

Thesis for the Degree of Master

A Study on Comparison of  
Bayesian Network Structure Learning  
Algorithm for Selecting Appropriate Model

by

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## 국 문 초 록

본 논문에서는 R의 bnlearn 패키지에서 제공하는 베이지안 네트워크 구조 학습 알고리즘 간의 성능을 비교하였다.

베이지안 네트워크 구조 학습 결과에 대한 성능 평가는 score를 이용하는 방법과, 목표 네트워크와 학습된 네트워크를 서로 비교하는 방법이 있다. 본 논문에서는 이 두 가지 방법으로 알고리즘별 성능을 비교했을 때, 결과가 서로 다를 수 있음을 확인하였다.

Topology에 따른 Synthetic Data를 생성, 이에 대하여 알고리즘별 성능을 비교하여, 목표 네트워크의 형태에 따라 적합한 알고리즘 선택을 할 수 있도록 객관적인 방향을 제시하고자 하였다.

그동안 베이지안 네트워크 데이터 생성기가 매우 고가이거나, 공개된 툴도 매우 사용하기 까다로웠기 때문에, 베이지안 네트워크 관련 실증 연구는 사례 데이터를 이용한 경우가 대부분이었다. 이에 따라 Bayesian Network 모델을 바탕으로 R에서 데이터를 생성할 수 있는 생성기를 제작하여 공개하였다.

## Abstract

In this paper, we compare the performance between the Bayesian network structure learning algorithm provided by **bnlearn** package in **R**.

The performance of the study results is evaluated by using a score method comparing between the target network and the learning network. In this paper, it was confirmed that algorithm specific performance test results using fore-mentioned methods are different.

Unlike most previous studies which generally used real data, synthetic data generated based on topology was used to compare performance of contrast-specific algorithm. The aim of this paper is to provide objective guidance of selecting suitable algorithm in accordance to target network.

Previous tools suffer from serious trade-off between cost and complexity, restricting most studies relevant to Bayesian network to using only real data. To address such problem, a data generator based on Bayesian network model using **R** is built and introduced.

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# Notations

$r_i$  : Number of states of the finite random variable  $X_i$ ,

$x_{ik}$  :  $k$ -th value of  $X_i$ .

$q_i = \prod_{X_j \in \prod_{X_i}} r_j$  : Number of possible configurations of the parent set  $\prod_{X_i}$  of  $X_i$ .

$\omega_{ij}$  :  $j$ -th configuration of  $\prod_{X_i}$  ( $1 \leq j \leq q_i$ ).

$N_{ijk}$  : Number of instances in the data  $T$  where the variable  $X_i$  takes its  $k$ -th value  $x_{ik}$  and the variables in  $\prod_{X_i}$  take their  $j$ -th configuration  $\omega_{ij}$ .

$N_{ij} = \sum_{k=1}^{r_i} N_{ijk}$  : Number of instances in the data  $T$  where the variables in  $\prod_{X_i}$  take their  $j$ -th configuration  $\omega_{ij}$ .

$N_{ik} = \sum_{j=1}^{q_i} N_{ijk}$  : Number of instances in the data  $T$  where the variable  $X_i$  takes its  $k$ -th value  $x_{ik}$ .

$\mathbf{N}$  : Total number of instances in the data  $T$ .

$\Theta_G = \{\Theta_i\}_{i=1,\dots,n}$  : Encodes parameters of a BN  $B$  with underlying DAG  $G$

$\Theta_i = \{\Theta_{ij}\}_{j=1,\dots,q_i}$  : Encodes parameters concerning only the variable  $X_i$  of  $X$  in  $B$

$\Theta_{ij} = \{\Theta_{ijk}\}_{k=1,\dots,r_i}$  : Encodes parameters for variable  $X_i$  of  $X$  in  $B$  given that its parents take their  $j$ -th configuration

# Chapter 1

## Introduction

### 1.1 Bayesian Network

Bayesian networks (BN) are graphical models where nodes represent random variables and arrows represent probabilistic dependencies between them (Kevin B. K. and Ann E. N., 2010).

A  $n$ -dimensional Bayesian network is a triple  $B = (X, G, \Theta)$  where:

- $X$  is a  $n$ -dimensional finite random vector each random variable  $X_i$  ranges over by a finite domain  $D_i$ . Henceforward, we denote the joint domain by  $D = \prod_{i=1}^n D_i$
- $G = (N, E)$  is a directed acyclic graph (DAG) with nodes  $N = \{X_1, \dots, X_n\}$  and edges  $E$  representing direct dependencies between the variables.
- $\Theta$  encodes the parameters  $\{\theta_{ijk}\}_{i \in 1, \dots, n, j \in D_{\Pi_{X_i}}, k \in D_i}$  of the network, where

$$\theta_{ijk} = P_B(X_i = x_{ik} \mid \prod_{X_i} = \omega_{ij}),$$

$\Pi_{X_i}$  denotes the set of parents of  $X_i$  in  $G$ ,  $D_{\Pi_{X_i}}$  denotes the joint domain of the variables in  $\Pi_{X_i}$ ,  $x_{ik}$  is the  $k$ -th value of  $X_i$  and  $\omega_{ij}$  is the  $j$ -th configuration of  $\Pi_{X_i}$ .

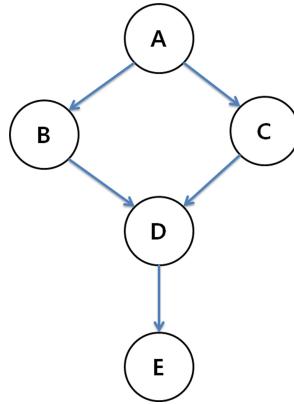


Figure 1.1:  $P(A, B, C, D, E) = P(A)P(B|A)P(C|A)P(D|B, C)P(E|D)$

A BN defines a unique joint probability distribution over  $X$  given by

$$P_B(X_1, \dots, X_n) = \prod_{i=1}^n P_B(X_i | \prod_{X_i}).$$

- A BN encodes the independence assumptions over the component random variables of  $X$ .
- An edge  $(j, i)$  in  $E$  represents a direct dependency of  $X_i$  from  $X_j$ .
- The set of all Bayesian networks with  $n$  variables is denoted by  $B_n$ .

