JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON MONDAY 07th APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

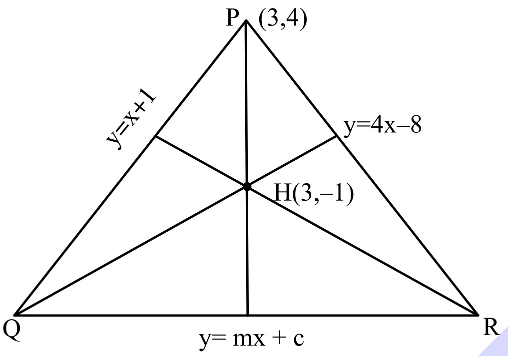
1. If the orthocentre of the triangle formed by the lines $y = x + 1$, $y = 4x - 8$ and $y = mx + c$ is at $(3, -1)$, then $m - c$ is :

(1) 0 (2) –2

(3) 4 (4) 2

Ans. (1)

Sol.



Solve line PQ & QR

Point Q $\left(\frac{1-c}{m-1}, \frac{1-c}{m-1}+1\right)$

$$m_{2H} = \frac{\frac{1-c}{m-1}+2}{\frac{1-c}{m-1}-3} = \frac{1-c+2m-2}{1-c-3m+3} = -\frac{1}{4} \qquad ...(1)$$

$\because m_{PH} = \frac{5}{0} \to \infty$

$\Rightarrow$ Slope of line QR (m) = 0

Put value of m in equation (1)

$$\frac{1-c-2}{1-c+3} = -\frac{1}{4} \Rightarrow c = 0$$

so $m - c = 0$ Ans.

2. Let $\vec{a}$ and $\vec{b}$ be the vectors of the same magnitude such that $\frac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|} = \sqrt{2}+1$. Then $\frac{|\vec{a}+\vec{b}|^2}{|\vec{a}|^2}$ is :

(1) $2+4\sqrt{2}$ (2) $1+\sqrt{2}$

(3) $2+\sqrt{2}$ (4) $4+2\sqrt{2}$

Ans. (3)

Sol. $$\frac{|\bar{a}+\bar{b}|+|\bar{a}-\bar{b}|}{|\bar{a}+\bar{b}|-|\bar{a}-\bar{b}|} = \sqrt{2}+1$$

Apply componendo and dividendo

$$\Rightarrow \frac{2|\bar{a}+\bar{b}|}{2|\bar{a}-\bar{b}|} = \frac{\sqrt{2}+2}{\sqrt{2}}$$

$$\Rightarrow |\bar{a}+\bar{b}| = \left(1+\sqrt{2}\right)|\bar{a}-\bar{b}|$$

$$\Rightarrow |\bar{a}+\bar{b}|^2 = \left(3+2\sqrt{2}\right)|\bar{a}-\bar{b}|^2$$

$$\Rightarrow 2|\bar{a}|^2 + 2\bar{a}.\bar{b} = \left(3+2\sqrt{2}\right)\left(2|\bar{a}|^2 - 2\bar{a}.\bar{b}\right)$$

$$\Rightarrow 2|\bar{a}|^2\left(2+2\sqrt{2}\right) = 2\bar{a}.\bar{b}\left(4+2\sqrt{2}\right)$$

$$\Rightarrow \frac{\bar{a}.\bar{b}}{|\bar{a}|^2} = \frac{2+2\sqrt{2}}{4+2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Now

$$\frac{|\bar{a}+\bar{b}|^2}{|\bar{a}|^2} = 1 + \frac{|\bar{b}|^2}{|\bar{a}|^2} + \frac{2\bar{a}.\bar{b}}{|\bar{a}|^2}$$

$$= 1 + 1 + 2\left(\frac{1}{\sqrt{2}}\right) = 2+\sqrt{2}$$

3. Let

$A = \{(\alpha, \beta) \in \mathbf{R} \times \mathbf{R} : |\alpha - 1| \le 4 \text{ and } |\beta - 5| \le 6\}$

and

$B = \{(\alpha, \beta) \in \mathbf{R} \times \mathbf{R} : 16(\alpha - 2)^2 + 9(\beta - 6)^2 \le 144\}$.

