

# FINAL JEE-MAIN EXAMINATION – APRIL, 2024

(Held On Tuesday 09<sup>th</sup> April, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

## MATHEMATICS

## TEST PAPER WITH SOLUTION

### SECTION-A

1. Let the line L intersect the lines

$$x - 2 = -y = z - 1, 2(x + 1) = 2(y - 1) = z + 1$$

$$\text{and be parallel to the line } \frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}.$$

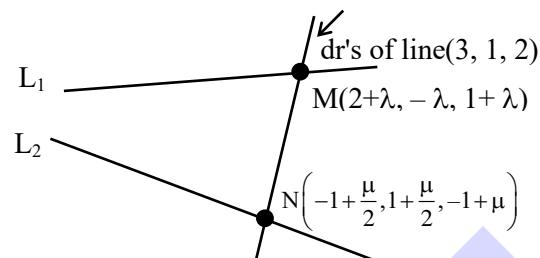
Then which of the following points lies on L ?

$$(1) \left(-\frac{1}{3}, 1, 1\right) \quad (2) \left(-\frac{1}{3}, 1, -1\right)$$

$$(3) \left(-\frac{1}{3}, -1, -1\right) \quad (4) \left(-\frac{1}{3}, -1, 1\right)$$

**Ans. (2)**

**Sol.**



$$L_1: \frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{1} = \lambda$$

$$L_2: \frac{x+1}{2} = \frac{y-1}{2} = \frac{z+1}{1} = \mu$$

dr of line MN will be

$$\langle 3+\lambda-\frac{\mu}{2}, -1-\lambda-\frac{\mu}{2}, 2+\lambda-\mu \rangle \text{ & it will be}$$

proportional to  $\langle 3, 1, 2 \rangle$

$$\therefore \frac{3+\lambda-\frac{\mu}{2}}{3} = \frac{-1-\lambda-\frac{\mu}{2}}{1} = \frac{2+\lambda-\mu}{2}$$

$$4\lambda + \mu = -6$$

$$4 + 3\lambda = 0$$

$$\Rightarrow \lambda = -\frac{4}{3} \text{ & } \mu = -\frac{2}{3}$$

$\therefore$  Coordinate of M will be  $\left(\frac{2}{3}, \frac{4}{3}, -\frac{1}{3}\right)$

and equation of required line will be.

$$\frac{x-\frac{2}{3}}{3} = \frac{y-\frac{4}{3}}{1} = \frac{z+\frac{1}{3}}{2} = k$$

So any point on this line will be

$$\left(\frac{2}{3} + 3k, \frac{4}{3} + k, -\frac{1}{3} + 2k\right)$$

$$\therefore \frac{2}{3} + 3k = -\frac{1}{3} \Rightarrow k = -\frac{1}{3}$$

$\therefore$  Point lie on the line for

$$k = -\frac{1}{3} \text{ is } \left(-\frac{1}{3}, 1, -1\right)$$

2. The parabola  $y^2 = 4x$  divides the area of the circle  $x^2 + y^2 = 5$  in two parts. The area of the smaller part is equal to :

$$(1) \frac{2}{3} + 5 \sin^{-1} \left( \frac{2}{\sqrt{5}} \right) \quad (2) \frac{1}{3} + 5 \sin^{-1} \left( \frac{2}{\sqrt{5}} \right)$$

$$(3) \frac{1}{3} + \sqrt{5} \sin^{-1} \left( \frac{2}{\sqrt{5}} \right) \quad (4) \frac{2}{3} + \sqrt{5} \sin^{-1} \left( \frac{2}{\sqrt{5}} \right)$$

**Ans. (1)**

**Sol.**  $y^2 = 4x$

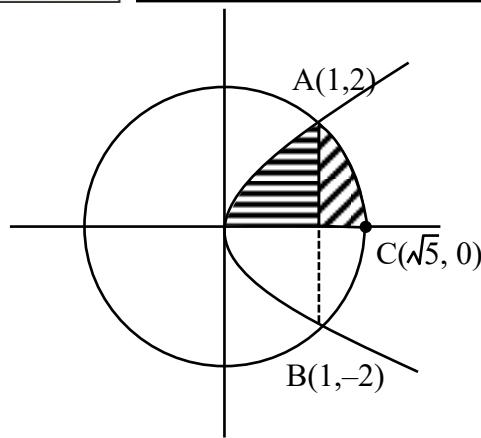
$$x^2 + y^2 = 5$$

$\therefore$  Area of shaded region as shown in the figure will be



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$$\begin{aligned}
A_1 &= \int_0^1 \sqrt{4x} dx + \int_1^{\sqrt{5}} \sqrt{5-x^2} dx \\
&= \frac{4}{3} \left[ x^{\frac{3}{2}} \right]_0^1 + \left[ \frac{x}{2} \sqrt{5-x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_1^{\sqrt{5}} \\
&= \frac{1}{3} + \frac{5\pi}{4} - \frac{5}{2} \sin^{-1} \left( \frac{1}{\sqrt{5}} \right)
\end{aligned}$$

