

# FINAL JEE-MAIN EXAMINATION – JANUARY, 2024

(Held On Saturday 27<sup>th</sup> January, 2024)

TIME : 3 : 00 PM to 6 : 00 PM

## MATHEMATICS

## TEST PAPER WITH SOLUTION

### SECTION-A

1. Considering only the principal values of inverse trigonometric functions, the number of positive real values of  $x$  satisfying  $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$  is :

- (1) More than 2
- (2) 1
- (3) 2
- (4) 0

**Ans.** (2)

**Sol.**  $\tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4}; x > 0$

$$\Rightarrow \tan^{-1} 2x = \frac{\pi}{4} - \tan^{-1} x$$

Taking tan both sides

$$\Rightarrow 2x = \frac{1-x}{1+x}$$

$$\Rightarrow 2x^2 + 3x - 1 = 0$$

$$x = \frac{-3 \pm \sqrt{9+8}}{8} = \frac{-3 \pm \sqrt{17}}{8}$$

$$\text{Only possible } x = \frac{-3 + \sqrt{17}}{8}$$

2. Consider the function  $f : (0, 2) \rightarrow \mathbb{R}$  defined by

$$f(x) = \frac{x}{2} + \frac{2}{x} \text{ and the function } g(x) \text{ defined by}$$

$$g(x) = \begin{cases} \min\{f(t)\}, & 0 < t \leq x \text{ and } 0 < x \leq 1 \\ \frac{3}{2} + x, & 1 < x < 2 \end{cases} \text{. Then}$$

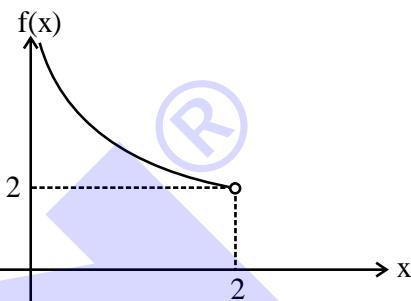
- (1)  $g$  is continuous but not differentiable at  $x = 1$
- (2)  $g$  is not continuous for all  $x \in (0, 2)$
- (3)  $g$  is neither continuous nor differentiable at  $x = 1$
- (4)  $g$  is continuous and differentiable for all  $x \in (0, 2)$

**Ans.** (1)

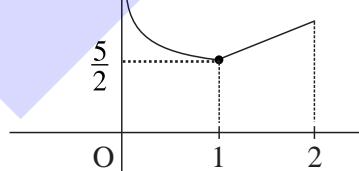
**Sol.**  $f : (0, 2) \rightarrow \mathbb{R}; f(x) = \frac{x}{2} + \frac{2}{x}$

$$f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

$\therefore f(x)$  is decreasing in domain.



$$g(x) = \begin{cases} \frac{x}{2} + \frac{2}{x} & 0 < x \leq 1 \\ \frac{3}{2} + x & 1 < x < 2 \end{cases}$$



3. Let the image of the point  $(1, 0, 7)$  in the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$  be the point  $(\alpha, \beta, \gamma)$ . Then which one of the following points lies on the line passing through  $(\alpha, \beta, \gamma)$  and making angles  $\frac{2\pi}{3}$

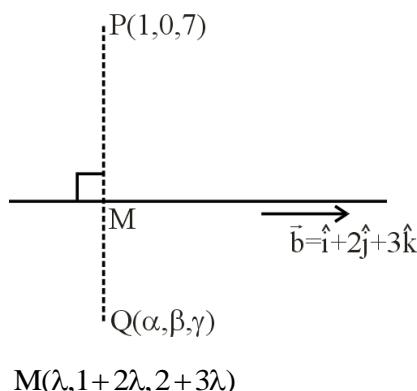
and  $\frac{3\pi}{4}$  with  $y$ -axis and  $z$ -axis respectively and an acute angle with  $x$ -axis ?

- (1)  $(1, -2, 1 + \sqrt{2})$
- (2)  $(1, 2, 1 - \sqrt{2})$
- (3)  $(3, 4, 3 - 2\sqrt{2})$
- (4)  $(3, -4, 3 + 2\sqrt{2})$

**Ans.** (3)



Sol.  $L_1: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$



$M(\lambda, 1+2\lambda, 2+3\lambda)$

$\vec{PM} = (\lambda - 1)\hat{i} + (1 + 2\lambda)\hat{j} + (3\lambda - 5)\hat{k}$

$\vec{PM}$  is perpendicular to line  $L_1$

$\vec{PM} \cdot \vec{b} = 0 \quad (\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

$\therefore M = (1, 3, 5)$

$\vec{Q} = 2\vec{M} - \vec{P}$  [M is midpoint of  $\vec{P}$  &  $\vec{Q}$ ]

$\vec{Q} = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$

$\vec{Q} = \hat{i} + 6\hat{j} + 3\hat{k}$

$\therefore (\alpha, \beta, \gamma) = (1, 6, 3)$

Required line having direction cosine (l, m, n)

$l^2 + m^2 + n^2 = 1$

$\Rightarrow l^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$

$l^2 = \frac{1}{4}$

$\therefore l = \frac{1}{2}$  [Line make acute angle with x-axis]

Equation of line passing through (1, 6, 3) will be

$\vec{r} = (\hat{i} + 6\hat{j} + 3\hat{k}) + \mu \left( \frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k} \right)$

