FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 13th April, 2023)   **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

31. Which of the following Maxwell's equations is valid for time varying conditions but not valid for static conditions :

(1) $\oint \vec{B}.\vec{dl} = \mu_0 I$ (2) $\oint \vec{E}.\vec{dl} = 0$

(3) $\oint \vec{E}.\vec{dl} = -\dfrac{\partial \phi_B}{\partial t}$ (4) $\oint \vec{D}.\vec{dA} = Q$
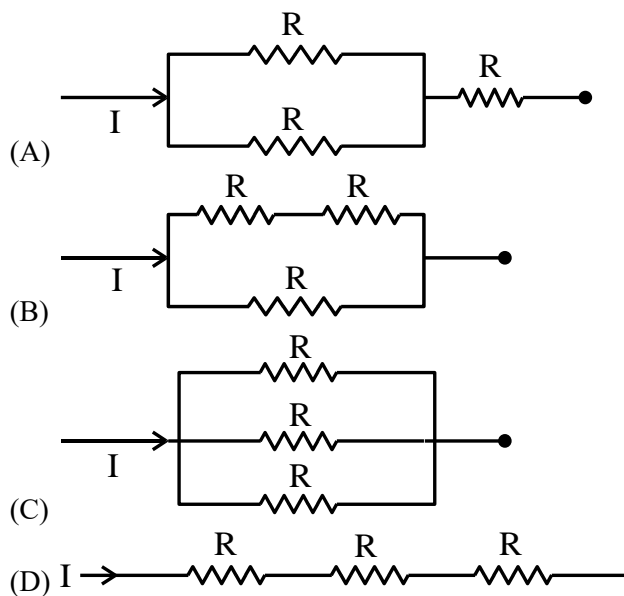
Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Based on equations of Maxwell

32. Different combination of 3 resistors of equal resistance R are shown in the figures.

The increasing order for power dissipation is:

(A)

(B)

(C)

(D) I →

(1) $P_A < P_B < P_C < P_D$

(2) $P_C < P_D < P_A < P_B$

(3) $P_B < P_C < P_D < P_A$

(4) $P_C < P_B < P_A < P_D$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $P = I^2 R$

$R_1 = \dfrac{3R}{2}, R_2 = \dfrac{2R}{3}, R_3 = \dfrac{R}{3}, R_4 = 3R$

Since i is same, hence $P \propto R$ so options (4) is correct

33. A vessel of depth 'd' is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be-

(1) $\dfrac{d\, n_1 n_2}{(n_1 + n_2)}$

(2) $\dfrac{d(n_1 + n_2)}{2n_1 n_2}$

(3) $\dfrac{d\, n_1 n_2}{2(n_1 + n_2)}$

(4) $\dfrac{2d(n_1 + n_2)}{n_1 n_2}$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. Formula used $d_{app} = \dfrac{d_1}{n_1} + \dfrac{d_2}{n_2}$

$$d_{app} = \dfrac{d}{2}\left[\dfrac{n_1 + n_2}{n_1 n_2}\right]$$

34. The source of time varying magnetic field may be

(A) a permanent magnet

(B) an electric field changing linearly with time

(C) direct current

(D) a decelerating charge particle

(E) an antenna fed with a digital signal

Choose the correct answer from the options given below:

(1) (D) only

(2) (C) and (E) only

(3) (A) only

(4) (B) and (D) only

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Source of time varying magnetic field may be

→accelerated or retarded charge which produces varying electric and magnetic fields.

