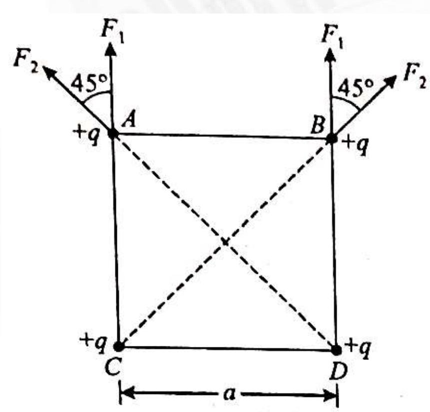
$$\Rightarrow \frac{f}{m} = R\left[\frac{\tau - fR}{mR^2}\right]$$

$$\Rightarrow f = \frac{\tau}{R} - f$$

$$\Rightarrow f = \frac{\tau}{2R} = \lambda R E_0$$



4.



$$F_2 = \frac{q^2}{4\pi\varepsilon_0\left(\sqrt{2a}\right)^2} = \frac{q^2}{4\pi\varepsilon_0 \times 2a^2}$$

Net force on side AB of the film is

$$F = 2F_1 + 2F_2 \cos 45$$

$$\Rightarrow F=\frac{2q^2}{4\pi\varepsilon_0 a^2}+\frac{2q^2}{4\pi\varepsilon_0 2\sqrt{2a^2}}$$

$$\Rightarrow F=\frac{q^2}{4\pi\varepsilon_0 a^2}\left(2+\frac{1}{\sqrt{2}}\right)$$

Force on AB due to surface tension = $2\sigma a$.

$$\frac{q^2}{4\pi\varepsilon_0 a^2}\left(2+\frac{1}{\sqrt{2}}\right)=2\sigma a$$

$$\Rightarrow a=\left[\frac{1}{4\pi\in_0}\left(1+\frac{1}{2\sqrt{2}}\right).\frac{q^2}{\sigma}\right]^{1/3}\text{......(1)}$$

Given that $a=k\left(\frac{q^2}{\sigma}\right)^{1/N}$ ....(2)

Comparing equation (1) and (2), we have N =3

Is and $k=\left[\frac{1}{4\pi\in_0}\left(1+\frac{1}{2\sqrt{2}}\right)\right]^{1/3}$

5 & 6. Mass of the complete sphere is $\frac{8M_0}{7}$,

Mass of in the cavity $\frac{M_0}{7}$

$$V=\frac{-GM}{2R^3}\left(3R^2-r^2\right)$$

Field of A:

$$B_A=\frac{G\left(\frac{8Mo}{7}\right)}{R^2}+\frac{-G\left(\frac{M_0}{7}\right)}{\left(\frac{3R}{2}\right)^2}$$

$$=\frac{8GM_0}{7R^2}-\frac{4GM_0}{7\times 9R^2}=\frac{GM_0}{7R^2}\left(8-\frac{4}{9}\right)=\frac{68GM_0}{7R^2\times 9}=\frac{68GM_0}{63R^2}$$

Potential at B:

$$V_B=\frac{-3}{2}\frac{G\left(\frac{8M_0}{7}\right)}{R}-\frac{-G\left(\frac{M_0}{7}\right)}{\left(\frac{R}{2}\right)}$$

$$=-\frac{24GM_0}{14R}+\frac{2GM_0}{7R}=-\frac{12}{7R}GM_0+\frac{2GM_0}{7R}=\frac{-10GM_0}{7R}$$

7&8. Where R denotes radius of earth, we have centripetal force = Gravitational force

$$=\frac{mv^2}{(R+h)}=\frac{GmM}{(R+h)^2}$$

$$=v^2=\frac{GM}{(R+h)}........(i)$$

But $v=\frac{v_e}{2}=$ Half of escape velocity from earth.

$$=v=\frac{1}{2}\sqrt{\frac{2GM}{R}}$$

Using equation. (i) and (ii)

$$\therefore\frac{2GM}{4R}=\frac{GM}{R+h}\Rightarrow 2R=R+h\Rightarrow R=h$$

$$h=R=6400km$$

(ii) Speed of satellite at surface of earth:

Let the satellite be stopped at P in its orbit. It falls freely and hits earth at Q with velocity v

Mechanical energy is conserved in its fall.

$$P.E.at(P)=P.E.at(Q)+K.E.at(Q)$$

$$-\frac{GmM}{2R}=-\frac{GmM}{R}+\frac{1}{2}mv^2$$

$$=\frac{GmM}{2R}=\frac{1}{2}mv^2\ or\ v=\sqrt{\frac{GM}{R}}=\sqrt{gR}$$

$$v=\sqrt{10\times 6400\times 10^3}=8000m/s$$

9. Electric field due to ring at location of dipole is given

$$E=\frac{KQx}{(R^2+x^2)^{3/2}},$$

We use

$$\frac{dE}{dx}=KQ\left[\frac{(R^2+x^2)^{3/2}-x.\frac{3}{2}(R^2+x^2)^{1/2}(2x)}{(R^2+x^2)^3}\right]$$

$$\frac{dE}{dx}=KQ\left[\frac{R^2+x^2-3x^2}{(R^2+x^2)^{5/2}}\right]$$

Force on dipole is given as

$$\Rightarrow F = P\frac{dE}{dx} = \frac{Qqa}{2\pi\varepsilon_0}\left[\frac{R^2 - 2x^2}{\left(R^2 + x^2\right)^{5/2}}\right]$$

