Mathematics

1. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to:

A. $\left\{\frac{1}{3}, -1\right\}$ B. $\left\{-\frac{1}{3}, 1\right\}$

C. $\left\{\frac{1}{3}, 1\right\}$ D. $\left\{-\frac{1}{3}, -1\right\}$

Ans. B.

Sol: Given that

$y = f(x) = x^3 - x^2 - 2x$

slope of tangent $\frac{dy}{dx} = f'(x) = 3x^2 - 2x - 2$

This tangent is parallel to line segment joining points (1, f(1)) and (–1, f(–1)) $\therefore m_1 = m_2$

$\Rightarrow \quad 3x^2 - 2x - 2 = \frac{f(-1) - f(1)}{-1 - 1}$

$\Rightarrow \quad 3x^2 - 2x - 2 \frac{(-1 - 1 + 2) - (1 - 1 - 2)}{-2}$

$\Rightarrow \quad 3x^2 - 2x - 2 = -1$

$\Rightarrow \quad 3x^2 - 2x - 1 = 0$

$\Rightarrow \quad (3x + 1)(x - 1) = 0$

$\Rightarrow \quad x = -\frac{1}{3}, 1$

2. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is:

A. $\frac{1}{192}$ B. $\frac{25}{32}$

C. $\frac{7}{32}$ D. $\frac{25}{192}$

Ans. B

Sol:

Given the probability of hitting a target independently by four persons are respectively

$P_1 = \frac{1}{2}, P_2 = \frac{1}{3}, P_3 = \frac{1}{4}$ and $P_4 = \frac{1}{8}$

Let four persons are A, B, C, D. Probability of Hitting target = 1 – (None of four person Hit the target)

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$= 1 - P(\bar{A}).P(\bar{B}).P(\bar{C}).P(\bar{D})$

$= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{7}{8}$

$= \frac{25}{32}$

3. The integral $\int \sec^{2/3} x \ \mathrm{cosec}^{4/3} x \ dx$ is equal to:

(Here C is a constant of integration)

A. $-3 \cot^{-1/3} x + C$ B. $-\frac{3}{4} \tan^{-4/3} x + C$

C. $-3 \tan^{-1/3} x + C$ D. $3 \tan^{-1/3} x + C$

Ans. C.

Sol: Let

$$I = \int \sec^{2/3} x \, \mathrm{cosec}^{4/3} x \, dx$$

$$I = \int \frac{dx}{(\sin x)^{4/3} (\cos x)^{2/3}}$$

$$I = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3} . \cos^2 x}$$

$$I = \int \frac{\sec^2 x \, dx}{(\tan x)^{4/3}}$$

put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$I = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + C$$

$$I = \frac{-3}{(\tan x)^{1/3}} + C$$

4. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1 - x^2}$, is surjective, then A is equal to:

A. R–[–1, 0) B. R–(–1, 0)

C. [0, ∞) D. R–{–1}

Ans. A

Sol: Given that

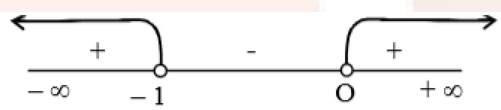
$$f(x) = \frac{x^2}{1 - x^2}$$

$$y = \frac{x^2}{1 - x^2}$$

$$\Rightarrow\ y - x^2 y = x^2$$

$$\Rightarrow\ x^2 = \frac{y}{1 + y}$$

$$\Rightarrow\ x = \sqrt{\frac{y}{1 + y}}\ \Rightarrow\ \frac{y}{1 + y} \ge 0$$

$\Rightarrow$ 

Range of y is R – [–1, 0)

For surjective function codomain = Range

∴ A is R – [–1, 0)

5. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is:

A. $p \leftrightarrow q$ B. $p \wedge q$

C. $\sim p \wedge \sim q$ D. $\sim p \vee \sim q$

Ans. C

Sol:

Given that,

$=\sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge q)$

$= c \vee (\sim p \wedge \sim q)$

So,

$= \sim p \wedge \sim q$

6. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, $-x+y+4=0$, then which one of the following point lies on the curve?

