

Concept Based Questions

ALGEBRA OF MATRICES

1. If two matrices A and B are of order $p \times q$ and $r \times s$ respectively, can be subtracted only, if
 1) $p = q$ 2) $p = q, r = s$
 3) $p = r, q = s$ 4) $p = r$

2. If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ij} = k$ for all $i=j$, then trace of A =
 1) nk 2) $n+k$ 3) $\frac{n}{k}$ 4) 1

3. If A and B are two matrices such that A has identical rows and AB is defined. Then AB has
 1) no identical rows 2) identical rows
 3) no identical columns
 4) cannot be determined

4. If $AB = O$, then
 1) $A = O$ 2) $B = O$
 3) A and B need not be zero matrices
 4) A and B are zero matrices

5. If A, B are two square matrices of order n and A and B commute (K be a real number). Then
 1) $A - KI, B - KI$ Commute
 2) $A - KI, B - KI$ are equal
 3) $A - KI, B - KI$ do not commute
 4) $A + KI, B - KI$ do not commute

6. If D_1 and D_2 are two 3×3 diagonal matrices then
 1) D_1, D_2 is a diagonal matrix
 2) $D_1 + D_2$ is a diagonal matrix
 3) $D_1^2 + D_2^2$ is a diagonal matrix
 4) 1, 2, 3 are correct

7. If $AB = AC$ then
 1) $B = C$ 2) $B \neq C$
 3) B need not be equal to C 4) $B = -C$

8. If $AB = AC \Rightarrow B = C$, then A is

- 1) non-singular 2) singular
 3) symmetric 4) Skew symmetric

9. If A and B are two matrices such that $A + B$ and AB are both defined then
 1) A and B are two matrices not necessarily of same order
 2) A and B are square matrices of same order
 3) A and B are matrices of same type
 4) A and B are rectangular matrices of same order

10. If $A = \begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix}$ then A is

- 1) Nilpotent 2) involutory
 3) Symmetric 4) Idempotent

11. A skew - symmetric matrix S satisfies the relation $S^2 + I = 0$, where I is a unit matrix then S is
 1) Idempotent 2) Orthogonal
 3) involutory 4) Nilpotent

12. If $A = \begin{bmatrix} 1 & 2-3i & 3+4i \\ 2+3i & 0 & 4-5i \\ 3-4i & 4+5i & 2 \end{bmatrix}$ then A is

- 1) Hermitian 2) Skew-Hermitian
 3) Symmetric 4) Skew-Symmetric

13. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true.

- 1) $AB = BA$ 2) Either A or B is a zero matrix
 3) Either A or B is an identity matrix
 4) $A = B$

- 14.** If B is an idempotent matrix and $AB = I - B$ then $AB =$
- I
 - 0
 - $-I$
 - B

- 15.** If A is a symmetric or skew-symmetric matrix then A^2 is
- symmetric
 - skew-symmetric
 - Diagonal
 - scalar or matrix

- 16.** Let A be a square matrix. consider

$$1) A + A^\top \quad 2) AA^\top \quad 3) A^\top A \quad 4) A^\top + A$$

- 17.** If a matrix A is both symmetric and skew-symmetric then A is
- I
 - 0
 - Both 1 and 2
 - Diagonal matrix

- 18.** If A is a skew-symmetric matrix and n is odd positive integer, then A^n is
- a symmetric matrix
 - skew-symmetric matrix
 - diagonal matrix
 - triangular matrix

- 19.** If A is a skew-symmetric matrix and n is even positive integer , then A^n is
- a symmetric matrix
 - skew-symmetric matrix
 - diagonal matrix
 - triangular matrix

- 20.** If $A = \begin{bmatrix} a_{ij} \end{bmatrix}_{3 \times 3}$ is a square matrix so that $a_{ij} = i^2 - j^2$, then A is a
- unit matrix
 - symmetric matrix
 - skew symmetric matrix
 - orthogonal matrix

- 21.** If A, B are two idempotent matrices and $AB = BA = 0$ then $A+B$ is
- Scalar matrix
 - Idempotent matrix

- 3) Diagonal matrix
4) Nilpotent matrix

- 22.** If A is a skew-symmetric matrix of order n , and C is a column matrix of order n , then $C^T AC$ is
- A Identity matrix of order n
 - A unit matrix of order one
 - A zero matrix of order one
 - A zero matrix of order n .

DETERMINANTS AND INVERSE OF A MATRIX

- 23.** A and B are square matrices of order 3×3 , A is an orthogonal matrix and B is a skew symmetric matrix. Which of the following statement is not true
- $|A| = \pm 1$
 - $|B| = 0$
 - $|AB| = 1$
 - $|AB| = 0$

- 24.** If $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ and A_p, B_p, C_p are cofactors of a_p, b_p, c_p then $a_1 B_1 + a_2 B_2 + a_3 B_3 =$
- 0
 - $2|A|$
 - $|A|^2$
 - $2|A|$

- 25.** If each element of a row of square matrix is doubled, the determinant of the matrix is
- not changed
 - doubled
 - multiplied by 4
 - multiplied by $1/2$

- 26.** If A is a 3×3 singular matrix then $A(\text{Adj } A) =$
- $|A|I$
 - $2I$
 - 0
 - ± 1

- 27.** If A and B are two non-singular matrices then $|\text{Adj}(AB)| =$
- $|\text{Adj}(B)||\text{Adj}A|$
 - $|\text{Adj}A||\text{Adj}B|$
 - Both (1) and (2)
 - $|A||B|$

38. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is a nonsingular matrix $\Rightarrow A^1 (Adj)$

- 1) $|A|A = 1$
- 2) I
- 3) $|A|I$
- 4) $|A|^2 I$

If A is a non singular matrix then which of the following is not true

- 1) $Adj A = |A|A^{-1}$
- 2) $(Adj A)^{-1} = \frac{1}{|A|} A$

$$39. \det(A^{-1}) = (\det A)^{-1}$$

$$40. \text{If } A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ then}$$

$(A_1, A_2, A_3 \dots)$ are cofactors

- 1) $\frac{\Delta^2}{2}$
- 2) 2Δ
- 3) Δ^2
- 4) Δ

31. Given $a_i^2 + b_i^2 + c_i^2 = 1 (i=1,2,3)$ and

$$a_i a_j + b_i b_j + c_i c_j = 0 (i \neq j, i, j = 1, 2, 3) \text{ then}$$

$$\text{the value of } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ is}$$

- 1) 0
- 2) $\frac{1}{2}$
- 3) ± 1
- 4) 2

32. If A, B are square matrices of order 3, then

- 1) $AB = 0 \Rightarrow |A| = 0 \text{ and } |B| = 0$
- 2) $AB = 0 \Rightarrow |A| = 0 \text{ or } |B| = 0$

$$33. (A+B)^{-1} = A^{-1} + B^{-1}$$

$$34. \text{adj}(AB) = (\text{adj}A)(\text{adj}B)$$

35. If the product of two non zero square matrices A and B of the same order is a zero matrix then

- 1) Both are singular
- 2) atleast one of $A \& B$ is singular
- 3) A is non-singular, but B is singular
- 4) A is singular but B is non-singular

- 1) A^{-1} equals
- 2) A^\top
- 3) AA^\top
- 4) I

36. The inverse of a symmetric matrix is (if exists)

- 1) Diagonal matrix
- 2) Symmetric matrix
- 3) Skew - symmetric matrix
- 4) Can't say

37. The inverse of a skew symmetric matrix (if it exists) is

- 1) a symmetric matrix
- 2) a skew symmetric matrix
- 3) a diagonal matrix
- 4) none of a matrix is unique

38. The inverse of a skew symmetric matrix of odd order is

- 1) a symmetric matrix
- 2) a skew symmetric matrix
- 3) diagonal matrix
- 4) does not exist

39. If A is a square matrix of order 3 then

$$40. \left| \text{adj}(Adj A^2) \right| =$$

$$1) |A|^2 \quad 2) |A|^4 \quad 3) |A|^8 \quad 4) |A|^{16}$$

LINEAR EQUATIONS

39. The system of equations which can be solved by cramer's rule have

- 1) unique solution
- 2) no solution
- 3) infinitely many solutions
- 4) two solutions

40. Let $AX = B$ be a system of non homogeneous equations and $\det A = 0$ then the system has

- 1) infinity solutions
- 2) unique solution
- 3) no solution
- 4) infinity solutions or no solution

41. The system $AX = B$ of $(n - a)$ equations in $(n - a)$ unknowns has infinitely many solutions if

- 1) $\det A \neq 0, (\text{adj } A)(B) = 0$
- 2) $|A| = 0, (\text{adj } A)(B) = 0$
- 3) $|A| = 0, (\text{adj } A)(B) \neq 0$
- 4) Identity matrix

42. The system of equations which can be solved by matrix inversion method have

- 1) unique solution
- 2) no solution
- 3) infinitely many solutions
- 4) two solutions

43. consider the system of equations

$$a_i x + b_i y + c_i z = 0 \text{ (where } i=1,2,3\text{), if}$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0, \text{ then the system has}$$

- 1) only one solution
- 2) one solution (0,0,0) and one more solution
- 3) no solution
- 4) infinite solutions

44. If A is invertible matrix and B is another matrix such that (AB) exists then

- 1) rank (AB) = rank (A)
- 2) rank (AB) = rank (B)
- 3) rank (AB) > rank (A)
- 4) rank (AB) > rank (B)

45. If A is a non zero column matrix of order $m \times 1$ and B is a non zero row matrix of order $1 \times n$ then rank of AB is

- 1) 0
- 2) 1
- 3) m
- 4) n

46. If $A_{3 \times 3} X_{3 \times 1} = D_{3 \times 1}$ is a consistent system of equations having unique solution then rank (A)

- 1) 3
- 2) 2
- 3) 1
- 4) 0

47. If $A = [a_{ij}]_{m \times n}$ is a matrix of rank r then

- 1) r = $\min \{m, n\}$
- 2) r < $\min \{m, n\}$
- 3) r $\leq \min \{m, n\}$
- 4) r = $\max \{m, n\}$

KEY

- 01) 3 02) 1 03) 2 04) 3 05) 1
06) 1 07) 3 08) 1 09) 2 10) 3
11) 2 12) 1 13) 1 14) 2 15) 1
16) 3 17) 2 18) 2 19) 1 20) 3
21) 2 22) 3 23) 3 24) 1 25) 2

EXERCISE-1

CRTQ & SPQ LEVEL-I

FORMATION OF MATRICES

C.R.T.Q.

- Class Room Teaching Questions
1. For 2×3 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$, then

A is equal to

$$1) \begin{bmatrix} 2, & 8 & \frac{9}{2} \\ 8 & 2 & 2 \end{bmatrix}, \quad 2) \begin{bmatrix} 2, & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$$

$$3) \begin{bmatrix} 2 & \frac{9}{2} & 8 \\ 8 & \frac{9}{2} & \frac{25}{2} \end{bmatrix}, \quad 4) \begin{bmatrix} 2 & \frac{25}{2} & 8 \\ \frac{9}{2} & \frac{9}{2} & 2 \end{bmatrix}$$

2. If $\begin{bmatrix} r+2 & 5 \\ -2 & r+1 \end{bmatrix} = \begin{bmatrix} 4 & y+3 \\ z & 3 \end{bmatrix}$, then

- 1) $r = y = z$
- 2) $r = -y = z$
- 3) $-r = y = z$

S.P.Q.

- Student Practice Questions

3. If $a_{ij} = \frac{1}{2}(3i-2j)$ and $A = [a_{ij}]_{2 \times 2}$, then
- A is equal to
- 1) $\begin{bmatrix} 1/2 & 2 \\ -1/2 & 1 \end{bmatrix}$
 - 2) $\begin{bmatrix} 1/2 & -1/2 \\ 2 & 1 \end{bmatrix}$
 - 3) $\begin{bmatrix} 2 & 2 \\ 1/2 & -1/2 \end{bmatrix}$
 - 4) $\begin{bmatrix} -2 & -2 \\ -1/2 & 1/2 \end{bmatrix}$



4. If $\begin{bmatrix} x+y & 3 \\ z-1 & 4x-z \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$,
 $(x+y+z+a) =$

- 1) 0 2) 1 3) 8 4) -1

SUM AND DIFFERENCE OF THE MATRICES AND SCALAR MULTIPLE OF A MATRIX

C.R.T.Q. Class Room Teaching Questions

5. If $A=2B=\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ and $2A-3B=\begin{pmatrix} -3 & 3 \\ 1 & -1 \end{pmatrix}$

then $B=$

- 1) $\begin{pmatrix} -5 & 7 \\ 5 & 1 \end{pmatrix}$
 2) $\begin{pmatrix} -5 & 7 \\ -5 & -1 \end{pmatrix}$
 3) $\begin{pmatrix} -5 & 7 \\ 5 & -1 \end{pmatrix}$

6. If $I=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and

$C=\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ then $C=$

- 1) $I \cos\theta + B \sin\theta$
 2) $I \sin\theta + B \cos\theta$
 3) $I \cos\theta - B \sin\theta$
 4) $-I \cos\theta + B \sin\theta$

7. If $A=\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 3 \\ 0 & 2 \end{pmatrix}$ and
 $2A+C=B$ then $C=$

- 1) $\begin{pmatrix} 6 & 7 \\ 4 & 12 \end{pmatrix}$
 2) $\begin{pmatrix} 2 & -1 \\ -4 & -8 \end{pmatrix}$
 3) $\begin{pmatrix} 3 & -1 \\ -4 & 8 \end{pmatrix}$
 4) $\begin{pmatrix} -6 & -7 \\ -4 & -12 \end{pmatrix}$

8. If $A=\begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$, $kA=\begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$ then the values of k, a, b are respectively.