10. Work done in ration of dipole is given as

$$W = U_f - U_i$$

$$\Rightarrow W = -PE\cos 180^0 + PE\cos 0^0$$

$$\Rightarrow W = 2PE$$

$$\Rightarrow W = 2(q)(2a)\left[\frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(R^2 + x^2\right)^{3/2}}\right]$$

$$\Rightarrow W = \frac{aqQx}{\pi\varepsilon_0\left(R^2 + x^2\right)^{3/2}}$$

11. $F_g = F_c$

$$\frac{G(2m)m}{d^2} = m\left(\frac{2d}{3}\right)w^2$$

$$\sqrt{\frac{3Gm}{d^3}} = \omega = \frac{2\pi}{T}$$

$$T = \sqrt{\frac{4\pi^2 d^3}{3GM}}$$

$$\frac{Lm}{L_{2m}} = \frac{I_1\omega}{I_2\omega} = \frac{m\left(\frac{2d}{3}\right)^2}{(2m)\left(\frac{d}{3}\right)^2} = 2$$

12. Orbital speed, $v = \sqrt{\frac{GM}{R}} \Rightarrow v\alpha\frac{1}{\sqrt{R}}$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{4} = 2$$

Angular momentum, L = mvR

$$\therefore \frac{L_1}{L_2} = \frac{m_1}{m_2}\times\frac{v_1}{v_2}\times\frac{R_1}{R_2} = \frac{2}{1}\times\frac{2}{1}\times\frac{1}{4} = 1$$

K.E of satellite, $K = \frac{GMm}{2R} \therefore \frac{K_1}{K_2} = \frac{m_1}{m_2}\times\frac{R_2}{R_1} = 2\times 4 = 8$

From Kepler's second law, $T^2 \propto R^3 \Rightarrow T\alpha R^{3/2}$

$$\therefore \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2} = \frac{1}{8}$$

13. $m\vartheta_1 r_1 = m\vartheta_2 r_2 \Rightarrow \vartheta v_1 = \vartheta_2 r_2 - (1)$

$$\frac{-GMm}{r_1} + \frac{1}{2}mV_1^2 = \frac{-GMm}{r_2} + \frac{1}{2}mv_2^2$$

$$\frac{-GMm}{r_1} + \frac{1}{2}m\frac{V_2^2 r_2^2}{r_1^2} = \frac{-GMm}{r_2} + \frac{1}{2}mv^2$$

$$\vartheta_2 = \sqrt{\frac{2GMr_1}{r_2(r_1+r_2)}} \rightarrow (2)$$

$$L = m\vartheta_2 r_2 = m\sqrt{\frac{2GMr_1}{r_2(r_1+r_2)}}\ r_2 = m\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}} \rightarrow (3)$$

14. (A,B,C,D) By an external force in case of SHM only equilibrium position changes Time period remains same. As speed of block at mean position is same, amplitude will be same in all cases. In case-4 equilibrium position $x_0 = 3mg/k$ which is maximum among all cases. Thus, all the given options are correct

15. (A,B,C) For the revolving charge we have

$T\cos\alpha = mg$

$T\sin\alpha = \frac{Kq^2}{r^2} + m\omega^2 r$

$\Rightarrow T > mg$ as well as $T > \frac{Kq^2}{r^2}$

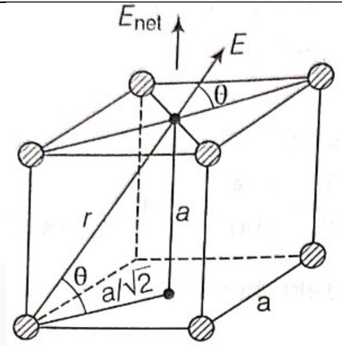
If no charge is there on the revolving ball, we use

$$T\sin\alpha = \frac{mv^2}{r}$$

Thus to maintain the angle, v must be increased as earlier electrostatic force was present which is now no longer present. Thus options (A), (B)and (C) are correct

16. Contribution to net filed from charges on same face is zero

For remaining four charges, distance



$$r=\sqrt{\frac{a^2}{2}+a^2}=\sqrt{\frac{3}{2}}a$$

Also,

$$\sin\theta=\frac{a}{r}$$

$$E_{net}=\frac{4kq}{r^2}\sin\theta=\frac{4kq}{r^2}.\frac{a}{r}$$

$$=\frac{4kqa}{r^3}=\frac{4kq\,a}{\left(\sqrt{\frac{3}{2}}a\right)^3}$$

$$=\frac{2^{7/2}}{3^{3/2}}.\frac{kq}{a^2}N/C$$

17. $E=\frac{GMm}{2a}\Rightarrow a=20l=\frac{r_p+r_a}{2}\Rightarrow r_a=36l,$ Now, $V_pr_p=V_ar_a\Rightarrow\frac{V_p}{V_a}=\frac{r_a}{r_p}=\frac{36l}{4l}=9$

