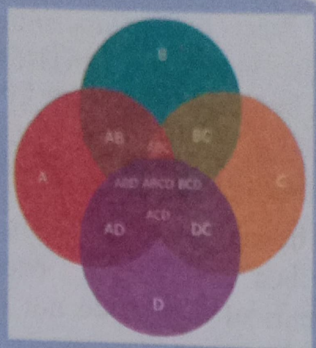
1.1 SETS



Internals of the Topic

- ROSTER
- SET-BUILDER FORM
- OPERATION ON SETS

CRT CONCEPT

☑ **Object :** In our mathematical language, every thing in this universe ,whether living or non living is called an object.

- **Set :** A set is a well defined collection of objects. The objects in a set are called its members or elements.
- **Well defined :** Well defined is for a given object, it is possible to determine, whether that object belongs to the given collection or not.

  The following collections constitute a set :

  1) The vowels in english alphabets : a,e,i,o,u
  2) All prime numbers
  3) All rivers flowing in india.
  4) The collection of all prime numbers less than20.
- **Not well defined :** The collection of all beautiful girls of india is not a set,since the term 'beautiful' is vague and it is not well defined.similarly 'rich persons', 'honest persons' ,'good players', 'young men' , 'yesterday', etc., do not form sets.
- **Notations :** The sets are usually denoted by capital letters A, B, C, etc. The members or elements of the set are denoted by lower-case letters a, b, c, etc.If x is a member of the set A, we write $x \in A$ (read as x belongs to A) and if x is not a member of the set A, we write $x \notin A$ (read as x does not belong to A). If x and y both belong to A, we write $x, y \in A$. some examples of sets used particularly in mathematics

  N : The set of all natural numbers

  I or **Z :** The set of all integers

  Q : The set of all rational numbers

  R : The set of all real numbers

  z^+ **:** The set of all positive integers

  Q^+ **:** The set of all positive rational numbers

  R^+ **:** The set of all positive real numbers
- **Representation of a Set :** Usually, sets are represented in the following two ways.
  1) Roster form or Tabular form.
  2) Set Builder form or Rule Method.

☑ **Roster form :** In this form, all elements of a set are listed, the elements are being separated by commas and are enclosed within brackets { } (curly brackets). For example, the set A of all odd natural numbers less than 10 in the roster form is written as A = {1,3,5,7,9}

1) In roster form, every element of the set is listed only once.
2) The order in which the elements are listed is immaterial.

Eg 1 : Each of the following sets denotes the same set $\{1, 2, 3\}$, $\{3, 2, 1\}$, $\{1, 3, 2\}$.

Eg 2 : Roster form or tabular form of set of all letters in the word 'MATHEMATICS' is given by { M,A,T,H,I,E,C,S}

Note : (i) All infinite sets cannot be described in the roster form

(ii) The set of real numbers cannot be described in this form , because these elements of the set do not follow any particular pattern.

☑ **Set - Builder form :** In this form, All the elements of a set possess a single common property or characterstic property which is not possessed by any element outside the set. Write a variable (say x) representing any member of the set followed by colon(:) or slash (/) which is follwed by a property satisfied by each member of the set. i.e., A set is denoted as { x : x satisfies p(x) where p(x) is the common property}.

For example, the set A of all prime numbers less than 10 in the set-builder form is written as A = {x / x is a prime number less than 10}

The symbol '/' stands for the words 'such that'. Sometimes, we use the symbol ':' in place of the symbol '/'.

Eg : Set builder form of {a,e,i,o,u} is V = { x : x is a vowel in english alphabet }

☑ **Classification (or) Types of Sets :**

Empty Set or Null Set or void set :

A set which has no elements is called the null set or empty set or void set. It is denoted by the symbol ϕ or { }. For example, each of the following is a null set.

Eg 1 : Let A = { x : 1< x <2, x is a natural number} then A is the empty set because there is no natural number between 1and 2.

Eg 2 : The set of all real numbers whose square is -1.

Eg 3 : The set of all rational numbers whose square is 2.

Note : A set consisting of atleast on element is called a non-empty set.

Singleton Set : A set having only on element is called a singleton set.

Eg 1 : $\{0\},\{\phi\}$ are singleton sets, whic contains only one element.

Eg 2 : Let $A=\{x : x \in N$ and $x^2-9=0$

then $A=\{3\}$, which is a singleton set.

But $\{x : x \in Z$ and $x^2-9=0\}$
= {-3 ,3}is not a singleton set.

Finite and Infinite Sets : A set which i empty or consists of finite number elements is called a finite set. Otherwise, is called an infinite set. For example, th set of all days in a week is a finite set. Whe as, the set of all integers, denoted by Z={. -2, -1, 0, 1, 2, ...} or {x / x is an integer}, an infinite set.

☑ **Cardinal Number (or) Order of a set :** The number of distinct elements in a fini set A is called the cardinal number of th set A and is denoted by n(A) or O(A) ($|A|$.

Eg : If A = { 2,4,6,8,10,12} then, n(A)=

Equal Sets : If A and B are two sets suc that every member of A is a member of and every member of B is a member of A then we say that A and B are equal, w write as A = B. Otherwise the sets are sai to be unequal and we write as $A \neq B$.

Eg 1 : A = {1,2,3 }, B = { 3,1,2 }
Then A=B

Eg 2 : A set does not change if one or mor elements of the set is repeated.

A= { 1,2,3} B = { 2,2,1,3,3} are equa sets.That is why we generally do not repea any element in describing a set.

Note : A = { 1,2,3} B = { 1,3,4}
Then $A \neq B$

Equivalent Sets : Two finite sets A and B are said to be equivalent, if $n(A) = n(B)$.

Clearly, equal sets are equivalent but equivalent sets need not be equal.

For example, the sets A={4, 5, 3, 2} and B={1, 6, 8, 9} are equivalent but are not equal.

Subset and Superset : Let A and B be any two sets. If every element of A is an element of B, then A is called a subset of B and we write $A \subseteq B$.

If $A \subseteq B$, then ,B is called superset of A and we write $B \supseteq A$.

Proper Subset : If $A \subseteq B$ and $A \neq B$ then A is called a proper subset of B and we write $A \subset B$ (read as A is a proper subset of B or B is a proper superset of A)

Eg : The set Q of rational numbers is proper subset of real number set R.

In two sets one is a subset of the other ,then the sets are called comparable sets.

Properties of subset :

1) Every set is a subset and a superset of itself.
2) The empty set is the subset of every set.
3) If A is a set with n(A)=m, then the number of subsets of A is 2^m and the number of proper subsets of A is $2^m - 1$.

