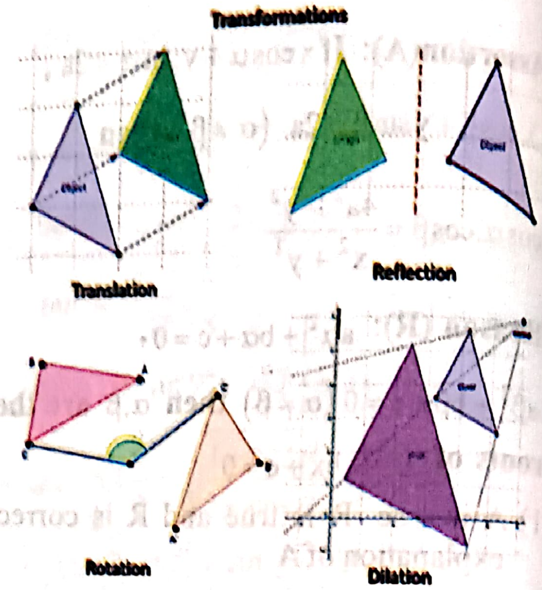
6.5 Transformations

Internals of the Topic

- **Sum To Product**
  $\sin C \pm \sin D$; $\cos C \pm \cos D$
- **Standard Results**
- **Product To Sum Rule**
- **Sum To Product Fule**
- **Elimination of 'θ'**
- **Componendo And Dividendo Property**



CRT CONCEPT

- The sum or difference of the trigonometric ratios are transforms into their products is said to be transformation between trigonometric ratios and vice versa.
- If A, B are any two angles and $A+B=C$, $A-B=D$ then the transformation of sum or difference of the trigonometric ratios are transforms into their products

i) Sin (A + B) + Sin (A - B)= 2 SinA CosB

ii) Sin (A + B) - Sin (A - B) = 2 CosA Sin B

iii) Cos (A + B) + Cos (A - B) = 2 CosA CosB

iv) Cos (A - B) - Cos (A + B) = 2 Sin A Sin B

- i) SinC+ Sin D = 2 Sin $\left(\frac{C+D}{2}\right)$ Cos $\left(\frac{C-D}{2}\right)$

ii) SinC - Sin D = 2 Cos $\left(\frac{C+D}{2}\right)$ $\left(\frac{C-D}{2}\right)$

iii) CosC+CosD=2 Cos $\left(\frac{C+D}{2}\right)$ Cos $\left(\frac{C-D}{2}\right)$

iv) CosC-CosD = -2Sin $\left(\frac{C+D}{2}\right)$ Sin $\left(\frac{C-D}{2}\right)$

$= 2\text{Sin}\left(\frac{C+D}{2}\right)\text{Sin}\left(\frac{D-C}{2}\right)$

- If $\cos x+\cos y=a, \sin x+\sin y=b$ then

i) $\tan\frac{x+y}{2}=\frac{b}{a}$

ii) $\sin(x+y)=\frac{2ab}{a^2+b^2}$

iii) $\cos(x+y)=\frac{a^2-b^2}{a^2+b^2}$

iv) $\tan(x+y)=\frac{2ab}{a^2-b^2}$

- If $\cos x-\cos y=a, \sin x-\sin y=b$ then

i) $\tan\frac{x+y}{2}=-\frac{a}{b}$

ii) $\sin(x+y)=\frac{-2ab}{a^2+b^2}$

iii) $\cos(x+y)=\frac{b^2-a^2}{b^2+a^2}$

iv) $\tan(x+y)=-\frac{2ab}{a^2-b^2}$

- If $\cos x-\cos y=a, \sin x+\sin y=b$ then

i) $\tan\frac{x-y}{2}=-\frac{a}{b}$

ii) $\sin(x-y)=-\frac{2ab}{a^2+b^2}$

iii) $\cos(x-y)=\frac{b^2-a^2}{b^2+a^2}$

iv) $\tan(x-y)=-\frac{2ab}{a^2-b^2}$

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ii) $\sin(x-y)=\frac{2ab}{a^2+b^2}$

iii) $\cos(x-y)=\frac{a^2-b^2}{a^2+b^2}$

iv) $\tan(x-y)=\frac{2ab}{a^2-b^2}$

- $\sin 9^\circ=\frac{1}{4}\left[\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}\right]$

$=\sqrt{\frac{4-\sqrt{10+2\sqrt{5}}}{8}}$

$=\frac{1}{4}\sqrt{8-2\sqrt{10+2\sqrt{5}}}=\cos 81^0$

- $\cos 9^\circ=\frac{1}{4}\left[\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}\right]$

$=\sqrt{\frac{4+\sqrt{10+2\sqrt{5}}}{8}}$

$=\frac{1}{4}\sqrt{8+2\sqrt{10+2\sqrt{5}}}=\sin 81^0$ 5(15, 65 3, 13

- $\cos x.\cos 2x.\cos 4x.........\cos(2^n x)$

$=\frac{1}{2^{n+1}}\frac{\sin(2^{n+1}x)}{\sin x}$

Proof: Let $C=\cos x.\cos 2x.\cos 4x.........\cos(2^n x)$

$S=\sin x.\sin 2x.\sin 4x...\sin(2^n x)$

$\therefore C\times S=\left[\cos x.\cos 2x.\cos 4x.........\cos(2^n x)\right]$

$\left[\sin x.\sin 2x.\sin 4x...\sin(2^n x)\right]$

$=\frac{1}{2^{n+1}}[2\sin x\cos x][2\sin 2x.\cos 2x]...$

$\left[2\sin(2^n x)\cos(2^n x)\right]$

$=\frac{1}{2^{n+1}}\sin 2x.\sin 4x...\sin(2^{n+1}x)$

$=\frac{1}{2^{n+1}}.\frac{1}{\sin x}.\left[\sin x.\sin 2x...\sin(2^n x)\right].\sin(2^{n+1}x)$

$\Rightarrow C\times S=\frac{1}{2^{n+1}}\left[\frac{\sin(2^{n+1}x)}{\sin x}\right].S$

$\therefore C=\frac{1}{2^{n+1}}\left[\frac{\sin(2^{n+1}x)}{\sin x}\right]$

- i) $\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+....\sin[\alpha+(n-1)\beta]$

$=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}$

ii) $\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)$ $+....\cos[\alpha+(n-1)\beta]$

$=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}$

- $\forall x\in R$, $\tan x+\frac{1}{2}\tan\frac{x}{2}+\frac{1}{2^2}\tan\frac{x}{2^2}+....$ $.....+\frac{1}{2^{n-1}}\tan\left(\frac{x}{2^{n-1}}\right)$

$= \frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right) - 2\cot 2x$

Proof: $\cot x - \tan x = 2\cot 2x$

$\frac{1}{2}\cot\frac{x}{2} - \frac{1}{2}\tan\frac{x}{2} = \frac{1}{2^2}\cot x$

Similarly, $\frac{1}{2^2}\cot\frac{x}{2^2} - \frac{1}{2^2}\tan\frac{x}{2^2} = \frac{1}{2}\cot\frac{x}{2}$

$\frac{1}{2^3}\cot\frac{x}{2^3} - \frac{1}{2^3}\tan\frac{x}{2^3} = \frac{1}{2^2}\cot\frac{x}{2^2}$

$\vdots \qquad \vdots$

$\frac{1}{2^{n-1}}\cot\frac{x}{2^{n-1}} - \frac{1}{2^{n-1}}\tan\frac{x}{2^{n-1}} = \frac{1}{2^{n-2}}\cot\frac{x}{2^{n-2}}$

adding all the above equations

$-\left[\tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \ldots + \frac{1}{2^{n-1}}\tan\frac{x}{2^{n-1}}\right] =$

$2\cot 2x - \frac{1}{2^{n-1}}\cot\frac{x}{2^{n-1}}$

$\therefore \tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \ldots + \frac{1}{2^{n-1}}\tan\frac{x}{2^{n-1}} =$

$\frac{1}{2^{n-1}}\cot\frac{x}{2^{n-1}} - 2\cot 2x$

- α, β are the solutions of, $a\cos\theta + b\sin\theta = c$, then

i) $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a}$

ii) $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$

iii) $\cos(\alpha+\beta) = \frac{a^2-b^2}{a^2+b^2}$

iv) $\tan(\alpha+\beta) = \frac{2ab}{a^2-b^2}$

S.E-1: In ΔABC, $\tan A + \tan B + \tan C \geq 3\sqrt{3}$, where A, B, C are acute angles.

Sol.: In ΔABC,

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$,

$A.M \geq G.M$

$\Rightarrow \frac{\tan A + \tan B + \tan C}{3} \geq \sqrt[3]{\tan A \tan B \tan C}$

$\Rightarrow \tan A \tan B \tan C \geq 3\sqrt[3]{\tan A \tan B \tan C}$

$\Rightarrow \tan^2 A \tan^2 B \tan^2 C \geq 27$

$\Rightarrow \tan A \tan B \tan C \geq 3\sqrt{3}$

$\Rightarrow \tan A + \tan B + \tan C \geq 3\sqrt{3}$

S.E-2: In ΔABC, prove that

$\cos A + \cos B + \cos C \leq \frac{3}{2}$.

Sol: Let $\cos A + \cos B + \cos C = x$

$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + 1 - 2\sin^2\frac{C}{2} = x$

$\Rightarrow 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) + 1 - 2\sin^2\frac{C}{2} = x$

$\Rightarrow 2\sin^2\frac{C}{2} - 2\cos\left(\frac{A-B}{2}\right)\sin\frac{C}{2} + x - 1 = 0$

this is quadratic in $\sin\frac{C}{2}$ which is real. So discriminant $D \geq 0$

$4\cos^2\left(\frac{A-B}{2}\right) - 4\times 2(x-1) \geq 0$

$\Rightarrow 2(x-1) \leq \cos^2\left(\frac{A-B}{2}\right)$

$\Rightarrow 2(x-1) \leq 1 \Rightarrow x \leq \frac{3}{2}$

Thus, $\cos A + \cos B + \cos C \leq \frac{3}{2}$

S.E-3: Find the least value of $\sec A + \sec B + \sec C$ in an acute angle triangle.

Sol.: In an acute angle triangle, $\sec A, \sec B, \sec C$ are positive.

Now $A.M. \geq H.M.$

$$\Rightarrow \frac{\sec A+\sec B+\sec C}{3} \geq \frac{3}{\cos A+\cos B+\cos C}$$

but in $\Delta ABC, \cos A+\cos B+\cos C \leq \frac{3}{2}$

$$\Rightarrow \frac{\sec A+\sec B+\sec C}{3} \geq 2$$

$$\Rightarrow \sec A+\sec B+\sec C \geq 6$$

S.E-4. In ΔABC, prove that

$$\operatorname{cosec}\frac{A}{2}+\operatorname{cosec}\frac{B}{2}+\operatorname{cosec}\frac{C}{2} \geq 6.$$

Sol: In ΔABC we know that $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \leq \frac{1}{8}$

Now $A.M \geq G.M$

$$\Rightarrow \frac{\operatorname{cosec}\frac{A}{2}+\operatorname{cosec}\frac{B}{2}+\operatorname{cosec}\frac{C}{2}}{3} \geq \left(\operatorname{cosec}\frac{A}{2}\operatorname{cosec}\frac{B}{2}\operatorname{cosec}\frac{C}{2}\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{\operatorname{cosec}\frac{A}{2}+\operatorname{cosec}\frac{B}{2}+\operatorname{cosec}\frac{C}{2}}{3} \geq \left(\frac{1}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{\operatorname{cosec}\frac{A}{2}+\operatorname{cosec}\frac{B}{2}\operatorname{cosec}\frac{C}{2}}{3} \geq (8)^{\frac{1}{3}}$$

$$\Rightarrow \operatorname{cosec}\frac{A}{2}+\operatorname{cosec}\frac{B}{2}+\operatorname{cosec}\frac{C}{2} \geq 6$$

S.E-5: The value of $\cot 70^0+4\cos 70^0$ is

Sol: $\cot 70^0+4\cos 70^0 =$

$$\frac{\cos 70^0+4\sin 70^0\cos 70^0}{\sin 70^0}$$

$$=\frac{\cos 70^0+2\sin 140^0}{\sin 70^0}$$

$$=\frac{\cos 70^0+2\sin\left(180^0-40^0\right)}{\sin 70^0}$$

$$=\frac{\sin 20^0+\sin 40^0+\sin 40^0}{\sin 70^0}$$

$$=\frac{2\sin 30^0\cos 10^0+\sin 40^0}{\sin 70^0}$$

$$=\frac{\sin 80^0+\sin 40^0}{\sin 70^0}=\frac{2\sin 60^0\cos 20^0}{\sin 70^0}=\sqrt{3}$$

S.E-6: The absolute value of the expression

$$\tan\frac{\pi}{16}+\tan\frac{5\pi}{16}+\tan\frac{9\pi}{16}+\tan\frac{13\pi}{16} \text{ is } \_\_\_$$

Sol: Let $\theta=\frac{\pi}{16} \Rightarrow 8\theta=\frac{\pi}{2}$

$y=\tan\theta+\tan 5\theta+\tan 9\theta+\tan 13\theta$

$\therefore y=(\tan\theta-\cot\theta)+(\tan 5\theta-\cot 5\theta)$ [as $\tan 13\theta=\tan(8\theta+5\theta)=-\cot 5\theta$ and

$\tan 9\theta=\tan(8\theta+\theta)=-\cot\theta$]

$=(\tan\theta-\cot\theta)+(\cot 3\theta-\tan 3\theta)$

$=2(\cot 6\theta-\cot 2\theta)$

$$\Rightarrow y=2\left[\frac{\cos 6\theta}{\sin 6\theta}-\frac{\cos 2\theta}{\sin 2\theta}\right]$$

$$=2\left[\frac{\sin 2\theta\cos 6\theta-\cos 2\theta\sin 6\theta}{\sin 6\theta\sin 2\theta}\right]$$

$$=-2\left[\frac{\sin 4\theta}{\cos 2\theta\sin 2\theta}\right]=-4\left(\because 6\theta=\frac{\pi}{2}-2\theta\right)$$

Hence, absolute value $=4$.