## 1.2 Bayesian Network Structure Learning

The problem of learning a BN given data  $T$  consists of finding the BN that best fits the data  $T$ . In order to quantify the fitting of a BN a scoring function

$\phi$  is considered.

Learning a Bayesian network is as follows:

Given a data  $T = \{y_1, \dots, y_n\}$  and a scoring function  $\phi$ , the problem of learning a Bayesian network is to find a Bayesian network  $B \in B_n$  that maximizes the value  $\phi(B, T)$ .

Bayesian network structure learning algorithms can be grouped into two categories by Marco S. (2010).

**Constraint-based algorithms** These algorithms learn the network structure by analyzing the probabilistic relations entailed by the Markov property of Bayesian networks with conditional independence tests and then constructing a graph which satisfies the corresponding d-separation statements. The resulting models are often interpreted as causal models even when learned from observational data (Pearl J. 1988).

**Score-based algorithms** The main idea behind score-based learning is to optimize the degree of match between the generated network and the observations. (Benjamin B. P., 2003) These algorithms assign a score to each candidate Bayesian network and try to maximize it with some heuristic search algorithm. The search problem of identifying a Bayesian network that has a relative posterior probability greater than a given constant is NP-complete. (D.M. Chickering, 1996) Greedy search algorithms (such as hill-climbing or TABU search) are a common choice, but almost any kind of search procedure can be used.

Traditionally, in searching for a Bayesian network structure, the set of states was the set of all possible Bayesian network structures, the representation was

a Directed Acyclic Graph(DAG) and the set of operators were various small local changes to a DAG, e.g. adding, removing or reversing an arc, as illustrated in Figure 1.2. (R. Daly and Q. Shen., 2007)

| Operator          | Before  | After   |
|-------------------|---|---|
| Insert_Arc(X, Y)  | X<br>  | Y<br>  |
| Delete_Arc(X, Y)  | X<br> → Y<br> | X<br> Y<br>   |
| Reverse_Arc(X, Y) | X<br> → Y<br> | X<br> ← Y<br> |

Figure 1.2: Basic Modification Operators in Searching for a Bayesian Network Structure

# Chapter 2

## Bayesian Network Structure Learning Algorithms in bnlearn Package

**bnlearn** is an R package which includes several algorithms for learning the structure of Bayesian networks with either discrete or continuous variables. Both constraint-based and score-based algorithms are implemented. (Marco S., 2010)

### 2.1 Constraint-based Algorithms

#### 2.1.1 Grow-Shrink (GS) Markov Blanket Algorithm

Based on the Grow-Shrink Markov blanket, the simplest Markov blanket detection algorithm (Margaritis D., 2003) used in a structure learning algo-

rithm.

The definition of a Markov blanket is as follows: for any variable  $X \in U$ , the Markov blanket  $BL(X) \subseteq U$  is any set of variables such that for any  $Y \in U - BL(X) - \{X\}$ ,  $X \perp Y | BL(X)$ . In other words,  $BL(X)$  completely "shields" (d-separates) variable  $X$  from any other variable outside  $BL(X) \cup \{X\}$ .

In a Bayesian network graph, the Markov blanket of a node includes its parents, children and other parents of all of its children.

---

**Algorithm.** The GS Markov Blanket Algorithm

1.  $S \leftarrow NULL$

2. **While**  $\exists Y \in U - \{X\}$

such that  $Y \not\perp X | S$ ,

**do**  $S \leftarrow S \cup \{Y\}$ . (Growing phase)

**End While**

3. **While**  $\exists Y \in S$

such that  $Y \perp X | S - \{Y\}$ ,

**do**  $S \leftarrow S - \{Y\}$ . (Shrinking phase)

**End While**

4.  $B(X) \leftarrow S$ .

---

GS, for the recovery of the Markov blanket of  $X$  is based on pairwise independent tests. It consists of two phases, a growing and a shrinking one. Starting from an empty set  $S$ , the growing phase adds variables to  $S$  as long as they are dependent with  $X$  given the current contents of  $S$ .

### 2.1.2 Incremental Association (IAMB) Algorithm

Based on the Incremental Association Markov blanket (IAMB) algorithm (Tsamardinos I. *et al.* 2003), which is based on a two-phase selection scheme (a forward selection followed by an attempt to remove false positives).

---

**Algorithm.** The IAMB Algorithm

1. (Forward phase)

$S \leftarrow \text{NULL}$

**While**  $S$  has changed

**Find** the feature  $X$  in  $V - S - \{T\}$  that maximizes  $f(X; T|S)$

**If** not  $I(X; T|S)$

**Add**  $X$  to  $S$

**End If**

**End While**

2. (Backward phase)

**Remove** from  $S$  all variables  $X$ , for which  $I(X; T|S - \{X\})$

### 3. Return $S$

---

IAMB consists of two phases, a forward and a backward one.

The Markov blanket of a variable of interest  $T$ , will be denoted as  $MB(T)$ .

An estimate of the  $MB(T)$  is kept in the set  $S$ . In the forward phase all variables that belong in  $MB(T)$  and possibly more (false positives) enter  $S$  while in the backward phase the false positives are identified and removed so that  $S = MB(T)$  in the end.

The heuristic used in IAMB to identify potential Markov blanket members in 'forward phase' is the following:

Start with an empty candidate set for the  $S$  and admit into it (in the next iteration) the variable that maximizes a heuristic function  $f(X; T|S)$ . Function  $f$  should return a non-zero value for every variable that is a member of the Markov blanket for the algorithm to be sound, and is typically a measure of association between  $X$  and  $T$  given  $S$ . In our experiments we used  $f$  as the Mutual Information similar to what was suggested in Margaritis D. and Thrun S. (1999), J. Cheng *et al.* (2002):  $f(X; T|S)$  is the Mutual Information between  $S$  and  $T$  given  $S$ . It is important that  $f$  is an informative and effective heuristic so that the set of candidate variables after 'forward phase' is as small as possible for two reasons: one is time efficiency (i.e. do not spend time considering irrelevant variables) and another is sample efficiency (do not require sample larger than what is absolutely necessary to perform conditional tests of independence).

