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A right Choice for the Real Aspirant

ICON Central Office – Madhapur – Hyderabad

MATHS : AREAS _ PYQ -2025 (ORIGINAL & REPLICAS)

1. The area (in sq. units) of the region $\{(x,y):0<y<2|x|+1, 0\le y\le x^2+1, |x|\le 3\}$ is **[28-01-25 S-1]**

a) $\frac{80}{3}$ b) $\frac{64}{3}$ c) $\frac{17}{3}$ d) $\frac{32}{3}$

2. The area of the region $A=\{(x,y):0\le y\le x|x|+1 \text{and} -1\le x\le 1\}$ in sq. units in

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) 2 d) $\frac{4}{3}$

3. The area of region $A=\{(x,y):(x-1)[x]\le y\le 2\sqrt{x}, 0\le x\le 2\}$ where [x] is G.I.F.

a) $\frac{8}{3}\sqrt{2}-\frac{1}{2}$ b) $\frac{4}{3}\sqrt{2}+1$ c) $\frac{8}{3}\sqrt{2}-1$ d) $\frac{4}{3}\sqrt{2}-\frac{1}{2}$

4. Let $f:R\to R$ be a twice differentiable function such that $f(x+y)=f(x)f(y)$ for all $x,y\in R$. If $f'(0)=4a$ and f satisfies $f''(x)-3af'(x)=0, a>0$, then the area of the region $R=\{(x,y)\,|\,0\le y\le f(ax), 0\le x\le 2\}$ is **[22-01-25, SHIFT-01]**

1) e^2+1 2) e^4-1 3) e^4+1 4) e^2-1

5. Let $f:R\to R$ be any differentiable functions such that $f(x+y)=f(x)f(y)\ \forall x,y\in R$. If $f^1(0)=3$ then area of the region $R=\{(x,y):x\le y\le f(x), 0\le x\le 2\}$ is

1) e^2+1 2) $\frac{e^2+7}{3}$ 3) $\frac{e^2-7}{3}$ 4) e

6. Given that f(x) is differentiable function of x such that f(x).f(y) = f(x)+f(y)+f(xy)-2 And f(2) = 5 then area of region R = {(x,y) : f(x) ≤ y ≤ x+1}is

1) $\frac{1}{6}$ 2) $\frac{2}{3}$ 3) $\frac{2}{3}$ 4) $\frac{4}{5}$

7. A function satisfies the relation $f(x+y)=f(x)+f(y)+xy(x+y), \forall x,y\in R$ $f^1(0)=-1$, then area of region $R=\{(x,y):0\le y\le f(x), -1\le x\le 2\}$ is

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{5}$

8. Let the area enclosed between the curves $|y|=1-x^2$ and $x^2+y^2=1$ be α. If $9\alpha=\beta\pi+\gamma$; β,γ are integers, then the value of $|\beta-\gamma|$ equals **[29-01-25-SHIFT-02]**

1) 27 2) 18 3) 15 4) 33

9. Given two closed curves $C_1:|x|=1-y^2$ and $C_2:|y|=1-x^2$. The area of the region which is outside the curve C_1 but inside the curve C_2 is equal to

1) $\frac{10\sqrt{5}-22}{3}$ 2) $\frac{5\sqrt{5}-11}{3}$ 3) $\frac{5\sqrt{5}-11}{6}$ 4) $\frac{20\sqrt{5}-44}{3}$

10. Given two closed curves $C_1:|x|+|y|=2$ and $C_2:|x|=2-y^2$. If the area of the region which is outside the curve C_1 but inside the curve C_2 is equal to α/β where α and β are coprime natural numbers then the value of $\alpha^\beta+\beta^\alpha$ is:

1) 4 2) 17 3) 57 4) 145

11. Let the function

$$f(x)=\begin{cases}-3ax^2-2, & x<1\\ a^2+bx, & x\geq 1\end{cases}$$

Be differentiable for all $x\in R$, where a>1, $b\in R$. If the area of the region enclosed by $y=f(x)$ and the line $y=-20$ is $\alpha+\beta\sqrt{3}, \alpha, \beta\in Z$, then the value of $\alpha+\beta$ is ______

[22-01-25, shift-01]

12. $f(x)=\begin{cases}e^{x^2+x}, & x>0\\ ax+b, & x\leq 0\end{cases}$ is differentiable at x= 0, If the area of region enclosed by y=f(x), & line y=-10, x=-10, and x-axis is $10\alpha+\beta, \alpha, \beta\in N$, value of $\alpha+\beta$ is__

$[1\leq\alpha,\beta\leq 9]$

13. If $f(x)=\begin{cases}ax^2+1, & x\leq 1\\ x^2+ax+b, & x>1\end{cases}$ is differentiable at x = 1, Area of the region y=f(x), x=2, and y=20 is α then $[\alpha]$ is $[.]$ $isG.I.F$.

14. Let the area of the region $\{(x,y): 2y\leq x^2+3, y+|x|\leq 3, y\geq|x-1|\}$ be A. Then 6A is equal to:

1)16 2)14 3)12 4) 4)18

[29-01-25-SHIFT-01]

15. Let the area of the region $\{(x,y): y\leq x^2+3, y+|x|\leq 4, y\geq|x-2|\}$ be P then P is equal to

1) $\frac{5\sqrt{5}+7}{6}$ 2) $\frac{43-5\sqrt{5}}{6}$ 3) $\frac{\sqrt{5}-7}{6}$ 4) $\frac{43+5\sqrt{5}}{6}$

16. Let the area of the region $\{(x,y)/ y\leq x^2+4, y+|x|\leq 6, y\geq|x-1|\}$ be P then 6P is equal to

1) 91 2) 92 3) 93 4) 94

17. The area of the region $\{(x,y): x^2+4x+2\leq y\leq|x+2|\}$ is equal to **[24-01-25-SHIFT-01]**

1)7 2) $\frac{24}{5}$ 3)5 4) $\frac{20}{3}$

18. If the area of the region bounded by the set of points

$S=\{(x,y): x^2+2nx+n^2-n\leq y\leq|x+n|, n\in N\}$ is of the form $\frac{(f(n))^{\frac{3}{2}}+g(n)+2}{6}$,

then $\lim_{n\to\infty}\frac{f(n)}{g(n)}=$ 1) $\frac{4}{3}$ 2) $\frac{2}{3}$ 3) $\frac{8}{3}$ 4) $\frac{1}{3}$

19. If the area of the larger portion bounded between the curves $x^2+y^2=25$ and $y=|x-1|$ is $\frac{1}{4}(b\pi+c), b, c \in \mathbb{N}$ then b + c is equal to ____ **[23-01-25, SHIFT-01]**

20. Let a be the Area of the larger region bounded by the curve $y^2=8x$ and the lines $y=x$ and $x=2$. Which lies in the first quadrant then the value of 30 is equal to ____

21. If the smaller area bounded between the curve $|x|^2+|y|^2=1$ and the line $x+y=1$ is $\frac{1}{4}(6\pi-c)$, b, c$\in N$ then $b+c$ is equal to

22. If the area bounded by the curves $x^2+y^2=15$, $4y=|4-x^2|$ and $x=0$ above x-axis is $b+c\sin^{-1}\left(\frac{4}{5}\right)$ then $b+c=$

23. If the Area of the portion bounded between the curves $x^2+y^2=1$ and the curve $|x|+|y|=1$ is $b\pi-c$ then $b+c=$

24. If the Area of the region bounded the curves $y=2^x$ and $y=|x+1|$ in the first quadrant is $\frac{K}{l}-\frac{1}{\log m}$ then $k+l+m=$___

25. The area of the region bounded by the curves $x(1+y^2)=1$ and $y^2=2x$ is: **[28-01-25, shift-02]**

1) $\frac{\pi}{2}-\frac{1}{3}$ **2)** $\frac{\pi}{4}-\frac{1}{3}$ **3)** $\frac{1}{2}\left(\frac{\pi}{2}-\frac{1}{3}\right)$ **4)** $2\left(\frac{\pi}{2}-\frac{1}{3}\right)$

26. The area bounded by the curves $x(1+y^2)=2$ and $x=y^2$ is

1) $\pi-\frac{2}{3}$ **2)** $\pi+\frac{2}{3}$ **3)** $\pi-\frac{1}{3}$ **4)** $2\left(\pi-\frac{2}{3}\right)$

27. The area of the region enclosed by the curves $y=e^x, y=|e^x-1|$ and y-axis is

1) $2\log_e 2-1$ 2) $1-\log_e 2$ 3) $1+\log_e 2$ 4) $\log_e 2$

[24-01-25-SHIFT-02]

28. The area of the region enclosed by the curves $y=e^x$, $y=|e^x-1|$ and $x=1$ is

1) $2\log_e^2-1$ 2) $2-\log_e^2$ 3) $1+\log_e^2$ 4) $\log_e^2$

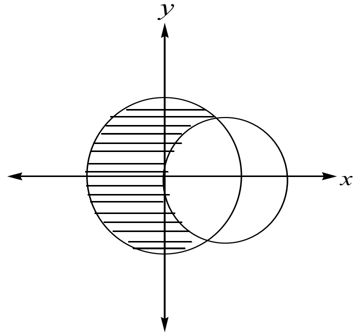
29. The area of the bounded by the curves $y=|2^x-1|$, $y=2-x$ and x-axis is

1) $\frac{1}{\log 2}+\frac{1}{2}$ 2) $\frac{1}{\log 2}-\frac{1}{2}$ 3) $\frac{1}{\log 2}+\frac{3}{2}$ 4) $\log 2-\frac{1}{2}$

30. The area of the region, inside the circle $\left(x-2\sqrt{3}\right)^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is **[22-01-25, SHIFT-1]**

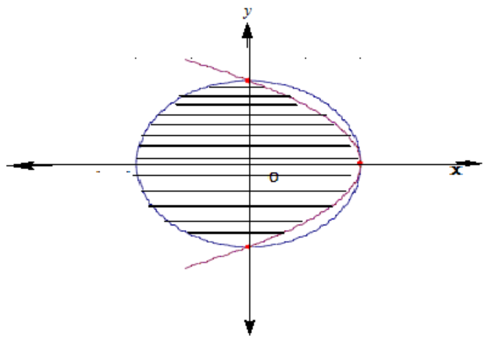
1) $6\pi-16$ 2) $3\pi+8$ 3) $3\pi-8$ 4) $6\pi-8$

31. The area of region enclosed between the curves $x^2+y^2\le 4$ and $\left(x-2\right)^2+y^2\ge 4$ is



1) $\frac{4\pi}{3}+2\sqrt{3}$ 2) $\frac{8\pi}{3}-\sqrt{3}$ 3) $\sqrt{3}-\frac{\pi}{3}$ 4) $\frac{8\pi}{3}-2\sqrt{3}$

32. The area of the portion of the circle $x^2+y^2=1$ which lies inside the parabola $y^2=1-x$ is



1) $\frac{\pi}{2}-\frac{2}{3}$ 2) $\frac{\text{[illegible]}}{2}+\frac{\text{[illegible]}}{3}$ 3) $\frac{\pi}{2}+\frac{4}{3}$ 4) $\frac{\pi}{2}-\frac{4}{3}$

