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Section → C

Assignment 1

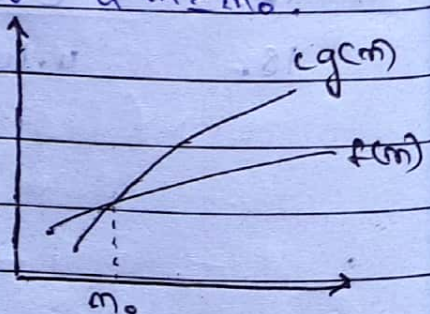
Q1. What do you understand by asymptotic notations. Define different Asymptotic notations with example.

Ans. → Asymptotic notations are the mathematical notations used to describe the running time of an algo when the i/p tends towards a particular value or a limiting value.

→ There are mainly 3 Asymptotic notations:-

(1) Big – O notation

- It represents the upper bound of running time of an algo.
- This notation is called as upper bound of the algo, or a worst case of an algo.
- $O(g(n)) = \{ f(n)$: there exist positive constants c & n_0 such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$, where $c > 0$ & $n \geq n_0 \}$.
- e.g.,
  $f(n) = 3 \log n + 100$
  $g(n) = \log n$
  $3 \log n + 100 <= c * \log(n)$
  $c = 150$ & $n > 2$
  (undefined at $n = 1$)



(II) Big Omega (Ω) Notation

- It represents the lower bound of the running time of an algo.
- This notation is known as lower bound of an algo, or best case of an algo.
- $\Omega(g(n)) = \{ f(n) :$ there exist positive constant c & n_0 such that $0 \leq cg(n) \leq f(n) \ \forall \ n, n \geq n_0$
- e.g.,

$f(n) = 3n + 2$

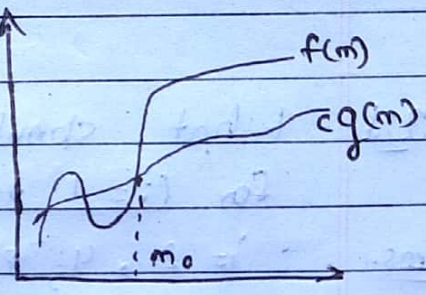
$c.g(n) \leq f(n)$

[c = constant, $g(n) = n$]

$cn \leq 3n + 2$

$cn - 3n \leq 2$

$n(c-3) \leq 2 \Rightarrow n \leq \frac{2}{c-3}$



if we assume $c = 4$, then $n_0 = 2$

$\boxed{c = 4, n_0 = 2}$

(III) Theta (Θ) notation

- It enclose the function from above & below. Since, it represents the upper & lower bound of running time of an algo.
- This is known as tight bounds of an algo, or an average case of algo.

- $\Theta(g(n)) = \{f(n)$: there exist positive constant c_1, c_2 & n_0 such that $0 \le c_1 * g(n) \le f(n) \le c_2 * g(n)$ $\forall$ $n \ge n_0$.
- e.g.,

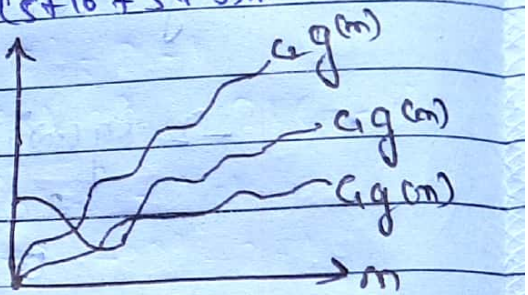
$f(n) = 5n^3 + 16n^2 + 3n + 8$

$5n^3 \le 5n^3 + 16n^2 + 3n + 8 \le (5+16+3+8)n^3$

$5n^3 \le f(n) \le 32n^3$

$\boxed{c_1 = 5,\ c_2 = 32,\ n_0 = 1}$

$f(n) \longleftrightarrow \Theta(n^3)$



Q2) What should be the time complexity:

for (i = 1 to n) { i = i * 2 }

Ans. $i = 2, 4, 8, 16, \ldots$ K^{th} term $\ldots n$

$T_n = ar^{n-1}$

$T_n = 1(2)^{K-1}$

$n = 2^{K-1}$

$\log_2 n = (K-1)\log_2 2$

$\boxed{K = \log_2 n + 1}$

$\Theta(n) = \log n$

Q3) $T(n) = \{3T(n-1)$ if $n > 0$, otherwise $1\}$.