S.E-7: If $\alpha=\frac{2\pi}{7}$, then

$\tan\alpha\tan 2\alpha+\tan 2\alpha\tan 4\alpha+\tan 4\alpha\tan\alpha=$

Sol: $\alpha=\frac{2\pi}{7} \Rightarrow 7\alpha=2\pi \Rightarrow \cos 7\alpha=1$

$\tan\alpha\tan 2\alpha+\tan 2\alpha\tan 4\alpha+\tan 4\alpha\tan\alpha$

$$=\frac{\sin\alpha\sin 2\alpha}{\cos\alpha\cos 2\alpha}+\frac{\sin 2\alpha\sin 4\alpha}{\cos 2\alpha\cos 4\alpha}+\frac{\sin 4\alpha\sin\alpha}{\cos 4\alpha\cos\alpha}$$

6.5 TRANSFORMATIONS

$$=\frac{\cos\alpha\sin2\alpha\sin4\alpha+\sin\alpha\cos2\alpha\sin4\alpha+\sin\alpha\sin2\alpha\cos4\alpha}{\cos\alpha\cos2\alpha\cos4\alpha}$$

$$=\frac{\cos\alpha\cos2\alpha\cos4\alpha-\cos(\alpha+2\alpha+4\alpha)}{\cos\alpha\cos2\alpha\cos4\alpha}$$

$$=1-\frac{1}{\cos\alpha\cos2\alpha\cos4\alpha}=1-\frac{1}{\left(\frac{1}{8}\right)}=-7$$

EXERCISE-I
CRTQ & SPQ LEVEL-I

PROBLEMS ON PRODUCT TO SUM COMPONENDO AND DIVIDENDO PROPERTY

C.R.T.Q Class Room Teaching Questions

1. $\sin A+\sin3A+\sin5A+\sin7A=$
   1) $4\sin A\cos2A\cos4A$
   2) $4\sin A\cos2A\cos3A$
   3) $4\cos A\sin2A\sin4A$
   4) $4\cos A\cos2A\sin4A$

2. $\sin\frac{\theta}{2}.\sin\frac{7\theta}{2}+\sin\frac{3\theta}{2}.\sin\frac{11\theta}{2}-\sin2\theta.\sin5\theta$
   1) 0 2) 1 3) -1 4) 2

3. $\cos48^\circ.\cos12^\circ=$
   1) $\frac{1-\sqrt5}{8}$ 2) $\frac{\sqrt5+3}{8}$
   3) $\frac{\sqrt5-1}{8}$ 4) $\frac{\sqrt5+1}{8}$

4. $\cos66^\circ+\sin84^\circ=$
   1) $\frac{\sqrt{15}-\sqrt3}{4}$ 2) $\frac{\sqrt{15}-3}{4}$
   3) $\frac{\sqrt{15}+\sqrt3}{4}$ 4) $\frac{\sqrt{15}+3}{4}$

5. $4\sin(420^\circ-\alpha)\cos(60^\circ+\alpha)=$
   1) $\sqrt3-2\sin2\alpha$ 2) $\sqrt3+2\sin2\alpha$
   3) $\sqrt3-2\cos2\alpha$ 4) $\sqrt3+2\cos2\alpha$

6. $\cot16^\circ.\cot44^\circ+\cot44^\circ.\cot76^\circ-\cot76^\circ.\cot16^\circ=$
   1) 3 2) 0 3) 1 4) 4

7. $\frac{\sin\alpha.\sin3\alpha+\sin3\alpha.\sin7\alpha+\sin5\alpha.\sin15\alpha}{\sin\alpha.\cos3\alpha+\sin3\alpha\cos7\alpha+\sin5\alpha.\cos15\alpha}=$
   1) $\sin(11\alpha)$ 2) $\cot(11\alpha)$
   3) $\cos(11\alpha)$ 4) $\tan(11\alpha)$

8. $\sin\alpha+\sin\beta=a,\ \cos\alpha+\cos\beta=b$
   $\Rightarrow\sin(\alpha+\beta)=$
   1) ab 2) a+b 3) $\frac{2ab}{a^2-b^2}$ 4) $\frac{2ab}{a^2+b^2}$

9. $\pi<\alpha-\beta<3\pi$, $\sin\alpha+\sin\beta=\frac{-21}{65}$, $\cos\alpha+\cos\beta=\frac{-27}{65}\Rightarrow\cos\left(\frac{\alpha-\beta}{2}\right)=$
   1) $\frac{-6}{65}$ 2) $\frac{-3}{\sqrt{130}}$ 3) $\frac{3}{\sqrt{130}}$ 4) $\frac{6}{65}$

10. $\cos x+\cos y=\frac{4}{5}$, $\cos x-\cos y=\frac{2}{7}$
    $\Rightarrow14\tan\left(\frac{x-y}{2}\right)+5\cot\left(\frac{x+y}{2}\right)=$
    1) 0 2) 1/4 3) 5/4 4) 3/4

11. The value of
    $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^{2015}+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^{2015}=$
    1) 0 2) $\cot^{2015}\left(\frac{A+B}{2}\right)$
    3) $\cot^{2015}\left(\frac{A-B}{2}\right)$ 4) $2\tan^{2015}\left(\frac{A+B}{2}\right)$

12. $\cos2B=\frac{\cos(A+C)}{\cos(A-C)}\Rightarrow\tan A,\tan B,\tan C$ are in
    1) A.P. 2) H.P 3) G.P. 4) A.G.P

13. $A+B+C=180^\circ\Rightarrow\cos2A+\cos2B+\cos2C=$
    1) $1-4\sin A\sin B\sin C$ 2) $1+4\sin A\sin B\sin C$
    3) $1+4\cos A\cos B\cos C$ 4) $-1-4\cos A\cos B\cos C$

14. $A+B+C=180^0 \Rightarrow$
$\cos^2 A+\cos^2 B+\cos^2 C=$
1) $1+2\cos A\cos B\cos C$
2) $1+2\sin A\sin B\sin C$
3) $1-2\cos A\cos B\cos C$
4) $1-2\sin A\sin B\sin C$

15. If $A+B+C=270^0$, then
$\cos 2A+\cos 2B+\cos 2C$
$+4\sin A\sin B\sin C=$
1) 0 2) 1 3) 2 4) 3

16. $A+B+C=0^0 \Rightarrow \sin A+\sin B+\sin C=$
1) $2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
2) $-2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
3) $4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
4) $-4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

17. $A+B+C=2S \Rightarrow \sin S+\sin(S-A)$
$+\sin(S-B)-\sin(S-C)=$
1) $4\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}$
2) $4\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$
3) $4\cos\frac{A}{2}.\sin\frac{B}{2}.\cos\frac{C}{2}$
4) $4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$

18. $\frac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)}=$
1) $\sin\frac{A}{2}.\cos\frac{B}{2}$ 2) $\sec\frac{A}{2}\operatorname{cosec}\frac{B}{2}$
3) $\tan\frac{A}{2}.\cot\frac{B}{2}$ 4) $2\sin\frac{A}{2}.\cos\frac{B}{2}$

S.P.Q. Student Practice Questions

19. $xy+yz+zx=1, \Rightarrow \frac{x}{1+x^2}+\frac{y}{1+y^2}+\frac{z}{1+z^2}=$
1) $\frac{2}{\sqrt{(1+x^2)(1-y^2)(1-z^2)}}$
2) $\frac{2}{\sqrt{(1-x^2)(1+y^2)(1-z^2)}}$
3) $\frac{2}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}}$
4) $\frac{2}{\sqrt{(1-x^2)(1+y^2)(1+z^2)}}$

20. $4\cos 6\theta\cos 4\theta\cos 2\theta=$
1) $\cos 12\theta+\cos 8\theta+\cos 4\theta+1$
2) $\cos 12\theta+\cos 8\theta-\cos 4\theta+1$
3) $\cos 12\theta-\cos 8\theta+\cos 4\theta+1$
4) $\cos 12\theta-\cos 8\theta-\cos 4\theta-1$

21. $2(1-2\sin^2\theta)\cos 4\theta=$
1) $\sin 6\theta+\cos 2\theta$ 2) $\sin 6\theta+\sin 2\theta$
3) $\cos 6\theta+\cos 2\theta$ 4) $\cos 6\theta+\sin 2\theta$

22. $\sin 48^0.\cos 78^0=$
1) $\frac{\sqrt{5}+1}{8}$ 2) $\frac{1+\sqrt{5}}{8}$
3) $\frac{1-\sqrt{5}}{8}$ 4) $\frac{\sqrt{5}-1}{8}$

23. $\sin 24^0+\cos 6^0=$
1) $\frac{\sqrt{15}+\sqrt{3}}{4}$ 2) $\frac{\sqrt{15}+3}{4}$
3) $\frac{\sqrt{15}-3}{4}$ 4) $\frac{\sqrt{15}-\sqrt{3}}{4}$

24. $\cos^2(45^0-\alpha)+\cos^2(15^0+\alpha)$
$-\cos^2(15^0-\alpha)=$
1) 0 2) 1 3) 1/2 4) 2

25. $\cos 20^0.\cos 40^0.\cos 60^0.\cos 80^0=$
1) 3/16 2) 1/32 3) 1/16 4) 1/8

26. $\frac{\sin A+\sin 5A+\sin 9A}{\cos A+\cos 5A+\cos 9A}=$
1) tan 3A
2) tan 5A
3) tan 4A
4) tan 2A

27. $\cos x+\cos y=\frac{1}{3},\sin x+\sin y=\frac{1}{4}$
$\Rightarrow\sin(x+y)=$
1) $\frac{7}{25}$ 2) $\frac{25}{24}$ 3) $\frac{25}{7}$ 4) $\frac{24}{25}$

28. $\frac{\cos^2 33^0-\cos^2 57^0}{\sin 21^0-\cos 21^0}=$
1) $-1/\sqrt{2}$
2) 1/2
3) $1/\sqrt{2}$
4) $-1/2$

29. $\sin x+\sin y=\frac{1}{4}$, $\sin x-\sin y=\frac{1}{5}$
$\Rightarrow 4\cot\left(\frac{x-y}{2}\right)=$
1) $5\cot\left(\frac{x-y}{2}\right)$ 2) $5\tan\left(\frac{x-y}{2}\right)$
3) $5\cot\left(\frac{x+y}{2}\right)$ 4) $5\tan\left(\frac{x+y}{2}\right)$

30. $\cos\theta=\frac{\cos\alpha-\cos\beta}{1-\cos\alpha.\cos\beta}\Rightarrow\tan^2\left(\frac{\theta}{2}\right)\tan^2\left(\frac{\beta}{2}\right)=$
1) $\tan\frac{\alpha}{2}$
2) $\tan^2\frac{\alpha}{2}$
3) $\cot\frac{\alpha}{2}$
4) $\cot^2\frac{\alpha}{2}$

31. $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}\Rightarrow\frac{\tan x}{\tan y}=$
1) b/a 2) a/b 3) 1 4) 0

32. $A+B+C=180^0\Rightarrow$
$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}=$
1) $1+2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
2) $1+2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
3) $1-2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
4) $1-2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

33. $A+B+C=270^0$
$\Rightarrow\sin 2A-\sin 2B+\sin 2C=$
1) $4\sin A\cos B\sin C$
2) $4\cos A\sin B\cos C$
3) $-4\sin A\cos B\sin C$
4) $-4\cos A\sin B\cos C$

34. $A+B+C=2S\Rightarrow\sin(S-A)+\sin(S-B)+\sin(S-C)-\sin S=$
1) $2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
2) $2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
3) $4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
4) $4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

35. $A+B+C=2S$, then $\sin S-\sin(S-A)-\sin(S-B)-\sin(S-C)=$
1) $2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
2) $2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
3) $-4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
4) $-4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

KEY

01) 4	02) 1	03) 2	04) 3	05) 1
06) 1	07) 4	08) 4	09) 2	10) 1
11) 1	12) 3	13) 4	14) 3	15) 2
16) 4	17) 2	18) 3	19) 3	20) 1
21) 3	22) 4	23) 1	24) 3	25) 3
26) 2	27) 4	28) 1	29) 3	30) 2
31) 2	32) 3	33) 3	34) 4	35) 4

