An IMPES Simulator for Two-Phase Slightly Compressible Flow Through Porous Media

Simulation Assignment II

Submitted in partial fulfillment of the requirements of

PGE 392K Num Simulation of Reservoirs

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Summary

Reservoir simulation is an essential tool for making decisions as it yields information pertaining to the processes occurring in the interior of an oil reservoir, thereby enabling the analysis of a number of recovery strategies in order to ensure optimal exploitation. As part of this assignment, I have written, validated and run a simulator for two phase, slightly compressible flow through porous media in 2 dimensions using a scheme that is implicit for pressure and explicit for saturations. The code is written in MATLAB and supports variable number of wells (pressure specified and rate specified), spatially varying reservoir thickness, porosity and anisotropic permeability. Validation is performed using a one dimensional Buckley-Leverett Problem involving the displacement of oil by water. The code is then used to analyze a field problem possessing a considerably greater degree of complexity.

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1. Introduction

The use and impact of numerical simulation are continuously growing. For the development of oil fields, it is only through numerical simulation that knowledge pertaining to the processes occurring in the interior of an oil reservoir can be obtained and that an analysis of the various recovery strategies in order to guarantee optimal exploitation can be made.

In order to simulate such a phenomenon, the governing differential equations of the underlying physical processes are first discretized over a mesh to produce a system of linear equations which is then solved. Ultimately the effectiveness of a reservoir simulator depends upon the expertise and insight of the user because numerous judgements must be made about the problem at hand before a simulator can be efficiently deployed.

In reservoir simulation a primary challenge is the accurate description of multiphase flow in highly heterogenous media and very complex geometries. This work, deals with the simpler case of two dimensional areal geometry and two phase flow - a problem that is considerably easier to solve and has a developed theoretical base.

2. Model Development

Model development begins with the volatile equations. If the reader is not familiar with these equations, it is advisable to consult any seminal text on reservoir simulation.

$$\frac{\partial}{\partial t}[\phi\frac{S_i}{B_i}] + \vec{\nabla}\cdot[\frac{\vec{u_i}}{B_i}] = 0 \tag{1}$$

$$\frac{\partial}{\partial t}[\int_V (\phi\frac{S_i}{B_i})\,dV] + \int_A (\frac{\vec{u_i}}{B_i}\cdot\vec{n})\,dA = \int_V q_i\,dV \tag{2}$$

$$V\frac{\partial}{\partial t}(\phi\frac{S_i}{B_i}) + \int_A (\frac{\vec{u_i}}{B_i}\cdot\vec{n})\,dA = \int_V q_i\,dV \tag{3}$$

$$V\frac{\phi}{B_j}\frac{\partial S_j}{\partial t} + V\frac{\phi S_j}{B_j}[c_f - \frac{1}{B_j}\frac{\partial B_j}{\partial P}]\frac{\partial P}{\partial t} + \int_A (\vec{n}\cdot\frac{\vec{u_j}}{B_j})\,dA = \int_V q_j\,dV \tag{4}$$

Eliminating the gaseous phase from the volatile oil equations we arrive at equation (1) which represents a mass balance over the entire domain for the two pseudo components - oil and water. Since the components are assumed to be immiscible, phase and component subscripts can be used interchangeably and there is no distinction between i and j as they can both assume values 1 and 2. Equation (1) represents the strong form of the material balance and yields the weak form upon integration over a finite volume - Eq. (2). We now invoke a continuum assumption throughout the cell volume and hence, the saturations, formation volume factors and porosities remaining constant over the cell volumes (cell-centered values) yields Eq. (3). This can be further simplified and expressed in terms of an appropriate pressure upon which the formation volume factor depends and recast as Eq. (4).

Summing equation (4) over phases j = 1,2 and recognizing that the sum of saturations of the two components must equal unity, we arrive at (5) where the total compressibility is

$$V_p c_t(P,S_1)\frac{\partial P}{\partial t} + B_1\int_A(\vec{n}\cdot\frac{\vec{u_1}}{B_1})\,dA + B_2\int_A(\vec{n}\cdot\frac{\vec{u_2}}{B_2})\,dA = B1\int_V q_1\,dV + B2\int_V q_2\,dV \tag{5}$$

$$c_t(P,S_1) = c_f - S_1(\frac{1}{B_1}\frac{\partial B_1}{\partial P}) - S_2(\frac{1}{B_2}\frac{\partial B_2}{\partial P}) \tag{6}$$

$$V_p\frac{\partial S_1}{\partial t} + V_p S_1[c_f - \frac{1}{B_1}\frac{\partial B_1}{\partial P}]\frac{\partial P}{\partial t} + B_1\int_A(\vec{n}\cdot\frac{\vec{u_1}}{B_1})\,dA = B1\int_V q_1\,dV \tag{7}$$

$$\vec{u_j} = -\vec{\vec{\lambda}}_j\cdot(\vec{\nabla}\gamma_j\phi_j) \tag{8}$$

$$\gamma_j = \rho_j|g| \tag{9}$$

$$\phi_j = \frac{P_j - P_{j_{datum}}}{\gamma_j} - D \tag{10}$$

$$P_c = P_2 - P_1 \tag{11}$$

expressed as (6). (5) and (7) together form a system of coupled, non-linear differential equations that expresses the motion of fluid through the reservoir. Applying Darcy's Law (8) to the flux terms in (5) and equating the fluxes across cell interfaces allows us to obtain the fully discretized form of the system to be solved. The use of an IMPES (implicit in pressure, explicit in saturation) scheme means that we must solve a system of linear equations to obtain the cell centered values of pressure and use a marching algorithm to compute saturations at the next time step. Equations (12)-(16) represent the terms that populate the mass matrix while (17) describes the forcing function. The explicit marching scheme for saturations is given by (18).

