JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON FRIDAY 24th JANUARY 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is (3,1) is :

(1) $48x + 25y = 169$ (2) $4x + 122y = 134$

(3) $25x + 101y = 176$ (4) $5x + 16y = 31$

Ans. **(1)**

Sol. Equation of chord with given middle point

$T = S_1$

$$\Rightarrow \frac{3x}{25}+\frac{y}{16}-1=\frac{9}{25}+\frac{1}{16}-1$$

$48x + 25y = 144 + 25$

$48x + 25y = 169$ Ans.

2. The function $f : (-\infty, \infty) \to (-\infty, 1)$, defined by $f(x)=\frac{2^x-2^{-x}}{2^x+2^{-x}}$ is :

(1) One-one but not onto

(2) Onto but not one-one

(3) Both one-one and onto

(4) Neither one-one nor onto

Ans. **(1)**

Sol. $f(x) = \frac{2^{2x}-1}{2^{2x}+1}$

$$=1-\frac{2}{2^{2x}+1}$$

$f'(x) = \frac{2}{\left(2^{2x}+1\right)^2}.2.2^{2x}.\ell n2$ i.e always +ve

so f(x) is ↑ function

$\therefore f(-\infty) = -1$

$f(\infty) = 1$

$\therefore f(x) \in (-1, 1) \neq$ co-domain

so function is one-one but not onto

3. If $\alpha > \beta > \gamma > 0$, then the expression

$$\cot^{-1}\left\{\beta+\frac{(1+\beta^2)}{(\alpha-\beta)}\right\}+\cot^{-1}\left\{\gamma+\frac{(1+\gamma^2)}{(\beta-\gamma)}\right\}+\cot^{-1}\left\{\alpha+\frac{(1+\alpha^2)}{(\gamma-\alpha)}\right\}$$ is equal to:

(1) $\frac{\pi}{2}-(\alpha+\beta+\gamma)$ (2) 3π

(3) 0 (4) π

Ans. **(4)**

Sol. $\Rightarrow \cot^{-1}\left(\frac{\alpha\beta+1}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{\beta\gamma+1}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{\alpha\gamma+1}{\gamma-\alpha}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$

$\Rightarrow\left(\tan^{-1}\alpha-\tan^{-1}\beta\right)+\left(\tan^{-1}\beta-\tan^{-1}\gamma\right)+\left(\pi+\tan^{-1}\gamma-\tan^{-1}\alpha\right)$

$\Rightarrow \pi$

4. Let $f : (0, \infty) \to \mathbf{R}$ be a function which is differentiable at all points of its domain and satisfies the condition $x^2f'(x) = 2xf(x) + 3$, with $f(1) = 4$. Then $2f(2)$ is equal to:

(1) 29 (2) 19

(3) 39

(4) 23

Ans. **(3)**

Sol. $x^2f'(x) - 2x\,f(x) = 3$

$$\left(\frac{x^2f'(x)-2xf(x)}{\left(x^2\right)^2}\right)=\frac{3}{\left(x^2\right)^2}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{f(x)}{x^2}\right)=\frac{3}{x^4}$$

Integrating both sides

$$\frac{f(x)}{x^2}=-\frac{1}{x^3}+C$$

$f(x) = -\frac{1}{x}+Cx^2$

put x = 1

$4 = -1 + C \Rightarrow C = 5$

$f(x) = -\frac{1}{x} + 5x^2$

Now $2 \times f(2) = 2 \times \left[-\frac{1}{2} + 5 \times 2^2\right]$

$= 39$

5. Let

$$A = \left\{x \in (0,\pi) - \left\{\frac{\pi}{2}\right\} : \log_{(2/\pi)}|\sin x| + \log_{(2/\pi)}|\cos x| = 2\right\}$$

and

$B = \left\{x \ge 0 : \sqrt{x}(\sqrt{x} - 4) - 3\left|\sqrt{x} - 2\right| + 6 = 0\right\}$. Then

$n(A \cup B)$ is equal to:

(1) 4 (2) 2

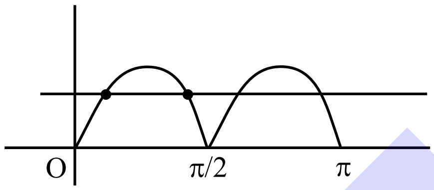
(3) 8 (4) 6

Ans. **(3)**

Sol. A : $\log_{2/\pi}|\sin x| + \log_{2/\pi}|\cos x| = 2$

$\Rightarrow \log_{2/\pi}(|\sin x \cdot \cos x|) = 2$

$\Rightarrow |\sin 2x| = \frac{8}{\pi^2}$



Number of solution 4

B : let $\sqrt{x} = t < 2$

Then $\sqrt{x}\left(\sqrt{x} - 4\right) + 3\left(\sqrt{x} - 2\right) + 6 = 0$

$\Rightarrow t^2 - 4t + 3t - 6 + 6 = 0$

$\Rightarrow t^2 - t = 0, t = 0, t = 1$

$x = 0, x = 1$

again let $\sqrt{x} = t > 2$

then $t^2 - 4t - 3t + 6 + 6 = 0$

$\Rightarrow t^2 - 7t + 12 = 0$

$\Rightarrow t = 3, 4$

$x = 9, 16$

Total number of solutions

$n(A \cup B) = 4 + 4 = 8$

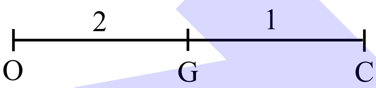
6. Let the position vectors of three vertices of a triangle be $4\vec{p} + \vec{q} - 3\vec{r}, -5\vec{p} + \vec{q} + 2\vec{r}$ and $2\vec{p} - \vec{q} + 2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p} + \vec{q} + \vec{r}}{4}$ and $\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}$ respectively, then $\alpha + 2\beta + 5\gamma$ is equal to:

(1) 3 (2) 1

(3) 6 (4) 4

Ans. **(1)**

Sol. We know that



O (orthocentre) $\frac{\vec{p} + \vec{q} + \vec{r}}{4}$

C (circum centre) $\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}$

C (centroid) $= \frac{\vec{p} + \vec{q} + \vec{r}}{3}$

by relation

$\Rightarrow 2\left(\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}\right) + \frac{\vec{p} + \vec{q} + \vec{r}}{4} = 3\left(\frac{\vec{p} + \vec{q} + \vec{r}}{3}\right)$

$\Rightarrow 8\left(\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}\right) = 3\left(\vec{p} + \vec{q} + \vec{r}\right)$

$\Rightarrow 8\alpha = 3,\ 8\beta = 3,\ 8\gamma = 3$

$\alpha = \frac{3}{8}, \beta = \frac{3}{8}, \gamma = \frac{3}{8}$

$\therefore \alpha + 2\beta + 3\gamma$

$\frac{3}{8} + \frac{6}{8} + \frac{15}{8} = \frac{24}{8} = 3$

7. Let [x] denote the gereatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x) = [x] + |x - 2|$, $-2 < x < 3$, is not continuous and not differentiable. Then m + n is equal to:

(1) 6 (2) 9

(3) 8 (4) 7

Ans. **(3)**

Sol. $f(x) = [x] + |x-2| \quad -2 < x < 3$

$$f(x) = \begin{cases} -x, & -2 < x < -1 \\ -x+1, & -1 \le x < 0 \\ -x+2, & 0 \le x < 1 \\ -x+3, & 1 \le x < 2 \\ x, & 2 \le x < 3 \end{cases}$$

So f(x) is not continuous at 4 points and not differentiable at 4 point

So m + n = 4 + 4 = 8

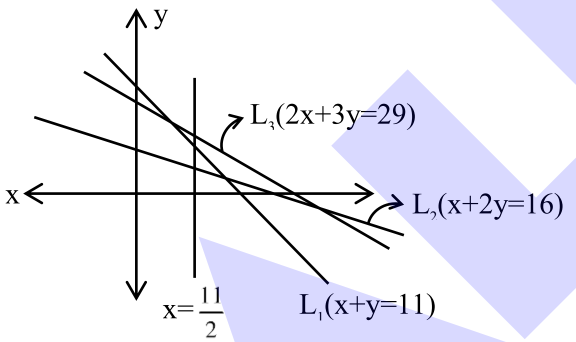
8. Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides x + y = 11, x + 2y = 16 and 2x + 3y = 29. Then the product of the smallest and the largest values of α is equal to :

(1) 22 (2) 44

(3) 33 (4) 55

Ans. **(3)**

Sol.