A. (2, –1) B. (–2, 1)

C. (–2, 2) D. (2, –2)

Ans. D

Sol: Given that

$y = x^3 + ax - b$

(1, – 5) lies on curve

$\therefore -5 = 1 + a - b$

$\Longrightarrow a - b = -6 \qquad \ldots(1)$

$$\frac{dy}{dx} = 3x^2 + a$$

Slope of tangent at $(1, -5)$

$$\Rightarrow \frac{dy}{dx} = 3 + a$$

This tangent is perpendicular to $-x + y + 4 = 0$

$\therefore (3 + a)(1) = -1$

$\Longrightarrow a = -4 \qquad \ldots(2)$

By (1) & (2) $a = -4$, $b = 2$

So, eq^n. of curve $y = x^3 - 4x - 2$

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(2, –2) lies on this curve

7. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3k$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to:

A. $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5k\right)$ B. $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5k\right)$

C. $-3\hat{i} + 9\hat{j} + 5k$ D. $3\hat{i} - 9\hat{j} - 5k$

Ans. A

Sol: Given that

$\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3k$

$\vec{\beta}_1$ is parallel to $\vec{\alpha}$

$\therefore \vec{\beta}_1 = \lambda\vec{\alpha}$

$\Rightarrow \vec{\beta}_1 = \lambda\left(3\hat{i} + \hat{j}\right)$

Given that $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$

$\Rightarrow \vec{\beta}_2 = \vec{\beta}_1 - \vec{\beta}$

$\Rightarrow \vec{\beta}_2 = \lambda\left(3\hat{i} + \hat{j}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$\Rightarrow \vec{\beta}_2 = \hat{i}\left(3\lambda - 2\right) + \hat{j}\left(\lambda + 1\right) - 3\hat{k}$

Also given that $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$

$\therefore \vec{\beta}_2 . \vec{\alpha}$

$\Rightarrow 3\left(3\lambda - 2\right) + \left(\lambda + 1\right) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

So, $\vec{\beta}_1 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$ and $\vec{\beta}_2 = \frac{-1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

$$\vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3/2 & 1/2 & 0 \\ 1/2 & 3/2 & -3 \end{vmatrix} = \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$$

8. If one end of a focal chord of the parabola, $y^2 = 6x$ is at (1, 4), then the length of this focal chord is:

A. 24 B. 20

C. 25 D. 22

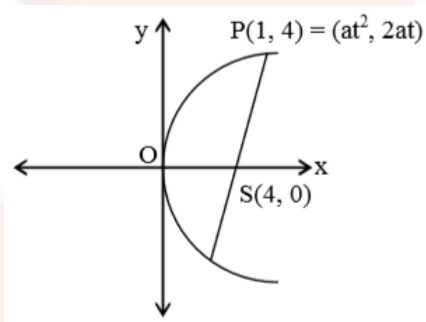
Ans. C

Sol:

Equation of given parabola is $y^2 = 16x$, its focus is (4, 0).

Parabola $y^2 = 16x$

$\{4a = 16 \Rightarrow a = 4\}$



One end $(at^2, 2at) = (1, 4)$

$\Rightarrow 2at = 4$

$\Rightarrow 2(4)t = 4$

$\Rightarrow t = 1/2$

Length of focal chord

$$= a\left(t + \frac{1}{t}\right)^2 = 4\left(\frac{1}{2} + 2\right)^2 = 25$$

9. If a tangent to the circle $x^2 + y^2 = 1$ intersects to coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ

A. $x^2 + y^2 - 2xy = 0$

B. $x^2 + y^2 - 2x^2y^2 = 0$

C. $x^2 + y^2 - 4x^2y^2 = 0$

D. $x^2 + y^2 - 16x^2y^2 = 0$

Ans. C.

