3. $\dfrac{[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{b}\ \bar{a}\ \bar{c}]}=$

1) 1 2) -1
3) 2 4) 3

4. If $\bar{p},\bar{q},\bar{r}$ are any three vectors. Which of the following statement is not true ?

1) $(\bar{q}\times\bar{r}).\bar{p}=\bar{p}.(\bar{q}\times\bar{r})$
2) $(\bar{p}\times\bar{q}).\bar{r}=\bar{p}.(\bar{q}\times\bar{r})$
3) $(\bar{p}\times\bar{q}).\bar{r}=(\bar{q}\times\bar{p}).\bar{r}$
4) $(\bar{p}\times\bar{q}).\bar{r}$ represents the volume of the Parallelopiped with coterminus edges $\bar{p},\bar{q},\bar{r}$

5. $[\bar{i}-\bar{j}\ \bar{k}]+[-\bar{i}-\bar{j}\ \bar{k}]+[\bar{i}-\bar{k}\ \bar{k}]=$

1) 0 2) 1 3) -1 4) 2

6. If $\bar{a},\bar{b},\bar{c}$ are the sides of a triangle ABC then $[\bar{a}\bar{b}\bar{c}]=$

1) 0 2) 1 3) - 1 4) 2

7. If $\bar{a},\bar{b},\bar{c}$ are three non-coplanar vectors, then $\dfrac{\bar{a}.(\bar{b}\times\bar{c})}{(\bar{c}\times\bar{a}).\bar{b}}+\dfrac{\bar{b}.(\bar{a}\times\bar{c})}{\bar{c}.(\bar{a}\times\bar{b})}=$

1) 1 2) 2 3) 3 4) 0

8. If $\bar{x}.\bar{a}=0,\bar{x}.\bar{b}=0,\bar{x}.\bar{c}=0$ for some non-zero vector $\bar{x}$, then

1) $[\bar{a}\ \bar{b}\ \bar{c}]=0$ 2) $[\bar{a}\ \bar{b}\ \bar{c}]\neq 0$
3) $[\bar{a}\ \bar{b}\ \bar{c}]=1$ 4) $[\bar{a}\ \bar{b}\ \bar{c}]=2$

9. If $\bar{r}.\bar{a}=0,\bar{r}.\bar{b}=0,\bar{r}.\bar{c}=0$ where $\bar{a},\bar{b},\bar{c}$ are non-coplanar, then

1) $\bar{r}=0$
2) $\bar{r}$ is perpendicular to $(\bar{a}\times\bar{b})$
3) $\bar{r}$ is perpendicular to $(\bar{b}\times\bar{c})$
4) $\bar{r}$ is perpendicular to $(\bar{c}\times\bar{a})$

10. I : No two skew lines intersects.
II : No two skew lines are parallel.
Which of above is correct.

1) only I 2) only II
3) Both I and II 4) Neither I nor II

11. If $\bar{a},\bar{b}$ and $\bar{c}$ are mutually perpendicular unit vectors, then $[\bar{a}\ \bar{b}\ \bar{c}]=$

1) 0 2) -1 3) 1 4) ±1

KEY

01) 4 02) 1 03) 2 04) 3 05) 1
06) 1 07) 4 08) 1 09) 1 10) 3
11) 3

EXERCISE-I

CRTQ & SPQ LEVEL-I

VOLUME OF TETRAHEDRON, PARALLELOPIPED, SKEW LINES, SCALAR TRIPLE PRODUCT, PROPERTIES

C.R.T.Q Class Room Teaching Questions

1. If $\bar{a}=\bar{i}-2\bar{j}-3\bar{k},\ \bar{b}=2\bar{i}+\bar{j}-\bar{k},\ \bar{c}=\bar{i}+3\bar{j}-2\bar{k}$ then $\bar{a}.(\bar{b}\times\bar{c})$

1) 10 2) -10 3) -20 4) 20

2. $(\bar{a}+2\bar{b}-\bar{c}).(\bar{a}-\bar{b})\times(\bar{a}-\bar{b}-\bar{c})=$

1) $-[\bar{a}\ \bar{b}\ \bar{c}]$ 2) $2[\bar{a}\ \bar{b}\ \bar{c}]$
3) $3[\bar{a}\ \bar{b}\ \bar{c}]$ 4) 0

3. $[\bar{i}\ \bar{j}\ \bar{k}]+[\bar{j}\ \bar{k}\ \bar{i}]+[\bar{k}\ \bar{i}\ \bar{j}]+[\bar{i}\ \bar{k}\ \bar{j}]+[\bar{j}\ \bar{i}\ \bar{k}]+[\bar{k}\ \bar{j}\ \bar{i}]$

1) 1 2) 2 3) 6 4) 0

4. If $[\bar{a}+2\bar{b}\ \ 2\bar{b}+\bar{c}\ \ 5\bar{c}+\bar{a}]=k[\bar{a}\bar{b}\bar{c}]$ then k =

1) 2 2) 4 3) 8 4) 12

5. If $\bar{a}=2\bar{i}-\bar{j},\ \bar{b}=4\bar{j}+\bar{k},\bar{c}=3\bar{k}$ then $(2\bar{a}+\bar{b}+\bar{c}).(-\bar{b}+2\bar{c})\times\bar{c}$

1) 48 2) 28 3) -28 4) -48

6. If $\left[\bar{i}+4\bar{j}+6\bar{k}\ \ 2\bar{i}+a\bar{j}+3\bar{k}\ \ \bar{i}+2\bar{j}-3\bar{k}\right]=18$ then a =

1) -4 2) 4 3) ±4 4) 1

7. The vectors $\bar{i}+4\bar{j}+6\bar{k}, 2\bar{i}+4\bar{j}+3\bar{k}$ and $\bar{i}+2\bar{j}+3\bar{k}$ form

1) right handed system
2) left handed system
3) cannot be decide
4) orthonormal triad

8. If $\bar{u},\bar{v},\bar{w}$ are three non coplanar vectors then $(\bar{u}+\bar{v}-\bar{w}).\{(\bar{u}-\bar{v})\times(\bar{v}-\bar{w})\}=$

1) $\bar{0}$ 2) $\bar{u}.(\bar{v}\times\bar{w})$

3) $\bar{u}.(\bar{w}\times\bar{v})$ 4) $3\bar{u}.(\bar{v}\times\bar{w})$

$\bar{a},\bar{b}$ and $\bar{c}$ are mutually perpendicular unit vectors then $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=$

1) 0 2) -1 3) 1 4) ±1

0. If $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=1$ then

$$\frac{\bar{a}.(\bar{b}\times\bar{c})}{(\bar{c}\times\bar{a}).\bar{b}}+\frac{\bar{b}.(\bar{c}\times\bar{a})}{(\bar{a}\times\bar{b}).\bar{c}}+\frac{\bar{c}(\bar{a}\times\bar{b})}{(\bar{b}\times\bar{c}).\bar{a}}$$

1) 3 2) 1 3) -1 4) 0

1. The volume of parallelo piped whose coterminus edges are $\bar{i}-\bar{j},\ 2\bar{i}+\bar{j}+\bar{k},\ \bar{i}-\bar{j}+3\bar{k}$ is _____

1) 3 2) -2 3) 9 4) -4

The volume of a parallelopiped whose edges are represented by $-12\bar{i}+\lambda\bar{k}$, $3\bar{j}-\bar{k}$ and $2\bar{i}+\bar{j}-15\bar{k}$ is 546 then $\lambda=$

