

**JEE-MAIN EXAMINATION – APRIL 2025**(HELD ON WEDNESDAY 2<sup>nd</sup> APRIL 2025)

TIME : 3:00 PM TO 6:00 PM

**MATHEMATICS****TEST PAPER WITH SOLUTION****SECTION-A**

1. If the image of the point  $P(1, 0, 3)$  in the line joining the points  $A(4, 7, 1)$  and  $B(3, 5, 3)$  is  $Q(\alpha, \beta, \gamma)$ , then  $\alpha + \beta + \gamma$  is equal to

- (1)  $\frac{47}{3}$       (2)  $\frac{46}{3}$   
 (3) 18      (4) 13

**Ans.** (2)**Sol.**  $P(1, 0, 3)$  $A(4, 7, 1), B(3, 5, 3)$ 

$$\text{Line } AB \Rightarrow \frac{x-3}{1} = \frac{y-5}{2} = \frac{z-3}{-2} = \lambda$$

Let foot of perpendicular of  $P$  on  $AB$  be

$$R \equiv (\lambda + 3, 2\lambda + 5, -2\lambda + 3)$$

$$\Rightarrow (\lambda + 3 - 1)(1) + (2\lambda + 5 - 0)(2) + (-2\lambda + 3 - 3)$$

$$(-2) = 0$$

$$\Rightarrow \lambda + 2 + 4\lambda + 10 + 4\lambda = 0$$

$$\Rightarrow \lambda = -\frac{4}{3}$$

$$\Rightarrow R \equiv \left( \frac{5}{3}, \frac{7}{3}, \frac{17}{3} \right)$$

$$Q \equiv \left( \frac{10}{3} - 1, \frac{14}{3} - 0, \frac{34}{3} - 3 \right) \equiv \left( \frac{7}{3}, \frac{14}{3}, \frac{25}{3} \right)$$

$$\Rightarrow \alpha + \beta + \gamma = \frac{7+14+25}{3} = \frac{46}{3}$$

2. Let  $f : [1, \infty) \rightarrow [2, \infty)$  be a differentiable function,

$$\text{If } 10 \int_1^x f(t) dt = 5xf(x) - x^5 - 9 \text{ for all } x \geq 1, \text{ then}$$

the value of  $f(3)$  is :

- (1) 18      (2) 32  
 (3) 22      (4) 26

**Ans.** (2)

$$\text{Sol. } 10 \frac{d}{dx} \int_1^x f(t) dt = \frac{d}{dx} (5xf(x) - x^5 - 9)$$

$$\Rightarrow 10f(x) = 5f(x) + 5x f'(x) - 5x^4$$

$$\Rightarrow f(x) + x^4 = x f'(x)$$

$$\Rightarrow y + x^4 = x \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} + y \left( -\frac{1}{x} \right) = x^3$$

$$\Rightarrow ye^{-\frac{1}{x}dx} = \int x^3 e^{-\frac{1}{x}dx} + c$$

$$\Rightarrow ye^{-\ell n|x|} = \int x^3 e^{-\ell n|x|} + c$$

$$\Rightarrow \frac{y}{|x|} = \int \frac{x^3}{|x|} + c$$

$$\Rightarrow \frac{y}{x} = \int x^2 + c$$

$$\Rightarrow \frac{y}{x} = \frac{x^3}{3} + c$$

Put  $x = 1$  in given equation

$$\Rightarrow 0 = 5f(1) - 1 - 9 \Rightarrow f(1) = 2$$

$$\Rightarrow \frac{2}{1} = \frac{1}{3} + c \Rightarrow c = \frac{5}{3}$$

$$\Rightarrow \frac{f(3)}{3} = \frac{27}{3} + \frac{5}{3}$$

$$\Rightarrow f(3) = 32$$

3. The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by  $\frac{21}{2}$ . Then the number of terms which are integers in the A.P. is :

- (1) 4      (2) 10  
 (3) 6      (4) 8

**Ans.** (1)

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Sol.  $a_2 + a_4 + \dots + a_n = 30 \quad \dots(1)$

$a_1 + a_3 + \dots + a_{n-1} = 24 \quad \dots(2)$

(1) - (2)

