Mathematics

61. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

A. 38 B. 23
C. 58 D. 35

Ans. A

Sol:

Given binomial is

$\left(x^2 + \frac{1}{x^3}\right)^n$, its $(r + 1)^{th}$ term, is

General term

$T_{r+1} = {}^{n}C_r\ x^{2n-2r}.x^{-3r}$

$\because\ 2n - 5r = 1 \Rightarrow 2n = 5r + 1$

$\therefore\ r = \dfrac{2n-1}{5}$

$\therefore$ Coeff. of $x = {}^{n}C_{\left(\frac{2n-1}{5}\right)} = {}^{n}C_{23}$

$\therefore\ \dfrac{2n-1}{5} = 23$ or $n - \left(\dfrac{2n-1}{5}\right) = 23$

$2n - 1 = 115$ $\quad\quad\quad\quad$ $n = 38$

62. If 5x + 9 = 0 is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is:

A. $\left(\frac{5}{3}, 0\right)$ $\quad\quad$ B. (-5,0)

C. $\left(-\frac{5}{3}, 0\right)$ $\quad\quad$ D. (5,0)

Ans. B

Sol:

Equation of given hyperbola is

$16x^2 - 9y^2 = 144$

$\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

$a = 3$

$b = 4$

So, the eccentricity of above equation

$e^2 = 1 + \dfrac{16}{9}$

$e = \dfrac{5}{3}$

$\therefore$ focus is $(-ae, 0) = (-5, 0)$

63. The angles A, B and C of a triangle ABC are in A.P. and a : b =1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq.cm) of this triangle is :

A. $4\sqrt{3}$ B. $\frac{4}{\sqrt{3}}$

C. $\frac{2}{\sqrt{3}}$ D. $2\sqrt{3}$

Ans. D

Sol:

It is given that the angles of a ΔABC are in AP

$\therefore\ 2B = A + C$

As we know sum of all angles of a triangle $= \pi$

& $A + B + C = \pi$

$B = \frac{\pi}{3}$

$\therefore\ A + C = \frac{2\pi}{3}\ \sigma$

$\frac{a}{b} = \frac{1}{\sqrt{3}}$ Given

$\frac{2R\sin A}{2R\sin B} = \frac{1}{\sqrt{3}}$

$\sin A = \frac{1}{2}$

$\therefore\ A = 30^\circ$

$\therefore\ a = 2,\ b = 2\sqrt{3},\ c = 4$

Area of ABC $= \frac{1}{2}$ ab sin C

$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 \times 1 = 2\sqrt{3}$

64. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in R, \left(x \neq \pm\sqrt{3}\right)$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line 2x + 6y – 11 = 0, then :

A. $|6\alpha + 2\beta| = 19$ B. $|2\alpha + 6\beta| = 19$

C. $|2\alpha + 6\beta| = 11$ D. $|6\alpha + 2\beta| = 9$

Ans. B

Sol:

Equation of given curve is

$$y = \frac{x}{x^2 - 3}, \ x \in R, \ (x \neq \pm\sqrt{3}) \qquad ...(i)$$

On differentiating eq. (i) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(x^2 - 3) - x(2x)}{(x^2 - 3)^2} = \frac{(-x^2 - 3)}{(x^2 - 3)^2}$$

It is given that tangent at a point (α, β) ≠ (0, 0) on it is parallel to the line

2x + 6y – 11 = 0

∴ Slope of this line $= -\frac{2}{6} = \left.\frac{dy}{dx}\right|_{(\alpha,\beta)}$

$$\left.\frac{dy}{dx}\right|(\alpha, \beta) = \frac{-\alpha^2 - 3}{\left(\alpha^2 - 2\right)^2}$$

Given

$$\frac{-\alpha^2 - 3}{\left(\alpha^2 - 3\right)^2} = -\frac{1}{3}$$

$$\Rightarrow \ \alpha = 0, \pm 3 \left(\alpha \neq 0\right)$$

Now, from equation (i)

$$\beta = \frac{\alpha}{\alpha^2 - 3} = \frac{1}{2} \text{ or } -\frac{1}{2}$$

then |6α + 2β| = 19

65. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

A. 8 B. 6

C. 5 D. 7

Ans. D

Sol:

Now, let n be the minim numbers of toss required to get at least one head, then required probability = 1 – (probability that on all 'n' toss we are getting tail)

$$-1-\left(\frac{1}{2}\right)^{n}$$

According to questions

$$1-\left(\frac{1}{2}\right)^{n}>\frac{99}{100}$$

$$\left(\frac{1}{2}\right)^{n}<\frac{1}{100}$$

$$\Rightarrow\ n=7$$

66. The negation of the Boolean expression $\sim s\vee(\sim r\wedge s)$ is equivalent to :

A. ~ s ^ ~ r B. r

C. s ^ r D. s V r

Ans. C

Sol:

The given Boolean expression is ~ s ∨ ((~r) ∧ s)

Now, the negation of given Boolean expression is

$$\sim(\sim s\vee(\sim r\wedge s))$$

$$s\wedge(r\vee\sim s)$$

$$(s\wedge r)\vee(s\wedge\sim s)$$

$$(s\wedge r)\wedge(s)$$

$$(s\wedge r)$$

67. The number of real roots of the equation $5+\left|2^x-1\right|=2^x(2^x-2)$ is :

A. 3 B. 1

C. 2 D. 4

Ans. B

Sol:

Given that

$5+\left|2^x-1\right|=2^x(2^x-2)$

Case I

$2^x \geq 1$

$5+2^x-1=2^x\left(2^x-2\right)$

Let $2^x=t$

$5+t-1=t(t-2)$

$t=4,-1(\text{rejected})$

$2^x=4$

$x=2$

only 1 solution

Case II

$2^x<1$

$5+1-2^x=2^x\left(2^x-2\right)$

$2^x=t$

$5+1-t=t(t-2)$

$0=t^2-t-6$

$0=(t-3)(t-2)$

$t=3,-2$

$2^x=3,\ 2^x=-2$

(rejected)

Therefore, number of real roots is one.

68. The tangent and normal to the ellipse $3x^2+5y^2=32$ at the point P(2,2) meet the x-axis at Q and R, respectively. Then the area (in sq. units)of the triangle PQR is :

A. $\frac{68}{15}$ B. $\frac{34}{15}$

C. $\frac{16}{3}$ D. $\frac{14}{3}$

Ans. A

Sol:

The equation of given ellipse is

$3x^2+5y^2=32$

On differentiating both side of above equation we get

$6x+10yy'=0$

$y'=\frac{-3x}{5y}$

$y'_{(2,2)}=-\frac{3}{5}$

Tangent $(y-2)=-\frac{3}{5}(x-2)$

$\Rightarrow Q\left(\frac{16}{3},0\right)$

Normal $(y-2)=\frac{5}{3}(x-2)$

$\Rightarrow R\left(\frac{4}{5},0\right)$

Area $=\frac{1}{2}(QR)\times 2=QR=\frac{68}{15}$

$\left[\because QR=\sqrt{\left(\frac{16}{3}-\frac{4}{5}\right)^2}=\sqrt{\left(\frac{68}{15}\right)^2}=\frac{68}{15}\text{ and height}=2\right]$

69. Let λ be a real number for which the system of linear equations

$x+y+z=6$

$4x+\lambda y-\lambda z=\lambda-2$

$3x+2y-4z=-5$

has infinitely many solutions. Then λ is a root of the quadratic equation :

A. $\lambda^2+\lambda-6=0$ B. $\lambda^2-\lambda-6=0$

C. $\lambda^2-3\lambda-4=0$ D. $\lambda^2+3\lambda-4=0$

Ans. B

Sol:

Given, system of linear equations

$x + y + z = 6$ …(i)

$4x + \lambda y – \lambda z = \lambda – 2$ …(ii)

and $3x + 2y – 4z = -5$…(iii)

has infinitely means solutions, then

$\Delta=0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix}=0$$

$\Rightarrow -8\lambda+24=0 \quad \Rightarrow \lambda=3$

On solving we get $\lambda=3$

$\therefore$For $\lambda = 3$, infinitely many solutions is obtained.

70. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x+y+z=3$ such that the foot of the perpendicular Q also lies on the plane $x-y+z=3$. Then the co-ordinates of Q are :

A. (2,0,1) B. (4,0,-1)

C. (1,0,2) D. (-1,0,4)

Ans. A

Sol:

Let a general point on the given line

$$\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}=\lambda$$

Let a point on the line is

$(2\lambda+1, -\lambda-1, +\lambda)$

the foot of perpendicular $Q(x_2, y_2, z_2)$ drawn from point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is given by

$$\frac{x_2-x_1}{a}=\frac{y_2-y_1}{b}=\frac{z_2-z_1}{c}$$

$$=-\frac{ax_1+by_1+cz_1+d}{a^2+b^2+c^2}$$

Foot of $\perp^r$ Q is given by

$$\frac{x-2\lambda-1}{1}=\frac{y+\lambda+1}{1}=\frac{z-\lambda}{1}=-\frac{(2\lambda-3)}{3}$$

$\because$ Q lies on $x+y+z=3$ and $x-y+z=3$

$\Rightarrow\ x+z=3$ and $y=0$

$\therefore\ y=0 \quad \Rightarrow \lambda+1=\dfrac{-2\lambda+3}{3} \quad \Rightarrow \lambda=0$

$\therefore$ Q is $(2, 0, 1)$

71. Let a, b and c be in G.P. with common ratio r, where $a\neq 0$ and $0<r\leq\frac{1}{2}$. If 3a,7b and 15c are the first three terms of an A.P., then the 4^{th} term of this A.P. is :

A. $\frac{2}{3}a$ B. $\frac{7}{3}a$

C. a D. 5a

Ans. C

Sol:

It is given that, the terms a, b, c are in GP with common ratio r, where $a\neq 0$ and $0<r\leq\frac{1}{2}$.

So, let, $b = ar$ and $c = ar^2$

Now, the terms 3a, 7b and 15c are the first three terms of an AP, then

$14b = 3a + 15c$

$14(ar) = 3a + 15(ar^2)$

$15r^2 - 14r + 3 = 0$

$\Rightarrow r = \frac{1}{3}, \frac{3}{5}$ (rejected)

Common difference $= 7b - 3a$

$= 7ar - 3a$

$= \frac{7a}{3} - 3a$

$= -\frac{2}{3}a$

4^{th} term is

$\Rightarrow \ 15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

72. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x+1|$, in the first quadrant is :

A. $\log_e 2 + \frac{3}{2}$

B. $\frac{3}{2} - \frac{1}{\log_e 2}$

C. $\frac{1}{2}$

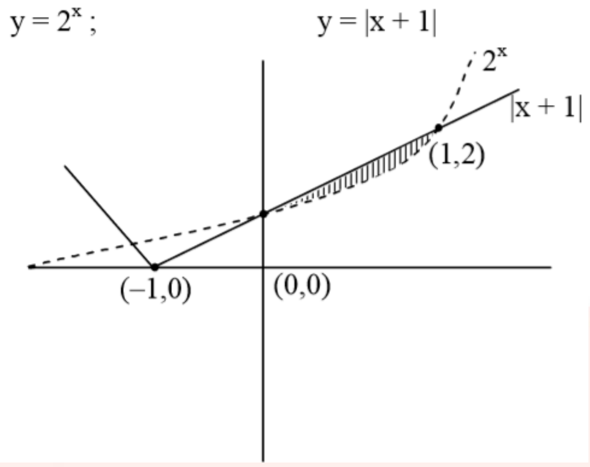
D. $\frac{3}{2}$

Ans. B

Sol:

Given, equation of curves

$y = 2^x$ and $y = |x + 1| = \begin{cases} x+1, & x \geq -1 \\ -x-1, & x < -1 \end{cases}$



Intersect points (1, 2)

Required Area $= \int_0^1 \left((x+1) - 2^x\right) dx$

$$= \left(\frac{x^2}{2} + x - \frac{2^x}{\log_e 2}\right)_0^1$$

$$= \left[\frac{1}{2} + 1 - \frac{2}{\log_e 2} + \frac{1}{\log_e 2}\right]$$

$$= \frac{3}{2} - \frac{1}{\log_e 2}$$

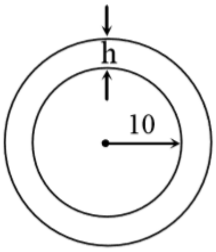
73. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

A. $\frac{5}{6\pi}$
B. $\frac{1}{18\pi}$
C. $\frac{1}{9\pi}$
D. $\frac{1}{36\pi}$

Ans. B

Sol:

Let the thickness of layer of ice is x cm, the volume of spherical ball (only ice layer) is



$V = \frac{4}{3}\pi\left((10+h)^3 - 10^3\right)$

On differentiating with respect to x

$\frac{dV}{dt} = 4\pi(10+h)^2 \frac{dh}{dt}$

$-50 = 4\pi\,(10+5)^2 \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{-1}{18\pi} \frac{cm}{min}$

74. If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c,$ where c is a constant of integration, then g(-1) is equal to :

A. 1

B. $-\frac{5}{2}$

C. $-\frac{1}{2}$

D. -1

Ans. B

Sol:

Given integral, $I = \int x^5 e^{-x^2} dx$

Let $x^2 = t$

$\Rightarrow \frac{1}{2}\int t^2 e^{-t} dt$

$= \frac{1}{2}\left[-t^2 e^{-t} + \int 2te^{-t} dt\right]$

$= \frac{-t^2 e^{-t}}{2} - te^{-t} - e^{-t}$

$= \left(-\frac{x^4}{2} - x^2 - 1\right) e^{-x^2} + c \quad ...(i)$

∵ It is given that,

$$I=\int x^5 e^{-x^2}dx=g(x).e^{-x^2}+C$$

By Eq. (i), comparing both sides, we get

$$g(x)=-\frac{1}{2}\left(x^4+2x^2+2\right)$$

$$\text{So, } g(-1)=-\frac{1}{2}(1+2+2)=-\frac{5}{2}$$

$$g(x)=-\frac{x^4}{2}-x^2-1$$

$$g(-1)=-\frac{1}{2}-1-1$$

$$=-\frac{5}{2}$$

75. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is:

A. 180 B. 210

C. 170 D. 190

Ans. C

Sol:

Required number of beams= ${}^{20}C_2-20=190-20=170$

76. If $\lim\limits_{x\to 1}\frac{x^2-ax+b}{x-1}=5$, then a+b is equal to:

A. -7 B. 1

C. 5 D. -4

Ans. A

Sol:

Given that

$$\lim_{x\to 1}\frac{x^2 - ax + b}{x-1} = 5$$

Since, limit exist and equal to 5 and denominator is zero at x = 1, so numerator $x^2 - ax + b$ should be zero at x = 1

So, $1 - a + b = 0 \Rightarrow a = 1 + b$ … (i)

Now,

'L' hospital rule

$2x - a = 5$

$2 - a = 5 \quad (\because x = 1)$

$a = -3 \qquad \ldots(ii)$

Put in (1)

$\therefore \; b = -4$

$a + b = -7$

77. Let $a_1, a_2\, a_3, \ldots..$ be an A.P. when $a_6=2$. Then the common difference of this A.P., which maximises the product $a_1a_4a_5$, is:

A. $\frac{8}{5}$ B. $\frac{2}{3}$

C. $\frac{6}{5}$ D. $\frac{3}{2}$

Ans. A

Sol:

Given, the terms a_1, a_2, a_3, … , are in AP. Let the common difference of this AP is ‘d’ and fist term a_1 = a, then

Common difference = d

$\therefore \; a + 5d = 2$

$a_1 . a_4 . a_5 = a(a+3d)\,(a+4d)$

$f(d) = (2-5d)(2-2d)(2-d)$

$f'(d)\, 0 \Rightarrow \; d = \frac{2}{3}, \frac{8}{5}$

$f''(d) < 0 \text{ at } d = \frac{8}{5}$

$\Rightarrow \; d = \frac{8}{5}$

78. Let $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y(0)=1. Then :

A. $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

B. $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

C. $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

D. $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

Ans. D

Sol:

Given differential equation is

$\frac{dy}{dx} + y(\tan x) = 2x + x^2 \tan x$

which is linear differential equation in the from of $\frac{dy}{dx} + Py = Q.$

Here, P = tam x and Q = 2x + x^2 tan x.

