FINAL JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Sunday 29th January, 2023) TIME : 9 : 00 AM to 12 : 00 NOON

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Match List I with List II :

List-I (Physical Quantity)		List-II (Dimensional Formula)	
A	Pressure gradient	I	$[M^0L^2T^{-2}]$
B	Energy density	II	$[M^1L^{-1}T^{-2}]$
C	Electric Field	III	$[M^1L^{-2}T^{-2}]$
D	Latent heat	IV	$[M^1L^1T^{-3}A^{-1}]$

Choose the **correct** answer from the options given below:

(1) A-III, B-II, C-I, D-IV

(2) A-II, B-III, C-IV, D-I

(3) A-III, B-II, C-IV, D-I

(4) A-II, B-III, C-I, D-IV

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Pressure gradient $= \frac{dp}{dx} = \frac{[ML^{-1}T^{-2}]}{[L]}$

$= [M^1L^{-2}T^{-2}]$

Energy density $= \frac{\text{energy}}{\text{volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$

$= [M^1\,L^{-1}\,T^{-2}]$

Electric field $= \frac{\text{Force}}{\text{charge}} = \frac{[MLT^{-2}]}{[A.T]}$

$= [M^1\,L^1\,T^{-3}\,A^{-1}]$

Latent heat $= \frac{\text{heat}}{\text{mass}} = \frac{[ML^2T^{-2}]}{[M]}$

$= [M^0\,L^2\,T^{-2}]$

2. In a cuboid of dimension $2L\times2L\times L$, a charge q is placed at the centre of the surface 'S' having area of 4 L^2. The flux through the opposite surface to 'S' is given by

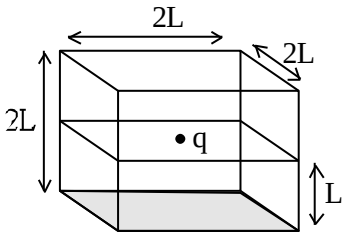
(1) $\frac{q}{12\varepsilon_0}$ (2) $\frac{q}{3\varepsilon_0}$

(3) $\frac{q}{2\varepsilon_0}$ (4) $\frac{q}{6\varepsilon_0}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\phi = \frac{Q/\varepsilon_0}{6}$



Flux passing through shaded face $= \frac{q}{6\varepsilon_0}$

3. Ratio of thermal energy released in two resistor R and 3R connected in parallel in an electric circuit is :

(1) 3 : 1 (2) 1 : 1

(3) 1 : 3 (4) 1 : 27

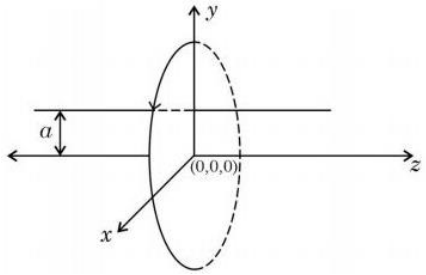
Official Ans. by NTA (1)

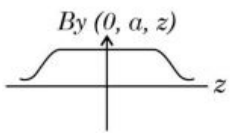
Allen Ans. (1)

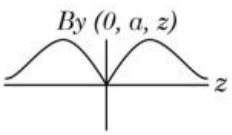
Sol. $H = \frac{V^2}{R}\times t$

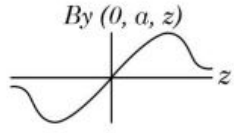
$\frac{H_1}{H_2} = \frac{\frac{V^2t}{R}}{\frac{V^2t}{3R}} = 3:1$

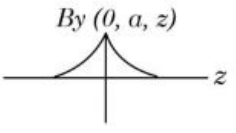
4. A single current carrying loop of wire carrying current I flowing in anticlockwise direction seen from +ve z direction and lying in xy plane in shown in figure. The plot of $\hat{j}$ component of magnetic field (By) at a distance 'a' (less than radius of the coil) and on yz plane vs z coordinate look like



(1) 

(2) 

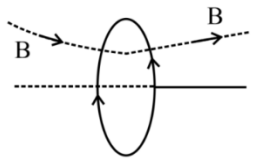
(3) 