Sol:

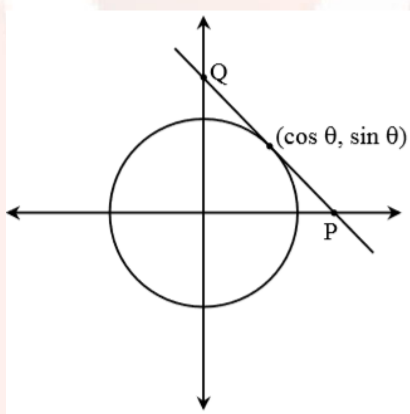
Equation of given circle is $x^2 + y^2 = 1$, then equation of tangent at the point $(\cos\theta, \sin\theta)$ on the given circle is

$x \cos\theta + y \sin\theta = 1$ …(i)

[∵ Equation of tangent at the point $P(\cos\theta, \sin\theta)$ the circle $x^2 + y^2 = r^2$ is $x \cos\theta + y \sin\theta = rI$]

Let the equation of tangent is $x \cos\theta + y \sin\theta = 1$ co-ordinates of P and Q are

$P\left(\frac{1}{\cos\theta}, 0\right)$ and $Q\left(0, \frac{1}{\sin\theta}\right)$



Let mid-point of P and Q is (h, k)

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so, $h = \dfrac{\dfrac{1}{\cos\theta} + 0}{2}$ and $k = \dfrac{0 + \dfrac{1}{\sin\theta}}{2}$

$\Rightarrow \ \cos\theta = \dfrac{1}{2h}$ and $\sin\theta = \dfrac{1}{2k}$

squaring and adding we get

$\dfrac{1}{4h^2} + \dfrac{1}{4k^2} = 1$

$\therefore$ locus $\dfrac{1}{4x^2} + \dfrac{1}{4k^2} = 1$

$\Rightarrow \ x^2 + y^2 - 4x^2y^2 = 0$

10. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then:

A. n = m –8 B. m = n = 78

C. m = n = 68 D. m + n = 68

Ans. B

Sol:

Since there are 8 males and 5 females. Out of these 13 members committee of 11 members is to be formed.

m = no. of ways the committee is formed with at least 6 males.

$= {}^8C_6 \times {}^5C_5 + {}^8C_7 \times {}^5C_4 + {}^8C_8 \times {}^5C_3 = 78$

n = no. of ways the committee is formed with atleast 3 female

$= {}^8C_8 \times {}^5C_3 + {}^8C_7 \times {}^5C_4 + {}^8C_6 \times {}^5C_5$

$= 10 + 40 + 28 = 78$

$\Longrightarrow$ m = n = 78

11. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane *y*– z + 5 = 0, also passes through the point:

A. $\left(\sqrt{2},1,4\right)$ B. $\left(-\sqrt{2},-1,-4\right)$

C. $\left(-\sqrt{2},1,-4\right)$ D. $\left(\sqrt{2},-1,4\right)$

Ans. A

Sol:

Let the equation of plane is

ax + by + cz = d

Since plane (i) passes through the points (0, -1, 0) and (0, 0, 1), then

-b = d and c = d

∴ Equation of plane becomes ax – dy + dz = d

Given that angle b/w them is π/4

$$\cos\theta = \left|\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right|$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{a^2+1+1}\sqrt{0+1+1}}$$

$$\Rightarrow a^2+2=4$$

$$\Rightarrow a=\pm\sqrt{2}$$

$\therefore$ eq^n. of plane $\pm\sqrt{2}\ x-y+z=1$

Now for – ve sign

$$-\sqrt{2}\left(\sqrt{2}\right)-1+4=1$$

$\therefore$ $\left(\sqrt{2},1,4\right)$ satisfy the eq^n. of plane.

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12. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is:

A. $\frac{\sqrt{5}}{2}$ B. $\frac{\sqrt{15}}{2}$

C. $\frac{2}{\sqrt{5}}$ D. $\frac{3}{\sqrt{5}}$

Ans. C

Sol:

Given equation of hyperbola, is

$$\frac{x^2}{24} - \frac{y^2}{b^2} = 1$$

Since, the equation of the normals of slope m to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, are given by