∴ Required Area = 2 A<sub>1</sub>

$$\begin{aligned}
&= \frac{2}{3} + \frac{5\pi}{2} - 5 \sin^{-1} \left( \frac{1}{\sqrt{5}} \right) \\
&= \frac{2}{3} + 5 \left( \frac{\pi}{2} - \sin^{-1} \frac{1}{\sqrt{5}} \right) \\
&= \frac{2}{3} + 5 \cos^{-1} \frac{1}{\sqrt{5}} \\
&= \frac{2}{3} + 5 \sin^{-1} \left( \frac{2}{\sqrt{5}} \right)
\end{aligned}$$

3. The solution curve, of the differential equation

$$2y \frac{dy}{dx} + 3 = 5 \frac{dy}{dx}$$
, passing through the point

- (0, 1) is a conic, whose vertex lies on the line :  
(1)  $2x + 3y = 9$       (2)  $2x + 3y = -9$   
(3)  $2x + 3y = -6$       (4)  $2x + 3y = 6$

Ans. (1)

Sol.  $(2y-5) \frac{dy}{dx} = -3$

$$(2y-5)dy = -3dx$$

$$2 \cdot \frac{y^2}{2} - 5y = -3x + \lambda$$

∴ Curve passes through (0, 1)

$$\Rightarrow \lambda = -4$$

∴ Curve will be

$$\left( y - \frac{5}{2} \right)^2 = -3 \left( x - \frac{3}{4} \right)$$

$$\therefore \text{Vertex of parabola will be } \left( \frac{3}{4}, \frac{5}{2} \right)$$

$$\therefore 2x + 3y = 9$$

4. A ray of light coming from the point P (1, 2) gets reflected from the point Q on the x-axis and then passes through the point R (4, 3). If the point S (h, k) is such that PQRS is a parallelogram, then hk<sup>2</sup> is equal to :

- (1) 80      (2) 90  
(3) 60      (4) 70

Ans. (4)

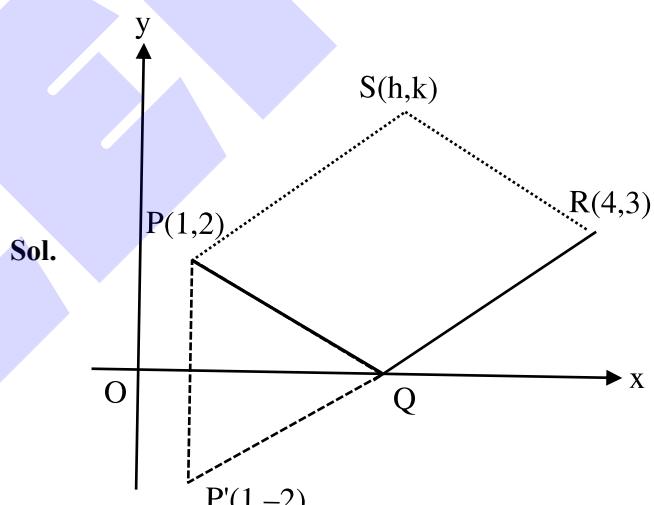


Image of P wrt x-axis will be P'(1, -2) equation of line joining P'R will be

$$y - 3 = \frac{5}{3}(x - 4)$$

Above line will meet x-axis at Q where

$$y = 0 \Rightarrow x = \frac{11}{5}$$

$$\therefore Q \left( \frac{11}{5}, 0 \right)$$

∴ PQRS is parallelogram so their diagonals will bisects each other



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$$\Rightarrow \frac{4+1}{2} = \frac{\frac{11}{5} + h}{2} \quad \& \quad \frac{2+3}{2} = \frac{k+0}{2}$$

$$\Rightarrow h = \frac{14}{5} \quad \& \quad k = 5$$

$$\therefore hk^2 = \frac{14}{5} \times 5^2 = 70$$

5. Let  $\lambda, \mu \in \mathbb{R}$ . If the system of equations

$$3x + 5y + \lambda z = 3$$

$$7x + 11y - 9z = 2$$

$$97x + 155y - 189z = \mu$$

has infinitely many solutions, then  $\mu + 2\lambda$  is equal to :

- |        |        |
|--------|--------|
| (1) 25 | (2) 24 |
| (3) 27 | (4) 22 |

**Ans. (1)**

$$\text{Sol. } 3x + 5y + \lambda z = 3$$

$$7x + 11y - 9z = 2$$

$$97x + 155y - 189z = \mu$$

$$93x + 155y + 31\lambda z = 93$$

$$97x + 155y - 189z = \mu$$

$$\begin{array}{r} - \\ - \\ \hline -4x + (31\lambda + 189)z = 93 - \mu \end{array}$$

$$1085x + 1705y - 1395z = 310$$

$$1067x + 1705y - 2079z = 11\mu$$

$$\begin{array}{r} - \\ - \\ \hline 18x + 684z = 310 - 11\mu \end{array}$$

$$-36x + 9(31\lambda + 189)z = 9(93 - \mu)$$

$$36x + 1368z = 2(310 - 11\mu)$$

$$(279\lambda + 3069)z = 1457 - 31\mu$$

for infinite solutions -

$$\lambda = \frac{-3069}{279} = \frac{-341}{31}$$

$$\mu = \frac{1457}{31}$$

$$\mu + 2\lambda = \frac{1457 - 682}{31} = \frac{775}{31} = 25$$

6. The coefficient of  $x^{70}$  in  $x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + \dots + x^{54}(1+x)^{46}$  is  ${}^{99}C_p - {}^{46}C_q$ .