- 1) -6, -12, -18
 2) -6, 4, 9
 3) -6, -4, -9
 4) -6, 12, 18

S.P.Q. Student Practice Questions

9. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 2X = \begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix} \Rightarrow X =$

1) $\begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix}$
 2) $\begin{bmatrix} 1 & \frac{3}{2} \\ 1 & \frac{5}{2} \end{bmatrix}$

3) $\begin{bmatrix} -2 & -3 \\ 2 & 5 \end{bmatrix}$
 4) $\begin{bmatrix} 1 & \frac{3}{2} \\ -1 & -\frac{5}{2} \end{bmatrix}$

10. The additive inverse of $\begin{pmatrix} 1 & 4 & -7 \\ -3 & 2 & 5 \\ 2 & 3 & -1 \end{pmatrix}$ is

- 1) $\begin{pmatrix} -1 & -4 & 7 \\ 3 & 2 & -5 \\ 2 & 3 & -1 \end{pmatrix}$
 2) $\begin{pmatrix} -1 & -4 & 7 \\ 3 & -2 & -5 \\ -2 & -3 & -1 \end{pmatrix}$
 3) not possible

- 4) $\begin{pmatrix} -5 & 1 & -4 & 7 \\ 3 & -3 & -2 & -5 \\ -2 & -3 & 1 & 1 \end{pmatrix}$

11. If $A=\begin{pmatrix} 9 & 1 \\ 4 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 5 \\ 6 & 11 \end{pmatrix}$ and

$3A+5B+2X=0$ then $X=$

- 1) $\begin{pmatrix} 16 & -14 \\ 21 & -32 \end{pmatrix}$
 2) $\begin{pmatrix} 16 & 14 \\ -21 & -32 \end{pmatrix}$
 3) $\begin{pmatrix} -16 & -14 \\ -21 & -32 \end{pmatrix}$
 4) $\begin{pmatrix} -16 & 14 \\ 21 & 32 \end{pmatrix}$

MULTIPLICATION OF MATRICES

C.R.T.Q. Class Room Teaching Questions

12. If (1 2 3) B=(3 4) then order of the matrix B is

- 1) 3×1
 2) 3×2
 3) 2×4
 4) 5×2

13. If a matrix has 13 elements, then the possible dimensions (orders) of the matrix are

- 1) 1×13 or 13×1
 2) 1×26 or 26×1
 3) 2×13 or 13×2
 4) 13×13

14. If $A=\begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ then

- 1) AB, BA exist and equal
 2) AB, BA exist and are not equal
 3) AB exist and BA does not exist
 4) AB does not exist and BA exist

23. A fruit shop sells 6 dozen bananas, 6 dozen mangoes, 6 dozen apples and 6 dozen grapes. The selling prices are Rs 60, Rs 40, Rs 40 and Rs 30 respectively. Using matrix algebra, find the value of the fruits in the shop.

- 1) (2,3) 2) (3,4)
3) (4,3) 4) (3,2)

S.P.Q.

Student Practice Questions

15. If $[m \ n] \begin{bmatrix} m \\ n \end{bmatrix} = [25]$ and $m < n$, then $(m, n) =$

- 1) (2,3) 2) (3,4)
3) (4,3) 4) (3,2)

16. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ and $A'' = 0$, then the minimum value of 'n' is

- 1) 2 2) 3 3) 4 4) 5

17. If $A = [x, y], B = \begin{bmatrix} a & h \\ h & b \end{bmatrix}, C = \begin{bmatrix} x \\ y \end{bmatrix}$, then $ABC =$

- 1) $(ax + hy + bx)y$
2) $(ax^2 + 2hxy + by^2)$
3) $(ax^2 - 2hxy + by^2)$
4) $(bx^2 - 2hxy + ay^2)$

18. If A = diagonal (3,3,3) then $A^4 =$

- 1) 12A 2) 81A
3) 684A 4) 27A

19. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then $A^5 =$

- 1) I 2) O 3) A 4) A^2

20. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then $A^2 - 4A$ is equal to

- 1) $2I_3$ 2) $3I_3$ 3) $4I_3$ 4) $5I_3$

21. If $\begin{pmatrix} 1 & -\tan\theta & 1 \\ \tan\theta & 1 & -\tan\theta \\ a & -b & a \end{pmatrix} =$

- 1) $a = 1, b = -1$
2) $a = \sec^2\theta, b = 0$
3) $a = 0, b = \sin^2\theta$
4) $a = \sin 2\theta, b = \cos 2\theta$

22. If $A = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ and

- $a^2 + b^2 + c^2 = 1$, then $A^2 =$

- 1) A 2) $2A$ 3) $3A$ 4) $4A$

$$23. \text{ If } [x \ y \ z] \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

24. The order of $[x \ y \ z]$ is

- 1) 3×1
2) 1×1
3) 1×3
4) 3×3

25. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ then $A^2 =$

- 1) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
2) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
3) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
4) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

26. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ then $A^5 =$

- 1) 243 2) 81A
3) 243A 4) 81I

27. If $[x \ y \ z] \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$

- 1) $[ax^2 + by^2 + cz^2 + 2hxy + 2gxz + 2fyz]$
2) $[ax^2 + by^2 + cz^2 + hxy + gxz + fyz]$
3) $[2ax^2 + 2by^2 + 2cz^2 + hxy + gxz + fyz]$
4) $[2ax^2 + 2by^2 + 2cz^2 + hxy + gxz + fyz]$

28. If $AB = A$ and $BA = B$

- 1) $A = 2B$ 2) $A^2 = A$ 3) $2A = B$ 4) $B^2 = B$

cannot be determined

22. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ then $A^6 + 2A + I =$

- 1) $\begin{bmatrix} 12 & 4 \\ 12 & 4 \end{bmatrix}$
 2) $\begin{bmatrix} 12 & -4 \\ 4 & 12 \end{bmatrix}$
 3) $\begin{bmatrix} 4 & 12 \\ 12 & 4 \end{bmatrix}$
 4) $\begin{bmatrix} 4 & 12 \\ -12 & -4 \end{bmatrix}$

23. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then
 $(A + BE)^T =$

- 1) $AI + BE$
 2) $A^2I + B^2E$
 3) $A^2I + 3AB^2E$
 4) $A^2I + 3A^2BE$

24. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then $A^{2016} =$

- 1) I
 2) O
 3) A
 4) A^2

25. If $P = \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & -1 \\ 5 & 0 \end{bmatrix}$, then $PQ =$

- 1) $\begin{bmatrix} 2 & -1 & 5 \\ 6 & -3 & 15 \\ 3 & -4 & 20 \end{bmatrix}$
 2) $\begin{bmatrix} 2 & -3 & 20 \end{bmatrix}$
 3) $\begin{bmatrix} 2 \\ -3 \\ 20 \end{bmatrix}$
 4) $[19]$

26. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ then $AB =$

- 1) A
 2) B
 3) I
 4) O

PROBLEMS BASED ON INDUCTION

C.R.T.Q.

Class From Teaching Questions

34. If $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$, then $A^n =$ _____, $n \in \mathbb{N}$

- 1) $\begin{bmatrix} 2^n x^n & 2^n x^n \\ 2^n x^n & 2^n x^n \end{bmatrix}$
 2) $\begin{bmatrix} 2^{n-1} x^n & 2^{n-1} x^n \\ 2^{n-1} x^n & 2^{n-1} x^n \end{bmatrix}$
 3) $\begin{bmatrix} 2^{n-2} x^n & 2^{n-2} x^n \\ 2^{n-2} x^n & 2^{n-2} x^n \end{bmatrix}$
 4) $\begin{bmatrix} 2^{n-1} x^{n-1} & 2^{n-1} x^{n-1} \\ 2^{n-1} x^{n-1} & 2^{n-1} x^{n-1} \end{bmatrix}$

35. If $A = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$ then $A^{2016} =$ _____

- 1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 2) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
 3) $\begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$
 4) $\begin{bmatrix} -i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & -i \end{bmatrix}$

36. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then

which one of the following holds for all $n \geq 1$ (by the principle of mathematical induction)

- 1) $A^n = nA + (n-1)I$
 2) $A^n = 2^{n-1}A + (n-1)I$
 3) $A^n = nA - (n-1)I$
 4) $A^n = 2^{n-1}A - (n-1)I$

S.P.Q.

Student Practice Questions

37. $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ if 'n' is

- 1) odd 2) any natural number
3) even 4) not possible

38. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then A^P where $P \in N$ is

- 1) $\begin{bmatrix} 2P & -4P \\ P & 1-2P \end{bmatrix}$
2) $\begin{bmatrix} 1+2P & -4P \\ P & 1-2P \end{bmatrix}$
3) $\begin{bmatrix} 1+2P & -4P \\ P & -P \end{bmatrix}$
4) $\begin{bmatrix} 1+2P & -4 \\ -P & 1-2P \end{bmatrix}$

39. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $n \in N$ then $A^n =$

- 1) $2^{n-1}A$
2) 2^nA
3) nA
4) $2n$

TRACE OF MATRIX

C.R.T.Q Class Room Teaching Questions

40. If $\text{Tr}(A) = 6 \Rightarrow \text{Tr}(4A) =$

- 1) $3/2$
2) 2
3) 12
4) 24

41. If $\text{Tr}(A)=3$, $\text{Tr}(B)=5$ then $\text{Tr}(AB) =$

- 1) 15
2) 5
3) $3/5$
4) Cannot say

42. If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}, \text{Tr}(BA) = \dots$$

- 1) 40
2) 45
3) 39
4) 5

S.P.Q. Student Practice Questions

43. If $\text{Tr}(A) = 2+i \Rightarrow \text{Tr}[(2-i)A] =$

- 1) $2+i$
2) $2-i$
3) 3
4) 5

44. If $\text{Tr}(A) = 8$, $\text{Tr}(B) = 6$, $\Rightarrow \text{Tr}(A - 2B) =$

- 1) -4
2) 4
3) 2
4) 11

TRANSPOSE AND PROPERTIES OF TRANSPOSE OF MATRIX

C.R.T.Q Class Room Teaching Questions

45. If A is a 3×4 matrix and B is matrix such that $A^T B$ and $B A^T$ are both defined then order of B is

- 1) 3×4
2) 4×3
3) 3×3
4) 4×4

46. $\begin{bmatrix} r+4 & 6 \\ 3 & r+3 \end{bmatrix} = \begin{bmatrix} 5 & r+2 \\ r+5 & 4 \end{bmatrix}$ then $r =$

- 1) 1
2) 2
3) 3
4) -1

47. If $3A + 4B^T = \begin{pmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{pmatrix}$ and

$$2B - 3A^T = \begin{pmatrix} -1 & 18 \\ 4 & -6 \\ -5 & -7 \end{pmatrix}$$

1) $\begin{pmatrix} 1 & 3 \\ -1 & 0 \\ -2 & -4 \end{pmatrix}$
2) $\begin{pmatrix} 1 & 3 \\ 1 & 0 \\ 2 & 4 \end{pmatrix}$
3) $\begin{pmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{pmatrix}$
4) $\begin{pmatrix} 1 & 3 \\ 1 & 0 \\ 2 & 4 \end{pmatrix}$

48. If $5A = \begin{pmatrix} 3 & -4 \\ 4 & x \end{pmatrix}$ and $AA^T = A^TA = I$ then $x =$

- 1) 3
2) -3
3) 2
4) -2

S.P.Q. Student Practice Questions

49. If $O(A) = 2 \times 3$, $O(B) = 3 \times 2$, and $O(C) = 3 \times 3$, which one of the following is not defined

- 1) $CB + A^T$
2) BAC
3) $C(A+B^T)^T$
4) $C(A+B^T)$
5. If the order of A is 4×5 and the order of B is 4×5 and the order of C is 7×3 , then the order of $(A^T B)^T C^T$ is
1) 4×5
2) 4×3
3) 4×3
4) 5×7
5) 3×7

51. If $3A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ then

1) $AA^T = A^TA = I$

2) $AA^T = A^TA = -I$

3) $AA^T = A^TA = 0$

4) $AA^T = A^TA = A$

52. Which of the following is not true, if A and B are two matrices each of order $n \times n$, then

1) $(A+B)^T = A^T + B^T$

2) $(A-B)^T = A^T - B^T$

3) $(AB)^T = A^T B^T$

4) $(ABC)^T = C^T B^T A^T$

SYMMETRIC, SKew SYMMETRIC MATRICES & SPECIAL TYPES OF MATRICES

C.R.T.Q Class Room Teaching Questions

53. If $A = \begin{bmatrix} 2 & x-3 & x-2 \\ 3 & -2 & -1 \\ 4 & -1 & -5 \end{bmatrix}$

is a symmetric matrix then $x =$
1) 0 2) 3 3) 6 4) 8

54. $\begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} = P+Q$, where P is a symmetric
and Q is a skew-symmetric then Q =

$\begin{pmatrix} 0 & -1 & 2 \\ \frac{1}{2} & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$

3) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

55. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ then A is

- 1) an idempotent matrix
- 2) nilpotent matrix
- 3) involuntary
- 4) orthogonal matrix

S.P.Q. Student Practice Questions

56. $A = \begin{bmatrix} x & -7 \\ 7 & y \end{bmatrix}$ is a skew-symmetric matrix, then $(x,y) =$

- 1) (1,-1)
- 2) (7,-7)
- 3) (0,0)
- 4) (14,-14)

