

- 4'. Then the moment of the force 'P' about 'O' is given by  $\vec{P} \times \vec{F}$   
 If  $\vec{a} = a_1\vec{i} + a_2\vec{m} + a_3\vec{n}$ ,  $\vec{b} = b_1\vec{i} + b_2\vec{m} + b_3\vec{n}$ ,  
 where  $\vec{i}, \vec{m}, \vec{n}$  form a right handed system of non-coplanar vectors, then

$$\vec{a} \cdot \vec{b} = \begin{vmatrix} \vec{m} & \vec{n} & \vec{i} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**Some useful Results:** For any vector  $\vec{a}$ ,

$$|\vec{a} \times \vec{i}|^2 + |\vec{a} \times \vec{j}|^2 + |\vec{a} \times \vec{k}|^2 = 2|\vec{a}|^2$$

$\vec{a}, \vec{b}, \vec{a} \times \vec{b}$  form a right handed system.

$\vec{a}, \vec{b}, \vec{b} \times \vec{a}$  form a left handed system.

If  $\vec{a}, \vec{b}, \vec{c}$  are mutually perpendicular vectors then  $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$  are also mutually perpendicular vectors.

If  $\vec{a}, \vec{b}, \vec{c}$  are in right handed system then  $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$  are also in right handed system.

4. If  $\vec{p} = \vec{a} + \vec{b}, \vec{q} = \vec{a} - \vec{b}, |\vec{a}| = |\vec{b}| = r$  then  $|\vec{p} \times \vec{q}| = |\vec{b}| = r$ , then  $|\vec{p} \times \vec{q}| =$

$$1) \sqrt{r^2 - (\vec{a} \cdot \vec{b})^2} \quad 2) 2\sqrt{r^2 - (\vec{a} \cdot \vec{b})^2}$$

$$3) \sqrt{r^2 - (\vec{a} \cdot \vec{b})^2} \quad 4) 0$$

5. Number of unit vectors perpendicular to two non zero vectors  $\vec{a}$  and  $\vec{b}$  are

- 1) one
- 2) two
- 3) infinite
- 4) three

6. If  $\vec{a}, \vec{b}, \vec{c}$  are three vectors such that

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$
 then

$$1) \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

$$2) \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$$

$$3) \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$$

$$4) \vec{a} \times \vec{b} = \vec{0}$$

Two planes are perpendicular to each other. One of them contains vectors  $\vec{a}, \vec{c}$  and other contains  $\vec{b}, \vec{d}$  then

$$(\vec{a} \times \vec{c}) \cdot (\vec{b} \times \vec{d}) =$$

$$1) 1 \quad 2) 0$$

$$3) [\vec{a} \vec{b} \vec{c}] \quad 4) [\vec{b} \vec{c} \vec{d}]$$

## Concept

Based Questions

If Magnitude of cross product and dot product of two non zero, non parallel vectors are equal then angle between two vectors is

$$1) \frac{\pi}{4} \quad 2) \frac{\pi}{2} \quad 3) \frac{3\pi}{2} \quad 4) \frac{\pi}{3}$$

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 =$$

$$1) |\vec{a}|^2 \quad 2) |\vec{b}|^2$$

$$3) |\vec{a}|^2 |\vec{b}|^2 \quad 4) |\vec{a}| |\vec{b}|$$

5. If  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$  and  $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$  then

$$1) \vec{a} \text{ is perpendicular to } \vec{b} - \vec{c}$$

$$2) \text{either } \vec{a} = 0 \text{ or } \vec{b} = \vec{c}$$

$$3) \vec{a} \text{ is parallel to } \vec{b} - \vec{c}$$

$$4) \vec{a} = \vec{b} = \vec{c}$$

8. Let  $\vec{a}, \vec{b}, \vec{c}$  are position vectors of a triangle ABC. Then length of perpendicular from vertex 'C' to AB is

$$1) \frac{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}{|\vec{a} - \vec{b}|} \quad 2) \frac{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}{|\vec{b} - \vec{c}|}$$

$$3) \frac{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}{|\vec{a} - \vec{c}|} \quad 4) |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$$

9. Perpendicular distance of any point  $\vec{a}$  on to the line  $\vec{r} = \vec{b} + t\vec{c}$  (t is scalar) is

$$1) \frac{(\vec{a} - \vec{b}) \times \vec{c}}{|\vec{c}|}$$

$$2) \frac{(\vec{a} - \vec{b}) \times \vec{c}}{|\vec{c}|}$$

$$3) \frac{|\vec{a} - \vec{b}|}{|\vec{c}|}$$

$$4) \frac{(\vec{a} - \vec{b}) \times \vec{c}}{|\vec{c}|}$$

# EXERCISE-1

## C.R.T.Q & S.P.Q LEVEL-1

### R.H & L.H SYSTEM

#### C.R.T.Q Class Room Teaching Questions

The vector  $\bar{a} = 1/7(2\bar{i} + 3\bar{j} + 6\bar{k})$ ,

$$\bar{b} = \frac{1}{7}(3\bar{i} - 6\bar{j} + 2\bar{k}),$$

$$\bar{c} = \frac{1}{7}(6\bar{i} + 2\bar{j} - 3\bar{k}) \text{ form}$$

1) a right handed system

2) a left handed system

3) an orthogonal system

4) an orthonormal system

$\bar{c}$  is a unit Vector orthogonal to  $\bar{a}, \bar{b}$  and  $\bar{a}, \bar{b}, \bar{c}$  are in R.H.S

and  $\bar{a}, \bar{b}, \bar{c}$  are in R.H.S

$\bar{a} = \bar{i} + \bar{j} + \bar{k}, \bar{b} = 2\bar{j} + 2\bar{k}$  then  $\bar{c} =$

$$1) \frac{\bar{i} + \bar{j}}{\sqrt{2}} \quad 2) \frac{\bar{j} + \bar{k}}{\sqrt{2}} \quad 3) \frac{\bar{i} - \bar{k}}{\sqrt{2}} \quad 4) \frac{\bar{k} - \bar{j}}{\sqrt{2}}$$

#### S.P.Q. Student Practice Questions

If the Vector  $\bar{c}, \bar{a} = x\bar{i} + y\bar{j} + z\bar{k}, \bar{b} = \bar{j}$  are such that  $\bar{a}, \bar{c}, \bar{b}$  form R.H.S then  $\bar{c} =$

$$1) z\bar{i} - x\bar{k} \quad 2) x\bar{i} - z\bar{k}$$

$$3) x\bar{j} - y\bar{k} \quad 4) y\bar{j}$$

The unit vector orthogonal to  $\bar{i} + \bar{k}, 2\bar{j} - \bar{k}$  and forming a right handed system with them is

$$1) 2\bar{i} + \bar{j} + 2\bar{k} \quad 2) \frac{2\bar{i} + \bar{j} + 2\bar{k}}{3} \quad 3) -\frac{2\bar{i} + \bar{j} + 2\bar{k}}{3} \quad 4) -\frac{2\bar{i} + \bar{j} + 2\bar{k}}{9}$$

#### PROBLEMS ON VECTOR PRODUCT

#### C.R.T.Q Class Room Teaching Questions

If  $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}, \bar{b} = 3\bar{i} + 4\bar{j} - \bar{k}$  then  $|\bar{a} \times \bar{b}| =$

$$1) 9 \quad 2) 3\sqrt{10} \quad 3) \sqrt{155} \quad 4) 5\sqrt{5}$$

6. If  $|\bar{a}|=2, |\bar{b}|=4, (\bar{a}, \bar{b})=\frac{\pi}{6}$  then  $|\bar{a} \times \bar{b}|^2 =$

$$1) 16 \quad 2) 2 \quad 3) 775 \quad 4) 36$$

then values of  $\lambda, \mu$  are

$$1) 3, 27 \quad 2) 3, \frac{27}{2} \quad 3) \frac{27}{2}, 3 \quad 4) 3, \frac{9}{2}$$

$$8. (\bar{i} + \bar{x}\bar{j} + 3\bar{k}) \times (\bar{i} - \bar{j} + \bar{k}) = 5\bar{i} + \bar{x}\bar{j} - 3\bar{k}$$

then  $\mathbf{x} =$

$$1) 2 \quad 2) 3 \quad 3) -1 \quad 4) 0$$

9. If  $|\bar{a}|=1, |\bar{b}|=2, (\bar{a}, \bar{b})=\frac{2\pi}{3}$  then

$$\{(\bar{a}+3\bar{b}) \times (3\bar{a}-\bar{b})\}^2 =$$

$$1) 425 \quad 2) 375 \quad 3) 325 \quad 4) 300$$

10. If  $\bar{a} = \bar{i} - 3\bar{j} + 2\bar{k}, \bar{b} = 2\bar{i} + \bar{j} - \bar{k}$  then the length of the component vector of  $\bar{a} \times \bar{b}$  along  $5\bar{i} - \bar{k}$  is

$$1) \sqrt{\frac{1}{13}} \quad 2) \sqrt{\frac{2}{13}} \quad 3) \sqrt{\frac{3}{13}} \quad 4) \sqrt{\frac{4}{13}}$$