1) -3 2) -2 3) 2 4) 3

If $\left[\bar{a}\,\bar{b}\,\bar{c}\right]=-4$ then the volume of the parallelopiped with coterminus edges $\bar{a}+2\bar{b},\ 2\bar{b}+\bar{c},\ 3\bar{c}+\bar{a}$ is (in cu units)

1) 32 2) -32 3) 8 4) 12

14. Volume of the tetrahedron with vertices at $(0,0,0),(1,0,0),(0,1,0)$ and $(0,0,1)$ is (cu units)

1) $\frac{1}{6}$ 2) $\frac{1}{4}$ 3) $\frac{1}{3}$ 4) $\frac{1}{2}$

15. If the volume of the tetrahedron with edges $2\bar{i}+\bar{j}+\bar{k}$, $\bar{i}+a\bar{j}+\bar{k}$ and $\bar{i}+2\bar{j}-\bar{k}$ is one cubic unit then $a=$

1) 1 2) -2 3) 2 4) -1

16. If $\left[\bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}\ \ \bar{a}\times\bar{b}\right]=\left[\bar{a}\,\bar{b}\,\bar{c}\right]^k$ then $k=$

1) 1 2) 2 3) 3 4) 4

17. The volume of tetrahedron with edges $\bar{i}\times\bar{j}\ \ \bar{j}\times\bar{k}\ \ \bar{k}\times\bar{i}$

1) 1 2) 1/6 3) 3 4) 1/3

18. The vector equation of the plane passing through the points $(1,-2,5),(0,-5,-1)$ and $(-3,5,0)$ is _____

1) $\left[\bar{r}-(\bar{i}-2\bar{j}+5\bar{k})\ \ -5\bar{i}-\bar{k}\ \ -3\bar{i}+5\bar{j}\right]=0$

2) $\left[\bar{r}\ \ -5\bar{i}-\bar{k}\ \ -3\bar{i}+5\bar{j}\right]=0$

3) $\left[\bar{r}-(\bar{i}-2\bar{j}+5\bar{k})\ \ -\bar{i}-3\bar{j}-6\bar{k}\ \ -4\bar{i}+7\bar{j}+5\bar{k}\right]=0$

4) $\left[\bar{r}-\bar{i}-3\bar{j}-6\bar{k}-4\bar{i}+7\bar{j}-5\bar{k}\right]=0$

19. The vector equation of the plane passing through the points $\bar{i}-2\bar{j}+\bar{k},\ 3\bar{k}-2\bar{j}$ and parallel to the vector $2\bar{i}+\bar{j}+\bar{k}$ is_____

1) $\left[\bar{r}-(\bar{i}-2\bar{j}+\bar{k})\ \ 3\bar{k}-2\bar{j}\ \ 2\bar{i}+\bar{j}+\bar{k}\right]=0$

2) $\left[\bar{r}-(\bar{i}-2\bar{j}+\bar{k})\ \ -\bar{i}+2\bar{k}\ \ 2\bar{i}+\bar{j}+\bar{k}\right]=0$

3) $\left[\bar{r}-(\bar{i}-2\bar{j}+\bar{k})\ \ 2\bar{k}-2\bar{j}\ \ \bar{i}+3\bar{j}+\bar{k}\right]=0$

4) $\left[\bar{r}-(\bar{i}-2\bar{j}+\bar{k})\ \ -\bar{i}+2\bar{k}\ \ \bar{i}+3\bar{j}+\bar{k}\right]=0$

3 SCALAR TRIPLE PRODUCT

20. The vector equation of the plane passing through $\bar{i}+\bar{j}+\bar{k}$ and parallel to the vectors $2\bar{i}+3\bar{j}-\bar{k}$, $\bar{i}+2\bar{j}+3\bar{k}$ is

1) $\left[\bar{r}-(\bar{i}+\bar{j}+\bar{k})\ \ 2\bar{i}+3\bar{j}-\bar{k}\ \ \bar{i}+2\bar{j}+3\bar{k}\right]=0$

2) $\left[\bar{r}\ \ 2\bar{i}+3\bar{j}-\bar{k}\ \ \bar{i}+2\bar{j}+5\bar{k}\right]=0$

3) $\left[\bar{r}-(\bar{i}+\bar{j}+\bar{k})\ \ \bar{i}+2\bar{j}-2\bar{k}\ \ \bar{j}+2\bar{k}\right]=0$

4) $\left[\bar{r}\ \ \bar{i}+2\bar{j}-2\bar{k}\ \ \bar{j}+2\bar{k}\right]=0$

21. The equation of the plane passing through the point with position vector $\bar{a}$ and perpendicular to $\bar{b}$ is ____

1) $\bar{r}.(\bar{a}\times\bar{b})=0$ 2) $\bar{r}=\bar{a}\times\bar{b}$

3) $\bar{r}=\bar{b}\times\bar{a}$ 4) $(\bar{r}-\bar{a})\bar{b}=0$

22. The perpendicular distance from origin to the plane passing through the points $2\bar{i}-2\bar{j}+\bar{k}$, $3\bar{i}+2\bar{j}-\bar{k}$, $3\bar{i}-\bar{j}-2\bar{k}$ is

1) $\frac{12}{\sqrt{30}}$ 2) $\frac{25}{\sqrt{110}}$

3) $\frac{10}{\sqrt{60}}$ 4) $\frac{15}{\sqrt{187}}$

23. The distance between the line $\bar{r}=2\bar{i}-2\bar{j}+3\bar{k}+\lambda(\bar{i}-\bar{j}+4\bar{k})$ and the plane $\bar{r}.(\bar{i}+5\bar{j}+\bar{k})=5$ is ____

1) $\frac{10}{3}$ 2) $\frac{3}{10}$ 3) $\frac{10}{3\sqrt{3}}$ 4) $\frac{10}{9}$

24. The shortest distance between the lines $\bar{r}=\bar{i}+2\bar{j}+3\bar{k}+s(2\bar{i}+3\bar{j}+4\bar{k})$ and $\bar{r}=(2\bar{i}+4\bar{j}+5\bar{k})+t(3\bar{i}+4\bar{j}+5\bar{k})$ is

1) $\frac{1}{6}$ 2) $\frac{1}{\sqrt{6}}$ 3) $\frac{1}{3}$ 4) $\frac{1}{\sqrt{3}}$

25. The lines $\bar{r}=\bar{a}+t\bar{b}$, $\bar{r}=\bar{c}+s\bar{d}$ are coplanar if

1) $(\bar{a}-\bar{b}).\bar{c}\times\bar{d}=0$

2) $(\bar{a}-\bar{c}).\bar{b}\times\bar{d}=0$

3) $(\bar{b}-\bar{c}).\bar{a}\times\bar{d}=0$

4) $(\bar{b}-\bar{d}).\bar{a}\times\bar{c}=0$

26. If $\bar{a},\bar{b},\bar{c}$ are three non coplanar vectors $\bar{p}=\frac{\bar{b}\times\bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]}$, $\bar{q}=\frac{\bar{c}\times\bar{a}}{[\bar{a}\ \bar{b}\ \bar{c}]}$, $\bar{r}=\frac{\bar{a}\times\bar{b}}{[\bar{a}\ \bar{b}\ \bar{c}]}$ then $(2\bar{a}+3\bar{b}+4\bar{c}).\bar{p}+(2\bar{b}+3\bar{c}+4\bar{a})\bar{q}+(2\bar{c}+3\bar{a}+4\bar{b}).\bar{r}$