→ An electric field varying linearly with time will not produce variable magnetic field as current will be constant

35. Two trains 'A' and 'B' of length 'l' and '$4l$' are travelling into a tunnel of length 'L' in parallel tracks from opposite directions with velocities 108 km/h and 72 km/h, respectively. If train 'A' takes 35s less time than train 'B' to cross the tunnel then, length 'L' of tunnel is :

(Given L = 60 l)

(1) 1200 m

(2) 2700 m

(3) 1800 m

(4) 900 m

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $\dfrac{60\ell + 4\ell}{20} - \dfrac{61\ell}{30} = 35$

$\Rightarrow \ell = \dfrac{1050}{35}$

$\Rightarrow L = 60\ell = \dfrac{1050}{35} \times 60 = 1800$ m

36. The ratio of powers of two motors is $\dfrac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of x will be

(1) 2

(2) 4

(3) 2.4

(4) 16

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Average Power = $\dfrac{total\ work\ done}{total\ time}$

So $P = \dfrac{mgh}{t}$

$\dfrac{P_1}{P_2} = \dfrac{\dfrac{m_1gh}{t_1}}{\dfrac{m_2gh}{t_2}} = \dfrac{m_1}{t_1}\dfrac{t_2}{m_2}$

$\dfrac{P_1}{P_2} = \dfrac{300 \times 2}{5 \times 50} = \dfrac{12}{5} = \dfrac{3\sqrt{x}}{\sqrt{x}+1}$

$12\sqrt{x} + 12 = 15\sqrt{x}$

$3\sqrt{x} = 12$

$x = 16$

37. A planet having mass 9 Me and radius 4Re, where Me and Re are mass and radius of earth respectively, has escape velocity in km/s given by: (Given escape velocity on earth

$V_e = 11.2 \times 10^3$ m/s)

(1) 67.2

(2) 16.8

(3) 33.6

(4) 11.2

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $V_p = \sqrt{\dfrac{2GM_p}{R_p}}$ $V_E = \sqrt{\dfrac{2GM_E}{R_E}}$

$\dfrac{V_P}{V_E} = \dfrac{\sqrt{\dfrac{2GM_P}{R_P}}}{\sqrt{\dfrac{2GM_E}{R_E}}} = \sqrt{\dfrac{R_E}{R_P} \times \dfrac{M_P}{M_E}}$

$V_P = \sqrt{\dfrac{1}{4} \times 9} \times V_E = \dfrac{3}{2}V_E$

$V_P = \dfrac{3}{2} \times 11.2$ km / sec

$= 16.8$ km / sec

38. The difference between threshold wavelengths for two metal surfaces A and B having work function $\phi_A = 9eV$ and $\phi_B = 4.5eV$ in nm is:

(Given, hc = 1242 eV nm)

(1) 264

(2) 138

(3) 276

(4) 540

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\lambda_A = \left(\dfrac{1242}{9}\right) = 138$ nm

$\lambda_B = \left(\dfrac{1242}{4.5}\right) = 276$ nm

$\lambda_B - \lambda_A = 138$ nm

39. A bullet 10 g leaves the barrel of gun with a velocity of 600 m/s. If the barrel of gun is 50 cm long and mass of gun is 3 kg, then value of impulse supplied to the gun will be :

(1) 12 Ns   (2) 6 Ns

(3) 36 Ns   (4) 3 Ns

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. By momentum conservation

$$0 = 3(-v) + 0.01(600 - v)$$

$$v \simeq 2m/s$$

Impulse on gun $= 3 \times 2 = 6$ Ns

40. Two charges each of magnitude 0.01 C and separated by a distance of 0.4 mm constitute an electric dipole. If the dipole is placed in an uniform electric field $\vec{E}$ of 10 dyne/C making 30^0 angle with $\vec{E}$, the magnitude of torque acting on dipole is :

(1) 4.0×10^{-10} Nm   (2) 2.0×10^{-10} Nm

(3) 1.0×10^{-8} Nm   (4) 1.5×10^{-9} Nm

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\left|\vec{P}\right| = qd$

$$= 0.01 \times 0.4 \times 10^{-3}$$
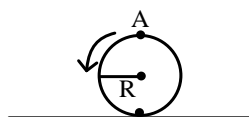
$$= 4 \times 10^{-6}$$

