- No. of elements in exactly one of the sets A,B,C $= n(A)+n(B)+n(C)-2n(A\cap B)-2n(B\cap C)-2n(A\cap C)+3n(A\cap B\cap C)$
- No. of elements in exactly two of the sets A,B,C $= n(A\cap B)+n(B\cap C)+n(C\cap A)-3n(A\cap B\cap C)$

## Concept Based Questions

1. **Which of the following is an empty set**
   1) $\{\phi\}$ 2) $\{0\}$
   3) $\{n\in N \text{ and } n<1\}$
   4) The set of all even prime numbers
2. **Let A and B be two sets such that $A\cup B=A$ Then $A\cap B$ is equal to**
   1) $\phi$ 2) B 3) A 4) $A\cup B$
3. **Which of the following is not correct?**
   1) $A\subseteq A^c$ if and only if $A=\phi$
   2) $A^c\subseteq A$ if and only if $A=X$, where X is the universal set
   3) If $A\cup B=A\cup C$, then $B=C$
   4) A = B is equivalent to $A\cup C=B\cup C$ and $A\cap C=B\cap C$
4. **If A and B are two sets then $(A-B)\cup(B-A)\cup(A\cap B)$ is**
   1) $A\cup B$ 2) $A\cap B$ 3) A 4) $B^1$
5. $A\cap(A\cup B)^c =$
   1) A 2) B 3) $\phi$ 4) A-B
6. **If $U=\{a,b,c,d,e,f,g,h\}$ and $A=\{a,b,c\}$ then complement of A is**
   1) $\{d,e,f\}$ 2) $\{d,e,f,g,h\}$
   3) $\{a,b,c\}$ 4) $\{d,e,g,h\}$

## KEY

1) 3 2) 2 3) 3 4) 1 5) 3 6) 2

## Hints & Solutions

1. $N = \{1, 2, 3, \ldots\ldots\ldots\}$
2. Since, $A\cup B=A \Rightarrow B\subseteq A \therefore A\cap B=$
3. $A^c$ satisfies (1) and (2) by definition, (4) also follows trivially.
   Assuming A to be any set other than the empty set, also $B=A$ and $C=\phi$, we have
   $A\cup B=A=A\cup C$
   But $B\neq C$, So (3) is incorrect
4. $(A-B)\cup(B-A)\cup(A\cap B)=A\cup B$
   Draw venn diagram
5. $A\cap(A\cup B)^c=A\cap(A^c\cap B^c)$
   $=(A\cap A^c)\cap(A\cap B^c)=\phi\cap(A\cap B^c)=$
6. Complement of A = U - A

# EXERCISE-I

## CRTQ & SPQ LEVEL-I

### ROSTER, SET-BUILDER FORM, OPERATION ON SETS

**C.R.T.Q** Class Room Teaching Questions

1. **Which of the following not a well defined collection of objects**
   1) The set of Natural Numbers
   2) Rivers of India
   3) Various kinds of Triangles
   4) Five most renowned Mathematicians of the world.
2. **Write the solution set of the equation $x^2+x-6=0$ in roster form**
   1) {2, -3} 2) {−1,−2}
   3) {1,2} 4) {−1,2}

3. Write the set A={1,4,9,16,25.....} in set builder form
   1) $\{x : x = n^2 \text{where} n \in N\}$
   2) $\{x : x = n^2 \text{where} n \in W\}$
   3) $\{x : x = n^2 \text{where} n \in Z\}$
   4) $\{x : x = n^2 \text{where} n \in Q\}$
4. Which of the following is not empty set
   1) A ={x : 1<x<2, x is a natural number between 1 and 2}
   2) $B = \{x : x^2 - 2 = 0 \text{ and } x \text{ is rational}\}$
   3) C = {x : x is even prime number >2}
   4) $D = \{x : x^2 = 0 \text{ and } x \text{ is integer}\}$
5. If A = {x/x is a letter in the word "ACCOUNTANCY"} then cardinality of A is
   1) 5 2) 6 3) 7 4) 8
6. Let $F_1$ be the set of all parallelograms, $F_2$ be the set of rectangles, $F_3$ be the set of rhombuses, $F_4$ be the set of squares and $F_5$ be the set of trapeziums in a plane then $F_1$=
   1) $F_2 \cap F_3$ 2) $F_2 \cup F_3 \cup F_4$
   3) $F_3 \cup F_4 \cup F_5$ 4) $F_3 \cap F_1$
7. If the set of factors of a whole number 'n' including 'n' itself but not '1' is denoted by F(n). If $F(16) \cap F(40) = F(x)$ then 'x' is
   1) 4 2) 8 3) 6 4) 10
8. If A is the set of the divisors of the number 15, B is the set of prime numbers smaller than 10 and C is the set of even numbers smaller than 9, then $(A \cup C) \cap B$ is the set
   1){1, 3, 5} 2){1, 2, 3}
   3){2, 3, 5} 4){2, 5}
9. Let A = {1,2,3,4,5,6}, B = {2,4,6,8} then A - B =
   1) {1,3,5} 2) {8} 3) {2,4,6} 4) $\phi$
10. Let $A = \{1,2,3,4\}$ and $B = \{2,3,4,5,6\}$, then $A \Delta B$ =
    1) $\{2,3,4\}$ 2) $\{1\}$ 3) $\{5,6\}$ 4) $\{1,5,6\}$
11. Let U={1,2,3,4,5,6}, A={2,3},B={3,4,5} then $A^1 \cap B^1$ = __
    1) {1,2} 2) {1,6} 3){1,5} 4){1,4}
12. In a class of 35 students,24 like to play cricket and 16 like to play football also each student likes to play at least one of the two games. How many students like to play both cricket and football?
    1) 3 2) 4 3) 5 4) 6
13. In a group of 70 people, 37 like coffee, 52 like tea and each person like atleast one of the two drinks. The number of persons liking both coffee and tea is
    1) 16 2) 13 3) 19 4) 20
14. If n(X)=28, n(Y)=32, $n(X \cup Y)$=50 then $n(X \cap Y)$=
    1) 6 2) 7 3) 8 4)10
15. If n(A)=50, n(B)=20 and $n(A \cap B) = 10$ then $n(A \Delta B)$ is
    1) 50 2) 60 3) 70 4) 40

**S.P.Q. Student Practice Questions**

16. The group of beautiful girls is
    1) a null set 2) A finite set
    3) not a set 4) Infinite set
17. Which of the following is the roster form of letters of word "SCHOOL"
    1) {s,h,o,l} 2) {s,c,h,o,l}
    3) {s,c,o,l} 4) {h,o,o,l}
18. Write the set {x : x is a positive integer and $x^2 < 40$} in the roaster form
    1) {1,2,3,4,5,6} 2) {1,2,3,4,5,6,7}
    3) {2,3,4,5,6,7} 4) {0,1,2,3,5,6}
19. Which of the following is finite
    1) A = {x : x is set of points on a line}
    2) B = {x : x $\in$ N and x is prime}
    3) C = {x : x $\in$ N and x is odd}
    4) D = {x : x $\in$ N and (x-1)(x-2)=0}

7. If $aN=\{ax:x\in N\}$ then $3N\cap 7N=$
1. 21N 2. 10N 3. 4N 4. 5N

8. If $A=\{1,2,3,4,5,6\}$, $B=\{1,2\}$, then $\frac{A}{\left(\frac{A}{B}\right)}=$
1) A 2) $\phi$ 3) $A\cap B$ 4) $A\cup B$

9. If $n(U)=700, n(A)=200, n(B)=300,$ $n(A\cap B)=100$, then $n(A'\cap B')$ is equal to
1) 400 2) 240 3) 300 4) 500

10. If $n(U)=48, n(A)=28, n(B)=33$ and $n(B-A)=12$, then $n(A\cap B)^C$ is
1) 27 2) 28 3) 29 4) 30

11. If $n(A\cap B^C)=5, n(B\cap A^C)=6,$ $n(A\cap B)=4$ then the value of $n(A\cup B)$ is
1) 18 2) 15 3) 16 4) 17

12. Let n(A-B)=25+X, n(B-A)=2X and $n(A\cap B)=2X$. If n(A)=2(n(B)) then 'X' is
1) 4 2) 5 3) 6 4) 7

## S.P.Q. Student Practice Questions

13. Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and foot ball, and 12 play foot ball and cricket. Eight play all the three games. The total number of members in the three athletic teams is
1) 43 2) 76 3) 49 4) 53

14. If sets A and B have 3 and 6 elements each, then the minimum number of elements in $A\cup B$ is
1) 3 2) 6 3) 9 4) 18

15. If $n(U)=60, n(A)=21, n(B)=43$ the greatest value of $n(A\cup B)$ and value of $n(A\cup B)$ are
1) 60, 43 2) 50, 36 3) 70, 44 4) 60,

16. If A={x:x is a multiple of 4} and B={x a multiple of 6} then $A\cap B$ consists multiples of
1) 16 2) 12 3) 8 4)

17. Two finite sets have m and n elements. total number of subsets of the firs is 48 more than the total numbe subsets of the second set. The value m and n are
1) 7,6 2) 7,6 3) 6,4 4)

18. If $A=]2,4[$ and $B=[3,5[$ then $A\cap B=$
1) [3, 4] 2) [3, 4]
3) [3, 4] 4) [2,5]

19. Let A be the set of non-negative integ I is the set of integers, B is the se non-positive integers, E is the set of e integers and P is the set of pri numbers then.
1) I-A=B 2) $A\cap B=\phi$
3) $E\cap P=\phi$ 4) $A\Delta B=I-$

20. In a class of 100 students, 55 stude have passed in Mathematices and students have passed in physics. No students who have passed in phys only is
1) 22 2) 33 3) 10 4)

21. 90 students take mathematics, 72 t science in a class of 120 students. If take neither Mathematics nor scie then number of students who take b the subjects is
1) 52 2) 110 3) 162 4) 10

22. A set A has 3 elements and another B has 6 elements. Then
1) $3\le n(A\cup B)\le 6$ 2) $3\le n(A\cup B)$
3) $6\le n(A\cup B)\le 9$ 4) $0\le n(A\cup B)$

23. In a survey of 400 students in a schoo 100 were listed as taking apple juice, 1 as taking orange juice and 75 were liste

of families =40,000 and $n(P\cap C)=5\%$.

$\therefore (ii)\,and\,(iii)$ are correct.

15. Minimum value of

$x=100-(30+20+25+15)=100-90=10.$

# EXERCISE-IV

## LEVEL-IV

### Assertion - Reason Type :

**Note :**

1) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

2) Statement-1 is true, Statement-2 is true, Statement-2 is not correct explanation for Statement-1.

3) Statement-1 is true, Statement-2 is false.

4) Statement-1 is false, Statement-2 is false.

1. **Statement-1:** If A = {x:x is a prime number} and B = {x: x $\in$N} then $A\cap B$ = {x: x is a prime number}=A.
**Statement-2:** If $A\subset B$ then $A\cap B$ = A.

2. **Statement-1:** If A={2,4,7,10}, B={1,2,3,4} then A-B={1,3,7,10}.
**Statement-2:** A-B={x:x $\notin$A and x $\in$B}.

3. **Statement-1:** A={x: 0<x<3, $x\in R$} and B = {x: $1\le x\le 5, x\in R$}then
A - B = { x : 0 <x< 1, $x\in R$}
**Statement-2:** $A\Delta B=(A-B)\cup(B-A)$.

4. **Statement-1:** If $n(B)=3$ then number of elements in power set of $B$= $2^3=8$
**Statement-2:** If $n(A)=m$ then number of elements in power set of $A$ = $2^m-1$

5. **Statement-1:** If X = {x : x is w number}, Y = {x : x is natural num then $X\cup Y$ = { x : x is whole number
**Statement-2:** If $Y\subset X$ then $Y\cup X=$

6. **Statement-1:** If A and B are disjoin sets then A - B = A
**Statement-2:** If A and B are disjoin sets then $A\cap B=\varnothing$

7. **Statement-1 :** If n (A) = 5 then num of proper subsets of $A$ = 31
**Statement-2 :** If n (A) = m then number of proper subsets of $B=2^$

8. Match the following sets for all A, B a

| | |
|---|---|
| i) $\left((A'\cup B')-A\right)'$ | a) A-B |
| ii) $\left[B'\cup(B'-A)\right]'$ | b) A |
| iii) $(A-B)-(B-C)$ | c) B |
| iv) $(A-B)\cap(C-B)$ | d) $(A\cap C)-B$ |

1) i-b, ii-c , iii-a, iv-d
2) i-b, ii-c, iii-d,
3) i-b, ii-c, iii-c, iv-d
4) i-d, ii-c, iii-a,

## KEY

| | | | | |
|---|---|---|---|---|
| 01) 1 | 02) 4 | 03) 2 | 04) 3 | 05) 1 |
| 06) 1 | 07) 1 | 08) 2 | | |

## Hints & Solutions

1. The set of Prime numbers are the sub of the Natural number set.
2. A-B= {7,10}. A-B={x:x$\in$A and x$\notin$B}.
3. A=(0,3), B=[1,5]
A-B=(0,1) = {x: 0<x<1, $x\in R$}
B-A=[3,5]= {x: $3\le x\le 5, x\in R$}
$A\Delta B=(A-B)\cup(B-A)=(0,1)\cup[3,5]$

