Sri Chaitanya IIT Academy., India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_STERLING BT	**JEE-MAIN**	**Date: 16-08-2025**
Time: 09:00AM to 12:00PM	**RPTM-02**	**Max. Marks: 300**

KEY SHEET

MATHEMATICS

1	**3**	2	**1**	3	**3**	4	**4**	5	**1**
6	**1**	7	**4**	8	**3**	9	**3**	10	**2**
11	**3**	12	**2**	13	**4**	14	**3**	15	**3**
16	**4**	17	**4**	18	**2**	19	**2**	20	**2**
21	**0**	22	**34**	23	**2**	24	**2**	25	**16**

PHYSICS

26	**3**	27	**4**	28	**3**	29	**2**	30	**3**
31	**4**	32	**3**	33	**4**	34	**3**	35	**3**
36	**3**	37	**1**	38	**2**	39	**3**	40	**3**
41	**4**	42	**3**	43	**2**	44	**2**	45	**3**
46	**3**	47	**10**	48	**40**	49	**15**	50	**40**

CHEMISTRY

51	**4**	52	**1**	53	**4**	54	**3**	55	**2**
56	**3**	57	**4**	58	**4**	59	**2**	60	**4**
61	**3**	62	**1**	63	**4**	64	**1**	65	**3**
66	**1**	67	**4**	68	**4**	69	**1**	70	**4**
71	**45**	72	**8**	73	**12**	74	**60**	75	**2**

SOLUTIONS

MATHEMATICS

1. $$\lim_{x\to 0}\frac{\tan^{-1}+x+\frac{1}{2}\left[\log_e(1+x)-\log_e(1-x)-2x\right]}{x^5}$$

$$=\lim_{x\to 0}\frac{\frac{1}{1+x^2}+\frac{1}{2}\left(\frac{1}{1+x}+\frac{1}{1-x}\right)-2}{5x^4}$$

$$=\lim_{x\to 0}\frac{\frac{1}{1+x^2}+\frac{1}{1-x^2}-2}{5x^4}$$

$$=\lim_{x\to 0}\frac{\frac{2}{1-x^4}-2}{5x^4}=2\lim_{x\to 0}\frac{\frac{x^4}{1-x^4}}{5x^4}=\frac{2}{5}$$

$$=\lim_{x\to 1}x^{\frac{2}{(1-x)}}=e^{=\lim\limits_{x\to 0}\left(\frac{2}{1-x}\right)(x-1)}=e^{-2}$$

So, both statements are correct.

2. $$\lim_{x\to\infty}\left(\frac{a(a+1)}{2}\tan^{-1}\frac{1}{a}+a^2-2\log_e a\right)$$

$$\lim_{x\to\infty}a^2\left(\frac{\left(1+\frac{1}{a}\right)}{2}\tan^{-1}\left(\frac{1}{a}\right)+1\frac{2}{a^2}\log_e\frac{1}{a}\right)$$

$$\lim_{x\to\infty}a^2 f\left(\frac{1}{a}\right)=f(1),$$

Where $f(x)=\frac{1}{2}(1+x)\tan^{-1}x+1+2x^2\log_e x$

Now, $f^1(x)=\frac{1}{2}\left(\frac{1+x}{1+x^2}+\tan^{-1}x\right)+4x\log_e x+2x$

$$\Rightarrow f^1(1)=\frac{1}{2}\left(1+\frac{\pi}{4}\right)+2=\frac{5}{2}+\frac{\pi}{8}$$

3. $$\beta=\lim_{x\to 0}\frac{e^{x^3}-\left(1-x^3\right)^{\frac{1}{3}}+\left(\left(1-x^2\right)^{\frac{1}{2}}-1\right)\sin x}{x\sin^2 x}$$

$$=\lim_{x\to 0}\frac{\left(x^3+\frac{x^3}{3}-\frac{x^3}{2}\right)+x^4(......)}{x^3}$$

$$=\lim_{x\to 0}\frac{x^3\left(1+\frac{1}{3}-\frac{1}{2}\right)}{x^3}$$ [Neglecting higher power of x]

$$=1+\frac{1}{3}-\frac{1}{2}=\frac{5}{6}$$

$$\therefore 6\beta=6\times\frac{5}{6}=5$$

4. $$=\lim_{n\to\infty}\sum_{r=0}^{n}\left(\frac{\tan\left(x/2^{r+1}\right)}{\cos\left(x/2^{r}\right)}\right)$$

$$=\lim_{n\to\infty}\sum_{r=0}^{n}\left(\frac{\sin\left(x/2^{r+1}\right)}{\cos\left(x/2^{r+1}\right)\cos\left(x/2^{r}\right)}\right)$$

$$=\lim_{n\to\infty}\sum_{r=0}^{n}\left(\frac{\sin\left(x/2r-x/2^{r+1}\right)}{\cos\left(x/2^{r+1}\right)\cos\left(x/2^{r}\right)}\right)$$

$$=\lim_{n\to\infty}\sum_{r=0}^{n}\left(\tan\frac{x}{2^r}-\tan\frac{x}{2^{r+1}}\right)$$

$$=\lim_{n\to\infty}\left(\tan x-\tan\frac{x}{2^{n+1}}\right)=\tan x$$

$$\therefore \lim_{x\to 0}\frac{e^x-e^{f(x)}}{\left(x-f(x)\right)}$$

$$=\lim_{x\to 0}\frac{e^x-e^{\tan x}}{x-\tan x}==\lim_{x\to 0}e^{\tan x}\frac{e^{x-\tan x}-1}{x-\tan x}=1$$

5. **(P)** $$\lim_{x\to 0}\left(\frac{f^2(a+x)}{f(a)}\right)^{\frac{1}{x}}=e^{\lim_{x\to\infty}\left(\frac{f^2(a+x)-f(a)}{f(a)}\right)\frac{1}{x}}$$

$$e^{\lim_{x\to 0}\frac{2f(a+x).f'(a+x)}{f(a)}}=e^{2.f^1(a)}=e^4$$

(Q) $$\lim_{x\to\frac{\pi^+}{2}}\frac{\cos\left(\tan^{-1}(\tan x)\right)}{x-\frac{\pi}{2}}=\lim_{x\to\frac{\pi+}{2}}\frac{\cos(x-\pi)}{x-\frac{\pi}{2}}$$

$$\lim_{x\to\frac{\pi^+}{2}} \frac{-\cos x}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi^+}{2}} \frac{\sin x}{1} = 1$$

