Numerical fracture experiments using nonlinear nonlocal models

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Joint work with
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Overview of the talk

- Why Peridynamics?
- Peridynamics as an alternative framework
- Introduction to bond-based peridynamics
- Numerical results
- Discussion and future works

Why peridynamics?

- Strain/stress can't be defined near crack
- Phase field introduces another field which is to be solved along with displacement field
- Handle situations where crack is not apriori known but is found by solving the evolution equation subjected to boundary conditions and initial conditions.

Key researchers in this area:
Stewart Silling & co., Florin Bobaru & co., Kaushik Dayal, Erdogan Madenci & co., John Foster, Olaf Weckner, Pablo Seleson, Erkan Oterkus

[1] Silling, S. A. (2000). Reformulation of elasticity theory for discontinuities and long-range forces. Journal of the Mechanics and Physics of Solids, 48(1), 175-209.
[2] Bobaru, F., & Hu, W. (2012). The meaning, selection, and use of the peridynamic horizon and its relation to crack branching in brittle materials. International journal of fracture, 176(2), 215-222.
[3] Silling, S. A., Weckner, O., Askari, E., & Bobaru, F. (2010). Crack nucleation in a peridynamic solid. International Journal of Fracture, 162(1-2), 219-227.
[4] Dayal, K., & Bhattacharya, K. (2006). Kinetics of phase transformations in the peridynamic formulation of continuum mechanics. Journal of the Mechanics and Physics of Solids, 54(9), 1811-1842.
[5] Silling, S. A., & Lehoucq, R. B. (2008). Convergence of peridynamics to classical elasticity theory. Journal of Elasticity, 93(1), 13.
[6] Agwai, A., Guven, I., & Madenci, E. (2011). Predicting crack propagation with peridynamics: a comparative study. International journal of fracture, 171(1), 65.

Peridynamics: Advantages

- Formulation completely bypasses the computation of derivatives of displacement field.
- A priori location of crack is not required.
- Only unknown field in the theory is displacement field.
- For smooth deformation the Peridynamics theory converges to the elastodynamics theory in the limit of nonlocal length scale going to zero.
- It has been shown numerically, as well as theoretically (for nonlinear model), that one recovers sharp-crack as the nonlocal length scale tends to zero.
- The elastic and fracture properties of the material fully determine the parameters in peridynamic constitutive law.

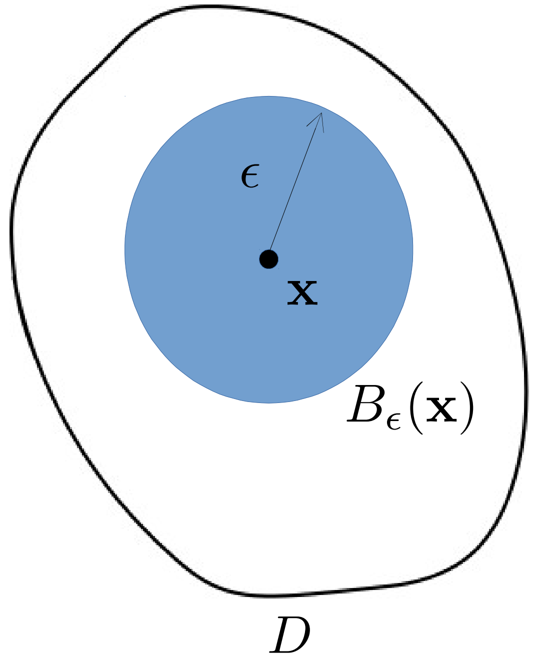
Peridynamics: Disadvantages

- No physical interpretation of the nonlocal length scale.
- While for smooth deformation, the peridynamics recovers elastodynamics solution, as nonlocal length scale goes to zero, it remains to show how the deformation behaves within the fracture zone in vanishing nonlocality.
- Need more thorough validation of the theory and show that it predicts the material behavior accurately for fairly large class of problems.
- Computationally expensive due to nonlocal nature of force. The computational complexity is of the form $O(Nr^\wedge d)$, where d is the dimension, r is the ratio of nonlocal length scale and mesh size, and N is the total number of mesh nodes.

Bond-based peridynamics

Displacement. Let $\mathbf{u}(\mathbf{x})$ denote the displacement of material point $\mathbf{x} \in D$. Throughout this talk we will make small-deformation assumption.

Nonlocal length scale. Let $\epsilon > 0$ be the nonlocal length scale. We refer to it as "Horizon". The material point $\mathbf{x} \in D$ interacts with all other points $\mathbf{y} \in B_\epsilon(\mathbf{x})$.



Bond strain. For any two material points $\mathbf{x}, \mathbf{y} \in D$, we define the bond strain as

$$S(\mathbf{y}, \mathbf{x}) = \frac{\mathbf{u}(\mathbf{y}) - \mathbf{u}(\mathbf{x})}{|\mathbf{y} - \mathbf{x}|} \cdot \frac{\mathbf{y} - \mathbf{x}}{|\mathbf{y} - \mathbf{x}|}$$

Here $|\mathbf{y} - \mathbf{x}|$ is the initial length of bond , $\mathbf{u}(\mathbf{y}) - \mathbf{u}(\mathbf{x})$ is the relative stretch after deformation, and $\frac{\mathbf{y}-\mathbf{x}}{|\mathbf{y}-\mathbf{x}|}$ is the unit vector pointing towards $\mathbf{y}$ from $\mathbf{x}$.

Internal forces

Internal force. Force at material point $\mathbf{x} \in D$ is given by the integration of the pairwise forces between the point $\mathbf{x}$ and the neighboring points $\mathbf{y} \in B_\epsilon(\mathbf{x})$.

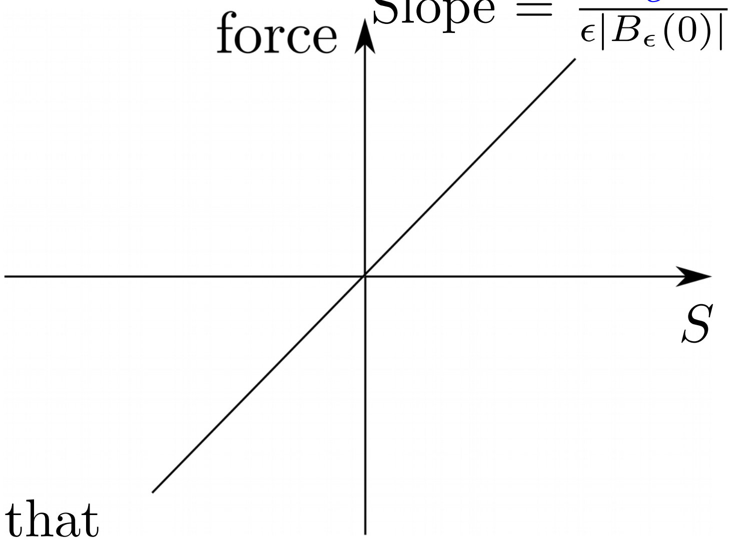
Suppose $\hat{\mathbf{f}}^\epsilon(\mathbf{y}, \mathbf{x})$ denotes the force applied on $\mathbf{x}$ from the neighboring point $\mathbf{y}$. Then total force at $\mathbf{x}$ is given by

$$\mathbf{f}^\epsilon(\mathbf{x}) = \int_{B_\epsilon(\mathbf{x})} \hat{\mathbf{f}}^\epsilon(\mathbf{y}, \mathbf{x}) d\mathbf{y}$$

Example 1: Linear force

Example 1. Linear peridynamic force

$$\hat{\mathbf{f}}^\epsilon(\mathbf{y},\mathbf{x}) = \frac{c}{\epsilon|B_\epsilon(\mathbf{0})|} S(\mathbf{y},\mathbf{x}) \frac{\mathbf{y}-\mathbf{x}}{|\mathbf{y}-\mathbf{x}|}$$



If displacement field is sufficiently smooth, we can show that

In 2-d/3-d $\quad \mathbf{f}^\epsilon(\mathbf{x}) = \nabla \cdot \mathbb{C}\nabla \mathbf{u} + O(\epsilon^2)$,

$$\mathcal{E}\mathbf{u} = \frac{\nabla \mathbf{u} + \nabla \mathbf{u}^T}{2}$$

$$\mathbb{C}_{ijkl} = 2\mu \left(\frac{\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}}{2} \right) + \lambda \delta_{ij}\delta_{kl}$$

$\lambda = \mu = \frac{c}{12}$ in 2-d, $\lambda = \mu = \frac{c}{20}$ in 3-d

In 1-d

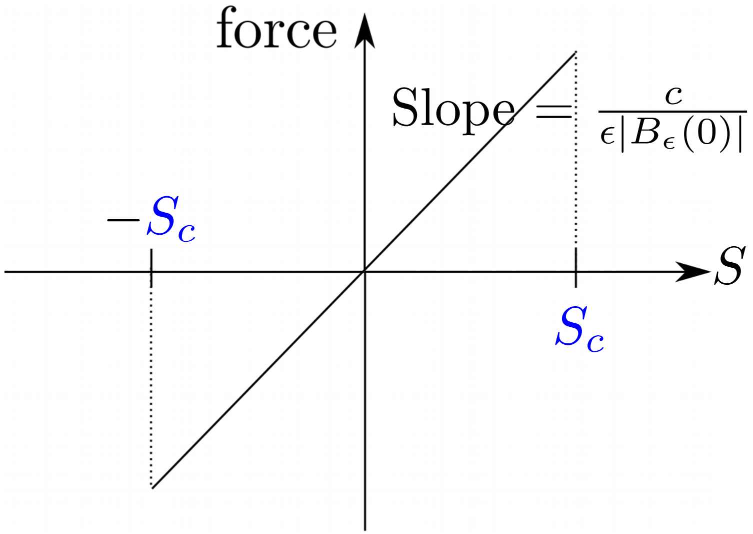
$$\mathbf{f}^\epsilon(\mathbf{x}) = \frac{\partial}{\partial \mathbf{x}} \left(\frac{c}{2} \frac{\partial \mathbf{u}(\mathbf{x})}{\partial \mathbf{x}} \right) + O(\epsilon^2)$$

Example 2: Adding fracture to linear force

Example 2. Adding fracture to the model

$$\hat{\mathbf{f}}^\epsilon(\mathbf{y},\mathbf{x}) = \mu(S(\mathbf{y},\mathbf{x}))\frac{c}{\epsilon|B_\epsilon(\mathbf{0})|}S(\mathbf{y},\mathbf{x})\frac{\mathbf{y}-\mathbf{x}}{|\mathbf{y}-\mathbf{x}|}$$

where $\mu(S) = 1$ if $|S| < S_c$ and $\mu(S) = 0$ if $|S| \geq S_c$.



