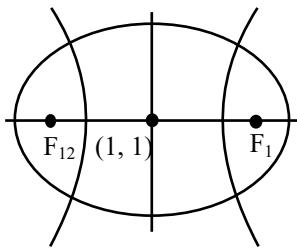


Sol.



$$e_1 = \sqrt{1 - \frac{75}{100}} = \frac{5}{10} = \frac{1}{2}$$

$$e_2 = 2$$

$$F_1(6, 1), F_2(-4, 1)$$

$$2ae_2 = 10 \Rightarrow a = \frac{5}{2} \Rightarrow 2a = 5$$

$$\Rightarrow \alpha = 5$$

$$4 = 1 + \frac{b^2}{a^2} \Rightarrow b^2 = 3a^2$$

$$b = \sqrt{3} \times \frac{5}{2}$$

$$\beta = 5\sqrt{3}$$

$$3\alpha^2 + 2\beta^2 = 3 \times 25 + 2 \times 25 \times 3 \\ = 225$$

7. Two vertices of a triangle ABC are A(3, -1) and B(-2, 3), and its orthocentre is P(1, 1). If the coordinates of the point C are (α, β) and the centre of the circle circumscribing the triangle PAB is (h, k) , then the value of $(\alpha + \beta) + 2(h + k)$ equals :

(1) 51

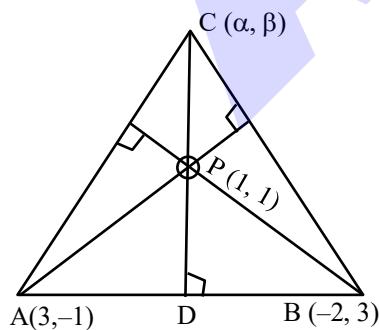
(2) 81

(3) 5

(4) 15

Ans. (3)

Sol.



$$M_{AB} = \frac{4}{-5} \Rightarrow M_{DP} = \frac{5}{4}$$

$$\text{Equation of PC is } y - 1 = \frac{5}{4}(x - 1) \dots\dots(1)$$

$$M_{AP} = \frac{2}{-2} = -1 \Rightarrow M_{BC} = +1$$

$$\text{Equation of BC is } y - 3 = (x + 2) \dots\dots(2)$$

On solving (1) and (2)

$$x + 4 = \frac{5}{4}(x - 1) \Rightarrow 4x + 16 = 5x - 5 \Rightarrow \alpha = 21$$

$$\Rightarrow \beta = y = x + 5 = 26$$

$$\alpha + \beta = 47$$

Equation of \perp bisector of AP

$$y - 0 = (x - 2) \dots\dots(3)$$

Equation of \perp bisector of AB

$$y - 1 = \frac{5}{4}\left(x - \frac{1}{2}\right) \dots\dots(4)$$

On solving (3) & (4)

$$(x - 3)4 = 5x - \frac{5}{2}$$

$$x = \frac{-19}{2} = h$$

$$y = \frac{-23}{2} = k$$

$$\Rightarrow 2(h + k) = -42$$

8. If the variance of the frequency distribution is 160, then the value of $c \in N$ is

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

(1) 5 (2) 8

(3) 7 (4) 6

Ans. (3)

Sol.

x	C	2C	3C	4C	5C	6C
f	2	1	1	1	1	1

$$\bar{x} = \frac{(2+2+3+4+5+6)C}{7} = \frac{22C}{7}$$



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$$\begin{aligned} \text{Var}(x) &= \frac{c^2(2+2^2+3^2+4^2+5^2+6^2)}{7} \\ &\quad - \left(\frac{22c}{7}\right)^2 \\ &= \frac{92c^2}{7} - c^2 \times \frac{484}{49} \\ &= \frac{(644-484)c^2}{49} = \frac{160c^2}{49} \\ 160 &= \frac{160 \times c^2}{49} \Rightarrow c = 7 \end{aligned}$$

9. Let the range of the function

$$f(x) = \frac{1}{2 + \sin 3x + \cos 3x}, x \in \text{IR} \text{ be } [a, b].$$

If α and β are respectively the A.M. and the G.M.

of a and b , then $\frac{\alpha}{\beta}$ is equal to :

- (1) $\sqrt{2}$ (2) 2
(3) $\sqrt{\pi}$ (4) π

Ans. (1)

$$\text{Sol. } f(x) = \frac{1}{2 + \sin 3x + \cos 3x}$$

$$\left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}} \right]$$

$$\frac{\alpha}{\beta} = \frac{a+b}{2\sqrt{ab}} = \frac{1}{2} \left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} \right)$$

$$= \frac{1}{2} \left(\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} + \sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}} \right)$$

$$= \frac{(2-\sqrt{2})+(2+\sqrt{2})}{2 \times \sqrt{2}} = \sqrt{2}$$

10. Between the following two statements :

Statement-I : Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Then the vector \vec{r} satisfying $\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$ and $\vec{a} \cdot \vec{r} = 0$ is of magnitude $\sqrt{10}$.