$$\left|\vec{\tau}\right| = PE\sin\theta$$

$$= 4 \times 10^{-6} \times 10 \times 10^{-5} \times \sin 30$$

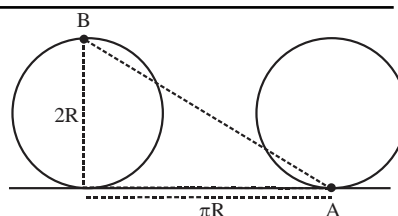$$= 4 \times 10^{-6-5+1} \times \frac{1}{2}$$

$$= 2 \times 10^{-10}$$

41. A disc is rolling without slipping on a surface. The radius of the disc is R. At t = 0, the top most point on the disc is A as shown in figure. When the disc completes half of its rotation, the displacement of point A from its initial position is

(1) $R\sqrt{(\pi^2 + 4)}$ (2) $R\sqrt{(\pi^2 + 1)}$

(3) 2 R   (4) $2R\sqrt{(1 + 4\pi^2)}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol.

Displacement $= \sqrt{(2R)^2 + (\pi R)^2} = R\sqrt{4 + \pi^2}$

42. Match List – I with List – II

List - I (Layer of atmosphere)	List – II (Approximate height over earth's surface)
(A) F_1 - Layer	(I) 10 km
(B) D – Layer	(II) 170 – 190 km
(C) Troposphere	(III) 100 km
(D) E-layer	(IV) 65 – 75 km

Choose the correct answer from the options given below:

(1) A – III, B – IV, C – I, D – II

(2) A – II, B – IV, C – III, D – I

(3) A – II, B – I, C – IV, D – III

(4) A – II, B – IV, C – I, D – III

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Based on Theory

43. The rms speed of oxygen molecule in a vessel at particular temperature is $\left(1 + \frac{5}{x}\right)^{\frac{1}{2}} \upsilon$, where υ is the average speed of the molecule. The value of x will be:(Take $\pi = \frac{22}{7}$)

(1) 28

(2) 27

(3) 8

(4) 4

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\sqrt{\frac{3RT}{M}} = \left(1 + \frac{5}{x}\right)^{\frac{1}{2}} \sqrt{\frac{8RT}{\pi M}}$

$$\Rightarrow \frac{3 \times 22}{7 \times 8} = 1 + \frac{5}{x}$$

$$\Rightarrow x = 28$$

44. A body of mass (5 ± 0.5) kg is moving with a velocity of (20 ± 0.4) m/s. Its kinetic energy will be

(1) (1000 ± 140) J

(2) (1000 ± 0.14) J

(3) (500 ± 0.14) J

(4) (500 ± 140) J

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $k = \frac{1}{2}mv^2$

$k = \frac{1}{2} \times 5 \times 400 = 5 \times 200 = 1000$ J

$\frac{\Delta k}{2k} = \frac{\Delta m}{m} + \frac{2\,\Delta v}{v} = \frac{0.5}{5} + \frac{2 \times 0.4}{20}$

$\Delta k = 1000 \left(\frac{1}{10} + \frac{4}{100} \right) = 1000 \left(\frac{10+4}{100} \right) = 140$ J

45. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is :

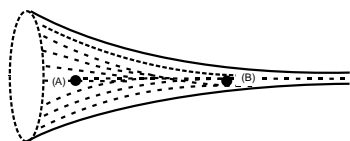(1) 4 : 3   (2) 3 : 4

(3) 16 : 9   (4) 9 : 16

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\frac{K_1}{K_2} = \frac{p_1^2}{2m_1} \times \frac{2m_2}{p_2^2} = \frac{m_2}{m_1} = \frac{16}{9}$

$\frac{m_1}{m_2} = \frac{9}{16}$

46.

The figure shows a liquid of given density flowing steadily in horizontal tube of varying cross-section. Cross sectional areas at A is 1.5 cm^2, and B is 25 mm^2, if the speed of liquid at B is 60 cm/s then $(P_A - P_B)$ is :

(Given P_A and P_B are liquid pressures at A and B points.

