Sri Chaitanya IIT Academy.,India.
A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI
A right Choice for the Real Aspirant
ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS-BT JEE-MAIN Date: 06-09-2025
Time: 09.00Am to 12.00Pm RPTM-09 Max. Marks: 300

KEY SHEET

MATHEMATICS

1	**2**	2	**3**	3	**1**	4	**1**	5	**1**
6	**3**	7	**1**	8	**2**	9	**3**	10	**2**
11	**2**	12	**1**	13	**4**	14	**1**	15	**4**
16	**1**	17	**4**	18	**1**	19	**1**	20	**3**
21	**2**	22	**18**	23	**488**	24	**2**	25	**8**

PHYSICS

26	**3**	27	**1**	28	**1**	29	**4**	30	**1**
31	**4**	32	**3**	33	**1**	34	**1**	35	**1**
36	**2**	37	**3**	38	**1**	39	**1**	40	**2**
41	**2**	42	**2**	43	**4**	44	**3**	45	**1**
46	**50**	47	**0**	48	**1**	49	**9**	50	**4**

CHEMISTRY

51	**2**	52	**4**	53	**2**	54	**3**	55	**1**
56	**4**	57	**4**	58	**1**	59	**4**	60	**4**
61	**4**	62	**4**	63	**3**	64	**3**	65	**1**
66	**4**	67	**2**	68	**1**	69	**3**	70	**4**
71	**3**	72	**5**	73	**3**	74	**4**	75	**4**

SOLUTION

MATHEMATICS

1. $\left|\vec{a}\right|=\left|\vec{b}\right| \Rightarrow 25+p^2+144=1+169+4q \ \Rightarrow p^2=4q+1$

P is odd integer where $p=(2k+1) \quad \Rightarrow (2k+1)^2=4q+1 \Rightarrow q=k(k+1)$

$\therefore 1 \le k(k+1) \le 100$

Number of possible values of k is 31 $\dfrac{(k+1)}{16}=\dfrac{(31+1)}{16}=2$

2. $(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a} \quad \Rightarrow \left[\begin{array}{l} \vec{a}.\vec{c}=0 \\ -\vec{b}\,.\,\vec{c}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right| \end{array}\right.$

$\Rightarrow \cos\theta=-\frac{1}{3} \Rightarrow \sin\theta=\frac{2\sqrt{2}}{3}$

3. Equation of line L_1 is $7\hat{i}+6\hat{j}+2\hat{k}+\lambda\left(-3\hat{i}+2\hat{j}+4\hat{k}\right)$

Equation of line L_2 is $5\hat{i}+3\hat{j}+4\hat{k}+\mu\left(2\hat{i}+\hat{j}+3\hat{k}\right)$

$\overrightarrow{CD}=2\hat{i}+3\hat{j}-2\hat{k}+\lambda\left(-3\hat{i}+2\hat{j}+4\hat{k}\right)-\mu\left(2\hat{i}+\hat{j}+3\hat{k}\right)$. Since it is parallel to $2\hat{i}-2\hat{j}-\hat{k}$

$\therefore \dfrac{2-3\lambda-2\mu}{2}=\dfrac{3+2\lambda-\mu}{-2}=\dfrac{-2+4\lambda-3\mu}{-1} \qquad \therefore \lambda=2, \mu=1$

$\overrightarrow{CD}=-6\hat{i}+6\hat{j}+3\hat{k} \quad \therefore \left|\overrightarrow{CD}\right|=9$

4. $p=(5k-6, 3k-10, 8k-14)$ Now $\overrightarrow{BP}.\overrightarrow{AC}=0$

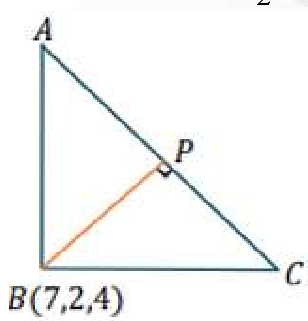
$5(5k-13)+3(3k-12)+8(8k-18)=0 \qquad k=\dfrac{245}{98}=\dfrac{5}{2}$

$p=\left(\dfrac{13}{2},-\dfrac{5}{2},6\right)$, Now length $BP=\dfrac{\sqrt{98}}{2}$

Now $A,C=\left(\dfrac{13}{2}+\dfrac{5}{\sqrt{98}}\cdot\dfrac{\sqrt{98}}{2},-\dfrac{5}{2}\pm\dfrac{3}{\sqrt{98}}\cdot\dfrac{\sqrt{98}}{2},6\pm\dfrac{8}{\sqrt{98}}\cdot\dfrac{\sqrt{98}}{2}\right) \qquad A=(9,-1,10) \ \ C=(4,-4,2)$

Equation of $BC\ \dfrac{x-7}{3}=\dfrac{y-2}{6}=\dfrac{z-4}{2}$

Equation of $AC\ \dfrac{x-7}{2}=\dfrac{y-2}{-3}=\dfrac{z-4}{6}$



5. **Statement 1:**

Let $\overrightarrow{OA},\overrightarrow{OB},\overrightarrow{OC},\overrightarrow{OP}$ are $\hat{a},\hat{b},\hat{c}$ and $\hat{d}$ respectively

