Sri Chaitanya IIT Academy.,India.

✪ A.P ✪ T.S ✪ KARNATAKA ✪ TAMILNADU ✪ MAHARASTRA ✪ DELHI ✪ RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_STERLING BT	RPTA-01	Date: 10-08-2025
Time: 09:00AM to 12:00PM	JEE-ADV_(2021-P1)	Max. Marks: 180

KEY SHEET

PHYSICS

1	**D**	2	**B**	3	**A**	4	**C**	5	**90**	6	**12**
7	**31**	8	**36**	9	**3**	10	**1**	11	**BD**	12	**BD**
13	**ABD**	14	**CD**	15	**ABCD**	16	**ABC**	17	**5**	18	**27**
19	**6**										

CHEMISTRY

20	**D**	21	**B**	22	**A**	23	**B**	24	**5**	25	**12**
26	**5**	27	**8**	28	**1**	29	**3**	30	**BC**	31	**ABD**
32	**ABD**	33	**BCD**	34	**ABCD**	35	**ACD**	36	**13**	37	**4**
38	**7**										

MATHEMATICS

39	**C**	40	**B**	41	**A**	42	**D**	43	**9**	44	**14**
45	**13**	46	**6**	47	**2**	48	**2025**	49	**ABC**	50	**AC**
51	**ABCD**	52	**ABD**	53	**AC**	54	**AB**	55	**6**	56	**3**
57	**0**										

SOLUTIONS

PHYSICS

01. We have $V=\frac{4}{3}\pi R^3$ differentiating w.r.t 'R'

$\frac{dV}{dR}=4\pi R^2\ldots\ldots\ldots(1)$

From definition of 'α'; $\alpha=\frac{dR}{R(\Delta\theta)}$......(2)

$\therefore dV=4\pi R^3\alpha(\Delta T)$

02. $W=\int pdV+\int pdV=0+\int pdV$

Cylinder Balloon Balloon

$=101.3\,kPa\times 0.6\,m^3=60.795\,kJ=60.8\,kJ$

AS, balloon pushes the atmosphere

$\therefore$ Work done by the atmosphere $=-60.8\,kJ$

03. $PV=RT;\left(P_0-\alpha V^2\right)V=RT$

For maximum temperature $\frac{dT}{dV}=0$

$\frac{dT}{dV}=\left(\frac{P_0}{R}-\frac{\alpha}{R}\times 3V^2\right)=0\Rightarrow V=\left(\frac{P_0}{3\alpha}\right)^{1/2}$

T is maximum at $V=\left(\frac{P_0}{3\alpha}\right)^{1/2}$

$\therefore\left(P_0-\alpha\times\frac{p_0}{3\alpha}\right)\left(\frac{p_0}{3\alpha}\right)^{1/2}=RT_{\max}\Rightarrow T_{\max}=\frac{2P_0}{3R}\times\left(\frac{P_0}{3\alpha}\right)^{\frac{1}{2}}$

04. Diatomic $\gamma=7/5;$

$\Delta Q:\Delta U:\Delta W=\gamma:1:(\gamma-1)$

05. Speed of train

$v=\left(\frac{75}{60}click/\sec\right)(20m)\left(\frac{18}{5}\right)=90km/hr$

06. $\Delta L=L\alpha\Delta T=(20m)\left(12\times10^{-6}\right)(50)=12mm$

7. F$\rightarrow$G is an isothermal process

For an isothermal process, PV = constant

$\therefore P_F V_F = P_G V_G \Rightarrow V_G = \dfrac{P_F V_F}{P_G}$

Here, $P_F = 32P_0, V_F = V_0, P_G = P_0$

$\therefore V_G = \dfrac{32P_0V_0}{P_0} = 32V_0$

F $\rightarrow$H is an adiabatic process.

For an adiabatic process, $PV^{\gamma} =$constant

$\therefore P_F V_F^{\gamma} = P_H V_H^{\gamma} \Rightarrow V_H = \left(\dfrac{P_F}{P_H}\right)^{1/\gamma} V_F$

Here, $\gamma = \dfrac{5}{3}, P_F = 32P_0$

$V_F = V_0, P_H = P_0 \therefore V_H = \left(\dfrac{32P_0}{P_0}\right)^{3/5} V_0 = (25)^{3/5} V_0 = 8V_0$

G$\rightarrow$E is an isobaric process.