Then a possible value to  $p + q$  is :

- |        |        |
|--------|--------|
| (1) 55 | (2) 61 |
| (3) 68 | (4) 83 |

**Ans. (4)**

$$\text{Sol. } x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + \dots$$

$$x^{54}(1+x)^{46}$$

$$\text{Coeff. of } x^{70} : {}^{98}C_{68} + {}^{97}C_{67} + {}^{96}C_{66} + \dots$$

$$+ {}^{47}C_{17} + {}^{46}C_{16}$$

$$= {}^{46}C_{30} + {}^{47}C_{30} + \dots {}^{98}C_{30}$$

$$= ({}^{46}C_{31} + {}^{46}C_{30}) + {}^{47}C_{30} + \dots {}^{98}C_{30} - {}^{46}C_{31}$$

$$= {}^{47}C_{31} + {}^{47}C_{30} + \dots {}^{98}C_{30} - {}^{46}C_{31}$$

....

$$= {}^{99}C_{31} - {}^{46}C_{31} = {}^{99}C_p - {}^{46}C_q$$

Possible values of  $(p + q)$  are 62, 83, 99, 46

$$\Rightarrow p + q = 83$$

7. Let

$$\int \frac{2 - \tan x}{3 + \tan x} dx = \frac{1}{2} (\alpha x + \log_e |\beta \sin x + \gamma \cos x|) + C$$

, where C is the constant of integration.

Then  $\alpha + \frac{\gamma}{\beta}$  is equal to :

- |       |       |
|-------|-------|
| (1) 3 | (2) 1 |
| (3) 4 | (4) 7 |

**Ans. (3)**

$$\text{Sol. } \int \frac{2 - \tan x}{3 + \tan x} dx = \int \frac{2 \cos x - \sin x}{3 \cos x + \sin x} dx$$

$$2 \cos x - \sin x = A(3 \cos x + \sin x) + B(\cos x - 3 \sin x)$$

$$3A + B = 2$$

$$A - 3B = -1$$



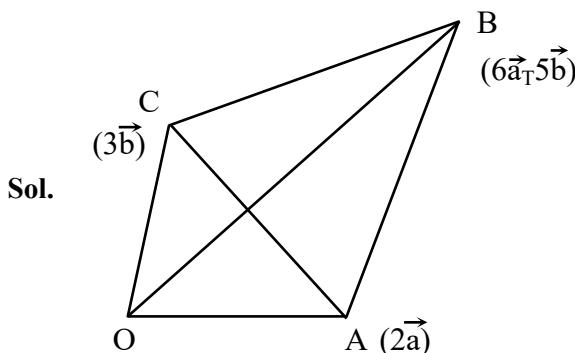
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10. Let  $\overrightarrow{OA} = 2\vec{a}$ ,  $\overrightarrow{OB} = 6\vec{a} + 5\vec{b}$  and  $\overrightarrow{OC} = 3\vec{b}$ , where O is the origin. If the area of the parallelogram with adjacent sides  $\overrightarrow{OA}$  and  $\overrightarrow{OC}$  is 15 sq. units, then the area (in sq. units) of the quadrilateral OABC is equal to :  
 (1) 38      (2) 40  
 (3) 32      (4) 35

**Ans. (4)**



**Sol.**

Area of parallelogram having sides

$$\overrightarrow{OA} \text{ & } \overrightarrow{OC} = |\overrightarrow{OA} \times \overrightarrow{OC}| = |2\vec{a} \times 3\vec{b}| = 15$$

$$6|\vec{a} \times \vec{b}| = 15$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \frac{5}{2} \dots\dots\dots(1)$$

Area of quadrilateral

$$OABC = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

$$= \frac{1}{2} |\overrightarrow{AC} \times \overrightarrow{OB}| = \frac{1}{2} |(3\vec{b} - 2\vec{a}) \times (6\vec{a} + 5\vec{b})|$$

$$= \frac{1}{2} |18\vec{b} \times \vec{a} - 10\vec{a} \times \vec{b}| = 14 |\vec{a} \times \vec{b}|$$

$$= 14 \times \frac{5}{2} = 35$$

11. If the domain of the function

$$f(x) = \sin^{-1} \left( \frac{x-1}{2x+3} \right)$$

then  $12\alpha\beta$  is equal to :