Here, $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = \frac{1}{2}$

And $\hat{d}.\hat{a} = \hat{d}.\hat{b} = \hat{d}.\hat{c} = \cos\alpha$

Let $\hat{d} = x\hat{a} + y\hat{b} + z\hat{c}$ …….. (1)

$\hat{d}.\hat{a} = x + \frac{y}{2} + \frac{z}{2} = \cos\alpha$ $\quad \hat{d}.\hat{b} = \frac{x}{2} + y + \frac{z}{2} = \cos\alpha$ $\quad \hat{d}.\hat{c} = \frac{x}{2} + \frac{y}{2} + z = \cos\alpha$

By adding above equations we get $2(x+y+z) = 3\cos\alpha$

Taking dot product with $\hat{d}$ in (1) we get

$x + y + z = \frac{1}{\cos\alpha}$ $\quad \Rightarrow \frac{3\cos\alpha}{2} = \frac{1}{\cos\alpha} \Rightarrow \cos^2\alpha = \frac{2}{3}$ $\quad \Rightarrow 3m_1 = 3\cos^2\alpha = 2$

Statement 2:

Note that $\overrightarrow{AB}^2 + \overrightarrow{CD}^2 = 3^2 + 11^2 = 130 = 7^2 + 9^2 = \overrightarrow{BC}^2 + \overrightarrow{DA}^2$

Since $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \vec{0}$,

$DA^2 = \left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}\right)^2 \Rightarrow AB^2 - BC^2 + CD^2 + 2\left(\overrightarrow{BC}^2 + \overrightarrow{AB}.\overrightarrow{BC} + \overrightarrow{BC}.\overrightarrow{CD} + \overrightarrow{CD}.\overrightarrow{AB}\right)$,

$\Rightarrow AB^2 - BC^2 + CD^2 + 2\left(\overrightarrow{AB} + \overrightarrow{BC}\right).\left(\overrightarrow{BC} + \overrightarrow{CD}\right)$,

i.e., $2\overrightarrow{AC}.\overrightarrow{BD} = 2\left(\overrightarrow{AB} + \overrightarrow{BC}\right).\left(\overrightarrow{BC} + \overrightarrow{CD}\right) = AD^2 + BC^2 - AB^2 - CD^2 = 0$

Hence both AC and BD are perpendicular to each other $\Rightarrow m_2 = \cos\theta = 0$.

6. $\cos\theta = \frac{\ell m + mn + n\ell}{\ell^2 + m^2 + n^2}$ …(i)

$x^3 + x^2 - 4x - 4 = 0 \Rightarrow \ell + m + n = -1 \Rightarrow \ell m + mn + n\ell = -4$

$(\ell + m + n)^2 = \ell^2 + m^2 + n^2 + 2(-4) \Rightarrow \ell^2 + m^2 + n^2 = 1 + 8 = 9$ $\quad \therefore \cos\theta = -\frac{4}{9}$

$\therefore$ acute angle between the lines is $\cos^{-1}\frac{4}{9}$

7. Let P be (x, y) $\quad \overrightarrow{PA} = (1-x)\hat{i} - y\hat{j}$; $\quad \overrightarrow{PB} = (-1-x)\hat{i} - y\hat{j}$

$\therefore \quad \left(\overrightarrow{PA} \cdot \overrightarrow{PB}\right) = \left((x-1)\hat{i} + y\hat{j}\right) \cdot \left((x+1)\hat{i} + y\hat{j}\right) = \left(x^2 - 1\right) + y^2$

also $\quad 3\left(\overrightarrow{OA} \cdot \overrightarrow{OB}\right) = 3\hat{i} \cdot \left(-\hat{i}\right) = -3$ $\quad$ hence $\left(\overrightarrow{PA} \cdot \overrightarrow{PB}\right) + 3\left(\overrightarrow{OA} \cdot \overrightarrow{OB}\right) = 0$

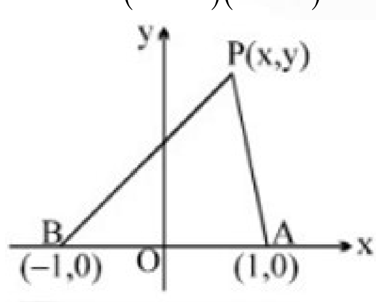
$x^2 - 1 + y^2 - 3 = 0 = 0$ $\quad x^2 + y^2 = 4$ ….. (1)

Which gives the locus of P i.e. P move on a circle with centre (0, 0) and radius 2.

now $\quad \left|\overrightarrow{PA}\right|^2 = (x-1)^2 + y^2$; $\left|\overrightarrow{PB}\right|^2 = (x+1)^2 + y^2$

$\therefore \quad \left|\overrightarrow{PA}\right|^2 \left|\overrightarrow{PB}\right|^2 = \left(x^2 + y^2 - 2x + 1\right)\left(x^2 + y^2 + 2x + 1\right)$

$= (5 - 2x)(5 + 2x)$ $\quad \left[\text{using } x^2 + y^2 = 4\right]$



$\therefore \quad \left|\overline{PA}\right|^2\left|\overline{PB}\right|^2 = 25-4x^2$ subject to $x^2+y^2=4$

$\left.\left|\overline{PA}\right|^2\left|\overline{PB}\right|^2\right|_{\min.} = 25-16=9;$ (when x = 2 or -2)

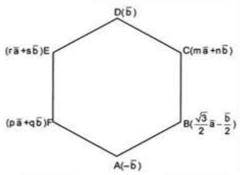
and $\left.\left|\overline{PA}\right|^2\left|\overline{PB}\right|^2\right|_{\max.} = 25-0=25$ (when x = 0)

$3 \le \left|\overline{PA}\right|\left|\overline{PB}\right| \le 5$

hence M = 5 and m = 3 $\Rightarrow \quad M^2+m^2=34$ Ans.