33. Let the straight line x = b divide the area enclosed by $y=(1-x)^2, y=0$ and $x=0$ into two parts $R_1\left(0\le x\le b\right)$ and $R_2\left(b\le x\le 1\right)$ such that $R_1-R_2=\frac{1}{4}$ then b =

1) $\frac{3}{4}$ 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

34. Consider the region $R=\left\{\left(x,y\right): x\le y\le 9-\frac{11}{3}x^2, x\ge 0\right\}$ The area of the largest rectangle of sides parallel to the coordinate axes and inscribed in R, is :

1) $\frac{821}{123}$ 2) $\frac{730}{119}$ 3) $\frac{567}{121}$ 4) $\frac{625}{111}$

[24-01-25-SHIFT-01]

35. The area of the region $\left\{\left(x,y\right)/x^2\le y\le 8-x^2, y\le 7\right\}$ is

1) 21 2) 20 3) 18 4) 24

36. The area of the region $s=\left\{(x,y)/y^2\le 8x\,y\ge\sqrt{2}x,\ x\ge 1\right\}$ is

1) $\frac{13\sqrt{2}}{6}$ 2) $\frac{11\sqrt{2}}{6}$ 3) $\frac{5\sqrt{2}}{6}$ 4) $\frac{19\sqrt{2}}{6}$

37. If the area of the region $\left[(x,y):-1\le x\le 1, 0\le y\le a+e^{|x|}-e^{-x}, a>0\right]$ is $\frac{e^2+8e+1}{e}$,then the value of a is: [23-01-25, SHIFT-02]

1)8 2)5 3)7 4)6

38. If the area of the region $\left\{(x,y):-\alpha\le x\le\alpha, 0\le y\le\frac{9}{2}+e^{|x|}-e^{-x}\right\}$ is

1) 1 2) 2 3) 3 4) 4

39. The area of the figure bounded by the curves y = |x-1| and y = 3-|x| is

1) 4 2) 5 3) 6 4) 7

40. The area region ecclosed by the curves $y=x^2-4x+4\,and\,y^2=16-8x$ is:

1) $\frac{8}{3}$ 2) $\frac{4}{3}$ 3) 8 4) 5

41. The area enclosed by the curves $y^2+4x=4\,and\,y-2x=2$ above the x-axis is:

1) $\frac{23}{2}$ 2) $\frac{22}{3}$ 3) $\frac{7}{3}$ 4) $\frac{25}{3}$

42. If the area of region $\left\{(x,y):|4-x^2|\le y\le x^2, y\le 4, x\le 0\right\}$ is $\left(\frac{80\sqrt{2}}{\alpha}-\beta\right),\alpha,\beta\in N$, then $\alpha+\beta$ is equal to _____ (02-04-2025 S-I)

43. The area of the region $\left\{(x,y):x^2\le y\le|x^2-4|, y\ge 1\right\}$ is

1) $\frac{3}{4}(4\sqrt{2}-1)$ 2) $\frac{4}{3}(4\sqrt{2}-1)$ 3) $\frac{3}{4}(4\sqrt{2}+1)$ 4) $\frac{4}{3}(4\sqrt{2}+1)$

44. Find the area of the region $\left\{(x,y):x^2\le y\le|x|\right\}$is

45. If the area of the region $\left\{(x,y):|x-5|\le y\le 4\sqrt{x}\right\}$is A, then 3A is equal to _______

(04-04-2025 S-I)

46. Let two area of the region bounded by b/w curves $y^2=4x$ and |y|=x is A then 3A is equal to

47. The area of the region bounded by the curve $y=\max\left\{|x|, x|x-2|\right\}$, the x-axis and the lines $x=-2$ and $x=4$ is equal to _______ (03-04-2025 S-I)

48. Find the area bounded by the curve y=x|x|,x-axis and ordinates x=-3 and x= 3

49. The area of the region $\left\{(x,y):|x-y|\le y\le 4\sqrt{x}\right\}$ is (03-04-2025 S-II)

1) 512 2) $\frac{1024}{3}$ 3) $\frac{512}{3}$ 4) $\frac{2048}{3}$

50. Find the area of the region $\left\{(x,y): x^2+y^2\le 1\le x+y\right\}$ is

1) $\frac{\pi^2}{5}$ 2) $\frac{\pi^2}{2}$ 3) $\frac{\pi^2}{3}$ 4) $\frac{\pi}{4}-\frac{1}{2}$

51. If the area of the region $\left\{(x,y):1+x^2\le y\le \min\{x+7,11-3x\}\right\}$ is A, then 3A is equal to (07-04-2025 S-II)

1)50 2)46 3)47 4)49

52. If the area of two region $\{(x,y):0\le x\le 3\}$, $0\le y\le \min\ \{x^2+2,2x+2\}\}$ be A then 12A is

53. Let the area of the bounded region $\left\{(x,y):0\le 9x\le y^2, y\ge 3x-6\right\}$ be A. Then 6A is equal to _____

54. The area of the region described by $\{(x,y): y^2\le 2x\ \ and\ \ y>4x-1\}$ A then 32A is

PRACTICE SHEET-1 (Areas & Differential equations)

1. A continuous function $f:R\to R$ satisfies the differential equation $f(x)=\left(1+x^2\right)\left(1+\int_0^x \frac{f^2(t)}{1+t^2}dt\right)$, then $\left(f(-1)+f(-2)\right)$ is equal to

1) $\frac{207}{119}$ **2)** $\frac{217}{119}$ **3)** $\frac{227}{119}$ **4)** $\frac{119}{207}$

2. Let $f:R^+\to R$ be a differentiable function where $f(x)=e-(x-1)(\ln x-1)+\int_1^x f(t)dt$.

Then identify which of the following is not true?

1) $f(e)=1+e^e$

2) f increases on R^+

3) $f(x)=0$ has exactly one real root in $(0,\infty)$

4) f decreases on R^+

3. The curve C passes through $(2,2)$ in which the portion of tangent included between the coordinate axes is bisected at point of contact. The curve C passes through

1) $\left(2,\frac{1}{2}\right)$ **2)** $\left(3,\frac{4}{3}\right)$ **3)** $\left(3,\frac{3}{4}\right)$ **4)** $\left(2,\frac{1}{4}\right)$

4. Let the differential equation be $x\frac{dy}{dx}-y=x^2\left(xe^x+e^x-1\right)$ for all $x\in R-\{0\}$ such that $y(1)=e-1$. If $y(2)=Ky(1)\left(y(1)+2\right)$, then the value of K is

1) 1 2) 2 3) 3 4) 4

5. Let $f:[1,\infty)\to R$ be a differentiable function such that $f(1)=\frac{1}{3}$ and $3\int_1^x f(t)dt=xf(x)-\frac{x^3}{3},x\in[1,\infty)$. Let e denotes the base of the natural logarithm. Then the value of $f(e)$ is equal to

1) $\frac{e^2+4}{3}$ 2) $\frac{\log_e 4+e}{3}$ 3) $\frac{4e^2}{3}$ 4) $\frac{e^2-4}{3}$

6. Consider a rectangle ABCD formed by the points $A(0,0), B(6,0), C(6,4), D(0,4)$. P(x,y) is a moving interior point of the rectangle moving in such a way that $d(P,AB)\le\min\{d(P,BC),d(P,CD),d(P,AD)\}$

Here $d(P,AB), d(P,BC), d(P,CD)$ and $d(P,AD)$ represent the distances of the point P from the sides AB,BC,CD and AD respectively. Area of the region representing all possible positions of the point P is equal to

1) 8 sq. units 2) 4 sq. units 3) 12 sq. units 4) 6 sq. units

7. If $\frac{dy}{dx}=x+\int_0^1 y(x)dx=0,y(0)=1,$ then $3\int_0^1 y(x)dx=$

1) 7 2) 5 3) 2 4) 11

8. Area bounded by the curve $f(x)=\cos^{-1}(\cos x),0\le x\le 2\pi$ with the tangent to the curve $g(x)=|\cos x|$ at $x=\pi$ is λ sq. units. The value of $[\lambda]$ (where $[.]$ represents greatest integer function) is equal to

1) 4 2) 6 3) 5 4) 3

9. Consider functions $f(x)$ and $g(x)$, both defined from $R\to R$ and are defined as $f(x)=2x-x^2$ and $g(x)=x^n$ where $n\in N$, If the area between $f(x)$ and $g(x)$ is $\frac{3}{5}$ sq. units , then n is a divisor of

1) 28 2) 21 3) 35 4) 45

10. If the area bounded by the curve $f(x)=\cos^{-1}\left(\sin\left(\left\{\frac{\pi}{2}\right\}-[\sin x]\right)\right)$ and $x-axis$ from $x=0$ to $x=2\pi$ is $k\pi$ sq. units, then the value of $3k$ is ([.] is greatest integer function and {.} is fractional part function)

1) 6 **2)** 3 **3)** 12 **4)** 15

11. Let $f:R\to R$ be a twice differentiable function such that $f(x+y)=f(x)f(y)$ for all $x,y\in R$. If $f'(0)=4\alpha$ and f satisfies $f''(x)-3\alpha f'(x)-f(x)=0, \alpha>0$, then the area of the region $R=\{(x,y): 0\le f(\alpha x), 0\le x\le 2\}$ is $\beta e^{\gamma}-1$. Then the value of $2\alpha+\beta+\gamma=$

1) 3 **2)** 4 **3)** 5 **4)** 6

12. The area (in sq. units) of the region $\{(x,y): 0\le y\le 2|x|+1, 0\le y\le x^2+1, |x|\le 3\}$ is A, then the value of 3A is equal to

1) 17 **2)** 32 **3)** 64 **4)** 80

13. If $f(x)=x-1$ and $g(x)=|f(|x|)-2|$, then the area (in sq. units) bounded by $y=g(x)$ and the curve $x^2-4y+8=0$ is equal to

1) $\frac{4}{3}(4\sqrt{2}-5)$ **2)** $\frac{4}{3}(4\sqrt{2}-3)$ **3)** $\frac{8}{3}(4\sqrt{2}-3)$ **4)** $\frac{8}{3}(4\sqrt{2}-5)$

14. If $(x^2+4y^2+4xy)dy=(2x+4y+1)dx$ has a solution $y=\ln|(f(x,y))|-\frac{3}{2\sqrt{2}}\ln|(g(x,y))|+c$, then

	COLUMN-I		COLUMN-II
A)	$f(0,0)$	P)	$3-2\sqrt{2}$
B)	$g(0,0)$	Q)	$\sqrt{2}-1$
C)	$f(1,-1)$	R)	2
D)	$g(\sqrt{2},0)$	S)	-1

1) A: R, B:P, C:Q , D: S **2)** A: R, B:Q, C:S , D: P
3) A: P, B:Q, C:R , D: S **4)** A: R, B:P, C:S , D:Q

15. Let $f:[-1,1]\to\left[-\frac{1}{2},\frac{1}{2}\right]$ and $f(x)=\frac{x}{1+x^2}$. The area (in sq. units) bounded by $y=f^{-1}(x)$, $x-axis$ between the ordinates $x=-\frac{1}{2}$ and $x=\frac{1}{2}$ is

1) $\frac{1}{2}\ln e$ **2)** $\ln\left(\frac{e}{2}\right)$ **3)** $\ln e$ **4)** $\frac{1}{2}\ln\left(\frac{e}{2}\right)$

