New chapter 8.

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One Port Network

Positive Real Function (PRF)

expression भनेको n/w design गर्नुपर्दा, ~~जसले~~ PRF हुनुपर्छ।

A function $F(s)$ is said to be positive Real Function (PRF) if and only if the following conditions satisfied. (Necessary and sufficient conditions for PRF)

1. All the poles and zeros must lie on the left half of s-plane.
2. The poles and zeros may lie on the $j\omega$ but the residue must be positive.
   (partial fraction गर्दा माथि आउने constants → Residue)
3. $\text{Real}\{(M_1 M_2 - N_1 N_2 \geq 0)\}$ for all ω

where

M_1 = even part of numerator (माथि)

N_1 = odd " "

M_2 = even part of denominator (तल)

N_2 = odd " "

* Properties of a PRF

1. If a function $F(s)$ is PRF then its $1/F(s)$ is also PRF.
2. The sum of two PRFs is also a PRF.
3. The subtraction of two PRFs is also a PRF.
4. The coefficients of the numerator and denominator of the function should be positive.
5. The poles are simple and may lie on $j\omega$ axis with positive residue.

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(6) The difference of highest powers of numerator and denominator should not exceed than unity (1).

(7) The difference of lowest powers of numerator and denominator should not exceed than unity (1).

Example

$$H(s) = \frac{s^2+9s+3}{s^2+7s+9}$$ check for positive realness (PRF)

Sol:

a) All the coefficients of the numerator and denominator are positive.

b) denominator → $\frac{-7 \pm \sqrt{7^2-4\times1\times9}}{2\times1}$

= no imaginary poles

no residue test necessary.

if imaginary poles are partial fraction & find residue which must have +ve.

c) $M_1M_2 - N_1N_2$

power.

even part of numerator $M_1 = s^2+3$

odd $N_1 = 9s$

even denominator $M_2 = s^2+9$

$N_2 = 7s$

$M_1M_2 - N_1N_2$

$= (s^2+3)(s^2+9) - 9s * 7s$

$= s^4 + 12s^2 + 27 - 63s^2$

$= s^4 - 51s^2 + 27$

Put $s = j\omega$

$= (j\omega)^4 - 51(j\omega)^2 + 27$

Real $= \omega^4 + 51\omega + 27$ For all ω = -ve or +ve

$\omega^4 + 51\omega + 27 > 0$.

CamScanner

NOTE ② (If power of denominator > 2 check ~~Routh's~~ Hurwitz.)

d) power difference is lesser than 1.

Hence,

The given function is PRF.

Q. $H(s) = \dfrac{s^3 + 5s^2 + 9s + 3}{s^3 + 4s^2 + 7s + 9}$ check for PRF.

soln

a) All the coefficients of numerator & denominator are positive.

b) (always denominator.)

The denominator polynomial is Hurwitz or not?

even part $= 4s^2 + 9$

odd part $= s^3 + 7s$

continue division or fraction.

$$4s^2+9 \,\big)\, s^3 + 7s \,\big(\, \frac{s}{4}$$

$$\underline{-\,s^3 \;-\; \frac{9s}{4}}$$

$$\frac{19s}{4} \,\big)\, 4s^2+9 \,\big(\, \frac{4s \times 4}{19}$$

$$\underline{\underline{4s^2}}$$

$$9 \,\big)\, \frac{19s}{4} \,\big(\, \frac{19}{4 \times 9} s$$

$$\underline{\frac{19s}{4}}$$

$$x$$

In the given continue fraction all quotients are positive so the given polynomial is Hurwitz.

{ No need to check imaginary. }

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c) even numerator $M_1 = 5s^2+3$

odd ~~even odd~~ $N_1 = s^3+9s$

even denominator $M_2 = 4s^2+9$

odd $N_2 = s^3 + 7s$

Now, $M_1M_2 - N_1N_2$

$= (5s^2+3)(4s^2+9) - (s^3+9s)(s^3+7s)$

$= 20s^4 + 45s^2 + 12s^2 + 27 - (s^6 + 7s^4 + 9s^4 + 63s^2)$

$= 20s^4 + 57s^2 + 27 - s^6 - 16s^4 - 63s^2$

$= -s^6 + 4s^4 - 6s^2 + 27$

put $s = j\omega$ $(-1)^6$

$= -(j\omega)^6 + 4(j\omega)^4 - 6(j\omega)^2 + 27$ $j^6 = (j^2)^3 = -1$

$= \omega^6 + 4\omega^4 + 6\omega^2 + 27$ $j^4 = (j^2)^2 = 1$

≥ 0 $j^2 = -1$

for all ω.

d) power difference is less than 1.

Hence, given function is positive real function.

1 → the coefficient

2 → the residue, poles lie in left real axis.

3 → $M_1M_2 - N_1N_2 \geq 0$

main point to test.

Q.

$$H(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)} = \frac{s^2+4s+3}{s^2+6s+8}$$

a) All the coefficients are +ve.

b) $s+2=0 \Rightarrow s=-2$

$s+4=0 \Rightarrow s=-4$

All poles lie on -ve real axis.

No residue or Hurwitz test required.

c) $M_1M_2 - N_1N_2$

$= (s^2+3)(s^2+8) - 4s \times 6s$

$= s^4 + 11s^2 + 24 - 24s^2$

$= s^4 - 13s^2 + 24$

put $s = j\omega$

$= (j\omega)^4 - 13(j\omega)^2 + 24$

$= \omega^4 + 13\omega^2 + 24$

> 0

for all ω.

d) power difference < 1.

Hence given function is PRF.

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Q. $Z(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$ check PRF!

$$= \frac{s^3+2s}{s^4+4s^2+3}$$

a) all coefficients are +ve.

b) poles are $s^2+1=0$ | $s^2+3=0$

$s^2 = \pm j\omega$ | $s = \pm j\sqrt{3}$

The poles are on $j\omega$ axis, so residue test is mendatory.

$$\frac{s(s^2+2)}{(s^2+1)(s^2+3)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+3}$$

or, $s^3+2s = (As+B)(s^2+3) + (Cs+D)(s^2+1)$

$= As^3 + 3As + Bs^2 + 3B + Cs^3 + Cs + Ds^2 + D$

$A+C=1 \Rightarrow A = 1-C$

$B+D=0 \Rightarrow B=-D$

$3A+C=2 \Rightarrow 3(1-C)+C=2$

$3B+D=0$; $3-3C+C=2$

$-3D+D=0$; $3-2C=2$

$-2D=0$; $-2C=-1$

$D=0$; $C=1/2$

$B=0$

$A = 1-1/2$

$= 1/2$

Here all residues are +ve.

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c) $M_1 M_2 - N_1 N_2$

$= 2s(s^4 + 4s^2 + 3) - s^3$

$= 2s^5 + 8s^3 + 6s - s^3$

$M_1 = 0$

$N_1 = s^3 + 2s$

$M_2 = s^4 + 4s^2 + 3$

$N_2 = 0$

$\therefore M_1 M_2 - N_1 N_2 = 0 \geq 0$ for all ω.

d) power difference = 1 for highest as well as lowest.