1) -6 2) 6 3) 3 4) 2

S.P.Q. Student Practice Questions

27. $(2\bar{i}-3\bar{j}+\bar{k}).(\bar{i}-\bar{j}+2\bar{k})\times(2\bar{i}+\bar{j}+\bar{k})$

1) -14 2) 14 3) -12 4) 12

28. $(\bar{a}+\bar{b}).(\bar{b}+\bar{c})\times(\bar{a}+\bar{b}+\bar{c})$

1) 0 2) $-[\bar{a}\ \bar{b}\ \bar{c}]$

3) $2[\bar{a}\ \bar{b}\ \bar{c}]$ 4) $[\bar{a}\ \bar{b}\ \bar{c}]$

29. $[\bar{i}\ \bar{j}\ \bar{k}]+[\bar{j}\ \bar{k}\ \bar{i}]+[\bar{i}\ \bar{k}\ \bar{j}]=$

1) 1 2) -1 3) 3 4) 10

30. If $[\bar{a}\bar{b}\bar{c}]\neq 0$ then $\frac{[\bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\ \ \bar{c}+\bar{a}]}{[\bar{a}\ \bar{b}\ \bar{c}]}$

1) 1 2) -1 3) 2 4) -3

If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors and $(\bar{a}+2\bar{b}+\bar{c}).(\bar{a}-\bar{b})\times(\bar{a}-\bar{b}-\bar{c}) = \lambda[\bar{a}\ \bar{b}\ \bar{c}]$ then $\lambda =$ ______

1) 3 2) 2 3) 7 4) 8

If $\bar{a} = 2\bar{i}+3\bar{j}$, $\bar{b} = \bar{i}+\bar{j}+\bar{k}$, $\bar{c} = 3\bar{i}+\lambda\bar{j}+2\bar{k}$ and $[\bar{a}\ \bar{b}\ \bar{c}] = -1$ then $\lambda =$

1) 4 2) 3 3) 2 4) 1

The vectors $2\bar{i}+3\bar{j}-2\bar{k}$, $2\bar{i}-\bar{j}+4\bar{k}$, $3\bar{i}+2\bar{j}$ forms

1) right handed system
2) left handed system
3) cannot be decide
4) orthonormal triad

If $\bar{a}, \bar{b}$ and $\bar{c}$ are three non-coplanar vectors then $(\bar{a}+\bar{b}+\bar{c}).\{(\bar{a}+\bar{b})\times(\bar{a}+\bar{c})\} =$

1) 0 2) $[\bar{a}\ \bar{b}\ \bar{c}]$
3) $-[\bar{a}\ \bar{b}\ \bar{c}]$ 4) 4

If $\bar{a}, \bar{b}, \bar{c}$ are three unit vectors, $\bar{a}$ is parallel to $\bar{b}$ and $\bar{c}$ then $[\bar{a}\ \bar{b}\ \bar{c}] =$

1) 0 2) -1 3) 1 4) 2

If the volume of tetrahedron with edges $\bar{i}+\bar{j}-\bar{k}$, $\bar{i}+a\bar{j}+\bar{k}$ and $\bar{i}+2\bar{j}-\bar{k}$ is $\frac{a}{6}$ then $a =$

1) 0 2) 1 3) 2 4) -1

The volume of the parallelopiped having coterminus edges $\bar{i}+\bar{j}+\bar{k}$, $\bar{i}-\bar{j}$, $\bar{i}+2\bar{j}-\bar{k}$ is

1) 2 2) 3 3) 5 4) 7

38. If $\bar{a}=2\bar{i}+3\bar{j}$, $\bar{b}=\bar{i}+\bar{j}+\bar{k}$, $\bar{c}=\lambda\bar{i}+4\bar{j}+2\bar{k}$ are coterminus edges of a parallelopiped of volume 2 cubic units then λ is

1) 1 2) 2 3) 3 4) 4

39. The volume of parallelopiped with vectors $\bar{a}+2\bar{b}-\bar{c}$, $\bar{a}-\bar{b}$, $\bar{a}-\bar{b}-\bar{c}$ as coterminus edges is $k[\bar{a}\ \bar{b}\ \bar{c}]$ then $|k|$ is

1) -2 2) 2 3) -3 4) 3

40. The volume of the tetrahedron with edges $\bar{i}+\bar{j}+\bar{k}, \bar{i}-\bar{j}+\bar{k}, \bar{i}+2\bar{j}-\bar{k}$ is (In cu units)

1) 2 2) $\frac{3}{2}$ 3) $\frac{1}{3}$ 4) $\frac{2}{3}$

41. The volume of the tetrahedron whose vertices are $(2,1,1), (1,-1,2), (0,1,-1)$ and $(1,-2-1)$ is ___ (in cu. units)

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{8}{3}$ 4) $\frac{5}{3}$

42. If $\bar{a},\bar{b},\bar{c}$ are vectors such that $[\bar{a}\ \bar{b}\ \bar{c}]=4$ then $[\bar{a}\times\bar{b}\ \ \bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}]$ is

1) 16 2) 64 3) 4 4) 8

43. The value of $[\bar{i}\times\bar{j}\ \ \bar{j}\times\bar{k}\ \ \bar{k}\times\bar{i}]$ is

1) 0 2) 1 3) -1 4) 2

44. The equation of the plane passing through the points $A=(2,3,-1)$, $B=(4,5,2)$, $C=(3,6,5)$ is

1) $3x+9y+4z+25=0$
2) $3x-9y+4z+25=0$
3) $3x-9y+4z-25=0$
4) $3x-9y-4z-25=0$

45. The cartesian equation of the plane passing through the points $4\bar{i}+\bar{j}-2\bar{k}, 5\bar{i}+2\bar{j}+\bar{k}$ and parallel to the vector $3\bar{i}-\bar{j}+4\bar{k}$ is __
 1) $5x-y-z+3=0$
 2) $2x-6y+5z-1=0$
 3) $2x-6y+z-15=0$
 4) $7x+5y-4z-41=0$

46. The equation of the plane passing through the points $A(3,-2,-1)$ and parallel to the vectors $(1,-2,4)\,and\,(3,2,-5)$ is
 1) $2x+17y+8z+36=0$
 2) $2x+17y+8z-36=0$
 3) $2x+17y-8z-36=0$
 4) $2x-17y-8z-36=0$

47. The vector equation of the plane containing the line $\bar{r}=\bar{a}+s\bar{b}$ and parallel to the line $\bar{r}=\bar{c}+t\bar{d}$ is
 1) $[\bar{r}-\bar{a}\ \ \bar{b}\ \ \bar{d}]=0$
 2) $[\bar{r}-\bar{b}\ \ \bar{c}\ \ \bar{d}]=0$
 3) $[\bar{r}-\bar{d}\ \ \bar{a}\ \ \bar{b}]=0$
 4) $[\bar{r}-\bar{c}\ \ \bar{a}\ \ \bar{d}]=0$

48. The perpendicular distance from origin to the plane passing through the points $2\bar{i}+\bar{j}+3\bar{k}, \bar{i}+3\bar{j}+2\bar{k}, 3\bar{i}+2\bar{j}+\bar{k}$ is
 1) $\sqrt{3}$ 2) $2\sqrt{3}$ 3) $\frac{\sqrt{3}}{2}$ 4) $\sqrt{\frac{3}{2}}$

