



## KEY SHEET

## PHYSICS

# CHEMISTRY

## MATHEMATICS



# SOLUTIONS

## PHYSICS

01.  $\vec{F} = q(\vec{V} \times \vec{B}) = 1.6 \times 10^{-13} N$

02. Kinetic Energy  $KE = Vq \Rightarrow \frac{P^2}{2m} = Vq \Rightarrow (Bqr)^2 = Vq2m \left[ \because r = \frac{mV}{Bq} \right]$   
 $B^2 q^2 \left( \frac{d}{2} \right)^2 = Vq2m \Rightarrow m = \frac{B^2 d^2 q}{8V}$

03.  $\left[ \frac{\mu_0 I_1 I_2}{2\pi} \ln \left( 1 + \frac{b}{a} \right) \hat{j} \right]$

Magnetic field  $(\vec{B})$  created by wire-I at a distance x from it, i.e..  $(\vec{B}) = \frac{\mu_0 I_1}{2\pi x} (-k)$

Force acting on a small element dx of wire -2, i.e

$$\overrightarrow{dF} = I_2 \overrightarrow{d} \times X \vec{B} \text{ or } \overrightarrow{dF} = I_2 \overrightarrow{d} \times X \left( \frac{\mu_0 I_1}{2\pi x} (-k) \right) = \frac{\mu_0 I_1 I_2}{2\pi x} dx \left[ \hat{i} \times (-\hat{k}) \right] (as \overrightarrow{d} \times -\overrightarrow{d} \times \hat{i}) = \left( \frac{\mu_0 I_1 I_2}{2\pi x} d \times \right) \hat{j};$$

$$\vec{F} = \int \overrightarrow{dF} = \left[ \frac{\mu_0 I_1 I_2}{2\pi} \int_a^{a+b} \frac{1}{x} d \times \right]$$

$$\hat{j} = \left[ \frac{\mu_0 I_1 I_2}{2\pi} \ln \left( 1 + \frac{b}{a} \right) \right] \hat{j}$$

$\vec{F}$  point upward as indicated by  $\hat{j}$

04.  $\therefore$  electric force on the slab

$$F = \frac{du}{d \times} = \frac{1}{2} \frac{\epsilon_0 b E^2}{d} (K-1)$$

$$Mg = \frac{\epsilon_0 b E^2}{2d} (K-1)$$

05. Particle is projected in x-y plane which is  $\perp$  er to the magnetic field. Since field is along z-axis.

Magnetic of velocity  $V = \sqrt{2}V_0$

$$\text{Radius of the circle } r = \frac{mV}{Bq} = \frac{\sqrt{2}mV_0}{Bq}$$

06. Force on the charged particle

$$\vec{F} = q \left[ \vec{E} + (\vec{V} \times \vec{B}) \right] = q \left[ E_0 \hat{k} - B_0 V_0 \hat{j} \right]$$

Here the magnetic force is along y-axis it causes the change in direction of velocity.

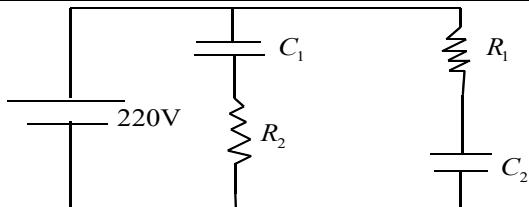
Electric force is along z-axis it accelerate the charged particle. So velocity along -axis increases continuously

$\therefore$  Path of the particles helix with non uniform pitch.