Density $\rho = 1000$ kg m^{-3}

A and B are on the axis of tube

(1) 175 Pa

(2) 27 Pa

(3) 135 Pa

(4) 36 Pa

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. From continuity theorem $A_1 V_1 = A_2 V_2$

$1.5 \times V_1 = 25 \times 10^{-2} \times 60$

$V_1 = \frac{25 \times 60 \times 10^{-2} \times 10}{1.5}$

$V_1 = 10$ cm / s

By Bernoulli's theorem

$P_1 + \frac{1}{2} \times 1000 \times (0.1)^2 = P_2 + \frac{1}{2} \times 1000 \times (0.6)^2$

$P_1 + 5 = P_2 + \frac{1}{2} \times 1000 \times 36 \times 10^{-2}$

$P_1 + 5 = P_2 + 180$

$P_1 - P_2 = 175$ Pa

47. Under isothermal condition, the pressure of a gas is given by $P = aV^{-3}$, where a is a constant and V is the volume of the gas. The bulk modulus at constant temperature is equal to

(1) $\frac{P}{2}$ (2) 3 P

(3) 2 P   (4) P

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $B = -\frac{dP}{dv / v}$

$Pv^3 = a$

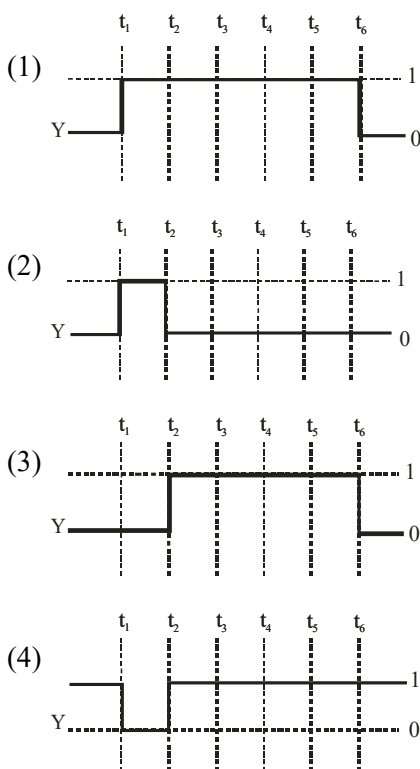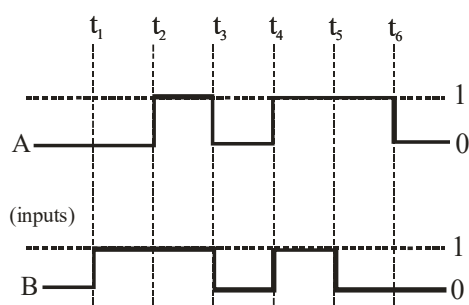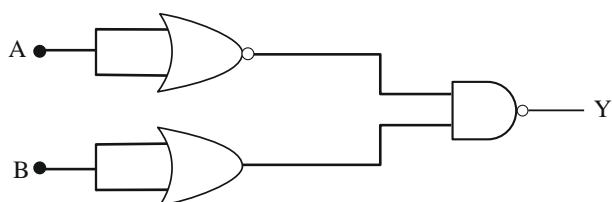Differentiating w.r.t to pressure

$v^3 + P3v^2 \frac{dv}{dP} = 0$

$v = -3 \frac{P dv}{dP} = 0$

$v = -3 \frac{P dv}{dP}$

$\frac{dP.v}{dv} = -3P$

$B = -\left(\frac{dPv}{dv} \right) = -(-3P) = 3P$

48. For the following circuit and given inputs A and B, choose the correct option for output 'Y'
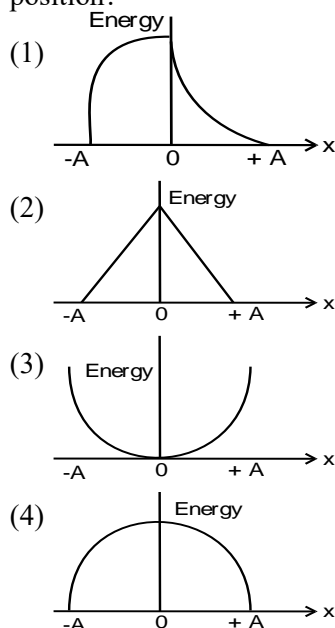

