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Worksheet 11

1) Red marbles - 9

Blue marbles - 1

900 Random draws

For one draw, $\mu = E(x) = p = \frac{9}{10}$

$Var = p(1-p) = \frac{9}{10}\left(1-\frac{9}{10}\right)$

$\sigma^2 = \frac{9}{100}$

For 900 Random draws $= N = (n\mu, n\sigma^2)$

$N = \left(\frac{9}{10} \times 900, \frac{9}{100} \times 900\right)$

$N = (810, 81)$

2) P(left handed) = 1% = 0.01

$\therefore \mu = p = 0.01$

$var = \sigma^2 = p(1-p) = 0.01(1-0.01)$

$= 0.01(0.99)$

$= 0.0099$

For 200 people $= N = (n\mu, n\sigma^2)$

$N = (200 \times 0.01, 200 \times 0.0099)$

$N = (2, 1.98)$

For 99.7% confidence interval

$\hat{\mu} \pm 3\hat{\sigma}$

$\hat{\mu} + 3\hat{\sigma} = 2 + 3 \times \sqrt{1.98} = 6.22$

$\hat{\mu} - 3\hat{\sigma} = 2 - 3 \times \sqrt{1.98} = -2.22.$

Therefore we are 99.7% sure that out of 200 people, $[0, 6] \rightarrow$ 0 to 6 people will be left handed

(3) Probability that one dart fall in wedge i

$P(w_i) = \frac{1}{20} = p$

$var = \sigma^2 = p(1-p)$

$= \frac{1}{20}\left(1 - \frac{1}{20}\right)$

$= \frac{19}{400}$

(a) For 100 darts

$E(x_i) = np = 100 \times \frac{1}{20} = 5$

$\sigma^2(x_i) = n\sigma^2 = 100 \times \frac{19}{400} = \frac{19}{4}$

(b) $N\left(5, \frac{19}{4}\right)$

95% confidence, upper bound $= \mu + 2\sigma$

$= 5 + 2\sqrt{\frac{19}{4}}$

$= 5+\sqrt{19}$

$= 5+4.358899$

$= 9.358899$

$\therefore$ 95% confidence that out of 100 darts ≤ 9 darts will fall in wedge i

(c) $E(Y_i)$ and $Var(Y_i) = ?$

$E(Y_i) = \sum y \, P(y_i = y)$

P(dart falls on red) $= \frac{1}{2}$

P(dart falls on black) $= \frac{1}{2}$

$E(Y_1) = \sum y \, P(Y_1 = y)$

$= 1 \times \frac{1}{2} + (-1) \times \frac{1}{2}$

$= 0$

$\therefore E(Y_i) = E(Y_1) + E(y_2) + \dots E(Y_{100})$

$= 0$

$Var(Y_i) = E(y_i^2) - (E(y_i))^2$

$= 1^2 \times \frac{1}{2} + (-1)^2 \frac{1}{2} - 0^2$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

(d) $N(n\mu, n\sigma^2)$

$N(100 \times 0, 100 \times 1)$

$= N(0, 100)$

(e) 99% confidence $= \mu \pm 3\sigma$

$\sigma = \sqrt{100} = 10$

$\mu + 3\sigma = 0 + 3 \times 10 = 30$

$\mu - 3\sigma = 0 - 3 \times 10 = 10$

$\therefore [0, 30] =$ Interval.

(4) $n =$ Sample Size $= ?$

$P(\text{color blind}) = 1\% = 0.01$

In n people atleast one person should be color blind.

Let $C =$ number of color blind person in a sample of n

$\therefore P(C > 0) \geq 95\%$ given

≥ 0.95

$P(C > 0) = 1 - P(C = 0) \geq 0.95$

$P(C = 0) \leq 0.05$

~~$P(C=0)$~~

From binomial distribution since n = infinite

$P(X=k) = {}^nC_k\, p^k (1-p)^{n-k}$

$P(C=0) = {}^nC_0\, p^0 (1-p)^{n-0} \leq 0.05$

$1 \times 1 \times (1-0.01)^n \leq 0.05$

$(0.99)^n \leq 0.05$

$\log(0.99)^n \leq \log(0.05)$

$n \log(0.99) \leq \log(0.05)$

$n \times -0.004365 \leq -1.30103$

$-n \leq 298$

$n \geq 298$

$\therefore n = 299$ for the probability that atleast one color blind person to be atleast 95%.

⑥ std error = 1% = $\frac{1}{100}$

⑦

$20\% \leq P \leq 40\%$

$0.2 \leq P \leq 0.4$

$P_{max} = 0.4$

$$std = \sqrt{\frac{P(1-P)}{n}}$$

$$\frac{1}{100} = \sqrt{\frac{0.4(1-0.4)}{n}}$$

$$\left(\frac{1}{100}\right)^2 = \frac{0.4 \times 0.6}{n}$$

$$n = 0.24 \times 10000$$

$$n = 2400$$

∴ Sample size we can use is 2500

⑧ 100,000 people

$n = 500$

194 enrolled in college

$$P = \frac{194}{500} = 0.388$$

$$std = \sigma = \sqrt{\frac{P(1-P)}{n}}$$

$$= \sqrt{\frac{0.388 \times (1-0.388)}{500}}$$

$$\sigma = 0.02179$$

95.5% confidence interval

$\mu \pm 2\sigma$

$= 0.388 \pm 2(0.02179)$

$= 0.388 \pm 0.04358$

$= [0.34442, 0.43158]$

So 95.5% confidence that between

[34442, 43158] enrolled in college.

That is between [34%, 43%]

(9) 1000 would be enough since what matters is the sample size and not the overall population size. Also demographics of Austin and Dallas are same.

(10) $n = 1000$

$\mu = 307$

$\sigma = 30$

For Nation wide, average doesnt change since we dont have true μ

$\therefore \hat{\mu} = 307$

std dev $= \hat{\sigma} = \frac{30}{\sqrt{1000}} = 0.95$

Distribution $N(307, 0.95^2)$