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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_STERLING BT **JEE-MAIN** **Date: 20-09-2025**

Time: 09:00AM to 12:00PM **RPTM-07** **Max. Marks: 300**

KEY SHEET

MATHEMATICS

1	**4**	2	**4**	3	**3**	4	**1**	5	**1**
6	**4**	7	**1**	8	**1**	9	**1**	10	**2**
11	**4**	12	**1**	13	**1**	14	**3**	15	**4**
16	**3**	17	**2**	18	**3**	19	**4**	20	**3**
21	**3**	22	**13**	23	**6**	24	**44**	25	**74**

PHYSICS

26	**3**	27	**3**	28	**4**	29	**4**	30	**4**
31	**2**	32	**1**	33	**1**	34	**1**	35	**3**
36	**1**	37	**3**	38	**1**	39	**1**	40	**4**
41	**1**	42	**2**	43	**2**	44	**4**	45	**2**
46	**20**	47	**15**	48	**4**	49	**1**	50	**3**

CHEMISTRY

51	**4**	52	**3**	53	**2**	54	**4**	55	**4**
56	**1**	57	**4**	58	**1**	59	**3**	60	**2**
61	**2**	62	**2**	63	**2**	64	**1**	65	**3**
66	**4**	67	**2**	68	**1**	69	**2**	70	**2**
71	**5**	72	**2**	73	**31**	74	**2**	75	**3**

SOLUTIONS
MATHEMATICS

1.

$Since\ A, B, C, D\ are\ coplanner$

$Hence\left[\overrightarrow{BA}\ \overrightarrow{CA}\ \overrightarrow{DA}\right]=0$

$$\begin{vmatrix} 4 & 3 & 2\lambda-3 \\ 7 & 5-\lambda & 2\lambda-4 \\ 6 & 0 & 2\lambda-6 \end{vmatrix}=0$$

$\lambda=2,3\ hence \sum_{\lambda\in S}(\lambda+2)^2=41$

2.

$\vec{a}=\alpha\hat{i}+2\hat{j}-\hat{k}, \vec{b}=-2\hat{i}+\alpha\hat{j}+\hat{k},$

$Area\ of\ \ parallelogram=\left|\hat{a}\times\hat{b}\right|$

$\hat{a}\times\hat{b}=\sqrt{(\alpha+2)^2+(\alpha-2)^2+(\alpha^2+4)^2}$

$Given\ \left|\hat{a}\times\hat{b}\right|=\sqrt{15(\alpha^2+4)}$

$so, 2\left(\alpha^2+4\right)+\left(\alpha^2+4\right)^2=15\left(\alpha^2+4\right)$

$\Rightarrow\left(\alpha^2+4\right)^2=13\left(\alpha^2+4\right)\Rightarrow\alpha^2+4=13\therefore\alpha^2=9$

$Now,\ 2\left|\vec{a}\right|^2+\left(\vec{a}.\vec{b}\right)\left|\vec{b}\right|^2$

$\left|\vec{a}\right|^2=\alpha^2+4+1=\alpha^2+5$

$\left|\vec{b}\right|^2=4+\alpha^2+1=\alpha^2+5$

$\vec{a}.\vec{b}=-2\alpha+2\alpha-1=-1$

$\therefore 2\left|\vec{a}\right|^2+\left(\vec{a}.\vec{b}\right)\left|\vec{b}\right|^2$

$2(\alpha^2+5)-1(\alpha^2+5)=\alpha^2+5=14$

3.

$\left(\vec{a}+\vec{b}\right) is\ perpendicular\ to\ \vec{c}, hence$

$\left(\vec{a}+\vec{b}\right).\vec{c}=0\Rightarrow\vec{a}.\vec{b}+\vec{b}.\vec{c}=0$

$\Rightarrow 5+1+2+5b_1+b_2+2=0\Rightarrow 5b_1+b_2=-10....(1)$

$Given, \frac{\vec{b}.\vec{a}}{\left|\vec{a}\right|}=\left|\vec{a}\right|$

$\left\{\because\vec{p}=r\hat{i}+s\hat{j}+t\hat{k}\Rightarrow\left|\vec{p}\right|=\sqrt{r^2+s^2+t^2}\right\}$

$\Rightarrow b_1+b_2+2=4\Rightarrow b_1+b_2=2....(2)$

$solving\ (1)\ and\ (2), b_1=-3, b_2=5$

$\therefore\left|\vec{b}\right|=\sqrt{9+25+2}=6$

4.

Give a vector $\vec{a}$ is coplanar with vector

$\vec{a}$ *is cooplanar with vector* $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ *and* $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ *and* $\vec{a}$ *is perpendicular to* $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, *and* $|\vec{a}| = \sqrt{10}$.

Thus $\vec{a} = \lambda\vec{b} + \mu\vec{c}$

$\Rightarrow \vec{a} = \lambda(2\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k}) \Rightarrow \vec{a} = \hat{i}(2\lambda + \mu) + \hat{j}(\lambda - \mu) + \hat{k}(\lambda + \mu)$

given that $\vec{a}$ *is perpendicular to* $\vec{d}$, *and we know that if*

two vector are perpendicular, then $\vec{a}.\vec{d} = 0$

$\Rightarrow 3(2\lambda + \mu) + 2(\lambda - \mu) + 6(\lambda + \mu) = 0$

$\Rightarrow 14\lambda + 7\mu = 0 \Rightarrow \mu = -2\lambda$

$\Rightarrow \vec{a} = (0)\hat{i} + 3\lambda\hat{j} + (-\lambda)\hat{k} \Rightarrow \vec{a} = \lambda(3\hat{j} - \hat{k})$

$\Rightarrow |\vec{a}| = \sqrt{10}|\lambda|$ *Given* $|\vec{a}| = \sqrt{10}$

$\Rightarrow |\lambda| = 1 \Rightarrow \lambda = 1 or -1$

Also if, $\vec{a}, \vec{b}$ & $\vec{c}$ *are coplanar, then* $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0$

Now $\left[\vec{a}\,\vec{b}\,\vec{c}\right] + \left[\vec{a}\,\vec{b}\,\vec{c}\right] + \left[\vec{a}\,\vec{b}\,\vec{c}\right]$

$= 0 + \left[\vec{a}\,\vec{b}\,\vec{c}\right] + \left[\vec{a}\,\vec{b}\,\vec{c}\right] = \left[\vec{a}\,\vec{b} + \vec{c}\,\vec{d}\right]$

On putting the value of the vectors for finding the box product we get

$$= \begin{vmatrix} 0 & 3\lambda & -\lambda \\ 3 & 0 & 2 \\ 3 & 2 & 6 \end{vmatrix} = -3\lambda(12) - \lambda(6) = -42\lambda = -42 or 42$$

5.