8. $\overline{AD} = 2\overline{AD}$

$\Rightarrow 2\bar{b} = 2\left[\left(m-\frac{\sqrt{3}}{2}\right)\bar{a}+\left(m+\frac{1}{2}\right)\bar{b}\right]$



$\Rightarrow m=\frac{\sqrt{3}}{2}, n=\frac{1}{2}$ $\quad \overline{ED}=\overline{AB}$ $\quad \Rightarrow \bar{b}(1-s)-r\bar{a}=\frac{\bar{b}}{2}+\frac{\sqrt{3}}{2}\bar{a}$

$\Rightarrow s=\frac{1}{2}$ and $r=-\frac{\sqrt{3}}{2}$ $\quad \overline{AF}=\overline{CD}$

$\Rightarrow p\bar{a}+(q+1)\bar{b}=(1-n)\bar{b}-m\bar{a}$

Putting values of m and n and comparing, we get

$q=-\frac{1}{2}$ and $p=-\frac{\sqrt{3}}{2}$

9. $\vec{r}=\left(\vec{a}\times\vec{b}\right)\sin x+\left(\vec{b}\times\vec{c}\right)\cos y+2\left(\vec{c}\times\vec{a}\right)$ $\quad \vec{r}\,.\left(\vec{a}.+\vec{b}+\vec{c}\right)=0$

$\Rightarrow \quad \left[\vec{a}\ \vec{b}\ \vec{c}\right](\sin x+\cos y+2)=0 \Rightarrow \quad \left[\vec{a}\ \vec{b}\ \vec{c}\right]\ne 0 \quad \Rightarrow \quad \sin x+\cos y=-2$

This is possible only when $\sin x=-1$ and $\cos y=-1$

For x^2+y^2 to be minimum $x=-\frac{\pi}{2}$ and $y=\pi$

$\Rightarrow$ minimum value of $\left(x^2+y^2\right)$ is = $\frac{\pi^2}{4}+\pi^2=\frac{5\pi^2}{4}$

10. Let the vector be given as $a\hat{i}+b\hat{j}+c\hat{k}$. For this vector to be coplanar with $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, we will have $a\hat{i}+b\hat{j}+c\hat{k}=p\left(\hat{i}+\hat{j}+2\hat{k}\right)+r\left(\hat{i}+2\hat{j}+\hat{k}\right)$

This gives, $a=p+r$ …..(i)

$b=p+2r$ …..(ii)

$c=2p+r$ …..(iii)

For the vector $a\hat{i}+b\hat{j}+c\hat{k}$ to be perpendicular to $\hat{i}+\hat{j}+\hat{k}$, we will have

$\left(a\hat{i}+b\hat{j}+c\hat{k}\right).\left(\hat{i}+\hat{j}+\hat{k}\right)=0 \Rightarrow a+b+c=0$ ….. (iv)

Adding equation (i) to (iii), we get $4p+4r=a+b+c \Rightarrow 4(p+r)=0 \Rightarrow p=-r$

Now with the help of (i), (ii) and (iii), we get

$a=0, b=r, c=p=-r$

Hence the required vector is $r\left(\hat{j}-\hat{k}\right)$

To be its unit vector $r^2+r^2=1 \Rightarrow r=\pm\dfrac{1}{\sqrt{2}}$

Hence the required unit vector is, $\pm\dfrac{1}{\sqrt{2}}\left(\hat{j}-\hat{k}\right)$.

11. $\Rightarrow \vec{b}+\vec{c}=-\vec{a} \quad \Rightarrow 48+48+\left|\vec{c}\right|^2=144$

$\Rightarrow \vec{c}^{\,2}=48 \qquad \Rightarrow \left|\vec{c}\right|=4\sqrt{3} \quad \therefore \dfrac{\left|\vec{c}\right|^2}{2}-\left|\vec{a}\right|=24-12=12$

Further

$\vec{a}+\vec{b}=-\vec{c} \qquad \Rightarrow 144+48+2\vec{a}.\vec{b}=48 \qquad \Rightarrow \vec{a}.\vec{b}=-72$

$\because \vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{a}\times\vec{b}+\vec{a}\times\vec{c}=0 \qquad \therefore \left|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}\right|=2.\sqrt{144.48-(72)^2}=48\sqrt{3}$

12. $S.D.=\dfrac{\left|\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}\right|}{\left|\vec{b}\right|}=\dfrac{\left|\left(\hat{i}+2\hat{j}+3\hat{k}\right)\times\left(\hat{i}+\hat{j}+\hat{k}\right)\right|}{\left|\hat{i}+\hat{j}+\hat{k}\right|}=\dfrac{\left|-\hat{i}+2\hat{j}-\hat{k}\right|}{\sqrt{3}}=\dfrac{\sqrt{6}}{\sqrt{3}}=\sqrt{2}$

13. $\left|\vec{a}+\vec{b}-\vec{c}\right|^2+\left|\vec{b}+\vec{c}-\vec{a}\right|^2+\left|\vec{c}+\vec{a}-\vec{b}\right|^2=36$

$\Rightarrow 3\left(\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2\right)-2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=36 \qquad \Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-\dfrac{9}{2}$

Also $\left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=0$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow \vec{c}=-\vec{a}-\vec{b}$

Hence $\vec{a},\vec{b},\vec{c}$ are coplanar and represents sides of triangle.

