Survey on Elliptic Curve Cryptography

A R PRASHANTH
computer science dept.
PES University
bangalore, India
prashanthathunt@gmail.com

***Abstract*—Elliptic Curve Cryptography has been studied since 1986 for academic and industrial purposes . elliptic Curve Cryptography provides more security on a shorter key size .It is mainly used in public key cryptography. It is widely used in Internet of Things (IoT) , in military communication , to exchange symmetric key between parties there are many applications . it takes less memory and is faster to compute . it can be used for low end devices . In this era everyone wants it to be fast , instant and also very secure hence Elliptic Curve Cryptography is coming up as it provides high security and smaller key size . still to find make it more efficient it has to be made hardware accelerate . it has to be made or built inside every hardware unit (if required) so that we get fast , efficient output . we need to have a light weight Elliptic Curve Cryptography as well for better efficiency . although it may provide little less security.**

In this survey paper we are going to discuss on, how the Elliptic Curve can be used in Cryptography , why it is important to use hardware accelerators to accelerate the speed of encryption and decryption process . we will see how the authentication scheme of Elliptic Curve cryptography. What are the various fields Elliptic Curve cryptography is been used like Elliptic Curve Diffie-Hellman (ECDH) and Elliptic Curve Digital Signature Algorithm (ECDSA) which is used in bitcoin currently

***Index Terms*—ECC, elliptic Curve, cryptography**

I. Introduction

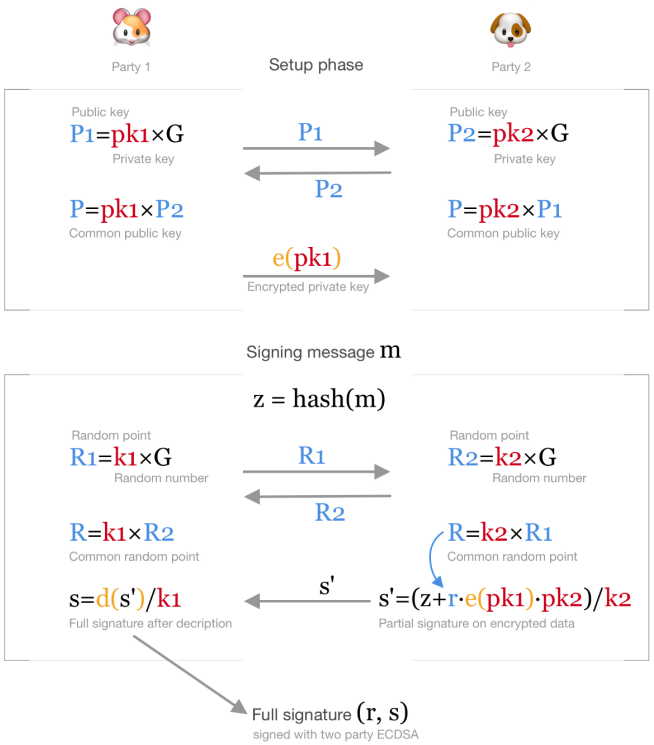
Elliptic curve cryptography uses elliptic curve theory to build a public key encryption technique which is fast , more efficient and smaller in key size . It is used mostly with a combination of encryption methods like Rivest–Shamir–Adleman (RSA) or Diffie-Hellman key exchange .a 160 bit ecc key can provide as much as 1024bit key size RSA/DSA that is almost 1:3 ratio as we go higher 224 bit of ecc provides 2048 bits of RSA/DSA which is almost 1:6 ratio as we increase the number a 521bit ecc give as much as security as a 15360 bit RSA which his 64 time more secure and dealing with 15360bit of number is a lot of computation to handle .

Key size		Security level (bits)	Ratio of cost
RSA/DSA	ECC		
1024	160	80	3:1
2048	224	112	6:1
3072	256	128	10:1
7680	384	192	32:1
15360	**521**	256	64:1

hence Elliptic curve cryptography is used to make it more secure and fast with lesser key size . The main objective of Elliptic curve cryptography let say cryptography in this case is to protect our data using different cryptography algorithms . so the cryptographic algorithm we use should be feasible should cost less than the original data cost for sending it . What I meant to say is that protecting the data should not cost more than the original data itself . Although Elliptic curve cryptography uses less power and it is very efficient than other methods we still need to make it more feasible and faster by using hardware accelerators which can give support at hardware level .