7. If a relation 'R' is defined by $R=\{(x,y)/2x^2+3y^2\le 6\}$, then the domain of 'R' is

1) $[-3,3]$ 2) $[-\sqrt{3},\sqrt{3}]$

3) $[-\sqrt{2},\sqrt{2}]$ 4) $[-2,2]$

8. Let A be a set of first ten natural numbers and R be a relation on A, defined by $(x,y)\in R \Leftrightarrow x+2y=10$, then domain of R is

1) {1,2,3,....., 10} 2) {2,4,6,8}

3) {1,2,3,4} 4) {2,4,6,8,10}

9. Let A={x,y,z} B={1,2}. Then the number of relations from A to B

1) $2^6$ 2) $2^3$ 3) $2^2$ 4) $2^7$

10. If P={1,2}, Then P x P x P is

1) {(1,1,1,),(1,1,2),(1,2,1),(1,2,2),(2,1,1), (2,1,2),(2,2,1),(2,2,2,)}

2) {(1,1,1),(1,2,2),(1,2,4)}

3) {(1,1,3)}

4) All the above

## S.P.Q. Student Practice Questions

11. If $A=\{x:x^2-5x+6=0\}, B=\{2,4\}$, $C=\{4,5\}$, then $A\times(B\cap C)$ is

1) $\{(2,4),(3,4)\}$

2) $\{(4,2),(4,3)\}$

3) $\{(2,4),(3,4),(4,4)\}$

4) $\{(2,2),(3,3),(4,4),(5,5)\}$

12. If $A=\{x:x^2-5x+6=0\}, B=\{1,2\}$ and $C=\{4,5\}$ then $(A-B)\times(A-C)=$

1) {(2,3)} 2) {(1,2)}

3) {(1,2),(2,3)} 4) {(3,2)(3,3)}

13. A={1,2,3,4,}, relation R on A is defined by R={(x,y) / x<y and $|x^2-y^2|<9$; x,y∈A} then R=

1) {(1,1) (2,2) (3,3) (4,4)}

2) {(2,1) (3,2) (3,2) (4,3)}

3) {(1,2) (1,3) (2,3) (3,5)}

4) {(1,2) (1,3) (2,3) (3,4)}

14. Let A={1,2,3,.....,14}. Define a relation R from A to A by $R=\{(x,y):3x-y=0;\ x,y\in A\}$. Then domain of R is

1) {3,6,9,12} 2) {3,6}

3) {1,2,3,.....,14} 4) {1,2,3,4}

15. The domain and range of relation $R=\{(x,y)/x,y\in N, x+2y=5\}$ is

1) {1, 3}, {2, 1} 2) {2, 1}, {3, 2}

3) {1, 3}, {1, 1} 4) {1, 2}, {1, 3}

16. If R ={(x,y): x,y ∈Z , $x^2+y^2\le 4$} is a relation in Z, then domain of R is

1) {0,1,2} 2) {0,-1,-2}

3) {-2,-1,0,1,2} 4) {1,2,3}

17. If R = {(x,y): x, y∈N, y is the remainder when x is divided by 7}. Then sum of all numbers in range of R

1) 14 2) 21 3) 28 4)12

18. Write the relation $R=\{(x,x^3):x$ is prime number less than 10} in roaster form

1) $R=\{(2,8),(3,27),(5,125),(7,343)\}$

2) $R=\{(2,4),(3,9),(5,25),(7,49)\}$

3) $R=\{(2,2),(3,3),(5,5),(7,7)\}$

4) $R=\{(2,8),(3,9),(5,25),(7,343)\}$

19. A relation R is defined in the set of integers I as follows $(x,y)\in R$ iff $x^2+y^2=9$, which of the following is true?

1) $R=\{(0,3),(0,-3),(3,0),(-3,0)\}$

2) Domain of R = {–3, 0, 3}

3) Range of R = {–3, 0, 3}

4) All the above

## TYPES OF RELATIONS

### C.R.T.Q Class Room Teaching Questions

20. Let R be a reflexive relation on a finite set A having n elements and let there be m ordered pairs in R, then

1) $m\geq n$ 2) $m\leq n$

3) $m=n$ 4) $m<n$

21. Let R be a reflexive relation on a set A and I be the identity relation on A. Then

1) $R\subset I$ 2) $I\subseteq R$

3) $R=I$ 4) all the above

22. Let A be the set of the children in a family. The relation 'x is a brother of y' The relation on A is

1) reflexive 2) symmetric

3) transitive 4) anti symmetric

23. Let $A=\{1,2,3\}$ and $R=\{(1,1),(1,3),(3,1),(2,2),(2,1),(3,3)\}$, then the relation R on A is

1) reflexive 2) symmetric

3) transitive 4) equivalence

24. Let $A=\{2,4,6,8\}$ and $R=\{(2,4),(4,2),(4,6),(6,4)\}$ then R is

1) reflexive 2) symmetric

3) transitive 4) anti symmetric

25. Let A ={1,2,3,4}, R={(2,2),(3,3),(4,4),(1,2)} be a relation on A. Then R is

1) reflexive 2) symmetric

3) transitive 4) Both 1 & 2

26. N is the set of natural numbers and relation R is defined on $N\times N$ as follows $(a,b)R(c,d)\Leftrightarrow a+d=b+c$. Then R is

1) reflexive only 2) symmetric only

3) transitive only 4) an equivalence relation

27. Let $R=\{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation on the set $A=\{1,2,3,4\}$. Then the relation R is

1) not symmetric 2) transitive

3) a function 4) reflexive

28. In the set $A=\{1,2,3,4,5\}$, a relation is defined by $R=\{(x,y):x,y\in A,x<y\}$. Then R is

1) reflexive 2) symmetric

3) transitive 4) equivalence

29. If $A=\{1,2,3\}$, the number of reflexive relation in A is

1) 9 2) 3 3) 64 4)

30. Let R and S be two non-void relations on set A which of the following statements is false.

1) R and S are transitive $\Rightarrow R\cup S$ is transitive

2) R and S are transitive $\Rightarrow R\cap S$ is transitive

3) R and S are symmetric $\Rightarrow R\cup S$ is symmetric

4) R and S are symmetric $\Rightarrow R\cap S$ is symmetric

### S.P.Q. Student Practice Questions

31. Two points A and B in a plane are related if OA=OB, where O is a fixed point. This relation is

1) partial order relation

2) equivalence relation

3) reflexive but not symmetric

4) reflexive but not transitive

32. Which one of the following relations on Z is equivalence relation?

1) $xR_1y \Leftrightarrow |x| = |y|$ 2) $xR_2y \Leftrightarrow x \geq y$

3) $xR_3y \Leftrightarrow x/y$ 4) $xR_4y \Leftrightarrow x < y$

33. $R = \{(a,b): a,b \in N, a+b \text{ is even}\}$ is

1) reflexive 2) Symmetric

3) both 1,2 4) none of 1,2

34. Let $X = \{1,2,3\}$ and

$R = \{(1,1),(2,2),(3,3),(2,3)\}$ be a relation on X. Then which one is not true

1) R is reflexive 2) R is transitive

3) R is antisymmetric 4) R is symmetric

35. Let $A = \{a,b,c\}$ and

$R = \{(a,a),(b,b),(a,b),(b,a),(b,c)\}$ be a relation on A, then R is

1) reflexive 2) symmetric

3) transitive 4) not reflexive

36. The relation R={(1,1)(2,2)(3,3)} on the set {1,2,3} is

1) Symmetric only 2) Reflexive only

3) Transitive only 4) An equivalence

37. Which of the following are not equivalence relations on I ?

1) aRb if $a+b$ is an even integer

2) aRb if $a-b$ is an even integer

3) aRb if $a < b$

4) aRb if $a = b$

38. Total number of equivalence relations defined in the set $S = \{a,b,c\}$ is

1) 5 2) 3! 3) $2^3$ 4) $3^3$

39. Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. The relation is

1) reflexive and symmetric only

2) an equivalence relation

3) reflexive only

4) reflexive and transitive only

40. In the set Z of all integers, which of the following relation R is not an equivalence relation?

1) $x R y$ : if $x \leq y$

2) $x R y$ : if $x = y$

3) $x R y$ : if $x-y$ is an even integer

4) $x R y$ : if $x \equiv y \pmod 3$

41. Which of the following is an equivalence relation ?

1) $x<y$ 2) $x>y$

3) $x-y$ is divisible by 5 4) x divides y

## INVERSE RELATION

### C.R.T.Q Class Room Teaching Questions

42. If the relation $R: A \to B$, where $A = \{1,2,3,4\}$ and $B = \{1,3,5\}$ is defined by $R = \{(x,y): x<y, x \in A, y \in B\}$, then $RoR^{-1}$ is

1) $\{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$

2) $\{(3,1),(5,1),(5,2),(5,3),(5,4)\}$

3) $\{(3,3),(3,5),(5,3),(5,5)\}$

4) $\{(3,5)\}$

43. R is a relation from {11,12,13} to {8,10,12} defined by y = x-3. Then $R^{-1}$ is

1) {(8,11),(10,13)}

2) {(11,8),(13,10)}

3{(10,11),(8,11)}

4) {(11,8),(10,13),(12,15)}

8. If P={x : x<3 ,x ∈N}, Q={x :x ≤3, x∈W}. then $(p\cup Q)x(p\cap Q)$. where W is the set of whole numbers
1) {(0,1),(0,2),(1,1),(1,2),(2,1),(2,2),(0,0),(3,3)}
2 {(0,1),(0,2),(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)}
3) {(0,1),(0,2),(1,1),(1,2),(2,1),(2,2),(0,3),(3,3)}
4) {(0,1),(0,2),(1,1),(1,2),(2,1),(2,2),(0,3),(0,0)}

9. The domain of the relation R defined by R={$(x,x+5): x\in\{0,1,2,3,4,5\}$}is
1) {1,2,3,4,5} 2) {0,1,2,6,7}
3) {5,6,7,8,9,10} 4) {0,1,2,3,4,5}

10. Range of R ={(x , y):x,y ∈z, x+3y=12}
1) $\{\pm 12,\pm 9,\pm 6,\pm 3,0\}$
2) {0,±3,±6,. ..} 3) Z 4) N

11. The relation R is defined by R={(a,b):b=$|a+1|$ and $|a|\le 3, a,b\in Z$} then range of R =
1) {-3,-2,-1,0,1,2} 2) {0,1,2,3,4}
3) {-3,-2,-1} 4) {0,-1,-2,-3}

## TYPES OF RELATIONS

### C.R.T.Q Class Room Teaching Questions

12. Let R be the relation on the set of all real numbers defined by a R b if $|a-b|\le 1$. Then R is
1) reflexive and symmetric
2) symmetric only
3) transitive only 4) anti - symmetric only

13. Let $R=\{(x,y): x,y\in A; x+y=5\}$ where A = {1,2,3,4,5} then
1) R is not reflexive, symmetric and not transitive
2) R is an equivalence relation
3) R is reflexive, symmetric but not transitive
4) R is not reflexive, not symmetric but transitive

14. On the set of natural numbers N, the relation R is defined by xRy iff x + y = 100 is
1) reflexive 2) not reflexive
3) equivalence 4) not symmetric

15. On the set of all vectors in space the relation R is defined by $\bar{a}R\bar{b}\Leftrightarrow\bar{a}.\bar{b}$ is scalar is
1) symmetric 2) not symmetric
3) not reflexive 4) both 2 and 3

16. If A={ 1, 2,3} Then a relation reflexive but not Symmetric on A is
1) {(1,1), (1,2)}
2) {(1,1),(1,2),(2,1),(2,2)}
3) {(1,1),(2,2),(3,3)}
4) {(1,1),(2,2),(3,3),(2,3)}

17. The correct statement of the following is
1) The relation 'less than' on Z is anti-symmetric
2) The relation 'sister of' on the members of a family is transitive
3) The relation 'relatively prime' on N is reflexive
4) The relation 'perpendicular' on a set of lines in a plane is transitive

18. If A ={ 1,2,3} ,R ={ (1,2),(1,1),(2,3)} Then minimum number of elements may be adjoined with the elements of R so that it may become transitive is
1) 0 2) 1 3) 2 4) 3

19. Let R be an equivalence relation defined on a set containing 6 elements. Then the minimum numbers of ordered pairs that R should contain
1) 6 2) 12 3) $6^6$ 4) 36

### S.P.Q. Student Practice Questions

20. R ={(a,b): a,b ∈ R, $a^2+b^2=1$ } is
1) reflexive 2) Symmetric
3) transitive 4) anti symmetric

21. { (x,y): x,y ∈ Z,x-y is divisible by 5} is
1) reflexive 2) Symmetric
3) both 1,2 4) not reflexive

## DOMAIN RANGE

### C.R.T.Q Class Room Teaching Questions

3. The relation R defined on $A=\{1,2,3\}$ by a Rb. if $|a^2-b^2|\le 5$, which of the following is false.

1) $R=\{(1,1),(2,2),(3,3),(2,1),(1,2),(2,3),(3,2)\}$

2) $R^{-1}=R$

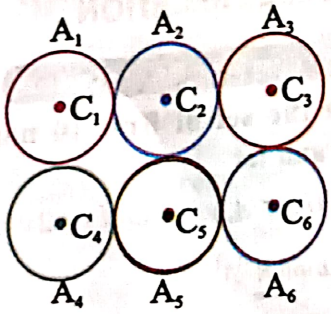
3) Domain of R = {1, 2, 3}

4) Range of R = {5}

## TYPES OF RELATIONS

### C.R.T.Q Class Room Teaching Questions

4. Let $A=\{A_1,A_2,A_3,A_4,A_5,A_6\}$ be the set of six unit circles with centres $C_1,C_2,C_3...C_6$ arranged as shown in the diagram.The relation R on A is defined by $(A_i,A_j)\in R\Leftrightarrow C_iC_j\le 2\sqrt{2}$ then




1) R is symmetric, transitive but not reflexive

2) R is only transitive

3) R is symmetric, reflexive but not transitive

4) R is neither reflexive nor transitive but is symmetric.