Option (3) satisfying for  $\mu = 4$

4. Let R be the interior region between the lines  $3x - y + 1 = 0$  and  $x + 2y - 5 = 0$  containing the origin. The set of all values of a, for which the points  $(a^2, a + 1)$  lie in R, is :

(1)  $(-3, -1) \cup \left(-\frac{1}{3}, 1\right)$

(2)  $(-3, 0) \cup \left(\frac{1}{3}, 1\right)$

(3)  $(-3, 0) \cup \left(\frac{2}{3}, 1\right)$

(4)  $(-3, -1) \cup \left(\frac{1}{3}, 1\right)$  

Ans. (2)

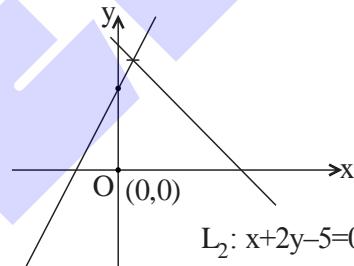
Sol.  $P(a^2, a + 1)$

$L_1: 3x - y + 1 = 0$

Origin and P lies same side w.r.t.  $L_1$

$\Rightarrow L_1(0) \cdot L_1(P) > 0$

$\therefore 3(a^2) - (a + 1) + 1 > 0$



$L_1: 3x - y + 1 = 0$

$\Rightarrow 3a^2 - a > 0$

$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right) \dots \dots \dots (1)$

$\text{Let } L_2: x + 2y - 5 = 0$

Origin and P lies same side w.r.t.  $L_2$

$\Rightarrow L_2(0) \cdot L_2(P) > 0$

$\Rightarrow a^2 + 2(a + 1) - 5 < 0$

$\Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a + 3)(a - 1) < 0$

$\therefore a \in (-3, 1) \dots \dots \dots (2)$

Intersection of (1) and (2)

$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$



5. The 20<sup>th</sup> term from the end of the progression

$$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$$

- (1) -118  
 (2) -110  
 (3) -115  
 (4) -100

**Ans.** (3)

**Sol.**  $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$

This is A.P. with common difference

$$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$$

$$-129\frac{1}{4}, \dots, 19\frac{1}{4}, 20$$

This is also A.P.  $a = -129\frac{1}{4}$  and  $d = \frac{3}{4}$

Required term =

$$-129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right)$$

$$= -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$$

6. Let  $f : R - \left\{-\frac{1}{2}\right\} \rightarrow R$  and  $g : R - \left\{-\frac{5}{2}\right\} \rightarrow R$  be

defined as  $f(x) = \frac{2x+3}{2x+1}$  and  $g(x) = \frac{|x|+1}{2x+5}$ . Then

the domain of the function fog is :

- (1)  $R - \left\{-\frac{5}{2}\right\}$   
 (2)  $R$   
 (3)  $R - \left\{-\frac{7}{4}\right\}$   
 (4)  $R - \left\{-\frac{5}{2}, -\frac{7}{4}\right\}$

**Ans.** (1)

**Sol.**  $f(x) = \frac{2x+3}{2x+1}; x \neq -\frac{1}{2}$

$$g(x) = \frac{|x|+1}{2x+5}, x \neq -\frac{5}{2}$$

Domain of  $f(g(x))$

$$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$$

$$x \neq -\frac{5}{2} \text{ and } \frac{|x|+1}{2x+5} \neq -\frac{1}{2}$$

$$x \in R - \left\{-\frac{5}{2}\right\} \text{ and } x \in R$$

$$\therefore \text{Domain will be } R - \left\{-\frac{5}{2}\right\}$$

7. For  $0 < a < 1$ , the value of the integral

$$\int_0^{\pi} \frac{dx}{1-2a \cos x + a^2}$$

$$(1) \frac{\pi^2}{\pi + a^2}$$

$$(2) \frac{\pi^2}{\pi - a^2}$$

$$(3) \frac{\pi}{1-a^2}$$

$$(4) \frac{\pi}{1+a^2}$$

**Ans.** (3)

**Sol.**  $I = \int_0^{\pi} \frac{dx}{1-2a \cos x + a^2}; 0 < a < 1$

$$I = \int_0^{\pi} \frac{dx}{1+2a \cos x + a^2}$$

$$2I = 2 \int_0^{\pi/2} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2 \cos^2 x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2(1+a^2) \cdot \sec^2 x}{(1+a^2)^2 \cdot \sec^2 x - 4a^2} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2 \cdot (1+a^2) \cdot \sec^2 x}{(1+a^2)^2 \cdot \tan^2 x + (1-a^2)^2} dx$$

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$$\Rightarrow I = \int_0^{\pi/2} \frac{2 \sec^2 x}{1+a^2} dx$$

$$= \frac{2}{1+a^2} \left[ \tan x \right]_0^{\pi/2}$$

$$\Rightarrow I = \frac{2}{(1-a^2)} \left[ \frac{\pi}{2} - 0 \right]$$

$$I = \frac{\pi}{1-a^2}$$

8. Let  $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$  and  $f''(x) > 0$  for all  $x \in (0, 3)$ . If  $g$  is decreasing in  $(0, \alpha)$  and increasing in  $(\alpha, 3)$ , then  $8\alpha$  is