(R) $\lim_{x\to\pi} \dfrac{\sin(1+\cos x)}{\cos\left(\frac{x}{2}\right)} = \lim_{x\to\pi} \dfrac{\cos(1+\cos x)(-\sin x)}{-\sin\left(\frac{x}{2}\right).\frac{1}{2}} = 0$

(S) $\lim_{x\to 0} \dfrac{xe^{\sin x} - e^x \sin^{-1}(\sin x)}{\sin^2 x - x\sin x}$

$$= \lim_{x\to 0} \frac{x\left(e^{\sin x} - e^x\right)}{\sin x(\sin x - x)} = \lim_{x\to 0} \frac{xe^x\left(e^{\sin x - x} - 1\right)}{\sin x(\sin x - x)} = 1$$

6.

$f(x) = \dfrac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$ is continuous at $x = 0$

$$\lim_{x\to 0} = \frac{3x - \left(\frac{3x}{\lfloor 3}\right)^3 + ... + \alpha\left(x - \frac{x^3}{\lfloor 3}...\right) - \beta\left(1 - \frac{(3x)^2}{\lfloor 2}...\right)}{x^3} = f(0)$$

$$\lim_{x\to 0} = \frac{-\beta + x(3+\alpha) + \frac{9\beta x^2}{\lfloor 2} + \left(\frac{-27}{\lfloor 3} - \frac{\alpha}{\lfloor 3}\right)x^3...}{x^3} = f(0)$$

for exist $\beta = 0, +\alpha = 0, -\dfrac{27}{\lfloor 3} - \dfrac{\alpha}{\lfloor 3} = f(0)$

$$\alpha = -3, -\frac{27}{6} - \frac{(-3)}{6} = f(0)$$

$$f(0) = \frac{-27+3}{6} = -4$$

7. Put $x = 2$ in $x^2 + px + q = 0$

$\Rightarrow 4 + 2p + q = 0 \Rightarrow -2p = (q+4)$

Now, $\lim_{x\to 2p^+} \dfrac{1 - \cos\left(x^2 - 4px + q2 + 8q + 16\right)}{(x-2p)^4}$

$$= \lim_{x\to 2p^+} \frac{1 - \cos\left[x^2 - 2x.2p + q^2 + 2.q.4 + 4^2\right]}{(x-2p)^4}$$

$$= \lim_{x\to 2p^+} \frac{1 - \cos(x-2p)^2}{(x-2p)^4}$$

$$= \lim_{x\to 2p^+} \frac{2\sin^2\left(\frac{(x-2p)^2}{2}\right)}{\frac{(x-2p)^4}{4}}.\frac{1}{4}$$

$$= \lim_{x\to 2p^+} 2\times 1\times\frac{1}{4}=\frac{1}{2}\left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right)$$

Now, $[1/2]=0$

8. $f(x)=x^3-6x^2+9x-3$

$f(x)=3x^2-12x+9=3(x-1)(x-3)$

$f(1)=1 f(3)=-3$

$$g(x)=\begin{bmatrix} f(x) & 0\le x\le 1 \\ 0 & 1\le x\le 3 \\ -1 & 3\le x\le 4 \end{bmatrix}$$

$g(x)$ is continuous

$$g^1(x)=\begin{bmatrix} 3(x-1)(x-3) & 0\le x\le 1 \\ 0 & 1\le x\le 3 \\ -1 & 30\le x\le 4 \end{bmatrix}$$

$g(x)$ is non-differentiable at $x=3$

9.

$$f(x)=\begin{cases} 8+2x, & -4\le x<-2 \\ x^2, & -2\le x<-1 \\ |x|, & 1\le x<2 \\ 8-x & 2\le x<4 \end{cases}$$

Clearly, $f(x)$ is not differentiable at $x=\{-2,-1,0,1,2\}$.

10. $f(x)=\dfrac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx-9}}$

For continuity at $x=0, \lim_{x\to 0} f(x)=f(0)$

Now, $\therefore \lim_{x\to 0} f(x)=\lim_{x\to 0}\dfrac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx-9}}$

$\Rightarrow p=3$ (To make in determinant form)

So, $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\dfrac{\left(3^7+3x\right)^{\frac{1}{7}}-3}{(729+qx)^{\frac{1}{3}}-9}$

$$\lim_{x\to 0}\frac{3\left[\left(1+\dfrac{x}{3^6}\right)^{\frac{1}{7}}-1\right]}{9\left[\left(1+\dfrac{q}{729}x\right)^{\frac{1}{3}}-1\right]}=\frac{1}{3}\cdot\frac{\frac{1}{7}\cdot\frac{1}{3^6}}{\frac{1}{3}\cdot\frac{q}{729}}$$

$f(0)=\dfrac{1}{7q}$

$\therefore$ Hence Option (2) is correct.

11. $f(x)=a+[13\sin x];x\in(0,\pi)$

Number of non-differentiable points = $2n-1=2(13)-1=25$

12. $f(x)=\sin\cos^{-1}\left(\dfrac{\left(1-2^x\right)^2}{1+(2x)^2}\right)=\sin\left(2tan^{-1}2^x\right)$

$f^1(x)=\cos\left(2\tan^{-1}2^x\right).2.\dfrac{1}{1+\left(2^x\right)^2}\times 2^x.\log_e 2$

13. $\left(x^2-4x+3\right)\left|\left(x^3-6x^2+11x-6\right)\right|$ is not differentiable at $x=2$

$f(x)$ is not differentiable at x=2, $\dfrac{3\pi}{4},\dfrac{5\pi}{4}$

14. $y=\dfrac{x^2}{2}+\dfrac{1}{2}x\sqrt{x^2+1}+\dfrac{1}{2}\ln\left(x+\sqrt{x^2+1}\right)$

$\Rightarrow y'=x+\sqrt{x^2+1}$

$\Rightarrow xy'+\log y'=x+\left(x+\sqrt{x^2+1}\right)+\log\left(x+\sqrt{x^2+1}\right)$