Equations (19), (20) and (21) describe relative mobility, total mobility and transmissibility respectively. Handling mobilities is often the most sensitive part of an IMPES formulation since they lend a skew - symmetric nature to the mass matrix and it may so happen that matrix A is not positive definite and the scheme is unstable. This work uses central weighting of mobilities for pressure and an upwind scheme for saturations.

$$A_{i,j}^{n+1} = V_{p_{ij}} c_{t_{ij}}^{n} + \Delta t \lambda_t^n [T_{x_{i+\frac{1}{2},j}} + T_{y_{i,j+\frac{1}{2}}} + T_{x_{i-\frac{1}{2},j}} + T_{y_{i,j-\frac{1}{2}}}] \tag{12}$$

$$A_{i+1,j}^{n+1} = -\Delta t \lambda_t^n \, T_{x_{i+\frac{1}{2},j}} \tag{13}$$

$$A_{i,j+1}^{n+1} = -\Delta t \lambda_t^n \, T_{y_{i,j+\frac{1}{2}}} \tag{14}$$

$$A_{i-1,j}^{n+1} = -\Delta t \lambda_t^n \, T_{x_{i-\frac{1}{2},j}} \tag{15}$$

$$A_{i,j-1}^{n+1} = -\Delta t \lambda_t^n \, T_{y_{i,j-\frac{1}{2}}} \tag{16}$$

$$\begin{aligned} B_{i,j} = {} & V_{p_{ij}} c_{t_{ij}}^{n} P_{ij}^{n} + \Delta t Q_{1_{ij}} + \Delta t Q_{2_{ij}} \\ & + \Delta t \lambda_2^n [(P_{c_{i+1,j}}^{n} - P_{c_i,j}^{n}) T_{x_{i+\frac{1}{2},j}} + (P_{c_{i,j+1}}^{n} - P_{c_i,j}^{n}) T_{y_{i,j+\frac{1}{2}}} + (P_{c_{i-1,j}}^{n} - P_{c_i,j}^{n}) T_{x_{i-\frac{1}{2},j}} + \\ & (P_{c_{i,j-1}}^{n} - P_{c_i,j}^{n}) T_{y_{i,j-\frac{1}{2}}}] - (\gamma_1^n \lambda_1^n + \gamma_2^n \lambda_2^n) \Delta t [(D_{i+1,j} - D_{i,j}) T_{x_{i+\frac{1}{2},j}} + \\ & (D_{i,j+1} - D_{i,j}) T_{y_{i,j+\frac{1}{2}}} + (D_{i-1,j} - D_{i,j}) T_{x_{i-\frac{1}{2},j}} + (D_{i,j-1} - D_{i,j}) T_{y_{i,j-\frac{1}{2}}}] \end{aligned} \tag{17}$$

$$\begin{aligned} S_{i,j}^{n+1} = {} & S_{i,j}^{n} + \Delta t \frac{Q_{1_{i,j}}}{V_{p_{i,j}}} - (c_f^n + c_1^n)(P_{i,j}^{n+1} - Pi, j^n) S_{i,j}^{n} \\ & + \lambda_1^n \frac{\Delta t}{V_{p_{i,j}}} \{ T_{x_{i+\frac{1}{2},j}} [(P_{i+1,j}^{n+1} - P_{i,j}^{n}) - \gamma_1^n (D_{i+1,j} - D_{i,j})] \\ & + T_{y_{i,j+\frac{1}{2}}} [(P_{i,j+1}^{n+1} - P_{i,j}^{n}) - \gamma_1^n (D_{i,j+1} - D_{i,j})] \\ & + T_{x_{i-\frac{1}{2},j}} [(P_{i-1,j}^{n+1} - P_{i,j}^{n}) - \gamma_1^n (D_{i-1,j} - D_{i,j})] \\ & + T_{y_{i,j-\frac{1}{2}}} [(P_{i,j-1}^{n+1} - P_{i,j}^{n}) - \gamma_1^n (D_{i,j-1} - D_{i,j})] \} \end{aligned} \tag{18}$$

$$\lambda_i^n = \frac{k_{r_i}(S_i)}{\mu_i} \tag{19}$$

$$\lambda_t^n = \lambda_1^n + \lambda_2^n = \frac{k_{r_1}(S_1)}{\mu_1} + \frac{k_{r_2}(S_2)}{\mu_2} \tag{20}$$