While constant c is determined by elastic properties of the material, the critical strain S_c will be determined by the fracture properties of the material.

In 3-d, given Young's modulus E and the critical energy release rate G_c, we have [1]

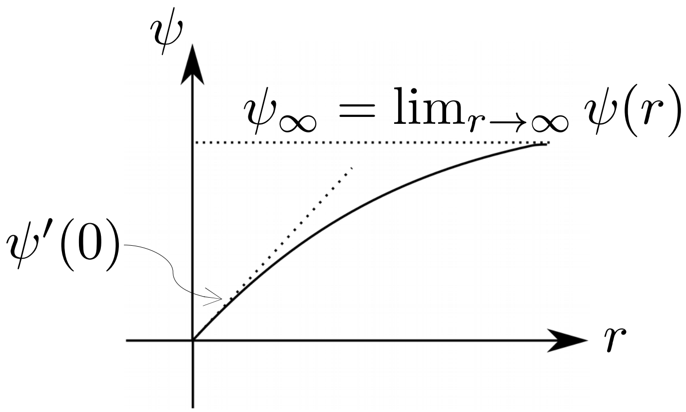
$$c = 8E, \quad S_c = \sqrt{\frac{10G_c}{c\pi\epsilon^5}}$$

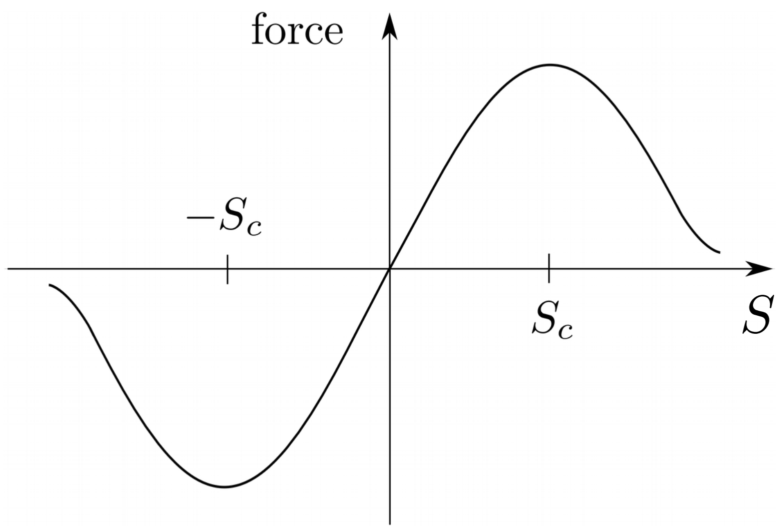
[1] Ren, B., Wu, C. T., & Askari, E. (2017). A 3D discontinuous Galerkin finite element method with the bond-based peridynamics model for dynamic brittle failure analysis. International Journal of Impact Engineering, 99, 14-25.

Example 3: Nonlinear force

Example 3. Nonlinear peridynamic model[1]

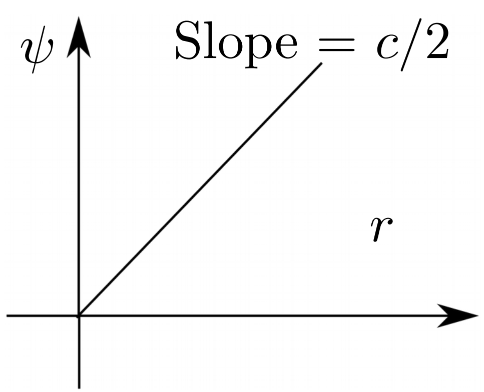
$$\hat{\mathbf{f}}^\epsilon(\mathbf{y},\mathbf{x}) = \frac{1}{\epsilon|B_\epsilon(\mathbf{0})|}\frac{\partial_S\psi(|\mathbf{y}-\mathbf{x}|S(\mathbf{y},\mathbf{x})^2)}{|\mathbf{y}-\mathbf{x}|}\frac{\mathbf{y}-\mathbf{x}}{|\mathbf{y}-\mathbf{x}|}$$





If function $r \mapsto \psi(r)$ is linear, i.e., $\psi(r) = cr/2$ then we get linear peridynamic model

$$\hat{\mathbf{f}}^\epsilon(\mathbf{y},\mathbf{x}) = \frac{c}{\epsilon|B_\epsilon(\mathbf{0})|}S(\mathbf{y},\mathbf{x})\frac{\mathbf{y}-\mathbf{x}}{|\mathbf{y}-\mathbf{x}|}$$



[1] Robert Lipton (2014) Dynamic brittle fracture as a small horizon limit of peridynamics Journal of Elasticity 117 no. 1 2150

Equation of motion

Equation of motion.

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) = \mathbf{f}^\epsilon(\mathbf{x};\mathbf{u}(t)) + \mathbf{b}(\mathbf{x},t), \qquad \forall \mathbf{x} \in D, t \in [0,T]$$

Boundary condition.

$$\mathbf{u}(\mathbf{x},t) = \mathbf{g}(\mathbf{x},t) \qquad \forall \mathbf{x} \in D_u, t \in [0,T]$$

$$\mathbf{b}(\mathbf{x},t) = \mathbf{f}_{ext}(\mathbf{x},t) \qquad \forall \mathbf{x} \in D_f, t \in [0,T]$$

Initial condition. $\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}), \dot{\mathbf{u}}(\mathbf{x},0) = \mathbf{v}_0(\mathbf{x})$ for all $\mathbf{x} \in D$

Weak form. Multiplying peridynamic equation by smooth test function $\tilde{\mathbf{u}}$ such that $\tilde{\mathbf{u}} = \mathbf{0}$ on D_u, and integrating over D, and using nonlocal integration by parts, gives

$$(\rho\ddot{\mathbf{u}}(t), \tilde{\mathbf{u}}) + a^\epsilon(\mathbf{u}(t), \tilde{\mathbf{u}}) = (\mathbf{b}(t), \tilde{\mathbf{u}})$$

where

$$a^\epsilon(\mathbf{u},\mathbf{w}) = \frac{1}{\epsilon|B_\epsilon(\mathbf{0})|}\int_D \left[\int_{B_\epsilon(\mathbf{x})} \psi'(|\mathbf{y}-\mathbf{x}|S(\mathbf{u})^2)|\mathbf{y}-\mathbf{x}|S(\mathbf{u})S(\mathbf{w})d\mathbf{y}\right] d\mathbf{x}$$

Well-posedness of nonlinear peridynamic model

- Using the fact that nonlinear peridynamic force is bounded and Lipschitz continuous with respect to displacement field $\boldsymbol{u} \in L^2_0(D)$, the existence of solutions over any finite time domain $[0, T]$ is shown [1].

- To prove existence of solutions in more regular spaces, we introduce boundary function ω into Peridynamic force. $\omega(\boldsymbol{x}) = 1$ in the interior and smoothly decays to 0 as $\boldsymbol{x}$ approaches boundary ∂D.

- To perform apriori error analysis of finite difference approximation, we consider Hölder space $C_0^{0,\gamma}(D)$, $\gamma \in (0, 1]$. In [2] we show existence of solutions in Hölder space $C_0^{0,\gamma}(D)$. In [3] we extend the results to state-based peridynamic models.

- For apriori error analysis of finite element approximation using continuous piecewise linear elements, we consider natural space $H^2(D) \cap H^1_0(D)$. In [4] we show existence of solutions in $H^2(D) \cap H^1_0(D)$.

[1] R. Lipton (2016) Cohesive dynamics and brittle fracture. Journal of Elasticity, 124(2), pp.143-191.

[2] P.K. Jha and R. Lipton (2018) Numerical analysis of nonlocal fracture models in Holder space. SIAM Journal on Numerical Analysis, 56(2), pp.906-941.

[3] P.K. Jha and R. Lipton (2019) Numerical convergence of finite difference approximations for state based peridynamic fracture models. Computer Methods in Applied Mechanics and Engineering, 351(1), 184 – 225.

[4] P.K. Jha and R. Lipton (2018) Finite element approximation of nonlocal fracture models. arXiv preprint arXiv:1710.07661. **Under review** in Discrete and Continuous Dynamical Systems Series B.

Well-posedness

Let W be either $C_0^{0,\gamma}(D)$ or $H^2(D) \cap H_0^1(D)$ space. We assume $\boldsymbol{u} \in W$ is extended by zero outside D. Domain D is assumed to be sufficiently smooth (precise details in [1,2]). Two key steps to show existence:

- Obtain Lipschitz bound on peridynamic force in W.
- Using Lipshitz bound, show local existence of unique solutions. Show that local existence of unique solutions can be repeatedly applied to get global existence of solutions for any time domain $(-T, T)$.