$I.F. = e^{\int \tan x\, dx} = e^{\ln \sec x} = \sec x$

$y.\sec x = \int (2x + x^2 \tan x)\sec x\, dx$

$y \sec x = x^2 \sec x + \lambda$

$\Rightarrow \ y = x^2 + \lambda \cos x$

$y(0) = 0 + \lambda = 1$

$\Rightarrow \ \lambda = 1$

$y = x^2 + \cos x$

$y' = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$

79. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to:

A. $3^{5/3} - 3^{1/3}$ B. $3^{7/6} - 3^{5/6}$

C. $3^{4/3} - 3^{1/3}$ D. $3^{5/6} - 3^{2/3}$

Ans. B

Sol:

Let

$$I = \int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$$

$$\int_{\pi/6}^{\pi/3} \frac{1}{\cos^{2/3} x \sin^{4/3}} dx$$

$$= \int \frac{\sec^2 x}{\tan^{4/3} x} dx$$

Let $\tan x = t$, $\sec^2 x\, dx = dt$

$$= \int \frac{dt}{t^{4/3}}$$

$$I = -3\left(t^{-1/3}\right)$$

$$= \left(-3(\tan x)^{\frac{-1}{3}}\right)_{\pi/6}^{\pi/3}$$

$$= 3\left(3^{\frac{1}{3}} - \frac{1}{3^{\frac{1}{6}}}\right)$$

$$= 3^{7/6} - 3^{5/6}$$

80. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6k$ from the straight line passing through the point (2, 3, -4) and parallel to the vector, $6\hat{i} + 3\hat{j} - 4k$ is:

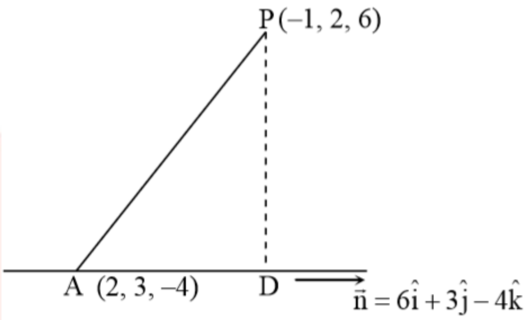
A. 7 B. 6

C. $2\sqrt{13}$ D. $4\sqrt{3}$

Ans. A

Sol:

Let point P whose position vector is $\left(-\hat{i}+2\hat{j}+6\hat{k}\right)$ and a straight line passing through A(2, 3, -4) parallel to the vector $n = 6\hat{i}+3\hat{j}-4\hat{k}$.



∵ Required distance = projection of line segment AP perpendicular to vector **n**.

$$AD = \left|\frac{\vec{AP}.n}{|\vec{n}|}\right| = \sqrt{61}$$

$$PD = \sqrt{AP^2 - AD^2}$$
$$= \sqrt{110-61}$$
$$= 7$$

81. Line are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines?

A. $\left(\frac{1}{4},\frac{1}{3}\right)$

B. $\left(\frac{1}{4},-\frac{1}{3}\right)$

C. $\left(-\frac{1}{4},\frac{2}{3}\right)$

D. $\left(-\frac{1}{4},-\frac{2}{3}\right)$

Ans. C

Sol:

Since, equation of line parallel to line
ax + by + c = 0 is ax + by + λ = 0
∴ Equation of line parallel to line
4x – 3y + 2 = 0 is 4x – 3y + λ = 0 …(i)

Now, distance of line (i) from the origin is

$\left|\frac{\lambda}{5}\right| = \frac{3}{5}$

$\lambda = \pm 3$

∴ required lines are 4x – 3y + 3 = 0 & 4x – 3y – 3 = 0

Now, from the given option the point $\left(-\frac{1}{4}, \frac{2}{3}\right)$ lies on the line 4x – 3y + 3 = 0.

82. Let $f(x) = \log_e(\sin x), (0 < x < \pi)$and $g(x) = \sin^{-1}(e^{-x}), (x \geq 0)$. If α is a positive real number such that a = (fog)' (α) and b = (fog) (α), then:

A. $a\alpha^2 + b\alpha + a = 0$

B. $a\alpha^2 + b\alpha - a = -2\alpha^2$

C. $a\alpha^2 - b\alpha - a = 1$

D. $a\alpha^2 - b\alpha - a = 0$

Ans. C

Sol: Given that

$f(x) = \log_e(\sin x), (0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x}), (x \geq 0)$

$f(g(x)) = -x \Rightarrow (f(g(x)))' = -1$

$f(g(\alpha)) = -\alpha = b \Rightarrow (f(g(\alpha)))' = -1 = a$

$\therefore\ b = -\alpha$

$a = -1$

Since, the value of a = -1 and b = -a, satisfy the quadratic equation (from the given options)

$a\alpha^2 - b\alpha = a = 1$.