$=x^2+x\sqrt{x^2+1}+\log\left(x+\sqrt{x^2+1}\right)=2y$

15. $f(x+1)=x+f(x)$

$\Rightarrow e^{f(x+1)}=e^x\times g(x)$

$\Rightarrow g(x+1)=e^x\times g(x)$

$\Rightarrow In\left(g(x+1)\right)=x+In\left(g(x)\right)$

$$\Rightarrow \frac{g^{'}(x+1)}{g(x+1)}-\frac{g^{'}(x)}{g(x)}=1 \Rightarrow \frac{g^{'}\left(\frac{1}{2}+1\right)}{g\left(\frac{1}{2}+1\right)}-\frac{g^{'}\left(\frac{1}{2}\right)}{g\left(\frac{1}{2}\right)}=1$$

$$\frac{g^{'}\left(\frac{1}{2}+2\right)}{g\left(\frac{1}{2}+2\right)}-\frac{g^{'}\left(1+\frac{1}{2}\right)}{g\left(1+\frac{1}{2}\right)}=1 \Rightarrow \frac{g^{'}\left(n+\frac{1}{2}\right)}{g\left(n+\frac{1}{2}\right)}-\frac{g^{'}\left(n-\frac{1}{2}\right)}{g\left(n-\frac{1}{2}\right)}=1$$

Adding $\frac{g^{'}\left(n+\frac{1}{2}\right)}{g\left(n+\frac{1}{2}\right)}-\frac{g^{'}\left(\frac{1}{2}\right)}{g\left(\frac{1}{2}\right)}=n$

16. $f(0)=\lim\limits_{x\to 0} f(x)=\dfrac{\left(\frac{a^x-1}{x}\right)^3}{\frac{\sin(x\log a)}{x\log a}.\log a.\log\left(1+x^2\log a^2\right)^{\frac{1}{x^2}}}=\dfrac{1}{2}\log a$

17. $\lim\limits_{x\to 0} f(x)=f(0)=3$

$\lim\limits_{x\to 0}\dfrac{b^2}{b\sqrt{a}\left(\sqrt{a+b^2x}+\sqrt{a}\right)}=3$

$\dfrac{b}{2a}=3$

18. $\lim\limits_{h\to 0}\dfrac{f(h)-\frac{1}{3}}{h}=\dfrac{4}{3}$

For limit to exists $f(0)=\frac{1}{3}, f^{'}(0)=\frac{4}{3}$

Put $x=y=0$ in functional relation, we get $\mu=f(0)=\frac{1}{3}$

$f(x)=\frac{4}{3}x+\frac{\lambda x^3}{3}+\frac{1}{3}$

$f(2)=\frac{25}{3}$

$8+8\lambda+1=25$

$\lambda=2$

19. $$f(x)=\begin{cases}\dfrac{x\left(3e^{\frac{1}{x}}+4\right)}{2-e^{\frac{1}{x}}}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$f'\left(0^{+}\right)=\lim_{h\to 0}\frac{3\left(1+\frac{1}{h}\right)+4}{2-1-\frac{1}{h}}=3$$

$$f^{1}\left(0^{-}\right)=\lim_{h\to 0}\frac{3\left(1-\frac{1}{h}\right)+4}{2-\left(1-\frac{1}{h}\right)}=-3$$, so not differentiable at $x=0$

20. $$\lim_{x\to 0}\left(\sin\left(\frac{2x^2}{a}\right)+\cos\left(\frac{3x}{b}\right)\right)^{ab/x^2}=f(0)$$

$$\Rightarrow e^3=e^{\lim\limits_{x\to 0}\left(\sin.\frac{2x^2}{a}+\cos.\frac{3x}{b}-1\right)\frac{ab}{x^2}}$$

$$\Rightarrow 3=ab\left(\frac{2}{a}-\frac{9}{2b^2}\right).$$

$\Rightarrow 4b^2-6b-9a=0$

Since b is real, we have $D\geq 0$

$\Rightarrow 36+144a\geq 0$

$$\Rightarrow a\geq -\frac{1}{4}$$

21. Given, $$\lim_{x\to 1}\left\{\frac{\sin(x-1)+a(1-x)}{(x-1)+\sin(x-1)}\right\}^{\frac{\left(1-\sqrt{x}\right)\left(1-\sqrt{x}\right)}{1-\sqrt{x}}}=\frac{1}{4}$$

$$\Rightarrow \lim_{x\to 1}\left\{\frac{\frac{\sin(x-1)}{(x-1)}-a}{1+\frac{\sin(x-1)}{(x-1)}}\right\}^{1+\sqrt{x}}=\frac{1}{4}$$

$$\Rightarrow\left(\frac{1-a}{2}\right)^2=\frac{1}{4}$$

$\Rightarrow (a-1)^2=1$

$\Rightarrow$ a=2 or 0

But for a=2, base limit approaches -1/2 and exponent approaches to 2 and since base cannot be negative, hence limit does not exist.

22. $f(x)=\begin{cases}3x^2+k\sqrt{x+1}, 0<x<1\\ mx^2+k^2, \quad x\ge 1\end{cases}$

Function is differentiable at $x=1$.

So, it must be continuous at $x=1$.