14. Let $\hat{i}$ be a unit vector in the direction of $\vec{b}, \hat{j}$ in the direction of $\vec{c}$. Note that $\vec{b}=\hat{i}$ and $\vec{c}=\hat{j}$

We have $\vec{b}\times\vec{c}=\left|\vec{b}\right|\left|\vec{c}\right|\sin\alpha\hat{k}=\sin\alpha\hat{k}$, where $\hat{k}$ is a unit vector perpendicular to $\vec{b}$ and $\vec{c}$.

$\Rightarrow \left|\vec{b}\times\vec{c}\right|=\sin\alpha \Rightarrow \hat{k}=\dfrac{\vec{b}\times\vec{c}}{\left|\vec{b}\times\vec{c}\right|}$

Any vector $\vec{a}$ can be written as a linear combination of $\hat{i},\hat{j}$ and $\hat{k}$.

Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$. Now $\vec{a}.\vec{b}=\vec{a}.\hat{i}=a_1, \vec{a}.\vec{c}=\vec{a}.\hat{j}=a_2$ and $\vec{a}.\dfrac{\vec{b}\times\vec{c}}{\left|\vec{b}\times\vec{c}\right|}=\vec{a}.\hat{k}=a_3$

Thus $\left(\vec{a}.\vec{b}\right)\vec{b}+\left(\vec{a}.\vec{c}\right)\vec{c}+\dfrac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left|\vec{b}\times\vec{c}\right|}\left(\vec{b}\times\vec{c}\right| \qquad =a_1\vec{b}+a_2\vec{c}+a_3\dfrac{\left(\vec{b}\times\vec{c}\right)}{\left|\vec{b}\times\vec{c}\right|}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}=\vec{a}$.

15.

$\overrightarrow{OB}\times\overrightarrow{OC}=\frac{1}{2}\overrightarrow{OB}\times\left(\overrightarrow{OB}-\lambda\overrightarrow{OA}\right)$

$=\frac{\lambda}{2}\left(\overrightarrow{OA}\times\overrightarrow{OB}\right)$

$\left|\overrightarrow{OB}\times\overrightarrow{OC}\right|=\frac{\lambda}{2}\left|\overrightarrow{OA}\right|\times\left|\overrightarrow{OB}\right|$ (Note $\overrightarrow{OA}$ & $\overrightarrow{OB}$ are perpendicular)

$\Rightarrow \frac{9\lambda}{2}=\frac{9}{2} \Rightarrow \lambda=1$ (given $\lambda>0$)

So $\overrightarrow{OC}=\frac{\overrightarrow{OB}-\overrightarrow{OA}}{2}=\frac{\overrightarrow{AB}}{2}$

M is mid point of AB

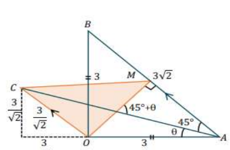
Note projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}=-\frac{3}{2}$

$\tan\theta=\frac{1}{3}$

Area of $\Delta ABC=\frac{9}{2}$

Acute angle between diagonals is

$\tan^{-1}\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)=\tan^{-1}2$



16. $\vec{a}\times(\vec{a}\times\vec{b})=(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}$

$\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)=-|\vec{a}|^2\left(\vec{a}\times\vec{b}\right)$

$\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\left(\vec{a}\times\vec{b}\right)\right)\right)\right)=-|\vec{a}|^2\left(\left(\vec{a}\times\vec{b}\right)\vec{a}-\left(\vec{a}\cdot\vec{a}\right)\vec{b}\right)$

$\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)\right)\right)=-|\vec{a}|^4\left(\vec{a}\times\vec{b}\right)$

Similarly, $\underbrace{\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}..............\times(\vec{a}}_{2026\text{ times}}\times(\vec{a}\times\vec{b}))=-|\vec{a}|^{2026}\left(\vec{a}\times\vec{b}\right)$

17. Let P and Q be (x_1,y_1) and (x_2,y_2)

$\therefore \overrightarrow{OP}\cdot\hat{i}=x_1=2$ and $\overrightarrow{OQ}\cdot\hat{i}=x_2=-2$

Let $y=f(x)=x^7-2x^5+5x^3+8x+5$

$\therefore y_1=f(x_1)=f(2)$ and $y_2=f(x_2)=f(-2)$

$\therefore\left|\overrightarrow{OP}+\overrightarrow{OQ}\right|=x_1\hat{i}+y_1\hat{j}+x_2\hat{i}+y_2\hat{j}=\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}=\sqrt{\left(f(2)+f(-2)\right)^2}$

$=\left(f(2)+f(-2)\right)\hat{j}$

So, magnitude of $\overrightarrow{OP}+\overrightarrow{OQ}=f(2)+f(-2)=10$ (from the given functional rule)

$\Rightarrow 2M=10 \Rightarrow M=5$ Ans.