Then

(1) $B \subset A$

(2) $A \cup B = \{(x, y) : -4 \le x \le 4, -1 \le y \le 11\}$

(3) neither $A \subset B$ nor $B \subset A$

(4) $A \subset B$

Ans. **(1)**

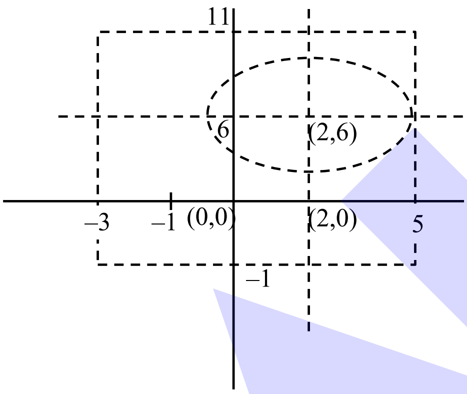
Sol. $A : |x-1| \le 4$ and $|y - 5| \le 6$

$\Rightarrow -4 \le x - 1 \le 4 \Rightarrow -6 \le y - 5 \le 6$

$\Rightarrow -3 \le x \le 5 \Rightarrow -1 \le y \le 11$

$B : 16(x-2)^2 + 9(y-6)^2 \le 144$

$$B : \frac{(x-2)^2}{9} + \frac{(y-6)^2}{16} \le 1$$



From Diagram $B \subset A$

4. If the range of the function $f(x) = \frac{5-x}{x^2 - 3x + 2}$, $x \ne 1, 2$, is $(-\infty, \alpha] \cup [\beta, \infty)$, then $\alpha^2 + \beta^2$ is equal to :

(1) 190 (2) 192

(3) 188 (4) 194

Ans. **(4)**

Sol. $y = \frac{5-x}{x^2 - 3x + 2}$

$yx^2 - 3xy + 2y + x - 5 = 0$

$yz^2 + (-3y + 1)x + (2y - 5) = 0$

Case I : If $y = 0$ (Accepted)

$\Rightarrow x = 5$

Case II : If $y \ne 0$

$D \ge 0$

$(-3y + 1)^2 - 4(y)(2y - 5) \ge 0$

$9y^2 + 1 - 6y - 8y^2 + 20y \ge 0$

$y^2 + 14y + 1 \ge 0$

$(y + 7)^2 - 48 \ge 0$

$|y + 7| \ge 4\sqrt{3}$

$\Rightarrow y + 7 \ge 4\sqrt{3}$ or $y + 7 \le -4\sqrt{3}$

$\Rightarrow y \ge 4\sqrt{3} - 7$ or $y \le -4\sqrt{3} - 7$

From Case I and Case II

$y \in (-\infty, -4\sqrt{3} - 7] \cup \left[4\sqrt{3} - 7, \infty\right)$

So $\alpha = -4\sqrt{3} - 7$

$\beta = 4\sqrt{3} - 7$

$\Rightarrow a^2 + b^2 = \left(-4\sqrt{3} - 7\right)^2 + \left(4\sqrt{3} - 7\right)^2$

$= 2(48 + 49)$

$= 194$

5. A bag contains 19 unbiased coins and one coin with head on both sides. One coin drawn at random is tossed and head turns up. If the probability that the drawn coin was unbiased, is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $n^2 - m^2$ is equal to :

(1) 80 (2) 60

(3) 72 (4) 64

Ans. **(1)**

Sol. $P(H) = \underbrace{\frac{19}{20}}_{\text{Selection of unbiased}} \times \underbrace{\frac{1}{2}}_{\text{Head occurs}} + \underbrace{\frac{1}{20}}_{\text{Selection of biased coin}} \times \underbrace{1}_{\text{Head occurs}}$

$$\text{Required probability} = \frac{\frac{19}{20} \times \frac{1}{2}}{\frac{19}{20} \times \frac{1}{2} + \frac{1}{20} \times 1} = \frac{19}{21}$$

$\therefore \frac{m}{n} = \frac{19}{21}$

$\Rightarrow n^2 - m^2 = 441 - 361 = 80$

6. Let a random variable X take values 0, 1, 2, 3 with $P(X = 0) = P(X = 1) = p$, $P(X = 2) = P(X = 3)$ and $E(X^2) = 2E(X)$. Then the value of $8p - 1$ is :

