Q2) Let E_1 be the event of choosing from bag I

Let E_2 be the event of choosing from bag II

Let A be the event of drawing a black ball

then, $P(E_1) = P(E_2) = \frac{1}{2}$

$P(A|E_1) = P(\text{drawing a black ball from bag I}) = \frac{6}{10}$

$P(A|E_2) = P(\text{drawing a black ball from bag II}) = \frac{3}{7}$

By using Bayes' theorem, the probability of drawing a black ball from bag I out of two bags,

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{6}{10}}{\frac{1}{2} \times \frac{6}{10} + \frac{1}{2} \times \frac{3}{7}} = \frac{\frac{6}{20}}{\frac{6}{20} + \frac{3}{14}}$$

$$= \frac{\frac{6}{20}}{\frac{72}{140}}$$

$$= \frac{\cancel{6} \times \cancel{140}^{\cancel{70}\,35}}{20 \times \cancel{72}_{\cancel{36}\,\cancel{18}\,3}}$$

$$= \frac{\cancel{35}\,7}{\cancel{20}_{4} \times 3}$$

$$= \frac{7}{12} = 0.5833$$

Q3) E = Man throws a die and reports that number obtained is a four.

$P(S_1)$ = Probability that four actually occur $= \frac{1}{6}$

$P(S_2)$ = Probability that four doesn't occur $= \frac{5}{6}$

$P(E|S_1)$ = Probability that man reports four and four actually occurred $= \frac{2}{3}$

$P(E|S_2)$ = Probability that man reports four and four doesn't occurred $= 1 - \frac{2}{3} = \frac{1}{3}$

By using Bayes' theorem, the probability that man reports four and number is actually four

$$P(S_1|E) = \frac{P(S_1)\,P(E|S_1)}{P(S_1)\,P(E|S_1) + P(S_2)\,P(E|S_2)}$$

$$= \frac{\frac{1}{6} \times \frac{2}{3}}{\frac{1}{6} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{3}}$$

$$= \frac{\frac{2}{18}}{\frac{2}{18} + \frac{5}{18}} = \frac{\frac{2}{18}}{\frac{7}{18}} = \frac{2 \times \cancel{18}}{\cancel{18} \times 7} = \frac{2}{7}$$

$$= 0.2857$$