Statement-II : In a triangle ABC, $\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$.

- (1) Both Statement-I and Statement-II are incorrect
(2) Statement-I is incorrect but Statement-II is correct
(3) Both Statement-I and Statement-II are correct
(4) Statement-I is correct but Statement-II is incorrect

Ans. (2)

Sol. $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$$\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{a} \times \vec{r} = \vec{a} \times \vec{b}; \vec{a} \cdot \vec{r} = 0$$

$$\Rightarrow \vec{a} \times (\vec{r} - \vec{b}) = \vec{0}$$

$$\Rightarrow \vec{a} = \lambda(\vec{r} - \vec{b})$$

$$\vec{a} \cdot \vec{a} = \lambda(\vec{a} \cdot \vec{r} - \vec{a} \cdot \vec{b})$$

$$14 = -7\lambda \Rightarrow \lambda = -2$$

$$\frac{-\vec{a}}{2} = \vec{r} - \vec{b} \Rightarrow \vec{r} = \vec{b} - \frac{\vec{a}}{2}$$

$$= \frac{2\vec{b} - \vec{a}}{2} = \frac{3\hat{i} + \hat{k}}{2}$$

Statement (I) is incorrect

$$\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$$

$$2A + 2B + 2C = 2\pi$$

$$\cos 2A + \cos 2B + \cos 2C$$

$$= -1 - 4 \cos A \cdot \cos B \cdot \cos C$$

$$\geq -1 - 4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= -\frac{3}{2}$$

Statement (II) is correct.



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11. $\lim_{x \rightarrow 2} \left(\frac{\int_{x^3}^{(\pi/2)^3} (\sin(2t^{1/3}) + \cos(t^{1/3})) dt}{\left(x - \frac{\pi}{2}\right)^2} \right)$ is equal

to :

- (1) $\frac{9\pi^2}{8}$ (2) $\frac{11\pi^2}{10}$
 (3) $\frac{3\pi^2}{2}$ (4) $\frac{5\pi^2}{9}$

Ans. (1)

Sol. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{0 - \{\sin(2x) + \cos(x)\} \cdot 3x^2}{2\left(x - \frac{\pi}{2}\right)}$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\{2\sin x \cos x + \cos x\} 3x^2}{2\left(x - \frac{\pi}{2}\right)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \left\{ \frac{2\sin x \sin\left(\frac{\pi}{2} - x\right)}{2\left(x - \frac{\pi}{2}\right)} + \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} \right\} 3x^2$$

$$= \left(1(1) + \frac{1}{2}\right) 3\left(\frac{\pi}{2}\right)^2$$

$$= \frac{9\pi^2}{8}$$

12. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3} + \frac{1}{2}x^{-2/5}\right)^9$ is :

- (1) 21/4 (2) 69/16
 (3) 63/16 (4) 19/4

Ans. (1)

Sol. $T_{r+1} = {}^9C_r (x^{2/3})^{9-r} \left(\frac{x^{-2/5}}{2}\right)^r$

$$= {}^9C_r \left(\frac{1}{2}\right)^r (r)^{\left(\frac{6-2r-2r}{3}\right)}$$

for coefficient of $x^{2/3}$, put $6 - \frac{2r}{3} - \frac{2r}{5} = \frac{2}{3}$

$$\Rightarrow r = 5$$

$$\therefore \text{Coefficient of } x^{2/3} \text{ is } {}^9C_5 \left(\frac{1}{5}\right)^5$$

For coefficient of $x^{-2/5}$, put $6 - \frac{2r}{3} - \frac{2r}{5} = -\frac{2}{5}$

$$\Rightarrow r = 6$$

$$\text{Coefficient of } x^{-2/5} \text{ is } {}^9C_6 \left(\frac{1}{2}\right)^6$$

$$\text{Sum} = {}^9C_5 \left(\frac{1}{2}\right)^5 + {}^9C_6 \left(\frac{1}{2}\right)^6 = \frac{21}{4}$$

