Mathematics

1. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is:

A.5:9:13 B. 5:6:7
C.3:4:5 D. 4:5:6

Ans. D

Given that
$a < b < c$ are in A.P.
$\angle C = 2\angle A$ (Given)
$\Rightarrow \sin C = \sin 2A$
$\Rightarrow \sin C = 2 \sin A. \cos A$

$$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A$$

$$\Rightarrow \frac{c}{a} = 2\frac{b^2 + c^2 - a^2}{2bc}$$

put $a = b - \lambda$, $c = b + \lambda$, $\lambda > 0$

$$\Rightarrow \lambda = \frac{b}{5}$$

$$\Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, c = b + \frac{b}{5} = \frac{6b}{5}$$

$\Rightarrow$ required ratio = 4 : 5 : 6

2. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbole at (4, 6) is:

A.2x-3y+10=0 B. 3x-2y=0
C.2x-y-2=0 D. x-2y+8=0

Ans. C

Let equation of hyperbola

$$\frac{x^2}{a^2} = \frac{y^2}{b^2} = 1$$

Hyperbola passing through the point $(4,\ 6)$

$$\frac{16}{a^2} - \frac{36}{b^2} \qquad ...(1)$$

$$a^2 = 1 + \frac{b^2}{a^2}$$

$$4 = 1 + \frac{b^2}{a^2}$$

$$b^2 = 3a^2 \qquad ...(ii)$$

from (i) & (ii)

$a^2 = 4$

$b^2 = 12$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

The equation of the tangent to the hyperbole at (4, 6) is

$$\frac{4x}{4} - \frac{6y}{12} = 1$$

$$x = \frac{y}{2} = 1$$

$2x - y - 2 = 0$

3. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to:

A.1- $\frac{11}{2^{20}}$ B. 2- $\frac{21}{2^{20}}$

C.2- $\frac{11}{2^{19}}$ D. 2- $\frac{3}{2^{17}}$

Ans. C

It is given that given that

$$S = \sum_{k=1}^{20} \frac{k}{2k}$$

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots. + \frac{20}{2^{20}} \qquad \ldots(i)$$

$$\frac{1}{2}S = \frac{1}{2^2} + \frac{2}{2^2} + \ldots + \frac{20}{2^{21}} \quad \ldots(ii)$$

Now solving the equation (i) and (ii) and subtracting equation (ii) from (i), we get,

$$\frac{1}{2}S - \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^{20}}\right) - \frac{20}{2^{21}}$$

$$\frac{1}{2}S = \frac{\frac{1}{2}\left(1 - \frac{1}{2^{20}}\right)}{1 - \frac{1}{2}} - \frac{20}{2^{21}}$$

$$\frac{1}{2}S = 1 - \frac{1}{2^{20}} - \frac{20}{2^{21}}$$

$$\frac{1}{2}S = 1 - \frac{1}{2^{20}} - \frac{10}{2^{20}}$$

$$\frac{1}{2}S = 1 - \frac{11}{2^{20}}$$

$$S = 2 - \frac{11}{2^{19}}$$

Thus, the sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to 2- $\frac{11}{2^{19}}$.

4. A student scores the following marks in five tests: 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is:

A. $\frac{100}{\sqrt{3}}$ B. $\frac{100}{3}$

C. $\frac{10}{\sqrt{3}}$ D. $\frac{10}{3}$

Ans. C

Let the marks in sixth tests is ‘x’ so mean

$$\frac{45+54+41+57+43+x}{6}=48$$

$$\frac{240+x}{6}=48$$

x = 48

Now, standard deviation of these marks

$$\sigma^2=\left(\frac{1}{6}\left(45^2+54^2+41^2+51^2+43^2+48^2\right)\right)-(43)^2$$

$$\sigma^2=\frac{1}{6}\times(14024)-(48)^2$$

$$\sigma^2=\frac{14024-((48)^2\times 6)}{6}$$

$$\sigma^2=\frac{14024-13824}{6}$$

$$\sigma^2=\frac{200}{6}=\frac{100}{3}$$

$$\sigma^2=\frac{10}{\sqrt{3}}$$

5. If a point R(4, y, 2) lies on the line segment joining the points P(2, — 3, 4) and Q(8, 0, 10), then the distance of R from the origin is:

A. $2\sqrt{14}$ B. $2\sqrt{21}$

C. $\sqrt{53}$ D. 6

Ans. A

The line equation segment joining the points P(2, — 3, 4) and Q(8, 0, 10) is given by

$$\frac{x-2}{6}=\frac{y+3}{3}=\frac{3-4}{6}=\lambda$$

Let point R (6λ + 2, 3λ –3, 6λ + 4)

given that $6\lambda + 2 = 4$
point R (4, –2, 6)
Distance between point R and origin $= \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$

6. The number of Integral values of m for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is :
A.1 B. infinitely many
C.2 D. 3

Ans. B

The equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$
Equation has no real solution, D<0

$4(1+3m)^2 - 4 \times (1+m^2)(1+8m) < 0$
$1 + 9m^2 + 6m - (1 + 8m + m^2 + 8m^3) < 0$
$1 + 9m^2 + 6m - 1 - 8m - m^2 - 8m^3 < 0$
$-8m^3 + 8m^2 - 2m < 0$
$8m^3 - 8m^2 + 2m > 0$
$m(4m^2 - 4m + 2) > 0$
$m[(2m-1)^2 + 1] > 0$
$m > 0$

$\Rightarrow$ Number of integral values of m are infinitely many

7. Let the number 2, b, c be in an A.P. and
$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det(A) ε [2,16], then c lies the interval:
A.$[3, 2+2^{3/4}]$ B. (2,3)

C.[4,6]   D. $(2+2^{3/4},4)$

Ans. C

Given, matrix $A=\begin{bmatrix}1 & 1 & 1\\ 2 & b & c\\ 4 & b^2 & c^2\end{bmatrix}$, so

$$\det(A)=\begin{vmatrix}1 & 1 & 1\\ 2 & b & c\\ 4 & b^2 & c^2\end{vmatrix}$$

on applying, $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

We get det (A) = $\begin{vmatrix}1 & 0 & 0\\ 2 & b-2 & c-2\\ 4 & b^2-4 & c^2-4\end{vmatrix}$

$$=\begin{vmatrix}b-2 & c-2\\ b^2-4 & c^2-4\end{vmatrix}$$

$$=\begin{vmatrix}b-2 & c-2\\ (b-2)(b+2) & (c-2)(c+2)\end{vmatrix}$$

[Taking common (b-2) from C_1 and (c-2) from C_2]

$|A| = (2-b)\,(b-c)\,(c-2)$

2, b, c, are in AP.

$$b=\frac{2+c}{2}$$

$$\text{Det}(A) = \frac{1}{4}(c-2)^3$$

$$2 \le \frac{1}{4}(c-2)^3 \le 16$$

$$8 \le (c-2)^3 \le 64$$

$$2 \le c-2 \le 4$$

$$4 \le c \le 6$$

8. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is:

A.5 B. 4

C.3 D. 2

Ans. B

The required probability of observing atleast one head = 1 – P (no head)

P (at least one hears) = 1 – [(no one heads) $\ge \frac{90}{100}$

$$1-\left(\frac{1}{2}\right)^n \ge \frac{9}{100}$$

$$\left(\frac{1}{2}\right)^2 \le \frac{10}{100} \Rightarrow 2^n \ge 10 \Rightarrow n \ge 4$$

The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is 4

9. The tangent to the parabola $y^2=4x$ at the point where it intersects the circle $x^2+y^2=5$ in the first quadrant, passes through the point:

A. $\left(\frac{3}{4},\frac{7}{4}\right)$ B. $\left(-\frac{1}{4},\frac{1}{2}\right)$

C. $\left(-\frac{1}{3},\frac{4}{3}\right)$ D. $\left(\frac{1}{4},\frac{3}{4}\right)$

Ans. A

Given equations of the parabola $y^2 = 4x$ …(i)

and circle $x^2 + y^2 = 5$….(ii)

So, for point of intersection of curves (i) and (ii), put $y^2 = 4x$ in Eq. (ii) we get

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x = -5$ $x = 1$

not possible $y = \pm 2$

Point in IQ (1, 2)

Tangent at (1, 2)

$2y = 4\left(\frac{x+1}{2}\right)$

$y = x + 1$

point $\left(\frac{3}{4},\frac{7}{4}\right)$ lies on tangent

10. If the system of linear equations

x-2y+kz=1

2x+y+z=2

3x-y-kz=3

has a solution (x, y, z),z≠0, then (x, y;) lies on the straight line whose equation is :

A.4x-3y-4=0 B. 3x-4y-1=0

C. $3x-4y-4=0$ D. $4x-3y-1=0$

Ans. A

Given system of linear equations

$x - 2y + kz = 1$ …. (i)

$2x + y + z = 2$ … (ii)

$3x - y - kz = 3$ … (iii)

for locus of (x, y)

equation (i) + (iii)

$4x - 3y = 4$

$4x - 3y - 4 = 0$

This is the required equation of the straight line in which point (x, y) lies.

11. Which one of the following statements is not a tautology?

A. $(p \wedge q) \rightarrow p$

B. $(p \vee q) \rightarrow (p \vee (\sim q))$

C. $p \rightarrow (p \vee q)$

D. $(p \wedge q) \rightarrow (\sim p) q$

Ans. B

p	q	$p \vee q$	$\vee$ q	$p \vee \sim q$	$(p \vee q) \rightarrow (p \vee \sim q)$
T	T	T	F	T	T
T	F	T	T	T	T
F	T	T	F	F	F
F	F	F	T	T	T

12. Let f:[-1,3] →R be defined as

$$F(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x<3\end{cases}$$

Where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at:

A. only one point

B. only three points

C. only two points

D. four or more points

Ans. B

$$\begin{cases}|x|+[x] & -1\le x<1\\ x+|x| & 1\le x<2\\ x+[x] & 2\le x\le 3\end{cases}$$

f×n is discontinuous

at x = 0, 1, 3

13. If $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}}=xf(x)(1+x^6)^{1/3}+C$

Where, C is a constant of integration, then the function $f(x)$ is equal to:

A. $-\frac{1}{2x^2}$ B. $-\frac{1}{2x^3}$

C. $-\frac{1}{6x^3}$ D. $\frac{3}{x^2}$

Ans. B

It is given that

$$I = \int \frac{dx}{x^3(1+2x^6)^{2/3}}$$

$$= \int \frac{dx}{x^3.x^4\left(\frac{1}{x^6}+1\right)^{2/3}} = \int \frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}}$$

$$I = \int \frac{x^{-7}dx}{(x^{-6}+1)^{2/3}} ------(i)$$

Now, putting t = x – 6 + 1 and differentiating it, we get,

$$dt = -6x^{-7}dx$$

$$-\frac{1}{6}dt = x^{-7}dx$$

So, now putting the value of dx in equation (i), we get,

$$I = -\frac{1}{6}\int 2^{-2/3}\,dt$$

$$I = -\frac{1}{6}\times 3(t)^{1/3} + c$$

$$I = -\frac{1}{2}\left(\frac{1+x^6}{x^6}\right)^{1/3} + c$$

$$I = -\frac{1}{2}(1+x^6)^{1/3} + c$$

So, the function *f*(x) is equal to:

$$xf(x) = \frac{1}{2x^2}$$

$$f(x) = -\frac{1}{2x^2}$$

14. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}|$ =r is possible if:

A. $0 < r \le \sqrt{\frac{3}{2}}$

B. $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$

C. $3\sqrt{\frac{3}{2}} < r \le 5\sqrt{\frac{3}{2}}$

D. $r \ge 5\sqrt{\frac{3}{2}}$

Ans. D

Given vectors are $a = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $b = \hat{i} - \hat{j} + \hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \hat{i}(2 + x) - \hat{j}(3 - x) + \hat{k}(-5)$$

$$\vec{a} \times \vec{b} = (2 + x)\hat{i} - +(x - 3)\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(2 + x)^2 + (x - 3)^2 + 25}$$

$$= \sqrt{2x^2 - 2x + 38}$$

$$\ge \sqrt{\frac{75}{2}}$$

$$\ge 5\sqrt{\frac{3}{2}}$$

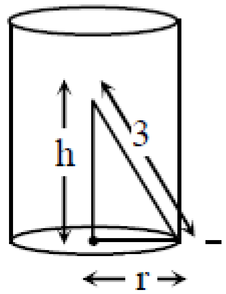
15. The height of a light circular cylinder of maximum volume inscribed in a sphere of radius 3 is:

A. $\frac{2}{3}\sqrt{3}$ B. $\sqrt{6}$

C. $\sqrt{3}$ D. $2\sqrt{3}$

Ans. D

Let a sphere of radius 3, which inscribed a right circular cylinder having radius r and height is h,



$h^2 + r^2 = 9$

$r^2 = 9 - h^2$

$V = \pi\, r^2(2h)$

$v = 2\pi h\,(9 - h^2)$

$v = 2\pi\,(9 - 4^3)$

$\frac{dv}{dx} = 0 \qquad h = \sqrt{3}$

$r = \sqrt{6}$

$V_{max} = \pi r^2(2h)$

$= \pi 6\left(2\sqrt{3}\right)$

$H = 2\sqrt{3} = 2\,(h)$

16. If f(1)=1, f'(1) =3, then the derivative of $f(f(f(x))) + (f(x))^2$ at x=1 is:

A.15 B. 33

C.12 D. 9

Ans. B

Let $y = f(f(f(x))) + (f(x))^2$ at x=1

On differentiating both sides with respect to x we get

$f\,(f(f(x))) + (f(x))^2$ at $x = 1$

$f'(f(f(x))) . f'(f(x)) . f'(x) + 2f(x) f'(x)$
$f'(f(f(x))) . f'(f(x)) . f'(x) + 2f(x) f'(1)$
$f'(f(f(1))) . f'(1) . f'(1) + 2f'(1)$
$f'(f(1) . f'(1) . f'(1) + 2f'(1)$
$3 \times 3 \times 3 + (2 \times 3)$ $f(1) = 1$ $f'(1) = 3$
$27 + 6 = 33$

17. Suppose that the points (h, k), (1, 2) and (-3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then k/h equals:

A. $-\frac{1}{7}$ B. $\frac{1}{3}$

C. 3 D. 0

Ans. B

Equation of line passes through (1, 2) and (–3, 4)

$(y-2) = \frac{4-2}{-3-1}(x-1)$

$(y-2) = -\frac{1}{2}(x-1)$

$x + 2y = 5$ … (i)

$\perp$ line $2x - y = \lambda \rightarrow$ passes through (4, 3)

$2x - y = 5$ …. (2) $\lambda = 5$

Intersection point of line (i) & line (ii) is (3, 1)

$\frac{k}{h} = \frac{1}{3}$

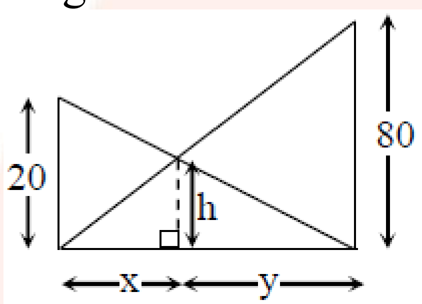
18. Two vertical poles of heights, 20 m and 80m apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is:

A.15 B. 12

C.16 D. 18

Ans. C

Let s first pole having height 20 m and second pole having height 80 m.