In backward conditioning we remove features that do not belong to the  $MB(T)$  one-by-one by testing whether a feature  $X$  from  $S$  is independent of  $T$  given the remaining  $S$ .

## 2.2 Score-Based Algorithms

### 2.2.1 Hill-Climbing (HC) Algorithm

A Hill-climbing is a greedy search on the space of the directed graphs. The optimized implementation uses score caching, score decomposability and score equivalence to reduce the number of duplicated tests.

---

**Algorithm.** The Hill-climbing(HC) Algorithm

1. **Current:** Make\_Node(Initial State)

2. **While**

**Neighbor:** a highest-valued successor of Current.State

**If** Neighbor.Value < Current.Value **Then**

**Return** Current.State

**End If**

Current  $\leftarrow$  Neighbor

**End While**

---

It is simply a loop that continually moves in the direction of increasing value. The algorithm does not maintain a search tree, so the data structure for the current node only needs to record the state and the value of the objective function. Hill-climbing does not look beyond the immediate neighbors of the current state. This resembles trying to find the top of Mount Everest in a thick fog while suffering from amnesia. (Russell S. J. and Norvig P., 2009)

### 2.2.2 TABU Search Algorithm

A modified Hill-climbing is able to escape local optima by selecting a network that minimally decreases the score function.

A variant of Hill-climbing called TABU search has gained popularity (Fred W. G. and Manuel L., 1997). This algorithm maintains a TABU list of  $k$  previously visited states that cannot be revisited, as well as improving efficiency when searching graphs. This list allows the algorithm to escape from some local minima.

---

**Algorithm.** The TABU Search Algorithm

1. Choose  $x \in X$  to start the process.
2. Find  $x' \in N(x)$  such that  $f(x') < f(x)$ .
3. If no such  $x'$  can be found,  $x$  is the local optimum and the method stops.
4. Otherwise, designate  $x'$  to be the new  $x$  and go to 2.

---

TABU search begins in the same way as ordinary local or neighborhood search, proceeding iteratively from one point solution to another until a chosen termination criterion is satisfied. Each  $x \in X$  has an associated neighborhood  $N(x) \subset X$ , and each solution  $x' \in N(x)$  is reached from  $x$  by an operation called 'move'.

As an initial point of departure, we may contrast TABU search with a simple descent method where the goal is to  $\min f(x)$  (or a corresponding ascent method where the goal is to  $\max f(x)$ ). Such method only permits moves to neighbor solutions that improve the current objective function value and ends when no improving solutions can be found. A pseudo-code of a generic descent method is presented in 'Algorithm'. The final  $x$  obtained by a descent method is called a local optimum, since it is at least as good or better than all solutions in its neighborhood. The evident shortcoming of a descent method is that such a local optimum in most cases will not be a global optimum, i.e., it usually will not minimize  $f(x)$  over all  $x \in X$ .

## 2.3 Hybrid Algorithms

### 2.3.1 Max-Min Hill-Climbing (MMHC) Algorithm

A hybrid algorithm which combines the Max-Min Parents and Children algorithm (to restrict the search space) and the Hill-Climbing algorithm (to find the optimal network structure in the restricted space). (Tsamardinos I. *et al.*, 2006)

The algorithm first identifies the parents and children set of each variable, then performs a greedy hill-climbing search in the space of Bayesian networks. The search begins with an empty graph. The edge addition, deletion, or direction reversal that leads to the largest increase in score is taken and the search continues in a similar fashion recursively.

### **2.3.2 More general 2-phase Restricted Maximization (RS-MAX2)**

A more general method is which Max-Min Hill-Climbing, uses any combination of constraint-based and score-based algorithms.

# Chapter 3

## The Comparison Methodology

### 3.1 The Number of Graphical Errors in the Learnt Structure

The comparison methodology used in this paper is similar to the method used in X.-w. Chen *et al.* (2006). The existence of the known network structures allows us to define three important terms which indicate the performance of the algorithm (in terms of the number of graphical errors in the learnt structure).

**C (Correct Arcs)** Edges present in the original network and in the learnt network structure.

**M (Missing Arcs)** Edges present in the original network but not in the learnt network structure.

**WO (Wrongly Oriented Arcs)** Edges present in the learnt network structure, but having opposite orientation when compared with the corre-

sponding edge in the original network structure.

**WC (Wrongly Corrected Arcs)** Edges not present in the original network but included in the learnt network structure.

## 3.2 Network Scores

The values of the BDe, the Log-likelihood (LL), the AIC, and the BIC are metrics for the learned networks. (Alexandra M. C., 2009) These measures can offer an idea of the quality of the networks from different points of view. In all four cases, the higher the value of the metric, the better the network. (D. Heckerman *et al.*, 1995, Silvia A. *et al.*, 2004).

### 3.2.1 Bayesian Scoring Functions

Compute the posterior probability distribution, starting from a prior probability distribution on the possible networks, conditioned to data  $T$ , that is,  $P(B|T)$ .

The best network is the one that maximizes the posterior probability.

Since the term  $P(T)$  is the same for all possible networks, in practice, for comparative purposes, computing  $P(B, T)$  is sufficient.

As it is easier to work in the logarithmic space, the scoring functions use the value  $\log(P(B, T))$  instead of  $P(B, T)$ .

