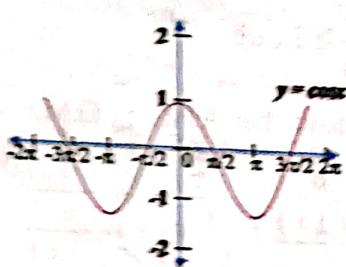
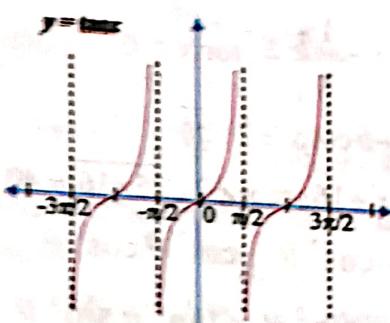
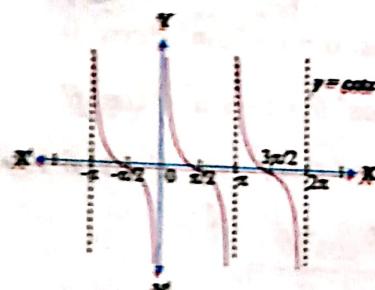


Graph of $\cos x$:Graph of $\tan x$:Graph of $\cot x$:
EXERCISE-I
C.R.T.Q & SPO **LEVEL-I**
**TRIGONOMETRIC RATIOS AND
VALUES, TRIGONOMETRIC
RELATIONS, MEASUREMENT OF
ANGLES**
C.R.T.Q

Class Room Teaching Questions

1. $-13\pi/6$ radians =
 1) -390° 2) -490° 3) -410° 4) -30°

2. $\cot(1358^\circ) + \tan(3608^\circ) =$
 1) -1 2) 0 3) 1 4) 2
3. $\frac{\sin(-660^\circ) \tan(1050^\circ) \sec(-420^\circ)}{\cos(225^\circ) \operatorname{cosec}(315^\circ) \cos(510^\circ)} =$
 1) $\frac{\sqrt{3}}{4}$ 2) $\frac{\sqrt{3}}{2}$ 3) $\frac{2}{\sqrt{3}}$ 4) $\frac{4}{\sqrt{3}}$
4. $\sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ =$
 1) 0 2) 2 3) 1 4) -1
5. $\log \tan 17^\circ + \log \tan 37^\circ + \log \tan 53^\circ + \log \tan 73^\circ =$
 1) 0 2) 1 3) 2 4) 3
6. $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ =$
 1) -1 2) 0 3) 1 4) 2
7. $\sec A + \tan A = 3 \Rightarrow \sec A =$
 1) $\frac{10}{3}$ 2) ~~$\frac{5}{3}$~~ 3) $\frac{2}{3}$ 4) $\frac{4}{3}$
8. $\sec \theta - \tan \theta = 3 \Rightarrow \theta$ lies in the quadrant
 1) I 2) II 3) III 4) IV
9. $\sin 10^\circ + \sin 20^\circ + \sin 30^\circ + \dots + \sin 360^\circ =$
 1) 0 2) 1 3) -1 4) 2
10. $3[\sin x - \cos x]^4 + 6[\sin x + \cos x]^2 + 4[\sin^6 x + \cos^6 x] =$
 1) 3 2) 6 3) 4 4) 13
11. $\frac{\sin^2 \alpha}{1 + \cot^2 \alpha} + \frac{\tan^2 \alpha}{(1 + \tan^2 \alpha)^2} + \cos^2 \alpha =$
 1) -1 2) 0 3) 1 4) 2
12. $1 - \frac{\sin^2 x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} - \frac{\sin x}{1 - \cos x} =$
 1) ~~$\sin x$~~ 2) ~~$\cos x$~~ 3) $\operatorname{cosec} x$ 4) $\operatorname{sec} x$
13. If A, B, C are the angles of a triangle ABC then
 $\cos\left(\frac{3A+2B+C}{2}\right) + \cos\left(\frac{A-C}{2}\right) =$
 1) 0 2) 1 3) $\cos A$ 4) $\cos C$

14. $\frac{\cot \theta + \operatorname{cosec} \theta - 1}{\cot \theta - \operatorname{cosec} \theta + 1} =$

1) $\frac{\sin \theta}{1 + \cos \theta}$

2) $\frac{\sin \theta}{1 - \cos \theta}$

3) $\frac{1 - \cos \theta}{\sin \theta}$

4) $\frac{\cos \theta}{1 - \sin \theta}$

15. $\sin^2(51^\circ - x) + \sin^2(39^\circ + x) =$

1) -1 2) 0 3) 1 4) 2

$$\sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} =$$

16. $\frac{\cos^2 \frac{\pi}{18} + \cos^2 \frac{\pi}{9} + \cos^2 \frac{7\pi}{18} + \cos^2 \frac{4\pi}{9}}{\sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9}} =$

1) 1 2) 2 3) 3 4) 4

17. If ABCD is a cyclic quadrilateral, then

$$\cos(180^\circ + A) + \cos(180^\circ - B) +$$

$$\cos(180^\circ - C) - \sin(90^\circ - D) =$$

1) -1 2) 0 3) 1 4) 2

18. $\left[\sin\left(\frac{\pi}{2} - x\right) + \sin(\pi - x) \right]^2 +$

$$\left[\cos\left(\frac{3\pi}{2} - x\right) + \cos(2\pi - x) \right]^2 =$$

1) 0 2) 2 3) 4 4) 8

19. $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3 \Rightarrow$

$$\cos \theta_1 + \cos \theta_2 + \cos \theta_3 =$$

1) 0 2) 1 3) 2 4) 3

20. $8 \sin^2 x + 3 \cos^2 x = 5 \Rightarrow \cot x =$

1) $\pm \frac{1}{\sqrt{2}}$ 2) $\pm \frac{1}{\sqrt{3}}$ 3) $\pm \sqrt{\frac{3}{2}}$ 4) $\pm \sqrt{\frac{2}{3}}$

21. $\sin x + \sin^2 x = 1 \Rightarrow \cos^2 x + \cos^4 x =$

1) 0 2) 1 3) 2 4) -1

22. If $\cos(x - y) = -1$, then $\cos x + \cos y =$

1) 0 2) 1 3) 2 4) -1

23. If $\sin x + \operatorname{cosec} x = 2$ then $\sin^8 x + \operatorname{cosec}^8 x =$

1) 1 2) 2 3) 3 4) 4

24. If $\sin x + \cos x = a$ then $|\sin x - \cos x| =$

1) $\sqrt{1-a^2}$ 2) $\sqrt{a^2-1}$

3) $\sqrt{2-a^2}$ 4) $\sqrt{a^2-2}$

25. $(1 + \sin A)(1 + \sin B)(1 + \sin C) = (1 - \sin A)$

$$(1 - \sin B)(1 - \sin C) = k \Rightarrow k =$$

1) $+\sin A \sin B \sin C$

2) $\pm \cos A \cos B \cos C$

3) $\pm \sec A \sec B \sec C$

4) $\pm \operatorname{cosec} A \operatorname{cosec} B \operatorname{cosec} C$

26. $\operatorname{cosec}^2 A \cot^2 A - \sec^2 A \tan^2 A -$

$$(\cot^2 A - \tan^2 A)(\sec^2 A \operatorname{cosec}^2 A - 1) =$$

1) 0 2) 1 3) -1 4) 2

27. $\tan^2 \theta + \sec \theta = 5 \Rightarrow \sec \theta =$

1) 3 2) 2 3) 1 4) -1

28. $\log \sin 1^\circ \cdot \log \sin 2^\circ \dots \log \sin 179^\circ$

1) 1 2) 0 3) -1 4) 2

29. If $\tan(\alpha + \beta) = \sqrt{3}$, $\tan(\alpha - \beta) = 1$ then

$$\tan 6\beta =$$

1) -1 2) 0 3) 1 4) 2

30. If $x = r \cos \theta \cos \phi$, $y = r \cos \theta \sin \phi$,

$$z = r \sin \theta \text{ then } x^2 + y^2 + z^2 =$$

1) r^2 2) y^2 3) x^2 4) z^2

31. $\sin^2 A \cos^2 B + \cos^2 A \sin^2 B$

$$+ \sin^2 A \sin^2 B + \cos^2 A \cos^2 B =$$

1) -1 2) 0 3) 1 4) 2

32. If $2 \sin^2 A - \cot^2 B = 1$ then $\sin A \sin B =$

1) $-\frac{1}{\sqrt{2}}$ 2) $\pm \frac{1}{\sqrt{2}}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{-1}{2\sqrt{2}}$

33. The value of

$$\sin^6 \theta + \cos^6 \theta + 3 \sin^2 \theta \cos^2 \theta$$

1) 0 2) 1 3) 2 4) 3

34. If $x \tan^2 120^\circ + 4 \cos^2 150^\circ = 9$ then $x =$

1) 3 2) 1 3) 2 4) 4

FOCUS TRACK

35. In ΔABC , right angled at C, then
 $\tan A + \tan B =$

- 1) $\frac{a^2}{bc}$ 2) $\frac{b^2}{ac}$ 3) $\frac{c^2}{ab}$ 4) $\frac{ab}{c}$

36. If $a = \sin 170^\circ + \cos 170^\circ$ then

- 1) $a > 0$ 2) $a < 0$ 3) $a = 0$ 4) $a = 1$

$$37. \tan \frac{\pi}{24} \tan \frac{3\pi}{24} \tan \frac{5\pi}{24} \tan \frac{7\pi}{24} \tan \frac{9\pi}{24} \tan \frac{11\pi}{24} \tan \frac{13\pi}{24} \tan \frac{15\pi}{24} =$$

- 1) 1 2) -1 3) $\sqrt{3}$ 4) $-\sqrt{3}$

38. $x = a \sec^3 \theta \tan \theta$, $y = b \tan^3 \theta \sec \theta$

then $\sin^2 \theta =$

- 1) $\frac{x}{a} - \frac{y}{b}$ 2) $\frac{x}{a} + \frac{y}{b}$ 3) $\frac{xy}{ab}$ 4) $\frac{ay}{bx}$

$$39. \text{If } \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1, \frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta = 1$$

then $\frac{x^2}{a^2} + \frac{y^2}{b^2} =$

- 1) 1 2) -1 3) 2 4) 3

40. In a ΔABC , if $\cot A \cot B \cot C > 0$ then the triangle is

- 1) acute angled 2) right angled
 3) obtuse angled 4) can't be decided

41. If α, β are complementary angles and

$\sin \alpha = \frac{3}{5}$, then $\sin \alpha \cos \beta - \cos \alpha \sin \beta =$

- 1) $\frac{7}{25}$ 2) $-\frac{7}{25}$ 3) $\frac{25}{7}$ 4) $-\frac{25}{7}$

$$42. 4a^2 \sin^2 \left(\frac{3\pi}{4} \right) - 3[\alpha \tan 225^\circ]^2 + [2a \cos 315^\circ]^2 =$$

- 1) 0 2) a 3) $\sqrt{2}a$ 4) a^2

43. $-11\frac{\pi}{3}$ radians =

- 1) -390° 2) -620° 3) -610° 4) -660°

S.P.Q.

Student Practice Questions

44. $\sin 4530^\circ =$

- 1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) $\frac{\sqrt{3}}{2}$ 4) $-\frac{\sqrt{3}}{2}$

45. $\frac{\sin 150^\circ - 5 \cos 300^\circ + 7 \tan 225^\circ}{\tan 135^\circ + 3 \sin 210^\circ} =$

- 1) 10 2) -5 3) -2 4) -3

46. $\cos 23^\circ \csc 67^\circ - \sin 23^\circ \sec 67^\circ =$

- 1) 0 2) 2 3) 1 4) -1

47. $\log \tan 18^\circ + \log \tan 36^\circ + \log \tan 54^\circ + \log \tan 72^\circ =$

- 1) $\log 4$ 2) $\log 3$ 3) $\log 2$ 4) 0

48. $\cos 5^\circ + \cos 24^\circ + \cos 175^\circ + \cos 204^\circ + \cos 300^\circ =$

- 1) $1/2$ 2) 1 3) $3/2$ 4) 2

49. $\cos \csc A + \cot A = \frac{2}{3} \Rightarrow \cos A =$

- 1) $\frac{5}{13}$ 2) $\frac{13}{5}$ 3) $-\frac{5}{13}$ 4) $-\frac{13}{5}$

50. If $\csc \theta - \cot \theta = 5$ then θ lies in the quadrant

- 1) I 2) II 3) III 4) IV

51. $\tan 20^\circ + \tan 40^\circ + \tan 60^\circ + \dots + \tan 180^\circ =$

- 1) 0 2) 1 3) 2 4) 3

52. $(\sin \alpha + \csc \alpha)^2 + (\sec \alpha + \cos \alpha)^2 =$

$k + \tan^2 \alpha + \cot^2 \alpha \Rightarrow k =$

- 1) 9 2) 7 3) 5 4) 3

53. $\frac{1}{(1+\cot^2 \alpha)^2} + \frac{\tan^2 \alpha}{(1+\tan^2 \alpha)^2} + \frac{1}{1+\tan^2 \alpha} =$

- 1) -1 2) 0 3) 1 4) 2

54. $\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = k \text{ then } k =$

- 1) 0 2) 1 3) 2 4) -1

55. In ΔABC , $\tan(A-B-C) =$

- 1) $\sin 2A$ 2) 1 3) $\tan 2A$ 4) 0

56. $\frac{1 + \cot \alpha + \csc \alpha}{1 - \cot \alpha + \cosec \alpha} =$

- 1) $\frac{\sin \alpha}{1 + \cos \alpha}$

- 2) $\frac{\sin \alpha}{1 - \cos \alpha}$

- 3) $\frac{1 + \cos \alpha}{\sin \alpha}$

- 4) $\frac{1 - \sin \alpha}{\cos \alpha}$

57. $\cos^2(125^\circ - x) - \cos^2(55^\circ + x) =$
 1) -1 ~~2) 0~~ 3) 1 4) 2

58. $\sin^2 \frac{\pi}{18} + \sin^2 \frac{2\pi}{18} + \sin^2 \frac{4\pi}{18} + \sin^2 \frac{8\pi}{18}$
 $+ \sin^2 \frac{7\pi}{18} + \sin^2 \frac{5\pi}{18} =$
 1) 1 2) 2 ~~3) 3~~ 4) 4

59. If A B C D is a quadrilateral then

$$\tan\left(\frac{A+B}{4}\right) =$$

1) $\cos\left(\frac{C-D}{4}\right)$ 2) $\cot\left(\frac{C-D}{4}\right)$
 3) $\cos\left(\frac{C+D}{4}\right)$ ~~4) cot~~ $\left(\frac{C+D}{4}\right)$

60. $\left[\cos\left(\frac{\pi}{2}-x\right)+\cos(\pi-x)\right]^2 + \left[\sin\left(\frac{3\pi}{2}-x\right)+\sin(2\pi-x)\right]^2 =$
 1) 0 ~~2) 2~~ 3) 1 4) 4

61. If $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 + \cos\theta_5 = 5$
 then
 $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 + \sin\theta_5 =$
 1) 3 2) 2 3) 1 ~~4) 0~~

62. $9\cos^2 x + 4\sin^2 x = 5 \Rightarrow \tan x =$
 1) ± 1 ~~2) ± 2~~ 3) ± 3 4) ± 4

63. $\cos x + \cos^2 x = 1 \Rightarrow \sin^8 x + 2\sin^6 x + \sin^4 x =$
 1) 0 ~~2) 1~~ 3) 2 4) -1

64. If $\cot(\alpha + \beta) = 0$ then $\sin(\alpha + 2\beta) =$
~~1) sin~~ α 2) $\cos\alpha$ 3) $\sin\beta$ ~~4) cos~~ β

65. If $\tan\alpha + \cot\alpha = 2$ then
 $\sqrt{\tan\alpha} + \sqrt{\cot\alpha} =$
 1) $\sqrt{2}$ 2) $2\sqrt{2}$ ~~3) 2~~ 4) $4\sqrt{2}$

66. $5\sin x + 4\cos x = 3 \Rightarrow 4\sin x - 5\cos x =$
 1) 4 ~~2) $4\sqrt{2}$~~ 3) $3\sqrt{2}$ 4) $\sqrt{2}$

67. If $k = (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) =$
 $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
 then $k =$
 1) 0 ~~2) ± 1~~ 3) ± 3 4) ± 4

68. The value of $\cot^2 \alpha \left(\frac{\sec \alpha - 1}{1 + \sin \alpha} \right) + \sec^2 \alpha \left(\frac{\sin \alpha - 1}{1 + \sec \alpha} \right)$ is
~~1) 0~~ 2) 1 3) 2 4) -2

69. $\sec^2 \theta + \tan \theta = 13 \Rightarrow \tan \theta =$
 1) 4 ~~2) 3~~ 3) -3 4) 5

70. $\log(\tan 1^\circ) \log(\tan 2^\circ) \log(\tan 3^\circ) \dots \log(\tan 45^\circ) =$
~~1) 0~~ 2) 1 3) -1 4) $\frac{1}{2}$

71. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha - \beta) = \frac{1}{2}$ then
 $\tan(\alpha + 2\beta) \tan(2\alpha + \beta) =$
~~1) 1~~ 2) -1 3) 0 4) 2

72. If $x = r \cos \alpha \cos \beta \cos \gamma$;
 $y = r \cos \alpha \cos \beta \sin \gamma$;
 $z = r \sin \alpha \cos \beta$; $\mu = r \sin \beta$ then
 $x^2 + y^2 + z^2 + \mu^2 =$
 1) r 2) $2r$ ~~3) r^2~~ 4) $4r^2$

73. $\sec^2 A \sec^2 B - \sec^2 A \tan^2 B - \tan^2 A \sec^2 B + \tan^2 A \tan^2 B =$
~~1) 1~~ 2) 0 3) -1 4) 2

74. $2\cos^2 B - 1 = \tan^2 A$, then $\cos A \cos B =$
 1) $\pm \frac{1}{2}$ 2) $\pm \frac{1}{3}$ ~~3) $\pm \frac{1}{\sqrt{2}}$~~ 4) ± 1

75. $\sin^4 \theta + 2\sin^2 \theta \left(1 - \frac{1}{\csc^2 \theta}\right) + \cos^4 \theta =$
~~1) 1~~ 2) 0 3) $\frac{1}{2}$ 4) -1