Hence the given function is PRF.

Q. Show that the given function is positive real.

$$Z(s) = \frac{2s^2 + 2s + 1}{s^3 + 2s^2 + s + 2}$$

$$= \frac{2s^2 + 2s + 1}{s^2(s+2) + 1(s+2)}$$

$$= \frac{2s^2 + 2s + 1}{(s^2+1)(s+2)}$$

a) all the coefficients are +ve.

b) $s^2 + 1 = 0 \Rightarrow s = \pm j1$

Residue test is required.

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$$\frac{2s^2+2s+1}{(s^2+1)(s+2)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+1}$$

$\Rightarrow$ $2s^2+2s+1 = As^2+A+Bs^2+2Bs+Cs+2C$

$A+B=2 \Rightarrow A = 2-B = 2-\cancel{5}/6 \Rightarrow 1$

$2B+C=2 \Rightarrow C = 2-2B \Rightarrow 2-2\times\cancel{5}/6 = 0$

$A+2C=1$

$\Rightarrow 2-B+2(2-2B)=1$

$2-B+4-4B=1$

$-5B=-5$

$B=1$

All the residus are +ve.

c) $M_1 = 2s^2+1 \qquad N_1 = 2s$

$M_2 = 2s^2+2 \qquad N_2 = s^3+s$

$$M_1M_2 - N_1N_2 = (2s^2+1)(2s^2+2) - 2s(s^3+s)$$

$$= 4s^4+6s^2+3-2s^4-2s^2$$

$$= 2s^4+4s^2+3$$

put $s=j\omega$

$$= 2(j\omega)^4+4(j\omega)^2+3$$

$$= 2\omega^4-4\omega^2+3$$

≥ 0

for all ω.

d) power difference = 1.

Hence, given function is PRF.

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Synthesis:

Designing network from the given immittance function
(immittance: impedance + admitance.)

Immittance = Impedance + Admittance.

LC, RC, RL.

Design methods (model)

- Foster method (partial fraction)
  - 1st Foster method (impedance)
  - Foster IInd method (admittance)
- Cauer method (continue fraction)
  - Cauer I form.
  - Cauer II form.

* LC network.

Properties of LC immittance function

(i) $Z_{LC}(s)$ or $Y_{LC}(s)$ is the ratio of odd to even or even to odd polynomials.

(ii) The poles and zeros are simple and lie on $j\omega$ axis.

(iii) The poles and zeros interlace on the $j\omega$-axis
↳ alternate. (o x o x)

(iv) The highest powers of numerator and denominator must differ by unit (1); the lowest power also differ by unity (1).

(v) There must be either a zero or pole at the origin and infinity.

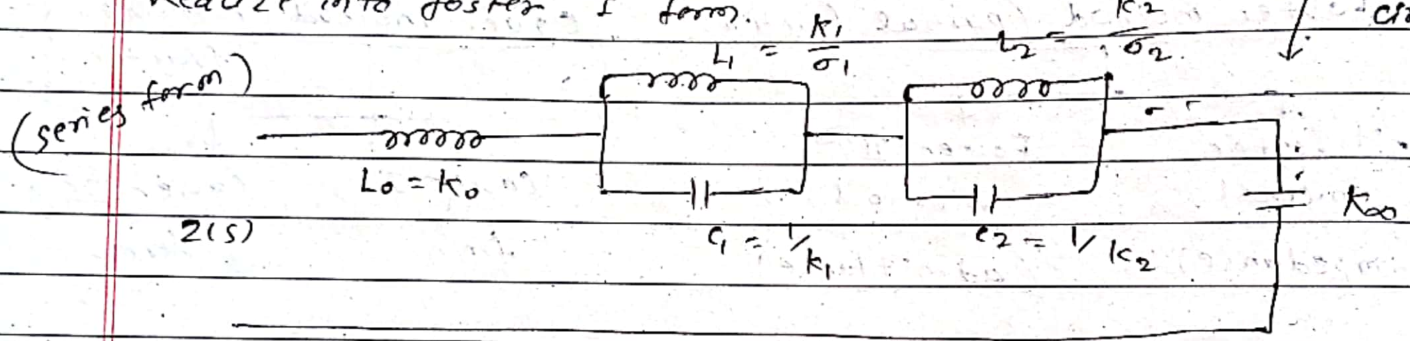
Foster I form for LC network (Impedance वाट find गर्न)

Do partial fraction of the given impedence function.

i.e,
$$Z(s) = K_0 s + \frac{K_1 s}{s^2+\sigma_1} + \frac{K_2 s}{s^2+\sigma_2} + \cdots + \frac{1}{K_\infty s}$$

be the result after partial fraction.

Realize into foster I form. equivalent circuit



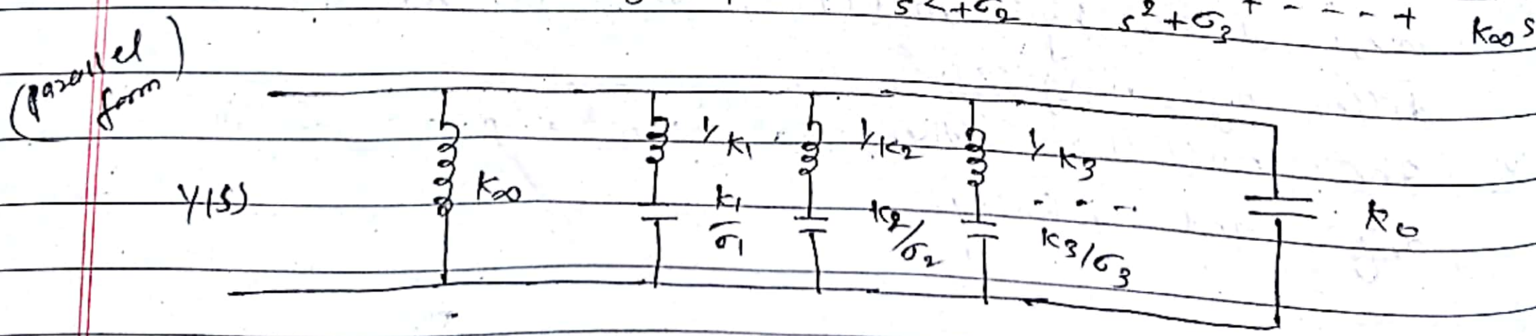
$L \to Ls$ $\quad LC \to \frac{Ks}{s^2+\sigma}$; $L = K/\sigma$, $C = 1/K$

$C \to \frac{1}{Cs}$

Foster II form. (From admittance)

Do partial fraction of admittance function.

i.e,
$$Y(s) = K_0 s + \frac{K_1 s}{s^2+\sigma_1} + \frac{K_2 s}{s^2+\sigma_2} + \frac{K_3 s}{s^2+\sigma_3} + \cdots + \frac{1}{K_\infty s}$$



General foster II form for LC n/w.