Theorem 1. ***Existence and uniqueness of solutions over finite time intervals***
Let $\boldsymbol{v}(t) = \dot{\boldsymbol{u}}(t)$, and $X = W \times W$. For any initial condition $x_0 \in X$, time interval $I_0 = (-T, T)$, and right hand side $\boldsymbol{b}(t)$ continuous in time for $t \in I_0$ such that $\boldsymbol{b}(t)$ satisfies $\sup_{t \in I_0} ||\boldsymbol{b}(t)||_W < \infty$, there is a unique solution $(\boldsymbol{u}(t), \boldsymbol{v}(t)) \in C^1(I_0; X)$ of Peridynamic equation of motion with initial condition x_0. Moreover, $(\boldsymbol{u}(t), \boldsymbol{v}(t))$ and $(\dot{\boldsymbol{u}}(t), \dot{\boldsymbol{v}}(t))$ are Lipschitz continuous in time for $t \in I_0$.

[1] P.K. Jha and R. Lipton (2018) Numerical analysis of nonlocal fracture models in Holder space. SIAM Journal on Numerical Analysis, 56(2), pp.906-941.
[2] P.K. Jha and R. Lipton (2019) Numerical convergence of finite difference approximations for state based peridynamic fracture models. Computer Methods in Applied Mechanics and Engineering, 351(1), 184 – 225.

Finite difference approximation

We approximate peridynamic equation using piecewise constant interpolation and central in time discretization. Let $\boldsymbol{u}_i^k$ denote the discrete displacement at mesh node $\boldsymbol{x}_i$ and time $t^k = k\Delta t$. We consider following piecewise constant function

$$\boldsymbol{u}_h^k(\boldsymbol{x}) = \sum_{i, \boldsymbol{x}_i \in D} \boldsymbol{u}_i^k \chi_{U_i}(\boldsymbol{x})$$

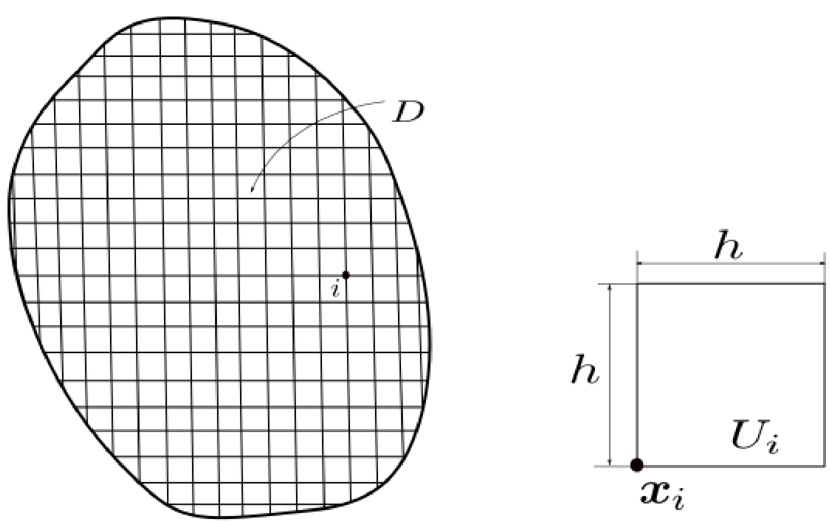
Discrete problem is

$$\frac{\boldsymbol{u}_h^{k+1} - 2\boldsymbol{u}_h^k + \boldsymbol{u}_h^{k-1}}{\Delta t^2} = \boldsymbol{f}_h^\epsilon(t^k) + \boldsymbol{b}_h^k,$$

where

$$\boldsymbol{f}_h^\epsilon(\boldsymbol{x}, t^k) = \sum_{i, \boldsymbol{x}_i \in D} \boldsymbol{f}^\epsilon(\boldsymbol{x}_i, t^k) \chi_{U_i}(\boldsymbol{x}),$$

$$\boldsymbol{b}_h(\boldsymbol{x}, t^k) = \sum_{i, \boldsymbol{x}_i \in D} \boldsymbol{b}(\boldsymbol{x}_i, t^k) \chi_{U_i}(\boldsymbol{x})$$



(a) (b)

Convergence of finite difference approximation

Error at time step k is defined as: $E^k = ||\boldsymbol{u}_h^k - \boldsymbol{u}(t^k)||$.

Theorem 1. *Let $\epsilon > 0$ be fixed. Let $(\boldsymbol{u}, \boldsymbol{v})$ be the solution of peridynamic equation. We assume $\boldsymbol{u}, \boldsymbol{v} \in C^2([0,T]; C^{0,\gamma}(D;\mathbb{R}^d))$. Then the finite difference scheme is consistent in both time and spatial discretization and converges to the exact solution uniformly in time with respect to the L^2 norm. If we assume the error at the initial step is zero then the error E^k at time t^k is bounded and satisfies*

$$\sup_{0\leq k\leq T/\Delta t} E^k \leq O\left(C_t \Delta t + C_s \frac{h^\gamma}{\epsilon^2}\right),$$

where constant C_s and C_t are independent of h and Δt. Constants C_t, C_s depend on the ϵ and Hölder norm of the exact solution.

[1] P.K. Jha and R. Lipton (2018) Numerical analysis of nonlocal fracture models in Holder space. SIAM Journal on Numerical Analysis, 56(2), pp.906-941.

[2] P.K. Jha and R. Lipton (2019) Numerical convergence of finite difference approximations for state based peridynamic fracture models. Computer Methods in Applied Mechanics and Engineering, 351(1), 184 – 225.

Finite element approximation

Let $V_h \subset H_0^1(D)$ be given by linear continuous interpolations over tetrahedral or triangular elements $\mathcal{T}_h$ where h denotes the size of finite element mesh. We assume elements are conforming and the mesh is shape regular.

For a continuous function $\boldsymbol{u}$ on $\bar{D}$, $\mathcal{I}_h(\boldsymbol{u})$ is the continuous piecewise linear interpolant on $\mathcal{T}_h$ and is given by

$$\mathcal{I}_h(\boldsymbol{u})(\boldsymbol{x}) = \sum_{T \in \mathcal{T}_h} \left[\sum_{i \in N_T} \boldsymbol{u}(\boldsymbol{x}_i)\phi_i(\boldsymbol{x}) \right].$$

Assuming that the size of each element in triangulation $\mathcal{T}_h$ is bounded by h, we have

$$||\boldsymbol{u} - \mathcal{I}_h(\boldsymbol{u})|| \leq ch^2 ||\boldsymbol{u}||_2, \qquad \forall \boldsymbol{u} \in H_0^2(D; \mathbb{R}^d).$$

Projection: Let $\boldsymbol{r}_h(\boldsymbol{u}) \in V_h$ is the projection of $\boldsymbol{u} \in H^2(D) \cap H_0^1(D)$ such that

$$||\boldsymbol{u} - \boldsymbol{r}_h(\boldsymbol{u})|| = \inf_{\tilde{\boldsymbol{u}} \in V_h} ||\boldsymbol{u} - \tilde{\boldsymbol{u}}||$$

Central difference time discretization

$(\boldsymbol{u}_h^k, \boldsymbol{v}_h^k)$ and $(\boldsymbol{u}^k, \boldsymbol{v}^k)$ denote the approximate and the exact solution at k^{th} step. Projection is denoted as $(\boldsymbol{r}_h(\boldsymbol{u}^k), \boldsymbol{r}_h(\boldsymbol{v}^k))$. Approximate initial condition $\boldsymbol{u}_0, \boldsymbol{v}_0$ by their projection $\boldsymbol{r}_h(\boldsymbol{u}_0), \boldsymbol{r}_h(\boldsymbol{v}_0)$ and set $\boldsymbol{u}_h^0 = \boldsymbol{r}_h(\boldsymbol{u}_0)$, $\boldsymbol{v}_h^0 = \boldsymbol{r}_h(\boldsymbol{v}_0)$.

For $k \geq 1$, $(\boldsymbol{u}_h^k, \boldsymbol{v}_h^k)$ satisfies, for all $\tilde{\boldsymbol{u}} \in V_h$,

$$\left(\frac{\boldsymbol{u}_h^{k+1} - \boldsymbol{u}_h^k}{\Delta t}, \tilde{\boldsymbol{u}} \right) = (\boldsymbol{v}_h^{k+1}, \tilde{\boldsymbol{u}}),$$

$$\left(\frac{\boldsymbol{v}_h^{k+1} - \boldsymbol{v}_h^k}{\Delta t}, \tilde{\boldsymbol{u}} \right) = (\boldsymbol{f}^\epsilon(\boldsymbol{u}_h^k), \tilde{\boldsymbol{u}}) + (\boldsymbol{b}_h^k, \tilde{\boldsymbol{u}}),$$

where we denote projection of $\boldsymbol{b}(t^k)$, $\boldsymbol{r}_h(\boldsymbol{b}(t^k))$, as $\boldsymbol{b}_h^k$. Combining the two equations delivers central difference equation for $\boldsymbol{u}_h^k$. We have

$$\left(\frac{\boldsymbol{u}_h^{k+1} - 2\boldsymbol{u}_h^k + \boldsymbol{u}_h^{k-1}}{\Delta t^2}, \tilde{\boldsymbol{u}} \right) = (\boldsymbol{f}^\epsilon(\boldsymbol{u}_h^k), \tilde{\boldsymbol{u}}) + (\boldsymbol{b}_h^k, \tilde{\boldsymbol{u}}), \qquad \forall \tilde{\boldsymbol{u}} \in V_h.$$

Convergence of approximation

Error at time step k is defined as: $E^k = ||\boldsymbol{u}_h^k - \boldsymbol{u}(t^k)||$.

Theorem 3. ***Convergence of Central difference approximation***
Let $(\boldsymbol{u}, \boldsymbol{v})$ be the exact solution of peridynamics equation and Let $(\boldsymbol{u}_h^k, \boldsymbol{v}_h^k)$ be the FE approximate solution. If $\boldsymbol{u}, \boldsymbol{v} \in C^2([0,T], H^2(D) \cap H_0^1(D))$, then the scheme is consistent and the error E^k satisfies following bound

$$\sup_{k \leq T/\Delta t} E^k = C_t \Delta t + C_s \frac{h^2}{\epsilon^2}$$

where constant C_t and C_s are independent of h and Δt and depends on the horizon and the norm of exact solution. Constant L/ϵ^2 is the Lipschitz constant of peridynamic force in L^2.