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $Y = \overline{\overline{A} \cdot B} = A + \overline{B}$

49. Which graph represents the difference between total energy and potential energy of a particle executing SHM Vs it's distance from mean position?
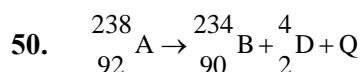

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. T.E. – P.E. = K.E.

$$K.E. = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

Which is the equation of downward parabola.

50.
$$_{92}^{238}A \rightarrow {}_{90}^{234}B + {}_{2}^{4}D + Q$$

In the given nuclear reaction, the approximate amount of energy released will be :

[Given, mass of $_{92}^{238}A = 238.05079 \times 931.5 \text{ MeV}/c^2$,

mass of $_{90}^{234}B = 234.04363 \times 931.5 \text{ MeV}/c^2$,

mass of $_{2}^{4}D = 4.00260 \times 931.5 \text{ MeV}/c^2$]

(1) 3.82 MeV   (2) 5.9 MeV

(3) 2.12 MeV   (4) 4.25 MeV

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $Q = \left(m_A - m_B - m_D\right) \times 931.5 \text{ MeV}$

$$= \left(238.05079 - 234.04363 - 4.00260\right) \times 931.5$$

$$\Rightarrow 4.25 \text{ Mev}$$

Section - B

51. The elastic potential energy stored in a steel wire of length 20 m stretched through 2 cm is 80 J. The cross sectional area of the wire is_____ mm^2.

(Given, $y = 2.0 \times 10^{11} \, \text{Nm}^{-2}$)

Official Ans. by NTA (40)

Allen Ans. (40)

Sol. Energy per unit volume $= \dfrac{1}{2} \, stress \times strain$

$$Energy = \frac{1}{2} \, stress \times strain \times volume$$

$$80 = \frac{1}{2} \times Y \times strain^2 \, A \times \ell$$

$$80 = \frac{1}{2} \times 2 \times 10^{11} \times \frac{\left(2 \times 10^{-2}\right)^2}{400} \times A \times 20$$

$$20 = \frac{10^{+7}}{20} \times A$$

$$40 \times 10^{-6} \, m^2 = A$$

$$A = 40 \, mm^2$$

52. A potential V_0 is applied across a uniform wire of resistance R. The power dissipation is P_1. The wire is then cut into two equal halves and a potential of V_0 is applied across the length of each half. The total power dissipation across two wires is P_2. The ratio $P_2 : P_1$ is $\sqrt{x} : 1$. The value of x is _____.

Official Ans. by NTA (16)

Allen Ans. (16)

Sol. $P = VI = I^2 R = \dfrac{V^2}{R}$

Now $R = \dfrac{\rho l}{A}$

If wire is cut in two equal half

$$R' = \frac{R}{2}$$

Initial $P_1 = \dfrac{V_0^2}{R}$

After $P_2 = \dfrac{V_0^2}{R'} \times 2 \Rightarrow \dfrac{V_0^2}{R} \times 4$

$$\frac{P_2}{P_1} = 4 = \frac{\sqrt{x}}{1}$$

$$x = 16$$

53. At a given point of time the value of displacement of a simple harmonic oscillator is given as

$y = A \cos (30^0)$. If amplitude is 40 cm and kinetic energy at that time is 200 J, the value of force constant is $1.0 \times 10^x \, \text{Nm}^{-1}$. The value of x is _____.