11. Given that

$$\bar{a} = 2\bar{i} + 3\bar{j} + 6\bar{k}, \bar{b} = 3\bar{i} - 6\bar{j} + 2\bar{k},$$

$$\bar{c} = 6\bar{i} + 2\bar{j} - 3\bar{k} \text{ then } \bar{a} \times \bar{b} =$$

$$1) 7\bar{c} \quad 2) \bar{c} \quad 3) 5\bar{c} \quad 4) 4\bar{c}$$

12. If  $\bar{a} \times \bar{i} + 2\bar{a} - 4\bar{i} - 2\bar{j} + \bar{k} = \bar{0}$  then  $\bar{a} =$

$$1) 2\bar{j} + \bar{k} \quad 2) 2\bar{j} - \bar{k}$$

$$3) 2\bar{i} + \bar{j} \quad 4) 2\bar{i} - \bar{j}$$

If  $\bar{a} = 2\bar{i} + 2\bar{j} + \bar{k}, \bar{a} \cdot \bar{b} = 14, \bar{a} \times \bar{b} = 3\bar{i} + \bar{j} - 8\bar{k}$ , then  $\bar{b} =$

$$1) 5\bar{i} - \bar{j} + 2\bar{k} \quad 2) 5\bar{i} + \bar{j} - \bar{k}$$

$$3) 5\bar{i} + \bar{j} + 2\bar{k} \quad 4) 5\bar{i} + \bar{j} - 2\bar{k}$$

If  $\bar{a} + \bar{b} + \bar{c} = \bar{0}$  then  $\bar{a} \times \bar{b} =$

$$1) \bar{c} \times \bar{b} \quad 2) \bar{b} \times \bar{c}$$

$$3) \bar{a} \times \bar{c} \quad 4) 2\bar{b} \times \bar{c}$$

15. If  $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$ ,  $\bar{b} = -\bar{i} + 2\bar{j} + \bar{k}$ ,  $\bar{c} = 3\bar{i} + \bar{j}$  and  $\bar{d}$  is normal to both  $\bar{a}$  and  $\bar{b}$ , then  $(\bar{c}, \bar{d})$

- 1)  $\cos^{-1}\left(\frac{4}{\sqrt{30}}\right)$     2)  $\sin^{-1}\left(\frac{4}{\sqrt{30}}\right)$   
 3)  $\cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$     4)  $\sin^{-1}\left(\frac{2}{\sqrt{30}}\right)$

16.  $|\bar{p}|=2$ ,  $|\bar{q}|=3$  then  $\frac{|\bar{p} \times \bar{q}|}{\sin(\bar{p}, \bar{q})} =$

- 1) 6    2) 3/2    3) 2/3    4) 1

**S.P.Q.**

Student Practice Questions

17. If  $(2\bar{i} + 4\bar{j} + 2\bar{k}) \times (2\bar{i} - \bar{j} + 5\bar{k}) = 16\bar{i} - 6\bar{j} + 2\bar{k}$  then the value of  $x$  is

- 1) 2    2) -2    3) 0    4) 3

18. If  $\bar{a} = 3\bar{i} - \bar{j} - 2\bar{k}$ ,  $\bar{b} = 2\bar{i} + 3\bar{j} + \bar{k}$  then

$$(\bar{a} + 2\bar{b}) \times (2\bar{a} - \bar{b}) =$$

- 1)  $25\bar{i} + 35\bar{j} - 55\bar{k}$     2)  $25\bar{i} - 35\bar{j} - 22\bar{k}$   
 3)  $-25\bar{i} - 35\bar{j} - 55\bar{k}$     4)  $-25\bar{i} + 35\bar{j} - 55\bar{k}$

19. The value of

- $\bar{i} \cdot [\bar{i} \times \bar{j}] + \bar{j} \cdot [\bar{j} \times \bar{k}] + \bar{k} \cdot [\bar{k} \times \bar{i}] =$

- 1) 0    2) 2    3) 4    4) 6

20. The value of  $|\bar{i} \times \bar{j} + \bar{j} \times \bar{k} + \bar{k} \times \bar{i}| =$

- 1) 0    2) 1    3)  $\sqrt{3}$     4)  $\sqrt{5}$

**PROPERTIES OF VECTOR PRODUCT**

**C.R.T.Q.**

Class Room Learning Questions

21.  $\bar{a}, \bar{b}$  are two vectors such that  $|\bar{a}|=3$ ,

$$|\bar{b}|=\frac{\sqrt{2}}{3}. \text{ If } \bar{a} \times \bar{b} \text{ is unit vector then } (\bar{a}, \bar{b}) =$$

- 1)  $\frac{\pi}{6}$     2)  $\frac{\pi}{4}$     3)  $\frac{\pi}{3}$     4)  $\frac{\pi}{2}$

22. If  $|\bar{a}|=2$ ,  $|\bar{b}|=7$  and  $\bar{a} \times \bar{b}=3\bar{i} + 2\bar{j} + 6\bar{k}$ , then  $(\bar{a}, \bar{b}) =$

- 1)  $30^\circ$     2)  $60^\circ$     3)  $45^\circ$     4)  $75^\circ$

23. The sine of the angle between the vectors  $\bar{i} + 3\bar{j} + 2\bar{k}$  and  $2\bar{i} - 4\bar{j} + \bar{k}$  is

- 1)  $\sqrt{\frac{155}{156}}$     2)  $\sqrt{\frac{115}{116}}$     3)  $\sqrt{\frac{115}{147}}$     4)  $\sqrt{\frac{157}{158}}$

24. If  $|\bar{a}|=2$ ,  $|\bar{b}|=7$  and  $\bar{a} \times \bar{b}=3\bar{i} + 2\bar{j} + 6\bar{k}$ , then  $\bar{a} \cdot \bar{b} =$

- 1)  $\sqrt{147}$     2)  $\sqrt{145}$     3)  $\sqrt{143}$     4)  $\sqrt{142}$

25. If  $\bar{a}=2\bar{i}+2\bar{j}+\bar{k}$ ,  $\bar{b}=5\bar{i}+\bar{j}+2\bar{k}$  then

- 1) 270    2) 120    3) 170    4) 110

$$|\bar{a} \times \bar{b}|^2 + (\bar{a} \cdot \bar{b})^2 =$$

- 1) 1    2)  $\frac{1}{2}$     3) 2    4)  $\frac{1}{4}$

26.  $\frac{|\bar{a} \times \bar{b}|^2 + (\bar{a} \cdot \bar{b})^2}{2a^2 b^2}$ , where  $|\bar{a}|=a$ ,  $|\bar{b}|=b$

- 1) 1    2)  $\frac{1}{2}$     3) 2    4)  $\frac{1}{4}$

**S.P.Q.** Student Practice Questions

27. If  $|\bar{a}|=2$ ,  $|\bar{b}|=5$  and  $|\bar{(a} \times \bar{b})|=8$ , then  $\bar{a} \cdot \bar{b} =$

- 1) 4    2) 6    3) 8    4) 10

28. If  $|\bar{a}|=3$ ,  $|\bar{b}|=4$  and  $\bar{a} \cdot \bar{b}=8$  then  $|\bar{a} \times \bar{b}| =$

- 1) 4    2) 6    3) 8    4)  $\sqrt{80}$

29. Given that  $\bar{a}=(1,1,1)$  and  $\bar{c}=(0,1,-1)$ . If  $\bar{a} \cdot \bar{b}=3$  and  $\bar{a} \times \bar{b}=\bar{c}$  then  $\bar{b} =$

- 1)  $\frac{1}{3}(5\bar{i} + 2\bar{j} + 2\bar{k})$     2)  $\frac{1}{3}(5\bar{i} - 2\bar{j} - 2\bar{k})$   
 3)  $\frac{1}{3}(5\bar{i} + 2\bar{j} - 2\bar{k})$     4)  $\frac{1}{4}(5\bar{i} + 2\bar{j} + 2\bar{k})$

30.  $\bar{a}=(\bar{i} + \bar{j} + \bar{k})$ ,  $\bar{a} \cdot \bar{b}=1$ ,  $\bar{a} \times \bar{b}=\bar{j} - \bar{k}$ , then  $\bar{b} =$

- 1)  $\bar{i} - \bar{j} + \bar{k}$     2)  $2\bar{j} - \bar{k}$     3)  $\bar{i}$     4)  $2\bar{j}$

31. Let  $\vec{a}$  and  $\vec{b}$  be two unit vectors then  $\vec{a} \times \vec{b}$  will also be a unit vector if.

1)  $\vec{a} = \vec{b}$

2)  $\vec{a} \perp \vec{b}$

3)  $\vec{a} \parallel \vec{b}$

4)  $\vec{a} = m\vec{b}, m > 1$

32.  $\vec{a} = \vec{i} + \vec{j}, \vec{b} = 2\vec{i} - \vec{k}$  &  $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ ,  $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$  the  $\vec{r}$  =