Hints & Solutions

1. Apply SinC+SinD formula
2. Use, 2SinA SinB = Cos(A-B) - Cos(A+B)
3. Multiply and divide by 2 and 2 CosA. CosB =
   Cos(A+B) + Cos(A-B)
4. Write Sin84⁰=Cos6⁰ and Apply CosC+CosD
5. Use, 2SinACosB = Sin(A+B) + Sin(A-B)
6. Write $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$ and use
   $\sin(A+B-C)$
   Formula, A=76, B = 16, C = 44
7. Use 2SinASinB = Cos(A-B)-Cos(A+B)
8. Apply transformations and
   $$\sin\theta=\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$
9. Squaring and addding given equations
10. Apply $\cos C+\cos D, \cos C-\cos D$
11. If n is odd then
    $$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n=0$$
12. Apply componendo and dividendo
13. Put A = B = C = 60^0 and verify
14. Put A = B = C = 60^0 and verify
15. Use Cos C+ Cos D formulae
16. Put A = B = 60^0 , C = -120^0 and verify
17. Apply SinC+SinD formulae
18. Put A=B=30^0 and verify
19. Put $x=\tan A, y=\tan B, z=\tan C$ and simplfy
20. Put $\theta=0$ and verify
21. Write $1-2\sin^2\theta=\cos2\theta$
22. Multiply and divide by 2 and
    $2\sin A\sin B=\cos(A-B)-\cos(A+B)$
23. Write $\cos6^0=\sin84^0$ and apply
    $\cos6^0=\sin84^0$
24. Put $\alpha=0^0$ and verify
25. Use $\cos\theta.\cos(60-\theta).\cos(60+\theta)$
    $=\frac{1}{4}\cos3\theta$
26. Apply SinC + SinD formulae
27. Apply transformations and
    $$\sin\theta=\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$
28. $\cos^2B-\cos^2A=\sin(A+B).\sin(A-B)$
29. Apply $\sin C+\sin D, \sin C-\sin D$
30. Given $\dfrac{\cos\theta}{1}=\dfrac{\cos\alpha-\cos\beta}{1-\cos\alpha.\cos\beta}$
    Apply componendo and dividendo
    if $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-b}{a+b}=\frac{c-d}{c+d}$
    $$\frac{1-\cos\theta}{1+\cos\theta}=\frac{1-\cos\alpha\cos\beta-\cos\alpha+\cos\beta}{1-\cos\alpha\cos\beta+\cos\alpha-\cos\beta}$$
    $$\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}=\frac{(1-\cos\alpha)(1+\cos\beta)}{(1+\cos\alpha)(1-\cos\beta)}$$

$$\tan^2\frac{\theta}{2}=\frac{2\sin^2\frac{\alpha}{2}.2\cos^2\frac{\beta}{2}}{2\cos^2\frac{\alpha}{2}.2\sin^2\frac{\beta}{2}}$$

$$=\tan^2\frac{\alpha}{2}.\cot^2\frac{\beta}{2}$$

$$\tan^2\frac{\theta}{2}=\tan^2\frac{\alpha}{2}.\frac{1}{\tan^2\frac{\beta}{2}}$$

$$\therefore \tan^2\frac{\theta}{2}.\tan^2\frac{\beta}{2}=\tan^2\frac{\alpha}{2}$$

31. Apply componendo and dividendo
32. Put A= B = C= 60^0 and verify
33. Put $A=120^0, B=60^0, C=90^0$ and verify
34. Apply $\sin C+\sin D$ formula
35. Apply transformations

EXERCISE-II
CRTQ & SPQ LEVEL-II

SUM TO PRODUCT
$\sin C\pm\sin D$; $\cos C\pm\cos D$

C.R.T.Q Class Room Teaching Questions

1. In a Quadrilateral ABCD, $\cos A.\cos B+\sin C\sin D=$
   1) $\cos C\cos D+\sin A\sin B$
   2) $\cos C\cos D-\sin A\sin B$
   3) $\sin C\sin D-\cos A\cos B$
   4) $\sin A+\sin B+\sin C+\sin D$

2. $\dfrac{(\cos\alpha-\cos3\alpha)(\sin8\alpha+\sin2\alpha)}{(\sin5\alpha-\sin\alpha)(\cos4\alpha-\cos6\alpha)}=$
   1) 1 2) -1 3) 2 4) -2

3. $\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.\sin\frac{7\pi}{14}.\sin\frac{9\pi}{14}.\sin\frac{11\pi}{14}.\sin\frac{13\pi}{14}=$
   1) 1/64 2) 3/64 3) 5/64 4) 7/64

10

4. $\dfrac{\sin3\theta-\sin\theta.\sin^2(2\theta)}{\sin\theta+\sin2\theta.\cos\theta}=\cos x\Rightarrow x=$
   1) 4θ 2) 2θ 3) θ 4) 3θ

5. $\cos\alpha+\cos\beta=a$, $\sin\alpha+\sin\beta=b$, $\alpha-\beta=2\theta\Rightarrow\dfrac{\cos3\theta}{\cos\theta}=$
   1) $3-a^2-b^2$ 2) $\dfrac{a^2+b^2}{4}$
   3) a^2+b^2-1 4) a^2+b^2-3

6. $\sin\alpha+\cos\alpha=m\Rightarrow\sin^6\alpha+\cos^6\alpha=$
   1) $\dfrac{4+3(m^2-1)^2}{4}$ 2) $\dfrac{4-3(m^2-1)^2}{4}$
   3) $\dfrac{3+4(m^2-1)^2}{4}$ 4) $\dfrac{4-3(m^2+1)^2}{4}$

7. $\cos x+\cos y+\cos z=0$ and $\sin x+\sin y+\sin z$, then $\cos^2\left(\dfrac{x-y}{2}\right)=$
   1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{3}{4}$ 4) 1

8. If $\cos(\theta-\alpha),\cos\theta,\cos(\theta+\alpha)$ are H.P. then $\cos\theta.\sec\frac{\alpha}{2}=$
   1) $-\sqrt{2}$ 2) $\sqrt{2}$ 3) $\pm\sqrt{2}$ 4) 1

9. $x\tan\left(\theta-\frac{\pi}{6}\right)=y\tan\left(\theta+\frac{2\pi}{3}\right)\Rightarrow\dfrac{x+y}{x-y}=$
   1) $\cos2\theta$ 2) $2\cos2\theta$
   3) $\sin2\theta$ 4) $2\sin2\theta$

10. $x=\cos55^0, y=\cos65^0, z=\cos175^0\Rightarrow xy+yz+zx=$
    1) $-\frac{3}{4}$ 2) $\frac{3}{4}$ 3) $\frac{3}{2}$ 4) $\frac{1}{2}$

S.P.Q. Student Practice Questions

11. If A, B, C, D be the angles of a quadrilateral, then

$$\frac{\tan A+\tan B+\tan C+\tan D}{\cot A+\cot B+\cot C+\cot D}=$$

1) tan A tan B tan C tan D
2) 0
3) 1
4) cot A cot B cot C cot D

12. $\dfrac{\cos 6\theta+6\cos 3\theta+15\cos 2\theta+10}{\cos 5\theta+5\cos 3\theta+10}=$

1) $\sin\theta$ 2) $\cos\theta$
3) $2\sin\theta$ 4) $2\cos\theta$

13. $K=\sin\left(\dfrac{\pi}{18}\right)\sin\left(\dfrac{5\pi}{18}\right)\sin\left(\dfrac{7\pi}{18}\right)\Rightarrow K=$

1) 1/4 2) 1/6 3) 1/8 4) 1/2

14. $\sin A+\sin B=\sqrt{3}(\cos B-\cos A)$

$\Rightarrow \sin 3A+\sin 3B=$

1) 0 2) 2 3) 1 4) -1

15. $\dfrac{\sin(n+1)A+2\sin nA+\sin(n-1)A}{\cos(n-1)A-\cos(n+1)A}=$

1) $\tan A/2$ 2) $\cot A/2$ 3) tan A 4) cot A

16. $\sin\alpha-\cos\alpha=\dfrac{1}{3}\Rightarrow \sin^4\alpha-\cos^4\alpha=$

1) $\dfrac{8}{\sqrt{17}}$ 2) $-\dfrac{\sqrt{17}}{9}$ 3) $\dfrac{\sqrt{17}}{9}$ 4) $-\dfrac{8}{\sqrt{17}}$

17. If $\cos x+\cos y+\cos z=0$

$=\sin x+\sin y+\sin z$ then $\tan(x-y)=$

1) $\sqrt{3}$ 2) $-\sqrt{3}$ 3) $\dfrac{1}{\sqrt{3}}$ 4) $-\dfrac{1}{\sqrt{3}}$

18. $\sin(y+z-x),\sin(z+x-y),$

$\sin(x+y-z)$ are in A.P. are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P

19. $x\cos\theta=y\cos\left(\theta+\dfrac{2\pi}{3}\right)=z\cos\left(\theta+\dfrac{4\pi}{3}\right)$

$\Rightarrow xy+yz+zx=$

1) –1 2) 1 3) 0 4) 2

KEY

01) 1	02) 1	03) 1	04) 2	05) 4
06) 2	07) 2	08) 3	09) 2	10) 1
11) 1	12) 4	13) 3	14) 1	15) 2
16) 2	17) 2	18) 1	19) 3	

Hints & Solutions

1. Use A+B+C+D =360, (A+B) = [360-(C+D)]

cos(A+B)=cos(C+D)

2. $\dfrac{2\sin 2\alpha\sin\alpha 2\sin 5\alpha\cos 3\alpha}{2\cos 3\alpha\sin 2\alpha 2\sin 5\alpha\sin\alpha}=1$

3. Write $\left(\sin\dfrac{\pi}{14}\sin\dfrac{3\pi}{14}\sin\dfrac{5\pi}{14}\right)^2$

$=\left(\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{3\pi}{7}\right)^2$

and use the formula

$\cos\theta\cos 2\theta\cos 4\theta.....\cos 2^{(n-1)}\theta=\dfrac{2\sin 2^{n-1}\theta}{2^n\sin\theta}$

4. Simplify, get $\cos 2\theta=x$, $\theta=65°$

5. squaring and adding,

$\cos(\alpha-\beta)=\dfrac{a^2+b^2}{2}-1=\cos 2\theta$

$\cos^2\theta=\dfrac{a^2+b^2}{4}$

6. $(\sin\alpha+\cos\alpha)^2=m^2$

$\Rightarrow \sin\alpha\cos\alpha=\dfrac{m^2-1}{2}$

Write $\sin^6\alpha+\cos^6\alpha=(\sin^2\alpha)^3+(\cos^2\alpha)^3$

$=1-3\sin^2\alpha\cos^2\alpha$

7. Squaring and adding

8. Apply H.P. condition

9. Write $\frac{x}{y}=\frac{\tan(\theta+2\pi/3)}{\tan(\theta-\pi/6)}$, apply by C & D

10. $2\cos A\cos B=\cos(A+B)+\cos(A-B)$

11. $A+B=2\pi-(C+D)$

$\sin(A+B)=-\sin(C+D)$
$\cos(A+B)=\cos(C+D)$

12. $\theta=0°$ and verify

13. $k=\sin 10°\sin(60°-10°)\sin(60°+10°)$

$=\frac{1}{4}\sin 30°=\frac{1}{8}$

14. Put A = -B and verify

15. $\frac{2\sin nA\cos A+2\sin nA}{2\sin A\sin nA}=\frac{1+\cos A}{\sin A}=\cot\frac{A}{2}$

16. $\sin^4\alpha-\cos^4$

$\alpha=(\sin^2\alpha-\cos^2\alpha)(\sin^2\alpha+\cos^2\alpha)$

$=-\cos 2\alpha$

17. squiring and adding we get

$\cos^2\left(\frac{x-y}{2}\right)=\frac{1}{4}$

$\cos\left(\frac{x-y}{2}\right)=\frac{1}{2}$, $\tan\left(\frac{x-y}{2}\right)=\sqrt{3}$

$\therefore \tan(x-y)=\frac{2(\sqrt{3})}{1-3}=-\sqrt{3}$

18. Use $b-a=c-b$, (A.P. condition)

19. Put $\theta=0$ and verify

EXERCISE–III
CRTQ & SPQ LEVEL-III

STANDARD RESULTS

C.R.T.Q Class Room Teaching Questions

1. If $\theta_1,\theta_2,\theta_3,.....\theta_n$ are in A.P. then $\frac{\sin\theta_1+\sin\theta_2+...+\sin\theta_n}{\cos\theta_1+\cos\theta_2+...+\cos\theta_n}=$

1) 0
2) $\tan(\theta_1+\theta_n)$
3) $\tan\left(\frac{\theta_1+\theta_n}{2}\right)$
4) $\tan\left(\frac{\theta_1-\theta_n}{2}\right)$

2. In a triangle ABC, $\cos A+\cos B+\cos$

1) < 1
2) > 1
3) > 1but not > 2
4) >1 but not >3/

S.P.Q. Student Practice Questions

3. In a triangle ABC

1) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\le\frac{1}{8}$
2) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}<\frac{1}{8}$
3) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}>\frac{1}{8}$
4) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\ge\frac{1}{8}$

4. If $\alpha=\frac{2\pi}{7}$, then

$\tan\alpha\tan 2\alpha+\tan 2\alpha\tan 4\alpha+\tan 4\alpha\tan\alpha$

1) -1
2) -3
3) -5
4) -7

PRODUCT TO SUM RULE

C.R.T.Q Class Room Teaching Questions

5. The value of the expression

$\frac{1-4\sin 10^0\sin 70^0}{2\sin 10^0}$

1) 1/2
2) 1
3) 2

6. If A, B, C, D are four angles then $\sin(A-B).\cos(A+B)+\sin(B-C)$ $.\cos(B+C)+\sin(C-D).\cos(C+D)$ $+\sin(D-A).\cos(D+A)=$

1) 0
2) 1
3) 4sinA.sinBsinCsinD
4) 4cosAcosBcosCcosD

7. The value of $\frac{3+\cot 76^0\cot 16^0}{\cot 76^0+\cot 16^0}$ is

1) $\tan 44^0$ 2) $\cot 46^0$
3) $\tan 2^0$ 4) $\tan 46^0$

S.P.Q. Student Practice Questions

8. Let $0<A, B<\frac{\pi}{2}$ satisfying the equations $3\sin^2 A+2\sin^2 B=1$ and $3\sin 2A-2\sin 2B=0$ then $A+2B=$

1) 0 2) $\pi/2$
3) $\pi/6$ 4) $3\pi/2$

9. Let $f(n)=2\cos nx, \forall n\in N$, then $f(1)f(n+1)-f(n)$ is equal to

1) $f(n+3)$ 2) $f(n+2)$
3) $f(n+1)f(2)$ 4) $f(n+2)f(2)$

SUM TO PRODUCT FULE

C.R.T.Q Class Room Teaching Questions

10. The minimum value of expression $\sin\alpha+\sin\beta+\sin\gamma$ where α,β,γ are positive real numbers satisfying $\alpha+\beta+\gamma=\pi$