49. The distance between the plane whose equation is $\bar{r}.(2\bar{i}+\bar{j}-3\bar{k})=5$ and the line whose equation is $\bar{r}=\bar{i}+\lambda(2\bar{i}+5\bar{j}+3\bar{k})$ is
 1) $\frac{3}{\sqrt{14}}$ 2) $\frac{5}{\sqrt{14}}$ 3) 5 4) 0

50. The shortest distance between the lines whose equations are $\bar{r}=t(\bar{i}+\bar{j}+\bar{k})$, $\bar{r}=\bar{k}+s(\bar{i}-2\bar{j}+3\bar{k})$ is
 1) 3 2) $\frac{3}{\sqrt{38}}$ 3) $\sqrt{\frac{3}{14}}$ 4) $\frac{2}{\sqrt{13}}$

51. The equation of the plane containing the lines $\bar{r}=\bar{a}+t\bar{b}$ and $\bar{r}=\bar{b}+s\bar{a}$ is
 1) $[\bar{r}\ \bar{a}\ \bar{b}]=0$ 2) $\bar{r}.\bar{a}=\bar{a}.\bar{b}$
 3) $\bar{r}.\bar{a}=\bar{r}.\bar{b}$ 4) $\bar{r}.\bar{b}=\bar{a}.\bar{b}$

52. If $\bar{a}',\bar{b}',\bar{c}'$ represents the reciprocal system of vectors of $\bar{a},\bar{b},\bar{c}$ then $\bar{a}\bar{a}'+\bar{b}\bar{b}'+\bar{c}\bar{c}'=$
 1) 0 2) 1 3) 2 4) 3

KEY

01) 3	02) 3	03) 4	04) 4	05) 4
06) 2	07) 2	08) 2	09) 4	10) 1
11) 3	12) 1	13) 2	14) 1	15) 2
16) 2	17) 2	18) 3	19) 2	20) 1
21) 4	22) 2	23) 3	24) 2	25) 2
26) 2	27) 3	28) 4	29) 1	30) 3
31) 1	32) 1	33) 1	34) 3	35) 1
36) 3	37) 3	38) 4	39) 4	40) 4
41) 3	42) 1	43) 2	44) 2	45) 4
46) 1	47) 1	48) 2	49) 1	50) 2
51) 1	52) 4			

Hints Solutions

1. $[\bar{a}\ \bar{b}\ \bar{c}]=\begin{vmatrix}1 & -2 & -3\\ 2 & 1 & -1\\ 1 & 3 & -2\end{vmatrix}$

2. $\begin{vmatrix}1 & 2 & -1\\ 1 & -1 & 0\\ 1 & -1 & -1\end{vmatrix}[\bar{a}\ \bar{b}\ \bar{c}]$

1) $2[\bar{a}\,\bar{b}\,\bar{c}]\bar{r}$ 2) $3[\bar{a}\,\bar{b}\,\bar{c}]\bar{r}$

3) $[\bar{a}\,\bar{b}\,\bar{c}]\bar{r}$ 4) $5[\bar{a}\,\bar{b}\,\bar{c}]\bar{r}$

If $\bar{a}=2\bar{i}-\bar{j}+\bar{k}$, $\bar{b}=\bar{i}+2\bar{j}-3\bar{k}$

$\bar{c}=3\bar{i}+\mu\bar{j}+5\bar{k}$ are coplanar then μ is a root of the equation

1) $x^2+3x=4$ 2) $x^2+2x=6$

3) $x^2+3x=6$ 4) $x+5=0$

If the vectors $a\bar{i}+\bar{j}+\bar{k}, \bar{i}+b\bar{j}+\bar{k}, \bar{i}+\bar{j}+c\bar{k}$ (a,b,c are not equal to 1) are coplanar, then

$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$........

1) 0 2) 3 3) 2 4) 1

If $\bar{a},\bar{b},\bar{c}$ are non coplanar vectors and λ is a real number then

$[\lambda(\bar{a}+\bar{b})\ \ \lambda^2\bar{b}\ \ \lambda\bar{c}]=[\bar{a}\ \ \bar{b}+\bar{c}\ \ \bar{b}]$ for

1) Exactly two values of λ

2) Exactly three values of λ

3) No value of λ

4) Exactly one value of λ

The volume of the parallelopiped with coterminus edges $l\bar{i}-5\bar{k}, \bar{i}-\bar{j}+m\bar{k}$ and $3\bar{i}-5\bar{j}$ is 8. Then 'l' and 'm' are related as

1) $3lm+2=0$

2) $lm+2=0$

3) $3lm-2=0$

4) $5lm+2=0$

Let $\bar{a}$ be a unit vector $\bar{b}=2\bar{i}+\bar{j}-\bar{k}$ and $\bar{c}=\bar{i}+3\bar{k}$ then maximum value of $[\bar{a}\,\bar{b}\,\bar{c}]$ is

1) -1 2) $\sqrt{10}+\sqrt{6}$

3) $\sqrt{10}-\sqrt{6}$ 4) $\sqrt{59}$

9. If $x(\bar{a}\times\bar{b})+y(\bar{b}\times\bar{c})+z(\bar{c}\times\bar{a})=\bar{r}$ and $[\bar{a}\,\bar{b}\,\bar{c}]=\frac{1}{8}$ then $x+y+z=$

1) $\bar{r}.(\bar{a}+\bar{b}+\bar{c})$

2) $4[\bar{r}.(\bar{a}+\bar{b}+\bar{c})]$

3) $8[\bar{r}.(\bar{a}+\bar{b}+\bar{c})]$ 4) 0

10. $\begin{vmatrix} \bar{a}.\bar{a} & \bar{a}.\bar{b} & \bar{a}.\bar{c} \\ \bar{b}.\bar{a} & \bar{b}.\bar{b} & \bar{b}.\bar{c} \\ \bar{c}.\bar{a} & \bar{c}.\bar{b} & \bar{c}.\bar{c} \end{vmatrix}=0$ then the vectors

$\bar{a}+\bar{b}, \bar{b}+\bar{c}, \bar{c}+\bar{a}$ are

1) Coplanar

2) Non coplanar

3) Unit vectors

4) Mutually perpendicular

11. For any three non-zero vectors $\bar{a},\bar{b},\bar{c}$, if $|(\bar{a}\times\bar{b}).\bar{c}|=|\bar{a}||\bar{b}||\bar{c}|$ then

$\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}=$

1) 1 2) 2 3) 0 4) 3

12. $\bar{a},\bar{b},\bar{c}$ are mutually perpendicular unit vectors and $\bar{d}$ is a unit vector equally inclined to each other of $\bar{a},\bar{b}$ and $\bar{c}$ at an angle of 60^0. Then $|\bar{a}+\bar{b}+\bar{c}+\bar{d}|^2=$

1) 4 2) 5 3) 6 4) 7

13. The lines $\bar{r}=\bar{i}+\bar{j}-\bar{k}+s(3\bar{i}-\bar{j})$ and $\bar{r}=4\bar{i}-\bar{k}+t(2\bar{i}+3\bar{k})$

1) intersect

2) do not intersect

3) are skew lines

4) cannot be determined

14. The shortest distance between the lines
$\bar{r}=3\bar{i}+5\bar{j}+7\bar{k}+\lambda(\bar{i}+2\bar{j}+\bar{k})$
$\bar{r}=-\bar{i}-\bar{j}+\bar{k}+\mu(7\bar{i}-6\bar{j}+\bar{k})$ is
1) $\frac{16}{5\sqrt{5}}$ 2) $\frac{26}{5\sqrt{5}}$
3) $\frac{46}{5\sqrt{5}}$ 4) $\frac{36}{5\sqrt{5}}$

