A Simulator for Single Phase Slightly Compressible Flow Through Porous Media

Simulation Assignment I

Submitted in partial fulfillment of the requirements of

PGE 392K Num Simulation of Reservoirs

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Summary

Reservoir simulation is an essential tool for making decisions as it yields information pertaining to the processes occurring in the interior of an oil reservoir, thereby enabling the analysis of a number of recovery strategies in order to ensure optimal exploitation. As part of this assignment, I have written, validated and run a simulator for single phase, slightly compressible flow through porous media in 2 dimensions. The code is written in MATLAB and supports variable number of wells (pressure specified and rate specified), spatially varying reservoir thickness, porosity and anisotropic permeability. Validation is performed using the analytical exponential integral solution for an infinite acting well. The code is then used to analyze a field problem possessing a considerably greater degree of complexity.

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1. Introduction

The use and impact of numerical simulation are continuously growing. For the development of oil fields, it is only through numerical simulation that knowledge pertaining to the processes occurring in the interior of an oil reservoir can be obtained and that an analysis of the various recovery strategies in order to guarantee optimal exploitation can be made.

In order to simulate such a phenomenon, the governing differential equations of the underlying physical processes are first discretized over a mesh to produce a system of linear equations which is then solved. Ultimately the effectiveness of a reservoir simulator depends upon the expertise and insight of the user because numerous judgements must be made about the problem at hand before a simulator can be efficiently deployed.

In reservoir simulation a primary challenge is the accurate description of multiphase flow in highly heterogenous media and very complex geometries. The current work however, deals with the simpler case of two dimensional geometries and single phase flow - a problem that is considerably easier to solve and is supplemented by a vast array of literature.

2. Model Development

We begin our analysis with the strong form of the equation for single phase flow:

$$\phi c_t \frac{dP}{dt} + \vec{\nabla} \bullet \vec{u}_2 = q$$

Integrating the equation over a cuboidal grid block we obtain:

$$\phi c_t V \frac{dP}{dt} + \int_A (\vec{u} \bullet \vec{n}) dA = \int_V q dV$$

Evaluating the flux term over each face of the grid block:

$$(\phi c_t)_{ij} \Delta x_{ij} \Delta y_{ij} h_{ij} \frac{dP_{ij}}{dt} + (u_x h \Delta y)\Big|_{i+1/2,j} - (u_x h \Delta y)\Big|_{i-1/2,j} + (u_y h \Delta x)\Big|_{i,j+1/2} - (u_y h \Delta x)\Big|_{i,j-1/2} = \Delta x_{ij} \Delta y_{ij} h_{ij} q_{ij}$$

Using finite differencing and applying Darcy's Law to relate the pressure to velocity and enforcing continuity of flux across each face, we obtain an expression for interface pressures:

$$P_{i+1/2,j} = \frac{\left(h\Delta y \frac{2k_x}{\mu \Delta x}\right)_{ij} P_{ij} - \left(h\Delta y \frac{2k_x}{\mu \Delta x}\right)_{i+1,j} P_{i+1,j}}{\left(h\Delta y \frac{2k_x}{\mu \Delta x}\right)_{ij} + \left(h\Delta y \frac{2k_x}{\mu \Delta x}\right)_{i+1,j}}$$

We define Transmissibilities and associate them with each face as follows:

$$(T_x)_{i+1/2,j} = 2\left(\frac{1}{\left(\frac{k_x h \Delta y}{\mu \Delta x}\right)_{ij}} + \frac{1}{\left(\frac{k_x h \Delta y}{\mu \Delta x}\right)_{i+1,j}} \right)^{-1}$$