- (1) 24  
(2) 0  
(3) 18  
(4) 20

**Ans. (3)**

**Sol.**  $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$  and  $f''(x) > 0 \forall x \in (0, 3)$

$\Rightarrow f'(x)$  is increasing function

$$g'(x) = 3 \times \frac{1}{3} \cdot f'\left(\frac{x}{3}\right) - f'(3-x)$$

$$= f'\left(\frac{x}{3}\right) - f'(3-x)$$

If  $g$  is decreasing in  $(0, \alpha)$

$$g'(x) < 0$$

$$f'\left(\frac{x}{3}\right) - f'(3-x) < 0$$

$$f'\left(\frac{x}{3}\right) < f'(3-x)$$

$$\Rightarrow \frac{x}{3} < 3-x$$

$$\Rightarrow x < \frac{9}{4}$$

$$\text{Therefore } \alpha = \frac{9}{4}$$

$$\text{Then } 8\alpha = 8 \times \frac{9}{4} = 18$$

9. If  $\lim_{x \rightarrow 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1-x)}{3 \tan^2 x} = \frac{1}{3}$ , then

$2\alpha - \beta$  is equal to :

- (1) 2  
(2) 7  
(3) 5  
(4) 1

**Ans. (3)**

**Sol.**  $\lim_{x \rightarrow 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1-x)}{3 \tan^2 x} = \frac{1}{3}$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{3 + \alpha \left[ x - \frac{x^3}{3!} + \dots \right] + \beta \left[ 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots \right] + \left( -x - \frac{x^2}{2} - \frac{x^3}{3} \dots \right)}{3 \tan^2 x} = \frac{1}{3}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{(3+\beta) + (\alpha-1)x + \left( -\frac{1}{2} - \frac{\beta}{2} \right)x^2 + \dots}{3x^2} \times \frac{x^2}{\tan^2 x} = \frac{1}{3}$$

$$\Rightarrow \beta + 3 = 0, \alpha - 1 = 0 \text{ and } -\frac{1}{2} - \frac{\beta}{2} = \frac{1}{3}$$

$$\Rightarrow \beta = -3, \alpha = 1$$

$$\Rightarrow 2\alpha - \beta = 2 + 3 = 5$$

10. If  $\alpha, \beta$  are the roots of the equation,  $x^2 - x - 1 = 0$  and  $S_n = 2023\alpha^n + 2024\beta^n$ , then

- (1)  $2S_{12} = S_{11} + S_{10}$   
(2)  $S_{12} = S_{11} + S_{10}$   
(3)  $2S_{11} = S_{12} + S_{10}$   
(4)  $S_{11} = S_{10} + S_{12}$

**Ans. (2)**

**Sol.**  $x^2 - x - 1 = 0$

$$S_n = 2023\alpha^n + 2024\beta^n$$

$$S_{n-1} + S_{n-2} = 2023\alpha^{n-1} + 2024\beta^{n-1} + 2023\alpha^{n-2} + 2024\beta^{n-2}$$

$$= 2023\alpha^{n-2}[1 + \alpha] + 2024\beta^{n-2}[1 + \beta]$$

$$= 2023\alpha^{n-2}[\alpha^2] + 2024\beta^{n-2}[\beta^2]$$

$$= 2023\alpha^n + 2024\beta^n$$

$$S_{n-1} + S_{n-2} = S_n$$

Put  $n = 12$

$$S_{11} + S_{10} = S_{12}$$

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11. Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m, n) from the point Q(-2, -3) is

- (1) 10
- (2) 6
- (3) 4
- (4) 8

**Ans.** (1)

**Sol.**  $2^m - 2^n = 56$

$$2^n(2^{m-n} - 1) = 2^3 \times 7$$

$$2^n = 2^3 \text{ and } 2^{m-n} - 1 = 7$$

$$\Rightarrow n = 3 \text{ and } 2^{m-n} = 8$$

$$\Rightarrow n = 3 \text{ and } m - n = 3$$

$$\Rightarrow n = 3 \text{ and } m = 6$$

$$P(6,3) \text{ and } Q(-2, -3)$$

$$PQ = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

Hence option (1) is correct

12. The values of  $\alpha$ , for which

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0, \text{ lie in the interval}$$

- (1) (-2, 1)
- (2) (-3, 0)
- (3)  $\left(-\frac{3}{2}, \frac{3}{2}\right)$
- (4) (0, 3)

**Ans.** (2)

**Sol.** 
$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2\alpha + 3) \left\{ \frac{7\alpha}{6} \right\} - (3\alpha + 1) \left\{ \frac{-7}{6} \right\} = 0$$

$$\Rightarrow (2\alpha + 3) \cdot \frac{7\alpha}{6} + (3\alpha + 1) \cdot \frac{7}{6} = 0$$

$$\Rightarrow 2\alpha^2 + 3\alpha + 3\alpha + 1 = 0$$

$$\Rightarrow 2\alpha^2 + 6\alpha + 1 = 0$$

$$\Rightarrow \alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$$

Hence option (2) is correct.

13. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

(1)  $\frac{5}{256}$  (2)  $\frac{5}{715}$

(3)  $\frac{3}{715}$  (4)  $\frac{3}{256}$

**Ans.** (3)

**Sol.** 
$$\frac{\frac{6}{15}C_4}{C_4} \times \frac{\frac{9}{11}C_4}{C_4} = \frac{3}{715}$$

Hence option (3) is correct.

14. The integral  $\int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1) \tan^{-1} \left( x^3 + \frac{1}{x^3} \right)}$  is equal to :

(1)  $\log_e \left( \left| \tan^{-1} \left( x^3 + \frac{1}{x^3} \right) \right| \right)^{1/3} + C$

(2)  $\log_e \left( \left| \tan^{-1} \left( x^3 + \frac{1}{x^3} \right) \right| \right)^{1/2} + C$

(3)  $\log_e \left( \left| \tan^{-1} \left( x^3 + \frac{1}{x^3} \right) \right| \right) + C$

(4)  $\log_e \left( \left| \tan^{-1} \left( x^3 + \frac{1}{x^3} \right) \right| \right)^3 + C$

**Ans.** (1)



**Sol.**  $I = \int \frac{x^8 - x^2}{(x^{12} + 3x^6 + 1) \tan^{-1}\left(x^3 + \frac{1}{x^3}\right)} dx$

Let  $\tan^{-1}\left(x^3 + \frac{1}{x^3}\right) = t$

$$\Rightarrow \frac{1}{1 + \left(x^3 + \frac{1}{x^3}\right)^2} \cdot \left(3x^2 - \frac{3}{x^4}\right) dx = dt$$

$$\Rightarrow \frac{x^6}{x^{12} + 3x^6 + 1} \cdot \frac{3x^6 - 3}{x^4} dx = dt$$

$$I = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln |t| + C$$

$$I = \frac{1}{3} \ln \left| \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) \right| + C$$

$$I = \ln \left| \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) \right|^{1/3} + C$$

Hence option (1) is correct

- 15.** If  $2\tan^2\theta - 5\sec\theta = 1$  has exactly 7 solutions in the interval  $\left[0, \frac{n\pi}{2}\right]$ , for the least value of  $n \in \mathbb{N}$

then  $\sum_{k=1}^n \frac{k}{2^k}$  is equal to :

(1)  $\frac{1}{2^{15}}(2^{14} - 14)$

(2)  $\frac{1}{2^{14}}(2^{15} - 15)$

(3)  $1 - \frac{15}{2^{13}}$

(4)  $\frac{1}{2^{13}}(2^{14} - 15)$

**Ans. (4)**

**Sol.**  $2\tan^2\theta - 5\sec\theta - 1 = 0$

$$\Rightarrow 2\sec^2\theta - 5\sec\theta - 3 = 0$$

$$\Rightarrow (2\sec\theta + 1)(\sec\theta - 3) = 0$$

$$\Rightarrow \sec\theta = -\frac{1}{2}, 3$$

$$\Rightarrow \cos\theta = -2, \frac{1}{3}$$

$$\Rightarrow \cos\theta = \frac{1}{3}$$

For 7 solutions  $n = 13$

So,  $\sum_{k=1}^{13} \frac{k}{2^k} = S$  (say)

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{13}{2^{13}}$$

$$\frac{1}{2}S = \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{12}{2^{13}} + \frac{13}{2^{14}}$$

$$\Rightarrow \frac{S}{2} = \frac{1}{2} \cdot \frac{1 - \frac{1}{2^{13}}}{1 - \frac{1}{2}} - \frac{13}{2^{14}} \Rightarrow S = 2 \left( \frac{2^{13} - 1}{2^{13}} \right) - \frac{13}{2^{13}}$$

- 16.** The position vectors of the vertices A, B and C of a triangle are  $2\hat{i} - 3\hat{j} + 3\hat{k}$ ,  $2\hat{i} + 2\hat{j} + 3\hat{k}$  and  $-\hat{i} + \hat{j} + 3\hat{k}$  respectively. Let  $l$  denotes the length of the angle bisector AD of  $\angle BAC$  where D is on the line segment BC, then  $2l^2$  equals :

(1) 49

(2) 42

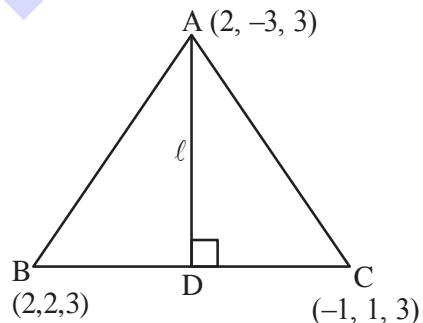
(3) 50

(4) 45

**Ans. (4)**

**Sol.**  $AB = 5$

$AC = 5$



$\therefore D$  is midpoint of BC

$$D\left(\frac{1}{2}, \frac{3}{2}, 3\right)$$

$$\therefore l = \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-3 - \frac{3}{2}\right)^2 + (3 - 3)^2}$$

$$l = \sqrt{\frac{45}{2}}$$

$$\therefore 2l^2 = 45$$



17. If  $y = y(x)$  is the solution curve of the differential equation  $(x^2 - 4)dy - (y^2 - 3y)dx = 0$ ,  
 $x > 2$ ,  $y(4) = \frac{3}{2}$  and the slope of the curve is never zero, then the value of  $y(10)$  equals :