57. $\begin{bmatrix} 1 & 6 \\ 7 & 2 \end{bmatrix} = P+Q$, where P is a symmetric & Q is a skew-symmetric then P =

1) $\begin{bmatrix} 1 & \frac{13}{2} \\ \frac{13}{2} & 2 \end{bmatrix}$

2) $\begin{bmatrix} 1 & -\frac{13}{2} \\ -\frac{13}{2} & 2 \end{bmatrix}$

3) $\begin{bmatrix} 1 & -\frac{1}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$

4) $\begin{bmatrix} 0 & \frac{13}{2} \\ \frac{13}{2} & 0 \end{bmatrix}$

- 1) an idempotent matrix
- 2) nilpotent matrix
- 3) an orthogonal matrix
- 4) symmetric

DETERMINANTS AND ITS PROPERTIES

C.R.T.Q Class Room Teaching Questions

59. If $A = \begin{bmatrix} a & c & b \\ b & a & c \\ c & b & a \end{bmatrix}$ then the cofactor of a

~~Sum A + A^T is~~

$$D - \left(2ab(b+c) - (b+c)^2 \right)$$

$$2) ab-b^2$$

$$3) a^2-bc$$

$$4) 2a(a+c)-(a+c)^2$$

62. The value of $\begin{vmatrix} 2+i & 2-i \\ 1+i & 1-i \end{vmatrix}$ is

{as $i^2 = -1\}$

1) A complex quantity

2) real quantity

3) 0

4) cannot be determined

$$63. \det \begin{bmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{bmatrix} =$$

$$1) -8 \quad 2) -6 \quad 3) -1 \quad 4) 0$$

$$64. \begin{vmatrix} a^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} =$$

$$1) abc \quad 2) 4abc \quad 3) 4abc^2 \quad 4) 2abc^2$$

$$65. \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & 2+\lambda \\ \lambda+1 & 2-\lambda & 2-4 \\ \lambda-3 & \lambda+4 & 32 \end{vmatrix}$$

$$= p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t \text{ then } t =$$

$$1) 16$$

$$2) 17$$

$$3) 18$$

$$4) 19$$

$$66. \text{ If } a, b, c \text{ are in A.P. then } \begin{vmatrix} x+1 & x+2 & x+\sigma \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} =$$

$$1) 1 \quad 2) 0 \quad 3) 2 \quad 4) 2$$

$$3) -1$$

67. If $a \neq 0, bc$ satisfy $\begin{vmatrix} a & 2b & 2c \\ b & a & c \\ c & b & a \end{vmatrix} = 0$ then

$$a^2bc =$$

$$2) 0$$

$$1) a+b+c$$

$$4) ab+bc-ac$$

$$3) b^2$$

$$4) ab+bc-ac$$

$$\log e^x$$

$$\log e^y$$

$$\log e^z$$

$$\log e^w$$

$$\log e^t$$

$$\log e^s$$

$$\log e^u$$

$$\log e^v$$

$$\log e^w$$

$$\log e^x$$

$$\log e^y$$

$$\log e^z$$

$$\log e^w$$

$$\log e^t$$

$$\log e^s$$

$$\log e^u$$

$$\log e^v$$

$$\log e^w$$

$$\log e^x$$

$$\log e^y$$

$$\log e^z$$

$$\log e^w$$

$$\log e^t$$

$$\log e^s$$

$$\log e^u$$

$$\log e^v$$

$$\log e^w$$

$$\log e^x$$

$$\log e^y$$

$$\log e^z$$

$$\log e^w$$

$$3) 16 \quad 4) 24$$

S.P.Q.**Student Practice Questions**

$$\text{If } A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

then cofactor of a_{21} is

- 1) $b^2 - ac$
2) $ac - b^2$
3) $a^2 - bc$
4) $bc - a^2$

72. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} x & y \\ y & x \end{bmatrix}$ then the cofactor of a_{21} in AB is

- 1) $-y - 4x$
2) $y + 4x$
3) $2x + 8y$
4) $-2x - 8y$

$$77. \begin{vmatrix} a-b & p-q & x-y \\ b-c & q-r & y-z \\ c-a & r-p & z-x \end{vmatrix} =$$

- 1) 0
2) 1
3) abc
4) xyz

$$78. \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} =$$

- 1) $(x+y+z)^3$
2) $2(x+y+z)^3$
3) $x+y+z$
4) $(x+y+z)^2$

79. If A, B, C are the angles of triangle

$$\begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix} =$$

ABC, then

- 1) 1
2) 0
3) -1
4) $\frac{3\sqrt{3}}{8}$

$$73. \det \begin{bmatrix} 1 & 29 & 32 \\ 3 & 68 & 87 \end{bmatrix} = \dots$$

- 1) 45
2) 64
3) 54
4) 32

$$74. \det \begin{bmatrix} 1990 & 1991 & 1992 \\ 1991 & 1992 & 1993 \\ 1992 & 1993 & 1994 \end{bmatrix} =$$

- 1) 1992
2) 1993
3) 1994
4) 0

$$75. \begin{vmatrix} (b+c)^2 & a^2 & b^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} =$$

- 1) $\lambda abc(a+b+c)^3$ then $\lambda = \dots$

$$76. \begin{vmatrix} 1+x & 2 & 3 \\ 1 & 2+x & 3 \\ 1 & 2 & 3+x \end{vmatrix} = 0 \text{ then } x =$$

- 1) 8
2) 16
3) -8
4) -16

$$77. \text{If } \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \dots$$

- 1) 0
2) 1
3) 2
4) -1

$$78. \log_a b \cdot \log_b a = \dots$$

- 1) ab
2) b/a
3) a/b
4) 0

84. If $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$ then $x =$

- 1) -1,-2 2) 1,2 3) 1,-2 4) -1,2

ADJOINT OF A MATRIX

C.R.T.Q. Class Room Teaching Questions

85. If $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$ then $|\text{adj } A| =$

- 1) 8 2) 16 3) 64 4) 128

86. $\text{Adj} \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix} \Rightarrow [a \ b] =$

- 1) $\begin{bmatrix} -4 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} -4 & -1 \end{bmatrix}$
 3) $\begin{bmatrix} 4 & 1 \end{bmatrix}$ 4) $\begin{bmatrix} 4 & -1 \end{bmatrix}$

87. If A is a 3×3 matrix and $|\text{adj } A| = 16$ then

$$|A| = \begin{cases} 1) 4 & 2) -4 & 3) \pm 4 & 4) 8 \\ 1) 4 & 2) 2 \times 3^8 & 3) 3^3 \times 2^4 & 4) 3^2 \times 2^8 \end{cases}$$

S.P.Q. Student Practice Questions

88. If $\det(A_{3 \times 3}) = 6$, then $\det(\text{adj } 2A) =$

- 1) 144 2) $2^2 \times 3^8$ 3) $3^3 \times 2^4$ 4) $3^2 \times 2^8$

94. If $|A| \neq 0$ and $(A-2I)(A-3I)=0$ then

$$A^{-1} = \begin{cases} 1) \frac{A-3I}{6} & 2) \frac{5I-A}{5} \\ 3) \frac{5A-I}{6} & 4) \frac{5I-A}{6} \end{cases}$$

S.P.Q. Student Practice Questions

89. If $P = \begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix}$, then $\text{adj}(P) =$

- 1) $\begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix} 2 \begin{bmatrix} 6 & -4 \\ -2 & 1 \end{bmatrix} 3 \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} 4 \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix}$
 2) $\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$

- 1) 5 2) 25 3) 125 4) 1/5

91. If A is a square matrix such that

A $\text{adj}(A) = \text{diag}(k, k)$ then $|\text{adj } A| =$

- 1) k 2) k^2 3) k^3
 4) k^4

INVERSE OF A MATRIX

C.R.T.Q. Class Room Teaching Questions

92. If $\begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ then $\alpha =$

- 1) 0 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$
 4) $\frac{3\pi}{4}$

93. The inverse of

$$\begin{bmatrix} 1 & a & b \\ 0 & x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -a & -b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is } x =$$

1) a 2) b 3) 0 4) 1

95. If $A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$ then $(B^{-1} A)^{-1} =$

- 1) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
 2) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$
 3) $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$
 4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

S.P.Q. Student Practice Questions

96. If $A = \begin{bmatrix} 2 & 2 \\ -2 & 3 \end{bmatrix}$ then $(B^{-1} A)^{-1} =$

- 1) $\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$
 2) $\begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$
 3) $\begin{bmatrix} 2 & 2 \\ -2 & 3 \end{bmatrix}$
 4) $\begin{bmatrix} -2 & 2 \\ 2 & 3 \end{bmatrix}$

96. The matrix A is such that $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$

$$\begin{bmatrix} -4 & 1 \\ 7 & 7 \end{bmatrix} \text{ then } A =$$

$$1) \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix} \quad 2) \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

$$3) \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \quad 4) \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$$

97. If A is a 3×3 non singular matrix and $|adj A| = |A|^x$. $|adj(adj A)| = |A|^y$, $|A^{-1}| = |A|^z$, then the values of x, y, z in descending order.

$$1) x, y, z \quad 2) z, y, x \\ 3) z, x, y \quad 4) y, x, z$$

98. If $A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 8 \end{bmatrix}$, then $A^{-1} =$

$$1) \begin{bmatrix} \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{10} & 0 \\ 0 & 0 & \frac{1}{8} \end{bmatrix} \quad 2) \begin{bmatrix} 9 & 0 & 0 \\ 0 & \frac{1}{10} & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$3) \begin{bmatrix} 9 & 0 & 0 \\ 0 & \frac{1}{10} & 0 \\ 0 & 0 & \frac{1}{8} \end{bmatrix} \quad 4) \begin{bmatrix} \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{10} & 0 \\ 0 & 0 & \frac{1}{8} \end{bmatrix}$$

99. If the value of a third order determinant is 11, then the value of the determinant of $A^{-1} =$

$$1) 11 \quad 2) 121 \quad 3) 1/11 \quad 4) 1/121$$

$$100. \text{ If } A = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ then } (Adj A)^{-1} =$$

RANK OF A MATRIX

C.R.T.Q. Class Room Teaching Questions

101. The rank of the matrix $A = \begin{bmatrix} -1 & 2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$ is

1) 3 2) 2 3) 1 4) 0

102. If I is a (9×9) unit matrix, then $\text{rank}(I) =$

1) 0 2) 3 3) 6 4) 9

103. The ranks of the matrices in descending order

$$A. \begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad B. \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad C. \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

S.P.Q. Student Practice Questions

104. The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is

1) 3 if $a=6$ 2) 1 if $a=-6$
3) 3 if $a=2$ 4) 2 if $a=-6$

$$105. \text{ The rank of } \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 \end{bmatrix} \text{ is :}$$

1) 0 2) 1 3) 2 4) 3

$$106. \text{ The Rank of } \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix} \text{ is }$$

1) 1 2) 2 3) 0 4) 3

SOLUTION OF SIMULTANEOUS EQUATIONS NON HOMOGENEOUS LINEAR EQUATIONS

C.R.T.Q. Class Room Teaching Questions

107. The solution of $2x+y+z=1, x-2y-3z=1,$

$3x+2y+4z=5$ is

1) 1,2,3 2) 1,2,-3
3) 1,-3,2 4) 1,3,2

108. The system of equations

$$2x + 6y + 11 = 0, 6y - 18z + 1 = 0$$

$$6x + 20y - 6z + 3 = 0$$

1) is consistent

2) has unique solution

3) is inconsistent

4) cannot be determined

109. The value of 'a' for which the equations $3x - y + az = 1$, $2x + y + z = 2$, $x + 2y - az = -1$ fail to have unique solution is

$$1) \frac{7}{2} \quad 2) -\frac{7}{2} \quad 3) \frac{2}{7} \quad 4) -\frac{2}{7}$$

110. The number of solutions of the equation $3x + 3y - z = 5$, $x + y + z = 3$, $2x + 2y - z = 3$

$$1) 1 \quad 2) 0 \quad 3) \text{infinite} \quad 4) \text{two}$$

S.P.Q.

Student Practice Questions

111. The number of solutions of the equations $2x - 3y = 5$, $x + 2y = 7$ is....

$$1) 1 \quad 2) 2 \quad 3) 4 \quad 4) 0$$

112. If the system of equations $x + y + z = 6$, $x + 2y + \lambda z = 0$, $x + 2y + 3z = 10$ has no solution, then $\lambda =$

$$1) 2 \quad 2) 3 \quad 3) 4 \quad 4) 5$$

113. The equations $x + 4y - 2z = 3$, $3x + y + 5z = 7$, $2x + 3y + z = 5$ have

- 1) A unique solution
- 2) Infinite number of solutions
- 3) No solution
- 4) Two solutions

114. If $x + y + z = 1$, $ax + by + cz = k$, $a^2x + b^2y + c^2z = k^2$ has unique solution then $x = \dots$

$$1) \frac{(k-b)(c-k)}{(a-b)(c-a)} \quad 2) \frac{(k-c)(a-k)}{(b-c)(c-a)}$$

$$3) \frac{(k-a)(b-k)}{(b-c)(c-a)} \quad 4) (k-a)(k-b)(k-c)$$

115. Solution of the system of equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2, (x, y, z) =$$

$$1) \frac{1}{(1,2,3)} \quad 2) (2,3,5)$$

$$3) \frac{1}{(3,2,5)} \quad 4) (5,3,2)$$

116. If the system of equations $2x - 3y + 4z = 0$, $5x - 2y - z = 0$, $21x - 8y + az = 0$ has infinity solution then $a =$

$$1) -5 \quad 2) -4 \quad 3) 2 \quad 4) 4$$

117. The real value of 'a' for which the equations $ax + y + z = 0$, $-x + ay + z = 0$, $-x - y + az = 0$ have non-zero solution

$$1) 1 \quad 2) 0 \quad 3) -1 \quad 4) \text{all the above}$$

118. If the system of equations $2x + 3ky + (3k+4)z = 0$

$$x + (k+4)y + (4k+2)z = 0,$$

$x + 2(k+4)y + (3k+4)z = 0$ has non trivial solution then $k =$

$$1) -8 \text{ or } \frac{1}{2} \quad 2) 8 \text{ or } -\frac{1}{2}$$

$$3) -4 \text{ or } \frac{1}{2} \quad 4) 4 \text{ or } -\frac{1}{2}$$

HOMOGENEOUS LINEAR EQUATION:

C.R.T.Q.