07.  $i = 2 \times 10^2 A$

$$P_{R_1} = i^2 R_1 = (2 \times 10^{-2})^2 \times 4 \times 10^3 = 1.6 W$$

08.  $Q_{C_1} = V_{R_1} \times C_1 = 80 \times 310^{-6} = 240 \mu C$

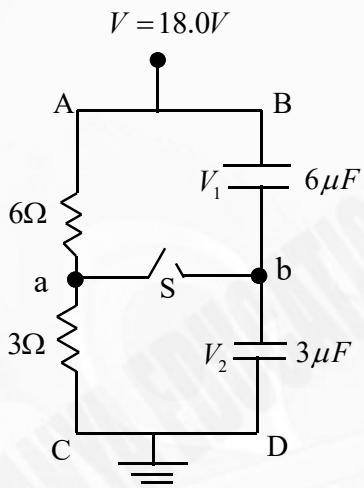


09.  $\int d\tau = \frac{1}{2} Il^2 B$

10.  $\frac{1}{2} Il^2 B = Kx l \sin 53^\circ$  find  $x$

11. When 'S' is open across capacitors

$$i = \frac{18}{9} = 2A$$



$$\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{3}{6} = \frac{1}{2}$$

$$V_A = V_a = 6(2) = 12; V_1 = \frac{1}{3}(18) = 6$$

$$V_a - V_C = 3(2) = 6; V_2 = \frac{2}{3}(18) = 12$$

$$V_a = 6 (\because V_C = 0); V_1 = V_B - V_a = 6$$

$$V_2 = V_b - V_D = 12$$

$$V_{ab} = V_a - V_b = 6 - 12 = -6$$

$$V_b = 12 (V_D = 0)$$

$$Q_1 = C_1 V_1 = 6(6) = 36 \mu C;$$

$$Q_2 = C_2 V_2 = 3(12) = 36 \mu C$$

When 'S' is closed

$$V_1^1 = 12V \therefore Q_1^1 = C_1 V_1^1 = (6)12 = 72 \mu C$$

$$V_1^1 = 6V \therefore Q_2^1 = C_2 V_2^1 = 3(6) = 18 \mu C$$

$\therefore$  Charge flown through 'S'

$$= Q_1^1 - Q_2^1 = 72 - 18 = 54 \mu C$$

12. initially it acts as balanced wheat stone bridge  
 $\therefore$  current through G is zero

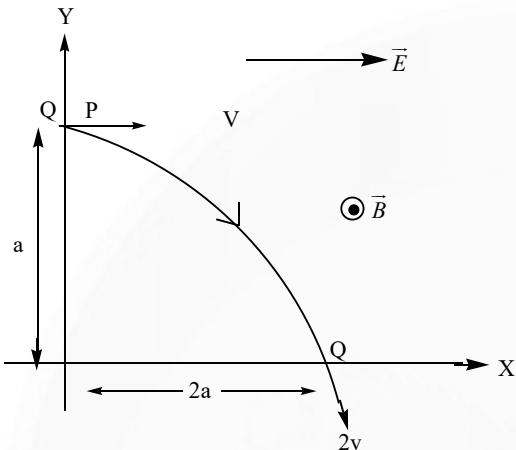


In steady state  $i = \frac{2}{4+1+5} = 0.2A$

P.D. across  $C_2 = V_G + V_{5\Omega} = I(G + R) = 0.2(1+5) = 1.2V$

13. From W-E theorems F.S =  $\Delta KE$

$$E.q[2a] = \frac{1}{2}m4v^2 - \frac{1}{2}mv^2$$



$$E.q.2a = \frac{3mv^2}{2} \Rightarrow E = \frac{3mv^2}{4qa}$$

Rate of work done by Electric field =  $\frac{dw}{dt} = P$

$$P = Fv = Eqv = \frac{3mv^2}{4qa}qv = \frac{3mv^2}{4a}$$

At (Q),  $P = \vec{F} \cdot \vec{v} = \vec{E}q \cdot \vec{v} = Eqv \neq 0$ , Hence  $\odot$  is wrong

At (Q),  $P = \vec{F} \cdot \vec{v} = \vec{E}q \cdot \vec{v} = 0$  Work done by uniform magnetic field is zero

$$14. R = \frac{mv \sin \theta}{Bq}; pitch = v \cos \theta T = v \cos \theta = \frac{2\pi m}{Bq}$$

$$15. Bqv = \frac{mv^2}{r} \Rightarrow Bq = \frac{mv}{r} = m\omega = m\left(\frac{2\pi}{T}\right)$$

$$[\because v = r\omega] \Rightarrow T = \frac{2\pi m}{Bq} \therefore a = \frac{T_1}{T_2} = 1$$

$$\Rightarrow pitch = P = (\theta \cos \theta) \left( \frac{2\pi m}{Bq} \right) \Rightarrow P \propto \cos \theta$$

$$C = \frac{P_1}{P_2} = \frac{\cos 30}{\cos 60} = \sqrt{3} = (3)^{1/2}$$

$$\text{As } Bqr_{\perp} = \frac{mv_{\perp}^2}{r}$$

$$\Rightarrow Bq = \frac{mv \sin \theta}{r}$$

$$\Rightarrow r = \frac{mv \sin \theta}{Bq}$$

$$\Rightarrow r \propto \sin \theta \Rightarrow \frac{r_1}{r_2} = b = \frac{\sin 30}{\sin 60} = \frac{1}{\sqrt{3}}$$