[1] P.K. Jha and R. Lipton (2018) Finite element approximation of nonlocal fracture models. arXiv preprint arXiv:1710.07661. **Under review** in Discrete and Continuous Dynamical Systems Series B.

Setting up peridynamic model

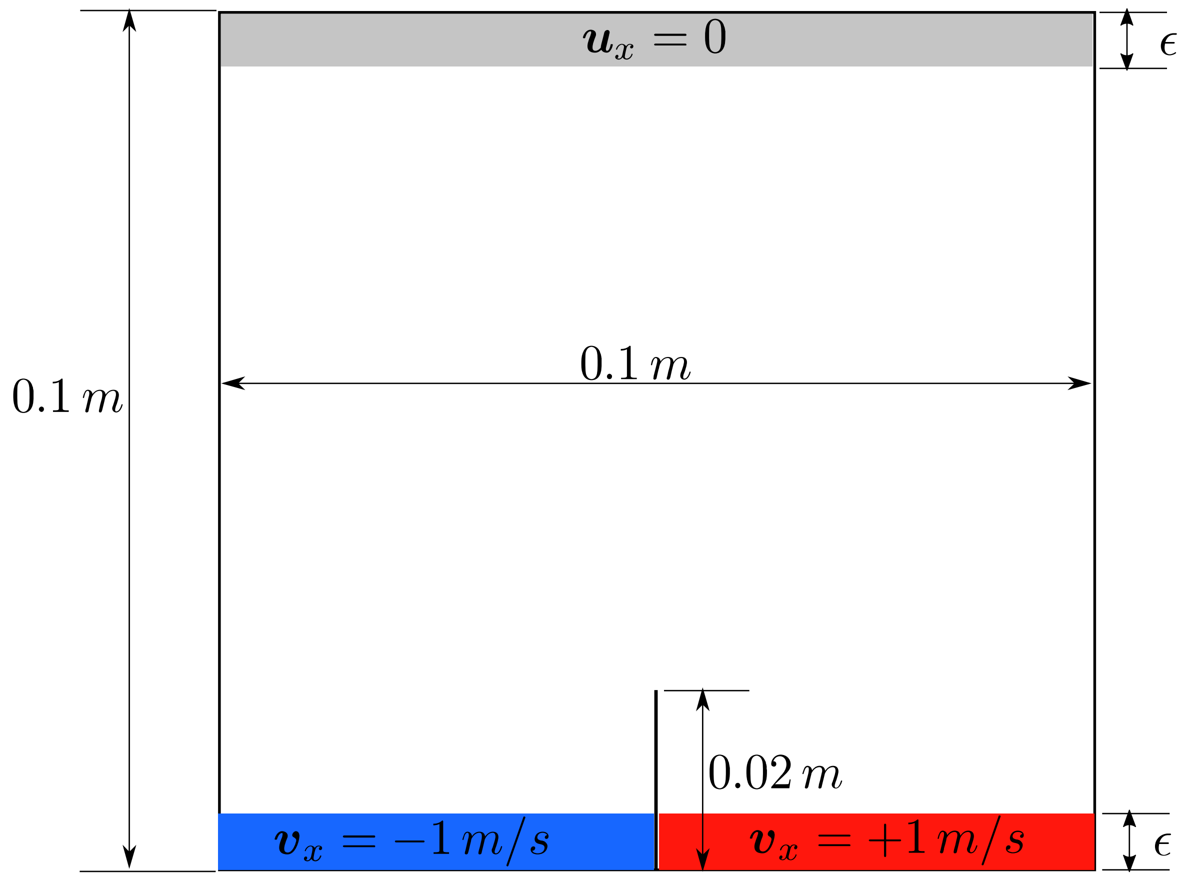
- Pairwise potential: $\psi(r) = c(1 - \exp[-\beta r^2])$
- Influence function: $J(r) = 1 - r$ for $0 \leq r < 1$ and $J(r) = 0$ for $r \geq 1$
- Critical strain: $S_c(\boldsymbol{y}, \boldsymbol{x}) = \dfrac{\pm \bar{r}}{\sqrt{|\boldsymbol{y} - \boldsymbol{x}|}}$, where $\bar{r}$ is the inflection point of function ψ
- We fix $\rho = 1200$ kg/m^3, bulk modulus $K = 3.24$ GPa, critical energy release rate $G_c = 500$ J/m^{-2}
- Using relation between nonlinear peridynamic model and linear elastic fracture mechanics[1], we find

$$c = 4712.4, \qquad \beta = 1.7533 \times 10^8, \qquad \bar{r} = \frac{1}{\sqrt{2\beta}} = 5.3402 \times 10^{-5}$$

[1] R. Lipton (2016) Cohesive dynamics and brittle fracture. Journal of Elasticity, 124(2), pp.143-191.

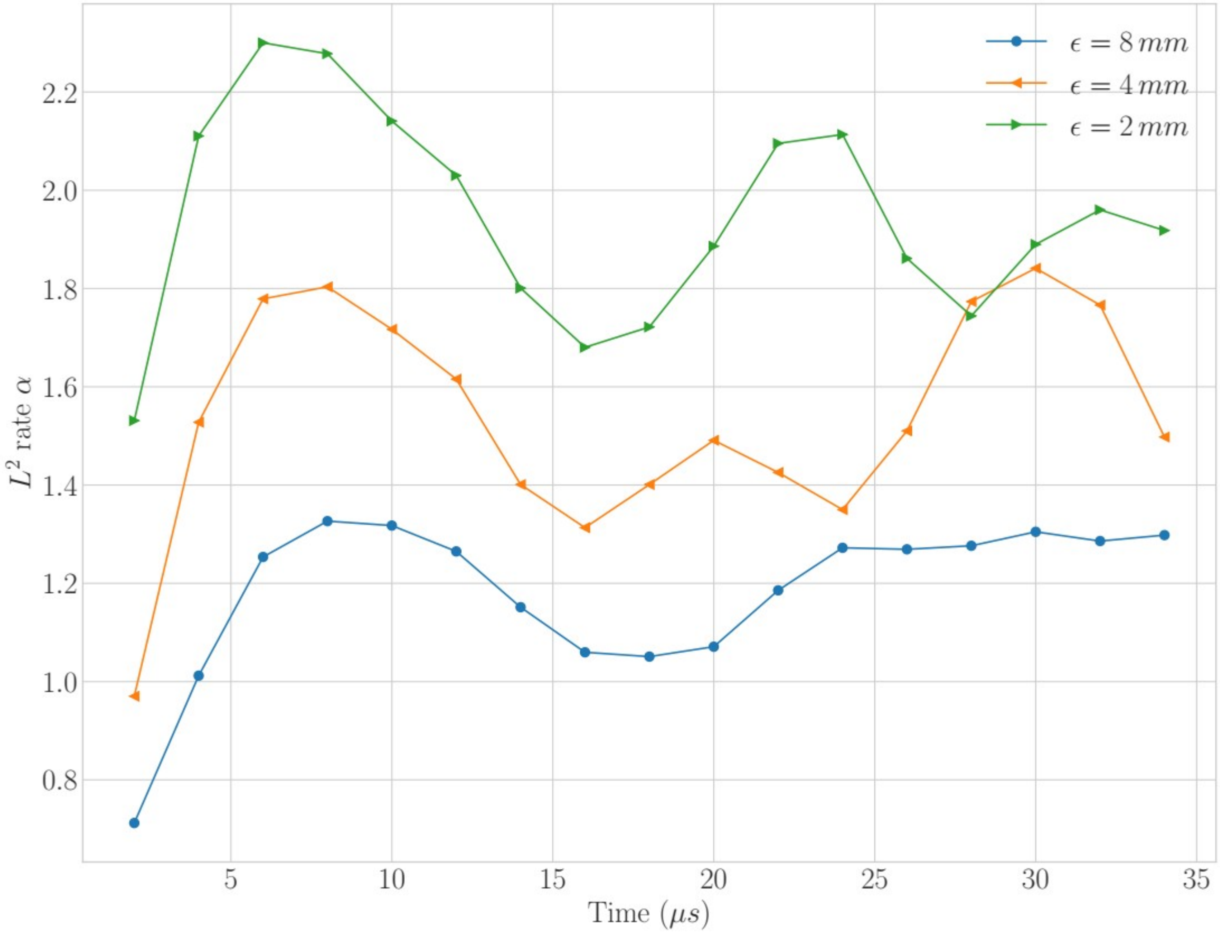
Mode-I crack propagation

- Final time $T = 34\,\mu$s, time step $\Delta t = 0.004\,\mu$s
- Uniform grid on square domain $D = [0, 0.1\,\mathrm{m}]^2$