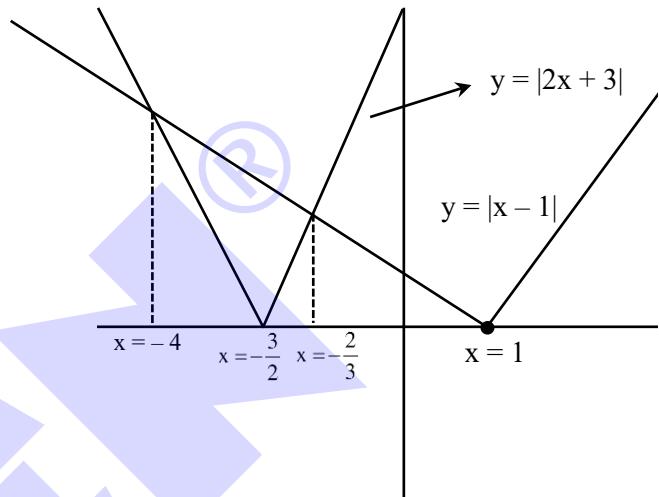
- (1) 36      (2) 24  
 (3) 40      (4) 32

**Ans. (4)**

**Sol.** Domain of  $f(x) = \sin^{-1} \left( \frac{x-1}{2x+3} \right)$  is

$$2x+3 \neq 0 \text{ & } x \neq \frac{-3}{2} \text{ and } \left| \frac{(x-1)}{2x+3} \right| \leq 1$$

$$|x-1| \leq |2x+3|$$



For  $|2x+3| \geq |x-1|$

$$x \in (-\infty, -4] \cup \left( -\frac{2}{3}, \infty \right)$$

$$\alpha = -4 \text{ & } \beta = -\frac{2}{3} : 12\alpha\beta = 32$$

12. If the sum of series

$$\frac{1}{1 \cdot (1+d)} + \frac{1}{(1+d)(1+2d)} + \dots + \frac{1}{(1+9d)(1+10d)}$$

is equal to 5, then  $50d$  is equal to :

- (1) 20      (2) 5  
 (3) 15      (4) 10

**Ans. (2)**

$$\frac{1}{1 \cdot (1+d)} + \frac{1}{(1+d)(1+2d)} + \dots + \frac{1}{(1+9d)(1+10d)} = 5$$

$$\frac{1}{(1+9d)(1+10d)} = 5$$



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$$\Rightarrow \int \frac{dt}{(-8)(t)} = \int \frac{dx}{5x}$$

$$\Rightarrow \frac{-1}{8} \ln|t| = \frac{1}{5} \ln|x| + \ln C$$

$$\Rightarrow -5 \ln|t| = 8 \ln|x| + \ln K$$

$$\Rightarrow \ln x^8 + \ln|t^5| + \ln K = 0$$

$$\Rightarrow x^8 |t^5| = C$$

$$\Rightarrow x^8 |1 - 4V^2|^5 = C$$

$$\Rightarrow x^8 \left| \frac{x^2 - 4y^2}{x^2} \right|^5 = C$$

$$\Rightarrow |x^2 - 4y^2|^5 = Cx^2$$

given  $y(1) = 0$

$$\Rightarrow |1|^5 = C \Rightarrow C = 1$$

$$\Rightarrow |x^2 - 4y^2|^5 = x^2$$

18. Let three vectors  $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}$ ,  $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$  from a triangle such that  $\vec{c} = \vec{a} - \vec{b}$  and the area of the triangle is  $5\sqrt{6}$ . if  $\alpha$  is a positive real number, then  $|\vec{c}|^2$  is :

(1) 16  
(3) 12

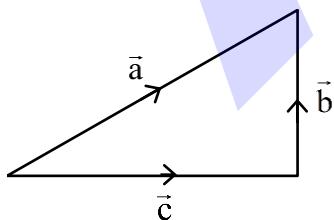
(2) 14  
(4) 10

**Ans. (2)**

**Sol.**  $\vec{c} = \vec{a} - \vec{b}$

$$\Rightarrow (x, y, z) = (\alpha - 5, 1, -2)$$

$$\Rightarrow x = \alpha - 5, y = 1, z = -2 \quad \dots\dots\dots(1)$$



Area of  $\Delta = 5\sqrt{6}$  (given)

$$\frac{1}{2} |\vec{a} \times \vec{c}| = 5\sqrt{6}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 4 & 2 \\ x & 1 & -2 \end{vmatrix} = 10\sqrt{6}$$

$$\Rightarrow |-10\hat{i} - \hat{j}(-2\alpha - 2x) + \hat{k}(\alpha - 4x)| = 10\sqrt{6}$$

$$\Rightarrow (2\alpha + 2\alpha - 10)^2 + (\alpha - 4\alpha + 20)^2 = 500$$

$$\Rightarrow (4\alpha - 10)^2 + (20 - 3\alpha)^2 = 500$$

$$\Rightarrow 25\alpha^2 - 80\alpha - 120\alpha = 0$$

$$\Rightarrow \alpha(25\alpha - 200) = 0$$

$$\Rightarrow \alpha = 8 \text{ (given } \alpha \text{ is +ve number)}$$

$$\Rightarrow x = \alpha - 5 = 3$$

$$|\vec{c}|^2 = x^2 + y^2 + z^2$$

$$= 9 + 1 + 4$$

$$= 14$$

19. Let  $\alpha, \beta$  be the roots of the equation

$$x^2 + 2\sqrt{2}x - 1 = 0. \text{ The quadratic equation,}$$

whose roots are  $\alpha^4 + \beta^4$  and  $\frac{1}{10}(\alpha^6 + \beta^6)$ , is :