83. If the line ax + y =c, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then |c| is equal to :

A. $\frac{1}{2}$ B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$ D. 2

Ans. C

Sol:

The equation of tangent of slope 'm' to the parabola $y^2 = 4ax$ is $y = mx + \frac{a}{m}$ and a line ax + by + c = 0 touches the circle

$x^2 + y^2 = r^2$, if $\frac{|c|}{\sqrt{a^2+b^2}} = r.$

Tangent to the curve

$y^2 = 4\sqrt{2x}$ is $y = mx + \frac{\sqrt{2}}{m}$

It is tangent to the circle $x^2 + y^2 = 1$

$\therefore \left|\frac{\sqrt{2}/m}{\sqrt{1+m^2}}\right| = 1 \Rightarrow m = \pm 1$

$\therefore$ tangent are $y = x + \sqrt{2}$ and $y = -x - \sqrt{2}$

Compare with $y = -ax + c$

$\Rightarrow a = \pm 1$ and $c = \pm\sqrt{2}$

84. The sum $1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \ldots$
$+ \frac{1^3+2^3+3^3+\ldots+15^3}{1+2+3+\ldots+15} - \frac{1}{2}(1+2+3+\ldots+15)$ is equal to :

A. 1240 B. 620

C. 660 D. 1860

Ans. B

Sol:

$$1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \ldots + \frac{1^3+2^3+3^3+\ldots+15^3}{1+2+3+\ldots+15} - \frac{1}{2}(1+2+3+\ldots+15)$$

Let $S_1 - S_2$

$$S_1 = \sum_{n=1}^{15} \frac{1^3 + 2^3 + \ldots.. + n^3}{1 + 2 + \ldots. + n} = \sum_{n=1}^{15} \frac{n(n+1)}{2}$$

$$S_2 = \frac{1}{2}\frac{15 \times 16}{2} = 16$$

Now, $S_1 - S_2$

$$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$$

$$= \frac{1}{2}\sum_{n=1}^{15} n^2 + \frac{1}{2}\sum_{n=1}^{15} n - 60$$

$$= \frac{1}{2} \times \frac{15 \times 16 \times 31}{6} + \frac{1}{2} \times \frac{15 \times 16}{2} - 60$$

$$= 620$$

85. The sum of the real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to:

A. -4 B. 0

C. 1 D. 6

Ans. B

Sol:

Given equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

After expanding,

$$x(-3x \times (x+2) - 2x(x-3)) + (-6)(2(x+2)$$
$$+3(x-3)) + (-1)(4x + 3(-3x))$$

$$\Rightarrow -5x^3 + 30x - 30 + 5x = 0$$

$$x^3 - 7x + 6 = 0$$

Since all roots are real

$$\therefore \text{Sum of roots} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3} = 0$$

$$\text{Sum of roots} = 0$$

86. If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots, x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, \ldots, (x_{50} - 4)^2$, is :

A. 525 B. 480

C. 400 D. 380

Ans. C

Sol:

It is given that both mean and standard deviation of 50 observations $x_1, x_2, x_3, \ldots x_{50}$ are equal to 16,

$$\text{Mean } (\mu) = \frac{\sum X_i}{50} = 16$$

$$\therefore \sum X_i = 16 \times 50$$

$$\text{and standard deviation} = (\sigma) = \sqrt{\frac{\sum X_i^2}{50} - (\mu)^2} = 16$$

$$\Rightarrow \frac{\sum X_i^2}{50} = 256 \times 2$$

$$\text{Required mean} = \frac{\sum (X_i - 4)^2}{50}$$

$$= \frac{\sum X_i^2 + 16 \times 50 - 8 \sum X_i}{50}$$

$$= 256 \times 2 + 16 - 8 \times 16$$

$$= 400$$

87. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, where $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$, then for all $x, y, 4x^2 - 4xy\cos\alpha + y^2$ is equal to :

