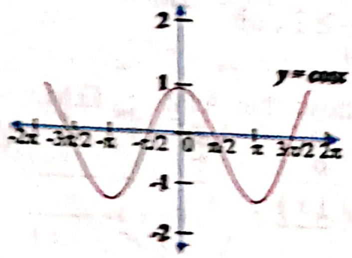
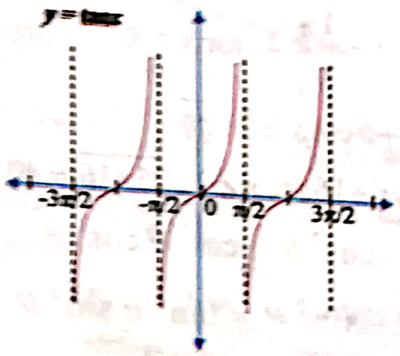
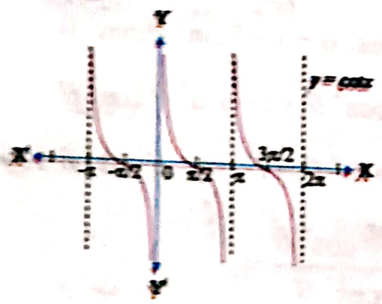
Graph of cos x:



Graph of tan x:



Graph of cot x:



EXERCISE-I

CRTQ & SPQ LEVEL-I

TRIGNOMETRIC RATIOS AND VALUES, TRIGNOMETRIC RELATIONS, MEASUREMENT OF ANGLES

C.R.T.Q Class Room Teaching Questions

1. $-13\pi/6$ radians =
   1) -390° 2) -490° 3) -410° 4) -30°

2. $Cot(1358^\circ)+\tan(3608^\circ)=$
   1) -1 2) 0 3) 1 4) 2

3. $\dfrac{\sin(-660^\circ)\tan(1050^\circ)\sec(-420^\circ)}{\cos(225^\circ)\operatorname{cosec}(315^\circ)\cos(510^\circ)}=$
   1) $\dfrac{\sqrt{3}}{4}$ 2) $\dfrac{\sqrt{3}}{2}$ 3) $\dfrac{2}{\sqrt{3}}$ 4) $\dfrac{4}{\sqrt{3}}$

4. $\sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ =$
   1) 0 2) 2 3) 1 4) -1

5. $\log\tan 17^\circ + \log\tan 37^\circ + \log\tan 53^\circ + \log\tan 73^\circ =$
   1) 0 2) 1 3) 2 4) 3

6. $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ =$
   1) -1 2) 0 3) 1 4) 2

7. $\sec A + \tan A = 3 \Rightarrow \sec A =$
   1) $\dfrac{10}{3}$ 2) $\dfrac{5}{3}$ 3) $\dfrac{2}{3}$ 4) $\dfrac{4}{3}$

8. $\sec\theta - \tan\theta = 3 \Rightarrow \theta$ lies in the quadrant
   1) I 2) II 3) III 4) IV

9. $\sin 10^\circ + \sin 20^\circ + \sin 30^\circ + \ldots\ldots + \sin 360^\circ =$
   1) 0 2) 1 3) -1 4) 2

10. $3[\sin x - \cos x]^4 + 6[\sin x + \cos x]^2 + 4[\sin^6 x + \cos^6 x] =$
    1) 3 2) 6 3) 4 4) 13

11. $\dfrac{\sin^2\alpha}{1+\cot^2\alpha} + \dfrac{\tan^2\alpha}{(1+\tan^2\alpha)^2} + \cos^2\alpha =$
    1) -1 2) 0 3) 1 4) 2

12. $1 - \dfrac{Sin^2 x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} - \dfrac{\sin x}{1-\cos x} =$
    1) sin x 2) cos x 3) cosec x 4) secx

13. If A, B, C are the angles of a triangle ABC then $\cos\left(\dfrac{3A+2B+C}{2}\right) + \cos\left(\dfrac{A-C}{2}\right) =$
    1) 0 2) 1 3) cosA 4) cosC

14. $\dfrac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=$

1) $\dfrac{\sin\theta}{1+\cos\theta}$ 2) $\dfrac{\sin\theta}{1-\cos\theta}$

3) $\dfrac{1-\cos\theta}{\sin\theta}$ 4) $\dfrac{\cos\theta}{1-\sin\theta}$

15. $\sin^2(51°-x)+\sin^2(39°+x)=$

1) -1 2) 0 3) 1 4) 2

16. $\dfrac{\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}}{\cos^2\frac{\pi}{18}+\cos^2\frac{\pi}{9}+\cos^2\frac{7\pi}{18}+\cos^2\frac{4\pi}{9}}=$

1) 1 2) 2 3) 3 4) 4

17. If ABCD is a cyclic quadrilateral, then $\cos(180°+A)+\cos(180°-B)+\cos(180°-C)-\sin(90°-D)=$

1) -1 2) 0 3) 1 4) 2

18. $\left[\sin\left(\dfrac{\pi}{2}-x\right)+\sin(\pi-x)\right]^2+\left[\cos\left(\dfrac{3\pi}{2}-x\right)+\cos(2\pi-x)\right]^2=$

1) 0 2) 2 3) 4 4) 8

19. $\sin\theta_1+\sin\theta_2+\sin\theta_3=3\Rightarrow$ $\cos\theta_1+\cos\theta_2+\cos\theta_3=$

1) 0 2) 1 3) 2 4) 3

20. $8\sin^2 x+3\cos^2 x=5\Rightarrow \cot x=$

1) $\pm\dfrac{1}{\sqrt{2}}$ 2) $\pm\dfrac{1}{\sqrt{3}}$ 3) $\pm\sqrt{\dfrac{3}{2}}$ 4) $\pm\sqrt{\dfrac{2}{3}}$

21. $\sin x+\sin^2 x=1\Rightarrow \cos^2 x+\cos^4 x=$

1) 0 2) 1 3) 2 4) -1

22. If $\cos(x-y)=-1$, then $\cos x+\cos y=$

1) 0 2) 1 3) 2 4) -1

23. If $\sin x+\operatorname{cosec} x=2$ then $\sin^8 x+\operatorname{cosec}^8 x=$

1) 1 2) 2 3) 3 4) 4

24. If $\sin x+\cos x=a$ then $|\sin x-\cos x|=$

1) $\sqrt{1-a^2}$ 2) $\sqrt{a^2-1}$

3) $\sqrt{2-a^2}$ 4) $\sqrt{a^2-2}$

25. $(1+\sin A)(1+\sin B)(1+\sin C)=(1-\sin A)(1-\sin B)(1-\sin C)=k\Rightarrow k=$

1) $+\sin A\sin B\sin C$

2) $\pm\cos A\cos B\cos C$

3) $\pm\sec A\sec B\sec C$

4) $\pm$ cosec A cosecB cosecC

26. $\operatorname{cosec}^2 A\cot^2 A-\sec^2 A\tan^2 A-(\cot^2 A-\tan^2 A)(\sec^2 A\operatorname{cosec}^2 A-1)=$

1) 0 2) 1 3) -1 4) 2

27. $\tan^2\theta+\sec\theta=5\Rightarrow\sec\theta=$

1) 3 2) 2 3) 1 4) -1

28. $\log\sin 1°.\log\sin 2°........\log\sin 179°$

1) 1 2) 0 3) -1 4) 2

29. If $\tan(\alpha+\beta)=\sqrt{3}$, $\tan(\alpha-\beta)=1$ then $\tan 6\beta=$

1) -1 2) 0 3) 1 4) 2

30. If $x=r\cos\theta\cos\phi$, $y=r\cos\theta\sin\phi$, $z=r\sin\theta$ then $x^2+y^2+z^2=$

1) r^2 2) y^2 3) x^2 4) z^2

31. $\sin^2 A\cos^2 B+\cos^2 A\sin^2 B+\sin^2 A\sin^2 B+\cos^2 A\cos^2 B=$

1) -1 2) 0 3) 1 4) 2

32. If $2\sin^2 A-\cot^2 B=1$ then $\sin A\sin B=$

1) $-\dfrac{1}{\sqrt{2}}$ 2) $\pm\dfrac{1}{\sqrt{2}}$ 3) $\dfrac{1}{\sqrt{2}}$ 4) $\dfrac{-1}{2\sqrt{2}}$

33. The value of $\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta$ is

1) 0 2) 1 3) 2 4) 3

34. If $x\tan^2 120°+4\cos^2 150°=9$ then $x=$

1) 3 2) 1 3) 2 4) 4

35. In ΔABC, right angled at C, then $\tan A + \tan B =$

1) $\frac{a^2}{bc}$ 2) $\frac{b^2}{ac}$ 3) $\frac{c^2}{ab}$ 4) $\frac{ab}{c}$

36. If $a = \sin 170^\circ + \cos 170^\circ$ then

1) a > 0 2) a < 0 3) a = 0 4) a = 1

37. $\tan\frac{\pi}{24}\tan\frac{3\pi}{24}\tan\frac{5\pi}{24}\tan\frac{7\pi}{24}\tan\frac{9\pi}{24}\tan\frac{11\pi}{24}\tan\frac{16\pi}{24} =$

1) 1 2) −1 3) $\sqrt{3}$ 4) $-\sqrt{3}$

38. $x = a\sec^3\theta\tan\theta$, $y = b\tan^3\theta\sec\theta$ then $\sin^2\theta =$

1) $\frac{x}{a}-\frac{y}{b}$ 2) $\frac{x}{a}+\frac{y}{b}$ 3) $\frac{xy}{ab}$ 4) $\frac{ay}{bx}$

39. If $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$, $\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1$ then $\frac{x^2}{a^2}+\frac{y^2}{b^2} =$

1) 1 2) -1 3) 2 4) 3

40. In a ΔABC, if $\cot A\cot B\cot C > 0$ then the triangle is

1) acute angled 2) right angled
3) obtuse angled 4) can't be decided

41. If α,β are complementary angles and $\sin\alpha = \frac{3}{5}$, then $\sin\alpha\cos\beta - \cos\alpha\sin\beta =$

1) $\frac{7}{25}$ 2) $-\frac{7}{25}$ 3) $\frac{25}{7}$ 4) $-\frac{25}{7}$

42. $4a^2\sin^2\left(\frac{3\pi}{4}\right) - 3[a\tan 225^\circ]^2 + [2a\cos 315^\circ]^2 =$

1) 0 2) a 3) $\sqrt{2a}$ 4) a^2

43. $-11\pi/3$ radians =

1) -390° 2) -620° 3) -610° 4) -660°

S.P.Q. Student Practice Questions

44. $\sin 4530^\circ =$

1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) $\frac{\sqrt{3}}{2}$ 4) $-\frac{\sqrt{3}}{2}$

45. $\frac{\sin 150^\circ - 5\cos 300^\circ + 7\tan 225^\circ}{\tan 135^\circ + 3\sin 210^\circ} =$

1) 10 2) −5 3) −2 4) $-3/2$

46. $\cos 23^\circ \operatorname{cosec} 67^\circ - \sin 23^\circ \sec 67^\circ =$

1) 0 2) 2 3) 1 4) -1

47. $\log\tan 18^\circ + \log\tan 36^\circ + \log\tan 54^\circ + \log\tan 72^\circ =$

1) log4 2) log3 3) log2 4) 0

48. $\cos 5^\circ + \cos 24^\circ + \cos 175^\circ + \cos 204^\circ + \cos 300^\circ =$

1) 1/2 2) 1 3) 3/2 4) 2

49. $\operatorname{cosec} A + \cot A = \frac{2}{3} \Rightarrow \cos A =$

1) $\frac{5}{13}$ 2) $\frac{13}{5}$ 3) $-\frac{5}{13}$ 4) $-\frac{13}{5}$

50. If $\operatorname{cosec}\theta - \cot\theta = 5$ then θ lies in the quadrant

1) I 2) II 3) III 4) IV

51. $\tan 20^0 + \tan 40^0 + \tan 60^0 + \ldots + \tan 180^0 =$

1) 0 2) 1 3) 2 4) 3

52. $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\sec\alpha + \cos\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha \Rightarrow k =$

1) 9 2) 7 3) 5 4) 3

53. $\frac{1}{(1+\cot^2\alpha)^2} + \frac{\tan^2\alpha}{(1+\tan^2\alpha)^2} + \frac{1}{1+\tan^2\alpha} =$

1) -1 2) 0 3) 1 4) 2

54. $\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = k$ then k=

1) 0 2) 1 3) 2 4) -1

55. In ΔABC, $\tan(A-B-C) =$

1) sin 2 A 2) 1 3) tan 2 A 4) 0

56. $\frac{1+\cot\alpha+\operatorname{cosec}\alpha}{1-\cot\alpha+\operatorname{cosec}\alpha} =$

1) $\frac{\sin\alpha}{1+\cos\alpha}$ 2) $\frac{\sin\alpha}{1-\cos\alpha}$ 3) $\frac{1+\cos\alpha}{\sin\alpha}$ 4) $\frac{1-\sin\alpha}{\cos\alpha}$

57. $\cos^2(125^\circ - x) - \cos^2(55^\circ + x) =$

1) -1 2) 0 3) 1 4) 2

58. $\sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \sin^2\frac{4\pi}{18} + \sin^2\frac{8\pi}{18} + \sin^2\frac{7\pi}{18} + \sin^2\frac{5\pi}{18} =$

1) 1 2) 2 3) 3 4) 4

59. **If A B C D is a quadrilateral then** $\tan\left(\frac{A+B}{4}\right) =$

1) $\cos\left(\frac{C-D}{4}\right)$ 2) $\cot\left(\frac{C-D}{4}\right)$

3) $\cos\left(\frac{C+D}{4}\right)$ 4) $\cot\left(\frac{C+D}{4}\right)$

60. $\left[\cos\left(\frac{\pi}{2}-x\right)+\cos(\pi - x)\right]^2 + \left[\sin\left(\frac{3\pi}{2}-x\right)+\sin(2\pi - x)\right]^2 =$

1) 0 2) 2 3) 1 4) 4

61. **If** $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 + \cos\theta_5 = 5$ **then** $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 + \sin\theta_5 =$

1) 3 2) 2 3) 1 4) 0

62. $9\cos^2 x + 4\sin^2 x = 5 \Rightarrow \tan x =$

1) ± 1 2) ± 2 3) ± 3 4) ± 4

63. $\cos x + \cos^2 x = 1 \Rightarrow \sin^8 x + 2\sin^6 x + \sin^4 x =$

1) 0 2) 1 3) 2 4) -1

64. **If** $\cot(\alpha+\beta) = 0$ **then** $\sin(\alpha + 2\beta) =$

1) $\sin\alpha$ 2) $\cos\alpha$ 3) $\sin\beta$ 4) $\cos\beta$

65. **If** $\tan\alpha + \cot\alpha = 2$ **then** $\sqrt{\tan\alpha} + \sqrt{\cot\alpha} =$

1) $\sqrt{2}$ 2) $2\sqrt{2}$ 3) 2 4) $4\sqrt{2}$

66. $5\sin x + 4\cos x = 3 \Rightarrow 4\sin x - 5\cos x =$

1) 4 2) $4\sqrt{2}$ 3) $3\sqrt{2}$ 4) $\sqrt{2}$

67. **If** $k = (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ **then k =**

1) 0 2) ± 1 3) ± 3 4) ± 4

68. **The value of** $\cot^2\alpha\left(\frac{\sec\alpha - 1}{1+\sin\alpha}\right) + \sec^2\alpha\left(\frac{\sin\alpha - 1}{1+\sec\alpha}\right)$ **is**

1) 0 2) 1 3) 2 4) -2

69. $\sec^2\theta + \tan\theta = 13 \Rightarrow \tan\theta =$

1) 4 2) 3 3) -3 4) 5

70. $\log(\tan 1^\circ)\log(\tan 2^\circ)\log(\tan 3^\circ)\ldots\log(\tan 45^\circ) =$

1) 0 2) 1 3) -1 4) $\frac{1}{2}$

71. **If** $\sin(\alpha+\beta) = 1,\ \sin(\alpha-\beta) = \frac{1}{2}$ **then** $\tan(\alpha + 2\beta)\tan(2\alpha + \beta) =$

1) 1 2) -1 3) 0 4) 2

72. **If** $x = r\cos\alpha\cos\beta\cos\gamma;$ $y = r\cos\alpha\cos\beta\sin\gamma;$ $z = r\sin\alpha\cos\beta;\ \mu = r\sin\beta$ **then** $x^2 + y^2 + z^2 + \mu^2 =$

1) r 2) 2r 3) r^2 4) $4r^2$

73. $\sec^2 A\sec^2 B - \sec^2 A\tan^2 B - \tan^2 A\sec^2 B + \tan^2 A\tan^2 B =$

1) 1 2) 0 3) -1 4) 2

74. $2\cos^2 B - 1 = \tan^2 A$, **then** $\cos A\cos B =$

1) $\pm\frac{1}{2}$ 2) $\pm\frac{1}{3}$ 3) $\pm\frac{1}{\sqrt{2}}$ 4) ± 1

75. $\sin^4\theta + 2\sin^2\theta\left(1 - \frac{1}{\operatorname{cosec}^2\theta}\right) + \cos^4\theta =$

1) 1 2) 0 3) $\frac{1}{2}$ 4) -1

76. **If** $x\cot^2 120^\circ + 4\sin^2 150^\circ = 3$, **then** $x =$

1) 2 2) 7 3) 6 4) 5

77. In a right angled triangle ABC, $\underline{|C} = 90°$, then $\cos^2 A + \cos^2 B =$

1) 2 2) 1 3) $\frac{1}{2}$ 4) $\frac{3}{4}$

78. If $x = \sin 130° + \cos 130°$ then

1) $x < 0$ 2) $x = 0$ 3) $x > 0$ 4) $x \geq 0$

79. $\cot\frac{\pi}{20}.\cot\frac{3\pi}{20}.\cot\frac{5\pi}{20}.\cot\frac{7\pi}{20}.\cot\frac{9\pi}{20}.\cot\frac{15\pi}{20} =$

1) 1 2) -1 3) $\sqrt{3}$ 4) $-\sqrt{3}$

80. If $x = a\sec^n\theta; y = b\tan^n\theta$ then $\left(\frac{x}{a}\right)^{2/n} - \left(\frac{y}{b}\right)^{2/n} =$

1) 0 2) -1 3) 1 4) 2

81. If $x = h + p\sec\alpha, y = k + q\,\mathrm{cosec}\,\alpha$ then $\left(\frac{p}{x-h}\right)^2 + \left(\frac{q}{y-k}\right)^2 =$

1) 1 2) -1 3) 0 4) 1/2

82. If A, B, C are angles of a triangle such that A is obtuse then

1) tanA tan B > 1 2) tan B tan C <1
3) tan C tan A > 1 4) tanA tan B tanC>1

83. If α, β are complementary angles, $\sin\alpha = \frac{3}{5}$, then $\cos\alpha\cos\beta - \sin\alpha\sin\beta =$

1) 1 2) 2 3) $\frac{4}{5}$ 4) 0

84. $a^2\cos^2\frac{2\pi}{3} - 4a^2\tan^2\frac{3\pi}{4} + 2a^2\sin^2\frac{2\pi}{3} =$

1) a^2 2) 0 3) $\frac{3a^2}{4}$ 4) $\frac{-9a^2}{4}$

85. If $\tan\theta = \frac{p}{q}$ then $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} =$

1) $\frac{2p}{p^2+q^2}$ 2) $\frac{2pq}{p^2+q^2}$

3) $\frac{p^2-q^2}{p^2+q^2}$ 4) $\frac{q^2-p^2}{p^2+q^2}$

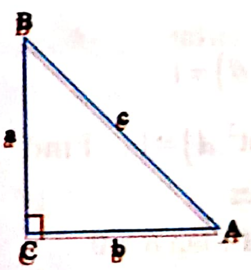
86. If $a\sec\theta + b\tan\theta = c$ then $(a\tan\theta + b\sec\theta)^2 =$

1) $a^2+b^2+c^2$ 2) $-a^2+b^2+c^2$
3) $a^2-b^2+c^2$ 4) $a^2+b^2-c^2$.

KEY

01) 1	02) 2	03) 3	04) 2	05) 1
06) 2	07) 2	08) 4	09) 1	10) 4
11) 3	12) 2	13)1	14) 2	15) 3
16) 1	17) 2	18) 2	19) 1	20) 3
21) 2	22) 1	23) 2	24) 3	25) 2
26) 1	27) 2	28) 2	29) 3	30) 1
31) 3	32) 2	33) 2	34) 3	35) 3
36) 2	37) 4	38) 4	39) 3	40) 1
41) 2	42) 4	43) 4	44) 2	45) 3
46) 1	47) 4	48) 1	49) 3	50) 2
51) 1	52) 2	53) 3	54) 3	55) 3
56) 3	57) 2	58) 3	59) 4	60) 2
61) 4	62) 2	63) 2	64) 4	65) 3
66) 2	67) 2	68) 1	69) 2	70) 1
71) 1	72) 3	73) 1	74) 3	75) 1
76) 3	77) 2	78) 3	79) 2	80) 3
81) 1	82) 2	83) 4	84) 4	85) 3
86) 2				

Hints & Solutions

1. $-13\times\dfrac{\pi}{6}\times\dfrac{180}{\pi}=-390^0$
2. $1358^0=8(180^0)-82^0$,
   $3608^0=20(180^0)+8^0$
3. Convert the corresponding values.
4. $\sin 48^0 \operatorname{cosec}48^0+\cos 48^0\sec 48^0=1+1=2$
5. $\because$ *If* $A+B=90^0\Rightarrow \tan A\tan B=1$
6. $\cos(204^0)=\cos(180^0+24^0)$
   $\cos(125^0)=\cos(180^0-55^0)$
7. Use $(\sec^2 A-Tan^2 A)=1$
8. Use $(\sec^2 A-Tan^2 A)=1$
9. $A+B=360^\circ\Rightarrow \sin A+\sin B=0$
10. Put $x=90^o$
11. Put $\alpha=45^o$
12. *Let* $x=90^0$
13. $A=B=C=60^0$
14. $1=\operatorname{cosec}^2\theta-\cot^2\theta$
15. $A+B=90^0\Rightarrow \sin^2 A+\sin^2 B=1$
16. If $A+B=90^0$ then $(\cos^2 A+\cos^2 B)=1$
    and $(\sin^2 A+\sin^2 B)=1$
17. $A+C=180^0$, $B+D=180^0$
18. Simplify
19. Put $\theta_1=\theta_2=\theta_3=90^\circ$
20. Divide the given equation by $\sin^2 x$ both sides
21. $\sin x=(1-\sin^2 x)\Rightarrow \sin x=\cos^2 x$
22. $x-y=\pi$
23. Put $x=90^0$
24. $\alpha=\dfrac{\pi}{4}$
25. $k^2=(1+\sin A)(1+\sin B)(1+\sin C)(1-\sin A)$
    $(1-\sin B)(1-\sin C)$
    $K^2=(1-\sin^2 A)(1-\sin^2 B)(1-\sin^2 C)$
    $K^2=Cos^2A.Cos^2B.Cos^2C$
    $K=\pm CosA\,CosB\,CosC$
26. *Put* $A=45^0$
27. $\tan^2\theta+\sec\theta=5$,
    $\Rightarrow(Sec^2\theta-1)+\sec\theta=5$,
    $\Rightarrow Sec^2\theta+\sec\theta-6=0$, $\sec\theta=2$ *or* -3
28. $\log(\sin 90^0)=0$
29. Let $(\alpha+\beta)=60^0$
    $(\alpha-\beta)=45^0\Rightarrow 2\beta=15^0\Rightarrow 6\beta=45^0$
30. Square and add
31. $\sin^2 A(\cos^2 B+\sin^2 B)+\cos^2 A(\sin^2 B+\cos^2 B)$ *or*
    Put $A=B=0^0$
32. Put $\cot B=\dfrac{\cos B}{\sin B}$
33. use $\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$
34. Substitute the corresponding values.
35. 