- (1)  $x^2 - 190x + 9466 = 0$   
(2)  $x^2 - 195x + 9466 = 0$   
(3)  $x^2 - 195x + 9506 = 0$   
(4)  $x^2 - 180x + 9506 = 0$

**Ans. (3)**

**Sol.**  $x^2 + 2\sqrt{2}x - 1 = 0$

$$\alpha + \beta = -2\sqrt{2}$$

$$\alpha\beta = -1$$

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$$

$$= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$$

$$= (8 + 2)^2 - 2(-1)^2$$

$$= 100 - 2 = 98$$

$$\alpha^6 + \beta^6 = (\alpha^3 + \beta^3)^2 - 2\alpha^3\beta^3$$

$$= ((\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta)^2 - 2(\alpha\beta)^3$$



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$$= (-2\sqrt{2}(8+3))^2 + 2$$

$$= (8)(121) + 2 = 970$$

$$\frac{1}{10}(\alpha^6 + \beta^6) = 97$$

$$x^2 - (98 + 97)x + (98)(97) = 0$$

$$\Rightarrow x^2 - 195x + 9506 = 0$$

20. Let  $f(x) = x^2 + 9$ ,  $g(x) = \frac{x}{x-9}$  and  $a = \text{fog}(10)$ ,  $b = \text{gof}(3)$ . If  $e$  and  $l$  denote the eccentricity and the length of the latus rectum of the ellipse  $\frac{x^2}{a} + \frac{y^2}{b} = 1$ , then  $8e^2 + l^2$  is equal to.

(1) 16

(2) 8

(3) 6

(4) 12

**Ans. (2)**

**Sol.**  $f(x) = x^2 + 9$     $g(x) = \frac{x}{x-9}$

$$a = f(g(10)) = f\left(\frac{10}{10-9}\right)$$

$$= f(10) = 109$$

$$b = g(f(3)) = g(9+9)$$

$$= g(18) = \frac{18}{9} = 2$$

$$E: \frac{x^2}{109} + \frac{y^2}{2} = 1$$

$$e^2 = 1 - \frac{2}{109} = \frac{107}{109}$$

$$\ell = \frac{2(2)}{\sqrt{109}} = \frac{4}{\sqrt{109}}$$

$$8e^2 + \ell^2 = \frac{8(107)}{109} + \frac{16}{109}$$

$$= 8$$

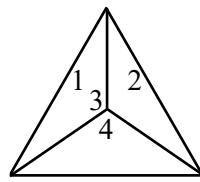
### SECTION-B

21. Let  $a$ ,  $b$  and  $c$  denote the outcome of three independent rolls of a fair tetrahedral die, whose

four faces are marked 1, 2, 3, 4. If the probability that  $ax^2 + bx + c = 0$  has all real roots is  $\frac{m}{n}$ ,  $\gcd(m, n) = 1$ , then  $m + n$  is equal to \_\_\_\_\_.

**Ans. (19)**

**Sol.**  $a, b, c \in \{1, 2, 3, 4\}$



Tetrahedral dice  
 $ax^2 + bx + c = 0$

has all real roots

$$\Rightarrow D \geq 0$$

$$\Rightarrow b^2 - 4ac \geq 0$$

Let  $b = 1 \Rightarrow 1 - 4ac \geq 0$  (Not feasible)

$$b = 2 \Rightarrow 4 - 4ac \geq 0$$

$$1 \geq ac \Rightarrow a = 1, c = 1,$$

$$b = 3 \Rightarrow 9 - 4ac \geq 0$$

$$\frac{9}{4} \geq ac$$

$$\Rightarrow a = 1, c = 1$$

$$\Rightarrow a = 1, c = 2$$

$$\Rightarrow a = 2, c = 1$$

$$b = 4 \Rightarrow 16 - 4ac \geq 0$$

$$4 \geq ac$$

$$\Rightarrow a = 1, c = 1$$

$$\Rightarrow a = 1, c = 2 \Rightarrow a = 2, c = 1$$

$$\Rightarrow a = 1, c = 3 \Rightarrow a = 3, c = 1$$

$$\Rightarrow a = 1, c = 4 \Rightarrow a = 4, c = 1$$

$$\Rightarrow a = 2, c = 2$$

$$\text{Probability} = \frac{12}{(4)(4)(4)} = \frac{3}{16} = \frac{m}{m}$$

$$m + n = 19$$



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22. The sum of the square of the modulus of the elements in the set

$$\{z = a + ib : a, b \in \mathbb{Z}, z \in \mathbb{C}, |z - 1| \leq 1, |z - 5| \leq |z - 5i|\}$$

is \_\_\_\_\_.