Ans. $T(n) = 3T(n-1)$

$T(n-1) = 3T(n-2)$

$T(n) = 3 \times 3T(n-2)$

$T(n-2) = 3T(n-3)$

$T(n) = 3 \times 3 \times 3T(n-3)$

$T(n) = 3^3 T(n-3)$

$T(n-3) = 3T(n-4)$

$T(n) = 3^3 \times 3T(n-4)$

$T(n) = 3^4 \times T(n-4)$

⋮

General form :-

$T(n) = 3^i T(n-i)$ ---(i) $[T(0) = 1]$

$T(n-i) = T(0)$

or $n - i = 0$

$\boxed{n = i}$

Putting $n = i$ in eq. (i) ;

$T(n) = 3^n T(n-n)$

$T(n) = 3^n \cdot T(0)$ $[T(0) = 1$, given$]$

$T(n) = 3^n$

$\boxed{T(n) = O(3^n)}$

Q4) $T(n) = \{ 2T(n-1) = 1$, if $n > 0$, otherwise $1 \}$.

Ans.) $T(n) = 2T(n-1) - 1$

$T(n-1) = 2T(n-2) - 1$

$T(n) = 2(2T(n-2) - 1) - 1$

$T(n) = 2^2 T(n-2) - 2 - 1$

$T(n-2) = 2T(n-3) - 1$

$T(n) = 2^2 (2T(n-3) - 1) - 2 - 1$

$T(n) = 2^3 T(n-3) - 2^2 - 2 - 1$

$T(n-3) = 2T(n-4) - 1$

$T(n) = 2^3 (2T(n-4) - 1) - 2^2 - 2 - 1$

$T(n) = 2^4 T(n-4) - 2^3 - 2^2 - 2 - 1$

⋮

General form :-

$T(n) = 2^i T(n-i) - (2^{i-1} + 2^{i-2} + \dots + 1)$

$T(n-i) = T(0)$

$n - i = 0$

$\boxed{n = i}$

$T(n) = 2^n T(0) - (1 + 2 + 2^2 + 2^3 + \dots 2^{n-1})$

$[T(0) = 1]$

$T(n) = 2^n (1 - (1 + 2 + 2^2 + \dots 2^{n-1}))$

$T(n) = 2^n - 1 \frac{(2^{n-1} - 1)}{2-1}$

$T(n) = 2^n - 2^{n-1} + 1$

$T(n) = 2^{n-1}(2-1) + 1$

$T(n) = 2^{n-1} + 1$

$\boxed{T(n) = O(2^n)}$

Q5 → What should be the T.C. of :-

```
int i=1, s=1;
while (s <= n)
{
    i++;
    s = s+i;
    printf ("#");
}
```

Ans)

No. of steps (K)	s	i
0	0	1
1	1	2
2	3	3
3	6	4
4	10	5
5	15	6
⋮	21	7
⋮	⋮	⋮
K step	n	

$T(n) = O(K)$

$S = 0, 1, 3, 6, 10, \ldots n$

$S_n = 1 + 3 + 6 + 10 + 15 + \ldots + n$

$S_n = 1 + 3 + 6 + 10 + \ldots + (n-1) + n$

$0 = 1 + 2 + 3 + 4 + 5 + \ldots - n$

$m = 1+2+3+4+\cdots$ K step

$m = \frac{K}{2}[2(1)+(K-1)1]$

$2m = K[2+K-1]$

$2m = K^2+K \Rightarrow 2m = \left(K+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$2m+\left(\frac{1}{2}\right)^2 = \left(K+1/2\right)^2$

$K+1/2 = \sqrt{2m+(1/2)^2}$

$K = \sqrt{2m+(1/2)^2} - 1/2$

$T(m) = T(K)$

$T(m) = T\left(\sqrt{2m+(1/2)^2} - 1/2\right)$

$\boxed{T(m) = O\sqrt{m}}$

Q6.) T.C. of :—

```
void function (int n)
{
    int i, count = 0;
    for(i=(n/2); i<=n; i++)
        for(j=1; j<=n; j=j*2)
            for(k=1; k<=n; k=k*2)
                count++
    int i, count = 0;
    for(i=1; i*i<=n; i++)
        count++
}
```