Equation of normal of hyperbola in slope form is

$$y = mx \pm \frac{m\left(a^2 + b^2\right)}{\sqrt{a^2 - b^2 m^2}}$$

$$\therefore\ 7\sqrt{3} = \frac{42m}{\sqrt{24 - 18m^2}}$$

$$\Rightarrow\ 72 - 54m^2 = 36m^2$$

$$\Rightarrow\ 72 - 90m^2$$

$$\Rightarrow\ m^2 = \frac{72}{90} = \frac{4}{5}$$

$$\Rightarrow\ m = \pm\frac{2}{\sqrt{5}}$$

$$m = \frac{2}{\sqrt{5}}$$

13. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to:

A. $2\sqrt{\frac{10}{3}}$ B. $4\sqrt{\frac{5}{3}}$

C. $2\sqrt{6}$ D. $\sqrt{6}$

Ans. C

Sol:

Given observations are -1, 0, 1 and k.

Also, the standard deviation of these four observations = $\sqrt{5}$

$$\text{S.D.} = \sqrt{\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2}$$

$$\text{Now mean } \bar{x} = \frac{-1+0+1+k}{4} = \frac{\cancel{-1}+0\cancel{+1}+k}{4}$$

$$\bar{x} = \frac{k}{4}$$

Given that S.D. = $\sqrt{5}$

$$\Rightarrow \sqrt{5} = \sqrt{\frac{1}{4}(1+0+1+k^2) - \frac{k^2}{16}}$$

$$\Rightarrow 5 = \frac{2+k^2}{4} - \frac{k^2}{16}$$

$$\Rightarrow 5 = \frac{4(2+k^2)-k^2}{16}$$

$$\Rightarrow 5 = \frac{8+4k^2-k^2}{16}$$

$$\Rightarrow 80 = 8 + 4k^2 - k^2$$

$$\Rightarrow 3k^2 = 72$$

$$\Rightarrow k^2 = 24$$

$$\Rightarrow k = \pm 2\sqrt{6}$$

$$k = 2\sqrt{6} \quad (\because k > 0)$$

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14. Let $S=\left\{\theta\in[-2\pi,2\pi]: 2\cos^2\theta+3\sin\theta=0\right\}$. Then the sum of the elements of S is:

A. $\frac{5\pi}{3}$ B. $\frac{13\pi}{6}$

C. π D. 2π

Ans. D

Sol: Given that

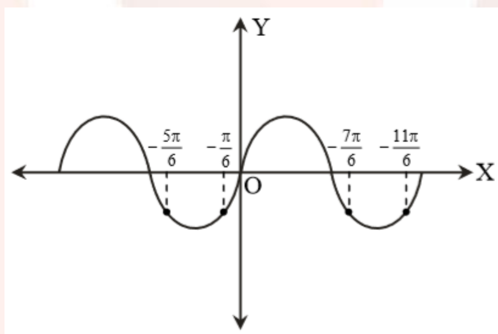
$2\cos^2\theta+3\sin\theta=0$

$\Rightarrow 2\left(1-\sin^2\theta\right)+3\sin\theta=0$

$\Rightarrow 2\sin^2\theta-3\sin\theta-2=0$

$\Rightarrow (2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\frac{1}{2}$



in $\theta\in[-2\pi,2\pi]$

$\Rightarrow \theta=-\frac{5\pi}{6},\frac{-\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$

Sum of all roots $=\frac{-5\pi-\pi+7\pi+11\pi}{6}$

$=2\pi$

15. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2\ (x\neq 0)$ with $y(1)=1$, is:

A. $y=\frac{3}{4}x^2+\frac{1}{4x^2}$ B. $y=\frac{4}{5}x^3+\frac{1}{5x^2}$

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C. $y=\frac{x^3}{5}+\frac{1}{5x^2}$ D. $y=\frac{x^2}{4}+\frac{3}{4x^2}$

