Sri Chaitanya IIT Academy.,India.

A.P T.S KARNATAKA TAMILNADU MAHARASTRA DELHI RANCHI

A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_STERLING BT **JEE-MAIN** **Date: 09-08-2025**

Time: 09:00AM to 12:00PM **RPTM-01** **Max. Marks: 300**

KEY SHEET

MATHEMATICS

1)	**3**	2	**4**	3)	**3**	4)	**4**	5)	**2**
6)	**4**	7)	**3**	8)	**1**	9)	**4**	10)	**2**
11)	**4**	12)	**4**	13)	**4**	14)	**3**	15)	**2**
16)	**1**	17)	**1**	18)	**3**	19)	**4**	20)	**2**
21)	**0**	22)	**1**	23)	**3**	24)	**5**	25)	**50**

PHYSICS

26	**4**	27	**2**	28	**3**	29	**1**	30	**2**
31	**3**	32	**2**	33	**4**	34	**4**	35	**2**
36	**2**	37	**4**	38	**3**	39	**1**	40	**1**
41	**1**	42	**3**	43	**4**	44	**1**	45	**4**
46	**5**	47	**270**	48	**1400**	49	**750**	50	**2**

CHEMISTRY

51	**4**	52	**2**	53	**2**	54	**1**	55	**4**
56	**1**	57	**4**	58	**3**	59	**3**	60	**4**
61	**3**	62	**2**	63	**2**	64	**4**	65	**1**
66	**2**	67	**2**	68	**1**	69	**1**	70	**3**
71	**9**	72	**5**	73	**16**	74	**2**	75	**4**

SOLUTIONS
MATHEMATICS

1. If $x \in I$ then $[x]=x$ and $\{x+r\}=0$ for any $r \in I$. Thus $f(x)=x$. If $x \in R \sim I$ then $[x]=$ integral part of x and $\{x+r\}=\{x\}$ for any $r=1,2,....1000$. Thus $f(x)=[x]+\{x\}=x$

2. $h(x)=f^{-1}(x)$ But $y=f(x)=(x+1)^2$

$\Rightarrow\ x=\sqrt{y}-1$ so $f^{-1}(x)=\sqrt{x}-1$. $g(x)=h(x+3)=\sqrt{x+3}-1$

3. $f(x)=0$ if $x \in I$ and for $x \in R-I$

$2(x-[x])<1+x-[x]$. Thus $f(x)<1/2$

4. We have the $\left(2\tan^{-1}(1/5)\right)=\dfrac{2\times\frac{1}{5}}{1-\frac{1}{25}}=\dfrac{2}{5}\times\dfrac{25}{24}=\dfrac{5}{12}$

So, that the given equation can be written is

$$17x^2-17x\tan\left(\pi/4-2\tan^{-1}(1/5)\right)-10=0 \Rightarrow 17x^2-17x\frac{1-(5/12)}{1+(5/12)}-10=0$$

$\Rightarrow 17x^2-7x-10=0\ \ \Rightarrow (x-1)(17x+10)=0$

$\therefore\ x=1$ is a root of the given equation.

5. We have from the given equation

$$\tan^{-1}\frac{(a+b)x}{x^2-ab}=\frac{\pi}{2}-\tan^{-1}\frac{(c+d)x}{x^2-cd} \Rightarrow \tan^{-1}\frac{(a+b)x}{x^2-ab}=\cot^{-1}\frac{(c+d)x}{x^2-cd}=\tan^{-1}\frac{x^2-cd}{(c+d)x}$$

$\Rightarrow (x^2-ab)(x^2-cd)=(a+b)(c+d)x^2 \Rightarrow x^4-x^2\sum ab+abcd=0$

6. Put $x=\cos y$ then $\cos^{-1}x=y$

$\Rightarrow\ 2\sin^{-1}\sqrt{\dfrac{1-\cos y}{2}}=y$

$2\cos^{-1}\sqrt{\dfrac{1+\cos y}{2}}=y$

7. We have $\sqrt{\dfrac{2-\sqrt{3}}{4}}=\sqrt{\dfrac{4-2\sqrt{3}}{8}}=\dfrac{\sqrt{3}-1}{2\sqrt{2}} \Rightarrow \sin^{-1}\sqrt{\dfrac{2-\sqrt{3}}{4}}=\sin^{-1}\dfrac{\sqrt{3}-1}{2\sqrt{2}}=\dfrac{\pi}{12}$