18. $F_1=F_g+F_2$

$$PA=\frac{GM\rho A}{2R^3}\left(R^2-r^2\right)+P_0A$$

$$PA=\frac{GM}{2R^3}\left(\frac{3M}{4\pi R^3}\right)A\left(R^2-r^2\right)+PoA$$

$$P=P_0+\frac{3GM^2}{8\pi R^6}\left(R^2-r^2\right)$$

$$at\ r=\frac{R}{2}$$

$$P=P_0+\frac{3GM^2}{8\pi R^6}\left(R^2-\frac{R^2}{4}\right)$$

$$P-P_0=\frac{3GM^2}{8\pi R^6}\left(\frac{3R^2}{4}\right)=\frac{9GM^2}{32\pi R^4}$$

$$dm=\rho dV=\rho Adx$$

$$dF_g=E\times dm$$

$$=\left(\frac{GMx}{R^3}\right)\rho Adx$$

$$\int dF_g=\frac{GM\rho A}{R^3}\int_r^R xAdx$$

$$F_g=\frac{GM\rho A}{2R^3}\left(R^2-r^2\right)$$

19. The direction of electric filed inside the cavity left ward is in direction and of constant magnitude given as

$$E_{\text{cavity}}=\frac{\rho a}{3\in_0}$$

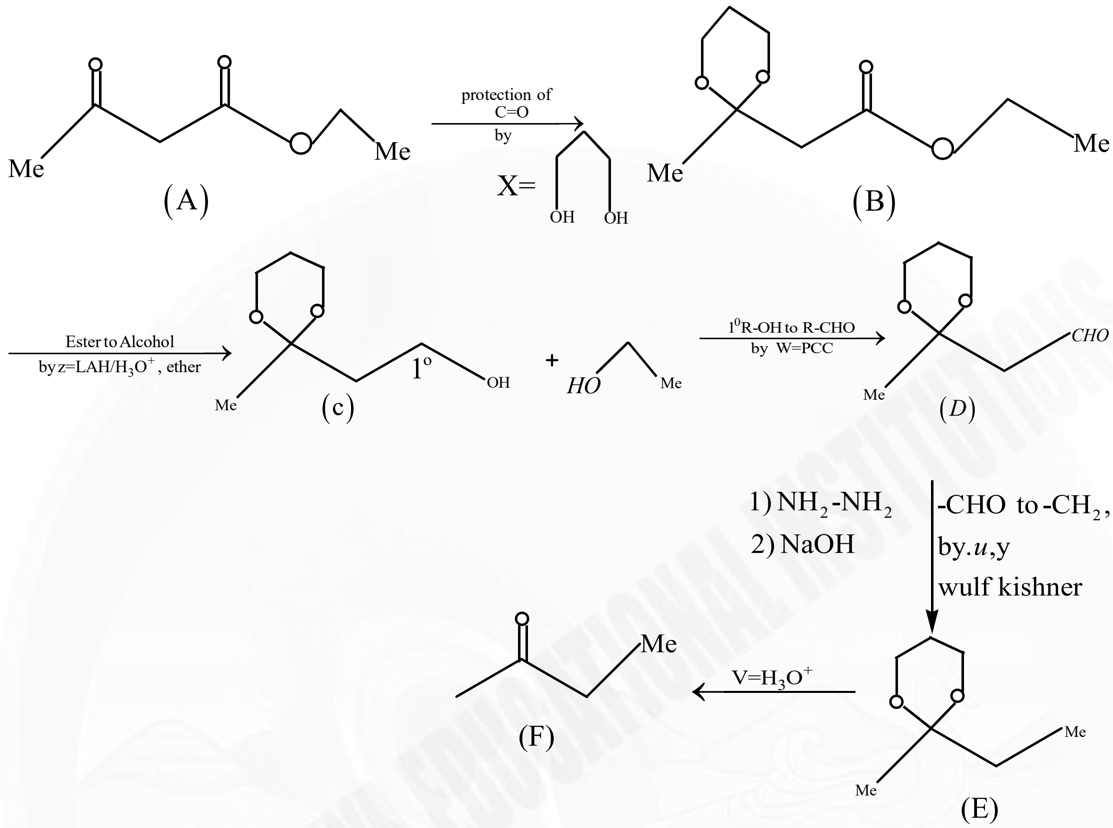
For touching the sphere again, electron must move a distance 2r cos 45^0 and time taken by electron for this is given as

$$t=\sqrt{\frac{2l}{a}}$$

$$\Rightarrow t=\sqrt{\frac{2r\cos 45^0}{eE/m}}=\sqrt{\frac{2\sqrt{2}r}{\frac{e}{m}\frac{\rho a}{3\varepsilon_0}}}$$

CHEMISTRY

20.