Note : If A,B,C are any three sets,then

i) $A \subseteq B, B \subseteq C \Rightarrow A \subseteq C$

ii) $A \subseteq B, B \subset C \Rightarrow A \subset C$

iii) $A \subset B, B \subseteq C \Rightarrow A \subset C$

iv) $A \subset B, B \subset C \Rightarrow A \subset C$

Power Set : The set of all subsets of a given set A is called power set of A and is denoted by $P(A)$. Clearly, if A has n elements, then its power set $P(A)$ contains exactly 2^n elements.

For example, if A={1, 2, 3}, then
$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1,3\}, \{2, 3\}, \{1, 2, 3\}\}$.

Universal Set : If there are some sets under consideration , then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set for those sets,We shall denote by U or μ.

Eg 1 : A = {1,2,3} B = { 2,3,4,5} C={6,7} Then we consider U={1,2,3,4,5,6,7}as its one of the universal sets.

Eg 2 : In the study of two dimensional geometry, the set of all points in the XY-plane is called universal set.

Disjoint sets : If two sets A and B are such that they do not have any elements in common i.e., $A \cap B = \phi$,then A,B are said to be disjoint sets.

Eg : A= { X : X is odd number},

B = {X : X is even number } then A,B have no common elements.

Venn Diagram : In order to express the relationship among sets in perspective,we represent them pictorially by means of diagrams is called Venn Diagram. In these diagrams ,the universal set is represented by a rectangular region and the subsets by circles inside the rectangle.We represent disjoint sets by disjoint circles and intersecting sets by intersecting circles.

OPERATIONS ON SETS :

Union of Two Sets :

The union of two sets A and B, written as $A \cup B$ (read as A union B), is the set consisting of all the elements which are either in A or in B or in both. Thus,

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Clearly (i) $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

(ii) $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$

For example, if A={a, b, c, d} and B = {c,d,e,f}, then $A\cup B=\{a, b, c, d, e, f\}$.

- **Intersection of Two Sets :** The intersection of two sets A and B, written as $A\cap B$ (read as A intersection B) is the set consisting of all the common elements of A and B. Thus,

$A\cap B=\{x:x\in A \text{ and } x\in B\}$

Clearly (i) $x\in A\cap B\Rightarrow x\in A$ and $x\in B$

(ii) $x\notin A\cap B\Rightarrow x\notin A$ or $x\notin B$.

For example, if A={a, b, c, d} and B = {c,d,e,f} then $A\cap B=\{c, d\}$.

- **Difference of Two Sets :**

If A and B are two sets, then their difference

$A-B$ or $A\setminus B$ (or) $\frac{A}{B}$ is defined as :

$A-B=\{x:x\in A \text{ and } x\notin B\}$

Similarly, $B-A=\{x:x\in B \text{ and } x\notin A\}$

For example, if A={1,2,3,4} and B={1,3,5,7,9}, then A–B={2,4} and $B-A=\{5,7,9\}$.

- **Important Results :**

In general

1. $A-B\neq B-A$
2. The sets A-B, B-A and $A\cap B$ are disjoint sets.
3. $A-B\subseteq A$ and $B-A\subseteq B$
4. $A-\phi=A$ and $A-A=\phi$

- **Symmetric Difference of Two sets :**

The symmetric difference of two sets A and B, denoted by $A\Delta B$, is defined as

$A\Delta B=(A-B)\cup(B-A)$

$=(A\cup B)-(A\cap B)$

Eg : If A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9} then

$A\Delta B=(A-B)\cup(B-A)$

$=\{2,4\}\cup\{7,9\}=\{2,4,7,9\}$

- **Complement of a Set :** If U is a universal set and A is a subset of U, then the complement of A is the set which contains those elements of U, which are not contained in A and is denoted by A' or A^c. Thus, $A'=\{x:x\in U \text{ and } x\notin A\}$

Eg : If U = {1, 2, 3, 4, ...} and A = {2, 4, 6, 8, ...}, then, A' = {1, 3, 5, 7, ...}.

Properties of complement sets

i) $U'=\phi$ ii) $\phi'=U$

iii) $A\cup A'=U$ iv) $A\cap A'=\phi$

v) $(A')'=A$, law of double complementation.

vi) $(A\cup B)'=A'\cap B'$ and

$(A\cap B)'=A'\cup B'$ are called demorgan laws

Algebra of sets :

i) **Idempotent Laws :** For any set A, we have

a) $A\cup A=A$ b) $A\cap A=A$

ii) **Commutative Laws :** For any two sets A and B, we have

a) $A\cup B=B\cup A$ b) $A\cap B=B\cap A$

iii) **Identity Laws :** For any set A, U is universal set, we have

a) $A\cup\phi=A$ b) $A\cap\phi=\phi$

c) $A\cup U=U$ d) $A\cap U=A$

iv) **Associative Laws :** For any three sets A, B and C, we have

a) $A\cup(B\cup C)=(A\cup B)\cup C$

b) $A\cap(B\cap C)=(A\cap B)\cap C$

v) **Distributive Laws :** For any three sets A, B and C, we have

a) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

For any three sets A, B and C

i) $A - (B \cup C) = (A - B) \cap (A - C)$

ii) $A - (B \cap C) = (A - B) \cup (A - C)$

- For any two sets A and B, we have

a) $P(A) \cap P(B) = P(A \cap B)$

b) $P(A) \cup P(B) \subseteq P(A \cup B)$,

where $p(A)$ is the power set of A.

- If A and B are any two sets then

i) $A \subset B \Rightarrow A \cup B = B,\ A \cap B = A$

ii) $A - B = A - (A \cap B) = (A \cup B) - B$

iii) $(A - B) \cup (A \cap B) = A$

iv) $(A - B) \cup (B - A) \cup (A \cap B) = A \cup B$

v) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

vi) $A \subseteq A \cup B,\ B \subseteq A \cup B,$
$A \cap B \subseteq A,\ A \cap B \subseteq B$

vii) $A - B = A \cap B'$

viii) $(A - B) \cup B = A \cup B$

ix) $(A - B) \cap B = \phi$

x) $A \subseteq B \Leftrightarrow B' \subseteq A'$

xi) $A - B = B' - A'$

xii) $A - B = B - A \Leftrightarrow A = B$

xiii) $A \cup B = A \cap B \Leftrightarrow A = B$

- **Properties on symmetric difference :**

A,B,C are any three sets

i) $A \Delta \phi = A$ ii) $A \Delta A = \phi$

iii) $A \Delta B = A \Delta C \Rightarrow B = C$

iv) $A \Delta B = B \Delta A$ v) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

vi) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

- **Some important results on cardinal numbers :**

If A, B and C are finite sets and U be the finite universal set, then

i) $n(A') = n(U) - n(A)$

ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

iii) $n(A \cup B) = n(A) + n(B)$,

where A and B are disjoint non-empty sets

iv) $n(A - B) = n(A \cap B') = n(A) - n(A \cap B)$

$= n(A \cup B) - n(B)$

v) $n(B - A) = n(A' \cap B) = n(B) - n(A \cap B)$

$= n(A \cup B) - n(A)$

vi) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

vii) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

viii) $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$

ix) $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

x) If $A_1, A_2, A_3 \ldots\ldots\ldots\ldots A_n$ are pair-wise disjoint sets, then

$n(A_1 \cup A_2 \cup A_3 \cup \ldots. \cup A_n)$

$= n(A_1) + n(A_2) + n(A_3) + \ldots. + n(A_n)$

- $n(A \Delta B)$ = number of elements which belong to exactly one of A or B.

