Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.Super60_NUCLEUS-BT | JEE-MAIN | Date: 02-08-2025
Time: 09.00Am to 12.00Pm | RPTM-04 | Max. Marks: 300

KEY SHEET

MATHEMATICS

1	**2**	2	**4**	3	**4**	4	**2**	5	**2**
6	**3**	7	**2**	8	**1**	9	**1**	10	**2**
11	**1**	12	**4**	13	**2**	14	**1**	15	**1**
16	**2**	17	**2**	18	**1**	19	**2**	20	**1**
21	**1**	22	**3**	23	**5**	24	**7**	25	**6**

PHYSICS

26	**4**	27	**3**	28	**2**	29	**2**	30	**2**
31	**3**	32	**2**	33	**2**	34	**2**	35	**1**
36	**4**	37	**1**	38	**3**	39	**4**	40	**3**
41	**4**	42	**4**	43	**1**	44	**4**	45	**4**
46	**635**	47	**360**	48	**2**	49	**500**	50	**8**

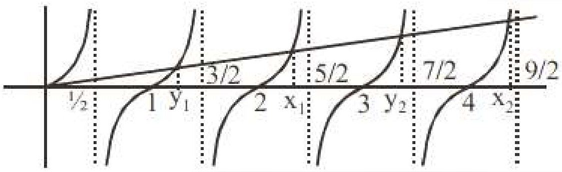
CHEMISTRY

51	**4**	52	**2**	53	**1**	54	**4**	55	**4**
56	**2**	57	**2**	58	**2**	59	**4**	60	**3**
61	**1**	62	**2**	63	**3**	64	**2**	65	**1**
66	**1**	67	**1**	68	**1**	69	**2**	70	**2**
71	**6**	72	**12**	73	**6**	74	**6**	75	**6**

SOLUTION

MATHEMATICS

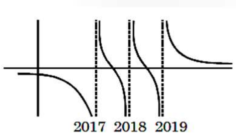
1. $f(x)=\dfrac{\sin \pi x}{x^2}$ $\quad f'(x)=\dfrac{2x\cos \pi x\left(\dfrac{\pi x}{2}-\tan \pi x\right)}{x^4}$



$\Rightarrow |x_n - y_n| > 1$ for every n

$x_1 > y_1$ $\quad x_n \in (2n, 2n+1/2)$ $\quad x_{n+1}-x_n > 2$

2. $f(x)=x$ $\quad g(x)=\dfrac{1}{x-2017}+\dfrac{1}{x-2018}+\dfrac{1}{x-2019}$



$$g'(x)=-\left(\frac{1}{(x-2017)^2}+\frac{1}{(x-2018)^2}+\frac{1}{(x-2019)^2}\right)$$

$$g'(2016)=-\left(1+\frac{1}{4}+\frac{1}{9}\right)=\frac{-49}{36}$$

3. Let $g(x)=f(x).f'(x)\Rightarrow g'(x)>0 \, in [a,b]$

4. $n<\sqrt{n^2+n+1}<n+1$ $\quad$ Hence $\left[\sqrt{n^2+n+1}\right]=n$

After rationalization $\lim\limits_{h\to\infty}\left[\dfrac{n+1}{\sqrt{n^2+n+1}+n}\right]=\dfrac{1}{2}$

5. $L=\lim\limits_{n\to\infty}\left(\dfrac{p^{1/n}+q^{1/n}}{2}\right)=e^{\lim\limits_{n\to\infty} n\left(\frac{p^{1/n}+q^{1/n}}{2}-1\right)}=e^{\lim\limits_{n\to\infty} n\left(\frac{\left(p^{1/n}-1\right)+\left(q^{1/n}-1\right)}{2}\right)}=e^{\lim\limits_{n\to\infty}\frac{1}{2}\left(\frac{\left(p^{1/n}-1\right)}{1/n}+\frac{\left(q^{1/n}-1\right)}{1/n}\right)}$

$=e^{\left(\frac{\ln p+\ln q}{2}\right)}=e^{\left(\ln\sqrt{pq}\right)}=\sqrt{pq}$

6. Hence $f(x)=\begin{cases}-\dfrac{1}{x}, & -1\le x<-\dfrac{1}{2}\\ 0, & -\dfrac{1}{2}\le x<\dfrac{1}{2}\\ \dfrac{1}{x}, & \dfrac{1}{2}\le x<\dfrac{3}{2}\\ \dfrac{2}{x}, & \dfrac{3}{2}\le x<\dfrac{5}{2}\end{cases}$

7. $f(0^+)=\lim\limits_{x\to 0^+}|x|^{\sin x}=e^{\lim\limits_{x\to 0^+}\sin x\ln|x|}$

$=e^{\lim\limits_{x\to 0^+}\frac{\ln|x|}{\operatorname{cosec} x}}=e^{\lim\limits_{x\to 0^+}\frac{1/x}{-\operatorname{cosec} x\cot x}}$ $\quad =e^{-\lim\limits_{x\to 0^+}\left(\frac{\sin x}{x}\right)\tan x}=e^{-1\times 0}=1$ $\quad \therefore f(0^-)=g(0)=1$

Let $g(x) = ax + b$ $\Rightarrow b = 1 \Rightarrow g(x) = ax + 1$

For $x > 0, f'(x) = e^{\sin x \ln(|x|)}\left[\cos x \ln(|x|) + \frac{\sin x}{x}\right]$

$$f'(1) = 1(0 + \sin 1) = \sin 1$$

$f(-1) = -a + 1 \Rightarrow a = 1 - \sin 1$

$g(x) = (1 - \sin 1)x + 1$

8. $y = \tan^{-1}\sqrt{\left(\frac{\sin\frac{x}{2} - \cos\frac{x}{2}}{\sin\frac{x}{2} + \cos\frac{x}{2}}\right)^2} = \tan^{-1}\left|\frac{\sin\frac{x}{2} - \cos\frac{x}{2}}{\sin\frac{x}{2} + \cos\frac{x}{2}}\right| \ldots\ldots\ldots(1)$