Given,

$\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$

$$\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right)\times\left(\vec{a}\times\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}$$

$$\Rightarrow\left(\left(\vec{b}-\vec{a}\right).\left(\vec{a}\times\vec{b}\right)\right)\left(\vec{a}+\vec{b}\right)-\left(\left(\vec{b}-\vec{a}\right).\left(\vec{a}+\vec{b}\right)\right)\left(\vec{a}\times\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}$$

$$\Rightarrow 0+\left(\left(\vec{a}-\vec{b}\right).\left(\vec{a}+\vec{b}\right)\right)\left(\vec{a}\times\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}....(1)\Rightarrow\left(\left(\vec{a}-\vec{b}\right).\left(\vec{a}+\vec{b}\right)\right)=8$$

$$8\left(\vec{a}-\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}\quad Now,\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\\lambda&2&-3\\1&-\lambda&2\end{vmatrix}$$

$$=(4-3\lambda)\hat{i}-(3\lambda+3)\hat{j}+(-\lambda^2-2)\hat{k}\Rightarrow\lambda=1$$

$$\therefore\vec{a}=\hat{i}+2\hat{j}-3\hat{k},\vec{b}=\hat{i}-\hat{j}+2\hat{k}$$

$$\Rightarrow\left(\vec{a}+\vec{b}\right)\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&1&-1\\0&3&-5\end{vmatrix}=2\hat{i}+10\hat{j}+6\hat{k}$$

$$\left|\left(\vec{a}+\vec{b}\right)\times(\vec{a}-\vec{b})\right|=\sqrt{4+100+36}$$

$$\left|\lambda\left(\vec{a}+\vec{b}\right)\times(\vec{a}-\vec{b})\right|^2=\lambda^2\left|\left(\vec{a}+\vec{b}\right)\times(\vec{a}-\vec{b})\right|^2=4+100+36=140$$

6.

$$\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}-\vec{c}}{2}$$

using the vector cross product we have,

$$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$$

$$so,(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=\frac{\vec{b}-\vec{c}}{2}$$

on comparing both side we get,

$$\vec{a}.\vec{c}=\frac{1}{2},\vec{a}.\vec{b}=\frac{1}{2}$$

$$\because\vec{b}.\vec{d}=\vec{a}.\vec{b}$$

$$\therefore\vec{b}.\vec{d}=\frac{1}{2}$$

using vector cross – product forulae we have

$$\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=\vec{a}.\left(\vec{b}\times(\vec{c}\times\vec{d})\right)$$

$$=\vec{a}.\left(\left(\vec{b}\times\vec{d}\right)\vec{c}-\left(\vec{b}\times\vec{c}\right)\vec{d}\right)\ \left(\because\vec{b}.\vec{c}=0\right)$$

$$=\left(\vec{a}\times\vec{c}\right)\left(\vec{b}.\vec{d}\right)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

7.

$$n = l + m$$

$$l^2 + m^2 = n^2 = (l+m)^2 \Rightarrow 2lm = 0$$

$$l^2 + m^2 = n^2 = 1$$

$$if\ l = 0 \Rightarrow 2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$$

$$m = n = \pm\frac{1}{\sqrt{2}}$$

$$And, if\ m = 0 \Rightarrow n = l = \pm\frac{1}{\sqrt{2}}$$

$$\therefore l^2 + m^2 = \frac{1}{2} \,\&\, l + m = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{2} + 2lm = \frac{1}{2}$$

$$\therefore l = 0, m = \frac{1}{\sqrt{2}}\ or\ l = \frac{1}{\sqrt{2}}, m = 0$$

so, direction cosines of two lines are

$$\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) and \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$\therefore \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$$

8.

$$x + 1 = 2y = -12z\ and\ x = y + 2 = 6z - 6$$

$$\frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}\ and\ \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$$

$$S.D = \frac{(\vec{b} - \vec{a}).(\vec{p} - \vec{q})}{\left|\vec{p} - \vec{q}\right|}$$

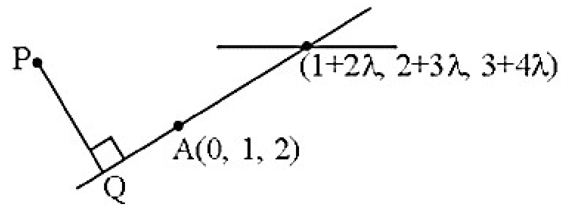
$$Here\ \vec{p} = \hat{i} + \frac{\hat{j}}{2} + \frac{-\hat{k}}{12} \,\&\, \vec{q} = \hat{i} + \hat{j} + \frac{\hat{k}}{6}\ and\ \vec{a} = \hat{i} \,\&\, \vec{b} = 2\hat{j} - \hat{k}$$

$$S.D = \left(-\hat{i} + \hat{j} - \hat{k}\right).\frac{(\vec{p} \times \vec{q})}{\left|\vec{p} \times \vec{q}\right|}$$

$$\vec{p} \times \vec{q} \equiv \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 1 & \frac{1}{2} & \frac{-1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix} \Rightarrow \vec{p} \times \vec{q} \equiv \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \Rightarrow \vec{p} \times \vec{q} \equiv 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$S.D = \frac{\left(-\hat{i} + 2\hat{j} - \hat{k}\right).\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)}{\sqrt{2^2 + 3^2 + 6^2}} = \left|\frac{-14}{7}\right| = 2$$

9.



$\overrightarrow{AB}.\vec{n}$

$\Rightarrow \left[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}\right].(2\hat{i}+\hat{j}-3\hat{k})$

$2+4\lambda+1+3\lambda-3-12\lambda=0$

$5\lambda=0 \Rightarrow \lambda=0$

$Line\,\overrightarrow{AB}, \vec{r}=\hat{j}+2\hat{k}+\mu(\hat{i}+\hat{j}+\hat{k})$

$General\ \ form: Q(\mu, 1+\mu, 2+\mu)$

$\therefore \overrightarrow{PQ}.\overrightarrow{AB}=0$

$(\mu-1)+(10+\mu)+\mu=0$

$3\mu=-9 \Rightarrow \mu=-3$

$\therefore dis\tan ce=\sqrt{16+49+9}=\sqrt{74}$

10.