(1) 0 (2) 2

(3) 1 (4) 3

Ans. **(2)**

Sol. $2p + 2q = \frac{1}{2}$

$p + q$

$E(x^2) = \sum_{i=0}^{3} x_i^2 p(x_i) = 0.p + 1.p + 4.q + 9q$

$= p + 13q$

$E(x) = \sum_{i=0}^{3} x_i^2 p(x_i) = 0.p + 1.p + 2q + 3q = p + 5q$

$p + 13q = 2(p + 5q)$

$p = 3q$

So, $q = \frac{1}{8}$ & $p = \frac{3}{8}$

So, $8p - 1 = 2$ Option (2)

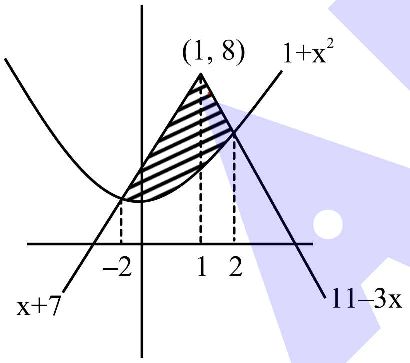
7. If the area of the region $\{(x, y) : 1 + x^2 \le y \le \min\{x + 7, 11 - 3x\}\}$ is A, then 3A is equal to

(1) 50 (2) 49

(3) 46 (4) 47

Ans. **(1)**

Sol.



$$A = \int_{-2}^{1} (x + 7 - x^2 - 1)\,dx + \int_{1}^{2} (11 + 3x - x^2 - 1)\,dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_{-2}^{1} + \left[10x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{1}^{2}$$

$= \frac{50}{3} \Rightarrow 3A = 50$ Option (1)

8. Let $f : \mathbf{R} \to \mathbf{R}$ be a polynomial function of degree four having extreme values at $x = 4$ and $x = 5$. If $\lim_{x \to 0} \frac{f(x)}{x^2} = 5$, then $f(2)$ is equal to :

(1) 12 (2) 10

(3) 8 (4) 14

Ans. **(2)**

Sol. $\lim_{x \to 0} \frac{f(x)}{x^2} = 5$

$\lim_{x \to 0} \frac{ax^4 + bx^3 + cx^2 + dx + e)}{x^2} = 5$

$c = 5$ and $d = e = 0$

$f(x) = ax^4 + bx^3 + 5x^2$

$f'(x) = 4ax^3 + 3bx^2 + 10x$

$= x(4ax^2 + 3bx + 10)$

has extremes at 4 and so $f'(4) = 0$ & $f'(5) = 0$

so $a = \frac{1}{8}$ & $b = \frac{-3}{2}$

so $f(2) = \frac{1}{8} \times 2^4 - \frac{3}{2} \times 2^3 + 5 \times 2^2$

$= 2 - 12 + 20 = 10$ Option (2)

9. The number of solutions of the equation

$$\cos 2\theta \cos\frac{\theta}{2} + \cos\frac{5\theta}{2} = 2\cos^3\frac{5\theta}{2}$$

in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is :

(1) 7 (2) 5

(3) 6 (4) 9

Ans. **(1)**

Sol. $\cos 2\theta \cos\frac{\theta}{2} + \cos\frac{5\theta}{2} = 2\cos^3\frac{5\theta}{2}$

$\frac{1}{2}\left(2\cos 2\theta \cos\frac{\theta}{2}\right) + \cos\frac{5\theta}{2}$

$= \frac{1}{2}\left(\cos\frac{15\theta}{2} + 3\cos\frac{5\theta}{2}\right)$

or solving

$\cos\frac{3\theta}{2} = \cos\frac{15\theta}{2}$

$\cos\frac{15\theta}{2} - \cos\frac{3\theta}{2} = 0$

$2\sin 3\theta \sin\frac{9\theta}{2} = 0$

$3\theta = n\pi$ or $\frac{9\theta}{2} = m\pi$

$\theta = \frac{n\pi}{3}$ $\quad \theta = \frac{2m\pi}{9}$

$\theta = \left\{-\frac{\pi}{2}, \frac{\pi}{3}, 0\right\}$

$\theta = \left\{-\frac{4\pi}{9}, \frac{-2\pi}{9}, \frac{4\pi}{9}, \frac{2\pi}{9}\right\}$

Option (1)