Class Room Learning Questions

119. If x, y, z not all zeros and the equation $x + y + z = 0$, $(1+a)x + (2+a)y - 8z = 0$ trivial solution then $a =$

$$1) 2 + \sqrt{15} \quad 2) \sqrt{15} \quad 3) -5 \pm 2\sqrt{2}$$

1. If $\begin{bmatrix} \lambda^2 - 2\lambda + 1 & \lambda - 2 \\ 1 - \lambda^2 + 3\lambda & 1 - \lambda^2 \end{bmatrix} = A\lambda^2 + B\lambda + C$

where A,B,C are matrices then $B+C=$

$$\begin{aligned} 1) \begin{bmatrix} -1 & -1 \\ 4 & 1 \end{bmatrix} & 2) \begin{bmatrix} 1 & -1 \\ 4 & 1 \end{bmatrix} \\ 3) \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix} & 4) \begin{bmatrix} -1 & -1 \\ -4 & 1 \end{bmatrix} \end{aligned}$$

8. If $A = \begin{bmatrix} 0 & x \\ y & 0 \end{bmatrix}$ and $A^3 + A = O$ then

- 1) $xy = 0$
- 2) $xy = \frac{1}{2}$
- 3) $xy = -1$
- 4) $xy = 1$

9. If $A = \begin{pmatrix} \cos^2 \alpha & \cos \alpha \sin \alpha \\ \cos \alpha \sin \alpha & \sin^2 \alpha \end{pmatrix}$ and

$$B = \begin{pmatrix} \cos^2 \beta & \cos \beta \sin \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{pmatrix}$$

are two matrices such that the product AB is the null matrix then $\alpha - \beta =$

- 1) 0
- 2) multiple of π
- 3) an odd multiple of $\frac{\pi}{2}$
- 4) can not be determined

10. If A,B are two square matrices such that $AB=B$; $BA=A$ and $n \in N$ then $(A+B)^n =$

$$1) 2^n(A+B) \quad 2) 2^{n-1}(A+B)$$

$$3) 2^{n+1}(A+B) \quad 4) 2^{n+2}(A+B)$$

- II. The number of 2×2 matrices that can be formed by using 1,2,3,4 without repetition is
 - 1) 24
 - 2) 12
 - 3) 6
 - 4) 256

SYMMETRIC MATRICES

C.R.T.Q Class Room Teaching Questions

12. If $[3x^2 + 10xy + 5y^2] = [x \ y] A \begin{bmatrix} x \\ y \end{bmatrix}$, and A is a symmetric matrix then A =

$$1) \begin{bmatrix} 3 & 10 \\ 10 & 5 \end{bmatrix} \quad 2) \begin{bmatrix} 10 & 3 \\ 5 & 10 \end{bmatrix} \quad 3) \begin{bmatrix} +3 & -5 \\ -5 & +5 \end{bmatrix} \quad 4) \begin{bmatrix} 3 & 5 \\ 5 & 5 \end{bmatrix}$$

13. A square matrix A is said to be nilpotent of index m. If $A^m = 0$ now, if for this A,

$$(I - A)^n = I + A + A^2 + A^3 + \dots + A^{m-1},$$

then n is equal to

- 1) 0
- 2) m
- 3) -m
- 4) -1

14. Let A be the set of all 3×3 skew-symmetric matrices whose entries are either $-1, 0$, or 1 if there are exactly three 0's, three 1's then the number of such matrices is

- 1) 3
- 2) 6
- 3) 8
- 4) 9

- The maximum number of different possible non-zero entries in a skew-symmetric matrix of order 'n' is

$$1) \frac{1}{2}(n^2 - n) \quad 2) \frac{1}{2}(n^2 + n)$$

$$3) n^2 \quad 4) (n^2 - n)$$

S.P.Q. Student Practice Questions

15. If $\begin{bmatrix} 1 & 3 & 0 \\ 1 & 0 & -2 \\ -4 & -4 & 4 \end{bmatrix} = A+B$ where A is symmetric matrix and B is skew-symmetric then $A - B =$

$$1) \begin{bmatrix} 1 & 3 & 0 \\ 1 & 0 & -2 \\ -4 & -4 & 4 \end{bmatrix} \quad 2) \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 4 \\ 3 & -1 & 2 \end{bmatrix}$$

16. If $\begin{bmatrix} 0 & 1 & -1 \\ 2 & 3 & 4 \\ -4 & 1 & 2 \end{bmatrix} = A+B$ where A is

$$1) \begin{bmatrix} 1 & 3 & 0 \\ 3 & 0 & -4 \\ 0 & -2 & 4 \end{bmatrix} \quad 2) \begin{bmatrix} 2 & -3 & 0 \\ 0 & 1 & 2 \\ 2 & 4 & 0 \end{bmatrix}$$

- SPECIAL TYPES OF MATRICES, SYMMETRIC & SKEW-MATRICES**

17. If $\begin{pmatrix} 2 & 4 \\ -1 & k \end{pmatrix}$ is an nilpotent matrix of index '2' then k =

- 1) 2 2) -2 3) 3 4) -3

DETERMINANTS

CRT.Q Class Room Teaching Questions

18. If $r^2 = a^2 + b^2 + c^2 + d^2 = ab + bc + cd$ then

$$\begin{vmatrix} r^2 & s^2 & t^2 \\ s^2 & r^2 & u^2 \\ t^2 & u^2 & r^2 \end{vmatrix} =$$

- 1) $3abc - 2^2b^2c^2$ 2) $2^2a^2b^2c^2 - 3abc$

- 3) $(abc - a^2b^2c^2)^2$ 4) 0

19. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then $\left| \frac{1}{x-y} \right| A^x =$

- 1) I 2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 3) 0

- 3) A 4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

20. If $A_x = \begin{bmatrix} x & x-1 \\ x-1 & x \end{bmatrix}$ then $|A_1| + |A_2| + \dots + |A_{2015}| =$

- 1) 0 2) $2015 \cdot 3 \cdot (2015)^2 \cdot 4 \cdot (2015)^3$

21. Matrix A is given by $A = \begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix}$ then the determinant of $A^{2015} - 6A^{2014}$ is

- 1) 2^{2016} 2) $(-11)2^{2015}$
 3) $-2^{2015} \times 7$ 4) $(-9)2^{2014}$

22. If $\begin{vmatrix} x^m & x^{m+2} & x^{m+4} \\ y^m & y^{m+2} & y^{m+4} \\ z^m & z^{m+2} & z^{m+4} \end{vmatrix} =$

$$=(x-y)(y-z)(z-x)\left(\frac{1}{x-y}+\frac{1}{y-z}\right)$$

then value of m is

- 1) -1 2) -2 3) 1 4) 2

23. If $\begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = \begin{vmatrix} 1 & -x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ then x =

- 1) $\sin x$ 2) $\cos x$
 3) $\sin x \cdot \cos x$ 4) $\sin -\cos x$

24. If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = k$, then $\begin{vmatrix} 5x & 25 & 2z \\ 5m & n & p \\ 3x & y & z \end{vmatrix} =$

- 1) $k/5$ 2) $2k$ 3) $3k$ 4) $6k$

25. If $\begin{vmatrix} a+b & b+c & c+a \\ c+a & a+b & b+c \\ b+c & c+a & a+b \end{vmatrix} = 0$ then the circuitous matrix whose elements of first column are a,b,c when 'b' equals

- 1) 5 2) 6 3) -2 4) 2

26. If x,y,z are integers in A.P lying between 1 and 9 and x≤y≤z and z is three digit numbers, then the value

- 1) $\begin{vmatrix} 5 & 4 & 3 \\ x & y & z \\ 5 & 4 & 3 \end{vmatrix}$
 2) $x+y+z$
 3) 0

27. If the entries in a 3×3 either 0 or 1, then the determinant of their determinants is

- 1) 1 2) 2 3) 3 4) 9
 28. If a,b,c are the p², q², r² terms in H.P. then

- 1) $\frac{1}{ab}$ 2) $\frac{1}{bc}$ 3) $\frac{1}{ca}$
 4) abc

29. The roots of $\begin{vmatrix} x & a & b & 1 \\ a & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & v & 1 \end{vmatrix} = 0$ are independent of :

1) λ, μ, v

2) a, b

3) λ, μ, v, a, b

4) 0, a

30. If $[.]$ denotes the greatest integer less than or equal to the real number under consideration, and $-1 \leq x < 0$; $0 \leq y < 1$; $1 \leq z < 2$, then the value of the

determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is

1) $[x]$

2) $[y]$

3) $[z]$

4) $[x]+[y]+[z]$

31. If α is the repeated root of quadratic equation $f(x)=0$ and $A(x), B(x)$ and $C(x)$ are polynomials of degree 3, 4 and 5 respectively, then

$\phi(x) = \begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A^1(\alpha) & B^1(\alpha) & C^1(\alpha) \end{vmatrix}$

divisible by

1) $f(x)$

2) $A(x)$

3) $B(x)$

4) $C(x)$

32. If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+2 & n^2+n \\ 2k-1 & n^2 & n^2+n+2 \end{vmatrix}$

and $\sum_{k=1}^n D_k = 48$, then 'n' equals

1) 4

2) 6

3) 8

4) 10

33. If $a+b+c=0$ and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ then $x =$

1) 0

2) $\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

3) $-\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

4) $0, \pm \sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

34. If $a, b, c > 0$ and $x, y, z \in R$ then the determinant:

$$\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^y+b^{-y})^2 & (b^y-b^{-y})^2 & 1 \\ (c^z+c^{-z})^2 & (c^z-c^{-z})^2 & 1 \end{vmatrix}$$
 is equal to :

1) $a^x b^y c^z$

2) $a^{-x} b^{-y} c^{-z}$

3) 0

4) $a^{2x} b^{2y} c^{2z}$

35. If $A = \begin{vmatrix} 1+\cos\alpha & 1+\sin\alpha & 1 \\ 1+\cos\beta & 1+\sin\beta & 1 \\ 1 & 1 & 1 \end{vmatrix} \neq 0$, then

1) $\alpha = \beta$

2) $\alpha \neq \beta + n\pi$, n being any integer

3) $\alpha \neq \beta + \pi/2$

4) $\alpha \neq \beta - \pi/2$

36. If ω is a cube root of unity then

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$
 has a factor

1) $a+b\omega+c\omega^2$

2) a-b-c

3) $a-b\omega^2-c\omega$

4) a+b-c

37. If $a = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$ then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a^3 \end{vmatrix} =$

- 1) purely real
- 2) purely imaginary
- 3) complex number
- 4) rational

38. $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$
then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} =$

- 1) 1
- 2) 2
- 3) 3
- 4) c

39. If $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r \\ p-x & q-y & r \end{vmatrix} = 0$ then the value of $\frac{p}{x} + \frac{q}{y} + \frac{r}{z}$ is

- 1) 0
- 2) 1
- 3) 2
- 4) $4pqr$

40. If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} a_1 + pb_1 & b_1 + qc_1 & c_1 + ra_1 \\ a_2 + pb_2 & b_2 + qc_2 & c_2 + ra_2 \\ a_3 + pb_3 & b_3 + qc_3 & c_3 + ra_3 \end{vmatrix}$
then $\Delta_1 =$

- 1) $\Delta(1+pqr)$
- 2) $\Delta(1+p+q+r)$
- 3) $\Delta(1-pqr)$
- 4) Δ

41. If $\begin{vmatrix} (1+x)^4 & (1+x)^2 & (1+x)^3 \\ (1+x)^4 & (1+x)^5 & (1+x)^6 \\ (1+x)^7 & (1+x)^8 & (1+x)^9 \end{vmatrix} = a_0 + a_1x + a_2x^2 + \dots$,
then, a_1 is equal to

- 1) 1
- 2) 2
- 3) 0
- 4) 3

42. If $y = \cos x, y_n = \frac{d^n(\cos x)}{dx^n}$ then

$$\begin{vmatrix} y_4 & y_5 & y_6 \\ y_7 & y_8 & y_9 \\ y_{10} & y_{11} & y_{12} \end{vmatrix} = \dots$$

- 1) 0
- 2) -cos x
- 3) cos x
- 4) sin x

43. The value of the determinant

$$\begin{vmatrix} \sin \theta & \cos \theta & \sin 2\theta \\ \sin \left(\theta + \frac{2\pi}{3}\right) & \cos \left(\theta + \frac{2\pi}{3}\right) & \sin \left(2\theta + \frac{4\pi}{3}\right) \\ \sin \left(\theta - \frac{2\pi}{3}\right) & \cos \left(\theta - \frac{2\pi}{3}\right) & \sin \left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix}$$