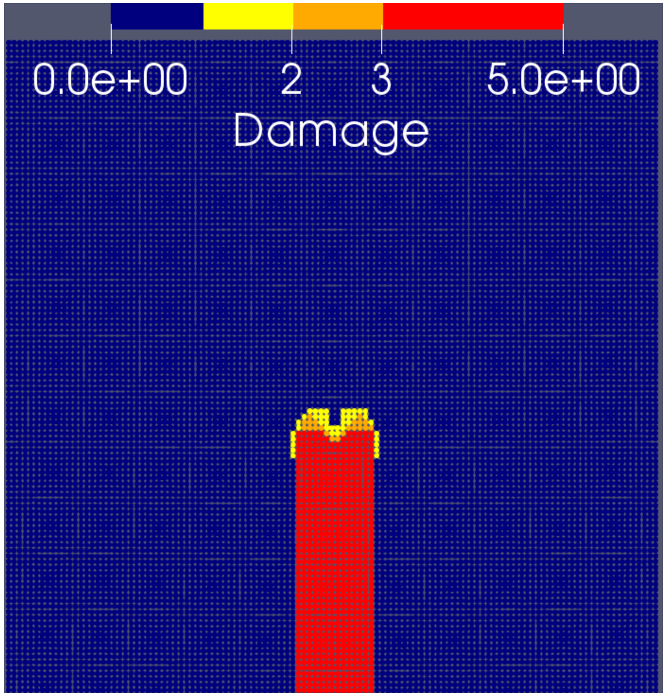


Convergence wrt mesh size

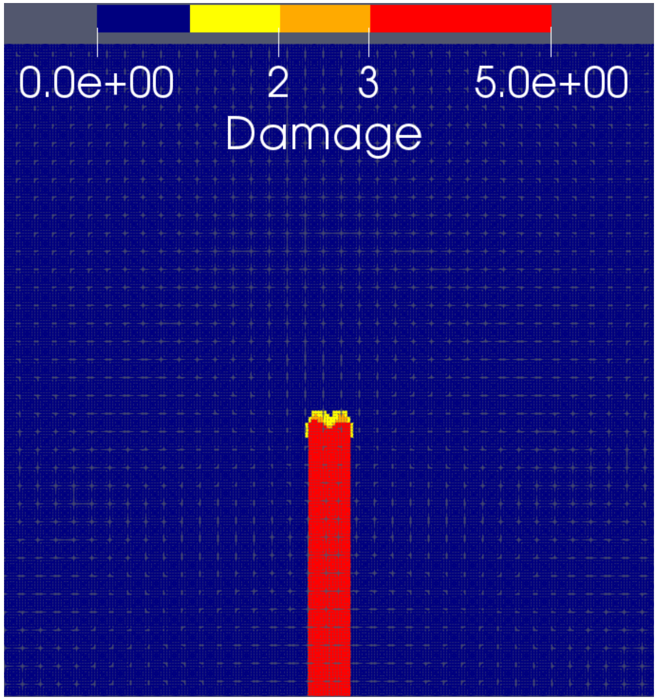
- Three set of horizons $\epsilon = 8, 4, 2$ mm. For each fixed ϵ, simulations were run with three different meshes of size $h = \epsilon/2, \epsilon/4, \epsilon/8$.



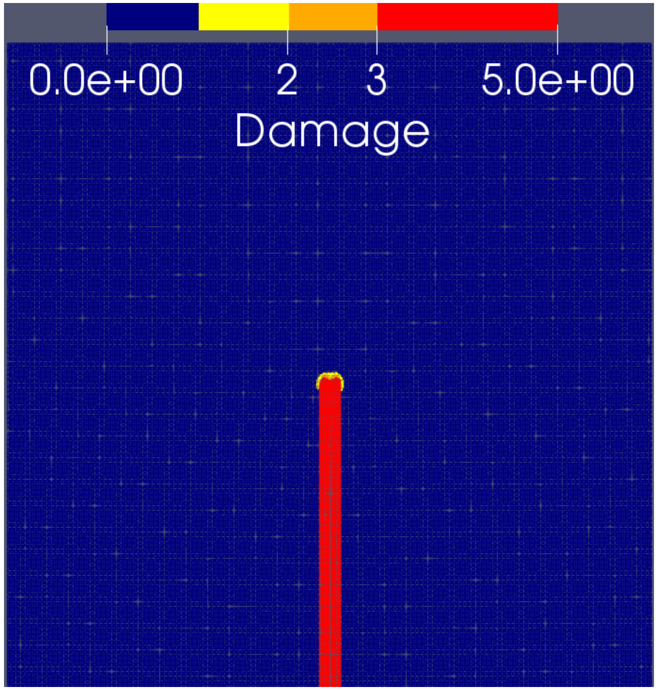
Localization of softening zone



$\epsilon = 8$ mm



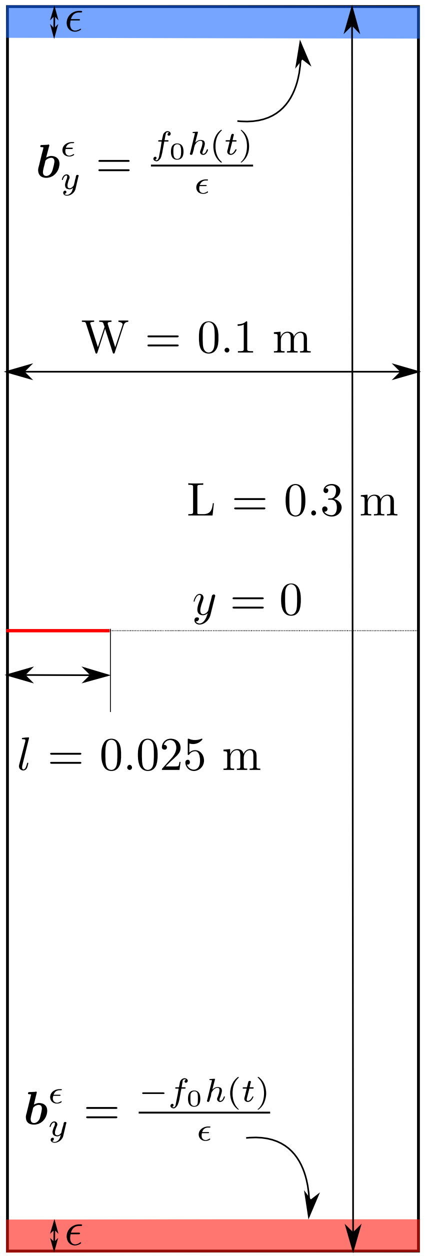
$\epsilon = 4$ mm



$\epsilon = 2$ mm

Mode I crack propagation: Setup

Goal: Localization of crack and convergence to classical fracture mechanics for simple mode-I crack propagation[1]



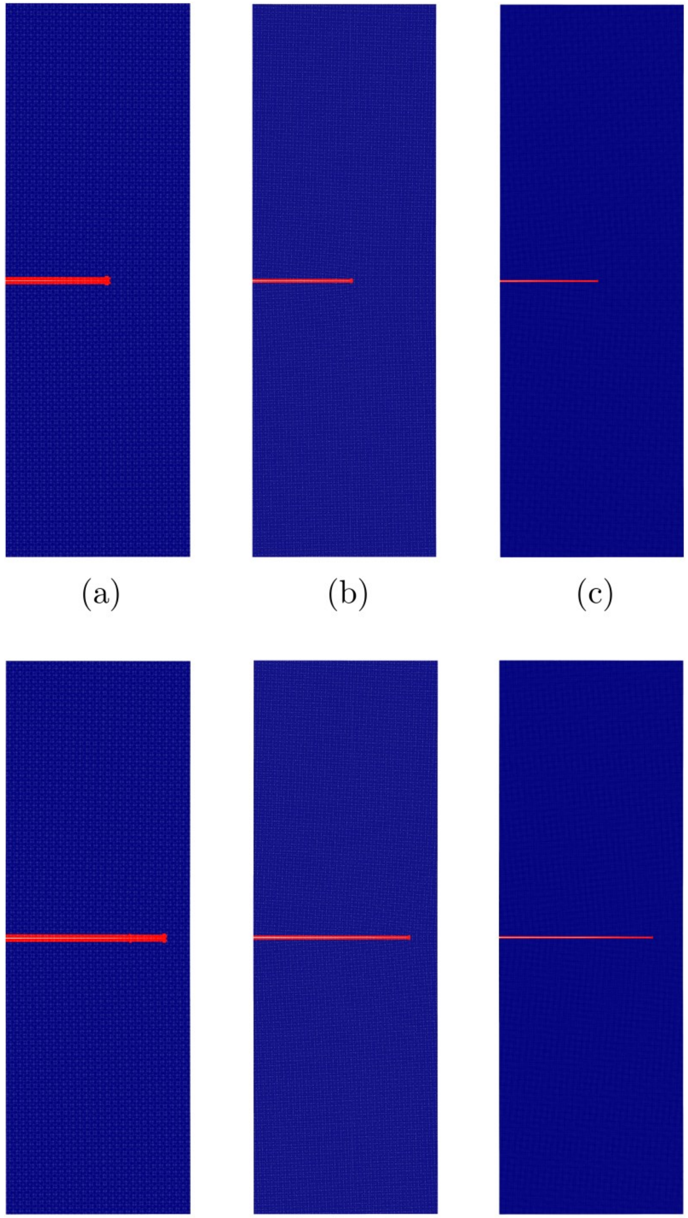
- Final time $T = 560\,\mu$s, time step $\Delta t = 0.02\,\mu$s
- Uniform grid on square domain $D = [0, 0.1\,\text{m}] \times [-0.15\,\text{m}, 0.15\,\text{m}]$
- Experiment with three different horizons $\epsilon = 2.5, 1.25, 0.625\,\text{mm}$
- Body force $\boldsymbol{b}^\epsilon(\boldsymbol{x}, t) = (0, f_0 h(t)/\epsilon)$ on top layer and $\boldsymbol{b}^\epsilon(\boldsymbol{x}, t) = (0, -f_0 h(t)/\epsilon)$ on bottom layer
- $h(t)$ is a step function such that $h(t) = t$ for $t \leq 350\,\mu$s and $h(t) = 1$ for $t > 350\,\mu$s
- Mesh size is fixed by relation $h = \epsilon/4$

[1] R. Lipton & P.K. Jha (2019). Classic dynamic fracture recovered as the limit of a nonlocal peridynamic model: The single edge notch in tension. Under review in International Journal of Fracture (FRAC).