We thus get the discretized equation:

$$
\begin{aligned}
&c_{t_{ij}} V_{pij}\left(P_{ij}^{n+1}-P_{ij}^{n}\right)= \\
&-\Delta t\left(\left(T_x\right)_{i+1/2,j}\left(P_{i+1,j}-P_{ij}\right)^{n+1}-\left(T_x\right)_{i-1/2,j}\left(P_{ij}-P_{i-1,j}\right)^{n+1}\right) \\
&-\Delta t\left(\left(T_y\right)_{i,j+1/2}\left(P_{i,j+1}-P_{ij}\right)^{n+1}-\left(T_y\right)_{i,j-1/2}\left(P_{ij}-P_{i,j-1}\right)^{n+1}\right) \\
&+\Delta t Q_{ij}^{n+1}
\end{aligned}
$$

We now form the matrices as follows:

$$
\begin{pmatrix}
T_{11} & -\Delta t T_{x3/2,1} & 0 & -\Delta t T_{y1,3/2} & 0 & 0 \\
-\Delta t T_{x3/2,1} & T_{21} & -\Delta t T_{x5/2,1} & 0 & -\Delta t T_{y2,3/2} & 0 \\
0 & -\Delta t T_{x5/2,1} & T_{31} & 0 & 0 & -\Delta t T_{y3,3/2} \\
-\Delta t T_{y1,3/2} & 0 & 0 & T_{12} & -\Delta t T_{x3/2,2} & 0 \\
 & & 0 & -\Delta t T_{x3/2,2} & T_{22} & -\Delta t T_{x5/2,2} \\
0 & 0 & -\Delta t T_{y3,3/2} & 0 & -\Delta t T_{x5/2,2} & T_{32}
\end{pmatrix}
\begin{pmatrix} P_{11} \\ P_{21} \\ P_{31} \\ P_{12} \\ P_{22} \\ P_{32} \end{pmatrix}^{n+1}
=
\begin{pmatrix} B_{11} \\ B_{21} \\ B_{31} \\ B_{12} \\ B_{22} \\ B_{32} \end{pmatrix}
$$

Where the diagonal terms are:

$$
T_{ij}+\Delta t\left(\left(T_x\right)_{i+1/2,j}+\left(T_x\right)_{i-1/2,j}\right)+\Delta t\left(\left(T_y\right)_{i,j+1/2}+\left(T_y\right)_{i,j-1/2}\right)=c_{tij}V_{pij}
$$

Wells are modeled using the equation:

$$
Q_{\ell}^{m}=J_{\ell}\left(P_{wf,\ell}^{m}-P_{i_{\ell},j_{\ell}}^{m}\right)
$$

Where the productivity index J is given by:

$$
J_{\ell}=\frac{2\pi k_{i_{\ell},j_{\ell}}h_{i_{\ell},j_{\ell}}}{\mu\left[\frac{1}{2}\ln\left[\frac{4A_{i_{\ell},j_{\ell}}}{\gamma C_A r_{w\ell}^{2}}\right]+\frac{1}{4}+s_{\ell}\right]}
$$