$\therefore 3+k\sqrt{2}=m+k^2$ ....(1)

Also, $f(x)=\begin{cases}6x+\dfrac{k}{2\sqrt{x+1}}, 0<x<1\\ 2mx, \quad x\ge 1\end{cases}$

$f'(1^-)=f'(1^+)$

$\therefore 6+\dfrac{k}{2\sqrt{2}}=2m$ ....(2)

Eliminating m from (1) and (2), we get

$3+k\sqrt{2}=3+\dfrac{k}{4\sqrt{2}}+k^2$

$\Rightarrow k^2+k\left(\dfrac{1}{4\sqrt{2}}-\sqrt{2}\right)=0$

$\Rightarrow k=\dfrac{7}{4\sqrt{2}}$, as k>0

$\Rightarrow m=3+\dfrac{7}{32}=\dfrac{103}{32}$

$\Rightarrow f'(x)=\begin{cases}6x+\dfrac{7}{8\sqrt{2}\sqrt{x+1}}, 0<x<1\\ \dfrac{103}{16}x, \quad x\ge 1\end{cases}$

$\Rightarrow f'(8)=\dfrac{103}{2}$ and $f'\left(\dfrac{1}{8}\right)=\dfrac{6}{8}+\dfrac{7}{8\sqrt{2}\sqrt{\dfrac{1}{8}+1}}=\dfrac{4}{3}$

23. $f(x)=x^2+Ax+B$

$f''(x)=2$

$\Rightarrow g(x)=2x(A+B+3)+Ax+2$

$f(x)=x^2-3x$

$g(x)=-3x+2$

24. $\therefore f(x)=\begin{cases} 3-2x^2, & x\in(-2,-1) \\ x^2, & x\in[1,0) \\ \dfrac{\sqrt{3}}{2}-1, & x\in[0,1) \\ \dfrac{1}{\sqrt{2}}-3+x^2, & x\in[1,2) \end{cases}$

25. $g^3(x)=g^2(x)-2g^2(x)+2g(x)x^3+g(x)-x^3=0$

$\left(g^2(x)-2g(x)+1\right)\left(g(x)-x^3\right)=0$

$\left(g(x)-1\right)^2\left(g(x)-x^3\right)=0$

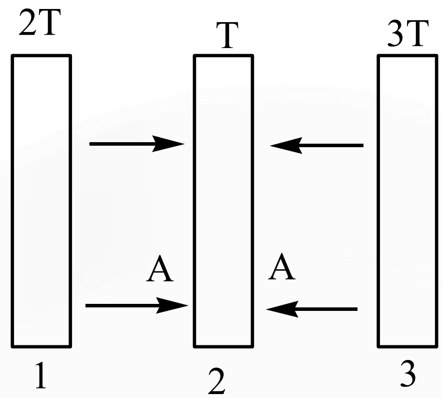
$g(x)=1$ or $g(x)=x^3$

Given g is invertible

$g(x)=x^3$

PHYSICS

26. Under steady conditions, the heat gained per second by a plate is equal to the heat released per second by the plate



$$\frac{\text{heat gained}}{\text{second}}[\text{by}(2)\text{ from}(1)]+\frac{\text{heat gained}}{\text{second}}$$

$$[\text{by}((2)\text{ from}(3)]=\frac{\text{heat gained}}{\text{second}}(\text{by } 2)$$

$$\sigma A(2T)^4+\sigma A(3T)^4=\sigma(2A)(T')^4$$

$$\therefore T'=\left[\frac{97}{2}\right]^{1/4}T$$

27. Consider a shell of thickness (dr) and of radii (r) and the temperature of inner and outer surface of this shell be T, (T-dT)

$\frac{dQ}{dt}=$ rate of flow of heat through it

$$\frac{KA[T-dT)-T]}{dr}=\frac{KAdT}{dr}$$

$$-4\pi Kr^2\frac{dT}{dr}(\because A=4\pi r^2)$$

28. is Kelvin-Planck's statement and (b) is clauss statement of second law of thermodynamics. Both the statements are completely equivalent.

29. For a Carnot engine, (i) there is absolutely no friction between the walls of cylinder and the piston.(ii)Woking substance is an ideal gas. In a real engine, these conditions cannot full filled and hence no heat engine working between the same two temperatures can have efficiency greater than that of Carnot engine.

30. The efficiency (η) of a cannot engine and the coefficient of performance (β) of a refrigerator are related as

$$\beta=\frac{1-\eta}{\eta}$$

Here $\eta = \frac{1}{10}$

$$\therefore \beta = \frac{1-\frac{1}{10}}{\left(\frac{1}{10}\right)} = 9.$$

Also, coefficient of performance (β) is given by $\beta = \frac{Q_2}{W}$, where Q_2 is the energy absorbed from the reservoir .

Or, $9 = \frac{Q_2}{10}$

$\therefore Q_2 = 90J$

31. Coefficient of performance,

$$\text{Cop } = \frac{T_1}{T_1 - T_2}$$

$$5 = \frac{273-20}{T_1-(273-20)} = \frac{253}{T_1-253}$$

$$5T_1 = (5\times 253) = 253$$

$$5T_1 = 253 + (5\times 253) = 1518$$

$$\therefore T_1 = \frac{1518}{5} = 303.6$$

$$T_1 = 303.6 - 273 = 30.6 \cong 31\ ^oC$$

32. Let P be the power radiated by the sun and R be radius of planet. Energy radiated by Planet.

$$= 4\pi R^2 \times \left(\sigma T^4\right)$$

For thermal equilibrium

$$\frac{p}{4\pi d^2}\times \pi R^2 = 4\pi R^2\left(\sigma T^4\right)$$

$$\therefore\ T^4 \alpha\, d^{-1/2}$$

Hence $n = \frac{1}{2}$

33. From Wein's law, $\lambda_m T$ =constant, where T is the temperature of black body and λ_m is the wavelength corresponding to maximum energy of emission. Energy distribution

Of black body radiation is given below:

i. U_1 and $U_1$are not zero because a black body emits radiations of nearly all wavelengths.

ii. Since U_1 corresponding to lower wave length U_2 corresponds to medium wave length
hence $U_2 > U_1$