    $\angle C=90^o$
    $\therefore \tan A+\tan B=\dfrac{a}{b}+\dfrac{b}{a}=\dfrac{a^2+b^2}{ab}=\dfrac{c^2}{ab}$
36. $a=\sin 170^o+\cos 170^o$
    $=\sin 10^o-\cos 10^o<0$
37. $A+B=90^0\Rightarrow \tan A\tan B=1$

38. $\frac{y}{x}=\frac{b\tan^3\theta\sec\theta}{a\sec^3\theta\tan\theta}=\frac{b}{a}\sin^2\theta$

39. by squaring and adding $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$

40. $\cot A\cot B\cot C>0$
$\Rightarrow \cot A,\cot B,\cot C$ are positive
$\Rightarrow \Delta ABC$ is acute angled

41. $\sin\alpha=\frac{3}{5},\cos\beta=\frac{3}{5},$
$\cos\alpha=\frac{4}{5},\sin\beta=\frac{4}{5}$

42. Given $=4a^2\left(\frac{1}{\sqrt{2}}\right)^2-3[a\times 1]^2+\left[2a\times\frac{1}{\sqrt{2}}\right]^2$

43. $-11\frac{\pi}{3}\times\frac{180}{\pi}=-660°$

44. $\sin(4530°)=\sin((12)360°+210°)$
$=\sin 210°=-\sin 30°=\frac{1}{2}$

45. Convert the corresponding values.

46. $\cos 23°\operatorname{cosec}(90°-23°)-\sin 23°\sec(90°-23°)$
$=1-1=0$

47. $\log(\tan 18°\tan 36°\tan 54°\tan 72°)=\log 1=0$
$[\because If\ A+B=90°\Rightarrow \tan A\tan B=1]$

48. $\alpha+\beta=180°$
$\Rightarrow \cos\alpha+\cos\beta=0$

49. Use $(\operatorname{cosec}^2\theta-\cot^2\theta)=1$

50. Use $(\operatorname{cosec}^2A-\cot^2A)=1$. Find the quadrant in which A lies

51. $A+B=180°\Rightarrow \tan A+\tan B=0$

52. Square and simplyfy

53. Put $\alpha=45°$

54. Put $A=0°$

55. $B+C=180°-A\Rightarrow \tan(A+A-180°)=\tan 2A$

56. Use $1=(coese^2\alpha-\cot^2\alpha)$

57. $(125°-x)+(55°+x)=180°$

58. If $(A+B)=\frac{\pi}{2}$ then $\sin^2A+\sin^2B=1$

59. $(A+B+C+D)=360°\Rightarrow\frac{A+B}{4}=\frac{\pi}{2}-\frac{C+}{4}$

60. $(\sin x-\cos x)^2+(-\cos x-\sin x)^2=2$

61. Put $\theta_1=\theta_2=\theta_3=0°$

62. Divide the given equation by $\cos^2x$ both sid

63. $\cos x=1-\cos^2x\Rightarrow \cos x=\sin^2x$

64. $\cot(\alpha+\beta)=0\Rightarrow\alpha+\beta=\frac{\pi}{2}$
$\Rightarrow\sin(\alpha+2\beta)=\sin(90°+\beta)=\cos\beta$

65. Put $\alpha=\frac{\pi}{4}$

66. $a\sin x+b\cos x=c$
$\Rightarrow b\sin x-a\cos x=\pm\sqrt{a^2+b^2-c^2}$

67. $K^2=(1+\sin A)(1+\sin B)(1+\sin C)$
$(1-\sin A)(1-\sin B)(1-\sin C)$
$K^2=(1-\sin^2A)(1-\sin^2B)(1-\sin^2C)$
$K^2=\cos^2A.\cos^2B.\cos^2C$
$K=\pm\cos A\cos B\cos C$

68. Put $\alpha=45°$; verify the options.

69. $1+\tan^2\theta+\tan\theta=13\Rightarrow\tan\theta=3 or -4$

70. $\log(\tan 45°)=\log 1=0$

71. $\alpha+\beta=90°,\alpha-\beta=30°\Rightarrow\alpha=60°,\beta=30°$

72. Squaring and adding

73. Put A=B=45°

74. $2\cos^2B=1+\tan^2A$

75. $\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta$

76. $x.\frac{1}{3}+4.\frac{1}{4}=3\Rightarrow x=6$

77. $A+B=90°\Rightarrow\cos^2A+\cos^2B=1$

78. $x=\sin 50°-\cos 50°$

79. $A+B=90°\Rightarrow\cot A\cot B=1$

80. $\sec^2\theta-\tan^2\theta=1$

81. $\sin^2\theta+\cos^2\theta=1$

82. A is obtuse
$\Rightarrow B+C<90°\Rightarrow\tan B\tan C<1$

83. $\sin\alpha=\frac{3}{5}, \cos\beta=\frac{3}{5}, \cos\alpha=\frac{4}{5}, \sin\beta=\frac{4}{5}$

84. $a^2\left(-\frac{1}{2}\right)^2-4a^2(-1)^2+2a^2\left(\frac{\sqrt{3}}{2}\right)^2$

85. Divide the Nr and Dr. with $\cos\theta$

86. Let $a\tan\theta+b\sec\theta=k$

$\therefore a^2-b^2=c^2-k^2 \Rightarrow k^2=c^2-a^2+b^2$

EXERCISE-II

CRTQ & SPQ LEVEL-II

TRIGNOMETRIC RELATIONS ELIMINATION OF 'θ' TRIGNOMETRIC RATIOS, VALUES

C.R.T.Q Class Room Teaching Questions

1. **If θ is not in 4th quadrant and $\tan\theta=-4/3$ then**

$5\sin\theta+10\cos\theta+9\sec\theta+16\operatorname{cosec}\theta+4\cot\theta=$

1) -1 2) $2/5$ 3) $4/5$ 4) 0

2. $f(x)=x^3-3x+5$ then $f\left(\sin\frac{3\pi}{2}\right)+f\left(\cos\frac{3\pi}{2}\right)=$

1) 10 2) 12 3) 14 4) 16

3. **If $\tan\theta, 2\tan\theta+2, 3\tan\theta+3$ are in G.P then the value of $\frac{7-5\cot\theta}{9-4\sqrt{\sec^2\theta-1}}$ is**

1) $\frac{12}{5}$ 2) $\frac{-33}{28}$ 3) $\frac{33}{100}$ 4) $\frac{12}{13}$

4. **In a triangle ABC, C = 90°, then the equation whose roots are tanA, tanB is**

1) $ab\,x^2+c^2x+ab=0$

2) $ab\,x^2+c^2x-ab=0$

3) $ab\,x^2-c^2x-ab=0$

4) $ab\,x^2-c^2x+ab=0$

5. **Which of the following is correct?**

1) $\sin 1^\circ>\sin 1$ 2) $\sin 1^\circ<\sin 1$

3) $\sin 1^\circ=\sin 1$ 4) $\sin 1^\circ=\frac{\pi}{180}\sin 1$

6. **If $a=\cos 3$ and $b=\sin 8$ then**

1) $a>0, b>0$ 2) $ab<0$ 3) $a>b$ 4) $ab>0$

7. $\operatorname{cosec}A=4p+\frac{1}{16p} \Rightarrow \operatorname{cosec}A+\cot A=$

1) 8p 2) $\frac{1}{8p}$

3) $-8p$ (or) $\frac{1}{8p}$ 4) $8p$ (or) $\frac{1}{8p}$

8. **If $\sqrt{\frac{1-\sin A}{1+\sin A}}=\sec A-\tan A$ then A lies in the quadrants**

1) I, II 2) II, III 3) I, IV 4) I, III

9. **If $\sin\theta+\cos\theta=m$ and $\sec\theta+\operatorname{cosec}\theta=n$, then n $(m+1)(m-1)=$**

1) m 2) n 3) 2m 4) 2n

10. If $1+\sin x+\sin^2x+.....\text{to}\,\infty=4+2\sqrt{3}, 0<x<\pi$ **and** $x\neq\frac{\pi}{2}$ **then** $x=$

1) 30°, 60° 2) 60°, 120°

3) 90°, 120° 4) 30°, 45°

11. $\cos^2 5^\circ+\cos^2 10^\circ+\cos^2 15^\circ+....+\cos^2 360^\circ=$

1) 18 2) 27 3) 36 4) 45

12. **If $x=a\cos^2\theta\sin\theta$ and $y=a\sin^2\theta\cos\theta$, then $\frac{(x^2+y^2)^3}{x^2y^2}=$**

1) a 2) a^3 3) a^2 4) a^5

13. $a\sec\theta+b\tan\theta=1, a\sec\theta-b\tan\theta=5$

$\Rightarrow a^2(b^2+4)=$

1) $3b^2$ 2) $9b^2$ 3) b^2 4) $4b^2$

14. **tanA = a tanB, sinA = b sinB** $\Rightarrow \frac{b^2-1}{a^2-1}=$

1) $\sin^2A$ 2) $\sin^3A$ 3) $\cos^2A$ 4) $\cos^3A$

15. If $\frac{\cos^2\theta}{a}=\frac{\sin^2\theta}{b}$ then $\frac{\cos^4\theta}{a}+\frac{\sin^4\theta}{b}=$

1) $\frac{1}{a+b}$ 2) $\frac{1}{(a+b)^2}$

3) $\frac{1}{a^2}+\frac{1}{b^2}$ 4) a + b

16. If $a=x\cos^2 A+y\sin^2 A$ then $(x-a)(y-a)+(x-y)^2\sin^2 A\cos^2 A=$

1) 0 2) 1 3) $xy+a^2$ 4) $xy-a^2$

17. If $a\cos^3\alpha+3a\cos\alpha\sin^2\alpha=m$ and $a\sin^3\alpha+3a\cos^2\alpha\sin\alpha=n$ then $(m+n)^{2/3}+(m-n)^{2/3}=$

1) $2a^2$ 2) $2a^{1/3}$ 3) $2a^{2/3}$ 4) $2a^3$

18. $\cos^4\alpha-\sin^4\alpha=a$ then $\frac{1-a}{1+a}=$

1) $\tan^2\alpha$ 2) $\cot^2\alpha$

3) $-\tan^2\alpha$ 4) $-\cot^2\alpha$

19. $a=\frac{1+\sin x}{1-\cos x+\sin x}\Rightarrow\frac{1+\cos x+\sin x}{2\sin x}=$

1) a 2) $\frac{1}{a}$ 3) a^2 4) $\frac{1}{a^2}$

20. Eliminate θ from $x=1+\tan\theta; y=2+\cot\theta$

1) $xy+1=x+y$ 2) $xy+2=2x+y$

3) $xy+1=2x+y$ 4) $xy+1=2y+x$

21. $f(x)=\sin^2 x+\operatorname{cosec}^2 x\Rightarrow$

1) $f(x)<1$ 2) $f(x)=1$

3) $1<f(x)<2$ 4) $f(x)\geq 2$

22. Which of the following is not possible ?

1) $\sin\theta=\frac{5}{7}$ 2) $\cos\theta=\frac{1+a^2}{1-a^2},|a|\neq 1$

3) $\tan\theta=100$ 4) $\sec\theta=\frac{5}{2}$

23. If $\theta=\frac{11\pi}{6}$, then $\cos\theta+\sin\theta=$

1) $\frac{\sqrt{3}+1}{\sqrt{2}}$ 2) $\frac{\sqrt{3}-1}{\sqrt{2}}$

3) $\frac{\sqrt{3}-1}{2}$ 4) $\frac{\sqrt{3}+1}{2}$

24. The value of $\sin\left(n\pi+(-1)^n\frac{\pi}{4}\right), n\in I$,

1) 0 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{-1}{\sqrt{2}}$ 4) $\frac{\sqrt{3}}{2}$

25. If $e^{(\sin^2 x+\sin^4 x+\sin^6 x+.....\infty)\log 2}=8$ a $0<x<\frac{\pi}{2}$ then $\frac{\cos x}{\cos x+\sin x}=$

1) $\frac{\sqrt{3}+1}{2}$ 2) $\frac{\sqrt{3}-1}{2}$

3) $\frac{2}{\sqrt{3}+1}$ 4) $\frac{2}{\sqrt{3}-1}$

26. If $0\leq x\leq\pi, 81^{\sin^2 x}+81^{\cos^2 x}=30$ then x

$1-\sin^2$

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{15}$ 4)

27. $\operatorname{cosec}^2\alpha=\frac{4xy}{(x+y)^2}$ $[(x,y)\neq(0,0)]$ is

1) Possible for x = y

2) Impossible for x = y

3) Possible for x = – y

4) Not possible

28. If $a\sin\theta+b\cos\theta=c$ then $\frac{a-b\tan\theta}{b+a\tan\theta}=$

1) $\frac{\pm\sqrt{b^2-c^2+a^2}}{c}$ 2) $\frac{\pm\sqrt{b^2+c^2-a^2}}{c}$

3) $\frac{\pm\sqrt{b^2-c^2-a^2}}{c}$ 4) $\frac{\pm\sqrt{a^2+b^2+c^2}}{b^2-c^2}$

S.P.Q. Student Practice Questions

29. If $\cos\theta=\frac{3}{5}$ and θ is not in the first quadrant, then
$\frac{5\tan(\pi+\theta)+4\cos(\pi+\theta)}{5\sec(2\pi-\theta)-4\cot(2\pi+\theta)}=$

1) $\frac{4}{5}$ 2) $-\frac{4}{5}$ 3) $\frac{5}{4}$ 4) $-\frac{5}{4}$

30. $f(x)=x^3-2x^2+3x-5$
$\Rightarrow f\left[\sin\left(\frac{5\pi}{2}\right)\right]+f\left[\sin\left(\frac{3\pi}{2}\right)\right]=$

1) 10 2) -10 11) 14 4) -14

31. cosA, sinA, cotA are in GP then $\tan^6 A-\tan^2 A=$
1) -1 2) 0 3) 1 4) 2

32. If $ABCD$ is a cydic quadrilateral such that $12\tan A-5=0$ and $5\cos B+3=0$, then $\cos C\tan D=$

1) $\frac{-16}{13}$ 2) $\frac{16}{13}$ 3) $\frac{-13}{16}$ 4) $\frac{23}{16}$

33. $x=\cos 1^\circ, y=\cos 1\Rightarrow$
1) $x=y$ 2) $x>y$
3) $x<y$ 4) $2x=y$

34. $a=\sec 2^\circ, b=\sec 2\Rightarrow$
1) $a=b$ 2) $a<b$ 3) $b<a$ 4) $2a=b$

35. $\tan\theta=P-\frac{1}{4p}\Rightarrow\sec\theta-\tan\theta=$

1) $2p$ (or) $\frac{1}{2p}$ 2) $\frac{1}{2p}$ (or) $-2p$

3) $-\frac{1}{2p}$ (or) $2p$ 4) $\frac{-1}{2p}$ (or) $-2p$

36. If $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$, then A lies in the quadrants
1) I, II 2) II, III 3) I, III 4) I, IV

37. $x=\tan\theta+\cot\theta, y=\cos\theta-\sin\theta\Rightarrow$

1) $x=y$ 2) $\frac{1-y^2}{2}=\frac{1}{x}$

3) $\frac{y^2-1}{2}=\frac{1}{x}$ 4) $\frac{1+y^2}{2}=\frac{1}{x}$

38. $1+\cos x+\cos^2 x+\ldots$ to $\infty=4+2\sqrt{3}$ then $x=$
1) 30° 2) 60° 3) 45° 4) 90°

39. $\sin^2 5^\circ+\sin^2 10^\circ+\ldots+\sin^2 180^\circ=$
1) 18 2) 27 3) 1 4) 0

40. If $x=a\cos^3\theta\sin^2\theta, y=a\sin^3\theta\cos^2\theta$ and $\frac{(x^2+y^2)^p}{(xy)^q}, (p,q\in N)$ is independent of θ then

1) $p+q=6$ 2) $4p=5q$

3) $4q=5p$ 4) $pq=16$

41. $a=\sec\theta-\tan\theta, b=\operatorname{cosec}\theta+\cot\theta\Rightarrow a=$

1) $\frac{b+1}{b-1}$ 2) $\frac{1+b}{1-b}$ 3) $\frac{b-1}{b+1}$ 4) $\frac{1-b}{1+b}$

42. $\cos A=a\cos B, \sin A=b\sin B\Rightarrow(b^2-a^2)\sin^2 B=$
1) $1+a^2$ 2) $2+a^2$ 3) $1-a^2$ 4) $2-a^2$

43. If $\frac{\sin x}{a}=\frac{\cos x}{b}=\frac{\tan x}{c}=k$ then $bc+\frac{1}{ck}+\frac{ak}{1+bk}$ is

1) $k\left(a+\frac{1}{a}\right)$ 2) $\frac{1}{k}\left(a+\frac{1}{a}\right)$ 3) $\frac{1}{k^2}$ 4) $\frac{a}{k}$

44. If $m\cos^2 A+n\sin^2 A=p$, then $\cot^2 A=$

1) $\frac{p+n}{m+p}$ 2) $\frac{p-n}{p-m}$

3) $\frac{p-n}{m-p}$ 4) $\frac{n+1}{m+p}$

45. If $a\sin^3 x + b\cos^3 x = \sin x\cos x$ and $a\sin x = b\cos x$ then $a^2+b^2=$
1) 0 2) 1 3) 2 4) 3

46. If $\sin\theta+\cos\theta=a$, then $\sin^4\theta+\cos^4\theta=$
1) $1-\frac{1}{2}(a^2+1)^2$ 2) $1-\frac{1}{2}(a^2-1)^2$
3) $1+\frac{1}{2}(a^2+1)^2$ 4) $1+\frac{1}{2}(a^2-1)^2$

47. If $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{x}{\tan\theta-\sec\theta+1}$ then $x=$
1) 0 2) 2
3) $\tan\theta-\sec\theta+1$ 4) $\tan\theta+\sec\theta-1$

48. If $\tan\theta+\sin\theta=m$, $\tan\theta-\sin\theta=n$, then $(m^2-n^2)^2=$
1) 16 mn 2) 4 mn 3) 32 mn 4) 8 mn

49. If $\sec\theta+\cos\theta=2$ then $\sin^2\theta+\tan^2\theta=$
1) 0 2) 1 3) 2 4) 4

50. Which of the following possible?
1) $\sin\theta=2$ 2) $\cos\theta=\frac{17}{5}$
3) $\tan\theta=2$ 4) $\sec\theta=\frac{4}{5}$

51. If $\theta=\frac{\pi}{21}$, then $\frac{\sin 23\theta-\sin 7\theta}{\sin 2\theta+\sin 14\theta}=$
1) 0 2) 1 3) -1 4) 2

52. The value of $\tan\left(n\pi+\frac{\pi}{3}\right), n\in I$ is
1) $\sqrt{3}$ 2) $1/\sqrt{3}$ 3) 0 4) $1/\sqrt{2}$

53. If $e^{(1+\sin^2 x+\sin^4 x+\ldots\infty)\log 2}=16$, then $\tan^2 x=$
1) 1 2) 2 3) 3 4) 4

54. If $2\sin x+5\cos y+7\sin z=14$ then $7\tan\frac{x}{2}+4\cos y-6\cos z=$
1) 4 2) -3 3) 11 4) 5

55. If $\sin\theta$ and $\cos\theta$ are the roots of $px^2+qx+r=0$ then $q^2-p^2=$
1) 0 2) $-2pr$ 3) $2qr$ 4) $2rp$

56. If $0\le x\le\pi$, $4^{\sin^2 x}+4^{\cos^2 x}=5$, then $x=$
1) $\frac{\pi}{6}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

KEY

01) 4	02) 2	03) 2	04) 4	05)
06) 2	07) 4	08) 3	09) 3	10)
11) 3	12) 3	13) 2	14) 3	15)
16) 1	17) 3	18) 1	19) 1	20)
21) 4	22) 2	23) 3	24) 2	25)
26) 1	27) 1	28) 1	29) 2	30)
31) 3	32) 1	33) 2	34) 3	35)
36) 4	37) 2	38) 1	39) 1	40)
41) 3	42) 3	43) 2	44) 3	45)
46) 2	47) 4	48) 1	49) 1	50)
51) 3	52) 1	53) 3	54) 3	55)
56) 4				

Hints & Solutions

1. θ lies in II quadrant, substitute the value
2. $\sin 270^\circ=-1$ & $\cos 270^\circ=0$
3. $(2\tan\theta+2)^2=\tan\theta(3\tan\theta+3)$
$\Rightarrow \tan^2\theta+5\tan\theta+4=0 \Rightarrow \tan\theta=-1,-4$

but $\tan\theta=-4$ only satisfies above condition

4. $\tan A=\frac{a}{b}$, $\tan B=\frac{b}{a}$ and $(a^2+b^2)=c^2$
Find sum and product of roots

5. $1>1^0$

6. 3^c lies in the second quadrant $\therefore \cos 3<0$
8^c lies in the second quadrant $\therefore \sin 8>0$
$\Rightarrow ab<0$

7. Use $(\operatorname{cosec}^2 A-\cot^2 A)=1$
If $\operatorname{cosec} A=\left(4P+\frac{1}{16P}\right)$
then $\cot A=\pm\left(4P-\frac{1}{16P}\right)$

8. $\sqrt{\cos^2 A}=\cos A$ then $A\in Q_1,Q_4$

9. Find m^2 and multiply

10. Use formula $S_\infty=\frac{a}{1-r}$ for GP

11. If $(A+B)=360^0$ or $(A+B)=180^0$
then $\cos^2 A=\cos^2 B$ and $(A+B)=90^0$
then
$(\cos^2 A+\cos^2 B)=1$

12. θ can be eliminated by substitution.

13 Given $(a\sec\theta+b\tan\theta)=1$
$\Rightarrow(a\sec\theta-b\tan\theta)=5$, eliminate θ

14. Put $A=30^0$ and $B=45^0$ and verify

15. Find $\tan\theta$ and simplify

16. Put $A=90^o$

17. $m+n=a(\cos\alpha+\sin\alpha)^3$
$m-n=a(\cos\alpha-\sin\alpha)^3$

18. $a=\cos^2\alpha-\sin^2\alpha$
$\Rightarrow 1-a=2\sin^2\alpha$ and $1+a=2\cos^2\alpha$

19. In R.H.S. of given equation multiply and divide by $(1+\cos x+\sin x)$

20. $x-1=\tan\theta, y-2=\cot\theta$
Multiply two equations

21. $f(x)=\left(\sin^2 x+\frac{1}{\sin^2 x}\right)\geq 2$
$\because\left(a+\frac{1}{a}\right)\geq 2$ for any real number $a>0$

22. for $a=2$, $\cos\theta=\frac{5}{-3}$ not possible

23. Put $\theta=330^0$ substitution

24. Put $n=0$

25. $\sin^2 x+\sin^4 x+\ldots\infty=\frac{\sin^2 x}{1-\sin^2 x}=\tan^2 x$
$\therefore\left(e^{\log_e 2}\right)^{\tan^2 x}=8\Rightarrow 2^{\tan^2 x}=2^3\Rightarrow\tan x=\sqrt{3}$
$\therefore$ G.E $=\frac{1}{1+\tan x}=\frac{1}{1+\sqrt{3}}=\frac{\sqrt{3}-1}{2}$

26. If $81^{\sin^2 x}=y\Rightarrow 81^{\cos^2 x}=\frac{81}{y}$
$\therefore y+\frac{81}{y}=30\Rightarrow y^2-30y+81=0$
$\Rightarrow y=27,3$

27. $\because \operatorname{cosec}\alpha\in(-\infty,-1]\cup[1,\infty)$

28. $G.E=\frac{a\cos\theta-b\sin\theta}{a\sin\theta+b\cos\theta}=\frac{\pm\sqrt{b^2-c^2+a^2}}{c}$

29. lies in fourth quandrant substitute the corresponding values.

30. $\sin\frac{3\pi}{2}=-1, \sin\frac{5\pi}{2}=1$

31. $\sin^2 A=\cos A.\cot A\ \Rightarrow\sin^3 A=\cos^2 A$
$\Rightarrow\tan^2 A=\operatorname{cosec} A$ and simplify

32. $A+C=180^\circ, B+D=180^\circ$

Given $\tan A=\frac{5}{12}>0 \therefore 0<A<\frac{\pi}{2} \Rightarrow \frac{\pi}{2}<C<\pi$

$\cos B=\frac{-3}{5}<0 \therefore \frac{\pi}{2}<B<\pi \Rightarrow 0<D<\frac{\pi}{2}$

$\therefore \cos C=\frac{-12}{13}; \tan D=\frac{4}{3}$

33. $1>1^0 \Rightarrow \cos 1<\cos 1^0$

34. $2=2^c=114^0$ (approx)

35. Use $\left(\sec^2\theta-\tan^2\theta\right)=1$

36. $\sqrt{\cos^2 A}=\cos A$ then $A\in Q_1, Q_4$

37. Eliminate θ

38. Use formula $S_\infty=\frac{a}{1-r}$ for G.P

39. $A+B=180^\circ \Rightarrow \sin^2 A=\sin^2 B$
and $A+B=90^\circ \Rightarrow \sin^2 A+\sin^2 B=1$

40. $\frac{(x^2+y^2)^p}{(xy)^q}=\frac{(a^2\sin^4\theta\cos^4\theta)^p}{(a^2\sin^5\theta\cos^5\theta)^q}$

$=\frac{a^{2p}(\sin\theta\cos\theta)^{4p}}{a^{2q}(\sin\theta\cos\theta)^{5q}}$

which is independent of θ, if 4p=5q

41. Put $\theta=45^0$ and verify the options

42. squaring and adding

43. $bc+\frac{1}{ck}+\frac{ak}{1+bk}$

$=\frac{\sin x}{k^2}+\frac{\cos x(1+\cos x)+\sin^2 x}{\sin x(1+\cos x)}$

$=\frac{a}{k}+\frac{1}{\sin x}=\frac{a}{k}+\frac{1}{ak}=\frac{1}{k}\left(a+\frac{1}{a}\right)$

44. divided with $\sin^2 A$

weget $\cot^2 A=\frac{p-n}{m-p}$

45. Put $\sin x=b$ *and* $\cos x=a$

$\therefore a^2+b^2=\cos^2 x+\sin^2 x=1$

46. Use $\left(a^4+b^4\right)=\left(a^2+b^2\right)^2-2a^2b^2$

47. In LHS divide the Num and Den by co

48. $2\tan\theta=(m+n)$;

$2\sin\theta=(m-n)$

Eliminate θ and use

$\left(\tan^2\theta-\sin^2\theta\right)=\tan^2\theta.\sin^2\theta$

49. Put θ=0

50. Range of $\tan\theta$ is R

51. Use $21\theta=\pi$

52. put n = 0

53. $\left(e^{\log 2}\right)^{\frac{1}{1-\sin^2 x}}=16 \Rightarrow 2^{\frac{1}{\cos^2 x}}=2^4$

$\Rightarrow \sec^2 x=4 \Rightarrow \tan^2 x=3$

54. put $x=\frac{\pi}{2}, y=0, z=\frac{\pi}{2}$.