- 1) 0
- 2) $\sin \theta$
- 3) $\cos \theta$
- 4) $\sin \theta + \cos \theta$

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0$$

- 1) ΔABC is equilateral
- 2) ΔABC is right angled isosceles
- 3) ΔABC is isosceles
- 4) ΔABC is a right angle

$$\begin{aligned} \Delta ABC, \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = \\ 1) \frac{1}{8R^3}(a-b)(b-c)(c-a) \\ 2) 8R^3 \\ 3) (a-b)(b-c)(c-a) \\ 4) \frac{1}{8R}(a-b)(a-c)(b-a) \end{aligned}$$

46. If $f(\theta) = \begin{vmatrix} \cos\theta & 1 & 0 \\ 1 & 2\cos\theta & 1 \\ 0 & 1 & 2\cos\theta \end{vmatrix}$ then range of $f(\theta)$ is

- 1) $[0,1]$ 2) $[-1,0]$ 3) $[-1,1]$ 4) $\left[0, \frac{1}{2}\right]$

S.P.Q. Student Practice Questions

47. If A and B are square matrices of

order 3 such that $|A|=-1, |B|=3$ then

$$|3AB| =$$

48. If $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & b^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$

Then $x =$

- 1) 1 2) 2 3) 3 4) 1/2

52. $\begin{vmatrix} 2 & a+b+c+d & ab+cd \\ a+b+c+d & 2(a+b)(c+d) & ab(c+d)+cd(a+b) \\ ab+cd & ab(c+d)+cd(a+b) & 2abcd \end{vmatrix}$

53. 1) abcd 2) 0 3) $a+b+c+d$ 4) 1
If $f(x) = \tan x$ and A, B, C are the angles of $\triangle ABC$, then

54. If $\Delta_1 = \begin{vmatrix} f(A) & f(\pi/4) & f(\pi/4) \\ f(\pi/4) & f(B) & f(\pi/4) \\ f(\pi/4) & f(\pi/4) & f(C) \end{vmatrix}$

- 1) 0 2) -2 3) 2 4) 1

55. Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then $\frac{\Delta_1}{\Delta_2}$ is equal to

- 1) $a_1a_2c_3$ 2) $a_1a_2a_3$ 3) $a_3b_2c_1$ 4) $a_1b_1c_1 + a_2b_2c_2 + a_3b_3c_3$

56. If the determinant $\begin{vmatrix} a & a+b & a+b+c \\ 3a & 4a+3b & 5a+4b+3c \\ 6a & 9a+6b & 11a+9b+6c \end{vmatrix}$

- a=1, b=ω and c=ω² then $\Delta =$
1) ω 2) $-\omega^2$ 3) $1+\omega+\omega^2$ 4) -1

57. Let $\begin{vmatrix} b-c & c-a & a-b \\ b'-c' & c'-a' & a'-b' \\ b''-c'' & c''-a'' & a''-b'' \end{vmatrix}$ is expressible as

- then the value of $5a+4b+3c+2d+e$ is equal to

- 1) 0 2) -16 3) 16 4) -11

58. If A, B, C are the angles of a $\triangle ABC$,

- 1) -1 2) 0 3) 1 4) 2

Let t be a positive integer and

59. $\Delta_1 = \begin{vmatrix} 2t-1 & m^2-1 & \cos^2(m^2) \\ m & c_i & 2^m \\ a'' & b'' & \cos^2(m) \\ 1 & m+1 & \cos(m^2) \end{vmatrix}$ then the value of $\sum_{i=0}^m \Delta_i$ is equal to

- 1) 2^m 2) 0 3) $2^m \cos^2(2^m)$ 4) m^2
3) $\frac{a^2+b^2+c^2}{16R^2\Delta}$ 4) 0

58. If $D = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ then $\frac{D}{(10!)^3} - 4 =$

- 1) 2900 2) 2800
3) 2700 4) 2600

59. The value of the determinant

$$\begin{vmatrix} \sin \alpha \cos \beta & \cos \alpha \cos \beta & -\sin \alpha \sin \beta \\ \sin \alpha \sin \beta & \cos \alpha \sin \beta & \sin \alpha \cos \beta \\ \cos \alpha & -\sin \alpha & 0 \end{vmatrix}$$

- 1) independent of α
2) Independent of β
3) independent of α and β
4) cannot be said

63. If $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} = 0$ and a, b, c

not a root of $ax^2 + 2bx + c = 0$, then

- 1) a, b, c are in A.P
2) a, b, c are in G.P
3) a, b, c are in H.P
4) a, b, c are in A.G.P

64. If $f(x) = \begin{vmatrix} 2\cos x & 1 & 0 \\ x - \frac{\pi}{2} & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$, then

$$f'(x) = \begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$$

$f(3x) - f(x) =$

1) $3x\lambda^2$ 2) $6x\lambda^2$
3) $x\lambda^2$ 4) $9x\lambda^2$

65. Let

$$\Delta(x) = \begin{vmatrix} \cos^2 x & \cos x \sin x & -\sin x \\ \cos x \sin x & \sin^2 x & \cos x \\ \cos x & -\cos x & 0 \end{vmatrix}$$

61. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is

- 1) 0 2) 2
3) 1 4) 3

62. The sum of two non integral roots of

$$\begin{vmatrix} x & 3 & 4 \\ 5 & x & 5 \\ 4 & 2 & x \end{vmatrix} = 0$$

- 1) 4 2) -4
3) -25 4) 25

63. If $\int_0^{\pi/2} [\Delta(x) + \Delta'(x)] dx$ equals

- 1) $\pi/3$ 2) $\pi/2$
3) 2π 4) $3\pi/2$

66. If $a_1, b_1, c_1, a_2, b_2, c_2$ and a_3, b_3, c_3 are

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

- 1) divisible by 2 but not necessarily by 4
2) divisible by 4 but not necessarily by 8
3) divisible by 8 but not necessarily by 4
4) divisible by 25

67. If $f(x) = \det \begin{bmatrix} x^n & \sin x & \cos x \\ n & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{bmatrix}$ the value of

- $$\frac{d^n}{dx^n}(f(x)) \text{ at } x=0 \text{ is}$$
- 1) -1 2) 0 3) 1 4) a^6

ADJOINT MATRIX

C.R.T.Q.

Class Room Teaching Questions

68. If A and B are square matrices of same order and A is non-singular, then for a positive integer 'n', $(A^{-1}BA)^n$ is equal to

- 1) $A^{-n}B^nA^n$ 2) $A^nB^nA^{-n}$
 3) $A^{-1}B^nA$ 4) $n(A^{-1}BA)$

69. If A is orthogonal matrix of order 3 then $\det(\text{adj}2A) =$

- 1) 4 2) 16 3) 27 4) 64

S.P.Q.

Student Practice Questions

70. A, B, C are cofactors of elements, a, b, c in

$\begin{bmatrix} a & b & c \\ 2 & 4 & 7 \\ -1 & 0 & 3 \end{bmatrix}$ then the value of $(2A+4B+7C)$ is equal to

- 1) 0 2) 2 3) -1 4) 4

71. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ then $\det(\text{adj}(\text{adj}A))$ is

- 1) $(14)^4$ 2) $(14)^3$ 3) $(14)^2$ 4) $(14)^1$

72. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, then $[A(\text{adj}A)A^{-1}]A =$

- 1) $2 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ 2) $\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$

- 3) $\begin{bmatrix} 0 & \frac{1}{6} & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{bmatrix}$ 4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

73. If $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $A = \frac{1}{2} \begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$,

then $SAS^{-1} =$

- 1) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ 2) $\frac{1}{2} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

- 3) $2 \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ 4) $3 \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

74. $A(\alpha, \beta) = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & e^\beta \end{bmatrix}$

$\Rightarrow [A(\alpha, \beta)]^{-1} =$

- 1) $A(-\alpha, \beta)$ 2) $A(-\alpha, -\beta)$
 3) $A(\alpha, -\beta)$ 4) $A(\alpha, \beta)$

75. A nontrivial solution of the system of equations $x + \lambda y + 2z = 0, 2x + \lambda z = 0,$

$2\lambda x - 2y + 3z = 0$ is given by $x:y:z =$

- 1) 1:2:-2 2) 1:-2:2
 3) 2:1:2 4) 2:1:-2

76. If ' ω ' is cube root of unity and $x + y + z = a, x + \omega y + \omega^2 z = b, x + \omega^2 y + \omega z = c$ then which of the following is correct

- 1) $x = \frac{a+b+c}{3}$ 2) $y = \frac{a+b\omega^2 + \omega c}{3}$
 3) $z = \frac{a+b\omega + \omega^2 c}{3}$ 4) All the above

77. If the trivial solution is the only solution of the system of equations $x - ky + z = 0, kx + 3y - kz = 0, 3x + y - z = 0$. Then the set of all values of k is

- 1) $\{2, -3\}$ 2) $R - \{2\}$
 3) $R - \{2, -3\}$ 4) $R - \{-3\}$

78. If the system of equations $ax + y + z = 0$, $x + by + z = 0$, $x + y + cz = 0$ ($a, b, c \neq 1$) has a non trivial solution then

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$$

- 1) 1 2) -1 3) 2 4) -2

79. The system of equations

$$(\cos 2\theta)x + 4y + 3z = 0, 2x + 7y + 7z = 0$$

has non trivial solution then

$$\sin 3\theta + 2 \cos 2\theta =$$

- 1) 2 2) -2 3) 0 4) 1

80. If the system of linear equations $x + 2y + 3z = \lambda x$, $3x + y + 2z = \lambda y$, $2x + 3y + z = \lambda z$ has non trivial solution then $\lambda =$

- 1) 6 2) 12 3) 18 4) 16

81. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$, $2x + 2y + z = 0$ posses a non zero solution is:

- 1) 3 2) 2 3) 1 4) zero

INVERSE MATRIX

C.R.T.Q

Class Room Teaching Questions

82. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and

$$10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}.$$

If B is the inverse of A , then α is :

- 1) -2 2) -1 3) 2 4) 5

83. A is an involuntary matrix given by

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

then the inverse of $\frac{A}{2}$ will be

- 1) $2A$ 2) $\frac{A^{-1}}{2}$ 3) $\frac{A}{2}$ 4) A^2

84. If $A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ then $A^{-1} + (A - al)(A - cl)$,

1) $\frac{1}{ac} \begin{bmatrix} a & b \\ 0 & -c \end{bmatrix}$ 2) $\frac{1}{ac} \begin{bmatrix} -a & b \\ 0 & c \end{bmatrix}$
 3) $\frac{1}{ac} \begin{bmatrix} c & -b \\ 0 & a \end{bmatrix}$ 4) $\frac{1}{ac} \begin{bmatrix} c & b \\ 0 & a \end{bmatrix}$

S.P.Q.

Student Practice Questions

85. Let $A_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

I) $A_\alpha A_\beta = A_{\alpha+\beta}$ II) $A_\alpha B_\beta = A_{\alpha\beta}$

III) $(A_\alpha)^{-1} = -A_\alpha$ IV) $(A_\alpha)^{-1} = A_\alpha$

Then which of the above statements are correct

- 1) only II and III 2) only II and IV
 3) only I and III 4) only I and IV

86. If the product of the matrix

$$B = \begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

with a matrix A has

inverse $C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$ then $A^{-1} =$

1) $\begin{bmatrix} -3 & -5 & 5 \\ 0 & 9 & 14 \\ 2 & 2 & 6 \end{bmatrix}$ 2) $\begin{bmatrix} -3 & 5 & 5 \\ 0 & 0 & 9 \\ 2 & 14 & 16 \end{bmatrix}$

3) $\begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}$ 4) $\begin{bmatrix} -3 & -3 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 16 \end{bmatrix}$

SOLUTIONS OF SIMULTANEOUS EQUATION

C.R.T.Q

Class Room Teaching Questions

87. If A, B, C are the angles of a triangle the system of equations $(\sin A)x + y + z = \cos A$, $x + (\sin B)y + z = \cos B$, $x + y + (\sin C)z = 1 - \cos C$ has
- 1) no solution 2) unique solution
 3) infinitely many solutions 4) finitely many solutions

PROPERTIES

C.R.T.Q

Class Room Teaching Questions

5. If A, B and C are $n \times n$ matrices and $\det(A) = 2, \det(B) = 3$ and $\det(C) = 5$, then the value of the $\det(A^2 BC^{-1})$ is equal to

$$1) \frac{6}{5} \quad 2) \frac{12}{5} \quad 3) \frac{18}{5} \quad 4) \frac{24}{5}$$

6. If $\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} \frac{1}{2} & 4 \\ \frac{1}{3} & 3 \end{vmatrix} + \begin{vmatrix} \frac{1}{4} & 4 \\ \frac{1}{9} & 3 \end{vmatrix} + \dots \infty =$

$$1) 1 \quad 2) -1 \quad 3) 0 \quad 4) \infty$$

7. The maximum and minimum values of (3×3) determinant whose elements belong to $\{0, 1, 2, 3\}$ is

$$1) \pm 9 \quad 2) \pm 15 \quad 3) \pm 54 \quad 4) \pm 32$$

8. Let $A = \begin{bmatrix} 1+x^2-y^2-z^2 & 2(xy+z) & 2(zx-y) \\ 2(xy-z) & 1+y^2-z^2-x^2 & 2(yz+x) \\ 2(zx+y) & 2(yz-x) & 1+z^2-x^2-y^2 \end{bmatrix}$
then $\det A$ is equal to

$$1) (1+xy+yz+zx)^3 \quad 2) (1+x^2+y^2+z^2)^3 \\ 3) (xy+yz+zx)^3 \quad 4) (1+x^3+y^3+z^3)^2$$