34. Condition in the derivation of mirror formula.

35. $\lambda_m T = \text{constant}$
$E = \sigma T^4$

36. A-Q : For $u = -f$

$$\frac{1}{v} + \frac{1}{-f} = \frac{1}{f}$$

$$\therefore v = \frac{f}{2}$$

And $M = -\frac{v}{u} = -\frac{f/2}{(-f)} = 0.5.$

B-S : $u = -2f$, so $v = -2f$

$$M = -\frac{v}{u} = -\left(\frac{-2f}{-2f}\right) = -1$$

C-P : In concave mirror, $u = -2f, v = -\infty$

$$\therefore M = -\frac{v}{u} = -\infty.$$

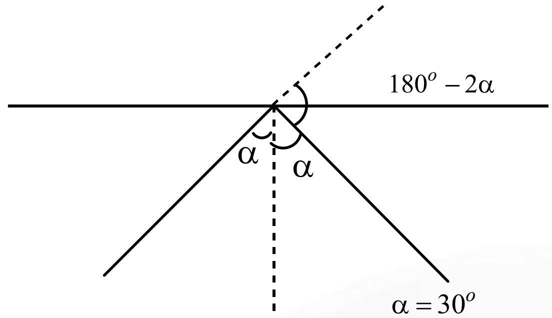
D-T : In convex mirror $u = -2f$

So $\frac{1}{v} + \frac{1}{-2f} = \frac{1}{f} \Rightarrow v = \frac{2f}{3}.$

Now $M = -\frac{v}{u} = \frac{1}{3}.$

37. $$\cos\left(180^o - 2\alpha\right) = \frac{\left(\frac{\hat{i}}{2} + \frac{\sqrt{3}}{2}\hat{j}\right) \cdot \left(\frac{\hat{i}}{2} + \frac{\sqrt{3}}{2}\hat{j}\right)}{\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\sqrt{\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)^2}}$$

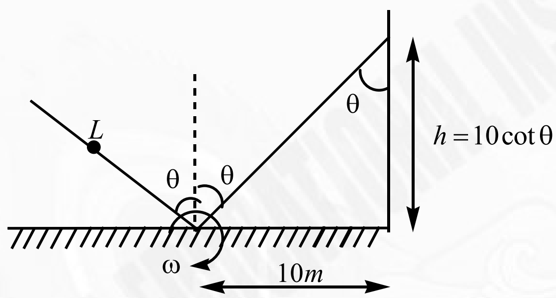
$$\therefore \cos(180^o - 2\alpha) = -\frac{1}{2}$$



$180^o - 2\alpha = 120^o$ $\alpha = 30^o$

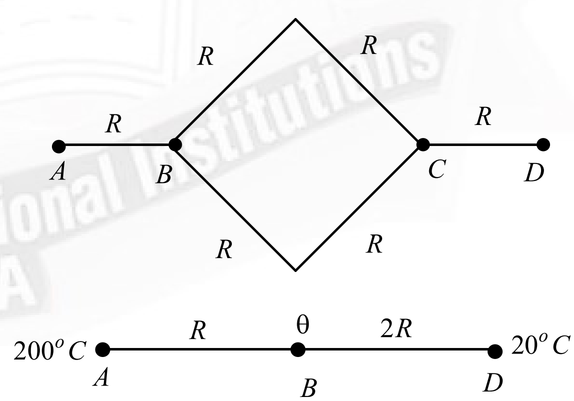
Option (a) is correct.

38. When mirror is rotated with angular speed ω_1 the reflected ray rotates with angular speed $2\omega(=36 rads^{-1})$



Speed of the spot $= \left|\frac{dh}{dt}\right| = \left|\frac{d}{dt}(10\cot\theta)\right| = \left|-10\cos ec^2\theta\frac{d\theta}{dt}\right| = \left|\frac{10}{(0.6)^2}\times 36\right| = 1000ms^{-1}$.

39. Let the thermal resistance of each rod be R. The two resistances connected along two paths from B to C are equivalent to 2R each and their parallel combination is R Effective



thermal resistance between B and D = 2R.

Temperature of interface $\theta = \frac{R_1\theta_2 + R_1\theta_1}{R_1 + R_2}$

$$\theta = \frac{R\times 20 + 2R\times 200}{R + 2R} = \frac{420}{3} = 140^o C.$$

40. According to Wien's law $\lambda_0 T_0 = \lambda T$

According to stefan's law $\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^4$

As $P = \frac{256}{81} P_0 \Rightarrow \lambda = \frac{3}{4}\lambda_0$

Wavelength shift $\Delta\lambda = \lambda - \lambda_0 = -\frac{\lambda_0}{4}$.

41. Let θ be the temperature of B

$$\frac{2KA(\theta-100)}{l} + \frac{\left(\frac{K}{2}\right)A(\theta-0)}{l} = 200$$

Substituting values $\theta = 800^o C$

Also from $B \to D, \frac{\frac{K}{2}A(\theta-0)}{l} = \frac{440}{5} \times 80$

$t = 800s1$.

42. At 70°C, the system attains steady state.
i.e., rate of heat generated = Rate of heat loss
or, 10 W = k (70-30)°C
(from Newton's law of cooling)

or K= (1/4) W/°C

At 50°C, rate of heat loss should be k (50-30)°C=5W, but rate of heat loss = (heat capacity) × (rate of cooling)

$$i.e., \frac{-dQ}{dt} = c\left(\frac{-dT}{dt}\right) - 5W = c\left[\frac{49.9-50}{60}\right]^o C/s1$$

$or \ \ 2993\ J/^o C$.

43. $\left(\frac{dQ}{dt}\right) \times \frac{1}{A} = K_A \frac{(50-30)}{3}$ (For slab A)

$= K_B = \frac{(50-20)}{3}$ (for slab B)

$2KA = 3KB$

$or,\ \ KA/KB = 3/2$.

44. From Newton's law cooling, we have rate of cooling,

$$\frac{dQ}{dt}=\frac{h}{ms}\left(T-T_0\right)$$

We know $m=V\cdot\rho$

So we have $\frac{dQ}{dt}=\frac{h}{ms}\left(T-T_0\right)=\frac{h\left(T-T_0\right)}{V\cdot\rho s}$

Since , h, $\left(T-T_0\right)$ and V are constant for both beaker.

$$\frac{dQ}{dt}\propto\frac{1}{\rho s}$$

We have given that

$\rho_A=8\times10^2 kgm^{-3}$,

$\rho_B=10^3 kgm^{-3}$,

$s_A=2000J\,kg^{-1}K^{-1}$

$s_B=4000J\,kg^{-1}K^{-1}$

$\rho_A s_A=16\times10^5$

And $\rho_B s_B=4\times10^6$

So, $\rho_A<\rho_B, s_A<s_B$

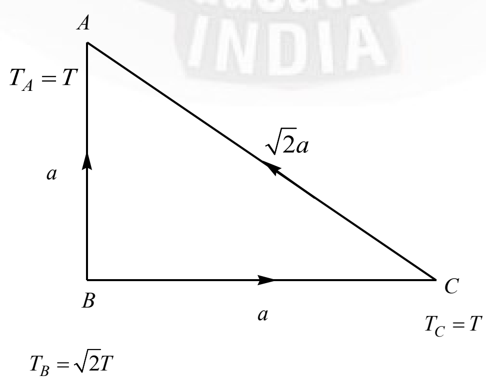
And $\rho_A s_A<\rho_B s_B$

$$\Rightarrow\frac{1}{\rho_A s_A}>\frac{1}{\rho_B s_B}$$

$$\Rightarrow\frac{dQ_A}{dt}>\frac{dQ_B}{dt}$$

So, for container B rate of cooling is smaller than the container A. Hence, graph of B lies above the graph of A and it is not a straight line (slope of A is greater than B).