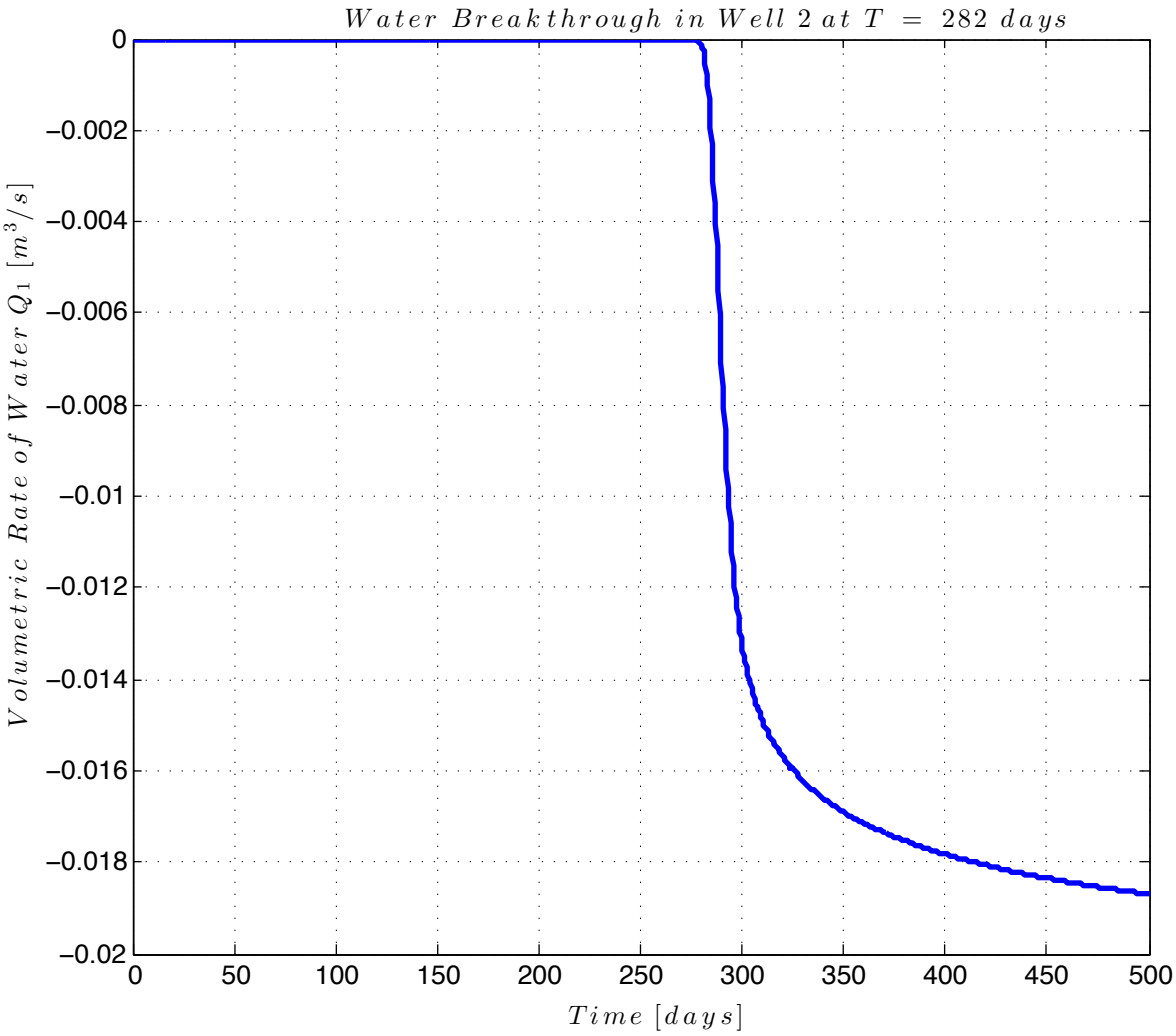
$$T_{x_{i+\frac{1}{2},j}} = \frac{2\alpha_{i,j} \, \alpha_{i+1,j}}{\alpha_{i,j} + \alpha_{i+1,j}} \;\; ; \qquad \alpha_{i,j} = (\frac{h\Delta y}{\Delta x})_{i,j} \tag{21}$$