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. General equation for displacement is given by

$$x = A \sin (\omega t + \phi)$$

at given time

$$\Rightarrow \omega t + \phi = 30^0$$

$$\Rightarrow x = 40 \times \frac{\sqrt{3}}{2} \Rightarrow 20\sqrt{3} \, cm$$

$$\Rightarrow A = 40 \, cm$$

$$\Rightarrow K.E = \frac{1}{2} k \left(A^2 - x^2\right) = 200$$

$$200 = \frac{1}{2} k \left(\frac{1600 - 1200}{100 \times 100}\right)$$
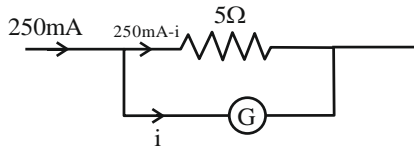
$$400 \times 100 \times 100 = k \times 400$$

$$k = 10^4$$

$$x = 4$$

54. When a resistance of 5Ω is shunted with a moving coil galvanometer, it shows a full scale deflection for a current of 250 mA, however when $1050 \, \Omega$ resistance is connected with it in series, it gives full scale deflection for 25 volt. The resistance of galvanometer is _____ Ω.
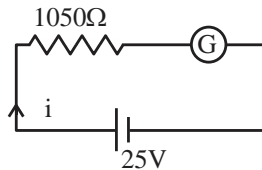
Official Ans. by NTA (50)

Allen Ans. (50)

Sol.

$$\frac{250\,mA \times 5}{5 + R_G} = i \qquad \text{........ (i)}$$

$$i = \frac{25}{1050 + R_G} \qquad \text{........ (ii)}$$

From (i) and (ii)

$$\frac{25}{1050 + R_G} = \frac{5}{4(5 + R_G)}$$

$$100(5 + R_G) = 1050 \times 5 + R_G \times 5$$

$$95\,R_G = 4750$$

$$R_G = 50\,\Omega$$

55. The radius of 2nd orbit of He^+ of Bohr's model is r_1 and that of fourth orbit of Be^{3+} is represented as r_2.

Now the ratio $\dfrac{r_2}{r_1}$ is x : 1. The value of x is _____

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $r \propto \dfrac{n^2}{z}$

$$\frac{r_{He^+}}{r_{Be^{3+}}} = \frac{2^2 \times 4}{2 \times 4 \times 4} = \frac{1}{2}$$

56. A thin infinite sheet charge and an infinite line charge of respective charge densities $+\sigma$ and $+\lambda$ are placed parallel at 5 m distance from each other. Points 'P' and 'Q' are at $\dfrac{3}{\pi}$ m and $\dfrac{4}{\pi}$ m perpendicular distance from line charge towards sheet charge, respectively. 'E_P' and 'E_Q' are the magnitudes of resultant electric field intensities at point 'P' and 'Q', respectively. If $\dfrac{E_P}{E_Q} = \dfrac{4}{a}$ for $2|\sigma| = |\lambda|$. Then the value of a is _____.
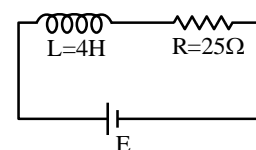
Official Ans. by NTA (6)

Allen Ans. (6)

Sol. $E_A = \dfrac{\lambda}{2\pi\varepsilon_0 r_A} - \dfrac{\sigma}{2\varepsilon_0} \left\{ r_A = \dfrac{3}{\pi} \right\}$

$$= \frac{1}{2\varepsilon_0}\left[\frac{\lambda}{3} - \sigma \right]$$

$$E_B = \frac{\lambda}{2\pi\varepsilon_0 r_A} - \frac{\sigma}{2\varepsilon_0} \left\{ r_B = \frac{4}{\pi} \right\}$$

$$= \frac{1}{2\varepsilon_0}\left[\frac{\lambda}{4} - \sigma \right]$$

$$\frac{E_A}{E_B} = \frac{4}{3}\left(\frac{\lambda - 3\sigma}{\lambda - 4\sigma} \right)$$

$$= \frac{4}{3}\left[\frac{2\sigma - 3\sigma}{2\sigma - 4\sigma} \right]$$

$$= \frac{4}{3}\left[\frac{-\sigma}{-2\sigma} \right]$$

$$= \frac{4}{6}$$

57. In the given figure, an inductor and a resistor are connected in series with a battery of emf E volt. $\dfrac{E^a}{2b}\,J/s$ represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\dfrac{b}{a}$ will be _____