1) 0 2) -3 3) Negative 4) possitive

11. The value of $\sin 47^0+\sin 61^0-\sin 11^0-\sin 25^0$ is

1) $\sin 7^0$ 2) $\cos 7^0$ 3) $\sin 14^0$ 4) $\cos 14^0$

12. If cosA+cosB+cosC = 0 then cos3A+cos3B+cos3C

1) 4cosAcos3Bcos3C 2) 12cosAcosBcosC
3) 8cosAcosBcosC 4) 10cosAcosBcosC

S.P.Q. Student Practice Questions

13. In a ΔABC, $\sin^4 A+\sin^4 B+\sin^4 C=$

1) $\frac{3}{2}+2\cos A\cos B\cos C+\frac{1}{2}\cos 2A\cos 2B\cos 2C$

2) $2\cos A\cos B\cos C+\frac{1}{2}\cos 2A\cos 2B\cos 2C$

3) $2\cos A\cos B\cos C+\frac{1}{2}\cos 2A\cos 2B$

4) $2\cos A\cos B\cos C+\cos 2A\cos 2B$

14. If $A+B+C=\pi$, then $\begin{vmatrix}\sin^2 A & \cot A & 1\\ \sin^2 B & \cot B & 1\\ \sin^2 C & \cot C & 1\end{vmatrix}=$

1) 0 2) 1 3) -1 4) 2

15. If $x=\sin\alpha+\sin\beta$, $y=\cos\alpha+\cos\beta$ then $\tan\alpha+\tan\beta=$

1) $\frac{8xy}{2(y^2-x^2)+(x^2+y^2)(x^2+y^2-2)}$

2) $\frac{4xy}{(y^2-x^2)+(x^2+y^2)(x^2+y^2-2)}$

3) $\frac{8xy}{(x^2+y^2)(x^2+y^2-2)}$ 4) $4xy$

ELIMINATION OF 'θ'

C.R.T.Q Class Room Teaching Questions

16. If $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=(a^2-b^2)$, and $\frac{ax\sin\theta}{\cos^2\theta}-\frac{by\cos\theta}{\sin^2\theta}=0$, then $(ax)^{2/3}+(by)^{2/3}=$

1) $(a^2-b^2)^{2/3}$ 2) $(a^2+b^2)^{2/3}$
3) $(a^2+b^2)^{3/2}$ 4) $(a^2-b^2)^{3/2}$

S.P.Q. Student Practice Questions

17. If $\tan\theta = \frac{p}{q}$ and $\theta = 3\phi\left(0<\theta<\frac{\pi}{2}\right)$, then $\frac{p}{\sin\phi} - \frac{q}{\cos\phi} =$

1) $2\sqrt{(p^2-q^2)}$ 2) $2\sqrt{(p^2+q^2)}$

3) $\sqrt{(p^2+q^2)}$ 4) $3\sqrt{(p^2+q^2)}$

COMPONENDO AND DIVIDENDO PROPERTY

C.R.T.Q Class Room Teaching Questions

18. The smallest positive value of x (in degrees) for which $\tan\left(x+100^0\right) = \tan\left(x+50^0\right).\tan x.\tan\left(x-50^0\right)$ is

1) 30^0 2) 45^0

3) 55^0 4) 15^0

KEY

01) 3	02) 4	03) 1	04) 4	05) 2
06) 1	07) 4	08) 2	09) 2	10) 4
11) 2	12) 2	13) 1	14) 1	15) 1
16) 1	17) 2	18) 1		

Hints & Solutions

1. $\sin\theta_1 + \sin\theta_2 + ... + \sin\theta_n = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\frac{\beta}{2}}.\sin\left(\frac{\theta_1+\theta_n}{2}\right)$

$\cos\theta_1 + \cos\theta_2 + ... + \cos\theta_n = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\frac{\beta}{2}}.\cos\left(\frac{\theta_1+\theta_n}{2}\right)$

where β is the common difference of angles

2. $\cos A + \cos B + \cos C$

$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} > 1$

as neither of of $\sin\frac{A}{2}, \sin\frac{B}{2}, \sin\frac{C}{2}$ is - or zero again

$\cos A + \cos B + \cos C$

$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$+1 - 2\sin^2\frac{C}{2} \le 2\sin\frac{C}{2}.1 + 1 - 2\sin^2\frac{C}{2}$

$1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{3}{2}$

$\Rightarrow \cos A + \cos B + \cos C \le \frac{3}{2}$

3. $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}$

4. $\sum \tan\alpha.\tan 2\alpha = 1 - \frac{1}{\cos\alpha.\cos 2\alpha.\cos 4\alpha}$

$= 1 - \frac{(8).\sin\alpha}{\sin 8\alpha} = -7$

5. $\frac{1-4\sin 10^0\cos 20^0}{2\sin 10^0}$

$= \frac{1-2\left[\sin 30^0 - \sin 10^0\right]}{2\sin 10^0} = 1$

6. $\frac{1}{2}[2\sin(A-B)\cos(A+B) + 2\sin(B-C)\cos(B+C) + 2\sin(C-D)\cos(C+D) + 2\sin(D-A)\cos(D+A)] =$

$\frac{1}{2}[\sin 2A - \sin 2B + \sin 2B - \sin 2C + \sin 2C - \sin 2D + \sin 2D - \sin 2A] = 0$

7. $\dfrac{3\sin 76^\circ \sin 16^\circ + \cos 76^\circ \cos 16^\circ}{\cos 76^\circ \sin 16^\circ + \sin 76^\circ \cos 16^\circ}$

$= \dfrac{2\sin 76^\circ \sin 16^\circ + \cos 60^\circ}{\sin 92^\circ}$

$= \dfrac{\cos 60^\circ - \cos 92^\circ + \cos 60^\circ}{\sin 92^\circ} = \dfrac{1-\cos 92^\circ}{\sin 92^\circ}$

$= \tan 46^\circ$

8. Given, $3\sin^2 A = \cos 2B$ and

$\sin 2B = \dfrac{3}{2}\sin 2A$

$\cos(A+2B) = \cos A.(3\sin^2 A) -$

$\sin A\left(\dfrac{3}{2}2\sin A\cos A\right) = 0$

$\therefore \cos(A+2B) = 0$

9. $f(n) = 2\cos nx \Rightarrow f(1)f(n+1) - f(n)$

$= 4\cos x\cos(n+1)x - 2\cos n\pi$

$= 2\cos(n+2)x = f(n+2)$

10. If $\sin\alpha + \sin\beta + \sin\gamma =$

$4\cos\dfrac{\alpha}{2}\cos\dfrac{\beta}{2}\cos\dfrac{\gamma}{2}$ but $\dfrac{\alpha}{2}+\dfrac{\beta}{2}+\dfrac{\gamma}{2}=\dfrac{\pi}{2}$

each of $\dfrac{\alpha}{2},\dfrac{\beta}{2},\dfrac{\gamma}{2}$ is less than $\dfrac{\pi}{2}$

$\therefore \cos\dfrac{\alpha}{2}, \cos\dfrac{\beta}{2}, \cos\dfrac{\gamma}{2}$ are +ve

$\therefore$ minimum value of $4\cos\dfrac{\alpha}{2}.\cos\dfrac{\beta}{2}.\cos\dfrac{\gamma}{2}$ is +ve

$\therefore$ the minimum value of $\sin\alpha + \sin\beta + \sin\gamma$ is +ve

11. The given expression

$= (\sin 47^\circ + \sin 61^\circ) - (\sin 11^\circ + \sin 25^\circ)$

$= 2\sin 54^\circ \cos 7^\circ - 2\sin 18^\circ \cos 7^\circ$

$= 2\cos 7^\circ[\sin 54^\circ - \sin 18^\circ]$

$= 2\cos 7^\circ 2\cos 36^\circ.\sin 18^\circ$

$= 2\cos 7^\circ 2.\dfrac{\sqrt{5}+1}{4} \times \dfrac{\sqrt{5}-1}{4} = \cos 7^\circ$

12. $\cos A + \cos B + \cos C = 0$

$\Rightarrow \cos^3 A + \cos^3 B + \cos^3 C$

$= 3\cos A\cos B\cos C$ and

$\cos 3A + \cos 3B + \cos 3C$

$= 4(\cos^3 A + \cos^3 B + \cos^3 C) -$

$3(\cos A + \cos B + \cos C)$

$= 4(3\cos A\cos B\cos C) - 3(0)$

$= 12\cos A\cos B\cos C$

13. L.H.S $= \left(\dfrac{1-\cos 2A}{2}\right)^2 + \left(\dfrac{1-\cos 2B}{2}\right)^2 + \left(\dfrac{1-\cos 2C}{2}\right)^2$

$= \dfrac{3}{4} + \dfrac{1}{4}(\cos^2 2A + \cos^2 2B + \cos^2 2C)$

$- \dfrac{1}{2}(\cos 2A + \cos 2B + \cos 2C)$

14. Apply $R_1 - R_2, R_2 - R_3$ to make two zeros.

Use $\sin^2 A - \sin^2 B$

$= \sin(A+B)\sin(A-B) = \sin C\sin(A-B)$

and $\cot A - \cot B = \dfrac{\sin(B-A)}{\sin A\sin B}$

15. We have, $\tan\left(\dfrac{\alpha+\beta}{2}\right) = \dfrac{x}{y}$,

$\sin(\alpha+\beta) = \dfrac{2xy}{x^2+y^2}$,

$\cos(\alpha+\beta) = \dfrac{y^2-x^2}{x^2+y^2}$

$\cos(\alpha-\beta) = \dfrac{x^2+y^2-2}{2}$.

Now $\tan\alpha + \tan\beta = \dfrac{2\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$

16. From the second relation, we get

$$\frac{\sin^2\theta\sin\theta}{\cos^2\theta\cos\theta}=\frac{by}{ax}$$

$$\tan^3\theta=by/ax \therefore \tan\theta=(by)^{1/3}/(ax)^{1/3}$$

$$\therefore \sin\theta=\frac{(by)^{1/3}}{\left[(ax)^{2/3}+(by)^{2/3}\right]^{1/2}}$$

$$\cos\theta=\frac{(ax)^{1/3}}{\left[(ax)^{2/3}+(by)^{2/3}\right]^{1/2}}$$

Putting for $\sin\theta$ and $\cos\theta$ in

$$\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2-b^2$$

17. From given relation

$$\frac{\sin\theta}{p}=\frac{\cos\theta}{q}=\sqrt{\frac{\sin^2\theta+\cos^2\theta}{p^2+q^2}}=\frac{1}{\sqrt{p^2+q^2}}$$

Now putting for p and q in the L.H.S. we have

$$\text{L.H.S.}=\sqrt{(p^2+q^2)}\left[\frac{\sin\theta}{\sin\phi}-\frac{\cos\theta}{\cos\phi}\right]$$

$$=\sqrt{(p^2+q^2)}\frac{\sin(\theta-\phi)}{\sin\phi\cos\phi}$$

$$=\sqrt{(p^2+q^2)}\,2\frac{\sin(3\phi-\phi)}{\sin 2\phi}=2\sqrt{p^2+q^2}$$

$$\because \theta=3\phi$$

18. Given

$$\frac{\sin(x+100^0).\cos(x-50^0)}{\cos(x+100^0).\sin(x-50^0)}=\frac{\sin(x+50^0).\sin x}{\cos(x+50^0).\cos x}$$

by Componendo & Dividendo,

$$\cos(2x+30^0).\cos(2x-20^0)=0$$

$$\Rightarrow x=30^0, 55^0$$

EXERCISE-IV

LEVEL-IV

Assertion and Reason Type Q-1 to Q-7

a) Assertion is True, Reason is True, Reason is a correct explanation for Assertion.

b) Assertion is True, Reason is a True, Reason is not a correct explanation for Assertion.

c) Asseration is True, Reason is False.

d) Asseration is False, Reason is True.