10. Let a_n be the n^{th} term of an A. P.

If $S_n = a_1 + a_2 + a_3 + \ldots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$, then a_n is equal to :

(1) 56 (2) 65

(3) 64 (4) 70

Ans. **(3)**

Sol. $S_n = 700 = \frac{n}{2}[2a + (n-1)d]$ ....(i)

$a_6 = 7 \Rightarrow a + 5d = 7$ .....(ii)

$S_7 = 7 \Rightarrow \frac{7}{2}(2a + 6d) = 7$

$a + 3d = 1$ .....(iii)

Solve (ii) and (iii)

$\frac{n}{2}(-16 + 3n - 3) = 700 \Rightarrow 3n^2 - 19n - 1400 = 0$

$(3n + 56)(n - 25) = 0$

$\therefore a_{25} = a + 24d = -8 + 24 \times 3$

$= -8 + 72$

$= 64$ Ans. $\rightarrow$ 3

11. If the locus of $z \in C$, such that

$\text{Re}\left(\frac{z-1}{2z+i}\right) + \text{Re}\left(\frac{\bar{z}-1}{2\bar{z}-i}\right) = 2,$

is a circle of radius r and center (a, b) then $\frac{15ab}{r^2}$ is equal to :

(1) 24 (2) 12

(3) 18 (4) 16

Ans. **(3)**

Sol. $\text{Re}\left(\frac{z-1}{2z+i}\right) + \text{Re}\left(\frac{\bar{z}-1}{2\bar{z}-i}\right) = 2$

Here, $\frac{z-1}{2z+i} = \overline{\left(\frac{\bar{z}-1}{2\bar{z}-i}\right)} = 2$

$= \text{Re}\left(\frac{z-1}{2z+i}\right) + \text{Re}\left(\frac{z-1}{2z+i}\right) = 2$

$= 2\,\text{Re}\left(\frac{z-1}{2z+1}\right) = 2 \Rightarrow \text{Re}\left(\frac{z-1}{2z+i}\right) = 1$

Let $z = x + iy$

$\text{Re}\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right) = 1 \Rightarrow \text{Re}\left[\frac{((x-1)+iy)(2x-i(y+1))}{(2x+i(2y+1)(2x-i(2y+1))}\right] = 1$

$\Rightarrow \frac{2x(x-1)+y(2y+1)}{4x^2+(2y+1)^2} = 1$

$\Rightarrow 2x^2 - 2x + 2y^2 + y = 4x^2 + 4y^2 + 1 + 4y$

$\Rightarrow 2x^2 + 2y^2 + 3y + 2x + 1 = 0$

$\Rightarrow x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$

$\text{centre} = \left(\frac{-1}{2}, \frac{-3}{4}\right), r = \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \frac{\sqrt{5}}{4}$

$a = \frac{-1}{2}, b = \frac{-3}{4}, r^2 = \frac{5}{16}$

$15\frac{ab}{r^2} = 15 \times \left(\frac{-1}{2}\right) \times \left(\frac{-3}{4}\right) \times \frac{16}{5} = 18$

12. Let the length of a latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be 10. If its eccentricity is the minimum value of the function $f(t) = t^2 + t + \frac{11}{12}$, $t \in \mathbf{R}$, then $a^2 + b^2$ is equal to :

(1) 125 (2) 126

(3) 120 (4) 115

Ans. (2)

Sol. Length of LR $= \frac{2b^2}{a} = 10 \Rightarrow \boxed{5a = b^2}$ .....(1)

$f(t) = t^2 + t + \frac{11}{12}$

$\frac{df(t)}{dt} = 2t + 1 = 0 \Rightarrow t = \frac{-1}{2}$

Min value of $f(t) = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + \frac{11}{12}$

$= \frac{1}{4} \frac{-1}{2} + \frac{11}{12} = \frac{3-6+11}{12} = \frac{8}{12} = \frac{2}{3} = e$

$e^2 = \frac{1-b^2}{a^2} \Rightarrow \frac{4}{9} = \frac{1-b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{1-4}{a} = \frac{5}{a} \Rightarrow \boxed{b^2 = \frac{5a^2}{a}}$ ......(2)

From (1) & (2)

