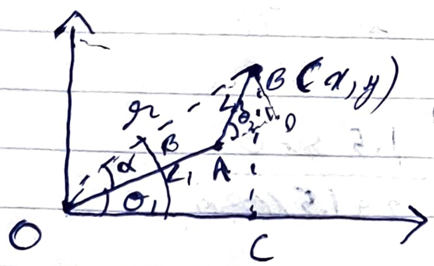
INVERSE KINEMATICS

Q2.

General expression for 2D 2 link arm:



$L_1 = 2m$

$L_2 = 1.5m$

~~β = θ1 + α~~ $\beta = \alpha + \theta_1$ $\tan \beta = \frac{y}{x}$

In ΔOAB, $r^2 = L_1^2 + L_2^2 - 2L_1L_2\cos(\pi - \theta_2)$

$r^2 = x^2 + y^2$

$x^2 + y^2 = L_1^2 + L_2^2 + 2L_1L_2\cos\theta_2$

$$\theta_2 = \cos^{-1}\left(\frac{r^2 - (L_1^2 + L_2^2)}{2L_1L_2}\right)$$

~~similarly~~, $\alpha = \cos^{-1} L_2^2 -$

In ΔOBD

$$\tan\alpha = \left(\frac{L_2\sin\theta_2}{L_1 + L_2\cos\theta_2}\right)$$

$$\theta_1 = \tan^{-1}\frac{y}{x} - \tan^{-1}\left(\frac{L_2\sin\theta_2}{L_1 + L_2\cos\theta_2}\right)$$

i) $(x,y) = (2,1)$

$r^2 = x^2 + y^2 = 4 + 1 = 5$

Plugging into eqn for θ_2

$$\theta_2 = \cos^{-1} \frac{5 - 6.25}{6} \approx 1.78$$

$$\theta_1 = \tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1.5 \sin\theta_2}{2 + 1.5 \cos\theta_2}$$

$= -0.25$

ii) $(3,0)$

$r^2 = 9$

$\Rightarrow \theta_2 = 1.09$

$\Rightarrow \theta_1 = -0.46$

iii) $(0, 3.4)$

$r^2 = 11.56$

$\Rightarrow \theta_2 = 0.48$

$\Rightarrow \theta_1 = 1.366$

iv) $(0, -2.5)$

$r^2 = 6.25$

$\Rightarrow \theta_2 = 1.57$

$\Rightarrow \theta_1 = -1.90$

v) $(4, 0)$

$r^2 = 16$

Let $\theta_1 = \cos^{-1}(f(r))$

where $f(r) = \frac{r^2 - 6.25}{6}$

$f(\sqrt{16}) = \frac{16 - 6.25}{6} = 1.625$

which ~~dos~~ doesn't lie in domain of $\cos^{-1}$ fn implying point is not reachable

vi) $(0, 0.2)$

$r^2 = 0.04$

$\theta_1 = \cos^{-1}(f(r))$

$f(r) = \frac{r^2 - 6.25}{6}$

$f(0.02) = \frac{0.04 - 6.25}{6} = -1.035 \notin$ domain of $\cos^{-1}$

point is unreachable

The arm can move in the area enclosed ~~by~~ between the circles of radii 0.5 & 3.5 centered at origin

$0.5 \leq r \leq 3.5$ which is also the domain for θ_1