Ans. D

Sol: The given differential equation is

$x\frac{dy}{dx}+2y=x^2\ (x\neq 0)$

$\Rightarrow \frac{dy}{dx}+\frac{2y}{x}=x$

$I.F.=e^{\int \frac{2}{x}dx}=e^{\log_e x^2}=x^2$

$\therefore$ Solution is

$\Rightarrow yx^2=\int x^2.x\ dx$

$\Rightarrow yx^2=\frac{x^4}{4}+C$

at $y(1)=1 \Rightarrow 1=\frac{1}{4}+C$

$\Rightarrow C=\frac{3}{4}$

$\Rightarrow yx^2=\frac{x^4}{4}+\frac{3}{4}$

$\Rightarrow y=\frac{x^2}{4}+\frac{3}{4x^2}$

16. The value of $\int_0^{\pi/2}\frac{\sin^3 x}{\sin x+\cos x}dx$ is:

A. $\frac{\pi-2}{4}$ B. $\frac{\pi-1}{4}$

C. $\frac{\pi-2}{8}$ D. $\frac{\pi-1}{2}$

Ans. B

Sol:

The property of definite integral

$\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}\,dx \qquad \ldots(i)$$

$$I = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\,dx$$

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos x + \sin x}\,dx \qquad \ldots(ii)$$

Adding (i) and (ii) we get

$$\Rightarrow\ 2I = \int_0^{\pi/2} \left(\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\right) dx$$

$$\Rightarrow\ 2I = \int_0^{\pi/2} \frac{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)}{(\sin x + \cos x)}\,dx$$

$$\Rightarrow\ 2I = \int_0^{\pi/2} (1 - \sin x \cos x)\,dx$$

$$\Rightarrow\ 2I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sin 2x\right) dx$$

$$\Rightarrow\ 2I = \int_0^{\pi/2} \left(x + \frac{\cos 2x}{4}\right)_0^{\frac{\pi}{2}}$$

$$\Rightarrow\ 2I = \left(\frac{\pi}{2} - \frac{1}{4}\right) - \left(\frac{1}{4}\right) = \frac{\pi}{2} - \frac{1}{2}$$

$$\Rightarrow\ I = \left(\frac{\pi - 1}{4}\right)$$

17. The value of

$\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is:

A. 3/2 B. $\frac{3}{2}(1 + \cos 20^\circ)$

C. $\frac{3}{4} + \cos 20^\circ$ D. 3/4

Ans. D

Sol: Given that

$\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$= \frac{1}{2}\left[2\cos^2 10° - 2\cos 10° \cos 50° + 2\cos^2 50°\right]$

$= \frac{1}{2}\left[(1 + \cos 20°) - (\cos 60° + \cos 40°) + (1 + \cos 100°)\right]$

$= \frac{1}{2}\left[2 - \cos 60° + \cos 20° + (\cos 100° - \cos 40°)\right]$

$= \frac{1}{2}\left[2 - \frac{1}{2} + \cos 20° + 2\sin 70° \sin(-30°)\right]$

$= \frac{1}{2}\left[\frac{3}{2} + \cos 20° - \sin 70°\right]$

$= \frac{1}{2}\left[\frac{3}{2} + \cos 20° - \sin(90° - 20°)\right]$

$= \frac{3}{4}$

18. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to:

A. $y(y^2 - 1)$ B. $y^3 - 1$

C. y^3 D. $y(y^2 - 3)$

Ans. C

Sol: The given equation is $x^2 + x + 1 = 0$

Root of eq^n. $x^2 + x + 1 = 0$ are α and β

$$\alpha, \beta = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$\Rightarrow \ \alpha = \omega, \beta = \omega^2$ (complex cube root of unity)

$$\Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \ \Delta = \begin{vmatrix} y & y & y \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix} \ (\because \ 1 + \omega + \omega^2 = 0)$$

$$\Rightarrow \ \Delta = y \begin{vmatrix} y & y & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$\Delta = y\left(y^2\right)$

$\Delta = y^3$

19. Let $f(x) = 15 - |x - 10|; x \in R$. Then the set of all vales of *x*, at which the function, $g(x) = f(f(x))$ is not differentiable, is:

A. {10} B. {10, 15}

C. {5, 10, 15} D. {5, 10, 15, 20}

Ans. C

Sol:

Given function is f(x) = 15 - |x - 10|, x ∈ R and g(x) = f(fx))