15. If $\bar{p}\,\bar{q}\,\bar{r}$ is reciprocal system of vector triad $\bar{a},\bar{b}$ and $\bar{c}$ then $[\bar{a}\,\bar{b}\,\bar{c}][\bar{p}\,\bar{q}\,\bar{r}]=$
1) 0 2) 1 3) 2 4) 3

16. Let $c_1=(1,0,0), c_2=(1,1,0), c_3=(1,1,1)$, then the reciprocal of $c_1=$
1) $\bar{i}+\bar{j}$ 2) $\bar{i}-\bar{j}$ 3) $\bar{j}-\bar{k}$ 4) $\bar{k}-\bar{i}$

17. If $\bar{a},\bar{b},\bar{c}$ and $\bar{p},\bar{q},\bar{r}$ are two sets of three non-coplanar vectors such that $\bar{a}.\bar{p}+\bar{b}.\bar{q}+\bar{c}.\bar{r}=3$ then $\bar{p}=...,\bar{q}=...,\bar{r}=...$
1) $\frac{\bar{a}\times\bar{b}}{[\bar{a}\,\bar{b}\,\bar{c}]},\frac{\bar{b}\times\bar{c}}{[\bar{a}\,\bar{b}\,\bar{c}]},\frac{\bar{c}\times\bar{a}}{[\bar{a}\,\bar{b}\,\bar{c}]}$
2) $\bar{b}\times\bar{c},\bar{c}\times\bar{a},\bar{a}\times\bar{b}$
3) $\frac{\bar{a}}{|\bar{a}|^2},\frac{\bar{b}}{|\bar{b}|^2},\frac{\bar{c}}{|\bar{c}|^2}$
4) $\frac{\bar{b}\times\bar{c}}{[\bar{a}\,\bar{b}\,\bar{c}]},\frac{\bar{c}\times\bar{a}}{[\bar{a}\,\bar{b}\,\bar{c}]},\frac{\bar{a}\times\bar{b}}{[\bar{a}\,\bar{b}\,\bar{c}]}$

S.P.Q. Student Practice Questions

18. If $\bar{a},\bar{b},\bar{c}$ form a left handed orthogonal system and $\bar{a}.\bar{a}=4,\bar{b}.\bar{b}=9,\bar{c}.\bar{c}=16$ then $[\bar{a}\,\bar{b}\,\bar{c}]=$
1) 24 2) −24 3) 12 4) −12

19. If $\bar{a}$ is a perpendicular to $\bar{b}$ and $\bar{c}$, $|\bar{a}|=2,|\bar{b}|=3,|\bar{c}|=4$ and the angle between $\bar{b}$ and $\bar{c}$ is $\frac{2\pi}{3}$, then $|[\bar{a}\,\bar{b}\,\bar{c}]|=$
1) 24 2) 12 3) $12\sqrt{3}$ 4) $24\sqrt{3}$

20. For any non zero, non collinear vectors $\bar{p}$ and $\bar{q}$ the value of $[\bar{i}\,\bar{p}\,\bar{q}]\bar{i}+[\bar{j}\,\bar{p}\,\bar{q}]\bar{j}+[\bar{k}\,\bar{p}\,\bar{q}]\bar{k}$ is ___
1) $\bar{0}$ 2) $2(\bar{p}\times\bar{q})$ 3) $\bar{q}\times\bar{p}$ 4) $\bar{p}\times\bar{q}$

21. If $\bar{a}=x\bar{i}+(x+1)\bar{j}+(x+2)\bar{k}$
$\bar{b}=(x+3)\bar{i}+(x+4)\bar{j}+(x+5)\bar{k}$
$\bar{c}=(x+6)\bar{i}+(x+7)\bar{j}+(x+8)\bar{k}$
are coplanar then $x=$
1) 0 2) 1 3) 3 4) All real values

22. If the vectors $\bar{a}=(1,a,a^2),\bar{b}=(1,b,b^2)$ and $\bar{c}=(1,c,c^2)$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ then $abc=$
1) 0 2) 1 3) -1 4) -2

23. $\bar{a}=\bar{i}-\bar{k},\bar{b}=x\bar{i}+\bar{j}+(1-x)\bar{k}$ and $\bar{c}=y\bar{i}+\lambda\bar{j}+(1+x-y)\bar{k}$ then $[\bar{a}\,\bar{b}\,\bar{c}]$ depends on
1) neither x nor y 2) both x and y
3) only x 4) only y

24. The volume of the parallelopiped whose sides are $\overline{OA}=(\lambda+2)\bar{i}+(\lambda+1)(\lambda+2)\bar{j}+\bar{k}$,
$\overline{OB}=(\lambda+3)\bar{i}+(\lambda+2)(\lambda+3)\bar{j}+\bar{k}$
$\overline{OC}=(\lambda+4)\bar{i}+(\lambda+3)(\lambda+4)\bar{j}+\bar{k}$ is
1) 2λ 2) 3λ 3) 4λ 4) 2

25. If $\bar{v}=2\bar{i}+\bar{j}-\bar{k}$ and $\bar{w}=\bar{i}$, $\bar{u}$ is a unit vector then maximum value of $[\bar{u}\,\bar{v}\,\bar{w}]$ is
1) -1 2) $\sqrt{10}+\sqrt{6}$ 3) $\sqrt{2}$ 4) $\sqrt{6}$

$\bar{a}=p(\bar{b}\times\bar{c})+q(\bar{c}\times\bar{a})+r(\bar{a}\times\bar{b})$ and if $\bar{a}.(\bar{a}+\bar{b}+\bar{c})=1$ then $[\bar{a}\ \bar{b}\ \bar{c}]=$

1) $p+q+r$ 2) $\dfrac{1}{p+q+r}$

3) $2(p+q+r)$ 4) $\dfrac{2}{p+q+r}$

If $\bar{a},\bar{b},\bar{c}$ are non coplanar vectors, then

$$\begin{vmatrix} \bar{a}.\bar{a} & \bar{a}.\bar{b} & \bar{a}.\bar{c} \\ \bar{b}.\bar{a} & \bar{b}.\bar{b} & \bar{b}.\bar{c} \\ \bar{c}.\bar{a} & \bar{c}.\bar{b} & \bar{c}.\bar{c} \end{vmatrix} =$$

1) $[\bar{a}\ \bar{b}\ \bar{c}]$ 2) $[\bar{a}\ \bar{b}\ \bar{c}]^2$ 3) 1 4) 0

If $\bar{a},\bar{b},\bar{c}$ are non zero vectors such that $|(\bar{a}\times\bar{b}).\bar{c}|=|\bar{a}||\bar{b}||\bar{c}|$ then $|\bar{a}+\bar{b}+\bar{c}|^2=$

1) 0 2) 1

3) $|\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2$ 4) -1

29. The lines $\bar{r}=\bar{i}-\bar{j}+\bar{k}+s(\bar{i}+2\bar{j}-3\bar{k})$ and $\bar{r}=(\bar{i}-2\bar{j}+3\bar{k})+t(-\bar{i}+\bar{j}+2\bar{k})$

1) Intersect
2) Do not intersect
3) Skew lines
4) Cannot be determine

30. The shortest distance between the lines through the points $(2,3,1),(4,5,2)$ and parallel to the vectors $(3,4,2),(4,5,3)$ respectively is