Point of intersection of $x = \frac{11}{2}$ with L_1 & L_3

gives, $\alpha_{min} = \frac{11}{2}$

and $\alpha_{max} = 6$

$\therefore \alpha_{min} \cdot \alpha_{max} = \frac{11}{2} \times 6 = 33$

9. In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to:

(1) 510 (2) 515

(3) 525 (4) 505

Ans. **(2)**

Sol. Let a & d are first term and common diff of an AP.

$$S_{40} = \frac{40}{2}[2a + 39d] = 1030 \quad \text{....(1)}$$

$$S_{12} = \frac{12}{2}[2a + 11d] = 57 \quad \text{....(2)}$$

by (1) & (2)

$$a = -\frac{7}{2} \qquad d = \frac{3}{2}$$

$$\therefore S_{30} - S_{10} = \frac{30}{2}[2a + 29d] - \frac{10}{2}[2a + 9d]$$

$= 20a - 390d$

$= 515$

10. If $7 = 5 + \frac{1}{7}(5 + \alpha) + \frac{1}{7^2}(5 + 2\alpha) + \frac{1}{7^3}(5 + 3\alpha) +$ .... ∞, then the value of α is:

(1) 1 (2) $\frac{6}{7}$

(3) 6 (4) $\frac{1}{7}$

Ans. **(3)**

Sol. Let $S = 5 + \frac{1}{7}(5 + \alpha) + \frac{1}{7^2}(5 + 2\alpha) + \ldots$

$$\frac{1}{7}S = \frac{1}{7}(5) + \frac{1}{7^2}(5 + \alpha) + \ldots \infty$$

$$\frac{6}{7}(S) = 5 + \frac{1}{7}\alpha\left(\frac{1}{1 - \frac{1}{7}}\right)$$

$$6 = 5 + \frac{\alpha}{6} \Rightarrow \alpha = 6$$

11. If the system of equations

x + 2y – 3z = 2

2x + λy + 5z = 5

14x + 3y + μz = 33

has infinitely many solutions, then λ + μ is equal to:

(1) 13 (2) 10

(3) 11 (4) 12

Ans. **(4)**

Sol. $D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 14 & 3 & \mu \end{vmatrix} = 0, \lambda\mu + 42\lambda - 4\mu + 107 = 0$

$D_1 = 2\lambda\mu + 99\lambda - 10\mu + 255$

$D_2 = 13 - \mu$

$D_3 = 5\lambda + 5$

$D_2 = 0 \Rightarrow \mu = 13 \ \& \ D_3 = 0 \Rightarrow \lambda = -1$

check & verify for these values D & $D_2 = 0$

12. Let (2, 3) be the largest open interval in which the function $f(x) = 2\log_e(x-2) - x^2 + ax + 1$ is strictly increasing and (b, c) be the largest open interval, in which the function $g(x) = (x-1)^3(x+2-a)^2$ is strictly decreasing. Then $100(a + b - c)$ is equal to:

(1) 280 (2) 360

(3) 420 (4) 160

Ans. **(2)**

Sol. $f'(x) = \frac{2}{x-2} - 2x + a \geq 0$

$f''(x) = \frac{-2}{(x-2)^2} - 2 < 0$

$f'(x) \downarrow$

$f'(3) \geq 0$

$2 - 6 + a \geq 0$

$a \geq 4$

$a_{min} = 4$

$g(x) = (x-1)^3(x+2-a)^2$

$g(x) = (x-1)^3(x-2)^2$

$g'(x) = (x-1)^3\, 2(x-2) + (x-2)^2\, 3(x-1)^2$

$= (x-1)^2(x-2)(2x-2+3x-6)$

$= (x-1)^2(x-2)(5x-8) < 0$

$x \in \left(\frac{8}{5}, 2\right)$

$100(a+b-c) = 100\left(4 + \frac{8}{5} - 2\right) = 360$

13. Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If 2A = 5B, then n is equal to:

(1) 22 (2) 21

(3) 20 (4) 19

Ans. **(2)**

Sol. $A = {}^{2n-1}C_{29} \quad B = {}^{2n-1}C_{11}$

$2\ {}^{2n-1}C_{29} = 5\ {}^{2n-1}C_{11}$

$$2\frac{(2n-1)!}{29!(2n-30)!} = 5\frac{(2n-1)!}{(2n-12)!11!}$$

$$\frac{1}{29\ldots12\cdot5} = \frac{1}{(2n-12)(2n-13)\ldots(2n-29)2}$$

$$\frac{1}{30\cdot29\ldots12} = \frac{1}{(2n-12)(2n-13)\ldots(2n-29)12}$$

$2n - 12 = 30$

$n = 21$

14. Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \vec{a} \times (\hat{i} - 2\hat{k})$ and $\vec{c} = \vec{b} \times \hat{k}$.

Then the projection of $\vec{c} - 2\hat{j}$ on $\vec{a}$ is:

(1) $3\sqrt{7}$

(2) $\sqrt{14}$

(3) $2\sqrt{14}$

(4) $2\sqrt{7}$

Ans. **(3)**

Sol. $\vec{b} = \vec{a} \times (\hat{i} - 3\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + 8\hat{j} + \hat{k}$$

$\vec{c} = \vec{b} \times \hat{k} = 8\hat{i} - 2\hat{j}$

$\vec{c} - 2\hat{j} = 8\hat{i} - 4\hat{j}$

Projection of $(\hat{i} - 2\hat{j})$ on $\vec{a}$

$$(\vec{c} - 2\hat{j}).\hat{a} = \frac{\langle 8,-4,0\rangle.\langle 3,-1,2\rangle}{\sqrt{14}}$$

$$= \frac{28}{\sqrt{14}} = 2\sqrt{14}$$

15. For some a, b, let

$$f(x)=\begin{vmatrix} a+\frac{\sin x}{x} & 1 & b \\ a & 1+\frac{\sin x}{x} & b \\ a & 1 & b+\frac{\sin x}{x}\end{vmatrix},\quad x\neq 0,$$

$\lim\limits_{x\to 0} f(x)=\lambda+\mu a+\nu b$. Then $(\lambda+\mu+\nu)^2$ is equal to:

(1) 25 (2) 9

(3) 36 (4) 16

Ans. (4)

Sol. $\lim\limits_{x\to 0} f(x)=\begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1\end{vmatrix}$

$= (a + 1)\,(2(b + 1) - b) + 1(ab - a(b + 1)) + ba$

$= (a + 1)\,(b + 2) - a + ab$

$= b + a + 2 = \lambda + \mu a + \nu b$

$\lambda = 2, \mu = 1, \nu = 1 \Rightarrow (\lambda + \mu + \nu)^2 = 16$

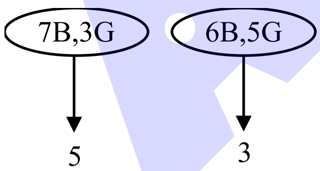
16. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to:

(1) 8575 (2) 9100

(3) 8925 (4) 8750

Ans. (3)

Sol.



C-I (3G & 2B) & (1G & 2B)

C-II (2G & 3B) & (2G & 1B)

C-III (1G & 4B) & (3G & 0B)

Total = C-I + C-II + C-III

$= {}^7C_2 \cdot {}^3C_3 \cdot {}^6C_2 \cdot {}^5C_1 + {}^7C_3 \cdot {}^3C_2 \cdot {}^6C_1\, {}^5C_2 + {}^7C_4 \cdot {}^3C_1 \cdot {}^6C_0 \cdot {}^5C_3$

= 8925

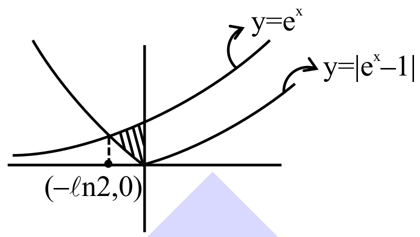
17. The area of the region enclosed by the curves $y = e^x$, $y = |e^x - 1|$ and y-axis is:

(1) $1 + \log_e 2$ (2) $\log_e 2$

(3) $2 \log_e 2 - 1$ (4) $1 - \log_e 2$

Ans. (4)

Sol.