$\frac{h}{x} = \frac{80}{x+y}$ $\frac{h}{y} = \frac{20}{x+y}$

$h = \frac{80}{1+(y/x)}$ ...(i) $h = \frac{20}{\left(\frac{x}{y}\right)+1}$

$h = \frac{80}{1+\frac{h}{20-h}}$ $\frac{x}{y}+1 = \frac{20}{h}$

$h = 4(20-h)$ $\frac{x}{y} = \frac{20}{h} - 1$

$h = 80 - 4h$ $\frac{x}{y} = \frac{20-h}{h}$

$5h = 80$ $\frac{x}{y} = \frac{h}{20-h}$

$h = 16$ put in ...(i)

19. Let f: R→R be a differentiable function satisfying f'(3) +f'(2) =0.

Then $\lim_{x\to}\left(\frac{1+f(3+x)-f(3)}{1+f(3+x)-f(2)}\right)^{\frac{1}{x}}$

Is equal to:

A. e^2 B. 1

C. e^{-1} D. e

Ans. B

$\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ Form $(1)^{\infty}$

$= e^{\lim_{x\to 0}} \frac{1}{x}\left[\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}-1\right]$

$= e^{\lim_{x\to 0}} \frac{1}{x}\left[\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right]$ Form $\left(\frac{0}{0}\right)$

$= e^{\lim_{x\to 0}} \left(\frac{f(3+z)-f(x)(2-z)+f(z)}{x}\right)\times\left(\frac{1}{1}\right)$

$= e^{\lim_{x\to 0}} \frac{f'(3+x)+f'(2-x)}{1}$

$= e^{f'(3)+f'(2)} = e^0 = 1$

$= e$

20. If three district numbers a, b, c are in G.P. and the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root, then which one of the following statements is correct?

A. d, e, f are in G.P

B. $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in G.P

C. d, e, f are in A.P

D. $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P

Ans. D

Given three distinct numbers a, b and c are in GP

$\therefore b^2 = ac$ …(i)

and the given quadratic equations

$ax^2 + 2bx + c = 0$ …(ii)

$dx^2 + 2ex + f = 0$ …(iii)

$(af - cd)^2 = (2ae - 2bd)(2bf - 2ec)$

$a^2f^2 + c^2d^2 - 2a + cd = 4aebf - 4ae^2c - 4b^2df + 4bdec$

$a^2f^2 + c^2d^2 + 4b^2e^2 + 2afcd - 4aebf - 4bdec = 0$

$(af + cd - 2be)^2 = 0$

$af + cd = 2be$

$$\frac{af}{b^2} + \frac{cd}{b^2} = \frac{2be}{b^2}$$

$$\frac{af}{ac} + \frac{cd}{ac} = 2\left(\frac{e}{b}\right)$$

$$\frac{f}{c} + \frac{d}{a} = 2\left(\frac{e}{b}\right)$$

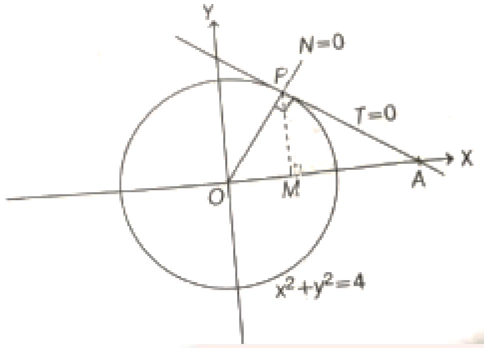
$\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in AP.

