



SEC: Sr.Super60(Incoming)_STERLING

WTA-32

Date: 18-05-2025

Time: 09:00AM to 12:00PM

JEE-ADV-2022_P1

Max: Marks: 180

KEY SHEET

MATHEMATICS

1	7	2	8	3	1	4	2	5	2	6	1
7	5	8	6	9	B,C,D	10	A,D	11	A,D	12	D
13	B,C	14	A,B,D	15	A	16	B	17	D	18	A

PHYSICS

19	2	20	2	21	8	22	1	23	2	24	8
25	2	26	2	27	ABD	28	AD	29	ABC	30	ACD
31	ABCD	32	BD	33	D	34	A	35	A	36	C

CHEMISTRY

37	1.75	38	3.33	39	1.60	40	8.75	41	6.50	42	2.33
43	1.20	44	1.25	45	AB	46	ACD	47	BD	48	ABD
49	AC	50	AC	51	C	52	D	53	B	54	C

SOLUTIONS MATHEMATICS

1. $f(x) = 2 \log x$

$$A = \int_0^1 (-x^3 + 6x^2 - 11x + 6 - 2 \log x) dx \\ = \frac{17}{4} \Rightarrow \frac{28}{17} \times \frac{17}{4} = 7$$

2. $[x]^2 = [y]^2$

$$\begin{aligned} [y] &= \pm 1 & \text{if } 1 < x < 2 \\ &= \pm 2 & 2 \leq x < 3 \\ &= \pm 3 & 3 \leq x < 4 \\ &\pm 4 & \text{if } 4 \leq x < 5 \\ &\pm 5 & x = 5 \end{aligned}$$

Plot the graph

Required area is $2(4) = 8$

3. $f(x) + f(3) = f(x+z)$

and $f(0) = 1$ and $f'(0) = 4$

$$\Rightarrow f(x) = 4x$$

$$\text{So area bounded} = \Delta = \int_0^4 (4x - x^2) dx$$

4. Conceptual.

5. Required area = $4 \int_0^1 \sqrt{x^2 - x^6} dx$

$$= 4x\sqrt{1-x^4} dx$$

$$\text{Put } x^2 = \sin \theta$$

$$2x dx = \cos \theta d\theta$$

$$\Rightarrow 2\cos \theta \cos \theta d\theta$$

$$\text{R. A} = \int_0^{\frac{\pi}{2}} 2\cos^2 \theta d\theta$$

6. Required area $4 \int_0^1 (1-x^{2/5}) dx = \frac{8}{7}$



7. At $x = 0 \quad y = 0$.

$$x + 5y - y^5 = 0$$

$$\Rightarrow 1 + 5y^1 - 5y^4 y^1 = 0$$

at $x = 0 \quad y = 0$

$$y^1 = \frac{1}{5}$$

Required tangent is $y = \frac{-x}{5}$

$$\therefore \text{Area} = \frac{1}{2} \times 5 \times 26 = 65$$

8. Conceptual.

9. (a) $f(x) = (x-a)(x-b)(x+c) = (x-a)x(x+c)$

Clearly option 1 is a correct.

$$(b) \int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx > 0$$

So, from the graph which is incorrect

$$(c) \int_a^b f(x) dx < 0 \quad \text{and} \quad \int_c^b f(x) dx < 0$$

But second term has large negative value

\therefore option c is incorrect

(d) \Rightarrow Clearly d is incorrect.

$$10. \quad I_n = \frac{1}{n} \sum_{r=2n}^{2n-1} \frac{\frac{r}{n}}{1 + \left(\frac{r}{n}\right)^2}$$

$$S_n = \frac{1}{n} \sum_{r=2n+1}^{3n} \frac{\left(\frac{r}{n}\right)^2}{\left(1 + \frac{r}{n}\right)^2}$$

$$\text{Let } f(x) = \frac{x}{1+x^2}$$

$$\Rightarrow f'(x) = \frac{1+x^2 - 2x^2}{(1+x^2)^2}$$

$\therefore f(x)$ is decreasing in $(2, 3)$



$$I_n > \int_2^3 f(x) dx$$

$$S_n < \int_2^3 f(x) dx$$

11. $\int_6^4 (a\sqrt{x} + bx) dx = 8$

$$\frac{2a}{3} + b = 1 \rightarrow 1$$

$$a + b = 2$$

$\therefore a, b, c, d$ are correct.