$Given\ l_1: \frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-2}{0}$

$l_2: \frac{x-1}{1}=\frac{y+\frac{3}{2}}{\frac{\alpha}{2}}=\frac{z+5}{2}$

$l_3: \frac{x-1}{-3}=\frac{y-\frac{1}{2}}{-2}=\frac{z-0}{4}$

$Slince\ l_1\ is\ \ perpendicular\ to\ l_2, so \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}}=0$

$Now\ angle\ between\ l_2\ \&\ l_3, \cos\theta=\frac{\left|1\times(-3)+(-2)\left(\frac{\alpha}{2}\right)+2\times 4\right|}{\sqrt{1+\frac{\alpha^2}{4}+4}\sqrt{9+4+16}}$

$\cos\theta=\frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}\ putting\ \alpha=3$

$\cos\theta=\frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}}=\frac{4}{29}=\theta=\cos^{-1}\left(\frac{4}{29}\right)\Rightarrow \theta=\sec^{-1}\left(\frac{29}{4}\right)$

11.

Given $\frac{x-\alpha}{2}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}$ line on the plane $x+2y-z=8$.

Let, $\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}=\varnothing$

and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}=q$

Any point on the first line can be considered as $(\phi+\alpha, 2\phi+1, 3\phi+1)$

and a point on the second line can be $(q\beta+4, 3q+6, 3q+7)$.

For interaection of the lines can be

$\phi+\alpha=q\beta+4....(1)$

$2\phi+1=3q+6....(2)$

$3\phi+1=3q+7....(3)$

For (2) & (3) $\phi=1, q=-1$

so, from (1) $\alpha+\beta=3$

Now, point of intersection is $(\alpha+1,3,4)$

it lies on the given plane, therfore,

$\alpha+1+2\times3-4=8 \Rightarrow \alpha=5 \Rightarrow \beta=-2$

from $\alpha+\beta=3$

Hence, $\alpha-\beta=5-(-2)=7$

12.

Given line can be written as

$$\frac{x+1}{a}=\frac{y-0}{1}=\frac{z-1}{a}$$

$$\frac{x+2}{3}=\frac{y-0}{1}=\frac{z-0}{\frac{3}{b}}$$

We know that line $\frac{x+x_1}{a_1}=\frac{y+y_1}{b_1}=\frac{z+z_1}{c_1}$ and

$\frac{x+x_2}{a_2}=\frac{y+y_2}{b_2}=\frac{z+z_2}{c_2}$ are coplanar if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0=\begin{vmatrix} -1 & 0 & -1 \\ a & 1 & a \\ 3 & 1 & \frac{3}{b} \end{vmatrix}=0$

$b \neq 0 \Rightarrow a \neq 0$ since, $ab \neq 0$

$$\Rightarrow -\left(\frac{3}{b}-a\right)-1(a-3)=0 \Rightarrow a-\frac{3}{b}, -a+3=0 \Rightarrow b=1$$

$\therefore b=1, a \in R-\{0\}$

13.

(A) *Both the lines pass through the point* $(7,11,15)$

$(B) < 2,3,4 >$ *are direction ratio of both the lines.*

Also the point $(1,2,3)$ *is common to both.*

$\therefore$ *The lines are coincident.*

$(C) < 5,4,-2 >$ *are direction ratios of both the lines.*

$\therefore$ *The lines are parallel.*

Also, $x = 2+5\lambda, y = -3+4\lambda, z = 5-2\lambda$

$$\therefore \quad \frac{2+5\lambda-7}{5} = \frac{-3+4\lambda-1}{4} = \frac{5-2\lambda-2}{-2}$$

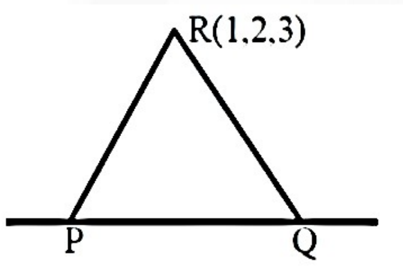
$$i.e \quad \lambda - 1 = \lambda - 1 = \frac{3-2\lambda}{-2}$$

$\therefore$ *No value of* λ.

Thus, the lines are parallel and different.

$(D) < 2,3,5 >$ *and* $< 3,2,5 >$ *are direction ratios of first and second line, respectively*

14.



$P(\lambda-3, 2\lambda+4, 2\lambda-1)$

$PR = 6$

$(8\lambda-4)^2 + (2\lambda+2)^2 + (2\lambda-4)^2 = 36$

$\lambda = 0,1$

Hence $P\ (-3,4,-1)$ & $Q\ (5,6,1)$

Centroid of $\Delta PQR = (1,4,1) \equiv (\alpha, \beta, \gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = 18$

15.

Area of quadrilateral = Area$(\Delta OAB) + Area(\Delta OBC)$

$$= \frac{1}{2}\left\{|\vec{a} \times (12\vec{a} + 4\vec{b}| + |\vec{b} \times (12\vec{a} + 4\vec{b})|\right\}$$

$$= 8\,|(\vec{a} \times \vec{b})|$$

$$Ratio = \frac{8\,|(\vec{a} \times \vec{b})|}{|(\vec{a} \times \vec{b})|} = 8$$

16.