3. Results and Discussion

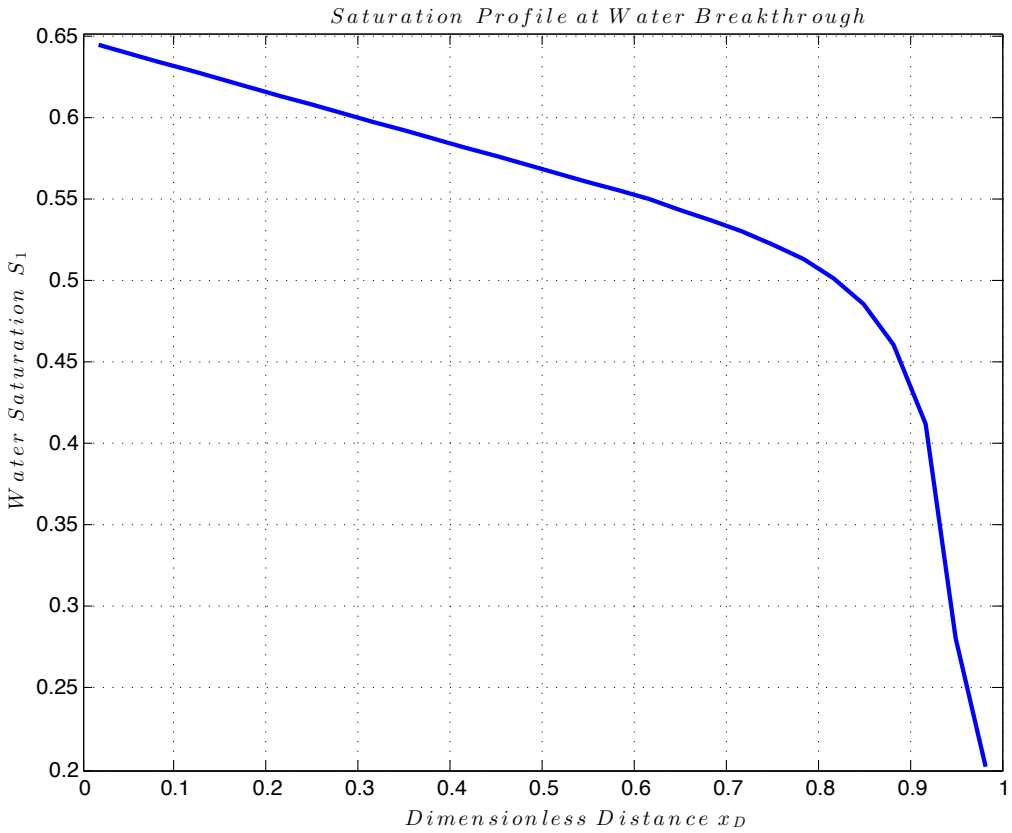
3.1. Validation

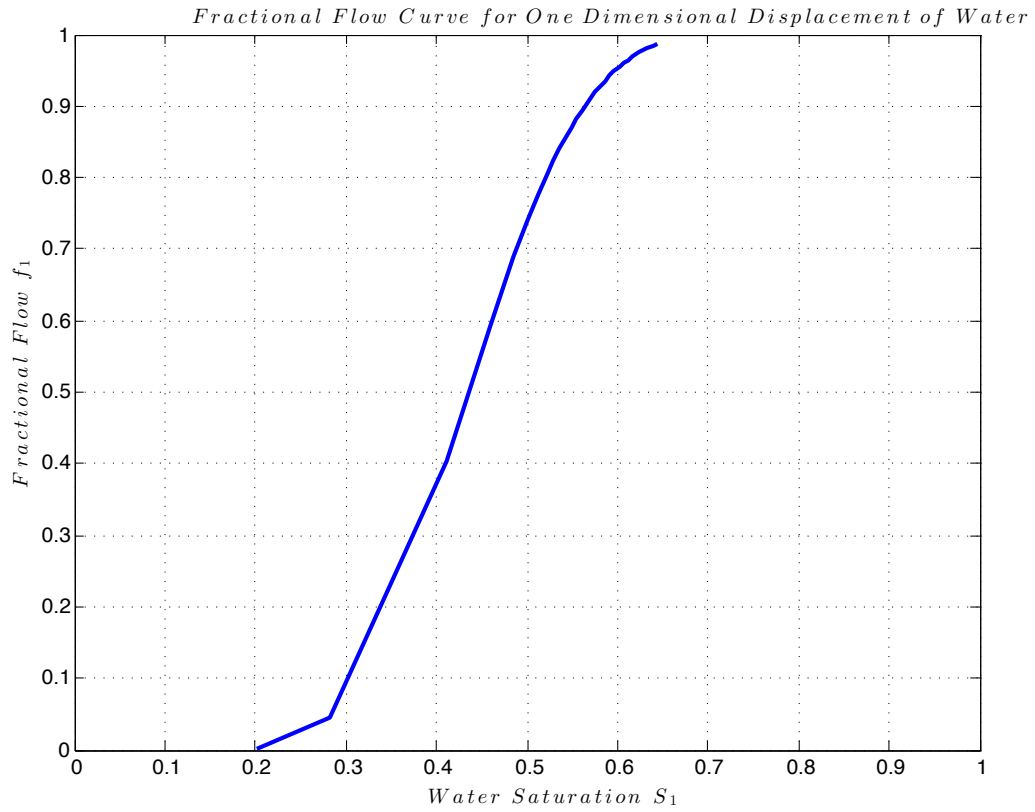
Parameter	
Height *h* (m)	10
x - dimension (m)	300
y - dimension (m)	10
Permeability	2
Aqueous Phase Viscosity *u* (mPa-s)	1
Oleic Phase Viscosity *u* (mPa-s)	5
Porosity Φ	0.3
Initial Pressure (MPa)	18
Initial Water Saturation	0.2
Total Compressibility	0.0001
Aqueous Phase Compressibility	0.00002
Oleic Phase Compressibility	0.00005
Total Simulation Time (days)	400
Time Step Size (days)	0.5
Wellbore Radius (m)	0.3
Skin Factor	0
Rate (m	11
Bottomhole Pressure (MPa)	9
Specific Weight of Water (kg/m	1000
Specific Weight of Oil (kg/m	800

The injector is situated at the left end of the oblong reservoir while the producer is at the far right end. A plot of the water production rate from well 2 indicated that water break-through at the producer occurred at 282 days. The saturation is plotted against distance from the injector at this time step, as is the fractional flow against saturation.



The results obtained from the simulation matched the Buckley Leverett analytic theory reasonably well. Examining the global mass balance calculations, the error was found to be of the order 10^{-16} for mass balance of water and 10^{-12} for mass balance of oil. The case was run with 30 gridblocks in the x direction and a single block in the y direction for a total of 30 degrees of freedom for Pressure and Saturation.