Localization of softening zone

$t = 460\,\mu$s

$t = 520\,\mu$s

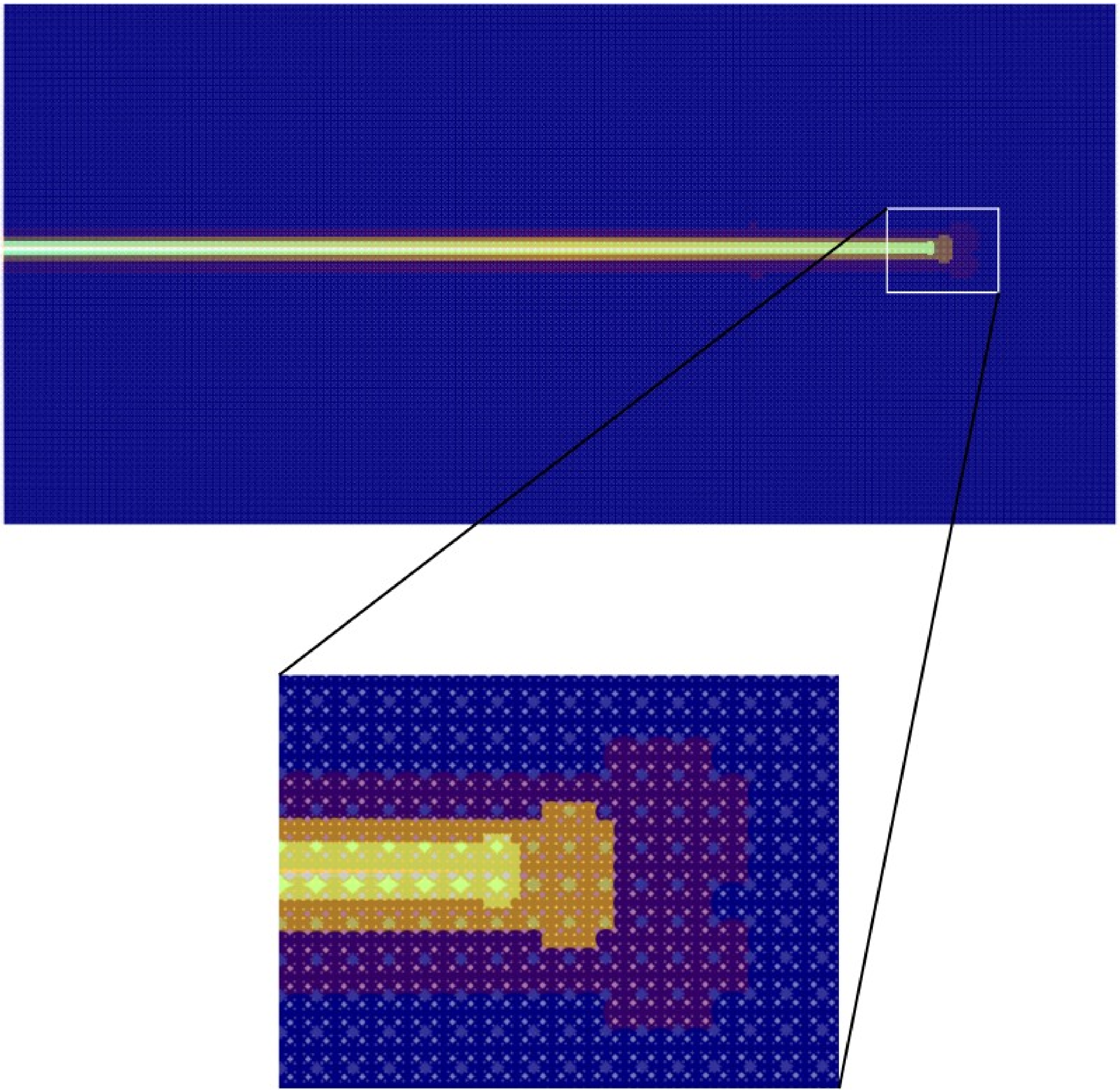


(a) (b) (c)

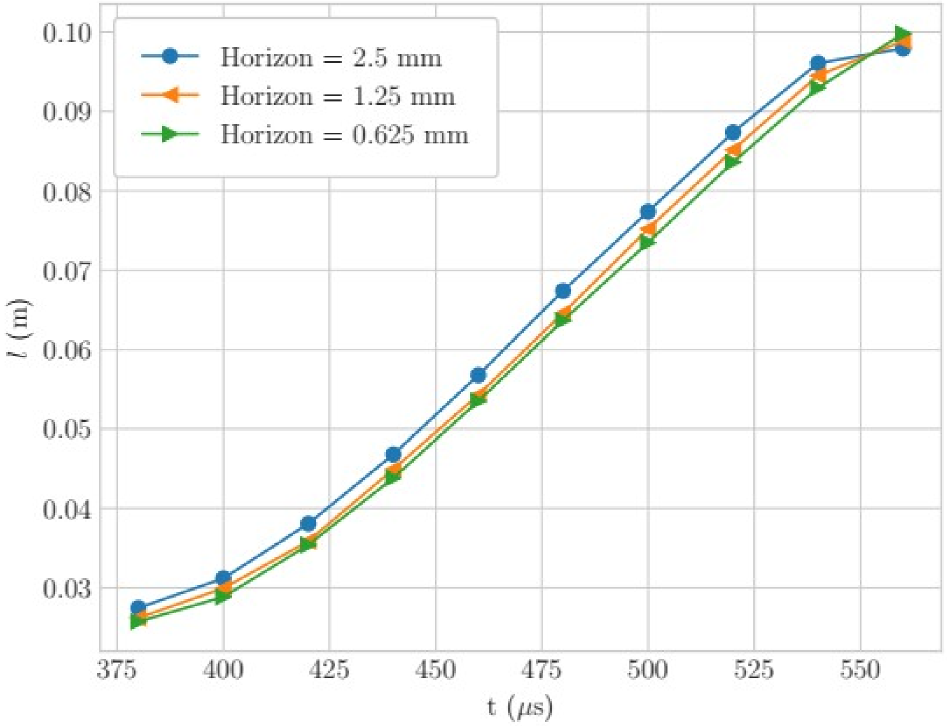
(d) (e) (f)

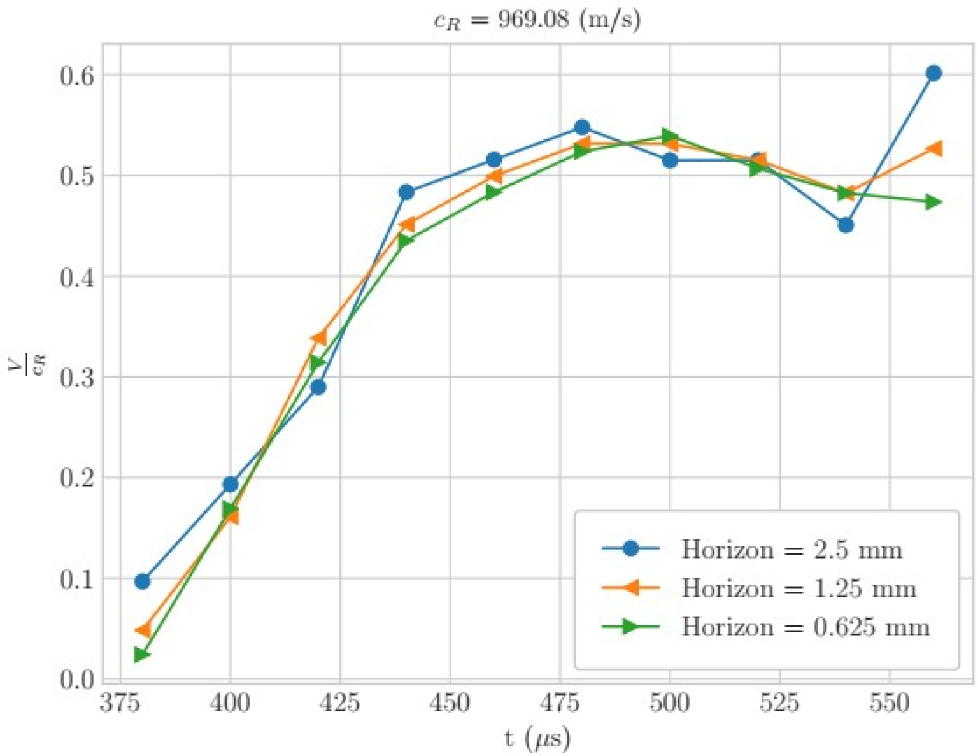
Localization of softening zone

$t = 520\,\mu$s



Crack tip location and velocity



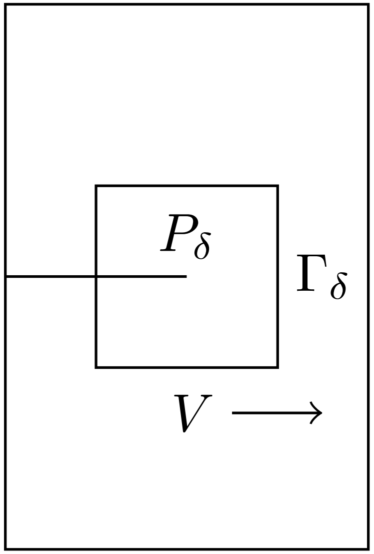


Energy into crack

The energy associated to crack is given by[1]