$\vec{a}.\vec{b} < 0,$

$\alpha t^2 - 12 + 6\alpha t < 0$

$\alpha t^2 + 6\alpha t - 12 < 0$

$\alpha < 0, d < 0$

$36\alpha^2 + 48\alpha < 0$

$12\alpha(3\alpha + 4) < 0$

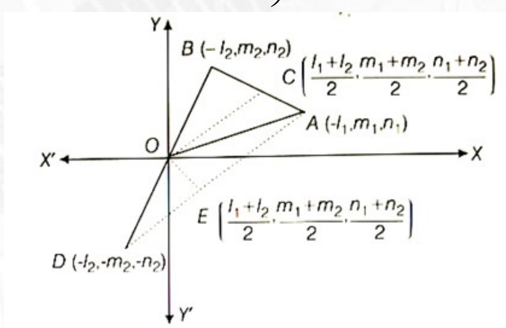
$\frac{-4}{3} < \alpha < 0$

also for a = 0, $\vec{a}.\vec{b} < 0$

hence $\alpha \in (\frac{-4}{3}, 0]$

17. Let OA and OB be two lines direction l_1, m_1, n_1 and l_2, m_2, n_2.

Let OA = OB = 1. Then, the coordinates of A and B are (l_1, m_1, n_1) and (l_2, m_2, n_2) respectively. Let OC be the bisector of $\angle AOB$. Then C is the mid point of AB and so its coordinates are $\left(\frac{l_1 + l_2}{2}, \frac{m_1 + m_2}{2}, \frac{n_1 + n_2}{2}\right)$



$\therefore$ *Direction ratio of OC are* $\frac{l_1 + l_2}{2}, \frac{m_1 + m_2}{2}, \frac{n_1 + n_2}{2}$

Now, $OC = \sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 + \left(\frac{m_1 + m_2}{2}\right) + \left(\frac{n_1 + n_2}{2}\right)^2}$

$OC = \frac{1}{2}$

$\sqrt{(l_1^2 + m_1^2 + n_1^2) + (l_2^2 + m_2^2 + n_2^2) + 2(l_1 l_2 + m_1 m_2 + n_1 n_2}$

$\Rightarrow \quad OC = \frac{1}{2}\sqrt{2 + 2\cos\theta} \quad [\because \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2]$

$\Rightarrow \quad OC = \frac{1}{2}\sqrt{2 + 2\cos\theta} = \cos\left(\frac{\theta}{2}\right)$

$\therefore$ *Direction* cos*ines of OC are*

$\frac{l_1 + l_2}{2(OC)}, \frac{m_1 + m_2}{2(OC)}, \frac{n_1 + n_2}{2(OC)}$

or, $\frac{l_1 + l_2}{2(\cos\frac{\theta}{2})}, \frac{m_1 + m_2}{2(\cos\frac{\theta}{2})}, \frac{n_1 + n_2}{(2\cos\frac{\theta}{2})}$

18.

Direction cosines l, m *and* n *of given vector* $\vec{v}$,

$l = \cos(60^\circ) = \frac{1}{2}$

$m = \cos(45^\circ) = \frac{1}{\sqrt{2}}$

$n = \cos(\gamma)$

$\vec{v} = \cos 60^\circ \hat{i} + \cos 45^\circ \hat{j} + \cos\gamma \hat{k} ...(1)$

we know,

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$

$\Rightarrow \cos\gamma = \frac{1}{2}$

$\gamma = 60^\circ$

Equation of plane is $\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y+1) + \frac{1}{2}(z-1) = 0$

$\Rightarrow \frac{x}{2} + \frac{y}{\sqrt{2}} + \frac{z}{2} = \frac{1}{2}$

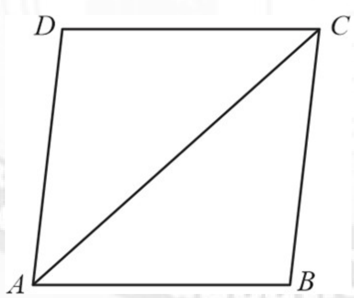
$\Rightarrow x + \sqrt{2}y + z = 1$

(a, b, c) *lies on it*

$\Rightarrow a + \sqrt{2}b + c = 1$

19. Given, ABCD be a quadrilateral, If E and F are the mid points of the diagonals AC and BD respectively and $\left(\overrightarrow{AB} - \overrightarrow{BC}\right) + \left(\overrightarrow{AD} - \overrightarrow{DC}\right) - k\overrightarrow{FE}$,

Now plotting the diagram we get,



Now let position vector of A,B, C and D are $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively

Now by mid point formula,

Position vector of E = $\frac{\overrightarrow{OC} + \overrightarrow{OA}}{2} = \frac{\vec{c} + \vec{a}}{2}$

And Position vector of F = $\frac{\vec{b} + \vec{d}}{2}$

Now, $\left(\overrightarrow{AB}-\overrightarrow{BC}\right)+\overrightarrow{AD}-\overrightarrow{DC}$

$\Rightarrow \vec{b}-\vec{a}-\left(\vec{c}-\vec{b}\right)+\vec{d}-\vec{a}-\left(\vec{c}-\vec{d}\right)$

$\Rightarrow 2\vec{b}-2\vec{a}-2\vec{c}+2\vec{d}$

$\Rightarrow 2\left(\vec{b}+\vec{d}\right)-2\left(\vec{a}+\vec{c}\right)$

$\Rightarrow 4\left[\frac{\vec{b}+\vec{d}}{2}-\frac{\vec{a}+\vec{c}}{2}\right]=4\left[\overrightarrow{OF}-\overrightarrow{OE}\right]$

$\Rightarrow 4\overrightarrow{EF}=-4\overrightarrow{FE}$

$\therefore k=-4$

20.

We know that by trangle law of vector addition,

$\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{b}+\vec{c}=-\vec{a}$

squaring both side we get

$\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\vec{b}\,.\,\vec{c}=\left|\vec{a}\right|^2$

given value $\left|\vec{a}\right|=6\sqrt{2},\left|\vec{b}\right|=2\sqrt{3}\ \&\ \vec{b}\,.\,\vec{c}=12$

we get $\left|\vec{c}\right|^2=36$

$\left|\vec{c}\right|=6$

Now, $S1:\left|\vec{a}\times\vec{b}+\vec{c}\times\vec{b}\right|-\left|\vec{c}\right|$

$\Rightarrow \left|(\vec{a}+\vec{c})\times\vec{b}\right|-\left|\vec{c}\right|$

$\Rightarrow 0-6=-6$

$S2:\vec{a}+\vec{b}+\vec{c}=0$

$\vec{a}+\vec{b}=-\vec{c}$

Squaring both side we get,

$\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\left|\vec{b}\right|\cos(\angle ACB)=\left|\vec{c}\right|^2$

$\Rightarrow 72+12-24\sqrt{6}\cos(\angle ACB)=36$

$\Rightarrow \cos(\angle ACB)=\frac{48}{24\sqrt{6}}=\frac{2}{\sqrt{6}}=\sqrt{\frac{2}{3}}$

21.