9. Let $ax^7+bx^6+cx^5+dx^4+ex^3+fx^2+gx+h$
 $= \begin{vmatrix} x+1 & x^2+2 & x^2+x \\ x^2+x & x+1 & x^2+2 \\ x^2+2 & x^2+x & x+1 \end{vmatrix}$ then

$$1) g=3 \text{ and } h=-5 \quad 2) g=-3 \text{ and } h=-5 \\ 3) g=3 \text{ and } h=9 \quad 4) g=-3, h=9$$

10. If $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$ and $\det A = 6$,

- If $B = \begin{bmatrix} p+x & q+y & r+z \\ a+x & b+y & c+z \\ a+p & b+q & c+r \end{bmatrix}$, then

- 1) $\det B = 6 \quad 2) \det B = -6 \\ 3) \det B = 12 \quad 4) \det B = -12$

11. Let $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and
 $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value

$$\frac{D_1}{D_2} \text{ where } b \neq 0 \text{ and } ad \neq bc, \text{ is}$$

$$1) -2 \quad 2) 0 \quad 3) -2b \quad 4) 2b$$

12. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix} \text{ and } \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are

$$1) \frac{\Delta_1}{\Delta_3} \text{ and } \frac{\Delta_2}{\Delta_3}$$

$$2) \frac{\Delta_2}{\Delta_1} \text{ and } \frac{\Delta_3}{\Delta_1}$$

$$3) \log\left(\frac{\Delta_1}{\Delta_3}\right) \text{ and } \log\left(\frac{\Delta_2}{\Delta_3}\right)$$

$$4) e^{\Delta_1/\Delta_3} \text{ and } e^{\Delta_2/\Delta_3}$$

13. The value of the determinant

$$\Delta = \begin{vmatrix} 1-a_1^3 b_1^3 & 1-a_1^3 b_2^3 & 1-a_1^3 b_3^3 \\ 1-a_1 b_1 & 1-a_1 b_2 & 1-a_1 b_3 \\ 1-a_2^3 b_1^3 & 1-a_2^3 b_2^3 & 1-a_2^3 b_3^3 \\ 1-a_2 b_1 & 1-a_2 b_2 & 1-a_2 b_3 \\ 1-a_3^3 b_1^3 & 1-a_3^3 b_2^3 & 1-a_3^3 b_3^3 \\ 1-a_3 b_1 & 1-a_3 b_2 & 1-a_3 b_3 \end{vmatrix}$$

$$1) 0$$

- 2) dependent only on a_1, a_2, a_3
3) dependent only on b_1, b_2, b_3
4) dependent on $a_1, a_2, a_3, b_1, b_2, b_3$

14. Let $\{D_1, D_2, D_3, \dots, D_n\}$ be the set of all third order determinants that can be formed with the distinct non-zero real numbers a_1, a_2, \dots, a_9 then

$$\begin{array}{ll} 1) \sum_{i=1}^n D_i = 1 & 2) \sum_{i=1}^n D_i = 0 \\ 3) D_i = D_j \text{ for all } i, j & 4) \text{None of the above} \end{array}$$

15. If

$$\Delta = \begin{vmatrix} \cos(\alpha_1 - \beta_1) & \cos(\alpha_1 - \beta_2) & \cos(\alpha_1 - \beta_3) \\ \cos(\alpha_2 - \beta_1) & \cos(\alpha_2 - \beta_2) & \cos(\alpha_2 - \beta_3) \\ \cos(\alpha_3 - \beta_1) & \cos(\alpha_3 - \beta_2) & \cos(\alpha_3 - \beta_3) \end{vmatrix}$$

then Δ equals

- 1) $\cos \alpha_1 \cos \alpha_2 \cos \alpha_3 \cos \beta_1 \cos \beta_2 \cos \beta_3$
- 2) $\cos \alpha_1 + \cos \alpha_2 + \cos \alpha_3 + \cos \beta_1 + \cos \beta_2 + \cos \beta_3$
- 3) $\cos(\alpha_1 - \beta_1) \cos(\alpha_2 - \beta_2) \cos(\alpha_3 - \beta_3)$
- 4) 0

16. If

$$\Delta(x) = \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} = A + Bx + Cx^2 + \dots$$

then B is equal to

- 1) 0
- 2) 1
- 3) 2
- 4) 4

S.P.Q.

Student Practice Questions

17. If $f(x) = \begin{bmatrix} \sin x & \csc x & \tan x \\ \sec x & x \sin x & x \tan x \\ x^2 - 1 & \cos x & x^2 + 1 \end{bmatrix}$ then

$\int_{-a}^a |f(x)| dx$ equals

- 1) 1
- 2) -1
- 3) 2a
- 4) 0

18. If $\begin{vmatrix} 1 & 1 & 1 \\ mC_1 & m+3C_1 & m+6C_1 \\ mC_2 & m+3C_2 & m+6C_2 \end{vmatrix} = 2^\alpha 3^\beta 5^\gamma$ then

$\alpha + \beta + \gamma$ is equal

- 1) 3
- 2) 5
- 3) 7
- 4) 0

19. Let $\Delta(x) = \begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix}$ and

$\int_0^2 \Delta(x) dx = -16$, where a, b, c, d are in A.P. then the common difference

- 1) 1
- 2) 2
- 3) 3
- 4) 4

$$\begin{vmatrix} \sqrt{13} + \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} + \sqrt{26} & 5 & \sqrt{10} \\ 3 + \sqrt{65} & \sqrt{15} & 5 \end{vmatrix} =$$

- 1) $15\sqrt{2} - 25\sqrt{3}$
- 2) $15\sqrt{5} - 25\sqrt{6}$
- 3) $25\sqrt{2} - 15\sqrt{3}$
- 4) 0

$$21. D_r = \begin{vmatrix} r-1 & n & 6 \\ (r-1)^2 & 2n^2 & 4n-2 \\ (r-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix} \Rightarrow \sum_{r=1}^n D_r =$$

- 1) nr
- 2) 0

$$3) \frac{n(n-1)}{2} - r^2$$

- 4) $2n-n^2$

22. The value of the determinant

$$\begin{vmatrix} 1 & e^{i\pi/3} & e^{i\pi/4} \\ e^{-i\pi/3} & 1 & e^{2i\pi/3} \\ e^{-i\pi/4} & e^{-2i\pi/3} & 1 \end{vmatrix}$$

where $i = \sqrt{-1}$, is

- 1) $2 + \sqrt{2}$
- 2) $-(2 + \sqrt{2})$
- 3) $-2 + \sqrt{3}$
- 4) $2 - \sqrt{3}$

23. A triangle has vertices $A_i(x_i, y_i)$ for $i = 1, 2, 3$. Then

$$\begin{vmatrix} x_2 - x_3 & y_2 - y_3 & y_1(y_2 - y_3) + x_1(x_2 - x_3) \\ x_3 - x_1 & y_3 - y_1 & y_2(y_3 - y_1) + x_2(x_3 - x_1) \\ x_1 - x_2 & y_1 - y_2 & y_3(y_1 - y_2) + x_3(x_1 - x_2) \end{vmatrix} = 0$$

means

- 1) medians of the triangle A_1, A_2, A_3 are concurrent;
- 2) the triangle A_1, A_2, A_3 is right angled at A_3 ;
- 3) the triangle A_1, A_2, A_3 is an equilateral triangle;
- 4) altitudes of the triangle A_1, A_2, A_3 are concurrent.

24. For a fixed positive integer n , let

$$D = \begin{vmatrix} (n-1)! & (n+1)! & (n+3)!/n(n+1) \\ (n+1)! & (n+3)! & (n+5)!/(n+2)(n+3) \\ (n+3)! & (n+5)! & (n+7)!/(n+4)(n+5) \end{vmatrix}$$

then $\frac{D}{(n-1)!(n+1)!(n+3)!}$ is equal to

- 1) - 8
- 2) - 16
- 3) - 32
- 4) - 64

25. If $f(x), g(x)$ and $h(x)$ are three polynomials of degree '2' and

$$\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}, \text{ then } \phi'(x) \text{ is}$$

- 1) a one degree polynomial
- 2) a three degree polynomial
- 3) a two degree polynomial
- 4) a constant

$$26. \text{ If } f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin(2x^2) \end{vmatrix}$$

then $f'(0) =$

- 1) 0
- 2) -1
- 3) -2
- 4) 2

$$27. \text{ If } f(x) = \begin{vmatrix} 1 & 2x & 3x^2 \\ 3 & a & 27 \\ 1 & 3 & 9 \end{vmatrix} \text{ and } \int_0^3 f(x) dx = 0,$$

then a is equal to

- 1) 3
- 2) 6
- 3) 9
- 4) any real number

$$28. \text{ If } g(x) = \begin{vmatrix} a^{-x} & e^{x \log_a a} & x^2 \\ a^{-3x} & e^{3x \log_a a} & x^4 \\ a^{-5x} & e^{5x \log_a a} & 1 \end{vmatrix} \text{ then}$$

- 1) $g(x) + g(-x) = 0$
- 2) $g(x) - g(-x) = 0$
- 3) $g(x).g(-x) = 0$
- 4) None of these

INVERSE OF A MATRIX

C.R.T.Q

Class Room Teaching Questions

$$29. \text{ If } P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ and}$$

$Q = PAP^T$ and $X = P^T Q^{2015} P$, then X is

$$1) \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix} \quad 2) \begin{bmatrix} 1 & 0 \\ 2015 & 1 \end{bmatrix}$$

$$3) \begin{bmatrix} 2015 & 1 \\ 0 & 1 \end{bmatrix} \quad 4) \begin{bmatrix} 1 & 1 \\ 0 & 2015 \end{bmatrix}$$

$$30. \text{ If } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and}$$

$$A^{-1} = \frac{1}{6}(A^2 + \alpha A + \beta I) \text{ then}$$

- 1) $\alpha = -6, \beta = 11$
- 2) $\alpha = 6, \beta = -11$
- 3) $\alpha = -6, \beta = -11$
- 4) $\alpha = 6, \beta = 11$

EXERCISE-IV

LEVEL-IV

These questions have Statement-I & statement-II. Each question has four choices (1),(2),(3),(4) out of which only one is correct. Choose the correct choice as per following.

- 1) Statement-I & statement-II both are true & statement-II is the correct explanation of Statement-1.
- 2) Statement-I and statement-II both are true & Statement-II is not explain statement-I.
- 3) Statement-I is true but Statement-II is false.
- 4) Statement-I is false but Statement-II is true.

1. Statement-I: Rank of matrix is defined for only square matrices.

Statement-II: Trace of matrices of a matrix is defined for square matrices only.

2. If $A = [a_{ij}]$ is a square matrix such that
- | | |
|---|--------------------------|
| List -I | List-II |
| A) $a_{ij} = 1$ if $i = j$
$= 0$ if $i \neq j$ | 1. Symmetric matrix |
| B) $a_{ij} = 0$ if $i \neq j$ | 2. Skew symmetric matrix |
| C) $a_{ij} = 0$ if $i > j$ | 3. Unit matrix |
| D) $a_{ij} = i^2 - j^2 \forall i, j$ | 4. Diagonal matrix |
| | 5. Upper triangular |

Correct match of List -I from List-II

	A	B	C	D
1)	3	4	5	1
2)	3	4	5	2
3)	3	5	4	2
4)	1	5	3	2

3. Statement-I: Let A be a square matrix given by

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 6 & 8 & 2 \\ 2 & -2 & 7 \end{pmatrix} = P + Q, P \text{ is symmetric}$$

matrix, Q is skew symmetric matrix then

$$Q \text{ is given by } \begin{pmatrix} 0 & -2 & 1 \\ 2 & 0 & 2 \\ -1 & -2 & 0 \end{pmatrix}$$

Statement-II: For every square matrix A, $A - A^T$ is skew symmetric.

4. Let

$$A = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}, B = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, i = \sqrt{-1} \text{ then}$$

Statement-I: $(A+B)(A-B) = A^2 - B^2$

Statement-II: $AB = BA$

5. Statement-I: The system of non-homogeneous equations $2x+3y+6z=8$, $x+2y+3z=5$, $x+y+3z=4$ has no solution.

Statement-II: $\det A = 0$ and

$$(adj A)B = 0 \text{ where}$$

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 2 & 3 \\ 1 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix}$$

6. Statement-I: Let A be an unknown matrix of order 2×2 and I be an identity matrix of order 2×2 , then $A^2 = I$, is satisfied by more than one matrix of order 2×2 .

Statement-II: All matrices of the form

$\begin{bmatrix} \pm\sqrt{1-\alpha\beta} & \alpha \\ \beta & \mp\sqrt{1-\alpha\beta} \end{bmatrix}$ are involuntary matrices for all real values of α and β such that $1 - \alpha\beta > 0$.

7. Statement-I: If A is matrix of order 3×3 , then

$$\underbrace{\det \det \dots \det}_{n \text{ times}} A = |A|^n$$

(Where $|A|$ represents determinant of A)

Statement-II: If A is a matrix of order $n \times n$, then $|\det A| = |A|^n$.

8. Statement-I: The determinant of a matrix $A = [a_{ij}]_{5 \times 5}$ is 0 where $a_{ij} + a_{ji} = 0$ for i and j.

Statement-II: The determinant of a skew-symmetric matrix of odd order is zero.

9. Statement-I: If

$$f(x) = \begin{vmatrix} -1 & \sin \theta \tan \theta & -1 \\ \theta & \sin x & x \\ 1 & \tan x - \tan \theta & 0 \end{vmatrix} \text{ then the value of } f'(\theta) \text{ is } \cos \theta$$

Statement-II: If $f(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix}$

$$\Rightarrow f'(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix}$$

where f_i, g_i, h_i ($i=1,2,3$) are polynomials in x.