Also $\cos^{-1}\dfrac{\sqrt{12}}{4}=\cos^{-1}\dfrac{\sqrt{3}}{2}=\dfrac{\pi}{6}$ And $\sec^{-1}\sqrt{2}=\pi/4$

So the given expression is equal to $\sin^{-1}\cot\left(\dfrac{\pi}{12}+\dfrac{\pi}{6}+\dfrac{\pi}{4}\right)=\sin^{-1}\cot\left(\dfrac{\pi}{2}\right)=0$

8. The given equation can be written as

$$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x \Rightarrow \tan^{-1}\frac{x-1+x+1}{1-(x-1)(x+1)}=\tan^{-1}\frac{3x-x}{1+3x^2}$$

$$\Rightarrow \frac{2x}{2-x^2}=\frac{2x}{1+3x^2} \Rightarrow x+3x^3=2x-x^3 \Rightarrow 4x^3-x=0 \Rightarrow x(4x^2-1)=0 \Rightarrow x=0, x=\pm\frac{1}{2}$$

None of which satisfies $1<x<\sqrt{2}$

9. $f(x)=\begin{cases} \left|[x-5]\right| & for\quad x<5 \\ \left[|x-5|\right] & for\quad x\ge 5 \end{cases}$

$fof\left(\frac{-7}{2}\right)=f\left(f(-3,5)\right)=f(9)=4 \quad = fof\left(\frac{9}{2}\right)=f\left(f\left(\frac{9}{2}\right)\right)=f(1)=4$

10. We have been given that

$f\left(g(x)\right)=x+3-\sqrt{x} \Rightarrow f\left(\sqrt{x}+1\right)=x+3-\sqrt{x}$ Put $\sqrt{x}+1=t$

$\sqrt{x}=t-1 \Rightarrow x=(t-1)^2 \Rightarrow f(t)=(t-1)^2+3-(t-1)=f(1)=3$

11. $\sin\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{33}{56}\right)=\sin\frac{\pi}{2}=1$

12. $f(x)=n-1; f(x)=(n-1)^2-(n-1)^2=0$

If $n-1<x<n,[x]=n-1$ and

$(n-1)^2 \le [x]^2 \le n^2-1 \Rightarrow 0 \le \left[x^2\right]-[x]^2 \le \left(n^2-1\right)-(n-1)^2$

$0 \le f(x) \le 2n-2$

The set of values of $f(x)$ is

$\{0,1,2,.........(2n-2)\}$

13. $=\tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$

$\because \ (\gamma-\alpha<0)$

$=\left(\tan^{-1}\alpha-\tan^{-1}\beta\right)+\left(\tan^{-1}\beta-\tan^{-1}\gamma\right)+\left(\tan^{-1}\gamma-\tan^{-1}\alpha\right)+\pi=\pi$

14. Replace x by $\frac{\pi}{2}-x \Rightarrow 2f(\cos x)+f\sin(x)=\left(\frac{\pi}{2}\right)-x \Rightarrow f(\sin x)=x-\frac{\pi}{6} \Rightarrow f(x)=\sin^{-1}\left(x-\frac{\pi}{6}\right)$

15. $\log_2^y=x(x-1)=x^2-x-\log_2^y=0 \Rightarrow x=\frac{1+\sqrt{1+4\log_2^y}}{2}$

16. $g(x)=f\left(\frac{4x-3}{6x-4}\right)=\frac{4\left(\frac{4x-3}{6x-4}\right)-3}{6\left(\frac{4x-3}{6x-4}\right)-4} \quad =\frac{16x-12-18x+12}{24x-18-24x+16}=\frac{-2x}{-2}=x$

17. $\sin^{-1}(-x)=-\sin^{-1}x,\cos^{-1}(-x)=\pi-\cos^{-1}x$

$\tan^{-1}(-x)=-\tan^{-1}x,\cot^{-1}(-x)=\pi-\cot^{-1}x$

18. Put $x=3 \Rightarrow f(3)+3f(8)=12$ …….(1)

Put $x=8 \Rightarrow f(8)+3f(3)=32$ …….(2)

$(1)+(2) \Rightarrow f(3)+f(8)=11$

19. $f(x)=2x-1; g(x)=\frac{x-\frac{1}{2}}{x-1}$

$fog(x)=fg(x)=2g(x)-1=\frac{x}{x-1}$

Here $x \ne 1$, Range $\ne$ co-domain

So, $fog(x)$ not onto function

$f^1(g(x))=\frac{1}{(x-1)^2}<0$

Which is decreasing function

So $fog(x)$ is one-one but not onto.