For Area $\int_{-\ell n2}^{0}\left[e^x-\left(1-e^x\right)\right]dx$

$\int_{-\ell n2}^{0}\left(2e^x-1\right)dx=\left[2e^x-x\right]_{-\ell n2}^{0}$

$= (2 - (1 + \ell n2))$

$= 1 - \ell n2$

18. The number of real solution(s) of the equation $x^2 + 3x + 2 = \min\{|x - 3|, |x + 2|\}$ is :

(1) 2

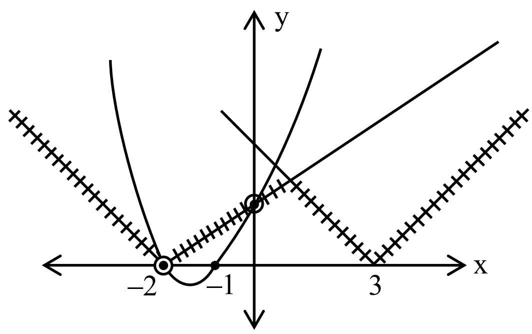
(2) 0

(3) 3

(4) 1

Ans. (1)

Sol.



Only 2 solutions.

19. Let A = $[a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1. Let E be the event that A is an invertible matrix. Then the probability P(E) is :

(1) $\frac{5}{8}$ (2) $\frac{3}{16}$

(3) $\frac{1}{8}$ (4) $\frac{3}{8}$

Ans. **(4)**

Sol. C-I $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \rightarrow 4$ ways

C-II $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ & $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \rightarrow 2$ ways

$P = \frac{\text{favourable}}{\text{total}} = \frac{6}{16} = \frac{3}{8}$

20. If the equation of the parabola with vertex $V\left(\frac{3}{2}, 3\right)$ and the directrix x + 2y = 0 is $\alpha x^2 + \beta y^2 - \gamma xy - 30x - 60y + 225 = 0$, then $\alpha + \beta + \gamma$ is equal to:

(1) 6

(2) 8

(3) 7

(4) 9

Ans. **(4)**

Sol. Equation of axis $y - 3 = 2\left(x - \frac{3}{2}\right)$

$y - 2x = 0$

foot of directrix

$y - 2x = 0$

& $\Rightarrow (0, 0)$

$2y + x = 0$

Focus = (3, 6)

$PS^2 = PM^2$

$(x-3)^2 + (y-6)^2 = \left(\frac{x+2y}{\sqrt{5}}\right)^2$

$4x^2 + y^2 - 4xy - 30x - 60y + 225 = 0$

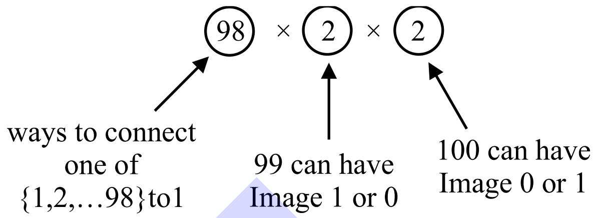
$\Rightarrow \alpha = 4, \beta = 1, \gamma = 4 \Rightarrow \alpha + \beta + \gamma = 9$

SECTION-B

21. Number of functions f : {1, 2, …, 100} → {0, 1}, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to ______.

Ans. **(392)**

Sol.

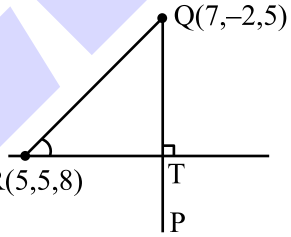


392 Ans.

22. Let P be the image of the point Q(7, –2, 5) in the line $L : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ and R(5, p, q) be a point on L. Then the square of the area of ΔPQR is ____

Ans. **(957)**

Sol.