$5a = \frac{5a^2}{a} \Rightarrow a = 9, \; b = \sqrt{45} = 3\sqrt{5}$

$\therefore$ a2 + b2 = 81 +45 = 126

13. Let y = y(x) be the solution of the differential equation $(x^2+1)y' - 2xy = (x^4 + 2x^2 + 1)\cos x$, $y(0) = 1$. Then $\int_{-3}^{3} y(x)dx$ is :

(1) 24 (2) 36
(3) 30 (4) 18

Ans. (1)

Sol. $(x^2+1)\frac{dy}{dx} - 2xy = (x^4 + 2x^2 + 1)\cos x$

$\frac{dy}{dx} - \left(\frac{2x}{x^2+1}\right)y = \frac{(x^2+1)^2 \cos x}{cx^2+1} = (x^2+1)\cos x$

(Linear D.E)

$P = \frac{-2x}{x^2+1}, Q = (x^2+1)\cos x$

$I.F = e^{\int Pdx} = e^{\int \frac{-2x}{x^2+1}dx} = \frac{1}{x^2+1}$

$y.\frac{1}{x^2+1} = \int (x^2+1)\cos x . \frac{1}{x^2+1} dx$

$\frac{y}{x^2+1} = \sin x + c \Rightarrow y\cos = 1 \Rightarrow c = 1$

$y = (x^2 + 1)(\sin x + 1)$

$\int_{-3}^{3} ydx = \int_{-3}^{3} (x^2+1)(\sin x + 1)$

$dx = \int_{-3}^{3} x^2 \sin x + x^2 \sin x + 1dx$

$\Rightarrow \int_{-3}^{3} x^2 \sin x dx + \int_{-3}^{3} x^2 dx + \int_{-3}^{3} \sin x dx + \int_{-3}^{3} 1dx$

$= 0 + 18 + 0 + 6 = 24$

14. If the equation of the line passing through the point $\left(0, -\frac{1}{2}, 0\right)$ and perpendicular to the lines $\vec{r} = \lambda(\hat{i} + a\hat{j} + b\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} - 6\hat{k}) + \mu(-b\hat{i} + a\hat{j} + 5\hat{k})$ is $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$, then a + b+ c+ d is equal to :

(1) 10 (2) 14
(3) 13 (4) 12

Ans. (2)

Sol. Line is $\perp^r$ to 2 line $\Rightarrow$ line will be parallel to

$(i + a\hat{j} + b\hat{k}) \times (-b\hat{i} + a\hat{j} + 5\hat{k})$

Parallel vector along the required line is

$\hat{i}(5a - ab) - \hat{j}(b^2 + 5) + \hat{k}(a + ab)$

Dr's of required line α (5a –ab), – $(b^2 + 5)$, (a + ab)

Also Dr's of required line α –2, d, –4

$\therefore \boxed{\frac{5a-ab}{-2} = \frac{-(b^2+5)}{d} = \frac{a+ab}{-4}}$ .....(1)

Also point $\left(0, \frac{-1}{2}, 0\right)$ will lie on $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$

$\frac{0-1}{-2} = \frac{\frac{-1}{2}+4}{d} = \frac{0-c}{-4} \Rightarrow d = 7, c = 2$

From (1) $\frac{5a-ab}{-2} = \frac{-b^2-5}{7} = \frac{a+ab}{-4}$

$\frac{5a-ab}{-2} = \frac{a+ab}{-4}; \frac{-b^2-5}{7} = \frac{a+ab}{-4}$

$-20a + 4ab = -2a - 2ab$	$4b^2 + 20 = 70 + 7ab$
$18a = 6ab$	$36 + 20 = 70 + 21a$
$\boxed{b = 3}$	$56 = 28a \Rightarrow \boxed{a = 2}$

$a + b + c + d = 2 + 3 + 2 + 7 = 14$

15. Let p be the number of all triangles that can be formed by joining the vertices of a regular polygon P of n sides and q be the number of all quadrilaterals that can be formed by joining the vertices of P. If p + q = 126, then the eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{n}=1$ is :

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$

(3) $\frac{\sqrt{7}}{4}$ (4) $\frac{1}{\sqrt{2}}$

Ans. (4)