1)  $-\vec{i} + \vec{j} + \vec{k}$

2)  $3\vec{i} - \vec{j} + \vec{k}$

3)  $3\vec{i} + \vec{j} - \vec{k}$

4)  $\vec{i} - \vec{j} - \vec{k}$

33. If  $(\vec{a}, \vec{b}) = \pi$  then  $|4\vec{a} \times 5\vec{b} + \vec{b} \times \vec{a}| =$

1) 0

2)  $19|\vec{a}||\vec{b}|$

3)  $|\vec{a}||\vec{b}|$

4)  $2|\vec{a}||\vec{b}|$

If  $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ ,  $\vec{b} = 2\vec{i} - 3\vec{j} + \vec{k}$ , then

$$\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} + \frac{\vec{b} \times \vec{a}}{|\vec{b} \times \vec{a}|} =$$

1)  $\vec{0}$

2)  $2\vec{i} + \vec{j} - 2\vec{k}$

3)  $\vec{i} + \vec{j} + 2\vec{k}$

4)  $\vec{i} + 2\vec{j} - \vec{k}$

35. If  $3\vec{i} \times (3\vec{i} - 4\vec{k}) + (\vec{i} + 2\vec{j}) \times \vec{k} =$

1)  $2\vec{i} + 11\vec{j}$

2)  $2\vec{i} - 11\vec{j}$

3)  $-2\vec{i} + 11\vec{j}$

4)  $\vec{i} + \vec{j} + \vec{k}$

36.  $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 169$  and  $|\vec{a}| = 13$  then  $|\vec{b}| =$

1) 16

2) 8

3) 1

4) 12

### VECTOR AREA, AREA

#### C.R.T.Q Class Room Testing Questions

37. The area of the triangle formed by the points whose position vectors are

$-\vec{3}\vec{i} + \vec{j}, 5\vec{i} + 2\vec{j} + \vec{k}$  and  $\vec{i} - 2\vec{j} + 3\vec{k}$  is

1)  $\sqrt{23}$  sq. units

2)  $\sqrt{21}$  sq. units

3)  $\sqrt{305}$  sq. units

4)  $\sqrt{33}$  sq. units

38. The area of the triangle formed by the points A(2,3,4), B(3,4,2) and C(4,2,3) is.

1)  $3\sqrt{3}$

2)  $\frac{3\sqrt{3}}{2}$

3)  $\frac{\sqrt{3}}{2}$

4)  $\frac{5\sqrt{3}}{2}$

39. If the adjacent sides of a parallelogram are  $\vec{i} + 2\vec{j} + 3\vec{k}$ , and  $-3\vec{i} - 2\vec{j} + \vec{k}$  then the area of the parallelogram is.

1)  $6\sqrt{5}$

2)  $7\sqrt{5}$

3)  $8\sqrt{5}$

4)  $5\sqrt{7}$

40. If ABCD is a quadrilateral such that

$\overline{AB} = \vec{i} + 2\vec{j}, \overline{AD} = \vec{j} + 2\vec{k}$  and

$\overline{AC} = 2(\vec{i} + 2\vec{j}) + 3(\vec{j} + 2\vec{k})$ . Then area

of the quadrilateral ABCD is

1)  $\frac{5\sqrt{21}}{2}$

2)  $\frac{3\sqrt{21}}{2}$

3)  $\frac{\sqrt{21}}{2}$

4)  $\frac{7}{2}$

41. The area of the parallelogram constructed on the Vectors  $\vec{a} = \vec{p} + 2\vec{q}$  and  $\vec{b} = 2\vec{p} + \vec{q}$  as sides, where  $\vec{p}, \vec{q}$  are unit Vectors forming an angle of  $60^\circ$  in square units is

1)  $\frac{3}{2}$

2)  $\frac{3\sqrt{3}}{2}$

3)  $\frac{3\sqrt{3}}{4}$

4)  $\frac{1}{2}$

42. If the area of the parallelogram whose adjacent sides are  $(3\vec{i} - 4\vec{j} + \lambda\vec{k})$  and  $(2\vec{j} - 4\vec{k})$  is  $\sqrt{436}$  sq. units and  $\lambda \geq 0$ , then  $\lambda =$

1) 0

2) 4

3) 1

4) 3

43. If the Vectors  $3\vec{i} + \vec{j} - 2\vec{k}, \vec{i} - 3\vec{j} + 4\vec{k}$  are diagonals of a quadrilateral then the Vector area is

1)  $\vec{i} + 7\vec{j} - 5\vec{k}$

2)  $\vec{i} - 7\vec{j} + 5\vec{k}$

3)  $-\vec{i} + 2\vec{j} + 5\vec{k}$

4)  $-\vec{i} - 7\vec{j} - 5\vec{k}$

44. If  $\overline{OA} = \vec{a}, \overline{OB} = 10\vec{a} + 2\vec{b}$  and  $\overline{OC} = \vec{b}$ , where A and C are non collinear points. Let p denote the area of the Quadrilateral OABC and q denote the area of a parallelogram with  $\overline{OA}$  and  $\overline{OC}$  as adjacent sides. The  $p/q =$

1) 4

2) 6

3) 8

4) 10

45. If  $\bar{a} = 2\bar{i} - 3\bar{j} + \bar{k}$ ,  $\bar{b} = -\bar{i} + \bar{k}$ ,  $\bar{c} = 2\bar{j} - \bar{k}$ , then the area of the parallelogram is having diagonals  $\bar{a} + \bar{b}$  and  $\bar{b} + \bar{c}$  (in sq.units) is

1)  $\sqrt{21}$    2)  $\frac{\sqrt{21}}{2}$    3)  $\sqrt{19}$    4)  $\frac{\sqrt{19}}{2}$

46. ABCD is a quadrilateral and let  $\overline{AB} = \bar{a}$ ,  $\overline{AD} = \bar{b}$ ,  $\overline{AC} = m\bar{a} + n\bar{b}$ .

Then the area of the quadrilateral is

1)  $(m+n)(\bar{a} \times \bar{b})$    2)  $2|m+n|(\bar{a} \times \bar{b})$

3)  $1/2|m+n|[\bar{a} \times \bar{b}]$    4)  $3(m+n)|\bar{a} \times \bar{b}|$

47. Area of rectangle having vertices A,

B, C and D with P.V's  $-\bar{i} + \frac{1}{2}\bar{j} + 4\bar{k}$ ,  $\bar{i} + \frac{1}{2}\bar{j} + 4\bar{k}$ ,  $\bar{i} - \frac{1}{2}\bar{j} + 4\bar{k}$  and  $\bar{i} - \frac{1}{2}\bar{j} + 4\bar{k}$  respectively is

1)  $\frac{1}{2}$    2) 1   3) 2   4) 4

**S.P.Q.** Student Practice Questions

48. If G is centroid of  $\Delta PQR$  where

$\overline{GP} = 2\bar{i} + \bar{j} + 3\bar{k}$ ,  $\overline{GQ} = \bar{i} - \bar{j} + 2\bar{k}$  then

the area of triangle PQR is

1)  $\sqrt{35}$    2)  $\frac{3\sqrt{35}}{2}$    3)  $\frac{\sqrt{35}}{2}$    4)  $\frac{5\sqrt{35}}{2}$

49. The Vector area of the triangle whose adjacent sides are  $2\bar{i} + 3\bar{j}$  and  $-2\bar{i} + 4\bar{j}$

1)  $5\bar{k}$    2)  $7\bar{k}$    3)  $9\bar{k}$    4)  $1\bar{k}$

50. The area of the triangle formed by the points A(1,2,3), B(2,3,1), C(3,1,2) is.

1)  $\frac{3\sqrt{3}}{2}$    2)  $3\sqrt{3}$    3)  $\sqrt{3}/2$    4)  $\sqrt{3}$

51. If  $\bar{a}, \bar{b}, \bar{c}$  are the vertices of a triangle

ABC then  $|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}| =$

- 1) Area of the triangle  $\Delta ABC$   
2) Two times Area of the triangle  $\Delta ABC$   
3) Three times Area of the triangle  $\Delta ABC$   
4) Four times Area of the triangle  $\Delta ABC$

52. If  $\bar{a}$  and  $\bar{b}$  are such that  $|\bar{a}| = 3$ ,  $|\bar{b}| = 2$ ,  $(\bar{a}, \bar{b}) = \pi/3$  then the area of the triangle with adjacent sides  $\bar{a} + 2\bar{b}$  and  $2\bar{a} + \bar{b}$  in sq. units is

1)  $3\sqrt{3}$    2)  $9\sqrt{3}$    3)  $\frac{9\sqrt{3}}{2}$    4)  $\frac{9}{2}$

53. If  $\bar{a} = -\bar{i} + \bar{k}$  and  $\bar{b} = 2\bar{j} - \bar{k}$  are the adjacent sides of a parallelogram then its vector area is