In the neighbourhood of $x = \frac{\pi}{6}, \sin\frac{x}{2} - \cos\frac{x}{2} < 0$

$$\therefore \left|\frac{\sin\frac{x}{2} - \cos\frac{x}{2}}{\sin\frac{x}{2} + \cos\frac{x}{2}}\right| = -\frac{\sin\frac{x}{2} - \cos\frac{x}{2}}{\sin\frac{x}{2} + \cos\frac{x}{2}}$$

$$= \frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}} = \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

From (1) $y = \frac{\pi}{4} - \frac{x}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

9. $|f'(x)| = \left(\frac{f(1) - f(0)}{1 - 0}\right) \le |f(x)| + |f(0)| \le 2$

$F(\alpha) = (f(\alpha))^2 + (f'(\alpha))^2 \le 1 + 4 = 5$

Similarly for $(F'(\beta)) \le 5$ for some $\beta \in (-1, 0)$ As $F(0) = 6$, there must be a point of local maxima for $F(x)$ in $(-1,1)$ and at the point of maxima,

$F(c) \ge 6, F'(c) = 0$ *and* $F''(c) \le 0$

10. Given $s^2 = (at^2 + 2bt + c) \ldots\ldots.(i)$

Or $s = \sqrt{at^2 + 2bt + c}$

Differentiating both sides with respect to 't' we get

$$\frac{ds}{dt} = \frac{(at + b)}{\sqrt{(at^2 + bt + c)}} = \frac{(at + b)}{s} = v$$

(say){From(i)}….(2)

Again differentiating both sides with respect to 't' then

$$\frac{d^2s}{dt^2} = \frac{s(a) - (at + b).\frac{ds}{dt}}{s^2} = \frac{as - (at + b).\frac{(at + b)}{s}}{s^2}$$

From (2) $= \frac{as^2 - (at + b)^2}{s^3}$ $= \frac{a(at^2 + 2bt + c) - (a^2t^2 + 2abt + b^2)}{s^3} = \frac{ac - b^2}{s^3}$

$\therefore$ Acceleration $\alpha \frac{1}{s^3}$

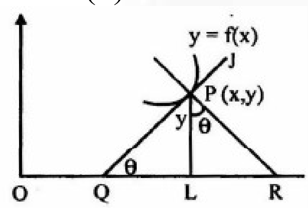
11. Given curve is $2x^2y^2 - x^4 = c......(1)$

Subnormal at $P(x,y) = yy_1 = y\frac{dy}{dx}....(2)$

From (1) $2\left(x^2.2y\frac{dy}{dx} + 2xy^2\right) - 4x^3 = 0$ $\Rightarrow \frac{dy}{dx} = \frac{x(x^2-y^2)}{x^2y}........(3)$

Now, $x(x - yy_1) = x^2 - xy\frac{dy}{dx} = x^2 - (x^2 - y^2)$

From (3)



$= y^2 \Rightarrow$ mean proportion $= \sqrt{x(x-yy_1)} = y$

12. $g'(x) = \left(f'(\tan x - 1)^2 + 3\right)2(\tan x - 1)\sec^2 x$

Since $f''(x) > 0 \Rightarrow f'(x)$ is increasing

So, $f'\left((\tan x - 1)^2 + 3\right) > f'(3) = 0\ \forall x \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Also, $(\tan x - 1) > 0$ for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

So, $g(x)$ is increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

13. If $f(x) = x^{1/x}$, then

$f'(x) = \frac{1}{x^2}\left[x^{1/x}(1 - \ln x)\right]$

f is decreasing if $x > e$ and f is increasing if $x < e$.

As $e < 3 < 4 < 5 < 6 < 8$

Max$\left\{3^{1/3}, 4^{1/4}, 5^{1/5}, 6^{1/6}, 8^{1/8}\right\} = 3^{1/3}$

Also $1 < 2 < e$ $\therefore$ Max$\left\{1, 2^{1/2}\right\} = 2^{1/2}$

But $2^{1/2} = 4^{1/4} < 3^{1/3}$ $\therefore$ The greatest number is $3^{1/3}$

14. $f'(x) = e^x\cos x - e^x \sin x$ $\therefore$ slope $m = e^x \cos x - e^x \sin x$

Now $\frac{dm}{dx} = e^x\cos x - e^x\sin x - e^x \sin x - e^x\cos x = 2e^x \sin x$

For maximum or minimum slope m, we have $\frac{dm}{dx} = 0 \Rightarrow x = n\pi, n \in I$

Again, $\frac{d^2m}{dx^2} = -2e^x\sin x - 2e^x\cos x$

Clearly, $\frac{d^2m}{dx^2} < 0$ if $x = 2k\pi$, hence maximum slope,

m occurs at $x = 2k\pi, k \in I$,

Hence, x=0 is the required answer.