10. Statement-I: The system of the equations $2x+4y+1=0, 4x+8y+3=0$ has no solution

Statement-II: The system of equations $a_1x+b_1y+c_1=0, a_2x+b_2y+c_2=0$ has no

solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

11. Statement-I: $A = \begin{bmatrix} 1 & 3 \\ 5 & 9 \end{bmatrix}$ is neither symmetric nor skew symmetric
- Statement-II: The matrix A cannot be expressed as a sum of symmetric and anti-symmetric matrices.

12. Statement-I: If a_1, a_2, \dots, a_n are in GP ($a_i > 0$ for all i) then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} = 0$$

Statement-II: The determinant of 3×3 matrix is zero when elements of each row are in A.P.

13. Statement-I: If a, b, c are distinct and x, y, z are not all zero given that $ax+by+cz \neq 0, bx+cy+az=0, cx+ay+bz=0$, then $a+b+c \neq 0$

Statement-II: $a^2 + b^2 + c^2 > ab + bc + ca$ if a, b, c are distinct.

14. Statement-I: There are only finitely many 2×2 matrices which commute with the

$$\text{matrix } \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

Statement-II: If A is non-singular, then it commutes with I, adj A and A^{-1}

15. Statement-I: If the system of equations

$3x-2y+z=0, \lambda x-14y+15z=0, x+2y-3z=0$ has nonzero solutions, then $\lambda =$

Statement-II: If the system of equation AX=0 has a nonzero solution then A is singular.

16. Let α, β, γ be the roots of the equation $x^3 + ax + b = 0$; $a, b, \in R$.

$$\text{Statement-I: } \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

Statement-II: Any cubic equation has at least one real root.

17. Statement I: If A is square matrix such that $A^2 = A$ then $(I+A)^3 - 7A = I$

Statement II: Let $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and $(A+I)^{50} - 50A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $a+b+c+d =$

$$18. \Delta = \begin{vmatrix} a^n & a^{n+1} & a^{n+2} \\ b^n & b^{n+1} & b^{n+2} \\ c^n & c^{n+1} & c^{n+2} \end{vmatrix} \quad (n \in N)$$

Statement-I: Factors of Δ are $(a-b)(b-c)(c-a)$

Statement-II: Taking the value of n=1 then the value of the above determinant $(abc)(a-b)(b-c)(c-a)$

INTEGER & NUMERICAL

ANSWER TYPE QUESTIONS

- 1.** $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$ and $A^8 + A^6 + A^4 + A^2 + I$;
 $V = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (where I is the 2×2 identity matrix), then the product of all elements of matrix V is _____
- 2.** If $\begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$ is an idempotent matrix and $f(x) = x - x^2$ and $bc = 1/4$, then the value of $1/f(a)$ is _____
- 3.** Let x be the solution set of the equation $A^x = I$. Where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \leq N$, then the minimum value of $\sum (\cos^x \theta + \sin^x \theta)$, $\theta \in R$. _____
- 4.** $A = \begin{bmatrix} 1 & \tan x \\ \tan x & 1 \end{bmatrix}$ and $f(x)$ is defined as $f(x) = \det(A^T A^{-1})$ then the value of $f(f(f(f(\dots f(x)))))$ is $(n \geq 2)$ _____
- 5.** The equation $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 4 & K \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has a solution for (x, y, z) besides $(0, 0, 0)$. Then the value of K is _____
- 6.** If A is an idempotent matrix satisfying, $(I - 0.4A^{-1}) = I - \alpha A$, where I is the unit matrix of the same order as that of A , then the value of $|9\alpha|$ is equal to _____

- 7.** Let $A = \begin{bmatrix} 3x^2 \\ 1 \\ 6x \end{bmatrix}$, $B = [a \quad b \quad c]$, and $C = \begin{bmatrix} (x+2)^2 & 5x^2 & 2x \\ 5x^2 & 2x & (x+2)^2 \\ 2x & (x+2)^2 & 5x^2 \end{bmatrix}$ be three matrices, where a, b, c and $x \in R$. Given that $\text{tr}(AB) = \text{tr}(C)x \in R$, where $\text{tr}(A)$ denotes trace of A . If $f(x) = a + bx + c$, then the value of $f(1)$ is _____
- 8.** Let A be the set of all 3×3 skew-symmetric matrices whose entries are either $-1, 0$ and three (-1) 's, then number of such matrices ϕ is _____
- 9.** Let $A = [a_{ij}]_{3 \times 3}$ be a matrix such that $AA^T = 4I$ and $a_{ij} + 2c_{ij} = 0$, where c_{ij} is cofactor of a_{ij} and I is the unit matrix order 3.

$$\begin{vmatrix} a_{11}+4 & a_{12} & a_{13} \\ a_{21} & a_{22}+4 & a_{23} \\ a_{31} & a_{32} & a_{33}+4 \end{vmatrix} = 5\lambda \begin{vmatrix} a_{11}+1 & a_{12} & a_{13} \\ a_{21} & a_{22}+1 & a_{23} \\ a_{31} & a_{32} & a_{33}+1 \end{vmatrix}$$

then the value of 10λ is _____
- 10.** Let S be the set which contains all possible values of l, m, n, p, q, r for which $A = \begin{bmatrix} I^2 - 3 & p & 0 \\ o & m^2 - 8 & q \\ r & 0 & n^2 - 15 \end{bmatrix}$ be a non singular idempotent matrix. Then the sum of all the elements of the set S is _____
- 11.** If A is a diagonal matrix of the set S is commutative with every square matrix of order 3×3 under multiplication and $\text{tr}(A) = 12$, then value of $|A|^{\frac{1}{12}}$ is _____

12. If A is a square matrix of order 3 such that $|A|=2$, then $\left(\left(\text{adj } A^{-1}\right)^{-1}\right)$ is _____

13. $m = [-3 \ 4] + n[4 \ -3] = [10 \ -11]$
 $\Rightarrow 3m + 7n = \underline{\hspace{2cm}}$

14. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and $A^2 = I$ then $nx = \underline{\hspace{2cm}}$

15. If $A = \begin{bmatrix} 2 & 4 & 5 \\ x & 5 & 6 \end{bmatrix}$ and $A^T = A$ then $x = \underline{\hspace{2cm}}$

16. If $A = \begin{bmatrix} -1 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}$ such that $A^T = -A$, then $x = \underline{\hspace{2cm}}$

17. If $5A = \begin{bmatrix} 3 & -4 \\ 4 & x \end{bmatrix}$ and $AA^T = A^TA = I$ then $x = \underline{\hspace{2cm}}$

18. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix, then

19. If $A = \begin{bmatrix} 1 & 3 & -5 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew-symmetric matrix, then $x = \underline{\hspace{2cm}}$

20. If $A = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 0 & 1 \end{bmatrix}$ then the trace of A is _____

21. If A is a skew-symmetric matrix, then trace of A is _____

22. If $A = \begin{bmatrix} x^2 & 6 & 8 \\ 3 & y^2 & 9 \\ 4 & 5 & z^2 \end{bmatrix}, B^T = \begin{bmatrix} 2x & 3 & 5 \\ 2 & 2y & 6 \\ 1 & 4 & 2z-3 \end{bmatrix}$ where x, y, z are real and trace of $B = \text{trace of } A^T$ then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} - xyz = \underline{\hspace{2cm}}$

23. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x-1)x(x+1) \end{vmatrix}$

then $f(100)$ is equal to

24. The parameter on which the value of the determinant

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$

depends upon how many parameters from a, p, x, d?

25. If $1, \omega, \omega^2$ are the cube roots of unity, then

$$\Delta = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$$

26. The value of the determinant

$$\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$$

is $A(10!11!12!).$ Find A.

27. For all values of A, B, C and P, Q, R, the value of

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}$$

28. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is

29. Let $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and
 $\Delta_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$, then $\Delta_1 \times \Delta_2$ can

be expressed as the sum of how many determinants?

30. If $A = \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$, then

$$B = \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \sin \phi \cos \phi & \sin^2 \phi \end{bmatrix} - f$$

and q and f differs by $\frac{\pi}{2}$. Then $|AB| =$

31. If for $AX = B$, $B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$ and

$$A^{-1} = \begin{bmatrix} 3 & -\frac{1}{2} & -\frac{1}{2} \\ -4 & \frac{3}{4} & \frac{5}{4} \\ 2 & -\frac{1}{4} & -\frac{3}{4} \end{bmatrix}$$

then X is equal to

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}, \text{ Find } x+y+z.$$

32. Let p a non-singular matrix $1 + p + p^2 + \dots + p^n = O$ (O denotes the null matrix), then $p^{-1} = p^k$. Find k/n.

33. If then

$$P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{then } P_{22} =$$

34. If $C = 2 \cos \theta$, then the value of determinant $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix}$ is

35. If $\Delta = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants,

$$\text{then } \frac{d}{dx}(\Delta_1) = A\Delta_2. \text{ Find } A.$$

36. If $a_1, a_2, a_3, \dots, a_n, \dots$ are in G.P. the value of the determinant

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$$

37. In the determinant the ratio of the co-factor to its minor the element -3 is A : 1. Find $-2A$.

38. If value of a third order determinant formed by the cofactors will be

$a_1x + b_1y + c_1z + d_1 = 0$,

$$a_2x + b_2y + c_2z + d_2 = 0$$

$$a_3x + b_3y + c_3z + d_3 = 0$$

Let us denote by $\Delta(a, b, c) \neq 0$, then the unique solution of the value of x in the

$$\frac{-A\Delta(bcd)}{\Delta(abc)}. \text{ Find } A.$$

If a matrix A is such that $A^2 + 2A + 7A + I = 0$, then A^{-1} equals _____.

$(A - 1)x + (5A + 1)y + 2Az = 0$. Find $x+y+z$?

For how many value(s) of λ in the closed interval $[-4, -1]$ is the matrix

$$\begin{bmatrix} 3 & -1+\lambda & 2 \\ 3 & -1 & \lambda+2 \\ \lambda+3 & -1 & 2 \end{bmatrix} \text{ singular?}$$

If 3, -2 are the Eigen value of a non-singular matrix A and $\|A\|=4$, then the Eigen values of $a\bar{\eta}(A)$ are surely where $\lambda \in \mathbb{R}$, find $a+\bar{\lambda}q$.

If the system of linear equations,

$$x+y+z=6$$

$$2x+2y+3z=10$$

$$3x+2y+4z=14$$

has more than two solutions, then $\mu - \lambda^2$ is equal to _____.

The number of all 3×3 matrices A, with entries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of AA^T is 3, is _____.

If $\det A = \begin{bmatrix} x & 1 \\ 1 & q \end{bmatrix}$, $x \in \mathbb{R}$ and $A^T = \begin{bmatrix} a_{ij} \end{bmatrix}$,

If $a_{11} = 100$, then a_{22} is equal to _____.

Let S be the set of all integer solutions (x, y, z) of the system of equations

$$x^2y^2z^2 = 4, 2x^2y^2z^2 = 9, 7x^2 + 14y^2 + 2z^2 =$$

such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then the number of elements in the set S is equal to _____.

If the system of equations

$x^2y^2z^2 = 9, 2x^2y^2z^2 = 8, x^2y^2 + 4z^2 = 24$, has infinitely many solutions, then x, y, z is equal to _____.

48. The sum of distinct values of λ for which the system of equations

$$(A - 1)x + (5A + 1)y + 2Az = 0$$

$$(A - 1)x + (4A - 2)y + (A + 2)z = 0$$

$$2x + (5A + 1)y + 3(A - 1)z = 0$$

has non-zero solutions, is _____.

49. Let I be an identity matrix of order 2×2 and

$$P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}. \text{ Then the value of } n \in \mathbb{N} \text{ for which}$$

$$P^n = 5I - 3P \text{ is equal to } _____.$$

50. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of MIM is seven, is _____.

51. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = \begin{bmatrix} q_{ij} \end{bmatrix}$ is a matrix satisfying $PQ = QP$, for some non-zero $k \in \mathbb{R}$. If

$$\frac{q_{13}}{q_{11}} = \frac{k}{8} \text{ and } |Q| = \frac{k^2}{2}, \text{ then } \alpha^2 + k^2 \text{ is equal to } _____.$$

52. Let $A = \begin{bmatrix} x & y & z \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $x = 2$. If $A^2 = I_3$, then the value of $x^2 + y^2 + z^2$ is _____.

$$53. \text{ Let } A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix} \text{ and}$$

$$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}, \text{ then } 13(a^2 + b^2)$$

54. If the system of equations

$$kx + y + 2z = 1$$

$$3x - y - 2z = 2$$

$$-2x - 2y - 4z = 3$$

has infinitely many solutions, then k is equal to _____.

55. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the

$$\text{equation } A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for some real number α and β , then $\beta - \alpha$ is equal to _____.

56. The total number of 3×3 matrices A

such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

57. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & \omega \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$

where $\omega = \frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to _____.

58. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries such that

$$A = XB, \quad \text{where } X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}, \quad \text{and}$$

$k \in R$. If

$$a_1^2 + a_2^2 (b_1^2 + b_2^2) \text{ and } (k^2 + 1)b_1^2 \neq -2b_1b_2,$$

Then the value of k is _____.

59. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det(A^{10} - (\text{Adj}(2A))^{10})$ is _____.

60. If l , $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ in arithmetic progression for a real number x , then the value of the determinant

$$\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$$

is equal to:

61. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that $AB = B$ and $a + d = 2021$, then the value of $ad - bc$ is equal to _____.