Let R(2λ + 1, 3λ – 1, 4λ)

2λ + 1 = 5

λ = 2

R(5, 5, 8)

let T(2λ + 1, 3λ – 1, 4λ)

$\overrightarrow{QT} = (2\lambda - 6)\hat{i} + (3\lambda + 1)\hat{j} + (4\lambda - 5)\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{QT} \cdot \vec{b} = 0$

4λ – 12 + 9λ + 3 + 16λ – 20 = 0

λ = 1

T(3, 2, 4)

QT = $\sqrt{33}$ RT = $\sqrt{29}$

$(\text{area of } \Delta PQR)^2 = \left(\frac{1}{2}\sqrt{29} . 2\sqrt{33}\right)^2$

= 957

23. Let y = y(x) be the solution of the differential equation $2\cos x\dfrac{dy}{dx} = \sin 2x - 4y\sin x,\ x \in \left(0, \dfrac{\pi}{2}\right)$.

If $y\left(\dfrac{\pi}{3}\right) = 0$, then $y'\left(\dfrac{\pi}{4}\right) + y\left(\dfrac{\pi}{4}\right)$ is equal to ____.

Ans. (1)

Sol. $\dfrac{dy}{dx} + 2y\tan x = \sin x$

I.F. = $e^{2\int \tan x\, dx} = \sec^2 x$

$y\sec^2 x = \int \dfrac{\sin x}{\cos^2 x}dx$

$= \int \tan x \sec x\, dx$

$= \sec x + C$

$C = -2$

$y = \cos x - 2\cos^2 x$

$y\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}} - 1$

$y' = -\sin x + 4\cos x\sin x$

$y'\left(\dfrac{\pi}{4}\right) = -\dfrac{1}{\sqrt{2}} + 2$

$y'\left(\dfrac{\pi}{4}\right) + y\left(\dfrac{\pi}{4}\right) = 1$

24. Let $H_1 : \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $H_2 : -\dfrac{x^2}{A^2} + \dfrac{y^2}{B^2} = 1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their eccentricities be $e_1 = \sqrt{\dfrac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to ______

Ans. (55)

Sol. $\dfrac{2b^2}{a} = 15\sqrt{2}$

$1 + \dfrac{b^2}{a^2} = \dfrac{5}{2}$

$a = 5\sqrt{2}$

$b = 5\sqrt{3}$

$\dfrac{2A^2}{B} = 12\sqrt{5}$

$2a.2B = 100\sqrt{10}$

$2.5\sqrt{2}.2B = 100\sqrt{10}$

$B = 5\sqrt{5}$

$A = 5\sqrt{6}$

$e_2^2 = 1 + \dfrac{A^2}{B^2}$

$= 1 + \dfrac{150}{125}$

$e_2^2 = 1 + \dfrac{30}{25}$

$25e_2^2 = 55$

25. If $\int \dfrac{2x^2 + 5x + 9}{\sqrt{x^2 + x + 1}}dx = x\sqrt{x^2 + x + 1} + \alpha\sqrt{x^2 + x + 1} + \beta\log_e\left|x + \dfrac{1}{2} + \sqrt{x^2 + x + 1}\right| + C$, where C is the constant of integration, then $\alpha + 2\beta$ is equal to ____

Ans. (16)

Sol. $2x^2 + 5x + 9 = A(x^2 + x + 1) + B(2x + 1) + C$

$A = 2 \qquad B = \dfrac{3}{2} \qquad C = \dfrac{11}{2}$

$2\int\sqrt{x^2 + x + 1}\,dx + \dfrac{3}{2}\int\dfrac{2x + 1}{\sqrt{x^2 + x + 1}}dx + \dfrac{11}{2}\int\dfrac{dx}{\sqrt{x^2 + x + 1}}$

$2\int\sqrt{\left(x + \dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2}dx + 3\sqrt{x^2 + x + 1} + \dfrac{11}{2}\int\dfrac{dx}{\sqrt{\left(x + \dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2}}$

$2\left(\dfrac{x + \frac{1}{2}}{2}\sqrt{x^2 + x + 1} + \dfrac{3}{8}\ell n\left(x + \dfrac{1}{2} + \sqrt{x^2 + x + 1}\right)\right) + 3\sqrt{x^2 + x + 1}$

$+\dfrac{11}{2}\ell n\left(x + \dfrac{1}{2} + \sqrt{x^2 + x + 1}\right) + C$

$\alpha = \dfrac{7}{2} \qquad \beta = \dfrac{25}{4}$

$\alpha + 2\beta = 16$