15. $AD = AB\cos\theta = 2R\cos\theta \quad \Rightarrow AE = AD\cos\theta = 2R\cos^2\theta$

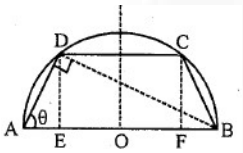
$\therefore EF = AB - 2AE = 2R - 4R\cos^2\theta$ and

$DE = AD\sin\theta = 2R\sin\theta\cos\theta$

Area of trapezium, $S = \frac{1}{2}(AB + CD) \times DE$

$= \frac{1}{2}\left(2R + 2R - 4R\cos^2\theta\right) \times 2R\sin\theta\cos\theta \qquad = 4R^2\sin^3\theta\cos\theta$

$\frac{dS}{d\theta} = 12R^2\sin^2\theta\cos^2\theta - 4R^2\sin^4\theta$



$= 4R^2\sin^2\theta\left(3\cos^2\theta - \sin^2\theta\right)$

For maximum area, $\frac{dS}{d\theta} = 0 \Rightarrow \tan^2\theta = 3 \quad \Rightarrow \tan\theta = \sqrt{3}\,(\because \theta \text{ is acute})$

$\therefore$ Maximum area $S_{\max} = \frac{3\sqrt{3}}{4}R^2$. For radius=2R $S_{\max} = \frac{3\sqrt{3}}{4}R^2$

16. The line $y = mx + c$ intercepts a length $\ell = \frac{4}{m^2}\sqrt{1+m^2}\sqrt{a(a - mc)}$

Also, the line is a normal if $c = -2am - am^3$

$\therefore \ell = \frac{4}{m^2}\sqrt{1+m^2}\sqrt{a\left(a + 2am^2 = am^4\right)} \qquad = \frac{4a}{m^2}\left(1+m^2\right)^{3/2}$

$\frac{d\ell}{dm} = \frac{4a}{m^3}\sqrt{1+m^2}\left(m^2 - 2\right)$. For least $\ell\,\frac{d\ell}{dm} = 0$

$\Rightarrow m = \pm\sqrt{2}$, Also $\frac{d^2\ell}{dm^2} > 0$ for $m = \pm\sqrt{2}$

$\therefore \ell$ is minimum when $m = \pm\sqrt{2} \quad \Rightarrow \ell_{\min} = \frac{4}{2}a\left(\sqrt{3}\right)^3 = 6\sqrt{3}a$

17. $f'(x) = 2a^2x^2 - 5ax + 3 = (ax - 1)(2ax - 3) = 0 \qquad x = \frac{1}{a}, \frac{3}{2a}$

If $a > 0$ the local maxima occurs at $x = \frac{1}{a}$ and minima at $x = \frac{3}{2a}$

minimum occurred $x = \frac{1}{a} = \frac{1}{3} \Rightarrow a = 3$ minima occurred $x = \frac{3}{2a} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) > 0 \Rightarrow \frac{2}{3} \times 9 \times \frac{1}{8} - \frac{5}{2} \times 3 \times \frac{1}{4} + \frac{3}{2} + b > 0;$

$\frac{6 - 15 + 12}{8} + b > 0 \Rightarrow \frac{3}{8} + b > 0 \Rightarrow b > -\frac{3}{8}$

If $a < 0$ then maxima shall occur at $x = \frac{3}{2}a$ and minima $x = \frac{1}{a}$

$\frac{3}{2a} = \frac{1}{3} \Rightarrow a = \frac{9}{2} > 0$ not admissible, Hence $b > -\frac{3}{8}$

18. $f(x)=\left(4\sin^2 x-1\right)^n\left(x^2-x+1\right)$

$f'(x)=n\left(4\sin^2 x-1\right)^{n-1}(8\sin x\cos x)\left(x^2-x+1\right)+\left(4\sin^2 x-1\right)^n(2x-1)$

$=\left(4\sin^2 x-1\right)^{n-1}\left[n\left(x^2-x+1\right)(4\sin 2x)\right]+(2x-1)\left(4\sin^2 x-1\right)$

19. We have

$f'(x)=6x^2-18ax+12a^2=6\left(x^2-3ax+4a^2\right)$ $\quad \therefore f'(x)=0\Rightarrow x=a,2a$

Now given $x_2=x_1^2\Rightarrow a>0$, so, $x_1=a$ is the point of local maximum and $x_2=2a$ is the point of local minimum $(\because a<2a)$

$\therefore 2a=a^2\Rightarrow a=2$ [$a\neq 0$ otherwise $f(x)$ is increasing function]

20. (p) $f(x)=\begin{cases} e^{-1/x^2}, & x\neq 0 \\ 0, & x=0 \end{cases}$

$\lim\limits_{x\to 0} e^{-1/x^2}=\lim\limits_{t\to\infty} e^{-t^2}=\lim\limits_{t\to\infty} e^{\frac{1}{t^2}=0}$

$y=x|x|=x^2, x\geq 0$ At $x=0$ $=x^2, x<0$ $f'(0^+)=2x=0$

$\frac{dy}{dx}$ exists at $x\in R$ $f'(0^-)=-2x=0$ $y=\sin^{-1}\sin x$

Continuous but non derivable at $x=(2n+1)\frac{\pi}{2}, n\in I$

$y=\begin{cases} x^2\sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$

$f'(0^+)=\lim\limits_{h\to 0}\frac{h^2\sin\frac{1}{h}-0}{h}=0$ $f'(0^-)=\frac{-h^2\sin\frac{1}{h}-0}{-h}=0$

$f(x)$ is derivable in R

21. $\beta=\lim\limits_{x\to 0}\frac{1+x^3-\left(1-\frac{x^3}{3}\right)+\left(1-\frac{x^2}{2}-1\right)x}{x^3}=\frac{5}{6}\Rightarrow 6\beta=5$

22. $y=[x]+|1-x|$ for $-1\leq x<3$

(i) for $-1\leq x<0, y=-1+1-x=-x$

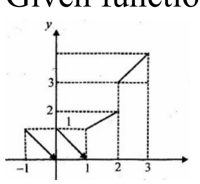
(ii) for $0\leq x<1, y=0+1-x=1-x$

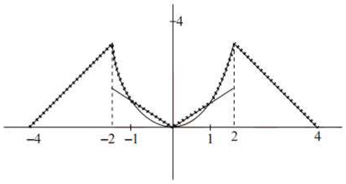
(iii) for $1\leq x<2, y=1+x-1=x$

(iv) For $2\leq x<3, y=2+x-1=x+1$.

Given function is discontinuous at $x=0,1,2$

Given function is not differentiable at $x=0,1,2$



23.