55. $\sin\theta+\cos\theta=\frac{-q}{p}, \sin\theta\cos\theta=\frac{r}{p}$

now $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$

56. x = 90°, verify

EXERCISE-III
CRTQ & SPQ LEVEL-III

TRIGNOMETRIC RELATIONS, VALUE

C.R.T.Q Class Room Teaching Questions

1. $\tan^2\alpha=1-p^2$, then

$\sec\alpha+\tan^3\alpha\,\text{cosec}\,\alpha=$

1) $(2+p^2)^{\frac{3}{2}}$ 2) $(1+p^2)^{\frac{3}{2}}$

3) $(2-p^2)^{\frac{3}{2}}$ 4) $(1-p^2)^{\frac{3}{2}}$

2. $\frac{\pi}{2}<\alpha<\pi \Rightarrow \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}+\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=$

1) $2\sec\alpha$ 2) $-2\sec\alpha$
3) $2\text{cosec}\,\alpha$ 4) $-2\text{cosec}\,\alpha$

3. If $\sin\beta$ is the G.M. between $\sin\alpha$ and $\cos\alpha$ then $(\cos\alpha-\sin\alpha)^2-2\cos^2\beta=$
1) 0 2) 1 3) 2 4) -1

4. If $\sin^2\theta=\frac{x^2+y^2+1}{2x}$ then x must be
1) -3 2) -2 3) 1 4) 3

5. $\sin x+\sin^2 x+\sin^3 x=1\Rightarrow \cos^6 x-4\cos^4 x+8\cos^2 x=$
1) 4 2) 2 3) 1 4) 0

S.P.Q. Student Practice Questions

6. The greatest among $(\sin 1+\cos 1)$, $(\sqrt{\sin 1}+\sqrt{\cos 1})$, $(\sin 1-\cos 1)$ and 1 is
1) $\sin 1+\cos 1$ 2) $\sqrt{\sin 1}+\sqrt{\cos 1}$
3) $\sin 1-\cos 1$ 4) 1

7. If $\tan^2\alpha\tan^2\beta+\tan^2\beta\tan^2\gamma+\tan^2\gamma\tan^2\alpha+2\tan^2\alpha\tan^2\beta\tan^2\gamma=1$ then the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$
1) 0 2) -1 3) 1 4) 2

8. Let $\theta\in\left(0,\frac{\pi}{4}\right)$ and $t_1=(\tan\theta)^{\tan\theta}$, $t_2=(\tan\theta)^{\cot\theta}$, $t_3=(\cot\theta)^{\tan\theta}$, $t_4=(\cot\theta)^{\cot\theta}$ then
1) $t_1>t_2>t_3>t_4$ 2) $t_4>t_3>t_1>t_2$
3) $t_3>t_1>t_2>t_4$ 4) $t_2>t_3>t_1>t_4$

9. Two arcs of same length of two different circles subtended angles of 25° and 75° at their centres respectively. Then the ratio of the radii of the circles is
1) 3 : 1 2) 1 : 3 3) 1 : 2 4) 2 : 1

ELIMINATION OF 'θ'

C.R.T.Q Class Room Teaching Questions

10. $\cos\theta+\cos^2\theta=1$, $a\sin^{12}\theta+b\sin^{10}\theta+c\sin^8\theta+d\sin^6\theta=1\Rightarrow\frac{b+c}{a+d}=$
1) 2 2) 3 3) 4 4) 6

11. $a\sin x=b\cos x=\frac{2c\tan x}{1-\tan^2 x}$ and $(a^2-b^2)^2=kc^2(a^2+b^2)\Rightarrow k=$
1) 1 2) 2 3) 3 4) 4

S.P.Q. Student Practice Questions

12. If $a\sin^2 x+b\cos^2 x=c$, $b\sin^2 y+a\cos^2 y=d$ and $a\tan x=b\tan y$ then $\frac{a^2}{b^2}=$
1) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$ 2) $\frac{(b-c)(b-d)}{(a-c)(a-d)}$
3) $\frac{(b-c)(d-b)}{(a-d)(c-a)}$ 4) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$

KEY

01) 3	02) 3	03) 4	04) 3	05) 1
06) 2	07) 3	08) 2	09) 1	10) 2
11) 4	12) 1			

Hints & Solutions

1. $\sec\alpha\left[1+\frac{\sin^3\alpha\cos\alpha}{\cos^3\alpha\sin\alpha}\right]$

$=\sec\alpha\left[1+\tan^2\alpha\right]=\sec^3\alpha$

$=(\sec^2\alpha)^{3/2}=(2-P^2)^{3/2}$

2. Simplifying we get $\frac{2}{\sqrt{\sin^2\alpha}}=2\operatorname{cosec}\alpha$
$\because \sin\alpha$ is positive in II quadrant.
3. $\sin^2\beta=\sin\alpha\cos\alpha$
4. $\sin^2\theta\le 1$
5. $\sin x\left(1+\sin^2 x\right)=\cos^2 x$
Squaring both sides we get
$\left(1-\cos^2 x\right)\left(2-\cos^2 x\right)^2=\cos^4 x$
Simplify further
6. $\sqrt{\sin 1}>\sin 1>\sin^2 1$
$\sqrt{\cos 1}>\cos 1>\cos^2 1$
Adding $\left(\sqrt{\sin 1}+\sqrt{\cos 1}\right)>\left(\sin 1+\cos 1\right)$
$>\left(\sin^2 1+\cos^2 1\right)$
$\Rightarrow\left(\sqrt{\sin 1}+\sqrt{\cos 1}\right)>\left(\sin 1+\cos 1\right)>1$
7. Let $\tan^2\alpha=x,\tan^2\beta=y,\tan^2\gamma=z$
$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma=\sum\frac{\tan^2\alpha}{1+\tan^2\alpha}=\sum\frac{x}{1+x}$$
$$=\frac{(x+y+z)+(xy+yz+zx+2xyz)+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)}$$
$$=\frac{1+x+y+z+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)}=1$$
8. In $\left(0,\frac{\pi}{4}\right)$, $\tan\theta<1$ and $\cot\theta>1$
$(\cot\theta)^{\cot\theta}>(\cot\theta)^{\tan\theta}>(\tan\theta)^{\tan\theta}>(\tan\theta)^{\cot\theta}$
$\therefore t_4>t_3>t_1>t_2$
9. Length of arc
10. $\cos\theta=1-\cos^2\theta \quad\Rightarrow\cos\theta=\sin^2\theta$
$\Rightarrow\cos^2\theta=\sin^4\theta \quad\Rightarrow\left(\sin^4\theta+\sin^2\theta\right)=1$
Cubing both sides we get the answer
11. Take $a\sin x=b\cos x=c\tan 2x=l$
$\Rightarrow\sin x=\frac{l}{a},\cos x=\frac{l}{b},\tan 2x=\frac{l}{c}$
Elimination l we get
$\left(a^2-b^2\right)^2=4c^2\left(a^2+b^2\right)\Rightarrow K=4$
12. $a\sin^2 x+b\cos^2 x=c$
$\Rightarrow a\tan^2 x+b=c\sec^2 x$
$\Rightarrow(a-c)\tan^2 x=c-b \quad\Rightarrow\tan^2 x=\frac{c-b}{a-c}$
similarly $\tan^2 y=\frac{d-a}{b-d}$
but $a\tan x=b\tan y\Rightarrow\frac{a^2}{b^2}=\frac{\tan^2 y}{\tan^2 x}$

EXERCISE-IV
LEVEL-IV

1. **Assertion A: In a right angled triangle $\sin^2 A+\sin^2 B+\sec^2 C=2$.**
Reason R: If α,β are complementary angles then $\sin^2\alpha+\sin^2\beta=1$
1) A, R are true and R is the correct explanation of A
2) A, R are true and R is not the correct expla nation of A
3) A is true, R is false
4) A is false, R is true
2. **Observe the following lists. Let $\frac{\sin x}{a}=\frac{\cos x}{b}=\frac{\tan x}{c}=k$.**

List - I	List-II
1) bc	a) $\frac{1}{b^2k^4}$
2) a^2+b^2	b) $\frac{1}{ak}$
3) $\frac{1}{ck}+\frac{ak}{1+bk}$	c) $\frac{a}{k}$
4) $a^2+b^2+c^2$	d) $\frac{1}{k^2}$

The correct match for List-I form List-II is
1) 1 - c, 2 - d, 3 - b, 4 - a
2) 1 - d, 2 - a, 3 - c, 4 - b
3) 1 - a, 2 - b, 3 - d, 4 - c
4) 1 - b, 2 - c, 3 - a, 4 - d

3. Observe the following lists.

List-I	List-II
I. $\sin^2\frac{2\pi}{3}+\cos^2\frac{5\pi}{6}-\tan^2\frac{3\pi}{4}$	a) $\frac{15}{2}$
II. $\sin^2\frac{11\pi}{6}+\cos^2\frac{7\pi}{6}-\tan^2\frac{5\pi}{4}$	b) $-\frac{1}{2}$
III. $\sin^2\frac{3\pi}{4}+\sec^2\frac{5\pi}{3}+\tan^2\frac{2\pi}{3}$	c) 0
IV. $\cos^2\frac{2\pi}{3}+\sin^2\frac{5\pi}{3}-\frac{3}{2}\tan^2\frac{3\pi}{4}$	d) 1/2

The correct match for List-I from List-II is

1) I-d, II-c, III-a, IV-b
2) I-a, II-b, III-c, IV-d
3) I-b, II-a, III-d, IV-c
4) I-c, II-d, III-b, IV-a

4. Satement I:The number of values of θ satisfying $\sec\theta+\cos\theta=1$ is 2.

Statement II: If $7\sin^2\alpha+3\cos^2\alpha=4$, then $\tan\alpha=\sqrt{3}$.

Which of the above statements is correct ?

1) only I is true
2) only II is true
3) Both I and II are true
4) Neither I nor II is true

5. If $\alpha=\cos 10^0-\sin 10^0$, $\beta=\cos 45^0-\sin 45^0$, $\gamma=\cos 70^0-\sin 70^0$ then the descending order of α,β,γ is

1) α,β,γ 2) γ,β,α
3) α,γ,β 4) β,α,γ

6. Match the following:

List - I	List - II
1) $3\tan x+27\cot x\ge$ $(x\in Q_1)$	a) 24
2) $5\sec^2 x+125\cos^2 x\ge$	b) 18
3) $16\operatorname{cosec}^2 x+9\sin^2 x\ge$	c) 50

1) 1-a, 2-b, 3-c 2) 1-c, 2-a, 3-b
3) 1-b, 2-c, 3-a 4) 1-c, 2-b, 3-a

7. A = tan1, B = tan2, C = tan3, then the descending order of A, B, C is

1) A, B, C 2) C, B, A
3) A, C, B 4) B, C, A

KEY

01) 4 02) 1 03) 1 04) 4 05) 1
06) 3 07) 3

Hints & Solutions

1. In a right angled triangle
$\sin^2 A+\sin^2 B+\sec^2 C\ne 2$
2. Simplify
3. Simplify
4. I. $(\sec\theta+\cos\theta)=1$ is not possible for $\theta\in R$
II. Divide given equation by $\cos^2\alpha$
5. α is positive, β is zero, γ is negative
6. $A.M\ge G.M$
$a\tan x+b\cot x\ge 2\sqrt{ab}$
7. $1=57°$; $2=114°$; $3=171°$ (Aprox)

INTEGER & NUMERICAL ANSWER TYPE QUESTIONS

1. The value of the expression $\dfrac{\tan^2 20^0 - \sin^2 20^0}{\tan^2 20^0 . \sin^2 20^0}$ is 1

2. Suppose that for some angles x and y, the equations $\sin^2 x + \cos^2 y = \dfrac{3a}{2}$ and $\cos^2 x + \sin^2 y = \dfrac{a^2}{2}$ hold simultaneously. The possible value of a is 1

3. If $0 < x < \dfrac{\pi}{4}$ and $\cos x + \sin x = \dfrac{5}{4}$, then the value of $16(\cos x - \sin x)^2$ is 7

4. The value of $3\dfrac{\sin^4 t + \cos^4 t - 1}{\sin^6 t + \cos^6 t - 1}$ is equal to 2

5. If $\sin\theta - \cos\theta = 1$, then the value of $\sin^3\theta - \cos^3\theta$ is 1

6. If $\sin\theta, \tan\theta, \cos\theta$ are in G.P. then $4\sin^2\theta - 3\sin^4\theta + \sin^6\theta =$ ____

7. Let $f(\theta) = \dfrac{1}{1+(\cot\theta)^x}$, and $S = \sum\limits_{\theta=1^0}^{89^0} f(\theta)$, then the value of $\sqrt{2S-8}$ is____

8. The minimum value of $\sqrt{(3\sin x - 4\cos x - 10)(3\sin x + 4(\cos x - 10))}$ is____

9. If $a \in (0,1)$, and $f(a) = (a^2 - a + 1) + \dfrac{8\sin^2 a}{\sqrt{a^2-a+1}} + \dfrac{27\cos ec^2}{\sqrt{a^2-a+1}}$, then the least value of $f(a)/2$ is 9

10. Minimum value of $\dfrac{\sec^4\alpha}{\tan^2\beta} + \dfrac{\sec^4\beta}{\tan^2\alpha}$, where $\alpha \neq \dfrac{\pi}{2}$, $\beta \neq \dfrac{\pi}{2}, 0 < \alpha$, $\beta < \dfrac{\pi}{2}$, is____

$-q^2\cot^2\theta = \ldots$

$\sqrt{81p^{-2} - q^{-2}}$ is____

12. $\cos 5^0 + \cos 24^0 + \cos 175^0 + \cos 204^0 + \cos 300^0 =$ ____

13. $\cos 1^0 + \cos 2^0 + \cos 3^0 + ... + \cos 179^0 =$ ____

14. $\tan 10^0 . \tan 20^0 .... \sin 80^0 =$ ____

15. $\cot\dfrac{\pi}{20}.\cot\dfrac{3\pi}{20}.\cot\dfrac{5\pi}{20}.\cot\dfrac{7\pi}{20}.\cot\dfrac{9\pi}{20} =$ ____

16. $\sin 120^0 \cos 150^0 - \cos 240^0 \sin 330^0 =$ ____

17. In $\Delta ABC, \cos\left(\dfrac{B+2C+3A}{2}\right) + \cos\left(\dfrac{A-B}{2}\right) =$ ____

18. If A, B, C, D are the angles of a cycl[ic] quadrilateral, then $\cos A + \cos B + \cos C + \cos D =$ ____

19. If α, β are complementary angles, then $\sin^2\alpha + \sin^2\beta =$ ____

20. If α, β are supplementary angles, then $\sin^2\alpha + \sin^2\beta =$ ____

21. If $180^0 < \theta < 270^0$ and $\sin\theta = -5/13$ then $5\cot^2\theta + 12\tan\theta + 13\cos ec\theta =$ ____

22. If $\sin(\alpha+\beta) = 1$, $\sin(\alpha-\beta) = 1/2$ the[n] that $(\alpha + 2\beta)\tan(2\alpha+\beta) =$ ____

23. $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \sin^2 x) + 1 =$ ____

24. If $(\sin\alpha + \cos ec\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = K + \tan^2\alpha + \cot^2\alpha$ then $K =$ ____

25. $\cot^2\theta\left(\dfrac{\sec\theta - 1}{1+\sin\theta}\right) + \sec^2\theta\left(\dfrac{\sin\theta - 1}{1 + s\sec\theta}\right) =$ ____

26. $\left(\dfrac{\sqrt{3}+2\cos A}{1-2\sin A}\right)^{-3} + \left(\dfrac{1+2\sin A}{\sqrt{3}-2\cos A}\right)^{-3} =$ ____

27. $\sin^4\theta + 2\sin^2\theta\left(1 - \dfrac{1}{\cos ec^2\theta}\right) + \cos^4\theta =$ ____

28. If $\sin\theta+\sin^2\theta=1$, then $\cos^2\theta+\cos^4\theta=$

29. If $\sin x+\sin^2 x=1$ then $\cos^8 x+2\cos^6 x+\cos^4 x=$____

30. If $\cos x+\cos^2 x=1$, then $\sin^{12} x+3\sin^{10} x+3\sin^{10} x+3\sin^8 x+\sin^6 x=$____

31. If $\sin\alpha+\operatorname{cosec}\alpha=2$, then $\sin^n\alpha+\operatorname{cosec}^n\alpha=$____

32. If $P_n=\cos^n x+\sin^n x$, then $2P_6-3P_4+1=$

33. If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$ then $\cos\theta_1+\cos\theta_2+\cos\theta_3=$____

34. If $\sin A,\cos A,\tan A$ form a G.P. then $\cot^6 A-\cot^2 A=$____

35. If $\sin\theta,\cos\theta,\tan\theta$ are in G.P. then $\cos^9\theta+\cos^6\theta+3\cos^5\theta-1=$____

36. If $\sin\alpha,\sin\beta,\sin\gamma$ are in A.P and $\cos\alpha,\cos\beta,\cos\gamma$ are in G.P. then

$$\frac{\cos^2\alpha+\cos^2\gamma-4\cos\alpha\cos\gamma}{1-\sin\alpha\sin\gamma}=\_\_\_\_$$

37. If $\tan^2\alpha\tan^2\beta+\tan^2\beta\tan^2\gamma+\tan^2\gamma\tan^2\alpha+2\tan^2\alpha\tan^2\beta\tan^2\gamma=1$ then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$____

38. If $\frac{\sin x+\cos x}{\cos^3 x}=a\tan^3 x+b\tan^2 x+c\tan x+d$ then $a+b+c+d=$____

39. $\sqrt{1-\sin^2 100^\circ}.\sec 100^\circ=$____

40. If $\sin\theta+\cos\theta=a$, and $\tan\theta+\cot\theta=b$ then $b\left(a^2-1\right)=$____

41. If $a\sin^3 x+b\cos^3 x=\sin x\cos x$ and $a\sin x=b\cos x$ then $a^2+b^2=$____

42. If $\operatorname{cosec}\theta-\sin\theta=m$, $\sec\theta-\cos\theta=n$, then $\left(m^2n\right)^{2/3}+\left(mn^2\right)^{2/3}=$____

43. If $a=x\cos^2\alpha+y\sin^2\alpha$ then $(x-a)(y-a)+(x-y)^2\sin^2\alpha\cos^2\alpha=$____

44. If $m^2+m_1^2+2mm_1\cos\theta=1=n^2+n_1^2+2nn_1\cos\theta$ and $mn+nA_1n_1+(mn_1+m_1n)\cos\theta=0$ then $\left(m^2+n^2\right)\sin^2\theta=$____

45. If $a=\cos\phi\cos\theta+\sin\phi\sin\theta\cos\delta$, $b=\cos\phi\sin\theta-\sin\phi\cos\theta\cos\delta$ and $c=\sin\phi\sin\delta$, then $a^2+b^2+c^2=$____

46. The absolute value of k, for which $(\cos x+\sin x)^2+k\sin x\cos x-1=0$ is an identity, is

47. If $\frac{\sin^4 A}{a}+\frac{\cos^4 A}{b}=\frac{1}{a+b}$, then the value of $\frac{\sin^8 A}{a^3}+\frac{\cos^8 A}{b^3}$ is equal to $\frac{a^p b^q}{(a+b)^r}$. Find value of p+q+r.

48. If $\sin^3 x\sin 3x=\sum_{m=0}^{n}C_m\cos mx$ where $c_0,c_1,c_2,\ldots\ldots\ldots c_n$ are constants and $c_n\neq0$, then the value of n is

49. For a positive integer n, let $f_n(\theta)=\left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec2\theta)(1+\sec4\theta)\ldots\ldots(1+\sec2^n\theta)$.

Then $7\,f_5\left(\frac{\pi}{128}\right)$ is ?

50. If $k=\sin\frac{\pi}{18}\cdot\sin\frac{5\pi}{18}\cdot\sin\frac{7\pi}{18}$, then the numberical value of 16k is

51. If $\sin x+\sin^2 x=1$, then the value of expression $\cos^{12} x+3\cos^{10} x+3\cos^8 x+\cos^6 x-1$ is equal to

52. If

$(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)$
$=(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$,
then each side is equal to

53. If 7 then the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is equal to ....

54. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of thesegments and A_0A_1, A_0A_2 and A_0A_4 is ....

55. $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]$

$-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]=$

56. The maximum value of $\cos\alpha_1, \cos\alpha_2, \ldots\ldots\ldots\cos\alpha_n$, under the restrictions $0\le\alpha_1,\alpha_2\ldots\ldots\ldots\ldots\alpha_n\le\frac{\pi}{2}$ and $\cot\alpha_1.\cot\alpha_2\ldots\ldots\ldots\cot\alpha_n=1$ is m, Find value of $2^{n/2}$ x m.

57. If $A=\sin^8\theta+\cos^{14}\theta$, then maximum value of A is

58. If θ is and acute angle and $\sin\theta=\frac{p-6}{8-p}$, minimum possible value of p is ?

59. If A, B, C be the angles of a triangle, then $\Sigma\frac{\cot A+\cot B}{\tan A+\tan B}=$

60. If

$\left|a\sin^2\theta+b\sin\theta\cos\theta+c\cos^2\theta-\frac{1}{2}(a+c)\right|\le\frac{1}{2}k,$

then k^2 is equal to $a^2+b^2+c^2$ + xab + ybc + zca. Find value of –x–y–z.

61. If $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\frac{x}{\cos\theta}=\frac{z}{\cos\left(\theta+\frac{2\pi}{3}\right)}$

then $x+y+z=$

62. Value of $\sqrt{3}\operatorname{cosec}20^0-\sec 20^0=$

Answers

1) 1	2) 1	3) 7	4) 2	5) 1
6) 1	7) 9	8) 7	9) 9	10) 8
11) 4	12) 0.5	13) 0	14) 1	15) 1
16) –1	17) 0	18) 0	19) 1	20) 1
21) 0	22) 1	23) 0	24) 7	25) 0
26) 0	27) 1	28) 1	29) 1	30) 1
31) 2	32) 0	33) 0	34) 1	35) 1
36) –2	37) 1	38) 4	39) –1	40) 2
41) 1	42) 1	43) 0	44) 1	45) 1
46) –2	47) 3	48) 6	49) 7	50) 2
51) 0	52) 1	53) 0	54) 3	55) 1
56) 1	57) 1	58) 6	59) 1	60) 2
61) 0	62) 4			

Hints & Solutions

1. $\tan^2 20^0-\sin^2 20^0=\tan^2 20^0\left(1-\cos^2 20\right.$

$=\tan^2 20^0\sin^2 20^0$

2. $\sin^2 x+\cos^2 y=\frac{3a}{2}$ .....(i)

and $\cos^2 x+\sin^2 y=\frac{a^2}{2}$ .....(ii)

Adding (i) and (ii), we get $2=\frac{3a}{2}+\frac{a^2}{2}$

or $a^2+3a-4=0$

or $(a+4)(a-1)=0$

$\therefore a=-4$ (rejected), $a=1$

Let $y=\cos x-\sin x\,(y>0)$

$y^2=1-2\sin x.\cos x$

$\cos x+\sin x=\dfrac{5}{4}$

$\therefore 1+2\sin x\cos x=\dfrac{25}{16}$

or $2\sin x.\cos x=\dfrac{9}{16}$

Now $y^2=1-2\sin x\cos x=1-\dfrac{9}{16}=\dfrac{7}{16}$

4. $\sin^4 t+\cos^4 t-1=\left(\sin^2 t+\cos^2 t\right)^2$

$-2\sin^2 t\cos^2 t-1=-2\sin^2 t\cos^2 t$

$\sin^6 t+\cos^6 t-1=\left(\sin^2 t+\cos^2 t\right)^3$

$-3\sin^2 t\cos^2 t-1=-3\sin^2 t\cos^2 t$

Hence, $3\dfrac{\sin^4 t+\cos^4 t-1}{\sin^6 t+\cos^6 t-1}=2$

5. $\sin\theta-\cos\theta=1$

or $\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta=1$

or $\sin\theta\cos\theta=0$

Now $\sin^3\theta-\cos^3\theta=1$

$\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)$

$=\left(\sin\theta-\cos\theta\right)^2+3\sin\theta\cos\theta$

$=1+3\sin\theta\cos\theta=1$

6. Given that $\tan^2\theta=\sin\theta\cos\theta$

$\therefore \sin\theta=\cos^3\theta$

Also, given expression can be rewritten as

$\sin^6\theta-3\sin^4\theta+3\sin^2\theta-1+1+\sin^2\theta$

$=\left(\sin^2\theta-1\right)^3+1+\sin^2\theta$

$=-\left(1-\sin^2\theta\right)^3+1+\sin^2\theta$

$=-\cos^6\theta+1+\sin^2\theta=-\sin^2\theta+1+\sin^2\theta=1$

7. We have, $f(\theta)=\dfrac{(\sin\theta)^x}{(\cos\theta)^x+(\sin\theta)^x}$

$\Rightarrow f(\theta)+f\left(\dfrac{\pi}{2}-\theta\right)=1$

$\therefore S=\sum_{\theta=1^0}^{89^0} f(\theta)=f(1^0)+f(2^0)+....+$

$f(88^0)+f(89^0)$

$=\left(\underbrace{1+1+1+.....+}_{44\text{ times}}\right)+\dfrac{1}{2}=44+\dfrac{1}{2}=\dfrac{89}{2}$

$\therefore 2S-8=81$

8. $f(x)=9\sin^2 x-16\cos^2 x-10$

$(3\sin x-4\cos x)-10(3\sin x+4\cos x)$

$+100=25\sin^2 x-60\sin x+84$

$=(5\sin x-6)^2+48$

The minimum value of $f(x)$ occurs when $\sin x=1$. Therefore, the minimum value of $\sqrt{f(x)}$ is 7.