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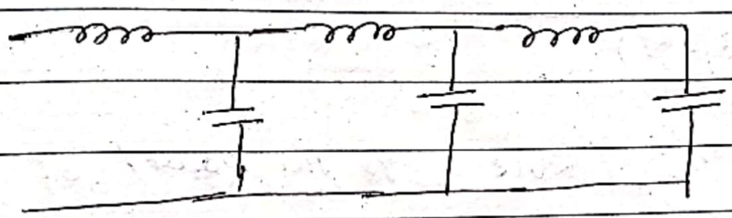
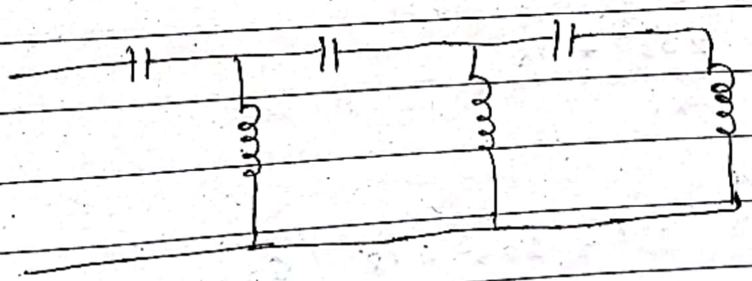
Foster I र II मा L र C को value लिने तरिका opposite हुन्छ।
capacitor को inductor र inductor को capacitor मा लिने।

Cauer I form



Cauer II form



Ladder form

Quotient अनुसार element हरु घट्दै जान्छ।

Q. Obtain the Foster I and II form of the LC impedance function.

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

Sol

For Foster I form.

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

Partial fraction: (Rule - If the power of denominator is greater than numerator, only otherwise divide by actual division)

$$= \frac{2(s^4+10s^2+9)}{s^3+4s}$$

$$= \frac{2s^4+20s^2+18}{s^3+4s}$$

$$s^3+4s \,\big)\, 2s^4+20s^2+18 \,\big(\, 2s$$
$$2s^4+8s^2$$
$$\overline{\quad 12s^2+18 \quad}$$

$$Z(s) = 2s + \frac{12s^2+18}{s^3+4s}$$

$$\frac{12s^2+18}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$$

$$\frac{12s^2+18}{s(s^2+4)} = \frac{A(s^2+4)+s(Bs+C)}{s(s^2+4)}$$

$$= As^2+4A+Bs^2+Cs$$

$$12s^2+18 = (A+B)s^2 + Cs + 4A$$

equating coefficients,

$A+B=12 \Rightarrow B = 12 - 9/2$

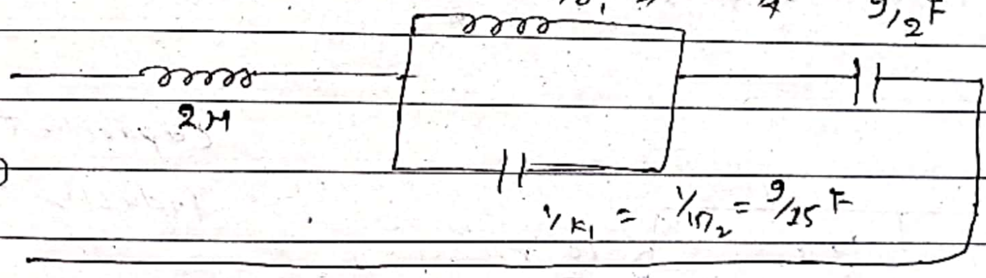
$C=0$ $\qquad = 15/2$

$4A=18$

$A = 9/2$

$$Z(s) = 2s + \frac{9/2}{s} + \frac{15/2\, s}{s^2+4}$$

$k_1/\sigma_1 \Rightarrow \frac{15/2}{4} \Rightarrow 15/8$ $\qquad 9/2\,F$



$1/k_1 = 1/\tfrac{15}{2} = 9/15\,F$

fig. Foster I form.

For Foster II form:

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

$$Y(s) = \frac{1}{Z(s)} = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)} = \frac{0.5\,s(s^2+4)}{(s^2+1)(s^2+9)}$$

partial fraction.

$$\frac{0.5\,s(s^2+4)}{(s^2+1)(s^2+9)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9}$$

$$\cancel{0.5\,s(s^2+4)} = \frac{(As+B)(s^2+9) + (Cs+D)(s^2+1)}{(s^2+1)(s^2+9)}$$

$$0.5s^3 + 2s = As^3 + 9As + Bs^2 + 9B + Cs^3 + Cs + Ds^2 + D$$

Equating,

$A + C = 0.5$ —— (1) $\quad 9A + C = 2$ —— (3)

$B + D = 0$ —— (2) $\quad 9B + D = 0$ —— (4)

solving,

$A = 3/16, \quad C = 5/16$

$B = 0, \quad D = 0$

$$Y(s) = \frac{3/16\ s}{s^2+1} + \frac{5/16\ s}{s^2+9}$$

No inductor & capacitor only. all mixes.

Capacitor मुनी छ ।
inductor माथि छ ।

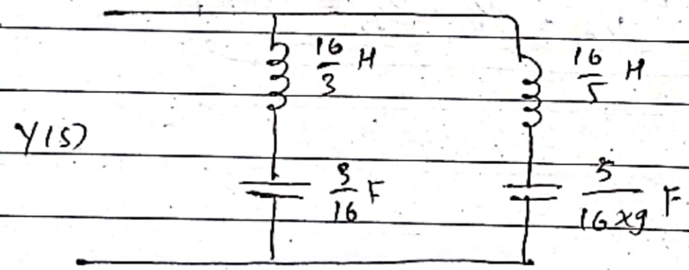


fig. Foster II form.

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Q. $Y(s) = \dfrac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}$

For the given admittance function, realize for foster II nd form of LC network.

$= \dfrac{(s^3+2s)(s^2+4)}{s^4+4s^2+3}$

$= \dfrac{s^5+6s^3+8s}{s^4+4s^2+3}$

power in numerator > power in denominator

Then by actual division

$$s^4+4s^2+3 \overline{)\, s^5+6s^3+8s \,}\ (s$$
$$\underline{-s^5 \mp 4s^3 \mp 3s}$$
$$2s^3+5s$$