Ans) Since, i is moving from 1 to $\sqrt{n}$ with linear growth so

$$\boxed{T(n) = O(\sqrt{n})}$$

Q7.) Time complexity of

```
void function (int n)
{
    int i, j, k, count = 0;
    for (i = n/2 ; i <= n ; i++)
        for (j = 1 ; j <= n ; j = j * 2)
            for (k = 1 ; k <= n ; k = k * 2)
                count ++;
}
```

Ans.) $O(n \log n \log n)$

$O(n (\log n)^2)$

Q8) Time complexity of

```
function (int n)
{ if (n == 1) return;
  for (i=1 to n)
  { for (j=1 to n)
    {
      printf ("*"); }
    }
    function (n-1); }
```

Ans) $T(n) = T(n-1) + n^2$ $\quad [T(n-1) = T(n-2) + (n-1)^2]$

$T(n) = T(n-2) + n^2 + (n-1)^2$

$[T(n-2) = T(n-3) + (n-2)^2]$

$T(n) = T(n-3) + n^2 + (n-1)^2 + (n-2)^2$

General Term :-

$T(n) = T(n-i) + n^2 + (n-1)^2 + (n-2)^2 + \dots (n-i)^2$

$T(n-i) = T(1)$

$n = i+1 \Rightarrow \boxed{n-1 = i}$

$T(n) = T(n-(n-1)) + n^2 + (n-1)^2 + (n-2)^2 + \dots + (n-(n-1))^2$

$T(n) = T(1) + n^2 + (n-1)^2 + (n-2)^2 + \dots + 1^2$

$T(n) = 1 + 1^2 + 2^2 + 3^2 + \dots + n^2$

$T(n) = \frac{n(n+1)(2n+1)}{6}$

$\boxed{T(n) = O(n^3)}$

Q9.) T.C. of :

```
void function (int n)
{
  for (i=0 ; i to n) {
  for (j=1 ; j <= n ; j=j+i)
    printf ("#");
  } }
```

Ans) $O(n\sqrt{n})$

Q10.) For the function n^k & a^n, what is the asymptotic relation b/w these function? Assume that $k >= 1$ & $a > 1$ are constants. Find out the value of c & n_0 for what relation holds.

Ans) If $c > 1$ then the exponential c^n for outgrows any term, so that answer is:
n^k is $O(c^n)$.

Q12.) Write the recurrence relation for recursive function that prints Fibonacci series. Solve recurrence relation to get T.C. of program. What will be the space complexity of this program & why?

Ans. $T(n) = T(n-1) + T(n-2) + c$

$T(n-2) \approx T(n-1)$

$T(n) = 2T(n-1) + c$

$T(n-1) = 2T(n-2) + c$

$T(n) = 2(2T(n-2) + c) + c$

$T(n) = 2^2 T(n-2) + 2c + c$

$T(n-2) = 2T(n-3) + c$

$T(n) = 2^3 (2T(n-3) + c) + 2c + c$

$T(n) = 2^3 T(n-3) + 2^2 c + 2c + c$

⋮

General Term :-

$T(n) = 2^i T(n-i) + (2^0 + 2^1 + 2^2 + \dots 2^{i-1})c$

$n - i = 0$

$\boxed{n = i}$

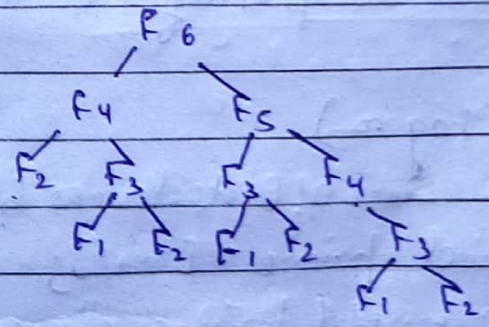
$T(n) = 2^n T(0) + (2^0 + 2^1 + 2^2 + \dots 2^{n-1})c$

$T(n) = 2^n (1) + 2^0 \frac{(2^{n-1})}{2-1} c$

$T(n) = 2^n (1+c) - c$

$\boxed{T(n) = O(2^n)}$

Fib. (6)



The max. depth is proportional to N, hence the space com. of Fibonacci recursive is O(n).