76. If $x \cot 120^\circ + 4 \sin^2 150^\circ = 3$, then $x =$
 1) 2 2) 7 ~~3) 6~~ 4) 5

77. In a right angled triangle ABC, $\angle C = 90^\circ$, then $\cos^2 A + \cos^2 B =$

- 1) 2 ~~2) 1~~ 3) $\frac{1}{2}$ 4) $\frac{3}{4}$

78. If $x = \sin 130^\circ + \cos 130^\circ$ then

- 1) $x < 0$ 2) $x = 0$ ~~3) $x > 0$~~ 4) $x \geq 0$

$$79. \cot \frac{\pi}{20} \cdot \cot \frac{3\pi}{20} \cdot \cot \frac{5\pi}{20} \cdot \cot \frac{7\pi}{20} \cdot \cot \frac{9\pi}{20} \cdot \cot \frac{15\pi}{20} =$$

- 1) 1 ~~2) -1~~ 3) $\sqrt{3}$ 4) $-\sqrt{3}$

80. If $x = a \sec^n \theta$; $y = b \tan^n \theta$ then

$$\left(\frac{x}{a}\right)^{\frac{2}{n}} - \left(\frac{y}{b}\right)^{\frac{2}{n}} =$$

- 1) 0 2) -1 ~~3) 1~~ 4) 2

81. If $x = h + p \sec \alpha$, $y = k + q \cosec \alpha$ then

$$\left(\frac{p}{x-h}\right)^2 + \left(\frac{q}{y-k}\right)^2 =$$

- ~~1) 1~~ 2) -1 3) 0 4) 1/2

82. If A, B, C are angles of a triangle such that A is obtuse then

- 1) $\tan A \tan B > 1$ ~~2) $\tan B \tan C < 1$~~
 3) $\tan C \tan A > 1$ 4) $\tan A \tan B \tan C > 1$

83. If α, β are complementary angles,

$$\sin \alpha = \frac{3}{5}, \text{ then}$$

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta =$$

- 1) 1 2) 2 3) $\frac{4}{5}$ ~~4) 0~~

$$84. a^2 \cos^2 \frac{2\pi}{3} - 4a^2 \tan^2 \frac{3\pi}{4} + 2a^2 \sin^2 \frac{2\pi}{3} =$$

- 1) a^2 2) 0 3) $\frac{3a^2}{4}$ ~~4) $\frac{-9a^2}{4}$~~

85. If $\tan \theta = \frac{p}{q}$ then $\frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta} =$

- 1) $\frac{2p}{p^2 + q^2}$ 2) $\frac{2pq}{p^2 + q^2}$
 3) $\frac{p^2 - q^2}{p^2 + q^2}$ 4) $\frac{q^2 - p^2}{p^2 + q^2}$

86. If $a \sec \theta + b \tan \theta = c$ then

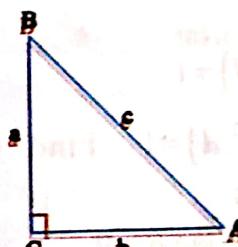
- $$(a \tan \theta + b \sec \theta)^2 =$$
- 1) $a^2 + b^2 + c^2$ ~~2) $-a^2 + b^2 + c^2$~~
 3) $a^2 - b^2 + c^2$ 4) $a^2 + b^2 - c^2$

KEY

01) 1	02) 2	03) 3	04) 2	05) 1
06) 2	07) 2	08) 4	09) 1	10) 4
11) 3	12) 2	13) 1	14) 2	15) 3
16) 1	17) 2	18) 2	19) 1	20) 3
21) 2	22) 1	23) 2	24) 3	25) 2
26) 1	27) 2	28) 2	29) 3	30) 1
31) 3	32) 2	33) 2	34) 3	35) 3
36) 2	37) 4	38) 4	39) 3	40) 1
41) 2	42) 4	43) 4	44) 2	45) 3
46) 1	47) 4	48) 1	49) 3	50) 2
51) 1	52) 2	53) 3	54) 3	55) 3
56) 3	57) 2	58) 3	59) 4	60) 2
61) 4	62) 2	63) 2	64) 4	65) 3
66) 2	67) 2	68) 1	69) 2	70) 1
71) 1	72) 3	73) 1	74) 3	75) 1
76) 3	77) 2	78) 3	79) 2	80) 3
81) 1	82) 2	83) 4	84) 4	85) 3
86) 2				


Hints & Solutions

1. $-13 \times \frac{\pi}{6} \times \frac{180}{\pi} = -390^\circ$
2. $1358^\circ = 8(180^\circ) - 82^\circ$,
 $3608^\circ = 20(180^\circ) + 8^\circ$
3. Convert the corresponding values.
4. $\sin 48^\circ \csc 48^\circ + \cos 48^\circ \sec 48^\circ = 1+1=2$
5. $\because A+B=90^\circ \Rightarrow \tan A \tan B=1$
6. $\cos(204^\circ) = \cos(180^\circ + 24^\circ)$
 $\cos(125^\circ) = \cos(180^\circ - 55^\circ)$
7. Use $(\sec^2 A - \tan^2 A) = 1$
8. Use $(\sec^2 A - \tan^2 A) = 1$
9. $A+B=360^\circ \Rightarrow \sin A + \sin B = 0$
10. Put $x=90^\circ$
11. Put $\alpha=45^\circ$
12. Let $x=90^\circ$
13. $A=B=C=60^\circ$
14. $1 = \cosec^2 \theta - \cot^2 \theta$
15. $A+B=90^\circ \Rightarrow \sin^2 A + \sin^2 B = 1$
16. If $A+B=90^\circ$ then $(\cos^2 A + \cos^2 B) = 1$
and $(\sin^2 A + \sin^2 B) = 1$
17. $A+C=180^\circ$, $B+D=180^\circ$
18. Simplify
19. Put $\theta_1=\theta_2=\theta_3=90^\circ$
20. Divide the given equation by $\sin^2 x$ both sides
21. $\sin x = (1 - \sin^2 x) \Rightarrow \sin x = \cos^2 x$
22. $x-y=\pi$

23. Put $x=90^\circ$
24. $\alpha = \frac{\pi}{4}$
25. $k^2 = (1+\sin A)(1+\sin B)(1+\sin C)(1-\sin A)(1-\sin B)(1-\sin C)$
 $K^2 = (1-\sin^2 A)(1-\sin^2 B)(1-\sin^2 C)$
 $K = \pm \cos A \cos B \cos C$
26. Put $A=45^\circ$
27. $\tan^2 \theta + \sec \theta = 5$,
 $\Rightarrow (\sec^2 \theta - 1) + \sec \theta = 5$,
 $\Rightarrow \sec^2 \theta + \sec \theta - 6 = 0$, $\sec \theta = 2$ or -3
28. $\log(\sin 90^\circ) = 0$
29. Let $(\alpha+\beta)=60^\circ$
 $(\alpha-\beta)=45^\circ \Rightarrow 2\beta=15^\circ \Rightarrow 6\beta=45^\circ$
30. Square and add
31. $\sin^2 A(\cos^2 B + \sin^2 B) + \cos^2 A(\sin^2 B + \cos^2 B)$ or
Put $A=B=0^\circ$
32. Put $\cot B = \frac{\cos B}{\sin B}$
33. use $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$
34. Substitute the corresponding values.
35. 
- $\angle C = 90^\circ$
 $\therefore \tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$
36. $a = \sin 170^\circ + \cos 170^\circ$
 $= \sin 10^\circ - \cos 10^\circ < 0$
37. $A+B=90^\circ \Rightarrow \tan A \tan B = 1$

38. $\frac{y}{x} = \frac{b \tan^3 \theta \sec \theta}{a \sec^3 \theta \tan \theta} = \frac{b}{a} \sin^2 \theta$

39. by squaring and adding $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

40. $\cot A \cot B \cot C > 0$

$\Rightarrow \cot A, \cot B, \cot C$ are positive

$\Rightarrow \Delta ABC$ is acute angled

41. $\sin \alpha = \frac{3}{5}, \cos \beta = \frac{3}{5},$

$\cos \alpha = \frac{4}{5}, \sin \beta = \frac{4}{5}$

42. Given $= 4a^2 \left(\frac{1}{\sqrt{2}} \right)^2 - 3[a \times 1]^2 + \left[2a \times \frac{1}{\sqrt{2}} \right]^2$

43. $-11 \frac{\pi}{3} \times \frac{180}{\pi} = -660^\circ$

44. $\sin(4530^\circ) = \sin((12)360^\circ + 210^\circ)$

$= \sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$

45. Convert the corresponding values.

46. $\cos 23^\circ \cosec(90^\circ - 23^\circ) - \sin 23^\circ \sec(90^\circ - 23^\circ)$
 $= 1 - 1 = 0$

47. $\log(\tan 18^\circ \tan 36^\circ \tan 54^\circ \tan 72^\circ) = \log 1 = 0$
 $[\because \text{If } A+B=90^\circ \Rightarrow \tan A \tan B = 1]$

48. $\alpha + \beta = 180^\circ$

$\Rightarrow \cos \alpha + \cos \beta = 0$

49. Use $(\cosec^2 \theta - \cot^2 \theta) = 1$

50. Use $(\cosec^2 A - \cot^2 A) = 1$. Find the quadrant in which A lies

51. $A + B = 180^\circ \Rightarrow \tan A + \tan B = 0$

52. Square and simplify

53. Put $\alpha = 45^\circ$

54. Put $A = 0^\circ$

55. $B + C = 180^\circ - A \Rightarrow \tan(A + A - 180^\circ) = \tan 2A$

56. Use $1 = (\cosec^2 \alpha - \cot^2 \alpha)$

57. $(125^\circ - x) + (55^\circ + x) = 180^\circ$

58. If $(A + B) = \frac{\pi}{2}$ then $\sin^2 A + \sin^2 B = 1$

59. $(A + B + C + D) = 360^\circ \Rightarrow \frac{A+B}{4} = \frac{\pi}{2} = \frac{C+D}{4}$

60. $(\sin x - \cos x)^2 + (-\cos x - \sin x)^2 = 2$

61. Put $\theta_1 = \theta_2 = \theta_3 = 0^\circ$

62. Divide the given equation by $\cos^2 x$ both sides

63. $\cos x = 1 - \cos^2 x \Rightarrow \cos x = \sin^2 x$

64. $\cot(\alpha + \beta) = 0 \Rightarrow \alpha + \beta = \frac{\pi}{2}$

$\Rightarrow \sin(\alpha + 2\beta) = \sin(90^\circ + \beta) = \cos \beta$

65. Put $\alpha = \frac{\pi}{4}$

66. $a \sin x + b \cos x = c$

$\Rightarrow b \sin x - a \cos x = \pm \sqrt{a^2 + b^2 - c^2}$

67. $K^2 = (1 + \sin A)(1 + \sin B)(1 + \sin C)$

$(1 - \sin A)(1 - \sin B)(1 - \sin C)$

$K^2 = (1 - \sin^2 A)(1 - \sin^2 B)(1 - \sin^2 C)$

$K^2 = \cos^2 A \cos^2 B \cos^2 C$

$K = \pm \cos A \cos B \cos C$

68. Put $\alpha = 45^\circ$; verify the options.

69. $1 + \tan^2 \theta + \tan \theta = 13 \Rightarrow \tan \theta = 3 \text{ or } -4$

70. $\log(\tan 45^\circ) = \log 1 = 0$

71. $\alpha + \beta = 90^\circ, \alpha - \beta = 30^\circ \Rightarrow \alpha = 60^\circ, \beta = 30^\circ$

72. Squaring and adding

73. Put $A = B = 45^\circ$

74. $2 \cos^2 B = 1 + \tan^2 A$

75. $\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta$

76. $x \cdot \frac{1}{3} + 4 \cdot \frac{1}{4} = 3 \Rightarrow x = 6$

77. $A + B = 90^\circ \Rightarrow \cos^2 A + \cos^2 B = 1$

78. $x = \sin 50^\circ - \cos 50^\circ$

79. $A + B = 90^\circ \Rightarrow \cot A \cot B = 1$

80. $\sec^2 \theta - \tan^2 \theta = 1$

81. $\sin^2 \theta + \cos^2 \theta = 1$

82. A is obtuse

$\Rightarrow B + C < 90^\circ \Rightarrow \tan B \tan C < 1$

83. $\sin \alpha = \frac{3}{5}, \cos \beta = \frac{3}{5}, \cos \alpha = \frac{4}{5}, \sin \beta = \frac{4}{5}$

84. $a^2 \left(-\frac{1}{2}\right)^2 - 4a^2 (-1)^2 + 2a^2 \left(\frac{\sqrt{3}}{2}\right)^2$

85. Divide the Nr and Dr. with $\cos \theta$

86. Let $a \tan \theta + b \sec \theta = k$

$$\therefore a^2 - b^2 = c^2 - k^2 \Rightarrow k^2 = c^2 - a^2 + b^2$$

EXERCISE-II

CRTQ & SPQ LEVEL-II

TRIGNOMETRIC RELATIONS ELIMINATION OF ' θ ' TRIGNOMETRIC RATIOS, VALUES

C.R.T.Q

Class Room Teaching Questions

1. If θ is not in 4th quadrant and $\tan \theta = -4/3$ then

$$5\sin \theta + 10\cos \theta + 9\sec \theta + 16\cosec \theta + 4\cot \theta =$$

1) -1 2) 2/5 3) 4/5 4) 0

2. $f(x) = x^3 - 3x + 5$ then $f\left(\sin \frac{3\pi}{2}\right) + f\left(\cos \frac{3\pi}{2}\right) =$
 1) 10 2) 12 3) 14 4) 16

3. If $\tan \theta, 2\tan \theta + 2, 3\tan \theta + 3$ are in GP
 then the value of $\frac{7 - 5\cot \theta}{9 - 4\sqrt{\sec^2 \theta - 1}}$ is

1) $\frac{12}{5}$ 2) $\frac{-33}{28}$ 3) $\frac{33}{100}$ 4) $\frac{12}{13}$

4. In a triangle ABC, $C = 90^\circ$, then the equation whose roots are $\tan A, \tan B$ is

1) $abx^2 + c^2x + ab = 0$

2) $abx^2 + c^2x - ab = 0$

3) $abx^2 - c^2x - ab = 0$

4) $abx^2 - c^2x + ab = 0$

5. Which of the following is correct?

1) $\sin 1^\circ > \sin 1$ 2) $\sin 1^\circ < \sin 1$

3) $\sin 1^\circ = \sin 1$ 4) $\sin 1^\circ = \frac{\pi}{180} \sin 1$

6.1 TRIGNOMETRIC RATIOS

6. If $a = \cos 3$ and $b = \sin 8$ then
 1) $a > 0, b > 0$ 2) $ab < 0$ 3) $a > b$ 4) $ab > 0$

7. $\cosec A = 4p + \frac{1}{16p} \Rightarrow \cosec A + \cot A =$

1) $8p$ 2) $\frac{1}{8p}$

3) $-8p$ (or) $\frac{1}{8p}$ 4) $8p$ (or) $\frac{1}{8p}$

8. If $\frac{1 - \sin A}{1 + \sin A} = \sec A - \tan A$ then A lies in the quadrants

1) I, II 2) II, III 3) I, IV 4) I, III

9. If $\sin \theta + \cos \theta = m$ and

$\sec \theta + \cosec \theta = n$, then n

$(m+1)(m-1) =$

1) m 2) n 3) $2m$ 4) $2n$

10. If $1 + \sin x + \sin^2 x + \dots \text{to } \infty = 4 + 2\sqrt{3}, 0 < x < \pi$

and $x \neq \frac{\pi}{2}$ then $x =$

1) $30^\circ, 60^\circ$ 2) $60^\circ, 120^\circ$
 3) $90^\circ, 120^\circ$ 4) $30^\circ, 45^\circ$

11. $\cos^2 5^\circ + \cos^2 10^\circ$

$+ \cos^2 15^\circ + \dots + \cos^2 360^\circ =$

1) 18 2) 27 3) 36 4) 45

12. If $x = a \cos^2 \theta \sin \theta$ and $y = a \sin^2 \theta \cos \theta$,

then $\frac{(x^2 + y^2)^3}{x^2 y^2} =$

1) a 2) a^3 3) a^2 4) a^5

13. $a \sec \theta + b \tan \theta = 1, a \sec \theta - b \tan \theta = 5$

$\Rightarrow a^2(b^2 + 4) =$

1) $3b^2$ 2) $9b^2$ 3) b^2 4) $4b^2$

14. $\tan A = a \tan B, \sin A = b \sin B \Rightarrow \frac{b^2 - 1}{a^2 - 1} =$

1) $\sin^2 A$ 2) $\sin^3 A$ 3) $\cos^2 A$ 4) $\cos^3 A$

15. If $\frac{\cos^2 \theta}{a} = \frac{\sin^2 \theta}{b}$ then $\frac{\cos^4 \theta}{a} + \frac{\sin^4 \theta}{b} =$

1) $\frac{1}{a+b}$

2) $\frac{1}{(a+b)^2}$

3) $\frac{1}{a^2} + \frac{1}{b^2}$

4) $a+b$

16. If $a = x\cos^2 A + y\sin^2 A$ then $(x-a)(y-a) + (x-y)^2 \sin^2 A \cos^2 A =$

1) 0 2) 1 3) $xy + a^2$ 4) $xy - a^2$

17. If $a\cos^3 \alpha + 3a\cos \alpha \sin^2 \alpha = m$ and $a\sin^3 \alpha + 3a\cos^2 \alpha \sin \alpha = n$ then

$(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}} =$

1) $2a^2$ 2) $2a^{\frac{1}{3}}$ 3) $2a^{\frac{2}{3}}$ 4) $2a^3$

18. $\cos^4 \alpha - \sin^4 \alpha = a$ then $\frac{1-a}{1+a} =$

1) $\tan^2 \alpha$

2) $\cot^2 \alpha$

3) $-\tan^2 \alpha$

4) $-\cot^2 \alpha$

19. $a = \frac{1+\sin x}{1-\cos x + \sin x} \Rightarrow \frac{1+\cos x + \sin x}{2\sin x} =$

1) a

2) $\frac{1}{a}$

3) a^2

4) $\frac{1}{a^2}$

20. Eliminate θ from

$x = 1 + \tan \theta, y = 2 + \cot \theta$

1) $xy + 1 = x + y$ 2) $xy + 2 = 2x + y$

3) $xy + 1 = 2x + y$ 4) $xy + 1 = 2y + x$

21. $f(x) = \sin^2 x + \operatorname{cosec}^2 x \Rightarrow$

1) $f(x) < 1$

2) $f(x) = 1$

3) $1 < f(x) < 2$

4) $f(x) \geq 2$

22. Which of the following is not possible?

1) $\sin \theta = \frac{5}{7}$

2) $\cos \theta = \frac{1+a^2}{1-a^2}, |a| \neq 1$

3) $\tan \theta = 100$

4) $\sec \theta = \frac{5}{2}$

23. If $\theta = \frac{11\pi}{6}$, then $\cos \theta + \sin \theta =$

1) $\frac{\sqrt{3}+1}{\sqrt{2}}$

2) $\frac{\sqrt{3}-1}{\sqrt{2}}$

3) $\frac{\sqrt{3}-1}{2}$

4) $\frac{\sqrt{3}+1}{2}$

24. The value of $\sin\left(n\pi + (-1)^n \frac{\pi}{4}\right), n \in I,$

1) 0 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{-1}{\sqrt{2}}$ 4) $\frac{\sqrt{3}}{2}$

25. If $e^{(\sin^2 x + \sin^4 x + \sin^6 x + \dots + \infty) \log 2} = 8$

$0 < x < \frac{\pi}{2}$ then $\frac{\cos x}{\cos x + \sin x} =$

1) $\frac{\sqrt{3}+1}{2}$

2) $\frac{\sqrt{3}-1}{2}$

3) $\frac{2}{\sqrt{3}+1}$

4) $\frac{2}{\sqrt{3}-1}$

26. If $0 \leq x \leq \pi, 81^{\sin^2 x} + 81^{\cos^2 x} = 30$ then x

1) $\frac{\pi}{6}$

2) $\frac{\pi}{4}$

3) $\frac{\pi}{15}$

4)

27. $\operatorname{cosec}^2 \alpha = \frac{4xy}{(x+y)^2} \quad [(x,y) \neq (0,0)]$ is

1) Possible for x = y

2) Impossible for x = y

3) Possible for x = -y

4) Not possible

28. If $a\sin \theta + b\cos \theta = c$ then $\frac{a-b\tan \theta}{b+a\tan \theta} =$

1) $\frac{\pm\sqrt{b^2 - c^2 + a^2}}{c}$

2) $\frac{\pm\sqrt{b^2 + c^2 - a^2}}{c}$

3) $\frac{\pm\sqrt{b^2 - c^2 - a^2}}{c}$

4) $\frac{\pm\sqrt{a^2 + b^2 + c^2}}{b^2 - c^2}$

S.P.Q.