#### BDe

D. Heckerman *et al.* (1995) proposed the Bayesian Dirichlet (BDe) score.

Given a directed acyclic graph (DAG)  $G$  such that  $P(G) > 0$  then  $\Theta_{ij}$  is Dirichlet for all  $\Theta_{ij}$  in  $\Theta_G$ . And given a Bayesian network  $B$ , data  $T$  can be seen as a multinomial sample of the joint space  $D$  with parameters

$$\Theta_D = \{\theta_{x_1, \dots, x_n}\}_{x_i=1, \dots, r_i, i \in 1, \dots, n}$$

$$\text{where } \theta_{x_1, \dots, x_n} = \prod_{i=1}^n \theta_{x_i} | \prod_{x_i}.$$

For any complete  $G$ , we have that  $P(G) > 0$ . Then  $\rho(\Theta_G|G) = \prod_{i=1}^n \rho(\Theta_i|G)$  (global parameter independence) and  $\rho(\Theta_i|G) = \prod_{j=1}^{q_i} \rho(\Theta_{ij}|G)$  for all  $i = 1, \dots, n$  (local parameter independence).

Given two DAGs  $G$  and  $G'$ , such that  $P(G) > 0$  and  $P(G') > 0$ , if  $X_i$  has the same parents in  $G$  and  $G_0$ , then  $\rho(\Theta_{ij}|G) = \rho(\Theta_{ij}|G')$  for all  $j = 1, \dots, q_i$ .

Suppose that  $\rho(\Theta_D|G)$  is Dirichlet with equivalent sample size  $N'$  for some complete  $G$  in  $D$ . Then, for any Bayesian network  $B$  in  $D$ ,

$$BDe(B, T) = P(B, T) = P(B) \times \prod_{i=1}^n \prod_{j=1}^{q_i} \left( \frac{\Gamma(N'_{ij})}{\Gamma(N_{ij} + N'_{ij})} \right) \times \prod_{k=1}^{r_i} \frac{\Gamma(N_{ijk} + N'_{ijk})}{\Gamma(N'_{ijk})}$$

$$\text{where } N'_{ijk} = N' \times P(X_i = x_{ik}, \prod_{X_i} = \omega_{ij}|G).$$

The equivalent sample size  $N'$  expresses the strength of our belief in the prior distribution.

### 3.2.2 Information-theoretic Scoring Functions

#### log-likelihood (LL)

The **log-likelihood (LL) Score** is defined in the following way:

$$LL(B|T) = \sum_{i=1}^n \sum_{j=1}^{q_i} \sum_{k=1}^{r_i} N_{ijk} \log\left(\frac{N_{ijk}}{N_{ij}}\right).$$

The LL score tends to favor complete network structures and it does not provide an useful representation of the independence assumptions of the learned network.

This phenomenon of overfitting is usually avoided in two different ways:

- By limiting the number of parents per network variable.
- By using some penalization factor over the LL score : AIC, BIC

### AIC and BIC

The measure of the quality of a BN can be computed in several different ways:

$$\phi(B|T) = LL(B|T) - f(N)|B|,$$

where  $f(N)$  is a non-negative penalization function.

- If  $f(N) = 1$ , we have the **Akaike Information Criterion (AIC)** scoring function:

$$AIC(B|T) = LL(B|T) - |B|.$$

- If  $f(N) = \frac{1}{2} \log(N)$ , we have the **Bayesian Information Criterion (BIC)** score.
- If  $f(N) = 0$ , we have the LL score.

# Chapter 4

## Data Generation with BN\_Data\_Generator in R

If given a Bayesian network model, then we can make a data set based on the model. However, it does not provide data by **bnlearn**, and in addition, it was difficult to find other functions. This makes it very hard work to create data in **R**. Other tools suffer from the trade-off between cost and complexity, restricting most studies relevant to Bayesian network to using only real data.

To address such problems, a data generator based on the Bayesian network model using R is built and introduced. At present, it exists as R functional form, but here are plans to make an R package. An update on the current status at [https://github.com/praster1/BN\\_Data\\_Generator](https://github.com/praster1/BN_Data_Generator). This generator was declared the GNU 2.0 license.

## 4.1 BN\_Data\_Generator Function in R

**Description** It based on a Bayesian network model to generates synthetic data.

**Usage** BN\_Data\_Generator (arcs, input\_Probs, n, node\_names)

### Arguments

Table 4.1: Argunemnts of BN\_Data\_Generator

| Argument    | Type     | Description                       |
|-------------|----------|-----------------------------------|
| arcs        | matrix   | A matrix that determines the arcs |
| input_Probs | list     | The conditional probabilities.    |
| n           | constant | sample size                       |
| node_names  | vector   | node names                        |

## 4.2 A Simple Example

Suppose we generate a data based on the model as show in Figure 4.1. This model is the "Bayesian network model of Asia Data Set by Lauritzen and Spiegelhalter" to be introduced in the next chapter.

It makes Arcs, input\_Probs, node\_names as follows:

---

```
R> arcs = rbind(  
# A S T L B E X D
```

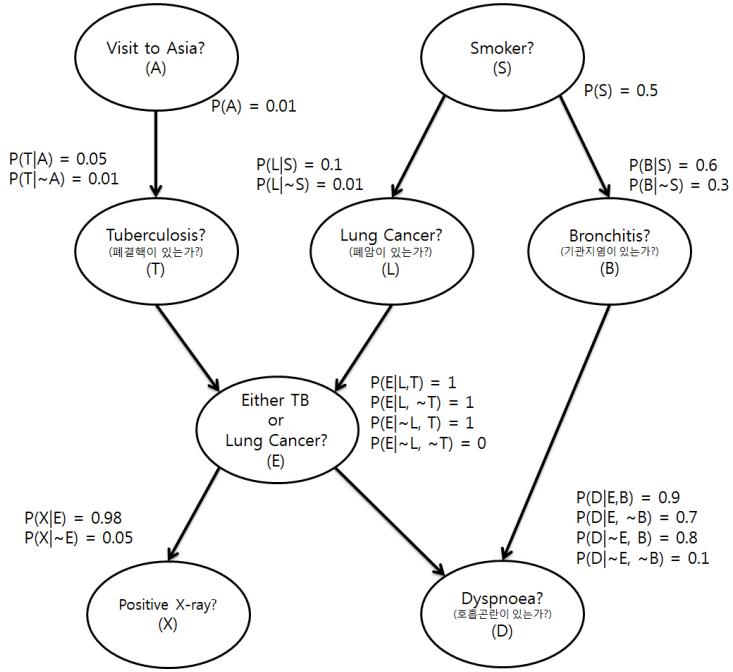


Figure 4.1: BN model of Asia Data Set by Lauritzen and Spiegelhalter

$c(0, 0, 1, 0, 0, 0, 0, 0, 0), \#A$

$c(0, 0, 0, 1, 1, 0, 0, 0, 0), \#S$

$c(0, 0, 0, 0, 0, 1, 0, 0, 0), \#T$

$c(0, 0, 0, 0, 0, 1, 0, 0, 0), \#L$

$c(0, 0, 0, 0, 0, 0, 0, 0, 1), \#B$

$c(0, 0, 0, 0, 0, 0, 1, 1), \#E$

$c(0, 0, 0, 0, 0, 0, 0, 0, 0), \#X$

$c(0, 0, 0, 0, 0, 0, 0, 0, 0)) \#D$

R> arc\_name = c("A", "S", "T", "L", "B", "E", "X", "D")

R> Probs = list(

$c(0.01), \#P(A)$

$c(0.5), \#P(S)$   
 $c(0.05, 0.01), \#P(T|A), P(T| \sim A)$   
 $c(0.1, 0.01), \#P(L|S), P(L| \sim S)$   
 $c(0.6, 0.3), \#P(B|S), P(B| \sim S)$   
 $c(1, 1, 1, 0), \#P(E|T, L), P(E| \sim T, L), P(E|T, \sim L), P(E| \sim T, \sim L)$   
 $c(0.98, 0.05), \#P(X|E), P(X| \sim E)$   
 $\#P(D|B, E), P(D| \sim B, E), P(D|B, \sim E), P(D| \sim B, \sim E)$   
 $c(0.9, 0.7, 0.8, 0.1))$

---

Suppose the sample size is 1000. If you type objects and sample size into BN\_Data\_Generator, then the data is generated.

---

```

R> n = 1000
R> res = BN_Data_Generator(arcs, Probs, n, arc_name)
R> data = res$data
R> head(data)

A S T L B E X D
1 N N N N N N N N N
2 N Y N N Y N N Y
3 N N N N N N N N N
4 N Y N N N N N N N
5 N N N N Y N N N
6 N Y N N Y N N Y

```

```
R> dim(data)
```

```
[1] 1000 8
```

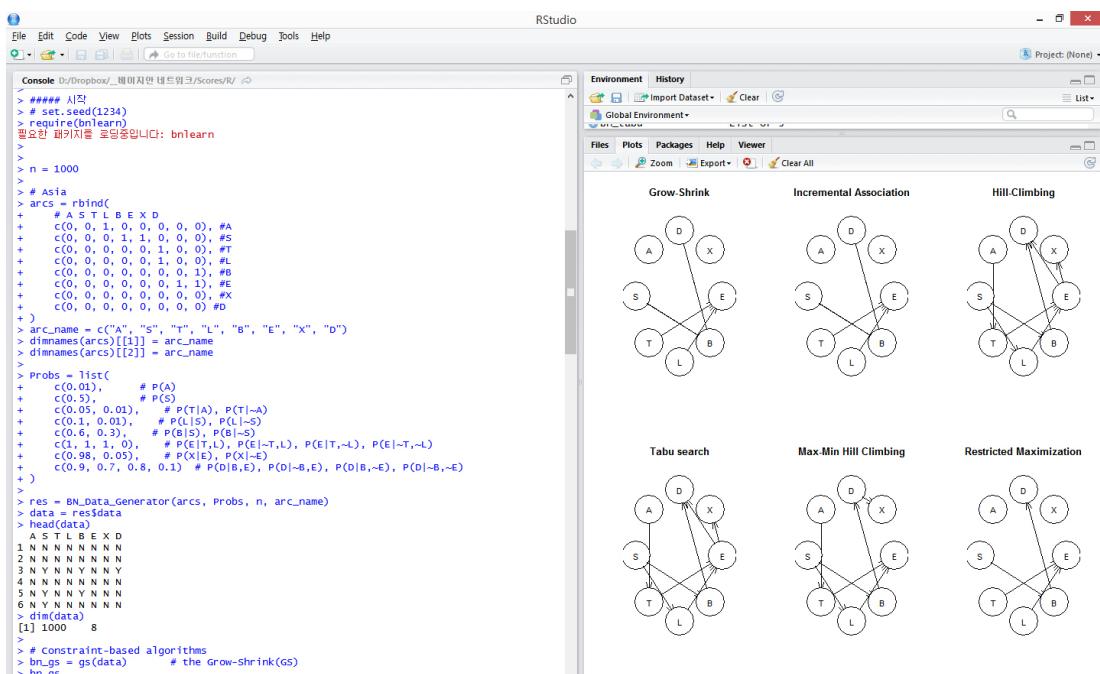


Figure 4.2: After make a data, execution results by bnlearn

# Chapter 5

## Simulation

Previous tools suffer from a serious trade-off between cost and complexity, restricting most studies relevant to Bayesian network to using only real data.

Therefore, in this paper, have been widely used so far, for the BN demonstration model, I tried to first apply the algorithm.

However, in order to measure the objective performance of the algorithm, it is necessary to try to analyze the synthetic data. Therefore, in this paper, by using the data generator BN that introduced, after generating the synthetic data in accordance with the topology, and algorithms were attempted to be applied to this.

In order to avoid the influence of chance, all experiments are repeated 100 times, and overall results are reported.

## 5.1 Real Data

### 5.1.1 Asia Data Set by Lauritzen and Spiegelhalter

**Description** Small synthetic data set from Lauritzen S. and Spiegelhalter D. (1988) about lung diseases (tuberculosis, lung cancer or bronchitis) and visits to Asia.