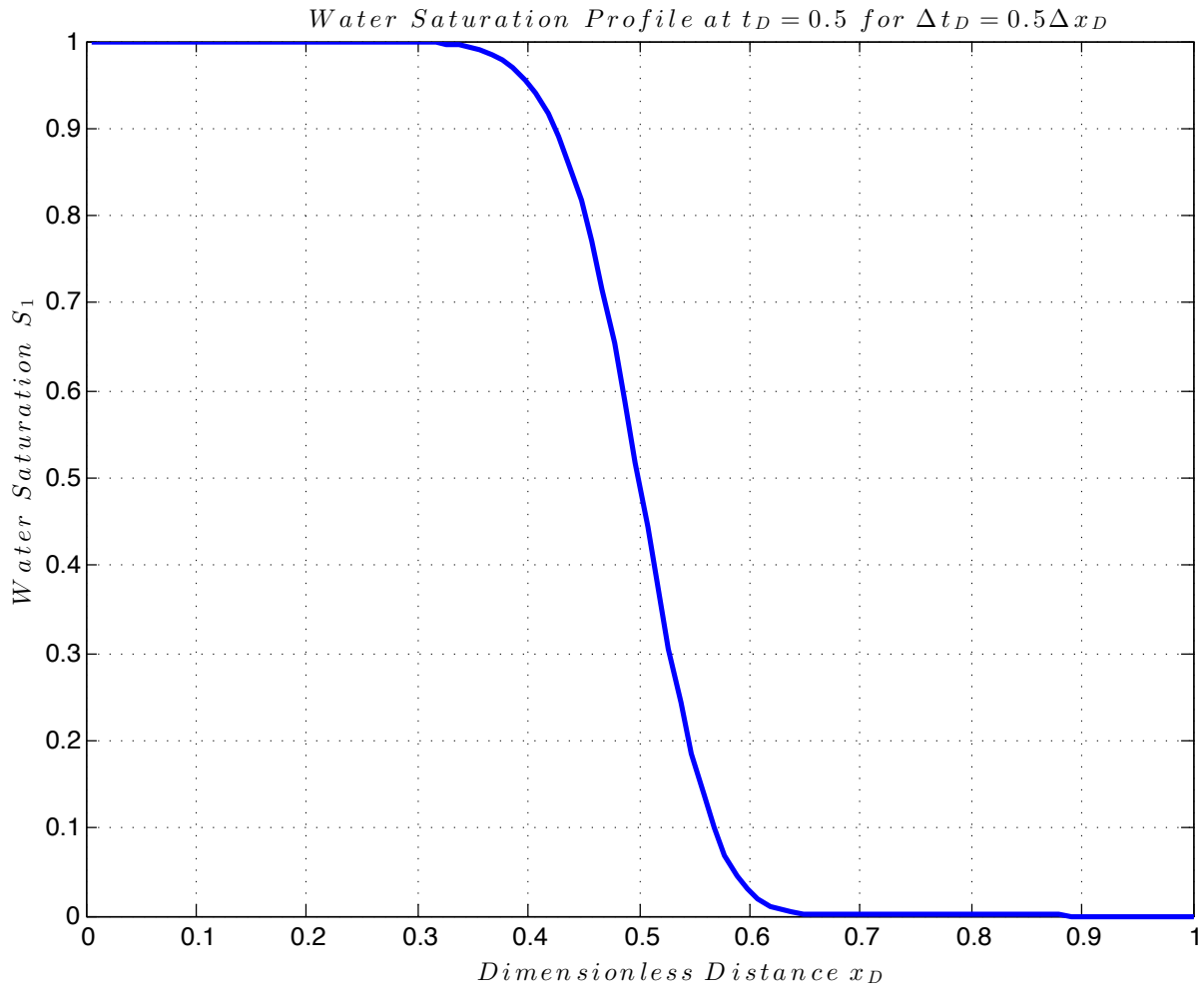

Saturation Profile at Water Breakthrough
Water Saturation S_1
Dimensionless Distance x_D
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Fractional Flow Curve for One Dimensional Displacement of Water
Fractional Flow f_1
Water Saturation S_1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

3.2. Study of Time Step Effects

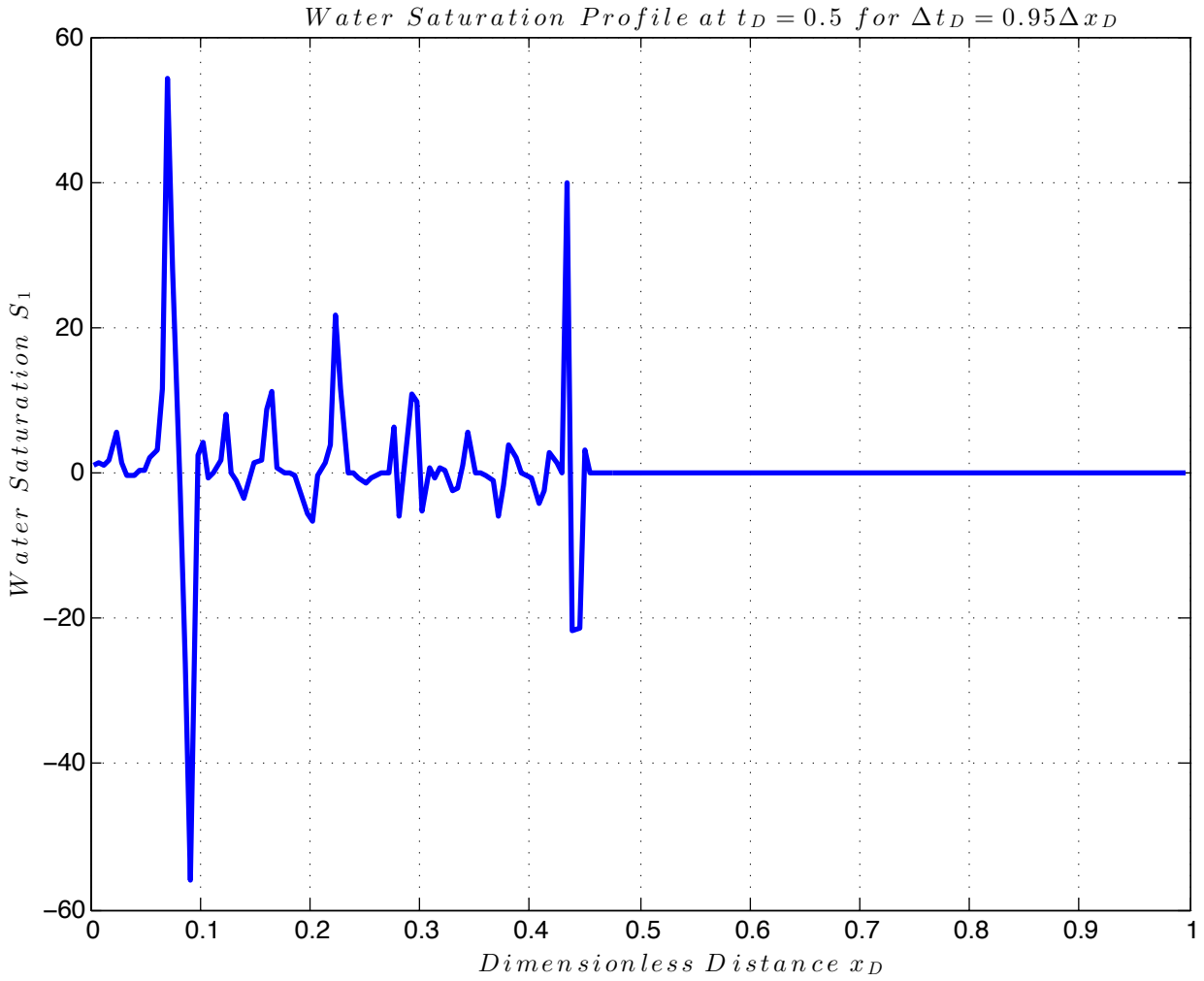
To maintain stability of the IMPES algorithm for this exercise the time step was fixed at 1 day and the end time of the simulation was 100 days. Cases were set up as detailed in the table below.

Δt	Length L [m]	Δx [m]	Injection Rate Q	t_{max}	Δt [days]
0.5	1000	10	50	100	1
1.5	3000	10	150		
0.95	1900	10	95		