**Ans. (9)**

**Sol.**  $|z - 1| \leq 1$

$$\Rightarrow |(x - 1) + iy| \leq 1$$

$$\Rightarrow \sqrt{(x - 1)^2 + y^2} \leq 1$$

$$\Rightarrow (x - 1)^2 + y^2 \leq 1 \quad \dots \dots \dots (1)$$

Also  $|z - 5| \leq |z - 5i|$

$$(x - 5)^2 + y^2 \leq x^2 + (y - 5)^2$$

$$-10x \leq -10y$$

$$\Rightarrow x \geq y \quad \dots \dots \dots (2)$$

Solving (1) and (2)

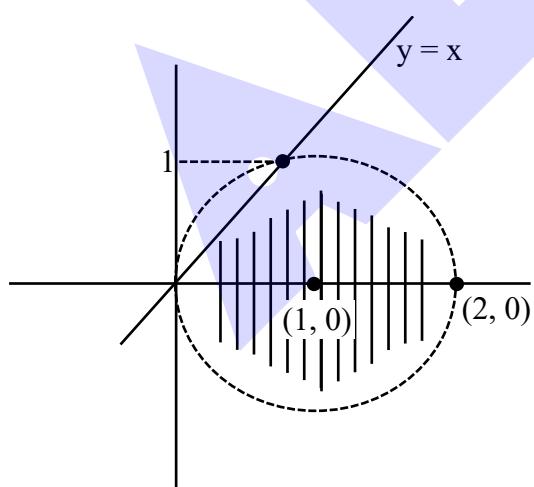
$$\Rightarrow (x - 1)^2 + x^2 = 1$$

$$\Rightarrow 2x^2 - 2x = 0$$

$$\Rightarrow x(x - 1) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 1$$

$$y = 0 \text{ or } y = 1$$



Given  $x, y \in I$

Points  $(0, 0), (1, 0), (2, 0), (1, 1), (1, -1)$  to find

$$|z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 + |z_5|^2$$

$$= 0 + 1 + 4 + 1 + 1 + 1 + 1 = 9$$

23. Let the set of all positive values of  $\lambda$ , for which the point of local minimum of the function

$$(1 + x(\lambda^2 - x^2)) \text{ satisfies } \frac{x^2 + x + 2}{x^2 + 5x + 6} < 0, \text{ be } (\alpha, \beta).$$

Then  $\alpha^2 + \beta^2$  is equal to \_\_\_\_\_.

**Ans. (39)**

**Sol.**  $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$

$$\Rightarrow \frac{1}{(x+2)(x+3)} < 0$$

$$\begin{array}{c} + \\ \text{---} \\ -3 \quad -2 \end{array}$$

$$x \in (-3, -2) \quad \dots \dots \dots (1)$$

$$f(x) = 1 + x(\lambda^2 - x^2)$$

Finding local minima

$$f'(x) = (\lambda^2 - x^2) + (-2x)x$$

$$\text{Put } f(x) = 0$$

$$\Rightarrow \lambda^2 = 3x^2$$

$$\Rightarrow x = \pm \frac{\lambda}{\sqrt{3}}$$

$$\begin{array}{c} - \\ \text{---} \\ -\frac{\lambda}{\sqrt{3}} \quad \frac{\lambda}{\sqrt{3}} \end{array}$$

Local min      Local max

We want local min

$$\Rightarrow x = \frac{-\lambda}{\sqrt{3}}$$

from (1)

$$x \in (-3, -2)$$



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$$-3 < \frac{-\lambda}{\sqrt{3}} < -2$$

$$3\sqrt{3} > \lambda > 2\sqrt{3}$$

$$\alpha = 2\sqrt{3}, \beta = 3\sqrt{3}$$

$$\alpha^2 + \beta^2 = 12 + 27 = 39$$

24. Let

$$\lim_{n \rightarrow \infty} \left( \frac{n}{\sqrt{n^4+1}} - \frac{2n}{(n^2+1)\sqrt{n^4+1}} + \frac{n}{\sqrt{n^4+16}} - \frac{8n}{(n^2+4)\sqrt{n^4+16}} \right. \\ \left. + \dots + \frac{n}{\sqrt{n^4+n^4}} - \frac{2n \cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}} \right) \text{ be } \frac{\pi}{k},$$

using only the principal values of the inverse trigonometric functions. Then  $k^2$  is equal to \_\_\_\_.