13. Let $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and A be a 2×2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta I = O$, then $2\beta - \alpha$ is equal to :

- (1) 16 (2) 2
 (3) 8 (4) 10

Ans. (4)

Sol. $BCB^{-1} = A$

$$\Rightarrow (BCB^{-1})(BCB^{-1}) = A \cdot A$$

$$\Rightarrow BCI CB^{-1} = A^2$$

$$\Rightarrow BC^2B^{-1} = A^2$$

$$\Rightarrow B^{-1}(BC^2B^{-1})B = B^{-1}(A \cdot A)B$$

From equation (1)

$$C^2 = A^{-1} \cdot A \cdot B$$

$$C^2 = B$$

$$\text{Also } AB^{-1} = A^{-1}$$

$$\Rightarrow AB^{-1} \cdot A = A^{-1} \cdot A = I$$

$$\Rightarrow A^{-1}(AB^{-1}A) = A^{-1}I$$

$$B^{-1}A = A^{-1}$$

Now characteristics equation of C^2 is

$$|C^2 - \lambda I| = 0$$

$$|B - \lambda I| = 0$$



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$$\Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ 1 & 5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(5-\lambda) - 3 = 0 \Rightarrow (\lambda^2 - 6\lambda + 5) - 3 = 0$$

$$\Rightarrow \lambda^2 - 6\lambda + 2 = 0$$

$$\Rightarrow \beta^2 - 6B + 2I = 0$$

$$\Rightarrow C^4 - 6C^2 + 2I = 0$$

$$\alpha = -6$$

$$\beta = 2$$

$$\therefore 2\beta - \alpha = 4 + 6 = 10$$

14. If $\log_e y = 3 \sin^{-1} x$, then $(1-x^2)y'' - xy'$ at $x = \frac{1}{2}$

is equal to :

$$(1) 9e^{\pi/6} \quad (2) 3e^{\pi/6}$$

$$(3) 3e^{\pi/2} \quad (4) 9e^{\pi/2}$$

Ans. (4)

Sol. $\ln(y) = 3 \sin^{-1} x$

$$\frac{1}{y} \cdot y' = 3 \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow y' = \frac{3y}{\sqrt{1-x^2}} \text{ at } x = \frac{1}{2}$$

$$\Rightarrow y' = \frac{3e^{3(\frac{\pi}{6})}}{\frac{\sqrt{3}}{2}} = 2\sqrt{3}e^{\frac{\pi}{2}}$$

$$\Rightarrow y'' = 3 \left(\frac{\sqrt{1-x^2}y' - y \frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)^2} \right)$$

$$\Rightarrow (1-x^2)y'' = 3 \left(3y + \frac{xy}{\sqrt{1-x^2}} \right)$$

$$\downarrow \text{at } x = \frac{1}{2}, y = e^{3\sin^{-1}(\frac{1}{2})} = e^{3(\frac{\pi}{6})} = e^{\frac{\pi}{2}}$$

$$(1-x^2)y'' \Big|_{x=\frac{1}{2}} = 3 \left(3e^{\frac{\pi}{2}} + \frac{\frac{1}{2}(e^{\frac{\pi}{2}})}{\sqrt{3}} \right)$$

$$= 3e^{\frac{\pi}{2}} \left(3 + \frac{1}{\sqrt{3}} \right)$$

$$(1-x^2)y'' - xy' \Big|_{x=\frac{1}{2}}$$

$$= 3e^{\frac{\pi}{2}} \left(3 + \frac{1}{\sqrt{3}} \right) - \frac{1}{2} \left(2\sqrt{3}e^{\frac{\pi}{2}} \right) = 9e^{\frac{\pi}{2}}$$