(4) 

Official Ans. by NTA (3)

Allen Ans. (3)

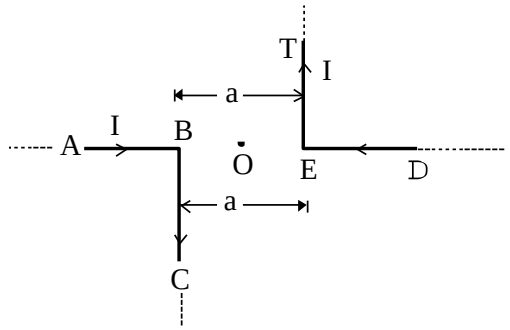
Sol.



$B_y = 0$ in plane of coil

B_y is opposite of each other in –z and +z positions.

5. The magnitude of magnetic induction at mid-point O due to current arrangement as shown in Fig will be :



(1) $\frac{\mu_0 I}{2\pi a}$ (2) 0

(3) $\frac{\mu_0 I}{4\pi a}$ (4) $\frac{\mu_0 I}{\pi a}$

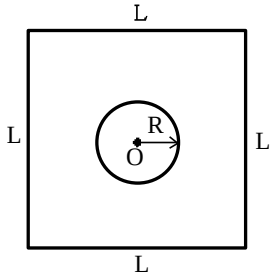
Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Magnetic field due to current in BC and ET are outward at point 'O'

$$B_0 = \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4\pi r} = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{\pi a}$$

6. Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side L (L >> R). The loops are coplanar and their centres coincide :



(1) $M = \frac{\sqrt{2}\mu_0 R^2}{L}$ (2) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$

(3) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$ (4) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $\phi = Mi$

$\phi = (BA)$

$$\phi = \pi R^2 \left(4 \frac{\mu_0}{4\pi} \frac{i}{\left(\frac{L}{2}\right)} \sqrt{2} \right)$$

$$\Rightarrow M = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

7. Which of the following are true?

A. Speed of light in vacuum is dependent on the direction of propagation.

B. Speed of light in a medium in independent of the wavelength of light.

C. The speed of light is independent of the motion of the source.

D. The speed of light in a medium is independent of intensity.

Choose the correct answer from the option given below :

(1) A and C only (2) B and D only

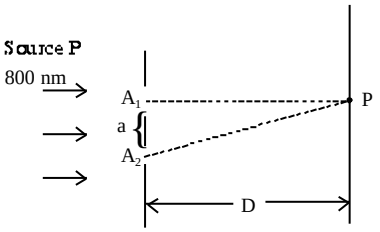
(3) B and C only (4) C and D only

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Speed of light does not depend on the motion of source as well as intensity.

8. In a Young's double slit experiment, two slits are illuminated with a light of wavelength 800 nm. The line joining A_1P is perpendicular to A_1A_2 as shown in the figure. If the first minimum is detected at P, the value of slits separation 'a' will be :



The distance of screen from slits D = 5 cm

(1) 0.4 mm (2) 0.5 mm

(3) 0.2 mm (4) 0.1 mm

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $A_2P - A_1P = \frac{\lambda}{2}$ (Condition of minima)

$$\sqrt{D^2 + a^2} - D = \frac{\lambda}{2}$$

$$D\left(1 + \frac{a^2}{D^2}\right)^{1/2} - D = \frac{\lambda}{2}$$

$$D\left(1 + \frac{1}{2} \times \frac{a^2}{D^2}\right) - D = \frac{\lambda}{2}$$

$$\frac{a^2}{2D} = \frac{\lambda}{2} \Rightarrow a = \sqrt{\lambda . D}$$

$$= \sqrt{800 \times 10^{-6} \times 50}$$

a= 0.2 mm

9. A stone is projected at angle 30º to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be :

(1) 1 : 2 (2) 1 : 4

(3) 4 : 1 (4) 4 : 3

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $$\frac{KE_{POP}}{KE_{top}} = \frac{\frac{1}{2}M\,(u)^2}{\frac{1}{2}M\,(u\cos 30^\circ)^2} = \frac{4}{3}$$