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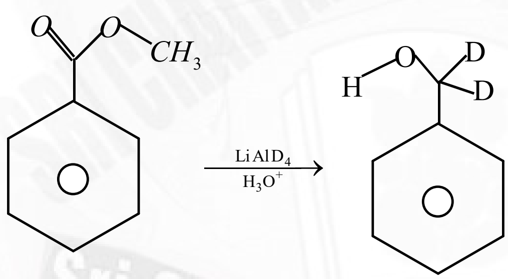
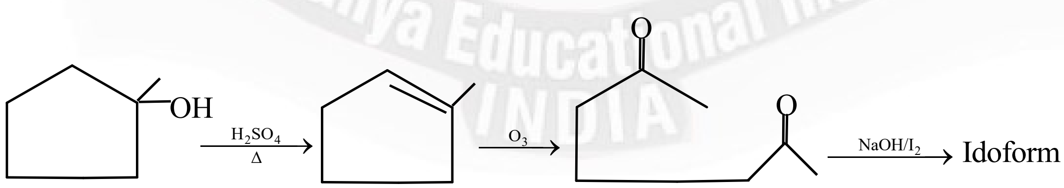


Ester reduced to Alcohol by Li Al H4

22. A: in acidic $K_2Cr_2O_7$ (non oxidizable) and gives alkene with H_2So_4, Δ i.e A is 3^o-alcohol



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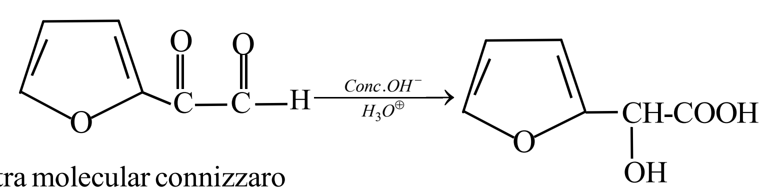
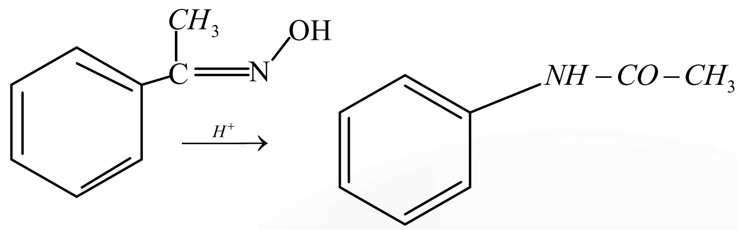


Intra molecular connizzaro

24 . 2

25.

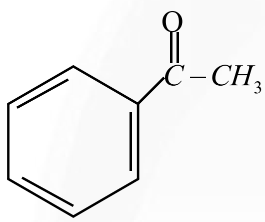


M.F=C_8H_9NO

m.wt $=135$

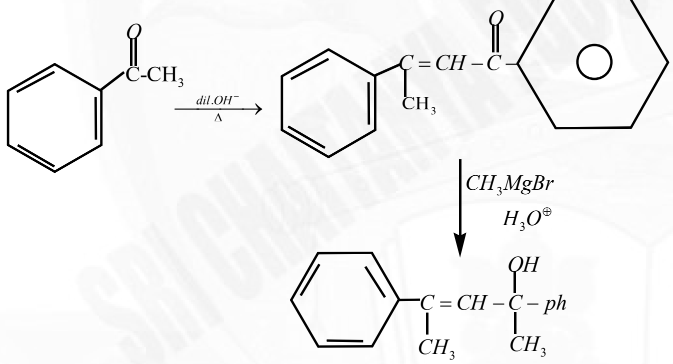
$$\frac{x}{2}=\frac{135}{2}=67.5$$

26.



No. of π bonds = 4

27.