9. Using $A.M.\geq G.M.$ (for positive numbers)

$\left(a^2-a+1\right)+\dfrac{8\sin^2 a}{\sqrt{a^2-a+1}}+\dfrac{27\operatorname{cosec}^2 a}{\sqrt{a^2-a+1}}$

$\geq 3\left[\left(a^2-a+1\right)\dfrac{8\sin^2 a}{\sqrt{\left(a^2-a+1\right)}}\dfrac{27\operatorname{cosec}^2 a}{\sqrt{\left(a^2-a+1\right)}}\right]^{\frac{1}{3}}$

$$=3\left(2^3 3^3\right)^{\frac{1}{3}}=3(6)=18$$

10. Let $a=\tan^2\alpha, b=\tan^2\beta$

Given expression be comes

$$\frac{(a+1)^2}{b}+\frac{(b+1)^2}{a}$$

$$=\frac{a^2+2a+1}{b}+\frac{b^2+2b+1}{a}$$

$$=\frac{a^2}{b}+\frac{1}{b}+\frac{b^2}{a}+\frac{1}{a}+2\left(\frac{a}{b}+\frac{b}{a}\right)$$

$$\geq 4.\sqrt[4]{\frac{a^2}{b}.\frac{1}{b}.\frac{b^2}{a.aa}}+2(2)$$

$$=4+4=8$$

11. We have $p\cos ec\theta+t\cot\theta=2$ .....(i)

And $(p\cos ec\theta)^2-(q\cot\theta)^2=5$

$\Rightarrow p\cos ec\theta-q\cot\theta=\frac{5}{2}$ .....(ii)

from (i) and (ii)

$$p\cos ec\theta=\frac{9}{4}, q\cot\theta=\frac{-1}{4}$$

Now $\cos ec^2\theta-\cot^2\theta=1$

$$=\frac{81}{16p^2}-\frac{1}{16q^2}=1$$

$$\Rightarrow 81p^{-2}-q^{-2}=16$$

$$=\sqrt{81p^{-2}-q^{-2}}=4$$

12. $G.E=\cos 5^0+\cos 24^0+\cos\left(180^0-5^0\right)$

$$+\cos\left(180^0+24^0\right)+\cos\left(360^0-60^0\right)$$

$$=\cos 5^0+\cos 24^0-\cos 5^0$$

$$-\cos 24^0+\cos 60^0=1/2$$

13. $\cos 1^0+\cos 2^0+\cos 3^0+.....+\cos 179^0$

$$=\left(\cos 1^0+\cos 179^0\right)+\left(\cos 2^0+\cos 178^0\right.$$

$$+....+\left(\cos 89^0+\cos 91^0\right)+\cos 90^0$$

$$=\left(\cos 1^0-\cos 1^0\right)+\left(\cos 2^0-\cos 2^0\right)+...$$

$$\left(\cos 89^0-\cos 89^0\right)+\cos 90^0=0$$

14. $G.E=\tan 10^0.\tan 20^0.\frac{1}{\sqrt{3}}.\tan 40^0.\tan$

$$\left(90^0-40^0\right).\sqrt{3}$$

$$\tan\left(90^0-20^0\right)\tan\left(90^0-10^0\right)$$

$$=\tan 10^0.\tan 20^0.\tan 40^0\cot 40^0\cot 20^0.10^0$$

15. $G.E=\cot\frac{\pi}{20}.\cot\frac{3\pi}{20}.\frac{\pi}{4}.\cot\left(\frac{\pi}{2}-\frac{3\pi}{20}\right)$

$$\cot\left(\frac{\pi}{2}-\frac{\pi}{20}\right)$$

$$=\cot\phi\frac{\pi}{10}.\cot\frac{3\pi}{20}.1\tan\frac{3\pi}{20}.\tan\frac{\pi}{20}=1$$

16. $\sin 120^0\cos 150^0-\cos 240^0\sin 330^0$

$$=\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)-\left(\frac{-1}{2}\right)\left(-\frac{1}{2}\right)=-\frac{3}{4}-\frac{1}{4}=$$

17. $G.E=\cos\left(\frac{B+2\left(180^0-A-B\right)+3A}{2}\right)$

$$+\cos\frac{A-B}{2}=\cos\left(\frac{360^0+A-B}{2}\right)+\cos\frac{A-B}{2}$$

$$=-\cos\frac{A-B}{2}+\cos\frac{A-B}{2}=0$$

8. ABCD is a cyclic quadrilateral

$\Rightarrow A+C=180^0, B+D=180^0$

$\Rightarrow C=180^0-A, D=180^0-B$

$\Rightarrow \cos C=\cos(180^0-A)=-\cos A,$

$\cos D=\cos(180^0-B)=-\cos B$

$\Rightarrow \cos A+\cos C=0, \cos B+\cos D=0$

$\Rightarrow \cos A+\cos B+\cos C+\cos D=0.$

9. $\alpha+\beta=90^0,$

$G.E=\sin^2\alpha+\sin^2(90^0-\alpha)$

$=\sin^2\alpha+\cos^2\alpha=1$

0. α,β are supplementary angles, i.e., $\alpha+\beta=180^0,$

$\therefore \sin^2\alpha+\cos^2\beta=\sin^2\alpha+\cos^2(180^0-\alpha)$

$=\sin^2\alpha+\cos^2\alpha=1$

1. $\theta E\ III;\ \sin\theta=\frac{-5}{13};$

$\therefore G.E.=5.\frac{144}{25}+\frac{12.5}{12}\ +13\left(\frac{-13}{5}\right)=0$

2. $\sin(\alpha+\beta)=1, \sin(\alpha-\beta)=1/2$

$\Rightarrow \alpha+\beta=\pi/2, \alpha-\beta=\pi/6$

$\Rightarrow \alpha=\pi/3, \beta=\pi/6.$

$\tan(\alpha+2\beta)\tan(2\alpha+\beta)$

$=\tan(2\pi/3)\tan(5\pi/6)=(-\sqrt{3})$

$(-1/\sqrt{3})=1.$

3. $G.E=2[(\sin^2x+\cos^2x)^3-3\sin^2x\cos^2x$

$(\sin^2x+\cos^2x)]-3$

$[(\sin^2x+\cos^2x)^2-2\sin^2x\cos^2x]+1$

$=2[1-3\sin^2x\cos^2x]-3(1-2\sin^2x\cos^2x)+1=0$

24. $L.H.S=\sin^2\alpha+\operatorname{cosec}^2\alpha+2+\cos^2\alpha$

$+\sec^2\alpha+2=7+\cot^2\alpha+\tan^2\alpha$

$\therefore K=7.$

25. $G.E.=\frac{\cot^2\theta(\sec^2\theta-1)+\sec^2\theta(\sin^2\theta-1)}{(1+\sin\theta)(1+\sec\theta)}$

$=\frac{\cot^2\theta.\tan^2\theta+\sec^2\theta(-\cos^2\theta)}{(1+\sin\theta)(1+\sec\theta)}=0$

26. Put $A=90^0$ in the given expression, we get;

$G.E=\left(\frac{\sqrt{3}+0}{1-2}\right)^{-3}+\left(\frac{1+2}{\sqrt{3}-0}\right)^{-3}$

$=-\left(\frac{1}{\sqrt{3}}\right)^3+\left(\frac{1}{\sqrt{3}}\right)^3=0$

27. $G.E=\sin^4\theta+2\sin^2\theta\cos^2\theta+\cos^4\theta$

$=(\sin^2\theta+\cos^2\theta)^2=1$

28. Given that

$\sin\theta=1-\sin^2\theta\Rightarrow\sin\theta=\cos^2\theta$

$\therefore \cos^2\theta+\cos^4\theta=\sin\theta+\sin^2\theta=1$

29. Given that $\sin x=1-\sin^2x=\cos^2x$

$G.E=(\cos^4x+\cos^2x)^2=(\sin x+\sin^2x)^2=1$

30. Given that $\cos x=\sin^2x;$

$G.E.=(\sin^4x+\sin^2x)^3=1$

31. Given that $\sin\alpha+\operatorname{cosec}\alpha=2$

$\Rightarrow \sin^2\alpha-2\sin\alpha+1=0$

$\Rightarrow \sin\alpha=1=\operatorname{cosec}\alpha$

$\therefore \sin^4\alpha+\operatorname{cosec}^4\alpha=2$

32. $2p_6-3p_4+1=2\left(1-3\sin^2x\cos^2x\right)-3$

$\left[\left(\sin^2x+\cos^2x\right)^3-2\sin^2x\cos^2x\right]+1$

$=2-6\sin^2x\cos^2x-3\left(1-2\sin^2x\cos^2x\right)+1=0$

33. $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$

$\Rightarrow\sin\theta_1=\sin\theta_2=\sin\theta_3=1$

$\Rightarrow\theta_1=\theta_2=\theta_3=90^0$

$\therefore\cos\theta_1+\cos\theta_2+\cos\theta_3=0$

34. $\cos^6A-\cot^2A=\dfrac{\cos^6A}{\sin^6A}-\dfrac{\cos^2A}{\sin^2A}$

$=\dfrac{\sin^4A}{\sin^6A}-\dfrac{\cos^2A}{\sin^2A}=\dfrac{1-\cos^2A}{\sin^2A}=1$

35. $\cos^2\theta=\sin\theta\tan\theta\Rightarrow\cos^3\theta=\sin^2\theta=1$
$-\cos^2\theta\Rightarrow\cos^3\theta+\cos^2\theta=1$

$\Rightarrow\cos^9\theta+\cos^6\theta+3\cos^5\theta=1$
[Taking cube of the both sides]

36. $\sin\alpha,\sin\beta,\sin\gamma$ are in A.P

$2\sin\beta=\sin\alpha+\sin\gamma$ .....(1)

$\cos\alpha,\cos\beta,\cos\gamma$ are in G.P

$\cos^2\beta=\cos\alpha\cos\gamma$

Now, $\dfrac{\cos^2\alpha+\cos^2\gamma-4\cos\alpha\cos\gamma}{1-\sin\alpha\sin\gamma}$

$=\dfrac{1-\sin^2\alpha+1-\sin^2\gamma-4\cos\alpha\cos\gamma}{1-\sin\alpha\sin\gamma}$

$=\dfrac{2-\left(\sin^2\alpha+\sin^2\gamma\right)-4\cos^2\beta}{1-\sin\alpha\sin\gamma}$

$=\dfrac{2-\left(4\sin^2\beta-2\sin\alpha\sin\gamma\right)-4\cos^2\beta}{1-\sin\alpha\sin\gamma}$

$=\dfrac{2-4(1)+2\sin\alpha\sin\gamma}{1-\sin\alpha\sin\gamma}$

$=\dfrac{2(\sin\alpha\sin\gamma-1)}{1-\sin\alpha\sin\gamma}=-2$

from (1)

$4\sin^2\beta=\sin^2\alpha+\sin^2\gamma+2\sin\alpha\sin\gamma$

37. Let $a=\tan\alpha, b=\tan\beta, c=\tan\gamma$.

Then $a^2b^2+b^2c^2+c^2a^2+2a^2b^2c^2=1$

$\Rightarrow a^2b^2+b^2c^2+c^2a^2+2a^2b^2c^2+1+$
$a^2+b^2+c^2-a^2b^2c^2-1+1+a^2+b^2+c$
$-a^2b^2c^2$

$\Rightarrow 1+a^2+b^2+c^2+a^2b^2+b^2c^2+c^2a^2$
$+a^2b^2c^2=2+a^2+b^2+c^2-a^2b^2c^2$

$=\left(1+a^2\right)\left(1+b^2\right)\left(1+c^2\right)$

$=2\left(a^2b^2+b^2c^2+c^2a^2+2a^2b^2c^2\right)$

$+a^2+b^2+c^2-a^2b^2c^2$

$=a^2+b^2+c^2+2a^2b+2b^2c^2$
$+2c^2a^2+3a^2b^2c^2$

$=a^2\left(1+b^2\right)\left(1+c^2\right)+b^2\left(1+c^2\right)\left(1+a^2\right)$
$\left(1+a^2\right)\left(1+b^2\right)$

$\Rightarrow\dfrac{a^2}{1+a^2}+\dfrac{b^2}{1+b^2}+\dfrac{c^2}{1+c^2}=1$

$\Rightarrow\dfrac{\tan^2\alpha}{1+\tan^2\alpha}+\dfrac{\tan^2\beta}{1+\tan^2\beta}+\dfrac{\tan^2\gamma}{1+\tan^2\gamma}=$

$\Rightarrow\sin^2\alpha+\sin^2\beta+\sin^2\gamma=1$

38. $\dfrac{\sin x+\cos x}{\cos^3x}=\tan x\sec^2x+\sec^2x$

$=\tan\left(1+\tan^2x\right)+1$

$+\tan^2x=\tan^3x+\tan^2x+\tan x+1$

$\Rightarrow a=1,b=1,c=1,d=1\Rightarrow a+b+c+d$

39. $G.E=\sqrt{\cos^2100^0}\sec100^0$

$=-\cos100^0.\sec100^0=-1$

40. $a^2=(\cos\theta+\sin\theta)^2=\left(\cos^2\theta+\sin^2\theta\right)$

$+2\sin\theta\cos\theta=1+2\sin\theta\cos\theta$

$\Rightarrow a^2-1=2\sin\theta\cos\theta$

$$\Rightarrow \frac{a^2-1}{2}=\sin\theta\cos\theta \quad .....(i)$$

$$b=\tan\theta+\cot\theta=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

$$=\frac{1}{\sin\theta\cos\theta}\Rightarrow \sin\theta\cos\theta=\frac{1}{b} \quad .....(ii)$$

From (i) & (ii); $\frac{a^2-1}{2}=\frac{1}{b}\Rightarrow b\left(a^2-1\right)=2$

41. Given equations are $a\sin x=b\cos x$ .....(i)

and $a\sin^3 x+b\cos^3 x=\sin x\cos x$ .....(ii)

From (ii);

$(a\sin x)\sin^2 x+b\cos^3 x=\sin x\cos x$

$\Rightarrow (b\cos x)\sin^2 x+b\cos^3 x=\sin x\cos x$

[From (i)]

$\Rightarrow b\cos x\left(\sin^2 x+\cos^2 x\right)=\sin x\cos x$

$\Rightarrow b=\sin x$

From (i) :

$a\sin x=b\cos x=\sin x\cos x\Rightarrow a=\cos x$

$\therefore a^2+b^2=\cos^2 x+\sin^2 x=1$

42. $\operatorname{cosec}\theta-\sin\theta=m\Rightarrow\frac{1-\sin^2\theta}{\sin\theta}=m$

$$\Rightarrow m=\frac{\cos^2\theta}{\sin\theta}$$

similarly, $n=\frac{\sin^2\theta}{\cos\theta}, \left(m^2n\right)^{2/3}+\left(mn\right)^{2/3}$

$$=\left(\frac{\cos^4\theta}{\sin^2\theta}\cdot\frac{\sin^2\theta}{\cos\theta}\right)+\left(\frac{\cos^2\theta}{\sin\theta}\cdot\frac{\sin^4\theta}{\cos^2\theta}\right)^{2/3}$$

$=\cos^2\theta+\sin^2\theta=1$

43. $x-a=x-x\cos^2\alpha-y\sin^2\alpha$

$=x\sin^2\alpha-y\sin^2\alpha=(x-y)\sin^2\alpha$

$y-a=y-x\cos^2\alpha-y\sin^2\alpha$

$=-x\cos^2\alpha+y\cos^2\alpha=-(x-y)\cos^2\alpha$

$\therefore (x-a)(y-a)+(x-y)\sin^2\alpha\cos^2\alpha=0$

44. Conceptual

45. $a^2+b^2+c^2=\cos^2\phi\cos^2\theta+\sin^2\phi\sin\theta\cos^2\delta$

$+\cos^2\phi\sin^2\theta+\sin^2\phi\cos^2\theta\cos^2\delta+\sin^2\phi\sin\delta$

$=\cos^2\phi+\sin^2\phi\cos^2\delta+\sin^2\phi\sin^2\delta$

$=\cos^2\phi+\sin^2\phi=1.$

46. Given, $(\cos x+\sin x)^2+k\sin x\cos x-1=0,\forall x$

$\Rightarrow\cos^2 x+\sin^2 x+2\cos x\sin x+k\sin x\cos x-1=0,\forall x$

$\Rightarrow(k+2)\cos x\sin x=0,\forall x\Rightarrow k+2=0\Rightarrow k=-2.$

47. It is given that $\frac{\sin^4 A}{a}+\frac{\cos^4 A}{b}=\frac{1}{a+b}$

$$\Rightarrow\frac{(1-\cos 2A)^2}{4a}+\frac{(1+\cos 2A)^2}{4b}=\frac{1}{a+b}$$

$\Rightarrow b(a+b)(1-2\cos 2A+\cos^2 2A)$

$\Rightarrow \{b(a+b)+a(a+b)\}\cos^2 2A+2(a+b)(a-b)$

$\cos 2A+a(a+b)+b(a+b)-4ab=0$

$\Rightarrow(a+b)^2\cos^2 2A+2(a+b)(a-b)\cos 2A+a(a+b)^2=0$

$\Rightarrow\{(a+b)\cos 2A+(a+b)\}^2-0$ or

$$\cos 2A=\frac{b-a}{b+a}$$

Hence,

$$\frac{\sin^8 A}{a^3}+\frac{\cos^8 A}{b^3}=\frac{(1-\cos 2A)^4}{16a^3}+\frac{(1+\cos 2A)^4}{16b^3}$$

$$=\frac{1}{16a^3}\left[1-\frac{b-a}{b+a}\right]^4+\frac{1}{16b^3}\left[1+\frac{b-a}{b+a}\right]^4$$

$$=\frac{16a^4}{16a^3(b+a)^4}+\frac{16a^4}{16b^3(b+a)^4}$$

$$=\frac{1}{(b+a)^4}(a+b)=\frac{1}{(a+b)^3}$$

p = q = 0, r =3; p + q + r = 3

48. $\sin^3 x\sin 3x=\frac{1}{4}(3\sin x-\sin 3x)\sin 3x$

$$=\frac{3}{8}.2\sin x\sin 3x-\frac{1}{8}.2\sin^2 3x$$

$$=\frac{3}{8}(\cos 2x-\cos 4x)-\frac{1}{8}(1-\cos 6x)$$

$$=\frac{1}{8}+\frac{3}{8}\cos 2x-\frac{3}{8}\cos 4x+\frac{1}{8}\cos 6x$$

....(i)

and $\sum_{m=0}^{n}C_m\cos mx=c_0+c_1\cos x+c_2\cos 2x$

$+c_3\cos 3x+........c_n\cos nx$ ....(ii)

Comparing both sides of (i) and (ii), we get n=6.

49. $f_n(\theta)=\frac{\sin(\theta/2)}{\cos(\theta/2)}\left[\frac{2\cos^2\theta/2}{\cos\theta}\cdot\frac{2\cos^2\theta}{\cos 2\theta}\cdot\frac{2\cos^2 2\theta}{\cos 4\theta}....\right]$

Combine first two factos,

$$f_n(\theta)=\frac{\sin\theta}{\cos\theta}\left[\frac{2\cos^2\theta}{\cos 2\theta}\cdot\frac{2\cos^2\theta}{\cos 4\theta}....\right]$$

Again, combine first two factors,

$$f_n(\theta)=\tan 2\theta\left[\frac{2\cos^2\theta}{\cos 4\theta}....\right]=\tan(2^n\theta)$$

$$\therefore f_2\left(\frac{\pi}{16}\right)=\tan\frac{4\pi}{16}=\left(\frac{\pi}{4}\right)=1$$

$$f_3\left(\frac{\pi}{32}\right)=\tan\frac{8\pi}{32}=\left(\frac{\pi}{4}\right)=1$$

$$f_4\left(\frac{\pi}{64}\right)=\tan\frac{16\pi}{64}=\tan\left(\frac{\pi}{4}\right)=1$$

$$f_5\left(\frac{\pi}{128}\right)=\tan 32\frac{\pi}{128}=\tan\left(\frac{\pi}{4}\right)=1.$$

50. We have $k=\sin\frac{\pi}{18}\cdot\sin\frac{5\pi}{18}\cdot\sin\frac{7\pi}{18}$

$$=\cos\left(\frac{\pi}{2}-\frac{\pi}{18}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{18}\right)$$

$$=\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}=\frac{\sin 2^3\frac{\pi}{9}}{2^3\sin\frac{\pi}{9}}=\frac{\sin\frac{8\pi}{9}}{8\sin\frac{\pi}{9}}$$

$$=\frac{\sin\left(\pi-\frac{\pi}{9}\right)}{8\sin\frac{\pi}{9}}=\frac{1}{8}.$$

Ans = 16 x 1/8 = 2

51. Since $\sin x+\sin^2 x=1$,

$\Rightarrow \sin x=1-\sin^2 x=\cos^2 x$ .....(i)

The given expression is

$=\cos^6 x(\cos^6 x+3\cos^4 x+3\cos^2 x+1)-1$

$=\cos^6 x(\cos^2 x+1)^3-1,(\because \sin x=\cos^2 x)$

$=(\sin^2 x\sin x)^3-1=1-1=0.$

52. If $L=M$, then $L^2=LM$ or ML,

Both $LM=ML$,

$\sec^2 4-\tan^2 A=1$

$\therefore L^2=M^2=1.$

53. We have

$$x\cos\theta=y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right.$$

$$\Rightarrow \cos\theta=\frac{k}{x},\cos\left(\theta+\frac{2\pi}{3}\right)=\frac{k}{y}$$

and $\cos\left(\theta+\frac{4\pi}{3}\right)=\frac{k}{z}$

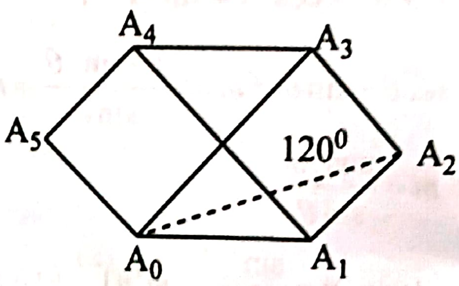
Hence

$$\frac{k}{x}+\frac{k}{y}+\frac{k}{z}=\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\left(\theta+\frac{4\pi}{3}\right.$$

$$=\cos\theta+\cos\left(\frac{\pi}{3}-\theta\right)-\cos\left(\frac{\pi}{3}+\theta\right.$$

$$=\cos\theta-2\cos\frac{\pi}{3}\cos\theta=0.$$

54. Each triangle is an equilateral trian



Hence $A_0A_1=1$

$A_0A_2^2=A_0A_1^2+A_1A_2^2-2A_0A_1A_1A_2\cos 120^0=1+1-2.1.1\left(-\right.$

$\Rightarrow A_0A_2=\sqrt{3}=A_0A_4$

$\therefore A_0A_1\times A_0A_2\times A_0A_4=1.\sqrt{3}.\sqrt{3}=3.$

55. Here

$3\left\{\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right\}-2\left\{\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi\right.$

$=3\{(-\cos\alpha)^4+(-\sin\alpha)^4\}-2\{\cos^6\alpha+\sin^6$

$=3\{\cos^2\alpha+\sin^2\alpha)^2-2\sin^2\alpha\cos^2$

$-2\{(\cos^2\alpha+\sin^2\alpha)^3-3\cos^2\alpha\sin^2\alpha(\cos^2\alpha+\sin^2$

$=3-6\sin^2\alpha\cos^2\alpha-2+6\sin^2\alpha\cos^2\alpha=3-2$

56. Here $(\cot\alpha_1)(\cot\alpha_2)......(\cot\alpha_n)=1$

$\cos\alpha_1\cos\alpha_2...\cos\alpha_n=\sin\alpha_1.\sin\alpha_2...\sin\alpha_n$

Now, $(\cos\alpha_1.\cos\alpha_2...\cos\alpha_n)^2$

$=(\cos\alpha_1.\cos\alpha_2...\cos\alpha_n)(\cos\alpha_1.\cos\alpha_2...\cos\alpha_n)$

$=(\cos\alpha_1.\cos\alpha_2...\cos\alpha_n)(\sin\alpha_1.\sin\alpha_2...\sin\alpha_n)$

$=\frac{1}{2^n}\sin 2\alpha_1.\sin 2\alpha_2...\sin 2\alpha_n.$

But each of $\sin 2\alpha_i \le 1$

$\therefore(\cos\alpha_1.\cos\alpha_2.....\cos\alpha_n)^2\le\frac{1}{2^n}.$

But each of $\cos\alpha_i$ is positive.

$\therefore\cos\alpha_1.\cos\alpha_2.....\cos\alpha_n\le\sqrt{\frac{1}{2^n}\frac{1}{2^{n-2}}}$

$2^{n/2}\times 1/2^{n/2}=1$

57. Given that $A=\sin^8\theta+\cos^{14}\theta$

we know that $(\sin^2\theta)^4\le\sin^2\theta$ and $(\cos^2\theta)^7\le\cos^2\theta$

Adding, we that $\sin^8\theta+\cos^{14}\theta\le 1$

or $0<\sin^8\theta+\cos^{14}\theta\le 1$.

Hence $0<A\le 1$.