Student Practice Questions

29. If $\cos \theta = \frac{3}{5}$ and θ is not in the first quadrant, then

$$\frac{5\tan(\pi+\theta)+4\cos(\pi+\theta)}{5\sec(2\pi-\theta)-4\cot(2\pi+\theta)} =$$

- 1) $\frac{4}{5}$ 2) $-\frac{4}{5}$ 3) $\frac{5}{4}$ 4) $-\frac{5}{4}$

30. $f(x) = x^3 - 2x^2 + 3x - 5$

$$\Rightarrow f\left[\sin\left(\frac{5\pi}{2}\right)\right] + f\left[\sin\left(\frac{3\pi}{2}\right)\right] =$$

- 1) 10 2) -10 11) 14 4) -14

~~Q.~~ cosA, sinA, cotA are in GP then
 $\tan^6 A - \tan^2 A =$

- 1) -1 2) 0 3) 1 4) 2

32. If ABCD is a cyclic quadrilateral such that
 $12\tan A - 5 = 0$ and $5\cos B + 3 = 0$, then
 $\cos C \tan D =$

- 1) $-\frac{16}{13}$ 2) $\frac{16}{13}$ 3) $-\frac{13}{16}$ 4) $\frac{23}{16}$

33. $x = \cos 1^\circ$, $y = \cos 1 \Rightarrow$

- 1) $x = y$ 2) $x > y$
 3) $x < y$ 4) $2x = y$

34. $a = \sec 2^\circ$, $b = \sec 2 \Rightarrow$

- 1) $a = b$ 2) $a < b$ 3) $b < a$ 4) $2a = b$

35. $\tan \theta = P - \frac{1}{4P} \Rightarrow \sec \theta - \tan \theta =$

- 1) $2P$ (or) $\frac{1}{2P}$ 2) $\frac{1}{2P}$ (or) $-2P$
 3) $-\frac{1}{2P}$ (or) $2P$ 4) $-\frac{1}{2P}$ (or) $-2P$

36. If $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$, then A lies in the quadrants

- 1) I, II 2) II, III 3) I, III 4) I, IV

37. $x = \tan \theta + \cot \theta$, $y = \cos \theta - \sin \theta \Rightarrow$

1) $x = y$ 2) $\frac{x-y^2}{2} = \frac{1}{x}$

3) $\frac{y^2-1}{2} = \frac{1}{x}$ 4) $\frac{1+y^2}{2} = \frac{1}{x}$

- 38) $1 + \cos x + \cos^2 x + \dots \text{to } \infty = 4 + 2\sqrt{3}$
 then $x =$

- 1) 30° 2) 60° 3) 45° 4) 90°

39. $\sin^2 5^\circ + \sin^2 10^\circ + \dots + \sin^2 180^\circ =$

- 1) 18 2) 27 3) 1 4) 0

40. If $x = a \cos^3 \theta \sin^2 \theta$, $y = a \sin^3 \theta \cos^2 \theta$ and $\frac{(x^2+y^2)^p}{(xy)^q}$, ($P, Q \in \mathbb{N}$) is independent of θ then

- 1) $p+q=6$ 2) $4p=5q$

- 3) $4q=5p$ 4) $pq=16$

- 41) $a = \sec \theta - \tan \theta$, $b = \cosec \theta + \cot \theta \Rightarrow a =$

- 1) $\frac{b+1}{b-1}$ 2) $\frac{1+b}{1-b}$ 3) $\frac{b-1}{b+1}$ 4) $\frac{1-b}{1+b}$

- 42) $\cos A = a \cos B, \sin A = b \sin B \Rightarrow (b^2 - a^2) \sin^2 B =$

- 1) $1+a^2$ 2) $2+a^2$ 3) $1-a^2$ 4) $2-a^2$

- 43) If $\frac{\sin x}{a} = \frac{\cos x}{b} = \frac{\tan x}{c} = k$ then

$bc + \frac{1}{ck} + \frac{ak}{1+bk}$ is

- 1) $k\left(a + \frac{1}{a}\right)$ 2) $\frac{1}{k}\left(a + \frac{1}{a}\right)$ 3) $\frac{1}{k^2}$ 4) $\frac{a}{k}$

- 44) If $m \cos^2 A + n \sin^2 A = p$, then $\cot^2 A =$

- 1) $\frac{p+n}{m+p}$ 2) $\frac{p-n}{p-m}$

- 3) $\frac{p-n}{m-p}$ 4) $\frac{n+1}{m+p}$

45. If $a \sin^3 x + b \cos^3 x = \sin x \cos x$

and $a \sin x = b \cos x$ then $a^2 + b^2 =$

1) 0

~~✓~~ 1

3) 2 4) 3

46. If $\sin \theta + \cos \theta = a$, then $\sin^4 \theta + \cos^4 \theta =$

1) $1 - \frac{1}{2}(a^2 + 1)^2$

~~✓~~ 2) $1 - \frac{1}{2}(a^2 - 1)^2$

3) $1 + \frac{1}{2}(a^2 + 1)^2$

4) $1 + \frac{1}{2}(a^2 - 1)^2$

47. If $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{x}{\tan \theta - \sec \theta + 1}$ then x

1) 0

2) 2

~~✓~~ 3) $\tan \theta - \sec \theta + 1$

4) $\tan \theta + \sec \theta - 1$

48. If $\tan \theta + \sin \theta = m$, $\tan \theta - \sin \theta = n$, then $(m^2 - n^2)^2 =$

~~✓~~ 1) 16 mn

2) 4 mn

3) 32 mn

4) 8 mn

49. If $\sec \theta + \cos \theta = 2$ then $\sin^2 \theta + \tan^2 \theta =$

~~✓~~ 1) 0

2) 1

3) 2 4) 4

50. Which of the following possible?

1) $\sin \theta = 2$

2) $\cos \theta = \frac{17}{5}$

3) $\tan \theta = 2$

4) $\sec \theta = \frac{4}{5}$

51. If $\theta = \frac{\pi}{21}$, then $\frac{\sin 23\theta - \sin 7\theta}{\sin 2\theta + \sin 14\theta} =$

1) 0

2) 1

3) -1 4) 2

52. The value of $\tan\left(n\pi + \frac{\pi}{3}\right)$, $n \in \mathbb{Z}$ is

~~✓~~ 1) $\sqrt{3}$

2) $\frac{1}{\sqrt{3}}$

3) 0

4) $\frac{1}{\sqrt{2}}$

53. If $e^{(1+\sin^2 x + \sin^4 x + \dots) \log 2} = 16$, then $\tan^2 x =$

1) 1

2) 2

~~✓~~ 3 4) 4

54. If $2 \sin x + 5 \cos y + 7 \sin z = 14$ then

~~✓~~ 1) $7 \tan \frac{x}{2} + 4 \cos y - 6 \cos z =$

1) 4

2) -3

~~✓~~ 3) 11

4) 5

55. If $\sin \theta$ and $\cos \theta$ are the roots of $px^2 + qx + r = 0$ then $q^2 - p^2 =$

1) 0

~~✓~~ 2) $-2pr$

3) $2qr$

4) $2p$

56. If $0 \leq x \leq \pi$, $4^{\sin^2 x} + 4^{\cos^2 x} = 5$, then $x =$

1) $\frac{\pi}{6}$

2) $\frac{\pi}{3}$

3) $\frac{\pi}{4}$

~~✓~~ 4) $\frac{\pi}{2}$

KEY

- | | | | | |
|-------|-------|-------|-------|-------|
| 01) 4 | 02) 2 | 03) 2 | 04) 4 | 05) 1 |
| 06) 2 | 07) 4 | 08) 3 | 09) 3 | 10) 1 |
| 11) 3 | 12) 3 | 13) 2 | 14) 3 | 15) 1 |
| 16) 1 | 17) 3 | 18) 1 | 19) 1 | 20) 1 |
| 21) 4 | 22) 2 | 23) 3 | 24) 2 | 25) 1 |
| 26) 1 | 27) 1 | 28) 1 | 29) 2 | 30) 1 |
| 31) 3 | 32) 1 | 33) 2 | 34) 3 | 35) 1 |
| 36) 4 | 37) 2 | 38) 1 | 39) 1 | 40) 1 |
| 41) 3 | 42) 3 | 43) 2 | 44) 3 | 45) 1 |
| 46) 2 | 47) 4 | 48) 1 | 49) 1 | 50) 1 |
| 51) 3 | 52) 1 | 53) 3 | 54) 3 | 55) 1 |
| 56) 4 | | | | |

Hints & Solutions

1. θ lies in II quadrant, substitute the values
2. $\sin 270^\circ = -1$ & $\cos 270^\circ = 0$
3. $(2 \tan \theta + 2)^2 = \tan \theta (3 \tan \theta + 3)$
 $\Rightarrow \tan^2 \theta + 5 \tan \theta + 4 = 0 \Rightarrow \tan \theta = -1$

but $\tan \theta = -4$ only satisfies above condition

4. $\tan A = \frac{a}{b}$, $\tan B = \frac{b}{a}$ and $(a^2 + b^2) = c^2$

Find sum and product of roots

5. $1 > 1^\circ$

6. 3° lies in the second quadrant $\therefore \cos 3 < 0$

8° lies in the second quadrant $\therefore \sin 8 > 0$

$$\Rightarrow ab < 0$$

7. Use $(\cosec^2 A - \cot^2 A) = 1$

If $\cosec A = \left(4P + \frac{1}{16P} \right)$

then $\cot A = \pm \left(4P - \frac{1}{16P} \right)$

8. $\sqrt{\cos^2 A} = \cos A$ then $A \in Q_1, Q_4$

9. Find m^2 and multiply

10. Use formula $S_\infty = \frac{a}{1-r}$ for GP

11. If $(A+B) = 360^\circ$ or $(A+B) = 180^\circ$

then $\cos^2 A = \cos^2 B$ and $(A+B) = 90^\circ$

then

$$(\cos^2 A + \cos^2 B) = 1$$

12. θ can be eliminated by substitution.

13. Given $(a \sec \theta + b \tan \theta) = 1$

$$\Rightarrow (a \sec \theta - b \tan \theta) = 5, \text{ eliminate } \theta$$

14. Put $A = 30^\circ$ and $B = 45^\circ$ and verify

15. Find $\tan \theta$ and simplify

16. Put $A = 90^\circ$

17. $m+n = a(\cos \alpha + \sin \alpha)^3$

$$m-n = a(\cos \alpha - \sin \alpha)^3$$

18. $a = \cos^2 \alpha - \sin^2 \alpha$

$$\Rightarrow 1-a = 2 \sin^2 \alpha \text{ and } 1+a = 2 \cos^2 \alpha$$

19. In R.H.S. of given equation multiply and divide by $(1+\cos x + \sin x)$

20. $x-1 = \tan \theta, y-2 = \cot \theta$

Multiply two equations

21. $f(x) = \left(\sin^2 x + \frac{1}{\sin^2 x} \right) \geq 2$

$$\because \left(a + \frac{1}{a} \right) \geq 2 \text{ for any real number } a > 0$$

22. for $a=2, \cos \theta = \frac{5}{-3}$ not possible

23. Put $\theta = 330^\circ$ substitution

24. Put $n = 0$

25. $\sin^2 x + \sin^4 x + \dots \infty = \frac{\sin^2 x}{1 - \sin^2 x} = \tan^2 x$

$$\therefore (e^{\log 2})^{\tan^2 x} = 8 \Rightarrow 2^{\tan^2 x} = 2^3 \Rightarrow \tan x = \sqrt{3}$$

$$\therefore GE = \frac{1}{1 + \tan x} = \frac{1}{1 + \sqrt{3}} = \frac{\sqrt{3} - 1}{2}$$

26. If $81^{\tan^2 x} = y \Rightarrow 81^{\cos^2 x} = \frac{81}{y}$

$$\therefore y + \frac{81}{y} = 30 \Rightarrow y^2 - 30y + 81 = 0$$

$$\Rightarrow y = 27, 3$$

27. $\because \cosec \alpha \in (-\infty, -1] \cup [1, \infty)$

28. $GE = \frac{a \cos \theta - b \sin \theta}{a \sin \theta + b \cos \theta} = \frac{\pm \sqrt{b^2 - c^2 + a^2}}{c}$

29. lies in fourth quadrant substitute the corresponding values.

30. $\sin \frac{3\pi}{2} = -1, \sin \frac{5\pi}{2} = 1$

31. $\sin^2 A = \cos A \cdot \cot A \Rightarrow \sin^3 A = \cos^2 A$
 $\Rightarrow \tan^2 A = \cosec A$ and simplify

32. $A + C = 180^\circ, B + D = 180^\circ$

Given $\tan A = \frac{5}{12} > 0 \therefore 0 < A < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < C < \pi$

$\cos B = \frac{-3}{5} < 0 \therefore \frac{\pi}{2} < B < \pi \Rightarrow 0 < D < \frac{\pi}{2}$

$\therefore \cos C = \frac{-12}{13}, \tan D = \frac{4}{3}$

33. $1 > 1^\circ \Rightarrow \cos 1 < \cos 1^\circ$

34. $2 = 2^c = 114^\circ$ (approx)

35. Use $(\sec^2 \theta - \tan^2 \theta) = 1$

36. $\sqrt{\cos^2 A} = \cos A$ then $A \in Q_1, Q_4$

37. Eliminate θ

38. Use formula $S_\infty = \frac{a}{1-r}$ for G.P

39. $A + B = 180^\circ \Rightarrow \sin^2 A = \sin^2 B$
and $A + B = 90^\circ \Rightarrow \sin^2 A + \sin^2 B = 1$

40.
$$\frac{(x^2 + y^2)^p}{(xy)^q} = \frac{(a^2 \sin^4 \theta \cos^4 \theta)^p}{(a^2 \sin^5 \theta \cos^5 \theta)^q}$$

$$= \frac{a^{2p} (\sin \theta \cos \theta)^{4p}}{a^{2q} (\sin \theta \cos \theta)^{5q}}$$

which is independent of θ , if $4p = 5q$

41. Put $\theta = 45^\circ$ and verify the options

42. squaring and adding

43. $bc + \frac{1}{ck} + \frac{ak}{1+bk}$

$$= \frac{\sin x}{k^2} + \frac{\cos x(1+\cos x) + \sin^2 x}{\sin x(1+\cos x)}$$

$$= \frac{a}{k} + \frac{1}{\sin x} = \frac{a}{k} + \frac{1}{ak} = \frac{1}{k} \left(a + \frac{1}{a} \right)$$

44. divided with $\sin^2 A$

we get $\cot^2 A = \frac{p-n}{m-p}$

45. Put $\sin x = b$ and $\cos x = a$

$$\therefore a^2 + b^2 = \cos^2 x + \sin^2 x = 1$$

46. Use $(a^4 + b^4) = (a^2 + b^2)^2 - 2a^2b^2$

47. In LHS divide the Num and Den by $\cos^2 \theta$

48. $2 \tan \theta = (m+n)$

$2 \sin \theta = (m-n)$

Eliminate θ and use

$$(\tan^2 \theta - \sin^2 \theta) = \tan^2 \theta \cdot \sin^2 \theta$$

49. Put $\theta = 0$

50. Range of $\tan \theta$ is R

51. Use $21\theta = \pi$

52. put $n = 0$

53. $(e^{\log 2})^{\frac{1}{1-\sin^2 x}} = 16 \Rightarrow 2^{\frac{1}{\cos^2 x}} = 2^4$

$$\Rightarrow \sec^2 x = 4 \Rightarrow \tan^2 x = 3$$

54. put $x = \frac{\pi}{2}, y = 0, z = \frac{\pi}{2}$.