12. Equation of normal is $y + x = \frac{7}{4}$

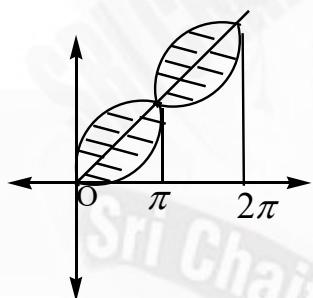
$$\therefore \text{Required area} = \int_{-3}^{\frac{3}{2}} \left(\frac{7}{4} - x \right) - (x^2 + 1) dx$$

13. Conceptual

14. Conceptual.

15. graph of

A $y = x + \sin x$ and its inverse is show in the graph



$y = x$ is function and inverse

$$\text{Required area} = 4 \int_0^{\pi} (x + \sin x) - (x) dx$$

$$4 \int_0^{\pi} \sin x dx$$

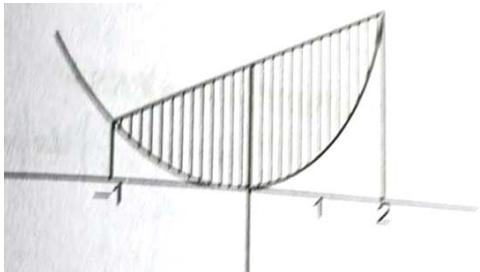
$$4(2) = 8$$

$$S = 2$$



16. The area = 2 unit

B) Area enclosed = $\int_0^{\pi} \sin x dx = 2$

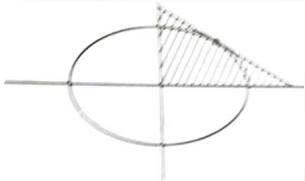


The line $y = x+2$ intersects $y = x^2$ at $x = -1$ and $x = 2$

The given region is shaded region area = $\frac{15}{2} - \int_{-1}^2 x^2 dx = \frac{9}{2}$

D) Here, $a^2 = 9, b^2 = 5, b^2 = a^2(1-e^2) \Rightarrow e^2 = \frac{4}{9} \Rightarrow \frac{2}{3}$

Equation of tangent at $\left(2, \frac{5}{3}\right)$ is $\frac{2x}{9} + \frac{y}{3} = 1$



x -intercept = $\frac{9}{2}$, y -intercepts = 3

Area = $4 \times \frac{9}{2} \times 3 \times \frac{1}{2} = 27$ sq.units

17. Consider the intervals for x

$[0,1)[1,2)[2,3)[3,4)$

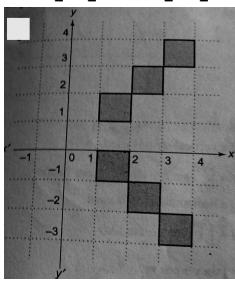
The values of y in the interval

$[0,1)[1,2)[2,3)[3,4)$

Required are 4 square units

18. $\lceil x^2 \rceil = \lceil y \rceil^2$, where $1 \leq x \leq 4$

$\Rightarrow \lceil x \rceil = \pm \lceil y \rceil$

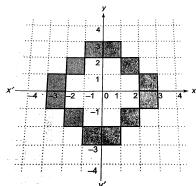


B) $\lceil |x| \rceil \lceil |y| \rceil = 2$



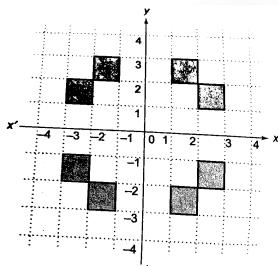
The graph is symmetrical about both the x-axis and y-axis

$$\text{For } x, y > 0; [x] + [y] = 2$$



$$\Rightarrow [x] = 0 \text{ and } [y] = 2, [x] = 1 \text{ and } [y] = 1 \text{ or } [x] = 2 \text{ and } [y] = 0$$

$$[\text{C}]: [|x|][|y|] = 2$$



The graph is symmetrical about both the x-axis and y-axis

$$\text{For } x, y > 0; [|x|][|y|] = 2 \Rightarrow [x] = 1 \text{ and } [y] = 2 \text{ or } [x] = 2 \text{ and } [y] = 1$$