$(a_2 - a_1) + (a_4 - a_3) \dots (a_n - a_{n-1}) = 6$

$\Rightarrow \frac{n}{2} d = 6 \Rightarrow nd = 12$

$a_n - a_1 = (n-1)d = \frac{21}{2}$

$\Rightarrow nd - d = \frac{21}{2} \Rightarrow 12 - \frac{21}{2} = d$

$\Rightarrow d = \frac{3}{2}, n = 8$

Sum of odd terms =  $\frac{4}{2} [2a + (4-1)3] = 24$

$\Rightarrow a = \frac{3}{2}$

A.P.  $\Rightarrow \frac{3}{2}, 3, \frac{9}{2}, 6, \frac{15}{2}, 9, \frac{21}{2}, 12$

no. of integer terms = 4

4. Let  $A = \{1, 2, 3, \dots, 10\}$  and R be a relation on A such that  $R = \{(a, b) : a = 2b + 1\}$ . Let  $(a_1, a_2), (a_2, a_3), (a_3, a_4), \dots, (a_k, a_{k+1})$  be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k, for which such a sequence exists, is equal to :

(1) 6

(2) 7

(3) 5

(4) 8

Ans. (3)

Sol.  $a = 2b + 1$

$2b = a - 1$

$R = \{(3, 1), (5, 2), \dots, (99, 49)\}$

Let  $(2m + 1, m), (2\lambda - 1, \lambda)$  are such ordered pairs.

According to the condition

$m = 2\lambda - 1 \Rightarrow m = \text{odd number}$

$\Rightarrow 1^{\text{st}}$  element of ordered pair (a, b)

$a = 2(2\lambda - 1) + 1 = 4\lambda - 1$

Hence  $a \in \{3, 7, \dots, 99\}$

$\Rightarrow \lambda \in \{1, 2, \dots, 25\}$

$\Rightarrow$  set of sequence

$\left\{ (4\lambda - 1, 2\lambda - 1), (2\lambda - 1, \lambda - 1), \left( \lambda - 1, \frac{\lambda - 2}{2} \right), \dots \right\}$

2<sup>nd</sup> element of each ordered pair =  $\frac{\lambda - 2^{r-2}}{2^{r-2}}$

For maximum number of ordered pairs in such sequence

$\frac{\lambda - 2^{r-2}}{2^{r-2}} = 1 \text{ or } 2 ; 1 \leq \lambda \leq 25$

$\lambda = 2^{r-1} \text{ or } \lambda = 3 \cdot 2^{r-2}$

**Case-I :**  $\lambda = 2r - 1$

$\lambda = 2, 2^2, 2^3, 2^4$

$r = 2, 3, 4, 5$

Hence maximum value of r is 5 when  $\lambda = 16$

**Case-II :**  $\lambda = 3 \cdot 2^{r-2}$

$\lambda = 3, 6, 12, 24$

$r = 2, 3, 4, 5$

Hence maximum value of r is 5 when  $\lambda = 24$

5. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is :

(1)  $\frac{4}{\sqrt{17}}$       (2)  $\frac{\sqrt{3}}{16}$

(3)  $\frac{3}{\sqrt{19}}$       (4)  $\frac{\sqrt{5}}{7}$

Ans. (1)

Sol.  $2b = \frac{1}{4}(2ae)$

$\frac{b}{a} = \frac{e}{4}$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{e^2}{16}}$

$e^2 \left( 1 + \frac{1}{16} \right) = 1$

$e = \frac{4}{\sqrt{17}}$



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6. The line  $L_1$  is parallel to the vector  $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$  and passes through the point  $(7, 6, 2)$  and the line  $L_2$  is parallel to the vector  $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$  and passes through the point  $(5, 3, 4)$ . The shortest distance between the lines  $L_1$  and  $L_2$  is :

- (1)  $\frac{23}{\sqrt{38}}$       (2)  $\frac{21}{\sqrt{57}}$   
 (3)  $\frac{23}{\sqrt{57}}$       (4)  $\frac{21}{\sqrt{38}}$