55. $\sin \theta + \cos \theta = \frac{-q}{p}, \sin \theta \cos \theta = \frac{r}{p}$

now $(\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cos \theta$

56. $x = 90^\circ$, verify

EXERCISE-III

CRTQ & SPQ

LEVEL-III

TRIGONOMETRIC RELATIONS, VALUE

C.R.T.Q

Class Room Teaching Questions

1. $\tan^2 \alpha = 1 - p^2$, then

$\sec \alpha + \tan^3 \alpha \cosec \alpha =$

1) $(2+p^2)^{\frac{3}{2}}$

2) $(1+p^2)^{\frac{3}{2}}$

3) $(2-p^2)^{\frac{3}{2}}$

4) $(1-p^2)^{\frac{3}{2}}$

2. $\frac{\pi}{2} < \alpha < \pi \Rightarrow \sqrt{\frac{1-\cos \alpha}{1+\cos \alpha}} + \sqrt{\frac{1+\cos \alpha}{1-\cos \alpha}} =$

1) $2\sec \alpha$

3) $2\cosec \alpha$

2) $-2\sec \alpha$

4) $-2\cosec \alpha$

3. If $\sin \beta$ is the G.M. between $\sin \alpha$ and $\cos \alpha$, then $(\cos \alpha - \sin \alpha)^2 - 2\cos^2 \beta =$
- 1) 0 2) 1 3) 2 4) -1

4. If $\sin^2 \theta = \frac{x^2 + y^2 + 1}{2x}$ then x must be
- 1) -3 2) -2 3) 1 4) 3

5. $\sin x + \sin^2 x + \sin^3 x = 1 \Rightarrow \cos^6 x - 4\cos^4 x + 8\cos^2 x =$
- 1) 4 2) 2 3) 1 4) 0

S.P.Q.**Student Practice Questions**

6. The greatest among $(\sin 1 + \cos 1)$, $(\sqrt{\sin 1} + \sqrt{\cos 1})$, $(\sin 1 - \cos 1)$ and 1 is
- 1) $\sin 1 + \cos 1$ 2) $\sqrt{\sin 1} + \sqrt{\cos 1}$
 3) $\sin 1 - \cos 1$ 4) 1
7. If $\tan^2 \alpha \tan^2 \beta + \tan^2 \beta \tan^2 \gamma + \tan^2 \gamma \tan^2 \alpha + 2 \tan^2 \alpha \tan^2 \beta \tan^2 \gamma = 1$ then the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$
- 1) 0 2) -1 3) 1 4) 2

8. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan \theta)^{\tan \theta}$,
 $t_2 = (\tan \theta)^{\cot \theta}$, $t_3 = (\cot \theta)^{\tan \theta}$,
 $t_4 = (\cot \theta)^{\cot \theta}$ then
- 1) $t_1 > t_2 > t_3 > t_4$ 2) $t_4 > t_3 > t_1 > t_2$
 3) $t_3 > t_1 > t_2 > t_4$ 4) $t_2 > t_3 > t_1 > t_4$

9. Two arcs of same length of two different circles subtended angles of 25° and 75° at their centres respectively. Then the ratio of the radii of the circles is
- 1) 3 : 1 2) 1 : 3 3) 1 : 2 4) 2 : 1

ELIMINATION OF 'θ'**C.R.T.Q****Class Room Teaching Questions**

10. $\cos \theta + \cos^2 \theta = 1$,
 $a \sin^{12} \theta + b \sin^{10} \theta + c \sin^8 \theta + d \sin^6 \theta = 1 \Rightarrow \frac{b+c}{a+d} =$
- 1) 2 2) 3 3) 4 4) 6
11. $a \sin x = b \cos x = \frac{2c \tan x}{1 - \tan^2 x}$ and
 $(a^2 - b^2)^2 = kc^2(a^2 + b^2) \Rightarrow k =$
- 1) 1 2) 2 3) 3 4) 4

S.P.Q.**Student Practice Questions**

12. If $a \sin^2 x + b \cos^2 x = c$,
 $b \sin^2 y + a \cos^2 y = d$ and $a \tan x = b \tan y$
 then $\frac{a^2}{b^2} =$
- 1) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$ 2) $\frac{(b-c)(b-d)}{(a-c)(a-d)}$
 3) $\frac{(b-c)(d-b)}{(a-d)(c-a)}$ 4) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$

KEY

- 01) 3 02) 3 03) 4 04) 3 05) 1
 06) 2 07) 3 08) 2 09) 1 10) 2
 11) 4 12) 1

Hints & Solutions

$$\begin{aligned} 1. \quad & \sec \alpha \left[1 + \frac{\sin^3 \alpha \cos \alpha}{\cos^3 \alpha \sin \alpha} \right] \\ &= \sec \alpha [1 + \tan^2 \alpha] = \sec^3 \alpha \\ &= (\sec^2 \alpha)^{3/2} = (2 - P^2)^{3/2} \end{aligned}$$

FOCUS TRACK

2. Simplifying we get $\frac{2}{\sqrt{\sin^2 \alpha}} = 2 \cos ec \alpha$
 $\because \sin \alpha$ is positive in II quadrant.

3. $\sin^2 \beta = \sin \alpha \cos \alpha$

4. $\sin^2 \theta \leq 1$

5. $\sin x(1 + \sin^2 x) = \cos^2 x$

Squaring both sides we get

$$(1 - \cos^2 x)(2 - \cos^2 x)^2 = \cos^4 x$$

Simplify further

6. $\sqrt{\sin 1} > \sin 1 > \sin^2 1$

$$\sqrt{\cos 1} > \cos 1 > \cos^2 1$$

$$\text{Adding } (\sqrt{\sin 1} + \sqrt{\cos 1}) > (\sin 1 + \cos 1)$$

$$> (\sin^2 1 + \cos^2 1)$$

$$\Rightarrow (\sqrt{\sin 1} + \sqrt{\cos 1}) > (\sin 1 + \cos 1) > 1$$

7. Let $\tan^2 \alpha = x, \tan^2 \beta = y, \tan^2 \gamma = z$

$$\begin{aligned} \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma &= \sum \frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sum \frac{x}{1+x} \\ &= \frac{(x+y+z) + (xy+yz+zx+2xyz) + xy+yz+zx+xyz}{(1+x)(1+y)(1+z)} \\ &= \frac{1+x+y+z+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)} = 1 \end{aligned}$$

8. In $\left(0, \frac{\pi}{4}\right)$, $\tan \theta < 1$ and $\cot \theta > 1$

$$(\cot \theta)^{\cot \theta} > (\cot \theta)^{\tan \theta} > (\tan \theta)^{\tan \theta} > (\tan \theta)^{\cot \theta}$$

$$\therefore t_4 > t_3 > t_1 > t_2$$

9. Length of arc

10. $\cos \theta = 1 - \cos^2 \theta \Rightarrow \cos \theta = \sin^2 \theta$
 $\Rightarrow \cos^2 \theta = \sin^4 \theta \Rightarrow (\sin^4 \theta + \sin^2 \theta) =$

Cubing both sides we get the answer

11. Take $a \sin x = b \cos x = c \tan 2x = l$

$$\Rightarrow \sin x = \frac{l}{a}, \cos x = \frac{l}{b}, \tan 2x = \frac{l}{c}$$

Elimination l we get

$$(a^2 - b^2)^2 = 4c^2(a^2 + b^2) \Rightarrow K = 4$$

$$\begin{aligned} 12. a \sin^2 x + b \cos^2 x &= c \\ \Rightarrow a \tan^2 x + b &= c \sec^2 x \\ \Rightarrow (a-c) \tan^2 x &= c-b \Rightarrow \tan^2 x = \frac{c-b}{a-c} \\ \text{similarly } \tan^2 y &= \frac{d-a}{b-d} \\ \text{but } a \tan x &= b \tan y \Rightarrow \frac{a^2}{b^2} = \frac{\tan^2 y}{\tan^2 x} \end{aligned}$$

EXERCISE-IV

LEVEL-IV

1. Assertion A: In a right angled triangle $\sin^2 A + \sin^2 B + \sec^2 C = 2$.

Reason R: If α, β are complementary angles then $\sin^2 \alpha + \sin^2 \beta = 1$

1) A, R are true and R is the correct explanation of A

2) A, R are true and R is not the correct explanation of A

3) A is true, R is false

4) A is false, R is true

2. Observe the following lists. Let

$$\frac{\sin x}{a} = \frac{\cos x}{b} = \frac{\tan x}{c} = k.$$

List - I

List-II

1) bc

a) $\frac{1}{b^2 k^4}$

2) $a^2 + b^2$

b) $\frac{1}{ak}$

3) $\frac{1}{ck} + \frac{ak}{1+bk}$

c) $\frac{a}{k}$

4) $a^2 + b^2 + c^2$

d) $\frac{1}{k^2}$

The correct match for List-I from List-II is

- ~~1~~ 1 - c, 2 - d, 3 - b, 4 - a
 2) 1 - d, 2 - a, 3 - c, 4 - b
 3) 1 - a, 2 - b, 3 - d, 4 - c
 4) 1 - b, 2 - c, 3 - a, 4 - d

3. Observe the following lists.

List-I

- | | |
|---|-------------------|
| I. $\sin^2 \frac{2\pi}{3} + \cos^2 \frac{5\pi}{6} - \tan^2 \frac{3\pi}{4}$ | a) $\frac{15}{2}$ |
| II. $\sin^2 \frac{11\pi}{6} + \cos^2 \frac{7\pi}{6} - \tan^2 \frac{5\pi}{4}$ | b) $-\frac{1}{2}$ |
| III. $\sin^2 \frac{3\pi}{4} + \sec^2 \frac{5\pi}{3} + \tan^2 \frac{2\pi}{3}$ | c) 0 |
| IV. $\cos^2 \frac{2\pi}{3} + \sin^2 \frac{5\pi}{3} - \frac{3}{2} \tan^2 \frac{3\pi}{4}$ | d) 1/2 |

List-II

6. Match the following:

List - I

List - II

1) $3 \tan x + 27 \cot x \geq$

a) 24

($x \in Q_1$)

2) $5 \sec^2 x + 125 \cos^2 x \geq$

b) 18

3) $16 \csc^2 x + 9 \sin^2 x \geq$

c) 50

1) 1-a, 2-b, 3-c

2) 1-c, 2-a, 3-b

~~3) 1-b, 2-c, 3-a~~

4) 1-c, 2-b, 3-a

7. $A = \tan 1^\circ, B = \tan 2^\circ, C = \tan 3^\circ$, then the descending order of A, B, C is

1) A, B, C

2) C, B, A

~~3) A, C; B~~

4) B, C, A

KEY

- 01) 4 02) 1 03) 1 04) 4 05) 1
06) 3 07) 3

Hints & Solutions

4. Statement I: The number of values of θ satisfying $\sec \theta + \cos \theta = 1$ is 2.

Statement II: If $7 \sin^2 \alpha + 3 \cos^2 \alpha = 4$, then $\tan \alpha = \sqrt{3}$.

Which of the above statements is correct?

- 1) only I is true
2) only II is true
3) Both I and II are true
~~4) Neither I nor II is true~~

5. If $\alpha = \cos 10^\circ - \sin 10^\circ$, $\beta = \cos 45^\circ - \sin 45^\circ$, $\gamma = \cos 70^\circ - \sin 70^\circ$ then the descending order of α, β, γ is

- ~~1) α, β, γ~~ 2) γ, β, α
3) α, γ, β 4) β, α, γ

- In a right angled triangle $\sin^2 A + \sin^2 B + \sec^2 C \neq 2$
- Simplify
- Simplify
- I. $(\sec \theta + \cos \theta) = 1$ is not possible for $\theta \in R$
II. Divide given equation by $\cos^2 \alpha$
- α is positive, β is zero,
 γ is negative
- $A.M \geq G.M$
 $a \tan x + b \cot x \geq 2\sqrt{ab}$
- $1 = 57^\circ; 2 = 114^\circ; 3 = 171^\circ$ (Aprox)

INTEGER & NUMERICAL

ANSWER TYPE QUESTIONS

1. The value of the expression $\frac{\tan^2 20^\circ - \sin^2 20^\circ}{\tan^2 20^\circ \cdot \sin^2 20^\circ}$ is ~~1~~
2. Suppose that for some angles x and y , the equations $\sin^2 x + \cos^2 y = \frac{3a}{2}$ and $\cos^2 x + \sin^2 y = \frac{a^2}{2}$ hold simultaneously. The possible value of a is ~~1~~
3. If $0 < x < \frac{\pi}{4}$ and $\cos x + \sin x = \frac{5}{4}$, then the value of $16(\cos x - \sin x)^2$ is ~~7~~
4. The value of $3 \frac{\sin^4 t + \cos^4 t - 1}{\sin^6 t + \cos^6 t - 1}$ is equal to ~~2~~
5. If $\sin \theta - \cos \theta = 1$, then the value of $\sin^3 \theta - \cos^3 \theta$ is ~~1~~
6. If $\sin \theta, \tan \theta, \cos \theta$ are in G.P. then $4\sin^2 \theta - 3\sin^4 \theta + \sin^6 \theta =$ _____
7. Let $f(\theta) = \frac{1}{1 + (\cot \theta)^x}$, and $S = \sum_{\theta=1^\circ}^{89^\circ} f(\theta)$, then the value of $\sqrt{2S - 8}$ is _____
8. The minimum value of $\sqrt{(3\sin x - 4\cos x - 10)(3\sin x + 4(\cos x - 10))}$ is ~~10~~
9. If $a \in (0, 1)$, and $f(a) = (a^2 - a + 1) + \frac{8\sin^2 a}{\sqrt{a^2 - a + 1}} + \frac{27\cosec^2}{\sqrt{a^2 - a + 1}}$, then the least value of $f(a)/2$ is ~~9~~
10. Minimum value of $\frac{\sec^4 \alpha}{\tan^2 \beta} + \frac{\sec^4 \beta}{\tan^2 \alpha}$, where $\alpha \neq \frac{\pi}{2}, \beta \neq \frac{\pi}{2}, 0 < \alpha, \beta < \frac{\pi}{2}$, is _____

- $-q^2 \cot^2 \theta =$ _____
- $\sqrt{81p^{-2} - q^{-2}}$ is _____
12. $\cos 5^\circ + \cos 24^\circ + \cos 175^\circ + \cos 204^\circ + \cos 300^\circ =$ _____
 13. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 179^\circ =$ _____
 14. $\tan 10^\circ \cdot \tan 20^\circ \dots \sin 80^\circ =$ _____
 15. $\cot \frac{\pi}{20} \cdot \cot \frac{3\pi}{20} \cdot \cot \frac{5\pi}{20} \cdot \cot \frac{7\pi}{20} \cdot \cot \frac{9\pi}{20} =$ _____
 16. $\sin 120^\circ \cos 150^\circ - \cos 240^\circ \sin 330^\circ =$ _____
 17. In ΔABC , $\cos\left(\frac{B+2C+3A}{2}\right) + \cos\left(\frac{A-B}{2}\right) =$ _____
 18. If A, B, C, D are the angles of a cyclic quadrilateral, then $\cos A + \cos B + \cos C + \cos D =$ _____
 19. If α, β are complementary angles, then $\sin^2 \alpha + \sin^2 \beta =$ _____
 20. If α, β are supplementary angles, then $\sin^2 \alpha + \sin^2 \beta =$ _____
 21. If $180^\circ < \theta < 270^\circ$ and $\sin \theta = -5/13$ then $5\cot^2 \theta + 12\tan \theta + 13\cosec \theta =$ _____
 22. If $\sin(\alpha + \beta) = 1, \sin(\alpha - \beta) = 1/2$ then that $(\alpha + 2\beta)\tan(2\alpha + \beta) =$ _____
 23. $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \sin^2 x) + 1 =$ _____
 24. If $(\sin \alpha + \cosec \alpha)^2 + (\cos \alpha + \sec \alpha)^2 = K + \tan^2 \alpha + \cot^2 \alpha$ then $K =$ _____
 25. $\cot^2 \theta \left(\frac{\sec \theta - 1}{1 + \sin \theta} \right) + \sec^2 \theta \left(\frac{\sin \theta - 1}{1 + \cosec \theta} \right) =$ _____
 26. $\left(\frac{\sqrt{3} + 2\cos A}{1 - 2\sin A} \right)^{-3} + \left(\frac{1 + 2\sin A}{\sqrt{3} - 2\cos A} \right)^{-3} =$ _____
 27. $\sin^4 \theta + 2\sin^2 \theta \left(1 - \frac{1}{\cosec^2 \theta} \right) + \cos^4 \theta =$ _____

28. If $\sin \theta + \sin^2 \theta = 1$, then $\cos^2 \theta + \cos^4 \theta =$ _____

29. If $\sin x + \sin^2 x = 1$ then $\cos^8 x + 2\cos^6 x + \cos^4 x =$ _____

30. If $\cos x + \cos^2 x = 1$, then $\sin^{12} x + 3\sin^{10} x + 3\sin^8 x + 3\sin^6 x + \sin^4 x =$ _____

31. If $\sin \alpha + \operatorname{cosec} \alpha = 2$, then

$$\sin^2 \alpha + \operatorname{cosec}^2 \alpha = \text{_____}$$

32. If $P_n = \cos^4 x + \sin^4 x$, then $2P_6 - 3P_4 + 1 =$ _____

33. If $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$ then $\cos \theta_1 + \cos \theta_2 + \cos \theta_3 =$ _____

34. If $\sin A, \cos A, \tan A$ form a GP. then
 $\cot^6 A - \cot^2 A =$ _____

35. If $\sin \theta, \cos \theta, \tan \theta$ are in GP. then
 $\cos^9 \theta + \cos^6 \theta + 3\cos^5 \theta - 1 =$ _____

36. If $\sin \alpha, \sin \beta, \sin \gamma$ are in A.P and $\cos \alpha, \cos \beta, \cos \gamma$ are in G.P. then

$$\frac{\cos^2 \alpha + \cos^2 \gamma - 4\cos \alpha \cos \gamma}{1 - \sin \alpha \sin \gamma} = \text{_____}$$

37. If $\tan^2 \alpha \tan^2 \beta + \tan^2 \beta \tan^2 \gamma + \tan^2 \gamma \tan^2 \alpha + 2\tan^2 \alpha \tan^2 \beta \tan^2 \gamma = 1$ then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$ _____

38. If $\frac{\sin x + \cos x}{\cos^3 x} = a \tan^3 x + b \tan^2 x + c \tan x + d$
then $a + b + c + d =$ _____

39. $\sqrt{1 - \sin^2 100^\circ} \cdot \sec 100^\circ =$ _____

40. If $\sin \theta + \cos \theta = a$, and $\tan \theta + \cot \theta = b$
then $b(a^2 - 1) =$ _____

41. If $a \sin^3 x + b \cos^3 x = \sin x \cos x$ and $a \sin x = b \cos x$ then $a^2 + b^2 =$ _____

42. If $\operatorname{cosec} \theta - \sin \theta = m$, $\sec \theta - \cos \theta = n$,
then $(m^2 n)^{2/3} + (mn^2)^{2/3} =$ _____

43. If $a = x \cos^2 \alpha + y \sin^2 \alpha$ then $(x-a)(y-a) + (x-y)^2 \sin^2 \alpha \cos^2 \alpha =$ _____

44. If $m^2 + m_1^2 + 2nm_1 \cos \theta = 1 = n^2 + n_1^2 + 2nn_1 \cos \theta$
and $mn + nA_1n_1 + (mn_1 + m_1n) \cos \theta = 0$
then $(m^2 + n^2) \sin^2 \theta =$ _____

45. If $a = \cos \phi \cos \theta + \sin \phi \sin \theta \cos \delta$, $b = \cos \phi \sin \theta - \sin \phi \cos \theta \cos \delta$ and $c = \sin \phi \sin \delta$, then
 $a^2 + b^2 + c^2 =$ _____

46. The absolute value of k , for which
 $(\cos x + \sin x)^2 + k \sin x \cos x - 1 = 0$ is an identity, is

47. If $\frac{\sin^4 A}{a} + \frac{\cos^4 A}{b} = \frac{1}{a+b}$, then the value of
 $\frac{\sin^8 A}{a^3} + \frac{\cos^8 A}{b^3}$ is equal to $\frac{a^p b^q}{(a+b)^r}$.
Find value of $p+q+r$.