1) $\dfrac{\sqrt{6}}{7}$ 2) $\dfrac{1}{\sqrt{6}}$ 3) $\dfrac{2}{\sqrt{3}}$ 4) 9

31. Let $\bar{p}=\dfrac{\bar{b}\times\bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]},\bar{q}=\dfrac{\bar{c}\times\bar{a}}{[\bar{a}\ \bar{b}\ \bar{c}]},\bar{r}=\dfrac{\bar{a}\times\bar{b}}{[\bar{a}\ \bar{b}\ \bar{c}]}$ and $\bar{a},\bar{b},\bar{c}$ being any three non-coplanar vectors then $\bar{p}.(\bar{a}+\bar{b})+\bar{q}.(\bar{b}+\bar{c})+\bar{r}.(\bar{c}+\bar{a})=$

1) -3 2) 0 3) 3 4) -2

32. If $\bar{a}^1,\bar{b}^1,\bar{c}^1$ is the reciprocal system of vector triad of $\bar{a},\bar{b}$ and $\bar{c}$, then $\bar{a}.\bar{b}^1+\bar{b}.\bar{c}^1+\bar{c}.\bar{a}^1=$

1) 0 2) 1 3) 2 4) 3

33. $\bar{a}\times\bar{a}^1+\bar{b}\times\bar{b}^1+\bar{c}\times\bar{c}^1=$

1) $\bar{0}$ 2) $\bar{a}$ 3) $\bar{b}$ 4) $\bar{c}$

34. $(\bar{a}+\bar{b}).\bar{a}^1+(\bar{b}+\bar{c}).\bar{b}^1+(\bar{c}+\bar{a})\bar{c}^1=$

1) 0 2) 1 3) 2 4) 3

KEY

01) 1	02) 3	03) 3	04) 1	05) 4
06) 3	07) 4	08) 4	09) 3	10) 1
11) 3	12) 4	13) 1	14) 2	15) 2
16) 2	17) 4	18) 2	19) 3	20) 4
21) 4	22) 3	23) 3	24) 4	25) 3
26) 2	27) 2	28) 3	29) 3	30) 2
31) 3	32) 1	33) 1	34) 4	

Hints Solutions

1. $\bar{a},\bar{b},\bar{c}$ are non zero vectors

$|\bar{a}\times\bar{b}.\bar{c}|=|\bar{a}||\bar{b}||\bar{c}|$

$\Leftrightarrow \bar{a}.\bar{b}=\bar{b}.\bar{c}=\bar{c}.\bar{a}=0$ (conceptual)

2. $|[\bar{a}\ \bar{b}\ \bar{c}]|=|(\bar{a}\times\bar{b}).\bar{c}|=|\bar{a}||\bar{b}|\sin(\bar{a},\bar{b})|\bar{c}|$

3. $\bar{r}=x\bar{a}+y\bar{b}+z\bar{c}$ ----------(1) take dot product with $\bar{b}\times\bar{c},\bar{c}\times\bar{a}$ and $\bar{a}\times\bar{b}$ and substitute the values of x, y, z in (1)

4. $[\bar{a}\ \bar{b}\ \bar{c}]=0\Rightarrow\mu=-4$

$$\sum \bar{a}\times\frac{(\bar{b}\times\bar{c})}{[\bar{a}\,\bar{b}\,\bar{c}]}=0$$

$$\Sigma(\bar{a}+\bar{b})\cdot\frac{(\bar{b}\times\bar{c})}{[\bar{a}\,\bar{b}\,\bar{c}]}=(1+0)+(1+0)+(1+0)=3$$

EXERCISE-III

CRTQ & SPQ LEVEL-III

SCALAR TRIPLE PRODUCT AND PROPERTIES

C.R.T.Q Class Room Teaching Questions

1. If $\bar{a}=-\bar{i}+\bar{j}+\bar{k}$ and $\bar{b}=2\bar{i}+\bar{k}$, then the vector $\bar{c}$ satisfying the conditions
i) coplanar with $\bar{a}$ and $\bar{b}$
ii) perpendicular to $\bar{b}$,
iii) $\bar{a}.\bar{c}=7$
1) $-\frac{3}{2}\bar{i}+\frac{5}{2}\bar{j}+3\bar{k}$ 2) $-3\bar{i}+5\bar{j}+6\bar{k}$
3) $-6\bar{i}+\bar{k}$ 4) $-\bar{i}+2\bar{j}+2\bar{k}$

2. If $\bar{a},\bar{b},\bar{c}$ are unit vectors such that $(\bar{a},\bar{b})=\alpha=(\bar{c},\bar{a}\times\bar{b})$ then $[\bar{c}\,\bar{a}\,\bar{b}]=$
1) $\frac{1}{2}\sin 2\alpha$ 2) $\sin 2\alpha$
3) $\frac{1}{4}\sin 2\alpha$ 4) 1

3. $\bar{\alpha}=a\bar{i}+b\bar{j}+c\bar{k}$, $\bar{\beta}=b\bar{i}+c\bar{j}+a\bar{k}$ and $\bar{\gamma}=c\bar{i}+a\bar{j}+b\bar{k}$ be three coplanar vectors with $\bar{\alpha}\neq\bar{\beta}\neq\bar{\gamma}$ and $\bar{r}=\bar{i}+\bar{j}+\bar{k}$ then $\bar{r}$ is perpendicular to
1) $\bar{\alpha}$ 2) $\bar{\beta}$
3) $\bar{\gamma}$ 4) $\bar{\alpha},\bar{\beta},\bar{\gamma}$

4. If $4\bar{a}+5\bar{b}+9\bar{c}=\bar{0}$ then $(\bar{a}\times\bar{b})\times[(\bar{b}\times\bar{c})\times(\bar{c}\times\bar{a})]=$
1) A vector perpendicular to the plane of $\bar{a},\bar{b}$ & $\bar{c}$
2) $4\bar{a}+5\bar{b}+9\bar{c}$
3) $\bar{0}$
4) $[\bar{a}\bar{b}\bar{c}]$

5. If $\bar{a},\bar{b},\bar{c}$ are non-coplanar vectors and $\bar{\alpha}=\bar{b}\times\bar{c},\bar{\beta}=\bar{c}\times\bar{a},\bar{r}=\bar{a}\times\bar{b}$ then
$$\begin{vmatrix}\bar{\alpha}.\bar{a} & \bar{\alpha}.\bar{b} & \bar{\alpha}.\bar{c}\\ \bar{\beta}.\bar{a} & \bar{\beta}.\bar{b} & \bar{\beta}.\bar{c}\\ \bar{r}.\bar{a} & \bar{r}.\bar{b} & \bar{r}.\bar{c}\end{vmatrix}=$$
1) 0 2) 1
3) $[\bar{\alpha}\,\bar{\beta}\,\bar{r}][\bar{a}\bar{b}\bar{c}]^2$ 4) $[\bar{a}\bar{b}\bar{c}]^3$

6. If $\bar{b}$ and $\bar{c}$ are any two non-collinear unit vectors and $\bar{a}$ is any vector, then
$$(\bar{a}.\bar{b})\bar{b}+(\bar{a}.\bar{c})\bar{c}+\frac{\bar{a}.(\bar{b}\times\bar{c})}{|\bar{b}\times\bar{c}|^2}(\bar{b}\times\bar{c})=$$
1) $\bar{a}$ 2) $\bar{b}$
3) $\bar{c}$ 4) $\bar{a}+\bar{b}+\bar{c}$