Not differentiable at $x=\pm 2, \pm 1, 0$

24. $\frac{dy}{dx}=2x-5\Big|_{x-x_1}=2 \qquad 2x_1=7\Rightarrow x_1=\frac{7}{2}$

$y_1=\frac{49}{4}-\frac{35}{2}+5 \qquad =\frac{49-70+20}{4}=-\frac{1}{4} \qquad y+\frac{1}{4}=2\left(x-\frac{7}{2}\right)\ \left(\frac{7}{2},-\frac{1}{4}\right)$

$4y+1=8x-28\Rightarrow 8x-4y-29=0$

Now check options $x=\frac{1}{8}, y=-7$ Put x = 1/8, we get y = -7

25. Given curves are $y=\left|x^2-1\right|........(1)$

$y=\left|x^2-3\right|......(2) \qquad y=\begin{Bmatrix} x^2-1 & x\le -1\, or \ge 1 \\ 1-x^2, & -1\le x\le 1\end{Bmatrix}....(3)$ And $y=\begin{Bmatrix} x^2-3, & x\le -\sqrt{3}\, or\, x\ge\sqrt{3} \\ 3-x^2, & -\sqrt{3}\le x\le\sqrt{3}\end{Bmatrix}....(4)$

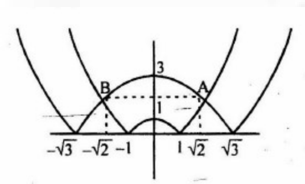
Equating the two values of y from(1) and (2) we get

$\left|x^2-1\right|=\left|x^2-3\right|$ Or $x^2-1=\pm\left(x^2-3\right)\Rightarrow x=\pm\sqrt{2}$

From(1), when $x=\pm\sqrt{2}, y=1$ Let $A\equiv\left(\sqrt{2},1\right)$ and $B\equiv\left(-\sqrt{2},1\right)$

Here A and B are the points of intersection of curves (1) and (2)

Angle of intersection between curves (1) and (2) at $A\left(\sqrt{2},1\right)$



From(3), $\left(\frac{dy}{dx}\right)_{at\left(\sqrt{2},1\right)}=(2x-0)=2\sqrt{2}=m_1(say)$

From(4), $\left(\frac{dy}{dx}\right)_{at\left(\sqrt{2},1\right)}=(-2x)=-2\sqrt{2}=m_2(say)$

Let θ be the acute angle between curves (1) and(2) at A, then

$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{2\sqrt{2}+2\sqrt{2}}{1-8}\right|=\frac{4\sqrt{2}}{7} \qquad \Rightarrow m=4\sqrt{2}$

PHYSICS

26. General wave equation along the +ve x-axis

$y = A\sin(\omega t - kx + \phi_0)$ $A = 1$ for initial phase ϕ_0 particle at X=0 approaches the mean. Below the mean.

Hence $\phi_0 = 2\pi - \frac{\pi}{6}$ $y = 1\sin\left(\omega t - kx + 2\pi - \frac{\pi}{6}\right)$ $y = \sin\left(\omega t - kx - \frac{\pi}{6}\right)$

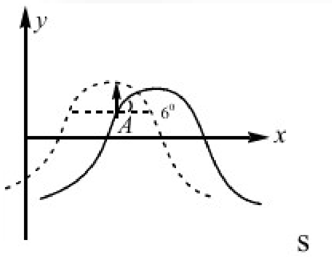
27. Transverse wave velocity $V = \sqrt{\frac{T}{\mu}}$

$100 = \sqrt{\frac{T}{0.01}}$ $T = 100N$ $T = mg\ \sin\theta$

$100 = m.10 \times \sin 30^0 \Rightarrow m = 20$

28. As shown in the curve, if wave is moving

Along $-x$ *axis*, V_p *is positive.*

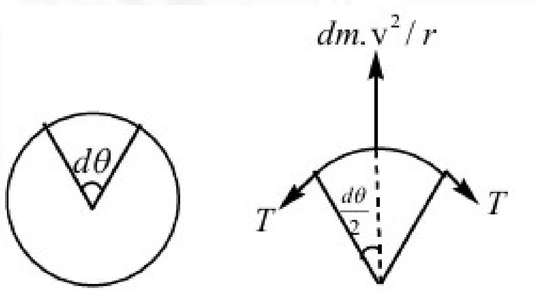


s

$\frac{V_p}{V_w} = -\tan\theta$ $|-\tan\theta| < 1$ $\therefore V_p < V_\omega$

29. $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2 f_2^2} = \frac{(3)^2(8)^2}{(2)^2(12)^2} = 1$

30. $dm.\omega^2 R = 2\sin\frac{d\theta}{2}$ $\mu R d\theta \omega^2 R 2T\frac{d\theta}{2}$

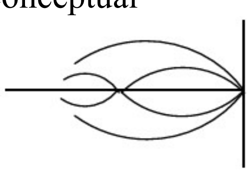


$\Rightarrow \mu\omega^2 R^2 = T$ $\Rightarrow V_w = \sqrt{\frac{T}{\mu}} = \sqrt{\omega^2 R^2} = \omega R$

Also speed of string is ωR

$\therefore$ The velocity of disturbance w.r.t. ground $= \omega R + \omega R = 2\omega R$.