5. The relation R on the set of natural numbers N is defined as

$xRy\Leftrightarrow x^2-4xy+3y^2=0$,

$x,y\in N$ then R is

1) reflexive but not symmetric and not tra

2) symmetric but not reflexive and not tra

3) transitive but not reflexive and not sym

4) equivalence relation

6. Which of the following relations transitive

1) $(a,b)\in R_1\Leftrightarrow a\le b,a,b\in z$

2) $(x,y)\in R_2\Leftrightarrow x$ divides $y$ if $x,y\in$

3) $(x,y)\in R_3\Leftrightarrow |x|+|y|=1$, for $x,y$

4) $(x_1,y_1)\in R_4\Leftrightarrow l_1$ parallel to $l_2$. where $l_1,l_2$ are lines

### S.P.Q. Student Practice Question

7. A relation R on the set of non ze complex numbers is defined by $z_1Rz_2\Leftrightarrow\frac{z_1-z_2}{z_1+z_2}$ is real, then $R$ is

1) Reflexive

2) Symmetric

3) Transitive

4) Equivalence

8. S is a relation over the set R of al numbers and it is given by (a, b) $\in$ S $\Leftrightarrow$ ab $\ge$ 0. Then S is

1) symmetric and transitive only

2) reflexive and symmetric only

3) a partial relation

4) an equivalance relation

## KEY

| | | | | |
|---|---|---|---|---|
| 01) 1 | 02) 2 | 03) 4 | 04) 3 | 05) |
| 06) 3 | 07) 4 | 08) 4 | | |

## Concept Based Questions

1. $f: A \to B$ is a function then A, B are respectivley.
   1) domain, range
   2) domain, co domain
   3) co domain, range
   4) range, domain

2. $f: A \to B$ then f(A) is called
   1) domain 2) co domain
   3) range 4) function

3. If $f: A \to B$ is a function then
   1) $f(A) = B$ 2) $f(A) \subset B$
   3) $f(A) \subseteq B$ 4) $B \subseteq f(A)$

4. If $f: A \to B$ is surjective then
   1) No two elements of A have the same image in B
   2) Every element in A has an image in B
   3) Every element of B has at least one pre-image in A
   4) A and B are finite non empty sets

5. A constant function $f: A \to B$ will be one-one if
   1) $n(A) = n(B)$ 2) $n(A) = 1$
   3) $n(B) = 1$ 4) $n(A) < n(B)$

6. If $f: A \to B$ is a constant function which is onto then B is
   1) a singleton set 2) a null set
   3) an infinite set 4) a finite set

7. If $n \geq 2$ then the number of surjections that can be defined from {1,2,3,....n} onto {1, 2} is
   1) $n^2 - n$ 2) $n^2$
   3) $2^n$ 4) $2^n - 2$

8. If f and g are functions such that $fog$ is onto then
   1) f is onto 2) g is onto 3) gof is onto
   4) Neither f nor g is onto

9. If f and g are functions such that fog is one - one then
   1) g must be onto
   2) g must be one - one
   3) f must be one-one
   4) f, g need not one-one

10. To have inverse for the function f, f should be
    1) one-one 2) onto
    3) one-one and onto 4) Identity function

11. If $f: A \to B$ is a bijection then $f^{-1}of =$
    1) $fof^{-1}$ 2) $f$ 3) $f^{-1}$ 4) $I_A$

12. $f: A \to B$ is a bijection then $fof^{-1} =$
    1) $I_A$ 2) $I_B$ 3) f 4) $f^{-1}$

13. Let $f(x) = ax^2 + bx + c$, where a,b,c are rational and $f: Z \to Z$ where $Z$ is the set of integers. Then $a+b$ is
    1) a negative integer
    2) an integer
    3) non integral rational number
    4) Real number

14. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$. Then
    1) $f(x+2) = f(x-2)$
    2) $f(2+x) = f(2-x)$
    3) $f(x) = f(-x)$
    4) $f(x) = -f(-x)$

15. The graph of odd function is
    1) symmetric about origin
    2) symmetric about y-axis
    3) parallel to x-axis
    4) parallel to y-axis

16. The graph of even function is
    1) symmetric in opp quadrtants
    2) symmetric about y-axis
    3) parallel to x-axis
    4) parallel to y-axis

# EXERCISE-I

## CRTQ & SPQ LEVEL-I

### REAL VALUED FUNCTIONS

#### C.R.T.Q Class Room Teaching Questions

1. Let f : {(1,1), (2,3), (0,-1), (-1,-3)} be a function from z to z defined by f(x) = ax+b, for some integers a,b then (a,b) =
   1) (-1,2) 2) (2,-1)
   3) (3,-2) 4) (0,3)

2. If $e^{f(x)}=\frac{10+x}{10-x}, x\in(-10,10)$ and $f(x)=k.f\left(\frac{200x}{100+x^2}\right)$ then $k$ =
   1) 0.5 2) 0.6
   3) 0.7 4) 0.8

3. If $f(x)=\frac{7^{1+\ln x}}{x^{\ln 7}}$ then f (2015) =
   1) 20 2) 7
   3) 2015 4) 100

4. If $f(x)=\frac{\cos^2 x+\sin^4 x}{\sin^2 x+\cos^4 x}$ for $x\in R$ then $f(2016)$ =
   1) 1 2) 2
   3) 3 4) 4

5. If $f=\{(-2,4),(0,6),(2,8)\}$ and $g=\{(-2,-1),(0,3),(2,5)\}$, then $\left(\frac{2f}{3g}+\frac{3g}{2f}\right)(0)$ =
   (1) 1/12 (2) 25/12
   (3) 5/12 (4) 13/12

6. If f(x) = sin (log x) then f(xy) + $f\left(\frac{x}{y}\right)$ -2f(x) cos (logy) =
   1) cos (logx) 2) sin (logy)
   3) cos (log (xy)) 4) 0

7. If $f(x+y,x-y)=xy$ then the arithmetic mean of $f(x,y)$ and $f(y,x)$ is
   1) $x$ 2) $y$
   3) 0 4) $xy$

#### S.P.Q. Student Practice Questions

8. f is defined by $f(x)=\begin{cases}x^2, 0\le x\le 2\\ 3x, 2\le x\le 10\end{cases}$ then f(2)=
   1) 9 2) 6
   3) 5 4) not defined

9. If f(x) = ax+b, where a and b are integers, f(–1) = –5 and f(3)=3, then a and b are equal to
   1) a = –3, b = –1 2) a = 2, b = –3
   3) a = 0, b = 2 4) a = 2, b = 3

10. If $f(x)=\begin{cases}x^2+1, x\le 0\\ 2x-1, 0<x<5\\ 4x+3, x\ge 5\end{cases}$ then $\frac{f(-3)+f(2)+f(5)}{f(1)}$ =
    1) 28 2) 36 3) 26 4) 34

11. If f = {(-1,3), (0,2), (1,1)} then the range of $f^2-1$ is
    1) {0, 8} 2) {0,3,8}
    3) {0,1,3} 4) {0,2,8}

12. $f:R\to R$ is a function defined by $f(x)=\frac{1}{2}$ then $f\left(-\frac{2}{5}\right)$ =
    1) 2 2) $-\frac{1}{2}$ 3) -2 4) $\frac{1}{2}$

13. If f(x) = $\sin\left(\frac{\pi}{3}[x]-x^2\right)$ then the value of $f\left(\sqrt{\frac{\pi}{3}}\right)$ is
    1) 1 2) -1 3) 0 4) $\frac{-3}{4}$

14. The function $y = f(x)$ such that $f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$

1) $2 - x^2$ 2) $x^2 - 2$
3) $x^2 + 4$ 4) $4x^2 - 2$

15. If $f = \{(a,1),(b,-2),(c,3)\}$, $g = \{(a,-2),(b,0),(c,1)\}$ then $f^2 + g^2 =$

1) $\{(a,-1),(b,-2),(c,4)\}$
2) $\{(a,3),(b,-2),(c,2)\}$
3) $\{(a,-4),(b,-4),(c,9)\}$
4) $\{(a,5),(b,4),(c,10)\}$

16. Which of the following relations are functions $f:\{(2,1),(3,1),(4,2)\}$
$g:\{(2,2),(2,4),(3,3),(4,4)\}$
$h = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

1) f, g 2) g, h 3) h, f 4) f, g, h

## EVEN AND ODD FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

17. Let $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$, then $f$ is

1) an odd funciton
2) an even function
3) both odd and even
4) neitheer odd nor even

18. Which of the following is an even function

1) $f(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}$ 2) $f(x) = \frac{a^x + 1}{a^x - 1}$

3) $f(x) = x\frac{a^x - 1}{a^x + 1}$

4) $f(x) = \log_2\left(x + \sqrt{x^2 + 1}\right)$

### S.P.Q Student Practice Questions

19. If $f(x) = ax^5 + bx^3 + cx + d$ is odd then
1) $a = 0$ 2) $b = 0$ 3) $c = 0$ 4) $d = 0$

20. A function whose graph is symmetrical about the y axis is given by

1) $f(x) = \sin\left[\log\left(x + \sqrt{x^2 + 1}\right)\right]$

2) $f(x) = \frac{\sec^4 x + \operatorname{cosec}^4 x}{x^3 + x^4 \cot x}$

3) $f(x+y) = f(x) + f(y) \forall x, y \in R$

4) $f(x) = x^2$

## PERIODIC FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

21. The period of $\cos x^2$ is

1) $2\pi$ 2) $\sqrt{2}\pi$ 3) $4\pi^2$ 4) does not exist

22. Period of $f(x) = e^{\cos\{x\}} + \sin \pi[x]$ is ($[\bullet]$ and $\{\bullet\}$ denote the greatest integer function and fractional part function respectively)

1) 1 2) 2 3) $\pi$ 4) $2\pi$

23. Let $f(x)$ be periodic and $k$ be a positive real number such that $f(x+k) + f(x) = 0$ for all $x \in R$. Then the period of $f(x)$ is

1) k 2) 2k 3) 4k 4) 8k

24. The period of $f(x) = \sqrt{x - [x]}$ is

1) no fundamental period 2) $\frac{1}{2}$
3) 1 4) 2

### S.P.Q Student Practice Questions

25. The period of $\operatorname{sgn}(x - [x])$ is

1) any real number 2) 0
3) 1 4) non periodic functio

26. The period of $x\cos x$ is

1) $2\pi$ 2) $\pi$ 3) $\frac{\pi}{2}$ 4) non period

## DOMAIN OF THE FUNCTION

### C.R.T.Q Class Room Teaching Questions

27. The domain of $f(x)=\frac{x^2+2x+1}{x^2-x-6}$

1) $R-\{3,-2\}$ 2) $R-\{-3,2\}$
3) $R-\{3,-2]$ 4) $R-(3,-2)$

28. The domain of $f(x)=\frac{1}{\log|x|}$ is

1) $R-\{0\}$ 2) $R-\{0,1\}$
3) $R-\{-1,0,1\}$ 4) $(-\infty,\infty)$

29. The domain of $f(x)=\frac{3^x+3^{-x}}{3^x-3^{-x}}$ is

1) $(-\infty,\infty)$ 2) $(-\infty,0)\cup(0,\infty)$
3) $(0,\infty)$ 4) $(0,1)$

30. The domain of the function $f(x)=\sqrt{\log_{16}x^2}$ is

1) $x=0$ 2) $|x|\geq 4$
3) $|x|\geq 1$ 4) $|x|\geq 2$

31. The domain of $f(x)=\frac{1}{[x]-x}$ is

1) R 2) Z 3) R - Z 4) Q-{0}

32. The domain of $f(x)=\sqrt{x-2}+\frac{1}{\log(4-x)}$ is

1) $[2,\infty)$ 2) $(-\infty,4)$
3) $[2,3)\cup(3,4)$ 4) $[3,\infty)$

33. The domain of $f(x)=e^{\sqrt{x}}+\cos x$ is

1) $(-\infty,\infty)$ 2) $[0,\infty)$
3) $(0,1)$ 4) $(1,\infty)$

34. The domain of $\log_a \sin^{-1}x$ is (a>0, $a\neq 1$)

1) $0<x\leq 1$ 2) $0\leq x\leq 1$
3) $0\leq x<1$ 4) $0<x<1$

35. The domain of $\cosh^{-1}5x$ is

1) R 2) $[0,\infty)$ 3) $\left(\frac{1}{5},\infty\right)$ 4) $\left[\frac{1}{5},\infty\right)$

36. For which Domain, the functions $f(x)=2x^2-1$ and $g(x)=1-3x$ are equal to

1) R 2) $\left\{\frac{1}{2},-2\right\}$ 3) $\left(\frac{1}{2},2\right)$ 4) $\left[\frac{1}{2},2\right]$

### S.P.Q. Student Practice Questions

37. Domain of $f(x)=\frac{x^2+3x+5}{x^2-5x+4}$ is

1) $R-\{1,4\}$ 2) $\{1,4\}$
3) $(1,4)$ 4) $(1,4]$

38. Domain of [x] + x is

1) R 2) z 3) R – Z 4) Q

39. Domain of $|x-1|$ is

1) $[1,\infty)$ 2) R 3) $[0,\infty)$ 4) Z

40. The domain of $f(x)=\sqrt{-x^2}$ is

1) $(0,\infty)$ 2) $(-\infty,0)$ 3) $\{0\}$ 4) $(1,\infty)$

41. The domain of $f(x)=\frac{1}{|x|+x}$ is

1) $(-\infty,0)$ 2) $(0,\infty)$
3) $(-\infty,1)$ 4) (-2,-1)