1)  $2\bar{i} + \bar{j} + 2\bar{k}$    2)  $-(2\bar{i} + \bar{j} + 2\bar{k})$

3)  $2\bar{i} - \bar{j} + 2\bar{k}$    4)  $2\bar{i} + \bar{j} - 4\bar{k}$

54.  $\bar{a} = \bar{i} - \bar{j} + 3\bar{k}$ ,  $\bar{b} = 2\bar{i} - 7\bar{j} + \bar{k}$ . Area of parallelogram whose adjacent sides are  $\bar{a}, \bar{b}$  is

1)  $15\sqrt{2}$    2) 20   3)  $\sqrt{350}$    4) 30

**GEOMETRICAL APPLICATION OF VECTOR PRODUCT**

**C.R.T.Q** Class Room Teaching Questions

55. A unit vector parallel to xy-plane and perpendicular to the vector  $4\bar{i} - 3\bar{j} + \bar{k}$  is

1)  $\pm \frac{1}{5}(3\bar{i} + 4\bar{k})$    2)  $\pm \frac{1}{5}(4\bar{i} + 3\bar{j})$

3)  $\pm \frac{1}{5}(3\bar{i} + 4\bar{j})$    4)  $\bar{i} + 2\bar{k}$

56. The unit Vectors orthogonal to  $-\bar{i} + 2\bar{k}$  and  $2\bar{k}$  and making equal angles with x and y axes are

1)  $\pm 1/3(2\bar{i} + 2\bar{j} - \bar{k})$    2)  $\pm 1/3(\bar{i} + \bar{j} - \bar{k})$

3)  $\pm 1/3(2\bar{i} - 2\bar{j} - \bar{k})$    4)  $\pm 1/4(\bar{i} + \bar{j} - \bar{k})$

51. If  $\bar{u} = \bar{a} + \bar{b}$ ,  $\bar{v} = \bar{a} - \bar{b}$ ,  $|\bar{a}| = 2$  then  
 $|\bar{u} \times \bar{v}| =$

1)  $\sqrt{16 - (\bar{a} \cdot \bar{b})^2}$       2)  $2\sqrt{16 - (\bar{a} \cdot \bar{b})^2}$

3)  $3\sqrt{16 - (\bar{a} \cdot \bar{b})^2}$       4) 0

Let  $\bar{\lambda} = \bar{a} \times (\bar{b} + \bar{c})$ ,  $\bar{\mu} = \bar{b} \times (\bar{c} + \bar{a})$ ,

- $\bar{v} = \bar{c} \times (\bar{a} + \bar{b})$  then

1)  $\bar{\lambda} + \bar{\mu} = \bar{v}$       2)  $\bar{\lambda}, \bar{\mu}, \bar{v}$  are coplanar

3)  $\bar{\lambda} + \bar{v} = 2\bar{\mu}$       4) None

59. If  $\bar{a}, \bar{b}, \bar{c}$  are non-coplanar vectors such that  $\bar{b} \times \bar{c} = \bar{a}$ ,  $\bar{c} \times \bar{a} = \bar{b}$  and  $\bar{a} \times \bar{b} = \bar{c}$ , then  $|\bar{a} + \bar{b} + \bar{c}| =$

1) 1      2) 2      3) 3      4)  $\sqrt{3}$

60. If  $\bar{a} \times \bar{i} = \bar{j}$  then  $\bar{a} \cdot \bar{i} =$

1) any scalar      2) 0      3) 1      4) 2

61. If  $\bar{r} \times \bar{a} = \bar{b} \times \bar{a}$ ,  $\bar{r} \times \bar{b} = \bar{a} \times \bar{b}$ ,  $\bar{a} \neq 0, \bar{b} \neq 0, \bar{b} \neq \lambda \bar{a}$ ,  $\bar{a}$  is not perpendicular to  $\bar{b}$  then  $\bar{r} =$

1)  $\bar{a} - \bar{b}$       2)  $\bar{a} + \bar{b}$       3)  $\bar{a} \times \bar{b} + \bar{a}$       4)  $\bar{a} \times \bar{b} + \bar{b}$

62. If  $\bar{r}$  satisfies  $\bar{r} \times (\bar{i} + 2\bar{j} + \bar{k}) = \bar{i} - \bar{k}$ , then for any scalar 't',  $\bar{r} =$

1)  $i + t(i + 2\bar{j} + \bar{k})$       2)  $\bar{j} + t(i + 2\bar{j} + \bar{k})$

3)  $\bar{k} + t(i + 2\bar{j} + \bar{k})$       4)  $\bar{i} - \bar{k} + t(i + 2\bar{j} + \bar{k})$

63. If  $\bar{r} = xi + y\bar{j} + zk$  then  $(\bar{r} \times \bar{i}) \cdot (\bar{r} \times \bar{j}) + xy$

1) 0      2) 1      3)  $\bar{r}$       4)  $|\bar{r}|$

64. Let  $\bar{A}, \bar{B}, \bar{C}$  be unit vectors suppose that  $\bar{A} \cdot \bar{B} = \bar{A} \cdot \bar{C} = 0$  and angle between  $\bar{B}$  and  $\bar{C}$  is  $\frac{\pi}{6}$ , then  $\bar{A} = k(\bar{B} \times \bar{C})$  and  $k =$

1)  $\pm 2$       2)  $\pm 4$       3)  $\pm 16$       4) 0

65.  $A(1,2,5)$ ,  $B(5,7,9)$ , and  $C(3,2,-1)$ , are given three points. A unit vector normal to the plane of the triangle ABC.

1)  $\frac{15\bar{i} + 16\bar{j} - 5\bar{k}}{\sqrt{506}}$       2)  $\frac{-15\bar{i} + 16\bar{j} + 16\bar{k}}{\sqrt{506}}$   
 3)  $\frac{-15\bar{i} + 16\bar{j} + 5\bar{k}}{\sqrt{506}}$       4)  $\frac{\bar{i} + \bar{j} + \bar{k}}{\sqrt{3}}$

66. A vector of length  $\sqrt{7}$  which is perpendicular to  $2\bar{j} - \bar{k}$  and  $-\bar{i} + 2\bar{j} - 3\bar{k}$  and makes obtuse angle with y-axis is

1)  $\frac{1}{\sqrt{5}}(4\bar{i} - \bar{j} + \sqrt{18}\bar{k})$   
 2)  $\frac{1}{\sqrt{3}}(4\bar{i} - \bar{j} - 2\bar{k})$   
 3)  $\frac{1}{\sqrt{3}}(-4\bar{i} + \bar{j} + 2\bar{k})$   
 4)  $\frac{1}{\sqrt{3}}(-4\bar{i} - \bar{j} + 2\bar{k})$

67. Let  $\bar{a} = \bar{i} + \bar{j}, \bar{b} = 2\bar{i} - \bar{k}$ . Then the point of intersection of the lines  $\bar{r} \times \bar{a} = \bar{b} \times \bar{a}$  and  $\bar{r} \times \bar{b} = \bar{a} \times \bar{b}$  is
- 1)  $3\bar{i} + \bar{j} - \bar{k}$       2)  $3\bar{i} - \bar{j} - \bar{k}$   
 3)  $3\bar{i} - 3\bar{j} - \bar{k}$       4)  $3\bar{i} + 3\bar{j} + \bar{k}$

68. The perpendicular distance of the point (6,-4,4) on to the line joining the points A(2,1,2), B(3,-1,4) is.

1) 1      2) 2      3) 3      4) 4

69. If  $\overline{AB} = \bar{b}$  and  $\overline{AC} = \bar{c}$  then the length of the perpendicular from A to the line BC is

1)  $\left| \frac{\bar{b} \times \bar{c}}{\bar{b} + \bar{c}} \right|$       2)  $\left| \frac{\bar{b} \times \bar{c}}{\bar{b} - \bar{c}} \right|$   
 3)  $\frac{1}{2} \left| \frac{\bar{b} \times \bar{c}}{\bar{b} - \bar{c}} \right|$       4)  $2 \left| \frac{\bar{b} \times \bar{c}}{\bar{b} - \bar{c}} \right|$

70. If the projection of Vector  $\overline{OA}$  on unit Vector  $\overline{OB}$  equals twice the area of  $\Delta OAB$  in magnitude, then  $\angle AOB$  in radian is

- 1) 0    2)  $\frac{\pi}{2}$     3)  $\frac{\pi}{4}$     4)  $\pi$   
 71. If  $\vec{a} = x\vec{i} - y\vec{j}$  and  $\vec{b} = y\vec{i} + x\vec{j}$ ,  $|\vec{a} \times \vec{b}| = 5$  then locus of  $(x, y)$  is  
 1) Hyperbola    2) Parabola  
 3) Ellipse    4) Circle

72. If  $\vec{a}$  and  $\vec{b}$  are unit vectors such that  $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$  then  $|\vec{a} - \vec{b}|^2 =$   
 1) 2    2)  $2 + \sqrt{2}$     3)  $2 - \sqrt{2}$     4)  $4\sqrt{2}$
- S.P.Q.** Student Practice Questions
73. The vector orthogonal to  $\vec{a} = 2\vec{i} + \vec{j} - 3\vec{k}$ ,  $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$  and having magnitude equal to 5 units is.