31. Conceptual



32. $f_1 = \frac{3V}{4\ell}$ $f_2 = \frac{5V}{4\ell}$

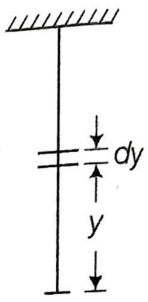
33. $y = e^{-\left(\sqrt{a}x+\sqrt{b}t\right)^2}$ $\frac{d}{dt}\left[\sqrt{a}x+\sqrt{b}t\right] = 0 \Rightarrow \frac{dx}{dt} = -\frac{\sqrt{b}}{\sqrt{a}}$

34. Mass of the differential element is $dm = \mu_0 y dy$

Mass of he string up to the length y is

$m = \int dm = \int_0^y \mu_0 y\, dy$ $\quad m = \frac{\mu_0 y^2}{2}$

Tension at the position of element is $T = mg = \frac{\mu_0 y^2}{2} g$



Speed of wave pulse at this location is $V = \sqrt{\frac{T}{\mu}}$

Or $V = \sqrt{\frac{\mu_0 y^2 g}{2\mu_0 y}} = \sqrt{y}\sqrt{\frac{g}{2}}$ $\quad \frac{dy}{dt} = \sqrt{y}\sqrt{\frac{g}{2}}$ $\quad or \quad t = \sqrt{\frac{8L_0}{g}}$

35. $du = \frac{1}{2}T\left(\frac{dy}{dx}\right)^2 dx; v_p = -v_w.slope$

36. $f = \frac{6v}{2L} = \frac{3v}{L}$ $\quad \lambda = \frac{V}{f} = \frac{L}{3} \Rightarrow L = 3\lambda$ $\quad \Delta p_0 = \left(\Delta p_0\right)_{\max} \sin kx \Rightarrow \sin kx = \frac{1}{\sqrt{2}}$

$\Rightarrow kx = \frac{\pi}{4} \Rightarrow x = \frac{x}{4\times\frac{2\pi}{\lambda}} = \frac{\lambda}{8}$ $\quad \Rightarrow \frac{\lambda}{8} = \frac{40}{2} = 20cm$ $\quad \Rightarrow L = 160cm \Rightarrow L = 3\lambda = 4.8m$

37. $V = \sqrt{\frac{F}{\mu}}$ $\quad \Rightarrow V_1 = \sqrt{\frac{F}{\mu}} and\, V_2 = \sqrt{\frac{F}{9\mu}} = \frac{V_1}{3}$ $\quad \therefore \frac{V_1}{V_2} = 3$

$A_1 = A_r = \left(\frac{V_2 - V_1}{V_1 + V_2}\right) A_i = \left(\frac{V_2 - 3V_2}{3V_2 - V_2}\right) A_i = -\frac{A_i}{2}$

$A_2 = A_t = \left(\frac{2V_2}{V_1 + V_2}\right) A_i = \frac{2V_2}{3V_2 + V_2}.A_i = \frac{A_i}{2}$ $\quad \therefore \left|\frac{A_1}{A_2}\right| = 1$

$l = \frac{1}{2}\rho\omega^2 A^2 V = \frac{1}{2}\frac{\mu}{s}\omega^2 A^2 V$

We do not have the information about density (and cross-sectional s of two ropes).

$SO, l_1 / l_2$ cannot be calculated. Power is $P = \frac{1}{2}\rho\omega^2 A^2 SV$

As $\mu = \rho s, P = \frac{1}{2}\mu\omega^2 A^2 V \Rightarrow P \propto \mu V A^2$ $\quad \therefore \frac{P_1}{P_2} = \frac{\mu_1}{\mu_2}.\frac{V_1}{V_2}\left(\frac{A_1}{A_2}\right)^2 = \frac{1}{9}.3(1) = \frac{1}{3}$ $\quad \therefore \frac{P_2}{P_1} = 3$

38. $k = 0.314 = \frac{2\pi}{\lambda}$ $\quad \lambda = 20m$ $\quad l = \frac{4\lambda}{2} = 40m$

39. Wave speed $= V = \frac{\omega}{K} = \frac{3000\pi}{7.5\pi} = 400 m/s \Rightarrow V = \sqrt{\frac{B}{\rho}} \Rightarrow (400)^2 = \frac{1.6\times10^5}{\rho}$

$\Rightarrow \rho = 1 kg/m^3 \qquad \Rightarrow \Delta p = BAK \Rightarrow 30 = 1.6\times10^5 \times A \times 7.5\pi$

$\Rightarrow A = \frac{10^{-4}}{4\pi} m \Rightarrow I = \frac{(\Delta P)^2}{2\rho V} = \frac{900}{2\times1\times400} = \frac{9}{8} W/m^2$

40. 9^{th} harmonic of closed pipe $= \frac{9V_1}{4\ell_1}$

4^{th} harmonic of open pipe $= \frac{2V_2}{\ell_2} \therefore \frac{9V_1}{4\ell_1} = \frac{2V_2}{\ell_2} \quad \therefore \frac{9}{4\ell_1}\sqrt{\frac{B}{P_1}} = \frac{2}{\ell_2}\sqrt{\frac{B}{P_2}} \quad \ell_2 = \frac{20}{9} cm$

41. Solids have higher value of bulk modulus than gases.

42. In resonance tube, the vibrations are set-up in the air column which only depends on the length of the resonance column and velocity of sound. The liquid in the tube only reflects the waves which upon superposition produce stationary waves. Therefore, if oil of density higher than that of water is used, there will be not effect on the frequency of waves set-up.

43. Only transverse waves can be polarised. Sound waves (mechanical wave) cannot ben polarised as they are longitudinal in nature whereas light waves can be polarised as they are transverse in nature.'

44. The speed of sound of sound in gaseous medium is given by

$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma\left(\frac{RT}{M}\right)}$ ....(1)

At constant temperature PV=constant ......(2)

If V is the volume of one of gas, then density of gas $\rho = \frac{M}{V}$ or $V = \frac{M}{\rho}$

Where M is the molecular weight of the gas.

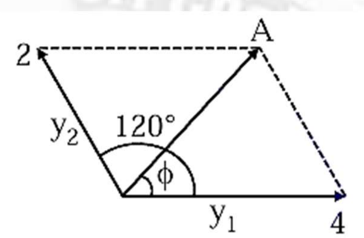
Therefore, Equation (2) becomes, $\frac{PM}{\rho} =$ constant

$\Rightarrow \frac{P}{\rho} =$ Constant as M is a constant $\Rightarrow v =$ constant

Thus, change in air pressure does not affect the speed of sound.