58. θ is an acute angle so $0^0\le\theta<90^0$

$\therefore 0\le\frac{p-6}{8-p}<1$

$\Rightarrow 0\le(p-6)<(8-p)\Rightarrow 6\le p<7.$

59. $\sum\frac{\cot A+\cot B}{\tan A+\tan B}=\sum\frac{\frac{\cot A}{\sin A}+\frac{\cos B}{\sin B}}{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}$

$\sum\frac{\sin B\cos A+\sin A\cos B}{\sin A\sin B}\frac{\cos A\cos B}{(\sin A\cos B+\cos A\sin B)}$

$\sum\cot A\cot B$

As we know if , $A+B+C=\pi$, then

$\cot A\cot B+\cot B\cot C+\cot C\cot A=1.$

60. $a\sin^2\theta+b\sin\theta\cos\theta+c\cos^2\theta-\frac{1}{2}(a+c)$

$=\frac{1}{2}[-a\cos 2\theta+b\sin 2\theta+c\cos 2\theta]$

$=\frac{1}{2}[b\sin 2\theta-(a-c)\cos 2\theta]$

$\because|b\sin 2\theta-(a-c)\cos 2\theta|\le\sqrt{b^2+(a-c)^2}$

$\because\left|\frac{1}{2}\{b\sin 2\theta-(a-c)\cos 2\theta\}\right|\le\frac{1}{2}\sqrt{b^2+(a-c)^2}$

$\Rightarrow\left|a\sin\theta+b\sin\theta\cos\theta+\cos^2\theta-\frac{1}{2}(a+c)\right|\le\frac{1}{2}\sqrt{b^2+(a-c)^2}$

ñ $K^2=a^2+b^2+c^2-2ac$

ñ $x+y+z=2+0+0=2$

61. We have

$\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\frac{x}{\cos\theta}=\frac{z}{\cos\left(\theta+\frac{2\pi}{3}\right)}=k$

$\Rightarrow x=k\cos\theta, y=k\cos\left(\theta-\frac{2\pi}{3}\right),$

$\Rightarrow z=k\cos\left(\theta+\frac{2\pi}{3}\right)$

$\Rightarrow x+y+z=k\left[\cos\theta+\cos\left(\theta-\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{2\pi}{3}\right)\right]$

$=k[(0)=0$

$\Rightarrow x+y+z=0.$

62. $\sqrt{3}\operatorname{cosec}20^0-\sec 20^0=\frac{\sqrt{3}}{\sin 20^0}-\frac{1}{\cos 20^0}$

$=\frac{\sqrt{3}\cos 20^0-\sin 20^0}{\sin 20^0\cos 20^0}=\frac{2\left[\frac{\sqrt{3}}{2}\cos 20^0-\frac{1}{2}\sin 20^0\right]}{\frac{2}{2}\sin 20^0\cos 20^0}$

$=\frac{4\cos(20^0+30^0)}{\sin 40^0}=\frac{4\cos 50}{\sin 40^0}=\frac{4\sin 40^0}{\sin 40^0}=4.$

6.2 Periodicity and Extreme Values

Internals of the Topic

- Periodicity
- Extreme values

CRT CONCEPT

- **Periodic function:** A real function $f: A \to R$ is said to be a periodic function if there exists a positive real number P such that $f(x+p)=f(x)$, $\forall x \in A$. Least value of P is known as the fundamental period of f.
- The period of $f(ax+b)=\frac{p}{|a|}$
- Let $g(x)=\frac{C_1f_1(x)\pm C_2f_2(x)}{C_3f_3(x)\pm C_4f_4(x)}$ (where f_1, f_2, f_3, f_4 are trigonometric functions, and C_1, C_2, C_3, C_4 are constants).

Then period of $g(x)$ = L.C.M. of period of f_1, f_2, f_3, f_4

Note:

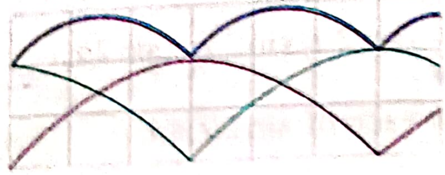
i) Numerator and Denominator can be extended to sum of any number of trignometric functions.

ii) If p and q are rational quantities. Then LCM of p and q exists

iii) If p and q are irrational, then LCM of p and q does not exist unless they have same irrational surd.

iv) LCM of rational and irrational is not possible.

$f(t)=|\sin t|+|\cos t|$



v) L.C. M $\left\{\frac{a}{b},\frac{c}{d},\frac{e}{f}\right\}=\frac{L.C.M\{a,c}{G.C.D\{b,d}$

- If 'f' is a periodic function with fu mental period 'P' then '$-f$' and $\frac{1}{f}$ also a periodic functions with perio
- A constant function is periodic but not have a fundamental period.
- Periods of trignometric functions given below:

Function	Period
1. $Sin(ax+b)$	$\frac{2\pi}{\lvert a\rvert}$
2. $Cos(ax+b)$	$\frac{2\pi}{\lvert a\rvert}$
3. $Tan(ax+b)$	$\frac{\pi}{\lvert a\rvert}$
4. $Cosec(ax+b)$	$\frac{2\pi}{\lvert a\rvert}$
5. $Sec(ax+b)$	$\frac{2\pi}{\lvert a\rvert}$
6. $Cot(ax+b)$	$\frac{\pi}{\lvert a\rvert}$

- Period of $\{x\}$ is 1. where $\{.\}$ denotes th fractional part.
- Standard results :

i) Period of $|\sin x|, |\cos x|, |\tan x|, |\cot x|, |\sec x|, |\operatorname{cosec} x|$ is π

ii) Period of $|\sin x|+|\cos x|, |\tan x|+|\cot x|$ is $\frac{\pi}{2}$

iii) Period of $|\sin x+\cos x|, |\sin x-\cos x|$

iv) Period of $|\tan x+\cot x|.|\tan x-\cot x|$ is $\frac{\pi}{2}$

v) If $n\in z^+$, the period of

$\sin^{2n}x+\cos^{2n}x$,

$\sec^{2n}x+\operatorname{cosec}^{2n}x, \tan^{2n}x+\cot^{2n}x$ is $\frac{\pi}{2}$

vi) Period of

$a\sin^{2n}x+b\cos^{2m}x, a\sec^{2n}x+b\operatorname{cosec}^{2m}x$,

$a\tan^{2n}x+b\cot^{2m}x$ is π $(n,m\in z^+)$

Maximum and minimum values :

i) Minimum value of $\sin x$ is -1

ii) Maximum value of $\sin x$ is 1

iii) Range of $\sin x$ is $[-1,1]$

- i) Minimum value of $\cos x$ is -1

  ii) Maximum value of $\cos x=1$

  iii) Range of $\cos x$ is $[-1,1]$

- Minimum value of

  $a\cos x+b\sin x+c$ is $c-\sqrt{a^2+b^2}$

- Maximum value of $a\cos x+b\sin x+c$ is

  $c+\sqrt{a^2+b^2}$

- Range of $a\cos x+b\sin x+c$ is

  $\left[c-\sqrt{a^2+b^2}, c+\sqrt{a^2+b^2}\right]$

- The minimum value of $a^2\sin^2\theta+b^2\operatorname{cosec}^2\theta$, $a^2\cos^2\theta+b^2\sec^2\theta$, $a^2\tan\theta+b^2\cot\theta$ is 2ab.

Concept Based Questions

1. If $f(x)$ is a periodic function with the period T then it can also have, the period.

   1) $\lambda T(\lambda\in N)$ 2) $2\lambda T(\lambda\in N)$

   3) $(2\lambda-1)T(\lambda\in N)$ 4) All above

2. If $f(x+k)+f(x)=0\forall x\in R$ and $k\in R^+$, then the period of f(x) is

   1) 0 2) $\frac{\pi}{2}$ 3) 2K 4) Does not exist

3. If g is periodic. Then

   1) fog will always be periodic.

   2) fog need not be periodic

   3) gof will always be periodc

   4) The period of g is same as gof.

4. If $f(x)=\sin x+\cos ax$ is a periodic function, then

   1) 'a' is a irrational 2) $a=0$ only

   3) 'a' is a rational number

   4) a has no

KEY

01) 4 02) 3 03) 1 04) 3

EXERCISE-I

CRTQ & SPQ LEVEL-I

PERIODICITY

C.R.T.Q Class Room Teaching Questions

1. Period of $\cos(x+2x+3x+...+nx)$ is

   1) $\frac{2\pi}{n(n+1)}$ 2) $\frac{4\pi}{n(n+1)}$

   3) $\frac{\pi}{n(n+1)}$ 4) $\frac{6\pi}{n(n+1)}$

2. Period of $\tan(x+4x+9x+...+n^2x)$ is

   1) $\frac{2\pi}{n(n+1)(2n+1)}$ 2) $\frac{4\pi}{n(n+1)(2n+1)}$

   3) $\frac{6\pi}{n(n+1)(2n+1)}$ 4) $\frac{8\pi}{n(n+1)(2n+1)}$

3. Period of $\cos\left(x+\frac{x}{3}+\frac{x}{3^2}+.........\right)$ is

   1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) π 4) $\frac{4\pi}{3}$

4. Period of $2\sin^3x-3\cos^3x$ is

   1) $\frac{\pi}{2}$ 2) π 3) 2π 4) 4π

5. Period of $\cos^6 x + \sin^6 x$ is

1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$ 4) 2π

6. Period of $\sin x \sin(120^0 - x)\sin(120^0 + x)$ is

1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) π 4) $\frac{\pi}{2}$

7. If $f(x)=\sin^2\left(\frac{\pi}{8}+\frac{x}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{x}{2}\right)$, then the period of f is

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) 2π

8. Sine function whose period is 6 is

1) $\sin\frac{2\pi x}{3}$ 2) $\sin\frac{\pi x}{3}$
3) $\sin\frac{\pi x}{6}$ 4) $\sin\frac{3\pi x}{2}$

9. The cosecant function whose period is 4 is

1) $cosec\frac{\pi x}{3}$ 2) $cosec\frac{2\pi x}{5}$
3) $cosec 2\pi x$ 4) $cosec\frac{\pi x}{2}$

10. Period of $\frac{2\sin 2x - 5\cos 2x}{7\cos x - 8\sin x}$ is

1) π 2) 2π 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{3}$

11. Period of $\frac{\sin(x+a)}{\cos x}$

1) $\frac{\pi}{2}$ 2) π 3) 2π 4) 3π

12. Period of $\sin\left(e^{\tan x}+e^{\cot x}\right)$ is

1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$ 4) 2π

S.P.Q. Student Practice Questions

13. Period of $\cot\left(\frac{5-7x}{2}\right)$ is

1) $\frac{\pi}{2}$ 2) $\frac{2\pi}{7}$
3) $\frac{\pi}{7}$ 4) $\frac{7\pi}{2}$

14. Period of $\tan(x + 2x + 3x + ...+ n$

1) $\frac{2\pi}{n(n+1)}$ 2) $\frac{4\pi}{n(n+1)}$
3) $\frac{\pi}{n(n+1)}$ 4) $\frac{6\pi}{n(n+1)}$

15. Period of $\cos(x + 4x + 9x+...+ n^2x$

1) $\frac{2\pi}{n(n+1)(2n+1)}$ 2) $\frac{4\pi}{n(n+1)(2n}$
3) $\frac{12\pi}{n(n+1)(2n+1)}$ 4) $\frac{8\pi}{n(n+1)(2n+}$

16. Period of $cosec\left(x+\frac{x}{2}+\frac{x}{4}+......\right)$ is

1) π 2) 2π 3) $\frac{\pi}{2}$ 4

17. Period of $6\cos^4 x - 7\sin^4 x$ is

1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$ 4)

18. The period of $\sin^3 x + \cos^3 x$ is

1) $\frac{\pi}{3}$ 2) π 3) 2π 4)

19. Period of $\cos x \cos(60^0 - x)\cos(60^0 + x$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{2\pi}{3}$ 4)

20. The tangent function whose period is

1) $\tan\frac{\pi x}{2}$ 2) $\tan\frac{\pi x}{3}$
3) $\tan\frac{3\pi x}{2}$ 4) $\tan\frac{2\pi x}{3}$

21. The cotangent function whose period is 3

1) cot 2x 2) cot 4 x
3) $\cot\frac{3x}{2}$ 4) $\cot\frac{x}{3}$

22. Period of $\tan 4x + \sec 4x$ is

1) 2π 2) $\frac{3\pi}{2}$ 3) π 4) $\frac{\pi}{2}$

23. The period of $\frac{\sin(2\pi x+a)}{\sin(2\pi x+b)}$ is

1) $\frac{a}{b}$ 2) 1 3) 2 4) π

24. The period of $\sin(\pi\sin\theta)$ is

1) π 2) $\frac{\pi}{2}$ 3) 2π 4) $\frac{\pi}{4}$

EXTREME VALUES

C.R.T.Q Class Room Teaching Questions

25. The range of $f(x)=-3\cos\sqrt{3+x+x^2}$

1) [-1,1] 2) [-2,2]
3) [-3,3] 4) [-4,4]

26. The minimum and maximum values of $8\cos 3x-15\sin 3x$ are

1) -7,7 2) -23,23
3) -17,17 4) -15,8

27. The maximum value of $\frac{3}{5\sin x-12\cos x+19}$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

28. The range of $\sin^6 x+\cos^6 x$

1) $\left[-\frac{1}{4},\frac{1}{4}\right]$ 2) $\left[\frac{1}{4},\frac{1}{2}\right]$

3) $\left[\frac{1}{4},1\right]$ 4) $\left[-\frac{1}{2},-\frac{1}{4}\right]$

29. The minimum and maximum values of $\sin^2(60^0-x)+\sin^2(60^0+x)$ are

1) $-\frac{1}{2},\frac{1}{2}$ 2) $\frac{1}{2},1$

3) $\frac{1}{2},\frac{3}{2}$ 4) $\frac{3}{2},2$

30. $\sin^2(60^0+x)-\sin^2(60^0-x)\in\left[-\frac{k}{2},\frac{k}{2}\right]\Rightarrow k=$

1) 3 2) 2 3) $\sqrt{3}$ 4) $\sqrt{2}$

31. $\sin\theta\sin(60^0-\theta)\sin(60^0+\theta)\in[-k,k]\Rightarrow k=$

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{3}{4}$

32. $16\cos^5 x-20\cos^3 x+5\cos x\in$

1) [-1,1] 2) $\left[-\frac{1}{4},\frac{1}{4}\right]$

3) $\left[-\frac{3}{4},\frac{3}{4}\right]$ 4) [–2, 2]

33. $3\sin^2 x-4\cos^2 x\in$

1) [0,3] 2) [-4,0]
3) [-3,4] 4) [-4, 3]

34. The maximum value of

$\cos x\left(\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}\right)$ is

1) 4 2) 3
3) 2 4) 1

35. $A+B=90^0\Rightarrow \cos A\cos B\in$

1) $\left[-\frac{1}{3},\frac{1}{3}\right]$ 2) $\left[-\frac{1}{2},\frac{1}{2}\right]$

3) $\left[-\frac{1}{4},\frac{1}{4}\right]$ 4) $\left[-\frac{3}{4},\frac{3}{4}\right]$

36. $A+B=90^0\Rightarrow \sin A+\sin B\in$

1) [-1,1] 2) $[-\sqrt{2},\sqrt{2}]$
3) $[-\sqrt{3},\sqrt{3}]$ 4) [-2,2]

37. Minimum value of $9\sec^2\theta+4\operatorname{cosec}^2\theta+7$ is

1) 32 2) 25 3) 20 4) 49

S.P.Q. Student Practice Questions

38. $16\sin x\cos x\cos 2x\cos 4x\cos 8x\in$

1) [-1,1] 2) $\left[-\frac{1}{4},\frac{1}{4}\right]$

3) $\left[-\frac{3}{4},\frac{3}{4}\right]$ 4) $\left[-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right]$

39. The minimum and maximum values of $2\sqrt{2}\cos x+\sin x$ are

1) - 1, 1 2) $-2\sqrt{2},2\sqrt{2}$
3) 1,8 4) -3,3

40. Maximum value of $\frac{1}{\sqrt{7}\sin x+\sqrt{29}\cos x+7}$

1) 13 2) 1 3) 1/13 4) 49

41. The minimum and maximum values of $\sin^4 x+\cos^4 x$ are

1) $\frac{1}{2},\frac{3}{2}$ 2) $\frac{1}{2},1$ 3) $1,\frac{3}{2}$ 4) 1,2

42. $\cos^2(60^0-x)+\cos^2(60^0+x)\in$

1) $\left[-\frac{1}{2},\frac{1}{2}\right]$ 2) $\left[\frac{1}{2},1\right]$
3) $\left[\frac{1}{2},\frac{3}{2}\right]$ 4) $\left[\frac{3}{2},2\right]$

43. $\cos^2(60^0+x)-\cos^2(60^0-x)\in\left[-\frac{m}{2},\frac{m}{2}\right]$ $\Rightarrow m=$

1) 1 2) 3 3) $-\sqrt{3}$ 4) $\sqrt{3}$

44. $\sin^2 x+4\sin x+5\in[k,5k]\Rightarrow k=$

1) 1 2) 2 3) 3 4) 4

45. $3\sin^2 x+4\cos^2 x\in$

1) [0,3] 2) [0,4] 3) [3,4] 4) [-4,-3]

46. The minimum value of $\sin x\left(\frac{1-\cos x}{\sin x}+\frac{\sin x}{1-\cos x}\right)$ is

1) 1 2) 2 3) 3 4) 4

47. $A+B=90^0\Rightarrow\sin A\sin B\in$

1) $\left[-\frac{1}{4},\frac{1}{4}\right]$ 2) $\left[-\frac{1}{3},\frac{1}{3}\right]$
3) $\left[-\frac{3}{4},\frac{3}{4}\right]$ 4) $\left[-\frac{1}{2},\frac{1}{2}\right]$

48. $A+B=90^0\Rightarrow\cos A-\cos B\in$

1) [-1,1] 2) $\left[-\sqrt{2},\sqrt{2}\right]$
3) $\left[-\sqrt{3},\sqrt{3}\right]$ 4) $[-2,2]$

49. The minimum value of $27\tan^2\theta+3\cot^2\theta$

1) 15 2) 18 3) 24 4)

KEY

01) 2	02) 3	03) 4	04) 3	05)
06) 2	07) 4	08) 2	09) 4	10)
11) 2	12) 2	13) 2	14) 1	15)
16) 1	17) 2	18) 3	19) 3	20)
21) 4	22) 4	23) 2	24) 3	25)
26) 3	27) 2	28) 3	29) 3	30)
31) 3	32) 1	33) 4	34) 3	35)
36) 2	37) 1	38) 1	39) 2	40)
41) 2	42) 3	43) 4	44) 2	45)
46) 2	47) 4	48) 2	49) 2	

Hints & Solutions

1. $p=\frac{2\pi}{|a|}=\frac{2\pi}{\frac{n(n+1)}{2}}=\frac{4\pi}{n(n+1)}$

2. $p=\frac{\pi}{|a|}=\frac{\pi}{\frac{n(n+1)(2n+1)}{6}}=\frac{6\pi}{n(n+1)(2n+1)}$

3. $1+\frac{1}{3}+\frac{1}{3^2}+.....=\frac{1}{1-1/3}=\frac{3}{2}$

$\therefore$ Period of $\cos\left(\frac{3}{2}x\right)=\frac{2\pi}{3/2}=\frac{4\pi}{3}$

4. By verification

5. When coefficients are equal and powe
are even then period is $\frac{\pi}{2}$

6. $\sin x\sin(120°-x)\sin(120°+x)=\frac{1}{4}\sin 3x$

$\therefore$ Period $= \frac{2\pi}{3}$

7. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

8. $\frac{2\pi}{|a|} = 6 \Rightarrow |a| = \frac{\pi}{3}$

9. $\frac{2\pi}{|a|} = 4$ 10. Conceptual

11. L.C.M of $(\pi, 2\pi)$

12. Cosa. Tanx + Sina,
$\therefore$ period is π

13. $p = \frac{\pi}{|a|} = \frac{\pi}{\frac{7}{2}} = \frac{2\pi}{7}$

14. $p = \frac{\pi}{|a|} = \frac{\pi}{\frac{n(n+1)}{2}} = \frac{2\pi}{n(n+1)}$

15. $p = \frac{2\pi}{|a|} = \frac{2\pi}{\frac{n(n+1)(2n+1)}{6}} = \frac{12\pi}{n(n+1)(2n+1)}$

16. $1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\ldots\ldots = \frac{1}{1-1/2} = 2$

$\therefore$ Period of $\operatorname{cosec}(2x) = \frac{2\pi}{2} = \pi$

17. When coefficients are different and power are even period is π

18. power is odd number.

19. $\cos x \cos(60° - x)\cos(60° + x) = \frac{1}{4}\cos 3x$

Period $= \frac{2\pi}{3}$

20. $\frac{\pi}{|a|} = \frac{2}{3}$ 21. $\frac{\pi}{|a|} = 3\pi$

22. L.C.M of $\left\{\frac{\pi}{4}, \frac{\pi}{2}\right\} = \frac{\pi}{2}$

23. L.C.M of (1, 1) 24. Period of $\sin\theta$

25. L.C.M of period of Tanx, Cotx is π

26. $f(x) \in -3[-1, 1]$

27. Min $= -\sqrt{a^2+b^2}$; Max $= \sqrt{a^2+b^2}$

28. Max.value of $\frac{3}{5\sin x - 12\cos x + 19} =$

$\frac{3}{19 - \sqrt{5^2 + (-12)^2}} = \frac{3}{19-13} = \frac{3}{6} = \frac{1}{2}$

29. $\sin^6 x + \cos^6 x = 1 - \frac{3}{4}\sin^2 2x$

30. If $f(x) = \sin^2(\alpha - x) + \sin^2(\alpha + x)$
for $\alpha = 60°, 120°, 240°, 300°$

range is $\left[\frac{1}{2}, \frac{3}{2}\right]$

31. If $f(x) = \sin^2(\alpha - x) - \sin^2(\alpha + x)$
for $\alpha = 60°, 120°, 240°, 300°$

range is $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

32. If

$f(\theta) = \sin\theta\sin(\alpha - \theta)\sin(\alpha + \theta) = \frac{1}{4}\sin 3\theta$

for $\alpha = 60°, 120°, 240°, 300°$, range is

$\left[-\frac{1}{4}, \frac{1}{4}\right]$

33. Put $\cos x = \pm 1$

34. Put $\cos^2 x = 1 - \sin^2 x$

35. $\frac{\cos^2 x}{1 - \sin x} + 1 - \sin x = 2$

36. $A + B = 90° \Rightarrow \cos A \cos B$

$= \frac{1}{2}\sin 2A \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

37. $(A+B) = 90° \Rightarrow \sin A + \sin B =$

$\sin A + \cos A \in \left[-\sqrt{2}, \sqrt{2}\right]$

38. $\sin 16x \in [-1, 1]$

39. $\text{Min} = -\sqrt{a^2+b^2}; \text{Max} = \sqrt{a^2+b^2}$
40. Find the minimum value of dr.
41. $sin^4x+cos^4x=1-\frac{1}{2}sin^2 2x$
42. If $f(x)=cos^2(\alpha-x)+cos^2(\alpha+x)$ for $\alpha=60°, 120°, 240°, 300°$, range is $\left[\frac{1}{2},\frac{3}{2}\right]$
43. If $f(x)=cos^2(\alpha-x)-cos^2(\alpha+x)$ for $\alpha=60°, 120°, 240°, 300°$ range is $\left[-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right]$
44. Put $\sin x=\pm 1$
45. $3\sin^2 x+4\cos^2 x=7\sin^2 x-4$
46. $(1-cosx)+(1+cosx)=2$
47. $(A+B)=90° \Rightarrow \sin A \sin B$ $=\frac{1}{2}\sin 2A \in \left[-\frac{1}{2},\frac{1}{2}\right]$
48. $A+B=90^0 \Rightarrow \cos A-\cos B=\cos A-\sin A$ $\Rightarrow\left[-\sqrt{a^2+b^2},+\sqrt{a^2+b^2}\right]$
49. A.M $\geq$ G.M

EXERCISE-II
CRTQ & SPQ LEVEL-II

PERIODICITY

C.R.T.Q Class Room Teaching Questions

1. The period of $f(x)=sinx+\{x\}$, where $\{x\}$ is fractional part of x is
   1) 0 2) 1 3) 2π 4) Does not exist
2. Period of $tan\left(\frac{3\pi x}{4}\right)-2sec\left(\frac{\pi x}{3}\right)+5sin\left(\frac{2\pi x}{5}\right)$ is
   1) 20 2) 30 3) 60 4) 80
3. The period of $f(x)=sin\frac{\pi x}{n!}-cos\frac{\pi x}{(n+1)!}$
   1) $(n+1)!$ 2) $2(n+1)!$
   3) $2(n)!$ 4) $\frac{(n+1)}{2!}$
4. The value of integer n for which the function $f(x)=\frac{sinnx}{sin(x/n)}$ has 4π as its period
   1) 2 2) 3 3) 4 4) 5
5. Let $f(x)=cos\sqrt{p}\,x$ where $p=[$ where $[x]$ is the integral part of x, the period of $f(x)$ is π, then $a\in$
   1) $[4,5]$ 2) $[4,5)$ 3) $(4,5]$ 4) $(4,5$
6. If the period of $\frac{\cos(\sin(nx))}{\tan\left(x/n\right)}, n\in N$ is 6π then $n=$
   1) 3 2) 2 3) 6 4)
7. The period of the functi $\left|\sin^3\frac{x}{2}\right|+\left|\cos^5\frac{x}{5}\right|$ is
   1) 2π 2) 10π 3) 8π 4) 5π
8. The period of the function $f(x)=[6x+7]+\cos\pi x-6x$, where $[.]$ denotes the greatest integer function,
   1) 3 2) 2π 3) 2 4) π
9. The function $\sin(x^2)+\cos\sqrt{x}$ is
   1) Periodic with period π
   2) Periodic with period 2π
   3) Periodic with period 0
   4) Not a Periodic function

.P.Q. Student Practice Questions

. Period of $sinx\,cos\left(x+\frac{\pi}{4}\right)$ is

1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$ 4) 2π

. Period of $sin\frac{3\pi x}{2}+cos\frac{\pi x}{2}$ is

1) 1 2) 2 3) 3 4) 4

. Period of $cos\frac{\pi x}{4}$-$2cosec\frac{\pi x}{6}+5tan\frac{\pi x}{3}$ is

1) 12 2) 24 3) 48 4) 6

. Period of $cos\frac{3x}{5}+2sin\frac{2x}{7}-5cot14x$ is

1) 7π 2) 14π 3) 28π 4) 70π

. If $n\in\mathbb{N}$, and the period of $\frac{\cos nx}{\sin\left(\frac{x}{n}\right)}$ is 4π, then n=