10. A block of mass m slides down the plane inclined at angle 30º with an acceleration $\frac{g}{4}$. The value of coefficient of kinetic friction will be :

(1) $\frac{2\sqrt{3}+1}{2}$ (2) $\frac{1}{2\sqrt{3}}$

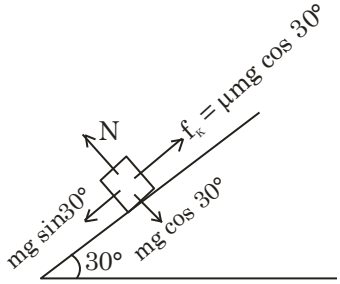
(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{2\sqrt{3}-1}{2}$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. Mg sin 30° - μmgcos 30° = ma

$$\frac{g}{2} - \frac{\sqrt{3}}{2}.\mu g = \frac{g}{4}$$



$$\frac{\sqrt{3}}{2}\mu = \frac{1}{4}$$

$$\mu = \frac{1}{2\sqrt{3}}$$

11. A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [Take g = 10 ms^{-2}]

(1) 3.4 ms^{-1} (2) 22.4 ms^{-1}

(3) 13 ms^{-1} (4) 17 ms^{-1}

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $$f_s = \frac{mv^2}{r}$$

For maximum speed in safe turning,

$f_s = f_s$ max $= \mu mg$

v_{max} (for safe turning $= \sqrt{\mu rg}$

$= \sqrt{0.34 \times 50 \times 10} \approx 13$ m/s

12. Two particles of equal mass 'm' move in a circle of radius 'r' under the action of their mutual gravitational attraction. The speed of each particle will be :

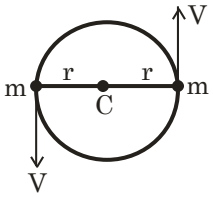
(1) $\sqrt{\frac{GM}{2r}}$ (2) $\sqrt{\frac{4GM}{r}}$

(3) $\sqrt{\frac{GM}{r}}$ (4) $\sqrt{\frac{GM}{4r}}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $$\frac{Gm^2}{4r^2} = \frac{mv^2}{r}$$



$$v = \sqrt{\frac{Gm}{4r}}$$

13. Surface tension of a soap bubble is 2.0×10^{-2} Nm^{-1}. Work done to increase the radius of soap bubble from 3.5 cm to 7 cm will be : [Take $\pi = \frac{22}{7}$]

(1) 0.72×10^{-4} J (2) 5.76×10^{-4} J

(3) 18.48×10^{-4} J (4) 9.24×10^{-4} J

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Surface area of soap bubble = $2 \times 4\pi R^2$

Work done = change in surface energy × T_S

$$= T_S \times 8\pi \times \left(R_2^2 - R_1^2\right)$$

$$= 2 \times 10^{-2} \times 8 \times \frac{22}{7} \times 49 \times \frac{3}{4} \times 10^{-4}$$

$= 18.48 \times 10^{-4}$ J

14. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : If dQ and dW represent the heat supplied to the system and the work done on the system respectively. Then according to the first law of thermodynamics $dQ = dU - dW$.

Reason R : First law of thermodynamics is based on law of conservation of energy.

In the light of the above statements, choose the correct answer from the option given below :

(1) A is correct but R is not correct

(2) A is not correct but R is correct

(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is not the correct explanation of A

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. First law of thermodynamics is based on law of conservation of energy and it can be written as $dQ = dU - dW$.

where dW is work done on the system

15. A bicycle tyre is filled with air having pressure of 270 kPa at 27ºC. The approximate pressure of the air in the tyre when the temperature increases to 36ºC is

(1) 270 kPa (2) 262 KPa

(3) 278 kPa (4) 360 kPa

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Taking volume constant : $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\Rightarrow P_2 = \frac{P_1}{T_1} \times T_2 = \frac{270 \times (309)}{300}$$

$$= 278\,\text{kPa}$$

16. A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of 30 m/s. If both trains emit sounds with frequency 300 Hz, (Speed of sound : 330 m/s) approximate difference of frequencies heard by the person will be :

(1) 33 Hz (2) 55 Hz

(3) 80 Hz (4) 10 Hz

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $f_1 = 300\left(\frac{330-0}{330-(-30)}\right) = 275$

$$f_2 = 300\left(\frac{330-0}{330-(30)}\right) = 330$$

$\Delta f = 330 - 275 = 55$ Hz.