20. $(fogoh)(x) = f(g(h(x)))$

$= f(g(x^2)) = f(3x^2+1) = \dfrac{3x^2-1}{6x^2+3}$

21. $\sin^{-1}2x + 2\left(\dfrac{\pi}{2} - \sin^{-1}x\right) = \pi + \sin^{-1}x$

$\sin^{-1}2x = 3\sin^{-1}x \Rightarrow 2x = \sin(3\sin^{-1}x) \Rightarrow 2x = 3x - 4x^3 \Rightarrow x - 4x^3 = 0$

$x(1-4x^2) = 0 \qquad x = 0, \dfrac{1}{2}, \dfrac{-1}{2}$

22. Since $|\sin^{-1}x| \le \pi/2 \Rightarrow x = y = z = 1$ and $3000(x+y+z) - \dfrac{816}{x^2+y^2+z^2} = 9000 - 272 = 8728$

23. Let $y = \dfrac{e^x - e^{-x}}{2} \Rightarrow e^{2x} - 1 = 2ye^x$

Therefore, $t^2 - 2yt - 1 = 0, t = e^x$

$\Rightarrow t = \dfrac{2y \pm \sqrt{4y^2+4}}{2} = y \pm \sqrt{y^2+1}$

$\Rightarrow x = \log\left(y + \sqrt{y^2+1}\right)$ (since $e^x > 0$)

$\therefore \ f^{-1}(x) = g(x) = \log\left(x + \sqrt{x^2+1}\right)$

$g\left(\dfrac{e^{1002}-1}{2e^{501}}\right) = \log\left(\dfrac{e^{1002}-1}{2e^{501}} + \dfrac{e^{1002}+1}{2e^{501}}\right)$

$= \log e^{501} = 501.$

24. $(K-2)x^2 + 8x + k + 4 > 12 - 4\pi + 4\pi - 12$

$(k-2)x^2 + 8x + K + 4 > 0 \forall x \in R$

$\therefore k - 2 > 0$ and $\Delta < 0$

25. Put $x = 1, y = 1 \Rightarrow f(1) = 2$

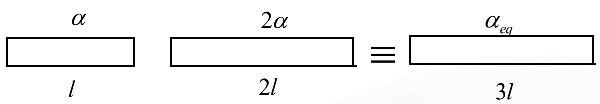
Put $y = \dfrac{1}{x} \Rightarrow f(x) = 1 \pm x^n$

$f(2) = 5 \Rightarrow n = 2 \qquad \therefore f(x) = 1 + x^2$

PHYSICS

26. $\frac{R_{100}-R_0}{R_\theta-R_0}=\frac{100-0}{\theta-0}\Rightarrow\frac{7.74-6.74}{6.53-6.74}=\frac{100}{\theta}\Rightarrow\ \theta=-21^0C$

27. $\Delta l_1+\Delta l_2=\Delta l$



$\Delta l_1+\Delta l_2=\Delta l\ \Rightarrow\alpha l\Delta T+(2\alpha)(2l)\Delta T=\alpha_{eq}(3l)\Delta T\ =\quad \alpha_{eq}=\frac{5}{3}\alpha=1.67\alpha$

28. If mass of the bullet is m,g heat absorbed by it to raise its temperature from 27^0C to $327^0C=\ \ mc\Delta T=m\times0.03\times(327-27)=9m$ cal

And heat required by the bullet to melt mL= $m\times6$

Total heat

$Q_1=(9m+6m)=15m$ cal $\quad=(15m\times4.2)J\qquad$ [as cal =4.2 J]

Now when bullet is stopped by the obstacle loss in its mechanical energy.

$ME=\frac{1}{2}(m\times10^{-3})v^2J\qquad$ [as $mg=m\times10^{-2}kg$]

As 25% of this energy is absorbed by the obstacle, the energy absorbed by the bullet.