(1)  $\frac{3}{1+(8)^{1/4}}$

(2)  $\frac{3}{1+2\sqrt{2}}$

(3)  $\frac{3}{1-2\sqrt{2}}$

(4)  $\frac{3}{1-(8)^{1/4}}$

**Ans. (1)**

**Sol.**  $(x^2 - 4)dy - (y^2 - 3y)dx = 0$

$$\Rightarrow \int \frac{dy}{y^2 - 3y} = \int \frac{dx}{x^2 - 4}$$

$$\Rightarrow \frac{1}{3} \int \frac{y - (y - 3)}{y(y - 3)} dy = \int \frac{dx}{x^2 - 4}$$

$$\Rightarrow \frac{1}{3} (\ln|y - 3| - \ln|y|) = \frac{1}{4} \ln|x - 2| + C$$

$$\Rightarrow \frac{1}{3} \ln \left| \frac{y - 3}{y} \right| = \frac{1}{4} \ln \left| \frac{x - 2}{x + 2} \right| + C$$

At  $x = 4$ ,  $y = \frac{3}{2}$

$$\therefore C = \frac{1}{4} \ln 3$$

$$\therefore \frac{1}{3} \ln \left| \frac{y - 3}{y} \right| = \frac{1}{4} \ln \left| \frac{x - 2}{x + 2} \right| + \frac{1}{4} \ln(3)$$

At  $x = 10$

$$\frac{1}{3} \ln \left| \frac{y - 3}{y} \right| = \frac{1}{4} \ln \left| \frac{2}{3} \right| + \frac{1}{4} \ln(3)$$

$$\ln \left| \frac{y - 3}{y} \right| = \ln 2^{3/4}, \forall x > 2, \frac{dy}{dx} < 0$$

as  $y(4) = \frac{3}{2} \Rightarrow y \in (0, 3)$

$$-y + 3 = 8^{1/4} \cdot y$$

$$y = \frac{3}{1+8^{1/4}}$$

18. Let  $e_1$  be the eccentricity of the hyperbola  $\frac{x^2}{16} - \frac{y^2}{9} = 1$  and  $e_2$  be the eccentricity of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$ , which passes through the foci of the hyperbola. If  $e_1 e_2 = 1$ , then the length of the chord of the ellipse parallel to the x-axis and passing through  $(0, 2)$  is :

(1)  $4\sqrt{5}$

(2)  $\frac{8\sqrt{5}}{3}$

(3)  $\frac{10\sqrt{5}}{3}$

(4)  $3\sqrt{5}$

**Ans. (3)**

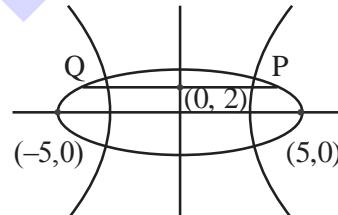
**Sol.**  $H: \frac{x^2}{16} - \frac{y^2}{9} = 1 \quad e_1 = \frac{5}{4}$

$$\therefore e_1 e_2 = 1 \Rightarrow e_2 = \frac{4}{5}$$

Also, ellipse is passing through  $(\pm 5, 0)$

$$\therefore a = 5 \text{ and } b = 3$$

$$E: \frac{x^2}{25} + \frac{y^2}{9} = 1$$



End point of chord are  $\left( \pm \frac{5\sqrt{5}}{3}, 2 \right)$

$$\therefore L_{PQ} = \frac{10\sqrt{5}}{3}$$

19. Let  $\alpha = \frac{(4!)!}{(4!)^{3!}}$  and  $\beta = \frac{(5!)!}{(5!)^{4!}}$ . Then :

(1)  $\alpha \in \mathbb{N}$  and  $\beta \notin \mathbb{N}$

(2)  $\alpha \notin \mathbb{N}$  and  $\beta \in \mathbb{N}$

(3)  $\alpha \in \mathbb{N}$  and  $\beta \in \mathbb{N}$

(4)  $\alpha \notin \mathbb{N}$  and  $\beta \notin \mathbb{N}$

**Ans. (3)**

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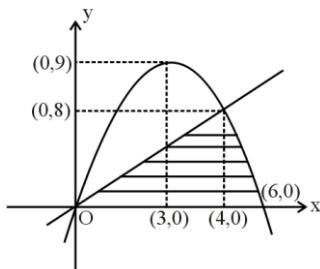


22. If the area of the region

$\{(x,y) : 0 \leq y \leq \min\{2x, 6x - x^2\}\}$  is A, then 12 A is equal to.....

**Ans. (304)**

**Sol.** We have



$$A = \frac{1}{2} \times 4 \times 8 + \int_{4}^{6} (6x - x^2) dx$$

$$A = \frac{76}{3}$$

$$12A = 304$$

23. Let A be a  $2 \times 2$  real matrix and I be the identity matrix of order 2. If the roots of the equation  $|A - xI| = 0$  be  $-1$  and  $3$ , then the sum of the diagonal elements of the matrix  $A^2$  is.....