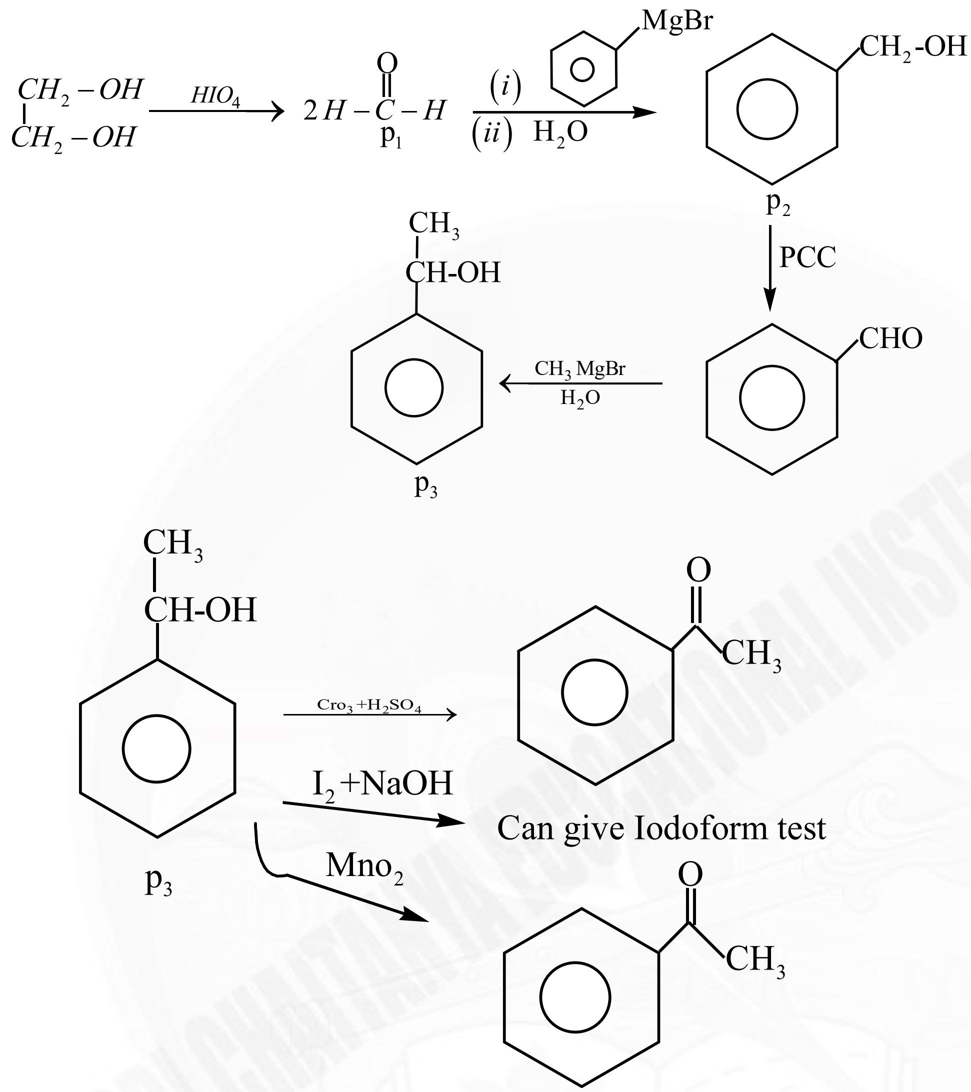
Stereo isomers$=2^n=2^2=4$

28. Rate = $k\left[ph-\overset{\overset{\displaystyle O}{\|}}{C}-H\right]^2\left[OH^-\right]^1$

$\therefore$ Order $=2+1=3$

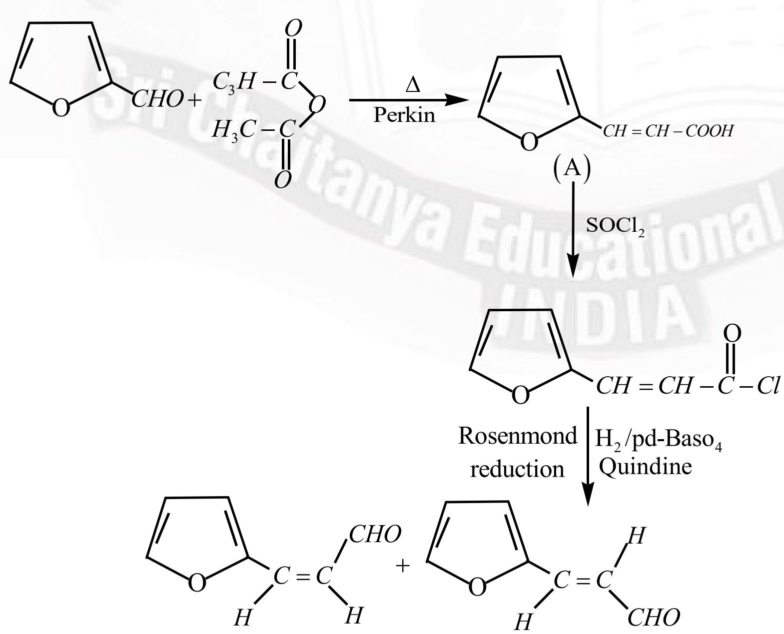
29. A,B,E, H, I, L

30.



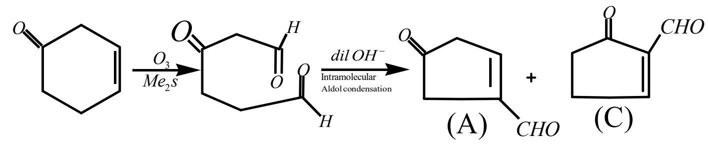
CH2-OH
CH2-OH
HIO4
2 H-C-H
O
p1
(i)
MgBr
(ii) H2O
CH2-OH
p2
PCC
CHO
CH3 MgBr
H2O
CH3
CH-OH
p3
Cro3 +H2SO4
CH3
I2+NaOH
Can give Iodoform test
Mno2

31.



CHO+
C3H-C
H3C-C
Δ
Perkin
CH=CH-COOH
(A)
SOCl2
CH=CH-C-Cl
Rosenmond
reduction
H2/pd-Baso4
Quindine
CHO
C=C
H
+
CHO

32.



