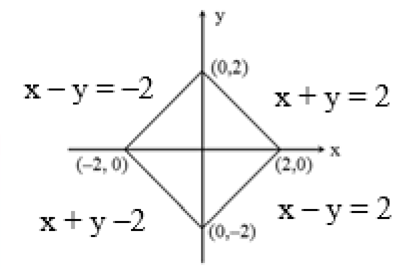
Mathematics

61. The region represented by $|x - y| \leq 2$ and $|x + y| \leq 2$ is bounded by a:
    A. rhombus of area $8\sqrt{2}$ sq. units
    B. square of side length $2\sqrt{2}$ unit
    C. square of area 16 sq. units
    D. rhombus of side length 2 units

Ans. B

Explanation:

Given that $|x - y| \leq 2$ and $|x + y| \leq 2$



Length of side = $\sqrt{2^2 + 2^2} = 2\sqrt{2}$

62. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$ also satisfy the equation
    A. $\sin x = |\sin y|$
    B. $\sin x = 2 \sin y$
    C. $2 \sin x = \sin y$
    D. $2|\sin x| = 3 \sin y$

Ans. A

Explanation:

Given that,

$$2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$$

$$\frac{3}{2}$$

So,

It is true only if sin x = 1 and |sin y| = 1

63. If α and β are the roots of the quadratic equation, x^2 + x sinθ – 2 sinθ = 0, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right).\left(\alpha-\beta\right)^{24}}$ is equal to:

A. $\frac{2^{12}}{\left(\sin\theta-8\right)^{6}}$

B. $\frac{2^{6}}{\left(\sin\theta+8\right)^{12}}$

C. $\frac{2^{12}}{\left(\sin\theta+8\right)^{12}}$

D. $\frac{2^{12}}{\left(\sin\theta-4\right)^{12}}$

Ans. C

Explanation: Given that

$x^2 + x\sin\theta - 2\sin\theta = 0$

has two roots α and β. Now

Sum of roots

$\alpha+\beta=-\sin\theta$

Product of roots

$\alpha\beta=-2\sin\theta$

Now, $\frac{\alpha^{12}+\beta^{12}}{\left(\frac{1}{\alpha^{12}}+\frac{1}{\beta^{12}}\right).\left(\alpha-\beta\right)^{24}}=\frac{\left(\alpha\beta\right)^{12}}{\left(\alpha-\beta\right)^{24}}$

$=\frac{\left(\alpha\beta\right)^{12}}{\left(\left(\alpha+\beta\right)^{2}-4\alpha\beta\right)^{12}}$

$=\left(\frac{\alpha\beta}{\left(\alpha+\beta\right)^{2}-4\alpha\beta}\right)^{12}$

Putting values from above

$=\left(\frac{-2\sin\theta}{\sin^{2}\theta+8\sin\theta}\right)^{12}$

$$= \left(\frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)} \right)$$

$$= \frac{2^{12}}{(\sin\theta+8)^{12}}$$

64. If a > 0 and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

A. $-\frac{1}{5}+\frac{3}{5}i$ B. $-\frac{1}{5}-\frac{3}{5}i$

C. $\frac{1}{5}-\frac{3}{5}i$ D. $-\frac{3}{5}-\frac{1}{5}i$

Ans. B

Explanation: Given that

$$z = \frac{(1+i)^2}{a-i} = \frac{(1+i)^2}{a-i} \times \frac{a+i}{a+i} = \frac{(1-1+2i)}{a^2+1} = \frac{2ai-2}{a^2+1}$$

magnitude

$$|z| = \sqrt{\left(\frac{-2}{a^2+1}\right)^2 + \left(\frac{2a}{a^2+1}\right)^2} = \sqrt{\frac{4(1+a^2)}{(a^2+1)^2}}$$

$$\sqrt{\frac{2}{5}} = \frac{2}{\sqrt{a^2+1}}$$

(squaring both side)

$$\frac{2}{5} = \frac{4}{1+a^2}$$

$$\Rightarrow a = 3$$

$$\therefore \quad \bar{z} = \frac{-2i(3-i)}{10}$$

$$\Rightarrow \frac{-1-3i}{5}$$

65.If y = y(x) is the solution of the differential equation dy/dx = (tanx – y) $\sec^2$x, x ∈ (–π/2, π/2), such that y (0) = 0,then y(–π/4) is equal to:

A. 1/2 – e B. e – 2

C. 2 + 1/e D. 1/e – 2

Ans. B

Explanation: Given differential equation is

$\frac{dy}{dx} = (\tan x - y)\sec^2 x$

$\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$

Let $\tan x = t$

$\Rightarrow \sec^2 x = \frac{dt}{dx}$

$\therefore \frac{dy}{dt} + y = t$ This is a Linear differrential equation

Now, $\text{If} = e^{\int \sec^2 \times dx} = e^{\tan x} = e^t$

$y.e^t = \int e^t + dt$

Now, solution

$y\, e^t = e^t(t-1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{=\tan x}$

$y(0) = 0 \quad \Rightarrow c = 1$

$y = \tan x - 1 + e^{-\tan x}$

So, $y\left(-\frac{\pi}{4}\right) = e - 2$

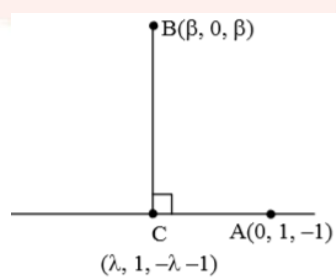
66. If the length of the perpendicular from the point (β, 0, β) (β ≠ 0) to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to

A. 2 B. 1

C. –2 D. –1

Ans. D

Explanation:



$$\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}=\lambda$$

A point on this line is $A(0, 1, -1)$

$\overrightarrow{AC}\,.\,\overrightarrow{BC}=0$

we get $\lambda=-\frac{1}{2}$

$\therefore\ C\equiv\left(-\frac{1}{2},1,-\frac{1}{2}\right)$

$$\frac{|\overrightarrow{BC}|=\sqrt{\frac{2}{3}}}{\left(\beta+\frac{1}{2}\right)^2+(1)^2+\left(\beta+\frac{1}{2}\right)^2=\sqrt{\frac{2}{3}}}$$

$\therefore\ \beta=0,-1$

$\beta=-1(\beta\neq 0)$

67.If

$$\Delta_1=\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} \text{ and } \Delta_2=\begin{vmatrix} x & \sin2\theta & \cos2\theta \\ -\sin2\theta & -x & 1 \\ \cos2\theta & 1 & x \end{vmatrix}, x\neq 0;$$

then for all $\theta\in\left(0,\frac{\pi}{2}\right)$:

A. $\Delta_1-\Delta_2=x(\cos2\theta-\cos4\theta)$
B. $\Delta_1+\Delta_2=-2x^3$
C. $\Delta_1+\Delta_2=-2(x^3+x-1)$
D. $\Delta_1-\Delta_2=-2x^3$

Ans. B

Explanation: Given That

$$\Delta_1=\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$= x(-x^2-1)-\sin\theta(-x\sin\theta-\cos\theta)+\cos\theta(-\sin\theta+x\cos\theta)$

$= x^3 - x + x\sin^2\theta+\sin\theta\cos\theta-\cos\theta\sin\theta+x\cos^2\theta$

$= -x^3-x+x(\sin^2\theta+\cos^2\theta) = -x^3-x+x$

$\Longrightarrow -x^3$

Similarly $\Delta_2 = -x^3$

Now, $\Delta_1 + \Delta_2 = - x^3 - x^3 - 2x^3$

68. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is:

A. 1/10 B. 1/17

C. 1/11 D. 1/12

Ans. C

Explanation:

	Family1	Family2
2 BOYS AND 2 GIRLS	BB	GG
	BG	BG
	BG	GB
	GB	GB
	GB	BG
	GG	BB
1 BOY AND 3 GIRLS	BG	GG
	GB	GG
	GG	BG
	GG	GB
4 GIRLS	GG	GG

Required probability = 1/11.

69. Which one of the following Boolean expressions is a tautology?

A. $(p \vee q) \wedge (\sim p \vee \sim q)$ B. $(p \vee q) \vee (p \vee \sim q)$

C. $(p \wedge q) \vee (p \wedge \sim q)$ D. $(p \vee q) \vee (p \vee \sim q)$

Ans. B

Explanation:

From options

$(p \vee q) \vee (p \vee \sim q) \equiv p \vee (q \vee \sim q) \to$ tautology

70. Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define g (A) = {$x \in R$: $f(x) \in A$}. If S = [0,4], then which one of the following statements is not true ?