Sol. Total trangles = ⇒ = $^hC_3$

Total auadrilaterals = $^hC_4 = q$

$^nC_3 + ^nC_4 = 126 \Rightarrow ^{n+1}C_4 = 126$

$\Rightarrow n+1=9 \Rightarrow n=8$

$\frac{x^2}{16}+\frac{y^2}{n}=1 \Rightarrow \frac{x^2}{16}+\frac{y^2}{8}=1$

$e=\sqrt{1-\frac{8}{16}}=\sqrt{\frac{8}{16}}=\frac{1}{\sqrt{2}}$

16. Consider the lines $L_1 : x-1 = y-2 = z$ and $L_2 : x-2 = y = z-1$. Let the feet of the perpendiculars from the point P(5,1, –3) on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A, then $4A^2$ is equal to :

(1) 139 (2) 147

(3) 151 (4) 143

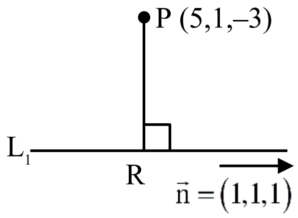
Ans. (2)

Sol. $L_1 : \frac{x-1}{1}=\frac{y-2}{1}=\frac{z-0}{2}$

Let Q (λ + 1, λ + 2, λ)

$\overrightarrow{PQ}=(\lambda-4,\lambda-1,\lambda+3)$

$\overrightarrow{PQ}.\vec{m}=0$



$\Rightarrow \lambda -4 + \lambda + 1, \lambda + 3 = 0$

$\Rightarrow 3\lambda = 0$

$\lambda = 0$

$\Rightarrow \boxed{Q(1,2,0)}$

$L_2 : \frac{x-2}{1}=\frac{y-0}{1}=\frac{z-1}{2}$

Let R (μ + 2, μ, μ + 1) $\overrightarrow{PR}=(\mu-3,\mu-1,\mu+4)$

$\overrightarrow{PR}.\vec{n}=0$

$\mu -3 + \mu -1 + \mu + 4 = 0$

$\neq \mu = 0$

$\boxed{R(2,0,1)}$

Area of ΔPQR (A) = $\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|$

$A=\frac{1}{2}\left|\left(-4\hat{i}+\hat{j}+3\hat{k}\right)\times\left(-3\hat{i}+\hat{j}+4\hat{k}\right)\right|$

$A=\frac{1}{2}\left|7\left(\hat{i}+\hat{j}+\hat{k}\right)\right|$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 1 & 3 \\ -3 & -1 & 4 \end{vmatrix}$$

$= 7\hat{i}+7\hat{j}+7\hat{k}$

$4A^2 = 49 \times 3 = 147$

17. The number of real roots of the equation $x|x-2|+3|x-3|+1=0$ is :

(1) 4 (2) 2

(3) 1 (4) 3

Ans. (3)

Sol. (I) $x < 2$

$-x^2 + 2x - 3x + 9 + 1 = 0$

$\Rightarrow x^2 + x - 10 = 0$

$\Rightarrow x=\frac{-1+\sqrt{41}}{2},\frac{-1-\sqrt{41}}{2}$

× √

(II) $2 \le x < 3$

$\Rightarrow x^2 - 2x - 3x + 9 + 1 = 0$

$\Rightarrow x^2 - 5x + 10 = 0$

$D < 0$

(III) $x \ge 3$

$x^2 -2x + 3x -9 + 2 = 0$

$\Rightarrow x^2 + x - 8 = 0$

$x=\frac{-1+\sqrt{32}}{2},\frac{-1-\sqrt{32}}{2}$

× ×

1 real roots

18. Let e_1 and e_2 be the eccentricities of the ellipse $\frac{x^2}{b^2}+\frac{y^2}{25}=1$ and the hyperbola $\frac{x^2}{16}-\frac{y^2}{b^2}=1$, respectively. If b < 5 and e_1e_2 =1, then the eccentricity of the ellipse having its axes along the coordinate axes and passing through all four foci (two of the ellipse and two of the hyperbola) is :