42. The domain of $f(x)=\sqrt{1-|x|}$ is

1) [-1,1] 2) (-1,1) 3) (0,1) 4) R

43. The domain of $f(x)=\log\{(x-3)(6-x)\}$ is

1) $(3,\infty)$ 2) $(3,6)$
3) $(0,\infty)$ 4) $(-\infty,\infty)$

44. The domain of $f(x)=\cot\frac{x}{3}$ is

1) $(-\infty,\infty)$ 2) $R-\{n\pi : n\in Z\}$
3) $R-\{3n\pi : n\in Z\}$ 4) $(0,\infty)$

45. The domain of $f(x)=\text{Tan}^{-1}(5x)$ is

1) $(-\infty,\infty)$ 2) $(0,\infty)$
3) $(-\infty,0)$ 4) $\left(-\frac{1}{5},\frac{1}{5}\right)$

46. The domain of $f(x)=\sqrt{x}\cot x$ is
1) R 2) $R-\{n\pi : n\in Z\}$
3) $R-\left\{(2n+1)\frac{\pi}{2}, n\in Z\right\}$ 4) $(0,\infty)$

47. Domain of $f(x)=\frac{|x|-x}{2x}$ is
1) R 2) $R-\{0\}$ 3) Z 4) N

48. Domain of $\frac{3^x}{x+1}$ is
1) R 2) $R-\{-1\}$
3) $(1,\infty)$ 4) $(-\infty,1)$

## RANGE OF THE FUNCTION

### C.R.T.Q Class Room Teaching Questions

49. The domain and range of the real function f defined by $f(x)=\frac{4-x}{x-4}$ is given by
1) Domain = R, Range = {–1, 1}
2) Domain = R – {1}, Range = R
3) Domain = R – {4}, Range = {–1}
4) Domain = R – {–4}, Range = {–1, 1}

50. Range of $f(x)=\frac{1}{1-2\cos x}$ is
1) $\left[\frac{1}{3},1\right]$ 2) $\left[-1,\frac{1}{3}\right]$
3) $(-\infty,-1]\cup\left[\frac{1}{3},\infty\right)$ 4) $\left[-\frac{1}{3},1\right]$

51. The range of $f(x)=x^2+x+1$ is
1) $\left[\frac{3}{4},\infty\right)$ 2) $[0,\infty)$
3) $[1,\infty)$ 4) $\left[\frac{1}{4},\infty\right)$

52. The domain and range of the functio f given by $f(x)=2-|x-5|$ is
1) Domain = $R^+$, Range = $(-\infty,1]$
2) Domain = R, Range = $(-\infty,2]$
3) Domain = R, Range = $(-\infty,2)$
4) Domain = $R^+$, Range = $(-\infty,2]$

53. $f=\{(x,\frac{x^2}{x^2+1}):x\in R\}$ ,be a function into R,range of 'f'
1) [0,1) 2) $(-\infty,\infty)$ 3) $(0,\infty)$ 4)

54. Range of the function $f(x)=\sqrt{[x]-x}$
1) R 2) {1} 3) {0} 4) (0,

55. Let $A=\{9,10,11,12,13\}$ and $f:A-$ be defined by $f(n)=$ highest pr factor of n, then its range is
1) {13} 2) {3, 5, 11, 13}
3) {11, 13} 4) {2, 3, 5, 11}

56. The range of $f(x)=\frac{x^2-x+1}{x^2+x+1}$ is
1) $\left[\frac{1}{3},3\right]$ 2) $\left[\frac{1}{2},2\right]$
3) [0,1] 4) [–1,1]

### S.P.Q. Student Practice Questions

57. Range of $\frac{|x-4|}{x-4}$ is
1) R – {4} 2) R
3) {–1, 1} 4) R – {–1, 1}

58. If x is positive, the values of $f(x)=-3\cos\sqrt{3+x+x^2}$ lie in the intervel
1) [ -1, 3] 2) [-3, 3]
3) [0 , 3] 4) [-3,0]

59. The range of $f(x)=\frac{\sin\pi[x^2-1]}{x^4+1}$ where [.] is greatest integer functio
1) R 2) [-1,1] 3) {0,1} 4)

60. The range of $f(x) = |x-2|+|x-12|$ is
1) $[2,\infty)$ 2) $(12,\infty)$
3) $[10,\infty)$ 4) $[14,\infty)$

61. The range of $f(x) = 3x^2+7x+10$ is
1) $\left[\frac{70}{3},\infty\right)$ 2) $\left[\frac{71}{12},\infty\right)$
3) $[0,\infty)$ 4) $\left(-\infty,\frac{70}{3}\right)$

62. The range of $f(x) = x-[x]$ is
1) $x = \{1,2,3...\}$ 2) $x \geq 0$
3) $x < 1$ 4) $0 \leq x < 1$

63. Range of $\sqrt{9-x^2}$ is
1) [0, 3] 2) [–3, 3] 3) [–3, 0] 4) R

64. Range of $f(x) = e^x$ is
1) $(0,\infty)$ 2) $[0,\infty)$
3) $(-\infty,\infty)$ 4) $[e,\infty)$

## TYPES OF FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

65. The equivalent function of $\log x^2$ is
1) $2\log x$ 2) $2\log|x|$
3) $|\log x^2|$ 4) $(\log x)^2$

66. The number of linear functions which map $[-1,1]$ to $[0,2]$ are
1) One 2) Two 3) Four 4) Three

67. If A = (3,81) and $f: A \to B$ is a surjection defined by $f(x) = \log_3 x$ then B =
1) [1,4] 2) (1,4] 3) (1,4) 4) $[1,\infty)$

68. Let $f(x) = \sin^2\frac{x}{2}+\cos^2\frac{x}{2}$ and $g(x) = \sec^2 x - \tan^2 x$. The two functions are equal over the set
1) $\phi$ 2) R
3) $R-\left\{x \middle| x=(2n+1)\frac{\pi}{2}, n \in Z\right\}$ 4) $R-\{0\}$

69. $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}, \forall x \in R$ is
1) one - one 2) onto
3) bijective 4) neither one one nor onto

70. If $f: Z \to Z$ is such that $f(x) = 6x-11$ then f is
1) injective but not surjective
2) surjective but not injective
3) bijective
4) neither injective nor surjective

71. $f: N \to A$ Where A = {0,1} defined by $f(x) = \begin{cases} 0 & \text{if } x \text{ is odd} \\ 1 & \text{if } x \text{ is even} \end{cases}$. Then f is
(1) one - one, onto (2) one-one, into
(3) many-one, onto (4) many-one, into

72. $f: (-\infty,\infty) \to (0, 1]$ defined by $f(x) = \frac{1}{x^2+1}$ is
1) one-one but not onto
2) onto but not one-one
3) bijective
4) neither one-one nor onto

73. The function $f: R \to R$ defined by $f(x) = 4^x + 4^{|x|}$ is
1) One - one and into
2) Many - one and into
3) One - one and onto
4) Many-one and onto

### S.P.Q. Student Practice Questions

74. If $f: [1,\infty) \to B$ defined by $f(x) = x^2 - 2x + 6$ is a surjection then B =
1) $[1,\infty)$ 2) $[5,\infty)$ 3) $[6,\infty)$ 4) $[2,\infty)$

75. $f: (-\infty,\infty) \to (-\infty,\infty)$ is defined by $f(x) = ax+b, a,b \in R \quad (a \neq 0)$ then f is
1) injective but not surjective
2) surjective but not injective
3) bijective
4) neither injective nor surjective

76. $f: Z \to Z$ defined as $f(x) = [x]$ then $f$ is
1) not a function 2) many-to-one function
3) into function 4) identity function

77. $f: Q \to Q$ is defined by $f(x) = 15x + 7$ is
1) injective only 2) surjective only
3) bijective
4) neither injective nor surjective

78. $f: (0, \infty) \to (0, \infty)$ defined by $f(x) = x^2$ is
1) one-one but not onto
2) onto but not one-one
3) bijective
4) neither one-one nor onto

79. Let $f: (0, \infty) \to [0, 2]$ be defined by $f(x) = \frac{2x}{1+x}$ then $f$ is
1) one-one but not onto
2) onto but not one-one
3) both one-one and onto
4) neither one-one nor onto

### NUMBER OF FUNCTIONS

80. The total number of functions from A to itself is 256, then $n(A) =$
1) 2 2) 3 3) 4 4) 5

81. If $B = \{1,2,3\}$ and $A = \{4,5,6,7,8\}$ then the number of surjections from A to B is
1) 81 2) 64 3) 48 4) 150

82. The number of one-one functions that can be defined from $A = \{1,2,3\}$ to $B = \{a,e,i,o,u\}$ is
1) $3^5$ 2) $5^3$ 3) $^5P_3$ 4) 5!

83. The number of possible many to one functions from $A = \{6, 36\}$ to $B = \{1, 2, 3, 4, 5\}$ is
1) 32 2) 25 3) 5 4) 20

84. If n (A) = 4 and n(B) = 6, then the number of surjections from A to B is
1) $4^6$ 2) $6^4$ 3) 0 4) 24

85. The number of bijections from the set A to itself when A contains 106 elements is
1) 106 2) 106! 3) 106! 4) 2106

86. The number of non-surjective mappings that can be defined from $A = \{1,2,3,4,5\}$ to $B = \{2,4,16,32,64\}$ is
1) 1024 2) 20 3) 505 4) 525

### NUMBER OF FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

87. The number of one-one functions that can be defined from $A = \{a,b,c,d\}$ to $B$ is 5040, then $n(B) =$
1) 7 2) 8 3) 9 4) 10

88. If $A = \{1,2,11,625\}$ then the condition to define a surjection from A to B is
1) $n(A) + n(B) = 20$ 2) $n(A) < n(B)$
3) $n(B) \le 5$ 4) $n(B) = 10$

89. If $A = \{1, 2, 3\}$, $B = \{1, 2\}$ then the number of functions from A to B are
1) 6 2) 8 3) 9 4) 32

### S.P.Q. Student Practice Questions

90. The number of non-bijective mappings that can be defined from $A = \{1,2,3\}$ to itself is
1) 21 2) 27 3) 6 4) 9

91. Let $A = \{1,2,3\}$ and $B = \{a,b,c\}$. If the number of functions from A to B is $m$ is number of one-one functions from A to B, then
1) $l$ is 9 2) $m$ is 9 3) $l$ is 27
4) $m$ is 16

92. The number of constant functions possible from R to B where $B = \{2,4,6,8,\ldots 24\}$ are
1) 24 2) 12 3) 8 4) 6

## COMPOSITE FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

**93.** If $f(x)=(a-x^n)^{\frac{1}{n}}$ then $fof(x)$ is

1) $x$ 2) $a-x$ 3) $x^n$ 4) $x^{\frac{-1}{n}}$

**94.** If $f(x)=\frac{x}{\sqrt{1+x^2}}$ then fofof (x) =

1) $\frac{x}{\sqrt{1+3x^2}}$ 2) $\frac{x}{\sqrt{1-x^2}}$

3) $\frac{2x}{\sqrt{1+2x^2}}$ 4) $\frac{x}{\sqrt{1+x^2}}$

### S.P.Q. Student Practice Questions

**95.** If $f(x) = 2x + 1$ and $g(x) = x^2 + 1$ then go(fof)(2) =

1) 112 2) 122 3) 12 4) 124

**96.** If $f(x)=[x], g(x)=x-[x]$ then which of the following functions is a zero function

1) $(f+g)(x)$ 2) $(fg)(x)$

3) $(f-g)(x)$ 4) $(fog)(x)$

**97.** Let $f(x)=\frac{Kx}{x+1}(x\neq -1)$ then the value of K for which $(fof)(x)=x$ is

1) 1 2) −1 3) 2 4) $\sqrt{2}$

**98.** If $f(x)=\frac{x}{\sqrt{1-x^2}}, g(x)=\frac{x}{\sqrt{1+x^2}}$ then $(fog)(x)=$

1) x 2) $\frac{x}{\sqrt{1+x^2}}$

3) $\sqrt{1+x^2}$ 4) 2x

**99.** If $f(x)=(1-x)^{1/2}$ and g(x) = $ln$(x) then the domain of (gof) (x) is

1) $(-\infty,2)$ 2) (-1,1)