$Y(s) = s + \dfrac{2s^3+5s}{s^4+4s^2+3}$

$Y(s) = s + \dfrac{2s^3+5s}{(s^2+1)(s^2+3)}$

Now, partial fraction

$\dfrac{2s^3+5s}{(s^2+1)(s^2+3)} = \dfrac{As+B}{s^2+1} + \dfrac{Cs+D}{s^2+3}$

$2s^3+5s = (As+B)(s^2+3) + (Cs+D)(s^2+1)$

$= As^3 + 3As + Bs^2 + Cs^3 + Cs + Ds^2 + D$

$= (A+C)s^3 + (B+D)s^2 + (3A+C)s + D$

Equating coefficients,

$A + C = 2$

$B + D = 0$

$3A + C = 5$

$D = 0$

$B = 0$

$A = 3/2$

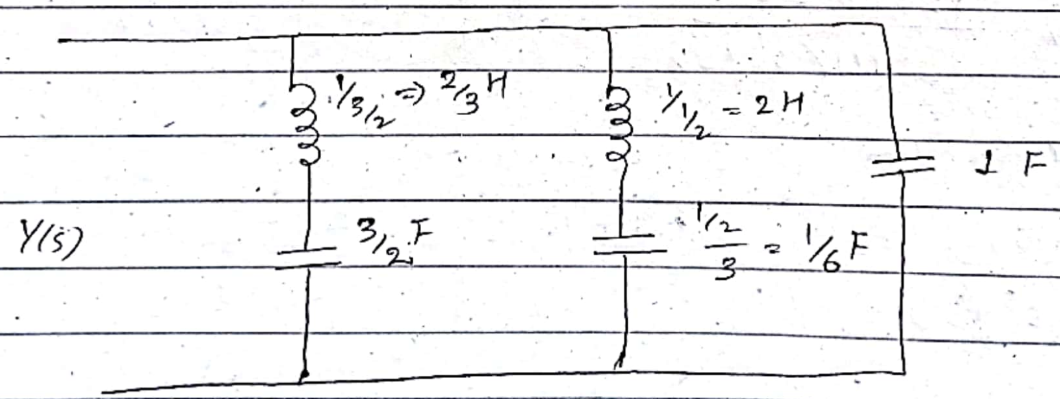
$C = 1/2$.

Then

$$Y(s) = s + \frac{3/2\, s}{s^2+1} + \frac{1/2\, s}{s^2+3}$$

$K_0 = 1$ $\quad K_1 = 3/2$ $\quad K_2 = 1/2$

$\sigma_1 = 1$ $\quad \sigma_2 = 3$



Q. Obtain Cauer I form of the given impedance function.

$$Z(s) = \frac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3}$$

Sol:

For cauer form, denominator highest power should be lesser. (always)

Rule : For cauer I form, arrange the given polynomials in the descending order.

(continue fraction method से solve करें)

$s^4 + 4s^2 + 3 \,)\, 2s^5 + 12s^3 + 16s \,(\, 2s \Leftarrow Ls \Rightarrow L = 2$

$\quad -(2s^5 + 8s^3 + 6s)$

$4s^3 + 10s \,)\, s^4 + 4s^2 + 3 \,(\, \frac{1s}{4}$

$\quad -(s^4 + \frac{10}{4}s^2)$

$\frac{3}{2}s^2 + 3 \,)\, 4s^3 + 10s \,(\, \frac{2}{3}s \times \cancel{4}$

$\quad -(4s^3 + 8s)$

$2s \,)\, \frac{3}{2}s^2 + 3 \,(\, \frac{3}{2}s \times \frac{1}{2}$

$\quad -\frac{3}{2}s^2$

$3 \,)\, 2s \,(\, \frac{2s}{3}$

$\quad -2s$

$\quad \times$

5- Quotient so must contain 5 elements in cauer I form.

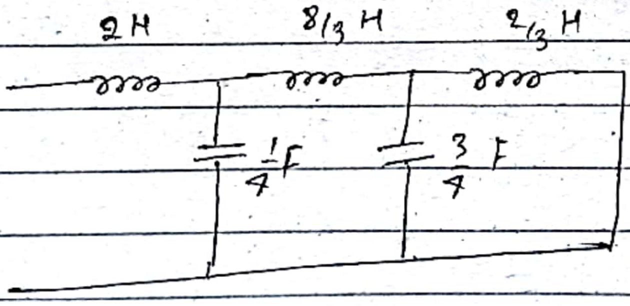


Fig. cauer I form.

Cauer II. Arrange the polynomials in the ascending order.

eg. $Z(s) = \dfrac{(s^2+1)(s^2+3)}{s(s^2+2)}$

sol $Z(s) = \dfrac{s^4+4s^2+3}{s^3+2s}$

$= \dfrac{3+4s^2+s^4}{2s+s^3}$

now continue fraction.

$2s+s^3 \overline{)\, 3+4s^2+s^4 \,} \left(\dfrac{3}{2s} \right.$ = capaciter. $\dfrac{1}{Cs} = \dfrac{1}{\frac{2}{3}s}$

$C = 2/3$

$3 + \dfrac{3s^2}{2}$

$\dfrac{5}{2}s^2+s^4 \overline{)\, 2s+s^3 \,} \left(\dfrac{2\times 2}{5\times 5} \right. = \dfrac{1}{\frac{5}{4}s}$

$2s + \dfrac{4s^3}{5}$

$\dfrac{s^3}{5} \overline{)\, \dfrac{5}{2}s^2+s^4 \,} \left(\dfrac{5\times 5}{2s} \right. = \dfrac{1}{\frac{2}{25}s}$

$\dfrac{5}{2}s^2$

$s^4 \overline{)\, \dfrac{s^3}{5} \,} \left(\dfrac{1}{5\times 5} \right. = \dfrac{1}{\frac{5}{1}s}$

$\dfrac{s^3}{5}$

×

eg.

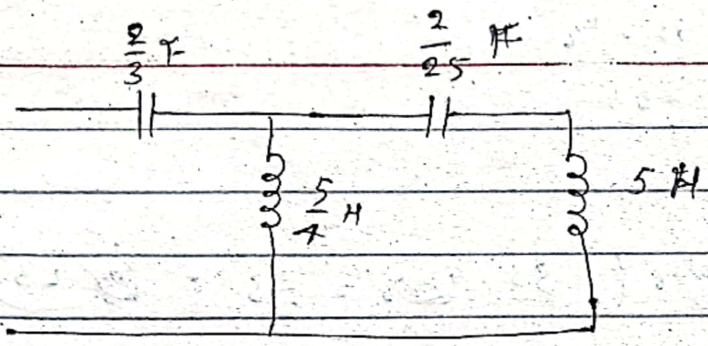


fig. Cauer II form.

Note: Cauer I → Descending order
Cauer II → Ascending order } Arrange.

(i) $Z(s) = \dfrac{6s^3 + 2s}{9s^4 + 4s^2 + 1/16}$ 2078/10/13

Find the first cauer network.

soln

$$9s^4 + 4s^2 + 1/16 \;\big)\; 6s^3 + 2s \;\big(\; \frac{6}{9}s$$
$$6s^3 + \frac{24s}{9} + \ldots$$

Since the order of denominator is greater than the numerator polynomial, we have to invert the given function and proceed with the continue fraction.