$|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2, 2(\vec{a}\times\vec{b}) = 3(\vec{c}\times\vec{a})$ *and angle between*

$\vec{b}\,\&\,\vec{c}$ *is given as* $\frac{2\pi}{3}$.

Now solving, $3(\vec{c}\times\vec{a}) + 2(\vec{b}\times\vec{a}) = 0$

$\Rightarrow (3\,\vec{c}\times 2\vec{b})\times\vec{a} = 0$

Means $(3\,\vec{c}\times 2\vec{b})\,\&\,\vec{a}$ *are parallel vector*,

$3\,\vec{c}\times 2\vec{b} = \lambda\vec{a}$

Now squering both sides we get,

$9|\vec{c}|^2 + 4|\vec{b}|^2 + 12(\vec{b}\,.\,\vec{c}) = \lambda^2|\vec{a}|^2$

$\Rightarrow 36 + 1 + 12 + \frac{1}{2}\times 2\left(\cos\left(2\frac{\pi}{3}\right)\right) = \lambda^2(31)$

$\lambda^2 = 1$

$\lambda = \pm 1$

value of λ *in* $(3\vec{c}\times 2\vec{b}) = \lambda\vec{a}$

$3\vec{c} + 2\vec{b} = \pm\vec{a}$ .....(1)

Now taking dot product with $\vec{b}$ *in above equation we get*

$3(\vec{b}\,.\,\vec{c}) + 2(\vec{b}\,.\,\vec{b}) = \pm\vec{a}\,.\,\vec{b}$

$\Rightarrow \vec{a}\,.\,\vec{b} = \pm\left(-\frac{3}{2} + \frac{1}{2}\right) = \pm(-1)$

$\left(\vec{a}\,.\,\vec{b}\right)^2 = 1$

Again taking $3\left(\vec{c}\times\vec{a}\right) = 2(\vec{a}\times\vec{b})$

$\left(\vec{c}\times\vec{a}\right)^2 = \frac{4}{9}\left(\vec{a}\times\vec{b}\right)^2$

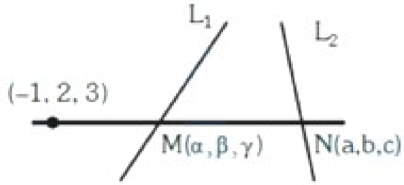
$\left(\vec{c}\times\vec{a}\right)^2 = \frac{4}{9}\times\frac{27}{4} = 3$

$\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\,.\,\vec{b}}\right)^2 = \frac{3}{1} = 3$

22.

$$M(3\lambda+1, 2\lambda+2, -2\lambda-1) \quad \therefore \alpha+\beta+\gamma = 3\lambda+2$$

$$N(-3\mu-2, 2\lambda+2, -2\mu+4\mu+1) \quad \therefore a+b+c = -\mu+1$$



$$\frac{3\lambda+2}{-3\mu-1}=\frac{2\lambda}{-2\mu}=\frac{-2\lambda-4}{4\mu-2}$$

$3\lambda\mu+2\mu=3\lambda\mu+\lambda$

$2\mu=\lambda$

$2\lambda\mu-\lambda=\lambda\mu+2\mu$

$\lambda\mu=\lambda+2\mu$

$\Rightarrow \lambda\mu=2\lambda$

$\Rightarrow \mu=2 \quad (\lambda\neq 0)$

$\therefore \ \lambda=4$

$\alpha+\beta+\gamma=14$

$a+b+c=-1$

$\alpha+\beta+\gamma+a+b+c=13$

23.

$$\vec{r_1}=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$$

$$\vec{r_2}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$$

If $\vec{r}=\vec{a}+\lambda\vec{b}$ *and* $\vec{r}=\vec{c}+\lambda\vec{d}$, *then shortest distance between two line is*

$$L=\frac{(\vec{a}-\vec{c}).(\vec{b}\times\vec{d})}{\left|\vec{b}\times\vec{d}\right|}$$

$$\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix}$$

$$\Rightarrow \vec{b}\times\vec{d}=8\hat{i}+8\hat{j}+4\hat{k}\Rightarrow \vec{b}\times\vec{d}=4(2\hat{i}+2\hat{j}+\hat{k})$$

$$\left|\vec{b}\times\vec{d}\right|=4.3$$

And, $\vec{a}-\vec{c}=\left((\alpha+4)\hat{i}+2\hat{j}+3\hat{k}\right)$

so, shortest distance is

$$\frac{\left((\alpha+4)\hat{i}+2\hat{j}+3\hat{k}\right).4\left(2\hat{i}+2\hat{j}+\hat{k}\right)}{4.3}=9\Rightarrow 2\left(\alpha+4\right)+4+3=27$$

$$\left(\alpha+4\right)=10$$

$$\alpha=6$$

24.

$l_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{i} + (4+2t)\hat{k}$

$l_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$

$DR\ of\ l_1 \equiv (1,2,2)$

$DR\ of\ l_2 \equiv (2,2,1)$

$$l_1 \times l_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$= i(2-4) - j(1-4) + k(2-4)$

$= -2i + 3j - 2k$

$Hence\ DR\ of\ l(line \perp to\ l_1\ \&\ l_2)$

$= (-2,3,-2)$

$\therefore l : \vec{r} = -2\mu\hat{i} + 3\mu\hat{j} - 2\mu\hat{k}$

$for\ intersection\ l\ \&\ l_1$

$3 + t = -2\mu$

$-1 + 2t = 3\mu$

$4 + 2t = -2\mu$

$\Rightarrow t = -1\ \&\ \mu = -1$

$\therefore point\ of\ intersction\ P \equiv (2,-3,2)$

$Let\ point\ on\ l_2\ be\ Q(3+2s, 3+2s, 2+s)$

$Given\ PQ = \sqrt{17} \Rightarrow (PQ)^2 = 17$

$\Rightarrow (2s+1)^2 + (6+2s)^2 + (s)^2 = 17$

$\Rightarrow 9s^2 + 28s + 20 = 0$

$\Rightarrow s = -2, -\dfrac{10}{9}$

$s \neq -2\ as\ point\ lies\ on\ 1^{st}\ octant$

$\therefore a = 3 + 2\left(-\dfrac{10}{9}\right) = \dfrac{7}{9}$

$b = 3 + 2\left(-\dfrac{10}{9}\right) = \dfrac{7}{9}$

$c = 2 + \left(-\dfrac{10}{9}\right) = \dfrac{8}{9}$

$\therefore 18(a+b+c) = 18\left(\dfrac{22}{9}\right) = 44$

25. A Vector in the direction of the required line can be obtained by cross product of

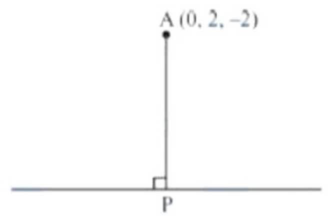
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$$