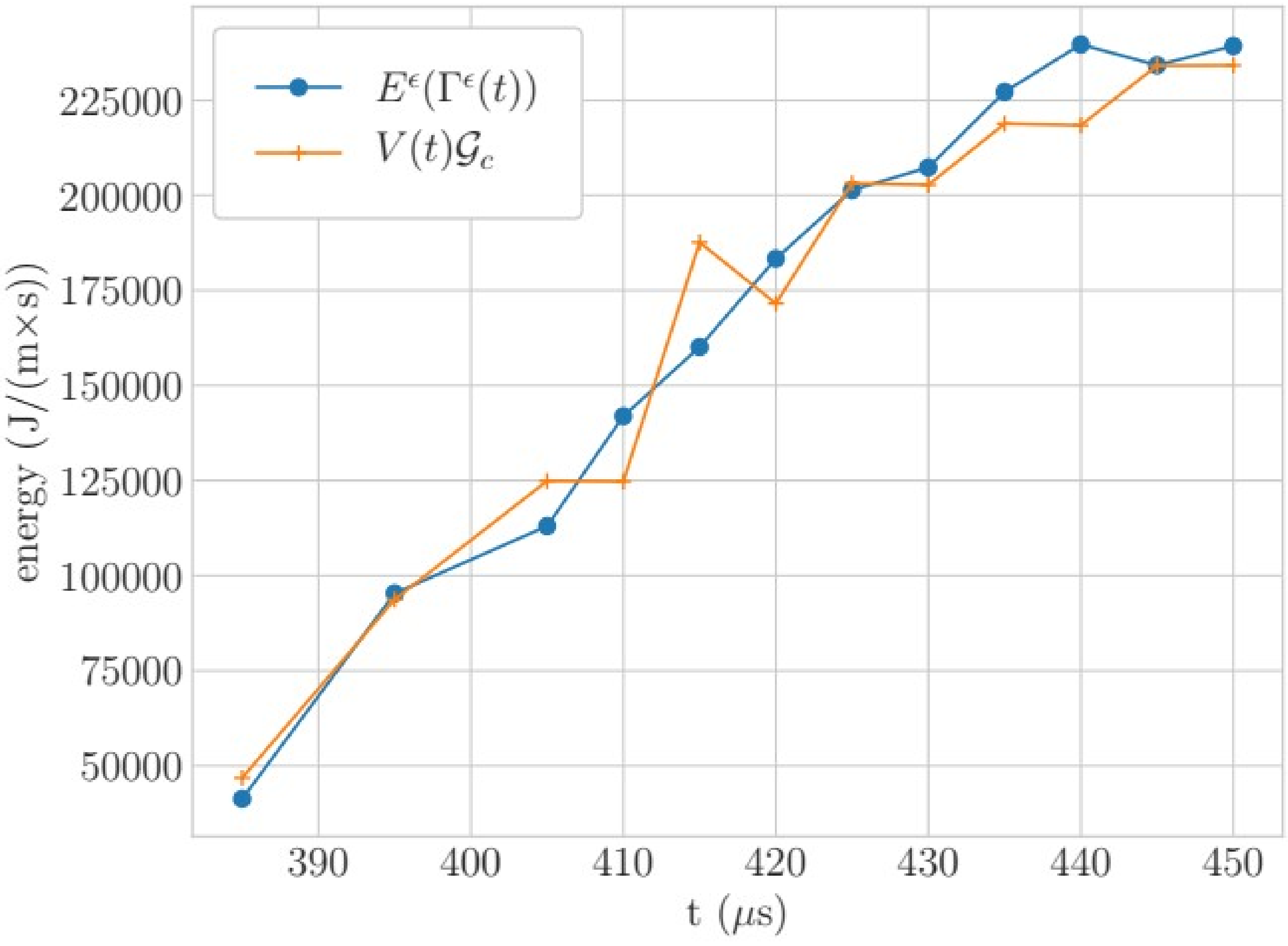
$$E(\Gamma_\delta(t)) = \frac{1}{|B_\epsilon(\mathbf{0})|} \int_{P^c_\delta(t)} \int_{P_\delta(t)\cap B_\epsilon(\boldsymbol{x})} \partial_S W(S(\boldsymbol{y},\boldsymbol{x};\boldsymbol{u}(t))) \frac{\boldsymbol{y}-\boldsymbol{x}}{|\boldsymbol{y}-\boldsymbol{x}|} \cdot (\dot{\boldsymbol{u}}(\boldsymbol{x},t) + \dot{\boldsymbol{u}}(\boldsymbol{y},t)) d\boldsymbol{y} d\boldsymbol{x}.$$

Here W is the peridynamic pairwise energy density. $P_\delta(t)$ is the rectangle domain with crack tip at its center. It is moving with tip. $P^c_\delta(t)$ is the complement of $P_\delta(t)$.



[1] R. Lipton & P.K. Jha (2019). Classic dynamic fracture recovered as the limit of a nonlocal peridynamic model: The single edge notch in tension. Under review in International Journal of Fracture (FRAC).

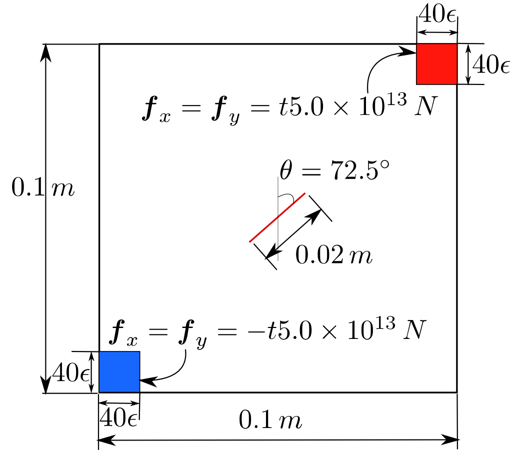
Energy into crack



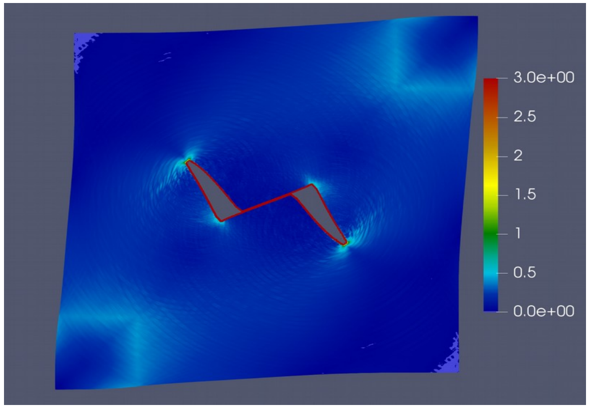
Mix mode crack propagation

Material properties are same as in the Mode-I problem. We set

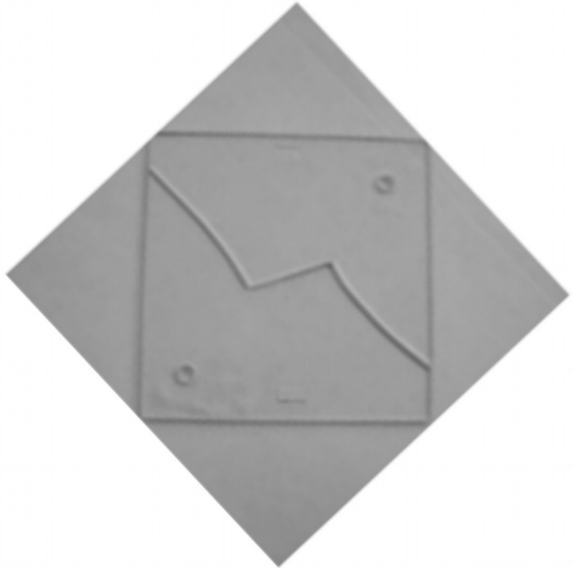
- Horizon $\epsilon = 0.5$ mm
- Mesh size $h = 0.125$ mm
- Final time $T = 140\,\mu$s
- Time step size $\Delta t = 0.004\,\mu$s



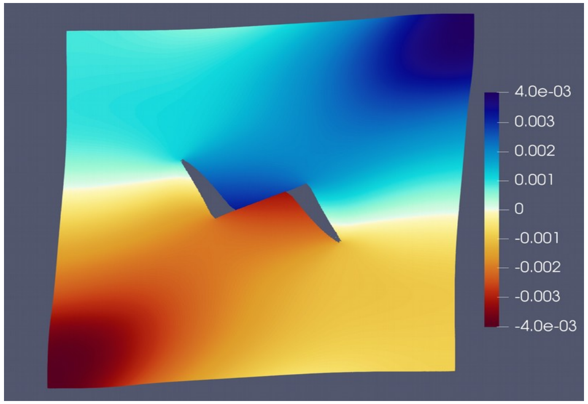
(a) Setup



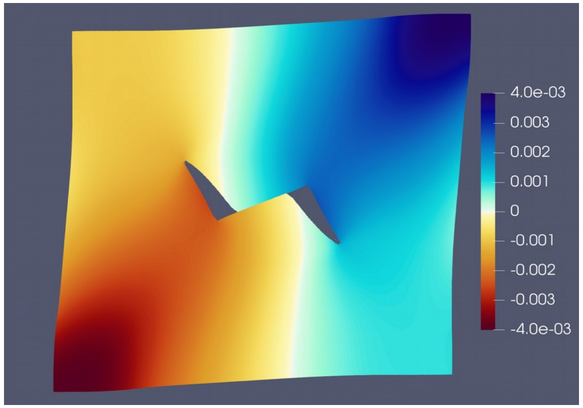
(b) Damage profile



(e) Experiment result [2]



(c) $\boldsymbol{u}_x$ plot



(d) $\boldsymbol{u}_y$ plot

[1] R. Lipton, R. Lehoucq, & P.K. Jha (2019) Complex fracture nucleation and evolution with nonlocal elastodynamics. Journal of Peridynamics and Nonlocal Modeling. April 2019.

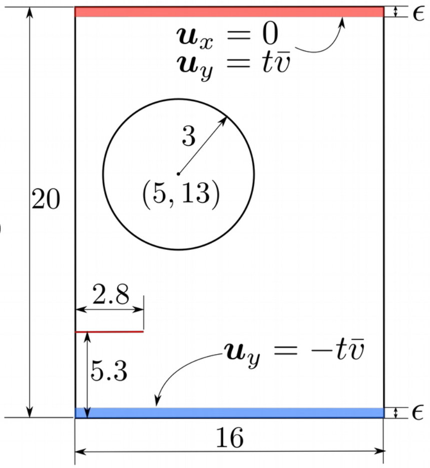
[2] M. R. Ayatollahi & M. R. M. Aliha (2009). Analysis of a new specimen for mixed mode fracture tests on brittle materials. Engineering Fracture Mechanics, 76(11), 1563-1573.

[3] E. Madenci et al (2018). A state-based peridynamic analysis in a finite element framework. Engineering Fracture Mechanics, 195, pp.104-128.