3) $(-\infty,1]$ 4) $(-\infty,1)$

## INVERSE OF A FUNCTION

### C.R.T.Q Class Room Teaching Questions

**100.** If $f(x)=\frac{e^x+e^{-x}}{2}$ then the inverse of f(x) is

1) $\log_e\left(x+\sqrt{x^2+1}\right)$ 2) $\log_e\sqrt{x^2+1}$

3) $\log_e\left(x+\sqrt{x^2-1}\right)$ 4) $\log_e\left(x-\sqrt{x^2-1}\right)$

**101.** If $f:\{1,2,3,.....\}\to\{0,\pm1,\pm2,....\}$ is defined by

$$f(n)=\begin{cases} n/2 & \text{if } n \text{ is even} \\ -\left(\frac{n-1}{2}\right) & \text{if } n \text{ is odd}\end{cases}$$

then $f^{-1}(-100)$ is

1) 100 2) 199 3) 201 4) 200

**102.** If $f(x)=Sin^{-1}\{3-(x-6)^4\}^{1/3}$

then $f^{-1}(x)=$

1) $6+\sqrt[4]{3+\sin^3 x}$ 2) $6+\sqrt[4]{3-\sin^3 x}$

3) $6+\sqrt[4]{3+\sin x}$ 4) $6+\sqrt[4]{3-\sin x}$

### S.P.Q. Student Practice Questions

**103.** $f:R\to R$ is a function defined by $f(x)=10x-7$. If $g=f^{-1}$ then $g(x)=$

1) $\frac{1}{10x-7}$ 2) $\frac{1}{10x+7}$

3) $\frac{x+7}{10}$ 4) $\frac{x-7}{10}$

**104.** $f:[0,\infty)\to[4,\infty)$ is defined by $f(x) = x^2 + 4$ then $f^{-1}(13) =$

1) 3 2) 2 3) 1 4) 4

**105.** If the function $f:R\to R$ be such that $f(x)=x-[x]$, where $[x]$ denotes the greatest integer less than or equal

# EXERCISE-II

## CRTQ & SPQ LEVEL-II

### REAL VALUED FUNCTIONS

**C.R.T.Q** Class Room Teaching Questions

1. Let $g(x)$ be a function defined on $[-1,1]$. If the area of the equilateral triangle with two of its vertices at $(0,0)$ and $(x,g(x))$ is $\sqrt{3}/4$, then the function $g(x)$ is

1) $g(x)=\pm\sqrt{1-x^2}$ 2) $g(x)=\sqrt{1-x^2}$

3) $g(x)=-\sqrt{1+x^2}$ 4) $g(x)=\sqrt{1+x^2}$

2. If $f:R\to R$ is defined by $f(x)=x-[x]-\frac{1}{2}$ for $x\in R$, where $[x]$ is the greatest integer not exceeding $x$, then $\left\{x\in R: f(x)=\frac{1}{2}\right\}=$

1) Z 2) N 3) $\phi$ 4) R

3. Suppose $f:[2,2]\to R$ is defined by

$$f(x)=\begin{cases}-1 & for\ -2\le x\le 0\\ x-1 & for\ 0\le x\le 2\end{cases}$$

then the $\{x\in(-2,2): x\le 0$ and $f(|x|)=x\}=$

1) $\{-1\}$ 2) $\{0\}$ 3) $\{-1/2\}$ 4) $\phi$

4. If $|\sin x+\cos x|=|\sin x|+|\cos x|$, then $x$ lies in

1) 1st quadrant only
2) 1st and 3rd quadrant only
3) 2nd and 4th quadrant only
4) 3rd and 4th quadrant only

**S.P.Q.** Student Practice Questions

5. If $f(x+ay,x-ay)=axy$, then $f(x,y)=$

1) $xy$ 2) $x^2-a^2y^2$

3) $\frac{x^2-y^2}{4}$ 4) $\frac{x^2-y^2}{a^2}$

6. If $f\left(2x+\frac{y}{8},2x-\frac{y}{8}\right)=xy$, then $f(m,n)+f(n,m)=0$

1) Only when m = n
2) Only when $m\neq n$
3) Only when m = - n
4) For all m and n

### EVEN AND ODD FUNCTIONS

**C.R.T.Q** Class Room Teaching Questions

7. Let $f(x)=\begin{cases}4 & x<-1\\ -4x & -1\le x\le 0\end{cases}$ If $f(x)$ is an even function on R then the definition of $f(x)$ on $(0,\infty)$ is

1) $f(x)=\begin{cases}4x & 0<x\le 1\\ 4 & x>1\end{cases}$

2) $f(x)=\begin{cases}4x & 0<x\le 1\\ -4 & x>1\end{cases}$

3) $f(x)=\begin{cases}4 & 0<x\le 1\\ 4x & x>1\end{cases}$

4) $f(x)=\begin{cases}4 & x<-1\\ -4x & -1\le x\le 0\end{cases}$

8. If $f(x)=\begin{cases}x^2\sin\frac{\pi x}{2} & |x|<1\\ x|x| & |x|\ge 1\end{cases}$

then $f(x)$ is

1) an even funciton
2) an odd function
3) a periodic function
4) neither odd nor even

9. $f(x)=\frac{\cos x}{\left[\frac{2x}{\pi}\right]+\frac{1}{2}}$, where $x$ is not an integral multiple of $\pi$ and $[\cdot]$ denotes the greatest integer function is
1) an odd function 2) even function
3) neither odd nor even
4) both even and odd

**S.P.Q. Student Practice Questions**

10. Let $f(x)=|x-2|+|x-3|+|x-4|$ and $g(x)=f(x+1)$. Then $g(x)$ is
1) an even function 2) an odd function
3) neither even nor odd
4) periodic

11. If $f(x)=sgn\{x\}$ (where $\{.\}$ denotes the fractional part of x), is
1) even function
2) odd function
3) neither even nor odd
4) constant function

## PERIODIC FUNCTIONS

**C.R.T.Q Class Room Teaching Questions**

12. Which of the following function is not periodic
1) $\frac{2^x}{2^{[x]}}$ 2) $\sin^{-1}(\{x\})$
3) $\sin^{-1}(\sqrt{\cos x})$ 4) $\sin^{-1}(\cos(x^2))$

13. Let $f(x)=nx+n-[nx+n]+\tan\frac{\pi x}{2}$, where $[x]$ is the greatest integer $\le x$ and $n\in N$. It is
1) a periodic function of period 1
2) a periodic function of period 4
3) not periodic
4) a periodic function of period 2

14. Let $f(x)=x(2-x), 0\le x\le 2$. If the definition of $f$ is extended over the set $R-[0,2]$ by $f(x+2)=f(x)$ then $f$ is a
1) periodic function of period 1
2) non periodic function
3) periodic function of period 2
4) periodic function of period 1/2

15. If $f$ is periodic, g is polynomial function and $f(g(x))$ is periodic and $g(2)=3, g(4)=7$ then $g(6)$ is
1) 13 2) 15 3) 11 4) 21

**S.P.Q. Student Practice Questions**

16. Period of $f(x)=[x]+[2x]+[3x]+...+[nx]-\frac{n(n+1)}{2}x$, where $n\in N$ is
1) n 2) 1 3) $\frac{1}{n}$ 4) 2

17. Let $f(x)=\cos 3x+\sin\sqrt{3}x$. Then $f(x)$ is
1) a periodic function of period $2\pi$
2) a periodic function of period $\sqrt{3}\pi$
3) not a periodic function
4) a periodic function of period $\pi$

18. The function $f(x)=x-[x]+\cos x$, where $[x]$ is the greatest integer less than or equal to $x$ is a
1) periodic function of indeterminate peri
2) periodic function of period $2\pi$
3) nonperiodic function
4) periodic function of period 1

19. If $f(x)=e^{x-[x]+|\cos\pi x|+|\cos 2\pi x|+\ldots+|\cos n\pi x|}$ then the period of $f(x)$ is

1) 1 2) $\frac{1}{n}$ 3) $\frac{2}{n}$

4) no fundamental period

20. If $f(x)$ and $g(x)$ are periodic functions with period 7 and 11, respectively. Then the period of $F(x)=f(x)g\left(\frac{x}{5}\right)-g(x)f\left(\frac{x}{3}\right)$ is

1) 177 2) 222 3) 433 4) 1155

## DOMAIN OF THE FUNCTION

### C.R.T.Q Class Room Teaching Questions

21. The domain of $f(x)=Sin^{-1}\left\{\log_3\left(\frac{x^2}{3}\right)\right\}$ is

1) $(-\infty,3]$ 2) $[3,\infty)$

3) $[-3,-1]\cup[1,3]$ 4) $(-9,-1)\cup(1,9)$

22. The domain of $f(x)=\log_x(9-x^2)$ is

1) $(-3,3)$ 2) $(0,\infty)$

3) $(0,1)\cup(1,\infty)$ 4) $(0,1)\cup(1,3)$

23. The domain of $f(x)=\sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$ is

1) $[0,1]$ 2) $[-1,1]$

3) $(-\infty,\infty)$ 4) $(-1,1)$

24. The domain of $f(x)=\cos(\log x)$ is

1) $(-\infty,\infty)$ 2) $(-1,1)$

3) $(0,\infty)$ 4) $(1,\infty)$

25. The domain of $f(x)=\cos^{-1}\left(\frac{2}{2+\sin x}\right)$ contained in $[0,2\pi]$ is

1) $\left[0,\frac{\pi}{2}\right]$ 2) $\left[\frac{\pi}{2},\pi\right]$

3) $[0,\pi]$ 4) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

26. The domain of the function defined by $f(x)={}^{(7-x)}P_{(x-3)}$ is

1) {3,7} 2) {3,4,5,6,7}

3) {3,4,5} 4) {1,2,3,4}

27. The domain of the function $f(x)=\frac{1}{\sqrt{\{\sin x\}+\{\sin(\pi+x)\}}}$ where $\{\bullet\}$ denotes the fractional part, is

1. $[0,\pi]$ 2. $(2n+1)\frac{\pi}{2}, n\in Z$

3. $(0,\pi)$ 4. $R-\left\{\frac{n\pi}{2}, n\in Z\right\}$

### S.P.Q. Student Practice Questions

28. The domain of $f(x)=\frac{1}{\sqrt{9-x^2}}+\sqrt{x^2-4}$ is

1) $(-4,-2)\cup(2,4)$ 2) $(-3,-2]\cup[2,3)$

3) $(-\infty,-3)\cup(2,\infty)$ 4) $(-\infty,\infty)$

29. Domain of $\frac{1}{\sqrt{[x]^2-[x]-2}}$

1) $R/[-1,3)$ 2) $(-\infty,-3)\cap[3,\infty)$

3) $[2,\infty)$ 4) $(-\infty,3]$

30. Domain of $\sqrt{\log_{10}\left(\frac{3-x}{x}\right)}$

1) $\left(0,\frac{3}{2}\right]$ 2) $\left(-\infty,\frac{3}{2}\right]$

3) (0, 3) 4) R

31. Domain of $f(x)=\log(x-[x])$

1) R 2) Z 3) R − Z 4) $(0,\infty)$

32. The domain of $\sqrt{\log_{\frac{x}{3}}^{e}+1}$ is

1) $\left(0,\frac{3}{e}\right]$ 2) $\left(-\infty,\frac{3}{e}\right]$

3) $\left[\frac{3}{e},\infty\right)$ 4) R

33. The domain of the function $f(x)=\sqrt{\frac{x-2}{x+2}}+\sqrt{\frac{1-x}{1+x}}$ is

1) R 2) [-2,2] 3) [-1,1] 4) $\phi$

34. The domain of $f(x)=\frac{1}{1-2Sinx}$ is

1) $(-\infty,\infty)$

2) $R-\left\{n\pi+(-1)^n\frac{\pi}{6}:n\in Z\right\}$

3) $R-\{n\pi:n\in Z\}$

4) $R-\left\{n\pi+(-1)^n\frac{\pi}{3}:n\in Z\right\}$

35. The domain of $f(x)=\sqrt{2-\log_3(x-1)}$ is

1) (2,12] 2) $(-\infty,10]$

3) (3,12] 4) (1,10]

36. The domain of $f(x)=\frac{\sqrt{2+x}+\sqrt{2-x}}{x}$ is

1) (-2,2) 2) $[-2,0)\cup(0,2]$

3) $[-2,2]$ 4) $(-\infty,2)$

37. Let f and g be two real functions given by
$f=\{(0,1),(2,0),(3,-4),(4,2),(5,1)\}$
$g=\{(1,0),(2,2),(3,-1),(4,4),(5,3)\}$
then the domain of $f.g$ is given by

1) $\{1,2,3,4\}$ 2) $\{1,2,3,4,5\}$

3) $\{0,1,2,3,4\}$ 4) $\{2,3,4,5\}$

38. The domain of the function f defined by $f(x)=\sqrt{4-x}+\frac{1}{\sqrt{x^2-1}}$ is equal to

1) $(-\infty,-1)\cup(1,4]$ 2) $(-\infty,-1]\cup(1,4]$

3) $(-\infty,-1]\cup[1,4]$ 4) $(-\infty,-1)\cup[1,4)$

39. Domain of $\sqrt{\frac{4-x^2}{[x]+2}}$

1) $-\infty,-2)\cup(2,\infty)$ 2) $R-\{-2\}$

3) $[-1,2)$ 4) $(-\infty,-2)\cup[-1,2]$

40. The set of all real numbers sati
$e^{\left(\frac{1}{x}-1\right)}<1$ is

1) $(0,\infty)$ 2) $(-\infty,0)\cup(1,$

3) $(-\infty,\infty)$ 4) (0,1)

## RANGE OF THE FUNCTION

### C.R.T.Q Class Room Teaching Questions

41. If $\alpha\in(0,\frac{\pi}{2})$, then $\sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to $(x\neq 0,-1)$

1) 2 2) 1

3) $2\tan\alpha$ 4) $2Sec^2\alpha$

42. The range of $f(x)=\sin^2x+\cos^4x$ is

1) $\left[\frac{1}{2},1\right]$ 2) $\left[\frac{3}{4},1\right]$ 3) [0,1] 4) $\left[0,\frac{1}{4}\right.$

43. The range of $f(x)=\text{Sin}^{-1}\left[\frac{1}{2}+x^2\right]$ is [ denotes greatest integer function)

1) $\left\{-\frac{\pi}{2},0,\frac{\pi}{2}\right\}$ 2) $\left\{0,\frac{\pi}{2}\right\}$

