Maths in ML

Linear Algebra

Basics of matrices (Intro)

- Linear Algebra is based on matrices
- Matrix is an array of numbers

$$A = \begin{bmatrix} -2 & 8 \\ 3 & 1 \end{bmatrix}$$

- Linear system:

$$2x + 3y = 8$$
$$x - 7y = -13$$

Linear, ∵ highest power is 1.

* Solving bigger systems (without matrices) is tedious and often impractical.

Uses of matrix

→ Dealing with large amount of informations
eg- Big Data, Machine learning, Artificial intelligence

→ Represent any type of info.
- Points in space
- Pixels on the screen
- Data of customers
- Population surveys
- Linear systems.

Matrix Notation

- Matrix Notation
- Size of a matrix
- Refer to elements in a Matrix

$A = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$ $\boxed{B = \begin{bmatrix} 2 & 1 & 6 \\ -3 & 2 & 4 \end{bmatrix}}$ $C = \begin{bmatrix} 5 \\ -2 \\ 0 \\ 9 \end{bmatrix}$ $D = [1 \; 4 \; 9$

$$\downarrow$$

$$2x + y = 6$$
$$-3x + 2y = 4$$

Dimension (order) of a matrix

Dimension = Rows × Columns.

$A = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$ = 2 rows × 2 columns [Square Matrix]
$\underline{2 \times 2}$

$B = \begin{bmatrix} 2 & 1 & 6 \\ -3 & 2 & 4 \end{bmatrix}$ = 2 rows × 3 columns
2×3

$C = \begin{bmatrix} 5 \\ -2 \\ 0 \\ 9 \end{bmatrix}$ = 4 rows × 1 column
$\underline{4 \times 1}$ (column Matrix)

$D = [1 \; 4 \; 9 \; 6]$ = 1 row × 4 columns
$\underline{1 \times 4}$ (Row Matrix)

— Rectangular Matrix

$E = [2]$ = 1 row × 1 column
$\underline{1 \times 1}$

Addressing Elements of a Matrix

$A = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 & 6 \\ -3 & 2 & 4 \end{bmatrix}$ $C = \begin{bmatrix} -5 \\ -2 \\ 0 \\ 9 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 4 & 9 & 6 \end{bmatrix}$

$a_{21} = 2$ $b_{13} = 6$ $c_{31} = 0$ $d_{12} = 4$

$a_{12} = 3$ $b_{23} = 4$ $c_{11} = -5$ $d_{14} = 6$

$b_{32} = \text{nil}$

Other ways of writing the elements's address

$= a_{2,2} = A[2,2] = a_{22} = 5$

Always it is rows × columns not ~~columns × rows~~

Solving Linear Systems in 2 unknowns.

$2x + y = 5$

$x - 2y = 0$

Substitution Method

$2x + y = 5 \quad (1)$

$x - 2y = 0 \quad (2)$

from (1)

$y = 5 - 2x$

$x - 2(5-2x) = 0$

$x - 10 + 4x = 0$

$5x = 10$

$x = \frac{10}{5} = 2.$

Substitue x value in any eq.

$x - 2y = 0$

$2 - 2y = 0$

$y = \frac{2}{2} = 1$

$\therefore (x,y) = (2,1).$

Elimination Method

$2x + y = 5 \quad -(1)$

$x - 2y = 0 \quad -(2)] \times 2$

$2x + 4y = 0$

$5y = 5$

$y = \frac{5}{5} = 1$

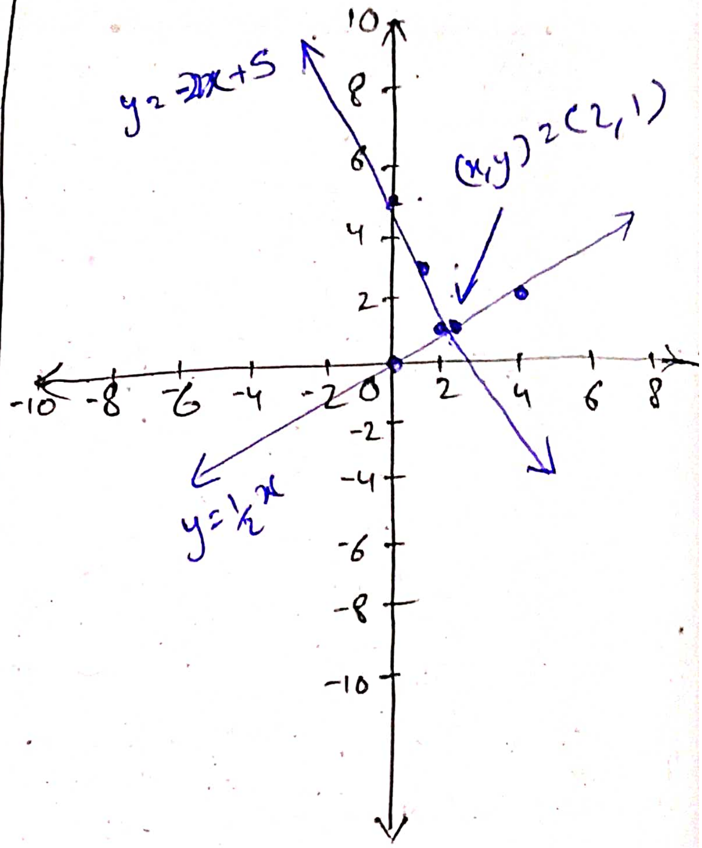
$2x + y = 5$

$2x + 1 = 5$

$x = \frac{4}{2} = 2$

$\therefore (x, y) = (2, 1).$

Graphical method



$y = mx + c$

m = slope of line

c = y intersecept

from eq (1)

$2x + y = 5$

$y = -2x + 5 \quad (1')$

from eq (2)

$x - 2y = 0$

$y = \frac{1}{2}x \quad (2')$

Solving linear Systems in 3 unknowns

$x - y + z = 4$ (1)

$2x + y + z = 7$ (2)

$-x - 2y + 2z = -1$ (3)

Eliminate 'x' from 1 and 2

$2x - 2y + 2z = 8$

$-(2x + y + z = 7)$

$-3y + z = 1$ —— (4)

Eliminate 'x' from 2 and 3

$2x + y + z = 7$ ~~5x2~~

$-x - 2y + 2z = -1$) × 2

$-2x - 4y + 4z = -2$

$-3y + 5z = 5$ —— (5)

$-3y + z = 1$

$z = 1 + 3y$ apply it in (5)

$-3y + 5(1 + 3y) = 5$

$-3y + 5 + 15y = 5$

$12y = 0$

$\boxed{y = 0.}$

apply value of y in any eq.

~~Eq~~ $z = 1 + 3y$

$z = 1 + 3 \times 0 = 1$

$\therefore \boxed{z = 1}$

Now apply value of y and z in any eq.

$x - y + z = 4$

$x - 0 + 1 = 4$

$x = 4 - 1 = 3$

$\therefore \boxed{x = 3}$

Hence, $(x, y, z) = (3, 0, 1)$