**Ans. (1)**

**Sol.**  $L_1 : (7\hat{i} + 6\hat{j} + 2\hat{k}) + \lambda(-3\hat{i} + 2\hat{j} + 4\hat{k})$   
 $L_2 : (5\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$

Distance between skew lines

$$= \frac{(2\hat{i} + 3\hat{j} - 2\hat{k}) \cdot (2\hat{i} + 17\hat{j} - 7\hat{k})}{\sqrt{342}} \\ = \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

7. Let  $(a, b)$  be the point of intersection of the curve  $x^2 = 2y$  and the straight line  $y - 2x - 6 = 0$  in the second quadrant. Then the integral  $I = \int_a^b \frac{9x^2}{1+5^x} dx$

is equal to :

- (1) 24      (2) 27  
 (3) 18      (4) 21

**Ans. (1)**

**Sol.**  $x^2 = 2y \text{ & } y = 2x + 6$   
 $x^2 = 4x + 12$

$$x^2 - 4x - 12 = 0 \Rightarrow \begin{cases} x = 6 & \text{if } x = -2 \\ y = 18 & y = 2 \end{cases}$$

$$\therefore (6, 18) \text{ & } (-2, 2)$$

Here  $(6, 18)$  Rejected because  $(a, b)$  lies in 2<sup>nd</sup> quadrant

$$\therefore a = -2 \text{ & } b = 2$$

$$\therefore I = \int_{-2}^2 \frac{9x^2}{1+5^x} dx = \int_{-2}^2 \frac{9 \cdot 5^x \cdot x^2}{1+5^x} dx$$

$$\therefore 2I = \int_{-2}^2 9x^2 dx = 18 \int_0^2 x^2 dx = 18 \left( \frac{x^3}{3} \right)_0^2$$

$$2I = 48$$

$$\therefore I = 24$$

8. If the system of equation

$$2x + \lambda y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \mu z = 9$$

has infinitely many solutions, then  $(\lambda^2 + \mu^2)$  is equal to :

- (1) 22      (2) 18

- (3) 26      (4) 30

**Ans. (3)**

**Sol.**  $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$

$$\Rightarrow 2(2\mu + 5) + \lambda(-4 - 3\mu) + 3(7) = 0$$

$$\Rightarrow 4\mu - 3\lambda\mu - 4\lambda + 31 = 0 \dots\dots(1)$$

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0$$

$$\Rightarrow 2(-17) + \lambda(1) + 5(7) = 0$$

$$\Rightarrow \lambda = -1$$

from equation (1)

$$4\mu + 3\mu + 4 + 31 = 0 \Rightarrow \boxed{\mu = -5}$$

$$\therefore \boxed{\lambda^2 + \mu^2 = 26}$$

9. If  $\theta \in \left[-\frac{7\pi}{6}, \frac{4\pi}{3}\right]$ , then the number of solutions of

$$\sqrt{3} \cosec^2 \theta - 2(\sqrt{3}-1) \cosec \theta - 4 = 0, \text{ is equal to}$$

- (1) 6      (2) 8

- (3) 10      (4) 7

**Ans. (1)**

**Sol.**  $\cosec \theta = \frac{2(\sqrt{3}-1) \pm \sqrt{4(3+1-2\sqrt{3})+16\sqrt{3}}}{2\sqrt{3}}$   
 $= \frac{2(\sqrt{3}-1) \pm \sqrt{16+8\sqrt{3}}}{2\sqrt{3}}$   
 $= \frac{2(\sqrt{3}-1) \pm (2+2\sqrt{3})}{2\sqrt{3}}$



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13.  $4 \int_0^1 \left( \frac{1}{\sqrt{3+x^2} + \sqrt{1+x^2}} \right) dx - 3 \log_e (\sqrt{3})$  is equal to :