(1) $\frac{4}{5}$ (2) $\frac{3}{5}$

(3) $\frac{\sqrt{7}}{4}$ (4) $\frac{\sqrt{3}}{2}$

Ans. **(2)**

Sol. $e_1^2=1-\frac{b^2}{25}$ $\quad e_2^2=1-\frac{b^2}{16}$

$\therefore\ e_1^2e_2^2=1$

$$\left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right)=1$$

$$\Rightarrow\ 2+\frac{b^2}{16}-\frac{b^2}{25}-\frac{b^2}{400}=1$$

$$\Rightarrow\ \frac{9b^2}{400}=\frac{b^4}{400}$$

$$b^2=9$$

$$\frac{x^2}{9}+\frac{y^2}{25}=1 \qquad \frac{x^2}{16}-\frac{y^2}{9}=0$$

$$e_1\sqrt{1-\frac{9}{25}} \qquad e_2=\frac{5}{4}$$

$$e_1=\frac{4}{5}$$

Focii : - $(0,\pm 4)$ $\quad(\pm 5,0)$

ellipse passing through all four foci

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

$$e=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

19. Let the system of equations

$x + 5y - z = 1$

$4x + 3y - 3z = 7$

$24x + y + \lambda z = \mu$

$\lambda,\mu\in R$, have infinitely many solutions. Then the number of the solutions of this system,

If x, y, z are integers and satisfy $7\le x+y+z\le 77$, is

(1) 3 (2) 6

(3) 5 (4) 4

Ans. **(1)**

Sol. For infinitely many solution

$\Delta = 0$

$$\begin{vmatrix}1 & 5 & -1\\ 4 & 3 & -3\\ 24 & 1 & \lambda\end{vmatrix}=0$$

$\Rightarrow 1\,(3\lambda + 3) - 5\,(4\lambda + 72) - 1\,(4 - 72) = 0$

$\Rightarrow -17\lambda + 3 - 4\times 72 - 4 = 0$

$\Rightarrow\ 17\lambda = -289$

$\Rightarrow \boxed{\lambda=-17}$

$\Delta 1 = 0$

$$\Rightarrow\begin{vmatrix}1 & 5 & -1\\ 7 & 3 & -3\\ \mu & 1 & -17\end{vmatrix}=0$$

$\Rightarrow 1\,(-51 + 3) - 5\,(-119 + 3\mu) - 1\,(7 - 3\mu) = 0$

$\Rightarrow\ -48 + 595 - 15\mu - 7 + 3\mu = 0$

$\Rightarrow\ 12\mu = 540$

$\boxed{\mu = 45}$

$x + 5y - z = 1$

$4x + 3y - 3z = 7$

$24x + y - 17z = 45$

Let $z = 1$

$x+5y=1+\lambda]\times 4$

$4x+3y=7+3\lambda$

$$\begin{array}{l}4x+20y=4+4\lambda\\ \underline{\ -\quad -\qquad\ -\quad -\ }\\ -17y=3-\lambda\end{array}$$

$$y=\frac{\lambda-3}{17},\ x=1+\lambda-\frac{5\lambda-15}{17}$$

$$=\frac{32\_12\lambda}{17}$$

$$7\le\frac{\lambda-3}{17}+\frac{32+12\lambda}{17}+\lambda\le 77$$

$$7\le\frac{30\lambda+29}{17}\le 77$$

$3\le\lambda\le 42$

$\lambda = 3, 20, 37$

20. If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is :

(1) 760 (2) 755

(3) 750 (4) 757

Ans. (4)

Sol. $ar + ar^3 + ar^5 = 21, \quad ar^7 + ar^9 + ar^{11} = 15309$

$\Rightarrow ar(1+r^2+r^4) = 21, \quad ar^7(1+r^2+r^4) = 15309$

____ (1) ____ (2)

Eq^n (2) $\div$ eq^n (1)

$$\Rightarrow \frac{a.r^7}{ar} = \frac{15309}{21} \Rightarrow r^6 = 729$$

$$\Rightarrow \frac{a.(r^9-1)}{r-1} = \frac{\frac{7}{91}(19683-1)}{2} = \frac{7\times 19682}{91\times 2}$$

$$= \frac{9841}{13} = 757$$

SECTION-B

21. If the function $f(x) = \frac{\tan(\tan x)-\sin(\sin x)}{\tan x - \sin x}$ is continuous at x = 0, then $f(0)$ is equal to _____.