1) 4 2) 3 3) 2 4) 1

. Period of $\sin\sqrt{x}+\cos\sqrt{x}$ =

1) π 2) 2π

3) 1 4) does not exist

EXTREME VALUES

.R.T.Q Class Room Teaching Questions

. $sin^2x+cos^4x\in$

1) $\left[\frac{1}{4},\frac{1}{2}\right]$ 2) $\left[\frac{3}{4},1\right]$

3) $\left[-1,-\frac{3}{4}\right]$ 4) $\left[-\frac{3}{4},\frac{3}{4}\right]$

7. **The minimum and maximum values of**

$sin\left(x+\frac{\pi}{4}\right)sin\left(x-\frac{\pi}{4}\right)$ **are**

1) -1, 1 2) $-\frac{1}{2},\frac{1}{2}$ 3) $-\frac{3}{2},\frac{3}{2}$ 4) $-\frac{1}{4},\frac{1}{4}$

18. **The minimum and maximum values of**

$cosx+3\sqrt{2}sin\left(x+\frac{\pi}{4}\right)+6$ **are**

1) 1, 11 2) 11,-1 3) 6,5 4) 5, 6

19. $(sinx+cosx)^2+cos^2\left(\frac{\pi}{4}+x\right)\in$

1) [0,1] 2) [0,2] 3) [1,2] 4) [0,3]

20. $A=\sin^8\theta+\cos^{14}\theta$, **then for all values of** θ

1) $0<A\le 1$ 2) $1<2A\le 3$

3) $A\ge 1$ 4) $0\le A\le\frac{1}{2}$

21. **For any real** θ **the maximum value of** $\cos^2(\cos\theta)+\sin^2(\sin\theta)$ **is**

1) 1 2) $1+\sin^2 1$

3) $1+\cos^2 1$ 4) $2+\cos^2 1$

22. If $5\cos x+12\cos y=13$, then the maximum value of $5\sin x+12\sin y$ is

1)12 2)$\sqrt{120}$ 3)$\sqrt{20}$ 4)13

S.P.Q. Student Practice Questions

23. $cos^2x+sin^4x\in$

1) $\left[\frac{1}{2},1\right]$ 2) $\left[1,\frac{3}{2}\right]$ 3) $\left[\frac{3}{2},2\right]$ 4) $\left[\frac{3}{4},1\right]$

24. $cos\left(x+\frac{\pi}{6}\right)cos\left(x-\frac{\pi}{6}\right)\in$

1) $\left[-\frac{1}{4},\frac{1}{4}\right]$ 2) $\left[-\frac{3}{4},\frac{3}{4}\right]$

3) $\left[-\frac{1}{4},\frac{3}{4}\right]$ 4) $\left[-\frac{1}{2},\frac{1}{2}\right]$

25. $5cos\theta+3cos\left(\theta-\frac{\pi}{3}\right)+8\in$

1) [-7,1] 2) [1,8] 3) [1,15] 4) [2,15]

26. $(sinx-cosx)^2+cos^2\left(\frac{\pi}{4}-x\right)\in$

1) [0,1] 2) [0,2] 3) [1,2] 4) [0,3]

27. For all values of θ, the values of $3-\cos\theta+\cos\left(\theta+\frac{\pi}{3}\right)$ lie in the interval

1) $[-2,3]$ 2) $[-2,1]$ 3) $[2,4]$ 4) $[1,5]$

KEY

01) 4	02) 3	03) 2	04) 1	05) 2
06) 3	07) 2	08) 3	09) 4	10) 2
11) 4	12) 2	13) 4	14) 3	15) 4
16) 2	17) 2	18) 1	19) 3	20) 1
21) 2	22) 2	23) 4	24) 3	25) 3
26) 3	27) 3			

Hints & Solutions

1. Does not exists

2. L.C.M of $\left\{\frac{4}{3},6,5\right\} \Rightarrow 60$

3. L.C.M of $\{2n!,2(n+1)!\}=2(n+1)!$

4. Period of $\sin nx=\frac{2\pi}{n}$,

Period of $\sin\frac{x}{n}=2n\pi$

$\therefore 2n\pi=4\pi \Rightarrow n=2$

5. $\frac{2\pi}{\sqrt{p}}=\pi$

6. The period of $\cos(\sin nx)$ is $\frac{\pi}{n}$

period of $\tan\left(\frac{x}{n}\right)$ is $n\pi$.

Thus, $6\pi=LCM\left(\frac{\pi}{n},\pi n\right)$

By checking $n=6$

7. The period of $\left|\sin^3\frac{x}{2}\right|=2\pi$

The period of $\left|\cos^5\frac{x}{5}\right|=5\pi$,

LCM is 10π

8. $f(x)=7+\cos\pi x-\{6x\}$

9. It is not a periodic function

10. Expand and L.C.M of $\{\pi,\pi\}=\pi$

11. L.C.M of $\left\{\frac{4}{3},4\right\}=4$

12. L.C.M of $\{8,12,3\}=24$

13. L.C.M of $\left\{\frac{10\pi}{3},7\pi,\frac{\pi}{14}\right\}=70\pi$

14. L.C.M. of $\left(\frac{2\pi}{n},2\pi n\right)=4\pi$

15. Here we have $\sqrt{x}$, so period does n ist

16. $\sin^2 x+\cos^4 x=1-\frac{1}{4}\sin^2 2x$

17. $\sin(A+B)\sin(A-B)=\sin^2 A-\text{si}$

18. $\cos x+3\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right]+$

$=3\sin x+4\cos x+6$

19. $(\sin x+\cos x)^2+\cos^2\left(\frac{\pi}{4}+x\right)=$

$2\sin^2\left(\frac{\pi}{4}+x\right)+\cos^2\left(\frac{\pi}{4}+x\right)$

20. Clearly we know that $A > 0$.

$\cos^{14}\theta \le \cos^2\theta \le 1$ and

$\sin^8\theta \le \sin^2\theta \le 1$

$\Rightarrow A \le 1$, Hence $0 < A \le 1$.

21. $\cos^2(\cos\theta)+\sin^2(\sin\theta)$

$+\sin^2(\cos\theta)-\sin^2(\cos\theta)$

$=1+\sin^2(\sin\theta)-\sin^2(\cos\theta)$, which is maximum when $\cos\theta=0$, $\sin\theta=1$

Maximum value is $1+\sin^2 1$.

22. Take 5sinx + 12siny = k and squaring and adding the both the equations.

23. $\cos^2 x+\sin^4 x=1-\frac{1}{4}\sin^2 2x$

24. $\cos\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)=\cos^2\frac{\pi}{6}-\sin^2 x$

$=\frac{3}{4}-\sin^2 x$

25. $5\cos\theta+3\left[\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta\right]+8=$

$\frac{13}{2}\cos\theta+\frac{3\sqrt{3}}{2}\sin\theta+8\in$

$\left[c-\sqrt{a^2+b^2},c+\sqrt{a^2+b^2}\right]$

26. $(\sin x-\cos x)^2+\cos^2\left(\frac{\pi}{4}-x\right)=$

$2\sin^2\left(\frac{\pi}{4}-x\right)+\cos^2\left(\frac{\pi}{4}-x\right)$

27. Given $3-\cos\theta+\cos\left(\theta+\frac{\pi}{3}\right)$

$=3-\cos\theta+\cos\theta\frac{1}{2}-\sin\theta.\frac{\sqrt{3}}{2}$

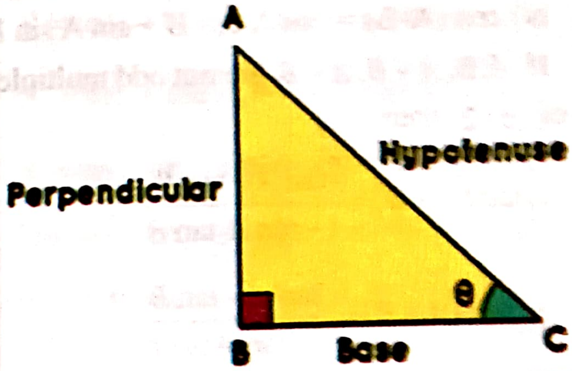
$=-\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta+3$

Which is in the form of $a\cos\theta+b\sin\theta+c$

$\text{Range}=\left[C-\sqrt{a^2+b^2},C+\sqrt{a^2+b^2}\right]$

$a=-\frac{1}{2}, b=-\frac{\sqrt{3}}{2}, c=3$

$\therefore \text{Range}=[2,4]$

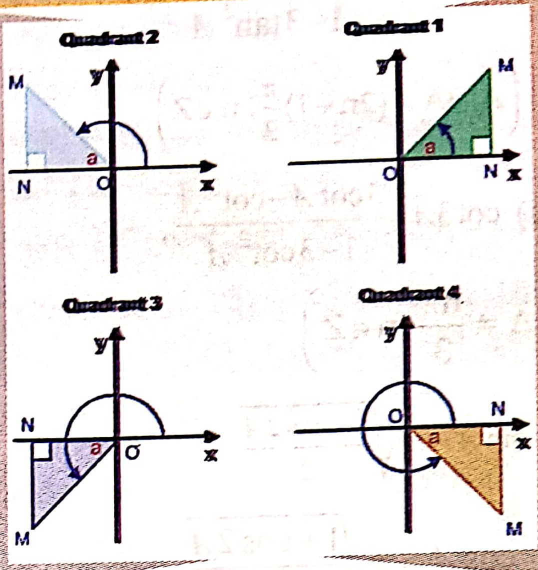


6.4 Multiple & Sub Multiple Angles

Internals of the Topic

- Multiple Angles And Half Angles Standard Angles
- Problems on $\sin\frac{A}{2}, \cos\frac{A}{2}, \tan\frac{A}{2}, \cos\frac{A}{2}$

  $\sin 2A, \cos 2A, \tan 2A, \cot 2A$
- Standard Results
- Componendo And Dividendo Property



CRT CONCEPT

- If A is an angle, then its integral multiples 2A,3A,4A,.... are called "Multiple angles of A.
- The multiples of A by fractions like $\frac{A}{2}, \frac{A}{3}, \ldots$ are called submultiple angles of A
- i) $\sin 2A = 2\sin A\cos A$

  ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$
  $= 1 - 2\sin^2 A$

  iii) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}\left(A, 2A \neq (2n+1)\frac{\pi}{2}, n\in Z\right)$

  iv) $\cot 2A = \frac{\cot^2 A - 1}{2\cot A}\left(A \neq \frac{n\pi}{2}, n\in Z\right)$
- i) $\sin 2A = \frac{2\tan A}{1+\tan^2 A}\left(A \neq (2n+1)\frac{\pi}{2}, n\in Z\right)$

  ii) $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}\left(A \neq (2n+1)\frac{\pi}{2}, n\in Z\right)$
- i) $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

  ii) $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$

  $= 2\cos^2\frac{A}{2} - 1 = 1 - 2\sin^2\frac{A}{2}$

  iii) $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}\left(\frac{A}{2}, A \neq (2n+1)\frac{\pi}{2}, n\in Z\right)$

  iv) $\cot A = \frac{\cot^2\frac{A}{2} - 1}{2\cot\frac{A}{2}}\left(A \neq n\pi, n\in Z\right)$
- i) $\sin A = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}\left(A \neq (2n+1)\pi, n\in Z\right)$

  ii) $\cos A = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}\left(A \neq (2n+1)\pi, n\in Z\right)$

SE-1: $3\sin x + 4\cos x = 5 \Rightarrow 6\tan\frac{x}{2} - 9\tan^2\frac{x}{2} =$

Sol: $3\sin x + 4\cos x = 5$

$$\Rightarrow 3\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + 4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) = 5$$

$$\Rightarrow 6\tan\frac{x}{2} - 9\tan^2\frac{x}{2} = 1$$

- i) $\sin 3A = 3\sin A - 4\sin^3 A$

  ii) $\cos 3A = 4\cos^3 A - 3\cos A$

  iii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A},$

  $\left(A, 3A \neq (2n+1)\dfrac{\pi}{2}, n \in Z\right)$

  iv) $\cot 3A = \dfrac{3\cot A - \cot^3 A}{1-3\cot^2 A},$

  $\left(A \neq \dfrac{n\pi}{3}, n \in Z\right)$

- i) $\sin A = \pm\sqrt{\dfrac{1-\cos 2A}{2}}$

  ii) $\cos A = \pm\sqrt{\dfrac{1+\cos 2A}{2}}$

  iii) $\tan A = \pm\sqrt{\dfrac{1-\cos 2A}{1+\cos 2A}}\left(A \neq (2n+1)\dfrac{\pi}{2}, n \in Z\right)$

  In above formulae, sign is based on quadrant.

- i) $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

  ii) $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

  iii)

  $\tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}\left(A \neq (2n+1)\pi, n \in Z\right)$

  In above formulae, sign is based on quadrant.

- i) $\tan\left(\dfrac{\pi}{4}+\dfrac{A}{2}\right) = \dfrac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}}$

  $= \dfrac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}} = \sqrt{\dfrac{1+\sin A}{1-\sin A}}$

  $= \dfrac{1+\sin A}{\cos A} = \dfrac{\cos A}{1-\sin A}$

  where $A \neq (4n+1)\dfrac{\pi}{2}, n \in Z$

  ii) $\tan\left(\dfrac{\pi}{4}-\dfrac{A}{2}\right) = \dfrac{1-\tan\frac{A}{2}}{1+\tan\frac{A}{2}}$

  $= \dfrac{\cos\frac{A}{2}-\sin\frac{A}{2}}{\cos\frac{A}{2}+\sin\frac{A}{2}} = \sqrt{\dfrac{1-\sin A}{1+\sin A}}$

  $= \dfrac{1-\sin A}{\cos A} = \dfrac{\cos A}{1+\sin A}$

  Where $A \neq (4n-1)\dfrac{\pi}{2}, n \in Z$

Standard results :

i) $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

$= 1 - \dfrac{1}{2}\sin^2 2\theta$

ii) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

$= 1 - \dfrac{3}{4}\sin^2 2\theta$

iii) $\cot A + \tan A = 2\operatorname{cosec} 2A\left(A \neq \dfrac{n\pi}{2}, n \in\right.$

iv) $\cot A - \tan A = 2\cot 2A\left(A \neq \dfrac{n\pi}{2}, n \in\right.$

v) $\tan\left(\dfrac{\pi}{4}+A\right) + \tan\left(\dfrac{\pi}{4}-A\right) = 2\sec 2A$

vi) $\tan\left(\dfrac{\pi}{4}+A\right) - \tan\left(\dfrac{\pi}{4}-A\right) = 2\tan 2A$

vii) $\tan A + 2\tan 2A + \ldots\ldots + 2^{n-1}\tan 2^{n-1}$

$+2^n \cot 2^n A = \cot$

(or)

$\cot A - \tan A - 2\tan 2A \ldots\ldots - 2^{n-1}\tan 2^{n-1}$

$= 2^n \cot 2^n A\left(A \neq \dfrac{n\pi}{2}, n \in Z\right)$

SE-2: If $\tan\dfrac{\pi}{7} + 2\tan\dfrac{2\pi}{7} + 4\tan\dfrac{4\pi}{7} + 8\cot\dfrac{8\pi}{7} = \tan$

then $\sin 7\theta =$

Sol: $\tan\dfrac{\pi}{7} + 2\tan\dfrac{2\pi}{7} + 4\tan\dfrac{4\pi}{7} + 8\cot\dfrac{8\pi}{7} = \cot\dfrac{\pi}{7}$

$$= \tan\left(\frac{\pi}{2}-\frac{\pi}{7}\right) = \tan\frac{5\pi}{14}$$

$$\therefore \theta = \frac{5\pi}{14}, \quad \sin 7\theta = \sin\frac{5\pi}{2} = 1$$

- If $\alpha = 60^\circ (or) 120^\circ (or) 240^\circ (or) 300^\circ$ then

i) $\sin\theta.\sin(\alpha-\theta).\sin(\alpha+\theta) = \boxed{\frac{1}{4}\sin 3\theta}$

ii) $\cos\theta.\cos(\alpha-\theta).\cos(\alpha+\theta) = \frac{1}{4}\cos 3\theta$

iii) $\tan\theta.\tan(\alpha-\theta).\tan(\alpha+\theta) = \tan 3\theta$

where $\theta, 3\theta \neq (2n+1)\frac{\pi}{2}, n \in Z$

iv) $\cot\theta.\cot(\alpha-\theta).\cot(\alpha+\theta) = \cot 3\theta$

where $\theta \neq \frac{n\pi}{3}, n \in Z$

v) $\tan\theta + \tan(\theta-\alpha) + \tan(\theta+\alpha) = 3\tan 3\theta$

where $\theta, 3\theta \neq (2n+1)\frac{\pi}{2}, n \in Z$

vi) $\cot\theta + \cot(\theta-\alpha) + \cot(\theta+\alpha) = 3\cot 3\theta$

where $\theta \neq \frac{n\pi}{3}, n \in Z$

SE-3: $x = \cot 6^\circ \cot 42^\circ, y = \tan 66^\circ \tan 78^\circ$ then $\frac{y}{x} =$

Sol: $\frac{y}{x} = \tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ$

$$= \frac{(\tan 6^\circ \tan 54^\circ \tan 66^\circ)(\tan 18^\circ \tan 42^\circ \tan 78^\circ)}{\tan 54^\circ \tan 18^\circ} = 1$$

- $$\left.\begin{array}{l} \tan\theta + \tan(60^\circ+\theta) + \tan(120^\circ+\theta) \\ \tan\theta + \tan(120^\circ+\theta) + \tan(240^\circ+\theta) \\ \tan\theta + \tan(240^\circ+\theta) + \tan(300^\circ+\theta) \\ \tan\theta + \tan(300^\circ+\theta) + \tan(60^\circ+\theta) \end{array}\right\} = 3\tan 3\theta$$

where $\theta, 3\theta \neq (2n+1)\frac{\pi}{2}, n \in Z$

- $$\left.\begin{array}{l} \cot\theta + \cot(60^\circ+\theta) + \cot(120^\circ+\theta) \\ \cot\theta + \cot(120^\circ+\theta) + \cot(240^\circ+\theta) \\ \cot\theta + \cot(240^\circ+\theta) + \cot(300^\circ+\theta) \\ \cot\theta + \cot(60^\circ+\theta) + \cot(300^\circ+\theta) \end{array}\right\} = 3\cot 3\theta$$

where $\theta \neq \frac{n\pi}{3}, n \in Z$

- $$\left.\begin{array}{l} \sin^2\theta + \sin^2(\alpha-\theta) + \sin^2(\alpha+\theta) \\ \cos^2\theta + \cos^2(\alpha-\theta) + \cos^2(\alpha+\theta) \end{array}\right\} = \frac{3}{2}$$

where $\alpha = 60^\circ (or) 120^\circ (or) 240^\circ (or) 300^\circ$

SE-4: *The value of* $\cos^2 10^\circ + \cos^2 50^\circ + \cos^2 70^\circ$ is

Sol.: $\cos^2 10^\circ + \cos^2 50^\circ + \cos^2 70^\circ$

$$= \cos^2 10^\circ + \cos^2(60^\circ - 10^\circ) + \cos^2(60^\circ + 10^\circ)$$

$$= \frac{3}{2}$$

- If $A + B = 60^\circ$ then

i) $\sin^2 A + \sin^2 B + \sin A \sin B = \frac{3}{4}$

ii) $\cos^2 A + \cos^2 B - \cos A \cos B = \frac{3}{4}$

- If $A - B = 60^\circ$ then

i) $\sin^2 A + \sin^2 B - \sin A \sin B = \frac{3}{4}$

ii) $\cos^2 A + \cos^2 B - \cos A \cos B = \frac{3}{4}$

SE-5: *The value of* $\frac{\sin^3 10^\circ + \cos^3 20^\circ}{\sin 10^\circ + \cos 20^\circ} =$

Sol.: $\frac{\sin^3 10^\circ + \cos^3 20^\circ}{\sin 10^\circ + \cos 20^\circ} = \frac{\cos^3 80^\circ + \cos^3 20^\circ}{\cos 80^\circ + \cos 20^\circ} =$

$$\cos^2 80^\circ + \cos^2 20^\circ - \cos 80^\circ \cos 20^\circ = \frac{3}{4}$$

- $$\left.\begin{array}{l} \sin^3\theta + \sin^3(60^\circ-\theta) - \sin^3(60^\circ+\theta) \\ \sin^3\theta - \sin^3(120^\circ-\theta) + \sin^3(120^\circ+\theta) \\ \sin^3\theta - \sin^3(240^\circ-\theta) + \sin^3(240^\circ+\theta) \\ \sin^3\theta + \sin^3(300^\circ-\theta) - \sin^3(300^\circ+\theta) \end{array}\right\} = -\frac{3}{4}\sin 3\theta$$

- $$\left.\begin{array}{l} \cos^3\theta - \cos^3(60^\circ-\theta) - \cos^3(60^\circ+\theta) \\ \cos^3\theta + \cos^3(120^\circ-\theta) + \cos^3(120^\circ+\theta) \\ \cos^3\theta + \cos^3(240^\circ-\theta) + \cos^3(240^\circ+\theta) \\ \cos^3\theta - \cos^3(300^\circ-\theta) - \cos^3(300^\circ+\theta) \end{array}\right\} = \frac{3}{4}\cos 3\theta$$

- $$\left.\begin{array}{l} \sin^3\theta - \sin^3(60^\circ+\theta) + \sin^3(120^\circ+\theta) \\ \sin^3\theta + \sin^3(120^\circ+\theta) + \sin^3(240^\circ+\theta) \\ \sin^3\theta + \sin^3(240^\circ+\theta) - \sin^3(300^\circ+\theta) \\ \sin^3\theta - \sin^3(60^\circ+\theta) - \sin^3(300^\circ+\theta) \end{array}\right\} = -\frac{3}{4}\sin 3\theta$$

$$\begin{vmatrix} \cos^3\theta - \cos^3(60^\circ+\theta)+\cos^3(120^\circ+\theta) \\ \cos^3\theta + \cos^3(120^\circ+\theta)+\cos^3(240^\circ+\theta) \\ \cos^3\theta + \cos^3(240^\circ-\theta)-\cos^3(300^\circ+\theta) \\ \cos^3\theta - \cos^3(60^\circ+\theta)-\cos^3(300^\circ+\theta) \end{vmatrix} = \frac{3}{4}\cos 3\theta$$

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots\sqrt{2+2\cos\theta}}}}$$

$$=2\cos\left(\frac{\theta}{2^n}\right)$$

Where 'n' is the number of square roots and $0<\theta<\pi$

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$$

Where 'n' is the number of square roots

SE-6: $\sqrt{2+\sqrt{2+\sqrt{2}}} =$

Sol: $n=3 \Rightarrow \sqrt{2+\sqrt{2+\sqrt{2}}} = 2\cos\left(\frac{\pi}{2^{3+1}}\right) = 2\cos\frac{\pi}{16}$

$$\cos\theta.\cos 2\theta.\cos 2^2\theta \ldots\ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n\sin\theta}$$

Where $\theta \neq n\pi$

SE-7: $\frac{\sin 8A}{\sin A} = K\cos A\cos 2A\cos 4A$. *Then K =*

Sol.: $\cos A\cos 2A\cos 4A = \frac{\sin 2^3 A}{2^3\sin A}$

$8\cos A\cos 2A\cos 4A = \frac{\sin 8A}{\sin A}$ $\therefore$ K = 8

$$\cos\left(\frac{\pi}{2n+1}\right)\cos\left(\frac{2\pi}{2n+1}\right)\cos\left(\frac{3\pi}{2n+1}\right)$$
$$\cos\left(\frac{4\pi}{2n+1}\right)\ldots\ldots\cos\left(\frac{n\pi}{2n+1}\right) = \frac{1}{2^n}$$

SE-8: *The value of*

$$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$$

Sol.: n = 7 ; Given $= \frac{1}{2^7} = \frac{1}{128}$

SE-9: $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} + \ldots\ldots$ *n terms*

$$= \frac{1}{2}\left(\tan 3^n x - \tan x\right)$$

Where $x \neq (2n+1)\frac{\pi}{2.3^n}, n \in Z$

Sol.:
$$\frac{2\cos x\sin x}{2\cos x\cos 3x} + \frac{2\cos 3x\sin 3x}{2\cos 3x\cos 9x} +$$
$$\frac{2\cos 9x\sin 9x}{2\cos 9x\cos 27x} + \ldots + \frac{2\cos 3^{n-1}x\sin 3^{n-1}x}{2\cos 3^{n-1}x\cos 3^n x}$$
$$= \frac{\sin 2x}{2\cos x\cos 3x} + \frac{\sin 6x}{2\cos 3x\cos 9x} +$$
$$\frac{\sin 18x}{2\cos 9x\cos 27x} + \ldots\ldots + \frac{\sin 2.3^{n-1}x}{2\cos 3^{n-1}x\cos 3^n x}$$
$$= \frac{\sin(3x-x)}{2\cos x\cos 3x} + \frac{\sin(9x-3x)}{2\cos 3x\cos 9x} +$$
$$\frac{\sin(27x-9x)}{2\cos 9x\cos 27x} + \ldots\ldots + \frac{\sin\left(3^n x - 3^{n-1}x\right)}{2\cos 3^{n-1}x\cos 3^n x}$$
$$= \frac{1}{2}(\tan 3x - \tan x) + \frac{1}{2}(\tan 9x - \tan 6x) +$$
$$\frac{1}{2}(\tan 27x - \tan x) + \ldots + \frac{1}{2}\left(\tan 3^n x - \tan 3^{n-1}x\right)$$
$$= \frac{1}{2}\left(\tan 3^n x - \tan x\right)$$

$$(1+\sec 2\theta)(1+\sec 2^2\theta)\ldots\ldots(1+\sec 2^n\theta)$$
$$= \frac{\tan(2^n\theta)}{\tan\theta}$$