$n(A \Delta B) = n\{(A - B) \cup (B - A)\}$

$= n(A) + n(B) - 2n(A \cap B)$

$= n(A \cup B) - n(A \cap B)$

- A and B are two sets and $n(A) = p, n(B) = q$, Then

(i) $\min\{n(A \cup B)\} = \max\{p, q\}$

(ii) $\max\{n(A \cup B)\} = p + q$,

(iii) $\min\{n(A \cap B)\} = 0$

(iv) $\max\{n(A \cap B)\} = \min\{p, q\}$

- No. of elements in exactly one of the sets A,B,C $= n(A)+n(B)+n(C)-2n(A\cap B)-2n(B\cap C)-2n(A\cap C)+3n(A\cap B\cap C)$
- No. of elements in exactly two of the sets A,B,C
$= n(A\cap B)+n(B\cap C)+n(C\cap A)-3n(A\cap B\cap C)$

Concept Based Questions

1. Which of the following is an empty set

1) $\{\phi\}$ 2) $\{0\}$

3) $\{n\in N \text{ and } n<1\}$

4) The set of all even prime numbers

2. Let A and B be two sets such that $A\cup B=A$ Then $A\cap B$ is equal to

1) ϕ 2) B 3) A 4) $A\cup B$

3. Which of the following is not correct?

1) $A\subseteq A^c$ if and only if $A=\phi$

2) $A^c\subseteq A$ if and only if $A=X$, where X is the universal set

3) If $A\cup B=A\cup C$, then $B=C$

4) A = B is equivalent to $A\cup C=B\cup C$ and $A\cap C=B\cap C$

4. If A and B are two sets then $(A-B)\cup(B-A)\cup(A\cap B)$ is

1) $A\cup B$ 2) $A\cap B$ 3) A 4) B^1

5. $A\cap(A\cup B)^C=$

1) A 2) B 3) ϕ 4) A-B

6. If $U=\{a,b,c,d,e,f,g,h\}$ and $A=\{a,b,c\}$ then complement of A is

1) $\{d,e,f\}$ 2) $\{d,e,f,g,h\}$

3) $\{a,b,c\}$ 4) $\{d,e,g,h\}$

KEY

1) 3 2) 2 3) 3 4) 1 5) 3 6) 2

Hints & Solutions

1. N = {1, 2, 3, ...}
2. Since, $A\cup B=A \Rightarrow B\subseteq A \therefore A\cap B=B$
3. A^c satisfies (1) and (2) by definition, (4) also follows trivially.
Assuming A to be any set other than the empty set, also $B=A$ and $C=\phi$, we have
$A\cup B=A=A\cup C$
But $B\neq C$, So (3) is incorrect
4. $(A-B)\cup(B-A)\cup(A\cap B)=A\cup B$
Draw venn diagram
5. $A\cap(A\cup B)^C=A\cap(A^C\cap B^C)$
$=(A\cap A^C)\cap(A\cap B^C)=\phi\cap(A\cap B^C)=\phi$
6. Complement of A = U - A

EXERCISE-I

CRTQ & SPQ LEVEL-I

ROSTER, SET-BUILDER FORM, OPERATION ON SETS

C.R.T.Q Class Room Teaching Questions

1. Which of the following not a well define collection of objects

1) The set of Natural Numbers
2) Rivers of India
3) Various kinds of Triangles
4) Five most renowned Mathematicians of the world.

2. Write the solution set of the equation $x^2+x-6=0$ in roster form

1) {2, -3} 2) $\{-1,-2\}$

3) {1,2} 4) $\{-1,2\}$

3. Write the set A={1,4,9,16,25......} in set builder form
   1) $\{x : x = n^2 \text{ where } n \in N\}$
   2) $\{x : x = n^2 \text{ where } n \in W\}$
   3) $\{x : x = n^2 \text{ where } n \in Z\}$
   4) $\{x : x = n^2 \text{ where } n \in Q\}$
4. Which of the following is not empty set
   1) A ={x : 1<x<2, x is a natural number between 1 and 2}
   2) $B = \{x : x^2 - 2 = 0$ and x is rational$\}$
   3) $C = \{x : x$ is even prime number $>2\}$
   4) $D = \{x : x^2 = 0$ and x is integer$\}$
5. If A = {x/x is a letter in the word "ACCOUNTANCY"} then cardinality of A is
   1) 5 2) 6 3) 7 4) 8
6. Let F_1 be the set of all parallelograms, F_2 be the set of rectangles, F_3 be the set of rhombuses, F_4 be the set of squares and F_5 be the set of trapeziums in a plane then $F_1 =$
   1) $F_2 \cap F_3$ 2) $F_2 \cup F_3 \cup F_4$
   3) $F_3 \cup F_4 \cup F_5$ 4) $F_3 \cap F_1$
7. If the set of factors of a whole number 'n' including 'n' itself but not '1' is denoted by F(n). If $F(16) \cap F(40) = F(x)$ then 'x' is
   1) 4 2) 8 3) 6 4) 10
8. If A is the set of the divisors of the number 15, B is the set of prime numbers smaller than 10 and C is the set of even numbers smaller than 9, then $(A \cup C) \cap B$ is the set
   1){1, 3, 5} 2){1, 2, 3}
   3){2, 3, 5} 4){2, 5}
9. Let A = {1,2,3,4,5,6}, B = {2,4,6,8} then A - B =
   1) {1,3,5} 2) {8} 3) {2,4,6} 4) ϕ
10. Let $A = \{1,2,3,4\}$ and $B = \{2,3,4,5,6\}$, then $A \Delta B =$
    1) {2,3,4} 2) {1} 3) {5,6} 4) {1,5,6}
11. Let U={1,2,3,4,5,6}, A={2,3},B={3,4,5} then $A^I \cap B^I =$ __
    1) {1,2} 2) {1,6} 3){1,5} 4){1,4}
12. In a class of 35 students,24 like to play cricket and 16 like to play football also each student likes to play at least one of the two games. How many students like to play both cricket and football?
    1) 3 2) 4 3) 5 4) 6
13. In a group of 70 people, 37 like coffee, 52 like tea and each person like atleast one of the two drinks. The number of persons liking both coffee and tea is
    1) 16 2) 13 3) 19 4) 20
14. If n(X)=28, n(Y)=32, $n(X \cup Y)=50$ then $n(X \cap Y) =$
    1) 6 2) 7 3) 8 4)10
15. If n(A)=50, n(B)=20 and $n(A \cap B) = 10$ then $n(A \Delta B)$ is
    1) 50 2) 60 3) 70 4) 40