Work done during an isobaric process

$W = P(V_f - V_i)$

$\therefore W_{GE} = P_0(V_0 - 32V_0) = -31P_0V_0$

G$\rightarrow$H is also an isobaric process.

$\therefore W_{GE} = P_0(8V_0 - 32V_0) = -24P_0V_0$

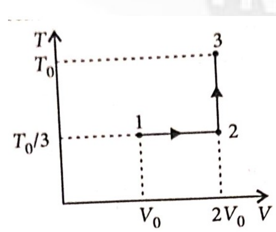
08. F $\rightarrow$H is an adiabatic process

Work done during an adiabatic process is

$W = \dfrac{P_f V_f - P_i V_i}{1-\gamma}$

$\therefore W_{FH} = \dfrac{(P_0)(8V_0)-(32P_0)(V_0)}{1-\dfrac{5}{2}} = \dfrac{-24P_0V_0}{-2/3} = 36P_0V_0$

09.



Process $1 \to 2$ is isothermal (temperature constant)

Process $2 \to 3$ is isochoric (volume constant)

(1) Work done in $1 \to 2 \to 3$ is isochoric (volume constant)

$$W = W_{1\to 2} + W_{2\to 3} = nRT\,1n\left(\frac{V_f}{V_i}\right) + 0 = \frac{RT_0}{3}1n\left(\frac{2V}{V_0}\right) = \frac{RT_0}{3}1n(2)$$

10. Change in internal energy (ΔU) in $1 \to 2 \to 3$,

$$\Delta U = \frac{f}{2}nR\left(T_f - T_i\right) = \frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right)$$

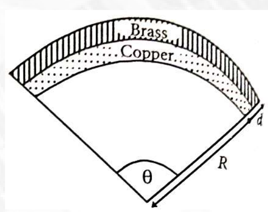
$$= \frac{3}{2}R \times \frac{2T_0}{3} = RT_0 \to (R)$$

For any system, first law of thermodynamics, for $1 \to 2 \to .3$,

$$\Delta Q = \Delta U + W = RT_0 + \frac{RT_0}{3}1n2$$

$$= \frac{RT_0}{3}(3 + 1n2)$$

11. Co-efficient of linear expansion of brass is greater than that of copper i.e. $\alpha_B > \alpha_C$.



$$\therefore L_B = L_0\left(1 + \alpha_B \Delta T\right)$$

Or $(R + d)\theta = L_0\left(1 + \alpha_B \Delta T\right)$

Again, $L_C = L_0\left(1 + \alpha_C \Delta T\right)$

Or $R\theta = L_0\left(1 + \alpha_C \Delta T\right)$

$$\therefore \frac{(R+d)\theta}{R\theta} = \frac{1 + \alpha_B \Delta T}{1 + \alpha_C \Delta T}$$

Or $\frac{R+d}{R} = \left(1 + \alpha_B \Delta T\right)\left(1 - \alpha_C \Delta T\right)$, by binomial theorem

Or $1 + \frac{d}{R} = 1 + \left(\alpha_B - \alpha_C\right)\Delta T -$ Smaller terms

Or $\frac{d}{R} = \left(\alpha_B - \alpha_C\right)\Delta T \; or \; R = \frac{d}{\left(\alpha_B - \alpha_C\right)\Delta T}$

$\therefore R \propto \frac{1}{\Delta T}$.Option (b) is correct

and $R \propto \frac{1}{|\alpha_B - \alpha_C|}$.Option (d) is correct.