- 1)  $\pm \frac{5\sqrt{3}}{3}(\vec{i} + \vec{j} + \vec{k})$     2)  $\pm \frac{5\sqrt{3}}{3}(\vec{i} + \vec{j} - \vec{k})$   
 3)  $\pm \frac{5\sqrt{3}}{3}(\vec{i} - \vec{j} + \vec{k})$     4)  $\pm \frac{5\sqrt{3}}{3}(-\vec{i} + \vec{j} + \vec{k})$

74. If the position Vectors of three points A, B, C are  $\vec{i} + \vec{j} + \vec{k}$ ,  $2\vec{i} + 3\vec{j} - 4\vec{k}$  and  $3\vec{i} + 2\vec{j} + \vec{k}$  respectively, then the unit vector perpendicular to the plane of the triangle ABC is

- 1)  $\frac{5\vec{i} - 10\vec{j} + 3\vec{k}}{\sqrt{134}}$     2)  $\frac{5\vec{i} - 10\vec{j} - 3\vec{k}}{\sqrt{134}}$   
 3)  $\frac{5\vec{i} + 10\vec{j} - 3\vec{k}}{\sqrt{134}}$     4)  $\frac{5\vec{i} + 10\vec{j} + 3\vec{k}}{\sqrt{134}}$

75. Given  $\vec{a} = \vec{i} + \vec{j} - \vec{k}$ ,  $\vec{b} = -\vec{i} + 2\vec{j} + \vec{k}$  and  $\vec{c} = -\vec{i} + 2\vec{j} - \vec{k}$ . A unit vector perpendicular to both  $\vec{a} + \vec{b}$  &  $\vec{b} + \vec{c}$  is

- 1)  $\frac{2\vec{i} + \vec{j} + \vec{k}}{\sqrt{6}}$     2)  $\vec{j}$     3)  $\vec{k}$     4)  $\frac{\vec{i} + \vec{j} + \vec{k}}{\sqrt{3}}$

76. The number of vectors of unit length perpendicular to the Vectors  $\vec{a} = (1, 1, 0)$  and  $\vec{b} = (0, 1, 1)$  is  
 1) 1    2) 2    3) 3    4) Infinite

77. The unit vector normal to the plane containing  $\vec{a} = \vec{i} - \vec{j} - \vec{k}$  and  $\vec{b} = \vec{i} + \vec{j} + \vec{k}$  is  
 1)  $\vec{j} - \vec{k}$     2)  $\frac{\vec{j} - \vec{k}}{\sqrt{2}}$     3)  $\frac{\vec{j} + \vec{k}}{\sqrt{2}}$     4)  $\frac{\vec{i} + \vec{j}}{\sqrt{2}}$

78. The sine of the angle between the vectors  $\vec{i} + 3\vec{j} + 2\vec{k}$ , and  $2\vec{i} - 4\vec{j} - \vec{k}$ ,  
 1) 7/5    2) 5/7    3) 7/6    4) 6/7
79. If  $\vec{a}$  and  $\vec{b}$  are unit vectors and  $|\vec{a} \times \vec{b}| = 1$ , then the angle between  $\vec{a}$  and  $\vec{b}$  is  
 1)  $\pi/4$     2)  $\pi/2$     3)  $\pi/3$     4)  $\pi$

80.  $|\vec{a}| = |\vec{b}| = 2$ ,  $\vec{p} = \vec{a} + \vec{b}$ ,  $\vec{q} = \vec{a} - \vec{b}$ , if  $|\vec{p} \times \vec{q}| = 2 \left[ K - (\vec{a} \cdot \vec{b})^2 \right]$  then K =  
 1) 16    2) 8    3) 4    4) 1
81. If  $\vec{i}, \vec{j}, \vec{k}$  is an orthonormal system of vectors,  $\vec{a}$  is a vector and  $\vec{a} \times \vec{i} + 2\vec{a} - 5\vec{j} = \vec{0}$  then  $\vec{a}$  =  
 1)  $2\vec{j} + \vec{k}$     2)  $2\vec{j} - \vec{k}$     3)  $2\vec{i} - \vec{j}$     4)  $2\vec{i} + \vec{j}$

82. If  $\vec{a} = 2\vec{i} + 2\vec{j} + 3\vec{k}$ ,  $\vec{b} = -\vec{i} + 2\vec{j} + \vec{k}$  and  $\vec{c} = 3\vec{i} + \vec{j} + \frac{17}{6}\vec{k}$  then  $\vec{a} + \vec{b}$  is parallel to  $\vec{c}$  if t is equal to  
 1)  $-\frac{4}{7}$     2) 6    3) -3    4) 2

83. If  $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar vectors such that  $\vec{a} \times \vec{b} = \vec{c}$ ,  $\vec{b} \times \vec{c} = \vec{a}$ ,  $\vec{c} \times \vec{a} = \vec{b}$  then  $|\vec{a}| + 2|\vec{b}| - 3|\vec{c}|$  is equal to  
 1) 1    2) 0    3) 2    4) 3

Let  $\bar{a}, \bar{b}, \bar{c}$  be three vectors such that  
Let  $\bar{a}, \bar{b}, \bar{c}$  and  $\bar{c} \times \bar{a} = \bar{b}$ , then

$$\bar{a} \times \bar{b} = \bar{c} \quad 2) \bar{c} \cdot \bar{a} = |\bar{b}|^2$$

$$1) \bar{a} \cdot \bar{b} = |\bar{c}|^2 \quad 4) \bar{a} \parallel \bar{b} \times \bar{c}$$

3)  $\bar{b} \cdot \bar{c} = |\bar{a}|^2$

3)  $\bar{b}$  be a unit vector perpendicular to unit vectors  $\bar{b}$  and  $\bar{c}$  and if the angle between  $\bar{b}$  and  $\bar{c}$  is  $\alpha$ , then

$$\bar{b} \times \bar{c} \text{ is } \quad 2) \pm (\operatorname{cosec} \alpha) \bar{a}$$

$$1) \pm (\sin \alpha) \bar{a} \quad 4) \pm \tan \alpha$$

3)  $\pm (\sin \alpha) \bar{a}$

3) If  $\bar{a} \times \bar{b} = \bar{b} \times \bar{c}$  then

$$1) \bar{b} \parallel \bar{a} - \bar{c} \quad 2) \bar{b} \parallel \bar{a} + \bar{c}$$

$$1) \bar{b} = \bar{a} \times \bar{c} \quad 4) \bar{b} = \bar{a} - \bar{c}$$

$$3) \bar{b} \parallel (\bar{a} + \bar{c})$$

If  $\bar{a} = 2\bar{i} - \bar{j} + 2\bar{k}$ , then the value of

$$|\bar{a} \times \bar{i}|^2 + |\bar{a} \times \bar{j}|^2 + |\bar{a} \times \bar{k}|^2 = \quad 4) 18$$

$$1) 2 \quad 2) 4 \quad 3) 6$$

1) If  $\bar{a}, \bar{b}, \bar{c}$  are the position vectors of

points lie on a line then  
 $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a} =$

$$1) \bar{0} \quad 2) 0 \quad 3) 1 \quad 4) \text{a}$$

1) If the vectors  $\bar{a}, \bar{b}, \bar{c}$  from the sides

$\overline{BC}, \overline{CA}, \overline{AB}$  of  $\triangle ABC$  then

$$1) \bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a} = 0$$

$$2) \bar{a} \times \bar{b} = \bar{b} \times \bar{c} = \bar{c} \times \bar{a}$$

$$3) \bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{c} = \bar{c} \cdot \bar{a} = \bar{0}$$

4)  $\bar{r} \times \bar{b} = \bar{b} \times \bar{c} = \bar{c} \times \bar{a} = \bar{0}$

1) Point of intersection of the lines  $\bar{r} \times \bar{a} = \bar{b} \times \bar{a}$  and  $\bar{r} \times \bar{b} = \bar{a} \times \bar{b}$  is

$$1) \bar{a} \quad 2) \bar{b} - \bar{a}$$

$$3) \bar{a} - \bar{b} \quad 4) \bar{a} + \bar{b}$$

## PHYSICAL APPLICATION OF CROSS PRODUCT

### C.R.T.Q

Class Room Teaching Questions

91. The moment of a force  $\bar{i} + \bar{j} + \bar{k}$  acting through the point  $A = -2\bar{i} + 3\bar{j} + \bar{k}$  about

the point  $B = \bar{i} + 2\bar{j} + 3\bar{k}$  is

- 1)  $3\bar{i} + \bar{j} + 4\bar{k}$
- 2)  $3\bar{i} - \bar{j} - 4\bar{k}$
- 3)  $3\bar{i} + \bar{j} - 4\bar{k}$
- 4)  $3\bar{i} + \bar{j} + 4\bar{k}$

92. The torque about the point  $3\bar{i} - \bar{j} + 3\bar{k}$  of a force  $4\bar{i} + 2\bar{j} + \bar{k}$  through the point