ALGEBRA

62. If $x > 0$ and $x, [x], \{x\}$ when $[.]$ denote greatest integer function and $\{\cdot\}$ denotes fractional part function, are in A.P then

63. All the values of k for which the quadratic polynomial $f(x) = -2x^2 + kx + k^2 + 5$ has two different zeros and only one of the possible interval (a, b) . Then the value of $a + 10b$ is _____.

64. Concentric circles of radii $1, 2, \dots, 100$ cm are drawn. The interior of the smallest circle coloured red and angular regions are coloured alternately green and red so that no two adjacent green and red so the colour. The total area of the same coloured green is equals to k . (1010π) sq.cm then the value of k is _____.

65. If $2 \log(x - 2y) = \log x + \log y$ then the numerical value of $\frac{x}{y}$ is _____

66. The number of the roots of the equation $2^x + 2^{x-1} + 2^{x-2} = 7^x + 7^{x-1} + 7^{x-2}$

67. The integral solutions of the equation $x^{-1} = (x - [x])(x - \{x\})$ where $[.]$ and $\{.\}$ denotes the greatest integer and fractional part function

68. Let three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}; B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ then $t_r(A) + t_r\left(\frac{ABC}{2}\right) + t_r\left(\frac{A(BC)^2}{4}\right) + t_r\left(\frac{A(BC)^3}{8}\right) + \dots + \infty =$

$$69. \begin{vmatrix} x^3 - a^3 & x^2 - b^2 & x^2 - c^2 \\ (x-a)^3 & (x-b)^3 & (x-c)^3 \\ (x+a)^3 & (x+b)^3 & (x+c)^3 \end{vmatrix} = 0 \text{ Where } a, b, c \text{ are non zero and distinct and } ab + bc + ca \leq 0 \text{ then the value of } x \text{ is }$$

70. If A & B are two 2×2 matrices such that $AB = BA$ and $f(x) = x^2 - (\text{Trace}(x))(x) + \det(x)$ (Where x is a square matrix) then $f(AB) - f(BA) = \underline{\hspace{2cm}}$

71. Let A be a 3×3 matrix such that $a_{11} = a_{33} = 2$ and all the other $a_{ij} = 1$.

Let $A^{-1} = xA^2 + yA + zI$ then find the value of $(x + y + z)$ where I is a unit matrix of order 3.

72. Find the value of

$$\begin{vmatrix} 2 & \alpha + \beta + \gamma + \delta & \alpha\beta + \gamma\delta \\ \alpha + \beta + \gamma + \delta & 2(\alpha + \beta)(\gamma + \delta) & \alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) \\ \alpha\beta + \gamma\delta & \alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) & 2\alpha\beta\gamma\delta \end{vmatrix}$$

73. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ then let us define a function $f(x) = \det.(A^T A^{-1})$ then what is the value of $\underbrace{f(f(f(\dots f(x))))}_{n \text{ times}}$ is ($n \geq 2$)

74. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{pmatrix}$ and $6A^{-1} = A^2 + \alpha A + \beta I$ then

$|\alpha + \beta|$ is equal to

75. The number of integral values of x satisfying the equation $|x| + \left| \frac{4-x^2}{x} \right| = \left| \frac{4}{x} \right|$ is

76. If the roots of the quadratic equation $ax^2 + bx - b = 0$ where $a, b \in R$ such that $ab > 0$ are α and β ,

then the value of $|\log_{|\beta-\alpha|} |\alpha - 1||$ is

$$77. \text{If the sum } \sqrt{1 + \frac{1}{1^2}} + \sqrt{1 + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{3^2}} + \dots + \sqrt{1 + \frac{1}{3^2} + \frac{1}{4^2} + \dots + \sqrt{1 + \frac{1}{(1999)^2} + \frac{1}{(2000)^2}}}$$

equal to $n - \frac{1}{n}$ where $n \in N$. Find n - 2000

$$78. \text{If the determinant } \begin{vmatrix} a+p & l+x & u+f \\ b+q & m+y & v+g \\ c+r & n+z & w+h \end{vmatrix}$$

splits into exactly K determinants of order 3, each element of which contains only one term, then the value of K, is

79. Let $f(x) = \left[\frac{1}{\cos\{x\}} \right]$ where $[y]$ and $\{y\}$ denote greatest integer and fractional part function respectively and $g(x) = 2x^2 - 3x(k+1) + k(3k+1)$. if $g(f(x)) < 0 \forall x \in R$ then find the number of integral value of K.

80. If $h(x) = Ax^5 + B \sin x + C \ln\left(\frac{1+x}{1-x}\right) + 7$,

Where A, B, C are non-zero real

constants and $h\left(\frac{-1}{2}\right) = 6$, then find the

$$\text{value of } h\left(\frac{\operatorname{sgn}(e^{-x})}{2}\right).$$

81. Let $f: R \rightarrow [0, \infty)$ be defined as

$$f(x) = \log_e \left(\sqrt{9x^2 - 12x + \lambda + 1} \right)$$

onto function where λ is a real parameter belong to $(0, 10)$. Find the greatest possible value of λ .

82. Let $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ be two roots of

$$(x+1)^n + x^n + 1 = 0 \text{ where } n > 2, n \in N.$$

Let α, β be roots of $x^2 + px + q = 0$. If α, β are also roots of $x^{2n} + p^n x^n + q^n = 0$ then the smallest positive integral value of n is _____.

83. If $\sum_{k=1}^{\infty} \frac{k^2}{5^k} = \frac{a}{b}$, where a, b are relatively prime positive integers, than the value of $b - 2a$ is equal to:

$$84. \text{ If } \frac{6}{5} \left(\log_a^a \right) \left(\log_b^b \right) \left(\log_c^c \right) - 3 \left(\log_{10} \frac{a}{10} \right) = 9 \log_{10}^a (a + \log_5^5)$$

(where $a, b > 0, b \neq 1$) then $\log_a^a = n + f, n$ is an integer and $f \in [0, 1)$, where $n = \underline{\hspace{2cm}}$

85. The set of real parameter 'a' for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all

real solutions, is given by $\left[\frac{m}{n}, \infty \right)$ where m and n are relatively prime positive integers, then the value of $(m+n)$ is equal to

86. If $\sum_{r=1}^{1003} (r^2 + 1)r! = a! - b.c!$ where $a, b, c \in \mathbb{N}$

then find the unit digit of least value of $(a+b+c)$.

87. Let $\alpha, \beta \in R$ if α, β^2 be the roots

quadratic equation $x^2 - px + 1 = 0$ if α^2, β be the roots of quadratic equation $x^2 - qx + 8 = 0$, then the value of (r, s) if $\frac{r}{s}$ be arithmetic mean of p and q, is

88. If x is real, then the maximum value of $y = 2(3-x)(x + \sqrt{x^2 + 4})$ is k, then $k - 10$

89. Given

$$\log_2(\log_{1/2}(\log_2 x)) = \log_3(\log_{1/3}(\log_3 y))$$

$= \log_5(\log_{1/5}(\log_5 z)) = 0 (x, y, z \in R^+)$ the cube of greatest of x, y, z is _____

90. Let P(a,b,c) be any point on the plane $3x + 2y + z = 7$ then the least value of $2(a^2 + b^2 + c^2)$ is _____

91. Total no. of values of P so the $x^2 - x - p = 0$ has integral roots where $P \in N$ and $10 \leq P \leq 100$ is equal to _____

92. Let $S_n = \sum_{k=1}^{4^n} (-1)^{\frac{k(k+1)}{2}} k^2$ then the digit in the 100th place of S_n is equal to _____

93. The number of solution(s) of the equation $e^x = x^2$ and $e^x = x^3$ are respectively m, n Then $m+n = \underline{\hspace{2cm}}$

94. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ upto ∞ terms = $\frac{\pi^2}{6}$,

then value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ upto ∞ term is $\frac{\pi^2}{n}$ then value of n is _____

95. Let $p(x) = 0$ be a polynomial equation of least possible degree, with rational coefficients, $\sqrt{7} + \sqrt{49}$ as one of its roots.

Then the product of all the roots of $p(x) = 0$ is K then $\frac{K}{7}$ is _____

The sum of the series

$$\sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \dots +$$

$$\sqrt{1 + \frac{1}{999^2} + \frac{1}{1000^2}} \text{ is equal to } \frac{100^k - 1}{1000},$$

where k = _____

97. For $a, b \in R$ if the equation

$ax^2 + bx + 6 = 0, (a > 0)$ does not have two distinct real roots, then the least value of $3a + b + 3 =$ _____

98. Let $p(x) = x^5 + x^2 + 1$ have roots x_1, x_2, x_3, x_4 and x_5 , $g(x) = x^2 - 2$, then the value of

$g(x_1)g(x_2)g(x_3)g(x_4)g(x_5) - 30g(x_1x_2x_3x_4x_5)$, is _____

$$99. \text{ If } S_\infty = \frac{1}{1.3.5.7.9} + \frac{1}{3.5.7.9.11} + \frac{1}{5.7.9.11.13} + \dots$$

then the value of $\frac{1}{120S_\infty}$ is _____

100. Sum of digits of least two digit positive integer of 'm' which satisfies

$$(m - [\sin x])! = 3!5!7! \text{ where } [.] \text{ & } |.|$$

represents G. I. F and modulus function

101. If A is a square matrix of order n such

that $|\text{adj}(\text{adj } A)| = |A|^9$, then the value of n can be _____

$$102. \text{ If } \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x & u \\ y & v \\ z & w \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}, \text{ then}$$

$(x+y+z)-(u+v+w)$, is _____

$$103. \text{ If } \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \text{ is the inverse of }$$

$$\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}, \text{ then } \frac{1}{x} =$$

$$104. \frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 2, \frac{3}{x} - \frac{4}{y} - \frac{2}{z} = 1, \text{ and}$$

$\frac{2}{x} + \frac{5}{y} - \frac{2}{z} = 3$, Then the value of y is _____

105. Suppose A is a 4×4 matrix such that

$$|A| = 4, \text{ find } \left| \frac{\text{adj } A}{16} \right|$$

106. Let a and b be constants and $f(x) = a\sin x + bx\cos x + 2x^2$. If $f(2) = 15$ then the value of $f(-2)$ is _____

$$107. \text{ If } A \text{ is a } 3 \times 3 \text{ matrix and } \det A = -2 \text{ then } \det(\text{adj } A) =$$

$$108. \text{ If } A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}, \text{ adj } A = xA^T \text{ then } x =$$

109. Number of triplets of a, b & c for which the system of equations, $ax - by = 2a - b$ and $(c+1)x + cy = 10 - a + 3b$ has infinitely many solutions and $x = 1, y = 3$ is one of the solutions, is:

110. Given the cubic equation

$$x^3 - 2kx^2 - 4kx + k^2 = 0.$$

If one root of the equation is less than 1, other root is in the interval (1, 4) and the 3rd root is greater

than 4, then the value of k lies in the interval $(a + \sqrt{b}, b(a + \sqrt{b}))$ where $a, b \in N$.

Find the value of $(a+b)^3 + (ab+2)^2 - 2000$

111. Let the equation $(a-1)x^2 = x(2b+3)$ be satisfied by three distinct value of x , where $a, b \in R$, if $f(x) = (a-1)x^2 + (2b+3)x^2 + 2x + 1$, and $f(g(x)) = 6x - 7$ where $g(x)$ is a linear function then find the value of $g(2)$ is equal to

112. If

$$\begin{aligned} & \sum_{r=1}^{\infty} \frac{8}{(2r-1)\sqrt{(2r+3)(2r+5)} + (2r+3)\sqrt{(2r+1)(2r-1)}} \\ &= \sqrt{a} + \sqrt{\frac{b}{3}} - \sqrt{c} \text{ where } a, b, c \in N \text{ then} \end{aligned}$$

the value of $\frac{a+b+c}{4}$ is equal to

113. If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and

$$B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

and I is an identity matrix of order 3 then answer the following questions.

$\det(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \dots \dots \dots 10$ terms) is equal to

114. Let A be the set of all 3×3 skew symmetric matrices whose entries are

either -1, 0 or 1. If there are exactly 0's, three 1's and three (-1)'s, then the number of such matrices is _____.

115. Let $A = \begin{bmatrix} 1 & m & n \\ p & q & r \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^2$,

$$(l-m)^2 + (p-q)^2 = 9,$$

$$(m-n)^2 + (q-r)^2 = 16,$$

$(n-l)^2 + (r-p)^2 = 25$, then the value (det. B) equals

116. Let B be a skew symmetric matrix of order 3×3 with real entries. Given $I - B$ and B are non-singular matrices. If $A = (I - B)(I - B)^{-1}$, where $\det.(A) > 0$, then the value of $\det.(2A) - \det.(\text{adj } A)$ is _____.

117. Consider a determinant of order 3 whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0'. Also $a_{ij} = a_{ji} \forall 1 \leq i, j \leq 3$. If n is number of such determinants, then find $n-10$.

118. Given the equations $x + [y] + \{z\} = 3.1$, 5.4 , value of $x = \underline{\hspace{2cm}}$ and $[x] + \{y\} + z = \underline{\hspace{2cm}}$ and $\{\cdot\}$ is Fractional part function.

119. Let A be greatest integral value of x satisfying $\frac{\log_{10} x + \log_{10} \pi}{1} > x$; B be least value of $\log_{10} x - \log_{10} (0.01)$, $x > 1$; expression C be the number of digits in 3^{40} (use $\log_{10} 3 = 4.77$), then the value of $C + B - A$ is _____.