$Q_2=\frac{1}{4}\times\frac{1}{2}mv^2\times10^{-3}\quad Q_2=Q_1\qquad v=410m/s$

29. In cooling 200 g of water from 25^0C to 10^0C heat to be extracted from water.

$Q_1=(mc\Delta T)_W=200\times1\times(25-10)\ \ =3000Cal$

And heat absorbed by m g ice at -14^0C to convert into water of 10^0C,

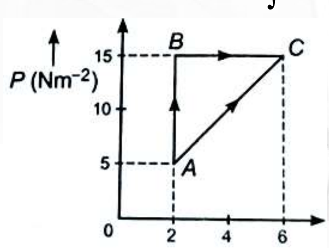
$Q_2=(mc\Delta T)_{ice}+mL+(mc\Delta T)_W$

i.e., $Q_2=m\left[0.5\left[0-(-14)\right]\right]+80+1(10-0)$=97m cal

According to given problem, $Q_2=Q_1$, i.e.,

97 m=3000, i.e.m=31g

30. As work done $W=\int P\,dv=$ are under P-V curve, so



$W_{ABC}=W_{AB}+W_{BC}$

$W_{ABC}=0+15\times4=60J$

And $W_{AC}=\frac{1}{2}(5+15)\times(6-2)=40J$

So work done along AC is least.

As according to first law of thermodynamics,

$dQ=dU+dW$

So for path AC

$\left(U_C - U_A\right) = dQ - dW = 200 - 40 = 160J$

So, $U_C = 160 + U_A = 160 + 10 = 170J$

31. $W_3 > W_2 > W_1$ and $\Delta U_2 = 0$ so $\Delta U_1 < \Delta U_2 < \Delta U_3$

32. $\frac{W}{Q} = \frac{W}{W + \Delta U} = \frac{nR\Delta T}{nR\Delta T + nC_1\Delta T} = \frac{nR\Delta T}{nR\Delta T + \left(\frac{5}{2}R\right)\Delta T} = \frac{2}{7}$

33. For adiabatic process $P\alpha T^{\frac{\gamma}{\gamma-1}}$ where $\gamma = \frac{7}{5}$

34. According to principle of calorimetry

$Q_{given} = Q_{used} \Rightarrow 0.2 \times S \times (150 - 40) = 150 \times 1 \times (40 - 27) + 25 \times (40 - 27) \Rightarrow 0.2 \times S \times 110 = 150 \times 13 + 25 \times 13$

Specific heat of aluminum $S = \frac{13 \times 25 \times 7}{0.2 \times 110} = 434 J/kg.^0C$

35. By ideal gas equation

$PV = nRT$

$n = \frac{PV}{RT} = \frac{10^5 \times 2000 \times 10^{-6}}{8.314 \times 300} = 0.08$

So, $n_h + n_0 = 0.08$ ………….(i)

As m=0.76g $\Rightarrow 2n_h + 32n_0 = 0.76 \Rightarrow n_h + 16n_0 = 0.38$ …………..(ii)

Subtracting (i) from (ii), we get

$15n_0 = 0.30 \Rightarrow n_0 = 0.02$

So, $n_h = 0.08 - n_0 = 0.08 - 0.02 = 0.06$

There fore, $\frac{n_h}{n_0} = \frac{0.06}{0.02} = \frac{3}{1}$

36. Average kinetic energy for diatomic gases

$K_{av} = \frac{5}{2}kT \quad \therefore \frac{(K_{av})_H}{(K_{av})_O} = \frac{(27 + 273)}{(27 + 273)} = 1$

37. Given, $n_1 = 2, n_2 = 4$

Specific heat of mixture at constant volume

$(C_v)_{mix} = \frac{n_1CV_1 + n_2CV_2}{n_1 + n_2} = \frac{2 \times \frac{5}{2} + 4 \times \frac{3}{2}R}{2 + 4} = \frac{11R}{6}$

Total internal energy of mixture $\Delta U = n(C_v)mix$ T$= \frac{11R}{6} \times T = 11RT$

38. In adiabatic process

$PV^\gamma =$ constant $\therefore p\left(\frac{m}{\rho}\right) =$ constant $\left(\because V = \frac{m}{\rho}\right)$

As mass is constant $\therefore P\alpha\rho^\gamma$

If P_i and P_f be the initial and final pressure of the gas and ρ_1 and ρ_f be the initial and final density of the gas. Then

$\frac{P_f}{P_i} = \left(\frac{\rho_f}{\rho_i}\right) = (32)^{7/5} \Rightarrow \frac{nP_i}{P_i} = \left(2^5\right)^{7/5} = 2^7 \Rightarrow n = 2^7 = 128$

39. From the conservation of energy change in potential energy =Heat energy

$\Rightarrow mgh = mc\Delta T \quad \Delta T = \frac{gh}{c} = \frac{10\times 63}{4200J/KgC} = 0.147^0 C$

40. $\frac{\Delta V}{V}\times 100 = 100\gamma\Delta T = 300\alpha\Delta T$

41. $dU = 0$

$dW = dQ$

42. According to 1st law of thermodynamics

$\Delta Q = \Delta U + W$

If $\Delta Q > 0, \Delta U < 0$ and $W > 0$ is also possible

Hence $\Delta T < 0$, so T decreases

Statement I is false.