15. The integral $\int_{1/4}^{3/4} \cos \left(2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right) dx$ is equal

to:

$$(1) -1/2$$

$$(2) 1/4$$

$$(3) 1/2$$

$$(4) -1/4$$

Ans. (4)

$$\text{Sol. } I = \int_{1/4}^{3/4} \cos \left(2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right) dx$$

$$\int_{1/4}^{3/4} \cos \left(2 \left(\tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) \right) dx$$

$$\int_{1/4}^{3/4} \frac{1 - \tan^2 \left(\tan^{-1} \sqrt{\frac{1+x}{1-x}} \right)}{1 + \tan^2 \left(\tan^{-1} \sqrt{\frac{1+x}{1-x}} \right)} dx$$

$$= \int_{1/4}^{3/4} \frac{1 - \left(\frac{1+x}{1-x} \right)}{1 + \left(\frac{1+x}{1-x} \right)} dx = \int_{1/4}^{3/4} \frac{-2x}{2} dx$$

$$= \int_{1/4}^{3/4} (-x) dx = - \left(\frac{x^2}{2} \right)_{1/4}^{3/4}$$

$$= -\frac{1}{2} \left[\frac{9}{16} - \frac{1}{16} \right]$$

$$= -\frac{1}{4}$$



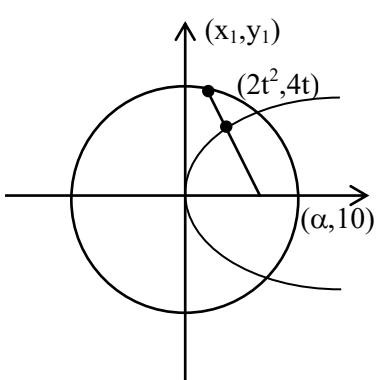
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SECTION-B

21. Consider the circle $C : x^2 + y^2 = 4$ and the parabola $P : y^2 = 8x$. If the set of all values of α , for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q) , then $(2q - p)^2$ is equal to _____.
Ans. (80)

Sol.



$$T = S_1$$

$$xx_1 + yy_1 = x_1^2 + y_1^2$$

$$\alpha x_1 = x_1^2 + y_1^2$$

$$\alpha(2t^2) = 4t^4 + 16t^2$$

$$\alpha = 2t^2 + 8$$

$$\frac{\alpha - 8}{2} = t^2$$

$$\text{Also, } 4t^4 + 16t^2 - 4 < 0$$

$$t^2 = -2 + \sqrt{5}$$

$$\alpha = 4 + 2\sqrt{5}$$

$$\therefore \alpha \in (8, 4 + 2\sqrt{5})$$

$$\therefore (2q - p)^2 = 80$$

22. Let the set of all values of p , for which

$f(x) = (p^2 - 6p + 8)(\sin^2 x - \cos^2 x) + 2(2-p)x + 7$ does not have any critical point, be the interval (a, b) . Then $16ab$ is equal to _____.
Ans. (252)

Sol. $f(x) = -(p^2 - 6p + 8) \cos 4x + 2(2-p)x + 7$

$$f'(x) = +4(p^2 - 6p + 8) \sin 4x + (4-2p) \neq 0$$

$$\sin 4x \neq \frac{2p-4}{4(p-4)(p-2)}$$

$$\sin 4x \neq \frac{2(p-2)}{4(p-4)(p-2)}$$

$$p \neq 2$$

$$\sin 4x \neq \frac{1}{2(p-4)}$$

$$\Rightarrow \left| \frac{1}{2(p-4)} \right| > 1$$

on solving we get

$$\therefore p \in \left(\frac{7}{2}, \frac{9}{2} \right)$$

$$\text{Hence } a = \frac{7}{2}, b = \frac{9}{2}$$

$$\therefore 16ab = 252$$

23. For a differentiable function $f : IR \rightarrow IR$, suppose

$$f'(x) = 3f(x) + \alpha, \text{ where } \alpha \in IR, f(0) = 1 \text{ and}$$

$$\lim_{x \rightarrow -\infty} f(x) = 7. \text{ Then } 9f(-\log_e 3) \text{ is equal to _____.}$$

$$\text{Ans. (61)}$$

$$\frac{dy}{dx} - 3y = \alpha$$

$$\text{If } y = e^{\int -3dx} = e^{-3x}$$

$$\therefore y - e^{-3x} = \int e^{-3x} \cdot \alpha dx$$

$$y e^{-3x} = \frac{\alpha e^{-3x}}{-3} + c$$

$$(* e^{3x})$$

$$y = \frac{\alpha}{-3} + C \cdot e^{3x}$$

on substituting $x = 0, y = 1$

$$x \rightarrow -\infty, y = 7$$

$$\text{we get } y = 7 - 6e^{3x}$$

$$9f(-\log_e 3) = 61$$



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24. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is _____.