Elliptic curve cryptography is mainly used in communication of data where privacy or we can say protecting the data is more important . But in this digital world everybody's first priority is privacy so we need to have a better faster and reliable system . In future there maybe a better solution for this problem but currently Elliptic curve cryptography combined with other methods like RSA or diffie hellman key exchange methods make the current know techniques feasible , more efficient which uses less computation power and provides higher security . such examples are Elliptic Curve Digital Signature Algorithm(ECDSA) which is currently used by bitcoin (a cryptocurrency invented by Satoshi Nakamoto in 2008 and it was started in 2009) to ensure the transaction are made to the rightful owners and the funds can only be spend by the owner and owner only .

It uses just 256 bits unsigned int that is 32 bytes Thanks to elliptic curve cryptography for providing far more security than a 15000+ bits rsa could provide . Elliptic Curve Digital signature algorithm is faster than RSA for signing and decryption . so it's a win-win situation .



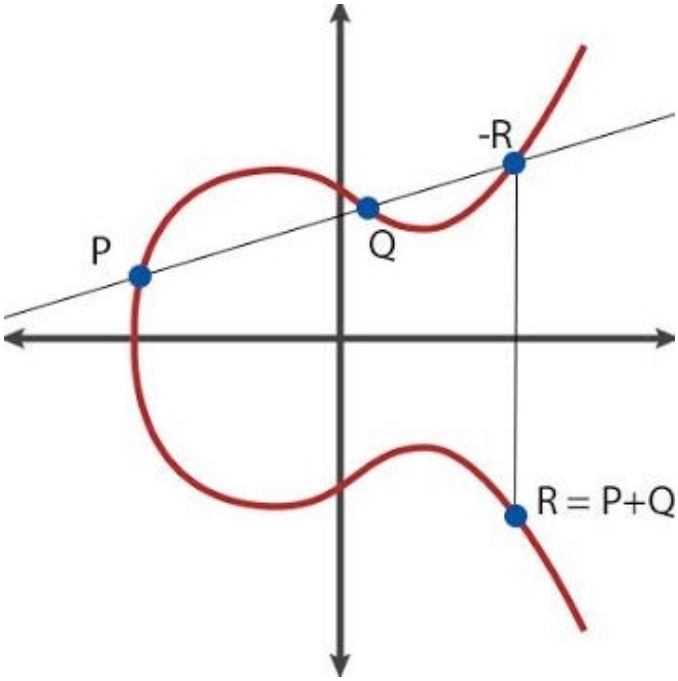
Another example is the Elliptic curve Diffie Hellman - it's the same as the Diffie Hellman protocol but uses the Elliptic curve for better security and it's faster too.

II. Algorithm used with security parameter

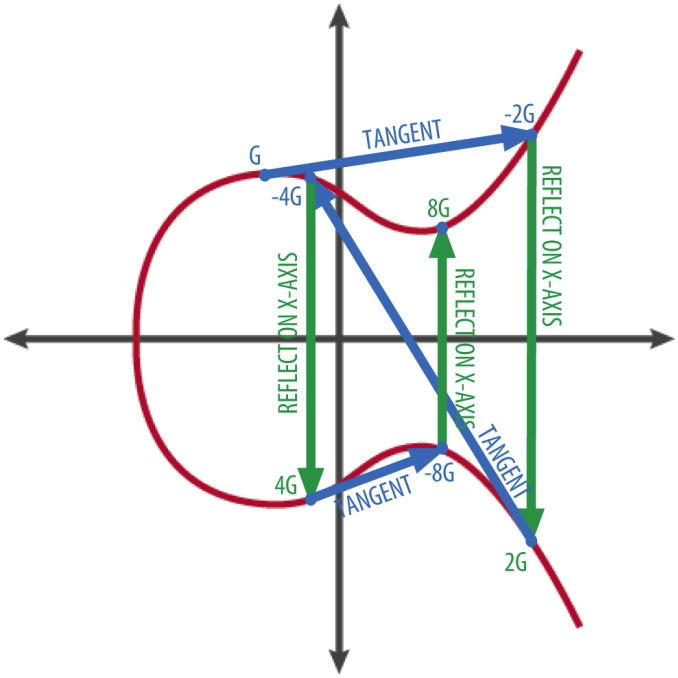
Weierstraß equation of elliptic curve E defined over K using affine coordinates is
E: y2 + a1xy + a3y = x3 + a2x2 + a4x + a6
where a1,a2,a3,a4,a5,a6 belongs to K
1st we will briefly talk about the mathematics of elliptic curve