Statement-2 is clear from Equation (1).

45.



$A = \sqrt{2^2 + 4^2 + 2\times2\times4\times\cos120^0} \qquad = \sqrt{12} = 2\sqrt{3}$

$\tan\phi = \frac{2\sin120^0}{4+2\cos120^0} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \qquad \phi = \frac{\pi}{6}$

46. $\rho_{mix} = \frac{m_1+m_2}{v_1+v_2} = \qquad \rho_{mix} = \frac{\rho_0 V_h + \rho_0 V_0}{V+4V} = \frac{\rho_H 4V + 16\rho_H^2}{5V}$

$\rho_{mix} = \frac{20v}{5r}\rho_H = 4\rho_H \quad V\alpha\sqrt{\frac{1}{\rho_H}} \to \frac{\vartheta_{mix}}{\vartheta_H} = \sqrt{\frac{\rho H}{4\rho H}} \qquad V_{mix} = \frac{1270}{2} = 635m/s$

47. Fundamental frequency in close/organ pipe $(f)\frac{v}{4\ell}$ $\qquad f_1 = \frac{v}{4\ell_1} \& f_2 = \frac{v}{4\ell_2}$

Beat= $(f_1 - f_2) = \frac{v}{4}\left(\frac{1}{\ell_1} - \frac{1}{\ell_2}\right) \qquad 15 = \frac{v}{4}\left(\frac{1}{1} - \frac{1}{1.2}\right)$

$v = \left(\frac{15\times4\times1.2}{0.2}\right) = 60\times6 = 360m/s$

48. The given equation can be written as

$x = \frac{a}{2}\cos[1.5+50.5]t + \frac{a}{2}\cos[50.5-1.5] \qquad x = \frac{a}{2}\cos[52t] + \frac{a}{2}\cos[49t]$

$Here, 2\pi f_1 \& 2\pi f_2 = 49 \qquad f_1 = \frac{52}{2\pi}, f_2 = \frac{49}{2\pi} \qquad \therefore f_{Beat} = f_1 - f_2 = \frac{3}{2\pi}Hz$

$\therefore T_{Beat} = \frac{1}{f_{Beat}} = \frac{2\pi}{3}\sec$

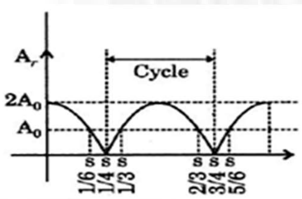
49. $\left[\frac{1}{2}\mu A^2\omega^2 v - \frac{1}{2}\mu\left(\frac{A}{\sqrt{2}}\right)^2\omega^2 v\right]\times t = MS\Delta T$

Where $\mu = 0.1kg/m \qquad A = \frac{1}{5}m, M = \frac{40}{21}kg, \omega = 40rad/\sec \text{ and } S = 4.2kJ/kgK$

50. $y_1 = A\sin\omega_1 t \quad y_2 = A\sin\omega_2 t \quad y_r = 2A\cos$

$y_r = 2A\cos\left\{\frac{\omega_2-\omega_1}{2}t\right\}\left\{\sin\frac{(\omega_2+\omega_1)}{2}t\right\}$

Resultant amplitude $A_r = 2A_0\left|\frac{\cos(\Delta\omega)t}{2}\right|$



$(\Delta\omega)\frac{t}{2} = \frac{\pi}{2} \Rightarrow \frac{1}{4}s \quad (\Delta\omega)\frac{t}{2} = \frac{\pi}{3} \Rightarrow t\frac{1}{6}s$

In one cycle intensity of $\frac{1}{2}s,$ the detector remain idle for $2\left(\frac{1}{4} - \frac{1}{6}\right)s = \frac{1}{6}\sec$

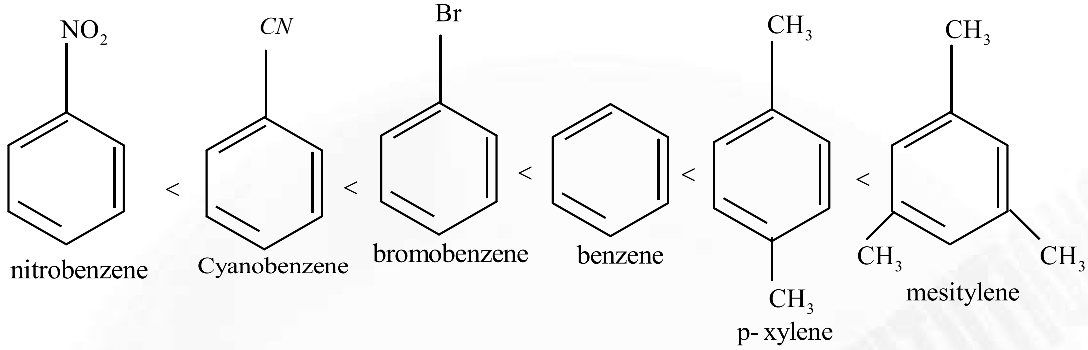
$\therefore in\frac{1}{2}\sec cycle, active\ time\ is\left(\frac{1}{2} - \frac{1}{6}\right) = \frac{1}{3}\sec$

$\therefore In\ 12\ \sec\ \text{interval}, active\ time\ is\ 12\times\frac{\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)} = 8\sec$

CHEMISTRY

51. The groups which direct the incoming group to meta position are called meta directing groups. Some examples of meta directing group are $-NO_2, -CN, -CHO, -COR, -COOH, -COOR, -SO_3H$, etc.