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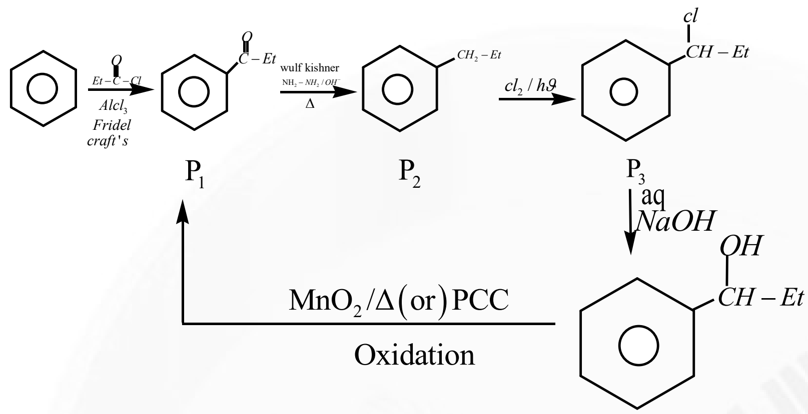

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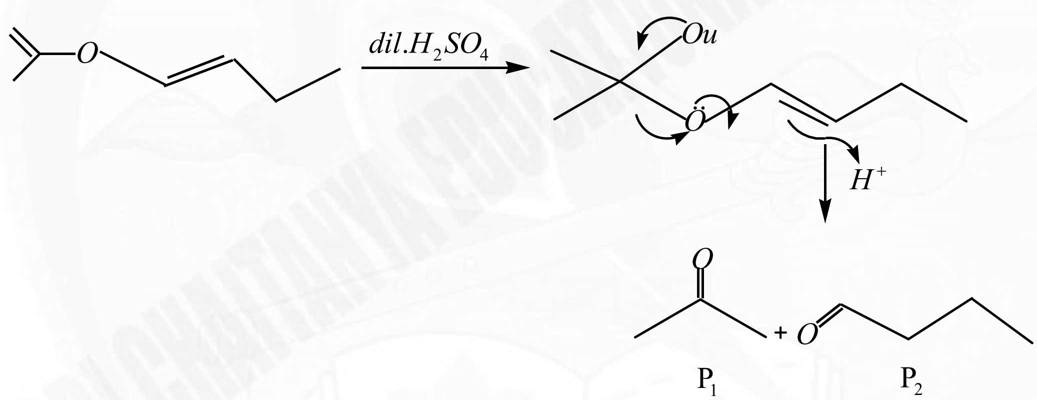
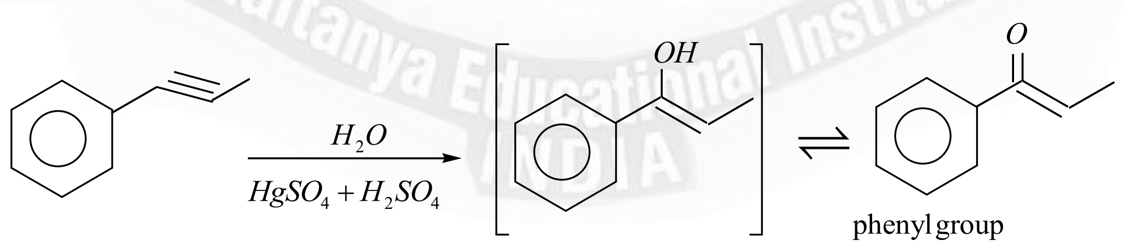


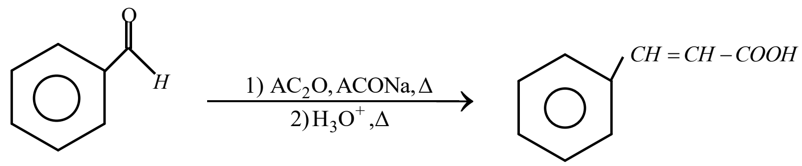
	P_1	P_2
Tollens	✗	✓
Iodoform	✓	✗
$2,4-DNP$	✓	✓
1% Alkaline $KMnO_4$	✗	✗

35.

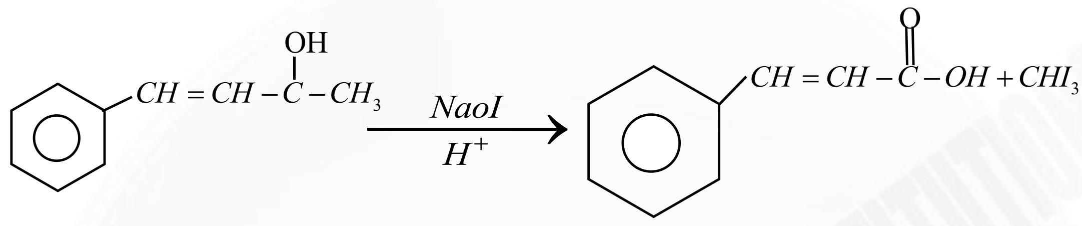


phenyl group
can stabilise
+ve charge by resonance

36.