$\text{Re}\,quired\ line,$

$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda'(-9\hat{i} - 9\hat{j} + 9\hat{k})$

$\vec{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$

Now distance of (0,2,-2)



P.V. of P $\equiv (5+\lambda)\hat{i} + (\lambda - 4)\hat{j} + (3-\lambda)\hat{k}$

$\overrightarrow{AP} = (5+\lambda)\hat{i} + (\lambda - 6)\hat{j} + (5-\lambda)\hat{k}$

$\overrightarrow{AP}\,.(\hat{i} + \hat{j} - \hat{k}) = 0$

$\overrightarrow{AP}$ $5 + \lambda + \lambda - 6 - 5 + \lambda = 0$

$\lambda = 2$

$|\overrightarrow{AP}| = \sqrt{49+16+9} = |\overrightarrow{AP}| = \sqrt{74}$

PHYSICS

26. Temperature coefficient of resistance = $1/R_t \dfrac{dR}{dt}$

$$1/R_0\,(1+\alpha t+\beta t^2)dt \times R_0\,(1+\alpha t+\beta t^2)$$

$$\frac{\alpha+2\,\beta t}{(1+\alpha t+\beta t^2)}$$

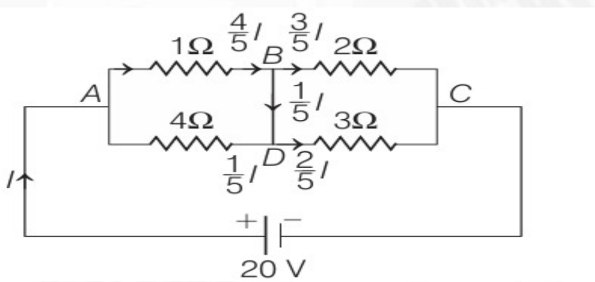
27. As ammeter must be connected in series of 20 Ω resistance and the voltmeter in parallel of 20 Ω resistance, the correct arrangement is as shown in figure (C).

28. An ideal voltmeter has infinite resistance therefore no current flows through the ammeter.

29. Zero current flows as it is a case of balanced Wheatstone bridge.

30. As potential drop is same across all three resistance thus $i_1 : i_2 : i_3 = 1/R_1 : 1/R_2 : 1/R_3$
$1/3 : 1/2 : 1/1 = 6 : 3 : 1$ same in the case with the power consumption.

31.



Since, the current flows in inverse ratio of the resistance of branch.

Now, total circuit resistance,

$$R_{eq} = (1\Omega \,||\, 4\Omega) + (2\Omega \,||\, 3\Omega)$$
$$= \left(\frac{1\times 4}{1+4}\right) + \left(\frac{2\times 3}{2+3}\right)$$
$$= \frac{4}{5} + \frac{6}{5}$$
$$= \frac{10}{5}\,\Omega = 2\Omega$$

So, current drawn from cell,

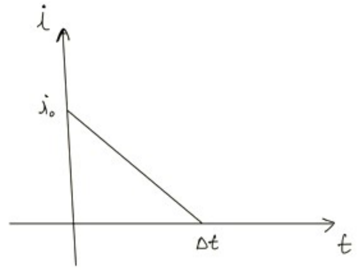
$$I = \frac{V}{R_{eq}} = \frac{20\text{V}}{2\Omega} = 10\text{ A}$$

Hence, current through BD arm is (refer to circuit diagram),

$$I_{BD} = \frac{I}{5} = \frac{10}{5}$$
$$= 2\text{A}$$

32. In first situation, the balancing length is at distance X from left end and 100-X from other end. In second case, balancing length is at a distance 100-X from left end. i.e.,
Shift is (100-X) – X = 20 cm X = 40 cm, 100-X = 60 cm
R/2 = 60/40 (by wheat stone bridge principle) or R = 3 Ω

33.



$q = \frac{1}{2}\Delta t\, i_0$

$i_0 = \frac{2q}{\Delta t}$ ---- (1)

$i = -\frac{i_0}{\Delta t}t + i_0$

$i = \left(1-\frac{t}{\Delta t}\right) i_0$

$H = \int i^2 R\, dt$

$H = R\int_0^{\Delta t}\left(1-\frac{t}{\Delta t}\right)^2 i_0^2\, dt$ – (2)

Soling (1) & (2)

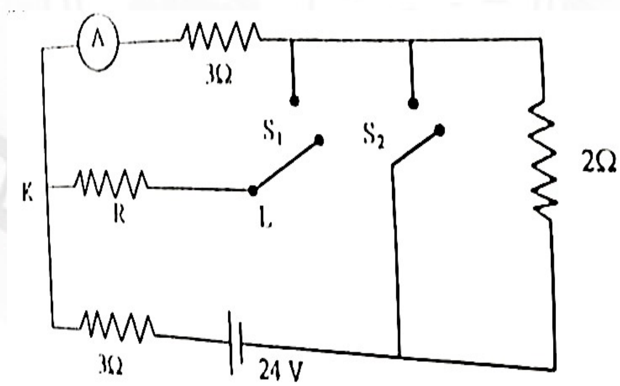
$H = \frac{4q^2R}{3\Delta t}$

34. A(p), B(p), C(p), D(p)

$$J\alpha\frac{1}{A}, E\alpha\frac{1}{A}, R\alpha\frac{1}{A}, dV\alpha\frac{1}{A}$$

35. When both switches are open then reading of ammeter is 24/8 = 3A When both switches are closed. $V_{AB} = 9$ V $\Rightarrow$ $V_{HG} = 15$ $\Rightarrow$ So current through R is 2A.