general properties of Elliptic Curve general formula of Elliptic curve is
E:y2 = x3 + ax + b

general Elliptic curve diagram



calculating multiples of G in an Elliptic curve

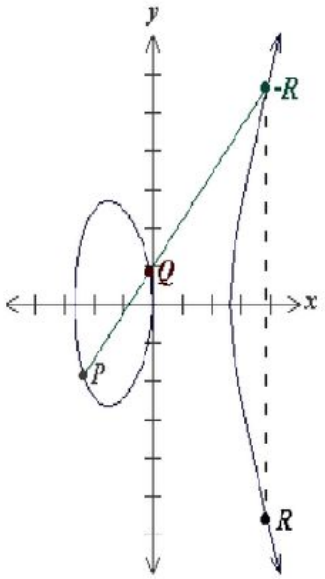


To calculate the multiple of G drew a tangent to at that point. See where it intersects the curve . Draw a parallel line in y axis see where it intersects the curve . There you go u have the the next multiple of G . It may look very simple because its the basic example . but as we make the equation complex it is practically impossible to trace 150bits length of multiple of G.

What makes it so secure is that it is easy to calculate multiples of G but where as going back is very hard . It is

Basic Properties:

- Equation of an elliptic curve:
  $y^2 = x^3 + ax + b$
- The equation is defined for no repeated factors.
- Elliptic curve groups are additive groups.
- The addition of any two points on curve is defined geometrically.
- Law of addition:
  P+Q = R.
- The point –R on the curve is reflected on x-axis to point R.



P (-2.35, -1.86)
Q (-0.1, 0.836)
-R (3.89, 5.62)
R (3.89, -5.62)

P + Q = R = (3.89, -5.62).

$y^2 = x^3 - 7x$

Source:
http://www.certicom.com/index.php?action=ecc_tutorial,ecc_tut_2_1

computationally very hard to go back from a multiple of G to G(starting point)

III. Working

Hardware implementation will has to be done in chip level or we can say nano level so that it can have high degree of performance then what we see using normal CPU or GPU or even TPUs. It should not be a separate unit from the rest of the unit or like a chip . It has to be mounted to the CPU or any kind of microprocessors so that there will no latency in sharing the data between our hardware accelerator and the output stream .