$W > 0$; $\int PdV > 0$

Therefore volume of system must increase during positive work done by the system.

Statement II is true.

43. The total translational kinetic energy of n moles of

Gas $= \frac{3}{2}nRT$ ($\because$ PV=nRT)=1.5PV

Yes, the molecules of a gas collide with each other and the velocities of the molecules change due to collision.

44. A) Process $A \to B$

This is an isobaric process, P=constant and volume (V) of the gas decreases. Therefore work is done on the gas.

$W = P(3V - V) = 2PV$

Also V decreases so temperature at B decreases o

$\therefore$ Internal energy U decreases.

From $Q = U + W$ as U and W decreases so Q decreases that means heat is lost.

B) Process $B \to C$

This is an isochoric process V=constant pressure decreases $P\alpha T$ So temperature also decreases $W = 0; \Delta U =$ negative so ΔQ negative

Hence heat is lost

C) Process $C \to D$

This isobaric, pressure P=constant V increases and $V\alpha T$ So T increases. Hence $\Delta W, \Delta U$ and ΔQ +ve so heat gained by the gas.

D) Process $D \to A$

Applying PV=nRT

For D $P(9V) = 1RT_D : T_D = \frac{9PV}{R}$

For A $3P(3V) = 1RT_A \quad \therefore \; T_A = \frac{9PV}{R}$

i.e., the process is isothermal $\quad \therefore \; \Delta U = 0$

Now $\Delta Q = \Delta U + W \quad \therefore \Delta Q = W$

As volume decrease in this process so W negative ie., wom done on the gas and ΔQ negative hence heat is lost. .

45. $a \to$ isobaric, $b \to$ isothermal, $c \to$ adiabatic, $d \to$ isochoric.

46. Elastic energy stored, $=\frac{Y}{2}(\text{strain})^2\times\text{area}\times\text{length}$ $\therefore$ Elastic energy stored, per unit length

$=\frac{Y}{2}(strain)^2\times area=\frac{Y}{2}(\alpha\Delta T)^2\times A\ \left[\because strain=\frac{\Delta l}{l}=\alpha\Delta T\right]$

$=\frac{10^{11}}{2}\times\left(10^{-5}\times10\right)^2\times10^{-2}=\frac{10^{11}}{2}\times\left(10^{-4}\right)^2\times10^{-2}=5J/m$

47. Let C be the specific heat capacity of liquid and L be the latent heat of vapoursisation.
From principle of calorimetry.
Heat lost = heat gain

$m_CS_C\Delta T=mC\Delta T+mL$

$m_CS_C(110-80)=5C(80-30)+5L$ ……..(1)

Where, m_C=mass of calorimeter

$S_C=sp$. Heat of calorimeter

Again, when 80gm liquid is poured and equilibrium temperature is 50^0C

$m_CS_C(80-50)=80C(50-30)$ ………(ii)

From equ. (i) & (ii)

1600C=250C+5L

$\therefore\ \frac{L}{C}=\frac{1350}{5}=270^0C$

48. Work done $=P\Delta V\Rightarrow\ 400=P\Delta V\ \Rightarrow\ 400=nR\Delta T$ [$\because\ P\Delta V=nR\Delta T$ at constant pressure]

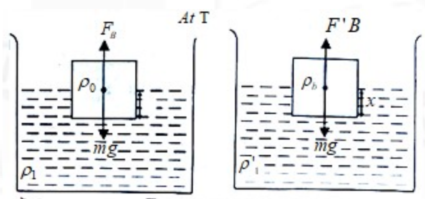
Now, $Q=nC_p\Delta T=n\frac{R\gamma}{\gamma-1}\Delta T=400\times\frac{\gamma}{\gamma-1}=400\times\frac{1.4}{0.4}=1400J$

49. $W=nR\Delta T=150J\quad[\because P\Delta V=nR\Delta T]$

$Q=\Delta U+W=\frac{f}{2}nR\quad\Delta T+nR\Delta T\Rightarrow Q=\left(\frac{f}{2}+1\right)nR\Delta T=\left(\frac{8}{22}+1\right)150=750J$

50. Initially, at temperature T bnoyant force

$F_B=mg$ or $Ax\rho_lg=AL\rho_bg$



At temperature $T+\Delta T$ the volume of the cube increases but the density of liquid decreases so depth upto which the cube is immersed in the liquid remains same.