**Ans. (32)**

$$\text{Sol. } \sum_{r=1}^{\infty} \frac{n}{\sqrt{n^4+r^4}} - \frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}$$

$$\sum_{r=1}^{\infty} \frac{\frac{1}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^4}} - \frac{2\left(\frac{1}{n}\right)\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\left(\frac{r}{n}\right)^4}}$$

$$\Rightarrow \int_0^1 \frac{dx}{\sqrt{1+x^4}} - \frac{2x^2 dx}{(1+x^2)\sqrt{1+x^4}}$$

$$\Rightarrow \int_0^1 \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx$$

$$\Rightarrow \int_0^1 \frac{\frac{1}{x^2}-1}{\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}} dx$$

$$\Rightarrow -\int_0^1 \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}} dx$$

$$x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2} dx = dt$$

$$\Rightarrow -\int_{\infty}^2 \frac{dt}{t\sqrt{t^2-2}}$$

$$\Rightarrow -\int_{\infty}^2 \frac{tdt}{t^2\sqrt{t^2-2}}$$

$$\text{take } t^2 - 2 = \alpha^2$$

$$t dt = \alpha d\alpha$$

$$\Rightarrow -\int_{\infty}^{\sqrt{2}} \frac{\alpha d\alpha}{(\alpha^2+2)\alpha}$$

$$\Rightarrow -\int_{\infty}^{\sqrt{2}} \frac{d\alpha}{\alpha^2+2}$$

$$\Rightarrow \left[ \frac{-1}{\sqrt{2}} \tan^{-1} \frac{\alpha}{\sqrt{2}} \right]_{\infty}^{\sqrt{2}}$$

$$\Rightarrow \frac{-1}{\sqrt{2}} \{ \tan^{-1} 1 \} + \frac{1}{\sqrt{2}} \tan^{-1} \infty$$

$$\Rightarrow \frac{1}{\sqrt{2}} \left\{ \frac{\pi}{2} - \frac{\pi}{4} \right\}$$

$$\Rightarrow \frac{\pi}{4\sqrt{2}} = \frac{\pi}{K}$$

$$\text{So } K = 4\sqrt{2}$$

$$K^2 = 32$$

25. The remainder when  $428^{2024}$  is divided by 21 is \_\_\_\_.

**Ans. (1)**

$$\text{Sol. } (428)^{2024} = (420 + 8)^{2024}$$

$$= (21 \times 20 + 8)^{2024}$$

$$= 21m + 8^{2024}$$

$$\text{Now } 8^{2024} = (8^2)^{1012}$$

$$= (64)^{1012}$$

$$= (63 + 1)^{1012}$$

$$= (21 \times 3 + 1)^{1012}$$

$$= 21n + 1$$

$\Rightarrow$  Remainder is 1.



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26. Let  $f:(0, \pi) \rightarrow \mathbb{R}$  be a function given by

$$f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0 < x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x|)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2} < x < \pi \end{cases}$$

Where  $a, b \in \mathbb{Z}$ . If  $f$  is continuous at  $x = \frac{\pi}{2}$ , then  $a^2 + b^2$  is equal to \_\_\_\_\_.

Ans. (81)

Sol. LHL at  $x = \frac{\pi}{2}$

$$\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} = \left(\frac{8}{7}\right)^0 = 1$$

RHL at  $x = \frac{\pi}{2}$

$$\lim_{x \rightarrow \frac{\pi}{2}} (1 + |\cot x|)^{\frac{b}{a}|\tan x|}$$

$$= e^{\lim_{x \rightarrow \frac{\pi}{2}} |\cot x| \frac{b}{a} |\tan x|} = e^{\frac{b}{a}}$$

$$\Rightarrow 1 = a - 8 = e^{\frac{b}{a}}$$

$$\Rightarrow a = 9, b = 0$$

$$\Rightarrow a^2 + b^2 = 81$$

27. Let  $A$  be a non-singular matrix of order 3. If  $\det(3\text{adj}(2\text{adj}((\det A)A))) = 3^{-13} \cdot 2^{-10}$  and  $\det(3\text{adj}(2A)) = 2^m \cdot 3^n$ , then  $|3m + 2n|$  is equal to \_\_\_\_\_.

Ans. (14)

Sol.  $|3 \text{adj}(2\text{adj}(|A|A))| = |3\text{adj}(2|A|^2 \text{adj}(A))|$   
 $= |3 \cdot 2^2 |A|^4 \text{adj}(\text{adj}(A))| = 2^6 3^3 |A|^{12} |A|^4$   
 $= 2^6 3^3 |A|^{16} = 2^{-10} 3^{-13}$   
 $\Rightarrow |A|^{16} = 2^{-16} 3^{-16} \Rightarrow |A| = 2^{-1} 3^{-1}$