48. If $\sin^3 x \sin 3x = \sum_{m=0}^n C_m \cos mx$ where
 $C_0, C_1, C_2, \dots, C_n$ are constants and
 $C_n \neq 0$, then the value of n is

49. For a positive integer n , let

$$f_n(\theta) = \left(\tan \frac{\theta}{2} \right) (1 + \sec \theta)(1 + \sec 2\theta)(1 + \sec 4\theta) \dots (1 + \sec 2^n \theta).$$

Then $7 f_5 \left(\frac{\pi}{128} \right)$ is ?

50. If $k = \sin \frac{\pi}{18} \cdot \sin \frac{5\pi}{18} \cdot \sin \frac{7\pi}{18}$, then the numerical value of $16k$ is

51. If $\sin x + \sin^2 x = 1$, then the value of expression
 $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$
is equal to

52. If

$$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C),$$

then each side is equal to

53. If 7 then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is equal to54. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of these segments and A_0A_1, A_0A_2 and A_0A_4 is

$$55. 3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4 (3\pi + \alpha) \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6 (5\pi - \alpha) \right] =$$

56. The maximum value of $\cos \alpha_1, \cos \alpha_2, \dots, \cos \alpha_n$, under the restrictions $0 \leq \alpha_1, \alpha_2, \dots, \alpha_n \leq \frac{\pi}{2}$ and $\cot \alpha_1 \cdot \cot \alpha_2 \cdots \cot \alpha_n = 1$ is m. Find value of $2^{n/2} \times m$.57. If $A = \sin^8 \theta + \cos^{14} \theta$, then maximum value of A is58. If θ is an acute angle and $\sin \theta = \frac{p-6}{8-p}$, minimum possible value of p is ?

59. If A, B, C be the angles of a triangle, then

$$\sum \frac{\cot A + \cot B}{\tan A + \tan B} =$$

60. If

$$\left| a \sin^2 \theta + b \sin \theta \cos \theta + c \cos^2 \theta - \frac{1}{2}(a+c) \right| \leq \frac{1}{2}k,$$

then k^2 is equal to $a^2 + b^2 + c^2 + xab + ybc + zca$. Find value of $-x-y-z$.

61. If $\frac{x}{\cos \theta} = \frac{y}{\cos \left(\theta - \frac{2\pi}{3} \right)} = \frac{z}{\cos \theta} = \frac{t}{\cos \left(\theta + \frac{2\pi}{3} \right)}$
then $x+y+z=$ 62. Value of $\sqrt{3} \csc 20^\circ - \sec 20^\circ =$ **Answers**

- | | | | | |
|--------|---------|-------|--------|-------|
| 1) 1 | 2) 1 | 3) 7 | 4) 2 | 5) 1 |
| 6) 1 | 7) 9 | 8) 7 | 9) 9 | 10) 8 |
| 11) 4 | 12) 0.5 | 13) 0 | 14) 1 | 15) 1 |
| 16) -1 | 17) 0 | 18) 0 | 19) 1 | 20) 1 |
| 21) 0 | 22) 1 | 23) 0 | 24) 7 | 25) 0 |
| 26) 0 | 27) 1 | 28) 1 | 29) 1 | 30) 1 |
| 31) 2 | 32) 0 | 33) 0 | 34) 1 | 35) 1 |
| 36) -2 | 37) 1 | 38) 4 | 39) -1 | 40) 2 |
| 41) 1 | 42) 1 | 43) 0 | 44) 1 | 45) 1 |
| 46) -2 | 47) 3 | 48) 6 | 49) 7 | 50) 2 |
| 51) 0 | 52) 1 | 53) 0 | 54) 3 | 55) 1 |
| 56) 1 | 57) 1 | 58) 6 | 59) 1 | 60) 2 |
| 61) 0 | 62) 4 | | | |

Hints & Solutions

$$1. \tan^2 20^\circ - \sin^2 20^\circ = \tan^2 20^\circ (1 - \cos^2 20^\circ) \\ = \tan^2 20^\circ \sin^2 20^\circ$$

$$2. \sin^2 x + \cos^2 y = \frac{3a}{2} \quad \dots \dots (i)$$

$$\text{and } \cos^2 x + \sin^2 y = \frac{a^2}{2} \quad \dots \dots (ii)$$

Adding (i) and (ii), we get $2 = \frac{3a}{2} + \frac{a^2}{2}$

$$\text{or } a^2 + 3a - 4 = 0$$

$$\text{or } (a+4)(a-1) = 0$$

$\therefore a = -4$ (rejected), $a = 1$

Let $y = \cos x - \sin x$ ($y > 0$)

$$y^2 = 1 - 2\sin x \cos x$$

$$\cos x + \sin x = \frac{5}{4}$$

$$\therefore 1 + 2\sin x \cos x = \frac{25}{16}$$

$$\text{or } 2\sin x \cos x = \frac{9}{16}$$

$$\text{Now } y^2 = 1 - 2\sin x \cos x = 1 - \frac{9}{16} = \frac{7}{16}$$

$$1. \quad \sin^4 t + \cos^4 t - 1 = (\sin^2 t + \cos^2 t)^2$$

$$-2\sin^2 t \cos^2 t - 1 = -2\sin^2 t \cos^2 t$$

$$\sin^6 t + \cos^6 t - 1 = (\sin^2 t + \cos^2 t)^3$$

$$-3\sin^2 t \cos^2 t - 1 = -3\sin^2 t \cos^2 t$$

$$\text{Hence, } 3 \frac{\sin^4 t + \cos^4 t - 1}{\sin^6 t + \cos^6 t - 1} = 2$$

$$5. \quad \sin \theta - \cos \theta = 1$$

$$\text{or } \sin^2 \theta + \cos^2 \theta - 2\sin \theta \cos \theta = 1$$

$$\text{or } \sin \theta \cos \theta = 0$$

$$\text{Now } \sin^3 \theta - \cos^3 \theta = 1$$

$$(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)$$

$$= (\sin \theta - \cos \theta)^2 + 3\sin \theta \cos \theta$$

$$= 1 + 3\sin \theta \cos \theta = 1$$

$$6. \quad \text{Given that } \tan^2 \theta = \sin \theta \cos \theta$$

$$\therefore \sin \theta = \cos^3 \theta$$

Also, given expression can be rewritten as

$$\sin^6 \theta - 3\sin^4 \theta + 3\sin^2 \theta - 1 + 1 + \sin^2 \theta$$

$$= (\sin^2 \theta - 1)^3 + 1 + \sin^2 \theta$$

$$= -(1 - \sin^2 \theta)^3 + 1 + \sin^2 \theta$$

$$= -\cos^6 \theta + 1 + \sin^2 \theta = -\sin^2 \theta + 1 + \sin^2 \theta = 1$$

$$7. \quad \text{We have, } f(\theta) = \frac{(\sin \theta)^x}{(\cos \theta)^x + (\sin \theta)^x}$$

$$\Rightarrow f(\theta) + f\left(\frac{\pi}{2} - \theta\right) = 1$$

$$\therefore S = \sum_{\theta=0}^{89^\circ} f(\theta) = f(1^\circ) + f(2^\circ) + \dots + f(89^\circ)$$

$$f(88^\circ) + f(89^\circ)$$

$$= \left(\underbrace{1+1+1+\dots+1}_{44 \text{ times}} \right) + \frac{1}{2} = 44 + \frac{1}{2} = \frac{89}{2}$$

$$\therefore 2S - 8 = 81$$

$$8. \quad f(x) = 9\sin^2 x - 16\cos^2 x - 10$$

$$(3\sin x - 4\cos x) - 10(3\sin x + 4\cos x)$$

$$+ 100 = 25\sin^2 x - 60\sin x + 84$$

$$= (5\sin x - 6)^2 + 48$$

The minimum value of $f(x)$ occurs when $\sin x = 1$. Therefore, the minimum value of $\sqrt{f(x)}$ is 7.

9. Using $A.M. \geq G.M.$ (for positive numbers)

$$(a^2 - a + 1) + \frac{8\sin^2 a}{\sqrt{a^2 - a + 1}} + \frac{27 \operatorname{cosec}^2 a}{\sqrt{a^2 - a + 1}}$$

$$\geq 3 \left[(a^2 - a + 1) \frac{8\sin^2 a}{\sqrt{(a^2 - a + 1)}} \frac{27 \operatorname{cosec}^2 a}{\sqrt{(a^2 - a + 1)}} \right]^{\frac{1}{3}}$$

$$= 3(2^3 3^3)^{\frac{1}{3}} = 3(6) = 18$$

10. Let $a = \tan^2 \alpha, b = \tan^2 \beta$

Given expression becomes

$$\frac{(a+1)^2}{b} + \frac{(b+1)^2}{a}$$

$$= \frac{a^2 + 2a + 1}{b} + \frac{b^2 + 2b + 1}{a}$$

$$= \frac{a^2}{b} + \frac{1}{b} + \frac{b^2}{a} + \frac{1}{a} + 2\left(\frac{a}{b} + \frac{b}{a}\right)$$

$$\geq 4 \sqrt[4]{\frac{a^2}{b} \cdot \frac{1}{b} \cdot \frac{b^2}{a} \cdot \frac{1}{a}} + 2(2)$$

$$= 4 + 4 = 8$$

11. We have $p \cosec \theta + q \cot \theta = 2$ (i)

And $(p \cosec \theta)^2 - (q \cot \theta)^2 = 5$

$$\Rightarrow p \cosec \theta - q \cot \theta = \frac{5}{2} \quad \dots\dots \text{(ii)}$$

from (i) and (ii)

$$p \cosec \theta = \frac{9}{4}, q \cot \theta = \frac{-1}{4}$$

Now $\cosec^2 \theta - \cot^2 \theta = 1$

$$= \frac{81}{16p^2} - \frac{1}{16q^2} = 1$$

$$\Rightarrow 81p^{-2} - q^{-2} = 16$$

$$= \sqrt{81p^{-2} - q^{-2}} = 4$$

12. $G.E = \cos 5^\circ + \cos 24^\circ + \cos(180^\circ - 5^\circ)$

$$+ \cos(180^\circ + 24^\circ) + \cos(360^\circ - 60^\circ)$$

$$= \cos 5^\circ + \cos 24^\circ - \cos 5^\circ$$

$$- \cos 24^\circ + \cos 60^\circ = 1/2$$

13. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 179^\circ$

$$= (\cos 1^\circ + \cos 179^\circ) + (\cos 2^\circ + \cos 178^\circ)$$

$$+ \dots + (\cos 89^\circ + \cos 91^\circ) + \cos 90^\circ$$

$$= (\cos 1^\circ - \cos 1^\circ) + (\cos 2^\circ - \cos 2^\circ) + \dots$$

$$(\cos 89^\circ - \cos 89^\circ) + \cos 90^\circ = 0$$

14. $G.E = \tan 10^\circ \cdot \tan 20^\circ \cdot \frac{1}{\sqrt{3}} \cdot \tan 40^\circ \cdot \tan$

$$(90^\circ - 40^\circ) \cdot \sqrt{3}$$

$$\tan(90^\circ - 20^\circ) \tan(90^\circ - 10^\circ)$$

$$= \tan 10^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cot 40^\circ \cot 20^\circ \cdot 10^\circ$$

15. $G.E = \cot \frac{\pi}{20} \cdot \cot \frac{3\pi}{20} \cdot \frac{\pi}{4} \cdot \cot \left(\frac{\pi}{2} - \frac{3\pi}{20}\right)$

$$\cot \left(\frac{\pi}{2} - \frac{\pi}{20}\right)$$

$$= \cot \phi \frac{\pi}{10} \cdot \cot \frac{3\pi}{20} \cdot 1 \tan \frac{3\pi}{20} \cdot \tan \frac{\pi}{20} = 1$$

16. $\sin 120^\circ \cos 150^\circ - \cos 240^\circ \sin 330^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{-1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{3}{4} - \frac{1}{4} = -1$$

17. $G.E = \cos \left(\frac{B + 2(180^\circ - A - B) + 3A}{2} \right)$

$$+ \cos \frac{A-B}{2} = \cos \left(\frac{360^\circ + A - B}{2} \right) + \cos \frac{A-B}{2}$$

$$= -\cos \frac{A-B}{2} + \cos \frac{A-B}{2} = 0$$

8. ABCD is a cyclic quadrilateral

$$\Rightarrow A + C = 180^\circ, B + D = 180^\circ$$

$$\Rightarrow C = 180^\circ - A, D = 180^\circ - B$$

$$\Rightarrow \cos C = \cos(180^\circ - A) = -\cos A,$$

$$\cos D = \cos(180^\circ - B) = -\cos B$$

$$\Rightarrow \cos A + \cos C = 0, \cos B + \cos D = 0$$

$$\Rightarrow \cos A + \cos B + \cos C + \cos D = 0.$$

9. $\alpha + \beta = 90^\circ$,

$$G.E. = \sin^2 \alpha + \sin^2(90^\circ - \alpha)$$

$$= \sin^2 \alpha + \cos^2 \alpha = 1$$

0. α, β are supplementary angles, i.e.,

$$\alpha + \beta = 180^\circ,$$

$$\therefore \sin^2 \alpha + \cos^2 \beta = \sin^2 \alpha + \cos^2(180^\circ - \alpha)$$

$$= \sin^2 \alpha + \cos^2 \alpha = 1$$

1. $\theta E III; \sin \theta = \frac{-5}{13}$;

$$\therefore G.E. = 5 \cdot \frac{144}{25} + \frac{12.5}{12} + 13 \left(\frac{-13}{5} \right) = 0$$

2. $\sin(\alpha + \beta) = 1, \sin(\alpha - \beta) = 1/2$

$$\Rightarrow \alpha + \beta = \pi/2, \alpha - \beta = \pi/6$$

$$\Rightarrow \alpha = \pi/3, \beta = \pi/6.$$

$$\tan(\alpha + 2\beta) \tan(2\alpha + \beta)$$

$$= \tan(2\pi/3) \tan(5\pi/6) = (-\sqrt{3})$$

$$(-1/\sqrt{3}) = 1.$$

3. $G.E. = 2[(\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x]$

$$(\sin^2 x + \cos^2 x)] - 3$$

$$[(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x] + 1$$

$$= 2[1 - 3 \sin^2 x \cos^2 x] - 3(1 - 2 \sin^2 x \cos^2 x) + 1 = 0$$

24. $L.H.S. = \sin^2 \alpha + \operatorname{cosec}^2 \alpha + 2 + \cos^2 \alpha$

$$+ \sec^2 \alpha + 2 = 7 + \cot^2 \alpha + \tan^2 \alpha$$

$$\therefore K = 7.$$

25. $G.E. = \frac{\cot^2 \theta (\sec^2 \theta - 1) + \sec^2 \theta (\sin^2 \theta - 1)}{(1 + \sin \theta)(1 + \sec \theta)}$

$$= \frac{\cot^2 \theta \cdot \tan^2 \theta + \sec^2 \theta (-\cos^2 \theta)}{(1 + \sin \theta)(1 + \sec \theta)} = 0$$

26. Put $A = 90^\circ$ in the given expression, we get;

$$G.E. = \left(\frac{\sqrt{3} + 0}{1 - 2} \right)^{-3} + \left(\frac{1 + 2}{\sqrt{3} - 0} \right)^{-3}$$

$$= -\left(\frac{1}{\sqrt{3}} \right)^3 + \left(\frac{1}{\sqrt{3}} \right)^3 = 0$$

27. $G.E. = \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta$

$$= (\sin^2 \theta + \cos^2 \theta)^2 = 1$$

28. Given that

$$\sin \theta = 1 - \sin^2 \theta \Rightarrow \sin \theta = \cos^2 \theta$$

$$\therefore \cos^2 \theta + \cos^4 \theta = \sin \theta + \sin^2 \theta = 1$$

29. Given that $\sin x = 1 - \sin^2 x = \cos^2 x$

$$G.E. = (\cos^4 x + \cos^2 x)^2 = (\sin x + \sin^2 x)^2 = 1$$

30. Given that $\cos x = \sin^2 x$;

$$G.E. = (\sin^4 x + \sin^2 x)^3 = 1$$

31. Given that $\sin \alpha + \operatorname{cosec} \alpha = 2$

$$\Rightarrow \sin^2 \alpha - 2 \sin \alpha + 1 = 0$$

$$\Rightarrow \sin \alpha = 1 = \operatorname{cosec} \alpha$$

$$\therefore \sin^4 \alpha + \operatorname{cosec}^4 \alpha = 2$$

32. $2p_6 - 3p_4 + 1 = 2(1 - 3\sin^2 x \cos^2 x) - 3$
 $\left[(\sin^2 x + \cos^2 x)^3 - 2\sin^2 x \cos^2 x \right] + 1$
 $= 2 - 6\sin^2 x \cos^2 x - 3(1 - 2\sin^2 x \cos^2 x) + 1 = 0$