45. The diagrammatic representation of the given problem is shown in figure. since , $T_B>T_A$ the heat will flow from B to A.
Similarly, heat will also flow from B to C and C to A



46. The thermal resistance is given by

$$\frac{x}{KA}+\frac{4x}{2KA}=\frac{x}{KA}+\frac{2x}{KA}=\frac{3x}{KA}$$

$$\therefore \frac{dQ}{dt}=\frac{\Delta T}{3x}=\frac{(T_2-T_1)}{3x}=\frac{1}{3}\left\{\frac{A(T_2-T_1)K}{x}\right\}$$

$$\therefore f=\frac{1}{3}$$

47. For part MN,

$u=-30cm\ f=-10cm$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-10}-\frac{1}{-30}=-\frac{1}{10}+\frac{1}{30}$$

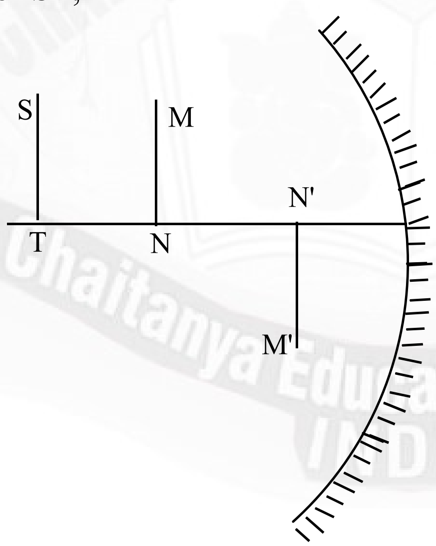
$$\frac{1}{v}=\frac{-3+1}{30}=\frac{-2}{30}$$

$V=-15\ cm$

$$m=\frac{I}{O}=-\frac{v}{u}$$

$$\Rightarrow \frac{1}{10}=-\frac{(-15)}{(-30)}=-\frac{1}{2}\Rightarrow I=5cm$$

For image of ST,



$u=--40cm$

$f=-10cm$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{(-40)}=\frac{1}{(-10)}$$

$$\frac{1}{v}=-\frac{1}{10}+\frac{1}{40}\ \frac{1}{v}=-\frac{-4+1}{40}=\frac{-3}{40}$$

$$=-\frac{40}{3}cm$$

For length of image of ST,

$$m=\frac{I}{O}=-\frac{v}{u}$$

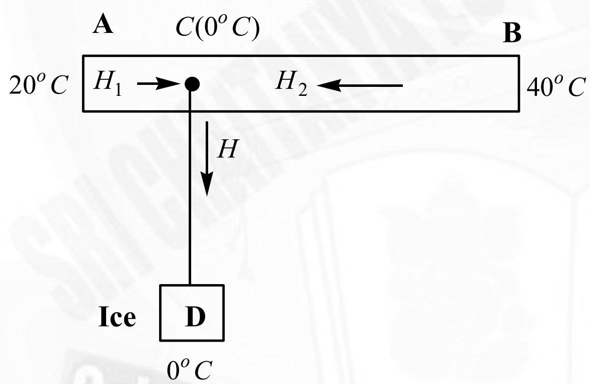
$$\frac{I}{10}=-\left(\frac{-40/3}{-40}\right)\Rightarrow I=\frac{10}{3}cm$$

Length of image of NT,

T' N' $=15-\frac{40}{3}=\frac{5}{3}cm$

Length of wire in image $=5\frac{10}{3}+\frac{5}{3}=10cm$.

48. Thermal resistance of $AC=\frac{L}{KA}=\frac{0.1}{336\times10^{-4}}=\frac{10^3}{36}=$ R (say).



Thermal resistance of $BC=\frac{0.2}{336\times10^{-4}}=2R$

Heat flow rates are $H_1=\frac{20}{R}:H_2=\frac{40}{2R}=\frac{20}{R}$

$$H=H_1+H_2=\frac{40}{R}=\frac{40\times336}{10^3}$$

$$=\frac{13440}{10^3}=13.44W$$

Rate of melting if ice $\frac{H}{L_f}=\frac{13.44/4.2}{80}$ g/s = 40 mg/s.

49. If θ is the temperature of outside, heat passing per second through the glass window,

$$\frac{dQ}{dt} = KA\frac{(\theta_1 - \theta_2)}{L}$$

$$= \frac{0.2 \times 1 \times (20 - \theta) cal}{0.2 \times 10^{-2}} = 100(200 - \theta)$$

And heat produced per second by the heater in the room

$$P = \frac{V_2}{R}\frac{J}{s} = \frac{V^2}{RJ}\frac{cal}{s}$$

$$= \frac{200 \times 200}{20 \times 4.2} = 476.2\frac{cal}{s}$$

Now as the temperature of the room is constant the heat produced per second by heater must be equal to the heat conducted through the glass window.

$100(20 - \theta) = 476.2 \Rightarrow \theta = 15.24^o C$.