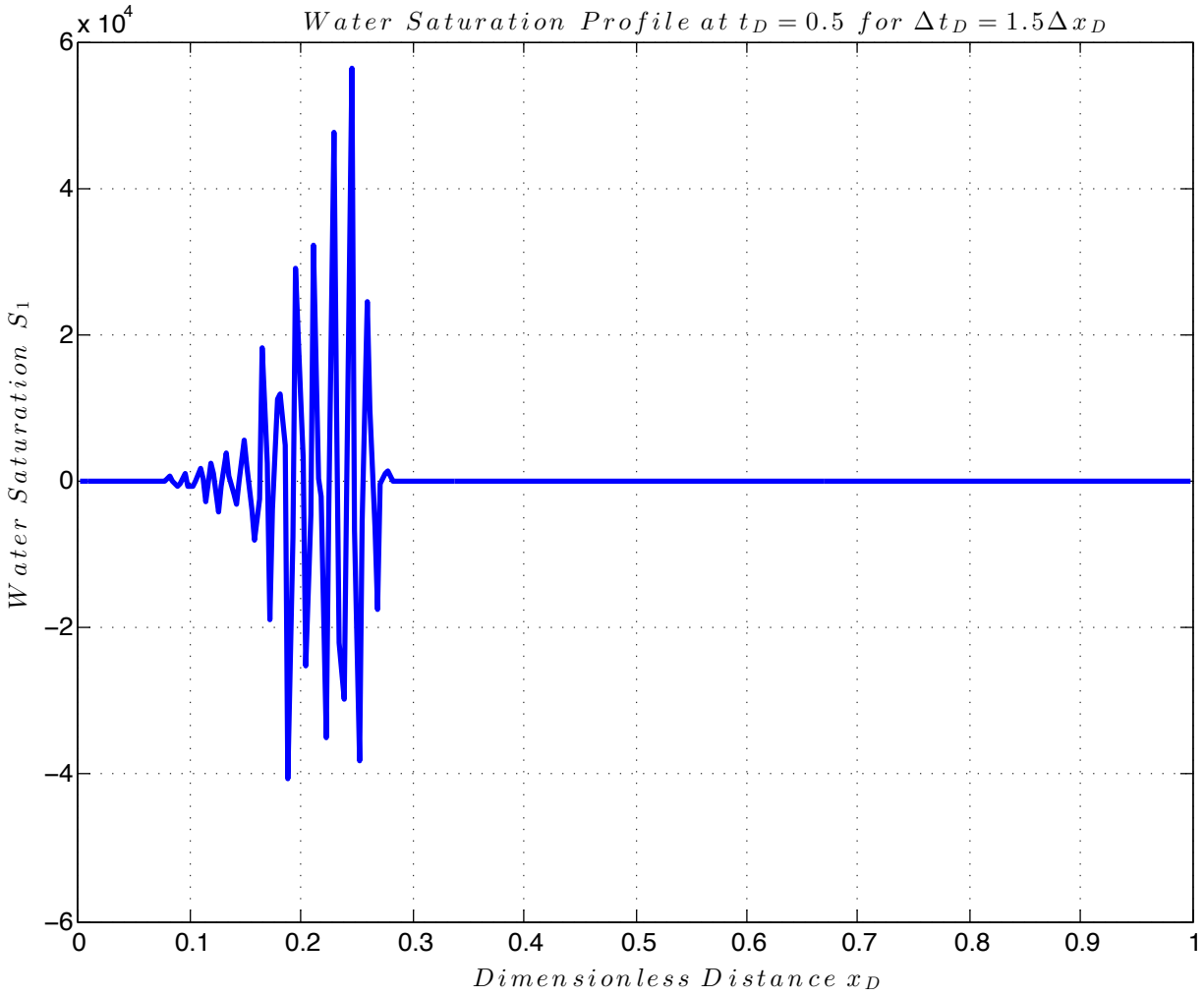


In the first case, the time step size was small enough for the IMPES algorithm to remain stable and yielded the expected saturation profile as seen below. The other two cas-

es saw saturation that was unbounded and oscillatory results were observed due to the numerical instability of the explicit saturation formulation with a large time step. As the time step was increased, the numerical oscillations became further amplified. These results demonstrated the sensitivity of an explicit discretization of the flow equations to time step size.