A. $g(f(S)) \neq S$ B. $f(g(S))= S$

C. $f(g(S)) \neq f(S)$ D. $g(f(S)) =g(S)$

Ans. D

Explanation:

Given that.

$f(x) = x^2$, $x \in R$.

g (A) = {$x \in R : f(x) \in A$} and S = [0,4]

g(s) = {$x \in R : f(x) \in S$}

= {$x \in R : 0 \leq x^2 \leq 4$}

= {$x \in R : -2 \leq x \leq 2$}

g(S) = [–2, 2]

f(g(S)) = [0,4] = S

f(S) = [0, 16] $\Longrightarrow$ $f(g(S)) \neq f(S)$

g(f(S)) = [–4, 4] $\neq$ g(S)

Therefore, g(f(s)) = g(s) is not true.

71. If $\lim_{x\to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is :

A. $\frac{3}{2}$ B. $\frac{8}{3}$

C. $\frac{3}{8}$ D. $\frac{3}{8}$

Ans. B

Explanation:

$$\lim_{x\to 1}\frac{x^4-1}{x-1}$$

$$=\lim_{x\to 1}\frac{(x^2)^2-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x^2-1^2)(x^2+1)}{(x-1)}$$

$$=\lim_{x\to 1}\frac{(x-1)(x+1)(x^2+1)}{(x-1)}$$

$$=\lim_{x\to 1}(x+1)(x^2+1)=4 \quad ...(i)$$

$$\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$$

$$=\frac{k^2+k^2+k^2}{2k}$$

$$=\frac{3k^2}{2k}$$

$$=\frac{3k}{2} \quad ...(ii)$$

Then,

from $(i)=(ii)$

$$4=\frac{3k}{2}$$

$$8=3k$$

$$\Rightarrow\ k=\frac{8}{3}$$

72.If a directrix of a hyperbola centred at the origin and passing through the point (4, –2 √3) is 5x = 4√5 and its eccentricity is e, then:

A. $4e^4 - 24e^2 + 27 = 0$ B. $4e^4 - 24e^2 + 35 = 0$
C. $4e^4 - 12e^2 - 27 = 0$ D. $4e^4 + 8e^2 - 35 = 0$

Ans. B

Explanation:

Let hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and Points (4, –2√3) passes through therefore

$$\frac{(4)^2}{a^2}-\frac{\left(-2\sqrt{3}\right)^2}{b^2}=1$$

$$\frac{16}{a^2}-\frac{12}{b^2}=1 \qquad (i) \qquad \therefore\ b^2=a^2\left(e^2-1\right)$$

Given $5x = 4\sqrt{5}$

$$x=\frac{4\sqrt{5}}{5}=\frac{a}{e}$$

Squaring both sides, we get,

$$\Rightarrow\ a^2=\frac{16}{5}e^2 \qquad ...(ii)$$

on solving equations (i) and (ii)

$$\Rightarrow 4e^4-24e^2+35=0$$

73. If $f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x} & , \ x<0 \\ q & , \ x=0 \\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , \ x>0\end{cases}$ is continuous at x = 0,

then the ordered pair (p, q) is equal to:

A. $\left(-\frac{3}{2},-\frac{1}{2}\right)$ B. $\left(-\frac{1}{2},\frac{3}{2}\right)$

C. $\left(-\frac{3}{2},\frac{1}{2}\right)$ D. $\left(\frac{5}{2},\frac{1}{2}\right)$

Ans. C

Explanation:

$$f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x} & , \ x<0 \\ q & , \ x=0 \\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , \ x>0\end{cases}$$

is continuous at x = 0,

So, $f(0^-) = f(0) = f(0^+)$ …(i)

$$RHL = f(0^+) \lim_{x\to 0^+} \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{1+x}-1}{x} = \frac{1}{2}$$

$$LHL = f(0^-) = \lim_{x\to 0^-} \frac{\sin(p+1)x+\sin x}{x}$$

$$=(p+1)+1$$

$$=p+2$$

from equations (i)

$$LHL = RHL = f(0)$$

$$P+2=q=\frac{1}{2}, \text{ so, } q=\frac{1}{2} \text{ and } P=-\frac{3}{2}$$

$$\Rightarrow (p,q)=\left(-\frac{3}{2},\frac{1}{2}\right)$$

74. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a,b) is equal to:

A. (28, 861) B. (28, 315)

C. (–21, 714) D. (–54, 315)

Ans. B

Explanation:

$(1+ax+bx^2)\ (1-3x)^{15}$

Coefficient of,

$x^3 = 1.\ {}^{15}C_2(-3)^2 + a.\ {}^{15}C_1(-3) + b.\ {}^{15}C_0 = {}^{15}C_2 \times 9 - 3a({}^{15}C_1) = 0$

$= {}^{15}C_2 \times 9 - 45a + b$

$\Rightarrow 945 - 459 + b = 0$...........(i)

Coefficient of,

$x^3 = {}^{15}C_2(-3)^3 + a.\ {}^{15}C_2(-3)^2 + b.\ {}^{15}C_1(-3) = 0$

$= -27 \times {}^{15}C_3 + 9a \times {}^{15}C_2 - 3b \times {}^{15}C_1 = 0$

$\Rightarrow -273 + 21a - b = 0$.........$(ii)$

Then,

from (i) + (ii)

$\Rightarrow 945 - 459 + b + (-273) + 21a - b = 0$

$\Rightarrow 945 - 459 + \cancel{b} + (-273) + 21a - \cancel{b} = 0$

$\Rightarrow -24a + 672 = 0$

$\Rightarrow a = 28$

$\Rightarrow b = 315$

75. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for:

A. exactly two values of K

B. no value of K.

C. exactly one value of K

D. infinitely many values of K

Ans. B

Explanation: Given,

$\mathbf{S_1} = x^2 + y^2 + 5Kx + 2y + K = 0$

$\mathbf{S_2} = 2(x^2 + y^2) + 2Kx + 3y - 1 = 0$

Equation of common chord

$S_1 - S_2 = 0$

$4kx + \frac{1}{2}y + k + \frac{1}{2} = 0 \quad ..(i)$

given $4x + 5y - k = 0$ ...(ii)

On comparing (i) & (ii) we get

$$\frac{4k}{4}=\frac{1}{10}=\frac{2k+1}{-2k}$$

$$k=\frac{1}{10}=\frac{k+1/2}{-k}$$

$$k=\frac{-5}{11}$$

$\Longrightarrow$ No. real value of k exist

76. The sum $\frac{3\times 1^3}{1^2}+\frac{5\times\left(1^3+2^3\right)}{1^2+2^2}+\frac{7\times\left(1^3+2^3+3^3\right)}{1^2+2^2+3^2}+......$ upto 10th term is :

A. 660 B. 680

C. 600 D. 620

Ans. A

Explanation: Given that

$$S=\sum_{n=1}^{10}(2n+1)\frac{6n^2(n+1)^2}{4n(n+1)(2n+1)}=\sum_{n=1}^{10}\frac{3n(n+1)}{2}=660$$

77. Let f : R → R be differentiable at c ∈R and f(c) = 0. If g(x) = |f(x)|, then at x = c, g is:

A. differentiable if f ' (c) = 0

B. differentiable if f ' (c) ≠ 0

C. not differentiable

D. not differentiable if f ' (c) = 0

Ans. A

Explanation:

f : R → R be differentiable at c ∈R and f(c) = 0. If g(x) = |f(x)|

$$g'(c) = \lim_{h\to 0} \frac{|f(c+h)| - |f(c)|}{h}$$

$$= \lim_{h\to 0} \frac{|f(c+h)|}{h} \quad (\because\ f(c) = 0)$$

$$= \lim_{h\to 0} \left|\frac{f(c+h) - f(c)}{h}\right| . \frac{|h|}{h}$$

$(\because\ f(c) = 0)$

$$= \lim_{h\to 0} \left|\frac{f(0+h) - f(0)}{h}\right| . \frac{|h|}{h}$$

$$= \lim_{h\to 0} |f'(c)| \frac{|h|}{h} = 0$$

if $f'(c) = 0$

That is, g (x) x = c if f '(c) = 0 is different.