3) $\left\{\frac{\pi}{2}\right\}$ 4) $\{0,\pi\}$

44. The range of
$x^2+4y^2+9z^2-6yz-3xz-2xy$ is

1) $\phi$ 2) R 3) $[0,\infty)$ 4) $(-\infty,0)$

45. The maximum possible domain and the corresponding range of $f(x)=(-1)^x$ are

1) $D_f=R,R_f=[-1,1]$ 2) $D_f=Z,R_f=\{1,-1\}$

3) $D_f=Z,R_f=[-1,1]$ 4) $D_f=R,R_f=\{-1,1\}$

46. The range of
$f(x)=\sin^{-1}x+\cos^{-1}x+\tan^{-1}x$ is

1) $(0,\pi)$ 2) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$

3) $\left[\frac{-\pi}{4},\frac{\pi}{4}\right]$ 4) $\left[0,\frac{3\pi}{4}\right]$

## S.P.Q. Student Practice Questions

47. The range of $f(x)=8\sqrt{2}\sin\sqrt{\frac{\pi^2}{16}-x^2}$ is

1) [-1,1] 2) [0, 1] 3) [0,8] 4) [0,4]

48. The range of $f(x)=\frac{x^4}{1+x^8}$ is

1) $[0,\infty)$ 2) $\left[0,\frac{1}{2}\right]$

3) [0, 1] 4) $(-\infty,\infty)$

49. The range of $f(x)=\frac{1-\tan x}{1+\tan x}$ is

1) $(-\infty,\infty)$ 2) $(-\infty,0)$

3) $(0,\infty)$ 4) $(-\infty,-1)\cup(-1,\infty)$

50. If $a^2+b^2+c^2=1$ then the range of ab+bc+ca is

1) $[1,\infty)$ 2) $\left[\frac{-1}{2},\infty\right]$

3) $\left(-\frac{1}{2},1\right)$ 4) $\left[\frac{-1}{2},1\right]$

51. Range of $f(x)=\sin^{-1}x+\sec^{-1}x$ is

1. $[-\pi/2,\pi/2]$ 2. $[0,\pi]-\{\pi/2\}$

3. $\{\pi/2\}$ 4. $\{\pi/1\}$

### TYPES OF FUNCTIONS

## C.R.T.Q Class Room Teaching Questions

52. Let A = [-1,1]= B then which of the following function from A to B is bijective function

1) $f(x)=\frac{x}{2}$ 2) $g(x)=|x|$

3) $h(x)=x^2$ 4) $k(x)=\sin\frac{\pi x}{2}$

53. If $f:R\to C$ is defined by $f(x)=e^{2ix}$ for $x\in R$ then, f is (Where C denotes the set of all Complex numbers)

1) One-one 2) Onto

3) One-one and Onto

4) neither one-one nor Onto

54. A function $f:N\to Z$ defined by

$$f(n)=\begin{cases}\frac{n-1}{2}, & \text{when 'n' is odd}\\ \frac{-n}{2}, & \text{when 'n' is even}\end{cases}$$ , is

1) one-one but not onto

2) onto but not one-one 3) one-one onto

4) neither one-one nor onto

55. M is the set of all $2\times2$ real matrices. $f:M\to R$ is defined by f(A)=det A for all A in M then f is

1) one-one but not onto

2) onto but not one-one

3) neither one-one nor onto

4) bijective

56. Let $f:R-\{n\}\to R$ be a function defined by $f(x)=\frac{x-m}{x-n}$ such that $m\neq n$ then

1) $f$ is one one into function

2) $f$ is one one onto function

3) $f$ is many one into funciton

4) $f$ is many one onto funcion

57. $f(x)=\begin{cases}0, if\ x\ is\ rational\\ x, if\ x\ is\ irrational\end{cases}$

$g(x)=\begin{cases}0, if\ x\ is\ irrational\\ x, if\ x\ is\ rational\end{cases}$

Then $f-g$ is

1) one-one and into

2) neither one-one nor onto

3) many one and onto 4) one-one and onto

## S.P.Q. Student Practice Questions

58. If f(x) = $|x-1|+|x-2|+|x-3|$ when 2<x<3 is

1) one one function only

2) an onto function only

3) into function 4) identify function

59. $f:(0,\infty)\to(0,\infty)$ defined by $f(x)=\begin{cases}2^x, x\in(0,1)\\ 5^x, x\in[1,\infty)\end{cases}$ is

1) one-one but not onto
2) onto but not one-one
3) neither one - one nor onto 4) bijective

60. The function $f:(-\infty,\infty)\to(-\infty,\infty)$ defined by $f(x)=e^{|x|}$ is

1) one-one but not onto
2) onto but not one-one
3) neither one - one nor onto
4) bijective

61. $f:C\to C$ is defined as $f(x)=\frac{ax+b}{cx+d}$, $bd\neq 0$ then f is a constant function when

1) $a=c$ 2) $b=d$
3) $ad=bc$ 4) $ab=cd$

62. $f:R\to R, f(x)=x|x|$ is

1) one-one but not onto
2) one-one onto
3) onto but not one-one
4) neitherone-one nor onto

63. Let $f:\{x,y,z\}\to[a,b,c]$ be a one-one func tion and only one of the conditions

(i) $f(x)\neq b$, (ii) $f(y)=b$, (iii) $f(z)\neq a$ is true then the function $f$ is given by the set

1) $\{(x,a),(y,b),(z,c)\}$
2) $\{(x,a),(y,c),(z,b)\}$
3) $\{(x,b),(y,a),(z,c)\}$
4) $\{(x,c),(y,b),(z,a)\}$

64. $f:N\to N$ where $f(x)=x-(-1)^x$ t

$f$ is

1) one-one and into
2) many-one and into
3) one-one and onto
4) many-one and onto

## COMPOSITE FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

65. If $f(x)$ and $g(x)$ are two function with $g(x)=x-\frac{1}{x}$ and $fog(x)=x^3-\frac{1}{x^3}$, then $f(x)=$

1) $x^3+3x$ 2) $x^2-\frac{1}{x^2}$
3) $1+\frac{1}{x^2}$ 4) $3x^2+\frac{3}{x^4}$

66. Let $f(x)=ax+b$ and $g(x)=cx+d$, $a\neq 0, c\neq 0$. Assume $a=1, b=2$. If $(fog)(x)=(gof)(x)$ for all x, what can you say about c and d

1) c and d both arbitrary
2) $c=1, d$ arbitrary
3) c arbitrary, $d=1$
4) $c=1, d=1$

67. If $f(x)=\sin^2 x$ and the composite functions $g\{f(x)\}=|\sin x|$, then the function $g(x)=$

1) $\sqrt{x-1}$ 2) $\sqrt{x}$ 3) $\sqrt{x+1}$ 4) $-\sqrt{x}$

68. If $f:R\to R$ and $g:R\to R$ are given by $f(x)=|x|$ and $g(x)=[x]$ for each $x\in R$, then $\{x\in R: g(f(x))\leq f(g(x))\}=$

1) $Z\cup(-\infty,0)$ 2) $(-\infty,0)$
3) Z 4) R

69. Let $g:R\to R$ be given by $g(x)=3+4x$. If $g^n(x)=gogo....og(x)$, and $g^n(x)=A+Bx$ then A and B are

1) $2^{n+1}-1,2^{n+1}$ 2) $4^n-1,4^n$
3) $3^n,3^n+1$ 4) $5^n-1,5^n$

## S.P.Q. Student Practice Questions

70. If $f(x)=\log_a x$ and $F(x)=a^x$, then $F[f(x)]$ is

1) $f[F(x)]$ 2) $f[F(2x)]$
3) $f|F(2x)|$ 4) $F(x)$

71. Suppose that $g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x$, then $f(x)$ is

1) $1+2x^2$ 2) $2+x^2$ 3) $1+x$ 4) $2+x$

72. Let $g(x)=1+x-[x]$ and

$$f(x)=\begin{cases}-1, & x<0\\ 0, & x=0,\\ 1, & x>0\end{cases}$$ then for all $x, f(g(x))=$

1) x 2)1 3) $f(x)$ 4) $g(x)$

73. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=2x+3$ and $g(x)=x^2+7$, then the values of x such that $g(f(x))=8$ are.

1) 1,2 2) -1,2 3) -1,-2 4) 1,-2

## INVERTIBILITY & INVERSE OF A FUNCTION

## C.R.T.Q Class Room Teaching Questions

74. Let 'f' be an injective function with domain {x, y, z} and range {1,2,3} such that exactly one of the follwowing statements is correct and the remaining are false $f(x)=1$, $f(y)\neq 1$, $f(z)\neq 2$ the value of $f^{-1}(1)$ is

1) x 2) y 3) z 4) x or z

75. If the function $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$ then $f^{-1}(x)$ is

1) $\left(\frac{1}{2}\right)^{x(x-1)}$ 2) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$
3) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ 4) $\frac{1}{2}\left(1\pm\sqrt{1+4\log_2 x}\right)$

## S.P.Q. Student Practice Questions

76. If $f(x)=1+x+x^2+x^3+....\infty$ for $|x|<1$ then $f^{-1}(x)=$

1) $\frac{x-1}{x+1}$ 2) $\frac{x+1}{x}$ 3) $\frac{x}{x-1}$ 4) $\frac{x-1}{x}$

77. Let $f(x)=\sin x+\cos x, g(x)=x^2-1$. Then $g(f(x))$ is invertible for $x\in$

1) $\left[-\frac{\pi}{2},0\right]$ 2) $\left[-\frac{\pi}{2},\pi\right]$
3) $\left[-\frac{\pi}{2},\frac{\pi}{4}\right]$ 4) $\left[0,\frac{\pi}{2}\right]$

## FUNCTIONAL TRANSFORMATIONS

## C.R.T.Q Class Room Teaching Questions

78. If $f(x)$ is a polynomial in $x(>0)$ satisfying the equation $f(x)+f(1/x)=f(x).f(1/x)$ and $f(2)=-7$, then $f(3)=$

1) −26 2) −27 3) −28 4) −29

79. A function $f:R\to R$ satisy the equation $f(x)f(y)-f(xy)=x+y$ for all $x,y\in R$ and f(y)>0, then

1) $f(x)=x+\frac{1}{2}$ 2) $f(x)=\frac{x}{2}+1$
3) $f(x)=\frac{x}{2}-1$ 4) $f(x)=x+1$

80. f is a function defined as $\sum_{k=1}^{n} f(a+k)=16(2^n-1)$ and $f(x+y)=f(x).f(y)$ and f(1) = 2 then integral value of a

1) 3 2) 0 3) 2 4) 1

# EXERCISE-III

## CRTQ & SPQ LEVEL-III

## REAL VALUED FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

1. If for nonzero x, $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-5$, where $a \neq b$, then $f(2)=$

1) $\frac{3(2b+3a)}{2(a^2-b^2)}$ 2) $\frac{3(2b-3a)}{2(a^2-b^2)}$

3) $\frac{3(3a-2b)}{2(a^2-b^2)}$ 4) $\frac{6}{a+b}$

2. If $f(x)=64x^3+\frac{1}{x^3}$ and a,b are the roots of $4x+\frac{1}{x}=3$, then

1) $f(a)=12$ 2) $f(b)=11$

3) $f(a)=f(b)$ 4) $f(a)\neq f(b)$

3. If $f(x)=\frac{9^x}{9^x+3}$ then

$f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+\ldots\ldots+\left(\frac{1995}{1996}\right)=$

1) 997 2) 997.5

3) 998 4) 998.5

4. If [x] stands for the greatest integer function, then $\left[\frac{1}{2}+\frac{1}{1000}\right]+\left[\frac{1}{2}+\frac{2}{1000}\right]+\ldots+\left[\frac{1}{2}+\frac{999}{1000}\right]=$

1) 498 2) 499

3) 500 4) 501

5. Let $f:[-3,3]\to R$ where $f(x)=x^3+\sin x+\left[\frac{x^2+2}{a}\right]$ be an odd function then the value of a is (where [.] represents greatest integer function)

1) less than 11 2)11

3) greater than 11 4)12

6. If $f(x+y)=f(x).f(y)$ for all re $x, y$ and $f(0)\neq 0$, then the funct $g(x)=\frac{f(x)}{1+\{f(x)\}^2}$ is

1) even function 2) odd function

3) odd if $f(x)>0$ 4) neither even nor

### S.P.Q. Student Practice Questions

7. Let $f(x)=\frac{2x(\sin x+\tan x)}{2\left[\frac{x+21\pi}{\pi}\right]-41}$, $x\neq n\pi$, then $f$ is (where [.] represents great integer function)

1) an odd function

2) an even function

3) both odd and even

4) neither odd nor even

8. If the real valued function $f(x)=\frac{a^x-1}{x^n(a^x+1)}$ is even, then n =

1) 2 2) $\frac{2}{3}$ 3) $\frac{1}{4}$ 4) 3

9. If f is an even function defined on the interval $(-5, 5)$ then find the total number of real values of x satisfying the equations $f(x)=f\left(\frac{x+1}{x+2}\right)$ are

1) 1 2) 2 3) 4 4) 8

10. Let the function $f(x)=3x^2-4x+8\log(1+|x|)$ be defined on the interval [0, 1]. The even extension of $f(x)$ of the interval [–1, 1] is