SE-10: *Find*

$$(1+\sec 20^\circ)(1+\sec 40^\circ)(1+\sec 80^\circ)$$

Sol.: $\theta = 10^\circ$, n = 3

$$\text{L.H.S} = \frac{\tan 2^3(10^\circ)}{\tan 10^\circ} = \cot^2 10^\circ$$

i) $\sqrt{1+\sin 2A} = |\cos A + \sin A|$

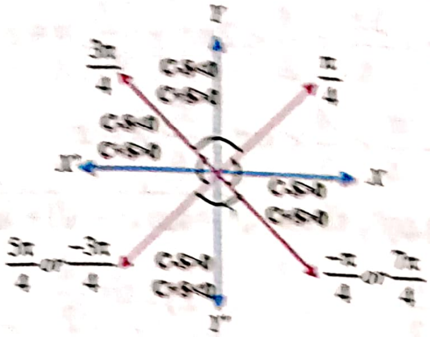
ii) $\sqrt{1+\sin A} = \left|\cos\frac{A}{2} + \sin\frac{A}{2}\right|$

iii) $\sqrt{1-\sin 2A} = |\cos A - \sin A|$

iv) $\sqrt{1-\sin A} = \left|\cos\frac{A}{2} - \sin\frac{A}{2}\right|$

Let $C=\cos\frac{A}{2}; S=\sin\frac{A}{2}$, then

$C+S=\pm\sqrt{1+\sin A},\ C-S=\pm\sqrt{1-\sin A}$



i) $c+s>0$ in $\left(-\frac{\pi}{4},\frac{3\pi}{4}\right)$

ii) $c+s<0$ in $\left(\frac{3\pi}{4},\frac{7\pi}{4}\right)$

iii) $c-s>0$ in $\left(-\frac{3\pi}{4},\frac{\pi}{4}\right)$

iv) $c-s<0$ in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$

SE-11: Find the value of

$$\frac{\sqrt{1+\sin 2A}+\sqrt{1-\sin 2A}}{\sqrt{1+\sin 2A}-\sqrt{1-\sin 2A}};$$

When $|\tan A|<1$ *and* $|A|$ *is acute.*

Sol.: $$\frac{\sqrt{1+\sin 2A}+\sqrt{1-\sin 2A}}{\sqrt{1+\sin 2A}-\sqrt{1-\sin 2A}}$$

$$=\frac{\sqrt{(\cos A+\sin A)^2}+\sqrt{(\cos A-\sin A)^2}}{\sqrt{(\cos A+\sin A)^2}-\sqrt{(\cos A-\sin A)^2}}$$

$$=\frac{|\cos A+\sin A|+|\cos A-\sin A|}{|\cos A+\sin A|-|\cos A-\sin A|}$$

$$=\frac{\cos A+\sin A+\cos A-\sin A}{\cos A+\sin A-(\cos A-\sin A)}$$

$\left(\because \text{If } A\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right) \text{ then } \cos A>\sin A\right)$

$$=\frac{2\cos A}{2\sin A}=\cot A$$

Important Values:

i) $\sin 7\frac{1}{2}^\circ=\sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{4\sqrt{2}}}=\frac{\sqrt{4-\sqrt{6}-\sqrt{2}}}{2\sqrt{2}}$

$=\cos 82\frac{1}{2}^\circ$

ii) $\sin 18^\circ=\frac{\sqrt{5}-1}{4}=\cos 72^\circ$

iii) $\sin 22\frac{1}{2}^\circ=\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}=\frac{1}{2}\sqrt{2-\sqrt{2}}=\cos 67\frac{1}{2}^\circ$

iv) $\sin 36^\circ=\frac{\sqrt{10-2\sqrt{5}}}{4}=\cos 54^\circ$

v) $\sin 54^\circ=\frac{\sqrt{5}+1}{4}=\cos 36^\circ$

vi) $\sin 67\frac{1}{2}^\circ=\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}=\frac{1}{2}\sqrt{2+\sqrt{2}}=\cos 22\frac{1}{2}^\circ$

vii) $\sin 72^\circ=\frac{\sqrt{10+2\sqrt{5}}}{4}=\cos 18^\circ$

viii) $\sin 82\frac{1}{2}^\circ=\sqrt{\frac{2\sqrt{2}+\sqrt{3}+1}{4\sqrt{2}}}$

$=\frac{\sqrt{4+\sqrt{6}+\sqrt{2}}}{2\sqrt{2}}=\cos 7\frac{1}{2}^\circ$

ix) $\tan 7\frac{1}{2}^\circ=(\sqrt{3}-\sqrt{2})(\sqrt{2}-1)=\cot 82\frac{1}{2}^\circ$

x) $\tan 18^\circ=\sqrt{4\sqrt{5}-8}=\cot 72^\circ$

xi) $\tan 22\frac{1}{2}^\circ=\sqrt{2}-1=\cot 67\frac{1}{2}^\circ$

xii) $\tan 36^\circ=\sqrt{5-2\sqrt{5}}=\cot 54^\circ$

xiii) $\tan 67\frac{1}{2}^\circ = \sqrt{2}+1 = \cot 22\frac{1}{2}^\circ$

xiv) $\tan 82\frac{1}{2}^\circ = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1) = \cot 7\frac{1}{2}^\circ$

Concept Based Questions

1. If $2\cos\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$, then
   1) $2n\pi + \frac{\pi}{4} < \frac{A}{2} < 2n\pi + \frac{3\pi}{4}$
   2) $2n\pi - \frac{\pi}{4} < \frac{A}{2} < 2n\pi - \frac{3\pi}{4}$
   3) $2n\pi - \frac{3\pi}{4} < \frac{A}{2} < 2n\pi + \frac{5\pi}{4}$
   4) $n\pi + \frac{\pi}{4} < \frac{A}{2} < n\pi + \frac{3\pi}{4}$

2. If $2\sin\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$, then $\frac{A}{2}$ lies between
   1) $2n\pi + \frac{\pi}{4}$ and $2n\pi + \frac{3\pi}{4}, n \in z$
   2) $2n\pi - \frac{\pi}{4}$ and $2n\pi + \frac{\pi}{4}, n \in z$
   3) $2n\pi - \frac{3\pi}{4}$ and $2n\pi - \frac{\pi}{4}, n \in z$
   4) $2n\pi - \frac{3\pi}{4}$ and $2n\pi + \frac{\pi}{4}, n \in z$

3. If $2\cos\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$, then $\frac{A}{2}$ lies between
   1) $2n\pi + \frac{\pi}{4}$ and $2n\pi + \frac{3\pi}{4}$
   2) $2n\pi - \frac{\pi}{4}$ and $2n\pi + \frac{\pi}{4}$
   3) $2n\pi - \frac{3\pi}{4}$ and $2n\pi - \frac{\pi}{4}$
   4) $2n\pi - \frac{5\pi}{4}$ and $2n\pi - \frac{\pi}{4}$

4. $2\sin\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$ then
   1) $2n\pi - \frac{\pi}{4} < \frac{A}{2} < 2n\pi + \frac{\pi}{4}, n \in z$
   2) $2n\pi + \frac{\pi}{4} < \frac{A}{2} < 2n\pi + \frac{3\pi}{4}, n \in z$
   3) $2n\pi + \frac{3\pi}{4} < \frac{A}{2} < 2n\pi + \frac{5\pi}{4}, n \in z$
   4) $n\pi - \frac{\pi}{4} < \frac{A}{2} < n\pi + \frac{\pi}{4}, n \in z$

5. If $A = 340^0$, then $\sqrt{1-\sin A} - \sqrt{1+\sin A} =$
   1) $2\cos\frac{A}{2}$ 2) $2\sin\frac{A}{2}$
   3) $-2\cos\frac{A}{2}$ 4) $-2\sin\frac{A}{2}$

6. $4\sin 27^0 =$
   1) $\sqrt{5+\sqrt{5}} + \sqrt{3-\sqrt{5}}$
   2) $\sqrt{5-\sqrt{5}} + \sqrt{3+\sqrt{5}}$
   3) $\sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}}$
   4) $\sqrt{5+\sqrt{5}} + \sqrt{3+\sqrt{5}}$

7. The range of all values of 'θ' such that $\sin 3\theta - \cos 3\theta > 0$
   1) $2n\pi + \frac{\pi}{6} < \theta < 2n\pi + \frac{5\pi}{6}$
   2) $\frac{2n\pi}{3} + \frac{\pi}{12} < \theta < \frac{2n\pi}{3} + \frac{5\pi}{12}$
   3) $2n\pi - \frac{\pi}{6} < \theta < 2n\pi + \frac{\pi}{6}$
   4) $\frac{n\pi}{3} - \frac{\pi}{6} < \theta < \frac{n\pi}{3} + \frac{\pi}{6}$

8. If $\sin^2 A = x$, then $\sin A \sin 2A \sin 3A \sin 4A$ is a polynomial in x, the sum of whose coefficients is
   1) 0 2) 40 3) 168 4) 336

KEY

01) 1	02) 1	03) 2	04) 1
05) 2	06) 3	07) 2	08) 1

Hints & Solutions

1. $C+S=\pm\sqrt{1+\sin A}$ & $C-S=\pm\sqrt{1-\sin A}$

For given data, $C+S>0$ & $C-S<0$

2. $C+S>0, C-S<0$
3. $C+S>0, C-S>0$
4. $C+S>0, C-S>0$
5. $C+S<0, C-S<0$

So, $C+S=-\sqrt{1+\sin A}$,

$C-S=-\sqrt{1-\sin A}$

$\Rightarrow S-C=\sqrt{1-\sin A}$

6. $\frac{A}{2}=27^{o}$, *then* $C+S>0, C-S>0$

We have $2\sin\frac{A}{2}=\sqrt{1+\sin A}-\sqrt{1-\sin A}$

Take, $\frac{A}{2}=27°$, $A=54°$

7. $2n\pi+\frac{\pi}{4}\le 3\theta\le 2n\pi+\frac{5\pi}{4}$
8. Let G.E be

$f(\sin A)=2\sin A(3\sin A-4\sin^3 A)$

$(1-2\sin^2 A)4\sin^2 A(1-\sin^2 A)$

Put sin A = 1, sum of the coefficients =

$f(1)=0$

EXERCISE-I

CRTQ & SPQ LEVEL-I

MULTIPLE ANGLES AND HALF ANGLES STANDARD ANGLES

C.R.T.Q Class Room Teaching Questions

1. $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=$

1) $\tan\frac{\theta}{2}$ 2) $\cot\frac{\theta}{2}$ 3) $\tan\theta$ 4) $\cot\theta$

2. $\cot\theta=\frac{2\tan 7½^{\circ}}{1-\tan^2 7½^{\circ}}$ then $\sin 3\theta=$

1) $\frac{-1}{\sqrt{2}}$ 2) $\frac{\sqrt{3}}{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{1}{2}\sqrt{2-\sqrt{2}}$

3. If $\tan\alpha=\sqrt{a}$, where a is rational number which is not a perfect square, then which of the following is a rational number.

1) $\sin 2\alpha$ 2) $\tan 2\alpha$ 3) $\cos 2\alpha$ 4) $\cot\alpha$

4. $\frac{\sin 12A}{\sin 4A}-\frac{\cos 12A}{\cos 4A}=$

1. 6 2. 4 3. 2 4. 1

5. $4\cos^3 40^0 - 3\sin 50^0 =$

1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{-\sqrt{3}}{2}$ 4) $\frac{-1}{2}$

6. If $x+\frac{1}{x}=2\cos 20^{o}$, then the value of $x^3+\frac{1}{x^3}=$

1) 1/4 2) 1/2 3) 1 4) 1/8

7. $\tan A=\frac{1-\cos B}{\sin B}\Rightarrow \tan 2A-\tan B=$

1) 0 2) 1 3) 1/2 4) 1/4

8. If $180^0<\theta<270^0, \cot\theta=\frac{4}{3}$, then $\sin\frac{\theta}{2}=$

1) $\frac{3}{\sqrt{10}}$ 2) $-\frac{2}{\sqrt{5}}$ 3) $-\frac{1}{\sqrt{5}}$ 4) $\frac{1}{\sqrt{5}}$

9. $\cos^6 A+\sin^6 A=1-k\sin^2(2A)\Rightarrow k=$

1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) $\frac{3}{4}$ 4) 1

10. $\sin 12^0\sin 48^0\sin 54^0=$

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{1}{8}$ 4) $\frac{1}{16}$

11. $\sin^2 160^0+\sin^2 140^0+\sin^2 100^0=$

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. $\frac{5}{2}$ 4. $\frac{7}{2}$

12. The value of

$\sin^2 46^0+\sin^2 14^0+\sin 46^0\sin 14^0=$

1) $\frac{1}{4}$ 2) $\frac{3}{4}$ 3) $\frac{5}{4}$ 4) $\frac{1}{2}$

13. $\cos^3 10^0 + \cos^3 110^0 + \cos^3 130^0 =$

1) $\frac{3}{4}$ 2) $\frac{3}{8}$ 3) $\frac{3\sqrt{3}}{8}$ 4) $\frac{3\sqrt{3}}{4}$

14. $\theta < \frac{\pi}{16}$, $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = k\cos\theta \Rightarrow k =$

1) 2 2) 4 3) 8 4) 16

15. $\frac{\sin x}{\sin x/8} =$

1) $8\cos\frac{x}{8}\cos\frac{x}{4}\cos\frac{x}{2}$

2) $8\cos\frac{x}{8}\sin\frac{x}{4}\sin\frac{x}{2}$

3) $8\sin\frac{x}{8}\sin\frac{x}{4}\sin\frac{x}{2}$

4) $8\sin\frac{x}{8}\sin\frac{x}{4}\cos\frac{x}{2}$

16. $\frac{\sin\theta - \sin 2\theta}{1-\cos\theta + \cos 2\theta} =$

1) $\tan\frac{\theta}{2}$ 2) $\cot\frac{\theta}{2}$

3) $\tan\theta$ 4) $-\tan\theta$

S.P.Q. Student Practice Questions

17. $\frac{1-\tan^2 7\frac{1}{2}^0}{2\tan 7\frac{1}{2}^0} =$

1) $2-\sqrt{3}$ 2) $2+\sqrt{3}$ 3) $\sqrt{2}-1$ 4) $\sqrt{2}+1$

18. Which of the following is rational number?

1) $\sin 15^0$ 2) $\cos 15^0$

3) $\sin 15^0 \cos 15^0$ Transformation 4) $\sin 15^0 \cos 75^0$

19. $\frac{\cos A - \cos 3A}{\cos A} + \frac{\sin A + \sin 3A}{\sin A} =$

1) 1 2) 2 3) 3 4) 4

20. $4\sin^3 75^0 - 3\cos 15^0 =$

1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

21. $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$ then $\cos 3\theta = K\left(a^3 + \frac{1}{a^3}\right)$ where K is equal to

1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) 1 4) $\frac{3}{2}$

22. $x = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \Rightarrow \frac{2x}{1-x^2} =$

1) $\sin\theta$ 2) $\cos\theta$ 3) $\tan\theta$ 4) $\cot\theta$

23. If $180^0 < \theta < 270^0, \sin\theta = -\frac{3}{5}$, then $\cos\frac{\theta}{2} =$

1) $\frac{-1}{\sqrt{10}}$ 2) $\frac{1}{\sqrt{10}}$ 3) $\frac{1}{10}$ 4) 10

24. $\sin^4\theta + \cos^4\theta = 1 - \frac{1}{2}f(\theta)$ then $f\left(\frac{\pi}{4}\right) =$

1) 1 2) 0 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

25. $\cos 12^0 \cos 24^0 \cos 36^0 \cos 48^0 \cos 72^0 \cos 84^0 =$

1) $\frac{1}{64}$ 2) $\frac{1}{32}$ 3) $\frac{1}{16}$

26. $\cot 15^0 + \cot 75^0 + \cot 315^0 =$

1) 0 2) 1 3) 2 4) 3

27. $\cos^2 25^0 + \cos^2 95^0 + \cos^2 145^0 =$

1) $\frac{1}{2}$ 2) $\frac{3}{2}$ 3) $\frac{3}{4}$ 4) $\frac{1}{\sqrt{2}}$

28. The value of $\cos^2 76^0 + \cos^2 16^0 - \cos 76^0 \cos 16^0 =$

1) $\frac{1}{4}$ 2) $\frac{3}{4}$ 3) $\frac{5}{4}$ 4) $\frac{1}{2}$

29. $\sin^3 10^0 + \sin^3 250^0 - \sin^3 310^0 =$

1) $-\frac{3}{8}$ 2) $\frac{3}{8}$ 3) $\frac{1}{8}$ 4) $\frac{3}{4}$

30. $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}} =$

1) $\cos\frac{\pi}{32}$ 2) $2\cos\frac{\pi}{32}$

3) $2\cos\frac{\pi}{64}$ 4) $\cos\frac{\pi}{64}$

31. $8\sin\theta\cos\theta.\cos 2\theta\cos 4\theta = \sin x \Rightarrow x =$

1) 16θ 2) 8θ 3) 4θ 4) 32θ

32. $\sec 72° - \sec 36° =$

1) 2 2) $\frac{1}{2}$ 3) 4 4) $\frac{1}{4}$

KEY

01) 3	02) 1	03) 3	04) 3	05) 4
06) 3	07) 1	08) 1	09) 3	10) 3
11) 2	12) 2	13) 3	14) 1	15) 1
16) 4	17) 2	18) 3	19) 4	20) 2
21) 1	22) 3	23) 1	24) 1	25) 1
26) 4	27) 2	28) 2	29) 1	30) 3
31) 2	32) 1			

Hints & Solutions

1. Use, $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$

2. $\frac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$

3. $\cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$

4. $\sin 12A = \sin 3(4A) = 3\sin 4A - 4\sin^3 4A$

$\cos 12A = \cos 3(4A) = 4\cos^3 4A - 3\cos 4A$

5. $4\cos^3 40° - 3\sin 50° = 4\cos^3 40° - 3\cos 40°$

$= \cos 3(40°) = \cos 120° = -\frac{1}{2}$

6. Cubing on both sides apply $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

7. Use, $1-\cos B = 2\sin^2\frac{B}{2}$ and

$\sin B = 2\sin\frac{B}{2}\cos\frac{B}{2}$

8. $\cos\theta = \frac{-4}{5}$, use $2\sin^2\frac{\theta}{2} = 1-\cos\theta$

9. $\cos^6 A + \sin^6 A = 1 - \frac{3}{4}\sin^2 2A$

10. Multiply and divided by $\sin 72^0$ and apply

$\sin\theta\sin(60°-\theta).\sin(60°+\theta) = \frac{1}{4}\sin 3\theta$

11. $\sin^2 20° + \sin^2 40° + \sin^2 80° = \frac{3}{2}$

12. $A + B = 46° + 14° = 60°$ then

$\sin^2 A + \sin^2 B + \sin A\sin B = \frac{3}{4}$

13. $\cos^3\theta + \cos^3(120° - \theta) + \cos^3(120° + \theta) = \frac{3}{4}\cos 3\theta$

14. n = 3, given $= 2\cos\left(\frac{8\theta}{2^3}\right) = 2\cos\theta$

15. $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= 4\sin\frac{x}{4}.\cos\frac{x}{4}.\cos\frac{x}{2}$

$= 8\sin\frac{x}{8}.\cos\frac{x}{8}.\cos\frac{x}{4}\cos\frac{x}{2}$

16. Use, $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$

17. $\frac{1-\tan^2\theta}{2\tan\theta} = \cot 2\theta$

18. $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

19. Use, $\sin 3A = 3\sin A - 4\sin^3 A$ and $\cos 3A = 4\cos^3 A - 3\cos A$

20. $\cos 3A = 4\cos^3 A - 3\cos A$

21. Cubing on both sides and apply $\cos 3A = 3\cos A - 4\cos^3 A$

22. $\tan^2\frac{A}{2} = \frac{1-\cos A}{1+\cos A}$

23. $\cos\theta = -\frac{4}{5}$; Use, $2\cos^2\frac{\theta}{2} = 1+\cos\theta$

24. $\sin^4\theta + \cos^4\theta = 1 - \frac{1}{2}.\sin^2 2\theta$

25. Use, $\cos\theta.\cos(60^\circ-\theta)\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$

26. $\cot\theta + \cot(60^\circ+\theta) + \cot(300^0+\theta) = 3\cot 3\theta$

27. $\cos^2\theta + \cos^2(120^0-\theta) + \cos^2(120^0+\theta) = \frac{3}{2}$

28. $A - B = 76^\circ - 16^\circ = 60^\circ$ then

$\cos^2 A + \cos^2 B - \cos A\cos B = \frac{3}{4}$

29. Use, $\sin^3\theta + \sin^3(240^\circ+\theta) -$

$\sin^3(300^\circ+\theta) = \frac{-3}{4}\sin 3\theta$

30. $\sqrt{2+\sqrt{2+\sqrt{2+....nterms}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$

31. $\cos\theta.\cos 2\theta.\cos 2^2\theta....\cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n\sin\theta}$

32. Subtitute the values

EXERCISE-II
CRTQ & SPQ LEVEL-II

PROBLEMS ON

$\sin\frac{A}{2}\cos\frac{A}{2}\tan\frac{A}{2}\cos\frac{A}{2}$

$\sin 2A, \cos 2A, \tan 2A, \cot 2A$

C.R.T.Q Class Room Teaching Questions

1. $\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ} = \lambda$ then $3\lambda^2 =$ Doubt

1) 16 2) 4 3) $4/\sqrt{3}$ 4) $16/3$

2. $\tan\theta = \frac{b}{a}\left(0<\theta<\frac{\pi}{4}\right) \Rightarrow \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} =$ common

1) $\frac{2\sin\theta}{\sqrt{\sin 2\theta}}$ 2) $\frac{2\cos\theta}{\sqrt{\cos 2\theta}}$

3) $\frac{2\cos\theta}{\sqrt{\sin 2\theta}}$ 4) $\frac{2\sin\theta}{\sqrt{\cos 2\theta}}$

3. If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$ then $\cos(\theta+\alpha)=\sqrt{1-\sin^2\alpha}$

$\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta) =$

1) $1-a^2-b^2$ 2) $1-2a^2-2b^2$

3) $2+a^2-b^2$ 4) $2-a^2-b^2$

4. The value of the expression CW

$\tan^6 20^\circ - 33\tan^4 20^\circ + 27\tan^2 20^\circ$ is

1) 0 2) 1 3) 2 4) 3

5. If $\theta \in Q_3$, then $2\cos^2\frac{\theta}{2} = 1+\cos\theta$

$\sqrt{4\sin^4\theta + \sin^2 2\theta} + 4\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right) =$

1) 2 2) -2 3) 0 4) 1

6. If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and CW

$\alpha - \beta = 2\theta$, then $\frac{\cos 3\theta}{\cos\theta} =$

1) a^2+b^2-2 2) a^2+b^2-3

3) $3-a^2-b^2$ 4) $(a^2+b^2)/4$

7. $\cos\alpha = \frac{2\cos\beta - 1}{2-\cos\beta}(0<\alpha,\beta<\pi)$. CW

$\alpha+\beta = \pi$, then $\tan\frac{\alpha}{2} =$

1) $3^{1/4}$ 2) $3^{1/2}$ 3) 3 4) 3^2

8. If $\sec\theta - \cos\theta = 1$, then $\tan^2\frac{\theta}{2} =$ CW

1) $\sqrt{5}+2$ 2) $\boxed{\sqrt{5}-2}$ 3) $2-\sqrt{5}$ 4) 0

9. If $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\alpha}{2}$, then $\cos\alpha =$ CW

1) $\frac{1-e\cos\theta}{\cos\theta+e}$ 2) $\frac{1+e\cos\theta}{\cos\theta+e}$

3) $\frac{1-e\cos\theta}{\cos\theta-e}$ 4) $\frac{\cos\theta-e}{1-e\cos\theta}$ CW

10. If $k = \tan 25^\circ$ then $\frac{k-1}{k+1} + \frac{k+1}{k-1} =$

1) $2\operatorname{cosec} 130^\circ$ 2) $2\operatorname{cosec} 45^\circ$

3) $2\sec 130^\circ$ 4) $2\sec 400^\circ$

11. $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}=$

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{3}{2}$ 4) $\frac{3}{4}$

12. $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}=$

1) $\frac{1}{4}$ 2) $\frac{1}{8}$ 3) $\frac{1}{16}$ 4) $\frac{1}{32}$

13. A quadratic equation whose roots are $\tan 22\frac{1}{2}^0$ and $\cot 22\frac{1}{2}^0$ is

CW

1) $x^2-2\sqrt{2}x+1=0$ 2) $2x^2-\sqrt{2}x+1=0$

3) $x^2+2\sqrt{2}x-1=0$ 4) $x^2-2\sqrt{2}x-1=0$

S.P.Q. Student Practice Questions

CW

14. $\cot 7\frac{1}{2}^\circ-\cot 37\frac{1}{2}^\circ-\cot 52\frac{1}{2}^\circ+\cot 82\frac{1}{2}^\circ=$

1) $4\sqrt{2}$ 2) 4 3) 2 4) 1

15. If A lies in the third quadrant and $3\tan A-4=0$ then $5\sin 2A+3\sin A+4\cos A=$

$\frac{84}{5}-\frac{12}{5}-\frac{12}{5}$

1) 0 2) $-\frac{24}{5}$ 3) $\frac{24}{5}$ 4) $\frac{48}{5}$

16. $2\sin^2\beta+4\cos(\alpha+\beta)\sin\alpha\sin\beta+\cos 2(\alpha+\beta)=$ Result