Hence R = 9/2 = 4.5Ω

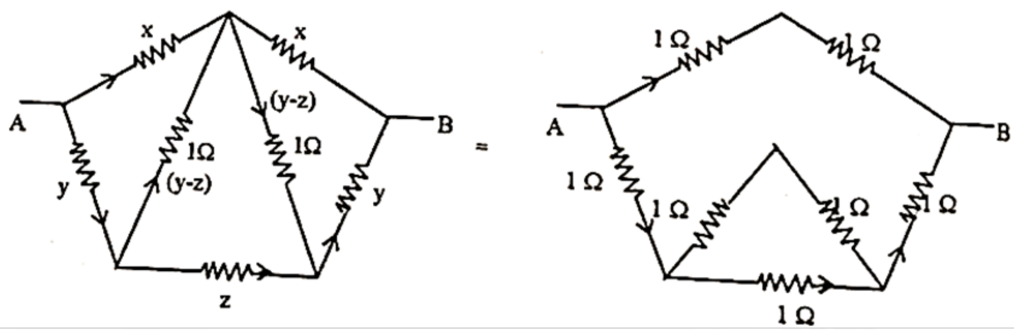


36. In the steady, no current flows through the capacitor

37.

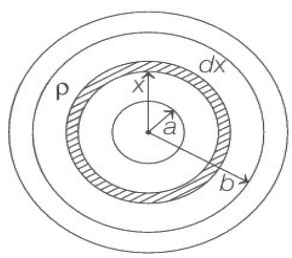
$$\therefore R_{equl} = \frac{\frac{8}{3}\times 2}{\frac{8}{3}+2}$$

$$= \frac{16}{14} = \frac{8}{7}\Omega.$$



38. Both are false (Conceptual).
39. Both (A) & (R) are true and the (R) is the correct explanation of the (A)
40. As shunt is connected across the ammeter the range of ammeter is increases and resistance of ammeter decreases.
41. Both are false (conceptual)
42.

For a elemental shell of radius x and thickness dx,



Resistance, $dR = \rho \frac{l}{A}$

$\Rightarrow \qquad dR = \rho \frac{dx}{4\pi x^2}$

So, resistance of complete arrangement is

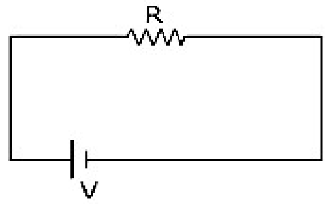
$$R = \int_a^b dR = \int_a^b \rho \frac{dx}{4\pi x^2} = \frac{\rho}{4\pi}\int_a^b x^{-2}dx$$

$$\Rightarrow R = \frac{\rho}{4\pi}\left(\frac{x^{-1}}{-1}\right)_a^b = \frac{\rho}{4\pi}\left(-\frac{1}{x}\right)_a^b$$

$$= \frac{\rho}{4\pi}\left(-\frac{1}{b}-\left(-\frac{1}{a}\right)\right)$$

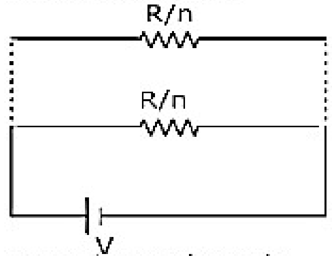
$$= \frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)\text{ohm}$$

43.



Initially $H = \frac{V^2}{R}$

Now after cutting



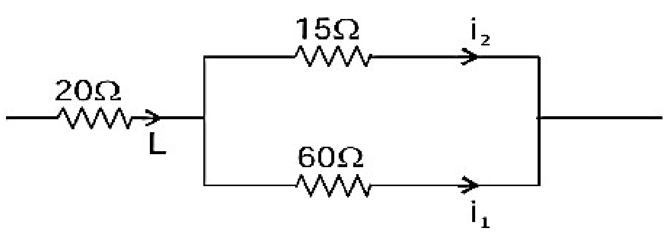
Power in one branch

$$= \frac{V^2}{R/n} = \frac{nV^2}{R}$$

$$\text{Total power} = \frac{nV^2}{R} + \frac{nV^2}{R} + \dots$$

$$= \frac{n^2V^2}{R}$$

44. $\frac{i}{2}R_g = \frac{i}{2}(20) \Rightarrow R_g = 20$



Given $\frac{15 \times i}{75} = 0.75$

Now $i_2 = \frac{60 \times i}{75} = \left[\frac{60 \times 0.75 \times 75}{15}\right]$

$= 3A$

45.

SECTION –B

46. As 5 Ω resistor is joined in parallel to series combination of 4 Ω and 6 Ω (i. e ,total resistance 10 Ω), V = constant.

And $i_1/ i_2 = R_2/R_1 = 10/5 = 2$

OR $i_2 = i_1/2$

Now heat produced per second in 5 Ω resistor

$H_1 = i_1^2 R_1 = i_1^2 \times 5 = 100\ Js^{-1}$

and for 4 Ω resistor

$H_2 = i_2^2 R_2 = (\frac{i_1}{2})^2 \times 4 = i_1^2$

Simplifying eqn. (i) and (ii), we get

$H_2/100 = 1/5$

Or $H_2 = \frac{1}{5} \times 100 = 20\ Js^{-1}$

47.

Let emf of cell = ε

Potential difference across the terminal of cell

$$V_1 = 1.25\text{ V}$$

Load resistance, $R_{L_1} = 5\Omega$

when load resistance, $R_{L_2} = 2\Omega$,

then $V_2 = 1\text{ V}$

I_1, I_2 be the current through load in the two above mentioned cases and r be internal resistance of cell.

As we know that,

By using Kirchhoff's voltage law,

$$\varepsilon - V = I_1 R_{L_1}$$

$$\varepsilon = I_1 R_{L_1} + V$$

$$\varepsilon = I_1 (R_{L_1} + r) \quad [\because V = I_1 r]$$

$$\Rightarrow \quad I_1 = \frac{\varepsilon}{R_{L_1} + r} = \frac{\varepsilon}{5 + r}$$

$$\because \quad V_1 = I_1 R_{L_1}$$

$$\therefore \quad 1.25 = \frac{\varepsilon}{5 + r} \times 5$$

$$\Rightarrow \quad \frac{\varepsilon}{5 + r} = 0.25 = \frac{25}{100} = \frac{1}{4}$$