1) $3x^2-4|x|+8\log(1+|x|)$

2) $3x^2-4x+8\log(1+|x|)$

3) $3x^2+4x-8\log(1+|x|)$

4) $3x^2-4x-8\log(1+|x|)$

11. If $f:R\to R$ is an invertible function such that $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y=-x$, then
1) $f(x)$ is odd
2) $f(x)$ and $f^{-1}(x)$ may not be symmetric about the line $y=x$
3) $f(x)$ may not be odd
4) $f^{-1}(x)$ may be odd

12. If $f:R\to R$ is a function satisfying the property $f(2x+3)+f(2x+7)=2,\forall x\in R$, then the period of $f(x)$ is
1) 2 2) 4
3) 8 4) 12

## PERIODIC FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

13. If the period of the function $f(x)=\sin\left(\sqrt{[n]}x\right)$ where [n] denotes the greatest integer less than or equal to n is $2\pi$, then
1) $1\le n<2$ 2) $1<n<2$
3) $1\le n\le 2$ 4) $0\le n\le 1$

14. The period of the function $f(x)=x[x]$ is
1) 1 2) 2
3) Non periodic 4) 4

15. The period of the function
$f(x)=\dfrac{|\sin x|-|\cos x|}{|\sin x+\cos x|}$ is
1) $\dfrac{\pi}{2}$ 2) $2\pi$
3) $\pi$ 4) $4\pi$

16. The period of the function
$f(x)=\cos\left(\dfrac{\pi x}{n!}\right)-\sin\left(\dfrac{\pi x}{(n+1)!}\right)$ is
1) $2(n+1)!$ 2) $2(n!)$
3) $(n+1)$ 4) $n!$

### S.P.Q. Student Practice Questions

17. The period of the function
$f(x)=|\sin 4x|+|\cos 4x|$ is
1) $\dfrac{\pi}{2}$ 2) $\dfrac{\pi}{8}$ 3) $\dfrac{\pi}{4}$ 4) $\dfrac{\pi}{3}$

18. If f (x) is an odd periodic function with period 2, then f (4) =
1) –4 2) 4 3) 2 4) 0

19. If f is periodic g is polynomial function and $f(g(x))$ is periodic and $g(2)=3, g(4)=7$ then $g(6)$ is
1)13 2) 15 3) 11 4)16

## DOMAIN OF THE FUNCTIONS

### C.R.T.Q Class Room Teaching Questions

20. The domain of the function
$f(x)=\sqrt{\log_{(x^2-1)}^{x}}$ is
1) $\left(\sqrt{2},\infty\right)$ 2) $(0,\infty)$
3) $(1,\infty)$ 4) R

21. The domain of $f(x)=\log_2\log_3\log_4 x$ is
1) $[4,\infty)$ 2) $(4,\infty)$
3) $(-\infty,4)$ 4) $[1,\infty)$

22. The domain of
$f(x)=\log_{10}\dfrac{x-5}{x^2-10x+24}-\sqrt[3]{x+5}$ is
1) $(4,5)$ 2) $(6,\infty)$
3) $(4,5)\cup(6,\infty)$ 4) $(4,5]\cup(6,\infty)$

23. The domain of $f(x)=\sqrt{e^{\sin^{-1}\left(\log_{16}^{x^2}\right)}}$ is
1) $\left[\dfrac{1}{4},4\right]$ 2) $\left[-4,\dfrac{-1}{4}\right]\cup\left[\dfrac{1}{4},4\right]$
3) $\left[-4,\dfrac{-1}{4}\right]$ 4) $\left[4,\dfrac{1}{4}\right]$

24. If $f(x)=\sqrt{3|x|-x-2}$ and $g(x)=\sin x$, then domain of $(f/g)(x)$ is

1) $\{2n\pi+\pi/2\}, n\in z$

2) $\left(2n\pi+\frac{7\pi}{6}, 2n\pi+\frac{11\pi}{6}\right), n\in Z$

3) $\left\{2n\pi+\frac{7\pi}{6}\right\}, n\in Z$

4) $\left(2n\pi+\frac{7\pi}{6}, 2n\pi+\frac{11\pi}{6}\right)\cup\left(2m\pi+\frac{\pi}{2}\right); n,m\in Z$

25. Let $f(x)=\frac{1}{2}-\tan\left(\frac{\pi x}{2}\right), -1<x<1$ and $g(x)=\sqrt{3+4x-4x^2}$, the domain of $(f+g)$ is

1) $\left[\frac{-1}{2},\frac{3}{2}\right]$ 2) $\left[\frac{1}{2},-1\right]$

3) $\left[\frac{-1}{2},1\right)$ 4) R

## S.P.Q. Student Practice Questions

26. The function

$f(x)=\cot^{-1}\left(\sqrt{(x+3)x}\right)+\cos^{-1}\left(\sqrt{x^2+3x+1}\right)$ is defined on the set S, then S is equal to

1) $\{-3,0\}$ 2) $[-3,0]$

3) $[0,3]$ 4) $(-3,0)$

27. If $b^2-4ac=0, a>0$, then the domain of $y=\log[ax^3+(a+b)x^2+(b+c)x+c]$ is

1) $R-\left\{-\frac{b}{2a}\right\}$

2) $R-\left\{\left\{-\frac{b}{2a}\right\}\cup\{x: x\geq -1\}\right\}$

3) $R-\left\{-\frac{b}{2a}\right\}\cup\{x>-1\}$

4) $R-\left\{\frac{b}{2a}\right\}$

28. If $f(x)$ is defined on $(0,1]$, then domain of $f(\sin x)$ is

1) $(2n\pi,(2n+1)\pi), n\in Z$

2) $\left((2n+1)\frac{\pi}{2},(2n+3)\frac{\pi}{2}\right); n\in Z$

3) $((n-1)\pi,(n+1)\pi), n\in Z$

4) $(n\pi,(2n+1)\pi) n\in Z$

29. Domain of $\sqrt{x(1-e^x)(x+2)(x-3)^2}$

1) $[-2,3]$

2) $(-2,0]$

3) $(-\infty,-2]\cup\{0,3\}$

4) $(-\infty,-2)\cup[0,3]$

30. The domain of $f(x)=\sqrt{x-\sqrt{1-x^2}}$ is

1) $\left[-1,\frac{-1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},1\right]$ 2) $\left(-\infty,\frac{-1}{2}\right]\cup\left[\frac{1}{\sqrt{2}},\infty\right)$

3) $[-1,1]$ 4) $\left[\frac{1}{\sqrt{2}},1\right]$

## RANGE OF THE FUNCTION

## C.R.T.Q Class Room Teaching Questions

31. If $x\in R$ and $P=\frac{x^2}{x^4-2x^2+4}$, then P lies interval

1) $\left[0,\frac{1}{2}\right]$ 2) $\left[\frac{3}{4},\frac{4}{5}\right]$

3) $\left[0,\frac{1}{3}\right]$ 4) $\left[0,\frac{1}{4}\right]$

32. The range of $f(x)=\log_e(3x^2-4x+5)$ is

1) $\left(-\infty,\log_e\frac{11}{3}\right]$ 2) $\left[\log_e\frac{11}{3},\infty\right)$

3) $\left(-\log_e\frac{11}{3},\log_e\frac{11}{3}\right)$ 4) $[1,\infty)$

33. The image of the interval $[1,3]$ under the mapping $f:R\to R$, given by $f(x)=2x^3-24x+107$ is

1) $[0,89]$ 2) $[75,89]$
3) $[0,75]$ 4) $[70,80]$

34. The range of $f(x)=\dfrac{e^x-e^{|x|}}{e^x+e^{|x|}}$ is

1) $[-1,0]$ 2) $(-1,0]$
3) $(-1,0)$ 4) $[-1,0)$

35. If $f(x)=\underset{m\to\infty}{Lt}\ \underset{n\to\infty}{Lt}\cos^{2m} n!\pi x$, then range of $f(x)$ is

1) $[0,1]$ 2) $\{0,1\}$
3) $(0,1)$ 4) $\{0\}$

36. The range of $f(x)=x^2+\dfrac{1}{x^2+1}$ is

1) $[1,\infty)$ 2) $[2,\infty)$
3) $[\frac{3}{2},\infty)$ 4) R

## S.P.Q. Student Practice Questions

37. The range of $y=\sin^{-1}\left(\dfrac{x^2}{1+x^2}\right)$ is

1) $\left(0,\dfrac{\pi}{2}\right)$ 2) $\left[0,\dfrac{\pi}{2}\right)$
3) $\left[0,\dfrac{\pi}{2}\right]$ 4) $[0,1]$

38. If $f:R\to R$ is defined by $f(x)=[2x]-2[x]$ for $x\in R$, where $[x]$ is the greatest integer not exceeding $x$, then the range of $f$ is

1) $\{x\in R:0\le x\le 1\}$ 2) $\{0,1\}$
3) $\{x\in R:x>0\}$ 4) $\{x\in R:x<0\}$

39. If $f(x)=ax^7+bx^3+cx-5\,(a,b,c$ are real constants) and $f(-7)=7$, then the range of $f(7)+17\cos x$ is

1) $[-34,0]$ 2) $[0,34]$
3) $[-34,34]$ 4) $\{-34,34\}$

40. The range of $\sin^{-1}\left[x^2+\dfrac{1}{2}\right]+\cos^{-1}\left[x^2-\dfrac{1}{2}\right]$, where $[.]$ denotes the greatest integer function, is

1) $\left\{\dfrac{\pi}{2},\pi\right\}$ 2) $\{\pi\}$
3) $\left\{\dfrac{\pi}{2}\right\}$ 4) $\left[\dfrac{\pi}{2},\pi\right]$

41. The range of $f(x)=\left[\sin x+\left[\cos x+\left[\tan x+[\sec x]\right]\right]\right]$, $x\in(0,\pi/4)$, where $[.]$ denotes the greatest integer function $\le x$, is

1) $\{0,1\}$ 2) $\{-1,0,1\}$ 3) $\{1\}$ 4) $\{0\}$

## TYPES OF FUNCTIONS

## C.R.T.Q Class Room Teaching Questions

42. The function $f:R\to B$ is defined by $f(x)=[x]+[-x]$ where $[\ .]$ is G.I.F is surjective then B =

1) R 2) $[0,1]$ 3) $[-1,0]$ 4) $\{-1,0\}$

43. If $A=\left\{x:\dfrac{-2}{5}\le x\le\dfrac{\pi-2}{5}\right\}$, $B=\{y:-1\le y\le 1\}$ and $f(x)=\cos(5x+2)$ then the mapping $f:A\to B$ is

1) One - one but not onto
2) Onto but not one - one
3) Both one - one and onto
4) Neither one - one nor onto

44. The functions $f:R\to R$ is defined by $f(x)=|(x-1)(x-2)|$ is

1) One one onto 2) One one into
3) Many one onto 4) Many one into

## S.P.Q. Student Practice Questions

45. Let S be the set of all triangles and $R^+$ be the set of positive real numbers. Then the function $f: S \to R^+, f(\Delta) = area\ of\ \Delta$, where $\Delta \in S$ is

1) injective but not surjective
2) surjective but not injective
3) injective as well as surjective
4) neither injective nor surjective

46. Let $f: R \to R$ is defined by $f(x) = 2x^3 + 2x^2 + 300x + 5\sin x$, then $f$ is

1) one-one onto 2) one-one into
3) many one onto 4) many one into

## COMPOSITE FUNCTION

## C.R.T.Q Class Room Teaching Questions

47. If $f(x) = \begin{cases} |x|, x \le 1 \\ 2-x, x > 1 \end{cases}$ then $f(f(x)) =$

1) $\begin{cases} 2-|x|, x < -1 \\ |x|, -1 \le x \le 1 \\ |2-x|, x > 1 \end{cases}$ 2) $\begin{cases} |x|, x < -1 \\ 2-|x|, -1 \le x \le 1 \\ |2-x|, x > 1 \end{cases}$

3) $\begin{cases} |2-x|, x < -1 \\ |x|, -1 \le x \le 1 \\ 2-|x|, x > 1 \end{cases}$ 4) Cannot say

48. If $f(x) = \sin^2 x + \sin^2(x+\pi/3) + \cos x \cos(x+\pi/3)$ and $g(5/4) = 1$ then $(gof)(x) =$

1) 1 2) 0
3) sinx 4) $-\cos x$

## S.P.Q. Student Practice Questions

49. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}gof(x) = 2x^2 - 5x + 2$, then which is $f(x)$

1) $2x-3$ 2) $-2x-3$
3) $x-3$ 4) $x+3$

## INVERSE FUNCTION

## C.R.T.Q Class Room Teaching Questions

50. If $f:(4,8) \to (5,9)$ is a function defi... by $f(x) = x + \left[\frac{x}{4}\right]$ where $[.]$ is G.I.F ...
$f^{-1}(x) =$

1) $1-x$ 2) $x-1$ 3) $x-3$ 4) ...

51. If the function $f:[2,\infty) \to [-1,...$ defined by $f(x) = x^2 - 4x + 3$ ...
$f^{-1}(x) =$

1) $2-\sqrt{x+1}$ 2) $2+\sqrt{x+1}$
3) $\frac{2-\sqrt{x+1}}{5}$ 4) $\frac{2+\sqrt{x+1}}{5}$

52. Let $f(x) = x^2 - x + 1, x \ge 1/2$, then ... solution of the equation $f^{-1}(x) = f(x)$

1) $x = 1$ 2) $x = 2$
3) $x = \frac{1}{2}$ 4) $x = 0$

## S.P.Q. Student Practice Questions

53. Let $f: R \to R$ be given by $f(x) = (x+1)^2 - 1, x \ge -1$ Then $f^{-1}(x) =$

1) $-1 + \sqrt{x+1}$
2) $-1 - \sqrt{x+1}$
3) does not exist because f is not one-one
4) does not exist because f is not onto

54. The inverse of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ is given by

1) $\log_e\left(\frac{x-1}{x+1}\right)^{-2}$ 2) $\log_e\left(\frac{x-2}{x-1}\right)^{1/2}$
3) $\log_e\left(\frac{x}{2-x}\right)^{1/2}$ 4) $\log_e\left(\frac{x-1}{3-x}\right)^{1/2}$

Reason : The composite function of two odd functions is an odd function

1) a 2) b 3) c 4) d

5. Assertion : If $\{x\}$ and $[x]$ represent fractional part and integral part of x then $[x]+\sum_{b=1}^{1000}\frac{\{x+b\}}{1000}=x$.