(A)
DU = 6



(B)
DU = 6

37.

$$CH_3-\overset{O}{\overset{||}{C}}-H+H_3C-\overset{O}{\overset{||}{C}}-H \xrightarrow{dil\,OH^-} CH_3-CH=CH-\overset{O}{\overset{||}{C}}-H$$

M. wt = 44 M. wt = 70

38.

$$CH_3-\overset{O}{\overset{||}{C}}-H+ph-CH_2-\overset{O}{\overset{||}{C}}-H \xrightarrow[\substack{Cross\\ Aldol}]{dil\,OH^{(-)}}$$

$$\underset{(2)}{CH_3-\overset{OH}{\underset{*}{CH}}-CH_2-\overset{O}{\overset{||}{C}}-H} + \underset{(4)}{ph-CH_2-\overset{OH}{\underset{*}{CH}}-\overset{*}{\underset{ph}{CH}}-CHO} + \underset{(4)}{CH_3-\overset{OH}{\underset{*}{CH}}-\overset{*}{\underset{ph}{CH}}-CHO}$$

$$\underset{(2)}{ph-CH_2-\overset{OH}{\underset{*}{CH}}-CH_2-\overset{O}{\overset{||}{C}}-H}$$

MATHEMATICS

39. $I=\int_0^{102}(x-1)(x-2).......(x-100)\times\left(\frac{1}{x-1}\right)+\left(\frac{1}{(x-2)}+....+\frac{1}{(x-100)}\right)dx$

$=\int_0^{102}\frac{d}{dx}((x-1)(x-2)....(x-100))dx$

$=\left[(x-1)(x-2)....(x-100)\right]_0^{102}=101!-100!$

40. Here, we have to prove that y = constant or derivative of y w.r.t x is zero

$y=\int\limits_{\frac{1}{8}}^{\sin^2 x}\sin^{-1}\sqrt{t}dt+\int\limits_{\frac{1}{8}}^{\cos^2 x}\cos^{-1}\sqrt{t}dt....(1)$

$\frac{dy}{dx}=\sin^{-1}\sqrt{\sin^2 x}2\sin x\cos x+\cos^{-1}\sqrt{\cos^2 x}(-2\cos x\sin x)$

$=2x\sin x\cos x-2x\sin x\cos x=0$ for all x

$\therefore$ the curve in equation (1) is a straight line parallel to the x – axis Now, since y is constant, it is independent of x- so let us select $x=\frac{\pi}{4}$, then

$y=\int_{1/8}^{1/2}\sin^{-1}\sqrt{t}dt+\int_{1/8}^{1/2}\cos^{-1}\sqrt{t}dt=\int_{1/8}^{1/2}\left(\sin^{-1}\sqrt{t}+\cos^{-1}\sqrt{t}\right)dt=\int\limits_{1/8}^{1/2}\frac{\pi}{2}dt$

$=\frac{\pi}{2}\left[\frac{1}{2}-\frac{1}{8}\right]=\frac{3\pi}{16}$

41. $\int_1^2\frac{dx}{(x-1)^2+1}+\int_2^3\frac{1}{(x-2)^2+1}dx=\frac{\pi}{2}$

42. x Replace with $x+\frac{1}{2}$

$F\left(x+\frac{1}{2}\right)+F(x+1)=3$

$\therefore F(x+1)=F(x)$

$\therefore\int_0^{1500}F(x)=1500\left[\int_0^{\frac{1}{2}}F(x)dx+\int\limits_{\frac{1}{2}}^{1}F(x)dx\right]$

$x=y+\frac{1}{2}$

$=1500\left[\int_0^{\frac{1}{2}}3\,dx\right]=\frac{9000}{4}$

43&44. We have $f(a)=\int_{a-1}^{a}\frac{1}{x}\cot^{-1}\left(\frac{x^2-x+1}{2x-3x^2}+\frac{x^2-x+1}{3-2x}\right)dx.....(1)$

$$x=\frac{1}{t}\Rightarrow dx=-\frac{1}{t^2}dt$$

$$f(a)=\int_{a}^{\frac{1}{a}}t\cot^{-1}\left(\frac{t^2-t+1}{2t-3}+\frac{t^2-t+1}{3t^2-2t}\right)\left(\frac{-1}{t^2}\right)dt=\int_{\frac{1}{a}}^{a}\frac{1}{t}\cot^{-1}\left(\frac{t^2-t+1}{2t-3}+\frac{t^2-t+1}{3t^2-2t}\right)dx$$

$$=\int_{\frac{1}{a}}^{a}\frac{1}{t}\left\{\pi-\cot^{-1}\left(\frac{t^2-t+1}{3-2t}+\frac{t^2-t+1}{2t-3t^2}\right)\right\}dt......(2)$$