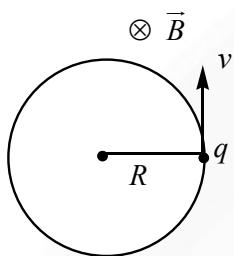
$$\therefore abc = 1 \& a = bc$$



$$16. \quad R = \frac{mv}{qB}; T = \frac{2\pi m}{q^B}$$

Rate of sweeping the area is

$$\frac{dA}{dt} = \frac{\pi R^2}{T} = \frac{\pi \left(\frac{mv}{qB}\right)^2}{\frac{2\pi m}{q^B}} = \frac{1}{2} \frac{mv^2}{qB}$$



$$17. \quad \text{Here } \vec{V} \text{ parallel to } \vec{B}$$

$$\therefore \frac{4}{2} = \frac{y}{3} = \frac{z}{-6}$$

$$\Rightarrow y = 6 \text{ and } z = -12$$

$$|\vec{B}| = \sqrt{16+36+144} = \sqrt{196} = 14T$$

$$18. \quad E = \frac{B^2 q^2 R^2}{2m} \Rightarrow B^2 = \frac{2mE}{q^2 R^2} \Rightarrow B = \frac{\sqrt{2mE}}{qR}$$

$$19. \quad T = \frac{2\pi m}{Bq} \text{ frequency } f = \frac{1}{T} = \frac{Bq}{2\pi m};$$



## CHEMISTRY

20.  $K_{eq} = \frac{k_f}{k_b} \Rightarrow \frac{[CH_3]^2}{[C_2H_6]}$   
 $\therefore [CH_3] = \frac{10^{-4}}{10} = 10^{-5} M$   
 $\frac{1.57 \times 10^{-3}}{K_b} = \frac{(10^{-5})^2}{1}$

21. CONCEPTUAL

22.  $r_1 = k[0.01]^a [0.01]^b = 6.93 \times 10^{-6}$  .....(i)  
 $r_2 = k[0.02]^a [0.01]^b = 1.386 \times 10^{-5}$  .....(ii)  
 $r_3 = k[0.02]^a [0.02]^b = 1.386 \times 10^{-5}$  .....(iii)

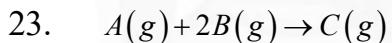
From data  $a=1; b=0;$

overall order = 1;  $k = 6.93 \times 10^{-4} \text{ sec}^{-1}$

$$6.93 \times 10^{-4} = \frac{1}{50 \times 60} \ln \frac{A_0}{A_t};$$

When  $[A]_0 = 0.5 \Rightarrow A_t = 0.0625$

rate of reaction  $= 6.93 \times 10^{-4} \times 0.0625$   
 $= 4.33 \times 10^{-5} \text{ Ms}^{-1}$



$$t=0 \quad 0.4 \quad 0.6$$

$$t=t \quad 0.3 \quad 0.4 \quad 0.1$$

$$r_1 = KP_A^1 P_B^2$$

$$r_1 = K \times 0.4 \times (0.6)^2$$

$$r_1 = K(0.3)(0.4)^2$$

$$\frac{r_1}{r_1} = 3$$

24.  $e^{-E_a/RT}$  is fraction of molecules having energy greater than activation energy ( $E_a$ ), it is unit less. So A has same unit of K. for 1<sup>st</sup> order reaction unit of K is s<sup>-1</sup>.

25.  $\ln \frac{K_2}{K_1} = \frac{E_a}{R} \left[ \frac{1}{T_1} - \frac{1}{T_2} \right]$

$$\ln 2 = \frac{E_a}{2cal} \left[ \frac{1}{290} - \frac{1}{300} \right]$$

$$E_a = \frac{2.303 \times 0.3 \times 2 \times 290 \times 300}{10} cal = 12K \text{ Cal}$$

26.  $E = \frac{nhC}{\lambda}$

$$7.2 \times 10^{-3} J/s = \frac{n \times 6.6 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}}$$

$$\text{No. of photon} = \frac{7.2 \times 10^{-3} \times 330 \times 10^9}{6.6 \times 10^{-34} \times 3 \times 10^8} = 120 \times 10^{14}$$

$$\text{Mole of CO formed} = 0.4 \times 20 \times 10^{-9} = 8 \times 10^{-9} \text{ mol/s}$$