PHYSICS

19. $R = 3 + 1 = 4\Omega, e = Bl\vartheta$

$$i = \frac{e}{R} \Rightarrow i = \frac{Bl\vartheta}{R}$$

$$10^{-3-1} = \frac{2 \times 10^5 \times 10^{-2}}{4} \vartheta \rightarrow \vartheta \frac{1}{5} \times 10^{-1}$$

$$\vartheta = 0.2 \times 10^{-1} = 0.02m$$

20. For falling $y \rightarrow \vartheta = \sqrt{2gy}$

$$l = 2x = 2\sqrt{\frac{y}{c}}$$

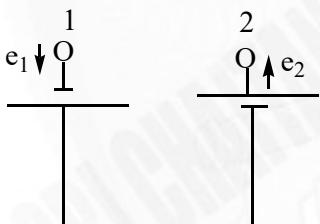
$$e = Bl\vartheta$$

$$e = B\sqrt{2gy} \cdot 2\sqrt{\frac{y}{c}} = By\sqrt{\frac{8a}{c}}$$

$$= 2By\sqrt{\frac{2a}{c}}$$

21. $e_1 = B2r2\vartheta$

$$e_2 = B4r.\vartheta$$

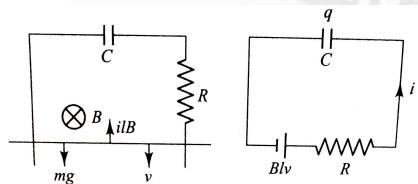


$$e = e_1 + e_2 = 8Br\vartheta$$

22. The rate of electrical energy consumed in the bulb = rate of loss of gravitational PE

$$\text{of the mass} = Mgv = 100W. \text{ Hence } M = \frac{100}{10 \times 10} = 1kg$$

23. By Newton's law $mg - ilB = m \frac{dv}{dt}$ (i)



$$\text{Using KVL } Blv = iR + \frac{q}{C} \quad (\text{ii})$$

Differentiating equation (ii) w.r.t time, we get

$$Bl \frac{dv}{dt} = R \frac{di}{dt} + \frac{i}{C} \quad (\text{iii})$$



Eliminating $\frac{dv}{dt}$ from equations (i) and (iii), we get

$$mg - ilB = \frac{m}{Bl} \left[R \frac{di}{dt} + \frac{i}{C} \right]$$

$$\Rightarrow mgBl - iB^2l^2 = m \left(R \frac{di}{dt} + \frac{mi}{C} \right) \quad (iv)$$

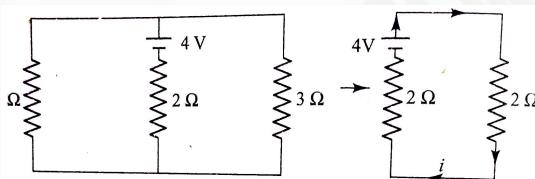
I will be maximum when $\frac{di}{dt} = 0$. Use this in equation (iv)

$$\Rightarrow mgBlC = i(B^2l^2C + m)$$

$$\Rightarrow i_{\max} = \frac{mgBlC}{m + B^2l^2C}$$

24. Induced emf : $B(\text{effective length})v = B2Rv$

25. Motional emf



$$e = Bvl$$

$$e = (2)(2)(1) = 4V$$

This acts as a cell of emf $E = 4$ V and internal resistance $r = 2\Omega$. The simple circuit can be drawn as follows

$$\text{Therefore, current through the connector } i = \frac{4}{2+2} = 1A$$

Magnetic force on connector

$$F = ilB = (1)(1)(2) = 2N \quad (\text{towards left})$$

Therefore, to keep the connector moving with a constant velocity, a force of 2N will have to be applied towards right.