78. If Q(0, –1 –3) is the image of the point P in the plane 3x – y + 4z = 2 and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

A. $\frac{\sqrt{65}}{2}$ B. $2\sqrt{13}$

C. $\frac{\sqrt{91}}{2}$ D. $\frac{\sqrt{91}}{24}$

Ans. C

Explanation: Given, Plane 3x – y + 4z = 2 and P(3, -1, -2)

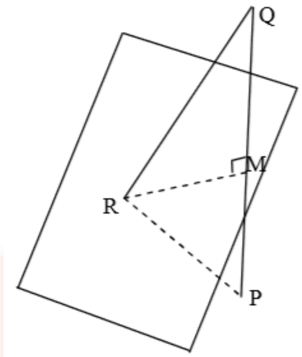


Image of Q in plane

$$\frac{x-0}{3}=\frac{y+1}{-1}=\frac{z+3}{+4}=\frac{-2(1-12-2)}{9+1+16}$$

$x=3,\ y=-2,\ z=1$

$P(3,-2,1),\quad Q(3,-2,1),\quad R(3,-2,1)$

Now area of ΔPQR

$$\text{Area}(\Delta PQR)=\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{QR}\right|=\frac{1}{2}\begin{vmatrix} i & j & k \\ 3 & -1 & 4 \\ 3 & 0 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left|\{i(-1)-j(3+2)+K(3)\}\right|$$

$$=\frac{1}{2}\left(\sqrt{1+81+9}\right)$$

$$=\frac{\sqrt{91}}{2}$$

79. If $\int \frac{dx}{\left(x^2-2x+10\right)^2}=A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$

where C is a constant of integration then :

A. $A=\frac{1}{54}$ and $f(x)=9(x-1)^2$

B. $A=\frac{1}{54}$ and $f(x)=3(x-1)$

C. $A=\frac{1}{81}$ and $f(x)=3(x-1)$

D. $A=\frac{1}{27}$ and $f(x)=9(x-1)$

Ans. B

Explanation:

$$\int \frac{dx}{(x^2-2x+10)^2} = \int \frac{dx}{((x-1)^2+9)^2}$$

Let $(x-1)^2 = 9\tan^2\theta$ ...(i)

$$\Rightarrow \tan\theta = \frac{x-1}{3}$$

On differentiating …(i)

$2(x - 1)\, dx = 18\tan\theta \sec^2\theta\, d\theta$

$$\therefore\ I = \int \frac{18\tan\theta\sec^2\theta\, d\theta}{2\times 3\tan\theta\times 81\sec^4\theta}$$

$$I = \frac{1}{27}\int \cos^2\theta\, d\theta = \frac{1}{27}\times\frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$I = \frac{1}{54}\left\{\theta + \frac{\sin 2\theta}{2}\right\} + c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{1}{2}\times\frac{2\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2}\right] + c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right] + c$$

So, $A = \dfrac{1}{54}$

$f(x) = 3(x - 1)$

80. The line x = y touches a circle at the point (1,1). If the circle also passes through the point (1, – 3), then its radius is

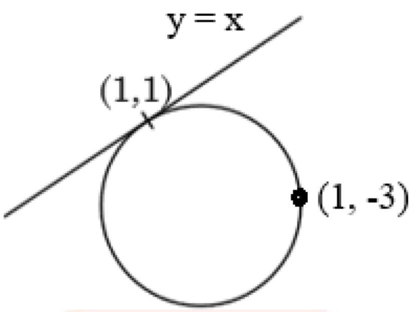
:

A. 3 B. 2

C. $2\sqrt{2}$ D. $3\sqrt{2}$

Ans. C

Explanation:



Equation of circle is given as

$S + \lambda L = 0$

$(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

Which is passes through (1, -3)