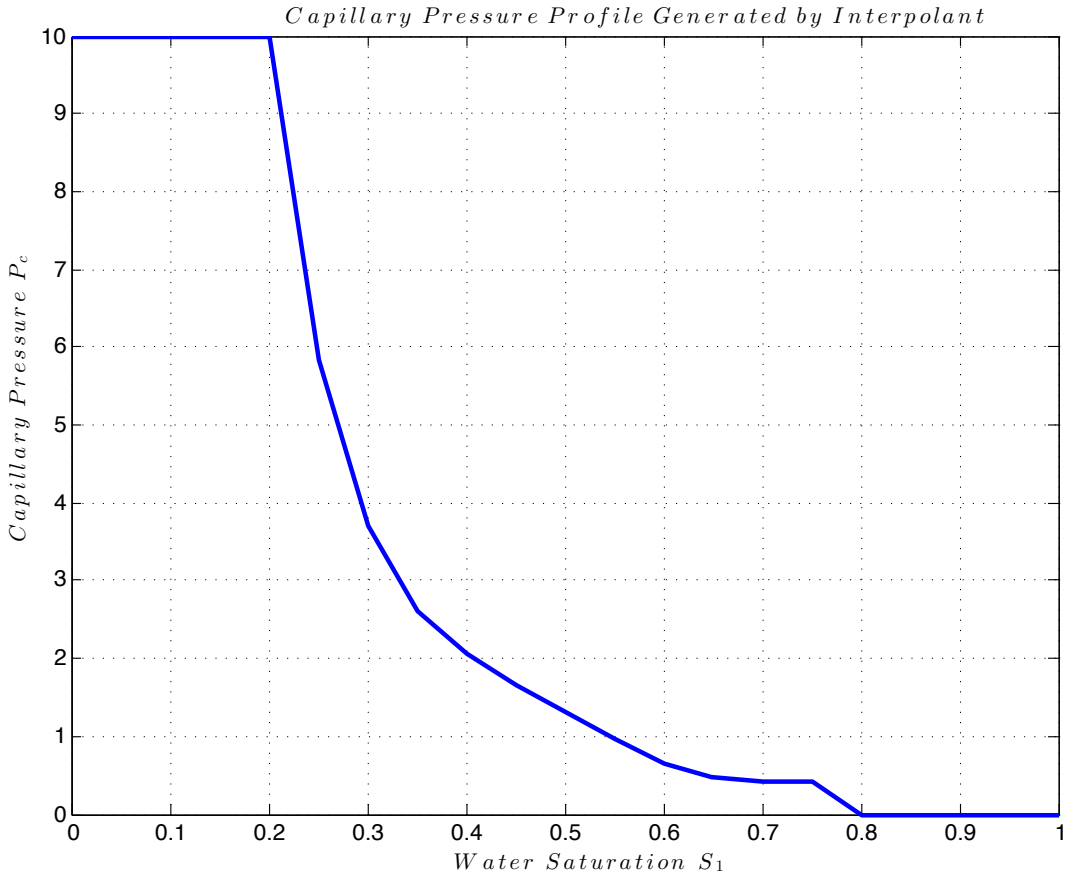


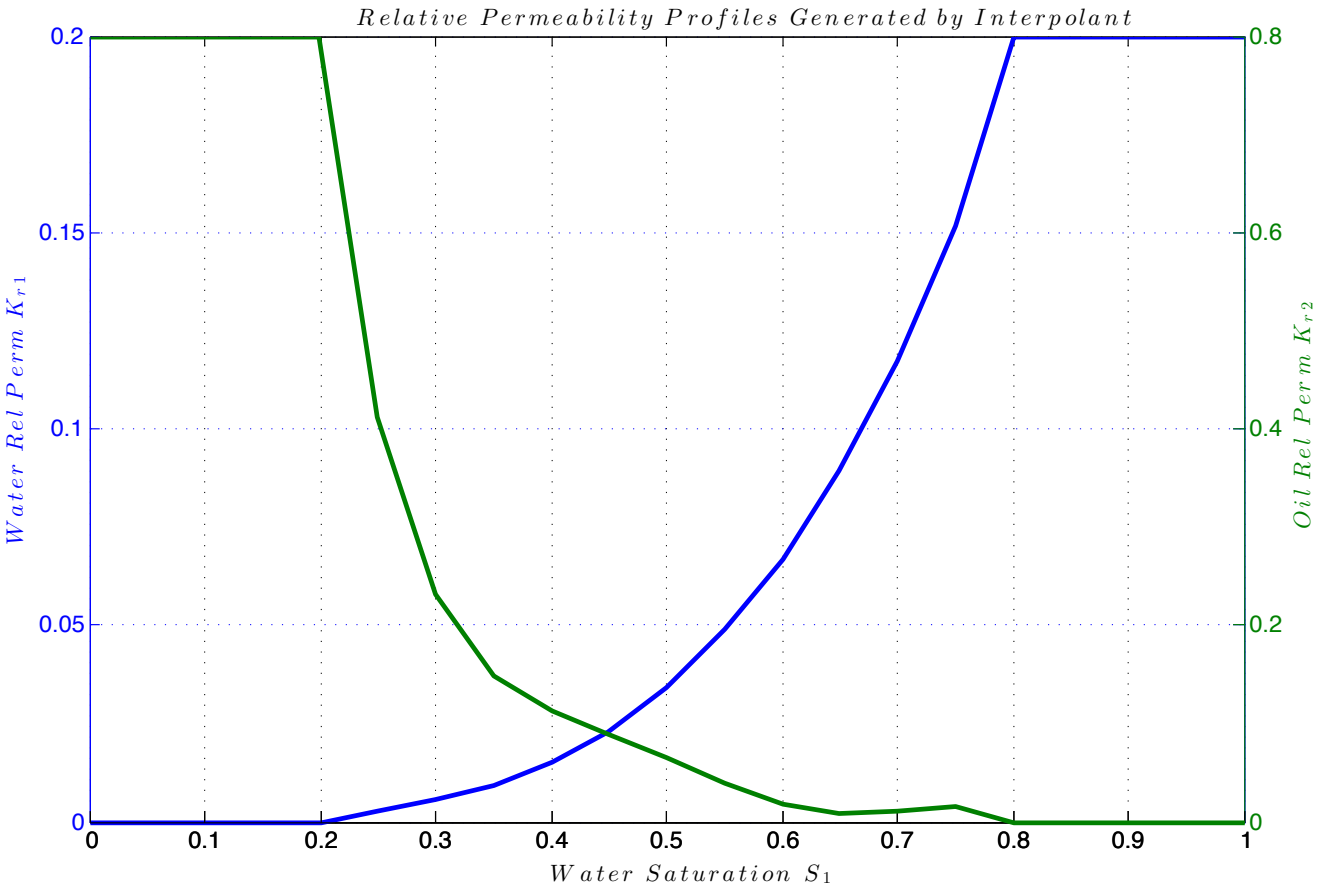
The global mass balance error was found to be of the order 10^{-16} for water and 10^{-12} for oil in all three cases. This demonstrates that the solution we obtain when such a scheme becomes unstable is numerically valid but physically intractable and hence the user must exercise informed judgement in avoiding such eventualities.



3.3. Weeks Island “S” Sand

A polynomial curve fitting routine was used with degree-5 polynomials to express the capillary pressure and oil and water relative permeabilities as functions of the water saturation. The interpolants (shown in fig) were then used as inputs to the model. Additionally all the given quantities were converted to the units used by the simulator - pressures in [MPa] and lengths in [m]. Some of the parameters used in the problem are summarized in the following table.


Capillary Pressure Profile Generated by Interpolant
Capillary Pressure P_c
Water Saturation S_1
0
1
2
3
4
5
6
7
8
9
10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