**Number of nodes** 8

**Number of arcs** 8

**Number of parameters** 18

Lauritzen S. and Spiegelhalter D. (1988) motivate this example as follows:  
"Shortness-of-breath (dyspnoea) may be due to tuberculosis, lung cancer or bronchitis, or none of them, or more than one of them. A recent visit to Asia increases the chances of tuberculosis, while smoking is known to be a risk factor for both lung cancer and bronchitis. The results of a single chest X-ray do not discriminate between lung cancer and tuberculosis, as neither does the presence or absence of dyspnoea."

Table 5.1: Comparison of scores and correct arcs via Asia data set

|             |        | Asia (Num of Nodes = 8) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|-------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                    |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                    | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -229814                 | 35.25    | -1113883 | 85.65    | -2218508 | 126.41   | C  | HC     | 677  | 0.55     | 716  | 0.37     | 735   | 0.48     |
|             | TABU   | -229806                 | 35.29    | -1113857 | 85.9     | -2218431 | 126.4    |    | TABU   | 655  | 0.73     | 677  | 0.72     | 703   | 0.76     |
|             | MMHC   | -249829                 | 41.68    | -1213021 | 117.02   | -2417769 | 191.84   |    | MMHC   | 461  | 0.55     | 503  | 0.48     | 514   | 0.59     |
|             | RSMAX2 | -252800                 | 43.81    | -1233095 | 121.09   | -2457976 | 173.35   |    | RSMAX2 | 400  | 0        | 400  | 0        | 400   | 0        |
| loglik      | HC     | -220520                 | 36.46    | -1102564 | 85.92    | -2206160 | 126.05   | M  | HC     | 122  | 0.52     | 83   | 0.38     | 65    | 0.48     |
|             | TABU   | -220505                 | 36.54    | -1102521 | 86.31    | -2206030 | 126.02   |    | TABU   | 122  | 0.52     | 83   | 0.38     | 65    | 0.48     |
|             | MMHC   | -241431                 | 43.03    | -1202710 | 117.97   | -2406616 | 192.19   |    | MMHC   | 339  | 0.55     | 297  | 0.48     | 286   | 0.59     |
|             | RSMAX2 | -244901                 | 44.97    | -1223631 | 121.63   | -2447783 | 173.8    |    | RSMAX2 | 400  | 0        | 400  | 0        | 400   | 0        |
| AIC         | HC     | -222238                 | 36.46    | -1104349 | 85.87    | -2208092 | 126.21   | WO | HC     | 1    | 0.1      | 1    | 0.1      | 0     | 0        |
|             | TABU   | -222226                 | 36.53    | -1104315 | 86.16    | -2207985 | 126.19   |    | TABU   | 23   | 0.51     | 40   | 0.62     | 32    | 0.66     |
|             | MMHC   | -242973                 | 43       | -1204336 | 117.81   | -2408295 | 192.1    |    | MMHC   | 0    | 0        | 0    | 0        | 0     | 0        |
|             | RSMAX2 | -246201                 | 44.97    | -1224936 | 121.62   | -2449114 | 173.8    |    | RSMAX2 | 0    | 0        | 0    | 0        | 0     | 0        |
| BIC         | HC     | -226454                 | 36.49    | -1110166 | 85.76    | -2215057 | 126.96   | WC | HC     | 72   | 1.22     | 96   | 1.12     | 224   | 1.69     |
|             | TABU   | -226449                 | 36.52    | -1110161 | 85.76    | -2215033 | 126.99   |    | TABU   | 112  | 1.43     | 170  | 1.46     | 292   | 2.02     |
|             | MMHC   | -246757                 | 42.97    | -1209634 | 117.34   | -2414348 | 191.83   |    | MMHC   | 202  | 0.2      | 218  | 0.58     | 244   | 0.83     |
|             | RSMAX2 | -249391                 | 44.97    | -1229189 | 121.6    | -2453913 | 173.82   |    | RSMAX2 | 0    | 0        | 8    | 0.39     | 50    | 0.87     |