12. For ideal gases, $C_P - C_V == R$

$\frac{C_P}{C_V} = \gamma$

$C_P = \left(1 + \frac{f}{2}\right)$

Monoatomic gas $C_V = \frac{3}{2}R; C_P = \frac{5}{2}R, \gamma = \frac{5}{3}$

Diatomic gas $C_V = \frac{5}{2}R; C_P = \frac{7}{2}R, \gamma = \frac{7}{5}$

(a) $C_P - C_V = R$, for all gases

(b) $C_P + C_V$ for diatomic gas $= \frac{12}{2}R = 6R$,

$C_P + C_V$ for monoatomic gas $= \frac{3}{2}R + \frac{5}{2}R = 4R$

$\therefore (b)$ is correct

(c) $C_P / C_V = \gamma = 1 + \frac{2}{f}$

For monoatomic gas, $\gamma = 1 + \frac{2}{3}$

For diatomic gas, $\gamma = 1 + \frac{2}{5}$

$\therefore$ For diatomic gas, it is less than that of monoatomic gas

(d) $C_P . C_V$ for monoatomic gas $= \frac{15}{4}R^2 = 3.75R^2$

For diatomic gas $\frac{35}{4}R^2 = 8.75R^2$

$\therefore (d)$ is correct

13. For hydrogen, $n_1 = 1, C_{V_1} = \frac{5}{2}R$

For helium, $n_2 = 1, C_{V_2} = \frac{3}{2}R$

For mixture of gases, $C_V = \dfrac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$$= \frac{1\times\frac{5}{2}R + 1\times\frac{3}{2}R}{1+1}$$

$C_P = C_V + R = 3R, \gamma_{mix} = \dfrac{C_p}{C_V} = \dfrac{3}{2}$

Also, $\gamma_{mix} = 1 + \dfrac{2}{f} \Rightarrow \dfrac{3}{2} = 1 + \dfrac{2}{f} \therefore f = 4$

$\therefore$ Average energy per mole $= \dfrac{1}{2} fRT = 2RT$

$M_{mix} = \dfrac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \dfrac{1\times 2 + 1\times 4}{1+1} = 3\, g/mol$

Speed of sound in a gas, $v = \sqrt{\dfrac{\gamma RT}{M}}$

For a given value of T, $v \propto \sqrt{\dfrac{\gamma}{M}}$

rms speed of a gas molecule at temperature T is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

For agiven value of T, $v_{rms} \propto \dfrac{1}{\sqrt{M}}$

$\dfrac{(v_{rms})He}{(v_{rms})H_2} = \sqrt{\dfrac{M_{H_2}}{M_{He}}} = \sqrt{\dfrac{2}{4}} = \dfrac{1}{\sqrt{2}}$

So, option $(a),(b)\, and\, (d)$ are corrects

14. Surface tension $= S, \gamma = 5/3$

$r_1 \to T_1 \to P_{a_1}$

$r_2 \to T_2 \to P_{a_2}$

For bubble the pressure is : $P_{gas} = P_a + \dfrac{4S}{r}$

As for isothermal condition

PV = Constant

So, $\left(P_{a_1} + \dfrac{4S}{r_1}\right)\left(\dfrac{4}{3}\pi r_1^3\right) = \left(P_{a_2} + \dfrac{4S}{r_2}\right)\left(\dfrac{4}{3}\pi r_2^3\right)$

$$\frac{r_1^3}{r_2^3}=\left(\frac{P_{a_2}+\frac{4S}{r_2}}{P_{a_1}+\frac{4S}{r_1}}\right)$$

Now, $P^{1-\gamma T\gamma}$ constant

$$\left(P_{a_2}+\frac{4S}{r_2}\right)^{1-\frac{5}{3}}T_2^{\frac{5}{3}}=\left(P_{a_1}+\frac{4S}{r_1}\right)^{1-\frac{5}{3}}T_1^{\frac{5}{3}}$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{5}{3}}=\left(\frac{P_{a_1}+\frac{4S}{r_1}}{P_{a_2}+\frac{4S}{r_2}}\right)^{\frac{-2}{3}};\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}}=\left(\frac{P_{a_2}+\frac{4S}{r_2}}{P_{a_1}+\frac{4S}{r_1}}\right)$$

15. The graph is a linear relation between $\frac{dN}{dV}$ and V

Area under the curve $(a)\, N=\int dN=\frac{aV_0}{2}$.......(1) $\therefore\ V_0=\frac{2N}{a}$

b) <V> = average velocity $<V>=\frac{\int V(dN)}{\int dN}$........(2)

but from graph $\frac{dN}{dV}=\left(\frac{a}{V_0}\right)V$ ....(2a)

$\therefore dN=\left(\frac{a}{V_0}\right)V(dV)$ ....(2b)

$$\therefore <V>=\frac{a}{NV_0}\int_0^{V_0}V^2(dV)=\frac{(V_0^3)a}{2V_0}$$

$$\therefore <V>=\frac{2V_0}{3}$$

C) $V_{rms}=\left[\frac{\int V^2(dN)}{\int dN}\right]^{1/2}$ ....(3)