21. The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2y^2=4$ and the x-axis form a triangle. The area of this triangle (in square units) is:

A. $\frac{2}{\sqrt{3}}$ B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{3}$ D. $\frac{4}{\sqrt{3}}$

Ans. A



So, equation of tangent (T = 0) is

$\sqrt{3}x + y = 4$

For point A, put y = 0, we get

$x = \frac{4}{\sqrt{3}}$

∵ Area of required ΔOPA = ½(OA) (PM)

$x = \frac{4}{\sqrt{3}}$

[∵ PM = y-coordinate of P]

$= \frac{2}{\sqrt{3}}$ sq unit .

22. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{2}}\right)^6$ is equal to 200, and x >1,then the value of x is:

A.10^4 B. 10

C.10^3 D. 100

Ans. B

It is given that binomial is $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{2}}\right)^6$

Since the fourth term in the expansion is 200,
So

$$\left(\left(\frac{1}{x^{1+\log_{10}^{x}}}\right)^{\frac{1}{2}}+x^{\frac{1}{12}}\right)^6 \text{ and } T_4 = 200$$

$$T_4 = {}^6C_3(x)^{-\frac{3}{2}(1+\log_{10}^{x})}.\left(x^{\frac{1}{12}}\right)^3 = 200$$

$$20.x^{-\frac{3}{2}(1+\log_{10}^{x})} + \frac{1}{4} = 200$$

$$x^{-\frac{3}{2}(1+\log_{10}^{x})+\frac{1}{4}} = 10$$

Taking, log on both side, we get,

$$\left(-\frac{3}{2}(1+\log_{10}^{x})+\frac{1}{4}\right)\log_{10}^{x} = 1$$

Now, let $\log_{10}^{x} = t$, we get,

$$-6(t^2+t)+\frac{t}{4} = 1$$

$-6(t^2 + t) + t = 4$

$-6t^2 - 6t + t = 4$

$6t^2 + 5t + 4 = 0$

$\Delta < 0$ and roots are imaginary.

(This is a bonus question)

23. Let $f(x)=\int_0^x g(t)dt$, where g is a non-zero even function. If $f(x+5)=g(x)$, then $\int_0^x f(t)dt$ equal:

A. $\int_5^{x+5} g(t)dt$ B. $\int_{x+5}^5 g(t)dt$

C. $5\int_{x+5}^5 g(t)dt$ D. $2\int_5^{x+5} g(t)dt$

Ans. B

Given $f(x)=\int_0^x g(t)\,dt$

On replacing x by (-x) we get

$f(-x)=\int_0^{-x} g(t)\,dt$

Now, put t = -u, so

$f(-x)=-\int_0^x g(-u)du=-\int_0^x g(u)\,du=-f(x)$

[$\because$ g is an even function]

$f(0)=\int_0^0 g(t)\,dt=0$

f(0) = 0 f(x) is odd function

give that g(x) is even function

f(x + 5) = f (–x + 5) = g(x) = g(–x)

$$I = \int_{0}^{x} f(t)\,dt$$

$$z = t + 5$$

$$I = \int_{5}^{x+5} f(3-5)\,dz$$

$$I = \int_{5}^{x+5} f(-(f-3))\,dz$$

$$I = \int_{5}^{x+5} f(5-3)\,dz$$

$$I = \int_{x+5}^{5} f(5-z)\,dz$$

$$I = \int_{x+5}^{5} g(z)\,dz$$

$$I = \int_{x+5}^{5} g(t)\,dt$$

24. Given that the slope of the tangent to a curve y=y(x) at any Point (x,y) is $\frac{2y}{x^2}$ If the curve passes through the center of the circle $x^2+y^2-2x-2y=0$,then its equation is:

A. $x^2 \log_e |y| = -2(x-1)$

B. $x \log_e |y| = -2(x-1)$

C. $x \log_e |y| = x-1$

D. $x \log_e |y| = 2(x-1)$

Ans. D

slope of the tangent to a curve y=y(x) at any Point (x,y) is given

$$\frac{dy}{dx} = \frac{2y}{x^2}$$
$$\frac{dy}{y} = \frac{2}{x^2}dx$$
Integrating both sides
$$\log_e |y| = -\frac{2}{x} + c$$
process through (1, 1)