- 1)  $5\bar{i} + 2\bar{j} + 4\bar{k}$ , is
- 2)  $\bar{i} + 2\bar{j} + 8\bar{k}$
- 3)  $\bar{i} - 2\bar{j} - 8\bar{k}$
- 4)  $-\bar{i} - 2\bar{j} - 8\bar{k}$

### S.P.Q.

Student Practice Questions

93. The moment about the point  $\bar{i} + 2\bar{j} + 3\bar{k}$  of a force represented by  $\bar{i} + \bar{j} + \bar{k}$  acting through the point  $2\bar{i} + 3\bar{j} + \bar{k}$  is

- 1)  $3\bar{i} - 3\bar{j}$
- 2)  $-3\bar{i} + 2\bar{j} + \bar{k}$
- 3)  $\bar{i} - 3\bar{j}$
- 4)  $\bar{i} - 3\bar{j} + \bar{k}$

### KEY

- 1) 4) 02) 4 03) 1 04) 3 05) 3
- 01) 1 07) 2 08) 1 09) 4 10) 2
- 06) 1 12) 3 13) 3 14) 2 15) 1
- 11) 1 17) 2 18) 4 19) 4 20) 3
- 16) 1 22) 1 23) 3 24) 1 25) 1
- 21) 2 27) 2 28) 4 29) 1 30) 3
- 26) 2 32) 3 33) 1 34) 1 35) 1
- 31) 2 37) 3 38) 2 39) 1 40) 1
- 36) 3 42) 1 43) 4 44) 2 45) 2
- 41) 2 47) 3 48) 1 49) 2 50) 1

10. If  $\bar{a} \times \bar{b} = \bar{c} \times \bar{d}$  and  $\bar{a} \times \bar{c} = \bar{b} \times \bar{d}$ , then

- 1)  $(\bar{a} + \bar{d}) = \lambda(\bar{b} - \bar{c})$
- 2)  $\bar{a} + \bar{d} = \lambda(\bar{b} + \bar{c})$
- 3)  $\bar{a} - \bar{b} = \lambda(\bar{c} + \bar{d})$
- 4)  $\bar{a} + \bar{b} + \bar{c} + \bar{d} = \bar{0}$

### VECTOR AREA, AREA

#### C.R.T.Q

Class Room Teaching Questions

11.  $\bar{a}, \bar{b}$  are such that  $|\bar{a}| = \sqrt{3}$ ,  $|\bar{b}| = 2$  and  $(\bar{a}, \bar{b}) = \frac{\pi}{3}$ . Then the area of the triangle with adjacent sides  $\bar{a} + 2\bar{b}$  and  $2\bar{a} + \bar{b}$  is
- 1)  $5\sqrt{3}$
  - 2) 15
  - 3)  $9/2$
  - 4)  $15/2$

12. ABCD is a quadrilateral with  $\overline{AB} = \bar{a}$ ,  $\overline{AD} = \bar{b}$ ,  $\overline{AC} = 2\bar{a} + 3\bar{b}$ . If the area of parallelogram ABCD is p times the area of the parallelogram with AB, AD as adjacent sides, then p is equal to
- 1) 5
  - 2)  $5/2$
  - 3) 1
  - 4)  $1/2$

#### S.P.Q.

Student Practice Questions

13. The area of the parallelogram is  $4\sqrt{29}$  sq.u. If one of adjacent side is  $3\bar{i} + 4\bar{j}$  then the other side is
- 1)  $\bar{i} - 4\bar{k}$
  - 2)  $2\bar{i} + 4\bar{k}$
  - 3)  $-2\bar{j} - 4\bar{k}$
  - 4)  $2\bar{i} - 4\bar{k}$

14. The area of the parallelogram whose adjacent sides are  $\bar{a}$  and  $\bar{b}$  is  $2\sqrt{10}$  sq.units. If  $\bar{a} = 3\bar{i} + 2\bar{j} + \bar{k}$  then  $\bar{b} =$
- 1)  $2\bar{i} + 3\bar{j} + \bar{k}$
  - 2)  $\bar{i} + \bar{j} + 2\bar{k}$
  - 3)  $3\bar{i} + 3\bar{j} + \bar{k}$
  - 4)  $3\bar{i} + \bar{k}$

### GEOMETRICAL APPLICATION OF CROSS PRODUCT

#### C.R.T.Q

Class Room Teaching Questions

15. If  $\bar{a} \times \bar{b} = \bar{c} \times \bar{d}$ ,  $\bar{a} \times \bar{c} = \bar{b} \times \bar{d}$  then

- 1)  $\bar{a} - \bar{d}$  is parallel to  $\bar{b} - \bar{c}$
- 2)  $\bar{a} - \bar{b}$  is parallel to  $\bar{c} - \bar{d}$
- 3)  $\bar{a} - \bar{c}$  is parallel to  $\bar{b} - \bar{d}$
- 4)  $\bar{a} + \bar{b}$  is parallel to  $\bar{c} + \bar{d}$

16. If  $\bar{a}, \bar{b}, \bar{c}$  be three vectors such that  $|\bar{a} + \bar{b} + \bar{c}| = 1$ ,  $\bar{c} = \lambda(\bar{a} \times \bar{b})$  and

$$|\bar{a}| = \frac{1}{\sqrt{2}}, \quad |\bar{b}| = \frac{1}{\sqrt{3}}, \quad |\bar{c}| = \frac{1}{\sqrt{6}},$$

then the angle between  $\bar{a}$  and  $\bar{b}$  is

- 1)  $\frac{\pi}{6}$
- 2)  $\frac{\pi}{4}$
- 3)  $\frac{\pi}{3}$
- 4)  $\frac{\pi}{2}$

17. If  $\bar{a} \times \bar{b} = \bar{c}$  and  $\bar{b} \times \bar{c} = \bar{a}$ , then

- 1)  $\bar{a}, \bar{b}, \bar{c}$  are orthogonal in pairs and  $|\bar{a}| = |\bar{c}|$  and  $|\bar{b}| = 1$
- 2)  $\bar{a}, \bar{b}, \bar{c}$  are orthogonal to each other
- 3)  $\bar{a}, \bar{b}, \bar{c}$  are orthogonal in pairs that  $|\bar{a}| \neq |\bar{c}|$
- 4)  $\bar{a}, \bar{b}, \bar{c}$  are orthogonal but  $|\bar{b}| \neq 1$

18.  $\bar{a} + 2\bar{b} + 3\bar{c} = \bar{0}$  and  $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a} = \ell (\bar{b} \times \bar{c})$ , then  $\ell =$

- 1) 2
- 2) 4
- 3) 6
- 4) 8

19. Let  $\bar{a}, \bar{b}, \bar{c}$  be three vectors satisfying  $\bar{a} \times \bar{b} = 2\bar{a} \times \bar{c}$ ,  $|\bar{a}| = |\bar{c}| = 1$ ,  $\bar{b} = 4$  and  $|\bar{b} \times \bar{c}| = \sqrt{15}$ . If  $\bar{b} - 2\bar{c} = \lambda \bar{a}$  then  $\lambda$  is

- 1) 1
- 2) -1
- 3) 2
- 4)  $\pm 4$

20. If  $\alpha, \beta$  are roots of the equation  $x^2 + 2x + 5 = 0$  and  $\bar{a} = (\alpha + \beta)\bar{i} + \alpha\beta\bar{j}$ ,  $\bar{b} = \alpha\beta\bar{i} + (\alpha + \beta)\bar{j} + (\alpha^2 + \beta^2)\bar{k}$  then  $\bar{a} \times \bar{b} =$

- 1)  $\bar{i} + 12\bar{j} + 12\bar{k}$
- 2)  $-30\bar{i} + 12\bar{j} - 5\bar{k}$
- 3)  $-30\bar{i} - 12\bar{j} - 21\bar{k}$
- 4)  $\bar{i} - 12\bar{j} + 29\bar{k}$

1. If  $\bar{x}$  and  $\bar{y}$  are two non-collinear vectors and ABC is a triangle with sides  $a, b, c$  satisfying

$$(20a - 15b)\bar{x} + (15b - 12c)\bar{y} + (12c - 20a)$$

$(\bar{x} \times \bar{y}) = \bar{0}$ , then the triangle ABC is

- 1) an acute angle triangle
- 2) an obtuse angle triangle
- 3) a right angle triangle
- 4) an isosceles triangle

2. The pairs  $\bar{a}, \bar{b}$  and  $\bar{c}, \bar{d}$  each determines a plane. Then the planes are parallel if

- 1)  $(\bar{a} \times \bar{c}) \times (\bar{b} \times \bar{d}) = \bar{0}$
- 2)  $(\bar{a} \times \bar{c}) \cdot (\bar{b} \times \bar{d}) = 0$
- 3)  $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = \bar{0}$
- 4)  $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) = 0$