Invert $\Rightarrow \dfrac{1}{Z(s)} = \dfrac{9s^4 + 4s^2 + 1/6}{6s^3 + 2s}$

continue fraction,

$$6s^3 + 2s \;\big)\; 9s^4 + 4s^2 + 1/6 \;\big(\; \frac{9s}{6} = \frac{3}{2}s = \frac{1}{LS}$$

$$\underline{-\,9s^4 \;-\; \frac{18s^2}{6}}$$

$$s^2 + 1/6 \;\big)\; 6s^3 + 2s \;\big(\; 6s$$

$$\underline{-\,6s^3 \;-\; s}$$

$$s \;\big)\; s^2 + 1/6 \;\big(\; s$$

$$\underline{-\,s^2}$$

$$1/6 \;\big)\; s \;\big(\; 6s$$

$$\underline{-\,s}$$

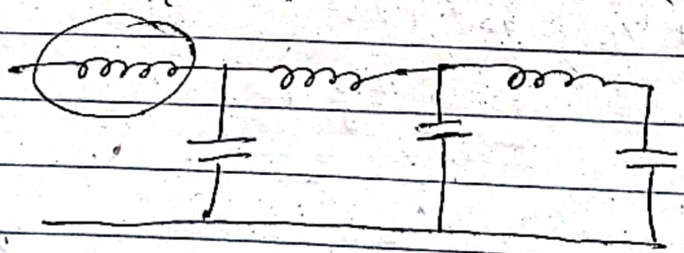
$$\times$$

Note:
invert गरेर आएको fraction मा
पहिलो element हटाएर
circuit बनाउने ।

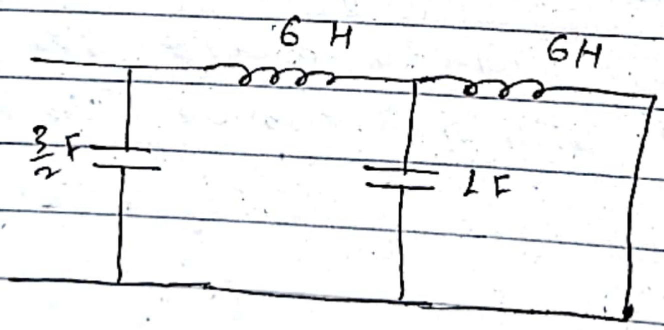
<u>Cauer I</u>.

हटाउने

Normal ckt:



$\Rightarrow$



Cauer II

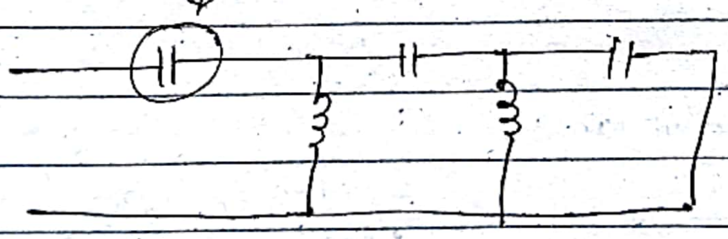
obtain the Cauer II for the given function.

$$Z(s) = \frac{8s^3 + 3s}{2s^4 + 10s^2 + 3}$$

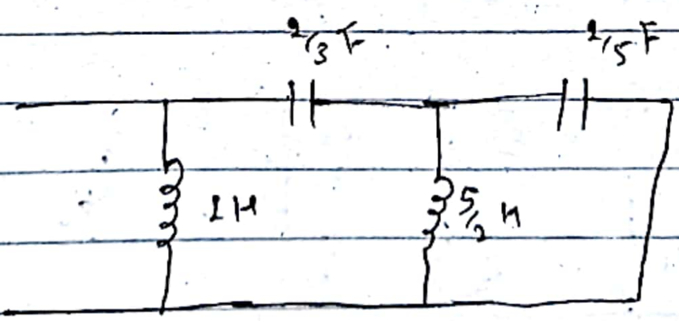
$\Rightarrow$ Inversing, $\frac{1}{Z(s)} = \frac{2s^4 + 10s^2 + 3}{8s^3 + 3s} = \frac{3 + 10s^2 + 2s^4}{3s + 8s^3}$

$$\begin{array}{l}
3s + 8s^3 \,\big)\, 3 + 10s^2 + 2s^4 \,\big(\, \frac{1}{s} = \frac{1}{Cs} \\
\qquad\qquad -3 + 8s^2 \\
\qquad\qquad\qquad 2s^2 + 2s^4 \,\big)\, 3s + 8s^3 \,\big(\, \frac{3}{2s} \\
\qquad\qquad\qquad\qquad 3s + 3s^3 \\
\qquad\qquad\qquad\qquad\qquad 5s^3 \,\big)\, 2s^2 + 2s^4 \,\big(\, \frac{2}{5s} \\
\qquad\qquad\qquad\qquad\qquad\qquad -2s^2 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2s^4 \,\big)\, 5s^3 \,\big(\, \frac{5}{2s} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -5s^3 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \times
\end{array}$$

Remove



so on $\Rightarrow$



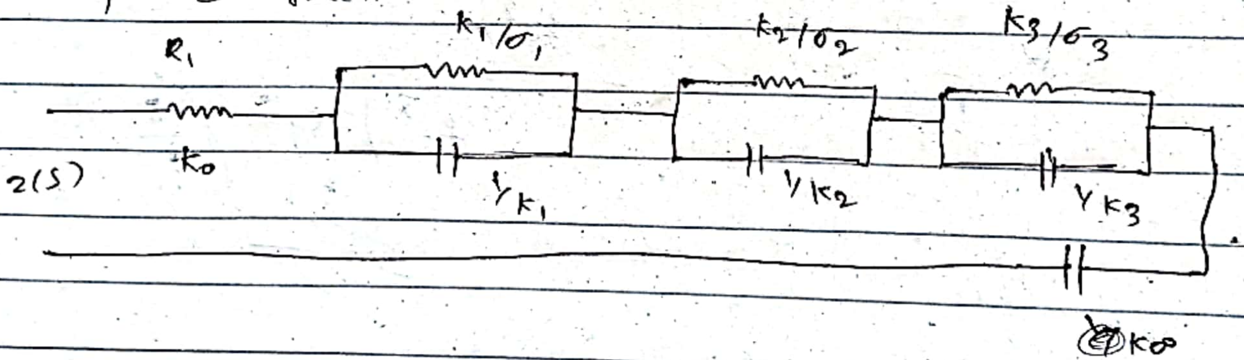
RC impedance or RL Admittance network.

properties:

1. poles and zeros lie on the negative real axis and they are in alternative form.
2. The singularity nearest to (or at) the origin must be a pole whereas the singularity nearest to (or at) the infinity is a zero.
3. The residue of the poles must be real and positive.

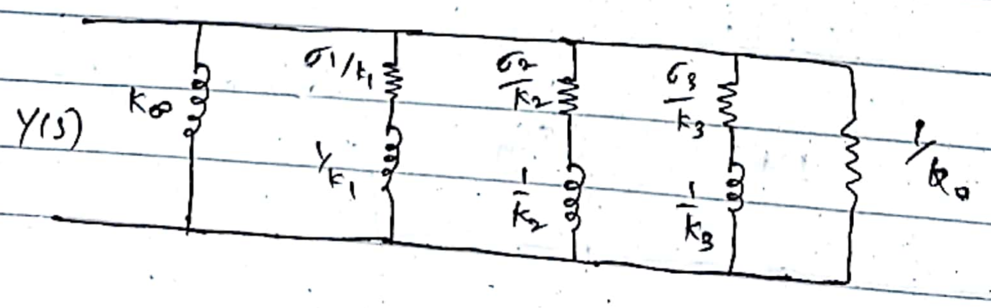
* Foster I^{st} form for RC impedance

let $Z(S) = K_0 + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \frac{K_3}{s+\sigma_3} + \cdots + \frac{1}{K_\infty s}$ is the partial fraction.