18. DR's of $\overrightarrow{OG}=(1,1,1)$

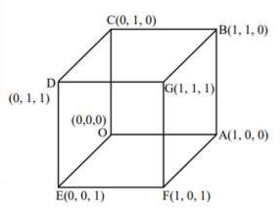
Dr's of $\overrightarrow{AC}=(-1,1,0)$

Equation of $\overrightarrow{OG}=\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

Equation of $\overrightarrow{AC}=\frac{x-1}{-1}=\frac{y}{1}=\frac{z}{0}$

$\overrightarrow{OA}=\hat{i}$

Normal of $\overrightarrow{OG}$ and $\overrightarrow{AC}$



$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&1\\-1&1&0\end{vmatrix}=\left(-\hat{i}-\hat{j}+2\hat{k}\right)\quad S.D.=\frac{\left|\hat{i}\left(-\hat{i}-\hat{j}+2\hat{k}\right)\right|}{\left|-\hat{i}-\hat{j}+2\hat{k}\right|}=\frac{1}{\sqrt{6}}$$

19. (A) Both the lines pass through the point (7, 11, 15)

(B) < 2, 3, 4 > are direction ratios of both the lines. Also the point (1, 2, 3) is common to both $\therefore$ The lines are coincident.

(C) < 5, 4 – 2 > are direction ratios of both the lines $\therefore$ The lines are parallel.

Also $x=2+5\lambda, y=-3+4\lambda, z=5-2\lambda \quad \therefore \quad \frac{2+5\lambda-7}{5}=\frac{-3+4\lambda-1}{4}=\frac{5-2\lambda-2}{-2}$

i.e. $\lambda - 1 = \lambda - 1 = \frac{3-2\lambda}{-2}$ $\therefore$ no value of λ

Thus the lines are parallel and different.

(D) < 2, 3, 5> and < 3, 2, 5 > are direction ratios of first and 2nd line respectively.

$\therefore$ The lines are not parallel

$x = 3 + 2\lambda, \quad y = -2 + 3\lambda, \quad z = 4 + 5\lambda$

$x = 3 + 3\mu, \quad y = -2 + 2\mu, \quad z = 7 + 5\mu$

Are parametric equations of the lines.

Solving $3 + 2\lambda = 3 + 3\mu$ and $-2 + 3\lambda = 2 + 2\mu$ We get $\lambda = \frac{12}{5}, \mu = \frac{8}{5}$

Now substituting these values in $4 + 5\lambda = 7 + 5\mu$

We get 4 + 12 = 7 + 8 i.e. 16 = 15 which is not true.

$\therefore$ The lines do not intersect , Hence the lines are skew.

20. A (*x, y, z*) Let P (0, 3, 2), Q (2, 0, 3), R(0, 0, 1)

AP = AQ = AR

$x^2 + (y-3)^2 + (z-2)^2 = (x-2)^2 + y^2 + (z-3)^2 = x^2 + y^2 + (z-1)^2$

In xy plane $z = 0$ So, $x^2 - 4x + 4 + y^2 + 9 = x^2 + y^2 + 1 \quad x = 3$

$9 + y^2 - 6y + 9 + 4 = x^2 + y^2 + 1$

So, A (3, 2, 0) also B (1, 4, -1) & C (2, 0, -2)

Now $AB = \sqrt{4+4+1} = 3$

$AC = \sqrt{1+4+4} = 3$

$BC = \sqrt{1+16+1} = \sqrt{18}$ AB = AC

isosceles Δ & $AB^2 + AC^2 = BC^2$ right angle Δ

Area of $\Delta ABC = \frac{1}{2} \times \text{base.height} \quad \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$

So only S_1 is true

21. $\left(\sqrt{a^2-4}\right) \tan A + a \tan B + \left(\sqrt{4+a^2}\right) \tan C = 6a$

From dot product in equality

$\left(a^2 - 4 + a^2 + 4 + a^2\right)\left(\tan^2 A + \tan^2 B + \tan^2 C\right) \ge 36a^2 \qquad \Rightarrow \tan^2 A + \tan^2 B + \tan^2 C \ge 12$

22. $\bar{c} = (2\alpha + \beta)\hat{i} + \hat{j}(\alpha + 2\beta) + \hat{k}(\beta - \alpha) \qquad \frac{\bar{c}.\left(\bar{a} + \bar{b}\right)}{\left|\bar{a} + \bar{b}\right|} = 3\sqrt{2}$

$\Rightarrow \quad \alpha + \beta = 2 \quad$ .... (1)

$\left(\bar{c} - \left(\bar{a} \times \bar{b}\right)\right).\left(\alpha\bar{a} + \beta\bar{b}\right) \qquad = \left|\bar{c}\right|^2 = \alpha^2\left|\bar{a}\right|^2 + \beta^2\left|b\right|^2 + 2\alpha\beta\left(\bar{a}.\bar{b}\right)$

$= 6\left(\alpha^2 + \beta^2 + \alpha\beta\right) \quad = 6\left(\alpha^2 + (2-\alpha)^2 + \alpha(2-\alpha)\right) = 6\left((\alpha-1)^2 + 3\right) \Rightarrow$ Min. value = 18

23. We have $\left(\overline{OP}\right)^2 = \left(e^t + e^{-t}\right)\left(\vec{a}\right)^2 + \left(e^t - e^{-t}\right)^2\left(\vec{b}\right)^2 + 2\left(e^t + e^{-t}\right)\left(e^t - e^{-t}\right)\left(\vec{a} \cdot \vec{b}\right)$