3. The vector equation of the line passing through the point  $\bar{i} - 2\bar{j} + \bar{k}$  and perpendicular to the vectors

$$2\bar{i} - 3\bar{j} - \bar{k}, \quad \bar{i} + 4\bar{j} - 2\bar{k}$$

$$1) \bar{r} = (\bar{i} - 2\bar{j} + \bar{k}) + t(\bar{i} - 7\bar{j} + \bar{k})$$

$$2) \bar{r} = (\bar{i} - 2\bar{j} + \bar{k}) + t(3\bar{i} + \bar{j} - 3\bar{k})$$

$$3) \bar{r} = (\bar{i} - 2\bar{j} + \bar{k}) + t(10\bar{i} + 3\bar{j} + 11\bar{k})$$

$$4) \bar{r} = \bar{i}$$

4. Equation of the plane containing the lines

$$\bar{r} = (\bar{i} - 2\bar{j} + \bar{k}) + t(\bar{i} + 2\bar{j} - \bar{k}),$$

$$\bar{r} = (\bar{i} + 2\bar{j} - \bar{k}) + s(\bar{i} + \bar{j} + 3\bar{k})$$

$$1) \bar{r} \cdot (7\bar{i} - 4\bar{j} - \bar{k}) = 14$$

$$2) \bar{r} \cdot (\bar{i} + 2\bar{j} - \bar{k}) = 10$$

$$3) \bar{r} \cdot (\bar{i} + \bar{j} + 3\bar{k}) = 20$$

$$4) \bar{r} \cdot (\bar{i} - 2\bar{j} + \bar{k}) = 27$$

25. A unit vector perpendicular to the lines

$$\frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \text{ and}$$

$$\frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3} \text{ is}$$

$$1) \frac{-\bar{i} + 7\bar{j} + 7\bar{k}}{\sqrt{99}}$$

$$2) \frac{-\bar{i} - 7\bar{j} + 5\bar{k}}{5\sqrt{3}}$$

$$3) \frac{-\bar{i} + 7\bar{j} + 5\bar{k}}{5\sqrt{3}} \quad 4) \frac{7\bar{i} - 7\bar{j} - \bar{k}}{\sqrt{99}}$$

26.  $\bar{a}$  and  $\bar{c}$  are unit vectors and  $|\bar{b}| = 4$  with  $\bar{a} \times \bar{b} = 2\bar{a} \times \bar{c}$ . The angle between  $\bar{a}$  and  $\bar{c}$  is  $\cos^{-1}\left(\frac{1}{4}\right)$ . Then

$$\bar{b} - 2\bar{c} = \lambda\bar{a}, \text{ if } \lambda \text{ is}$$

- 1) 1
- 2)  $\frac{1}{4}$
- 3) -4
- 4)  $-\frac{1}{4}$

27. If  $\bar{a}$  is a non zero vector and  $\bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{c}, \bar{a} \times \bar{b} = \bar{a} \times \bar{c}$ , then

- 1)  $\bar{a} \parallel \bar{b}$
- 2)  $\bar{b} = \bar{c}$
- 3)  $\bar{b} \neq \bar{c}$
- 4)  $\bar{a} \neq \bar{b}$  and  $\bar{a} = \bar{c}$

### S.P.Q.

Student Practice Questions

28. If A (1, 2, 3), B (2, 3, 1), C (3, 1, 2) then the length of the altitude through C is

- 1) 3
- 2)  $3\sqrt{3}$
- 3)  $3\sqrt{2}$
- 4)  $3/\sqrt{2}$

29. The angle between two diagonals of the parallelogram with Vectors  $2\bar{i} + \bar{j}$  and  $\bar{k} - 2\bar{j}$  as adjacent sides is.

- 1)  $\frac{\pi}{2}$
- 2)  $\frac{\pi}{6}$
- 3)  $\frac{\pi}{3}$
- 4)  $\frac{\pi}{4}$

30. If  $\bar{p}, \bar{q}$  are two non-collinear and non-zero vectors such that

$$(b - c)\bar{p} \times \bar{q} + (c - a)\bar{p} + (a - b)\bar{q} = 0$$

where a, b, c are the lengths of the sides of triangle then the triangle is

- 1) right angled
- 2) obtuse angled
- 3) equilateral
- 4) isosceles

31. If the position vectors of the three points A, B, C, are  $\bar{i} + \bar{j} + \bar{k}$ ,  $2\bar{i} + 3\bar{j} - 4\bar{k}$  and  $7\bar{i} + 4\bar{j} + 9\bar{k}$ , then the unit vector perpendicular to the plane of the triangle ABC is

- 1)  $(31\bar{i} - 38\bar{j} - 9\bar{k}) / 2486$

- 2)  $(31\bar{i} - 38\bar{j} + 9\bar{k}) / \sqrt{2486}$

- 3)  $(31\bar{i} - 38\bar{j} - 9\bar{k}) / \sqrt{2486}$

- 4)  $(31\bar{i} + 38\bar{j} + 9\bar{k}) / 2486$

4.  $\bar{A} = 2\bar{i} + \bar{k}$ ,  $\bar{B} = \bar{i} + \bar{j} + \bar{k}$  and  $\bar{C} = 4\bar{i} - 3\bar{j} + 7\bar{k}$ , then the Vector  $\bar{R}$  satisfying the conditions  $\bar{R} \times \bar{B} = \bar{C} \times \bar{B}$  and  $\bar{R} \cdot \bar{A} = 0$
- 1) (-1, -8, 2)
  - 2) (1, -8, -2)
  - 3) (-2, 8, 4)
  - 4) (1, 0, -2)
5. Let  $\bar{a} = -\bar{i} - \bar{k}$ ,  $\bar{b} = -\bar{i} + \bar{j}$  and  $\bar{c} = \bar{i} + 2\bar{j} + 3\bar{k}$  be three given vectors. If  $\bar{r}$  is a vector such that  $\bar{r} \times \bar{b} = \bar{c} \times \bar{b}$  and  $\bar{r} \cdot \bar{a} = 0$  then  $\bar{r} \cdot \bar{b} =$
- 1) -7
  - 2) 9
  - 3) -8
  - 4) 5
6. If  $\bar{r} \times \bar{b} = \bar{c} \times \bar{b}$ ,  $\bar{r} \cdot \bar{a} = 0$ ,  $\bar{a} = 2\bar{i} + 3\bar{j} - \bar{k}$ ,  $\bar{b} = 3\bar{i} - \bar{j} + \bar{k}$ ,  $\bar{c} = \bar{i} + \bar{j} + 3\bar{k}$  then  $\bar{r} =$
- 1)  $\frac{1}{2}(\bar{i} + \bar{j} + \bar{k})$
  - 2)  $2(\bar{i} + \bar{j} + \bar{k})$
  - 3)  $2(-\bar{i} + \bar{j} + \bar{k})$
  - 4)  $\frac{1}{2}(\bar{i} - \bar{j} + \bar{k})$
7. If  $\bar{a}$  and  $\bar{b}$  are unit vectors and  $\bar{c}$  satisfies  $2(\bar{a} \times \bar{b}) + \bar{c} = \bar{b} \times \bar{c}$  then the maximum value of  $|(\bar{a} \times \bar{c}) \cdot \bar{b}|$  is
- 1)  $\frac{1}{2}$
  - 2) 2
  - 3) 1
  - 4) 3

**S.P.Q.**

Student Practice Questions

8. If  $\bar{a}$  and  $\bar{b}$  are vectors such that  $|\bar{a} + \bar{b}| = \sqrt{29}$  and  $\bar{a} \times (2\bar{i} + 3\bar{j} + 4\bar{k}) = (2\bar{i} + 3\bar{j} + 4\bar{k}) \times \bar{b}$  then possible value of  $(\bar{a} + \bar{b}) \cdot (-7\bar{i} + 2\bar{j} + 3\bar{k})$  is
- 1) 0
  - 2) 3
  - 3) 4
  - 4) 8
9. If  $\bar{a} = \bar{i} + \bar{j} + \bar{k}$  and  $\bar{b} = 3\bar{i} + 4\bar{j}$  and  $\bar{a} \times \bar{b} = x\bar{i} + y\bar{j} + \sin \theta \bar{k}$  then  $(x, y, \theta) =$
- 1)  $(4, 3, n\pi + (-1)^n \frac{\pi}{2}, n \in \mathbb{Z})$
  - 2)  $(-4, 3, n\pi + (-1)^n \frac{\pi}{2}, n \in \mathbb{Z})$