26. $de = B\vartheta dx = \frac{\mu_0}{2\pi} \frac{i_o}{x} \vartheta dx$

$$e = \int de = \frac{\mu_0}{2\pi} i_o \vartheta \int_a^b \frac{dx}{x}$$

$$e = \frac{\mu_0}{2\pi} i_o \vartheta \ln(x) \Big|_a^b = \frac{\mu_0}{2\pi} i_o \vartheta \ln\left(\frac{b}{a}\right)$$

$$F = \frac{p}{\vartheta} = \frac{e^2}{R\vartheta} = \frac{\vartheta}{R} \left(\frac{\mu_0}{2\pi} i_o \ln\left(\frac{b}{a}\right) \right)^2$$

27. Conceptual



28. PQ does not cut lines of force so $e_{pq} = 0$

$$\text{In RQ } e = Bl\theta = Ba \quad (a w)$$

$$e = Ba^2\omega$$

$$29. \phi = BA = B_o t \pi (r_o t)^2$$

$$\rightarrow e = \frac{d\phi}{dt} = B_o \pi r_0^2 3t^2$$

$\rightarrow e \propto t^2$ so, parabola

$$\rightarrow \frac{B_o}{r_o} = \frac{e}{\pi r_0^3 3t^2}$$

$$\frac{de}{dt} = B_o \pi r^2 6t$$

$$\frac{de}{dt} \propto t$$

$$30. \text{ Potential difference} = \frac{1}{2} B \omega l^2$$

$$\text{From energy conservation: } mg \frac{l}{2} \sin \theta = \frac{1}{2} I \omega^2, \text{ where } I = \frac{ml^2}{3}$$

$$mg \frac{l}{2} \sin \theta = \frac{1}{2} \frac{mi^2}{3} \omega^2$$

$$\omega = \sqrt{\frac{3g}{l} \sin \theta}$$

$$e = \frac{2}{2} Bl^2 \sqrt{\frac{3g \sin \theta}{l}}$$

$$e \propto B; e \propto l^{3/2}; e \propto \sin \theta^{1/2}$$

$$31. \text{ Due to rotation, } emf = \frac{Br^2 \omega}{2}$$

Due to translation indeed $emf = Bvr$

Where r is the separation.

$$32. i = \frac{dq}{dt} = \frac{d}{dt}(CvBl) = CBl \frac{dv}{dt} = CBla$$

$$\therefore F - CB^2 l^2 a = ma$$

$$\Rightarrow a = \frac{F}{m + B^2 l^2 C}$$

\Rightarrow emf increases

\Rightarrow charge increases



33. We know that $e = -\frac{d\phi}{dt} = -A \frac{dB}{dt}$. If we take area vector in the upward direction, then anticlockwise direction will be positive. From 0 to t_1 and t_5 to t_6 , dB/dt is +ve. Hence induced emf e is -ve. So induced current will be in clockwise direction. From t_2 to t_4 , dB/dt is -ve. Hence induced emf e is +ve. So induced current will be in anticlockwise direction. From t_1 to t_2 and t_4 to t_5 , dB/dt is zero. Hence, no emf is induced. Induced emf or current is maximum from 0 to t_1 and t_5 to t_6 because here magnitude of dB/dt is maximum.

34. Conceptual

35. Since $\phi = 2t$

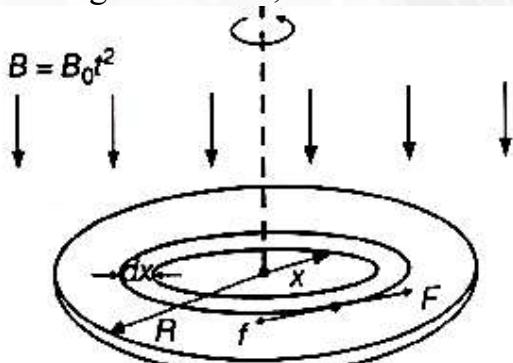
$$E = \frac{d\phi}{dt} = 2v$$

$$i = \frac{E}{R} = \frac{2}{2} = 1A$$

$$\Delta q = iot = 1 \times 2 = 2c$$

$$H = i^2 Rt = 1^2 \times 2 \times 2 = 4J$$

36. Assume the disc to be made of a number of infinitesimal concentric rings. Consider one ring of radius x, thickness dx .