Q11) What is the T.C. of code given?

```
void fun(int n) {
    int j=1, i=0;
    while (i < n) {
        i = i+j;
        j++; } }
```

Ans) i = 0, 1, 3, 6, 10, 15, ---

j = 1, 2, 3, 4, 5, 6, ---

So, i will go n till n & general formula for k^{th} term is $n = \frac{k(k+1)}{2}$

$\therefore$ **T.C = $O(\sqrt{n})$**

Q13) Write programs which have T.C.:

(i) $n \log n$

→
```
void fun()
{
  int i, j;
  for (i=1; i<=n; i++)
  {
    for (j=0; j<=n; j=j*2)
      printf("#");
    printf("\n"); } }
```

(ii) n^3

→
```
void fun (int n)
{ int i, j, k;
  for(i=0; i<=n; i++)
  { for (j=0; j<=n; j++)
    {
      for( k=0; k<=n; k++)
        printf ("#"); } } }
```

(iii) $\log(\log n)$

→
```
void SieveOfEratosthenes (int n)
{ bool prime [n+1];
  memset( prime, true, sizeof(prime));
  for (int p=2; p*p<=n; p++)
{ if (prime[p] == true)
    {
```

```
for (int i = p*p ; i <= n ; i += p)
    prime[i] = false;
}  }
for(int p = 2 ; p <= n ; p++)
    if (prime[p])
        cout << p << endl;
    }
```

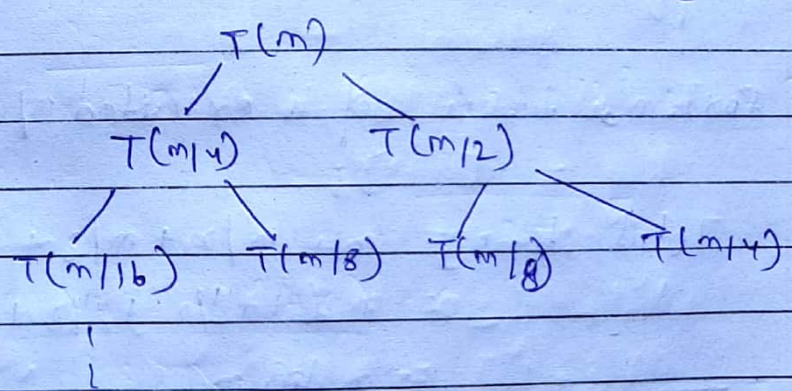
Q14) $T(n) = T(n/4) + T(n/2) + cn^2$

Sol) $T(1) = c$

$n = n/2$

$T(n/2) = T(n/8) + T(n/4) + c(n^2/4)$

$T(n) = T(n/4) + 2T(n/16) + c(n^2/16 + n^2/4 + n^2)$



$$T(n) = c\left[n^2 + \frac{5n^2}{16} + \frac{25n^2}{256} + \cdots\right]$$

$$T(n) = n^2 c\left[1 + \frac{5}{16} + \frac{5^2}{16^2} + \cdots\right]$$

$$\boxed{T(n) = O(n^2)}$$

Q15) T. C. of :-

```
int fun (int n)
{
  for(int i=1; i<=n; i++)  ——— n
  { for (int j=1; j<n; j+=i)
    {
      // Some O(1) task
    }
  }}
```

Ans. For i = 1, inner loop is executed n times
For i = 2, inner loop is executed n/2 times
For i = 3, inner loop is executed n/3 times
⋮
For i = n, inner loop is executed n/n times.

Total time = $n + n/2 + n/3 + \cdots n/2$
$= n(1 + 1/2 + 1/3 + \cdots 1/n)$
$= n \log n$

$\boxed{T(n) = O(n \log n)}$

Q16) T. C. of :-

```
for (int i=2; i<=n; i=pow(i,k))
{
  // some O(1) expressions
}
```

where, k is a constant.