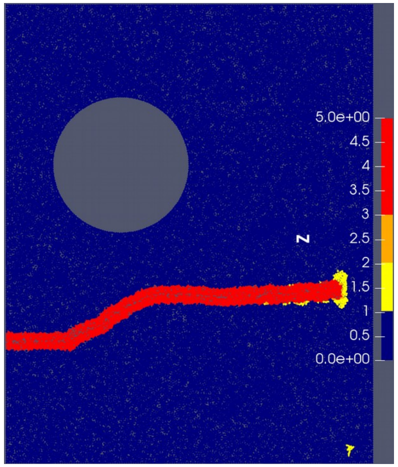
Crack-void interaction

Material properties are same as in the Mode-I problem. We set

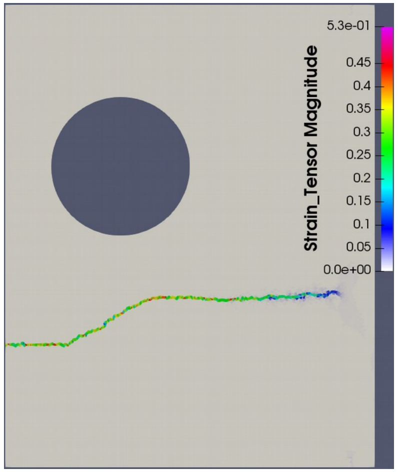
- Horizon $\epsilon = 0.4$ mm
- Mesh size $h = 0.1$ mm
- Final time $T = 800\,\mu$s
- Time step size $\Delta t = 0.004\,\mu$s



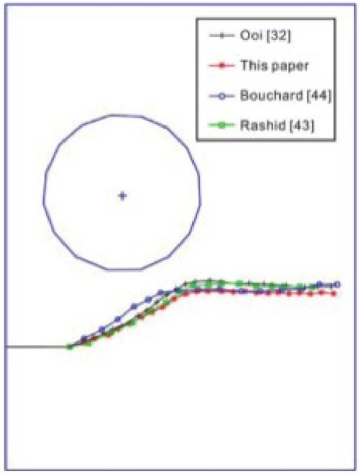
(a) Setup (units in mm)



(b) Damage profile



(c) Magnitude of symmetric gradient of displacement



(d) Numerical experiment results using FEM, Boundary element method [2]

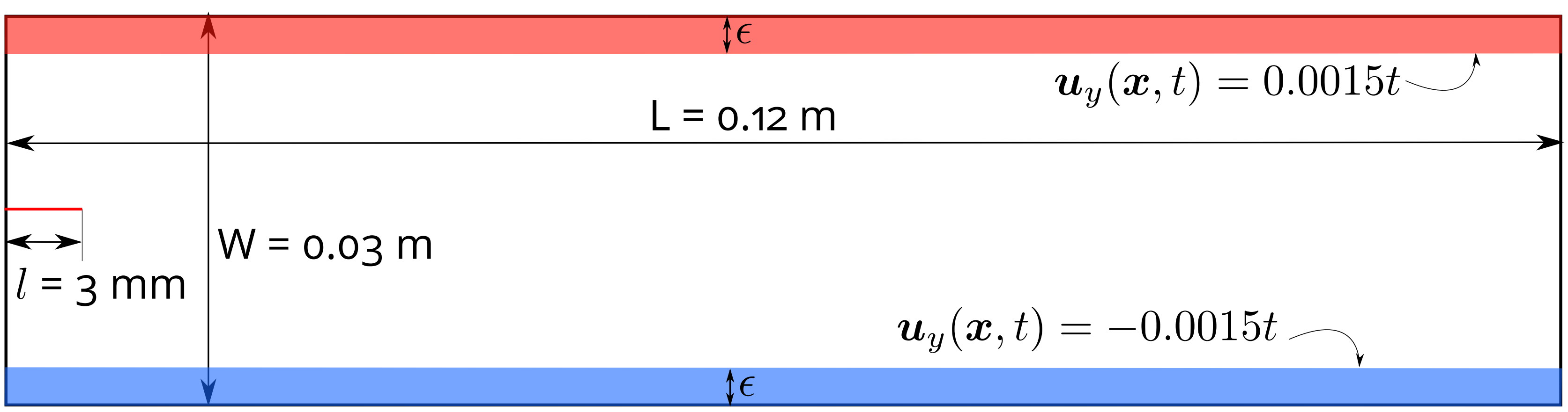
[1] P.K. Jha, P. Diehl & R. Lipton (2019). Nodal finite element approximation of nonlocal fracture models. ***In preparation***.

[2] S. Dai,C. Augarde, C. Du & D. Chen (2015). A fully automatic polygon scaled boundary finite element method for modelling crack propagation. Engineering Fracture Mechanics, 133, 163-178.

Wave reflection effect on crack velocity

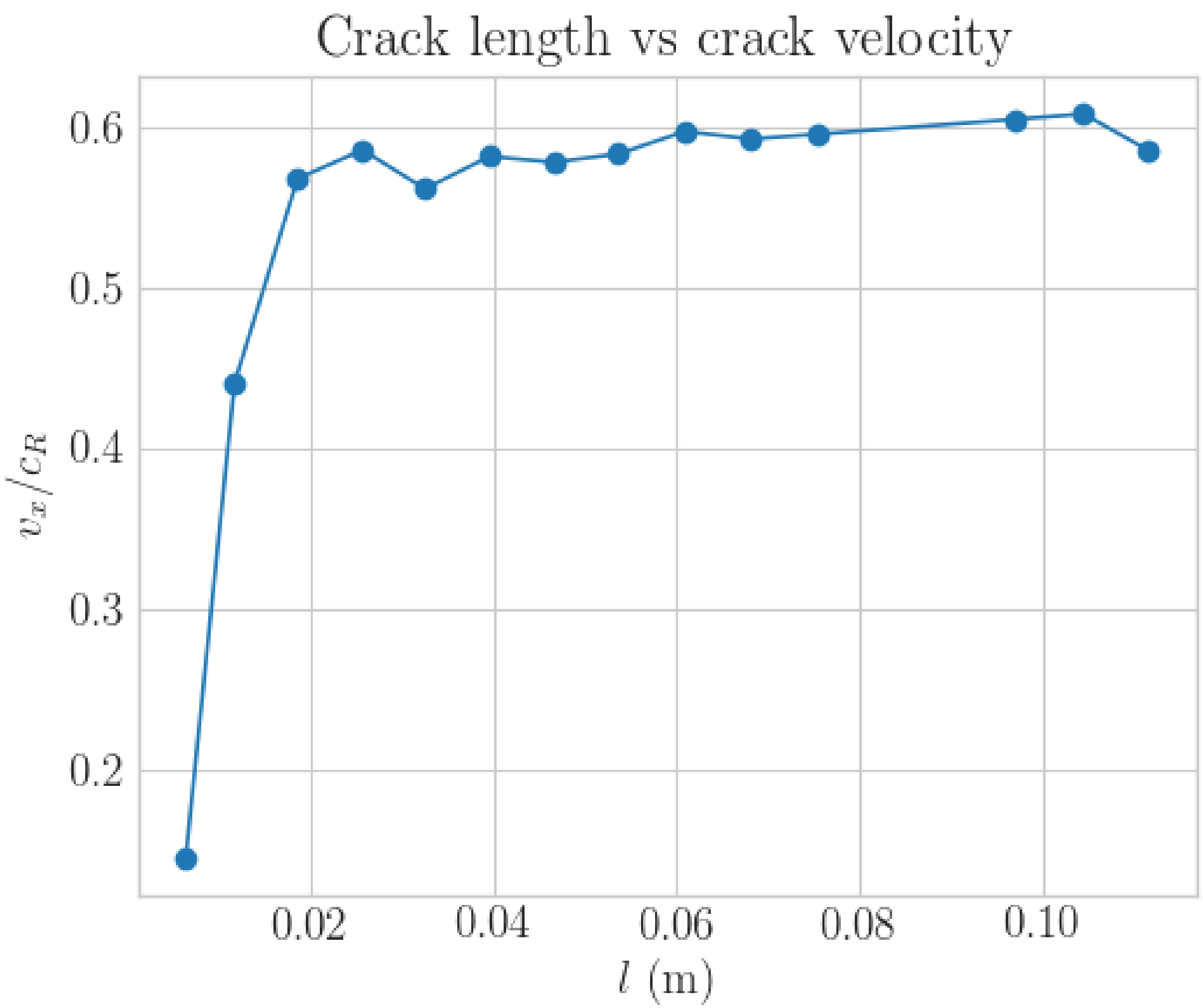
We consider a softer material with shear modulus $G = 35.2$ kPa, density $\rho = 1011$ kg/m^3, and critical energy release rate $G_c = 20$ J/m^{-2}. Poisson ratio is fixed to $\mu = 0.25$. Domain is $D = [0, 0.12\,\text{m}] \times [0, 0.03\,\text{m}]$.

- Horizon $\epsilon = 0.6$ mm, mesh size $h = 0.15$ mm
- Time $T = 1.1$ s, $\Delta t = 2.2\,\mu$s



Wave reflection effect on crack velocity

- Max crack length = 0.12 m
- Rayleigh wave speed $c_R = 5.502$ m/s



Ongoing and future works

- In [1] we show that the classical kinetic relation is embedded in peridynamics and we have $\lim_{\epsilon \to 0} J(t) = G_c$, where $J(t)$ is the nonlocal J-integral. In LEFM, the classical kinetic relation for the crack velocity is postulated. In contrast, we obtain the classical kinetic relation from the Peridynamics in the limit of vanishing nonlocality.

- Open source computational library for nonlocal modeling. This is a joint work with Patrick Diehl (LSU) and Robert Lipton (LSU).

- Study of granular material using nonlinear nonlocal model.

[1] R. Lipton & P.K. Jha (2019). Classic dynamic fracture recovered as the limit of a nonlocal peridynamic model: The single edge notch in tension. Under review in International Journal of Fracture (FRAC).

Thank you!