$$\text{Now } |3\text{adj}(2A)| = |3 \cdot 2^2 \text{adj}(A)|$$

$$= 2^6 3^3 |A|^2 = 2^{-m} 3^{-n}$$

$$\Rightarrow 2^6 3^3 2^{-2} 3^{-2} = 2^{-m} 3^{-n}$$

$$\Rightarrow 2^{-m} 3^{-n} = 2^4 3^1$$

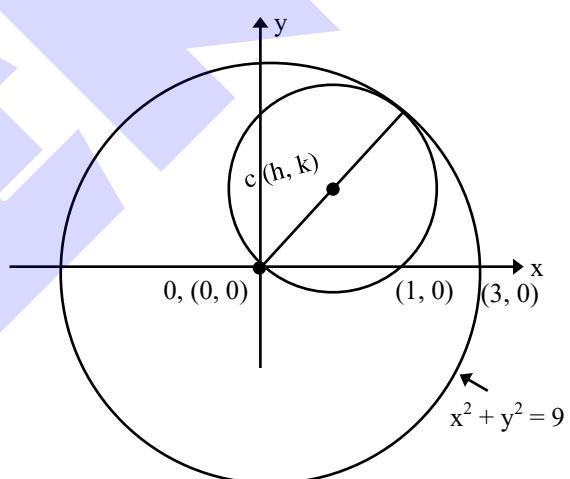
$$\Rightarrow m = -4, n = -1$$

$$\Rightarrow |3m + 2n| = |-12 - 2| = 14$$

28. Let the centre of a circle, passing through the point  $(0, 0), (1, 0)$  and touching the circle  $x^2 + y^2 = 9$ , be  $(h, k)$ . Then for all possible values of the coordinates of the centre  $(h, k)$ ,  $4(h^2 + k^2)$  is equal to \_\_\_\_\_.

Ans. (9)

Sol.



$$(x - h)^2 + (y - k)^2 = h^2 + k^2$$

$$x^2 + y^2 - 2hx - 2ky = 0$$

∴ passes through  $(1, 0)$

$$\Rightarrow 1 + 0 - 2h = 0$$

$$\Rightarrow h = 1/2$$

$$\therefore OC = \frac{OP}{2}$$

$$\sqrt{\left(\frac{1}{2}\right)^2 + k^2} = \frac{3}{2}$$



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$$\frac{1}{4} + k^2 = \frac{9}{4}$$

$$k^2 = 2$$

$$k = \pm \sqrt{2}$$

∴ Possible coordinate of

$$c(h, k) \left( \frac{1}{2}, \sqrt{2} \right) \left( \frac{1}{2}, -\sqrt{2} \right)$$

$$4(h^2 + k^2) = 4 \left( \frac{1}{4} + 2 \right) = 4 \left( \frac{9}{4} \right) = 9$$

29. If a function  $f$  satisfies  $f(m + n) = f(m) + f(n)$  for all  $m, n \in \mathbb{N}$  and  $f(1) = 1$ , then the largest natural number  $\lambda$  such that  $\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$  is equal to \_\_\_\_\_.

**Ans. (1010)**

**Sol.**  $f(m + n) = f(m) + f(n)$

$$\Rightarrow f(x) = kx$$

$$\Rightarrow f(1) = 1$$

$$\Rightarrow k = 1$$

$$f(x) = x$$

Now

$$\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$$

$$\Rightarrow \sum_{k=1}^{2022} (\lambda + k) \leq (2022)^2$$

$$\Rightarrow 2022\lambda + \frac{2022 \times 2023}{2} \leq (2022)^2$$

$$\Rightarrow \lambda \leq 2022 - \frac{2023}{2}$$

$$\Rightarrow \lambda \leq 1010.5$$

∴ largest natural no.  $\lambda$  is 1010.

30. Let  $A = \{2, 3, 6, 7\}$  and  $B = \{4, 5, 6, 8\}$ . Let  $R$  be a relation defined on  $A \times B$  by  $(a_1, b_1) R (a_2, b_2)$  if and only if  $a_1 + a_2 = b_1 + b_2$ . Then the number of elements in  $R$  is \_\_\_\_\_.

**Ans. (25)**

**Sol.**  $A = \{2, 3, 6, 7\}$

$$B = \{4, 5, 6, 8\}$$

$$(a_1, b_1) R (a_2, b_2)$$

$$a_1 + a_2 = b_1 + b_2$$

- |                     |                     |
|---------------------|---------------------|
| 1. (2, 4) R (6, 4)  | 2. (2, 4) R (7, 5)  |
| 3. (2, 5) R (7, 4)  | 4. (3, 4) R (6, 5)  |
| 5. (3, 5) R (6, 4)  | 6. (3, 5) R (7, 5)  |
| 7. (3, 6) R (7, 4)  | 8. (3, 4) R (7, 6)  |
| 9. (6, 5) R (7, 8)  | 10. (6, 8) R (7, 5) |
| 11. (7, 8) R (7, 6) | 12. (6, 8) R (6, 4) |
| 13. (6, 6) R (6, 6) |                     |

× 2

Total  $24 + 1 = 25$



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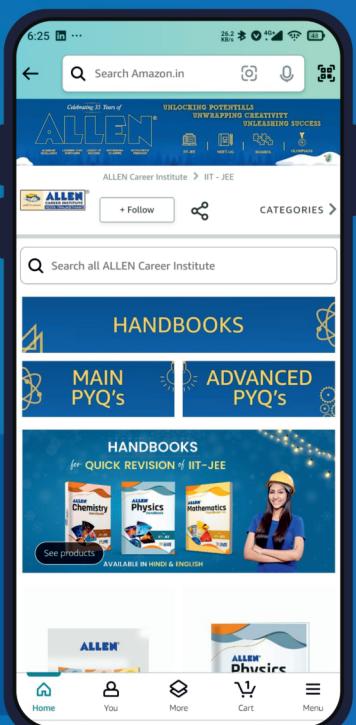
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