Ans. (70)
Sol. $N = a b c$

(i) All distinct digits

$$a + b + c = 14$$

$$a \geq 1$$

$$b, c \in \{0 \text{ to } 9\}$$

by hit & trial : 8 cases

(6, 5, 3) (8, 6, 0) (9, 4, 1)

(7, 6, 1) (8, 5, 1) (9, 3, 2)

(7, 5, 2) (8, 4, 2)

(7, 4, 3) (9, 5, 0)

(ii) 2 same, 1 diff $a = b ; c$

$$2a + c = 14$$

by values :

$$\begin{cases} (3, 8) \\ (4, 6) \\ (5, 4) \\ (6, 2) \\ (7, 0) \end{cases} \quad \text{Total } \frac{3!}{2!} \times 5 - 1$$

= 14 cases

(iii) all same :

$$3a = 14$$

$$a = \frac{14}{3} \times \text{rejected}$$

0 cases

Hence, Total cases :

$$8 \times 3! + 2 \times (4) + 14$$

$$= 48 + 22$$

$$= 70$$

25. Let $A = \{(x, y) : 2x + 3y = 23, x, y \in \mathbb{N}\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____.

Ans. (24)

Sol. $2x + 3y = 23$

$$x = 1 \quad y = 7$$

$$x = 4 \quad y = 5$$

$$x = 7 \quad y = 3$$

$$x = 10 \quad y = 1$$

A B

(1, 7) 1

(4, 5) 4

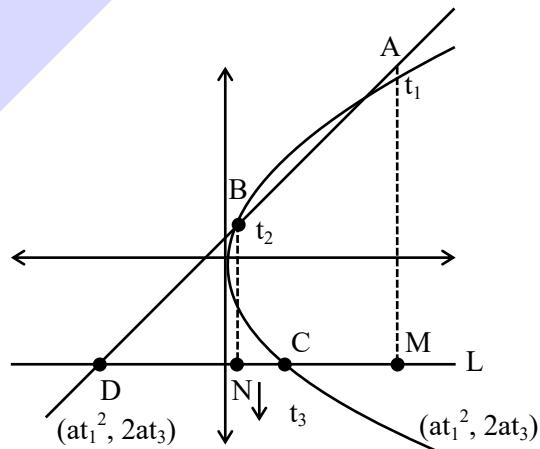
(7, 3) 7

(10, 1) 10

The number of one-one functions from A to B is equal to 4!

26. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L.

Then $\left(\frac{AM \cdot BN}{CD} \right)^2$ is equal to _____.

Ans. (36)
Sol.

Sol.

$$m_{AB} = m_{AD}$$

$$\Rightarrow \frac{2}{t_1 + t_2} = \frac{2a(t_1 - t_3)}{at_1^2 - \alpha}$$

$$\Rightarrow at_1^2 - \alpha = a(t_1^2 - t_1 t_3 + t_1 t_2 - t_2 t_3)$$

$$\Rightarrow \alpha = a(t_1 t_3 + t_2 t_3 - t_1 t_2)$$

$$AM = |2a(t_1 - t_3)|, BN = |2a(t_2 - t_3)|,$$

$$CD = |at_3^2 - \alpha|$$


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$$\begin{aligned} CD &= \left| at_3^2 - a(t_1t_3 + t_2t_3 - t_1t_2) \right| \\ &= a \left| t_3^2 - t_1t_3 - t_2t_3 + t_1t_2 \right| \\ &= a \left| t_3(t_3 - t_1) - t_2(t_3 - t_1) \right| \end{aligned}$$

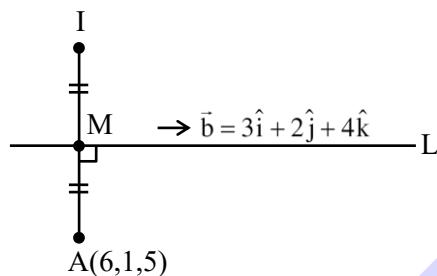
$$CD = a |(t_3 - t_2)(t_3 - t_1)|$$

$$\left(\frac{AM \cdot BN}{CD} \right)^2 = \left\{ \frac{2a(t_1 - t_3) \cdot 2a(t_2 - t_3)}{a(t_3 - t_2)(t_3 - t_1)} \right\}^2$$

$$16a^2 = 16 \times \frac{9}{4} = 36$$

27. The square of the distance of the image of the point $(6, 1, 5)$ in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is _____.