$\left(\left(\vec{a}\right)^2 = \left|\vec{a}\right|^2 = 9, \left(\vec{b}\right)^2 = \left|\vec{b}\right|^2 = 16 \text{ and } \vec{a} \cdot \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\frac{2\pi}{3}\right)$

$\Rightarrow \left|\overline{OP}\right|^2 = 9\left(e^t + e^{-t}\right)^2 + 16\left(e^t - e^{-t}\right)^2 + 2\left(e^{2t} - e^{-2t}\right) \cdot 3 \cdot 4 \cdot \left(-\frac{1}{2}\right) = 13e^{2t} + 37e^{-2t} - 14$

Now $\frac{d}{dt}\left|\overrightarrow{OP}\right|^2=0\Rightarrow 26e^{2t}-74e^{-2t}=0\Rightarrow e^{4t}=\frac{37}{13}$, so $e^{2t}=\frac{\sqrt{37}}{\sqrt{13}}$

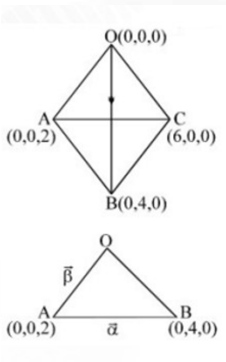
$\therefore \quad \left|\overrightarrow{OP}\right|^2_{\min}=13\left(\frac{\sqrt{37}}{\sqrt{13}}\right)+37\left(\frac{\sqrt{13}}{\sqrt{37}}\right)-14\Rightarrow\left|\overrightarrow{OP}\right|^2_{\min}=2\sqrt{481}-14$

$\Rightarrow \quad \left|\overrightarrow{OP}\right|_{\min}=\sqrt{2}\sqrt{\sqrt{481}-7}=\sqrt{2}\sqrt{\sqrt{a}-b}$, so a = 481, b=7

Hence $(a+b)=488$ Ans.

24. If I is the in centre of the tetrahedron then I together with 4 faces of the tetrahedron forms 4 smaller tetrahedrons.]

If S_1, S_2, S_3 and S_4 are the areas of the plane faces then the volume V of the tetrahedron is given by $V=\frac{1}{3}r\left(S_1+S_2+S_3+S_4\right)$



$\therefore \quad r=\frac{3V}{S_1+S_2+S_3+S_4}$ ….. (1)

$$V=\frac{1}{6}\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\frac{1}{6}\begin{vmatrix}0&0&2\\0&4&0\\6&0&0\end{vmatrix}=\left|\frac{1}{6}-2\left(0-24\right)\right|=8$$

$\vec{\alpha}=0\hat{i}+4\hat{j}-2\hat{k}$; $\vec{\beta}=2\hat{k}$

$S_1=\frac{1}{2}\left|\vec{\alpha}\times\vec{\beta}\right|=\frac{1}{2}\left|8\left(\hat{k}\times\hat{j}\right)\right|=4$(Plane OAB)

||| 1y other areas are 6, 12, 14

Hence from (1) $r=\frac{3\cdot 8}{4+6+12+14}=\frac{3\cdot 8}{36}=\frac{2}{3}\Rightarrow 3r=2$

25. $\left|\vec{a}_1\times\vec{a}_2+\vec{a}_2\times\vec{a}_3+....+\vec{a}_{n-1}\times\vec{a}_n\right|$

$=\left|\vec{a}_1.\vec{a}_2+\vec{a}_2.\vec{a}_3+....+\vec{a}_{n-1}.\vec{a}_n\right|$

Let $\left|\vec{a}_1\right|=\left|\vec{a}_2\right|......=\left|\vec{a}_n\right|=\lambda$ (as centre is origin)

More over angle between 2 consecutive $\vec{a}_i$'s is $\frac{2\pi}{n}$

Hence given equation reduces to

$$(n-1)\lambda^2\sin\left(\frac{2\pi}{n}\right)=(n-1)\lambda^2\cos\left(\frac{2\pi}{n}\right)$$

$$\Rightarrow \tan\left(\frac{2\pi}{n}\right)=1\Rightarrow\frac{2\pi}{n}=\frac{\pi}{4}\Rightarrow n=8$$

PHYSICS

26. $v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times G \times \frac{4}{3}\pi R^3 d}{R}}$

$\sqrt{2gh} = \sqrt{\frac{8}{3}G\pi d R^2} \quad \Rightarrow R = \sqrt{\frac{3gh}{4\pi G d}}$

27. $\omega_{sp} = \omega_s - \omega_p \Rightarrow \frac{1}{T_{sp}} = \frac{1}{T_s} + \frac{1}{T_p}$

28. $v_s = \sqrt{\frac{GM}{R}}$

From conservation of angular momentum $10muR = 11mv''R/2$

From conservation of energy $\frac{1}{2}11mv'^2 - G\frac{11mM}{R} = \frac{1}{2}11mV''^2 - \frac{G(11mM)}{R/2}$

From conservation of linear momentum $121V^{1^2} = 100u^2 + V^2$

After solving those equation we will get $V = \sqrt{\frac{58GM}{R}}$

29. $V = \frac{2}{x} - \frac{3}{xy} + \frac{4}{2}$

Then $\vec{E} = \frac{-\partial v}{\partial x}\hat{i} - \frac{-\partial v}{\partial y}\hat{j} - \frac{-\partial v}{\partial z}\hat{k}$

So $\vec{E} = \left(\frac{2}{x^2} - \frac{3}{yx^2}\right)\hat{i} - \frac{3}{xy^2}\hat{j} + \frac{4}{z^2}\hat{k}$

So $\vec{E}\, at\, (1,1,1) = -\hat{i} - \hat{j} + 4\hat{k}$

So force = $m\vec{E} = 2\sqrt{26}N$

30. SOL: Use law of conservation of linear momentum and law of conservation of energy

31.