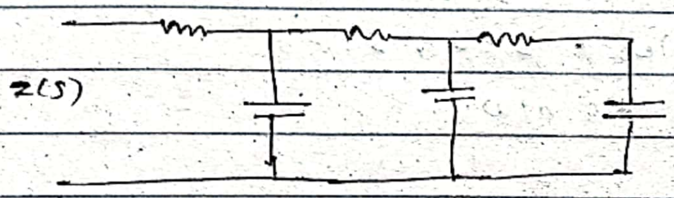
* Foster II^{nd} form (From admittance)

let $Y(S) = K_0 + \frac{K_1}{S+\sigma_1} + \frac{K_2}{S+\sigma_2} + \cdots + \frac{1}{K_\infty S}$ is the partial fraction.

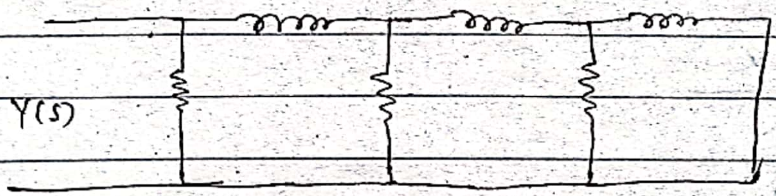


* Cauer Ist form :

If the function is an impedance function (continue fraction method)

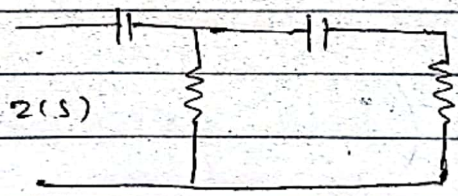


But if the function is an admittance function

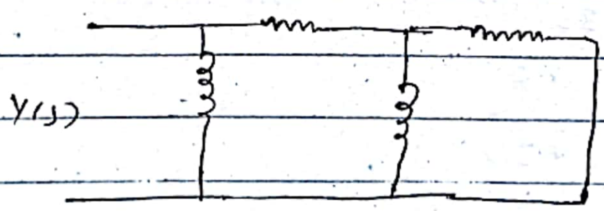


* Cauer IInd form

If impedance



If admittance,



Q. obtain the Foster I and Foster II form of the given function.

$$F(s) = \frac{3(s+2)(s+4)}{s(s+3)}$$

poles → $s=0$, $s=-3$.

RC n/w.

(i) For foster I.

assume $F(s)$ as impedance

(Rule: denominator power should higher than numerator)

$$F(s) = \frac{3(s+2)(s+4)}{s(s+3)}$$

$$= \frac{3(s^2+6s+8)}{s^2+3s}$$

$$= \frac{3s^2+18s+24}{s^2+3s}$$

$$\begin{array}{r|l} s^2+3s \;) & 3s^2+18s+24 \; (\; 3 \\ & \underline{-3s^2+9s} \\ & \quad 9s+24 \end{array}$$

$$F(s) = 3 + \frac{9s+24}{s^2+3s}$$

$$= 3 + \frac{9s+24}{s(s+3)}$$

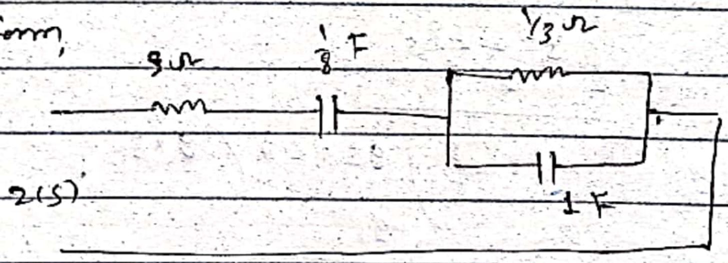
$$\frac{9s+24}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$$A = \left.\frac{9s+24}{s+3}\right|_{s=0} = 8$$

$$B = \left.\frac{9s+24}{s}\right|_{s+3=0,\ s=-3} = \frac{9\times -3+24}{-3} = 1$$

$$F(s) = 3 + \frac{8}{s} + \frac{1}{s+3} = k_0 + \frac{1}{k_\infty s} + \frac{k_1}{s+\sigma_1}$$

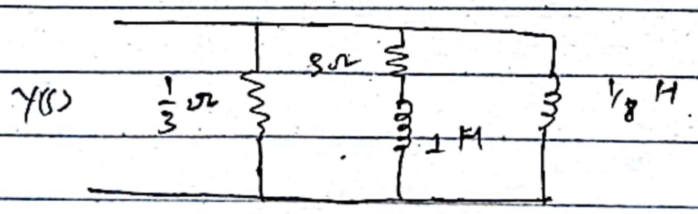
Foster I form,



(ii) For foster II

assume F(s) as admittance.

$$F(s) = 3 + \frac{8}{s} + \frac{1}{s+3}$$



* Cauer I.

$$z(s) = \frac{3(s+2)(s+4)}{s(s+3)} = \frac{3s^2 + 18s + 24}{s^2 + 3s} \quad \left(\text{decreasing order}\right)$$

$$s^2 + 3s \;\big)\; 3s^2 + 18s + 24 \;\big(\; 3$$

$$\underline{3s^2 + 9s}$$

$$9s + 24 \;\big)\; s^2 + 3s \;\big(\; \frac{s}{9}$$

$$\underline{s^2 + \frac{24}{9}s}$$

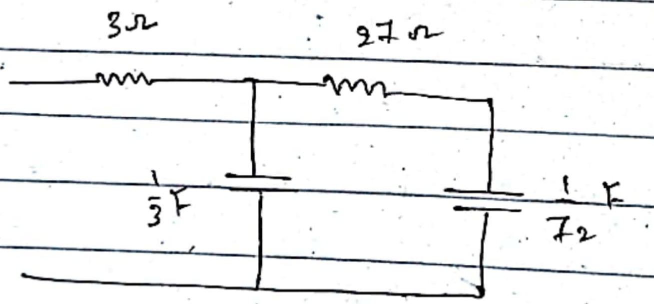
$$\frac{s}{3} \;\big)\; 9s + 24 \;\big(\; 9 \times 3$$

$$\underline{9s}$$

$$24 \;\big)\; \frac{s}{3} \;\big(\; \frac{s}{3 \times 24}$$

$$\underline{\frac{s}{3}}$$

$$\times$$



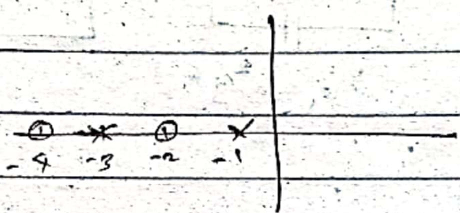
$\cancel{Z(s) = \frac{6s^3+2s}{9s^4+9s^2+1/6}}$ (2013 spring)

Realize the following RC driving point impedance function in (i) Foster I form and in