- 3)  $(-3, -4, n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z})$
- 4)  $(3, 4, n\pi + (-1)^n \frac{\pi}{2}, n \in \mathbb{Z})$
10. Given  $\bar{a} + m\bar{b} + n\bar{c} = 0$ . The value of  $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}$  is  $\bar{0}$  then
- 1)  $m+n=1$
  - 2)  $m+n=-1$
  - 3)  $m+mn=1$
  - 4)  $m-2n=1$
11. Let  $\bar{u} = \bar{i} + \bar{j}$ ,  $\bar{v} = \bar{i} - \bar{j}$ ,  $\bar{w} = \bar{i} + 2\bar{j} + 3\bar{k}$ . If  $\bar{n}$  is a unit vector such that  $\bar{u} \cdot \bar{n} = 0$ ,  $\bar{v} \cdot \bar{n} = 0$  then  $|\bar{w} \cdot \bar{n}| =$
- 1) 0
  - 2) 1
  - 3) 2
  - 4) 3
12. Let  $\bar{a}(2, 1, -1)$ ,  $\bar{b}(1, 2, 1)$ ,  $\bar{c}(2, -1, 3)$  and  $\bar{d}(3, -1, 2)$  be four vectors. The projection of the vector  $\bar{a} + \bar{c}$  on the vector  $(\bar{b} - \bar{d}) \times \bar{c}$  is
- 1)  $\sqrt{2}$
  - 2)  $\sqrt{3}$
  - 3)  $\sqrt{6}$
  - 4)  $\sqrt{7}$
13. Let  $\bar{a}, \bar{b}$  be two non collinear unit vectors. If  $\bar{\alpha} = \bar{a} - (\bar{a} \cdot \bar{b})\bar{b}$ ,  $\bar{\beta} = \bar{a} \times \bar{b}$  then
- 1)  $|\bar{\alpha}| = |\bar{\beta}|$
  - 2)  $|\bar{\alpha}|^2 = |\bar{\beta}|^2$
  - 3)  $|\bar{\beta}|^2 = |\bar{\alpha}|$
  - 4)  $|\bar{\alpha}| = 2|\bar{\beta}|$

**VECTOR AREA, AREA****C.R.T.Q.**

Class Room Teaching Questions

14. P and Q are the mid-points of the non-parallel sides BC and AD of a trapezium ABCD, then  $\Delta APD =$
- 1)  $2\Delta CQB$
  - 2)  $\frac{1}{2}\Delta CQB$
  - 3)  $\Delta CQB$
  - 4)  $4\Delta CQB$

**S.P.Q.**

Student Practice Questions

15. For any four points P, Q, R, S,  $|\overrightarrow{PQ} \times \overrightarrow{RS} - \overrightarrow{QR} \times \overrightarrow{PS} + \overrightarrow{RP} \times \overrightarrow{QS}|$  is equal to 4 times the area of the triangle
- 1) PQR
  - 2) QRS
  - 3) PRS
  - 4) PQS

2. Arrange the following in ascending order

- A)  $|\bar{a} \times \bar{b}|$  if  $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}$ ,  $\bar{b} = 3\bar{i} + 4\bar{j} - \bar{k}$
  - B)  $|\bar{a} \times \bar{b}|$  if  $\bar{a} = \bar{i} + \bar{j} + \bar{k}$ ,  
 $\bar{b} = 2\bar{i} - 3\bar{j} + \bar{k}$
  - C)  $|\bar{a} \times \bar{b}|$  if  $|\bar{a}|=2$ ,  $|\bar{b}|=4$ ,  $(\bar{a}, \bar{b}) = \pi/6$
  - D)  $|\bar{a} \times \bar{b}|$  if  $|\bar{a}|=5$ ,  $|\bar{b}|=6$ ,  $|\bar{a} \cdot \bar{b}|=24$
- 1) B,A,C,D      2) D,A,C,B  
 3) C,B,A,D      4) B,C,A,D

3. Assertion (A): Three points with position vectors  $\bar{a}, \bar{b}, \bar{c}$  are collinear if

$$\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a} = \bar{0}$$

Reason (R): Three points A, B, C are collinear iff  $\overline{AB} = t \overline{BC}$ , where t is scalar.

- 1) Both A and R are true and R is the correct explanation of A.
- 2) Both A and R are individually true and R is not the correct explanation of A
- 3) A is true but R is false
- 4) A is false but R is true

4. Statement-1: If vertices of a triangle ABC are  $A(\bar{a}), B(\bar{b}), C(\bar{c})$  then length of altitude through A is

$$\frac{|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|}{|\bar{b} - \bar{c}|}$$

Statement-2: Area of triangle is

$$\Delta = \frac{1}{2} |\overline{AB} \times \overline{AC}|$$

- 1) Statement-1 is true, statement-2 is true, Statement-2 is a correct explanation for Statement-1.
- 2) Statement-1 is true, statement-2 is true, Statement-2 is not a correct explanation for statement-1.
- 3) Statement-1 is true, statement-2 is false
- 4) Statement-1 is false, statement-2 is true

5.

**Statement-1:** The perpendicular distance from  $(1, 4, -2)$  to the line joining  $(2, 1, -2), (0, -5, 1)$  is  $\frac{3\sqrt{26}}{7}$ .

**Statement-2:** The perpendicular distance from a point P to the line joining the points A, B is  $\frac{|\overline{AP} \times \overline{AB}|}{|\overline{AB}|}$

- 1) Statement-1 is true, statement-2 is true, statement-2 is a correct explanation for statement-1.
- 2) Statement-1 is true, statement-2 is true, Statement-2 is not a correct explanation for statement-1.
- 3) Statement-1 is true, statement-2 is false
- 4) Statement-1 is false, statement-2 is true

6.

**Statement-1:** The torque about the point  $2\bar{i} + \bar{j} - \bar{k}$  of a force represented by  $4\bar{i} + \bar{k}$  acting through the point  $\bar{i} - \bar{j} + 2\bar{k}$  is  $-2\bar{i} + 13\bar{j} + 8\bar{k}$

**Statement-2:** The torque of a force  $\bar{F}$  about a point P is  $\bar{r} \times \bar{F}$  where  $\bar{r}$  is the vector from the point P to any point  $\bar{a}$  on the line of action L of  $\bar{F}$ .

- 1) Statement-1 is true, statement-2 is true, statement-2 is a correct explanation for statement-1.
- 2) Statement-1 is true, statement-2 is true, statement-2 is not a correct explanation for statement-1.
- 3) Statement-1 is true, statement-2 is false
- 4) Statement-1 is false, statement-2 is true

Let the vectors  $\overrightarrow{PQ}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$  and  $\overrightarrow{UP}$  represent the sides of a regular hexagon.

**Statement-I:**  $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq \vec{0}$

**Statement-II:**  $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$  and  $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$ . Then

- 1) I is true, II is true and II is correct explanation of I
- 2) I is true, II is true and II is not correct explanation of I
- 3) I is true, II is false
- 4) I is false, II is true

**Assertion(A):** If  $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d}$  and  $\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \times \overrightarrow{d}$  then  $\overrightarrow{b} - \overrightarrow{c}$  is parallel to  $\overrightarrow{a} - \overrightarrow{d}$

**Reason (R):** If cross product of two non-zero vectors is zero vector then those two vectors are parallel

- 1) Both A and R are true and R is the correct explanation of A
- 2) Both A and R are true but R is not correct explanation of A
- 3) A is true but R is false
- 4) A is false but R is true

**Statement-I:** Let  $\overrightarrow{P}$  be position vector of point P and Let the line passing through the point A with position vector  $\overrightarrow{a}$  and parallel to the vector  $\overrightarrow{b}$  then perpendicular distance from P

$$\text{to a line is } = \frac{|\overrightarrow{AP} \times \overrightarrow{b}|}{|\overrightarrow{b}|}$$

**Statement-II:** Vector equation of the line passing through two points A and B with position vector  $\overrightarrow{a}$  and  $\overrightarrow{b}$  is  $(\overrightarrow{r} - \overrightarrow{a}) \times (\overrightarrow{b} - \overrightarrow{a}) = \vec{0}$

Which of the above is / are True

- 1) only I is true
- 2) only II is true
- 3) both I and II true
- 4) neither I nor II are true

10. **Assertion(A):** A vector perpendicular to both  $\vec{i} + \vec{j} + \vec{k}$  and  $2\vec{i} + \vec{j} + 3\vec{k}$  is  $2\vec{i} - \vec{j} - \vec{k}$

**Reason (R):** Every vector perpendicular to plane containing  $\overrightarrow{a}, \overrightarrow{b}$  is equal to  $\overrightarrow{a} \times \overrightarrow{b}$

- 1) Both A and R are true and R is the correct explanation of A
- 2) Both A and R are true but R is not correct explanation of A
- 3) A is true but R is false
- 4) A is false but R is true

11. **Statement-I:** If  $\overrightarrow{a}, \overrightarrow{b}$  are two unit vectors  $(\overrightarrow{a}, \overrightarrow{b}) = \theta$ . Then  $|\overrightarrow{a} \times \overrightarrow{b}|$  is maximum when  $\theta$  is maximum.

**Statement-II:** If  $\overrightarrow{a}, \overrightarrow{b}$  are two unit vectors  $(\overrightarrow{a}, \overrightarrow{b}) = \theta$  then  $|\overrightarrow{a} \cdot \overrightarrow{b}|$  is maximum when  $\theta$  is minimum

- 1) only I is true
- 2) Only II is true
- 3) Both I and II are true
- 4) Neither I nor II are true

## KEY

- |       |       |       |       |       |
|-------|-------|-------|-------|-------|
| 01) 4 | 02) 3 | 03) 1 | 04) 1 | 05) 1 |
| 06) 1 | 07) 3 | 08) 1 | 09) 3 | 10) 3 |
| 11) 2 |       |       |       |       |