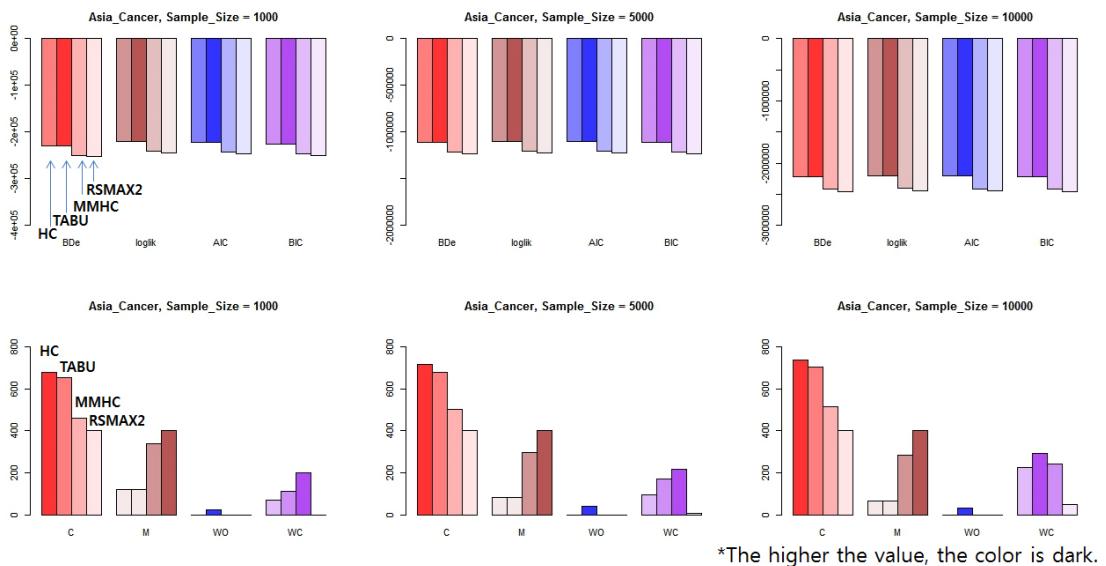


Figure 5.1: Comparison of scores and correct arcs via Asia data set

### 5.1.2 Insurance Evaluation Network Data Set

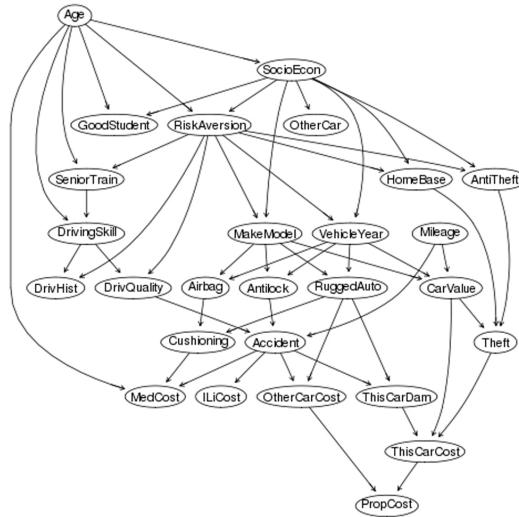


Figure 5.2: Bayesian network model of Insurance Evaluation Network Data Set

**Description** Insurance is a network for evaluating car insurance risks.

**Number of nodes** 27

**Number of arcs** 52

**Number of parameters** 984

Binder J. *et al.* (1997) motivate this example. This network for estimating the expected claim costs for a car insurance policyholder.

Table 5.2: Comparison of scores and correct arcs via Insurance data set

|             | Insurance (Num of Nodes = 27) |          |        |          |         |           |         |      |          |      |          |       |          |
|-------------|-------------------------------|----------|--------|----------|---------|-----------|---------|------|----------|------|----------|-------|----------|
| Sample Size | 1000                          |          | 5000   |          | 10000   |           |         | 1000 |          | 5000 |          | 10000 |          |
|             | Sum.                          | Std.Dev. | Sum.   | Std.Dev. | Sum.    | Std.Dev.  |         | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC                            | -1427953 | 113.01 | -6730331 | 189.16  | -13298470 | 246.4   | C    | HC       | 1864 | 1.57     | 2226  | 0.85     |
|             | TABU                          | -1422716 | 120.52 | -6708058 | 190.97  | -13294810 | 244.2   |      | TABU     | 1961 | 1.98     | 2543  | 0.79     |
|             | MMHC                          | -1644392 | 548.92 | -7281377 | 826.67  | -14112976 | 1680.81 |      | MMHC     | 1146 | 1.82     | 1457  | 1.17     |
|             | RSMAX2                        | -1735658 | 397.81 | -8293044 | 1714.16 | -16408869 | 2342.35 |      | RSMAX2   | 862  | 1.42     | 977   | 1.77     |
| loglik      | HC                            | -1347816 | 118.57 | -6603841 | 195.33  | -13134593 | 257.01  | M    | HC       | 2253 | 1.11     | 1738  | 0.79     |
|             | TABU                          | -1341071 | 128.83 | -6580889 | 194.64  | -13143085 | 241.01  |      | TABU     | 2155 | 1.39     | 1642  | 0.77     |
|             | MMHC                          | -1590326 | 585.75 | -7192400 | 852.4   | -14000007 | 1736.44 |      | MMHC     | 3526 | 2.2      | 2849  | 1.34     |
|             | RSMAX2                        | -1687201 | 417.78 | -8223919 | 1734.08 | -16330085 | 2362.44 |      | RSMAX2   | 3761 | 1.31     | 3497  | 1.05     |
| AIC         | HC                            | -1383888 | 112.68 | -6652052 | 190.99  | -13198494 | 248.77  | WO   | HC       | 1083 | 1.16     | 1236  | 0.73     |
|             | TABU                          | -1378131 | 120.12 | -6629457 | 191.37  | -13197529 | 243.3   |      | TABU     | 1084 | 1.56     | 1015  | 0.72     |
|             | MMHC                          | -1607916 | 566.08 | -7218618 | 841.94  | -14033715 | 1714.51 |      | MMHC     | 528  | 1.39     | 894   | 0.76     |
|             | RSMAX2                        | -1701145 | 409.22 | -8241628 | 1727.25 | -16350189 | 2356.64 |      | RSMAX2   | 577  | 0.99     | 726   | 1.67     |
| BIC         | HC                            | -1472404 | 105.57 | -6809152 | 185.18  | -13428868 | 242.07  | WC   | HC       | 1810 | 2.28     | 2096  | 1.49     |
|             | TABU                          | -1469072 | 108.51 | -6787720 | 191.17  | -13393809 | 256.04  |      | TABU     | 1756 | 2.49     | 1906  | 1.52     |
|             | MMHC                          | -1651080 | 519.05 | -7304052 | 808.56  | -14155238 | 1635.76 |      | MMHC     | 1098 | 2.35     | 1494  | 2.45     |
|             | RSMAX2                        | -1735362 | 389.69 | -8299334 | 1705.3  | -16422667 | 2335.81 |      | RSMAX2   | 1120 | 2.03     | 1220  | 1.69     |