**Ans. (10)**

**Sol.**  $|A - xI| = 0$

Roots are  $-1$  and  $3$

Sum of roots  $= \text{tr}(A) = 2$

Product of roots  $= |A| = -3$

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{We have } a + d = 2$$

$$ad - bc = -3$$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

$$\text{We need } a^2 + bc + bd + d^2$$

$$= a^2 + 2bc + d^2$$

$$= (a + d)^2 - 2ad + 2bc$$

$$= 4 - 2(ad - bc)$$

$$= 4 - 2(-3)$$

$$= 4 + 6$$

$$= 10$$

24. If the sum of squares of all real values of  $\alpha$ , for which the lines  $2x - y + 3 = 0$ ,  $6x + 3y + 1 = 0$  and  $\alpha x + 2y - 2 = 0$  do not form a triangle is p, then the greatest integer less than or equal to p is .....

**Ans. (32)**

$$2x - y + 3 = 0$$

$$6x + 3y + 1 = 0$$

$$\alpha x + 2y - 2 = 0$$

Will not form a  $\Delta$  if  $\alpha x + 2y - 2 = 0$  is concurrent with  $2x - y + 3 = 0$  and  $6x + 3y + 1 = 0$  or parallel to either of them so

Case-1: Concurrent lines

$$\left| \begin{array}{ccc} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{array} \right| = 0 \Rightarrow \alpha = \frac{4}{5}$$

Case-2 : Parallel lines

$$-\frac{\alpha}{2} = \frac{-6}{3} \text{ or } -\frac{\alpha}{2} = 2$$

$$\Rightarrow \alpha = 4 \text{ or } \alpha = -4$$

$$P = 16 + 16 + \frac{16}{25}$$

$$[P] = \left[ 32 + \frac{16}{25} \right] = 32$$

25. The coefficient of  $x^{2012}$  in the expansion of  $(1-x)^{2008}(1+x+x^2)^{2007}$  is equal to

**Ans. (0)**

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**Sol.**  $(1-x)(1-x)^{2007}(1+x+x^2)^{2007}$   
 $(1-x)(1-x^3)^{2007}$   
 $(1-x)(^{2007}C_0 - ^{2007}C_1(x^3) + \dots)$

General term

$$(1-x)((-1)^r {}^{2007}C_r x^{3r})$$

$$(-1)^r {}^{2007}C_r x^{3r} - (-1)^{r+1} {}^{2007}C_r x^{3r+1}$$

$$3r = 2012$$

$$r \neq \frac{2012}{3}$$

$$3r + 1 = 2012$$

$$3r = 2011$$

$$r \neq \frac{2011}{3}$$

Hence there is no term containing  $x^{2012}$ .

So coefficient of  $x^{2012} = 0$

- 26.** If the solution curve, of the differential equation

$$\frac{dy}{dx} = \frac{x+y-2}{x-y}$$
 passing through the point  $(2, 1)$  is

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta} \log_e\left(\alpha + \left(\frac{y-1}{x-1}\right)^2\right) = \log_e |x-1|,$$

then  $5\beta + \alpha$  is equal to

**Ans. (11)**

**Sol.**  $\frac{dy}{dx} = \frac{x+y-2}{x-y}$

$$x = X + h, y = Y + k$$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

$$\begin{cases} h+k-2=0 \\ h-k=0 \end{cases} \Rightarrow h=k=1$$

$$Y = vX$$

$$v + \frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow X - \frac{dv}{dX} = \frac{1+v^2}{1-v}$$

$$\frac{1-v}{1+v^2} dv = \frac{dX}{X}$$

$$\tan^{-1} v - \frac{1}{2} \ln(1+v^2) = \ln |X| + C$$

As curve is passing through  $(2, 1)$

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2} \ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right) = \ln |x-1|$$

$$\therefore \alpha = 1 \text{ and } \beta = 2$$

$$\Rightarrow 5\beta + \alpha = 11$$

- 27.** Let  $f(x) = \int_0^x g(t) \log_e\left(\frac{1-t}{1+t}\right) dt$ , where  $g$  is a continuous odd function.

If  $\int_{-\pi/2}^{\pi/2} \left(f(x) + \frac{x^2 \cos x}{1+e^x}\right) dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$ , then  $\alpha$  is equal to.....

**Ans. (2)**

**Sol.**  $f(x) = \int_0^x g(t) \ln\left(\frac{1-t}{1+t}\right) dt$

$$f(-x) = \int_0^{-x} g(t) \ln\left(\frac{1-t}{1+t}\right) dt$$

$$f(-x) = - \int_0^x g(-y) \ln\left(\frac{1+y}{1-y}\right) dy$$

$$= - \int_0^x g(y) \ln\left(\frac{1-y}{1+y}\right) dy \quad (g \text{ is odd})$$

$$f(-x) = -f(x) \Rightarrow f \text{ is also odd}$$

Now,

$$I = \int_{-\pi/2}^{\pi/2} \left(f(x) + \frac{x^2 \cos x}{1+e^x}\right) dx \quad \dots\dots(1)$$

$$I = \int_{-\pi/2}^{\pi/2} \left(f(-x) + \frac{x^2 e^x \cos x}{1+e^x}\right) dx \quad \dots\dots(2)$$

$$2I = \int_{-\pi/2}^{\pi/2} x^2 \cos x \, dx = 2 \int_0^{\pi/2} x^2 \cos x \, dx$$