1. **Assertion (A): If A, B, C are the angles of a triangle such that angle A is obtuse then $\tan B\tan C>1$**

   Reason (R) : In any triangle ABC

   $$\tan A=\frac{\tan B+\tan C}{\tan B\tan C-1}$$

   1) a 2) b 3) c 4) d

2. **Assertion (A) :**

   $$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}=\tan A$$

   Reason (R) :

   $\sin(A+B)+\sin(A-B)=\sin A$ **and**

   $\cos(A+B)+\cos(A-B)=\cos A$

   1) a 2) b 3) c 4) d

3. **Assertion (A): If**

   $x=\sin(\alpha-\beta)\sin(\gamma-\delta)$,

   $y=\sin(\beta-\gamma)\sin(\alpha-\delta)$

   $z=\sin(\gamma-\alpha)\sin(\beta-\delta)$

   then $x+y+z=0$

   Reason (R):

   $2\sin A\sin B=\cos(A-B)+\cos(A+B)$

   1) a 2) b 3) c 4) d

4. Assertion(A): $a=\tan\theta, b=\tan 2\theta$, $a\neq 0, b\neq 0$ and $\tan\theta+\tan 2\theta=\tan 3\theta$ then $a+b=0$

Reason(R): If $A+B=C$, then $\tan A-\tan B-\tan C=\tan A\tan B\tan C$

1) a 2) b 3) c 4) d

5. Assertion(A): In ΔABC, $\sum\frac{\cos A}{\sin B\sin C}=2$

Reason(R): In ΔABC,

$\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

1) a 2) b 3) c 4) d

6. Assertion(A): If $A+B+C=180^0$ then $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$

Reason(R): If $A+B+C=180^0$ then $\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$

1) a 2) b 3) c 4) d

7. Assertion(A): If $x+y+z=xyz$ then

$$\sum\left(\frac{2x}{1-x^2}\right)=\pi\left(\frac{2x}{1-x^2}\right)$$

Reason(R): If $\tan A+\tan B+\tan C=\tan A\tan B\tan C$, then $A+B+C=n\pi,\ n\in z$

1) a 2) b 3) c 4) d

8. Statement I : If $A+B+C=\pi$ and $\cos A=\cos B\cos C$ then $\cot B\cot C=\frac{1}{3}$

Statement II: If $5\sin B=\sin(2A+B)$ then $2\tan(A+B)=3\tan A$

1) only I is true
2) only II is true
3) Both I and II are true
4) Neither I nor II are true

9. Statement I: If $A+B+C=\frac{3\pi}{2}$ then $\cos 2A+\cos 2B+\cos 2C+4\sin A\sin B\sin C=0$

Statement II : If $A+B+C=0$, then $\sin 2A+\sin 2B+\sin 2C+4\sin A\sin B\sin C=0$

1) only I is true 2) only II is true
3) Both I and II are true
4) Neither I nor II are true

10. Statement I: $4\left(\sin 24^0+\cos 6^0\right)=\sqrt{15}+\sqrt{3}$

Statement II: $\sin 24^0\cos 6^0-\cos 24^0\cos 6^0=\frac{1}{4}$

1) only I is true 2) only II is true
3) Both I and II are true
4) Neither I nor II are true

11. Statement I : If $A+B+C=180^0$ then $\cos 2A+\cos 2B+\cos 2C=1-4\cos A\cos B\cos C$

Statement II : If $A+B+C=0^0$ then $\cos A+\cos B+\cos C=-1+4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

1) only I is true 2) only II is true
3) Both I and II are true
4) Neither I nor II are true

12. Statement I : $\cos x+\cos y=\frac{1}{3}$, $\sin x+\sin y=\frac{1}{4}\Rightarrow\cos(x+y)=\frac{-7}{25}$

Statement II: $\sin x+\sin y=\frac{1}{4}$, $\sin x-\sin y=\frac{1}{5}$ then $4\cot\left(\frac{x-y}{2}\right)=5\cot\left(\frac{x+y}{2}\right)$

1) only I is true
2) only II is true
3) Both I and II are true
4) Neither I nor II are true

13. Statement I: If $A+B+C=180^0$ then $\sin^2 A+\sin^2 B+\sin^2 C=2(1+\cos A\cos B\cos C)$

Statement II : If $A+B+C=180^0$ then $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$

1) only I is true
2) only II is true
3) Both I and II are true
4) Neither I nor II are true

14. $A=\cos 6^0 \sin 24^0 \cos 72^0$

$$B=\frac{\cos^2 33^0-\cos^2 57^0}{\sin 21^0-\cos 21^0}$$

$$C=\frac{\sin 9^0 \cos 9^0}{\sin 48^0 \sin 12^0}$$

Arrangement in descending order is

1) A, B, C 2) B, A, C
3) A, C, B 4) C, A, B

15. If $A=\sin 45^0 \sin 12^0$;

$B=\cos 45^0 \cos 12^0$

$C=\cos 66^0+\sin 84^0$ then descending order of these value is

1) C, A, B 2) C, B, A
3) A, C, B 4) A, B, C

KEY

01) 4 02) 3 03) 3 04) 1 05) 2 06) 1
07) 1 08) 2 09) 2 10) 3 11) 2 12) 2
13) 3 14) 4 15) 2

Hints & Solutions

1. If A is obtuse then $\tan B\tan C<1$
2. $\sin(A+B)+\sin(A-B)=2\sin A\cos B$
   $\cos(A+B)+\cos(A-B)=2\cos A\cos B$
3. $2\sin A\sin B=\cos(A-B)-\cos(A+B)$
4. $A=3\theta, B=\theta, C=2\theta$
   $3\theta-\theta=2\theta\Rightarrow\tan(3\theta-\theta)$
   $=\tan 2\theta\Rightarrow\tan 3\theta(ab)=0$
   $\Rightarrow\tan 3\theta=0(\text{since } ab\neq 0)\Rightarrow a+b=0$
5. apply $\sin C+\sin D$ formulae
6. apply $\cos C+\cos D$ formulae
   $x=\tan A, y=\tan B, z=\tan C$
   $\Rightarrow A+B+C=n\pi$
7. i) $\cos A=\cos B.\cos C$
   $\Rightarrow -\cos(B+C)=\cos B.\cos C$
   $\Rightarrow 2\cos B\cos C=\sin B\sin C$
   $\Rightarrow \cot B\cot C=1/2$
   ii) Write $\frac{\sin B}{\sin(2A+B)}=\frac{1}{5}$ and apply componendo and dividendo
8. i) Use cosC+cosD formulae
   ii) Use sinC+sinD formulae
9. (i) Use sinC+sinD formulae
   (ii) Apply transformations
10. Apply cosC+cosD formulae
11. i) Apply transformations and use
   $$\cos\theta=\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$
   ii) Apply transformations and take ratio
12. Apply transformations
13. Apply transformations
14. Apply transformations
15. Apply transformations

INTEGER & NUMERICAL
ANSWER TYPE QUESTIONS

9 1. If $f(\theta)=\dfrac{1-\sin 2\theta+\cos 2\theta}{2\cos 2\theta}$, then value of $8f(11^0).f(34^0)$ is____

6 2. If $f(n)=2(7\cos x+24\sin x)(7\sin x-24\cos x)$, for every $x\in R$, then maximum value of $(f(x))^{1/4}$ is____

3. In a triangle $ABC, \angle C=\frac{\pi}{2}$, if $\tan\left(\frac{A}{2}\right)$ and $\tan\left(\frac{B}{2}\right)$ are the rots of the equation $ax^2+bx+c=0(a\neq 0)$, then the value of $\frac{a+b}{c}$ (where a,b,c are sides of Δ opposite to angles A,B,C respectively) is 1

6 4. If $x,y\in R$ satisfies $(x+5)^2+(y-12)^2=(14)^2$, then the minimum value of $\sqrt{x^2+y^2}$ is____

5. Suppose x and y are real numbers such that $\tan x+\tan y=42$ and $\cot x+\cot y=49$. Then the prime number by which the value of $\tan(x+y)$ is not divisible by is 5

6. Let $0\le a,b,c,d\le \pi$, where b and c are not complementary, such that $2\cos a+6\cos b+7\cos c+9\cos d=0$ and $2\sin a-6\sin b+7\sin c-9\sin d=0$, then the value of $3\dfrac{\cos(a+d)}{\cos(b+c)}$ is____

7. Suppose A and B are two angles such that $A,B\in(0,\pi)$, and satisfy $\sin A+\sin B=1$ and $\cos A+\cos B=0$. Then the value of 12 $\cos 2A+4\cos 2B$ is 4

8. α and β are the positive acute angles and satisfying equations $5\sin 2\beta=3\sin 2\alpha$ and $\tan\beta=3\tan\alpha$ siultaneously. Then the value of $\tan\alpha+\tan\beta$ is 4

9. The absolute value of the expression $\tan\frac{\pi}{16}+\tan\frac{5\pi}{16}+\tan\frac{9\pi}{16}+\tan\frac{13\pi}{16}$ is____

10. The greatest integer less than or equal to $\dfrac{1}{\cos 290^0}+\dfrac{1}{\sqrt{3}\sin 250^0}$ is____

11. The maximum value of $y=\dfrac{1}{\sin^6 x+\cos^6 x}$ is____

12. The maximum value of $\cos^2(45^0+x)+(\sin x-\cos x)^2$ is____

13. The value of $\operatorname{cosec}10^0+\operatorname{cosec}50^0-\operatorname{cosec}70^0$ is____

14. Number of triangles ABC if $\tan A=x$, $\tan B=x+1$ and $\tan C=1-x$ is____

15. If $\log_{10}\sin x+\log_{10}\cos x=-1$ and $\log_{10}(\sin x+\cos x)=\dfrac{(\log_{10}x)-1}{2}$, then the value of '$x/3$' is____

16. The value of $\dfrac{\sin 1^0+\sin 3^0+\sin 5^0+\sin 7^0}{\cos 1^0.\cos 2^0.\sin 4^0}$ is____

17. In a triangle ABC, if $A-B=120^0$ and $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{32}$, then the value of $8\cos C$ is____

18. If $\dfrac{\tan x}{2}=\dfrac{\tan y}{3}=\dfrac{\tan z}{5}$, $x+y+z=\pi$ and $\tan^2 x+\tan^2 y+\tan^2 z=\dfrac{38}{K}$ then $K=$____

19. If $\sin^3 x\cos 3x+\cos^3 x\sin 3x=3/8$, then the value of $8\sin 4x$ is____

20. The value of $\operatorname{cosec}\frac{\pi}{18}-4\sin\frac{7\pi}{18}$ is_____

21. If $\tan x+\tan 2x+\tan 3x=\tan x\tan 2x\tan 3x$ then value of $|\sin 3x+\cos 3x|$ is_____

22. $16\left(\cos\theta-\cos\frac{\pi}{8}\right)\left(\cos\theta-\cos\frac{3\pi}{8}\right)$
$\left(\cos\theta-\cos\frac{5\pi}{8}\right)\left(\cos\theta-\cos\frac{7\pi}{8}\right)=\lambda\cos 4\theta,$
then the value of λ is_____

23. If $\frac{\tan(\ln 6).\tan(\ln 2).\tan(\ln 3)}{\tan(\ln 6)-\tan(\ln 2)-\tan(\ln 3)}=K,$
then the value of K is_____

24. If $\cot(\theta-\alpha),3\cot\theta,\cot(\theta+\alpha)$ are in A.P and θ is not an integral multiple of $\frac{\pi}{2}$, then the value of $\frac{4\sin^2\theta}{3\sin^2\alpha}=$_____

25. The value of $\frac{2\sin x}{\sin 3x}+\frac{\tan x}{\tan 3x}$ is_____

26. If $\cot^2 A.\cot^2 B=3$, then the value of $(2-\cos 2A)(2-\cos 2B)$ is_____

27. The value of $f(x)=x^4+4x^3+2x^2-4x+7$, when $x\cot\frac{11\pi}{8}$ is_____

28. The value of $\sin^2 12^0+\sin^2 21^0+\sin^2 39^0+\sin^2 48^0-\sin 9^0-\sin^2 18^0$ is_____

29. Given that, $f(n\theta)=\frac{2\sin 2\theta}{\cos 2\theta-\cos 4n\theta}$, and $f(\theta)+f(2\theta)+f(3\theta)+....+f(n\theta)=\frac{\sin\lambda\theta}{\sin\theta\sin\mu\theta}$, then the value of $\mu-\lambda$ is_

30. The positive integer value of $n>3$ satisfying the equation
$\frac{1}{\sin\left(\frac{\pi}{n}\right)}=\frac{1}{\sin\left(\frac{2\pi}{n}\right)}+\frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

31. $\tan 15^0+\tan 75^0=$_____

32. $\tan 203^0+\tan 22^0+\tan 203^0\tan 22^0=$

33. $\frac{\tan 80^0-\tan 10^0}{\tan 70^0}=$_____

34. $\frac{\cos 13^0-\sin 13^0}{\cos 13^0+\sin 13^0}+\frac{1}{\cot 148^0}=$_____

35. If $\tan(\pi/4+\theta)+\tan(\pi/4-\theta)=3$, then $\tan^2(\pi/4+\theta)+\tan(\pi/4-\theta)=$_____

36. If $A+B=45^0$ then $(1+\tan A)(1+\tan B)=$

37. If $\alpha-\beta=3\pi/4$ then $(1-\tan\alpha)(1+\tan\beta)=$__

38. If $A+B+C=0$, then $\cot A\cot B+\cot B\cot C+\cot C\cot A=$_____

39. If $A+B+C=\pi/2$, then $\tan A\tan B+\tan B\tan C+\tan A\tan C=$_____

40. If $A=35^0,B=15^0$ and $C=40^0$, then $\tan A.\tan B+\tan B.\tan C+\tan C.\tan A=$

41. If an acute-angled triangle, $\cot B\cot C+\cot A\cot C+\cot A\cot B=$_____

42. If $A+B+C=90^0$ then $\frac{\cot A+\cot B+\cot C}{\cot A\cot B\cot C}=$_____

43. If $A+B+C=\pi/2$, then $\Sigma\frac{\cos(B+C)}{\cos B\cos C}=$__

44. In $\Delta ABC, \tan\frac{A}{2}.\tan\frac{B}{2}.\tan\frac{C}{2}+\tan\frac{C}{2}.\tan\frac{A}{2}=$ ___

45. If $A+B+C=180^0$ then $\Sigma\frac{\cot A+\cot B}{\tan A+\tan B}=$ ___

46. $\frac{\sin^3 A+\sin 3A}{\sin A}+\frac{\cos^3 A-\cos 3A}{\cos A}=$ ___

47. $\frac{\cot x}{\cot x-\cot 3x}+\frac{\tan x}{\tan x-\tan 3x}=$ ___

48. $3\sin x+4\cos x=5\Rightarrow 6\tan\frac{x}{2}-9\tan^2\frac{x}{2}=$ ___

49. $(\sin 3A+\sin A)\sin A+(\cos 3A-\cos A)\cos A=$ ___

50. If 'θ' is the III quadrant then $\sqrt{4\sin^2\theta+\sin^2 2\theta}+4\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=$ ___