(1)  $2 + \sqrt{2} + \log_e (1+\sqrt{2})$

(2)  $2 - \sqrt{2} - \log_e (1+\sqrt{2})$

(3)  $2 + \sqrt{2} - \log_e (1+\sqrt{2})$

(4)  $2 - \sqrt{2} + \log_e (1+\sqrt{2})$

**Ans. (2)**

$$\begin{aligned} \text{Sol. } & 4 \int_0^1 \frac{1}{\sqrt{3+x^2} + \sqrt{1+x^2}} dx - 3 \ln \sqrt{3} \\ &= 4 \int_0^1 \frac{\sqrt{3+x^2} - \sqrt{1+x^2}}{(3+x^2)-(1-x^2)} dx - \frac{3}{2} \ln 3 \\ &= 2 \left[ \left\{ \frac{x}{2} \sqrt{3+x^2} + \frac{3}{2} \ln \left( x + \sqrt{3+x^2} \right) \right\}_0^1 \right. \\ &\quad \left. - \left\{ \frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \ln(x + \sqrt{1+x^2}) \right\}_0^1 \right] - \frac{3}{2} \ln 3 \\ &= 2 \left[ \left\{ \frac{1}{2} \sqrt{4} + \frac{3}{2} \ln(1+\sqrt{4}) \right\} - \left\{ 0 + \frac{3}{2} \ln \sqrt{3} \right\} \right. \\ &\quad \left. - \left\{ \frac{1}{2} \sqrt{2} + \frac{1}{2} \ln(1+\sqrt{2}) \right\} + \left\{ 0 + \frac{1}{2}(0) \right\} \right] - \frac{3}{2} \ln 3 \\ &= 2 \left[ 1 + \frac{3}{2} \ln 3 - \frac{3}{4} \ln 3 - \frac{1}{\sqrt{2}} - \frac{1}{2} \ln(1+\sqrt{2}) \right] - \frac{3}{2} \ln 3 \\ &= 2 + 3 \ln 3 - \frac{3}{2} \ln 3 - \sqrt{2} - \ln(1+\sqrt{2}) - \frac{3}{2} \ln 3 \\ &= 2 - \sqrt{2} - \ln(1+\sqrt{2}) \end{aligned}$$

14. If  $\lim_{x \rightarrow 0} \frac{\cos(2x) + a \cos(4x) - b}{x^4}$  is finite, then  $(a+b)$  is equal to :

(1)  $\frac{1}{2}$  (2) 0

(3)  $\frac{3}{4}$  (4) -1

**Ans. (1)**

**Sol.**  $\lim_{x \rightarrow 0} \frac{\cos 2x + a \cos 4x - b}{x^4} = \text{finite}$

$$L = \frac{\left\{ 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{4} \dots \right\} + a \left\{ 1 - \frac{(4x)^2}{2} + \frac{(4x)^4}{4} \dots \right\} - b}{x^4}$$

$$L = \frac{(1+a-b) - x^2(2+8a) + x^4 \left( \frac{2}{3} + \frac{32}{3}a \right) + x^6 \dots}{x^4}$$

$$\therefore 1 + a - b = 0 \text{ and } 2 + 8a = 0 \Rightarrow a = -\frac{1}{4}$$

$$b = a + 1$$

$$= -\frac{1}{4} + 1 = \frac{3}{4}$$

$$\therefore a + b = -\frac{1}{4} + \frac{3}{4} = \frac{1}{2}$$

15. If  $\sum_{r=0}^{10} \left( \frac{10^{r+1} - 1}{10^r} \right) \cdot {}^{11}C_{r+1} = \frac{\alpha^{11} - 11^{11}}{10^{10}}$ , then  $\alpha$  is

equal to :

- (1) 15 (2) 11  
(3) 24 (4) 20

**Ans. (4)**

$$\begin{aligned} \text{Sol. } & \sum_{r=0}^{10} \left( \frac{10^{r+1} - 1}{10^r} \right) {}^{11}C_{r+1} \\ &= \sum_{r=0}^{10} \left( 10 - \frac{1}{10^r} \right) {}^{11}C_{r+1} \\ &= 10 \sum_{r=0}^{10} {}^{11}C_{r+1} - 10 \sum \left( {}^{11}C_{r+1} \left( \frac{1}{10} \right)^{r+1} \right) \\ &= 10 \left[ {}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_{11} \right] \\ &\quad - 10 \left[ {}^{11}C_1 \left( \frac{1}{10} \right)^1 + {}^{11}C_2 \left( \frac{1}{10} \right)^2 + \dots + {}^{11}C_{11} \left( \frac{1}{10} \right)^{11} \right] \end{aligned}$$