CW

1) $\sin 2\alpha$ 2) $\cos 2\alpha$

3) $\tan 2\alpha$ 4) $\cot 2\alpha$

17. $32\sin^6 15^0-48\sin^4 15^0+18\sin^2 15^0=$

1) 1 2) 2 3) 3 4) -1

18. $(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2=k\sin^2\left(\frac{\alpha-\beta}{2}\right)\Rightarrow k=$

1) 4 2) 3 3) $\frac{3}{2}$ 4) $\frac{1}{4}$

19. $\cos^4\alpha-\sin^4\alpha=a$ then $\frac{1-a}{1+a}=$

1) $\tan^2\alpha$ 2) $\cot^2\alpha$

3) $-\tan^2\alpha$ 4) $-\cot^2\alpha$

20. α and β are acute angles and $\cos 2\alpha=\frac{3\cos 2\beta-1}{3-\cos 2\beta}$, then $\tan\alpha\cot\beta=$

1) $\sqrt{3}$ 2) $\sqrt{2}$ 3) 1 4) -1

21. If $\tan\frac{\theta}{2}=\operatorname{cosec}\theta-\sin\theta$, then $\cos^2\frac{\theta}{2}=$

1) $\frac{\sqrt{3}-1}{4}$ 2) $\frac{\sqrt{5}+1}{4}$

3) $\frac{\sqrt{3}+1}{4}$ 4) $\frac{\sqrt{5}-1}{4}$

22. If $2\tan\frac{\alpha}{2}=\tan\frac{\beta}{2}$ then $\frac{3+5\cos\beta}{5+3\cos\beta}=$

1) $\cos\alpha$ 2) $\sin\alpha$ 3) $\tan\alpha$ 4) $\cot\alpha$

23. $\operatorname{cosec}\theta=\frac{p+q}{p-q}\Rightarrow\cot\left(\frac{\theta}{2}+\frac{\pi}{4}\right)=$

CW

1) $\sqrt{\frac{p}{q}}$ 2) $\sqrt{\frac{q}{p}}$

3) $\sqrt{pq}$ 4) pq

24. $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=$

1) $\frac{1}{2}$ 2) $\frac{3}{2}$ 3) $\frac{1}{4}$ 4) $\frac{3}{4}$

25. $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}=$

1) $\frac{1}{16}$ 2) $\frac{1}{8}$ 3) $\frac{3}{4}$ 4) $\frac{1}{4}$

26. The quadratic equation whose roots are $\sin^2 18^0, \cos^2 36^0$

1) $16x^2-12x+1=0$

2) $x^2-12x+1=0$

3) $16x^2-12x-1=0$

4) $16x^2+12x+1=0$

KEY

01) 1	02) 2	03) 2	04) 4	05) 1
06) 2	07) 1	08) 2	09) 4	10) 3
11) 3	12) 4	13) 1	14) 1	15) 1
16) 2	17) 1	18) 1	19) 1	20) 2
21) 2	22) 1	23) 2	24) 2	25) 1
26) 1				

Hints & Solutions

1. $\lambda=\frac{1}{\sin 20^\circ}-\frac{1}{\sqrt{3}\cos 20^\circ}=\frac{\sqrt{3}\cos 20^\circ-\sin 20^\circ}{\sqrt{3}.\cos 20^\circ.\sin 20^\circ}=\frac{4}{\sqrt{3}}$

2. $\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}=\frac{2a}{\sqrt{a^2-b^2}}=\frac{2}{\sqrt{1-\frac{b^2}{a^2}}}$

3. $\cos(\alpha-\beta)=\cos((\theta+\alpha)-(\theta+\beta))$

4. $\tan 60^\circ=\tan 3(20^\circ)=\frac{3\tan 20^\circ-\tan^3 20^\circ}{1-3\tan^2 20^\circ}$

5. Use, $\sin^2 2\theta=4\sin^2\theta\cos^2\theta$ and $2\cos^2\theta=1+\cos 2\theta$

6. $a^2+b^2=4\cos^2\theta$

7. $\tan^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{1+\cos\alpha}$

8. $\cos^2\theta+\cos\theta-1=0$ and use $\tan^2\frac{\theta}{2}=\frac{1-\cos\theta}{1+\cos\theta}$

9. $\cos\alpha=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$

10. $\tan\left(\frac{\pi}{4}+A\right)+\tan\left(\frac{\pi}{4}-A\right)=2\sec 2A$

11. Use, $\sin^4\theta+\cos^4\theta=1-\frac{1}{2}\sin^2 2\theta$

12. $\cos\theta.\cos 2\theta.\cos 2^2\theta.........\cos 2^{n-1}\theta=\frac{\sin 2^n\theta}{2^n\sin\theta}$

13. $\alpha+\beta=2\sqrt{2},\ \alpha\beta=1$

14. $\left(\cot 7\frac{1}{2}^\circ+\cot 82\frac{1}{2}^\circ\right)-\left(\cot 37\frac{1}{2}^\circ+\cot 52\frac{1}{2}^\circ\right)$

$\frac{2}{\sin 15^\circ}-\frac{2}{\sin 75^\circ}=4\sqrt{2}$

15. $\tan A=4/3,\cos A=-3/5,\sin A=-4/5$

16. Use, $\cos 2\theta=2\cos^2\theta-1$,

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ and

$\cos(\alpha+\beta).\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta$

17. $\sin 3\theta=3\sin\theta-4\sin^3\theta$

put $\theta=15^\circ$ and squaring on both sides

18. Expand

19. $\cos^4\alpha-\sin^4\alpha=\cos 2\alpha$

20. $\tan^2\alpha=\frac{1-\cos 2\alpha}{1+\cos 2\alpha}$

21. $\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=\frac{1-\sin^2\theta}{\sin\theta}\Rightarrow 2\sin^2\frac{\theta}{2}=\cos^2\theta$

$\Rightarrow 2\left(1-\cos^2\frac{\theta}{2}\right)=\left(2\cos^2\frac{\theta}{2}-1\right)^2$

$\Rightarrow 4\cos^4\frac{\theta}{2}-2\cos^2\frac{\theta}{2}-1=0\Rightarrow\cos\frac{\theta}{2}=\frac{\sqrt{5}+1}{4}$

22. $4\tan^2\frac{\alpha}{2}=\tan^2\frac{\beta}{2}$,

use $\cos\beta=\frac{1-\tan^2\beta/2}{1+\tan^2\beta/2}$

23. $\cot\left(\frac{\theta}{2}+\frac{\pi}{4}\right)=\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

24. Use $\sin^4\theta+\cos^4\theta=1-\frac{1}{2}\sin^2 2\theta$

25. $\cos\theta.\cos2\theta.\cos2^2\theta$..........

$\cos2^{n-1}\theta=\frac{\sin2^n\theta}{2^n\sin\theta}$

26. $\alpha=\sin^2 18^\circ, \beta=\cos^2 36^\circ$

$x^2-(\alpha+\beta)x+\alpha\beta=0$; α,β are roots

EXERCISE-III

CRTQ & SPQ LEVEL-III

MULTIPLE ANGLES, HALF ANGLES

C.R.T.Q Class Room Teaching Questions

1. If $\alpha\in Q_4$ and $\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}=\frac{-1}{2}$ squaring then $\sin2\alpha=$

1) $\frac{3\sqrt{7}}{8}$ 2) $\frac{3\sqrt{7}}{16}$

3) $\frac{-3\sqrt{7}}{4}$ 4) $\frac{-3\sqrt{7}}{8}$

2. If $0<A<B<\pi$, $\sin A+\sin B=\sqrt{\frac{3}{2}}$ and $\cos A+\cos B=\frac{1}{\sqrt{2}}$, then A=

1) 15^0 2) 30^0 3) 45^0 4) $22\frac{1^0}{2}$

3. If $\cos\alpha+\cos\beta=0=\sin\alpha+\sin\beta$, then $\cos2\alpha+\cos2\beta$ is equal to squaring subtract

1) $-2\sin(\alpha+\beta)$ 2) $-2\cos(\alpha+\beta)$

3) $2\sin(\alpha+\beta)$ 4) $2\cos(\alpha+\beta)$

4. If $\sin\beta$ is geometric mean between $\sin\alpha$ and $\cos\alpha$, then $\cos2\beta=$ f→k

1) $2\sin^2\left(\frac{\pi}{4}-\alpha\right)$ or $2\cos^2\left(\frac{\pi}{4}+\alpha\right)$

2) $2\sin^2\left(\frac{\pi}{3}-\alpha\right)$ or $2\cos^2\left(\frac{\pi}{3}+\alpha\right)$

3) $\sin^2\left(\frac{\pi}{4}-\alpha\right)$ or $\cos^2\left(\frac{\pi}{4}+\alpha\right)$

4) $\sin^2\left(\frac{\pi}{3}-\alpha\right)$ or $\cos^2\left(\frac{\pi}{3}+\alpha\right)$

5. If $f\left(\frac{2\tan x}{1+\tan^2 x}\right)=\frac{(\cos2x+1)(\sec^2 x+2\tan x)}{2}$ then $f(4)=$

1) 1 2) 3 3) 0 4) 5

6. If $\tan x=\frac{2b}{a-c}, a\neq c$,

$y=a\cos^2 x+2b\sin x.\cos x+c\sin^2 x$

$z=a\sin^2 x-2b\sin x.\cos x+c\cos^2 x$, then

1) $y=z$ 2) $y-z=a+c$

3) $y-z=a-c$ 4) $y-z=(a-c)^2+4b^2$

7. $\frac{\sin3\alpha}{\cos2\alpha}<0$ if α lies in

1) $\left(\frac{13\pi}{48},\frac{14\pi}{48}\right)$ 2) $\left(\frac{14\pi}{48},\frac{18\pi}{48}\right)$

3) $\left(\frac{18\pi}{48},\frac{23\pi}{48}\right)$ 4) $\left(0,\frac{\pi}{2}\right)$

8. If $\frac{\tan3A}{\tan A}=k$, then $\frac{\sin3A}{\sin A}=$

1) $\frac{3k}{k-1}, k\in R$ 2) $\frac{2k}{k-1}, k\in\left(\frac{1}{3},3\right)$

3) $\frac{2k}{k-1}, k\notin\left(\frac{1}{3},3\right)$ 4) $\frac{k-1}{2k}, k\notin\left(\frac{1}{3},3\right)$

S.P.Q. Student Practice Questions

9. If $\alpha,\beta,\gamma,\delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity K the value of

$4\sin\left(\frac{\alpha}{2}\right)+3\sin\left(\frac{\beta}{2}\right)+2\sin\left(\frac{\gamma}{2}\right)+\sin\left(\frac{\delta}{2}\right)=$

1) $2\sqrt{1-K}$ 2) $2\sqrt{1+K}$

3) $2\sqrt{K}$ 4) $\sqrt{K+1}$

10. If α is an acute anlge and $\sin\left(\frac{\alpha}{2}\right)=\sqrt{\frac{x-1}{2x}}$, then $\tan\alpha$ is

CW

1) $\sqrt{\frac{x-1}{x+1}}$ 2) $\frac{\sqrt{x-1}}{x+1}$

3) $\sqrt{x^2-1}$ 4) $\sqrt{x^2+1}$

11. If $\cos(\theta-\alpha),\cos\theta,\cos(\theta+\alpha)$ are in H.P, then $\cos\theta\sec\frac{\alpha}{2}=$

1) $\pm\frac{1}{\sqrt{2}}$ 2) $\pm\sqrt{2}$ 3) ± 1 4) $\pm\frac{1}{2}$

12. If $\cos x-\frac{\cot\beta\sin x}{2}=\frac{\sqrt{3}}{2}$, then $\tan\frac{x}{2}=$

1) $\tan\frac{\beta}{2}\tan 15^\circ$ 2) $\tan\frac{\beta}{2}$

3) $\tan 15^\circ$ 4) $2+\sqrt{3}$

13. If $\sin(\theta/2)=a,\cos(\theta/2)=b$, then $(1+\sin\theta)/(3\sin\theta+4\cos\theta+5)=$

1) $(a+b)^2/(a+3b)^2$ 2) $(a+b)^2/(3a+b)^2$

3) $(a-b)^2/(a-3b)^2$ 4) $(a-b)^2/(3a-b)^2$

14. If $\tan\frac{\alpha}{2}$, $\tan\frac{\beta}{2}$ are the roots of $8x^2-26x+15=0$ then $\cos(\alpha+\beta)=$

1) $\frac{627}{725}$ 2) $\frac{547}{715}$ 3) $\frac{-547}{715}$ 4) $\frac{-627}{725}$

15. If $\cos\theta=\cos\alpha\cos\beta$, then $\tan\frac{\theta+\alpha}{2}\tan\frac{\theta-\alpha}{2}$ is equal to

1) $\tan^2(\alpha/2)$ 2) $\tan^2(\beta/2)$

3) $\tan^2(\theta/2)$ 4) $\cot^2(\beta/2)$

STANDARD ANGLES

C.R.T.Q Class Room Teaching Questions

16. Value of $\sin^2 11\frac{1}{4}^\circ+\cos^4 11\frac{1}{4}^\circ$ is

CW

1) $\frac{14-\sqrt{2}}{16}$ 2) $\frac{7+2\sqrt{2}}{16}$

3) $\frac{14+\sqrt{2}}{16}$ 4) $\frac{7-2\sqrt{2}}{16}$

17. Value of $\sin^6 7\frac{1}{2}^\circ+\cos^6 7\frac{1}{2}^\circ$ is

1) $\frac{3\sqrt{3}+2}{8}$ 2) $\frac{3\sqrt{3}-5}{16}$

3) $\frac{10+3\sqrt{3}}{16}$ 4) $\frac{5+3\sqrt{3}}{16}$

18. The value of $\tan\frac{\pi}{16}+2\tan\frac{\pi}{8}+4$ is equal to

CW

1) $\cot\frac{\pi}{8}$ 2) $\cot\frac{\pi}{16}$ 3) $\cot\frac{\pi}{16}-4$ 4) 0

S.P.Q. Student Practice Questions

19. $\cos 9^\circ-\sin 9^\circ=$

1) $-\frac{\sqrt{5-\sqrt{5}}}{2}$ 2) $\frac{5+\sqrt{5}}{4}$

3) $\frac{1}{2}\sqrt{5-\sqrt{5}}$ 4) $\sqrt{5-\sqrt{5}}$

20. $4\cos 36^\circ+\cot 7\frac{1}{2}^\circ=$

1) $1+\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{5}+\sqrt{6}$

2) $1+\sqrt{2}-\sqrt{3}+\sqrt{4}-\sqrt{5}+\sqrt{6}$

3) $1-\sqrt{2}+\sqrt{3}-\sqrt{4}+\sqrt{5}-\sqrt{6}$

4) $1+\sqrt{2}-\sqrt{3}+\sqrt{4}+\sqrt{5}-\sqrt{6}$

STANDARD RESULTS

C.R.T.Q Class Room Teaching Questions

21. $1+\operatorname{cosec}\frac{\pi}{4}+\operatorname{cosec}\frac{\pi}{8}+\operatorname{cosec}\frac{\pi}{16}=$

1) $\cot\frac{\pi}{8}$ 2) $\cot\frac{\pi}{16}$

3) $\cot\frac{\pi}{32}$ 4) $\operatorname{cosec}^2\frac{\pi}{}$

22. $\cos\left\{\dfrac{2\pi}{2^{64}-1}\right\}\cos\left\{\dfrac{2^2\pi}{2^{64}-1}\right\}......\cos\left\{\dfrac{2^{64}\pi}{2^{64}-1}\right\}=$

1) $\dfrac{1}{16^{16}}$ 2) $\dfrac{1}{8^8}$ 3) $\dfrac{1}{32^{32}}$ 4) $\dfrac{1}{64^{64}}$

23. If $f_n(x)=$ CW

$\dfrac{\sin x}{\cos 3x}+\dfrac{\sin 3x}{\cos 3^2 x}+\dfrac{\sin 3^2 x}{\cos 3^3 x}+......+\dfrac{\sin 3^{n-1} x}{\cos 3^n x}$

Then $f_2\left(\pi/4\right)+f_3\left(\pi/4\right)=$

1) 0 2) 1 3) –1 4) 2

24. Let $f_n(\theta)=\tan\dfrac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)$ $(1+\sec 4\theta)......(1+\sec 2^n\theta)$, then

$f_2\left(\dfrac{\pi}{16}\right)+f_3\left(\dfrac{\pi}{32}\right)+f_4\left(\dfrac{\pi}{64}\right)+f_5\left(\dfrac{\pi}{128}\right)=$

1) 0 2) 2 3) 4 4) 8

25. The acute angle of a rhombus whose side is a mean proportional between its diagonals is

1) 15° 2) 20° 3) 30° 4) 80°

COMPONENDO AND DIVIDENDO PROPERTY

C.R.T.Q Class Room Teaching Questions

26. If $\cos\theta_1=2\cos\theta_2$, then $\tan\dfrac{\theta_1-\theta_2}{2}\tan\dfrac{\theta_1+\theta_2}{2}$ is equal to

1) $\dfrac{1}{3}$ 2) $-\dfrac{1}{3}$ 3) 1 4) –1

S.P.Q. Student Practice Questions

27. $\cos\theta=\dfrac{a\cos\varphi+b}{a+b\cos\varphi}\Rightarrow\dfrac{\tan\frac{\theta}{2}}{\tan\frac{\varphi}{2}}=$

1) $\dfrac{a-b}{a+b}$ 2) $\dfrac{a+b}{a-b}$

3) $\sqrt{\dfrac{a+b}{a-b}}$ 4) $\sqrt{\dfrac{a-b}{a+b}}$

KEY

01) 4	02) 1	03) 2	04) 1	05) 4
06) 3	07) 1	08) 3	09) 2	10) 3
11) 2	12) 1	13) 1	14) 4	15) 2
16) 3	17) 3	18) 2	19) 3	20) 1
21) 3	22) 1	23) 3	24) 3	25) 3
26) 2	27) 4			

Hints & Solutions

1. Squaring on both sides
$\Rightarrow 1+\sin\alpha=\dfrac{1}{4}\Rightarrow\sin\alpha=\dfrac{-3}{4},\cos\alpha=\dfrac{\sqrt{7}}{4}$

2. squaring and adding, we get $B=A+\dfrac{\pi}{2}$

3. $(\cos\alpha+\cos\beta)^2-(\sin\alpha+\sin\beta)^2=0$
$\Rightarrow(\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta)-$
$(\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta)=0$
$\Rightarrow\cos 2\alpha+\cos\beta=-2\cos(\alpha+\beta)$

4. $\sin^2\beta=\sin\alpha\cos\alpha$,
$\cos 2\beta=1-2\sin\alpha\cos\alpha=(\sin\alpha-\cos\alpha)^2$

5. $f\left(\dfrac{2\tan x}{1+\tan^2 x}\right)=\dfrac{2\cos^2 x(1+\tan x)^2}{2}$
$=\dfrac{1+\tan^2 x+2\tan x}{1+\tan^2 x}=1+\dfrac{2\tan x}{1+\tan^2 x}$
$\Rightarrow f(x)=1+x$, whenever defined
$\therefore f(4)=5$

6. $y-z=(a-c)\cos 2x+2b(\sin 2x)$
then substitute $\tan x=\dfrac{2b}{a-c}$

7. $\dfrac{\sin 3\alpha}{\cos 2\alpha}<0$ for $\alpha\in\left(\dfrac{\pi}{4},\dfrac{\pi}{3}\right)$

8. $\frac{\tan 3A}{\tan A}=k \Rightarrow \tan^2 A=\frac{k-3}{3k-1}>0$

and $\frac{\sin 3A}{\sin A}=3-4\sin^2 A=3-\frac{4\tan^2 A}{1+\tan^2 A}$

9. $\beta=180^0-\alpha,\ \gamma=360^0+\alpha,$
$\delta=540^0-\alpha$

then $4\sin\frac{\alpha}{2}+3\sin\frac{\beta}{2}+2\sin\frac{\gamma}{2}+\sin\frac{\delta}{2}$

$=4\sin\frac{\alpha}{2}+3\cos\frac{\alpha}{2}-2\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}$

$=2\sqrt{1+\sin\alpha}=2\sqrt{1+k}$

10. $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{1-2\sin^2\frac{\alpha}{2}}$

$\tan\alpha=\frac{2\sqrt{\frac{x-1}{2x}}\sqrt{1-\left(\frac{1-x}{2x}\right)^2}}{1-2\left(\frac{x-1}{2x}\right)}=\sqrt{x^2-1}$

11. $\frac{1}{\cos(\theta-\alpha)},\frac{1}{\cos\theta},\frac{1}{\cos(\theta+\alpha)}$ are in A.P

$\frac{2}{\cos\theta}=\frac{1}{\cos(\theta-\alpha)}+\frac{1}{\cos(\theta+\alpha)}$ and simplify

12. Write $\cos x$ and $\sin x$ in terms of $\tan\frac{x}{2}$.

13. $\tan\theta/2=a/b$, write $\sin\theta$ and $\cos\theta$ in term of $\tan\theta/2$

14. $\tan\frac{(\alpha+\beta)}{2}=\frac{26/8}{1-15/8}=-\frac{26}{7}$

$\Rightarrow\cos(\alpha+\beta)=-\frac{627}{725}$

15. $\tan\frac{\theta+\alpha}{2}\tan\frac{\theta-\alpha}{2}$

$=\frac{\tan^2(\theta/2)-\tan^2(\alpha/2)}{1-\tan^2(\theta/2)\tan^2(\alpha/2)}$

$=\frac{\frac{1-\cos\theta}{1+\cos\theta}-\frac{1-\cos\alpha}{1+\cos\alpha}}{1-\frac{1-\cos\theta}{1+\cos\theta}\cdot\frac{1-\cos\alpha}{1+\cos\alpha}}$

$=\frac{2(\cos\alpha-\cos\theta)}{2(\cos\alpha+\cos\theta)}=\frac{\cos\alpha(1-\cos\beta)}{\cos\alpha(1+\cos\beta)}=\tan^2\frac{\beta}{2}$

16. Use $\sin^2\theta=\frac{1-\cos 2\theta}{2}$ and

$\cos^2\theta=\frac{1+\cos 2\theta}{2}$

17. $\sin^6\theta+\cos^6\theta=1-\frac{3}{4}\sin^2 2\theta$,

put $\theta=7\frac{1}{2}^{\circ}$

18. $\tan\theta=\cot\theta-2\cot 2\theta$,

put $\theta=\frac{\pi}{16},\frac{\pi}{8}$

19. $(\cos 9^\circ-\sin 9^\circ)^2=1-\sin 18^\circ$

20. use $\cot\frac{\theta}{2}=\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$, put $\theta=15^\circ$

21. Use, $\operatorname{cosec}\theta=\cot\frac{\theta}{2}-\cot\theta$

22.

$\cos\theta.\cos 2\theta.\cos 2^2\theta....\cos 2^{n-1}\theta=\frac{\sin 2^n\theta}{2^n\sin\theta}$

23. $f_2(x)+f_3(x)$

$=\frac{1}{2}(\tan 9x-\tan x)+\frac{1}{2}(\tan 27x-\tan x)$

$f_2(\pi/4)+f_3(\pi/4)=\frac{1}{2}(1-1)+\frac{1}{2}(-1-1)=-1$

24. $f_n(\theta)=\tan 2^n\theta$

25. Let $BC=a$ and $\angle ABC=\alpha$

given that $BC=\sqrt{AC\times BD}$.

26. $\tan\left(\frac{\theta_1-\theta_2}{2}\right)\tan\left(\frac{\theta_1+\theta_2}{2}\right)=$

$\frac{\sin\left(\frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\theta_1-\theta_2}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$

$=\frac{\sin^2\frac{\theta_1}{2}-\sin^2\frac{\theta_2}{2}}{\cos^2\frac{\theta_1}{2}-\sin^2\frac{\theta_2}{2}}$

$=\frac{1-\cos\theta_1-(1-\cos\theta_2)}{1+\cos\theta_1-(1-\cos\theta_2)}=-\frac{1}{3}$

27. Apply componendo and dividendo

EXERCISE-IV
LEVEL-IV

1. **Assertion(A):** If $x\cos\alpha + y\sin\alpha = 2a$, $x\cos\beta + y\sin\beta = 2a$ $(\alpha \neq \beta)$ then $\cos\alpha.\cos\beta = \frac{4a^2 - y^2}{x^2 + y^2}$

Reason (R): $a\alpha^2 + b\alpha + c = 0$, $a\beta^2 + b\beta + c = 0$ $(\alpha \neq \beta)$ then α, β are the roots of $ax^2 + bx + c = 0$

1) A is true, R is true and R is correct explanation of A
2) A is true, R is true and R is not correct explanation of A
3) A is true, R is false
4) A is false, R is true.

2. **Assertion(A):** If $\sin\alpha + \cos\alpha = m$ then $\sin^6\alpha + \cos^6\alpha = \frac{4 - 3(m^2 - 1)^2}{4}$

Reason(R): $0 \leq \sin 2\alpha \leq 1$, $\forall \alpha \in I$

1) Both A and R are true but R is not the correct explanation of A.
2) Both A and R are true but R is the correct explanation of A.
3) Both A and R are false
4) A is true but R is false

3. **Assertion(A) :** $\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\tan 8\theta + 16\cot 16\theta = \cot\theta$

Reason (R): $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

1) A is true, R is true and R is correct explanation of A
2) A is true, R is true and R is not correct explanation of A
3) A is true, R is false
4) A is false, R is true

KEY

01) 1 02) 4 03) 1

Hints & Solutions

1. α, β are roots of $x\cos\theta + y\sin\theta = 2a$ and simplify

2. $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

take $a = \sin^2\alpha, b = \cos^2\alpha$

3. $\tan A + 2\tan 2A + \ldots\ldots\ldots + 2^{n-1}\tan 2^{n-1}A + 2^n\cot 2^n A = \cot A$