The given equation is

f(x) = 15 – |x – 10|

g(x) = f [f(x)] = 15 – | f(x) – 10 |

= 15 – | 15 – | x – 10 | – 10 |

= 15 – | 5 – | x – 10 ||

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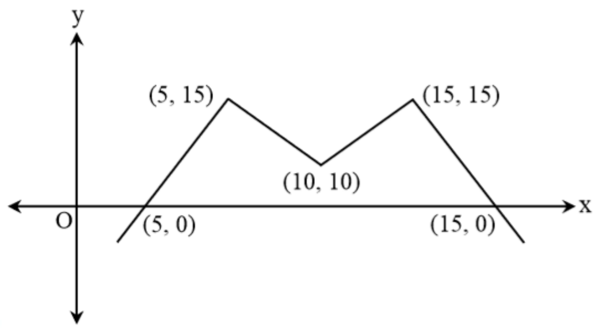


$\therefore$ g(x) is not differentiable at x = 5, 10, 15

20. All the point in the set $S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\}\left(i = \sqrt{-1}\right)$ lie on a:

A. circle whose radius is $\sqrt{2}$.
B. circle whose radius is 1.
C. straight line whose slope is –1.
D. straight line whose slope is 1.

Ans. B

Sol: The given set $S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\}\left(i = \sqrt{-1}\right)$

Let $\frac{\alpha + i}{\alpha - i} = z$

$\Rightarrow \left|\frac{\alpha + i}{\alpha - i}\right| = |z|$

$\Rightarrow |z| = 1$

$\Rightarrow$ Circle of radius $= 1$

21. The area (in sq. units) of the region $A = \left\{(x, y) : x^2 \le y \le x + 2\right\}$ is:

A. $\frac{9}{2}$ B. $\frac{13}{6}$

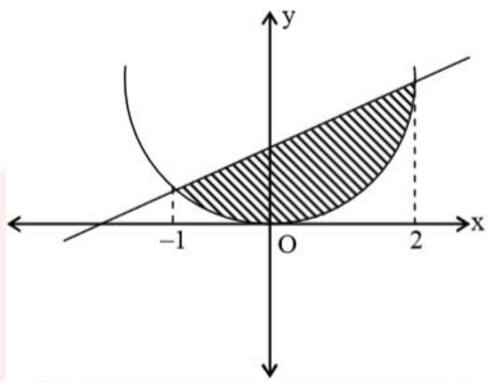
C. $\frac{10}{3}$ D. $\frac{31}{6}$

Ans. A

Sol:

Given region is A = $\{(x, y) : x^2 \le y \le x + 2\}$

Now, the region is shown in the following graph.



For intersecting points

$x^2 \le y \le x+2$

$x^2 = y;\ y = x+2$

$\Rightarrow\ x^2 = x+2$

$\Rightarrow\ x = 2, -1$

So, area $= \int_{-1}^{2} \{(x+2) - x^2\} dx = \frac{9}{2}$

22. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ……….. be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to:

A. (50, 50 + 46A) B. (50, 50 + 45A)

C. (A, 50 + 45A) D. (A, 50 + 46A)

Ans. D

Sol:

The formula of sum of first n terms of AP, ie, $S_n = \frac{n}{2}[2a + (n-1)d]$

Given that the sum of the first n terms

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$S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$T_n = 50n + \left(\frac{n(n-7)}{2}\right)A - 50(n-1) - \left(\frac{(n-1)(n-8)}{2}\right)A$

$= 50 + \frac{A}{2}\left[n^2 - 7n - n^2 + 9n - 8\right]$

= 50 + A(n – 4)

Now, $d = T_n - T_{n-1}$

= 50 + A(n – 4) – 50 – A(n – 5) = A

and $T_{50} = 50 + 46A$

$(d, A_{50}) = (A, 50 + 46A)$

23. If the function *f* defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by $f(x) = \begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \frac{\pi}{4} \\ k, & x = \frac{\pi}{4} \end{cases}$

is continuous, then k is equal to:

A. $\frac{1}{\sqrt{2}}$ B. 2

C. 1 D. $\frac{1}{2}$

Ans. D

Sol:

Given function is $f(x) = \begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \frac{\pi}{4} \\ k, & x = \frac{\pi}{4} \end{cases}$