32. Sign of q_1 and q_2 opposite because no point of zero field between them

q_2 smaller in magnitude

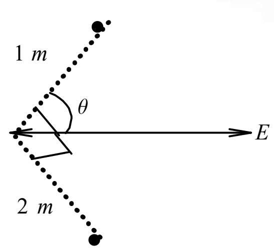
field between q_1 and q_2 is –ve so q_2 must be +ve and q_1 must be –ve.

33. $\overline{F} = q\overline{E} \Rightarrow F = qE$

$W = FS\cos\theta$

$W = qE \times 2 \times \cos(90-\theta) = qE(1)\cos\theta$

$\Rightarrow \tan\theta = \frac{1}{2} \Rightarrow \cos\theta = \frac{2}{\sqrt{5}}$



$(1) \Rightarrow W = qE.\frac{2}{\sqrt{5}}$

$qE = ma$

$(1) \Rightarrow W = ma.\frac{2}{\sqrt{5}} \Rightarrow m = \frac{\sqrt{5}W}{2a}$

37. $E = \sqrt{E_1^2 + E_2^2} \qquad = \frac{k}{8}\sqrt{(3+4)} \qquad = \sqrt{7}\frac{k}{8}$

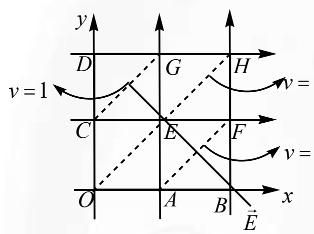
38. Here to solve this question we can use principle of superposition. The given structure can be considered as combination of two as shown in figure.

$\vec{E}$ at 0 (due to given structure)

$= \vec{E}_{at\,0}(due\ to\ I) - \vec{E}_{at\,0}(due\ to\ II)$

$E = \frac{dq}{4\pi\varepsilon_0 R^2}$ towards $dl \qquad = \frac{\frac{Q}{2\pi R} \times dl}{4\pi\varepsilon_0 R^2} = \frac{Q\,dl}{8\pi^2\varepsilon_0 R^3}$

40. OEH is an equipotential surface, the uniform E.F. must be per-perpendicular to it pointing from higher to lower potential as shown



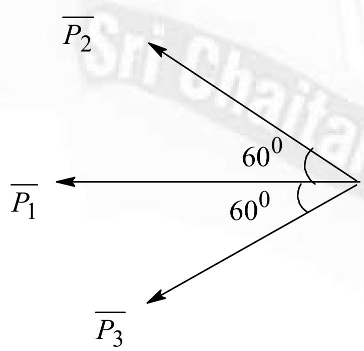
Hence, $E = \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right)$

$E = \frac{(V_E - V_B)}{EB} = \frac{0-(-2)}{\sqrt{2}} = \sqrt{2}V/m$

$\therefore \bar{E} = E.\bar{E} = \sqrt{2}\frac{(\hat{i} - \hat{j})}{\sqrt{2}} = (\hat{i} - \hat{j})V/m$

41. $\frac{mV^2}{R} = 2\cos 30^0 \frac{GM^2}{(\sqrt{3}R)^2} = \frac{GM^2}{\sqrt{3}R^2} \qquad V = \sqrt{\frac{GM}{\sqrt{3}R}}$

42.

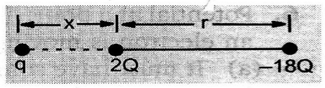


$p = p_1 + p_2\cos 60^0 + p_3\cos 60^0 = 2p = 4Qa$

$E_{equi} = \frac{Qa}{\pi \in_0 x^3}$

45.

When two charges are of same sign, then point of zero $\vec{E}$ in between the charges while if two charges are of opposite signs, then



the point of zero $\vec{E}$ lies outside the line joining the charges and nearer to charge having smaller magnitude.

So, according to given situation we have to place a test charge q as shown in the figure.

Let us find the value of x by considering that test charge is in equilibrium, then

$$\frac{q \times 2Q}{4\pi\varepsilon_0 x^2} = \frac{18Q \times q}{(r+x)^2}$$

$$x = \frac{r}{2}$$

Now, let us consider the equilibrium of 2Q, then

$$\frac{q \times 2Q}{4\pi\varepsilon_0 (r/2)^2} = \frac{2Q \times 18Q}{4\pi\varepsilon_0 (r^2)}$$

$$q = \frac{9}{2}Q$$

For system to be in equilibrium, $q = \frac{-9Q}{2}$

For $x = \frac{r}{2}$ and $q = -\frac{9Q}{2}$, we can check that $-18Q$ is also in equilibrium. So, the entire system is in equilibrium.