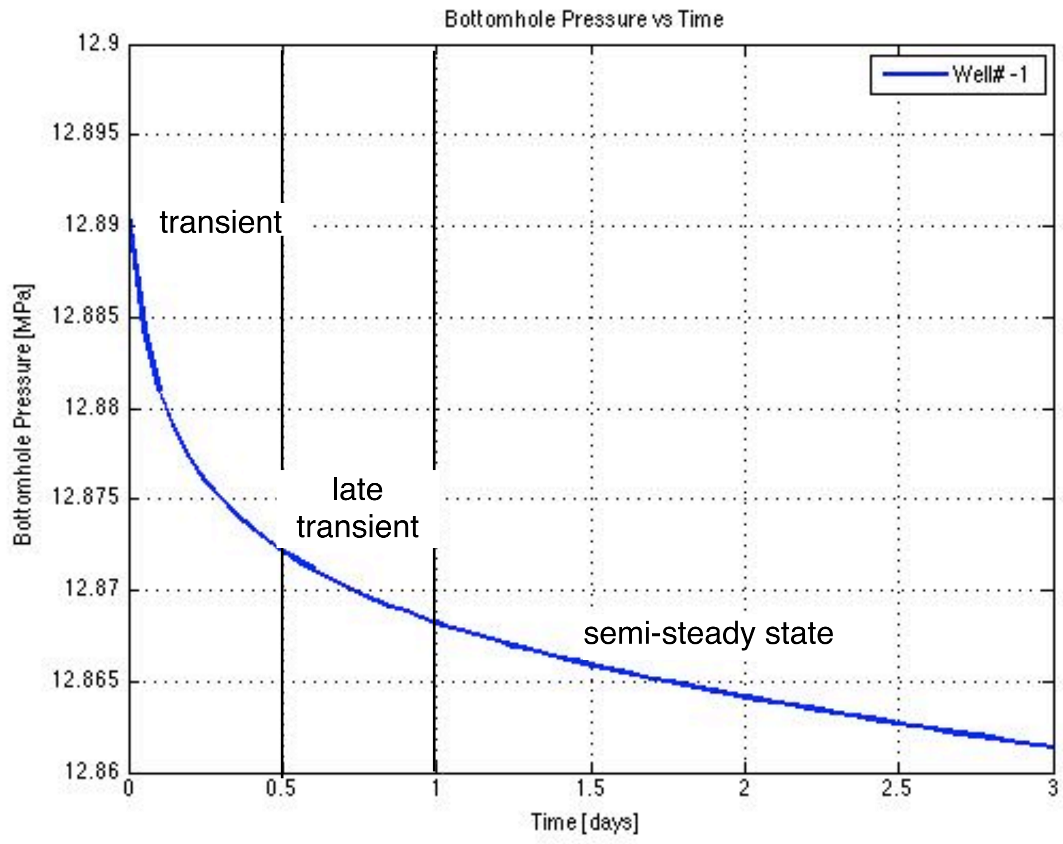
3. Results and Discussion

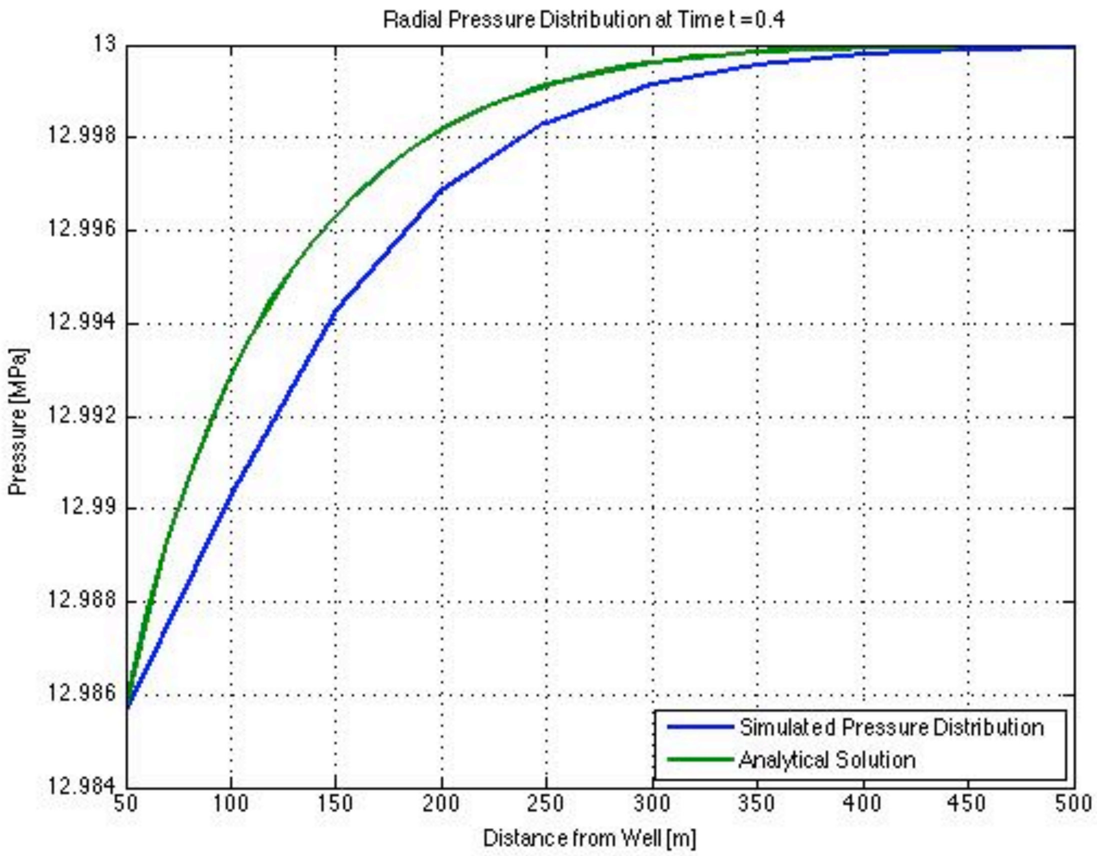
3.1. Validation Case

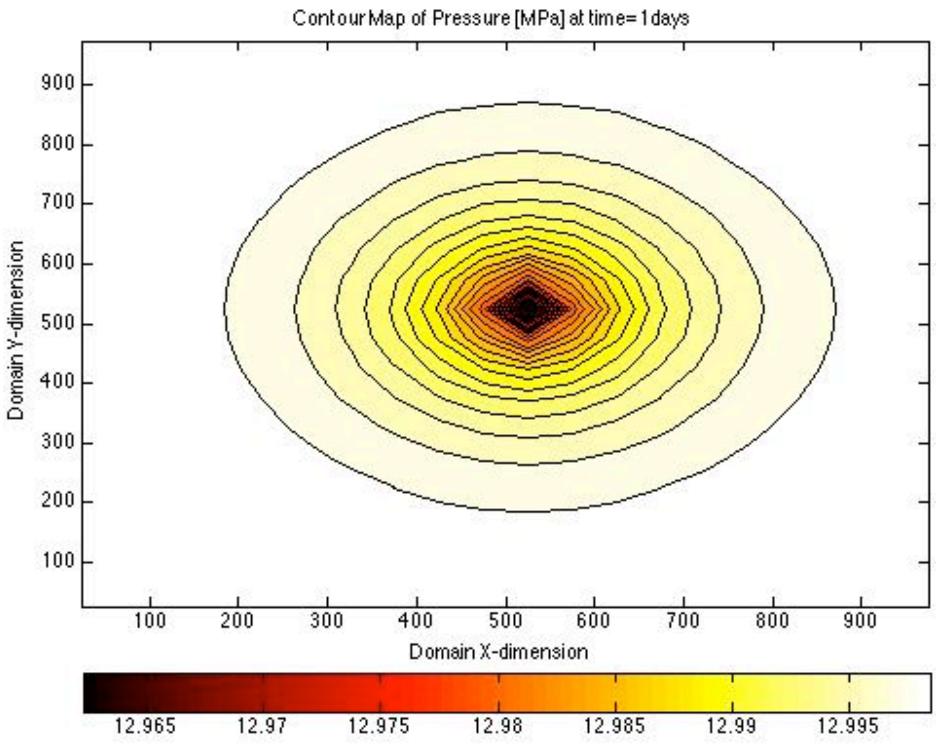
Parameter	
Height *h* (m)	5
x - dimension (m)	1000
y - dimension (m)	1000
Permeability *k* (um^2)	0.096
Viscosity *u* (mPa-s)	1
Porosity *Φ*	0.1
Initial Pressure (MPa)	13
Total Compressibility c_t (GPa^{-1})	2.6
Total Simulation Time (days)	3
Time Step Size (days)	0.05
Wellbore Radius (m)	0.1
Skin Factor	3
Rate (m^3/day)	3

The well is situated at the center of the square domain and is an injector. The flowing well pressure was plotted against time and the transient, late transient and semi-steady state flow regimes have been identified and plotted in the figure below.