$0 + 16 + \lambda (-3, - 1) = 0$

$16 + \lambda \times 4 = 0$

$\Rightarrow \lambda = -4$

Now equation of circle

$\therefore\ (x-1)^2 + (y-1)^2 - 4(x-y) = 0$

$x^2 + y^2 - 6x + 2y + 2 = 0$

so, radius = $\sqrt{9+1-2}$

$r = 2\sqrt{2}$

81. The value of $\int_0^{2\pi} [\sin 2x(1+\cos 3x)]\,dx,$ where [t] denotes the greatest integer function is:

A. 2π B. π

C. -2π D. $-\pi$

Ans. D

Explanation:

$$I = \int_0^{2\pi} \left[\sin 2x\left(1 + \cos 3x\right)\right] dx \qquad \ldots(i)$$

As we know that

$$\int_0^a f(x) = \int_0^a f(a-x)\,dx$$

$$\therefore I = \int_0^{2\pi} [-\sin 2x - \sin 2x.\cos 3x]dx$$

$$= \int_0^{2\pi} \left[-\sin x(1+\cos 2x)\right]dx \quad ...(ii)$$

By (i) + (ii)

$$2I = \int_0^{2\pi} + dx$$

$$2I = -(x)_0^{2\pi} = -2\pi$$

$$I = -\pi$$

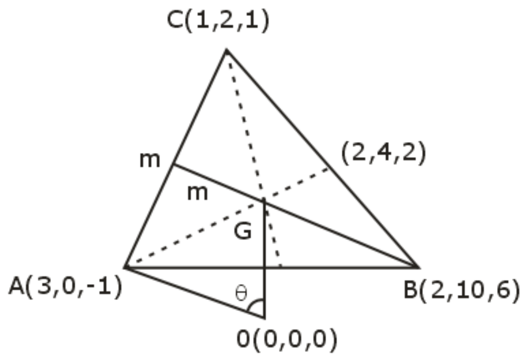
82. Let A (3,0, –1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then cos (∠GOA (O being the origin) is equal to :

A. $\frac{1}{\sqrt{15}}$ B. $\frac{1}{6\sqrt{10}}$

C. $\frac{1}{\sqrt{30}}$ D. $\frac{1}{2\sqrt{15}}$

Ans. A

Explanation:



G is the centroid of ΔABC

$G \equiv (2,4,2)$

$\overrightarrow{OG} = \vec{2}\hat{i} + 4\hat{j} + 2\hat{k}$

$\overrightarrow{OA} = 3\hat{i} - \hat{k}$

We know that,

$$\cos(\angle GOA) = \frac{\overrightarrow{OG} \cdot \overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|}$$

$$\Rightarrow \frac{6-2}{\sqrt{4+16+4}.\sqrt{9+1}} = \frac{4}{\sqrt{24} \times \sqrt{10}}$$

$$= \frac{4}{4\sqrt{15}}$$

$$= \frac{1}{\sqrt{15}}$$

83. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

A. 10 B. 9

C. 12 D. 7

Ans. A

Explanation:

Given that system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu, (\lambda, \mu \in R),$

$x + 3y + \lambda z - \mu = a(x + y + z - 5) + b(x + 2y + 2z - 6)$

comparing coefficients we get

$a + b = 1$ ...(i)

and $a + 2b = 3$ ...(ii)

On solving (i) and (ii)

$a = -1$ and $b = 2$

$(a, b) \equiv (-1, 2)$

So, $x + 3y + \lambda z - \mu = x + 3y + 3z - \lambda$

$\Rightarrow \mu = 7, \lambda = 3$

84. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated is :

A. 36 B. 60

C. 72 D. 48

Ans. B

Explanation:

a_1	a_2	a_3	a_4	a_5	a_6

digit 0, 1, 2, 5, 7, 9

$(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 11K$

so (1, 2, 9) (0, 5, 7)

Now number of ways to arranging them

$= 3! \times 3! \times 3! \times 2 \times 2$

$= 6 \times 6 + 6 \times 4$

$= 6 \times 10$

$= 60$

85. If for some $x \in R$, the frequency distribution of the marks

obtained by 20 students in a test is:

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x–5$	$x^2 – 3x$	x

then the mean of the marks is

A. 3.0 B. 2.8

C. 2.5 D. 3.2

Ans. B

Explanation:

$(x + 1)^2 + (2x - 5) + (x^2 – 3x) + x = 20$

$\Rightarrow x^2 + x – 12 = 0$

$\Rightarrow x = 3$

$$\text{Mean} = \frac{\sum_{i=1}^{4} n_i x_i}{\sum_{i=1}^{4} n_i} = \frac{2(x+1)^2 + 3(2x-5) + 5(x^2 - 3x) + 7x}{20} = 2.8$$

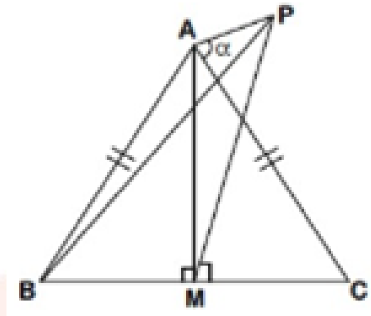
86. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is:

A. $\frac{100}{3\sqrt{3}}$ B. 25

C. 20 D. $10\sqrt{5}$

Ans. C

Explanation:



ΔAPM

$$\frac{h}{AM} = \frac{1}{3\sqrt{2}}$$

ΔBPM

$$\frac{h}{BM} = \frac{1}{\sqrt{7}}$$

ΔABM

$$\therefore AM^2 + MB^2 = (100)^2$$

$$\Rightarrow 18h^2 + 7h^2 = 100 \times 100$$

$$\Rightarrow h^2 = 4 \times 100$$

$$\Rightarrow h = 20$$

87. If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots\ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to:

A. 38 B. 98

C. 76 D. 64

Ans. C

Explanation:

Given that,

$a_1+a_4+a_{10}+a_{13}+a_{16}=114$

$\Rightarrow 3(a_1+a_{16})=114$

$\Rightarrow a_1+a_{16}=\frac{114}{3}$

$\Rightarrow a_1+a_{16}=38$

S0,

$a_1+a_6+a_{11}+a_{16}=2(a_1+a_{16})$

$\Rightarrow 2\times 38=76$

88. Let f(x) = e^x – x and g(x) = x^2 – x, $\forall$ x ∈ R. Then the set of all x ∈ R, where the function h(x) = (fog) (x) is increasing, is :

A. $[a, \infty)$

B. $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

C. $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$

D. $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

Ans. D

Explanation: Given that

f(x) = e^x – x and g(x) = x^2 – x, $\forall$ x ∈ R

h(x) = f(g(x))

If f(g(x)) is increasing functions.

h'(x) = f '(g(x)) g'(x) and f '(x) = e^x –1

h'(x) = ($e^{g(x)}$ – 1) g'(x)

$h'(x)=\left(e^{x^2-x}-1\right)(2x-1)\geq 0$

Case: 1

$e^{x^2-x} \leq 1$ and $2x-1 \leq 0$

$\Rightarrow x \in \left[0, \frac{1}{2}\right]$ ...(i)

Case: 2

$e^{x^2-x} \geq 1$ and $2x-1 \geq 0$

$\Rightarrow\ x \in [1, \infty) \quad ...(ii)$

from (i) and (ii)

$x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$

89. $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + ... + \frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to:

A. $\frac{4}{3}(2)^{3/4}$
B. $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$
C. $\frac{4}{3}(2)^{4/3}$
D. $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$

Ans. B

Explanation: Given that

$$\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + ... + \frac{(2n)^{1/3}}{n^{4/3}}\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{n+r}{n}\right)^{1/3} \qquad \left(\frac{r}{n} \to x \text{ and } \frac{1}{n} \to dx\right)$$

$$= \int_0^1 (1+x)^{1/3}\,dx = \left[\frac{3}{4}(1+x)^{4/3}\right]_0^1 = \frac{3}{4}(2)^{4/3} - \frac{3}{4}$$

90. If the line x – 2y = 12 is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

A. $8\sqrt{3}$
B. $12\sqrt{2}$
C. 5
D. 9

Ans. D

Explanation:

Given: line x – 2y = 12

Given: ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Given: point $\left(3, \frac{-9}{2}\right)$

Tangent at (3, – 9/2)

$$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$$

Comparing with x – 2y = 12

$$\frac{3}{a^2} = \frac{9}{4b^2} = \frac{1}{12}$$

$\Rightarrow$ a = 6 and b = $3\sqrt{3}$

length of latus rectum $= \frac{2b^2}{a} = 9$