50. $V_{IX} = 2V_M - V_0$

$V_{IX} = 2(-5\hat{i}) - 10\hat{i}$

$V_{IX} = -20\hat{i}$

$V_{IY} = 10\hat{j}$

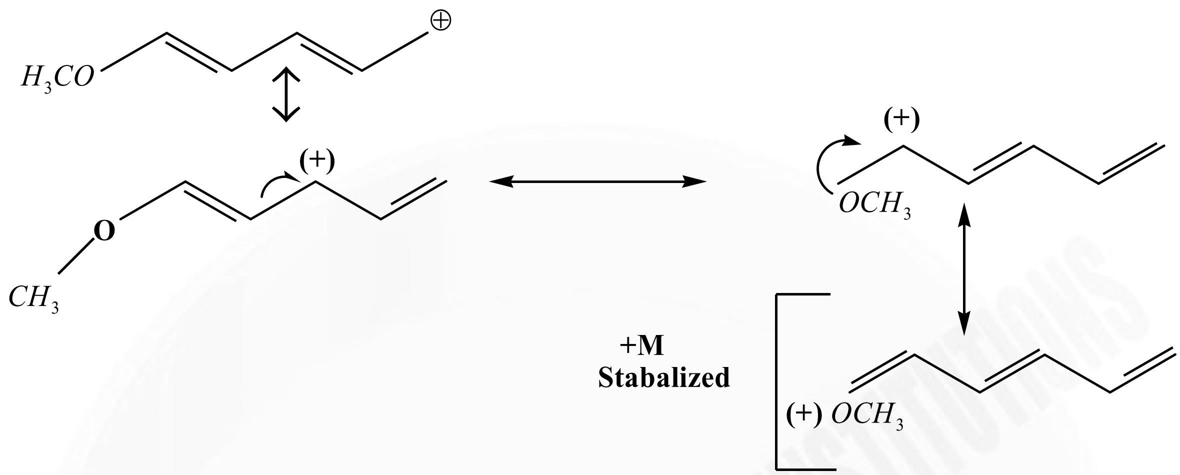
$\hat{V}_I = -20\hat{i} + 10\hat{j}$

$V_{IO} = (-20\hat{i} + 10\hat{j}) - (10i)$

$V_{IO} = -30\hat{i} + 10\hat{j}$

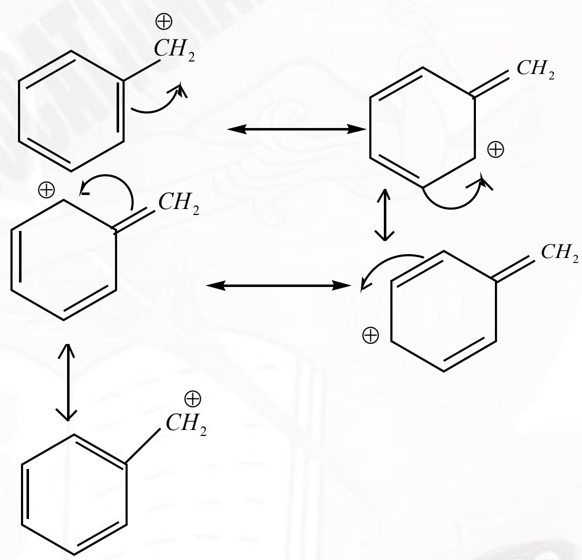
CHEMISTRY

51.

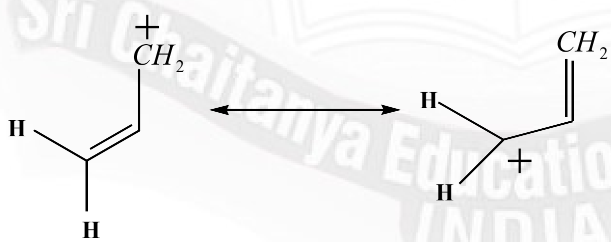


This molecule have more number of resonating structure as well as + M stabilization

52.



(A) and (B) are in resonance. (A)



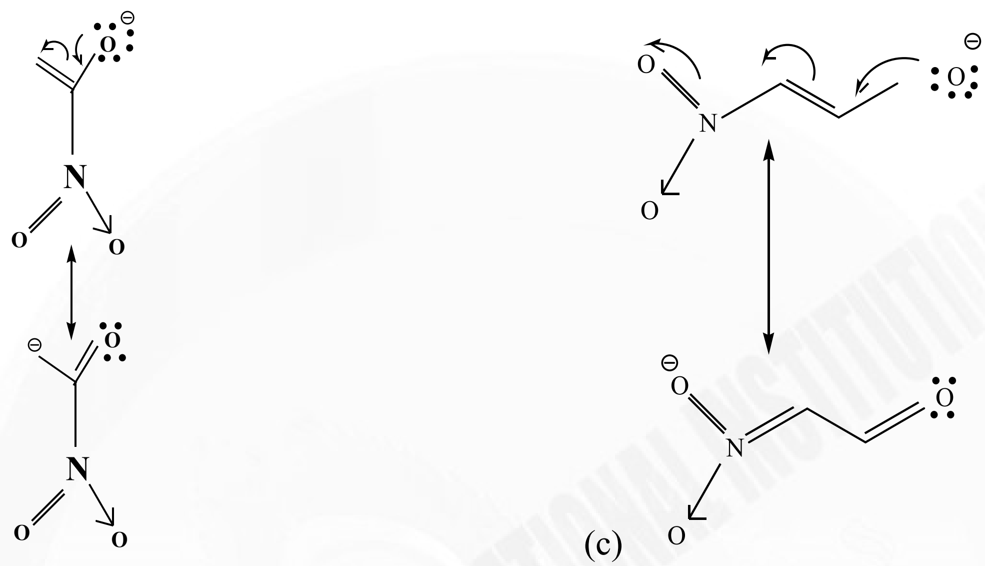
(B)

(D) Movement of C = C πe^- will destabilize the +ve charged carbon

53. Acidic strength $\alpha -I$, – M effect. Due to strong – I, and – M effect of three $-\ COOCH_3$ groups, it has most acidic hydrogen.

54. Basicity order can be determined by the cummulative effect of the factors on the electron density of concerned atom.

55.