S.P.Q. Student Practice Questions

16. The group of beautiful girls is
    1) a null set 2) A finite set
    3) not a set 4) Infinite set
17. Which of the following is the roster form of letters of word "SCHOOL"
    1) {s, h, o, l} 2) {s, c, h, o, l}
    3) {s, c, o, l} 4) {h, o, o, l}
18. Write the set {x : x is a positive integer and $x^2 < 40$} in the roaster form
    1) {1,2,3,4,5,6} 2) {1,2,3,4,5,6,7}
    3) {2,3,4,5,6,7} 4) {0,1,2,3,5,6}
19. Which of the following is finite
    1) A = {x : x is set of points on a line}
    2) B = {x : x $\in$ N and x is prime}
    3) C = {x : x$\in$Nand x is odd}
    4) D = {x : x $\in$ N and (x-1)(x-2)=0}

20. Which of the following pairs of sets are equal

1) A={x : x is letter of word "ALLOY"} B={x : x is letter of word "LOYAL"}
2) A={-2,-1,0,1,2},B={1,2}
3) A={0},B={x : x >5 and x<15}
4) A={x : x-5=0}B={x : x^2=25}

21. Let $\{1,\{2,3\}\}$ then the number of subsets of A is

1) 2 2) 4 3) 8 4) 0

22. How many elements has P(A),if $A=\phi$

1) 1 2) 2 3) 3 4) 0

23. A = {2,4,6,8}B={6,8,10,12} then $A\cup B$

1) {2,4,6,8,12} 2) {2,4,6,8,10,12}
3) {6,8} 4) {2,4}

24. If $A=\{2,3,4,8,10\}$, $B=\{3,4,5,10,12\}$, $C=\{4,5,6,12,14\}$ then $(A\cup B)\cap(A\cup C)=$

1) $\{2,3,4,5,8,10,12\}$ 2) $\{2,4,8,10,12\}$
3) $\{3,8,10,12\}$ 4) $\{2,8,10\}$

25. Let A={a,e,i,o,u}, B={a,i,k,u} then A-B

1){a,e} 2) {e,i}
3) {e, o} 4) {e,i,o}

26. Let U={1,2,----,10}, A={1,3,5,7,9} then A^{1}=

1) {2,4,6,8,10} 2) {1,3,5,7,9}
3) {1,3,2,4} 4) A

27. In a committee 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. The number of persons speaking at least one of these two languages is

1) 60 2) 40 3) 38 4) 58

28. If A and B are two sets such that $n(A)=70$, $n(B)=60$ and $n(A\cup B)=110$, then $n(A\cap B)$ is equal to

1) 240 2) 50 3) 40 4) 20

KEY

01) 4	02) 1	03) 1	04) 4	05)
06) 2	07) 2	08) 3	09) 1	10)
11) 2	12) 3	13) 3	14) 4	15)
16) 3	17) 2	18) 1	19) 4	20)
21) 2	22) 1	23) 2	24) 1	25)
26) 1	27) 1	28) 4		

Hints Solutions

1. For determining a mathematicians r renowned may vary from person to pe

2. $x^2+x-6=0 \Rightarrow x=2,-3$

3. $1^2=1, 2^2=4, 3^2=9$.... all are square of natural numbers.

4. 1) $A=\phi$ because there are no natural numbers between 1 and 2.

2) $B=\phi$ because $x^2=2 \Rightarrow x=\sqrt{2}$ not a rational number

3) $c=\phi$ because there is only one eve prime 2

5. Different letters of the word ACCOUNTANCY is {A,C,O,U,N,T,' Cardinality of A=7.

6. Since every rectangle, rhombus and squar a parallelogram so $F_1=F_2\cup F_3\cup F_4\cup F_1$

7. $F(16)=\{2,4,8,16\}$, $F(40)=\{2,4,8,20,40\}$

$F(16)\cap F(40)=\{2,4,8\}=F(8)$

$F(x)=F(8)\Rightarrow x=8$

8. $A=\{1,3,5,15\}$, $B=\{2,3,5,7\}$, $C=\{2,4,6,$

$\therefore A\cup C=\{1,2,3,4,5,6,7,8,15\}$

$(A\cup C)\cap B=\{2,3,5\}$

9. A-B= {1,2,3,4,5,6}-{2,4,6,8,}={1,3,5}

10. $A\Delta B=\left\{x:x\in\frac{A}{B}\text{ or }x\in\frac{B}{A}\right\}$

11. A^1= U-A={1,4,5,6}, B^1=U-B={1,2,6}

$A^1\cap B^1$={1,6}

12. n(C)=24, n(F)=16, n(C∪F)=35

n(C∩F)=n(c)+n(F)-n(C∪F)

$=24+16-35=40-35=5$

13. $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

we have, $70=37+52-n(A\cap B)$

14. n(X∩Y)=n(x) +n(Y)-n(X U Y)

$=28+32-50=10$

15. $n(A\Delta B)=n(A\cup B)-n(A\cap B)$

$=n(A)+n(B)-2n(A\cap B)=50$

16. Beautiful is a relative term but not well defined.

17. elements not repeated and denoted by small letters.

18. $1^2=1,2^2=4,3^2=9,4^2=16,5^2=25,6^2=36$

all are <40

19. x-1=0,x-2=0 x=1,2

20. A={a,l,o,y} B={a,l,o,y}

21. no of subsets = 2^n

22. n(A)=0, $n[p(A)]=2^n=2^0=1$

23. All the elements in A and B.

24. $A\cup(B\cap C)$

25. A-B= {a,e,i,o,u} -{a,i,k,u}={e,o}

26. A^1=U-A

27. $n(S\cup F)=n(S)+n(F)-n(S\cap F)$

$\Rightarrow n(S\cup F)=20+50-10=60$

28. $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

EXERCISE-II

CRTQ & SPQ LEVEL-II

ROSTER, SET-BUILDER FORM, OPERATION ON SETS

C.R.T.Q Class Room Teaching Questions

1. Let X = {1, 2, 3, 4, 5, 6}. If n represent any member of X, then roster form of $n\in X$ but $2n\notin X$

1) {2, 3, 5, 6} 2) {5, 6}
3) {4, 5, 6} 4) {3}

2. Two finite sets have m and n elements. If total number of subsets of the first set is 56 more than that of the total number of subsets of the second. The values of *m* and *n* respectively are

1) 7,6 2) 6,3 3)5,1 4) 8, 7

3. If $A=\{8^n-7n-1:n\in N\}$ and $B=\{49(n-1):n\in N\}$ then

1) $A\subset B$ 2) $B\subseteq A$
3) A = B 4) $A\subseteq B$

4. If $A=\{\phi,\{\phi\}\}$ then the power set of A is

1) A 2) $\{\phi,\{\phi\},A\}$
3) $\{\phi,\{\phi\},\{\{\phi\}\},A\}$ 4) $\{\phi\}$