27. Rate of formation of  $C_2H_6 = 0.8 \times 20 \times 10^{-9} = 16 \times 10^{-9} mol / s$

28.  $\frac{N}{N_0} = \left(\frac{1}{x}\right)^n$ ; where n=no.of halves;

$$\frac{N}{N_0} = \frac{1}{10} = \left(\frac{1}{x}\right)^n = n = 4$$

Total time  $= n \times t_{1/2} = 4740 = 4 = 4 \times t_{1/2}$ ;

$t_{1/2} = 1185$  years

29.  $r = k^1(O_3)[O] = k^1(O_3)K_C \frac{[O_3]}{[O_2]} = k [O_3]^2 [O_2]^{-1}$

30. CONCEPTUAL

31. (A)  $K = Ae^{-E_a/RT}$  as  $T \downarrow, K \downarrow$  rate of reaction  $\downarrow$

(B) Rate of reaction  $= k[A]^0 = K = \text{constant}$

(C) As surface area  $\uparrow$  No. of molecules will be more to react per unit time, more surface site available for the reaction.

(D) Average rate defined for macroscopic time and instantaneous rate defined for microscopic time.

32. For an elementary reaction  $aA + bB \longrightarrow cC + dD$  rate law is always  $r = K[A]^a[B]^b$  but not vice versa.

For a complex reaction  $aA + bB \longrightarrow cC + dD$  rate law may or may not be  $r = K[A]^a[B]^b$

33. (A) In a rate law reactant, product or catalyst may be appeared.

(B) We can not

(C) In an elementary reaction only positive integer stoichiometric coefficient appears.

(D) A zero order reaction is always complex

34. (A)  $K = Ae^{-E_a/RT}$

As  $T \uparrow, K \uparrow$  so if  $T \rightarrow \infty$ , then  $K \rightarrow A$  or as  $E_a \uparrow, K \uparrow$  so if  $E_a \rightarrow 0$ , then  $K \rightarrow A$

(B) Catalyst does not change  $\Delta H$  of reaction.

(C) A negative catalyst decrease rate of reaction by increasing activation energy.

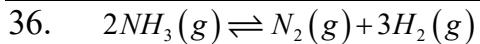
35.  $A + B \longrightarrow C$

(A) If it is an elementary reaction then it will be bimolecular reaction.

(B) It may be exothermic eq.  $NH_3(g) + HCl(g) \longrightarrow NH_4Cl(s)$

(C) It may be heterogeneous

(D) It may be photochemical reaction



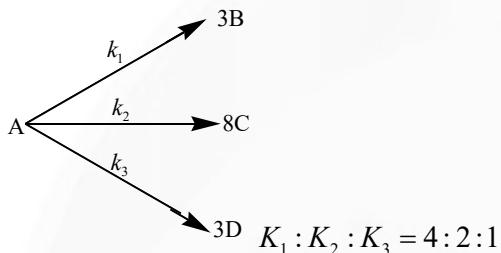
$$\text{Rate} = K[NH_3]^0$$

$$\text{Rate} = 2$$

$$\text{Rate} = \frac{1}{3} \times \frac{\Delta[H_2]}{\Delta t} \Rightarrow \frac{\Delta[H_2]}{\Delta t} = 6$$

37.  $t_{1/2} = \frac{[A_0]}{2K_A} = \frac{[A_0]}{2 \times 5K} = \frac{200 \text{ min.}}{2 \times 5 \times 400} = \frac{60 \text{ sec.}}{20} = 3$

38.



Mole of A remain after 45 days

$$= \frac{N_0}{2^n} = \frac{N_0}{T} = \frac{1}{2^{45/15}} = \frac{1}{2^3} = \frac{1}{8}$$

Moles of A convert into product =  $\frac{7}{8}$  mol

$$\text{Moles of } [C] = \frac{K_2}{K_1 + K_2 + K_3} \times \frac{7}{8} \times \frac{8}{1} = 2$$



## MATHEMATICS

39. Let  $I = \int e^x \left( \frac{x^4 + 2}{(1+x^2)^{5/2}} \right) dx$

$$= \int e^x \left( \frac{1}{(1+x^2)^{1/2}} + \frac{1-2x^2}{(1+x^2)^{5/2}} \right) dx$$

$$= \int e^x \left( \frac{1}{(1+x^2)^{1/2}} - \frac{x}{(1+x^2)^{3/2}} + \frac{x}{(1+x^2)^{3/2}} + \frac{1-2x^2}{(1+x^2)^{5/2}} \right) dx$$

$$= \frac{e^x}{(1+x^2)^{1/2}} + \frac{xe^x}{(1+x^2)^{3/2}} + C = \frac{e^x \{1+x^2+x\}}{(1+x^2)^{3/2}} + C$$

Hence, (d) is the correct answer.