Reason: $\{x\}=x-[x]$ and $[x+I]=[x]+I$, where I is integer

1) a 2) b 3) c 4) d

## STATEMENTS

1) only I is true
2) only II is true
3) both I and II are true
4) neither I nor II true

6. Statement I: The product of two odd functions is an even function.
Statement II: A constant function is always a bijection.

7. Statement I : Every strictly montonic function is one one
Statement II : The function $f:R^+\to R$ defined by $f(x)=5+x^2$ is one one.

8. Let $f(x)=\sec^{-1}\left[1+\cos^2 x\right]$ where [.] denotes the greatest integer function
I : Domain of f(x) is R
II : Range of f(x) is $\{\sec^{-1}1, \sec^{-1}2\}$

9. Statement I : The range of the function $f(x)=\frac{\sin([x]\pi)}{x^2+x+1}$ is {0}
Statement II : The range of the function $f(x)=\frac{x-[x]}{1+x-[x]}$ is $\left[0,\frac{1}{2}\right)$.

10. If Q denotes the set of all rational numbers and $f\left(\frac{p}{q}\right)=\sqrt{p^2-q^2}$ for any $\frac{p}{q}\in Q$, then observe the following statements

I. $f\left(\frac{p}{q}\right)$ is real for each $\frac{p}{q}\in Q$

II. $f\left(\frac{p}{q}\right)$ is a complex number for each $\frac{p}{q}\in Q$

Which of the following is correct ?
1) Both I and II are true
2) I is true, II is false
3) I is false, II is true
4) Both I and II are false

## MATCHINGS

11. If $f:R\to R$ is defined by

$$f(x)=\begin{cases}x+4 & \text{for } x<-4\\ 3x+2 & \text{for } -4\le x<4\\ x-4 & \text{for } x\ge 4\end{cases}$$

then the correct matching of List I from List II is

| List - I | List - II |
|---|---|
| A) $f(-5)+f(-4)$ | i) 14 |
| B) $f(\lvert f(-8)\rvert)$ | ii) 4 |
| C) $f(f(-7)+f(3))$ | iii) –11 |
| D) $f(f(f(f(0))))+1$ | iv) –1 |
| | v) 1 |
| | vi) 0 |

1) A - iii; B - vi; C - ii; D - v
2) A - iii; B - iv; C - ii; D - v
3) A - iv; B - iii; C - ii; D - i
4) A - iii; B - vi; C - v; D - ii

12. Match the following

| List - I | List - II |
|---|---|
| I. $f:R\to R$ defined by $f(x)=ax+b$ is | a) injection but not surjection |
| II. $f:R\to R$ defined by $f(x)=[x]$ is | b) surjection but not injection |
| III. $f:R\to[0,\infty)$ defined by $f(x)=\lvert x\rvert$ is | c) bijection |
| IV. $f:N\to N$ defined by $f(x)=x^3$ is | d) neither injection nor surjection |

1) c, d, a, b
2) c, d, b, a
3) d, c, b, a
4) d, c, a, b

13. If $f, g, h, \phi$ are functions from $R \to R$ then

| List - I | List - II |
|---|---|
| I. $f(x)=\sin^3 x+2\sin x+\sin 3x$ | a) even function |
| II. $g(x)=\cos 3x+\sin^4 x+3\cos x$ | b) odd function |
| III. $h(x)=\sin 3x+\cos 2x+\sin^2 x$ | c) both even and odd function |
| IV. $\phi(x)=\frac{\tan[x]\pi}{1+\sin[x]\pi}$ | d) neither even nor odd function |

1) I-b; II-a; III-c; IV-d
2) I-a;II-b;III-d;IV-c
3) I-a;II-b;III-c;IV-d
4) I-b;II-a;III-d;IV-c

14. Let R denote the set of all real numbers and $R^+$ denote the set of all positive real numbers. For the subsets A and B of R define $f: A \to B$ by $f(x)=x^2$ for $x \in A$. Observe the two lists given below.

| List - I | List - II |
|---|---|
| I. f is one-one and onto if | a) $A=R^+, B=R$ |
| II. f is one-one but not onto if | b) $A=B=R$ |
| III. f is onto but not one-one if | c) $A=R, B=R^+$ |
| IV. f is neither one-one nor onto if | d) $A=B=R^+$ |

The correct matching of List I to List II is
1) I-a; II-b; III-c; IV-d
2) I-d;II-b;III-a;IV-c
3) I-d;II-a;III-c;IV-b
4) I-d;II-b;III-c;IV-a

15. Observer the following lists

| List - I Functions | List - II Domain |
|---|---|
| I. $f(x)=\sqrt{\sin^{-1}(\log_2 x)}$ | a) $\{-1,1\}$ |
| II. $g(x)=\sqrt{\cos(\sin x)}$ | b) $\{1\}$ |
| III. $h(x)=\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ | c) $\{-1\}$ |
| IV. $f(x)=g(x)+h(x)$ | d) $[1, 2]$ |
| | e) R |

The correct matching of List I to List II is
1) I-b; II-a; III-c; IV-d 2) I-a;II-b;III-d;IV-c
3) I-d;II-e;III-a;IV-a 4) I-b;II-a;III-d;IV-c

## KEY

| | | | | |
|---|---|---|---|---|
| 01) 3 | 02) 4 | 03) 1 | 04) 1 | 05) 1 |
| 06) 1 | 07) 3 | 08) 3 | 09) 3 | 10) 3 |
| 11) 1 | 12) 2 | 13) 4 | 14) 3 | 15) 3 |

## Hints & Solutions

1. $\cos^{-1} x$ is defined for $-1 \le x \le 1$
2. $f(-x)=-f(x)$ 3. Denominator $\neq 0$
4. $f(-x)=-f(x)$ 7. $f'(x)>0$, increasing on $R^+$
9. $\sin n\pi = 0, n \in z$
10. Choose numerical valumes, verify
11. Substitute values
15. $\sqrt{f(x)}$ is defined for f(x) has to be define and $f(x) \ge 0$

## INTEGER & NUMERICAL ANSWER TYPE QUESTIONS

1. Let $f$ be a real-valued invertible function such that $f\left(\frac{2x-3}{x-2}\right)=5x-2, x \neq 2$. Then the value of $f^{-1}(13)$ is____

2. The number of values of $x$ for which $\left|\left|x^2-x+4\right|-2\right|-3\Big| = x^2+x-12$ is____

3. Let $f(x)=3x^2-7x+c$, where $c$ is a variable coefficient and $x>\frac{7}{6}$. Then the value of $[c]$ such that $f(x)$ touches $f^{-1}(x)$ is (where [.] represents greatst integer function)____

4. The number of integral values of $x$ for which $\frac{\left(2^{\frac{\pi}{\tan^{-1}x}}-4\right)(x-4)(x-10)}{x!-(x-1)!}<0$ is____

5. If $f(x)=\begin{cases} x\cos x+\log_e\left(\frac{1+x}{1-x}\right); & x\neq 0 \\ a; & x=0 \end{cases}$ is odd, then $a=$____

6. The number of integers in the range of the function $f(x)=\left|\frac{4\left(\sqrt{\cos x}-\sqrt{\sin x}\right)\left(\sqrt{\cos x}+\sqrt{\sin x}\right)}{(\cos x+\sin x)}\right|$ is____

7. Let $a>2$ be a constant. The three are just 18 positive integers satisfying the inequality $(x-a)(x-2a)\left(x-a^2\right)<0$, then the value of $a$ is____

8. The number of integers in the domain of function, satisfying $f(x)+f\left(x^{-1}\right)=\frac{x^3+1}{x}$, is____

9. If a polynomial function $f(x)$ satisfies $f(f(f(x)))=8x+21$, where p and q are real numbers, then p+q is equal to____

10. If $f(x)$ is an odd function, $f(1)=3$, and $f(x+2)=f(x)+f(2)$, then the value of $f(3)$ is____

11. Let $f:R\to R$ be a continuous function satisfying $f(x)+f(-x)=0;\forall x$ If $f(-3)=2$ and $f(5)=4$ in $[-5,5]$, the minimum number of roots of the equa $f(x)=0$ is____

12. The number of integral values of $x$ for w the function $\sqrt{\sin x+\cos x}+\sqrt{7x-x^2-}$ defined is____

13. Suppose that $f$ is an even, periodic func with period 2, and that $f(x)=x$ for al in the interval $[0,1]$. The value $[10f(3,14)]$ is (where [.] represents greatest integer function)____

14. If $f(x)=\sqrt{4-x^2}+\sqrt{x^2-1}$, then t maximum value of $(f(x))^2$ is____

15. The function $f(x)=\frac{x+1}{x^3+1}$ can be writt as the sum of an even function $g(x)$ and a odd function $h(x)$. Then the value of $|g(0$ is____

16. If $T$ is the period of the functio $f(x)=[8x+7]+[\tan 2\pi x+\cot 2\pi x]-8x$ (where [.] denotes the greatest intege functional, then the value of $1/T$ is____

17. If $a,b$, and $c$ are non-zero rational numbers, then the sum of all the possible values of $\frac{|a|}{a}+\frac{|b|}{b}+\frac{|c|}{c}$ is____

18. An even polynomial function $f(x)$ satisfies a relation $f(2x)\left(1-f\left(\frac{1}{2x}\right)\right)+f(16x^2y)$ $=f(-2)-f(4xy)\forall x,y\in R-\{0\}$ and $f(4)=-255$, $f(0)=1$. Then the value of $\left|\frac{f(2)+1}{2}\right|$ is____

19. If $f(x)=\sin^2 x+\sin^2\left(x+\frac{\pi}{3}\right)+\cos x\cos\left(x+\frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right)=1$, then $(gof)(x)$ is____

20. Let $E=\{1,2,3,4\}$ and $F=\{1,2\}$. If $N$ is the number of onto functions from E to F, then the value of N/2 is____

21. The function $f$ is continuous and has the property $f(f(x))=1-x$. Then the value of $f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)$ is____

22. The number of integral values of $x$ satisfying the inequality $\left(\frac{3}{4}\right)^{6x+10-x^2}<\frac{27}{64}$ is____

23. A function $f$ from integers to integers is defined as $f(x)=\begin{cases}x+3, & n\in \text{odd}\\ x/2, & n\in \text{even}\end{cases}$ suppose $K\in$ odd and $f(f(f(K)))=27$. Then the sum of digits of $K$ is____

24. If $\theta$ is the fundamental period of the function $f(x)=\sin^{99}x+\sin^{99}\left(x+\frac{2\pi}{3}\right)+\sin^{99}\left(x+\frac{4\pi}{3}\right)$, then the complex number $z=|z|(\cos\theta+i\sin\theta)$ lies in the quadrant number____

25. If $x=\frac{4}{9}$ satisfies the equation $\log_a(x^2-x+2)>\log_a(-x^2+2x+3)$ then the sum of all possible distinct values of $[x]$ is (where [ . ] represents the greatest integer function)____

26. If $4^x-2^{x+2}+5+\|b-1|-3|=|\sin y|, x,y,b\in R$, then the possible value of $b$ is____

27. If $f:N\to N$, and $x_2>x_1=f(x_2)>f(x_1)\forall x_1$, $x_2\in N$ and $f(f(x))=3n\forall x\in N$, then $f(2)=$____

28. The number of integral values of a for which $f(x)=\log\left(\log_{1/3}\left(\log_7(\sin x+a)\right)\right)$ is defined for every real value of $x$ is____

29. Let $f(x)=\sin^{23}x-\cos^{22}x$ and $g(x)=1+\frac{1}{2}\tan^{-1}(x)$. then the number of values of $x$ in the interval $[-10\pi,8\pi]$ satisfying the equation $f(x)=\text{sgn}(g(x))$ is____

30. Suppose that $f(x)$ is a function of the form $f(x)=\frac{ax^8+bx^6+cx^4+dx^2+15x+1}{x}$, $(x\neq 0)$. If $f(5)=2$, then the value of $|f(-5)|/4$ is____

31. The number of linear functions which map from $[-1,1]$ onto $[0,2]$ is____

32. If $f(x)=\frac{\cos^2 x+\sin^4 x}{\sin^2 x+\cos^4 x}$ for $x\in R$ then $f(2002)=$____

33. If $e^{f(x)}=\frac{10+x}{10-x}, x\in(-10,10)$ and $f(x)=kf\left(\frac{200x}{100+x^2}\right)$, then $k=$____

34. For any integer $n\geq 1$, the number of positive divisors of $n$ is denoted by $d(n)$. Then for a prime $p, d(d(d(p^7)))=$____

35. The natural number a for which $\sum_{k=1}^{n}f(a+k)=16(2^n-1)$ whre the function $f$ satisfies the relation $f(x+y)=f(x).f(y)$ for all natural numbers $x,y$ and further $f(1)=2$ is____