51. If $x\cos\alpha=y\cos(2\pi/3+\alpha)=z\cos(4\pi/3+\alpha)$, then $xy+yz+zx=$ ___

52. If $\log_2(\sin x)-\log_2(\cos x)-\log_2(1-\tan x)-\log_2(1+\tan x)=-1$ then $\tan 2x=$ ___

53. $\tan x+\tan\left(x+\frac{\pi}{3}\right)+\tan\left(x+\frac{2\pi}{3}\right)=3\Rightarrow\tan 3x=$ ___

54. $\tan 9^0-\tan 27^0-\tan 63^0+\tan 81^0=$ ___

55. $\sin^2 12^0+\sin^2 21+\sin^2 3a+\sin^2 48^0-\sin^2 9^0-\sin^2 18^0=$ ___

56. $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}=$ ___

57. $1+\cos 2x+\cos 4x+\cos 6x-4\cos x\cos 2x\cos 3x=$ ___

58. $\cos 40^0+\cos 80^0+\cos 160^0=$ ___

59. $2.\cos\frac{\pi}{13}.\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=$ ___

60. If $\cos(x-y)=3.\cos(x+y)$, then $\cot x.\cot y=$ ___

61. If $\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)}+\frac{\cos(\theta_3+\theta_4)}{\cos(\theta_3-\theta_4)}=0$, then $\tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4=$ ___

62. If $\frac{x}{\tan(\theta+\alpha)}=\frac{y}{\tan(\theta+\beta)}=\frac{z}{\tan(\theta+\gamma)}$ then $\frac{x+y}{x-y}\sin^2(\alpha-\beta)+\frac{y+z}{y-z}\sin(\beta-\gamma)+\frac{z+x}{z-x}\sin^2(\gamma-\alpha)=$ ___

63. If $\cos A=3/4$, then the value of $16\cos^2(A/2)-32\sin(A/2)\sin(5A/2)$ is ___

64. $\sin A+\sin B=\sqrt{3}(\cos B-\cos A)\Rightarrow\sin 3A+\sin 3B=$ 0

65. If $\sin^3 x\sin 3x=\sum_{m=0}^{n}c_m\cos^m x$ where $c_0,c_1,.....,c_n$ are constants and $c_x\neq 0$, then $x=$ ___

66. $A+B=C\Rightarrow\cos^2 A+\cos^2 B+\cos^2 C-2\cos A\cos B\cos C=$ 1

67. If $A+B+C=270^0$, then $\cos 2A+\cos 2B\cos 2C+\cos 2A+\cos 2B\cos 2C+4\sin A\sin B\sin C=$ ___

68. The vaue of the determinant $\begin{vmatrix}\sin^2 13^0 & \sin^2 77^0 & \tan^2 135^0\\ \tan^2 77^0 & \tan 135^0 & \sin^2 13^0\\ \tan 135^0 & \sin^2 13^0 & \sin^2 77^0\end{vmatrix}=$ ___

69. If $\sin\alpha,\sin\beta,\sin\gamma$ are in A.P and $\cos\alpha,\cos\beta,\cos\gamma$ are in G.P. then $\frac{\cos^2\alpha+\cos^2\gamma-4\cos\alpha\cos\gamma}{1-\sin\alpha\sin\gamma}=$ 2

70. If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)$

$\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16}\sin(10^\circ)$, then

$16 + \alpha^{-1}$ is equal to ________

Answers

1) 4	2) 5	3) 1	4) 1	5) 5	6) 7
7) 8	8) 4	9) 4	10) 2	11) 4	12) 3
13) 6	14) 0	15) 4	16) 4	17) 7	18) 3
19) 4	20) 2	21) 1	22) 2	23) 1	24) 2
25) 1	26) 3	27) 6	28) 1	29) 1	30) 7
31) 4	32) 1	33) 2	34) 0	35) 7	36) 2
37) 2	38) 1	39) 1	40) 1	41) 1	42) 1
43) 2	44) 1	45) 1	46) 3	47) 1	48) 1
49) 0	50) 2	51) 0	52) 1	53) 1	54) 4
55) 1	56) 4	57) 0	58) 0	59) 0	60) 2
61) –1	62) 0	63) 3	64) 0	65) 6	66) 1
67) 1	68) 0	69) –2	70) 8		

Hints & Solutions

1. $f(\theta) = \frac{1-\sin 2\theta + \cos 2\theta}{2\cos 2\theta}$

$= \frac{(\cos\theta - \sin\theta)^2 + (\cos^2\theta - \sin^2\theta)}{2(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$

$= \frac{\cos\theta}{\cos\theta + \sin\theta} = \frac{1}{1+\tan\theta}$

$f(11^\circ)f(34^\circ) = \frac{1}{(1+\tan 11^\circ)} \times \frac{1}{(1+\tan 34^\circ)}$

$= \frac{1}{(1+\tan 11^\circ)} \times \frac{1}{(1+\tan(45^\circ - 11^\circ))}$

$= \frac{1}{(1+\tan 11^\circ)} \times \frac{(1+\tan 11^\circ)}{2} = \frac{1}{2}$

$\therefore 8 f(11^\circ) f(34^\circ) = 8\left(\frac{1}{2}\right) = 4$

2. $f(x) = 2(7\cos x + 24\sin x)(7\sin x - 24\cos x)$

Let $r\cos\theta = 7, r\sin\theta = 24$

$\therefore r^2 = 625; \tan\theta = \frac{24}{7}$

$\therefore f(x) = 2r\cos(x-\theta) \times r\sin(x-\theta)$

$= r^2(\sin 2(x-\theta))$

$f(x)_{max} = 25^2$ or $(f(x))^{1/4} = 5$

3. $\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right) = \frac{-b}{a}$

$\tan\left(\frac{A}{2}\right) \times \tan\left(\frac{B}{2}\right) = \frac{c}{a}$

$A + B = 90^\circ$ or $\frac{A+B}{2} = 45^\circ$

$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \frac{\frac{-b}{a}}{1-\frac{c}{a}}$

or $1 - \frac{c}{a} = \frac{-b}{a} \Rightarrow$ or $a + b = c$ or $\frac{a+b}{c} = 1$

4. Let $x + 5 = 14\cos\theta$ and $y - 12 = 14\sin\theta$

$\therefore x^2 + y^2 = (14\cos\theta - 5)^2 + (14\sin\theta + 12)^2$

$= 196 + 25 + 144 + 28(12\sin\theta - 5\cos\theta)$

$= 365 + 28(12\sin\theta - 5\cos\theta)$

$\therefore \sqrt{x^2+y^2}\Big|_{min} = \sqrt{365 - 28 \times 13}$

$= \sqrt{365 - 364} = 1$

5. $\cot x + \cot y = 49$

or $\frac{1}{\tan x} + \frac{1}{\tan y} = 49$ or $\frac{\tan y + \tan x}{\tan x \tan y} = 49$

or $\tan x \tan y = \dfrac{\tan x + \tan y}{49} = \dfrac{42}{49} = \dfrac{6}{7}$

$\therefore \tan(x+y) = \dfrac{42}{1-(6/7)} = \dfrac{42}{1/7} = 294$

6. From the given equations, we have

$(2\cos a + 9\cos d)^2 = (6\cos b + 7\cos c)^2$

and $(2\sin a - 9\sin d)^2 = (6\sin b - 7\sin c)^2$

Adding, we have

$36\cos(a+d) = 84\cos(b+c)$

or $\dfrac{\cos(a+d)}{\cos(b+c)} = \dfrac{7}{3}$

7. Since $\cos A + \cos B = 0$

$\Rightarrow A+B=\pi \quad \therefore B = \pi - A$

$\sin A + \sin(\pi - A) = 1 \Rightarrow 2\sin A = 1$

$\Rightarrow A = 30^0$

8. $5\times\dfrac{2\tan\beta}{1+\tan^2\beta} = 3\times\dfrac{2\tan\alpha}{1+\tan^2\alpha}$

or $\dfrac{5\tan\beta}{1+\tan^2\beta} = \dfrac{3\tan\alpha}{1+\tan^2\alpha}$ .....(i)

substituting $\tan\beta = 3\tan\alpha$, we have

$\dfrac{5\times 3\tan\alpha}{1+9\tan^2\alpha} = \dfrac{3\tan\alpha}{1+\tan^2\alpha}$

or $5+5\tan^2\alpha = 1+9\tan^2\alpha$

or $4\tan^2\alpha = 4$ or $\tan\alpha = 1$, i.e., $\tan\beta = 3$

9. Let $\theta = \dfrac{\pi}{16}$ or $\theta = \dfrac{\pi}{2}$

$y = \tan\theta + \tan 5\theta + \tan 9\theta + \tan 13\theta$

$\therefore y = (\tan\theta - \cot\theta) + (\tan 5\theta - \cot 5\theta)$

[As $\tan 13\theta = \tan(8\theta + 5\theta) = -\cot 5\theta$ and

$\tan 9\theta = \tan(8\theta + \theta) = -\cot\theta$]

$= (\tan\theta - \cot\theta) + (\cot 3\theta - \tan 3\theta)$

$= \dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2 3\theta - \sin^2 3\theta}{\sin 3\theta \cos 3\theta}$

$\Rightarrow y = 2\left[\dfrac{\cos 6\theta}{\sin 6\theta} - \dfrac{\cos 2\theta}{\sin 2\theta}\right]$

$\Rightarrow 2\left[\dfrac{\sin 2\theta\cos 6\theta - \cos 2\theta \sin\theta}{\sin 6\theta \sin 2\theta}\right]$

$\Rightarrow -2\left[\dfrac{\sin 4\theta}{\cos 2\theta \sin 2\theta}\right] = -4$

$\left(\because 6\theta = \dfrac{\pi}{2} - 2\theta\right)$

Hence, absolute value is 4.

10. $\cos 290^0 = \sin 20^0; \sin 250^0$

$= -\sin 70^0 = -\cos 20^0$

$\Rightarrow \dfrac{1}{\sin 20^0} - \dfrac{1}{\sqrt{3}\cos 20^0} = \dfrac{\sqrt{3}\cos 20^0 - \sin 20^0}{\sqrt{3}\sin 20^0 \cos 20^0}$

$= \dfrac{2[\sin 60^0 \cos 20^0 - \sin 20^0 \cos 60^0]}{\sqrt{3}\sin 20^0 \cos 20^0}$

$= \dfrac{4\sin 40^0}{\sqrt{3}\sin 40^0} = \dfrac{4\sqrt{3}}{3}$

Hence, the greatest integer less than or equal to is 2.

11. $\sin^6 x + \cos^6 x \Rightarrow (\sin^2 x + \cos^2 x)$

$(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)$

$\Rightarrow 1 - 3\sin^2 x\cos^2 x = 1 - \dfrac{3(\sin 2x)^2}{4}$

$\Rightarrow y = \dfrac{4}{4-3(\sin 2x)^2} \Rightarrow y_{max} = \dfrac{4}{4-3(1)} = 4$

12. $\cos^2(45^0 + x) + (\sin x - \cos x)^2$

$= \left[\dfrac{\cos x}{\sqrt{2}} - \dfrac{\sin x}{\sqrt{2}}\right]^2 + (\sin x - \cos x)^2$

$= \dfrac{3}{2}(1 - \sin 2x) = \dfrac{3}{2}(1-(-1))$

Hence, maximum value $= \dfrac{3}{2}(1-(-1)) = 3$

13. $\dfrac{1}{\sin 10^0} + \dfrac{1}{\sin 50^0} - \dfrac{1}{\sin 70^0}$

$$=\frac{1}{\cos 80^0}+\frac{1}{\cos 40^0}-\frac{1}{\cos 20^0}$$

$$=\frac{\cos 40^0\cos 20^0+\cos 80^0\cos 20^0-\cos 40^0\cos 80^0}{\cos 20^0\cos 40^0\cos 80^0}$$

$$=8\left[\cos 20^0\left(\cos 40^0+\cos 80^0\right)-\cos 40^0\cos 80^0\right]$$

$$=8\left[2\cos 20^0\cos 60^0\cos 20^0-\cos 40^0\cos 80^0\right]$$

$$=4\left[2\cos^2 20^0-2\cos 40^0\cos 80^0\right]$$

$$=4\left[1+\cos 40^0-\left(\cancel{\cos} 20^0+\cos 40^0\right)\right]$$

$$=4\times\frac{3}{2}=6$$

14. In $\Delta ABC, \tan A+\tan B+\tan C$
$=\tan A\tan B\tan C$
$\Rightarrow x+x+1+1-x=x(1+x)(1-x)$
or $2+x=x-x^3$
or $x^3=-2\Rightarrow x=-2^{1/3}$
$\Rightarrow \tan A=x<0\Rightarrow A$ is obtuse
$\Rightarrow \tan B=x+1=1-2^{1/3}<0$
Hence, A and B are obtuse, which is not possible in a triangle. Hence, no such triangle can exist.