33. $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$
 $\Rightarrow \sin \theta_1 = \sin \theta_2 = \sin \theta_3 = 1$
 $\Rightarrow \theta_1 = \theta_2 = \theta_3 = 90^\circ$
 $\therefore \cos \theta_1 + \cos \theta_2 + \cos \theta_3 = 0$

34. $\cos^6 A - \cot^2 A = \frac{\cos^6 A}{\sin^6 A} - \frac{\cos^2 A}{\sin^2 A}$
 $= \frac{\sin^4 A}{\sin^6 A} - \frac{\cos^2 A}{\sin^2 A} = \frac{1 - \cos^2 A}{\sin^2 A} = 1$

35. $\cos^2 \theta = \sin \theta \tan \theta \Rightarrow \cos^3 \theta = \sin^3 \theta = 1$
 $- \cos^2 \theta \Rightarrow \cos^3 \theta + \cos^2 \theta = 1$
 $\Rightarrow \cos^9 \theta + \cos^6 \theta + 3\cos^5 \theta = 1$
[Taking cube of the both sides]

36. $\sin \alpha, \sin \beta, \sin \gamma$ are in A.P

$$2\sin \beta = \sin \alpha + \sin \gamma \quad \dots \dots (1)$$

$\cos \alpha, \cos \beta, \cos \gamma$ are in GP

$$\cos^2 \beta = \cos \alpha \cos \gamma$$

Now,
$$\frac{\cos^2 \alpha + \cos^2 \gamma - 4\cos \alpha \cos \gamma}{1 - \sin \alpha \sin \gamma}$$

$$= \frac{1 - \sin^2 \alpha + 1 - \sin^2 \gamma - 4\cos \alpha \cos \gamma}{1 - \sin \alpha \sin \gamma}$$

$$= \frac{2 - (\sin^2 \alpha + \sin^2 \gamma) - 4\cos^2 \beta}{1 - \sin \alpha \sin \gamma}$$

$$= \frac{2 - (4\sin^2 \beta - 2\sin \alpha \sin \gamma) - 4\cos^2 \beta}{1 - \sin \alpha \sin \gamma}$$

$$= \frac{2 - 4(1) + 2\sin \alpha \sin \gamma}{1 - \sin \alpha \sin \gamma}$$

$$= \frac{2(\sin \alpha \sin \gamma - 1)}{1 - \sin \alpha \sin \gamma} = -2$$

from (1)

$$4\sin^2 \beta = \sin^2 \alpha + \sin^2 \gamma + 2\sin \alpha \sin \gamma$$

37. Let $a = \tan \alpha, b = \tan \beta, c = \tan \gamma$.

$$\begin{aligned} \text{Then } a^2 b^2 + b^2 c^2 + c^2 a^2 + 2a^2 b^2 c^2 &= 1 \\ \Rightarrow a^2 b^2 + b^2 c^2 + c^2 a^2 + 2a^2 b^2 c^2 + 1 + a^2 + b^2 + c^2 - a^2 b^2 c^2 - 1 + 1 + a^2 + b^2 + c^2 - a^2 b^2 c^2 &= 1 + a^2 + b^2 + c^2 + a^2 b^2 + b^2 c^2 + c^2 a^2 \\ + a^2 b^2 c^2 &= 2 + a^2 + b^2 + c^2 - a^2 b^2 c^2 \\ &= (1 + a^2)(1 + b^2)(1 + c^2) \end{aligned}$$

$$= 2(a^2 b^2 + b^2 c^2 + c^2 a^2 + 2a^2 b^2 c^2)$$

$$+ a^2 + b^2 + c^2 - a^2 b^2 c^2$$

$$= a^2 + b^2 + c^2 + 2a^2 b + 2b^2 c^2$$

$$+ 2c^2 a^2 + 3a^2 b^2 c^2$$

$$= a^2(1 + b^2)(1 + c^2) + b^2(1 + c^2)(1 + a^2)$$

$$(1 + a^2)(1 + b^2)$$

$$\Rightarrow \frac{a^2}{1 + a^2} + \frac{b^2}{1 + b^2} + \frac{c^2}{1 + c^2} = 1$$

$$\Rightarrow \frac{\tan^2 \alpha}{1 + \tan^2 \alpha} + \frac{\tan^2 \beta}{1 + \tan^2 \beta} + \frac{\tan^2 \gamma}{1 + \tan^2 \gamma} =$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 1$$

38.
$$\frac{\sin x + \cos x}{\cos^3 x} = \tan x \sec^2 x + \sec^2 x$$

$$= \tan(1 + \tan^2 x) + 1$$

$$+ \tan^2 x = \tan^3 x + \tan^2 x + \tan x + 1$$

$$\Rightarrow a = 1, b = 1, c = 1, d = 1 \Rightarrow a + b + c + d = 4$$

39.
$$G.E = \sqrt{\cos^2 100^\circ} \sec 100^\circ$$

$$= -\cos 100^\circ \cdot \sec 100^\circ = -1$$

40.
$$a^2 = (\cos \theta + \sin \theta)^2 = (\cos^2 \theta + \sin^2 \theta)$$

$$+ 2\sin \theta \cos \theta = 1 + 2\sin \theta \cos \theta$$

$$\Rightarrow a^2 - 1 = 2\sin \theta \cos \theta$$

$$\Rightarrow \frac{a^2 - 1}{2} = \sin \theta \cos \theta \quad \dots \dots \text{(i)}$$

$$b = \tan \theta + \cot \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta \cos \theta} \Rightarrow \sin \theta \cos \theta = \frac{1}{b} \quad \dots \dots \text{(ii)}$$

$$\text{From (i) \& (ii); } \frac{a^2 - 1}{2} = \frac{1}{b} \Rightarrow b(a^2 - 1) = 2$$

41. Given equations are $a \sin x = b \cos x \dots \dots \text{(i)}$

and $a \sin^3 x + b \cos^3 x = \sin x \cos x \dots \dots \text{(ii)}$

From (ii);

$$(a \sin x) \sin^2 x + b \cos^3 x = \sin x \cos x$$

$$\Rightarrow (b \cos x) \sin^2 x + b \cos^3 x = \sin x \cos x$$

[From (i)]

$$\Rightarrow b \cos x (\sin^2 x + \cos^2 x) = \sin x \cos x$$

$$\Rightarrow b = \sin x$$

From (i);

$$a \sin x = b \cos x = \sin x \cos x \Rightarrow a = \cos x$$

$$\therefore a^2 + b^2 = \cos^2 x + \sin^2 x = 1$$

$$42. \csc \theta - \sin \theta = m \Rightarrow \frac{1 - \sin^2 \theta}{\sin \theta} = m$$

$$\Rightarrow m = \frac{\cos^2 \theta}{\sin \theta}$$

$$\text{similarly, } n = \frac{\sin^2 \theta}{\cos \theta}, (m^2 n)^{2/3} + (mn)^{2/3}$$

$$= \left(\frac{\cos^4 \theta}{\sin^2 \theta} \cdot \frac{\sin^2 \theta}{\cos \theta} \right) + \left(\frac{\cos^2 \theta}{\sin \theta} \cdot \frac{\sin^4 \theta}{\cos^2 \theta} \right)^{2/3}$$

$$= \cos^2 \theta + \sin^2 \theta = 1$$

$$43. x - a = x - x \cos^2 \alpha - y \sin^2 \alpha$$

$$= x \sin^2 \alpha - y \sin^2 \alpha = (x - y) \sin^2 \alpha$$

$$y - a = y - x \cos^2 \alpha - y \sin^2 \alpha$$

$$= -x \cos^2 \alpha + y \cos^2 \alpha = -(x - y) \cos^2 \alpha$$

$$\therefore (x - a)(y - a) + (x - y) \sin^2 \alpha \cos^2 \alpha = 0$$

44. Conceptual

$$45. a^2 + b^2 + c^2 = \cos^2 \phi \cos^2 \theta + \sin^2 \phi \sin \theta \cos^2 \delta$$

$$+ \cos^2 \phi \sin^2 \theta + \sin^2 \phi \cos^2 \theta \cos^2 \delta + \sin^2 \phi \sin^2 \delta$$

$$= \cos^2 \phi + \sin^2 \phi \cos^2 \delta + \sin^2 \phi \sin^2 \delta$$

$$= \cos^2 \phi + \sin^2 \phi = 1.$$

$$46. \text{Given, } (\cos x + \sin x)^2 + k \sin x \cos x - 1 = 0, \forall x$$

$$\Rightarrow \cos^2 x + \sin^2 x + 2 \cos x \sin x + k \sin x \cos x - 1 = 0, \forall x$$

$$\Rightarrow (k+2) \cos x \sin x = 0, \forall x \Rightarrow k+2=0 \Rightarrow k=-2.$$

$$47. \text{It is given that } \frac{\sin^4 A}{a} + \frac{\cos^4 A}{b} = \frac{1}{a+b}$$

$$\Rightarrow \frac{(1-\cos 2A)^2}{4a} + \frac{(1+\cos 2A)^2}{4b} = \frac{1}{a+b}$$

$$\Rightarrow b(a+b)(1-2\cos 2A + \cos^2 2A)$$

$$\Rightarrow \{b(a+b) + a(a+b)\} \cos^2 2A + 2(a+b)(a-b)$$

$$\cos 2A + a(a+b) + b(a+b) - 4ab = 0$$

$$\Rightarrow (a+b)^2 \cos^2 2A + 2(a+b)(a-b) \cos 2A + a(a+b)^2 = 0$$

$$\Rightarrow \{(a+b) \cos 2A + (a+b)\}^2 - 0 \text{ or}$$

$$\cos 2A = \frac{b-a}{b+a}$$

Hence,

$$\frac{\sin^8 A}{a^3} + \frac{\cos^8 A}{b^3} = \frac{(1-\cos 2A)^4}{16a^3} + \frac{(1+\cos 2A)^4}{16b^3}$$

$$= \frac{1}{16a^3} \left[1 - \frac{b-a}{b+a} \right]^4 + \frac{1}{16b^3} \left[1 + \frac{b-a}{b+a} \right]^4$$

$$= \frac{16a^4}{16a^3(b+a)^4} + \frac{16a^4}{16b^3(b+a)^4}$$

$$= \frac{1}{(b+a)^4} (a+b) = \frac{1}{(a+b)^3}$$

$$p = q = 0, r = 3; p + q + r = 3$$

$$48. \sin^3 x \sin 3x = \frac{1}{4} (3 \sin x - \sin 3x) \sin 3x$$

$$= \frac{3}{8} \cdot 2 \sin x \sin 3x - \frac{1}{8} \cdot 2 \sin^2 3x$$

$$= \frac{3}{8} (\cos 2x - \cos 4x) - \frac{1}{8} (1 - \cos 6x)$$

$$= \frac{1}{8} + \frac{3}{8} \cos 2x - \frac{3}{8} \cos 4x + \frac{1}{8} \cos 6x$$

....(i)

$$\text{and } \sum_{n=0}^{\infty} C_n \cos nx = c_0 + c_1 \cos x + c_2 \cos 2x$$

$$+ c_3 \cos 3x + \dots \dots c_n \cos nx \quad \dots \dots \text{(ii)}$$

Comparing both sides of (i) and (ii), we get $n=6$.

$$49. \quad f_n(\theta) = \frac{\sin(\theta/2)}{\cos(\theta/2)} \left[\frac{2\cos^2\theta/2}{\cos\theta} \cdot \frac{2\cos^2\theta}{\cos 2\theta} \cdot \frac{2\cos^2 2\theta}{\cos 4\theta} \dots \right]$$

Combine first two factors,

$$f_n(\theta) = \frac{\sin\theta}{\cos\theta} \left[\frac{2\cos^2\theta}{\cos 2\theta} \cdot \frac{2\cos^2\theta}{\cos 4\theta} \dots \right]$$

Again, combine first two factors,

$$f_n(\theta) = \tan 2\theta \left[\frac{2\cos^2\theta}{\cos 4\theta} \dots \right] = \tan(2^n\theta)$$

$$\therefore f_2\left(\frac{\pi}{16}\right) = \tan \frac{4\pi}{16} = \left(\frac{\pi}{4}\right) = 1$$

$$f_3\left(\frac{\pi}{32}\right) = \tan \frac{8\pi}{32} = \left(\frac{\pi}{4}\right) = 1$$

$$f_4\left(\frac{\pi}{64}\right) = \tan \frac{16\pi}{64} = \tan\left(\frac{\pi}{4}\right) = 1$$

$$f_5\left(\frac{\pi}{128}\right) = \tan 32 \frac{\pi}{128} = \tan\left(\frac{\pi}{4}\right) = 1.$$

$$50. \quad \text{We have } k = \sin \frac{\pi}{18} \cdot \sin \frac{5\pi}{18} \cdot \sin \frac{7\pi}{18} \\ = \cos\left(\frac{\pi}{2} - \frac{\pi}{18}\right) \cos\left(\frac{\pi}{2} - \frac{5\pi}{18}\right) \cos\left(\frac{\pi}{2} - \frac{7\pi}{18}\right)$$

$$= \cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9} = \frac{\sin 2^3 \frac{\pi}{9}}{2^3 \sin \frac{\pi}{9}} = \frac{\sin \frac{8\pi}{9}}{8 \sin \frac{\pi}{9}}$$

$$= \frac{\sin\left(\pi - \frac{\pi}{9}\right)}{8 \sin \frac{\pi}{9}} = \frac{1}{8}.$$

$$\text{Ans} = 16 \times 1/8 = 2$$

$$51. \quad \text{Since } \sin x + \sin^2 x = 1,$$

$$\Rightarrow \sin x = 1 - \sin^2 x = \cos^2 x \quad \dots \text{(i)}$$

The given expression is

$$= \cos^6 x (\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1) - 1 \\ = \cos^6 x (\cos^2 x + 1)^3 - 1, (\because \sin x = \cos^2 x) \\ = (\sin^2 x \sin x)^3 - 1 = 1 - 1 = 0.$$

$$52. \quad \text{If } L = M, \text{ then } L^2 = LM \text{ or } ML, \\ \text{Both } LM = ML, \\ \sec^2 4 - \tan^2 A = 1 \\ \therefore L^2 = M^2 = 1.$$

53. We have

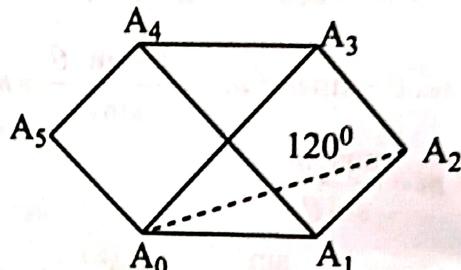
$$x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right) \\ \Rightarrow \cos \theta = \frac{k}{x}, \cos\left(\theta + \frac{2\pi}{3}\right) = \frac{k}{y}$$

$$\text{and } \cos\left(\theta + \frac{4\pi}{3}\right) = \frac{k}{z}$$

Hence

$$\frac{k}{x} + \frac{k}{y} + \frac{k}{z} = \cos \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \left(\theta + \frac{4\pi}{3}\right) \\ = \cos \theta + \cos\left(\frac{\pi}{3} - \theta\right) - \cos\left(\frac{\pi}{3} + \theta\right) \\ = \cos \theta - 2 \cos \frac{\pi}{3} \cos \theta = 0.$$

54. Each triangle is an equilateral triangle.



$$\text{Hence } A_0 A_1 = 1$$

$$A_0 A_2^2 = A_0 A_1^2 + A_1 A_2^2 - 2 A_0 A_1 A_1 A_2 \cos 120^\circ = 1 + 1 - 2, 1, 1 (-)$$

$$\Rightarrow A_0 A_2 = \sqrt{3} = A_0 A_4$$

$$\therefore A_0 A_1 \times A_0 A_2 \times A_0 A_4 = 1 \cdot \sqrt{3} \cdot \sqrt{3} = 3.$$

55. Here

$$3 \left\{ \sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4 (3\pi + \alpha) \right\} - 2 \left\{ \sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6 (5\pi + \alpha) \right\} \\ = 3 \left\{ (-\cos \alpha)^4 + (-\sin \alpha)^4 \right\} - 2 \left\{ \cos^6 \alpha + \sin^6 \alpha \right\} \\ = 3 \left\{ \cos^2 \alpha + \sin^2 \alpha \right\}^2 - 2 \sin^2 \alpha \cos^2 \alpha \\ - 2 \left[(\cos^2 \alpha + \sin^2 \alpha)^3 - 3 \cos^2 \alpha \sin^2 \alpha (\cos^2 \alpha + \sin^2 \alpha) \right] \\ = 3 - 6 \sin^2 \alpha \cos^2 \alpha - 2 + 6 \sin^2 \alpha \cos^2 \alpha = 3 - 2 = 1$$

56. Here $(\cot \alpha_1)(\cot \alpha_2) \dots (\cot \alpha_n) = 1$
 $\cos \alpha_1 \cos \alpha_2 \dots \cos \alpha_n = \sin \alpha_1 \sin \alpha_2 \dots \sin \alpha_n$
 Now, $(\cos \alpha_1 \cos \alpha_2 \dots \cos \alpha_n)^2$
 $= (\cos \alpha_1 \cos \alpha_2 \dots \cos \alpha_n)(\cos \alpha_1 \cos \alpha_2 \dots \cos \alpha_n)$
 $= (\cos \alpha_1 \cos \alpha_2 \dots \cos \alpha_n)(\sin \alpha_1 \sin \alpha_2 \dots \sin \alpha_n)$
 $= \frac{1}{2^n} \sin 2\alpha_1 \sin 2\alpha_2 \dots \sin 2\alpha_n.$

But each of $\sin 2\alpha_i \leq 1$

$$\therefore (\cos \alpha_1 \cos \alpha_2 \dots \cos \alpha_n)^2 \leq \frac{1}{2^n}.$$

But each of $\cos \alpha_i$ is positive.

$$\therefore \cos \alpha_1 \cos \alpha_2 \dots \cos \alpha_n \leq \sqrt{\frac{1}{2^n} \frac{1}{2^{n-2}}}$$

$$2^{n/2} \times 1 / 2^{n/2} = 1$$

57. Given that $A = \sin^8 \theta + \cos^{14} \theta$
 we know that $(\sin^2 \theta)^4 \leq \sin^2 \theta$ and
 $(\cos^2 \theta)^7 \leq \cos^2 \theta$

Adding, we get $\sin^8 \theta + \cos^{14} \theta \leq 1$
 $0 < \sin^8 \theta + \cos^{14} \theta \leq 1.$