7. $\bar{a},\bar{b},\bar{c}$ three non-zero vectors $|(\bar{a}\times\bar{b}).\bar{c}|=|\bar{a}||\bar{b}||\bar{c}|$ holds iff
1) $\bar{a}.\bar{b}=0,\bar{b}.\bar{c}=0$
2) $\bar{b}.\bar{c}=0,\bar{c}.\bar{a}=0$
3) $\bar{c}.\bar{a}=0,\bar{a}.\bar{b}=0$
4) $\bar{a}.\bar{b}=\bar{b}.\bar{c}=\bar{c}.\bar{a}=0$

8. The unit vector which is orthogonal to the vector $3\bar{i}+2\bar{j}+6\bar{k}$ and is coplanar with the vectors $2\bar{i}+\bar{j}+\bar{k}$ and $\bar{i}-\bar{j}+\bar{k}$ is
1) $\frac{2\bar{i}-6\bar{j}+\bar{k}}{\sqrt{41}}$ 2) $\frac{2\bar{i}-3\bar{j}}{\sqrt{13}}$
3) $\frac{3\bar{j}-\bar{k}}{\sqrt{10}}$ 4) $\frac{4\bar{i}+3\bar{j}-3\bar{k}}{\sqrt{34}}$

9. If $\bar{r}$ is a unit vector such that $\bar{r}=x(\bar{b}\times\bar{c})+y(\bar{c}\times\bar{a})+z(\bar{a}\times\bar{b})$, then $|(\bar{r}.\bar{a})(\bar{b}\times\bar{c})+(\bar{r}.\bar{b})(\bar{c}\times\bar{a})+(\bar{r}.\bar{c})(\bar{a}\times\bar{b})|=$

1) $[\bar{a}\ \bar{b}\ \bar{c}]$ 2) 1

3) $|[\bar{a}\ \bar{b}\ \bar{c}]|$ 4) 0

S.P.Q. Student Practice Questions

10. If $\bar{a}$ and $\bar{b}$ are two mutually perpendicular unit vectors and the vectors $x\bar{a}+x\bar{b}+z(\bar{a}\times\bar{b}), \bar{a}+(\bar{a}\times\bar{b})$ and $z\bar{a}+z\bar{b}+y(\bar{a}\times\bar{b})$ lie in a plane, then z is

1) A.M. of x and y 2) G.M. of x and y

3) H,M, of x and y 4) Equal to zero

11. If $(\bar{a},\bar{b})=\frac{\pi}{6}, \bar{c}$ is a perpendicular to $\bar{a}$ and $\bar{b}, |\bar{a}|=3, |\bar{b}|=4, |\bar{c}|=6$ then $|[\bar{a}\ \bar{b}\ \bar{c}]|=$

1) $12\sqrt{3}$ 2) $48\sqrt{3}$ 3) 36 4) 72

12. If $\bar{a}=x\bar{i}+12\bar{j}-\bar{k}, \bar{b}=2\bar{i}+2x\bar{j}+\bar{k}$ and $\bar{c}=\bar{i}+\bar{k}$ and given that the vectors $\bar{a},\bar{b},\bar{c}$ form a right handed system, then the range of x is

1) R-[-3,2] 2) (-4,3)

3) $R-(-3,2)$ 4) (-2,3)

13. If $\bar{a},\bar{b},\bar{c}$ are position vectors of three non-collinear points A, B, C respectively, the shortest distance of A from BC is

1) $\bar{a}.(\bar{b}-\bar{c})$

2) $\sqrt{|\bar{b}-\bar{a}|^2-\left\{\frac{\bar{a}.\bar{b}}{|\bar{c}|}\right\}^2}$ 3) $|\bar{b}-\bar{a}|$

4) $\sqrt{|\bar{b}-\bar{a}|^2-\left\{\frac{(\bar{a}-\bar{b})^2(\bar{c}-\bar{b})^2}{|\bar{c}-\bar{b}|^2}\right\}}$

14. If $\bar{a},\bar{b},\bar{c}$ are three non-coplanar, non-zero vectors, then $(\bar{a}.\bar{a})(\bar{b}\times\bar{c})+(\bar{a}.\bar{b})(\bar{c}\times\bar{a})+(\bar{a}.\bar{c})(\bar{a}\times\bar{b})=$

1) $[\bar{a}\ \bar{b}\ \bar{c}]\bar{c}$ 2) $[\bar{b}\ \bar{c}\ \bar{a}]\bar{a}$

3) $[\bar{c}\ \bar{a}\ \bar{b}]\bar{b}$ 4) $\bar{0}$

15. Let $\bar{a}=\alpha_1\bar{i}+\alpha_2\bar{j}+\alpha_3\bar{k}, \bar{b}=\beta_1\bar{i}+\beta_2\bar{j}+\beta_3\bar{k}$ and $\bar{c}=\gamma_1\bar{i}+\gamma_2\bar{j}+\gamma_3\bar{k}, |\bar{a}|=2\sqrt{2}$, $\bar{a}$ makes an angle $\frac{\pi}{3}$ with the plane of $\bar{b}$, $\bar{c}$ and the angle between $\bar{b},\bar{c}$ is $\frac{\pi}{6}$, then $\begin{vmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{vmatrix}^n$ is equal to (n is even natural number)

1) $\left(\frac{|\bar{b}||\bar{a}|}{6}\right)^{n/2}$ 2) $\left(\frac{\sqrt{3}|\bar{b}||\bar{c}|}{\sqrt{2}}\right)^n$

3) $\frac{|\bar{b}||\bar{c}|^{n/2}}{\sqrt{3}\ 2^n}$ 4) $\left(\frac{\sqrt{2}|\bar{b}||\bar{a}|}{\sqrt{3}}\right)^n$

16. $\bar{a}=\bar{i}+\bar{j}+\bar{k}, \bar{b}=2\bar{i}+\bar{k}, \bar{c}=\bar{i}+2\bar{j}+3\bar{k}$ then the ascending order of

A) $[\bar{a}\times\bar{b}\ \ \bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}]$

B) $[\bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\ \ \bar{c}+\bar{a}]$

C) $[\bar{a}\ \bar{b}\ \bar{c}][\bar{a}^1\ \bar{b}^1\ \bar{c}^1]$

D) $[\bar{a}-\bar{b}\ \ \bar{b}-\bar{c}\ \ \bar{c}-\bar{a}]$

1) A, B, C, D 2) B, D, C, A

3) C, B, A, D 4) D, B, C, A

If $\bar{u},\bar{v},\bar{w}$ are non co-planar vectors and p,q are real numbers then the equality $[3\bar{u}\ p\bar{v}\ p\bar{w}]-[p\bar{v}\ \bar{w}\ q\bar{u}]-[2\bar{w}\ q\bar{v}\ q\bar{u}]=0$ holds for

1) exactly two values of (p, q)
2) more than two but not all values of (p, q)
3) all values of (p,q)
4) exactly one value of (p,q)

If $\bar{a},\bar{b},\bar{c}$ are non co-planar vectors and λ is a real number then

$$\left[\lambda(\bar{a}+\bar{b})\ \lambda^2\bar{b}\ \lambda\bar{c}\right]=\left[\bar{a}\ \bar{b}+\bar{c}\ \bar{b}\right]$$ for

1) exactly two values of λ
2) exactly three values of λ
3) no value of λ
4) exactly one value of λ