52. $D < F < B < E < A < C$



53. Conceptual

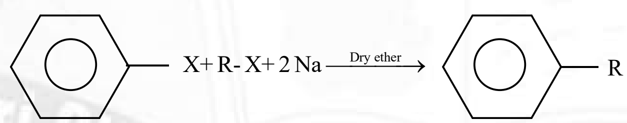
54. Inversion configuration occurs in S_N2 reaction and S_N1 reaction proceeds via carbocation intermediate, it produces racemic mixture. Hence, both statements are true.

55. KCN is predominantly ionic and provides cyanide ions in solution. Although both carbon and nitrogen atoms are in a position to donate electron pairs, the attack takes place mainly through carbon atom and not through nitrogen atom since C-C bond is more stable than C-N bond. However, AgCN is mainly covalent in nature and nitrogen is free to donate electron pair forming isocyanide as the main product.

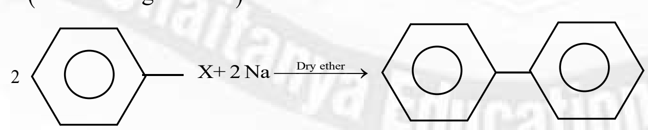
56. Conceptual

57. Rate =k[Nu] [Substrate]
High Concentration of strong nucleophilic reagent with secondary alkyl halides which do not have bulky substituents will follow S_N2 mechanism.
Ethanol is a weak nucleophilic. So, it favour S_N1 mechanism. Ethanol present in excess amount so there is no significance change in rate of reaction. So reaction only depend on concentration of alkyl halide hence favour S_N1 mechanism.

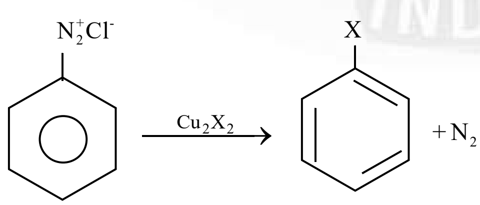
58.

(A) 

(Wurtz- fittig reaction)

(B) 

(Fittig reaction)

(C) 

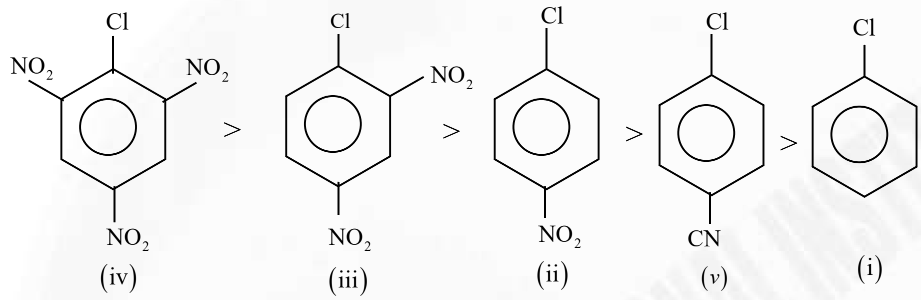
(Sandmeyer reaction)

(D) $CH_3 - Br + AgF \rightarrow CH_3F + AgBr$

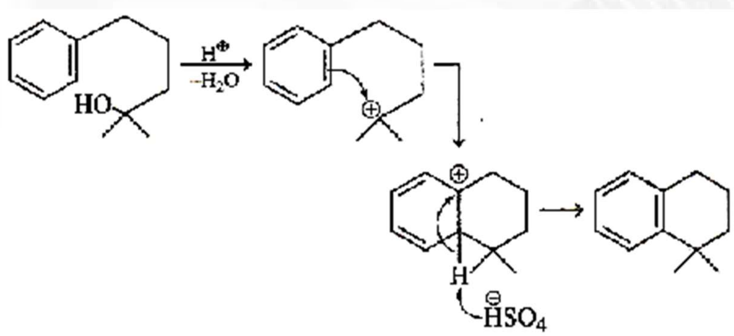
(Swart's Reaction)

59. The density increases with increase in number of carbon atoms, halogen atoms and atomic mass of halogen atoms. Higher the molecular mass of the compound, higher will be its density. Here the correct order of density is $D > C > B > E > A$

60. Electron withdrawing groups such as $-NO_2, -CN$ etc. at ortho and para positions increase the reactivity of haloarene towards nucleophilic substitution reactions. Hence, the correct order is



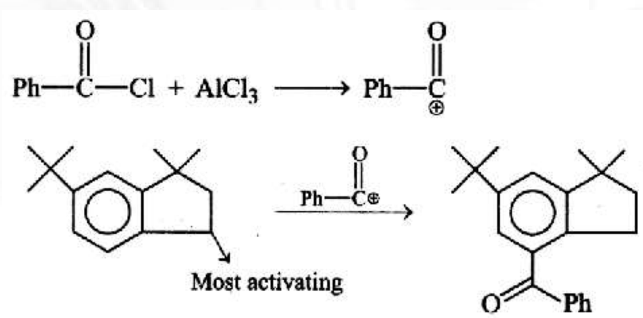
61.



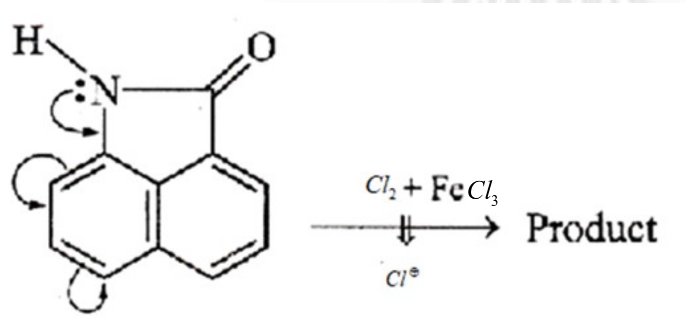
62. Conceptual

63. Conceptual

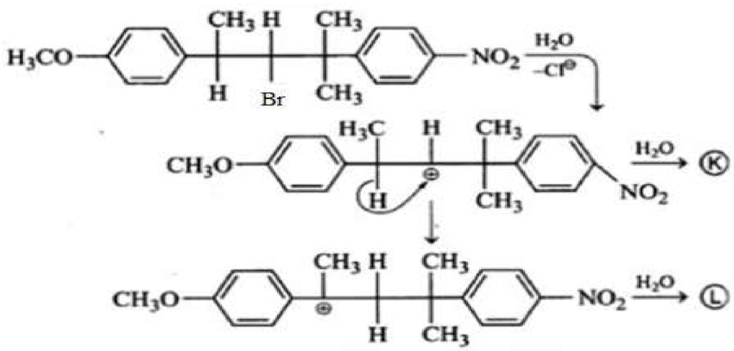
64.