$0 = -2 + c \quad c = 2$

$$\log_e |y| = -\frac{2}{x} + 2$$

$x \log_e |y| = -2 + 2x$

$x \log_e |y| = 2(x - 1)$

25. If $z=\frac{\sqrt{3}}{2}+\frac{i}{2}(i=\sqrt{-1})$, then $(1+iz+z^5+iz^8)^9$ is equal to:

A. 1 B. 0

C. -1 D. $(-1+2i)^9$

Ans. C

$$z = \frac{\sqrt{3}}{2} + \frac{i}{2} = -i\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -i\omega$$
$$(i + iz + z^5 + iz^8)^9 = (1 + \omega - i\omega^2 + i\omega^2)^9 = (1 + \omega^2)^9 = (-\omega^2)^9 = (-\omega^{18}) = -1$$

26. Let $f(x) = a^x$ $(a>0)$ be written as $f(x) = f_1(x)+f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y)+f_1(x-y)$ equals:

A. $2f_1(x+y)f_2(x-y)$

B. $2f_1(x)f_2(y)$

C. $2f_1(x+y)f_1(x-y)$
D. $2f_1(x)f_1(y)$

Ans. D

Given function $f(x) = a^x$ $(a>0)$ is written as sum of an even and odd function $f_1(x)$ and $f_2(x)$ respectively.

Clearly, $f_1(x) = \frac{a^x + a^{-x}}{2}$ and $f_2(x) = \frac{a^x - a^{-x}}{2}$

$$f(x) = \underbrace{\left(\frac{f(x)+f(-x)}{2}\right)}_{\text{even}} + \underbrace{\left(\frac{f(x)-f(-x)}{2}\right)}_{\text{odd}}$$

$$f_1(x) = \frac{f(x)+f(-x)}{2}$$

$$f_1(x+y) + f_1(x-y) = \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{y-x}}{2}$$

$$= \frac{1}{2}\left[a^x\left(a^y + a^{-y}\right) + a^{-x}\left(a^{-x} + a^{-y}\right)\right]$$

$$= \frac{1}{2}\left(a^x + a^{-x}\right)\left(a^y + a^{-y}\right)$$

$$= \frac{1}{2}\left(\frac{a^x + a^{-x}}{2}\right)\left(\frac{a^y + a^{-y}}{2}\right)$$

$= 2f1(x)\ f1(y)$

27. Let $S(\alpha)=\{(x,y):y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is the area of the region $S(\alpha)$. If for a $\lambda, 0<\lambda<4$, $A(\lambda):A(4)=2:5$, then λ equals:

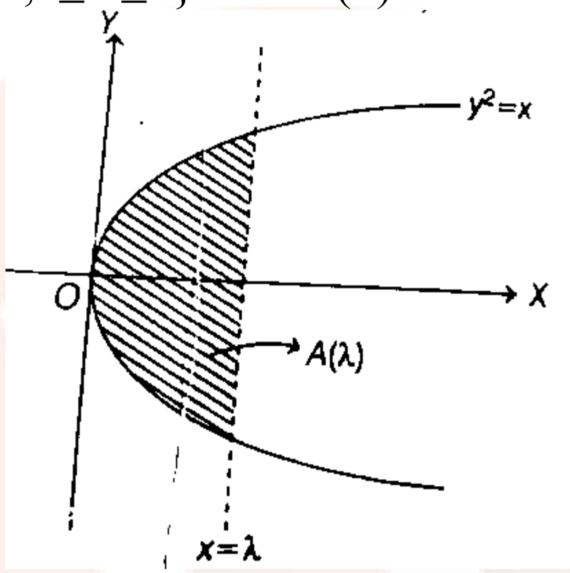
A. $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ B. $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