Hence $0 < A \leq 1.$

58. θ is an acute angle so $0^\circ \leq \theta < 90^\circ$

$$\therefore 0 \leq \frac{p-6}{8-p} < 1$$

$$\Rightarrow 0 \leq (p-6) < (8-p) \Rightarrow 6 \leq p < 7.$$

59. $\sum \frac{\cot A + \cot B}{\tan A + \tan B} = \sum \frac{\frac{\cot A}{\sin A} + \frac{\cos B}{\sin B}}{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}$
 $\sum \frac{\sin B \cos A + \sin A \cos B}{\sin A \sin B} \cdot \frac{\cos A \cos B}{(\sin A \cos B + \cos A \sin B)}$
 $\sum \cot A \cot B$

As we know if, $A + B + C = \pi$, then

$$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1.$$

60. $a \sin^2 \theta + b \sin \theta \cos \theta + c \cos^2 \theta - \frac{1}{2}(a+c)$
 $= \frac{1}{2}[-a \cos 2\theta + b \sin 2\theta + c \cos 2\theta]$
 $= \frac{1}{2}[b \sin 2\theta - (a-c) \cos 2\theta]$
 $\therefore |b \sin 2\theta - (a-c) \cos 2\theta| \leq \sqrt{b^2 + (a-c)^2}$
 $\therefore \left| \frac{1}{2} \{b \sin 2\theta - (a-c) \cos 2\theta\} \right| \leq \frac{1}{2} \sqrt{b^2 + (a-c)^2}$
 $\Rightarrow \left| a \sin \theta + b \sin \theta \cos \theta + c \cos^2 \theta - \frac{1}{2}(a+c) \right| \leq \frac{1}{2} \sqrt{b^2 + (a-c)^2}$
 $\text{If } K^2 = a^2 + b^2 + c^2 - 2ac$
 $\text{If } x + y + z = 2 + 0 + 0 = 2$
61. We have
- $$\frac{x}{\cos \theta} = \frac{y}{\cos \left(\theta - \frac{2\pi}{3} \right)} = \frac{z}{\cos \left(\theta + \frac{2\pi}{3} \right)} = k$$
- $\Rightarrow x = k \cos \theta, y = k \cos \left(\theta - \frac{2\pi}{3} \right),$
- $\Rightarrow z = k \cos \left(\theta + \frac{2\pi}{3} \right)$
- $\Rightarrow x + y + z = k \left[\cos \theta + \cos \left(\theta - \frac{2\pi}{3} \right) + \cos \left(\theta + \frac{2\pi}{3} \right) \right]$
- $= k[(0) = 0]$
- $\Rightarrow x + y + z = 0.$
62. $\sqrt{3} \cosec 20^\circ - \sec 20^\circ = \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$
- $$= \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ} = \frac{2 \left[\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ \right]}{2 \sin 20^\circ \cos 20^\circ}$$
- $$= \frac{4 \cos(20^\circ + 30^\circ)}{\sin 40^\circ} = \frac{4 \cos 50^\circ}{\sin 40^\circ} = \frac{4 \sin 40^\circ}{\sin 40^\circ} = 4.$$

6.2

Periodicity and Extreme Values

Internals of the Topic

- ◆ Periodicity
- ◆ Extreme values



CRT CONCEPT

Periodic function: A real function $f: A \rightarrow R$ is said to be a periodic function if there exists a positive real number P such that $f(x+p) = f(x)$, $\forall x \in A$. Least value of P is known as the fundamental period of f .

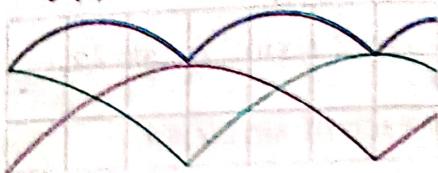
- The period of $f(ax+b) = \frac{p}{|a|}$
- Let $g(x) = \frac{C_1f_1(x) \pm C_2f_2(x)}{C_3f_3(x) \pm C_4f_4(x)}$ (where f_1, f_2, f_3, f_4 are trigonometric functions, and C_1, C_2, C_3, C_4 are constants).

Then period of $g(x) = \text{L.C.M. of period of } f_1, f_2, f_3, f_4$

Note:

- i) Numerator and Denominator can be extended to sum of any number of trigonometric functions.
- ii) If p and q are rational quantities. Then LCM of p and q exists
- iii) If p and q are irrational, then LCM of p and q does not exist unless they have same irrational surd.
- iv) LCM of rational and irrational is not possible.

$$f(t) = |\sin t| + |\cos t|$$



v) L.C.M $\left\{ \frac{a}{b}, \frac{c}{d}, \frac{e}{f} \right\} = \frac{\text{L.C.M}\{a, c, e\}}{\text{G.C.D}\{b, d, f\}}$

- If ' f ' is a periodic function with fundamental period ' P ' then ' $-f$ ' and $\frac{1}{f}$ also a periodic functions with period P .
- A constant function is periodic but not have a fundamental period.
- Periods of trigonometric functions given below:

Function	Period
1. $\sin(ax+b)$	$\frac{2\pi}{ a }$
2. $\cos(ax+b)$	$\frac{2\pi}{ a }$
3. $\tan(ax+b)$	$\frac{\pi}{ a }$
4. $\operatorname{Cosec}(ax+b)$	$\frac{2\pi}{ a }$
5. $\operatorname{Sec}(ax+b)$	$\frac{2\pi}{ a }$
6. $\operatorname{Cot}(ax+b)$	$\frac{\pi}{ a }$

- Period of $\{x\}$ is 1. where $\{\cdot\}$ denotes fractional part.
- Standard results :
 - i) Period of $|\sin x|, |\cos x|, |\tan x|, |\cot x|, |\sec x|, |\cosec x|$ is π
 - ii) Period of $|\sin x| + |\cos x|, |\tan x| + |\cot x|$ is $\frac{\pi}{2}$
 - iii) Period of $|\sin x + \cos x|, |\sin x - \cos x|$ is π

- iv) Period of $|\tan x + \cot x|, |\tan x - \cot x|$ is $\frac{\pi}{2}$
- v) If $n \in \mathbb{Z}^+$, the period of $\sin^{2n} x + \cos^{2n} x$, $\sec^{2n} x + \operatorname{cosec}^{2n} x, \tan^{2n} x + \cot^{2n} x$ is $\frac{\pi}{2}$
- vi) Period of $a\sin^{2n} x + b\cos^{2n} x, a\sec^{2n} x + b\operatorname{cosec}^{2n} x, a\tan^{2n} x + b\cot^{2n} x$ is π ($n, m \in \mathbb{Z}^+$)

Maximum and minimum values :

- i) Minimum value of $\sin x$ is -1
 ii) Maximum value of $\sin x$ is 1
 iii) Range of $\sin x$ is $[-1, 1]$
 i) Minimum value of $\cos x$ is -1
 ii) Maximum value of $\cos x$ is 1
 iii) Range of $\cos x$ is $[-1, 1]$

Minimum value of

$$a\cos x + b\sin x + c \text{ is } c - \sqrt{a^2 + b^2}$$

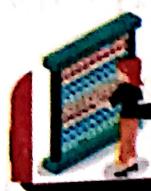
Maximum value of $a\cos x + b\sin x + c$ is

$$c + \sqrt{a^2 + b^2}$$

Range of $a\cos x + b\sin x + c$ is

$$\left[c - \sqrt{a^2 + b^2}, c + \sqrt{a^2 + b^2} \right]$$

The minimum value of $a^2 \sin^2 \theta + b^2 \operatorname{cosec}^2 \theta, a^2 \cos^2 \theta + b^2 \sec^2 \theta, a^2 \tan \theta + b^2 \cot \theta$ is $2ab$.



Concept Based Questions

1. If $f(x)$ is a periodic function with the period T then it can also have the period λT ($\lambda \in \mathbb{N}$)
 1) λT ($\lambda \in \mathbb{N}$) 2) $2\lambda T$ ($\lambda \in \mathbb{N}$)
 3) $(2\lambda - 1)T$ ($\lambda \in \mathbb{N}$) 4) All above
2. If $f(x+k) + f(x) = 0 \forall x \in \mathbb{R}$ and $k \in \mathbb{R}^+$, then the period of $f(x)$ is
 1) 0 2) $\frac{\pi}{2}$ 3) $2k$ 4) Does not exist

3. If g is periodic. Then
 1) fog will always be periodic.
 2) fog need not be periodic
 3) gof will always be periodic
 4) The period of g is same as gof .
4. If $f(x) = \sin x + \cos ax$ is a periodic function, then
 1) 'a' is a irrational 2) $a=0$ only
 3) 'a' is a rational number
 4) a has no

KEY

01) 4 02) 3 03) 1 04) 3

EXERCISE-I

CRTQ & SPO LEVEL-I

PERIODICITY

C.R.T.Q

Class Room Teaching Questions

1. Period of $\cos(x + 2x + 3x + \dots + nx)$ is
 1) $\frac{2\pi}{n(n+1)}$ 2) $\frac{4\pi}{n(n+1)}$
 3) $\frac{\pi}{n(n+1)}$ 4) $\frac{6\pi}{n(n+1)}$
2. Period of $\tan(x + 4x + 9x + \dots + n^2 x)$ is
 1) $\frac{2\pi}{n(n+1)(2n+1)}$ 2) $\frac{4\pi}{n(n+1)(2n+1)}$
 3) $\frac{6\pi}{n(n+1)(2n+1)}$ 4) $\frac{8\pi}{n(n+1)(2n+1)}$
3. Period of $\cos\left(x + \frac{x}{3} + \frac{x}{3^2} + \dots\right)$ is
 1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) π 4) $\frac{4\pi}{3}$
4. Period of $2 \sin^3 x - 3 \cos^3 x$ is
 1) $\frac{\pi}{2}$ 2) π 3) 2π 4) 4π

FOCUS TRACK

5. Period of $\cos^6 x + \sin^6 x$ is

- 1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$ 4) 2π

6. Period of $\sin x \sin(120^\circ - x) \sin(120^\circ + x)$ is

- 1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) π 4) $\frac{\pi}{2}$

7. If $f(x) = \sin^2\left(\frac{\pi}{8} + \frac{x}{2}\right) \cdot \sin^2\left(\frac{\pi}{8} - \frac{x}{2}\right)$,
then the period of f is

- 1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) 2π

8. Sine function whose period is 6 is

- 1) $\sin \frac{2\pi x}{3}$ 2) $\sin \frac{\pi x}{3}$
3) $\sin \frac{\pi x}{6}$ 4) $\sin \frac{3\pi x}{2}$

9. The cosecant function whose period is 4 is

- 1) $\operatorname{cosec} \frac{\pi x}{3}$ 2) $\operatorname{cosec} \frac{2\pi x}{5}$
3) $\operatorname{cosec} 2\pi x$ 4) $\operatorname{cosec} \frac{\pi x}{2}$

10. Period of $\frac{2\sin 2x - 5\cos 2x}{7\cos x - 8\sin x}$ is

- 1) π 2) 2π 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3}$

11. Period of $\frac{\sin(x+a)}{\cos x}$

- 1) $\frac{\pi}{2}$ 2) π 3) 2π 4) 3π

12. Period of $\sin(e^{\tan x} + e^{\cot x})$ is

- 1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$ 4) 2π

S.P.Q.

Student Practice Questions

13. Period of $\cot\left(\frac{5-7x}{2}\right)$ is

- 1) $\frac{\pi}{2}$

- 2) $\frac{2\pi}{7}$

- 3) $\frac{\pi}{7}$

- 4) $\frac{7\pi}{2}$

14. Period of $\tan(x + 2x + 3x + \dots + nx)$ is

- 1) $\frac{2\pi}{n(n+1)}$

- 2) $\frac{4\pi}{n(n+1)}$

- 3) $\frac{\pi}{n(n+1)}$

- 4) $\frac{6\pi}{n(n+1)}$

15. Period of $\cos(x + 4x + 9x + \dots + n^2 x)$ is

- 1) $\frac{2\pi}{n(n+1)(2n+1)}$ 2) $\frac{4\pi}{n(n+1)(2n+1)}$

- 3) $\frac{12\pi}{n(n+1)(2n+1)}$ 4) $\frac{8\pi}{n(n+1)(2n+1)}$

16. Period of $\operatorname{cosec}\left(x + \frac{x}{2} + \frac{x}{4} + \dots\right)$ is

- 1) π 2) 2π 3) $\frac{\pi}{2}$

17. Period of $6\cos^4 x - 7\sin^4 x$ is

- 1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$

18. The period of $\sin^3 x + \cos^3 x$ is

- 1) $\frac{\pi}{3}$ 2) π 3) 2π

19. Period of $\cos x \cos(60^\circ - x) \cos(60^\circ + x)$

- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{2\pi}{3}$

20. The tangent function whose period is

- 1) $\tan \frac{\pi x}{2}$

- 2) $\tan \frac{\pi x}{3}$

- 3) $\tan \frac{3\pi x}{2}$

- 4) $\tan \frac{2\pi x}{3}$

21. The cotangent function whose period is 3π

- 1) $\cot 2x$

- 2) $\cot 4x$

- 3) $\cot \frac{3x}{2}$

- 4) $\cot \frac{x}{3}$

22. Period of $\tan 4x + \sec 4x$ is

- 1) 2π 2) $\frac{3\pi}{2}$ 3) π 4) $\frac{\pi}{2}$

23. The period of $\frac{\sin(2\pi x+a)}{\sin(2\pi x+b)}$ is

- 1) $\frac{a}{b}$ 2) 1 3) 2 4) π

24. The period of $\sin(\pi \sin \theta)$ is

- 1) π 2) $\frac{\pi}{2}$ 3) 2π 4) $\frac{\pi}{4}$

EXTREME VALUES

C.R.T.Q

Class Room Teaching Questions

25. The range of $f(x) = -3\cos\sqrt{3+x+x^2}$

- 1) $[-1, 1]$ 2) $[-2, 2]$
3) $[-3, 3]$ 4) $[-4, 4]$

26. The minimum and maximum values of $8\cos 3x - 15\sin 3x$ are

- 1) -7, 7 2) -23, 23
3) -17, 17 4) -15, 8

27. The maximum value of

$$\frac{3}{5\sin x - 12\cos x + 19}$$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

28. The range of $\sin^6 x + \cos^6 x$

- 1) $[-\frac{1}{4}, \frac{1}{4}]$ 2) $[\frac{1}{4}, \frac{1}{2}]$
3) $[\frac{1}{4}, 1]$ 4) $[-\frac{1}{2}, -\frac{1}{4}]$

29. The minimum and maximum values of $\sin^2(60^\circ - x) + \sin^2(60^\circ + x)$ are

- 1) $-\frac{1}{2}, \frac{1}{2}$ 2) $\frac{1}{2}, 1$
3) $\frac{1}{2}, \frac{3}{2}$ 4) $\frac{3}{2}, 2$

30. $\sin^2(60^\circ + x) - \sin^2(60^\circ - x) \in \left[-\frac{k}{2}, \frac{k}{2}\right] \Rightarrow k =$

- 1) 3 2) 2 3) $\sqrt{3}$ 4) $\sqrt{2}$

31. $\sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) \in [-k, k] \Rightarrow k =$

- 1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{3}{4}$

32. $16\cos^5 x - 20\cos^3 x + 5\cos x \in$

- 1) $[-1, 1]$ 2) $[-\frac{1}{4}, \frac{1}{4}]$
3) $[-\frac{3}{4}, \frac{3}{4}]$ 4) $[-2, 2]$

33. $3\sin^2 x - 4\cos^2 x \in$

- 1) $[0, 3]$ 2) $[-4, 0]$
3) $[-3, 4]$ 4) $[-4, 3]$

34. The maximum value of

$$\cos x \left(\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} \right)$$

- 1) 4 2) 3
3) 2 4) 1

35. $A + B = 90^\circ \Rightarrow \cos A \cos B \in$

- 1) $[-\frac{1}{3}, \frac{1}{3}]$ 2) $[-\frac{1}{2}, \frac{1}{2}]$
3) $[-\frac{1}{4}, \frac{1}{4}]$ 4) $[-\frac{3}{4}, \frac{3}{4}]$

36. $A + B = 90^\circ \Rightarrow \sin A + \sin B \in$

- 1) $[-1, 1]$ 2) $[-\sqrt{2}, \sqrt{2}]$
3) $[-\sqrt{3}, \sqrt{3}]$ 4) $[-2, 2]$

37. Minimum value of

- 9 $\sec^2 \theta + 4 \operatorname{cosec}^2 \theta + 7$ is
- 1) 32 2) 25 3) 20 4) 49

S.P.Q. Student Practice Questions

38. $16 \sin x \cos x \cos 2x \cos 4x \cos 8x \in$

- 1) $[-1, 1]$ 2) $[-\frac{1}{4}, \frac{1}{4}]$
3) $[-\frac{3}{4}, \frac{3}{4}]$ 4) $[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}]$

FOCUS TRACK

39. The minimum and maximum values of $2\sqrt{2}\cos x + \sin x$ are
 1) -1, 1 2) $-2\sqrt{2}, 2\sqrt{2}$
 3) 1, 8 4) -3, 3

40. Maximum value of $\frac{1}{\sqrt{7}\sin x + \sqrt{29}\cos x + 7}$
 1) 13 2) 1 3) 1/13 4) 49

41. The minimum and maximum values of $\sin^4 x + \cos^4 x$ are
 1) $\frac{1}{2}, \frac{3}{2}$ 2) $\frac{1}{2}, 1$ 3) $1, \frac{3}{2}$ 4) 1, 2

42. $\cos^2(60^\circ - x) + \cos^2(60^\circ + x) \in$
 1) $[-\frac{1}{2}, \frac{1}{2}]$ 2) $[\frac{1}{2}, 1]$
 3) $[\frac{1}{2}, \frac{3}{2}]$ 4) $[\frac{3}{2}, 2]$

43. $\cos^2(60^\circ + x) - \cos^2(60^\circ - x) \in [-\frac{m}{2}, \frac{m}{2}]$
 $\Rightarrow m =$
 1) -1 2) 3 3) $-\sqrt{3}$ 4) $\sqrt{3}$

44. $\sin^2 x + 4\sin x + 5 \in [k, 5k] \Rightarrow k =$
 1) 1 2) 2 3) 3 4) 4

45. $3\sin^2 x + 4\cos^2 x \in$
 1) [0, 3] 2) [0, 4] 3) [3, 4] 4) [-4, -3]

46. The minimum value of
 $\sin x \left(\frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x} \right)$ is
 1) 1 2) 2 3) 3 4) 4