16. The area enclosed by the curves $y=\log_e\left(x+e^2\right), x=\log_e\left(\frac{2}{y}\right)$ and $x=\log_e 2$, above the line $y=1$ is $\left(1+\beta e+\gamma\log_e 2\right)$ where $\beta,\gamma\in I$, then $|\beta-\gamma|$ is equal to

1) 2 **2)** 3 **3)** 4 **4)** 5

17. The area of region $\left\{(x,y):xy\le 8, 1\le y\le x^2\right\}$ is $\left(\lambda\log_e 2-\frac{11+\alpha}{3}\right)$ sq. units, where $\lambda,\alpha\in N$. Then the value of $\left[\frac{\lambda}{\alpha}\right]$(where [.] represents greatest integer function) is equal to

1) 3 **2)** 5 **3)** 6 **4)** 4

18. Let S be the set of all points $x,y\in(0,1)$ such that $\left[\log_2\frac{1}{x}\right]$ and $\left[\log_5\frac{1}{y}\right]$ are both even (where [.] represents the greatest integer function). Then the area of the region S is equal to (in sq.units)

1) $\frac{4}{9}$ **2)** $\frac{5}{9}$ **3)** $\frac{2}{9}$ **4)** $\frac{8}{9}$

19. If $u=u(x)$ and $v=v(x)$ be differentiable functions for $x>0$ such that $xu'+v=0, xv'+u=0, u(1)=0, v(1)=2$, then $\int_0^1\frac{u}{v}dx=$

1) 1 **2)** $\frac{\pi}{4}$ **3)** $\frac{\pi}{2}-1$ **4)** $\frac{\pi}{4}-1$

20. **Statement-I** : Let $f: R \to R$, $f(x) = x + \sin x$. If $f^{-1}(x)$ is the inverse function of $f(x)$, then $\int_0^{\pi} f^{-1}(x)\,dx = \frac{\pi^2}{2} - 1$

Statement-II : The graph of $y = f^{-1}(x)$ is the image of the graph of $y = f(x)$ in the line $x - y = 0$

1) Statement-I is true, Statement-II is true
2) Statement-I is true, Statement-II is false
3) Statement-I is false, Statement-II is true
4) Statement-I is false, Statement-II is false

21. The slope of the tangent to a curve $C: y = y(x)$ at any point (x, y) on it is $\frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{e^{2\alpha}}{2}\right)$, then $e^{2\alpha}$ is equal to $\frac{l}{2}\left(\frac{11 + m\sqrt{2}}{11 - n\sqrt{2}}\right)$. The value of $(l + m + n) =$ ______________

22. Let $f:[1,4] \to R$ be a twice differentiable function on $(1,4)$ such that $f(x) = e^{-x}\int_3^x \left(3t^2 + 2t + 4f'(t)\right)dt$. If $f'(2) = \frac{\alpha e^{\beta} - 64}{\left(e^{\beta} - 4\right)^2}$, then $\frac{\alpha}{\beta}$ is equal to ______

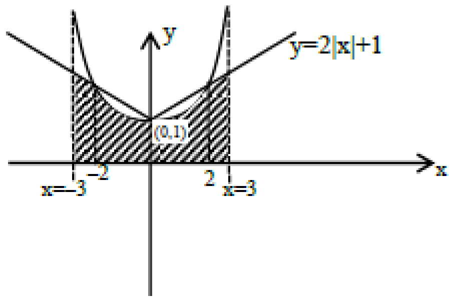
23. Let $g(x) = \int_x^{x^2 + \frac{\pi}{3}} 2\cos^2 \mu\, d\mu$ for all $x \in R$ and $f: \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continuous function. For $t \in \left[0, \frac{1}{2}\right]$, if $g'(t) + 4$ is the area of the region bounded by $x = 0, y = 0, y = f(x)$ and $x = t$, then $f(0)$ is ________

24. Let $f(x) = \frac{x^3}{24} + \frac{x^2}{8} + \frac{13x}{12} + 1$ and $g(x)$ is the inverse function of $f(x)$ .If the area bounded by the curve $g(x)$, the x-axis and the ordinates $x = -1$ and $x = 4$, is A sq. units, then the value of 3A is equal to

25. Let $f: R \to R$ be a thrice differentiable odd function satisfying $f'(x) \geq 0, f''(x) = f(x)$, $f(0) = 0, f'(0) = 2$, then the value of $6f\left(\log_e 2\right)$ is equal to

PYQ 2025 : KEY & SOLUTIONS

1. **Key**: b



Sol:

$$Area = 2\left[\int_0^2 (x^2+1)dx + \int_2^3 (2x+1)dx\right]$$

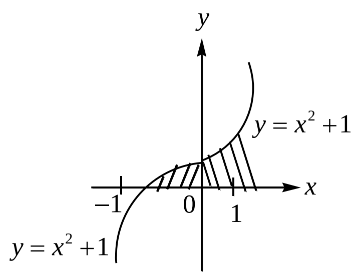
$$\Rightarrow \frac{64}{3} \therefore (2)$$

2. Key: c

Sol.: $\int_{-1}^{0} (-x^2+1)\,dx + \int_{0}^{1} (x^2+1)\,dx$

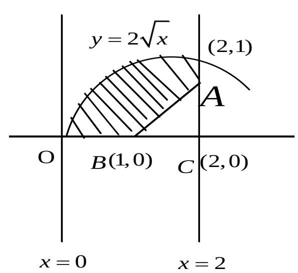
$= \left(\frac{-x^3}{3}+x\right)_0^{-1} + \left(\frac{x^3}{3}+x\right)_0^{1}$

$= 2$



3. Key: a

Sol.: $y \ge (x-1)[x]$, $y \le 2\sqrt{x}$



$y = \left\{2\sqrt{x},\ 0 < x < 2\right\}$

Area = Area of curve – Area of the Triangle ABC.

$= \int_0^2 2\sqrt{x}\,dx - \frac{1}{2}$

$= \left[\frac{2x^{3/2}}{3/2}\right]_0^2 - \frac{1}{2} = \frac{8}{3}\sqrt{2} - \frac{1}{2}$

4. Key : 4

Sol : $f(x) = p^x \Rightarrow f'(x) = p^x \log p$

$f^{'}(0) = p^0.\log p = 4a \Rightarrow p = e^{4a}$

$f(x) = e^{4ax}$

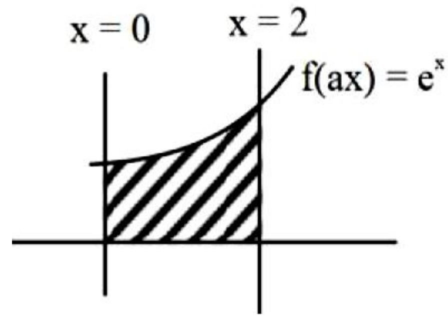
$f^{''}(x) - 3af^{'}(x) - f(x) = 0$

$16a^2e^{4ax} - 12a^2e^{4ax} = 0 \Rightarrow 4a^2 - 1 = 0 \Rightarrow a = \frac{1}{2}$

$f(x) = e^{2x}$

$0 \le y \le f(1/2) = e^x$

$R.A = \int_0^2 e^x.dx = e^2 - e^0 = e^2 - 1$



5. Key : 3

Sol: Let $f(x) = a^x \Rightarrow f^1(x) = a^x \log a$

$f^1(0) = 3$

$\Rightarrow \log a = 3 \Rightarrow a = e^3$

$f(x) = e^{3x}$

Req area = $\int_0^2 (e^{3x} - x)dx$

$$= \left(\frac{e^{3x}}{3} - \frac{x^2}{2}\right)_0^2 = \left(\frac{e^6}{3} - 2\right) - \left(\frac{1}{3}\right)$$

$$= \frac{e^6}{3} - \frac{7}{3}$$

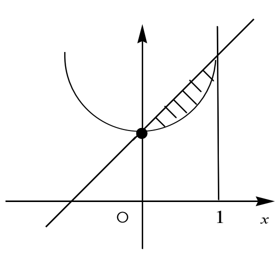
$$= \frac{e^6 - 7}{3}$$

6. Key : 1

Sol: We have $f(x)f(y) = f(x) + f(y) + f(xy) - 2$

$f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

$\Rightarrow f(x) = x^n + 1$



$\therefore f(x) = x^2 + 1$

$x^2+1=x+1$

$\Rightarrow x(x-1)=0, x=0, x=1$

$$A=\int_0^1 (x+1)-(x^2+1)=\left(\frac{x^2}{2}-\frac{x^3}{3}\right)_0^1$$

$$=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$$

7. Key : 2

Sol: $f(x+y)=f(x)+f(y)+xy(x+y)$

Diffrentiate w.r to x keeping 'y' constant

$f^1(x+y)=f^1(x)+2xy+y^2$

Pu t x= 0

$f^1(y)=f^1(0)+y^2$

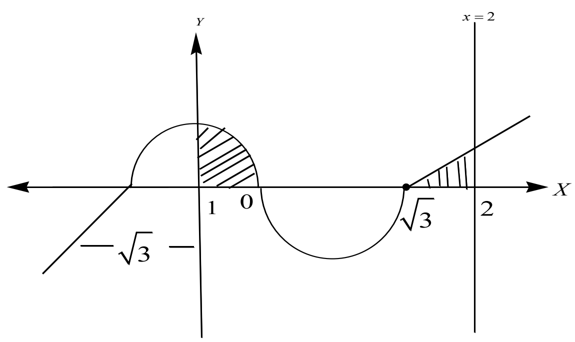
$f^1(x)=x^2-1$

$$f(x)=\frac{x^3}{3}-x+c\Rightarrow f(0)=0$$

$$f(x)=\frac{x^3}{3}-x$$

$y=0\Rightarrow x^3-3x=0$

$x=0, x=\pm\sqrt{3}$

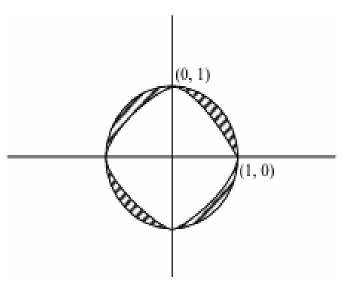


$$A=\int_{-1}^{0}\left(\frac{x^3}{3}-x\right)dx+\int_{\sqrt{3}}^{2}\left(\frac{x^3}{3}-x\right)dx$$

$$A=\frac{1}{3}units$$

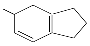
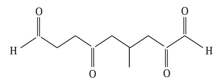
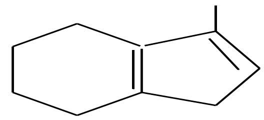
8. Key: 4

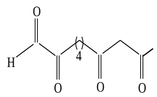
Sol.:



$C_1 : |y| = 1 - x^2$

$C_2 : x^2 + y^2 = 1 \quad \therefore$ Required Area.

=  [Area of circle in 1st quad. ] = 

 $\therefore \; 3\alpha = 3\pi - 8 \;\; \therefore \; 9\alpha = 9\pi - 24 \;\; \therefore \; \beta = 9, \gamma = -24$

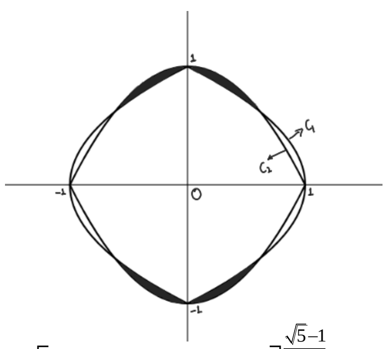
$\therefore \; |\beta - \gamma| = 33$

9. Key : 1

Sol. The given curves can be obtained by graphical transformations to standard parabolas. The required area is the shaded region in the figure.