5. The smallest set A such that $A\cup\{1,2\}=\{1,2,3,5,9\}$ is

1) $\{2,3,5\}$ 2) $\{3,5,9\}$
3) $\{1,2,5,9\}$ 4) $\{1,2\}$

6. If sets A and B are defined as

$A=\{(x,y):y=e^x,x\in R\}$

$B=\{(x,y):y=x,x\in R\}$, then

1) $B\subset A$ 2) $A\subset B$
3) $A\cap B=\phi$ 4) $A\cup B=A$

7. If $aN = \{ax : x \in N\}$ then $3N \cap 7N =$

1. 21N 2. 10N 3. 4N 4. 5N

8. If $A = \{1,2,3,4,5,6\}$, $B = \{1,2\}$, then $\dfrac{A}{\left(\frac{A}{B}\right)} =$

1) A 2) ϕ 3) $A \cap B$ 4) $A \cup B$

9. If $n(U) = 700, n(A) = 200, n(B) = 300,$ $n(A \cap B) = 100$, then $n(A' \cap B')$ is equal to

1) 400 2) 240 3) 300 4) 500

10. If $n(U) = 48, n(A) = 28, n(B) = 33$ and $n(B - A) = 12$, then $n(A \cap B)^C$ is

1) 27 2) 28 3) 29 4) 30

11. If $n(A \cap B^C) = 5, n(B \cap A^C) = 6,$ $n(A \cap B) = 4$ then the value of $n(A \cup B)$ is

1) 18 2) 15 3) 16 4) 17

12. Let n(A-B)=25+X, n(B-A)=2X and $n(A \cap B) = 2X$. If n(A)=2(n(B)) then 'X' is

1) 4 2) 5 3) 6 4) 7

S.P.Q. Student Practice Questions

13. Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and foot ball, and 12 play foot ball and cricket. Eight play all the three games. The total number of members in the three athletic teams is

1) 43 2) 76 3) 49 4) 53

14. If sets A and B have 3 and 6 elements each, then the minimum number of elements in $A \cup B$ is

1) 3 2) 6 3) 9 4) 18

15. If $n(U) = 60, n(A) = 21, n(B) = 43$ then greatest value of $n(A \cup B)$ and leas value of $n(A \cup B)$ are

1) 60, 43 2) 50, 36 3) 70, 44 4) 60, 38

16. If A={x:x is a multiple of 4} and B={x:x i a multiple of 6} then $A \cap B$ consists of a multiples of

1) 16 2) 12 3) 8 4) 4

17. Two finite sets have m and n elements. Th total number of subsets of the first se is 48 more than the total number o subsets of the second set. The values o m and n are

1) 7,6 2) 7,6 3) 6,4 4) 7,4

18. If A =]2,4[and B = [3,5[then $A \cap B =$

1) [3, 4] 2) [3, 4]
3) [3, 4] 4) [2,5]

19. Let A be the set of non-negative integers I is the set of integers, B is the set o non-positive integers, E is the set of eve integers and P is the set of prim numbers then.

1) I-A=B 2) $A \cap B = \phi$
3) $E \cap P = \phi$ 4) $A \Delta B = I - \{0\}$

20. In a class of 100 students, 55 student have passed in Mathematices and 6 students have passed in physics. No o students who have passed in physic only is

1) 22 2) 33 3) 10 4) 45

21. 90 students take mathematics, 72 tak science in a class of 120 students. If 1 take neither Mathematics nor scienc then number of students who take bot the subjects is

1) 52 2) 110 3) 162 4) 100

22. A set A has 3 elements and another se B has 6 elements. Then

1) $3 \leq n(A \cup B) \leq 6$ 2) $3 \leq n(A \cup B) \leq$
3) $6 \leq n(A \cup B) \leq 9$ 4) $0 \leq n(A \cup B) \leq$

23. In a survey of 400 students in a schoo 100 were listed as taking apple juice, 15 as taking orange juice and 75 were liste

as taking both apple as well as orange juice. then how many students were taking neither apple juice nor orange juice are

1) 120 2) 220 3) 225 4) 150

KEY

01) 3	02) 2	03) 1	04) 3	05) 2
06) 3	07) 1	08) 3	09) 3	10) 1
11) 2	12) 2	13) 1	14) 2	15) 1
16) 2	17) 3	18) 2	19) 4	20) 4
21) 1	22) 3	23) 3		

Hints Solutions

1. $A=\{n/n\in X \text{ but } 2n\notin X\}\Rightarrow n=4,5,6$

2. $2^m-2^n=56$

3. $8^n=(7+1)^n$

$={}^nc_0 7^n+{}^nc_1 7^{n-1}+........+{}^nc_{n-2}7^2+{}^nc_{n-1}7+{}^nc_n$

$={}^nc_0 7^n+{}^nc_1 7^{n-1}+........+{}^nc_{n-2}49+7n+1$

$8^n-7n-1=49\left[{}^nc_0 7^{n-2}+{}^nc_1 7^{n-3}+....+{}^nc_{n-2}\right]$

8^n-7n-1 is a multiple of 49 for all $n\in N$

$\therefore$ A contains elements which are multiple of 49 and clearly B contains all multiples of 49.