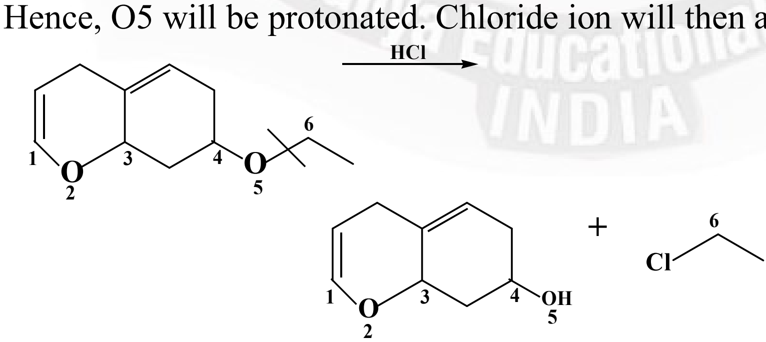


(B) (c)

Conjugation and – I effect of $-NO_2$ extended conjugation and – M, - I effect of $-NO_2$

56. Among the substituents attached to the given compounds, fluorine has maximum electronegativity, so it will pull electron pair towards itself. In option (b), the two F group are attached opposite to each other, thus net dipole moment will cancel out and reduce its polarity. In option (d), the F groups are attached in slightly opposite direction. Thus, this also decreases its polarity. But in option (c), the compound has the two F groups along same direction. Thus, net dipole moment will increase in this direction and therefore, it will exhibit maximum polarity.

57. The lone pair of electrons on O2 is involved in resonance with C = C. Hence O2 will not be protonated. The lone pair of electrons on O5 is not involved in resonance with C = C. Hence, O5 will be protonated. Chloride ion will then attack least substituted C atom (C6)



58. In II and IV, both -CH_3 groups are at 60^o. In IV, there is more repulsion between -CH_3 and – H than that in II between – H and – H. Potential energy $\alpha \frac{1}{Stability}$ Stability order I > III > IV > II So, Potential energy: II > IV > III > I

59. Let the hydrocarbon be C_xH_y.

$$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

Before Combustion: 10 mL 55mL

After Combustion: 0 $55 - 10\left(x+\frac{y}{4}\right)$ $10x$

Volume of $CO_2, 10x = 40;$ $x=4$

$$55-10\left(x+\frac{y}{4}\right)=0;$$

$\therefore$ Hydrocarbon is C_4H_6

60. The order of substitution in different alkanes is:

$$3^o > 2^o > 1^o$$

Thus, the bromination of 2-methylbutane mainly gives 2-bromo – 2-methylbutane.

$$\begin{array}{c} \qquad\qquad\quad CH_3 \\ \qquad\qquad\quad | \\ CH_3-CH_2-CH-CH_3 \end{array} \xrightarrow{Br_2}$$

2-methylbutane.

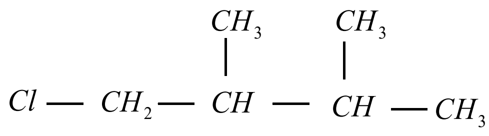
$$\begin{array}{c} \qquad\qquad\quad CH_3 \\ \qquad\qquad\quad | \\ CH_3-CH_2-CH-CH_2Br \end{array} + \begin{array}{c} \qquad\qquad\quad CH_3 \\ \qquad\qquad\quad | \\ CH_3-CH_2-C-CH_3 \\ \qquad\qquad\quad | \\ \qquad\qquad\quad Br \end{array}$$

1-Bromo-2methylbutane (Minor) 2-Bromo2-methylbutane (Major)

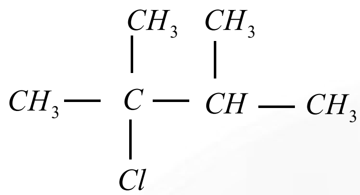
61.

$$\begin{array}{ccccccc} & & 1^o & & 1^o & & \\ & & CH_3 & & CH_3 & & \\ 1^o & & | & & | & & 1^o \\ CH_3 & — & CH & — & CH & — & CH_3 \\ & & 3^o & & 3^o & & \end{array}$$

Since it contains only two types of H-atoms, hence it will give only two monochlorinated compounds, viz.



1-Chloro-2, 3-dimethylbutane

and 

2-Chloro-2, 3-dimethylbutane

62. Both are correct & correct explanation. A reaction which involves free radical R.I are homopolar.

63. A false R – true. Inductive Effect $\alpha \frac{1}{Distance}$

64. A false R true. No resonance structure is real & due to resonance n – o bond length is same

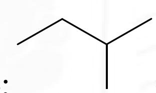
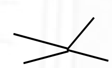
65. Both Statement – I & Statement – II are correct. Due to the presence of α has it shows hyper conjugation

66. A – S; B – T; C – Q; D – P. By wurtz reaction biphenyl is not prepared

67. $CH_3COONa + NaOH \xrightarrow[\Delta]{CaO} CH_4$

68. Hexane

69. B < A < C < D

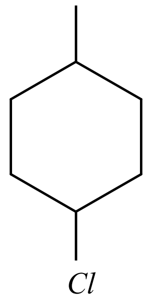
M.pt of $C_2H_6 = 101k$, $C_3H_8 = 85.3$;  = 113.1k  = 256.4

70. More + I effect

71. $2CH_3COONa_{\,(or)} \xrightarrow{electrolysis} C_2H_6 + 2CO_2$ At ano se

72. $Al + C \rightarrow Al_4C_3 \xrightarrow{H_2O} CH_{4(g)}$

73.

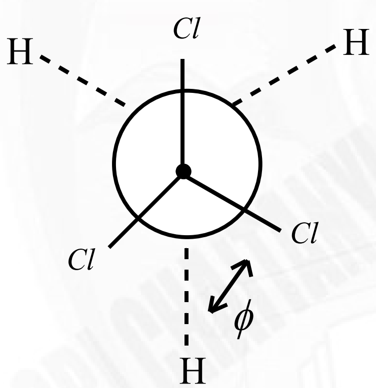


(2) No. of stereoisomer $= 2^n = 2^2 = 4$ No. of stereoisomer $= 2^n = 2^2 = 4$

$\Rightarrow$ Total number of isomers = 1 + 1 + 4 + 2 = 12

74. Staggered from is produced when the rearrangement of atoms or group takes place by an

angle of 60^o. 1.1.1 trichloroethane $\left(CCl_3 - CH_3\right)$ in eclipsed from on rotation by 60^o gives staggered from.



Dihedral angle $(\phi) = 60^o$

(Newman staggered)

75. No. of compounds which shows $\mu = 0$ are 2