Point of intersection of the parabolas in first quadrant is $\left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}-1}{2}\right)$

So required area $= 4\int_0^{\frac{\sqrt{5}-1}{2}} \left(1 - x^2 - \sqrt{1-x}\right) dx$



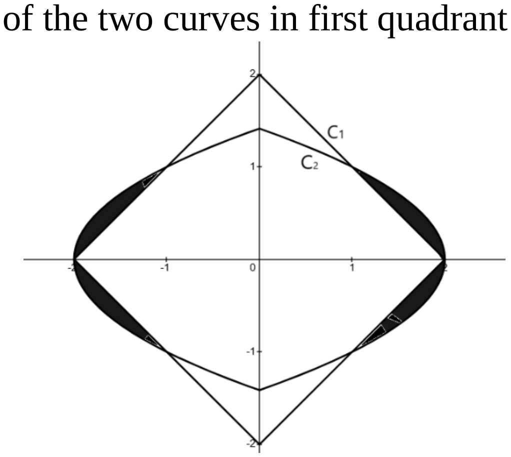
$$= 4\left[x - \frac{x^3}{3} - \frac{(1-x)^{3/2}}{3/2}\right]_0^{\frac{\sqrt{5}-1}{2}} = \frac{10\sqrt{5} - 22}{3}$$

10. Key : 2

Sol. The required area is the shaded region in the figure.

The point of intersection of the two curves in first quadrant is (1, 1), therefore, Required area

$= 4\int_1^2 \left(\sqrt{2-x} - (2-x)\right) dx = \frac{2}{3}$



11. Key: 34

Sol: f is continuous and differentiable at x = 1

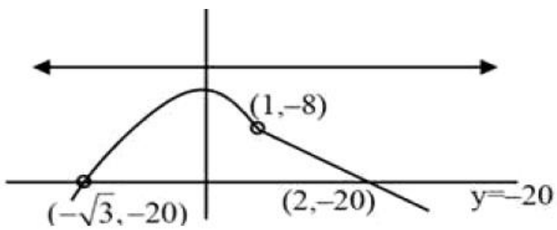
$-3a-2=a^2+b$ and $-6a=b$

$\Rightarrow a=1\ or\ 2$, $\because a>1, a=2, b=-12$

$$\therefore\ f(x)=\begin{cases}-6x^2-2, x<1\\ 4-12x,\ x\geq 1\end{cases}$$

$$R.A=\int_{-\sqrt{3}}^{1}\left[\left(-6x^2-2\right)-(-20)\right]dx+(Arae\ of\ \Delta ABC)$$

$$=(-2x^3+18x)_{-\sqrt{3}}^{1}+\frac{1}{2}(1)(12)=22+12\sqrt{3}=\alpha+\beta\sqrt{3}\qquad \therefore \alpha+\beta=22+12=34$$



12. Key: 9

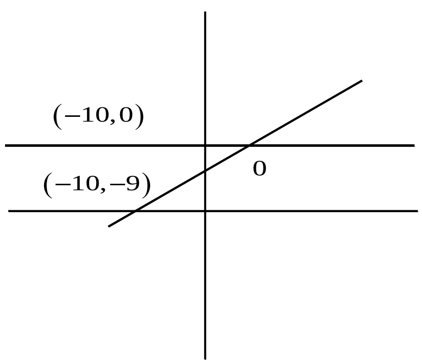
Sol: For f(x) to be continuous at x = 0 , we have

$$f(0^{-1})=f(0^{+})\Rightarrow 1+0=a(0)+b\Rightarrow b=1$$

$$f^1(0^-)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{e^{h^2+h}-b}{h}$$

$$=\lim_{h\to 0}\frac{e^{h^2+h}-1}{h}=\lim_{h\to 0}\frac{e^{h^2+h}-1}{h(h+1)}(h+1)=1$$

$\therefore f^1(0^+)=a,$ hence a = 1



required area is $\frac{1}{2}\times 10\times 9=45$

13. Key : 23

Sol: $f(x)=\begin{cases}ax^2+1\ x\leq 1\\ x^2+ax+b\ x>1\end{cases}$ is differentiable at x= 1 then f(x) is continuous at x= 1

$$f(1^+)=f(1^-)\Rightarrow a+1=1+a+b\Rightarrow b=0$$

Also $f(1^+) = f(1^-) \Rightarrow a+1 = 1+a+b \Rightarrow b=0$

We must have $f^1(1^-) = f^1(1^+) \Rightarrow 2a = 2+a \Rightarrow$ a= 2

Req. Area = $\int_2^4 \left(24 - \left(x^2+x\right)\right).dx = \frac{70}{3}$

14. Key:2

Solution:

$3 - x = \frac{x^2+3}{2} \Rightarrow x^2 + 2x - 3 = 0 \qquad \Rightarrow (x+3)(x-1) = 0 \qquad \Rightarrow x = 1, -3$

$y = -x+1 \;\&\; y = \frac{x^2+3}{2} \qquad \Rightarrow -x+1 = \frac{x^2+3}{2} \Rightarrow x^2 + 2x + 1 = 0$

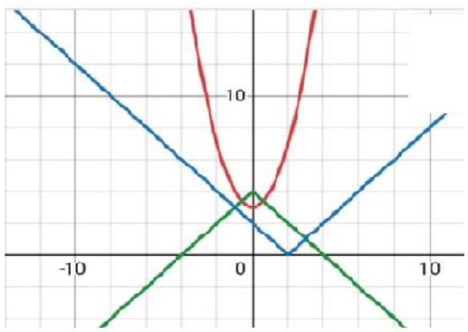
$\Rightarrow (x+1)^2 = 0 \qquad \Rightarrow x = -1 \qquad \Rightarrow (-1, 2)$

A = Area = $2\sqrt{2} \times \sqrt{2} - 2\left(\int_0^1 \left((3-x) - \left(\frac{x^2+3}{3}\right)\right)dx\right) \qquad = 4 - 2\left\{\frac{3x}{2} - \frac{x^2}{2} - \frac{x^3}{6}\Big|_0^1\right\}$

$= 4 - 2\left(\frac{3}{2} - \frac{1}{2} - \frac{1}{6}\right) \qquad = 4 - 2 \times \frac{5}{6} = 4 - \frac{5}{3} = \frac{7}{3} \qquad 6A = 6 \times \frac{7}{3} = 14$

15. Key: 2

Sol:



1. $y + |x| \le 4$

$x \ge 0 \Rightarrow y \le 4 - x$

$x < 0 \Rightarrow y < 4 - x$

2. $y \ge |x-2|$

$x \ge 2 \Rightarrow y \ge x - 2$

$x < 2 \Rightarrow y \ge 2 - x$

3. $y \le x^2 + 3$

$x^2 \ge y - 3$

Area = $3\sqrt{2} \times \sqrt{2} - 2\int_0^{\frac{\sqrt{5}-1}{2}} \left[4 - x - \left(x^2+3\right)\right]dx$

$= 6 - 2\int_0^{\frac{\sqrt{5}-1}{2}} \left[1 - x - x^2\right]dx$

$$= 6-2\left[x-\frac{x^2}{2}-\frac{x^3}{3}\right]_0^{\frac{\sqrt{5}-1}{2}}$$

$$= \frac{43-5\sqrt{5}}{6}$$

16. Key: 1

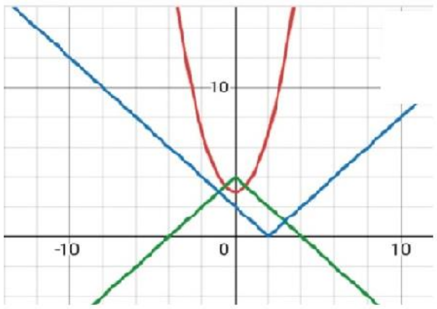
Sol:

$y \le x^2+4$ $\quad$ $y+|x| \le 6$ $\quad$ $y=|x-1|$

$x^2 \ge y-4$, $\quad$ $x \ge 0 \Rightarrow y \le 6-x$, $\quad$ $x \ge 1 \Rightarrow y \ge x-1$

$x<0 \Rightarrow y \le 6+x$ $\quad$ $x<1 \Rightarrow y \ge 1-x$

Area = $\frac{5}{\sqrt{2}}\times\frac{7}{\sqrt{2}}$ - $2\int_0^1\left[6-x-\left(x^2+4\right)\right]dx$



$$=35/2-2\int_0^1\left(2-x-x^2\right)dx$$

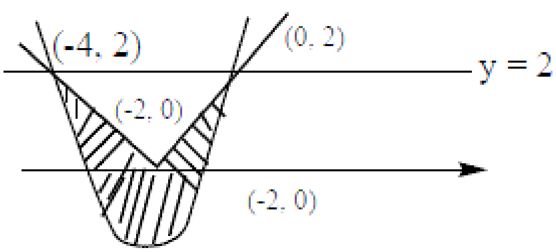
$$=35/2-2\left[2x-\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1$$

$$P = 35/2-2\left[2-\frac{1}{2}-\frac{1}{3}\right]$$

$$6P=91$$

17. Key:4

Solution:



$$\text{Re}.A = \int_{-4}^{0}\left(2-\left(x^2+4x+2\right)\right)dx-\frac{1}{2}\times 2\times 4$$

$$= -\frac{1}{3}\left(x^3\right)_{-4}^{0}-2\left(x^2\right)_{-4}^{0}-4 \quad = 32-\frac{64}{3}-4=\frac{20}{3}$$

18. Key:2

Solution: Translating origin to (-n,0), the given region is described by

$X^2-n \le y \le |X|$ So the area bounded symmetric about y-axis and is bounded between the curves y= X^2-n and y=$|X|$ between x=$-\frac{(1+\sqrt{4n+1})}{2}$ and x=$\frac{(1+\sqrt{4n+1})}{2}$. Therefore the area is

$$2\left(\int_0^{\frac{(1+\sqrt{4n+1})}{2}}\left(x-(x^2-n)\right)dx\right)=\frac{(4n+1)^{\frac{3}{2}}+6n+1}{6}$$

giving f(n)=4n+1 and g(n)=6n-2

19.