From $(2a)\&(3)$, $V_{rms}=\frac{V_0}{\sqrt{2}}$

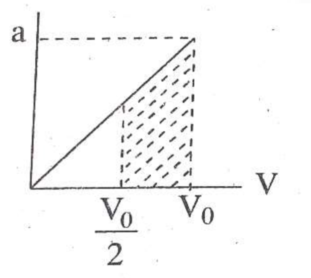
d) from equation (2a), shaded area

$$\int dN=\Delta N=\int_{V_0}\left(\frac{a}{V_0}\right)V(dV)=\frac{a}{V_0}\left(\frac{V^2}{2}\right)_{V_0/2}^{V_0}$$

$$= \frac{a}{2V_0}\left(\frac{3V_0^2}{4}\right)$$

$$= \frac{3aV_0}{8} = \frac{3N}{4}$$

$$\therefore \Delta N_X = \frac{3N}{4}$$



16. $dQ = -dU$

$$\therefore\ nCdT = -nC_V dT$$

$$C = -C_V$$

$$C = -\frac{R}{(\gamma-1)}$$

b) $dQ = dU + dW = -dU$

$$\therefore\ \ 2dU + dW = 0 \quad .....(i)$$

$$\therefore\ \ 2nC_V dT + pdV = 0$$

$$\Rightarrow\ 2n\left(\frac{R}{\gamma-1}\right)dT + pdV = 0$$

$$\Rightarrow \frac{2nRdT}{\gamma-1} + \frac{nRT}{V}.dV = 0\ \left(p = \frac{nRT}{V}\right)$$

Dividing this equation by nRT, we have

$$\frac{2}{\gamma-1}\frac{dT}{T} + \frac{dV}{V} = 0$$

Integrating, We get

$$\frac{2}{\gamma-1}\ln T + \ln V = C$$

$$\ln\left[T^{\frac{2}{\gamma-1}}V\right] = C$$

$$T^{\frac{2}{\gamma-1}}V = e^C = \text{ constant}$$

We can also write

$$TV^{\frac{\gamma-1}{2}} = \text{constant}$$

c) $TV^{\frac{\gamma-1}{2}} =$ constant

$$\therefore \qquad T_f V_f^{\frac{\gamma-1}{2}} = T_i V_i^{\frac{\gamma-1}{2}}$$

$$\Rightarrow \qquad T_f = \left(\frac{V_i}{V_f}\right)^{\frac{\gamma-1}{2}} T_i$$

$$= \left(\frac{1}{\eta}\right)^{\frac{\gamma-1}{2}} T_0 = \eta^{\frac{1-\gamma}{2}} T_0$$

Now from equation (i), we have

$$W = -2\Delta U$$

$$= -2nC_V \Delta T$$

$$= -2(1)\left(\frac{R}{\gamma-1}\right)\left(T_f - T_i\right)$$

$$= \frac{2R}{\gamma-1}\left(T_i - T_f\right) = \frac{2R}{\gamma-1}\left[T_0 - \eta^{\frac{1-\gamma}{2}} T_0\right]$$

Thus, $W = \dfrac{2RT_0}{\gamma-1}\left(1 - \eta^{\frac{1-\gamma}{2}}\right)$

17. $A_F \Delta l = V_0 \gamma_{app} \Delta t$

$$\Delta l = \frac{V_0\left(\gamma_R - 3\alpha\right)t}{A_0\left(1+\beta t\right)}$$

18. If calorimeter is closed then heat gained = heat lost

$5\,S(80-30) + 5L = W(110-80) \quad ......(i)$

Where, S = Specific heat of liquid

L = Latent heat of liquid,

W = Water equivalent of calorimeter

Now, when 80 g liquid is poured, and equilibrium temperature is 50^0C, then

Heat gained = heat lost

$80 \times S \times (50-30) = (W \times 30)$

$80 \times S \times 20 = (W \times 30)$ ……... (ii)

Now, from equation (i) and (ii) we get, L/S = 270°C

19. Heat lost by water at 20°C = $ms_W \Delta T$

$H = 5 \times 1 \times 20 = 100\,kcal$

At first, ice at (-20°C) will take heat to change into ice at 0°C.