The results obtained from the simulation matched the analytical solution reasonably well. The contour of pressures also showed the expected symmetry of the solution. Examining the global mass balance calculations, the error was found to be of the order 10^{-12} which is expected due to accumulation of round-off, truncation and floating point arithmetic error. The case was run with 20 gridblocks in both x and y directions for a total of 400 degrees of freedom for Pressure.


Bottomhole Pressure vs Time
Bottomhole Pressure [MPa]
Time [days]
Well# -1
transient
late transient
semi-steady state
12.9
12.895
12.89
12.885
12.88
12.875
12.87
12.865
12.86
0
0.5
1
1.5
2
2.5
3

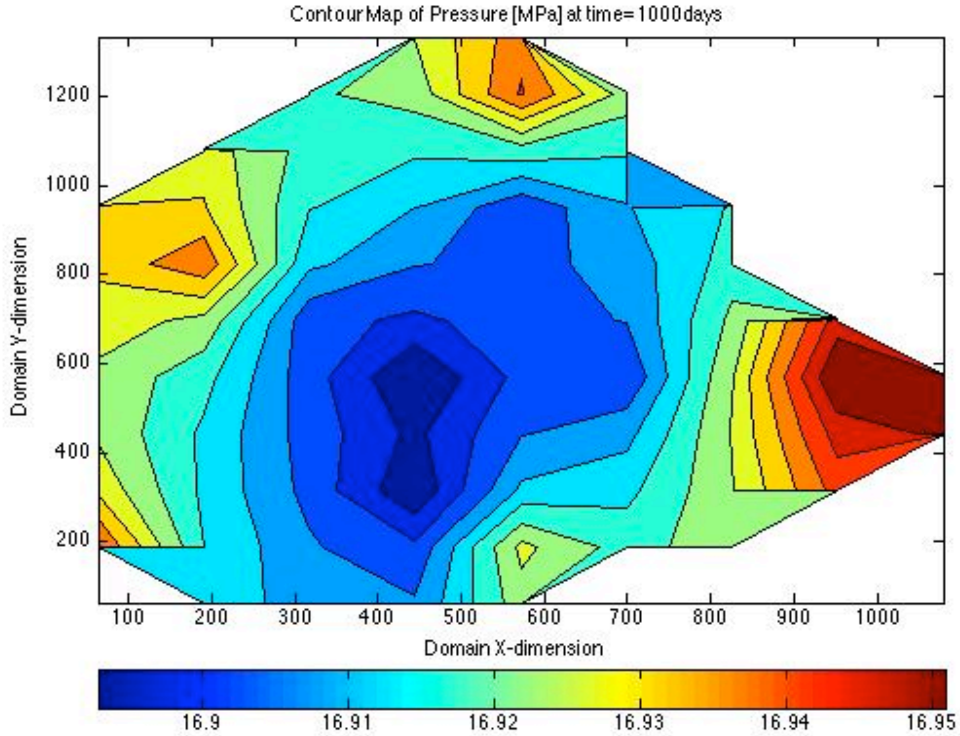

Radial Pressure Distribution at Time t =0.4
Pressure [MPa]
Distance from Well [m]
Simulated Pressure Distribution
Analytical Solution
13
12.998
12.996
12.994
12.992
12.99
12.988
12.986
12.984
50
100
150
200
250
300
350
400
450
500



3.2. Application to Ritchie Field

The Ritchie field was discretized manually and the data was input to the reservoir. A separate module was written to accept the highly heterogenous height function. It is usually more convenient from the point of view of implementation to perform curve fitting with such field data and use the resulting analytic functions as input to the code. Alternatively, to reduce the computational cost of calculating exponentials, it is common practice to perform table lookup implemented with a constant time complexity data structure like a hash-table.