dx having mass dm . Then the frictional force on this infinitesimal elements is

$$df = \mu(dm)g$$

$$\{\because dN = (dm)g\}$$

$$\text{where } dm = (2\pi x dx) \frac{M}{\pi R^2} = \left(\frac{2M}{R^2}\right)(x dx)$$

infinitesimal torque due to this frictional force is

$$d\tau = (df)x$$

$$\Rightarrow d\tau = \mu \left(\frac{2M}{R^2} x dx \right) gx$$

$$\Rightarrow d\tau = \frac{2\mu Mg}{R^2} x^2 dx$$



$$\Rightarrow \tau = \int d\tau = \frac{2\mu Mg}{R^2} \int_0^R x^2 dx$$

$$\Rightarrow \tau = \frac{2\mu Mg}{R^2} \left(\frac{R^3}{3} \right) = \frac{2}{3} (\mu MgR)$$

This expression is independent of τ . So

(B) \rightarrow (p)

Now, let us calculate the torque due to the varying magnetic field.

The varying magnetic field which will be tangential to the disc (or the infinitesimal element). If dq be the charge on the infinitesimal element, then

$$dq = \left(\frac{Q}{\pi R^2} \right) (2\pi x dx) = \left(\frac{2Q}{R^2} \right) x dx$$

Electrostatic force on this element in the presence of tangential electric field E_t is

$$dF = (dq)E_t$$

So, if $d\tau_m$ is the torque due to the magnetic field, then

$$\tau_m = \int d\tau_m = \int x dF = \int \left(\frac{2Q}{R^2} x^2 dx \right) E_t \quad \dots\dots(1)$$

From Faraday's Laws, we know that

$$\left| \oint \vec{E}_t \cdot d\vec{\ell} \right| = \left| \frac{d\phi_B}{dt} \right|$$

$$\Rightarrow (2\pi x)E_t = (\pi x^2) \frac{dB}{dt}$$

$$\Rightarrow E_t = \frac{x}{2} (2B_0 t) = (B_0 t)x$$

$$\Rightarrow \tau_m = \frac{(2QB_0)t}{R^2} \int_0^R x^3 dx$$

$$\Rightarrow \tau_m = \frac{(2QB_0)t}{R^2} \left(\frac{R^4}{4} \right) = \left(\frac{QB_0 R^2}{2} \right) t$$

$$\Rightarrow \tau_m = \left(\frac{QB_0 R^2}{2} \right) t \quad \dots\dots(2)$$

The disc will start rotating $t = t_0$, when

$$\tau_m = \tau_f$$



$$\Rightarrow \left(\frac{QB_0 R^2}{2} \right) t_0 = \frac{2}{3} \mu M g R$$

$$\Rightarrow t_0 = \frac{4}{3} \left(\frac{\mu M g}{QB_0 R} \right)$$

So, we get (A) \rightarrow (s)

Also, from (2), we observe that torque due to magnet field at $t = 0$ is zero

$$\text{at } t = t_0 \text{ is } \left(\frac{QB_0 R^2}{2} \right) t_0 = \frac{2}{3} \mu M g R$$

$$\text{at } t = 3t_0 \text{ is } \frac{3}{2} (QB_0 R^2)$$

$$t_0 = 2\mu M g R$$

So, (C) \rightarrow (p, r, t)

Let us calculate the net torque τ on the disc at t ($> t_0$)

$$\tau = \tau_m - \tau_f = \left(\frac{QB_0 R^2}{2} \right) t - \frac{2}{3} (\mu M g R)$$

$$\Rightarrow I\alpha = \left(\frac{QB_0 R^2}{2} \right) t - \frac{2}{3} (\mu M g R)$$

$$\Rightarrow I \frac{d\omega}{dt} = \left(\frac{QB_0 R^2}{2} \right) t - \frac{2}{3} (\mu M g R)$$

$$\Rightarrow \frac{1}{2} M R^2 \frac{d\omega}{dt} = \left(\frac{QB_0 R^2}{2} \right) t - \frac{2}{3} (\mu M g R)$$

$$\Rightarrow d\omega = \frac{QB_0}{M} \int_{t_0}^{2t_0} t dt - \frac{4\mu g}{3R} \int_{t_0}^{2t_0} dt$$

$$\Rightarrow \omega = \left(\frac{QB_0}{2M} \right) (3t_0^2) - \left(\frac{4\mu g}{3R} \right) t_0$$