$$\Rightarrow \quad 4\varepsilon = 5 + r \quad \ldots(i)$$

and $V_2 = I_2 R_{L_2} = \frac{\varepsilon}{2 + r} 2 = 1$

$$\Rightarrow \quad 2\varepsilon = 2 + r \quad \ldots(ii)$$

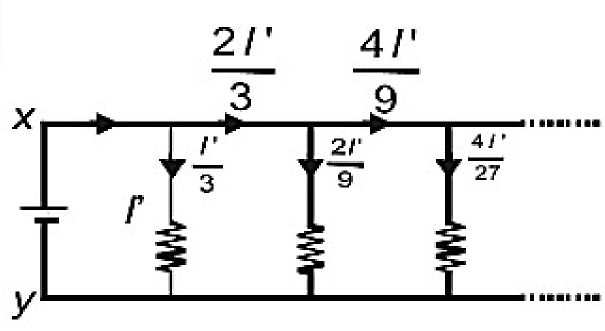
On subtracting Eq. (i) and Eq. (ii), we get

$$2\varepsilon = 3$$

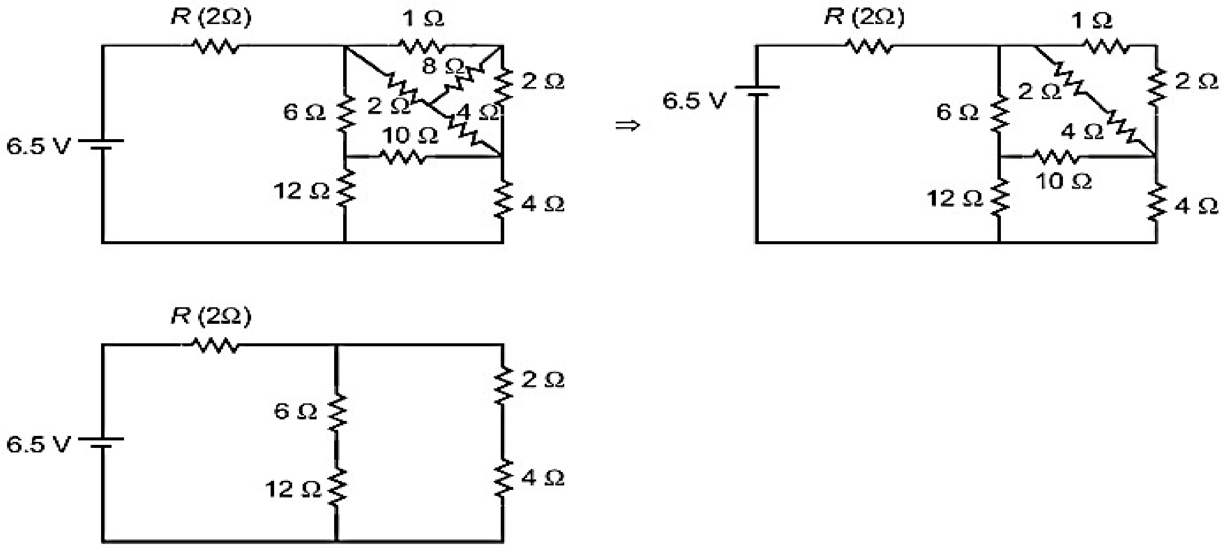
$$\varepsilon = \frac{3}{2} = 1.5\text{ V} = \frac{15}{10}\text{ V}$$

Hence, $x = 15$

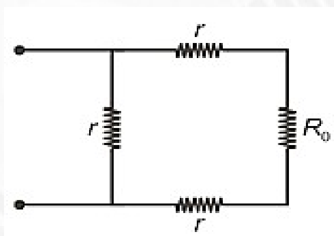
48. $R_{xy} = 3\ \Omega$ current distribution is as shown



49. The circuit is shown below



50. The equivalent resistance is independent of number of repetitions if the equivalent of this circuit equals to R_0



$$\Rightarrow R_0 = \frac{r(2R + R_0)}{3R + R_0}$$

$$\Rightarrow R_0 = (\sqrt{3} - 1)r$$

CHEMISTRY

51. Lactose is a disaccharide of $\alpha - D -$ Glucose and $\beta - D -$Galactose

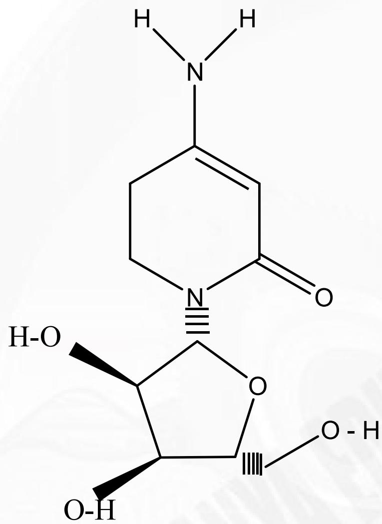
52. Vitamin A,D,E,K are fat soluable

53. DNA bases $\rightarrow$ G C A T

$A = T \rightarrow$ two H - bonds

$G \equiv C \rightarrow$ 3 H – Bonds

54. Cytidine is



55. Fructose $\xrightarrow[P/HI]{HCN/H_3O^+} 2 - Methyl\ hexane$

56. Compound with Hemiacetal Linkage Shows Mutarotation.

57. Anomers differs at functional group carbon

58. $15.07(x) + 52.8(1-x) = +80.2$

$97.9x = 27.4$

$x = 0.28$

$= 28\%$

59. Weight of glycine in decapepide $= \frac{958 \times 47}{100}$

Moles of glycine $\frac{450}{75} = 6$

60. $\frac{2.34 + 9.60}{2} = 5.97$

61. Sodium cyanide and sodium sulphide into HCN and H2S which are volatile.
NaCN also white colour

62. Sucrose is a non reducing disacharide Sugar

63. Conceptual

64. Conceptual

65. 5D represents Configuration. D (or) t represents dentrorotatory

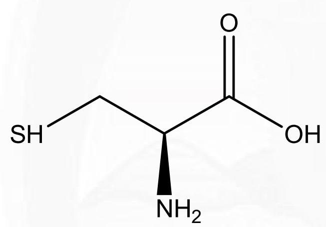
66. Maltose forms α Glycoside Linkage between C_1 & C_4 of glucose molecule

67. Conceptual

68. Enzyme Catalysis

69. DNA bases are G, C, A, T

70. Cystine



71. 5 moles of HIO_4 required

72. Aromatic aldehydes do not give bendict test

73. Molecular formula of Glucosazone is $C_{18}H_{22}N_4O_4$ (358)

74. $x = 4,\ y = 6,\ z = 5$

75. 4