VOLUME OF TETRAHEDRON, PARALLELOPIPED

C.R.T.Q Class Room Teaching Questions

19. A tetrahedron of volume V=5 has three of its vertices at the points $A(2,1,-1), B(3,0,1)$ and $C(2,-1,3)$. The fourth vertex D lies on the y-axis. Then D is

1) $(0,8,0)$
2) $(0,-7,0)$
3) $(0,8,0)$ or $(0,-7,0)$
4) $(0,7,0)$

20. The ratio between volume of tetrahedron and the volume of the tetrahedron formed by joining the centroids of faces is in

1) 27 : 1
2) 17 : 1
3) 7 : 1
4) 1 : 27

21. If V is the volume of the parallelopiped having three coterminus edges as $\bar{a},\bar{b}$ and $\bar{c}$ then the volume of the parallelopiped having three coterminus edges as

$\bar{\alpha}=(\bar{a}.\bar{a})\bar{a}+(\bar{a}.\bar{b})\bar{b}+(\bar{a}.\bar{c})\bar{c}$,

$\bar{\beta}=(\bar{a}.\bar{b})\bar{a}+(\bar{b}.\bar{b})\bar{b}+(\bar{b}.\bar{c})\bar{c}$,

$\bar{\gamma}=(\bar{a}.\bar{c})\bar{a}+(\bar{b}.\bar{c})\bar{b}+(\bar{c}.\bar{c})\bar{c}$ is

1) V^3 2) $3V$ 3) V^2 4) $2V$

S.P.Q. Student Practice Questions

22. If $\dfrac{1}{(x-11)(x-12)(x-13)}=\dfrac{A}{x-11}+\dfrac{B}{x-12}+\dfrac{C}{x-13}$ then the volume of the parallelopiped whose adjacent sides are $A\bar{i}+B\bar{j}+2C\bar{k}$, $2A\bar{i}-B\bar{j}$, $\bar{i}-3B\bar{j}+4C\bar{k}$ is (in cu. units)

1) 5 2) 4 3) 3 4) 2

23. Let $\bar{a},\bar{b},\bar{c}$ be the position vectors of the points A,B,C respectively and α,β and γ be the inclinations between $\bar{b}\,\bar{c};\bar{a},\bar{b}$ and $\bar{a},\bar{c}$ if the volume of the tetrahedron OABC is V then

1) $V^2=\dfrac{a^2b^2c^2}{36}\begin{vmatrix}1&\cos\beta&\cos\gamma\\\cos\beta&1&\cos\alpha\\\cos\gamma&\cos\alpha&1\end{vmatrix}$

2) $V^2=\dfrac{a^2b^2c^2}{6}\begin{vmatrix}1&\cos\beta&\cos\gamma\\\cos\beta&1&\cos\alpha\\\cos\gamma&\cos\alpha&1\end{vmatrix}$

3) $V^2=\dfrac{a^2b^2c^2}{36}\begin{vmatrix}0&\cos\beta&\cos\gamma\\\cos\beta&0&\cos\alpha\\\cos\gamma&\cos\alpha&0\end{vmatrix}$

4) $V^2=\dfrac{a^2b^2c^2}{36}\begin{vmatrix}1&\sin\beta&\sin\gamma\\\sin\beta&1&\sin\alpha\\\sin\gamma&\sin\alpha&1\end{vmatrix}$

2. List-I | List-II

A) If $\bar{a},\bar{b}$ and $\bar{c}$ are three mutually perpendicular vectors where $|\bar{a}|=|\bar{b}|=2,\ |\bar{c}|=1$ then $[\bar{a}\times\bar{b}\ \ \bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}]$ is — (p) 12

B) If $\bar{a}$ and $\bar{b}$ are two unit vectors inclined at $\frac{\pi}{3}$, then $16\left[\bar{a}\ \bar{b}+\bar{a}\times\bar{b}\ \ \bar{b}\right]$ is — (q) 0

C) If $\bar{b}$ and $\bar{c}$ are orthogonal unit vectors and $\bar{b}\times\bar{c}=\bar{a}$ then $\left[\bar{a}+\bar{b}+\bar{c}\ \ \bar{a}+\bar{b}\ \ \bar{b}+\bar{c}\right]$ is — (r) 16

D) If $[\bar{x}\ \bar{y}\ \bar{a}]=[\bar{x}\ \bar{y}\ \bar{b}]=[\bar{x}\ \bar{y}\ \bar{c}]=0$ and each vector being a non-zero vector then $\left[\bar{a}\ \bar{b}\ \bar{c}\right]=$ — (s) 1

1) $A\to p,B\to r,c\to q,d\to s$

2) $A\to p,B\to s,c\to q,d\to r$

3) $A\to r,B\to p,c\to s,d\to q$

4) $A\to r,B\to q,c\to s,d\to p$

3. Statement-I :

$\left[2\bar{a}+3\bar{b}\ \ 3\bar{b}+5\bar{c}\ \ 5\bar{c}+7\bar{a}\right]=135\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

Statement-II : If $\bar{a},\bar{b},\bar{c}$ are non coplanar $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$ is non zero, then

1) Only I is true

2) Only II is true

3) Both I & II are true

4) Neither I nor II are ture

4. Volume of the parallelopiped formed by the vector $\bar{a}\times\bar{b},\ \bar{b}\times\bar{c}$ and $\bar{c}\times\bar{a}$ is 36 sq units. Match the following lists.

List-I | List-II

A) Volume of the parallelopiped formed by vectors $\bar{a},\bar{b},\bar{c}$ is — (p) 0

B) Volume of tetrahedron formed by vectors $\bar{a},\bar{b}$ and $\bar{c}$ is — (q) 12

C) Volume of parallelopiped formed by the vectors $\bar{a}+\bar{b},\bar{b}+\bar{c},\bar{c}+\bar{a}$ — (r) 6

D) Volume of the parallelopiped formed by the vector $\bar{a}-\bar{b}$, $\bar{b}-\bar{c}$ and $\bar{c}-\bar{a}$ is — (s) 1

1) $A\to s,B\to q,C\to p,D\to r$

2) $A\to r,B\to s,C\to q,D\to p$

3) $A\to s,B\to r,C\to q,D\to q$

4) $A\to r,B\to q,C\to p,D\to s$

5. Let $\bar{r}$ be any vector satisfying $\bar{r}.\bar{a}=\bar{r}.\bar{b}=\bar{r}.\bar{c}=0$ for given non zero vectors $\bar{a},\bar{b}$ and $\bar{c}$

Statement-I : $\left[\bar{a}-\bar{b}\ \ \bar{b}-\bar{c}\ \ \bar{c}-\bar{a}\right]=0$

Statement-II : $\left[\bar{a}\ \bar{b}\ \bar{c}\right]=0$

1) Statement-I and Statement-II are true but Statement-II is the correct explanation for Statement-I

2) Statement-I and Statement-II are true but Statement-II is not the correct explanation for Statement-I

3) Statement-I is true and Statement-II false

4) Statement-I is false and Statement-II true

6. Statement-I : If $\bar{a},\bar{b},\bar{c}$ are non coplanar then $\bar{a},\bar{a}+\bar{b},\bar{a}+\bar{b}+\bar{c}$ are also non coplanar

Statement-II: $\left[\bar{a}\ \ \bar{a}+\bar{b}\ \ \bar{a}+\bar{b}+\bar{c}\right]=\left[\bar{a}\ \bar{b}\ \bar{c}\right]$ then

1) Only I is true 2) Only II is true

3) Both I&II are true 4) Neither I nor II are true