46. M = mass of complete sphere = $\frac{4}{3}\pi R^3 \rho$

Mass of the removed part = $\frac{4}{3}\pi R^3 \rho \ /8$ $\qquad F_2 = \frac{GMm}{9R^2} - \frac{GMm}{50R^2}$ $\qquad F_1 = \frac{GMm}{9R^2}$

47. $W = \vec{F}.\vec{S}$ $\qquad$ If $\vec{F}$ is $\perp \vec{S}$ then $W = 0$

$y + \frac{2}{3}x = 5$ $\qquad y = -\frac{2}{3}x + 5$

$m_1 = -\frac{2}{3}$ $\qquad \vec{S} = -2\hat{i} + 3\hat{j}$ $\qquad m_2 = \frac{3}{2}$ $\qquad m_1 \times m_2 = -1$

48. $\frac{qEL}{2} - \frac{mgL}{2} = 0$

49. Energy of the skylab in the first orbit $= -\frac{GMm}{4R}$

Total energy required to place the skylab into the orbit of radius 2R from the surface of the earth

$$= -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = \frac{3GMm}{4R}$$

$$= \frac{3gR^2m}{4R} = \frac{3}{4}mgR$$

Energy of the skylab in the second orbit $= -\frac{GMm}{6R}$

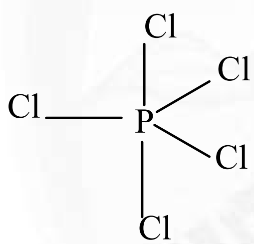
Energy needed to shift the skylab from the first orbit to the second orbit

$$= \frac{GMm}{4R} - \frac{GMm}{6R} = \frac{GMm}{R} \times \frac{2}{24} = \frac{mgR}{12}$$

50.

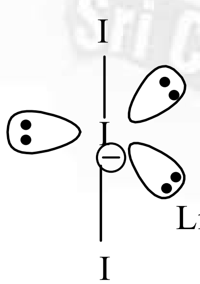
CHEMISTRY

51. $Na_2O < MgO < Al_2O_3 < SiO_2 < P_2O_5 < SO_3 < Cl_2O_7$
52. Fact based
53. Smallest size cation H^+
Smallest size anion F^-
54. XO_3 $O.N$ of $X = +6$
55. $F - Be - F$ sp
H_3O^+ SP^3
SOF_4 SP^3d
XeF_6 SP^3d^3
56. In MOT O_2 No of unpaired electrons = 2
57. Bond order $\alpha \dfrac{1}{\text{Bond dissociation energy}}$
58. X_eF_4 is planar as well as Non polar
59.



Sp^3d
$\left(S, Px, Py, Pz, dz^2\right)$

60. Bond angle
a & e 90°
e & e 120°
a & a 180°
61. $SF_6, XeOF_4$ & XeF_4 all have SP^3d^2 hybridisation
62.



Linear

$F - Be - F$
SP
Linear

63. CH_4 & CF_4 both bond angle is same
64. Fact based $\left(Sncl_2\right)$
65. Due to back bonding

$R_3Si-0-H \longrightarrow R_3Si-0^- + H^+$

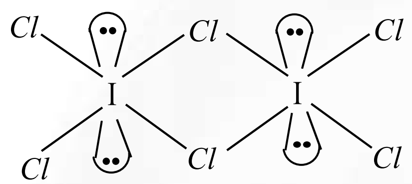
66. With increase in s character in an orbital, bond length will decrease

$d_{N-N(Bond\ length)}$ in $N_2H_4 > d_{N-N}$ in N_2F_4

$d_{C-C(Bond\ length)}$ in $C_2H_6 > d_{C-C}$ in C_2F_6

$d_{O-O(Bond\ length)}$ in $O_2H_2 > d_{O-O}$ in O_2F_2

67.



68. bond order of $O_2 = 2(paramagnetic)$ $O_2^{2-} = 1(diamagnetic)$

69. Bond order of He_2 is zero (does not exist)

70. bond order of $N_2 = 3$, $O_2^+ = 2.5, CO = 3$

71. Bond order

O_2	2
N_2	3
O_2^+	2.5

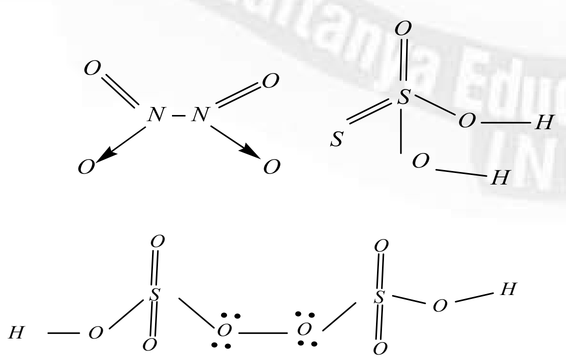
72. $O_2 \longrightarrow Paramagnetic$ $O_2^-(17\bar{e}) \longrightarrow Paramagnetic$

$O_2^+ \longrightarrow Paramagnetic$ $N_2^+(13\bar{e}) \longrightarrow Paramagnetic$

73.

$$\left.\begin{matrix} N_2(14E) \\ F_2(18E) \\ H_2(2E) \end{matrix}\right] Diamagraph$$

74.



75. NO, NO_2, BCl_3 **and** H_2SO_4 Do not follow Octet rule