$H = ms_{ice}\Delta T = (2\,kg) \times 0.5 \times 20 = 20\,kcal$

$\therefore$ After this, heat available $= (100 - 20) = 80\,kcal$

This heat will now be gained by ice at 0°C to melt into water at 0°C. Let m kg of ice melt

$\therefore m \times 80 = 80 \therefore m = 1\,kg.$

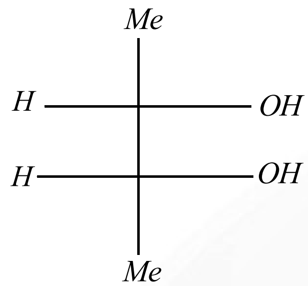
Out of 2 kg of ice, 1 kg of ice melts into water and 1 kg of ice remains unmelted in container

$\therefore$ Amount of water in container $= 5 + 1 = 6\,kg.$

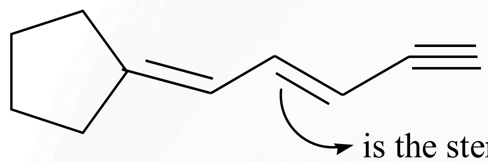
CHEMISTRY

20. A, B, C molecular formula $-C_8H_{14}$, D-molecular formula $-C_8H_{12}$

21.



22.



is the stereogenic centre

23. Pure (+) enantiomer $[\alpha]_+ = +8.4^0$

Observed specific rotation $[\alpha]_{obj} = +3.2^0$

Enantiomeric excess (ee)=$\dfrac{[\alpha]_{obj}}{[\alpha]_{pure}}\times 100$

$$= \frac{3.2}{8.4}\times 100$$
$$= 38.095\%$$

$P_{(+)} + P_{(-)} = 100\% \qquad \longrightarrow (1)$

$P_{(+)} - P_{(-)} = ee$

$P_{(+)} - P_{(-)} = 38.095\% \qquad \longrightarrow (2)$

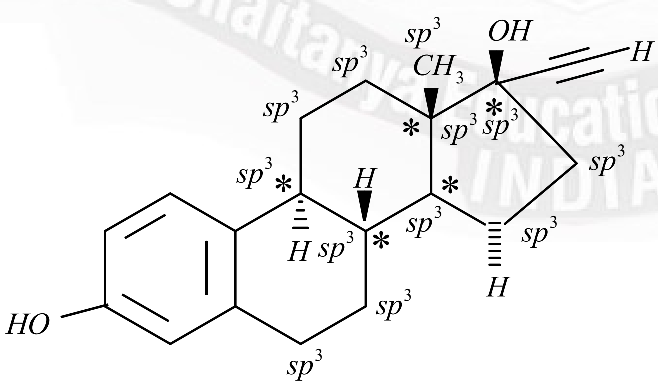
On solving (1) & (2)

$2P_{(+)} = 138.095$

$P_{(+)} = 138.095 \ P_{(+)} = 69.04$

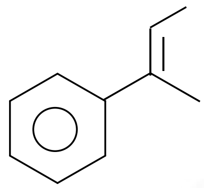
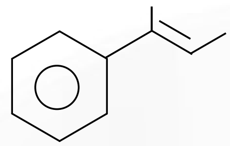
$\therefore P_{(-)} = 30.95\%$

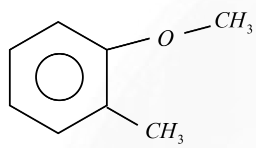
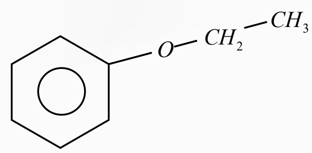
24 & 25



26. conceptual

27. conceptual

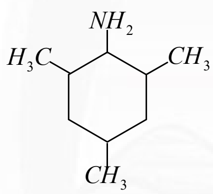
28.   identical

29.   metamers

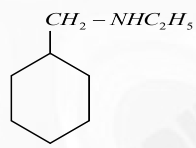
30. A & B are identical; (B & C) (A & C) are diastereomers

31. IUPAC rules

32.



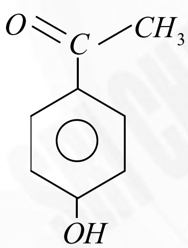
I



IV

I & IV are functional isomers

33.