Ans. (62)



Sol.

$$\text{Let } M(3\lambda + 1, 2\lambda, 4\lambda + 2)$$

$$\vec{AM} \cdot \vec{b} = 0$$

$$\Rightarrow 9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0$$

$$\Rightarrow 29\lambda = 29$$

$$\Rightarrow \lambda = 1$$

$$M(4, 2, 6), I(2, 3, 7)$$

$$\text{Required Distance} = \sqrt{4+9+49} = \sqrt{62}$$

Ans. 62

$$\begin{aligned} 28. \quad &\text{If } \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+1012} \right) \\ &- \left(\frac{1}{2 \cdot 1} + \frac{1}{4 \cdot 3} + \frac{1}{6 \cdot 5} + \dots + \frac{1}{2024 \cdot 2023} \right) \\ &= \frac{1}{2024}, \text{ then } \alpha \text{ is equal to-} \end{aligned}$$

Ans. (1011)

$$\begin{aligned} \text{Sol. } &\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+2012} \right) \\ &- \left\{ \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots + \left(\frac{1}{2023} - \frac{1}{2024} \right) \right\} = \frac{1}{2024} \\ &\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+2012} \right) \\ &- \left\{ \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right) + \dots + \frac{1}{2023} \right. \\ &\quad \left. - \frac{1}{2024} - 2 \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2022} \right) \right\} = \frac{1}{2024} \\ &\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+2012} \right) \\ &- \left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{2023} \right) \\ &\quad + \frac{1}{2024} + \left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{1011} \right) = \frac{1}{2024} \\ &\Rightarrow \frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+2012} \\ &= \frac{1}{1012} + \frac{1}{1013} + \dots + \frac{1}{2023} \\ &\Rightarrow \alpha = 1011 \end{aligned}$$

29. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2 \sin^{-1} x + 3 \cos^{-1} x = \frac{2\pi}{5}$, is _____.

Ans. (0)

$$\text{Sol. } 2 \sin^{-1} x + 3 \cos^{-1} x = \frac{2\pi}{5}$$

$$\Rightarrow \pi + \cos^{-1} x = \frac{2\pi}{5}$$

$$\Rightarrow \cos^{-1} x = \frac{-3\pi}{5}$$

Not possible

Ans. 0



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30. Consider the matrices : $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$

and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m , for which the

system of equations $AX = B$ has a negative solution
(i.e., $x < 0$ and $y < 0$), be the interval (a, b) .

Then $8 \int_a^b |A| dm$ is equal to _____.

Ans. (450)

Sol. $A = \begin{pmatrix} 2 & -5 \\ 3 & m \end{pmatrix}$, $B = \begin{pmatrix} 20 \\ m \end{pmatrix}$

$$X = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$2x - 5y = 20 \quad \dots(1)$$

$$3x + my = m \quad \dots(2)$$

$$\Rightarrow y = \frac{2m - 60}{2m + 15}$$

$$y < 0 \Rightarrow m \in \left(\frac{-15}{2}, 30 \right)$$

$$x = \frac{25m}{2m + 15}$$

$$x < 0 \Rightarrow m \in \left(\frac{-15}{2}, 0 \right)$$

$$\Rightarrow m \in \left(\frac{-15}{2}, 0 \right)$$

$$|A| = 2m + 15$$

Now,

$$8 \int_{\frac{-15}{2}}^0 (2m + 15) dm = 8 \left\{ m^2 + 15m \right\}_{\frac{-15}{2}}^0$$

$$\Rightarrow 8 \left\{ - \left(\frac{225}{4} - \frac{225}{2} \right) \right\}$$

$$= 8 \times \frac{225}{4} = 450$$




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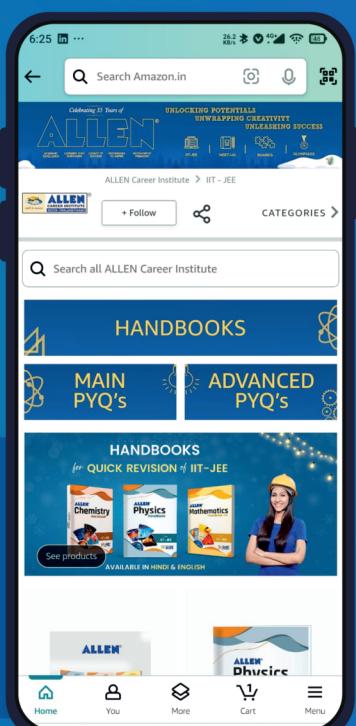
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