40. Let  $I = \int e^{(x \sin x + \cos x)} \cdot \left( \frac{x^4 \cos^3 x - x \sin x + \cos x}{x^2 \cos^2 x} \right) dx$

$$= \int (x \cdot e^{(x \sin x + \cos x)} \cdot x \cos x) dx - \int e^{(x \sin x + \cos x)} \cdot \left( \frac{x \sin x - \cos x}{(x \cos x)^2} \right) dx$$

Applying integration by parts

$$\begin{aligned} &= \left\{ x \cdot e^{(x \sin x + \cos x)} - \int e^{(x \sin x + \cos x)} dx \right\} \\ &\quad - \left\{ e^{(x \sin x + \cos x)} \cdot \frac{1}{x \cos x} - \int e^{(x \sin x + \cos x)} dx \right\} \\ &= e^{(x \sin x + \cos x)} \left( x - \frac{1}{x \cos x} \right) + C \end{aligned}$$

Hence, © is the correct answer.

41. Note that  $\sec^{-1} \sqrt{1+x^2} = \tan^{-1} x : \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right) = 2 \tan^{-1} x$

For  $x > 0$

$$\Rightarrow I = \int \frac{e^{\tan^{-1} x}}{1+x^2} \left\{ (\tan^{-1} x)^2 + 2 \tan^{-1} x \right\} dx,$$

Put  $\tan^{-1} x = t$

$$= \int e^t (t^2 + 2t) dt = e^t \cdot t^2 = e^{\tan^{-1} x} (\tan^{-1} x)^2 + C$$

Hence, © is the correct answer.

42.  $= \int \frac{1-7 \cos^2 x}{\sin^7 x \cos^2 x} dx = \int \left( \frac{\sec^2 x}{\sin^7 x} - \frac{7}{\sin^7 x} \right) dx$

$$= \int \frac{\sec^2 x}{\sin^7 x} dx - \int \frac{7}{\sin^7 x} dx = I_1 + I_2$$

Now,  $I_1 = \int \frac{\sec^2 x}{\sin^7 x} dx = \frac{\tan x}{\sin^7 x} + 7 \int \frac{\tan x \cdot \cos x}{\sin^8 x} dx$

$$= \frac{\tan x}{\sin^7 x} + I_2$$

$$\therefore I_1 + I_2 = \frac{\tan x}{\sin^2 x} + C \Rightarrow f(x) = \tan x$$



43. Let  $P = \sin^3 x \cos^3 x$

$$\begin{aligned}\frac{dP}{dx} &= 3\sin^2 x \cos^4 x - 3\sin^4 x \cos^2 x \\ &= 3\sin^2 x(1 - \sin^2 x)\cos^2 x - 3\sin^4 x \cos^2 x \\ &= 3\sin^2 x \cos^2 x - 6\sin^4 x \cos^2 x \\ P &= 3I_{2,2} - 6I_{4,2}\end{aligned}$$

$$I_{4,2} = \frac{1}{6}(-P + 3I_{2,2})$$

44. Let  $P = \sin^5 x \cos^3 x$

$$\begin{aligned}\frac{dP}{dx} &= 5\sin^4 x \cos^4 x \cos^4 x - 3\sin^6 x \cos^2 x \\ &= 5\sin^4 x(1 - \sin^2 x)\cos^2 x - 3\sin^6 x \cos^2 x \\ &= 5\sin^4 x \cos^2 x - 8\sin^6 x \cos^2 x \\ \therefore P &= 5I_{4,2} - 8I_{6,2}\end{aligned}$$

$$\therefore I_{4,2} = \frac{1}{5}(P + 8I_{6,2})$$

45.  $\therefore f(x) = 0 \Rightarrow \underbrace{(x^2 - 4x + 8)}_{D>0} \underbrace{(x^2 - 4x + 17)}_{D<0} = 0$

$\therefore$  Equation has two distinct and two imaginary roots.