Official Ans. by NTA (25)

Allen Ans. (25)

Sol. $E = \dfrac{1}{2}LI^2$

Rate of energy storing $= \dfrac{dE}{dt} = LI\dfrac{dI}{dt}$

Now we Know for $R-L$ circuit

$I = \dfrac{E}{R}\left(1 - e^{-t\frac{R}{L}}\right)$

So $\dfrac{dI}{dt} = \dfrac{E}{L}e^{-\frac{tR}{L}}$

$\dfrac{dE}{dt} = \dfrac{E^2}{R}\left(1 - e^{-\frac{tR}{L}}\right)\left(e^{-t\frac{R}{L}}\right)$

Time at which rate of power storing will be max,

$t = \dfrac{L}{R\ln 2}$

So $\dfrac{dE}{dt} = \dfrac{E^2}{R}\left(1 - \dfrac{1}{2}\right)\times\dfrac{1}{2}$

$\Rightarrow \dfrac{E^2}{4R} = \dfrac{E^2}{100} = \dfrac{E^2}{2\times 50}$

$a = 2, b = 50$

So $\dfrac{b}{a} = 25$

58. A fish rising vertically upward with a uniform velocity of 8 ms⁻¹, observes that a bird is diving vertically downward towards the fish with the velocity of 12 ms⁻¹. If the refractive index of water is $\dfrac{4}{3}$, then the actual velocity of the diving bird to pick the fish, will be _____ ms⁻¹.

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $\dfrac{V_{b/f}}{\dfrac{4}{3}} = \dfrac{-8}{\dfrac{4}{3}} + \dfrac{(-v)}{1}$

$\Rightarrow \dfrac{-12}{\dfrac{4}{3}} = \dfrac{-8}{\dfrac{4}{3}} + \dfrac{(-v)}{1}$

$\Rightarrow v = 3\,m/s$

59. A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of moving sphere is $\pi : 22$ then, the value of its angular speed will be _____ rad/s.
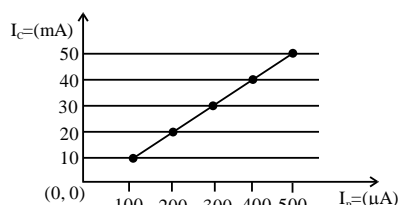
Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $L = (I_{com})(\omega)$ and $K = \dfrac{1}{2}(I_{com})(\omega^2) + \dfrac{1}{2}MV_{com}^2$

$L = \dfrac{2}{5}MR^2\dfrac{V_{com}}{R}$ $\quad K = \dfrac{1}{2}\left(\dfrac{2}{5}MR^2\right)\dfrac{V_{com}^2}{R^2} + \dfrac{1}{2}MV_{com}^2$

$L = \dfrac{2MRV_{com}}{5}$ $\quad K = \dfrac{7}{10}MV_{com}^2$

Ratio $\dfrac{L}{K} = \dfrac{4}{7}\dfrac{R}{V_{com}} = \dfrac{\pi}{22} \Rightarrow \omega = \dfrac{4}{7}\times\dfrac{22}{22}\times 7 = 4$

60. From the given transfer characteristic of a transistor in CE configuration, the value of power gain of this configuration is 10^x, for $R_B = 10\,k\Omega$, and $R_C = 1\,k\Omega$. The value of x is _____.

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Power gain

$\Rightarrow A_v \cdot A_1 = B\dfrac{R_C}{R_B}\cdot B = B^2\dfrac{R_C}{R_B}$

$= \left(\dfrac{(20-10)\times 10^{-3}}{(200-100)\times 10^{-6}}\right)\times\dfrac{1\times 10^3}{10\times 10^3} = 10^3$

Hence $x = 3$