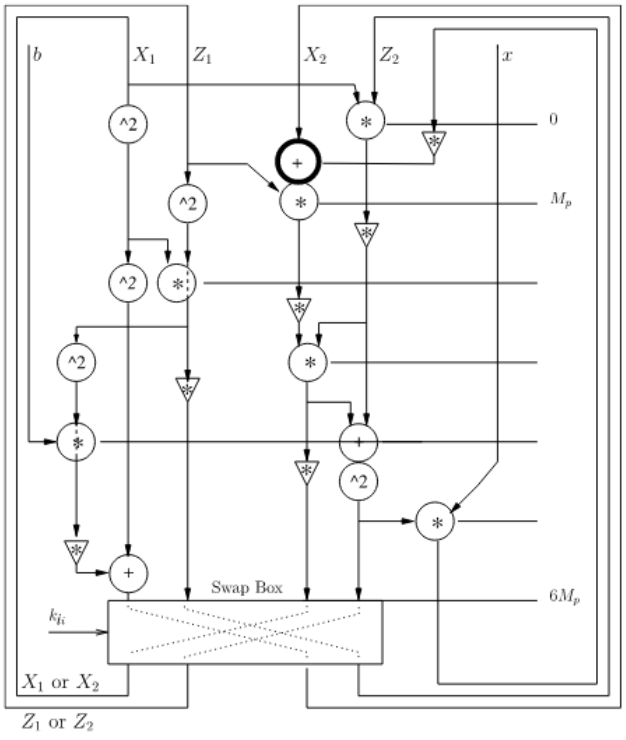


Fig. 2. Algorithm Scheduler

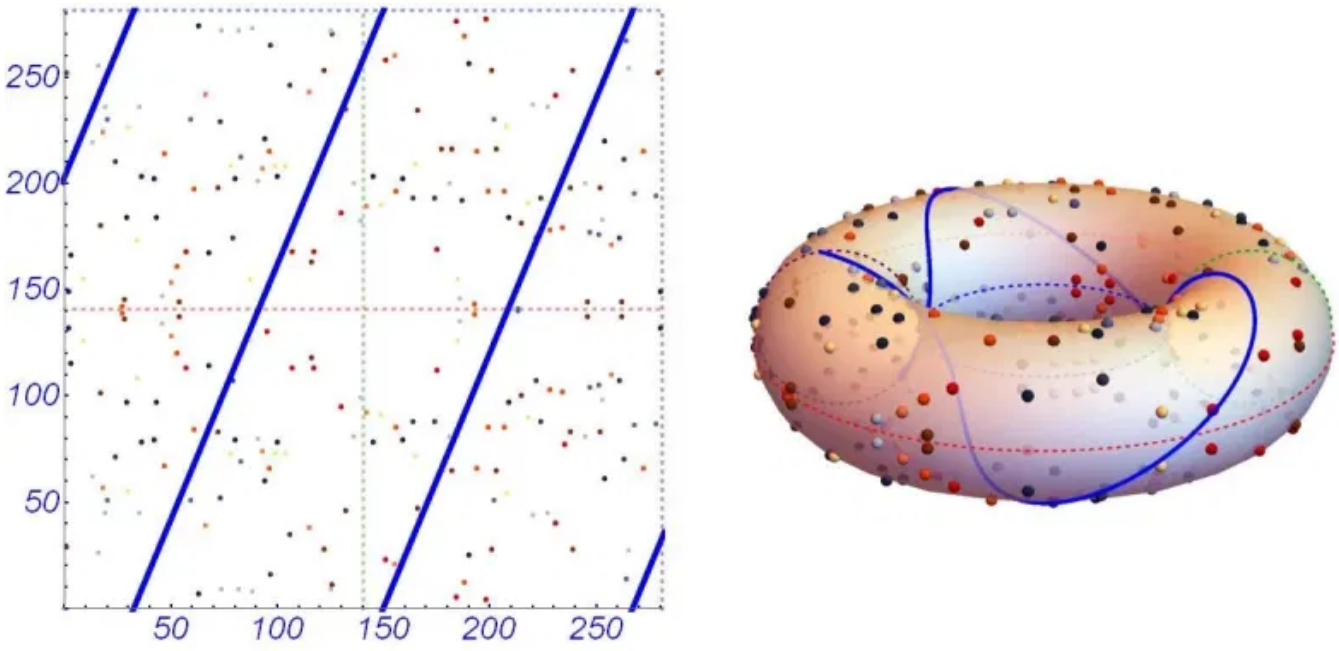


Fig. 1. Practical Elliptic curve

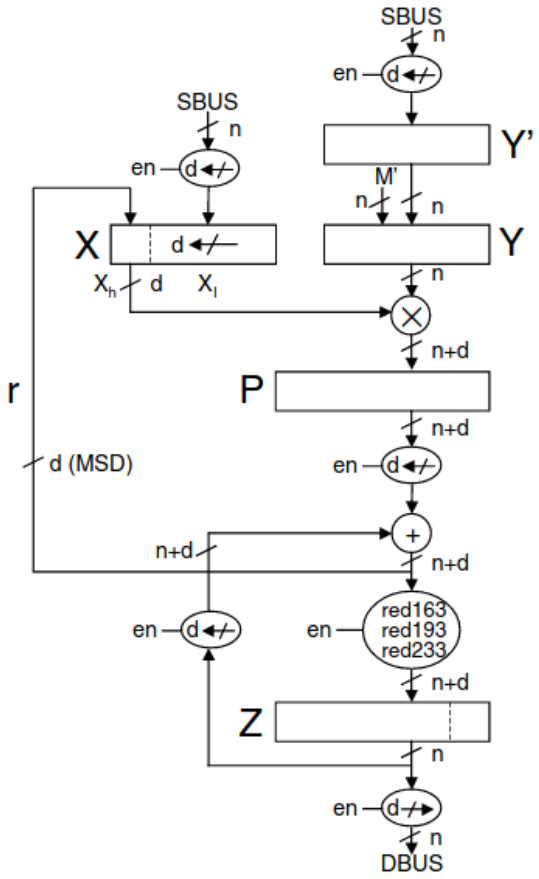


Fig. 3. Multiplier architecture

Those are the working diagram of hardware implementation of Elliptic curve cryptography

IV. Veriations with respect to main algorithm

There are in total 3 algorithms will be proposed in this paper

Algorithm 1. Left-to-right binary scalar multiplication

Input: $P \in E$, $k = (1, k_{l-2}, \ldots, k_1, k_0)_2$
Output: $Q = k \cdot P$
  $Q \leftarrow P$
  for i from $l-2$ downto 0 **do**
    $Q \leftarrow 2Q$
    if $k_i = 1$ **then** $Q \leftarrow Q + P$

Algorithm 2. Left-to-right binary NAF scalar multiplication

Input: $P \in E$, $k = (k_l, k_{l-1}, \ldots, k_0)_2$, $k_i \in \{-1, 0, 1\}$
Output: $Q = k \cdot P$
  $Q \leftarrow 0$
  for i from l downto 0 **do**
    $Q \leftarrow 2Q$
    if $k_i = 1$ **then** $Q \leftarrow Q + P$
    if $k_i = -1$ **then** $Q \leftarrow Q - P$

Algorithm 3. Montgomery ladder scalar multiplication

Input: $P \in E$, $k = (1, k_{l-2}, \ldots, k_1, k_0)_2$
Output: $x(P_1) = x(k \cdot P)$
$P_1 \leftarrow P$, $P_2 \leftarrow 2P$
for i from $l-2$ downto 0 **do**
if $k_i = 1$ **then** $x(P_1) \leftarrow x(P_1 + P_2)$, $x(P_2) \leftarrow x(2P_2)$
else $x(P_2) \leftarrow x(P_1 + P_2), x(P_1) \leftarrow x(2P_1)$

V. Applications

In day to day scenario public key cryptography is most important as people are demanding for privacy of their personal data. As the digit world is increasing their demand on data , the cybersecuiry plays a important role in it to protect the data of individual . So every application which has to communicate over a an unreliable network need public key cryptography to maintain data privacy .