Key:77

Sol: $x^2+y^2=25$, $y=|x-1|$

By solving PI=(-3,4)(4,3)

$$Area=\int_{-3}^{4}\sqrt{25-x^2}-\frac{1}{2}.4.4-\frac{1}{2}3.3$$

$$=\left(\frac{x}{2}\sqrt{25-x^2}+25\sin^{-1}\frac{x}{5}\right)_{-3}^{4}-8-\frac{9}{2}$$

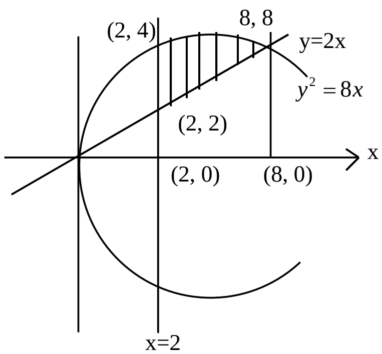
$$=\left(2.3+\frac{25}{2}\sin^{-1}\frac{4}{5}\right)-\left(-\frac{3}{2}.4-\frac{25}{2}\sin^{-1}\frac{3}{5}\right)-8-\frac{9}{2}$$

$$=6+6-8-\frac{9}{2}+\frac{25}{2}\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{3}{5}\right)=\frac{-1}{2}+\frac{25\pi}{4}$$

$$\text{Required area}=25\pi+\frac{1}{2}-\frac{25\pi}{4}=\frac{1}{4}(75\pi+2)$$

b+c=77

20. Key: 22



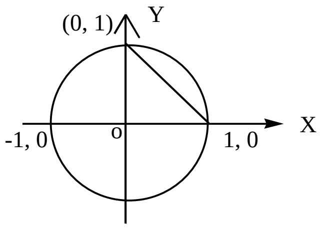
Sol: Solving we get (0, 0) (3, 3)

$x=2\,xy^2=3x \Rightarrow (2,\pm 4)$

$$\int_2^3\left(2\sqrt{2}\sqrt{x}-x\right)dx=\frac{22}{3}=a \Rightarrow 3a=22$$

21. Key: 3

Sol:



$$x^2+y^2=1 \Rightarrow y=\sqrt{1-x^2}$$

$$x+y=1 \Rightarrow y=1-x$$

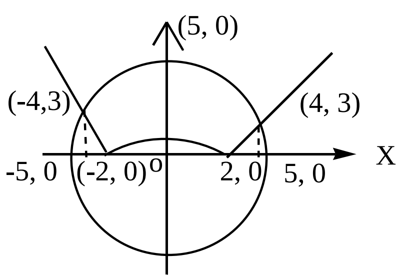
$$RA=\int_0^1 \sqrt{1-x^2}-(1-x)$$

$$=\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x+\frac{x^2}{2}-x\Big]_0^1$$

$$\frac{\sqrt{3}}{4}+\frac{1}{2}-1=\frac{\pi}{4}-\frac{1}{2}$$

22. Key:29

Sol:



$$x^2+y^2=25, 4y=\pm\left(4-x^2\right)$$

$$\Rightarrow 4y+4+y^2=25 \Rightarrow (y+2)^2=25 \Rightarrow y=3,-7$$

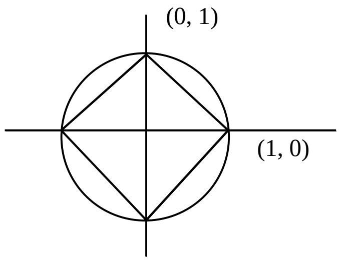
$$y=3 \Rightarrow x=\pm 4$$

$$=(4,3)(-4,3)$$

$$RA=2\left[\int_0^4 \sqrt{25-\lambda^2}\,dx-\frac{1}{4}\int_0^2\left(4-x^2\right)-\frac{1}{4}\int_2^4\left(x^2-\cos\right)dx\right]=4+25\sin^{-1}\left(\frac{4}{5}\right)$$

23. Key: 3

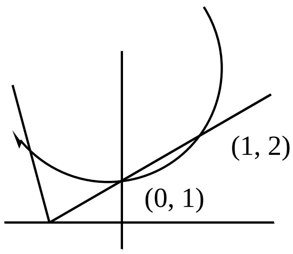
Sol:



$$\pi r^2 - 4\frac{1}{2}11 = \pi - 2$$

24. Key: 7

Sol:



$$RA = \int_0^1 (2+1) - 2^x\,dx = \frac{x^2}{2} + x - \frac{2^x}{\ln 2}\Bigg]_0^1 = \frac{3}{2} - \frac{1}{\ln 2}$$

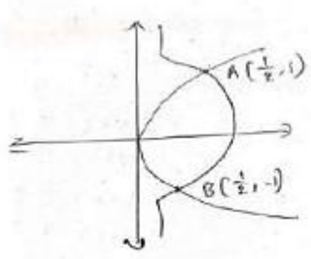
25. ANS: 1

SOL: Given curves

$$x\left(1+y^2\right) = 1 \ and \ y^2 = 2x, y^2 = \frac{1}{x} - 1$$

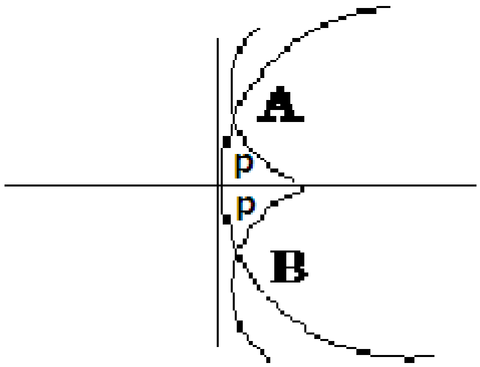
$$2x = \frac{1}{x} - 1 \Rightarrow x = \frac{1}{2}, -1(\text{Re}\ jected), y = \pm 1$$

Required Area $= \int_{-1}^{1}\left(\frac{1}{1+y^2} - \frac{y^2}{2}\right)dy = \tan^{-1} y - \frac{y^3}{6}\Bigg|_{-1}^{1} = \frac{\pi}{2} - \frac{1}{3}$



26. ANS: 1

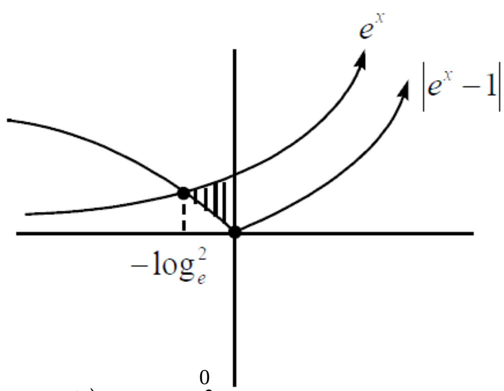
Sol:



use graph; req area = 2P

27. Key: 2

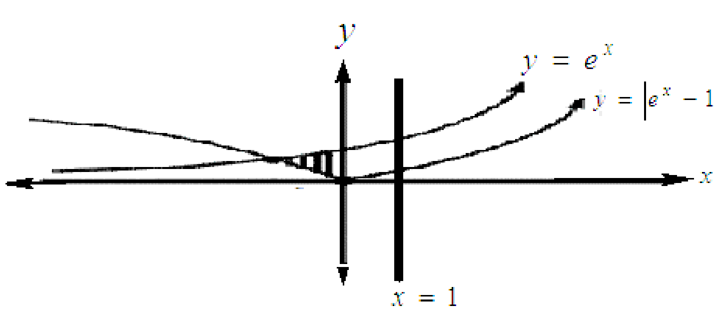
Sol: $y=e^{x}, y=|e^{x}-1|$



$$A=\int_{-\log_e^2}^{0}\left(e^{x}-\left(1-e^{x}\right)\right)dx=\int_{-\log_e^2}^{0}(2e^{x}-1)dx=2(e^{x})_{-\log_e^2}^{0}-(x)_{-\log_e^2}^{0}=1-\log_e^2$$

28. Key : 2

Sol:



$$A=\int_{-\log_e^2}^{0}\left(e^x-(1-e^x)\right)dx+\int_0^1(e^x-(e^x-1))dx$$

$$A=(1-\log_e^2)+(x)_0^1=2-\log_e^2$$

29. Key : 2

Sol:

$$A=\int_0^1(2^x-1)dx+\int_1^2(2-x)dx$$

$$A=\left(\frac{2^x}{\log 2}-x\right)_0^1+\left(2x-\frac{x^2}{2}\right)_1^2$$

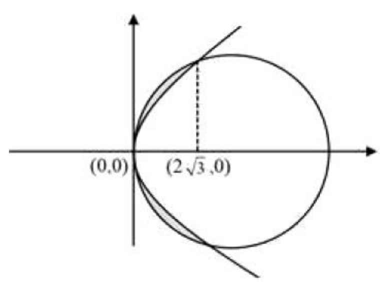
$$A=\left(\frac{2}{\log 2}-1\right)-\left(\frac{1}{\log 2}\right)+(4-2)-\left(2-\frac{1}{2}\right)$$

$$=\frac{1}{\log 2}-\frac{1}{2}$$

30. Key : 1

Sol: area $=2\left[\frac{1}{4}\pi\left(2\sqrt{3}\right)^2-\left(\int_0^{2\sqrt{3}}\sqrt{2\sqrt{3}}\times\sqrt{x}.dx\right)\right]$

$$=2\left[3\pi-\sqrt{2\sqrt{3}}\left[\frac{x^{3/2}}{3/2}\right]_0^{2\sqrt{3}}\right]=2\left[3\pi-\frac{2\sqrt{2\sqrt{3}}(2\sqrt{3})^{3/2}}{3}\right]=2\left[3\pi-\frac{3\times 2^3}{3}\right]=6\pi-16$$



31. Key : 1

Sol: P.I of (1) .(2)

X = 1

Req area = $\frac{1}{2}\pi(2)^2+2\left[\int_0^1\left(\sqrt{4-x^2}dx-\sqrt{4-(x-2)^2}dx\right)\right]$

$$=2\pi+2\left[\sqrt{3}-\frac{\pi}{3}\right]=\frac{4\pi}{3}+2\sqrt{3}$$

32. Key : 3

Sol: req area = $= 2\int_0^1 \sqrt{1-x} + 2\left(\frac{1}{4}\pi(1)^2\right)$

$$= \frac{\pi}{2} + \frac{4}{3}$$

33. Key: 2

Sol: $R_1 + R_2 = \int_0^1 (1-x)^2 dx = \frac{1}{3}$

$$R_1 - R_2 = \frac{1}{4}$$

$$2R_1 = \frac{1}{3} + \frac{1}{4}$$

$$\Rightarrow 2R_1 = \frac{7}{12}$$

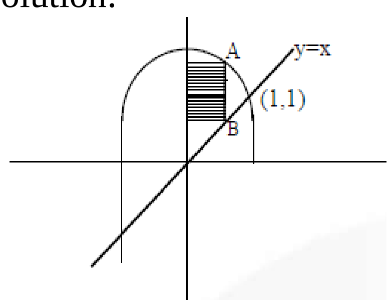
$$R_1 = \frac{7}{24}$$

$$\int_0^b (1-x)^2 dx = \frac{7}{24}$$

$$b = \frac{1}{2}$$

34. Key:3

Solution:



$$A = \left(t, 9 - \frac{11t^2}{3}\right)$$

$$B = (t,t)$$

$$A = Area = t\left(9 - \frac{11t^2}{3} - t\right) = 9t - t^2 - \frac{11t^3}{3}$$

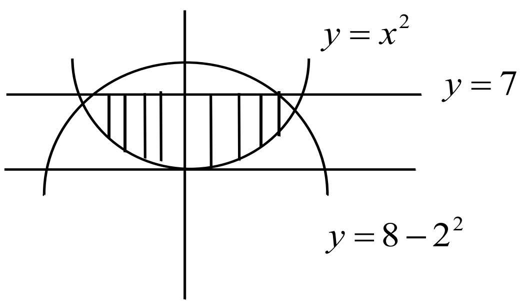
$$\frac{dA}{dt} = 9 - 2t - 11t^2 = 0$$

$\Rightarrow t=-1(or)\, t=\frac{9}{11}$

Largest Area we get at $t=\frac{9}{11}$

$A=\frac{9}{11}\left(9-\frac{11}{13}.\frac{81}{121}-\frac{9}{11}\right)=\frac{9}{11}\times\frac{63}{11}=\frac{567}{121}$

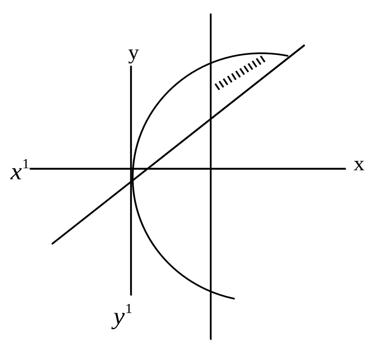
35. Key: 2



Solution: Required area $=2\int_0^1(7-x^2)dx+2\int_1^2(8-x^2-x^2)$=20

36. Key:2

Solution:



Required area $=\int_1^4\left(\sqrt{8}x-\sqrt{2}x\right)dx=\frac{11\sqrt{2}}{6}$.