We generate the pressure contours for the Ritchie field after 1000 days of operation, for which the time step was taken to be 1 day. Though it is hard to exercise intuition in a problem of such great complexity and heterogeneity, the contours agree with the proposed well locations, and the lowest field pressures are seen in the central region which has the

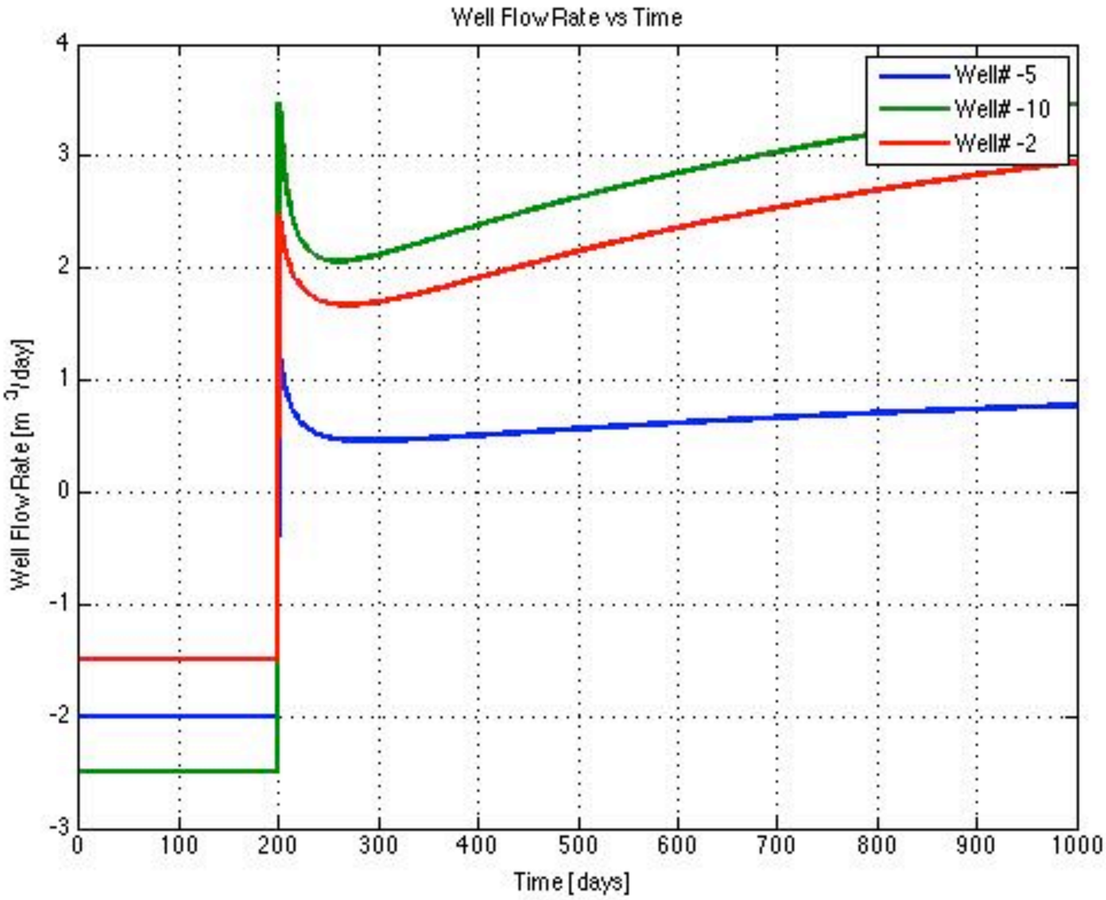


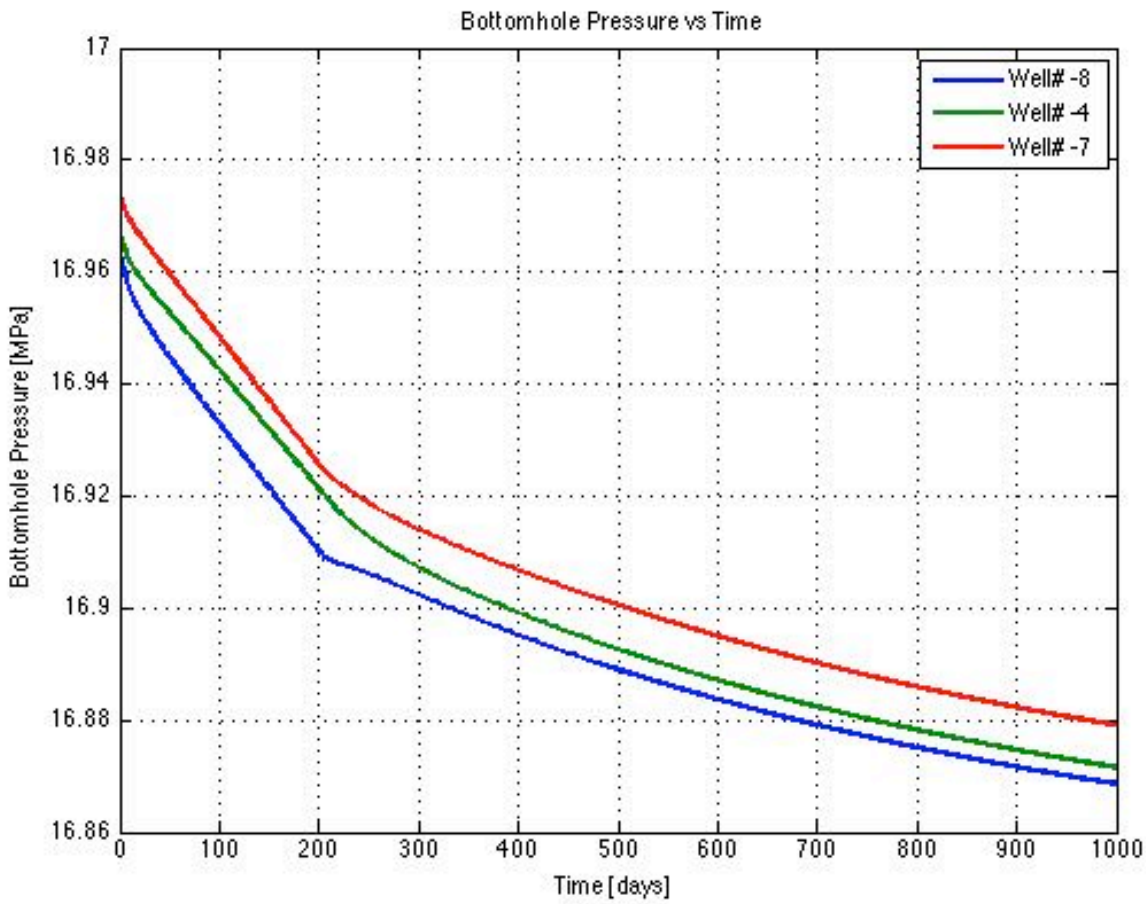
producing wells. Wells 1,2,10,9 and 5 can be clearly seen around the periphery of the field as centers of high pressure. The blank white region surrounding the contours indicates that the specified cells have been successfully disconnected from the matrix.

We also plot the well flow rates for three of the wells that are initially producers but are converted to constant bottomhole pressure injectors after 200 days. The graph is indicative of the flow rates that need to be maintained in order to achieve 17 MPa bottomhole pressure. The figure also demonstrates that the scheduling module is working correctly.

We have also plotted the flowing well pressure profiles for three injector wells. The effect of some of the wells being converted to producers is clearly noticeable in the form of discontinuities in the curves at time t = 200 days.

The global mass balance for the Ritchie field problem was observed to have error of order 10-11 which is expected as a result of accumulated floating point arithmetic error, truncation error and round off error.


Well FlowRate vs Time
Well# -5
Well# -10
Well# -2
Well FlowRate [m 3/day]
Time [days]
4
3
2
1
0
-1
-2
-3
0
100
200
300
400
500
600
700
800
900
1000


Bottomhole Pressure vs Time
Well# -8
Well# -4
Well# -7
Bottomhole Pressure [MPa]
Time [days]
17
16.98
16.96
16.94
16.92
16.9
16.88
16.86
0
100
200
300
400
500
600
700
800
900
1000