15. Given $\log_{10}\left(\frac{\sin 2x}{2}\right)=-1$

or $\frac{\sin 2x}{2}=\frac{1}{10}$ or $\sin 2x=\frac{1}{5}$

Also $\log_{10}(\sin x+\cos x)=\frac{\log_{10}\left(\frac{x}{10}\right)}{2}$

or $\log_{10}(\sin x+\cos)^2=\log_{10}\left(\frac{x}{10}\right)$

or $1+\sin 2x=\frac{x}{10}$ or $1+\frac{1}{5}=\frac{x}{10}$ or $\frac{6}{5}=\frac{x}{10}$

or $\frac{x}{3}=4$

16. $\frac{2\sin 4^0\cos 3^0+2\sin 4^0\cos 1^0}{\cos 1^0\cos 2^0\sin 4^0}$

$$=\frac{2\sin x^0\left(\cos 3^0+\cos 1^0\right)}{\cos 1^0\cos 2^0\sin x^0}$$

$$=\frac{4\cos 2^0\cos 1^0}{\cos 1^0\cos 2^0}=4$$

17. $2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{16}$

$$\Rightarrow\left[\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right]\sin\frac{C}{2}=\frac{1}{16}$$

or $\sin\frac{C}{2}\left(\frac{1}{2}-\sin\frac{C}{2}\right)+\frac{1}{16}=0$

or $\sin^2\frac{C}{2}-\frac{1}{2}\sin\frac{C}{2}+\frac{1}{16}=0$

or $\left(\frac{1}{4}-\sin\frac{C}{2}\right)^2=0$

or $\sin\frac{C}{2}=\frac{1}{4}$

or $\cos C=1-2\sin\frac{C}{2}=1-\frac{1}{8}=\frac{7}{8}$

18. $\tan x=2t, \tan y=3t, \tan z=5t$
Also $x+y+z=\pi$
$\therefore \tan x+\tan y+\tan z=\tan x\tan y\tan z$

$\Rightarrow t^2=\frac{1}{3}$ $\quad 10t = 30t^3$ $\quad t^2=\frac{1}{3}$

$\Rightarrow \tan^2 x+\tan^2 y+\tan^2 z=t^2(4+4+25)=38$

$K=3$ $\quad (2t)^2+(3t)^2+5t^2$

19. $4\sin^3 x\cos 3x+4\cos^3 x\sin 3x=\frac{3}{2}$

or $(3\sin x-\sin 3x)\cos 3x+(3\cos x+\cos 3x)$

$\sin 3x=\frac{3}{2}$

or $3[\sin x\cos 3x+\cos x\sin 3x]=\frac{3}{2}$

or $\sin 4x=\frac{1}{2}$

20. $\operatorname{cosec}10^0-4\sin 70^0$

$=\frac{1-4\sin 70^0\sin 10^0}{\sin 10^0}$

$=\frac{1-2(\cos 60^0-\cos 80^0)}{\sin 10^0}=\frac{1-2\left(\frac{1}{2}-\sin 10^0\right)}{\sin 10^0}$

$=\frac{1-1+2\sin 10^0}{\sin 10^0}=2$

21. $\tan x+\tan 2x+\tan 3x=\tan x\tan 2x\tan 3x$

$\Rightarrow x+2x+3x=n\pi, n\in z$

$\Rightarrow x=n\pi/6, n\in z$

$|\sin 3x+\cos 3x|=1$

22. $16\left(\cos\theta-\cos\frac{\pi}{8}\right)\left(\cos\theta-\cos\frac{3\pi}{8}\right)$

$\left(\cos\theta-\cos\frac{5\pi}{8}\right)\left(\cos\theta-\cos\frac{7\pi}{8}\right)$

$=16\left(\cos\theta-\cos\frac{\pi}{8}\right)\left(\cos\theta-\cos\frac{7\pi}{8}\right)$

$\times\left(\cos\theta-\cos\frac{3\pi}{8}\right)\left(\cos\theta-\cos\frac{5\pi}{8}\right)$

$=16\left(\cos\theta-\cos\frac{\pi}{8}\right)\left(\cos\theta+\cos\frac{\pi}{8}\right)$

$\times\left(\cos\theta-\cos\frac{3\pi}{8}\right)\left(\cos\theta+\cos\frac{3\pi}{8}\right)$

$=16\left(\cos^2\theta-\cos^2\frac{\pi}{8}\right)\left(\cos^2\theta-\cos^2\frac{3\pi}{8}\right)$

$=16\left(\cos^2\theta-\cos^2\frac{\pi}{8}\right)\left(\cos^2\theta-\sin^2\frac{\pi}{8}\right)$

$=16\left(\cos^4\theta-\cos^2\theta+\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)$

$=16\left(\cos^4\theta-\cos^2\theta+\frac{1}{8}\right)$

$=16\left(\frac{-\sin^2 2\theta}{4}+\frac{1}{8}\right)=16\left(\frac{1-2\sin^2 2\theta}{8}\right)$

$=\frac{16\cos 4\theta}{8}$

23. $\ln 6=\ln 3+\ln 2$

if $\gamma=\alpha+\beta$, then $\tan\gamma-\tan\alpha-\tan\beta$

$=\tan\gamma.\tan\alpha.\tan\beta$

$\Rightarrow\frac{\tan(\ln 6).\tan(\ln 2).\tan(\ln 3)}{\tan(\ln 6)-\tan(\ln 2)-\tan(\ln 3)}=1$

24. Given $\cot(\theta-\alpha), 3\cot\theta, \cot(\theta+\alpha)$ are in A.P.

$\Rightarrow 6\cot\theta=\cot(\theta-\alpha)+\cot(\theta+\alpha)$

$\Rightarrow\frac{6\cos\theta}{\sin\theta}=\frac{\sin 2\theta}{\sin(\theta-\alpha)\sin(\theta-\alpha)}$

$=6\cos\{\sin^2\theta-\sin^2\alpha\}=2\sin^2\theta\cos\theta$

$=3(\sin^2\theta-\sin^2\alpha)=\sin^2\theta$

$\Rightarrow 2\sin^2\theta=3\sin^2\alpha$

$\therefore\frac{2\sin^2\theta}{3\sin^2\alpha}=1\ \Rightarrow\frac{4\sin^2\theta}{3\sin^2\alpha}=2$

25. $\frac{2\sin x}{\sin 3x}+\frac{\tan x}{\tan 3x}$

$=\frac{2\sin x}{\sin x.(3-4\sin^2 x)}+\frac{\tan x}{\frac{(3\tan x-\tan^3 x)}{(1-3\tan^2 x)}}$

$=\frac{2}{3-4\sin^2 x}+\frac{(1-3\tan^2 x)}{(3-\tan^2 x)}=\frac{2}{3-4\sin^2 x}$

$$+\frac{1-\left(\frac{3\sin^2 x}{1-\sin^2 x}\right)}{3-\left(\frac{\sin^2 x}{1-\sin^2 x}\right)}$$

$$=\frac{2}{3-4\sin^2 x}+\frac{(1-4\sin^2 x)}{(3-4\sin^2 x)}=\frac{(3-4\sin^2 x)}{(3-4\sin^2 x)}=1$$

26. $(2-\cos 2A)(2-\cos 2B)$

$=(1+2\sin^2 A)(1+2\sin^2 B)=1+2$

$(\sin^2 A+\sin^2 B)+4\sin^2 A\sin^2 B$ .....(i)

Now, $\cot^2 A\cot^2 B=3$

$\Rightarrow \cot^2 A\cot^2 B=3\sin^2 A\sin^2 B$

$\Rightarrow (1-\sin^2 A)(1-\sin^2 B)=3\sin^2 A\sin^2 B$

$\Rightarrow \sin^2 A+\sin^2 B+2\sin^2 A\sin^2 B=1$

from (i), we get

$(2-\cos 2A)(2-\cos 2B)=1+2=3$

27. $x=\cot\frac{11\pi}{8}=\cot\left(\pi+\frac{3\pi}{8}\right)=\cot\frac{3\pi}{8}=\sqrt{2}-1$

$\Rightarrow (x+1)^2=2 \qquad \therefore x^2+2x-1=0$

Now, $f(x)=x^4+4x^3+2x^2-4x+7$

$=x^2(x^2+2x-1)+2x^3+3x^2-4x+7$

$=0+2x^3+3x^2-4x+7$

$=2x(x^2+2x-1)-x^2-2x+7$

$=-x^2-2x+7=-(x^2+2x-1)+6$

28. $\sin^2 12^0+\sin^2 21^0+\sin^2 39^0+\sin^2 48^0$
$-\sin^2 9^0-\sin^2 18^0$

$=\sin^2 12^0+\sin^2 21^0+(\sin^2 39^0-\sin^2 9^0)$

$+(\sin^2 48^0-\sin^2 18^0)$

$=1-(\cos^2 12^0-\sin^2 21^0)+\sin 48^0\sin 30^0$

$+\sin 66^0\sin 30^0$

$=1-\cos 33^0\cos 9^0+\frac{1}{2}\times 2\sin 57^0\cos 9^0$

$=1-\cos 33^0\cos 9^0+\cos 33^0\cos 9^0=1$

29. $f(n\theta)=\frac{2\sin 2\theta}{\cos 2\theta-\cos 4n\theta}$

$=\frac{2\sin 2\theta}{2\sin(2x+1)\theta\sin(2n-1)\theta}$

$=\frac{\sin(2x+1)\theta-(2x+1)\theta}{\sin(2x+1)\theta\sin(2x-1)\theta}$

$\sin(2x+1)\theta\cos(2x-1)\theta$

$=\frac{-\cos(2x+1)\theta\sin(2x-1)\theta}{\sin(2x+1)\theta\sin(2x-1)\theta}$

$=\cot(2x-1)\theta-\cot(2x+1)\theta$

$\therefore f(\theta)+f(2\theta)+f(3\theta)+....+$

$f(x\theta)=\cot\theta-\cot(2x+1)\theta$

$=\frac{\sin(2x+1)\theta\cos\theta-\cos(2x+1)\theta\sin\theta}{\sin(2x+1)\theta\sin\theta}$

$=\frac{\sin 2n\theta}{\sin(2n+1)\theta\sin\theta}$

30. $\frac{1}{\sin\frac{\pi}{n}}-\frac{1}{\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$

or $\frac{\sin\frac{3\pi}{n}-\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}=\frac{1}{\sin\frac{2\pi}{n}}$

or $\frac{\left(2\sin\frac{\pi}{n}\cos\frac{2\pi}{n}\right)}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}\sin\frac{2\pi}{n}=1$

or $\sin\frac{4\pi}{n}=\sin\frac{3\pi}{n}\Rightarrow\frac{4\pi}{n}+\frac{3\pi}{n}=\pi=n=7$

31. $\tan 15^0+\tan 75^0=\tan(45^0-30^0)+\tan(45^0+30^0)$

$=\frac{\tan 45^0-\tan 30^0}{1+\tan 45^0\tan 30^0}+\frac{\tan 45^0+\tan 30^0}{1-\tan 45^0\tan 30^0}$

$=\frac{1-1/\sqrt{3}}{1+1/\sqrt{3}}+\frac{1+1/\sqrt{3}}{1-1/\sqrt{3}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$

$=\frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}=2\frac{(3+1)}{3-1}=4$

32. $\tan(203^06+22^0)=\frac{\tan 203^0+\tan 22^0}{1-\tan 203^0\tan 22^0}=1$

$\Rightarrow \tan(180^0+45^0)\Rightarrow\frac{\tan 203^3+\tan 22^0}{1-\tan 203^0\tan 22^0}$

$\Rightarrow \tan 203^0+\tan 22^0+\tan 203^0\tan 22^0=1$

33. $\tan 70^0=\tan(80^0-70^0)=\frac{\tan 80^0-\tan 10^0}{1+\tan 80^0\tan 10^0}$

$=\frac{\tan 80^0-\tan 10^0}{2}\Rightarrow\frac{\tan 80^0-\tan 10^0}{\tan 70^0}=2$

34. $\frac{\cos 13^0-\sin 13^0}{\cos 13^0+\sin 13^0}+\frac{1}{\cot 148^0}$

$=\frac{1-\tan 13^0}{1+\tan 13^0}+\tan 148^0$

$=\tan(45^0-13^0)+\tan(180^0-32^0)$

$=\tan 32^0-\tan 32^0=0$

35. $G.E=\tan^2\left(\frac{\pi}{4}+\theta\right)+\tan^2\left(\frac{\pi}{4}-\theta\right)$

$=\left[\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)\right]^2$

$=2\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{\pi}{4}-\theta\right)=3^2-2=7$

36. Given $A+B=\frac{\pi}{4}\Rightarrow\frac{\tan A+\tan B}{1-\tan A\tan B}=1$

$\Rightarrow \tan A+\tan B+\tan A\tan B+1=1+1$

$(1+\tan A)(1+\tan B)=2$

37. Given $\alpha-\beta=\frac{3\pi}{4}\Rightarrow\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=-1$

$\Rightarrow \tan\alpha-\tan\beta+\tan\alpha\tan\beta=-1$

$=1-\tan\alpha+\tan\beta-\tan\alpha\tan\beta=1+1$

$\Rightarrow(1-\tan\alpha)(1+\tan\beta)=2$

38. $A+B+C=0\Rightarrow A+B=-C$

$\Rightarrow\frac{\cot A\cot B-1}{\cot A+\cot B}=-\cot C$

$\Rightarrow\Sigma\cot A\cot B=1$

39. Given $A+B+C=\frac{\pi}{2}\Rightarrow A+B=\frac{\pi}{2}-C$

$\Rightarrow\frac{\tan A+\tan B}{1-\tan A\tan B}=\cot C=\frac{1}{\tan C}$

$\Rightarrow\tan A\tan C+\tan B\tan C+\tan A\tan B=1$

40. $A+B+C=35^0+15^0+40^0=90^0$

$\Rightarrow\tan A\tan B+\tan B\tan C+\tan C\tan A=1$

41. Conceptual

42. $\frac{\cot A+\cot B+\cot C}{\cot A\cot B\cot C}=\frac{\cot A\cot B\cot C}{\cot A\cot C\cot C}=1$

43. $\Sigma\frac{\cos(B+C)}{\cos B\cos C}=\Sigma\frac{\cos B\cos C-\sin B\sin C}{\cos B\cos C}$