$\therefore A\subset B$

4. no of subsets $=2^n$

5. Since $A\cup\{1,2\}=\{1,2,3,5,9\}$ {given}

$\Rightarrow A=\{3,5,9\}$ atleast

6. The graph of $y=e^x$ *and* $y=x$ do not intersect

7. $3N=\{3x:x\in N\},7N=\{7x:x\in N\}$

$\Rightarrow 3N\cap 7N=21N$

8. $\frac{A}{B}=A-B=\{3,4,5,6\}$, $\frac{A}{\left(\frac{A}{B}\right)}=\{1,2\}$

9. $n(A'\cap B')=n(A\cup B)'=n(U)-n(A\cup B)$

10. $n(U)=48, n(A)=28, n(B)=33$,

$n(B-A)=12$

$n(A\cap B)=n(B)-n(B-A)=33-12=21$

$n(A\cap B)^C=n(U)-n(A\cap B)=48-21=27$

11. $n(A\cap B^C)=5, n(B\cap A^C)=6$,

$n(A\cap B)=4$

$n(A)=n(A\cap B)+n(A\cap B^C)=4+5=9$

$n(B)=n(A\cap B)+n(B\cap A^C)=6+4=10$

$n(A\cup B)=n(A)+n(B)-n(A\cap B)$

12. $n(A-B)=25+x, n(B-A)=2x$,

$n(A\cap B)=2x$,

$n(A)=n(A-B)+n(A\cap B)$

$=25+x+2x=25+3x$

$n(B)=n(B-A)+n(A\cap B)=2x+2x=4x$

$n(A)=2n(B)\Rightarrow 25+3x=2(4x)$

$\Rightarrow 5x=25\Rightarrow x=5$

13. $n(C)=21$, $n(H)=26$, $n(F)=29$,

$n(H\cap C)=14$, $n(H\cap F)=15$

$n(F\cap C)=12$, $n(F\cap C\cap H)=8$ Total no.of players $=n(C\cup H\cup F)=43$

14. $n(A\cup B)\geq \max\{3,6\}=6$

15. $n(U)=60, n(A)=21, n(B)=43$

Greatest value of $n(A\cup B)=n(U)=60$

Least value of $n(A\cup B)=n(B)=43$

16. $A=\{4,8,12,16,20,24,.....\}$

$B=\{6,12,18,24,.....\}$ $\therefore A\subset B=\{12,24,....\}$

$=\{x:x$ is a multiple of $12\}$

17. $2^m-2^n=48\Rightarrow m=6,n=4$

18. Consider common part

19. $A\Delta B = (A-B)\cup(B-A)$

$=\{1,2,3,...\}\cup\{...-3,-2,-1\}$

$=\{....-3,-2,-1,1,2,3....\} = I-\{0\}$

20. $n(M)=55, n(P)=67, n(M\cup P)=100$,

$n(M\cup P)=n(M)+n(P)-n(M\cap P)$

$\Rightarrow 100=55+67-n(M\cap P)$

$\therefore n(M\cap P)=122-100=22.$

$n(P\ only)=n(P)-n(M\cap P) = 67-22=45$

21. $n(M)=90, n(S)=72, n(M^1\cap S^1)=10$

$\Rightarrow n(M\cup S)=120-10=110$

$n(M\cap S)=n(M)+n(S)-n(M\cup S)$

$=90+72-110=52$

22. $n(A)=p$ and $n(B)=p$ then

$\min\{n(A\cup B)\}=\max\{p,q\}$

$\max\{n(A\cup B)\}=p+q,$

23. $n(A^1\cap B^1)=n(U)-n(A\cup B)$

EXERCISE-III

CRTQ & SPQ LEVEL-III

OPERATION ON SETS

C.R.T.Q Class Room Teaching Questions

1. **Let A and B be two sets then** $(A\cup B)^C\cup(A^C\cap B)=$

1) A^C 2) B^C 3) ϕ 4) U

2. **A set contains $(2n+1)$ elements. The number of subsets of this set containing more than n elements is equal to**

1) 2^{n-1} 2) 2^n 3) 2^{n+1} 4) 2^{2n}

3. **From 50 students taking examin in Mathematics, Physics and Chen each of the students has passed in a one of the subject, 37 p Mathematics, 24 Physics an Chemistry. At most 19 pa Mathematics and Physics, at mo Mathematics and Chemistry ar most 20 Physics and Chemistry. the largest possible number that have passed all three examinations**

1) 16 2) 14 3) 18 4)

4. **Which is the simplified representa of** $(A^1\cap B^1\cap C)\cup(B\cap C)\cup(A\cap$ **where A,B,C are subsets of set X**

1) A 2) B

3) C 4) $X\cap(A\cup B\cup C)$

5. **The set** $(A\cup B\cup C)\cap(A\cap B'\cap C')'\cap$ **is equal to**

1) $B\cap C'$ 2) $A\cap C$

3) $B\cup C'$ 4) $A\cap C'$

6. **If** $P=\{x\in R: f(x)=0\}$ **and** $Q=\{x\in R: g(x)=0\}$ **then** $P\cup Q$ **is**

1) $\{x\in R: f(x)+g(x)=0\}$

2) $\{x\in R: f(x).g(x)=0\}$

3) $\{x\in R; (f(x))^2+(g(x))^2=0\}$

4) $\{x\in R; x>1\}$

7. **Suppose** $A_1, A_2 ..., A_{30}$ **are thirty sets e with five elements and** $B_1, B_2,...B_n$ **are n each with three elements such th** $\bigcup_{i=1}^{30} A_i=\bigcup_{j=1}^{n} B_j=S$. **If each element of belongs to exactly ten of the** A_i's **and exa 9 of the** B_j's**, then the value of** n **is**

1) 15 2) 135 3) 45 4)

. If $aN=\{ax/x\in N\}$ and $bN\cap cN=dN$, where $b,c\in N$ are relatively prime, then

1) $d=bc$ 2) $c=bd$
3) $b=cd$ 4) none

S.P.Q. Student Practice Questions

. A survey show that in a city that 63% of the citizens like tea where as 76% like coffee. If x% like both tea and coffee, then

1) $x=63$ 2) $x=39$
3) $50\le x\le 63$ 4) $39\le x\le 63$

0. An investigator interviewed 100 students to determine their preferences for the three drinks: milk (M), coffee(C) and tea (T). He reported the following : 10 students had all the three drinks M,C,T; 20 had M and C only: 30 had C and T;25 had M and T;12 had M only;5 had C only;8 had T only. Then how many did not take any of the three drinks

1) 20 2) 3 3) 36 4) 42

1. In a college of 300 students , every students reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is

1) atleast 30 2) atmost 20
3) exactly 25 4) atmost10

2. In a class of 55 students the numbers of students studying different subjects are 23 in mathematics, 24 in physics, 19 in chemistry, 12 in mathematics and physics, 9 in mathematics and chemistry 7 in physics and chemistry and 4 in all the three subjects. the numbers of students who have taken exactly one subject is

1) 6 2) 13 3) 16 4) 22

3. Out of 800 boys in a school. 224 played cricket 240 played hockey and 336 played basketball. of the total, 64 played both bas-ketball and hockey, 80 played cricket and basketball and 40 played cricket and hoc-key, 24 played all the three games. The num-bers of boys who did not play any game is

1) 128 2) 216 3) 240 4) 160

14. In a certian town 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car. 2000 families own both a car and a phone. Consider the following statements in this regard.

i. 10% families own both a car and a phone

ii. 35% families own either a car or a phone

iii. 40,000 families live in the town.

Which of the above statements are correct?

1) i and ii 2) i and iii
3) ii and iii 4) i,ii and iii

15. In a battle 70% of the combatants lost one eye, 80% an ear, 75% an arm, 85% a leg, x% lost all the four limbs the minimum value of x is

1) 10 2) 12 3) 15 4) 5

KEY

01) 1	02) 4	03) 2	04) 3	05) 1
06) 2	07) 3	08) 1	09) 4	10) 1
11) 3	12) 4	13) 4	14) 3	15) 1

Hints & Solutions

1. $(A\cup B)^C\cup(A^C\cup B)$

$=(A^C\cap B^C)\cup(A^C\cup B)$

$=(A^C\cup A^C)\cap(A^C\cup B)\cap(B^C\cup A^C)\cap(B^C\cup B)$

$= A^c \cap \left[A^c \cup \left(B \cap B^c\right)\right] \cap U$

$= A^c \cap \left(A^c \cup \phi\right) \cap U$

$= A^c \cap A^c \cap U = A^c \cap U = A^c$

2. Let the original set contains 2n+1 elements, then subsets of this set containing more than n elements means subsets containing (n+1) elements, (n+2) elements .... (2n+1) elements.