37. Key:2

Sol: $\left|\int\limits_{-1}^{1}\left(a+e^{|x|}-e^{-x}\right)dx\right|$

$=\left|\int\limits_{-1}^{0}\left(a+\cancel{e^{-x}}-\cancel{e^{-x}}\right)dx+\int\limits_{0}^{1}\left(a+e^{x}-e^{-x}\right)dx\right|$

$=\left|a(1)+\left(ax+e^{x}+e^{-x}\right)_0^1\right|$

$=\left|a+a+e+e^{-1}-2\right|$

$$=\left|2a+e+\frac{1}{e}-2\right|$$

$$=\left|2a+\frac{e^2-2e+1}{e}\right|=\left|\frac{e^2+8e+1}{e}\right|$$

$$2a=\left|\frac{-10e}{e}\right|\ a=5$$

38. Key : 2

Sol: area $=\left|\int_{-\alpha}^{\alpha}\left(\frac{9}{2}+e^{|x|}-e^{x}\right)dx\right|$

$$=\left|\int_{-\alpha}^{0}\left(\frac{9}{2}+e^{-x}-e^{-x}\right)dx+\int_{\alpha}^{0}\left(\frac{9}{2}+e^{-x}-e^{-x}\right)dx\right|$$

$$=\left|\frac{9}{2}\alpha+\left(\frac{9}{2}\alpha+e^{\alpha}+e^{-\alpha}\right)-2\right|$$

$$\left|9\alpha+\frac{e^{2\alpha}+1-2e^{\alpha}}{e^{\alpha}}\right|$$

Given area = $\frac{e^{2\alpha}+16e^{\alpha}}{e^{\alpha}}+1$

$$9\alpha+\frac{e^{2\alpha}+1-2e^{\alpha}}{e^{\alpha}}=\frac{e^{2\alpha}+16e^{\alpha}+1}{e^{\alpha}}$$

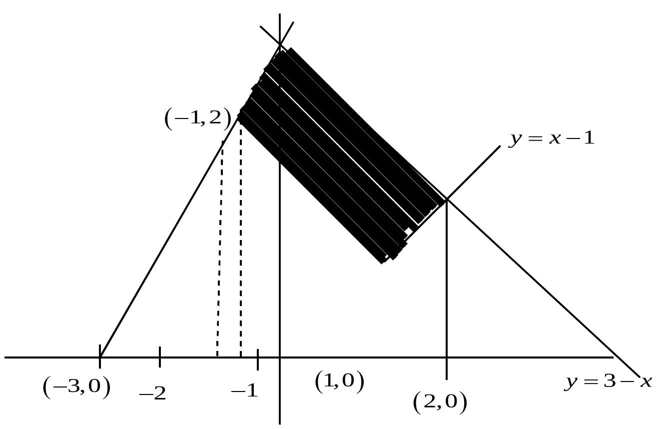
$$9\alpha=\frac{18e^{\alpha}}{e^{\alpha}}$$

$$9\alpha=18$$

$$\alpha=2$$

39. Key : 1

Sol:



required area = $\int_{-1}^{0}(3+x-(-x+1))dx+\int_{0}^{1}(3-x-(-x+1))dx+\int_{0}^{2}(3-x-(x-1))dx$

$=\int_{-1}^{0}(2+2x)dx+\int_{0}^{1}(2)dx+\int_{1}^{2}(4-2x)dx=4sq.units$

40. **KEY: 1**

SOL: $y=x^2-4x+4$ and $y^2=16-8x$

$y=(x-2)^2$ and $y^2=-8(x-2)$

Vertices of both parabolas are shifted to origin

$\therefore Y=X^2$ and $\therefore Y^2=-8X$

$\therefore$ area = $\left|\frac{16}{3}ab\right|$

$=\frac{16}{3}\left(\frac{1}{4}\times 2\right)=\frac{8}{3}$

41. Key: 3

Sol: $y^2=-4(x-1)$

Vertex = (1, 0)

Line $y=2+2x$

Point of intersection of two curves is

$(2+2x)^2=4-4x$

After solving we get $(-3,-4)\,and\,(0,2)$

Required area = $\int_{-1}^{0}(line)+\int_{0}^{1}(curve)$

$=\left(\frac{1}{2}\times 1\times 2\right)+\int_{0}^{1}2\sqrt{1-x}\,dx\;=1+\frac{4}{3}=\frac{7}{3}$

42.

Key: 22

Sol: $A=\int_{0}^{4}\sqrt{4+y}\;dy-\int_{0}^{2}\sqrt{4-y}\;dy-\int_{2}^{4}\sqrt{y}dy$

$=\left(\frac{(4+y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4+\left(\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2-\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$

$=\frac{80\sqrt{2}}{3}-16=\frac{40\sqrt{2}}{3}-16$

$\alpha=6,\beta=16$

$\alpha+\beta=22$

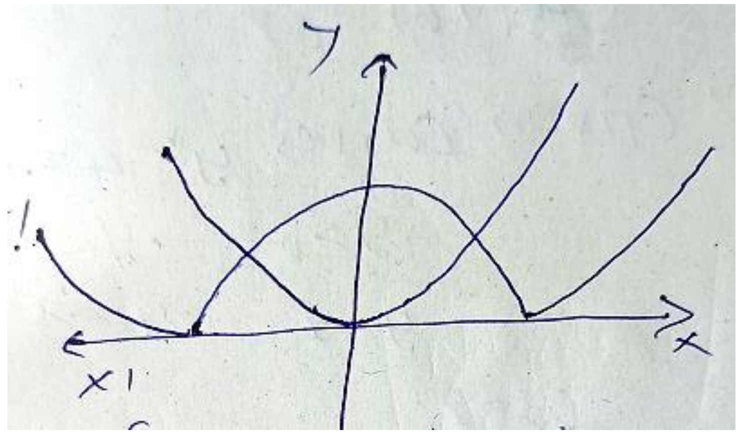
43.

Key: 2

Sol: given curve

$x^2 = y \quad y = |x^2 - 4|$

Area $A = 2\left[\int_0^1 (3 - x^2)dx + \int_1^{\sqrt{2}} (4 + 2x^2)dxa\right]$



$$= 2\left[\left(3x - \frac{x^3}{3}\right)_0^1 + \left(4x - \frac{2x^3}{3}\right)_1^{\sqrt{2}}\right]$$

$$= 2\left[\left(3 - \frac{1}{3}\right) - 0\right] + \left[\left(4\sqrt{2} - \frac{4\sqrt{2}}{3}\right) - \left(4 - \frac{2}{3}\right)\right]$$

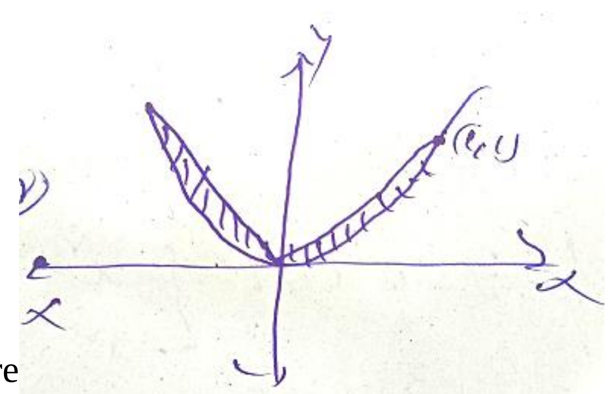
$$= 2\left[\frac{8}{3} + \frac{18\sqrt{2}}{3} - \frac{10}{3}\right]$$

$$= \frac{2}{3}[8\sqrt{2} - 2] = \frac{4}{3}(4\sqrt{2} - 1)$$

44. Key: $\frac{1}{3} = 0.333$

Sol: given equation of curve $x^2 = y$ -----(1)

Y = |x| -----(2)



Are

$A : 2\int_0^1 (x - x^2)dx$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$$

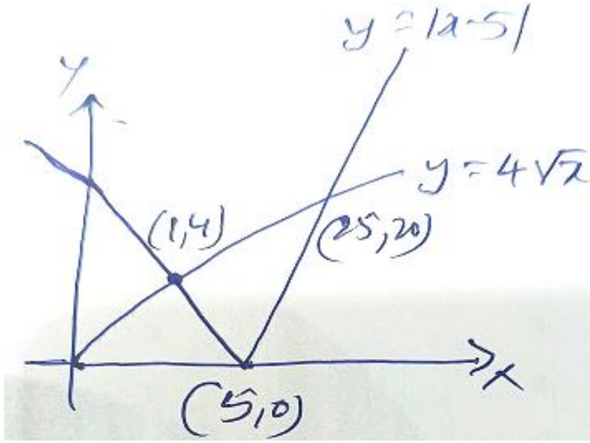
45. KEY: 368

SOL:
$$A=\int_1^{25} 4\sqrt{x}dx-\frac{1}{2}x:x:-\frac{1}{2}\times 22\times 20$$

$$A=\left[\frac{4x^{3/2}}{\frac{3}{2}}\right]^{25}-8-200$$

$$A=\frac{8}{3}(125-1)-208$$

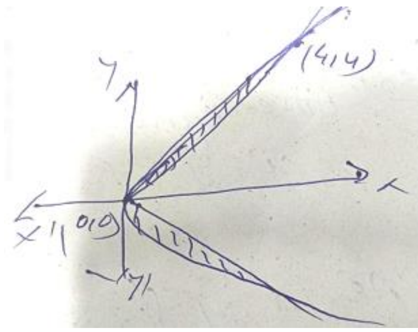
$$A=\frac{368}{3}\Rightarrow 3A=368$$



46.
Key: 16

Sol: given curves $y^2=4x$ ---(1)

$$y\ge 0,\, y<0$$
$$y=x \quad y=-x$$
$$x^2-4x=0\Rightarrow x=0,4$$
$$(x,4)=(0,0)(4,\pm 4)$$



$$A=2\int_0^4 (y-y_2)dx$$

$$=2\int_0^4 (2\sqrt{x}-x)dx$$

$$=2\left[2\frac{x^{3/2}}{3/2}-\frac{x^2}{2}\right]_0^4$$

$$A=2\left[\frac{4}{3}8-8\right]=\frac{16}{3}$$

$$3A=16$$

47.
Key: 12

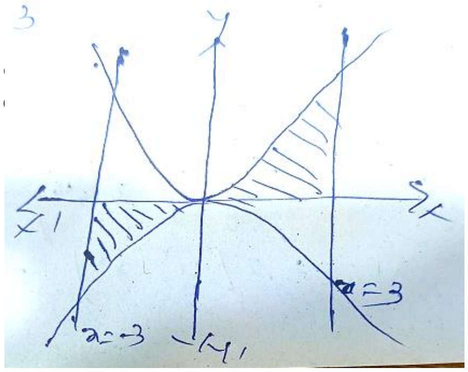
Sol: required area = $\frac{1}{2}\times 2\times 2\times\frac{1}{2}\times 3\times 3\times\frac{1}{2}\times 1\times 11=12$

48.