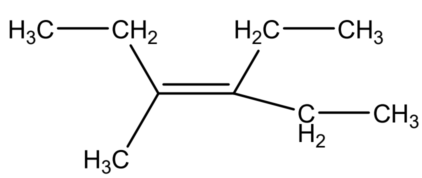
molecular formula $C_8H_8O_2$

34.

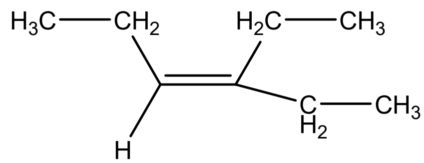
A) $\underset{F}{H_2C}-\underset{F}{CH_2}$ B) $H_3C-CH_2-CH_2-Cl$

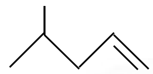
C) $\underset{OH}{CH_2}-\underset{OH}{CH_2}$ D) $\underset{COOH}{CH_2}-CH_2-COOH$

35. A)



C)

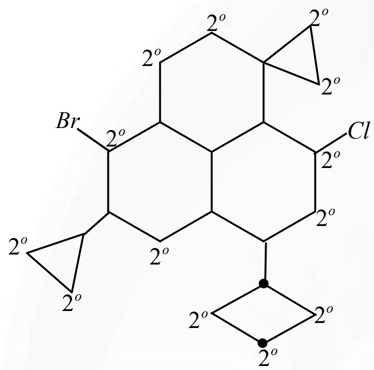




No chiral carbon

D)

36.



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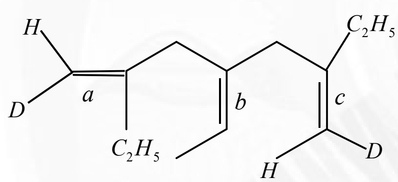


38. $C_5H_{10}O$

1) $CH_3-CH_2-CH_2-CH_2-CHO$

$$\begin{array}{c}CH_3-CH_2-CH-CHO\\ \qquad\quad\ |\\ \qquad\quad CH_3\end{array}$$

2)

$$\begin{array}{c}CH_3-CH-CH_2-CHO\\ |\qquad\qquad\\ CH_3\qquad\qquad\end{array}$$

3)

$$\begin{array}{c}CH_3\\ |\\ H_3C-C-CHO\\ |\\ CH_3\end{array}$$

4)

$$\begin{array}{c}CH_3-C-CH_2-CH_2-CH_3\\ ||\qquad\qquad\\ O\qquad\qquad\end{array}$$

5)

$$\begin{array}{c}CH_3-C-CH-CH_3\\ ||\quad ||\\ O\quad CH_3\end{array}$$

6)

$$\begin{array}{c}CH_5-CH_2-C-CH_2-CH_3\\ ||\\ O\end{array}$$

7)

MATHEMATICS

39. As function is symmetrical about x=30 and has exactly three real roots, implies one root must be 30 and others must be 30-d and 30+d respectively.

Hence $\alpha+\beta+\gamma=30+(30-d)+(30+d)=90$

40. $\left|[|x|]-1\right|>5 \Rightarrow \left|[|x|]\right|>6 \Rightarrow \left|[|x|]\right|\ge 7$

$\Rightarrow \left|[|x|]\right|\ge 7 \Rightarrow |x|\ge 7 \Rightarrow x\in(-\infty,-7]\cup[7,\infty)$

41. $[x+1]=0 \Rightarrow 0\le x+1<1 \Rightarrow x\in[-1,0)$

42. Domain of $\cos^{-1}x$ is $D_1=[-1,1]$

Domain of $\operatorname{cosec}^{-1}x$ is $D_2=(-\infty,-1]\cup[1,\infty)$

$\therefore$ Domain of $f(x)=D_1\cap D_2=\{-1,1\}$

$\therefore f(-1)=\cos^{-1}(-1)+\operatorname{cosec}^{-1}(-1)=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

$f(1)=\cos^{-1}(1)+\operatorname{cosec}^{-1}(1)=0+\frac{\pi}{2}=\frac{\pi}{2}$

$\therefore$ range of $f(x)=\left\{\frac{\pi}{2}\right\}$

43.& 44. Given, $f(x)+2f\left(\frac{1}{x}\right)+3f\left(\frac{x}{x-1}\right)=x-----(i)$