Ans. $\boxed{O(\log(\log n))}$

Q18) Arrange in inc. order of rate of growth

(a) 100, $\log\log n$, $\log n$, $\sqrt{n}$, n, $n\log n$, n^2, 2^n, 2^{2n}, 4^n, $n!$

(b) 1, $\log(\log(n))$, $\sqrt{\log n}$, $\log n$, $\log(2n)$, $\log(n!)$, $2\log(n)$, $2n$, $2n$, $4n$, $n\log(n)$, n^2, $2(2^n)$, $n!$

(c)

(c) 96, $\log_8 n$, $\log_2 n$, $\log(n!)$, $5n$, $n\log_6 n$, $n\log_2 n$, $8n^2$, $7n^3$, 8^{2n}, $n!$

Q19) Write Linear Search pseudo code ----

Ans
```
LinearSearch ( A, Key)
    comp ← 0 , f ← 0
    for i = 1 to A. length
    comp ← comp + 1
      if A[i] == Key
        print "Element found"
        f = 1
    if f == 0
        print "Element not found"
    print comp
```

Q20) Write pseudocode for ----------.

Ans) Iterative Method of Insertion Sort →

```
InsertionSort (A)
for j = 2 to A.length
    key = = A[j]
    i = j - 1
    while i > 0 & A[i] > key
        A[i+1] = A[i]
        i = i - 1
    A[i+1] = key
```

Recursive Method →

```
InsertionSort (A, n)
  if n ≤ 1
      return
  InsertionSort (A, n-1)
    Key = [n-1];
    j = n-2;
  while j ≥ 0 and A[j] > key
    A[j+1] = A[j]
      j = j - 1
  A[j+1] = key
```

→ Insertion sort considers one i/p element per iteration & produces a partial solution without considering future elements that's why it is called online sorting.

Other sorting algos that have been discussed in lecture are:—

- Bubble sort
- Selection Sort
- Quick Sort
- Merge Sort
- Heap sort
- Counting Sort

Q21) Complexity of all sorting ------

Ans)

	Best Case	Average Case	Worst case
Bubble Sort	$\Omega(N)$	$\Theta(N^2)$	$O(N^2)$
Selection Sort	$\Omega(N^2)$	$\Theta(N^2)$	$O(N^2)$
Insertion Sort	$\Omega(N)$	$\Theta(N^2)$	$O(N^2)$
Merge Sort	$\Omega(N \log N)$	$\Theta(N \log N)$	$O(N \log N)$
Heap Sort	$\Omega(N \log N)$	$\Theta(N \log N)$	$O(N \log N)$
Quick Sort	$\Omega(N \log N)$	$\Theta(N \log N)$	$O(N^2)$
Counting Sort	$\Omega(N+K)$	$\Theta(N+K)$	$O(N+K)$

Q22) Divide all sorting algo into ------

Ans)

	In Place	Stable	Online
Bubble Sort	Yes	Yes	Yes
Insertion Sort	Yes	Yes	Yes
Selection Sort	Yes	No	Yes
Merge Sort	No	Yes	Yes
Quick Sort	Yes	No	Yes
Heap Sort	Yes	No	Yes
Count Sort	No	Yes	Yes

Q23) Write recursive / iterative ——

Ans) Linear Search →

```
LinearSearch (A, key)
found ← 0
for i = 1 to N
    if A[i] == key
        found ← 1
        print "Element found"
        break
if found == 0
    print "Element Not found"
```

Time complexity — O(n)
Space complexity — O(1)

Binary Search (Iterative) →

```
BinarySearch (A, beg, end, key)
while beg ≤ end
    mid = beg + (end - beg)/2
    if mid == key
        return mid
    if A[mid] < key
        beg = mid + 1
    if A[mid] > key
        end = mid - 1
return -1
```

Time complexity – $O(\log_2 n)$
Space complexity – $O(1)$

Binary Search (Recursive) →

```
Binary_Search (A, beg, end, key)
    if end ≥ beg
        mid = (beg + end) / 2
        if A[mid] == item
            return mid + 1
        else if A[mid] < item
    return Binary_Search (A, mid + 1, end, key)
    else
    return Binary_Search (A, beg, mid - 1, End)
        return -1.
```

Time Complexity – $O(\log n)$
Space Complexity – $O(1)$

Q24) Write recurrence relation for binary recursive Search.

Ans $T(n) = T(n/2) + c$