Ans. (2)

Sol.
$$\lim_{x\to 0}\frac{\frac{\tan(\tan x)-\tan x}{\tan^3 x}\frac{\tan^3 x}{x^3}+\frac{\tan x-\sin x}{x^3}+\frac{\sin x-\sin(\sin x)}{\sin^3 x}\frac{\sin^3 x}{x^3}}{\frac{\tan x-\sin x}{x^3}}$$

$$= \frac{\frac{1}{3}+\frac{1}{2}+\frac{1}{6}}{\frac{1}{2}} = 2$$

22. If $\int\left(\frac{1}{x}+\frac{1}{x^3}\right)\left(\sqrt[23]{3x^{-24}+x^{-26}}\right)dx$

$$= -\frac{\alpha}{3(\alpha+1)}\left(3x^{\beta}+x^{\gamma}\right)^{\frac{\alpha+1}{\alpha}}+C,\ x>0,$$

$(\alpha,\beta,\gamma \in Z)$, where C is the constant of integration, then $\alpha+\beta+\gamma$ is equal to_____.

Ans. (19)

Sol. $$\int\left(\frac{1}{x^2}+\frac{1}{x^4}\right)\left(\frac{3}{x}+\frac{1}{x^3}\right)^{\frac{1}{23}}dx$$

using $t = \frac{3}{x}+\frac{1}{x^3} \Rightarrow dt = -3\left(\frac{1}{x^2}+\frac{1}{x^4}\right)dx$

$$\int\frac{t^{1/23}dt}{-3} = \frac{t^{24/23}}{\left(\frac{24}{23}\right)(-3)}+C$$

$\Rightarrow \alpha = 23\ \beta = -1\ \gamma = -3$

$\alpha+\beta+\gamma = 19$

23. For $t > -1$, let α_t and β_t be the roots of the equation
$$\left((t+2)^{\frac{1}{7}}-1\right)x^2+\left((t+2)^{\frac{1}{6}}-1\right)x+\left((t+2)^{\frac{1}{21}}-1\right)=0.$$

If $\lim_{t\to -1^+}\alpha_t = a$ and $\lim_{t\to -1^+}\beta_t = b$, then $72(a+b)^2$ is equal to _____.

Ans. (98)

Sol. $$a+b = \lim_{t\to -1^+}(\alpha+\beta) = \lim_{t\to -1^+} -\frac{(t+2)^{\frac{1}{6}}-1}{(t+2)^{\frac{1}{7}}-1}$$

let $t+2 = y$

$$a+b = \lim_{y\to 1^+}\frac{y^{1/6}-1}{y^{1/7}-1} = \frac{7}{6}$$

$$72(a+b)^2 = 72\ \frac{49}{36} = 98$$

24. Let the lengths of the transverse and conjugate axes of a hyperbola in standard form be 2a and 2b, respectively, and one focus and the corresponding directrix of this hyperbola be (–5,0) and 5x + 9 = 0, respectively. If the product of the focal distances of a point $(\alpha, 2\sqrt{5})$ on the hyperbola is p, then 4p is equal to ____.

Ans. (189)

Sol. $PF_1.PF_2 = (e\alpha - a)(e\alpha + a)$

$$P = e^2\alpha^2 - a^2 = \frac{25}{9}.9.\frac{9}{4}-9 = \frac{189}{4}$$

$$\left.\begin{array}{l} ae = 5 \\ \frac{a}{e} = 9/5 \end{array}\right\} \begin{array}{l} a = 3 \\ e = 5/3 \\ b = 4 \end{array}$$

$$\frac{x^2}{9}-\frac{y^2}{16} = 1$$

$$\frac{\alpha^2}{9}-\frac{4(5)}{16} = 1 \Rightarrow \alpha^2 = 9\left(\frac{36}{16}\right)$$

25. The sum of the series

$2\times 1\times {}^{20}C_4 - 3\times 2\times {}^{20}C_5 + 4\times 3\times {}^{20}C_6 - 5\times 4\times {}^{20}C_7 + \ldots + 18\times 17\times {}^{20}C_{20}$, is equal to

Ans. (34)

Sol. $(1-x)^{20} = {}^{20}C_0 - {}^{20}C_1x + {}^{20}C_2x^2 \ldots\ldots + {}^{20}C_{20}x^{20}$

$$\frac{(1-x)^{20}}{x^2} = \frac{{}^{20}C_0}{x^2} - \frac{{}^{20}C_1}{x} + {}^{20}C_2 - {}^{20}C_3x + {}^{20}C_4x^2 \ldots.$$

Diff twice and put x = 1

$= 6 - {}^{20}C_1(2) + A$

$A = 40 - 6 = 34$