Replacing x by $\frac{1}{x},\frac{x}{x-1},\frac{x-1}{x},\frac{1}{1-x},1-x$ respectively (i), we get

$f\left(\frac{1}{x}\right)+2f(x)+3f\left(\frac{1}{1-x}\right)=\frac{1}{x}-----(ii)$

$f\left(\frac{1}{x-1}\right)+2f\left(\frac{x-1}{x}\right)+3f(x)=\frac{1}{x-1}-----(iii)$

$f\left(\frac{x-1}{x}\right)+2f\left(\frac{x}{x-1}\right)+3f(1-x)=\frac{x-1}{x}-----(iv)$

From (i), (ii), (iii), (iv), (v) and (vi);

$f\left(\frac{1}{1-x}\right)+2f(1-x)+3f\left(\frac{1}{x}\right)=\frac{1}{1-x}-----(v)$

$f(1-x)+2f\left(\frac{1}{1-x}\right)+3f\left(\frac{n-1}{n}\right)=1-x-----(vi)$

We get $f(x)=\frac{2x^3+x^2+5x-2}{24x(x-1)}$

45 & 46.

$$Tr = \tan^{-1}\left(\frac{3}{9r^2+3r-1}\right) = \tan^{-1}\left(\frac{(3r+2)-(3r-1)}{1+(3r-1)(3r+2)}\right)$$

$$Tr = \tan^{-1}(3r+2) = \tan^{-1}(3r-1)$$

$$S = \sum Tr$$

$$= \tan^{-1}(5) - \tan^{-1}(2) + \tan^{-1}(8) - \tan^{-1}(5) + \tan^{-1}(32) - \tan^{-1}(29)$$

$$S = \tan^{-1}(32) - \tan^{-1}(2) = \tan^{-1}\left(\frac{6}{13}\right) = \cot^{-1}\left(\frac{13}{6}\right)$$

47 & 48.

From the graphs

49. $(A)\cos\left(\tan^{-1}\left(\tan(4-\pi)\right)\right) = \cos(4-\pi) = \cos(\pi-4) = -\cos 4 > 0$

$(B)\sin\left(\cot^{-1}\left(\cot(4-\pi)\right)\right) = \sin(4-\pi) = -\sin 4 > 0\,(as \sin 4 < 0)$

$(C)\tan\left(\cos^{-1}\left(\cos(2\pi-5)\right)\right) = \tan(2\pi-5) = -\tan 5 > 0\,(as \tan 5 < 0)$

$(D)\cot\left(\sin^{-1}\left(\sin(\pi-4)\right)\right) = \cot(\pi-4) = -\cot 4 < 0$

50. $\because (fof)(x)$ is one-one, onto

$\therefore f(x)$ is one-one, onto

51. $f(x) = \sqrt{\sin(\sin x)} + \sqrt{\sin(\cos x)}$

Calculation for domain :

$\sin(\sin x) \geq 0$ and $\sin(\cos x) \geq 0$

$\Rightarrow \sin x \geq 0$ and $\cos x \geq 0 \Rightarrow x \in \left[0, \frac{\pi}{2}\right] \Rightarrow x \in \bigcup_{n \in z}\left[2k\pi, 2k\pi + \frac{\pi}{2}\right]$

As $f(x)$ is periodic with period $'2\pi', f(x)$ is a many to one function

Calculation for range :

$$f'(x) = \frac{1}{2\sqrt{\sin(\sin x)}}\cos(\sin x)\cos x - \frac{1}{2\sqrt{\sin(\cos x)}}\cos(\cos x)\sin x$$

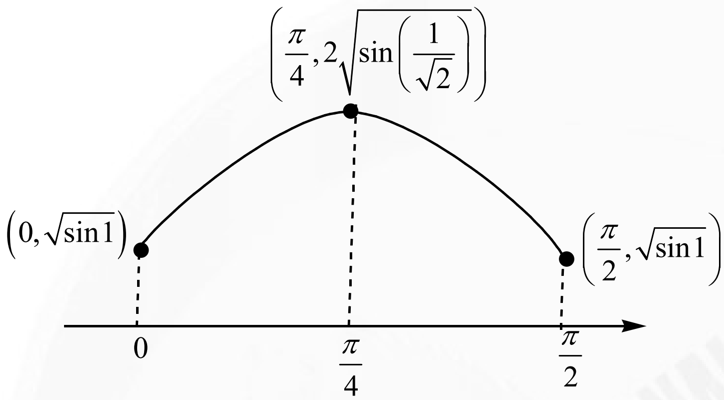
$$= \frac{\sqrt{\sin(\cos x)}\cos(\sin x)\cos x - \cos(\cos x)\sin x\sqrt{\sin(\sin x)}}{2\sqrt{(\sin x).\sin(\cos x)}}$$