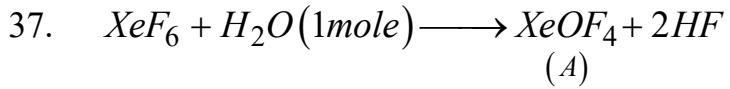
Substituting $t_0 = \frac{4}{3} \left(\frac{\mu M g}{QB_0 R} \right)$ in (3), we get

$$\omega = \frac{8}{9} \left(\frac{M \mu^2 g h 2}{QB_0 R^2} \right)$$

So, (D) \rightarrow (q)



CHEMISTRY



$$P = 5, Q = 1, R = 1, \quad \frac{P+Q+R}{4} = \frac{7}{4} = 1.75$$

38. $x = 6, y = 4$

$$\frac{x+y}{3} = \frac{10}{3} = 3.33$$



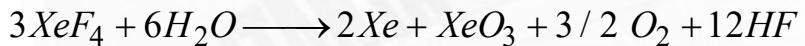
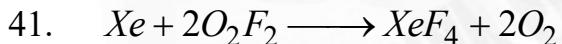
$$\therefore \frac{2x+y+z}{5} = \frac{8}{5} = 1.60$$

40.

	XeF_4	XeF_5^+	XeF_5^-	XeF_6	XeF_8^{2-}
Bond pair	4	5	5	6	8

Lone pairs	2	1	2	1	1
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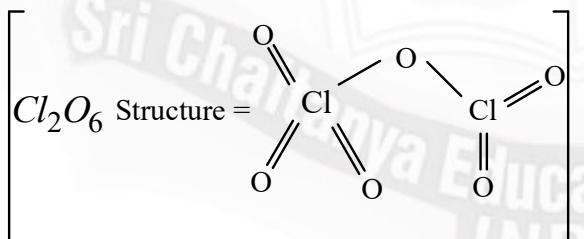
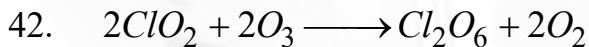
$$\therefore P = 28, Q = 7 \text{ and } \frac{P+Q}{4} = \frac{35}{4} = 8.75$$



$$\therefore X = \text{moles of } XeO_3 = 1$$

$$Y = \text{moles of HF} = 12$$

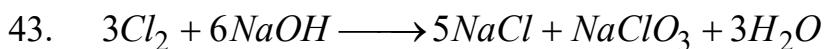
$$\text{and } \frac{x+y}{2} = \frac{1+12}{2} = \frac{13}{2} = 6.50$$



$$A = \text{No.of } \pi - \text{bonds} = 5$$

$$B = \text{Average oxidation state of Cl in P} = 6$$

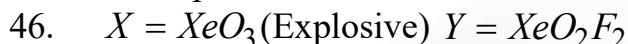
$$\frac{2B-A}{3} = \frac{12-5}{3} = \frac{7}{3} = 2.33$$



$$x = 6, y = 5 \text{ and } \frac{x}{y} = 1.20$$

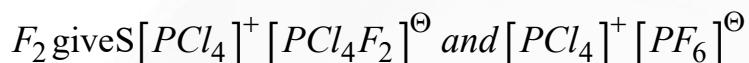
44. % of Cl_2 available = $\frac{3.55 \times N \times V}{W} = \frac{3.55 \times 0.25 \times 5}{3.55}$
= 1.25

45. Conceptual



47. Conceptual

48. In polar solvents, PCl_5 on reaction with



49. Conceptual



51. Conceptual

52. Conceptual

53. Conceptual

54. Conceptual