$\therefore\ F_{B'}=mg$

Or, $A'x\rho'\ell g=AL\rho bg$

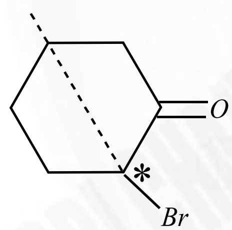
Now, $A'=A(1+2\alpha\Delta T)$

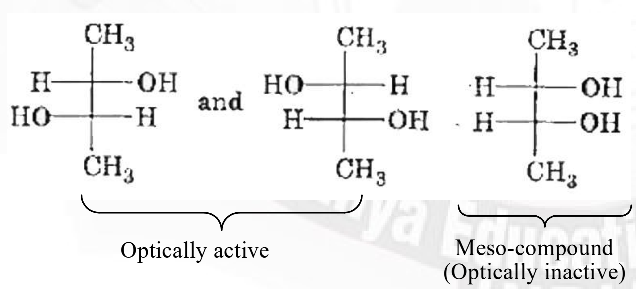
$\rho'\ell=\rho_\ell(1-\gamma\Delta T)$

$\therefore\ x\rho\ell(1+2\alpha\Delta T)(1-\gamma\Delta T)=L\rho_b$

$\Rightarrow x\rho_\ell(1+2\alpha\Delta T)(1-\gamma\Delta T)=x\rho_\ell$ [from eq. (i)]$\Rightarrow1+2\alpha\Delta T-\gamma\quad\Delta T=1\Rightarrow\gamma=2\alpha$ or $\gamma_l=2\alpha a$

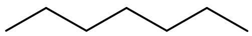
CHEMISTRY

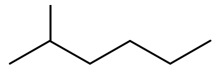
51. Double bond is given more preference over halogen atoms.
52. Classification of organic compounds.
53. Metameres are the isomers having same molecular formula and same functional group but different alkyl (or) aryl groups on either side of functional group
54. IUPAC rules.
55. All alkenes do not show geometrical isomerism and rotation about C=C is restricted.
56. IUPAC rules
57. The given compound shows both geometrical and optical isomerism.
58. The carbon which is directly attached to benzene ring is alpha carbon.
59. Number stereo isomers is given by $2^n = 2^3$
60. Fully eclipsed conformer is least stable due to repulsions.
61. Classification based on functional groups.
62. In compound 3 due to plane of symmetry it is meso form.
63. In Anti conformer intramolecular hydrogen bonding is not possible.
64. 

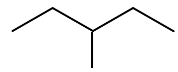
65. Chair form is free from angle strain and torsional strain. Therefore it is most stable.
66. 

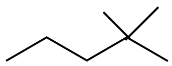
67. Priority order according to IUPAC rules.
68. IUPAC rules
69. Other than alkanes other functional group compounds also shows chain isomerism.
70. Pent-1-ene and pentan-2-one are not isomers.

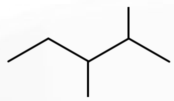
71.

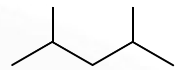

heptane


2-methyl hexane

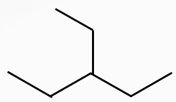

3-methyl hexane

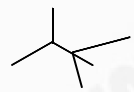

2,2 -Dimethyl pentane


2,3-Dimethyl pentane

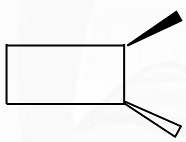
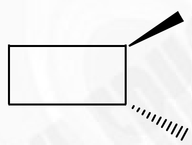
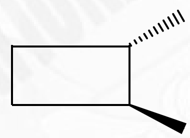
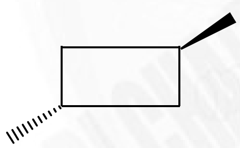
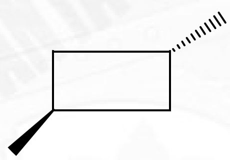
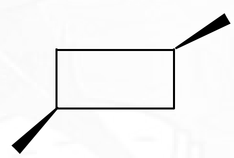

2,4-Dimethyl hexane


3,3-Dimethyl pentane


3-Ethyl pentane


2,2-Trimethyl butane

72.

73. 2,6 Dimethyl -2,5-diether acid.

74. The given compounds differ in IUPAC names.

75. four structures are possible, one in CH_2 group 3 in benzene ring.