(ii) Cauer I form

$$Z(s) = \frac{s^2+6s+8}{s^2+4s+3}$$

$$= \frac{(s+2)(s+4)}{(s+1)(s+3)}$$



soln

For foster I form (partial fraction)

since the highest power of numinator and denominator are equal.

$$s^2+4s+3 \overline{) s^2+6s+8 } (1$$

$$\underline{s^2+4s+3}$$

$$2s+5$$

$$Z(s) = 1 + \frac{2s+5}{s^2+4s+3}$$

$$= 1 + \frac{2s+5}{(s+1)(s+3)}$$

$$\frac{2s+5}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$

$$A = \left.\frac{2s+5}{s+3}\right|_{s+1=0}$$

$s=-1$

$$= \frac{-2+5}{-1+3}$$

$$= 3/2$$

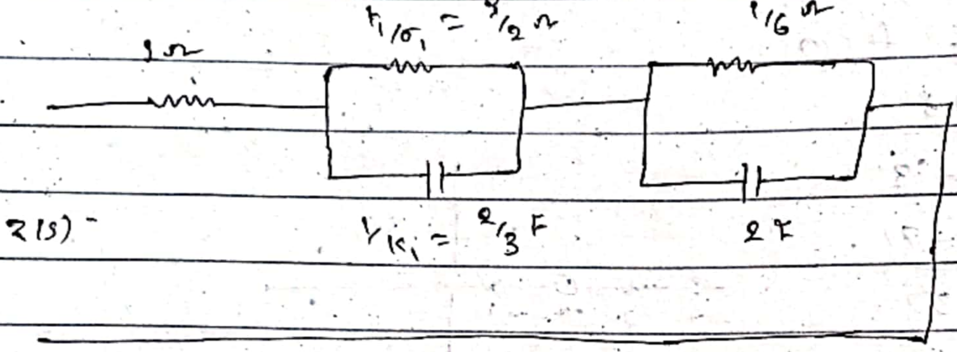
$$B = \left.\frac{2s+5}{s+1}\right|_{s+3=0}$$

$s=-3$

$$= \frac{-6+5}{-3+1}$$

$$= 1/2$$

$$y(s) = 1 + \frac{3/2}{s+1} + \frac{1/2}{s+3}$$

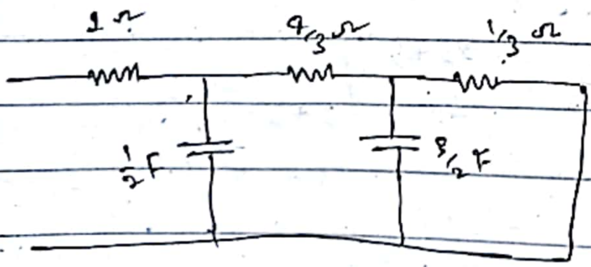


Foster I form.

For cauer I form (continue fraction):

$$y(s) = \frac{s^2+6s+8}{s^2+4s+3} \quad \text{(Descending order)}$$

$$s^2+4s+3 \overline{)\, s^2+6s+8 \,(}\ 1$$
$$\underline{s^2+4s+3}$$
$$2s+5 \overline{)\, s^2+4s+3 \,(}\ \frac{s}{2}$$
$$\underline{s^2+\frac{5s}{2}}$$
$$\frac{3}{2}s+3 \overline{)\, 2s+5 \,(}\ \frac{4}{3}$$
$$\underline{2s+4}$$
$$1 \overline{)\, \frac{3}{2}s+3 \,(}\ \frac{3}{2}s$$
$$\underline{\frac{3}{2}s}$$
$$3 \overline{)\, 1 \,(}\ 1/3$$
$$\underline{1}$$
$$\times$$



cauer I form.

RL Impedance / RC Admittance ckt.

- Opposite of RC impedance. (near the origin zero instead of poles)

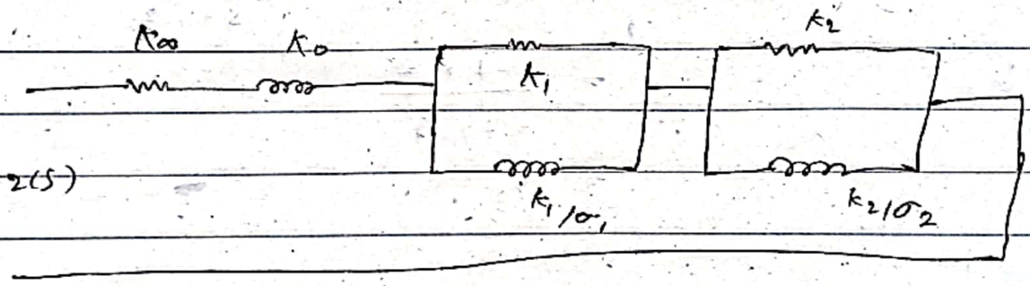
* Foster I of RL (Partial fraction)

Let $Z(s) = K_0 + \frac{K_\infty}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \cdots$

$$\frac{Z(s)}{s} = K_0 + \frac{K_\infty}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \cdots$$

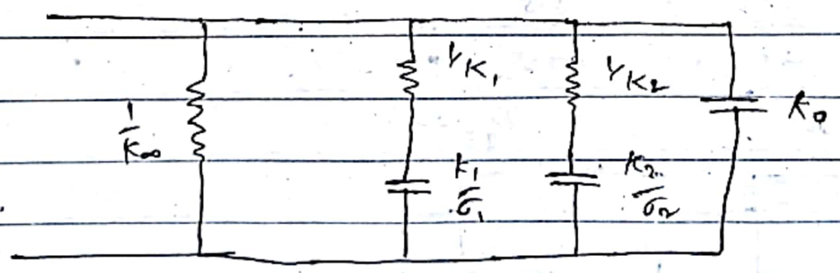
Partial factor

$$Z(s) = K_0 s + K_\infty + \frac{K_1 s}{s+\sigma_1} + \frac{K_2 s}{s+\sigma_2} + \frac{K_3 s}{s+\sigma_3} + \cdots$$



Foster II. $Y(s) = K_0 s + K_\infty + \frac{K_1 s}{s+\sigma_1} + \frac{K_2 s}{s+\sigma_2} + \cdots$

(admittance):



2015 spring.