$\therefore$ Required number of subsets

$= {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \ldots\ldots + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1}$

$= {}^{2n+1}C_{n} + {}^{2n+1}C_{n-1} + \ldots\ldots + {}^{2n+1}C_{1} + {}^{2n+1}C_{0.}$

$= {}^{2n+1}C_{0} + {}^{2n+1}C_{1} + \ldots\ldots + {}^{2n+1}C_{n-1} + {}^{2n+1}C_{n.}$

$= \frac{1}{2}\left[(1+1)^{2n+1}\right] = \frac{1}{2}\left(2^{2n+1}\right) = 2^{2n}$.

3. The given conditions can be expressed as

$n(M \cup P \cup C) = 50, n(M) = 37, n(p) = 24,$

$n(C) = 43, n(M \cap P) \leq 19, n(M \cap C) \leq 29$

and $n(P \cap C) \leq 20$.

$n(M \cup P \cup C) =$

$n(M) + n(P) + n(C) - n(M \cap P)$

$-n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$

$\Rightarrow n(M \cap P \cap C) \leq n(M \cap P) + n(M \cap C)$

$+n(P \cap C) - 54$

Therefore, the number of students is at most 19+29+20-54=14

4. $(A^1 \cap B^1 \cap C) \cup (B \cap C) \cup (A \cap C) = C$

draw venn diagram.

5. $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = (A \cup B \cup C)$

$\cap (A' \cup B \cup C) \cap C' =$

$[(A \cap A') \cup (B \cup C)] \cap C' = (B \cup C) \cap C'$

$= (B \cap C') \cup (C \cap C') = B \cap C'$

6. $f(x).g(x) = 0 \Rightarrow$ either

$f(x) = 0$ or $g(x) = 0$.

7. $S = \bigcup_{i=1}^{30} A_i \Rightarrow n(S) = \frac{1}{10}(5 \times 30) = 15$

Again, $S = \bigcup_{j=1}^{n} B_j \Rightarrow n(S) = \frac{1}{9}(3 \times n) = \ldots$

Thus $\frac{n}{3} = 15 \Rightarrow n = 45$

8. We have $bN = \{bx | x \in N\}$ = the set positive integral multiples of b

$cN = \{cx | x \in N\}$ = the set of positive integral multiples of c.

$\therefore bN \cap cN$ = the set of positive integral multiples of bc $= bcN$ [$\because$ b and c relatively prime] Hence, $d = bc$

9. Let the population of the city be 100 Let A denote the set of citizens who like tea and B denote the set of citizens who like coffee.

$\therefore n(A) = 63$ and $n(B) = 76$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) \leq 100 \Rightarrow 63 + 76 - n(A \cap B) \leq 100$

$\Rightarrow 63 + 76 - n(A \cap B) \leq 100$

$\Rightarrow 39 \leq n(A \cap B) \rightarrow (1)$

Also $n(A \cap B) \leq n(A)$ and

$n(A \cap B) \leq n(B)$

$\Rightarrow n(A \cap B) \leq 63$ and $n(A \cap B) \leq 76$

$\Rightarrow n(A \cap B) \leq 63 \rightarrow (2)$

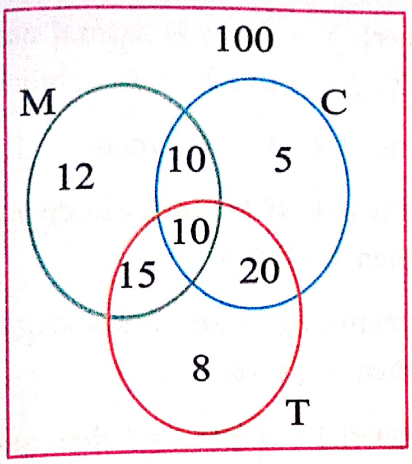
From (1) and (2) : $39 \leq n(A \cap B) \leq 63$

$\Rightarrow 39 \leq x \leq 63$

10. $S = 100$. The numbers can be read from the fig, number of people who did not take any drink

$= 100 - \{12 + 5 + 8 + 10 + 20 + 15 + 10\}$

$= 100 - 80 = 20$



11. If n is the required number of newspapers then $n\times 60=300\times 5\Rightarrow n=25$

12. $n(M)=23, n(P)=24, n(C)=19,$

$n(M\cap C)=9,$

$n(P\cap C)=7, n(M\cap P\cap C)=4.$

We have to find ,

$n(M\cap P'\cap C'),$

$n(P\cap M'\cap C'), n(C\cap M'\cap P')$

Now

$n(M\cap P'\cap C')=n\left[M\cap(P\cup C)'\right]$

$=n(M)-n[M\cap(P\cup C)]$

$=n(M)-n[(M\cap P)\cup(M\cup C)]$

$=n(M)-n(M\cap P)-n$

$(M\cap C)+n(M\cap P\cap C)$

$=23-12-9+4=27-21=6.$

$n(P\cap M'\cap C')=n\left[P\cap(M\cup C)'\right]$

$=n(P)-n[P\cap(M\cup C)]$

$=n(P)-n[(P\cap M)\cup(P\cap C)]$

$=n(P)-n[P\cap M]-$

$n(P\cap C)+n(P\cap M\cap C)$

$=24-12-7+4=9,$

$n(C\cap M'\cap P')=$

$n(C)-n(C\cap P)-$

$n(C\cap M)+n(C\cap P\cap M)$

$=19-7-9+4=23-16=7$

13. $n(C)=224, n(H)=240, n(B)=336.$

$n(H\cap B)=64,$

$n(B\cap C)=80, n(H\cap C)=40,$

$n(C\cap H\cap B)=24$

$n(C^C\cap H^C\cap B^C)=n\left[(C\cup H\cup B)^C\right]=$

$n(U)-n(C\cup H\cup B)$

$$=800-\left[\begin{array}{l}n(C)+n(H)+n(B)-n(H\cap C)\\ -n(H\cap B)-n(C\cap B)+n(C\cap H\cap B)\end{array}\right]$$

$=800-[224+240+336-64-80-40+24]$

$=800-[824-184]=984-824=160.$

14. $n(P)=25\%, n(C)=15\%$

$n(P^C\cap C^C)=65\%,\ n(P\cap C)=2000$

Since $n(P^c\cap C^c)=65\%\Rightarrow n(P\cup C)^C$

$=65\%\Rightarrow n(P\cup C)=35\%$, Now

$n(P\cup C)=n(P)+n(C)-n(P\cap C)$

$\Rightarrow 35=25+15-n(P\cap C)$

$\Rightarrow n(P\cap C)=40-35=5.$

Thus $n(P\cap C)=5\%.$

But $n(P\cap C)=2000$

$\therefore 5\%$ of the total $=2000\Rightarrow$

Total numbers of families

$=\dfrac{2000\times 100}{5}=40000.$

Since $n(P\cup C)=35\%$ and total number of families =40,000 and $n(P\cap C)=5\%$.

