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Worksheet 10

1) $\begin{pmatrix}3\\4\\0\end{pmatrix}$ $\begin{pmatrix}4\\-3\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\1\end{pmatrix}$

Any two vectors are orthonormal if their dot products is 0 and they are unit vectors

$\begin{pmatrix}3\\4\\0\end{pmatrix} \cdot \begin{pmatrix}4\\-3\\0\end{pmatrix} = 12-12+0=0$

$\begin{pmatrix}3\\4\\0\end{pmatrix} \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} = 0+0+0=0$

$\begin{pmatrix}4\\-3\\0\end{pmatrix} \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} = 0+0+0=0$

Length of $\begin{pmatrix}3\\4\\0\end{pmatrix} = \sqrt{3^2+4^2+0^2} = 5$

Length of $\begin{pmatrix}4\\-3\\0\end{pmatrix} = \sqrt{4^2+3^2+0^2} = 5$

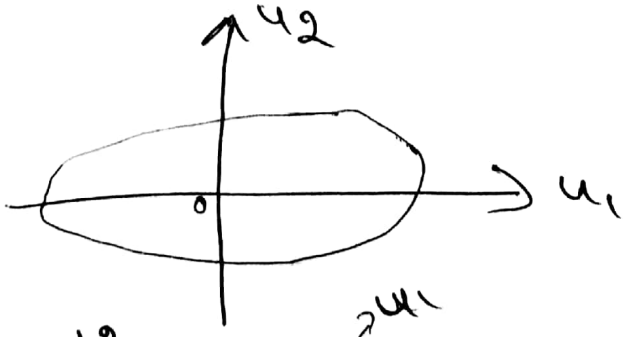
Length of $\begin{pmatrix}0\\0\\1\end{pmatrix} = \sqrt{0^2+0^2+1^2} = 1$

Therefore $\begin{pmatrix}3\\4\\0\end{pmatrix}$, $\begin{pmatrix}4\\-3\\0\end{pmatrix}$ and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ vectors are not orthonormal.

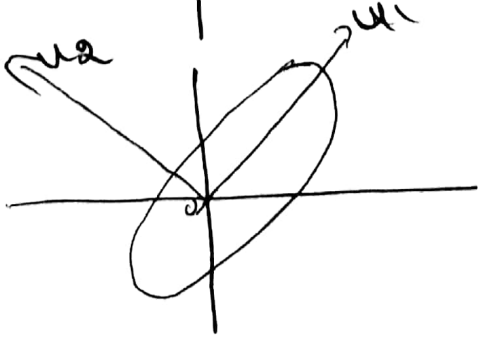
2) u_1 → First Eigen vector

u_2 → Second Eigen vector

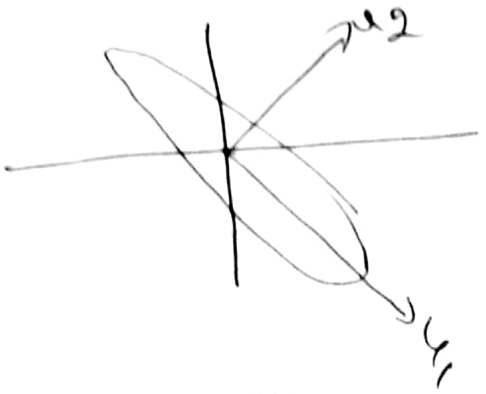
a)



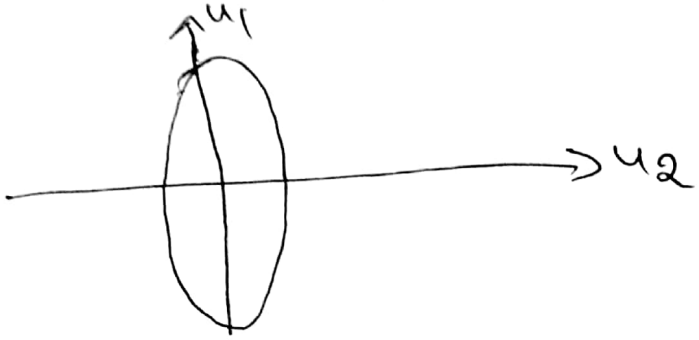
b)



c)



d)



③ $u_1, u_2 \in R^d$

$\|u_1\| = \|u_2\| = 1 \Rightarrow$ unit vectors

$u_1 \cdot u_2 = 0 \Rightarrow u_1 \& u_2$ are orthonormal.

$$U = \begin{pmatrix} \uparrow & \uparrow \\ u_1 & u_2 \\ d & \downarrow \end{pmatrix}$$

(a) $U \rightarrow$ Dimension $(d \times 2)$

$U^T \rightarrow$ Dimension $(2 \times d)$

$U U^T \Rightarrow$ Dimension $[\ ]_{d \times 2} [\ ]_{2 \times d}$

$= d \times d$

$u_1 u_1^T \rightarrow$ Dimension $= [\ ]_{d \times 1} [\ ]_{1 \times d}$

$= d \times d$

(3) (b) $x \rightarrow (u_1 \cdot x, u_2 \cdot x)$

It is projection of x on vector U

$x \rightarrow (u_1 \cdot x)\, u_1 + (u_2 \cdot x)\, u_2$

It is projection of x on vector U and then Reconstruction of x from the projected x on U

$x \rightarrow u^T x$

It is projection of x on vector U

$x \rightarrow u u^T x$

It is projection of x on vector U and then Reconstruction of x from the projected x on U