C. $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$ D. $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$

Ans. C

Given,

$S(\alpha) = \{(x,y): y^2 \le x, 0 \le x \le \alpha\}$ and A (α) is area of the region $S(\alpha)$



$$\frac{A(\lambda)}{A(4)} = \frac{2}{5}$$

$$\frac{\int_0^{\lambda} \sqrt{x}\, dx}{\int_0^{4} \sqrt{x}\, dx} = \frac{2}{5}$$

$$\frac{\lambda^{3/2}}{4^{3/2}} = \frac{2}{5}$$

$$\lambda^{3/2} = \frac{2}{5} \times 8$$

$\lambda = \left(\frac{16}{5}\right)^{2/3}$

$\lambda = 4\left(\frac{2}{5}\right)^{2/3}$

$\lambda = 4\left(\frac{4}{25}\right)^{1/3}$

28. In an ellipse, with center at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at (0, $5\sqrt{3}$), then the length of its latus rectum is:

A.10 B. 8

C.5 D. 6

Ans. C3

Given that 2 b – 2a = 10

b – a = 5 ...(i)

given that

One of the focus of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is one y-axis (0, 5 √3)

2be = 10√3

be = 5√3

$(b^2 - a^2) = 75$

(b – a) (b + a) = 75

5(b + a) = 75

b + a = 15 … (ii)

from equation (i) & equation (2)

b = 10

a = 5

length of lotus rectum

$= \frac{2a^2}{b}$

$= \frac{2 \times 25}{10} = 5$

29. The vector equation of the plane through the line of intersection of the planes x+y+z=1 and 2x+3y+4z=5 which is perpendicular to the plane x-y+z=0 is:

A. $\vec{r} \cdot (\hat{i} - \hat{k})$+2=0

B. $\vec{r} \cdot (\hat{i} - \hat{k})$-2=0

C. $\vec{r}$ x $(\hat{i} - \hat{k})$+2=0

D. $\vec{r}$ x $(\hat{i} - \hat{k})$+2=0

Ans. A

Since, equation of planes passes through the line of intersection of the planes

x + y + z = 1 and 2x + 3y + 4z = 5

Now, the equation of required plane

(x + y + z –1) + 1 (2x + 3y + 4z – 5) =0

x(1 + 2λ) + y (1 + 3λ) + 3 (1 + 4λ) – 5λ– = 0 … (i)

x – y + 3 = 0 … (ii)

Plane (1) & (2) are perpendicular to each other

(1) (1 + 2λ) + (–1) (1 + 3λ) + (1) (1 + 4λ) = 0

1 + 2λ –1 – 3λ + 1 + 4λ = 0

3λ = –1

$\lambda = -\frac{1}{3}$

put in equation (i)

$$\frac{x}{3} - \frac{z}{3} + \frac{2}{3} = 0$$

$$x - z + 2 = 0$$

$$\vec{r} \cdot \left(\hat{i} - \hat{k}\right) = -2$$

30. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is :

A.306 B. 310

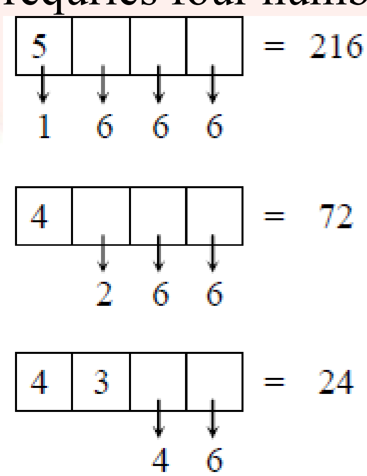
C.360 D. 288

Ans. B

Following are the cases in which the 4-digit numbers strictly greater than 4321 can be formed using digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed)

Given digits are 0, 1, 2, 3, 4, 5

requries four number greater than 4321



total case = 22

{substract two case 4320 & 4321}

required total numbers = 216 + 72 + 22

$= 310$