$\therefore (ii)\, and\, (iii)$ are correct.

15. Minimum value of

$x=100-(30+20+25+15)=100-90=10$.

EXERCISE-IV
LEVEL-IV

Assertion - Reason Type :

Note :

1) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

2) Statement-1 is true, Statement-2 is true, Statement-2 is not correct explanation for Statement-1.

3) Statement-1 is true, Statement-2 is false.

4) Statement-1 is false, Statement-2 is false.

1. Statement-1: If A = {x:x is a prime number} and B = {x: x $\in$N} then $A\cap B$ = {x: x is a prime number}=A.
Statement-2: If $A\subset B$ then $A\cap B$ = A.

2. Statement-1: If A={2,4,7,10}, B={1,2,3,4} then A-B={1,3,7,10}.

Statement-2: A-B={x:x $\notin$A and x $\in$B}.

3. Statement-1: A={x: 0<x<3, $x\in R$} and B = {x: $1\le x\le 5, x\in R$}then A - B = { x : 0 <x< 1,$x\in R$}
Statement-2: $A\Delta B = (A-B)\cup(B-A)$.

4. Statement-1: If $n(B)=3$ then number of elements in power set of B= $2^3=8$
Statement-2: If $n(A)=m$ then number of elements in power set of A = 2^m-1

5. Statement-1: If X = {x : x is wh... number}, Y = {x : x is natural numb...} then $X\cup Y$ = { x : x is whole number }...
Statement-2: If $Y\subset X$ then $Y\cup X=X$

6. Statement-1: If A and B are disjoint sets then A - B = A
Statement-2: If A and B are disjoint sets then $A\cap B=\varnothing$

7. Statement-1 : If n (A) = 5 then numb... of proper subsets of A = 31
Statement-2 : If n (A) = m then number of proper subsets of $B=2^m$...

8. Match the following sets for all A, B and...

i) $((A^1\cup B^1)-A)^1$	a) A-B
ii) $[B^1\cup(B^1-A)]^1$	b) A
iii) $(A-B)-(B-C)$	c) B
iv) $(A-B)\cap(C-B)$	d) $(A\cap C)-B$

1) i-b, ii-c , iii-a, iv-d 2) i-b, ii-c, iii-d, iv...

3) i-b, ii-c, iii-c, iv-d 4) i-d, ii-c, iii-a, iv...

KEY

01) 1 02) 4 03) 2 04) 3 05) 1
06) 1 07) 1 08) 2

Hints Solutions

1. The set of Prime numbers are the subs... of the Natural number set.
2. A-B= {7,10}. A-B={x:x$\in$A and x$\notin$B}.
3. A=(0,3), B=[1,5]
A-B=(0,1)= {x: 0<x<1, $x\in R$}
B-A=[3,5]= {x: $3\le x\le 5, x\in R$}
$A\Delta B=(A-B)\cup(B-A)=(0,1)\cup[3,5]$

4. $n[P(A)] = 2^n$
5. The set of natural numbers are the sub set of the whole numbers set
6. For disjoint sets no common elements
7. Number of proper sub sets = $2^m - 1$
8. Draw venn diagram

INTEGER & NUMERICAL ANSWER TYPE QUESTIONS

1. **Let $X = \{n \in N : 1 \le n \le 50\}$. If $A = \{n \in X : n$ is a multiple of 2$\}$ and $B = \{n \in X : n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is____**

2. **Let A = {n $\in$ N : n is a 3-digit number} B = {9k + 2 : k $\in$ N} and C = {9k + l : k $\in$ N} for some l $(0 < l < 9)$. If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to______.**

3. **If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \ne 0,$ then the value of expression $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is_**

4. **A function f(x) is given by $f(x) = \frac{5^x}{5^x + 5'}$ then the sum of the series $f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + ... + f\left(\frac{39}{20}\right)$ is equal to :**

Answers

01) 29.00	02) 5
03) 2	04) 4

Hints & Solutions

1. $n(A) = 25$

$n(B) = 7$

$n(A \cap B) = 3$

$n(A \cup B) = 25 + 7 - 3 = 29$

2. B and C will contain three digit numbers of the form 9k + 2 and 9k + ℓ respectively. We need to find sum of all elements in the set $B \cup C$ effectively.

Now, $S(B \cup C) = S(B) + S(C) - S(B \cap C)$

where S(k) denotes sum of elements of set k.

Also, B = {101, 109,............., 992}

$\therefore S(B) = \frac{100}{2}(101 + 992) = 54650$

Case-I : If $\ell = 2$ then $B \cap C = B$

$\therefore S(B \cup C) = S(B)$

which is not possible as given sum is $274 \times 400 = 109600$.

Case-II : If $\ell \ne 2$ then $B \cap C = \phi$

$\therefore S(B \cup C) = S(B) + S(C) = 400 \times 274$

$\Rightarrow 54650 + \sum_{k=11}^{110} 9k + \ell = 109600$

$$\Rightarrow 9\sum_{k=11}^{110} k+\sum_{k=11}^{110} \ell = 54950$$

$$\Rightarrow 9\left(\frac{100}{2}(11+110)\right)+\ell(100)=54950$$

$$\Rightarrow 54450+100\ell=54950$$

$$\Rightarrow \ell=5$$

3. $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}$ .....(1)

replace x by $\frac{1}{x}$

$af\left(\frac{1}{x}\right)+\alpha f(x)=\frac{b}{x}+\beta x$ .....(2)

$(1)+(2)$

$$(a+\alpha)f(x)+(a+\alpha)f\left(\frac{1}{x}\right)$$

$$=x(b+\beta)+(b+\beta)\frac{1}{x}$$

$$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=\frac{b+\beta}{a+\alpha}=\frac{2}{1}=2$$

4. $f(x)=\frac{5^x}{5^x+5}$; $f(2-x)=\frac{5}{5^x+5}$

$$f(x)+f(2-x)=1$$

$$\Rightarrow f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+\ldots+f\left(\frac{39}{20}\right)$$

$$=\left(f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)\right)+\ldots+$$

$$\left(f\left(\frac{19}{20}\right)+f\left(\frac{21}{20}\right)+f\left(\frac{20}{20}\right)\right)$$

$$=19+\frac{1}{2}=\frac{39}{2}$$