47. $A + B = 90^\circ \Rightarrow \sin A \sin B \in$
 1) $[-\frac{1}{4}, \frac{1}{4}]$ 2) $[-\frac{1}{3}, \frac{1}{3}]$
 3) $[-\frac{3}{4}, \frac{3}{4}]$ 4) $[-\frac{1}{2}, \frac{1}{2}]$

48. $A + B = 90^\circ \Rightarrow \cos A - \cos B \in$
 1) [-1, 1] 2) $[-\sqrt{2}, \sqrt{2}]$
 3) $[-\sqrt{3}, \sqrt{3}]$ 4) [-2, 2]

49. The minimum value of $27\tan^2 \theta + 3\cot^2 \theta$
 1) 15 2) 18 3) 24 4) 3

KEY

01) 2	02) 3	03) 4	04) 3	05)
06) 2	07) 4	08) 2	09) 4	10)
11) 2	12) 2	13) 2	14) 1	15)
16) 1	17) 2	18) 3	19) 3	20)
21) 4	22) 4	23) 2	24) 3	25)
26) 3	27) 2	28) 3	29) 3	30)
31) 3	32) 1	33) 4	34) 3	35)
36) 2	37) 1	38) 1	39) 2	40)
41) 2	42) 3	43) 4	44) 2	45)
46) 2	47) 4	48) 2	49) 2	

Hints & Solutions

1. $p = \frac{2\pi}{|a|} = \frac{2\pi}{n(n+1)} = \frac{4\pi}{2n(n+1)}$

2. $p = \frac{\pi}{|a|} = \frac{\pi}{n(n+1)(2n+1)} = \frac{6\pi}{6n(n+1)(2n+1)}$

3. $1 + \frac{1}{3} + \frac{1}{3^2} + \dots = \frac{1}{1-1/3} = \frac{3}{2}$

\therefore Period of $\cos\left(\frac{3}{2}x\right) = \frac{2\pi}{3/2} = \frac{4\pi}{3}$

4. By verification

5. When coefficients are equal and powers are even then period is $\frac{\pi}{2}$

6. $\sin x \sin(120^\circ - x) \sin(120^\circ + x) = \frac{1}{4} \sin 3x$

- ∴ Period = $\frac{2\pi}{3}$
7. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
8. $\frac{2\pi}{|a|} = 6 \Rightarrow |a| = \frac{\pi}{3}$
9. $\frac{2\pi}{|a|} = 4$ 10. Conceptual
11. L.C.M of $(\pi, 2\pi)$
12. Cosa, Tanx + Sina,
∴ period is π
13. $p = \frac{\pi}{|a|} = \frac{\pi}{\frac{7}{2}} = \frac{2\pi}{7}$
14. $p = \frac{\pi}{|a|} = \frac{\pi}{\frac{n(n+1)}{2}} = \frac{2\pi}{n(n+1)}$
15. $p = \frac{2\pi}{|a|} = \frac{2\pi}{\frac{n(n+1)(2n+1)}{6}} = \frac{12\pi}{n(n+1)(2n+1)}$
16. $1 + \frac{1}{2} + \frac{1}{2^2} + \dots = \frac{1}{1-1/2} = 2$
 \therefore Period of cosec(2x) = $\frac{2\pi}{2} = \pi$
17. When coefficients are different and power are even period is π
18. power is odd number.
19. $\cos x \cos(60^\circ - x) \cos(60^\circ + x) = \frac{1}{4} \cos 3x$
 $\text{Period} = \frac{2\pi}{3}$
20. $\frac{\pi}{|a|} = \frac{2}{3}$ 21. $\frac{\pi}{|a|} = 3\pi$
22. L.C.M of $\left\{\frac{\pi}{4}, \frac{\pi}{2}\right\} = \frac{\pi}{2}$
23. L.C.M of (1, 1) 24. Period of $\sin \theta$
25. L.C.M of period of Tanx, Cotx is π

26. $f(x) \in [-3, 1]$
27. Min = $-\sqrt{a^2 + b^2}$; Max = $\sqrt{a^2 + b^2}$
28. Max. value of $\frac{3}{5\sin x - 12\cos x + 19} =$
 $\frac{3}{19 - \sqrt{5^2 + (-12)^2}} = \frac{3}{19 - 13} = \frac{3}{6} = \frac{1}{2}$
29. $\sin^6 x + \cos^6 x = 1 - \frac{3}{4} \sin^2 2x$
30. If $f(x) = \sin^2(\alpha - x) + \sin^2(\alpha + x)$
for $\alpha = 60^\circ, 120^\circ, 240^\circ, 300^\circ$
range is $\left[\frac{1}{2}, \frac{3}{2}\right]$
31. If $f(x) = \sin^2(\alpha - x) - \sin^2(\alpha + x)$
for $\alpha = 60^\circ, 120^\circ, 240^\circ, 300^\circ$
range is $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$
32. If
 $f(0) = \sin 0 \sin(\alpha - 0) \sin(\alpha + 0) = \frac{1}{4} \sin 30$
for $\alpha = 60^\circ, 120^\circ, 240^\circ, 300^\circ$, range is
 $\left[-\frac{1}{4}, \frac{1}{4}\right]$
33. Put $\cos x = \pm 1$
34. Put $\cos^2 x = 1 - \sin^2 x$
35. $\frac{\cos^2 x}{1 - \sin x} + 1 - \sin x = 2$
36. $A + B = 90^\circ \Rightarrow \cos A \cos B$
 $= \frac{1}{2} \sin 2A \in \left[-\frac{1}{2}, \frac{1}{2}\right]$
37. $(A + B) = 90^\circ \Rightarrow \sin A + \sin B =$
 $\sin A + \cos A \in [-\sqrt{2}, \sqrt{2}]$
38. $\sin 16x \in [-1, 1]$

39. $\text{Min} = -\sqrt{a^2 + b^2}; \text{Max} = \sqrt{a^2 + b^2}$

40. Find the minimum value of dr.

41. $\sin^4 x + \cos^4 x = 1 - \frac{1}{2} \sin^2 2x$

42. If $f(x) = \cos^2(\alpha - x) + \cos^2(\alpha + x)$
for $\alpha = 60^\circ, 120^\circ, 240^\circ, 300^\circ$, range is

$$\left[\frac{1}{2}, \frac{3}{2} \right]$$

43. If $f(x) = \cos^2(\alpha - x) - \cos^2(\alpha + x)$
for $\alpha = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

range is $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} \right]$

44. Put $\sin x = \pm 1$

45. $3\sin^2 x + 4\cos^2 x = 7\sin^2 x - 4$

46. $(1-\cos x) + (1+\cos x) = 2$

47. $(A+B) = 90^\circ \Rightarrow \sin A \sin B$

$$= \frac{1}{2} \sin 2A \in \left[-\frac{1}{2}, \frac{1}{2} \right]$$

48. $A+B = 90^\circ \Rightarrow \cos A - \cos B = \cos A - \sin A$
 $\Rightarrow \left[-\sqrt{a^2 + b^2}, +\sqrt{a^2 + b^2} \right]$

49. A.M \geq G.M

EXERCISE-II

CRTQ & SPQ LEVEL-II

PERIODICITY

C.R.T.Q

Class Room Teaching Questions

1. The period of $f(x) = \sin x + \{x\}$, where $\{x\}$ is fractional part of x is

- 1) 0 2) 1 3) 2π 4) Does not exist

2. Period of $\tan\left(\frac{3\pi x}{4}\right) - 2\sec\left(\frac{\pi x}{3}\right) + 5\sin\left(\frac{2\pi x}{5}\right)$ is

- 1) 20 2) 30 3) 60 4) 80

3. The period of $f(x) = \sin\frac{\pi x}{n!} - \cos\frac{\pi x}{(n+1)!}$

1) $(n+1)!$

2) $2(n+1)!$

3) $2(n)!$

4) $\frac{(n+1)}{2!}$

4. The value of integer n for which the function

$f(x) = \frac{\sin nx}{\sin(x/n)}$ has 4π as its period

- 1) 2 2) 3 3) 4 4) 5

5. Let $f(x) = \cos\sqrt{p}x$ where $p = [x]$ where $[x]$ is the integral part of x. If the period of f(x) is π , then $a \in$

- 1) $[4, 5]$ 2) $[4, 5)$ 3) $(4, 5]$ 4) $(4, 5)$

6. If the period of $\frac{\cos(\sin(nx))}{\tan(x/n)}$, $n \in N$

is 6π then $n =$

- 1) 3 2) 2 3) 6 4)

7. The period of the function

$$\left| \sin^3 \frac{x}{2} \right| + \left| \cos^5 \frac{x}{5} \right|$$

- 1) 2π 2) 10π 3) 8π 4) 5π

8. The period of the function

$f(x) = [6x+7] + \cos \pi x - 6x$, where $[.]$

denotes the greatest integer function,

- 1) 3 2) 2π 3) 2 4) π

9. The function $\sin(x^2) + \cos\sqrt{x}$ is

- 1) Periodic with period π
 2) Periodic with period 2π
 3) Periodic with period 0
 4) Not a Periodic function

S.P.Q.**Student Practice Questions**

1. Period of $\sin x \cos\left(x + \frac{\pi}{4}\right)$ is
 1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$ 4) 2π
2. Period of $\sin\frac{3\pi x}{2} + \cos\frac{\pi x}{2}$ is
 1) 1 2) 2 3) 3 4) 4
3. Period of $\cos\frac{\pi x}{4} - 2\operatorname{cosec}\frac{\pi x}{6} + 5\tan\frac{\pi x}{3}$ is
 1) 12 2) 24 3) 48 4) 6
4. Period of $\cos\frac{3x}{5} + 2\sin\frac{2x}{7} - 5\cot 14x$ is
 1) 7π 2) 14π 3) 28π 4) 70π
5. If $n \in \mathbb{N}$, and the period of $\frac{\cos nx}{\sin\left(\frac{x}{n}\right)}$ is 4π , then $n =$
 1) 4 2) 3 3) 2 4) 1
6. Period of $\sin\sqrt{x} + \cos\sqrt{x} =$
 1) π 2) 2π
 3) 1 4) does not exist

EXTREME VALUES**C.R.T.Q****Class Room Teaching Questions**

5. $\sin^2 x + \cos^4 x \in$
 1) $\left[\frac{1}{4}, \frac{1}{2}\right]$ 2) $\left[\frac{3}{4}, 1\right]$
 3) $\left[-1, -\frac{3}{4}\right]$ 4) $\left[-\frac{3}{4}, \frac{3}{4}\right]$
7. The minimum and maximum values of $\sin\left(x + \frac{\pi}{4}\right) \sin\left(x - \frac{\pi}{4}\right)$ are
 1) -1, 1 2) $-\frac{1}{2}, \frac{1}{2}$ 3) $-\frac{3}{2}, \frac{3}{2}$ 4) $-\frac{1}{4}, \frac{1}{4}$

5.2 PERIODICITY AND EXTREME VALUES**18. The minimum and maximum values of**
 $\cos x + 3\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) + 6$ are

- 1) 1, 11 2) 11, -1 3) 6, 5 4) 5, 6

$(\sin x + \cos x)^2 + \cos^2\left(\frac{\pi}{4} + x\right) \in$

- 1) [0, 1] 2) [0, 2] 3) [1, 2] 4) [0, 3]

$A = \sin^8 \theta + \cos^{14} \theta$, then for all values of θ

- 1)
- $0 < A \leq 1$
- 2)
- $1 < 2A \leq 3$
-
- 3)
- $A \geq 1$
- 4)
- $0 \leq A \leq \frac{1}{2}$

21. For any real θ the maximum value of $\cos^2(\cos \theta) + \sin^2(\sin \theta)$ is

- 1) 1 2)
- $1 + \sin^2 1$
-
- 3)
- $1 + \cos^2 1$
- 4)
- $2 + \cos^2 1$

22. If $5\cos x + 12\cos y = 13$, then the maximum value of $5\sin x + 12\sin y$ is

- 1) 12 2)
- $\sqrt{120}$
- 3)
- $\sqrt{20}$
- 4) 13

S.P.Q.**Student Practice Questions**

23. $\cos^2 x + \sin^4 x \in$

- 1)
- $\left[\frac{1}{2}, 1\right]$
- 2)
- $\left[1, \frac{3}{2}\right]$
- 3)
- $\left[\frac{3}{2}, 2\right]$
- 4)
- $\left[\frac{3}{4}, 1\right]$

24. $\cos\left(x + \frac{\pi}{6}\right) \cos\left(x - \frac{\pi}{6}\right) \in$

- 1)
- $\left[-\frac{1}{4}, \frac{1}{4}\right]$
- 2)
- $\left[-\frac{3}{4}, \frac{3}{4}\right]$
-
- 3)
- $\left[-\frac{1}{4}, \frac{3}{4}\right]$
- 4)
- $\left[-\frac{1}{2}, \frac{1}{2}\right]$

25. $5\cos\theta + 3\cos\left(\theta - \frac{\pi}{3}\right) + 8 \in$

- 1) [-7, 1] 2) [1, 8] 3) [1, 15] 4) [2, 15]

26. $(\sin x - \cos x)^2 + \cos^2\left(\frac{\pi}{4} - x\right) \in$

- 1) [0, 1] 2) [0, 2] 3) [1, 2] 4) [0, 3]

FOCUS TRACK

JEE - MAIN

27. For all values of θ , the values of

$$3-\cos\theta+\cos\left(\theta+\frac{\pi}{3}\right)$$

- lie in the interval
 1) $[-2, 3]$ 2) $[-2, 1]$ 3) $[2, 4]$ 4) $[1, 5]$

KEY

- 01) 4 02) 3 03) 2 04) 1 05) 2
 06) 3 07) 2 08) 3 09) 4 10) 2
 11) 4 12) 2 13) 4 14) 3 15) 4
 16) 2 17) 2 18) 1 19) 3 20) 1
 21) 2 22) 2 23) 4 24) 3 25) 3
 26) 3 27) 3

Hints & Solutions

1. Does not exists

2. L.C.M of $\left\{\frac{4}{3}, 6, 5\right\} \Rightarrow 60$

3. L.C.M of $\{2n!, 2(n+1)!\} = 2(n+1)!$

4. Period of $\sin nx = \frac{2\pi}{n}$,

Period of $\sin \frac{x}{n} = 2n\pi$

$$\therefore 2n\pi = 4\pi \Rightarrow n = 2$$

$$5. \frac{2\pi}{\sqrt{p}} = \pi$$

6. The period of $\cos(\sin nx)$ is $\frac{\pi}{n}$

period of $\tan\left(\frac{x}{n}\right)$ is $n\pi$.

$$\text{Thus, } 6\pi = LCM\left(\frac{\pi}{n}, \pi n\right)$$

By checking $n = 6$

7. The period of $\left|\sin^3 \frac{x}{2}\right| = 2\pi$

The period of $\left|\cos^5 \frac{x}{5}\right| = 5\pi$,

LCM is 10π

$$8. f(x) = 7 + \cos \pi x - \{6x\}$$

9. It is not a periodic function

10. Expand and L.C.M of $\{\pi, \pi\} = \pi$

$$11. \text{L.C.M of } \left\{\frac{4}{3}, 4\right\} = 4$$

$$12. \text{L.C.M of } \{8, 12, 3\} = 24$$

$$13. \text{L.C.M of } \left\{\frac{10\pi}{3}, 7\pi, \frac{\pi}{14}\right\} = 70\pi$$

$$14. \text{L.C.M. of } \left(\frac{2\pi}{n}, 2\pi n\right) = 4\pi$$

15. Here we have \sqrt{x} , so period does not exist

$$16. \sin^2 x + \cos^4 x = 1 - \frac{1}{4} \sin^2 2x$$

$$17. \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$$

$$18. \cos x + 3\sqrt{2} \left[\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right] \\ = 3\sin x + 4\cos x + 6$$

$$19. (\sin x + \cos x)^2 + \cos^2\left(\frac{\pi}{4} + x\right) =$$

$$2\sin^2\left(\frac{\pi}{4} + x\right) + \cos^2\left(\frac{\pi}{4} + x\right)$$

20. Clearly we know that $A > 0$.

$$\cos^{14} \theta \leq \cos^2 \theta \leq 1 \text{ and}$$

$$\sin^8 \theta \leq \sin^2 \theta \leq 1$$

$$\Rightarrow A \leq 1, \text{ Hence } 0 < A \leq 1.$$

21. $\cos^2(\cos \theta) + \sin^2(\sin \theta)$

$$+ \sin^2(\cos \theta) - \sin^2(\cos \theta)$$

$$= 1 + \sin^2(\sin \theta) - \sin^2(\cos \theta), \text{ which is maximum when } \cos \theta = 0, \sin \theta = 1$$

Maximum value is $1 + \sin^2 1$.

22. Take $5\sin x + 12\sin y = k$ and squaring and adding the both the equations.

23. $\cos^2 x + \sin^4 x = 1 - \frac{1}{4} \sin^2 2x$

24. $\cos\left(x + \frac{\pi}{6}\right)\cos\left(x - \frac{\pi}{6}\right) = \cos^2 \frac{\pi}{6} - \sin^2 x$

$$= \frac{3}{4} - \sin^2 x$$

25. $5\cos \theta + 3\left[\frac{1}{2}\cos \theta + \frac{\sqrt{3}}{2}\sin \theta\right] + 8 =$

$$\frac{13}{2}\cos \theta + \frac{3\sqrt{3}}{2}\sin \theta + 8 \in$$

$$\left[c - \sqrt{a^2 + b^2}, c + \sqrt{a^2 + b^2}\right]$$

26. $(\sin x - \cos x)^2 + \cos^2\left(\frac{\pi}{4} - x\right) =$

$$2\sin^2\left(\frac{\pi}{4} - x\right) + \cos^2\left(\frac{\pi}{4} - x\right)$$

27. Given $3 - \cos \theta + \cos\left(\theta + \frac{\pi}{3}\right)$

$$= 3 - \cos \theta + \cos \theta \frac{1}{2} - \sin \theta \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2}\cos \theta - \frac{\sqrt{3}}{2}\sin \theta + 3$$

Which is in the form of
 $a\cos \theta + b\sin \theta + c$

$$\text{Range} = \left[C - \sqrt{a^2 + b^2}, C + \sqrt{a^2 + b^2}\right]$$

$$a = \frac{1}{2}, b = \frac{\sqrt{3}}{2}, c = 3$$

$$\therefore \text{Range} = [2, 4]$$