$$= 10 \left[ 2^{11} - 1 \right] - 10 \left[ \left( 1 + \frac{1}{10} \right)^{11} - 1 \right]$$

$$= 10(2)^{11} - 10 - \frac{11^{11}}{10^{10}} + 10$$

$$= \frac{(20)^{11} - 11^{11}}{10^{10}}$$

$$\therefore \alpha = 20$$

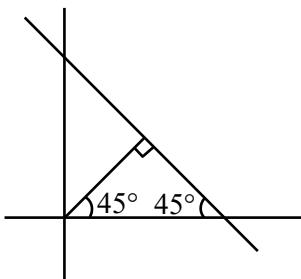


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Sol.  $\frac{x}{-c} + \frac{y}{-c/b} = 1$



$$\therefore \text{area of triangle} = \frac{1}{2} \left| \frac{c^2}{b} \right| = 48$$

$$\left| \frac{c^2}{b} \right| = 96$$

$$\therefore -c = -\frac{c}{b}$$

$$\Rightarrow b = 1 \quad \therefore c^2 = 96$$

$$\therefore b^2 + c^2 = 97$$

20. Let  $A$  be a  $3 \times 3$  real matrix such that  $A^2(A - 2I) - 4(A - I) = O$ , where  $I$  and  $O$  are the identity and null matrices, respectively. If  $A^5 = \alpha A^2 + \beta A + \gamma I$ , where  $\alpha, \beta$  and  $\gamma$  are real constants, then  $\alpha + \beta + \gamma$  is equal to:

(1) 12

(2) 20

(3) 76

(4) 4

Ans. (1)

Sol.  $A^3 - 2A^2 - 4A + 4I = 0$

$$A^3 = 2A^2 + 4A - 4I$$

$$A^4 = 2A^3 + 4A^2 - 4A$$

$$= 2(2A^2 + 4A - 4I) + 4A^2 - 4A$$

$$A^4 = 8A^2 + 4A - 8I$$

$$A^5 = 8A^3 + 4A^2 - 8A$$

$$= 8(2A^2 + 4A - 4I) + 4A^2 - 8A$$

$$A^5 = 20A^2 + 24A - 32I$$

$$\therefore \alpha = 20, \beta = 24, \gamma = -32$$

$$\therefore \alpha + \beta + \gamma = 12$$

## SECTION-B

21. Let  $y = y(x)$  be the solution of the differential equation  $\frac{dy}{dx} + 2y \sec^2 x = 2\sec^2 x + 3\tan x \cdot \sec^2 x$  such that  $y(0) = \frac{5}{4}$ . Then  $12 \left( y\left(\frac{\pi}{4}\right) - e^{-2} \right)$  is equal to \_\_\_\_\_.

Ans. (21)

Sol. I.F.  $= e^{\int 2\sec^2 x dx}$   
 $= e^{2\tan x}$

Solution of diff. eq.

$$y \cdot e^{2\tan x} = \int e^{2\tan x} (2\sec^2 x + 3\tan x \cdot \sec^2 x) dx$$

$$y \cdot e^{2\tan x} = \int e^{2\tan x} \cdot (2\sec^2 x) dx + \int e^{2\tan x} \cdot (3\tan x \cdot \sec^2 x) dx$$

$$y \cdot e^{2\tan x} = e^{2\tan x} \cdot 2\tan x - \int e^{2\tan x} \cdot 2\sec^2 x \cdot 2\tan x dx + \int e^{2\tan x} \cdot 3\tan x \cdot \sec^2 x dx$$

$$y \cdot e^{2\tan x} = 2\tan x \cdot e^{2\tan x} - \int e^{2\tan x} \cdot \tan x \sec^2 x dx$$

$$y \cdot e^{2\tan x} = 2\tan x \cdot e^{2\tan x} - \frac{\tan x \cdot e^{2\tan x}}{2} + \frac{e^{2\tan x}}{4} + C$$

$$y = 2\tan x - \frac{\tan x}{2} + \frac{1}{4} + Ce^{-2\tan x}$$

$$x = 0, y = \frac{5}{4}$$

$$c = 1$$

$$y\left(\frac{\pi}{4}\right) = \frac{7}{4} + e^{-2}$$

$$\text{Then } 12 \left( y\left(\frac{\pi}{4}\right) - e^{-2} \right) = 12 \left( \frac{7}{4} \right) = 21$$