∵ Function f(x) is continuous, so it is continuous at $x = \frac{\pi}{4}$.

$$\lim_{x \to \frac{\pi}{4}} \left(\frac{\sqrt{2}\cos x - 1}{\cot x - 1} \right) = f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$$

using L-Hospital Rule

$$\Rightarrow \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}(-\sin x)}{-\operatorname{cosec}^2 x} = k$$

$$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3 = k$$

$$\Rightarrow k = \frac{1}{2}$$

24. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies $f(x + y) = f(x)\,f(y)$ for all natural numbers x, y and $f(1) = 2$. Then the natural number 'a' is:

A. 2 B. 4

C. 3 D. 16

Ans. C

Sol:

Given $f(1) = 2$ and $f(x + y) = f(x) \cdot f(y)$

at $x = 1, y = 1 \Rightarrow f(2) = f(1) \cdot f(1) = 2^2$

$x = 2, y = 1 \Rightarrow f(3) = f(2) \cdot f(1) = 2^3$

....

....

$f(n) = 2^n$

Now $\sum_{k=1}^{10} f(a+k) = 16\left(2^{10}-1\right)$

$\Rightarrow \ f(a+1)+f(a+2)+.....+f(a+10) = 16\left(2^{10}-1\right)$

$\Rightarrow \ 2^{a+1}+2^{a+2}+.....+2^{a+10} = 16\left(2^{10}-1\right)$

$\Rightarrow \ 2^{a}+\left[2^{1}+2^{2}+.....+2^{10}\right] = 16\left(2^{10}-1\right)$

$\Rightarrow \ 2^{a}\left[\frac{2\left(2^{10}-1\right)}{2-1}\right] = 16\left(2^{10}-1\right)$

$\Rightarrow \ 2^{a+1} = 16$

$\Rightarrow \ a = 3$

25. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set $S = \{x \in R : f(x) = f(0)\}$ contains exactly:

A. four rational numbers.
B. two irrational and two rational numbers.
C. two irrational and one rational number.
D. four irrational numbers.

Ans. C

Sol:

The non-zero four degree polynomial f(x) has extremum points at x = -1, 0, 1 so we can assume f'(x) = a(x + 1)(x - 0)(x - 1) = ax $(x^2 - 1)$ where, a is non-zero constant.

Four degree polynomial function f(x) have local extreme points at x = –1, 0, 1

$\therefore\ f'(x) = \lambda(x+1)(x-0)(x-1) = \lambda(x^3 - x)$

$\Rightarrow\ f(x) = \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + K$

Now, $f(x) = f(0)$

$\Rightarrow\ \frac{x^4}{4} - \frac{x^2}{2} = 0$

$\Rightarrow\ x = 0, \pm\sqrt{2}$

Two irrational and one rational number.

26. Let p, q$\in$R. If $2-\sqrt{3}$ is a root of the quadratic equation, x^2 + px + q = 0, then:

A. $q^2 + 4p + 14 = 0$
B. $p^2 - 4q + 12 = 0$
C. $q^2 - 4p - 16 = 0$
D. $p^2 - 4q - 12 = 0$

Ans. D

Sol:

If one root of equaion

$x^2 + px + q = 0$ is $2 - \sqrt{3}$

then other root will be $2 + \sqrt{3}$

$\therefore$ equation $x^2 - 4x + 1 = 0$

So, sum of roots = -p = 4 ⇒ p = -4

and product of roots = q = 4 – 3 ⇒ q = 1

Now, from options $p^2 - 4q - 12 = 16 - 4 - 12 = 0$

27. If

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdot\ldots\ldots\ldots\ldots\cdot\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix},$$

then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is:

A. $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ B. $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$

C. $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$ D. $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$

Ans. C

Sol:

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\ldots\ldots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & 1+2+3+\ldots.+n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$$

$$\Rightarrow 1+2+3+\ldots.+(n-1)=78$$

We know that,

$$\Rightarrow \frac{n(n-2)}{2}=78$$

$$\Rightarrow \frac{n^2-2n}{2}=78$$

$$\Rightarrow n^2-2n=78\times 2$$

$$\Rightarrow n^2-2n=156$$

$$\Rightarrow n=13$$

Now, inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ i.e. $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}$

is $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

28. Slope of a line passing through P(2, 3) and intersecting the line, $x + y = 7$ at a distance of 4 units from P, is:

A. $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ B. $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

C. $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ D. $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

Ans. A

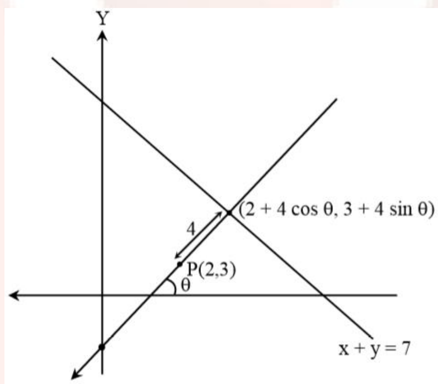
Sol:

The distance of a point (x_1, y_1) from the line ax + y + c = 0 is

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|.$$

Let any point on the line is P(2 ±4 cos θ, 3 ± 4 sin θ)

it also lie on line x + y = 7



$\therefore\ (2 \pm 4\cos\theta) + (3 \pm 4\sin\theta) = 7$

$\Rightarrow\ (\sin\theta + \cos\theta) = \pm\frac{1}{2}$

$\Rightarrow\ (\sin\theta + \cos\theta)^2 = \frac{1}{4}$

$\Rightarrow\ 1 + \sin 2\theta = \frac{1}{4}$

$\Rightarrow\ \sin 2\theta = -\frac{3}{4}$

$\Rightarrow\ \frac{2\tan\theta}{1 + \tan^2\theta} = -\frac{3}{4}$

$\Rightarrow\ 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow\ \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} = \frac{8 - 2\sqrt{7}}{6} = \frac{(1-\sqrt{7})^2}{1-7} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$

29. If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point, P, then the distance of P from the origin is:

A. 9/2 B. $2\sqrt{5}$

C. $\sqrt{5}/2$ D. 7/2

Ans. A

Sol:

Equation of given plane is

x + 2y + 3z = 15

Line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4} = k(\text{say})$ any point on this line P(2k + 1, 3k – 1, 4k + 2)

This point P lies on plane x + 2y + 3z = 15

∴ (2k + 1) + 2(3k – 1) + 3(4k + 2) = 15

⟹ 20k + 5 = 15

$\Longrightarrow 20k = 10$

$\Rightarrow$ 1/2 $\therefore$ $P\left(2, \frac{1}{2}, 4\right)$

Distance of P from origin is

$= \sqrt{4 + \frac{1}{4} + 16} = \frac{9}{2}$

30. If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6 (x > 0)$ is 20×8^7. Then a value of x is:

A. 8^3 B. 8

C. 8^{-2} D. 8^2

Ans. D

Sol:

Given binomial $\left(\frac{2}{x} + x^{\log_8 x}\right)^6 (x > 0)$

Since, general term in the expansion of $(x + a)^n$ is $T_{r+1} = {}^nC_r x^{n-r} a^r$

$\Rightarrow T_4 = 20 \times 8^7$

$\Rightarrow {}^6C_3 \left(\frac{2}{x}\right)^3 \left(x^{\log_g x}\right)^3 = 20 \times 8^7$

$\Rightarrow \frac{160}{x^3} x^{3\log_8 x} = 20 \times 8^7$

$\Rightarrow x^{3\log_8 x - 3} = 8^6$

$\Rightarrow x^{\log_2 x - 3} = 8^6 = 2^{18}$

$\Rightarrow \log_2\left(x^{\log_2 x - 3}\right) = \log_2 2^{18}$

$\Rightarrow (\log_2 x - 3)(\log_2 x) = 18$

Let $\log_2 x = t$

$\Rightarrow t^2 - 3t - 18 = 0$

$\Rightarrow t = 6, -3$

$\Rightarrow \log_2 x = 6 \quad \Rightarrow x = 2^6 = 8^2$

$\Rightarrow \log_2 x = -3 \quad \Rightarrow x = 2^{-3} = 1/8$

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