17. If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be :

Given : Earth's radius = 6.4×10^6 m.

(1) 32 km (2) 28 km

(3) 36 km (4) 64 km

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Maximum line of sight distance between two antennas, $d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$d_M = 2 \times \sqrt{2 \times 6.4 \times 10^6 \times 80} = 64 \text{ km}$$

18. The threshold wavelength for photoelectric emission from a material is 5500Å. Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a

A. 75 W infra-red lamp

B. 10 W infra-red lamp

C. 75 W ultra-violet lamp

D. 10 W ultra-violet lamp

Choose the correct answer from the options given below :

(1) B and C only (2) A and D only

(3) C only (4) C and D only

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\lambda < 5500$ Å for photoelectric emission

$\lambda_{uv} < 5500$Å

19. If a radioactive element having half-life of 30 min is undergoing beta decay, the fraction of radioactive element remains undecayed after 90 min. will be :

(1) $\frac{1}{8}$ (2) $\frac{1}{16}$

(3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\frac{N}{N_0}=\left(\frac{1}{2}\right)^{t/t\frac{1}{2}}=\left(\frac{1}{2}\right)^{\frac{90}{30}}$

$\frac{N}{N_0}=\left(\frac{1}{2}\right)^3=\frac{1}{8}$

20. Which of the following statement is not correct in the case of light emitting diodes?

A. It is a heavily doped p-n junction.

B. It emits light only when it is forward biased.

C. It emits light only when it is reverse biased.

D. The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.

Choose the correct answer from the options given below :

(1) C and D (2) A

(3) C (4) B

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. LED works in forward biasing and light energy maybe slightly less or equal to band gap.

SECTION-B

21. A radioactive element $^{242}_{92}X$ emits two α-particles, one electron and two positrons. The product nucleus is represented by $^{234}_{P}Y$. The value of P is ____

Official Ans. by NTA (87)

Allen Ans. (87)

Sol. $P=92-2-2+1-1-1$

$P=92-5$

P = 87

22. Two simple harmonic waves having equal amplitudes of 8 cm and equal frequency of 10 Hz are moving along the same direction. The resultant amplitude is also 8 cm. The phase difference between the individual waves is _____ degree.

Official Ans. by NTA (120)

Allen Ans. (120)

Sol. $2A\cos\left(\frac{\Delta\phi}{2}\right)=A$

$\cos\left(\frac{\Delta\phi}{2}\right)=\frac{1}{2}$

$\frac{\Delta\phi}{2}=60^\circ$

23. A body cools from 60°C to 40°C in 6 minutes. If, temperature of surroundings is 10°C. Then, after the next 6 minutes, its temperature will be ____ °C.

Official Ans. by NTA (28)

Allen Ans. (28)

Sol. By average form of Newton's law of cooling

$\frac{20}{6}=k(50-10)$ … (i)

$\frac{40-T}{6}=K\left(\frac{40+T}{2}-10\right)$ ….(ii)

From equation (i) and (ii)

$\frac{20}{40-T}=\frac{40}{10+T/2}$

$10+\frac{T}{2}=80-2T$

$\frac{5T}{2}=70\Rightarrow T=28^\circ C$

24. A solid sphere of mass 2kg is making pure rolling on a horizontal surface with kinetic energy 2240 J. The velocity of centre of mass of the sphere will be ____ ms^{-1}.

Official Ans. by NTA (40)

Allen Ans. (40)

Sol. $KE=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$

$2240=\frac{1}{2}2(v)^2+\frac{1}{2}\frac{2}{5}(2)R^2.\left(\frac{v}{R}\right)^2$

$2240=v^2+\frac{2}{5}v^2$

$\Rightarrow v=40\,m/s$

25. A 0.4 kg mass takes 8s to reach ground when dropped from a certain height 'P' above surface of earth. The loss of potential energy in the last second of fall is ____ J. [Take g = 10 m/s^2]

Official Ans. by NTA (300)

Allen Ans. (300)

Sol. Displacement is 8^{th} sec.