Performaces of RSA cryptographic algorithms Vs ECC on 90MHz pentium chip

Security level	Encrypt (blks/sec)	Decrypt (blks/sec)	Create Key (sec)
512 bit	370	42	0.45
768 bit	189	15	1.5
1024 bit	116	7	3.8

(b)

Security level	Encrypt (blks/sec)	Decrypt (blks/sec)	Create Key (sec)
512 bit	1020	125	0.26
768 bit	588	42	0.59
1024 bit	385	23	1.28

VI. Conclusion

Considering the performance of the hardware accelerator we can say that it can be used for every communication devices which is in need of data privacy . which provides you higher performance. The only way now is to make the Elliptic curve cryptography more faster sacrificing flexibility is to make it in hardware level . We saw that hardware support can really boost the performance of Elliptic curve cryptography . Is Elliptic curve cryptography quantum resistant ? In fact yes . so Elliptic curve cryptography is not going anywhere for at-least few decades so it is better to make it in hardware level so that it can out-stand and can be used by everyone without worrying about the performance or the cost of encryption over the cost of data . To truly minimise the cost of encryption hardware implementation is necessary .

Acknowledgment

Thanking the PES Institution and therefore the Teachers for his or her support, Guidance and encouragement.

References

[1] G.B. Agnew, T. Beth, R.C. Mullin, S.A. Vanstone, Arithmetic operations in GF(2^m), Journal of Cryptology 6 (1) (1993) 3–13.