22. If the sum of the first 10 terms of the series  $\frac{4.1}{1+4.1^4} + \frac{4.2}{1+4.2^4} + \frac{4.3}{1+4.3^4} + \dots$  is  $\frac{m}{n}$ , where  $\gcd(m, n) = 1$ , then  $m + n$  is equal to \_\_\_\_\_.

Ans. (441)

Sol.  $T_r = \frac{4.r}{1+4.r^4}$

$$T_r = \frac{4.r}{(2r^2 + 2r + 1)(2r^2 - 2r + 1)}$$

$$T_r = \frac{(2r^2 + 2r + 1) - (2r^2 - 2r + 1)}{(2r^2 + 2r + 1)(2r^2 - 2r + 1)}$$



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$$T_r = \frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1}$$

$$T_1 = \frac{1}{1} - \frac{1}{5}$$

$$T_2 = \frac{1}{5} - \frac{1}{13}$$

⋮

$$T_{10} = \frac{1}{181} - \frac{1}{221}$$

$$S_{10} = 1 - \frac{1}{221} = \frac{220}{221} = \frac{m}{n}$$

$$m + n = 441$$

23. If  $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$ , then  $(x - y)^2 + 3y^2$  is equal to \_\_\_\_\_.

**Ans. (3)**

$$\text{Sol. } y = \cos\left(\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{x}{2}\right)$$

$$y = \frac{1}{2} \times \frac{x}{2} - \sqrt{1 - \frac{1}{4}} \sqrt{1 - \frac{x^2}{4}}$$

$$4y = x - \sqrt{3} \sqrt{4 - x^2}$$

$$3(4 - x^2) = x^2 + 16y^2 - 8xy$$

$$12 - 3x^2 = x^2 + 16y^2 - 8xy$$

$$4x^2 + 16y^2 - 8xy = 12$$

$$x^2 + 4y^2 - 2xy = 3$$

$$x^2 + y^2 - 2xy - 3y^2 = 3$$

$$(x - y)^2 + 3y^2 = 3$$

24. Let A(4, -2), B(1, 1) and C(9, -3) be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is \_\_\_\_\_.

**Ans. (3)**

$$\text{Sol. Area of } \triangle ABC = \frac{1}{2} \begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix}$$

$$= 6 \text{ square units}$$

$$\text{Maximum area of AFDE} = \frac{1}{2} \times 6 = 3 \text{ sq. units}$$

25. If the set of all  $a \in \mathbb{R} - \{1\}$ , for which the roots of the equation  $(1 - a)x^2 + 2(a - 3)x + 9 = 0$  are positive is  $(-\infty, -\alpha] \cup [\beta, \gamma)$ , then  $2\alpha + \beta + \gamma$  is equal to \_\_\_\_\_.

**Ans. (7)**

**Sol.** Both the roots are positive

$$D \geq 0$$

$$4(a - 3)^2 - 4 \times 9(1 - a) \geq 0$$

$$a^2 - 6a + 9 - 9 + 9a \geq 0$$

$$a^2 + 3a \geq 0$$

$$a(a + 3) \geq 0$$

$$a \in (-\infty, -3] \cup [0, \infty) \quad \dots\dots(i)$$

$$-\frac{b}{2a} > 0$$

$$\frac{2(a-3)}{2(a-1)} > 0$$

$$a \in (-\infty, 1) \cup (3, \infty) \quad \dots\dots(ii)$$

$$f(0) = 9 > 0$$

Equation (i)  $\cap$  (ii)

$$a \in (-\infty, -3] \cup [0, 1)$$

$$2\alpha + \beta + \gamma - 6 + 0 + 1 = 7$$



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