For $x \in \left(0, \frac{\pi}{4}\right), \cos x > \sin x \Rightarrow \cos(\sin x) > \cos(\cos x)\, and \sin(\cos x) > \sin(\sin x)$

$\Rightarrow f'(x) > 0 \ \forall x \in \left(0, \frac{\pi}{4}\right)$

Similarly $f'(x) < 0 \, \forall x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Now graph of $f(x)$ is



$\Rightarrow$ Range : $\left[\sqrt{\sin 1}, 2\sqrt{\sin\frac{1}{\sqrt{2}}}\right]$

51. $f(x) = \frac{x^2+4x+3}{x^2+7x+14}, g(x) = \frac{x^2-5x+10}{x^2+5x+20}$

As $x^2+5x+20>0$, $x^2-5x+10>0$ and $x^2-7x+14>0 \ \forall x \in R$

$\Rightarrow g(x) > 0 \forall x \in R$ and $g(x)f(x)$ is well defined for all $x \in R$

$g(x) = \frac{x^2-5x+10}{x^2+5x+10} = 1 - 10\frac{x+1}{x^2+5x+20}$

$\Rightarrow g(x) \le 1 \forall x \ge -1$ and $g(x) \ge 1 \forall x \le -1$

Maximum of $f(x)$ is 2

As $f(x) \le 2 \Leftrightarrow \frac{x^2+4x+3}{x^2+71x}+14 \le 2$

$\Leftrightarrow (x+5)^2 \ge 0$

And it attained at x=-5

Also we can see that max of g(x) is 3

(Observe $g(x) \le 3 \Leftrightarrow (x+5)^2 \ge 0$)

Which attained at x=-5

$\Rightarrow \Rightarrow \max g(x) f(x) = 3^2 = 9$, at x=-5

52. $\therefore y=(x-a)^2+a \Rightarrow x=a+\sqrt{y-a}$

$f^{-1}(x)=a+\sqrt{x-a}$

$f(x)=f^{-1}(x) \Rightarrow (x-a)^4=x-a \Rightarrow x=a$ or a+1

Other solution is 5043 or 5045.

53. $\tan^{-1}x+\tan^{-1}\frac{1}{y}=\tan^{-1}3$

$\Rightarrow \frac{xy+1}{y-x}=3 \Rightarrow y=\frac{3x+1}{3-x} \therefore y>0, x>0$

$\Rightarrow 3-x>0 \Rightarrow 0<x<3 \,\&\, x\in I$

So x=1,2⇒y=2,7

54. $A=\frac{-\pi}{12},\ B=\frac{3\pi}{4},\ 0<C<\frac{\pi}{4}$

55. $f(1)+f(2)+........+f(n)=f(n).f(n+1)\forall n\in N$

For $n=1, f(1)=f(1), f(2) \Rightarrow f(2)=1$

Similary $f(4)=2$, $f(6)=3$, -----, $f(2n)=n$

Hence $f(2)+f(4)+.....+f(100)=1+2+----+50=\frac{50(51)}{2}=1275$

$S(S(1275))=S(15)=6$

56. $f(x)=[x]+2008\frac{\{x\}}{2008}$ $\qquad \because \{x+r\}=\{x\}$

$=x$

$\therefore f(3)=3$

57. Taking $f(n)=g(n)2^{n^2}$, recursion transforms to $g(n+2)-2g(n+1)+g(n)=n.16^{-n-1}$

Summing from 0 to n-1,

$g(n+1)-g(n)=\frac{1-(15n+1)16^{-n}}{15^2}$

$g(n)=\frac{15n-32+(15n+2)16^{-n+1}}{15^3}$

So $f(n)=\frac{1}{15^3}\left(15n+2+(15n-32)16^{n-1}\right)2^{(n-2)^2}$