1. Symbolic calculation in matlab (for problem 1)

```
% matlab script file
% declare math variable x in matlab
syms x
% define function qext
qext(x) = 12*x^2 + cos(5*x) + 100*x*sin(10*x);
disp('external heat function')
qext
% integrate qext function
Q1(x) = int(qext, 0, x);
disp('Q1 function')
Q1
\% integrate Q1 function
Q2(x) = int(Q1, 0, x);
disp('Q2 function')
Q2
% get value of Q2 function at x = 1
disp('vaue of Q2 function at x = 1')
eval(Q2(1))
```

2. Multiple plots with labels (for problem 1)

```
% matlab script file
% get discrete points to plot functions
x = 0:0.01:1;
qext = @(x) 12*x.^2 + cos(5*x) + 100*x.*sin(10*x);
Q2 = Q(x) x.^4 - x.*sin(10*x) -2*cos(5*x).*cos(5*x)/5 - cos(5*x)/25 + 11/25;
% exact solution
T = Q(x) 100*x + ...; % complete this formula
% use 'DisplayName' to assign label to curve
plot(x, qext(x), 'r+', 'DisplayName', 'External heat')
hold on
plot(x, T(x), 'bo', 'DisplayName', 'Temperature')
hold on
y1 = 0*x + 80; % line y = 80
y2 = 0*x + 40; % line y = 40
plot(x, y1, 'g', 'DisplayName', 'Line y = 80')
hold on
plot(x, y2, 'k', 'DisplayName', 'Line y = 40')
legend() % this line tells matlab to add labels to the curve
```

**3. Roots problem.** In **Problem 1**, you will plot the functions T(x) and the horizontal lines y = 80 and y = 40. Look at the plot and find the x at which T and y = 80 lines intersect; this should help you in verifying your results for **Problem 2**. Similarly, find x at which T and y = 40 intersect; this should help you in verifying results for **Problem 3**.

To check your results in **Problem 4**, look at the plot for T and locate graphically the point x at which function T is maximum.

**4. Derivative of the temperature function** can be computed from the exact solution  $T(x) = 100x + 200xQ_2(1) - 200Q_2(x)$ . We will have

$$\frac{\mathrm{d}T(x)}{\mathrm{d}x} = 100 + 200Q_2(1) - 200Q_1(x),\tag{1}$$

where we used the fact that, since  $Q_2(x) = \int_0^x Q_1(y) dy$ ,  $dQ_2(x)/dx = Q_1(x)$ .