Key: 18

Sol: $y = x|x| = \begin{cases} x^2 & x \ge 0 \\ -x^2 & x < 0 \end{cases}$

$$A = 2\int_0^3 x^2 dx = 2\left[\frac{x^3}{3}\right]_0^3 = 18$$



49.

Key: 2

Sol: $|x - y| \le y \le 4\sqrt{x}$

$y = |x - y|$

Now $y^2 = (x - y)^2$

$y = \frac{x}{2}$ and $x = 0$

Now area = $\int_0^{64} \left(4\sqrt{x} - \frac{x}{2}\right) dx$

$$= \left[\frac{4x^{3/2}}{3/2} - \frac{x^2}{4}\right]_0^{64} = \frac{8}{3}.8^3 - \frac{64^2}{4} = 64^2\left(\frac{1}{12}\right) = \frac{1024}{3}$$

50.

Key: 4

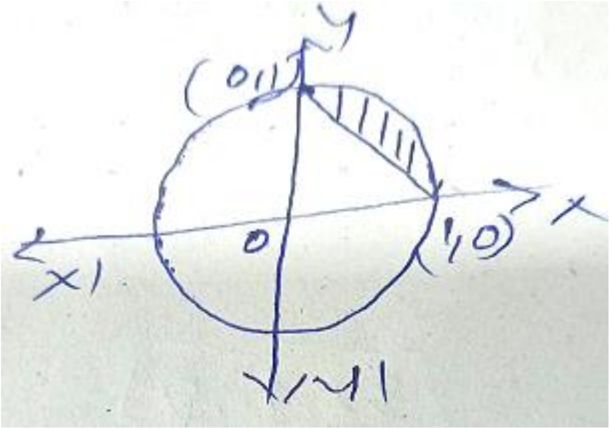
Sol: given curves

$x^2 + y^2 = 1$ ----(1)

X+y= 1-----(2)

X=1-y

$$(1-y)^2 + y^2 = 1$$
$$v + y^2 - 2y + y^2 = x$$
$$2y^2 - 2y = 0$$
$$2y(y-1) = 0$$
$$y = 0,1$$
$$x = 1,0$$



Area = $\int_0^1 \sqrt{1-x^2}\,dx - \int_0^1 (1-x)dx$

$$\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1$$
$$= 0 + \frac{1}{2}\frac{\pi}{2} - 1 + \frac{1}{2}$$
$$= \frac{\pi}{4} - \frac{1}{2}$$

51.

KEY: 1

Sol: $A = \int_{-2}^{1}(x+7-x^2-1)dx + \int_{1}^{2}(11+3x-x^2-1)dx$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_{-2}^{1} + \left[10x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_1^2$$
$$= \frac{50}{3} \Rightarrow 3A = 50$$

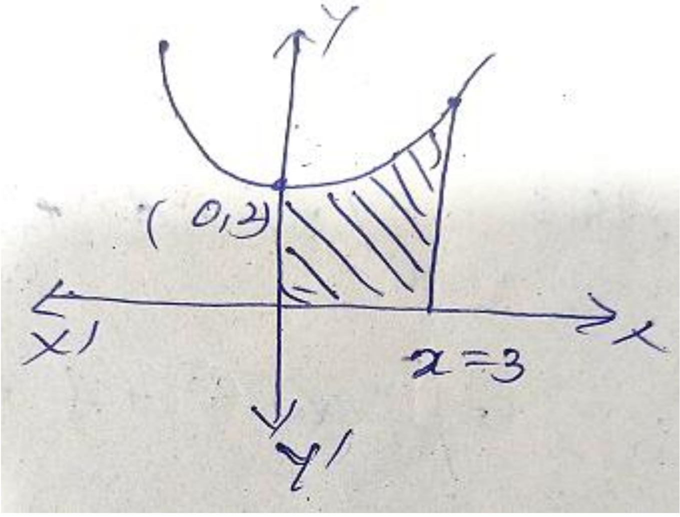
52.

Key: 164

$$A = \int_0^2 (x^2+2)dx + \int_2^3 (2x+2)dx$$
$$= \left(\frac{x^3}{3} + 2x\right)_0^2 + \left(x^2 + 2x\right)_2^3$$

Sol: $= \left(\frac{8}{3} + 4\right) + (15-8)$

$$= \frac{8}{3} + 4 + 7 = 11 + \frac{8}{3} = \frac{41}{3}$$
$$12A = 12\left(\frac{41}{3}\right) = 164$$



53. Key: 15

Sol: $0 \le 9x \le y^2, y \ge 3x-6$

A= required area = $\left[\int_0^1 \left(-3\sqrt{x}\right)dx - \int_0^1 (3x-6)dx\right]$

$$A = -3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^1 \left|-\left(\frac{3x^2}{2}-6x\right)\right|_0$$

$$A = -2(1-0) - \left[\frac{3}{2}-6\right]$$

$$A = -2-\frac{3}{2}+6 = \frac{5}{2} sq.units$$

$$\therefore 6A = 6\times\frac{5}{2} = 15$$

54.

Key : 9

Sol: $y^2 = 2x$ and $y = 4x-1$

$$y = 2y^2 - 1$$

$$2y^2 - y - 1 = 0$$

$$y = -1, -\frac{1}{2}$$

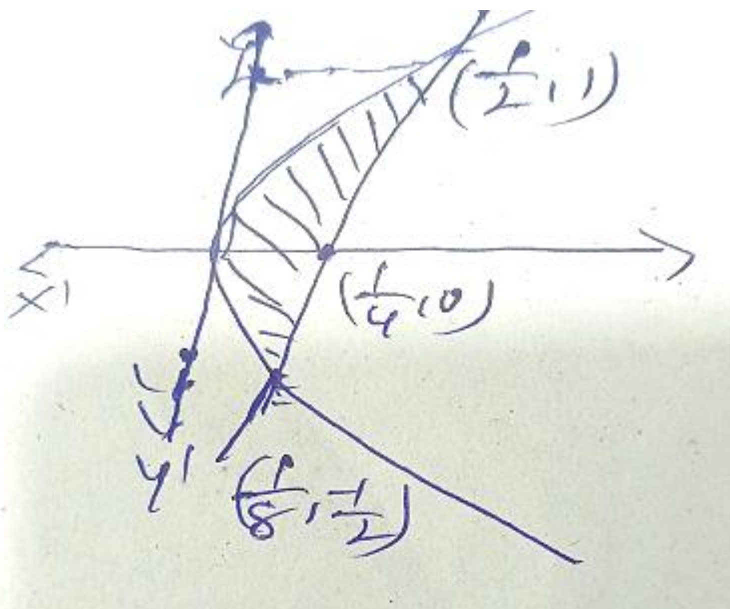
$$x = \frac{1}{2}, \frac{1}{8}$$

$$(x, y) = \left(\frac{1}{2}, 1\right)\left(\frac{1}{8}, \frac{-1}{2}\right)$$

$$A = \int_{-\frac{1}{2}}^{1} (x_1 - x_2)dy$$

Area $= \int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{2}-\frac{y^2}{2}\right)dy$

$$A = \left(\frac{(y+1)^2}{4}-\frac{y^3}{6}\right)_{-\frac{1}{2}}^{1} = \frac{9}{32}$$



32A=9

PRACTICE SHEET-1

1	**1**	2	**4**	3	**2**	4	**4**	5	**3**
6	**1**	7	**1**	8	**1**	9	**1**	10	**2**
11	**2**	12	**3**	13	**1**	14	**4**	15	**2**
16	**1**	17	**2**	18	**2**	19	**3**	20	**3**
21	**21**	22	**20**	23	**1**	24	**16**	25	**9**

1. $\frac{f(x)}{1+x^2} = 1 + \int_0^x \frac{f^2(t)}{1+t^2}dt \qquad (f(0)=1)$

Differentiating both the sides with respect x,

$$\frac{(1+x^2)f'(x) - f(x).2x}{(1+x^2)^2} = \frac{f^2(x)}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} - \frac{2x}{(1+x^2)}y = y^2 \Rightarrow \frac{1}{y^2}\frac{dy}{dx} + \frac{2x}{1+x^2}\left(-\frac{1}{y}\right) = 1 \Rightarrow y = \frac{-3(1+x^2)}{x^3+3x-3} = f(x)$$

$$\therefore f(-1) = \frac{-3(1+1)}{-1-3-3} = \frac{-6}{-7} = \frac{6}{7} \qquad f(-2) = \frac{15}{17}$$

$$f(-1) + f(-2) = \frac{6}{7} + \frac{15}{17} = \frac{102+105}{7\times 17} = \frac{207}{119}$$

2. $f^1(x) = -\left(\frac{x-1}{x}\right) - (\ln x - 1) + f(x) \Rightarrow f'(x) - f(x) = \frac{1}{x} - \ln x$

$f(x) = \ln x + Ce^x$

$Put\ x = 1, \Rightarrow f(1) = e \Rightarrow C = 1 \qquad \therefore f(x) = e^x + \ln x$

$(i) f(e) = 1 + e^e$

$(ii) f'(x) = e^x + \frac{1}{x} > 0$ for $x \in R^+$ implies $f(x)$ increases for $x \in R^+$

$(iii) f(0^+) = -\infty, f(\infty) = \infty, f(x) = 0$ has exactly one real root in $(0, \infty)$

3. Equation of tangent at (x,y) on the curve C is $\quad Y - y = \frac{dy}{dx}(X - x)$

According to the question given the equation of curve C is $\quad xy = k$

Curve passes through $(2,2)$ hence $k = 4$

Equation of curve is $xy = 4$

If passes through $\left(3, \frac{4}{3}\right)$

4. General solution is $\frac{y}{x} = \int \left(e^x(x+1) - 1\right)dx$

$\frac{y}{x} = xe^x - x + C \qquad$ As $y(1) = e - 1 \Rightarrow C = 0$

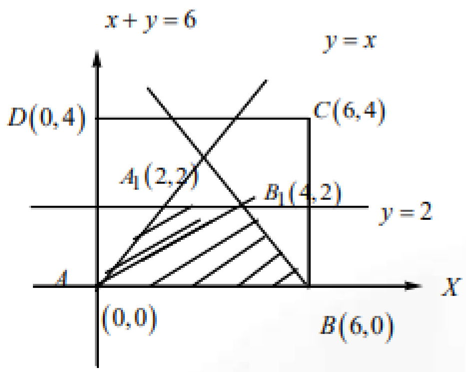
$x + \frac{y}{x} = xe^x \qquad 2 + \frac{y(2)}{2} = 2e^2 \Rightarrow y(2) = 4e^2 - 4$

$y(1) = e - 1$ (Given). Hence, $y(2) = 4(e+1)(e-1) = 4y(1)(y(1)+2) \quad \therefore K = 4$

5. On differentiation, we get

$3f(x)=xf^{1}(x)+f(x)-x^{2} \quad \Rightarrow f^{1}(x)-\frac{2}{x}f(x)=x$

$f(x)=x^{2}\left(\ln x+\frac{1}{3}\right) \quad f(e)=e^{2}\left(1+\frac{1}{3}\right)=\frac{4e^{2}}{3}$



6.