$=\Sigma(1-\tan B\tan C)$

$=\Sigma 1-\Sigma\tan B\tan C=3-1=2$

$[\therefore A+B+C=\pi/2\Rightarrow\Sigma\tan B\tan C=1]$

44. $A+B+C=\pi\Rightarrow\frac{A}{2}+\frac{B}{2}=\frac{\pi}{2}-\frac{C}{2}$

$\Rightarrow\frac{\tan(A/2)+\tan(B/2)}{1-\tan(A/2)\tan(B/2)}=\cot\frac{C}{2}=\frac{1}{\tan(C/2)}$

$\Rightarrow\Sigma\tan\frac{A}{2}-\tan\frac{B}{2}=1$

45. $\Sigma\frac{\cot A-\cot B}{\tan A+\tan B}=\Sigma\left[\frac{|\tan A+1|(\tan B)}{\tan A+\tan B}\right]$

$=\Sigma\left[\frac{1}{\tan A\tan B}\right]$

$$= \Sigma(\cot A \cot B) = \cot A \cot B + \cot B \cot C + \cot C \cot A = 1.$$

6. $$G.E = \frac{\sin^3 A + 3\sin A - 4\sin^3 A}{\sin A} + \frac{\cos^3 A - 4\cos^3 A + 3\cos A}{\cos A}$$

$$= 3 - 3\sin^2 A + 3 - 3\cos^2 A = 3$$

47. $$G.E = \frac{\frac{1}{\tan x}}{\frac{1}{\tan x} - \frac{1}{\tan 3x}} + \frac{\tan x}{\tan x - \tan 3x}$$

$$= \frac{\tan 3x}{\tan 3x - \tan x} - \frac{\tan x}{\tan 3x - \tan x} = 1$$

48. Put $\tan\frac{x}{2} = t$, $3\sin x + 4\cos x = 5$

$$\Rightarrow 3\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right) = 5$$

$$\Rightarrow 6t + 4(1-t^2) = 5(1+t^2)$$

$$\Rightarrow 9t^2 - 6t + 1 = 0 \Rightarrow 6t - 9t^2 = 1$$

$$\Rightarrow 6\tan\frac{x}{2} - 9\tan^2\frac{x}{2} = 1$$

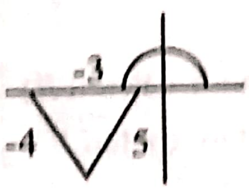
49. Given $\tan A = 4/3$, A is in the III quadrant.

$5\sin 2A + 3\sin A + 4\cos A = 5$

$(2\sin A\cos A) + 3\sin A + 4\cos A$

$$= 10\left(\frac{-4}{3}\right)\left(\frac{-3}{5}\right) + 3\left(\frac{-4}{5}\right) + 4\left(\frac{-3}{5}\right)$$

$$= \frac{24}{5} - \frac{12}{5} - \frac{12}{5} = 0$$



50. $$\sqrt{4\sin^4\theta + (\sin 2\theta)^2} + 4\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$\sqrt{4\sin^4\theta + 4\sin^2\theta\cos^2\theta} + 2\left[2\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right]$$

$$= \sqrt{4\sin^2\theta[\sin^2\theta + \cos^2\theta]} + 2$$

$$\left[1 + \cos\left(\frac{\pi}{2} - \theta\right)\right] = -2\sin\theta + 2 + 2\sin\theta$$

51. Given $x\cos\alpha = y\cos\left(\frac{2\pi}{3} + \alpha\right) =$

$$z\cos\left(\frac{4\pi}{3} + \alpha\right) = K(\text{say})$$

$$\Rightarrow \frac{K}{x} = \cos\alpha; \frac{K}{y} = \cos\left(\frac{2\pi}{3} + \alpha\right); \frac{K}{z}$$

$$= \cos\left(\frac{4\pi}{3} + \alpha\right)$$

$$\therefore \frac{K}{x} + \frac{K}{y} + \frac{K}{z} = 0 \Rightarrow K\left(\frac{xy + yz + zx}{xyz}\right) = 0$$

$$\Rightarrow xy + yz + zx = 0$$

52. Given equation becomes

$$\log\left[\frac{\sin x}{\cos x(1 - \tan x)(1 + \tan x)}\right] = -1$$

$$\Rightarrow \frac{\tan x}{1 - \tan^2 x} = 2^{-1} \Rightarrow \tan 2x = 1$$

53. $$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x} = 3$$

$$\Rightarrow \frac{\tan x + (\tan x + \sqrt{3})(1 + \sqrt{3}\tan x) + (\tan x - \sqrt{3})(1 - \sqrt{3}\tan x)}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \frac{\tan x - 3\tan^3 x + \tan x + \sqrt{3} + \sqrt{3}\tan^2 x + 3\tan x + \tan x - \sqrt{3} - \sqrt{3}\tan^2 x + 3\tan x}{1 - 3\tan^2 x} = 3$$

$\Rightarrow \frac{9\tan x - 3\tan^3 x}{1-3\tan^2 x} = 3$

$\Rightarrow \tan 3x = 1$

4. $G.E = (\tan 9^0 + \cot 9^0) - (\tan 27^0 + \cot 27^0)$

$= \frac{1}{\cos 9^0 \sin 9^0} - \frac{1}{\sin 27^0 \cos 27^0}$

$= \frac{2}{\sin 18^0} - \frac{2}{\sin 54^0} = \frac{2.4}{\sqrt{5}-1} - \frac{2.4}{\sqrt{5}+1}$

$= 4$

5. $\sin^2 12^0 + \sin^2 21 + \sin^2 39 + \sin^2 48^0$

$-\sin^2 9^0 - \sin^2 18^0$

$= \frac{1-\cos 24^0}{2} + \frac{1-\cos 42^0}{2} + (\sin^2 39^0 - \sin^2 9^0)$

$+(\sin^2 48^0 - \sin^2 18^0)$

$= 1 - \frac{1}{2}(\cos 42^0 + \cos 24^0) + \sin 48^0 . \sin 30^0$

$+ \sin 66^0 . \sin 30^0$

$= 1 - \frac{1}{2}[\cos(90^0 - 48^0) + \cos(90^0 - 66^0)]$

$+ \sin 48^0 \left(\frac{1}{2}\right) + \sin 66^0 \left(\frac{1}{2}\right)$

$= 1 - \frac{1}{2}(\sin 48^0 + \sin 66^0) + \frac{1}{2}$

$(\sin 66^0 + \sin 48^0) = 1$

6. $\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0} = \frac{2\left(\frac{1}{2}\cos 10^0 - \frac{\sqrt{3}}{2}\sin 10^0\right)}{\sin 10^0 . \cos 10^0}$

$= \frac{2\sin(30^0 - 10^0)}{\frac{1}{2}\sin 20^0} = 4$

57. $1 + \cos 2x + \cos 4x + \cos 6x$

$-4\cos x \cos 2x \cos 3x$

$= 2\cos^2 x + 2\cos 5x \cos x - 4\cos x \cos 2x$

$\cos 3x = 2\cos x[\cos x + \cos 5x]$

$-4\cos x \cos 2x \cos 3x = 2\cos x$

$[2\cos 3x \cos 2x] - 4\cos x \cos 2x \cos 3x = 0$

58. $G.E = 2\cos 60^0 \cos 20^0 + \cos(180^0 - 20^0)$

$= 2.\frac{1}{2}.\cos 20^0 - \cos 20^0 = 0$

59. $G.E = \cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\left(\pi - \frac{10\pi}{13}\right)$

$+ \cos\left(\pi - \frac{8\pi}{13}\right) = 0$

60. Given, $\frac{3}{1} = \frac{\cos(x-y)}{\cos(x+y)} \Rightarrow \frac{3+1}{3-1}$

$\frac{\cos(x-y)+\cos(x+y)}{\cos(x-y)-\cos(x+y)}$

$\Rightarrow \frac{4}{2}\frac{2\cos x \cos xy}{2\sin x \sin y} = \cot x \cot y = 2$

61. Given, $\frac{\cos(\theta_1 - \theta_1)}{\cos(\theta_1 + \theta_2)} = \frac{-\cos(\theta_3 - \theta_4)}{\cos(\theta_3 - \theta_4)}$

$\Rightarrow \frac{\cos(\theta_1 - \theta_2) + \cos(\theta_1 + \theta_2)}{\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)}$

$= \left[\frac{-\cos(\theta_3 + \theta_4) + \cos(\theta_3 - \theta_4)}{-\cos(\theta_3 + \theta_4) - \cos(\theta_3 - \theta_4)}\right]$

$\Rightarrow \frac{2\cos\theta_1 \cos\theta_2}{2\sin\theta_1 \sin\theta_2} = \frac{2\sin\theta_3 \sin\theta_4}{-2\cos\theta_3 \cos\theta_4}$

$\Rightarrow \cot\theta_1 \cot\theta_2 = -\tan\theta_3 \tan\theta_4$

$\Rightarrow \tan\theta_1 \tan\theta_2 \tan\theta_3 \tan\theta_4 = -1.$

63. The given expression is equal to

$8(1+\cos A) - 16(\cos 2A - \cos 3A)$

$= 8(1+\cos A) - 16[2\cos^2 A - 1 -$

$\cos A(4\cos^2 A - 3)]$

$= 8\left(1+\frac{3}{4}\right) - 16\left[2\times\frac{9}{16} - 1\frac{-3}{4}\left(4\times\frac{9}{16} - 3\right)\right]$

$= 14 - (18 - 16 - 27 + 36) = 3$

64. $\sin A+\sin B=\sqrt{3}(\cos B-\cos A)$

$=\sqrt{3}\cos A+\sin A=\sqrt{3}\cos B-\sin B$

$\Rightarrow \cos A(\sqrt{3}/2)+\sin A(1/2)$

$=\cos B(\sqrt{3}/2)-\sin B(1/2)$

$\Rightarrow \cos(A-\pi/6)=\cos(B+\pi/6)$

$\Rightarrow A-\pi/6=\pm(B+\pi/6)\Rightarrow A=-B$

or $A-B=\pi/3$

65. In the expansion of $\sin^3 x\sin 3x$, we get the heighst power term is $\cos^6 x$. $\therefore x=6$

66. $A+B+C\Rightarrow \cos(A+B)=\cos C$

$\Rightarrow \cos A\cos B-\sin A\sin B=\cos C$

$\Rightarrow \cos A\cos B-\cos C=\sin A\sin B$

$\Rightarrow (\cos A\cos B-\cos C)^2=\sin^2 A\sin^2 B$

$\Rightarrow \cos^2 A\cos^2 B+\cos^2 C-2\cos A\cos B\cos C$

$=(1-\cos^2 A)(1-\cos^2 B)$

$\Rightarrow \cos^2 A\cos^2 B+\cos^2 C-2\cot A\cos B\cos C$

$=1-\cos^2 A-\cos^2 B+\cos^2 A\cos^2 B$

$\Rightarrow \cos^2 A+\cos^2 B+\cos^2 C-2\cos A\cos B\cos C=1$

67. $\cos 2A+\cos 2B+\cos 2C+4\sin A\sin B\sin C$

$=2\cos(A+B)\cos(A-B)+\cos 2C$

$+4\sin A\sin B\sin C$

$=-2\sin C\cos(A-B)+1-2\sin^2 C$

$+4\sin A\sin B\sin C$

$=1-2\sin C[\cos(A-B+\sin C]$

$+4\sin A\sin B\sin C$

$=1-2\sin C[\cos(A-B)-\cos(A+B)]$

$+4\sin A\sin B\sin C$

$=1-4\sin A\sin B\sin C+4\sin A\sin B\sin C=1$

68. The given determinant is

$$=\begin{vmatrix}\sin^2 13^0 & \cos^2 13^0 & -1\\ \cos^2 13^0 & -1 & \sin^2 13^0\\ -1 & \sin^2 13^0 & \cos^2 13^0\end{vmatrix}$$

$$=\begin{vmatrix}0 & \cos^2 13^0 & -1\\ 0 & -1 & \sin^2 13^0\\ 0 & \sin^2 13^0 & \cos^2 13^0\end{vmatrix}=0$$

(Applying $C_1\to C_1+C_2+C_3$)

69. From the given conditions we have

$2\sin\beta=\sin\alpha+\sin\gamma$ .....(i)

$\cos^2\beta=\cos\alpha\cos\gamma$ .....(ii)

Squaring (i),

$4\sin^2\beta=\sin^2\alpha+\sin^2\gamma+2\sin\alpha\sin\gamma$

using (ii), $4(1-\cos\alpha\cos\gamma)=1-\cos^2\alpha-$

$-\cos^2\gamma+2\sin\alpha\sin\gamma$

$\Rightarrow \cos^2\alpha+\cos^2\gamma-4\cos\alpha\cos\gamma=2$

$(\sin\alpha\cos\gamma-1)\Rightarrow \dfrac{\cos^2\alpha+\cos^2\gamma-4\cos\alpha\cos\gamma}{1-\sin\alpha\sin\gamma}$

$=-2$

70. $\sin 10^0\left(\frac{1}{2}.2\sin 20^0\sin 40^0\right).\sin 10^0$

$\sin(60^0-10^0)\sin(60^0+10^0)$

$\sin 10^0\frac{1}{2}(\cos 20^0-\cos 60^0).\frac{1}{4}\sin 30^0$

$\frac{1}{2}.\frac{1}{4}.\frac{1}{2}\sin 10^0\left(\cos 20^0-\frac{1}{2}\right)$

$=\frac{1}{32}(2\sin 10^0\cos 20^0-\sin 10^0)$

$=\frac{1}{32}(\sin 30^0-\sin 10^0-\sin 10^0)$

$=\frac{1}{32}\left(\frac{1}{2}-2\sin 10^0\right)$

$=\frac{1}{64}(1-4\sin 10^0)$

$=\frac{1}{64}-\frac{1}{16}\sin 10^0$

Hence $\alpha=\frac{1}{64}$

$16+\alpha^{-1}=80$