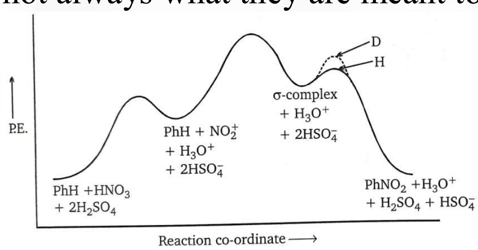
65.



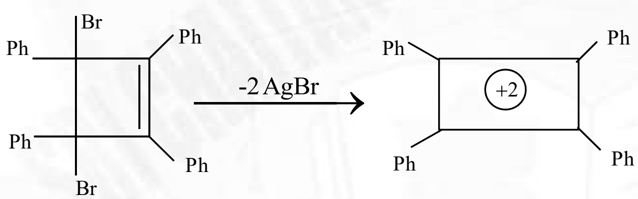
66.



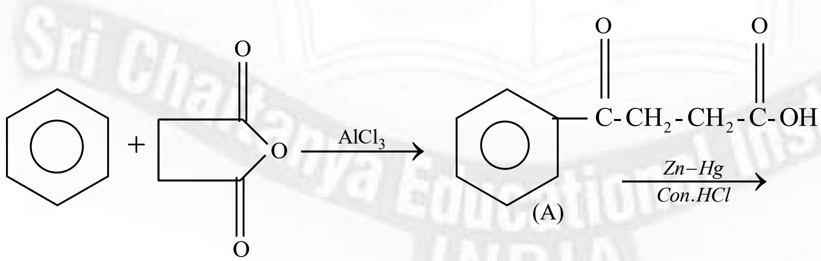
67. We are now in a position to draw possible energy profiles. Since the C-D bond is stronger than the C-H bond, the P.E. of the C_6D_6 reactant system will be lower than that C_6D_6 reactant system. This will also be the case for the corresponding product systems. Because the most important difference between the two systems is the step leasing to fission of the C-H and C-D bonds, Both energy profiles may be combined and only this difference is then shown. The student should carefully compare the energy profile drawn here with the one given in the text. This again illustrates the point that energy profiles are not always what they are meant to represent.



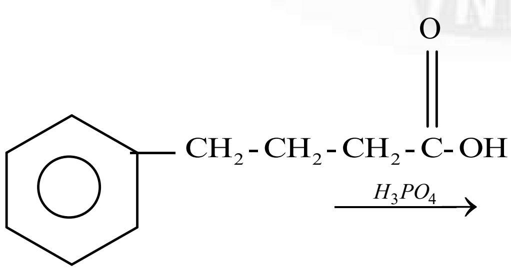
68.



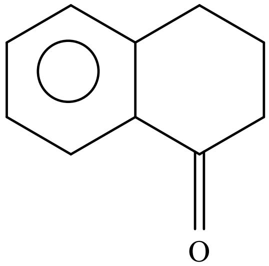
69. Both ring will be aromatic after π-bond breaking



70. (A)

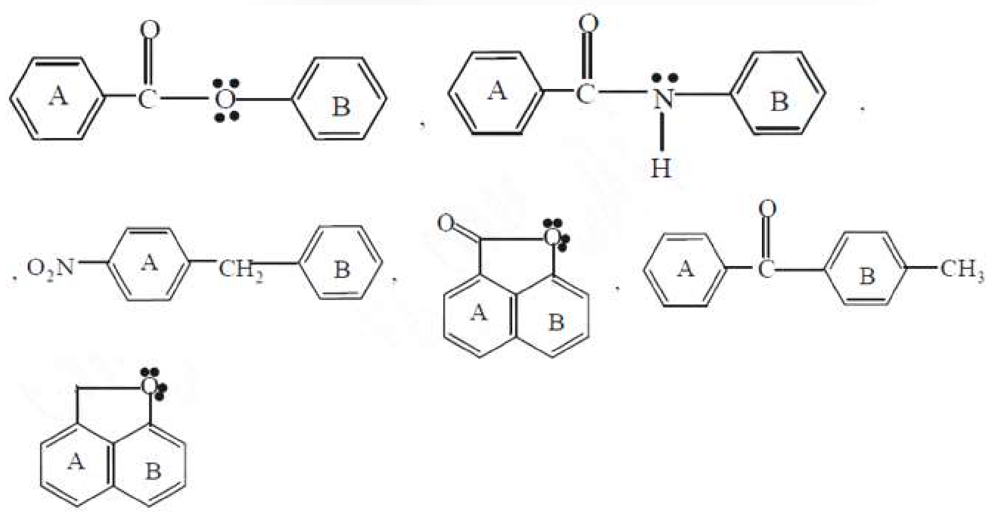


(B)

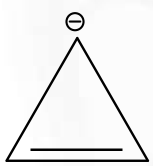


(C) (Intramolecular fridel- craft reaction)

71.



72.



Except this all are aromatic

73. 2mole with each $-\overset{O}{\overset{\|}{C}}-Cl$, once mole with $-OH$ and one mole with $-SH$

74. a,b,c,d,e,f will give benzene

75. A= C_6H_6, $S = C_6Cl_6, J = C_6H_6Cl_6$