[2] T. Akishita, Fast simultaneous scalar multiplication on elliptic curve with montgomery form, in: Selected Areas in Cryptography (SAC), LNCS 2259, 2001, pp. 255–267.

[3] ANSI, ANSI X9.62 The elliptic curve digital signature algorithm (ECDSA). Available from: <http://www.ansi.org>.

[4] B. Ansari, M. Anwar Hasan, High performance architecture of elliptic curve scalar multiplication, Tech. Report CACR 2006-01, 2006. Available from: <http://www.cacr.math.uwaterloo.ca/techreports/2006/cacr2006-01.pdf>.

[5] G. Bai, G. Chen, H. Chen, Fast scalar multiplications of elliptic curve cryptosystems over binary fields, in: SKLOIS Information Security and Cryptology (CISC), 2005, pp. 315–323.

[6] J.-C. Bajard, L. Imbert, C. Negre, T. Plantard, Efficient multiplication in GF(p^k) for elliptic curve cryptography, in: IEEE Symposium on Computer Arithmetic (ARITH-16), 2003, pp. 181–187.

[7] S. Bajracharya, C. Shu, K. Gaj, T. El-Ghazawi, Implementation of elliptic curve cryptosystems over GF(2^n) in optimal normal basis on a reconfigurable computer, in: Field-Programmable Logic and Applications (FPL), LNCS 3203, 2004, pp. 1001–1005.

[8] L. Batina, G. Bruin-Muurling, S.B. Örs, Flexible hardware design for RSA and elliptic curve cryptosystems, in: The Cryptographer's Track at RSA Conference (CT-RSA), LNCS 2964, 2004, pp. 250–263.

[9] L. Batina, S.B. Örs, B. Preneel, J. Vandewalle, Hardware architectures for public key cryptography, Elsevier Integration, the VLSI Journal, special issue on Embedded Cryptographic Hardware, 34 (2003) 1–2, pp. 1–64.

[10] M. Bednara, M. Daldrup, J. Teich, J. von zur Gathen, J. Shokrollahi, Tradeoff analysis of FPGA based elliptic curve cryptography, in: IEEE Symposium on Circuits and Systems (ISCAS), 5, 2002, pp. 797–800.

Table 1
Performance summary of state of the art

Name, Ref./Computation, IrrPol	Device/Size	Frequency/Time	Multiplier/Remarks
Ansari et al. [4] 2^m, 163-bit, NIST	XC2V2000 8300 luts	100 MHz 41 µs	MSB pipelined $D = 41$ 7 bRAMs, No final inv.
Ansari et al. [4] 2^m, 163-bit, NIST	0.18 µm CMOS 36000 gates	167 MHz 21 µs	MSB pipelined $D = 41$ No final inv.
Sozzani et al. [75] 2^m, 163-bit, NIST	0.13 µm CMOS 0.59 mm^2	417 MHz 30 µs	2 Mults $D = 8$
Saqib et al. [67] 2^m, 191-bit, trino.	XCV3200E 18314 slices	9.99 MHz 56 µs	Parallel Karatsuba 24 bRAMs, No final inv.
Shu et al. [70] 2^m, 163-bit, NIST	XCV2000E-7 25763 luts	68.9 MHz 48 µs	MSD 6 mult $D = 32,8$
Shu et al. [70] 2^m, 233-bit, NIST	XCV2000E-7 35800 luts	67.9 MHz 89 µs	MSD 6 mult $D = 32,8$
Orlando et al. [61] p, 192-bit, NIST	XCV1000E-8 11416 luts	40 MHz 3 ms	MMM, Booth, $D = 4$ 35 bRAMs
McIvor et al. [48] p, 256-bit, Any	XC2VP125-7 15755 slices	39.5 MHz 3.84 ms	MMM 256 Mults 18×18
Jarvinen et al. [39] 2^m, 163-bit, NIST	XC2V8000-5 18079 slices	90.2 MHz 106 µs	Generator
Grabbe et al. [27] 2^m, 233-bit, NIST	XC2V6000 19440 luts	100 MHz 130 µs	Hybride KOA Generator

Fig. 4. Performance Table on different hardware