A. $4\sin^2\alpha - 2x^2y^2$

B. $4\cos^2\alpha + 2x^2y^2$

C. $2\sin^2\alpha$

D. $4\sin^2\alpha$

Ans. D

Sol: Given that

$$\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha, \text{ where } -1 \le x \le 1,$$

$$-2 \le y \le 2 \text{ and } x \le \frac{y}{2}$$

$$\therefore \cos^{-1}\left(x\frac{y}{2} + \sqrt{1-x^2}\sqrt{1-(y/2)^2}\right) = \alpha$$

$$\left[\because \cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right) |x|, |y| \le \text{ and } x + y \ge 0\right]$$

$$x\frac{y}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$$

$$\left(\cos\alpha - \frac{xy}{2}\right) = \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}$$

Squaring both sides

$$x^2 + \frac{y^2}{4} - xy\cos\alpha = 1 - \cos^2\alpha$$

$$= \sin^2\alpha$$

88. If z and w are two complex numbers such that |zw| = 1 and arg(z)-arg(w)=$\frac{\pi}{2}$, then :

A. $z\overline{w} = \frac{-1+i}{\sqrt{2}}$

B. $z\overline{w} = \frac{1-i}{\sqrt{2}}$

C. $\overline{z}w = -i$

D. $\overline{z}w = i$

Ans. C

Sol:

It is given that, there are two complex numbers z and w, such that $|zw| = 1$ and $\arg(z) – \arg(w) = \pi/2$

$\therefore |z||w| = 1$ [$\because |z_1 – z_2| = |z_1||z_2|$]

and $\arg(z) = \frac{\pi}{2} + \arg(w)$

Let $|z| = r$, then $|w|=1/r$ …(i)

$|zw| = 1$

$|z||w| = 1$

Let $w = \frac{1}{r}e^{i\theta}$

then $z = re^{i\left(\theta + \frac{\pi}{2}\right)}$

$\bar{z}w = e^{-i\left(\theta + \frac{\pi}{2}\right)} . e^{i\theta} = e^{-i(\pi/2)} = -i$

and $z\bar{w} = e^{-i\left(\theta + \frac{\pi}{2}\right)} . e^{i\theta} = e^{-i\pi/2} = i$

89. If the plane $2x – y + 2z +3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$ respectively, then the maximum value of $\lambda + \mu$ is equal to:

A. 13 B. 9

C. 5 D. 15

Ans. A

Sol:

Equation of given plane are

$2x – y + 2z + 3 = 0$ …(i)

$4x – 2y + 4z + \lambda$ …(ii)

and $2x – y + 2z + \mu = 0$ …(iii)

$\because$ Distance between two parallel planes

$ax + by + cz + d_1 = 0$, and

$ax + by + cz + d_2 = 0$ is

distance = $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

Distance between planes (i) and (ii)

(i) $\frac{|\lambda - 6|}{\sqrt{16+4+16}} = \left|\frac{\lambda - 6}{6}\right| = \frac{1}{3}$

$|\lambda - 6| = 2$

$\lambda = 8, 4$

Distance between planes (i) and (iii)

(ii) $\frac{|\mu - 3|}{\sqrt{4+4+1}} = \frac{2}{3}$

$|\mu - 3| = 2$

$\mu = 5, 1$

$\therefore\ (\mu + \lambda)_{max} = 13$

90. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y – axis and lie in the first quadrant, is:

A. $x = \sqrt{1+4y}, y \geq 0$

B. $y = \sqrt{1+4x}, x \geq 0$

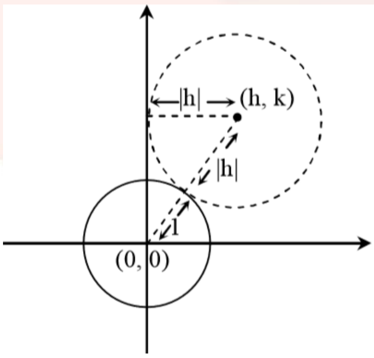
C. $x = \sqrt{1+2y}, y \geq 0$

D. $y = \sqrt{1+2x}, x \geq 0$

Ans. D

Sol:

Let (h, k) be the centre of the circle and radius r = h, as circle touch the Y. axis and other circle $x^2 + y^2 = 1$ whose centre (0, 0) and radius is 1.



$\sqrt{h^2 + k^2} = 1 + |h|$

$h > 0$ and $k > 0$ for first quadrant.

$h^2 + k^2 = 1 + h^2 + 2|h|$

$k^2 = 1 + 2|h|$

$y^2 = 1 + 2x$