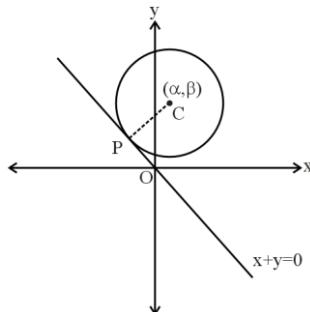


$$\begin{aligned} I &= \left( x^2 \sin x \right)_0^{\pi/2} - \int_0^{\pi/2} 2x \sin x dx \\ &= \frac{\pi^2}{4} - 2 \left( -x \cos x + \int \cos x dx \right)_0^{\pi/2} \\ &= \frac{\pi^2}{4} - 2(0+1) = \frac{\pi^2}{4} - 2 \Rightarrow \left( \frac{\pi}{2} \right)^2 - 2 \\ \therefore \alpha &= 2 \end{aligned}$$

- 28.** Consider a circle  $(x - \alpha)^2 + (y - \beta)^2 = 50$ , where  $\alpha, \beta > 0$ . If the circle touches the line  $y + x = 0$  at the point P, whose distance from the origin is  $4\sqrt{2}$ , then  $(\alpha + \beta)^2$  is equal to.....

**Ans. (100)**

Sol.



$$S: (x - \alpha)^2 + (y - \beta)^2 = 50$$

$$CP = r$$

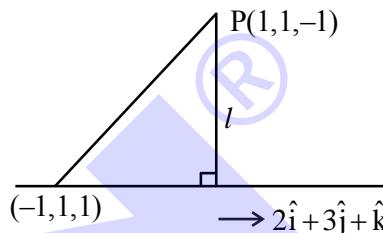
$$\left| \frac{\alpha + \beta}{\sqrt{2}} \right| = 5\sqrt{2}$$

$$\Rightarrow (\alpha + \beta)^2 = 100$$

- 29.** The lines  $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$  and  $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$  intersect at the point P. If the distance of P from the line  $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$  is  $l$ , then  $14l^2$  is equal to.....

**Ans. (108)**

$$\begin{aligned}
 \text{Sol. } & \frac{x-2}{1} = \frac{y}{-1} = \frac{z-7}{8} = \lambda \\
 & \frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1} = k \\
 \Rightarrow & \lambda + 2 = 4k - 3 \\
 \Rightarrow & -\lambda = 3k - 2 \\
 \Rightarrow & k = 1, \lambda = -1 \\
 8\lambda + 7 &= k - 2 \\
 \therefore P &= (1, 1, -1)
 \end{aligned}$$



Projection of  $2\hat{i} - 2\hat{k}$  on  $2\hat{i} + 3\hat{j} + \hat{k}$  is

$$= \frac{4-2}{\sqrt{4+9+1}} = \frac{2}{\sqrt{14}}$$

$$\therefore l^2 = 8 - \frac{4}{14} = \frac{108}{14}$$

$$\Rightarrow 14l^2 = 108$$

- 30.** Let the complex numbers  $\alpha$  and  $\frac{1}{\bar{\alpha}}$  lie on the circles  $|z - z_0|^2 = 4$  and  $|z - z_0|^2 = 16$  respectively, where  $z_0 = 1 + i$ . Then, the value of  $100|\alpha|^2$  is .....

Ans. (20)

$$\text{Sol. } |z - z_0|^2 = 4$$

$$\Rightarrow (\alpha - z_0)(\bar{\alpha} - \bar{z}_0) = 4$$

$$\Rightarrow \alpha\bar{\alpha} - \alpha\bar{z}_0 - z_0\bar{\alpha} + |z_0|^2 = 4$$

$$\Rightarrow |\alpha|^2 - \alpha \bar{z} - z \bar{\alpha} = 2$$

$$|z - z_+|^2 = 16$$



$$\Rightarrow \left( \frac{1}{\bar{\alpha}} - z_0 \right) \left( \frac{1}{\alpha} - \bar{z}_0 \right) = 16$$