$S_8=0+\frac{1}{2}\times10\times(2\times8-1)$

$S_8=5\times15$

$\Delta U=0.4\times10\times5\times15$

$\Delta U=20\times15=300$

26. A tennis ball is dropped on to the floor from a height of 9.8 m. It rebounds to a height 5.0 m. Ball comes in contact with the floor for 0.2s. The average acceleration during contact is ____ ms^{-2}. [Given g = 10 ms^{-2}]

Official Ans. by NTA (120)

Allen Ans. (120)

Sol. $v_i = \sqrt{2gh_i}$

$= \sqrt{2\times10\times9.8}\downarrow$

$= 14 m/s\downarrow$

$v_f = \sqrt{2gh_f}$

$= \sqrt{2\times10\times5}\uparrow$

= 10 m/s↑

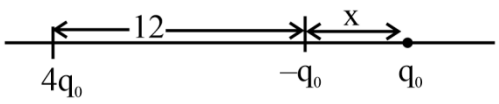
$\left|\vec{a}_{avg}\right| = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{24}{0.2} = 120\, m/s^2$

27. A point charge $q_1 = 4q_0$ is placed at origin. Another point charge $q_2 = -q_0$ is placed at x = 12 cm. Charge of proton is q_0. The proton is placed on x-axis so that the electrostatic force on the proton in zero. In this situation, the position of the proton from the origin is _____ cm.

Official Ans. by NTA (24)

Allen Ans. (24)

Sol.



$\frac{q_0}{x^2} = \frac{4q_0}{(x+12)^2}$

$x+12 = 2x$

$x = 12$

Distance from origin = x + 12 = 24 cm.

28. In a metre bridge experiment the balance point in obtained if the gaps are closed by 2Ω and 3Ω. A shunt of $X\Omega$ is added to 3Ω resistor to shift the balancing point by 22.5 cm. The value of X is ____

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\frac{2}{\left(\frac{3x}{3+x}\right)} = \frac{40+22.5}{60-22.5} = \frac{62.5}{37.5} = \frac{5}{3}$

$\frac{6}{5} = \frac{3x}{3+x}$

$6+2x = 5x \Rightarrow x = 2$

29. A certain elastic conducting material is stretched into a circular loop. It is placed with its plane perpendicular to a uniform magnetic field B = 0.8 T. When released the radius of the loop starts shrinking at a constant rate of 2 cm^{-1}. The induced emf in the loop at an instant when the radius of the loop is 10 cm will be ____ mV.

Official Ans. by NTA (10)

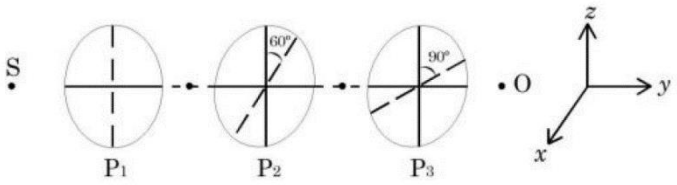
Allen Ans. (10)

Sol. $EMF = \frac{d}{dt}\left(B\pi r^2\right)$

$= 2B\pi r\frac{dr}{dt} = 2\times\pi\times0.1\times0.8\times2\times10^{-2}$

$= 2\pi\times1.6$ **= 10.06 [round off 10.06 = 10]**

30. As shown in figures, three identical polaroids P_1, P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at angle of 60º and 90º with respect to axis of P_1. The source S has an intensity of $256\frac{W}{m^2}$. The intensity of light at point O is $-----\frac{W}{m^2}$.



Official Ans. by NTA (24)

Allen Ans. (24)

Sol. By first polaroid P1 intensity will be halved then P2 and P3 will make intensity $\cos^2(60°)$ and $\cos^2(30°)$ times respectively.

Intensity out = $\frac{256}{2}\times\frac{1}{4}\times\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{256\times3}{2\times4\times4}$ **= 24**