46.  $f(x) = (x^2 - 4x - 17)(x^2 - 4x + 8)$

$$= \{(x-2)^2 - 21\} \{(x-2)^2 + 4\}$$

$$\therefore (f(x))_{\min} = (-21)(4) = -84$$

Which occurs at  $x = 2$

47. Here,  $2f'(x) = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} + \frac{f(x-h) - f(x)}{-h} \right)$

$$\begin{aligned}&= \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x-h)}{h} \right) \quad \dots(i) \\ \therefore 2f'(0) &= \lim_{h \rightarrow 0} \left( \frac{f(h) - f(0)}{h} + \frac{f(-h) - f(0)}{-h} \right) \\ &= \lim_{h \rightarrow 0} \frac{f(h) - f(-h)}{h} \quad \dots(ii)\end{aligned}$$

Now by given relation, we have

$$f(h) - f(-h) = \frac{f(x+h) - f(x-h)}{-h} \text{ and } f(0) = 1$$

From Eqs. (i) and (ii), we have  $\frac{f'(x)}{f(x)} = 2010$

$$\Rightarrow f(x) = e^{2010x}, f(0) = 1$$

$\therefore \{f(x)\}$  is non-periodic

48. Here,  $\int f(g(x)) \cos x dx = \int f(\log(\sin x)) \cdot \cos x dx$

$$\begin{aligned}&= \int e^{2010 \log(\sin x)} \cdot \cos x dx \\ &= \int (\sin x)^{2010} \cdot \cos x dx\end{aligned}$$



$$= \frac{(\sin x)^{2011}}{2011} + C$$

$$\therefore h(x) = \frac{(\sin x)^{2011}}{2011}$$

$$\Rightarrow h\left(\frac{\pi}{2}\right) = \frac{1}{2011}$$

49. Conceptual

$$50. \int (\sin 3\theta + \sin \theta) \cos \theta e^{\sin \theta} d\theta = (-4 \sin^3 \theta - 12 \cos^2 \theta - 20 \sin \theta + 32) + F$$

$$51. \int \frac{1}{(x^{5/6} + 5x^{7/6}) \cdot \sqrt{1+4x^{1/3}}} dx = -6 \tan^{-1}(x^{-1/3} + 4)^{1/2} + C = 6 \cot^{-1}(x^{-1/3} + 4)^{1/2} + C$$

$$52. \int \csc^5 x dx = -\frac{\csc^3 x \cot x}{4} - \frac{3 \csc x \cot x}{8} + \frac{3}{8} \log |\csc x - \cot x| + K$$

$$53. \int \frac{\cos x (1 + 4 \cos 2x)}{\sin x + 4 \sin x \cos^2 x} dx = \log |\sin x| + \frac{1}{2} \log |\sin^2 x + 5 \cos^2 x| + c,$$

$$54. \int \sin^{-1} x \cos^{-1} x dx = \int \left[ \frac{\pi}{2} \sin^{-1} x - (\sin^{-1} x)^2 \right] dx \\ = \frac{\pi}{2} \left( x \sin^{-1} x + \sqrt{1-x^2} \right) - \left( x (\sin^{-1} x)^2 + 2 \sin^{-1} x \sqrt{1-x^2} - 2x \right) + C$$

[Integrating by parts]

$$= \sin^{-1} x \left[ \frac{\pi}{2} x - x \sin^{-1} x - 2 \sqrt{1-x^2} \right] + \frac{\pi}{2} \sqrt{1-x^2} + 2x + C$$

$$\therefore f^{-1}(x) = \sin^{-1} x, f(x) = \sin x$$

55. Conceptual

$$56. \int \frac{dx}{(x^2 + 1)^2} = \frac{1}{2} \tan^{-1} x + \frac{1}{2} \frac{x}{x^2 + 1} + C$$

$$57. \int e^{x^3+x^2-1} (3x^4 + 2x^3 + 2x) dx \\ = \int \underbrace{x^2}_{I} e^{x^3+x^2-1} (3x^2 + 2x) dx + \int e^{x^3+x^2-1} 2x dx \\ = x^2 e^{x^3+x^2-1} - \int 2x e^{x^3+x^2-1} dx + \int e^{x^3+x^2-1} 2x dx \\ = x^2 e^{x^3+x^2-1} + C \\ \Rightarrow f(1) = e \text{ and } f(-1) = \frac{1}{e}$$