$d(P,AB)=y,\ d(P,BC)=6-x,\ d(P,CD)=4-y,\ d(P,AD)=x$

We must have $y\le 6-x,\ y\le 4-y,\ y\le x,$

$\Rightarrow x+y\le 6,\ y\le 2,\ y\le x$

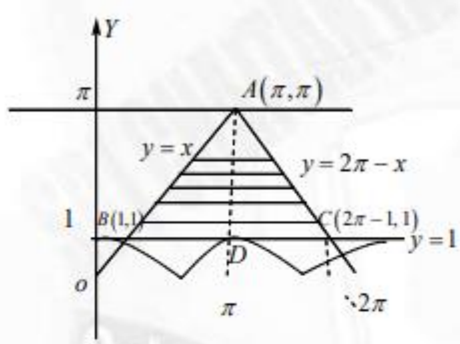
Shaded region represents the required area. This area is equal to the area of trapezium ABB_1A_1 $ar(ABB_1A_1)=\frac{1}{2}(6+2)2=8$ sq. units

7. $\int_0^1 y(x)dx=c \to \frac{dy}{dx}=x+c \quad y=\int (x+c)dx=\frac{(x+c)^2}{2}+d$

$y(0)=1\to d=1-\frac{c^2}{2} \qquad y(x)=1+\frac{(x+c)^2-c^2}{2}=\frac{x^2+2cx+2}{2}$

$c=\int_0^1 y(x)dx=\frac{1}{2}\int_0^1 (x^2+2cx+2)dx \qquad \Rightarrow 2c=\frac{1}{3}+c+2\to c=\frac{7}{3}$

8.



Here $AD=(\pi-1),\ B(1,1),\ C(2\pi-1,1)$

$BC=(2\pi-1-1)=2(\pi-1)$

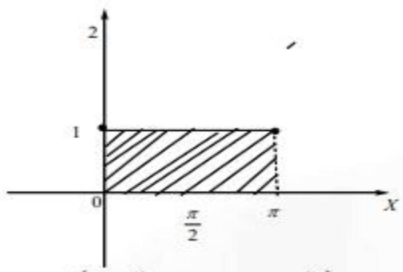
Required area $=\frac{1}{2}.2(\pi-1)(\pi-1)=(\pi-1)^2$ sq. unit $\qquad =\left[(\pi-1)^2\right]=[4.586]=4$

9. $A=\int_0^1\left(2x-x^2-x^n\right)dx \quad =\left(x^2-\frac{x^3}{x}-\frac{x^{n+1}}{n+1}\right)_0^1=1-\frac{1}{3}-\frac{1}{n+1}=-\frac{2}{3}-\frac{1}{n+1}$

$\frac{2}{3}-\frac{1}{n+1}=\frac{3}{5}\Rightarrow\frac{1}{n+1}=\frac{2}{3}-\frac{3}{5}=\frac{1}{15}$

$n=14$

10.



$f(x)=\cos^{-1}\left(\sin\left(\frac{\pi}{2}-1-[\sin x]\right)\right)=\cos^{-1}\cos(1+[\sin x]) \quad f(x)=1+[\sin x]\forall x\in R$

Required area $=\pi$ sq. units $\quad\therefore k=1$

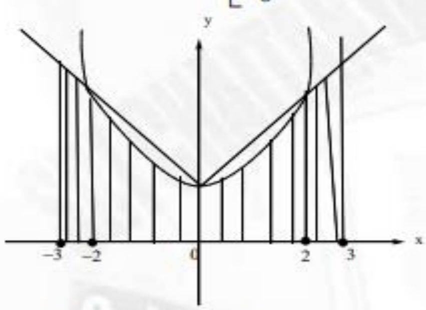
11. $f(x+y)=f(x)f(y)\Rightarrow f(x)=e^{4\alpha x}$

$f''(x)3\alpha f'(x)-f(x)=0\Rightarrow 16\alpha^2-12\alpha^2-1=0$

$\alpha=\frac{1}{2}(given\ \alpha>0) \qquad \therefore f(x)=e^{2x}$

Required area $=\int_0^2 e^x dx=e^2-1 \ \beta=1, \gamma=2$

12. Required area $2\left[\int_0^2\left(x^2+1\right)dx+\int_2^3(2x+1)dx\right]=\frac{64}{3}$ sq. units



13. $g(x)=|f(|x|)-2|=||x|-1-2|=||x|-3|$

$x^2=4(y-2)$ On solving $y=3-x, y=\frac{x^2+8}{4}$

$\therefore 12-4x=x^2+8\Rightarrow x^2+4x-4=0\Rightarrow x=2\sqrt{2}-2$

Area $=2\int_0^{2\sqrt{2}-2}\left(3-x-\frac{x^2}{4}-2\right)dx=\frac{4}{3}\left(4\sqrt{2}-5\right)$

14. $(x+2y)^2\frac{dy}{dx}=\left(2(x+2y)+1\right)$

Let $u=x+2y\Rightarrow\frac{du}{dx}=1+2\frac{dy}{dx} \ \ \frac{1}{2}\left(\frac{du}{dx}-1\right)=\frac{2u+1}{u^2}$

$$\frac{du}{dx}-1=\frac{2(2u+1)}{u^2}\Rightarrow\frac{du}{dx}=\frac{u^2+4u+2}{u^2}$$

On solving we get $y=\ln\left((x+2y)^2+4(x+2y)+2\right)-\frac{3}{\sqrt{2}}\log\left|\frac{x+2y+2-\sqrt{2}}{x+2y+2+\sqrt{2}}\right|+C$

$$f(x,y)=(x+2y)^2+4(x+2y)+2 \quad g(x,y)=\frac{x+2y+2-\sqrt{2}}{x+2y+2+\sqrt{2}}$$

$$f(0,0)=2,g(0,0)=3-2\sqrt{2},f(1,-1)=-1,g\left(\sqrt{3},0\right)=\sqrt{2}-1$$

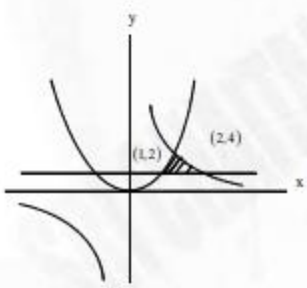
15. Req. Area $=A=2\int_0^{\frac{1}{2}}f^{-1}(x)dx$ Let $f^{-1}(x)=t\Rightarrow x=f(t),\,dx=f'(t)dt$

$$A=2\int_0^1 tf'(t)dt=2\left(f(1)-\int_0^1\frac{t}{1+t^2}dt\right)=1-\ln 2$$

16. Required area $\int_{e-e^2}^{0}\left(\ln\left(x+e^2\right)-1\right)dx+\int_0^{\ln 2}\left(2e^{-x}-1\right)dx \quad =1+e-\ln 2$

which implies $\beta-\gamma=2$

17.



Required area $\int_1^4\left(\frac{8}{y}-\sqrt{y}\right)dy$

18. We must have $x\in\left(\frac{1}{2},1\right]\cup\left(\frac{1}{8},\frac{1}{4}\right]\cup\left(\frac{1}{32},\frac{1}{16}\right]\cup\ldots\ldots$

and $y\in\left(\frac{1}{5},1\right]\cup\left(\frac{1}{125},\frac{1}{25}\right]\cup\left(\frac{1}{325},\frac{1}{625}\right]\cup\ldots\ldots$ so

$$S=\left(\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\ldots\ldots\right)\left(\frac{4}{5}+\frac{4}{125}+\frac{4}{325}+\ldots\ldots\right)=\frac{\frac{1}{2}}{\left(1-\frac{1}{4}\right)}\times 4\left(\frac{\frac{1}{5}}{1-\frac{1}{25}}\right)\quad=\frac{2}{3}\times\frac{5\times 4}{24}=\frac{5}{9}$$

19. Solution: $xu'+v=0$ ....(1)

$xv'+u=0$ ....(2)

$(1)+(2)\ \rightarrow x\frac{d}{dx}(u+v)+u+v=0\ \rightarrow x(u+v)=A$

$x=1, u=0, v=2 \rightarrow A=2 \rightarrow u+v=\frac{2}{x}$ ....(3)

$(1)-(2) \rightarrow \frac{xd}{dx}(u-v)=u-v \quad \rightarrow \int \frac{d(u-v)}{u-v}=\int \frac{dx}{x}$

$\ln(u-v)=\ln x+\ln B \quad u-v=Bx$

$x=1, u=0, v=2 \rightarrow B=-2, u-v=-2x$ ....(4)

$(3),(4) \rightarrow u=\frac{1}{x}-x, v=\frac{1}{x}+x \int_0^1 \frac{u}{v}dx=\int_0^1 \frac{1-x^2}{1+x^2}dx \quad =\int_0^1\left(\frac{2}{1+x^2}-1\right)dx \quad =\frac{\pi}{2}-1$

20. Conceptual

21. $\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$

On solving we get $y=\frac{e^{2x}}{2}-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}e^x}{3}\right)+C$

This curve is passing through $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$ which implies $C=\sqrt{2}\left[\frac{\pi}{4}\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)\right]$

Again, this curve is passing through $\left(\alpha,\frac{e^{2\alpha}}{2}\right)$ which will give $e^{2\alpha}=\frac{9}{2}\left(\frac{11+6\sqrt{2}}{11-6\sqrt{2}}\right)$

Here, $l=9$, $m=6$, $n=6$

22. $f(x)=e^{-x}\left(x^3+x^2+4f(x)-36\right)$

$f(x)=\frac{x^3+x^2-36}{e^x-4} \qquad f'(2)=\frac{40e^2-64}{\left(e^2-4\right)^2} \;\; \frac{\alpha}{\beta}=20$

23. $g'(x)=2\cos^2\left(x^2+\frac{\pi}{3}\right).2x-2\cos^2 x$

$g'(x)=4\times 2\cos\left(x^2+\frac{\pi}{3}\right)2x.x+4\cos^2\left(x^2+\frac{\pi}{3}\right).1+4\cos x.\sin x$

$=4\cos^2\left(x^2+\frac{\pi}{3}\right)-16x^2\cos\left(x^2+\frac{\pi}{3}\right)\sin\left(x^2+\frac{\pi}{3}\right)$ ATQ, $g'(t)+4=\int_0^t f(x)dx$

Differentiating w.r.t 't' $g''(t)=f(t)$

$g''(0)=f(0)=4\cos^2\frac{\pi}{3}=4\times\frac{1}{4}=1$

24. Conceptual

25. $f'(x)f''(x)=f'(x)f(x)$

On solving we get $f'(x)=\sqrt{\left(f(x)\right)^2+4}$

After solving $f(x)=e^x-e^{-x} \quad 6f\left(\log_e 2\right)=6\times\frac{3}{2}=9$