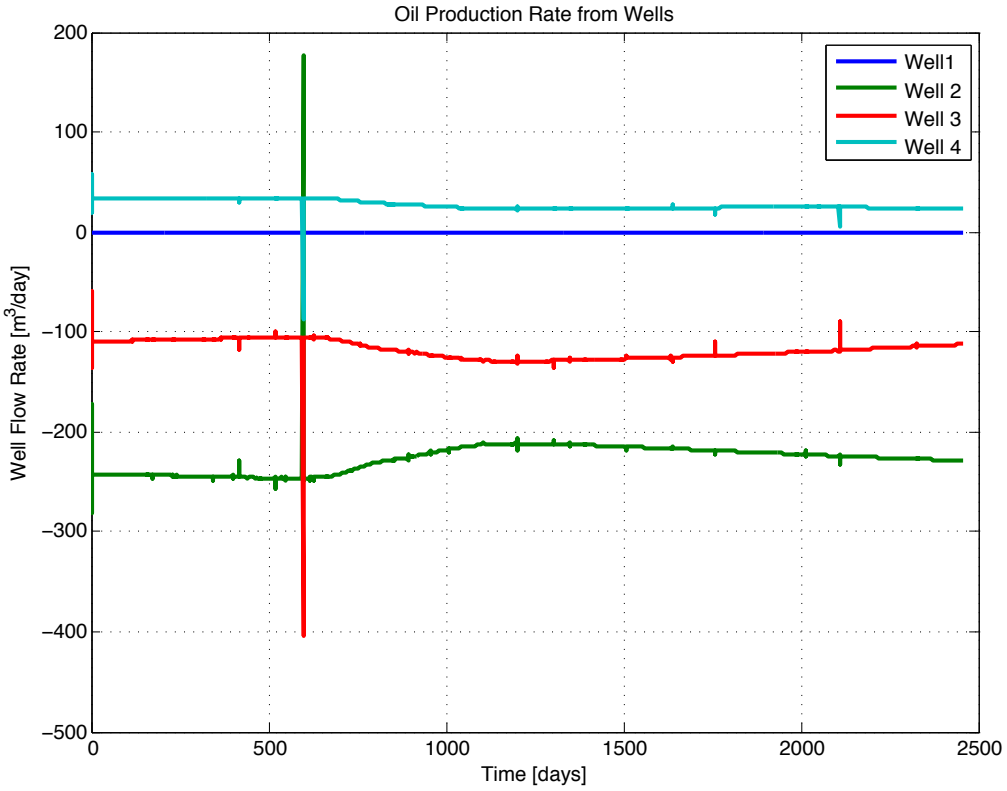

Relative Permeability Profiles Generated by Interpolant
Water Rel Perm K_{r1}
Oil Rel Perm K_{r2}
Water Saturation S_1
0
0.05
0.1
0.15
0.2
0.2
0.4
0.6
0.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

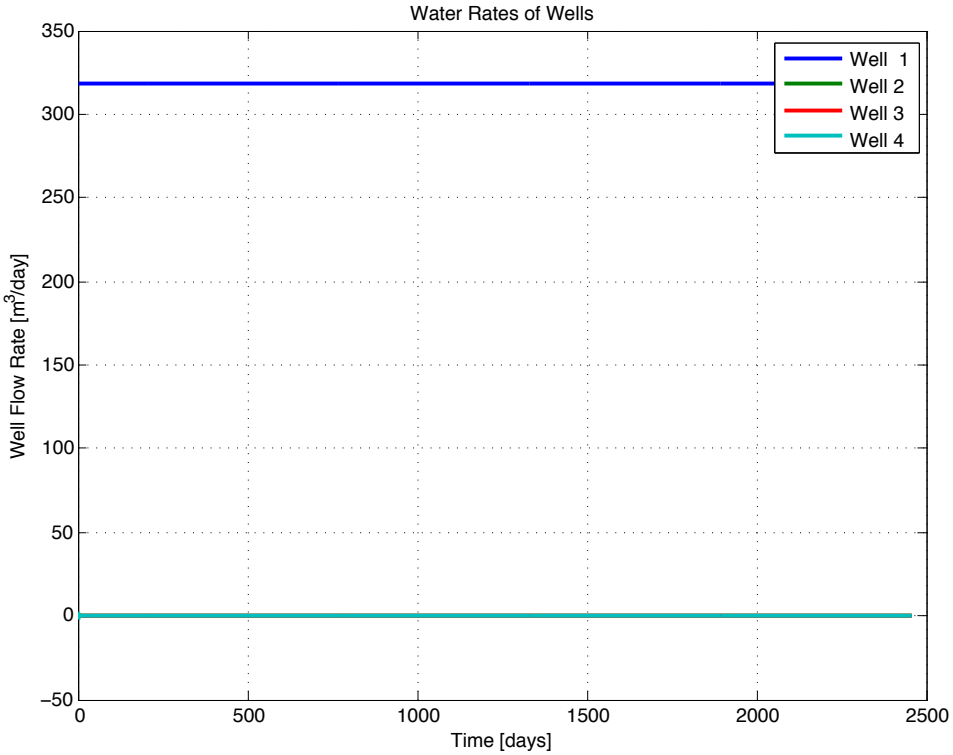
Parameter	
Height *h* (m)	60.96
Permeability	1.8
Aqueous Phase Viscosity *u* (mPa-s)	1
Oleic Phase Viscosity *u* (mPa-s)	5
Porosity *Φ*	0.26
Initial Pressure (MPa)	41.37
Initial Water Saturation	0.2
Total Compressibility	0.0001
Aqueous Phase Compressibility	0.00002
Oleic Phase Compressibility	0.00005
Time Step Size (days)	0.25
Wellbore Radius (m)	0.3
Skin Factor	0
Injection Rate (m	318
Bottomhole Pressure (MPa)	35.16
Specific Weight of Water (kg/m	1000
Specific Weight of Oil (kg/m	800

The amount of oil in place at the beginning of the water flood is calculated in (22).

$$OOIP = N_{activecells} \cdot H_{res} \cdot \Delta x \cdot \Delta y \cdot \phi = 286 \cdot 60.96 \cdot 25.4^2 \cdot 0.26 = 2,339,600.8\ m^3 \qquad (22)$$

The oil and water rates, the average water saturations and the cumulative oil production are also plotted as functions of time. Due to paucity of time, I was unable to run the code for long enough to address the questions in part (c). I shall submit these results at the time of the viva if you chose to accept them at said time.


Oil Production Rate from Wells
Well1
Well 2
Well 3
Well 4
Well Flow Rate [m3/day]
Time [days]
200
100
0
−100
−200
−300
−400
−500
0
500
1000
1500
2000
2500


Water Rates of Wells
Well 1
Well 2
Well 3
Well 4
Well Flow Rate [m3/day]
Time [days]
350
300
250
200
150
100
50
0
−50
0
500
1000
1500
2000
2500