$$\Rightarrow (1 - \bar{\alpha}z_0)(1 - \alpha\bar{z}_0) = 16|\alpha|^2$$

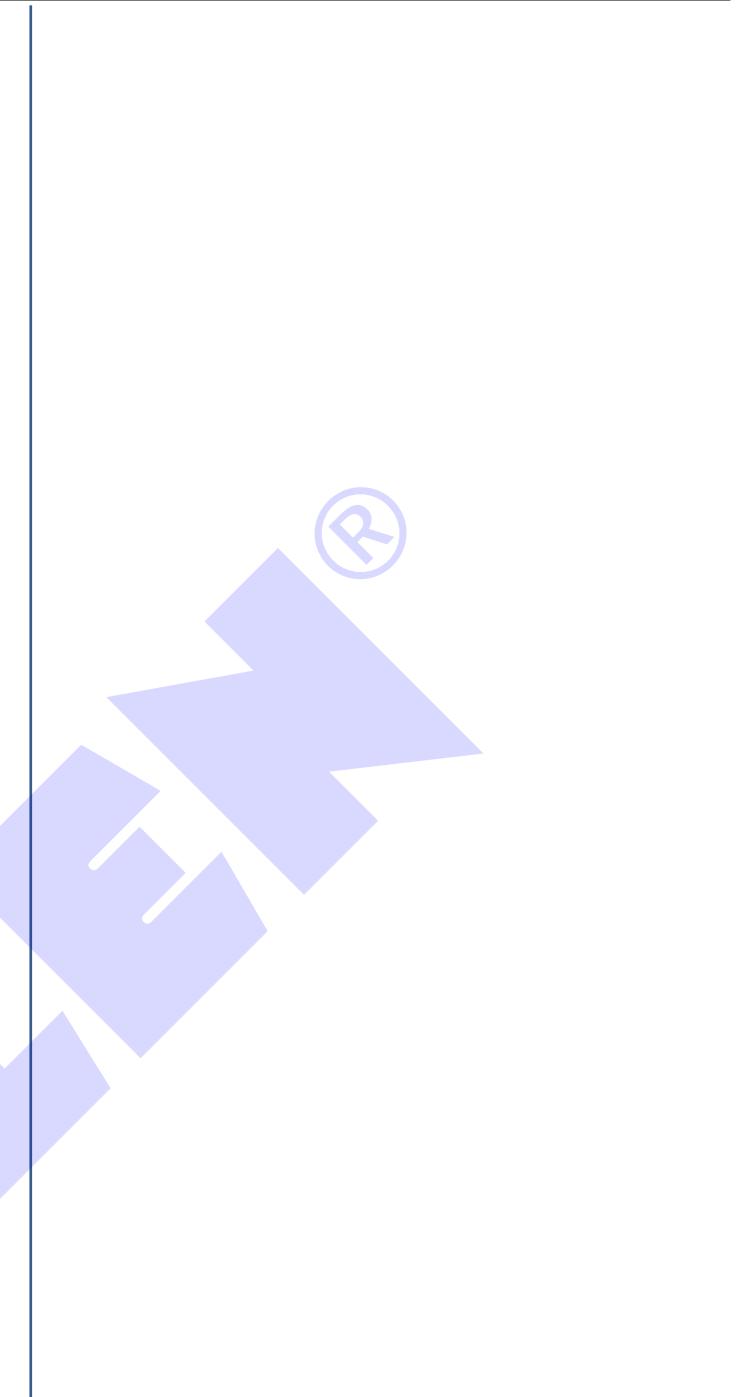
$$\Rightarrow 1 - \bar{\alpha}z_0 - \alpha\bar{z}_0 + |\alpha|^2|z_0|^2 = 16|\alpha|^2$$

$$\Rightarrow 1 - \bar{\alpha}z_0 - \alpha\bar{z}_0 = 14|\alpha|^2 \dots\dots\dots(2)$$

From (1) and (2)

$$\Rightarrow 5|\alpha|^2 = 1$$

$$\Rightarrow 100|\alpha|^2 = 20$$



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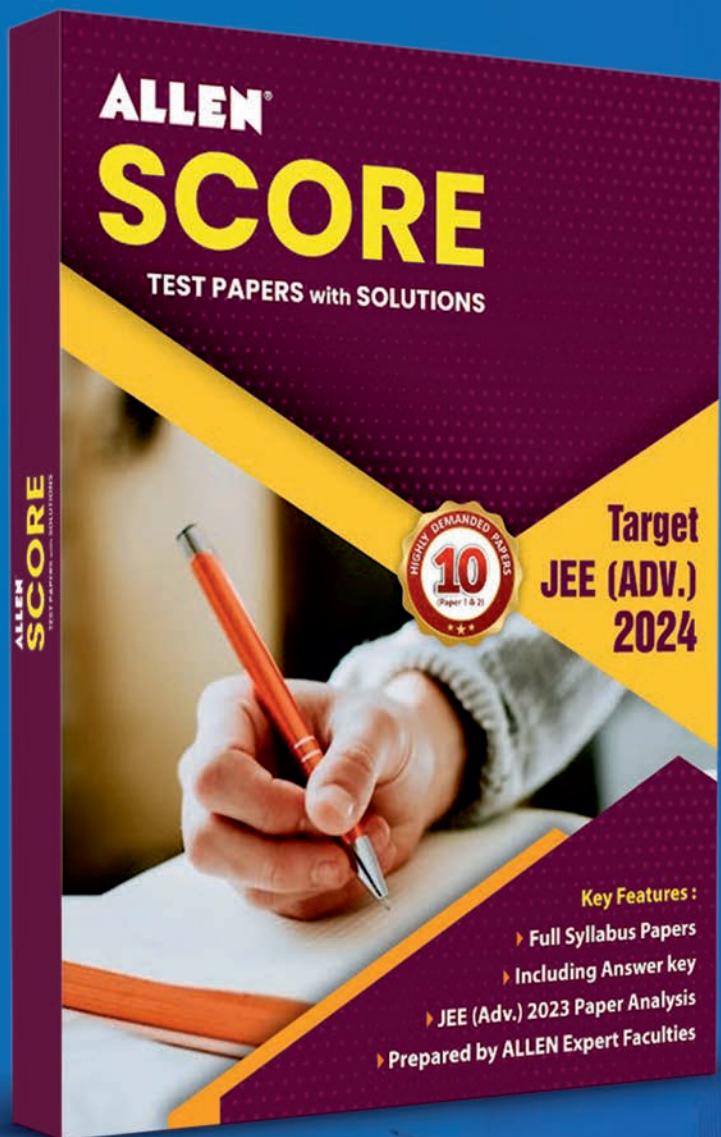


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