Synthesis the foster I and cauer II forms of RC driving function.

$$Z_{RC} = \frac{(s+1)(s+4)}{s(s+2)(s+5)}$$

Sol^n

Foster I form:

$$Z_{RC} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$A = \left.\frac{(s+1)(s+4)}{(s+2)(s+5)}\right|_{s=0} \qquad B = \left.\frac{(s+1)(s+4)}{s\cancel{(s+2)}(s+5)}\right|_{\substack{s+2=0 \\ s=-2}}$$

$$= \frac{4}{10} \qquad = \frac{-1 \times \cancel{2}}{-\cancel{2} \times 3}$$

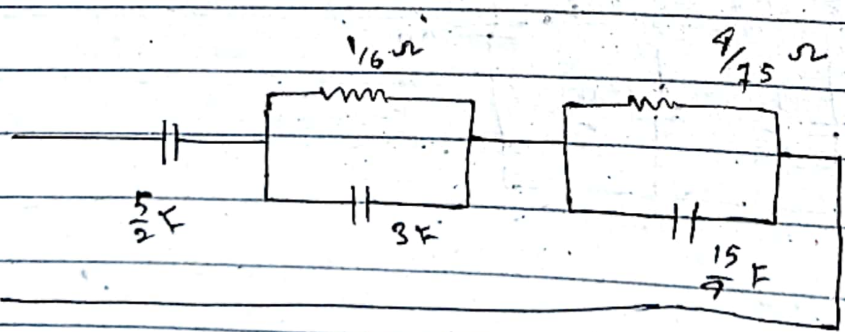
$$= \frac{2}{5} \qquad = 1/3$$

$$C = \left.\frac{(s+1)(s+4)}{s(s+2)}\right|_{\substack{s+5=0 \\ s=-5}}$$

$$= \frac{-4 \times -1}{-5 \times -3}$$

$$= 4/15$$

$$Z_{RC} = \frac{2/5}{s} + \frac{1/3}{s+2} + \frac{4/15}{s+5}$$



foster I form.

Cauer II form:

$$= \frac{(s+1)(s+4)}{s(s+2)(s+5)} = \frac{s^2+5s+4}{s(s^2+7s+10)} = \frac{s^2+5s+4}{s^3+7s^2+10s}$$

Arrange in ascending order,

$$= \frac{4+5s+s^2}{10s+7s^2+s^3}$$

$$10s+7s^2+s^3 \,\big)\, 4+5s+s^2 \,\big(\, \frac{4}{10s}$$

$$\cancel{4} + \frac{28}{10}s + \frac{4s^2}{10}$$

$$- \quad - \quad -$$

$$\frac{11}{5}\cancel{\frac{22}{10}}s + \frac{3}{5}\cancel{\frac{6}{10}}s^2 \,\big)\, 10s + 7s^2 + 5s^3 \,\big(\, \frac{2}{11}$$

$$10s + \frac{6s^2}{55}$$

$$- \quad -$$

$$\frac{379}{55}s^2 + s^3 \,\big)\, \frac{11}{5}s + \frac{3}{5}s^3 \,\big($$

RL Impedance $Z(s)$

properties:

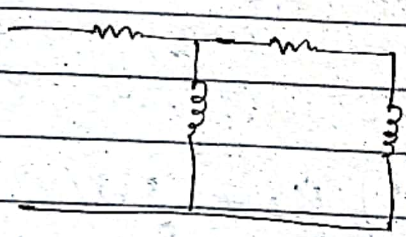
1. poles and zeros are located on the negative real axis.
2. The singularity nearest to the origin is a zero and the singularity nearest to (or at) the infinity is a pole.
3. The residue of the poles must be real and +ve.

Rule for foster I and II

$\frac{Z(s)}{s}$ · start with dividing $Z(s)$ or $Y(s)$ by s.

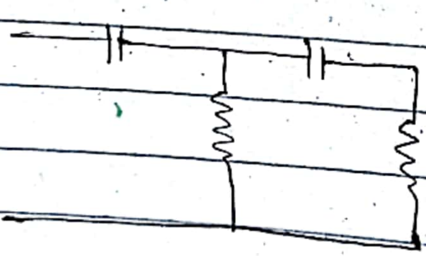
~~or multiply right side by s~~

Cauer I, for RL



RL impedance can be synthesized for cauer I

Cauer II for RL



RL admittance can be synthesized for cauer II

Q. Obtain the foster forms for the given impedance function.

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}$$

Foster I:

$$\frac{Z(s)}{s} = \frac{2(s+1)(s+3)}{s(s+2)(s+6)}$$

$$\frac{Z(s)}{s} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+6}$$

$$A = \left.\frac{2(s+1)(s+3)}{(s+2)(s+6)}\right|_{s=0}$$

$$= \frac{\cancel{2} \times 1 \times \cancel{3}}{2 \times \cancel{6}_2}$$

$$= \frac{1}{2}$$

$$B = \left.\frac{2(s+1)(s+3)}{s(s+6)}\right|_{s=-2}$$

$$= \frac{\cancel{2} \times -1 \times 1}{-\cancel{2} \times \cancel{4}}$$

$$= \frac{1}{4}$$

$$C = \left.\frac{2(s+1)(s+3)}{s(s+2)}\right|_{s=-6}$$

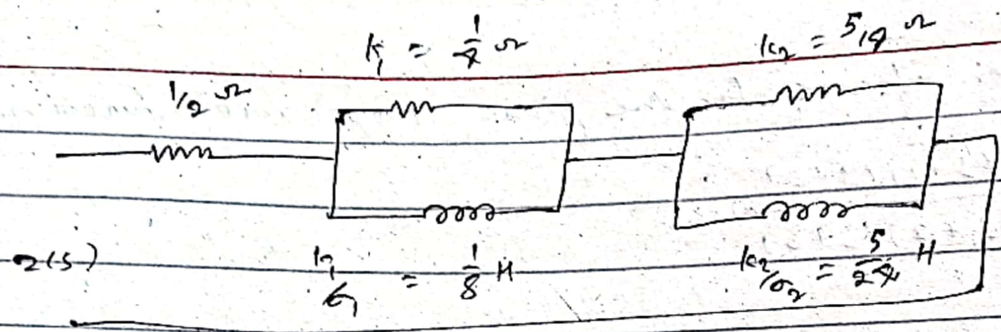
$$= \frac{\cancel{2} \times -5 \times -\cancel{3}}{-\cancel{6}_2 \times -\cancel{4}_2}$$

$$= \frac{5}{\cancel{4}}$$

$$\frac{Z(s)}{s} = \frac{1/2}{s} + \frac{1/4}{s+2} + \frac{5/8}{s+6}$$

$$Z(s) = \frac{1}{2} + \frac{\frac{1}{4}s}{s+2} + \frac{\frac{5}{4}s}{s+6}$$

$$= K_0 + \frac{K_1 s}{s+\sigma_1} + \frac{K_2 s}{s+\sigma_2}$$



Foster II :

$$Y(s) = \frac{(s+2)(s+6)}{2(s+1)(s+3)}$$

$$\frac{Y(s)}{s} = \frac{0.5(s+2)(s+6)}{s(s+1)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$A = \left.\frac{0.5(s+2)(s+6)}{(s+1)(s+3)}\right|_{s=0}$$

$$= \frac{0.5 \times 2 \times 6}{1 \times 3}$$

$$= 2.$$

$$B = \left.\frac{0.5(s+2)(s+6)}{s(s+3)}\right|_{s=-1}$$

$$= \frac{0.5 \times 1 \times 5}{-1 \times 2}$$

$$= -\frac{5}{4}$$

$$C = \left.\frac{0.5(s+2)(s+6)}{s(s+1)}\right|_{s=-3}$$

$$= \frac{0.5 \times -1 \times 3}{-3 \times -2}$$

$$= -\frac{1}{4}$$

Here Residue of poles are negative. So we cannot synthesized. foster II