On equation (1) + equation (2), We get

$$2f(a)=\int_{\frac{1}{a}}^{a}\frac{\pi}{t}=\pi\left(Ln\,a-Ln\left(\frac{1}{a}\right)\right)=2\pi Lna$$

$$f(a)=\pi Lna$$

Now $g(a)=\int_{Ln\frac{1}{a}}^{Ln\,a}\left(\begin{array}{c}\frac{|x^2-3x+2|-|(x+1)(x+2)|}{|x+1|+|x-1|}+1\\ \text{odd funciton i.e } f(x)=-f(x)\end{array}\right)dx$

$$g(a)=\int_{Ln\left(\frac{1}{a}\right)}^{\ln a}1.dx=\ln a-\ln\left(\frac{1}{a}\right)=2\ln a.$$

Now $f(200)\frac{\pi}{2}-g(50)=\pi\ln(200)-\pi\ln(50)=\pi\ln 4=3.\frac{\pi}{3}\ln 4.$

45&46. $\int_{0}^{100\pi}\left(\left[\cot^{-1}x\right]+\left[\mathrm{Tan}^{-1}x\right]\right)dx$

$$=\cot 1+100\pi-\tan 1$$

$$=100\pi+\frac{1-\tan^2 1}{\tan^2 1}$$

$$=100\pi+2\cot 2$$

47&48. $s_1=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\cos^2 x=\frac{\pi}{8}$

$$s_2=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\cos^2 x.|4x-\pi|\,dx=\frac{\pi^2}{32.}$$

49. We know that $\frac{1}{e}+\frac{1}{f(e)}=1; f(e)=\frac{e}{e-1}$

50. $f(x)=\left(7\tan^6 x-3\tan^2 x\right).\sec^2 x$

$$\therefore \int_0^{\pi/4} f(x)\,dx=\int_0^1\left(7t^6-3t^2\right)dt=\left(t^7-t^3\right)_0^1=0$$

Now $\int_0^{\pi/4} x\,f(x)\,dx=\int_0^1\left(7t^6-3t^2\right)\tan^{-1} t\,dt$

$$=\left(\tan^{-1} t.\left(t^7-t^3\right)\right)_0^1-\int_0^1\left(t^7-t^3\right)\frac{1}{1+t^2}dt$$

$$=\int_0^1\frac{t^3\left(1-t^4\right)}{1+t^2}dt=\int_0^1 t^3\left(1-t^2\right)dt=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

51. $f(2-x)=f(2+x), f(4-x)=f(4+x)$

$$f(4+x)=f(4-x)=f(2+2-x)=f(2-(2-x))=f(x)$$

$\therefore$ 4 is a period of f (x)

$$\int_0^{50} f(x)\,dx=\int_0^{48} f(x)\,dx+\int_0^2 f(x)\,dx=12\int_0^4 f(x)\,dx+\int_0^2 f(x)\,dx$$

$$=12\left(\int_0^2 f(x)\,dx+\int_0^2 f(4-x)\,dx\right)+5$$

$$=12\left(\int_0^2 f(x)\,dx+\int_0^2 f(4+x)\,dx\right)+5$$

$$=24\int_0^2 f(x)\,dx+5=125$$

52. $u_1=\frac{\pi}{2}, u_2=2.\frac{\pi}{2}, u_3=3.\frac{\pi}{2}......$

53. $\sqrt{x}=t$

$$I_1=2\int_0^{102}\{t\},$$

$$x^2=t$$

$$I_2=\int_0^{102}\frac{\{t\}}{2}dt$$

$$\therefore \frac{I_1}{I_2}=4, I_1=100$$

54. $I_n=\int_0^{\pi}\left(\frac{\sin nx}{\left(1+\pi^x\right)\sin x}+\frac{\pi^x \sin x}{\left(1+\pi^x \sin x\right)}\right)dx$

$$=\int_0^{\pi}\frac{\sin nx}{\sin x}dx$$

$$I_{n+2}-I_2=0,\ I_1=\pi, I_2=0$$

55. We have $f(x)=x^3-\frac{3x^2}{2}+x+\frac{1}{4}; I=\int_{1/4}^{3/4} f(f(x))dx=\int_{1/4}^{3/4} f(f(1-x))dx$

Now, $f(1-x)=(1-x)^3-\frac{3}{2}(1-x)^2+1-x+\frac{1}{4}$

$=1-x^3-3x+3x^2-\frac{3}{2}(1+x^2-2x)+1-x+\frac{1}{4}$

$f(x)+f(1-x)=1------(1)$

$\Rightarrow f(f(x))+f(1-f(x))=1-----(2)(x\to f(x))$

Now $I=\int_{1/4}^{3/4} f(f(x))dx-----(3)$

56. $\int_{-3}^{5} g(x)dx=\int_{-3}^{5} f(x-1)dx+\int_{-3}^{5} f(x+1)dx$

$=2\frac{1}{2}(2)(1)=2$ (from the graphs)

57. $\int_0^2 \sqrt[3]{x^2+2x}\ dx=\int_0^2\left((x+1)^2-1\right)^{1/3} dx$

$x+1=y\Rightarrow dx=dy$

$\Rightarrow \int_1^3 \left(y^2-1\right)^{1/3} dy$

$\therefore \text{Req.}=bf(b)-a.f(a)=2.3-0.1=6$