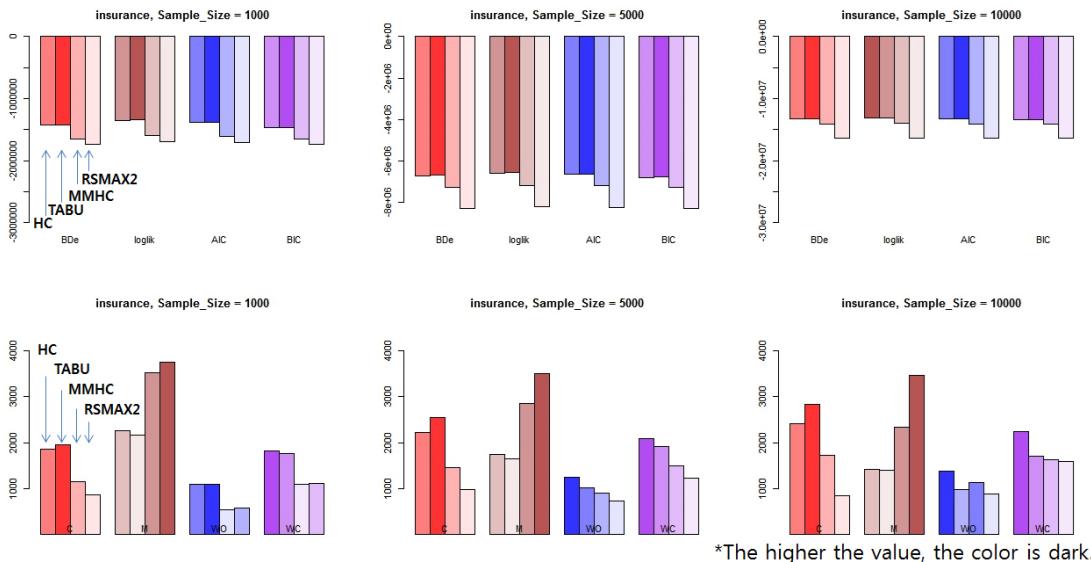


Figure 5.3: Comparison of scores and correct arcs via Insurance data set

### 5.1.3 ALARM Monitoring System Data Set

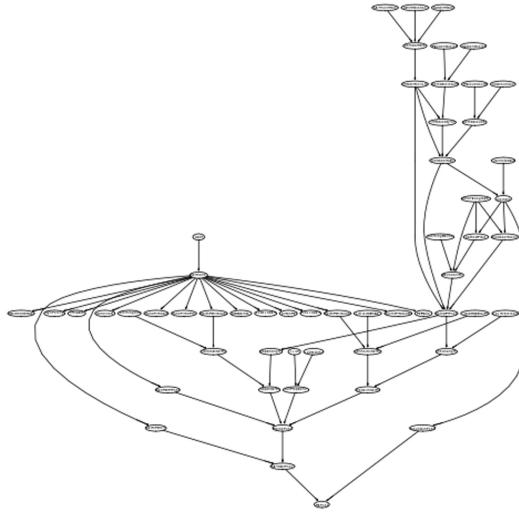


Figure 5.4: Bayesian network model of ALARM Monitoring System Data Set

**Description** The ALARM ("A Logical Alarm Reduction Mechanism") is a Bayesian network designed to provide an alarm message system for patient monitoring.

**Number of nodes** 37

**Number of arcs** 46

**Number of parameters** 509

Beinlich I. *et al.* (1989) motivate this example. ALARM (A Logical Alarm Reduction Mechanism) is a diagnostic application used to explore probabilistic reasoning techniques in belief networks. ALARM implements an alarm message system for patient monitoring; it calculates probabilities for a differential diagnosis based on available evidence.

Table 5.3: Comparison of scores and correct arcs via ALARM data set

|             |        | ALARM (Num of Nodes = 37) |          |          |          |           |          |    |        |      |          |      |          |       |          |
|-------------|--------|---------------------------|----------|----------|----------|-----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                      |          | 5000     |          | 10000     |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                      | Std.Dev. | Sum.     | Std.Dev. | Sum.      | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -1178527                  | 132.62   | -5580032 | 259.61   | -11006608 | 381.65   | C  | HC     | 2048 | 1.73     | 2283 | 0.43     | 2270  | 0.96     |
|             | TABU   | -1177619                  | 133.2    | -5580099 | 246.64   | -11005176 | 380.57   |    | TABU   | 2426 | 1.8      | 2709 | 1.05     | 2686  | 1.26     |
|             | MMHC   | -1673651                  | 649.54   | -7510018 | 2077.76  | -13885378 | 7417.41  |    | MMHC   | 1051 | 1.77     | 1421 | 1.75     | 1925  | 3.09     |
|             | RSMAX2 | -1735940                  | 439.39   | -8318508 | 2058.09  | -16586602 | 3307.55  |    | RSMAX2 | 633  | 1.65     | 867  | 0.92     | 858   | 1.19     |
| loglik      | HC     | -1099997                  | 130.61   | -5464607 | 260.53   | -10860805 | 370.7    | M  | HC     | 900  | 1.06     | 498  | 0.4      | 468   | 0.69     |
|             | TABU   | -1099325                  | 130.77   | -5465405 | 244.34   | -10861471 | 371.47   |    | TABU   | 878  | 0.97     | 501  | 0.36     | 468   | 0.69     |
|             | MMHC   | -1617451                  | 672.21   | -7426574 | 2093.3   | -13790813 | 7449.01  |    | MMHC   | 2460 | 1.68     | 1480 | 1.04     | 1268  | 1.56     |
|             | RSMAX2 | -1682617                  | 453.09   | -8242927 | 2079.53  | -16508716 | 3325.33  |    | RSMAX2 | 3286 | 1.6      | 3162 | 1.21     | 2949  | 1.05     |
| AIC         | HC     | -1135950                  | 129.48   | -5509035 | 253.54   | -10923175 | 374.81   | WO | HC     | 1652 | 1.37     | 1819 | 0.46     | 1862  | 0.9      |
|             | TABU   | -1134574                  | 130.77   | -5508543 | 244.04   | -10921249 | 372.87   |    | TABU   | 1296 | 1.52     | 1390 | 0.96     | 1446  | 1.27     |
|             | MMHC   | -1634699                  | 655.42   | -7451159 | 2085     | -13816106 | 7435.45  |    | MMHC   | 1089 | 1.38     | 1699 | 2.05     | 1407  | 1.98     |
|             | RSMAX2 | -1698026                  | 443.88   | -8266708 | 2064.51  | -16527481 | 3318.41  |    | RSMAX2 | 681  | 1.6      | 571  | 0.98     | 793   | 0.71     |
| BIC         | HC     | -1224175                  | 130.11   | -5653808 | 251.23   | -11148029 | 416.51   | WC | HC     | 2498 | 2.58     | 2306 | 1.9      | 2714  | 1.56     |
|             | TABU   | -1221071                  | 133.9    | -5649112 | 244.7    | -11136759 | 427.99   |    | TABU   | 2314 | 2.48     | 2032 | 2.08     | 2452  | 2.25     |
|             | MMHC   | -1677023                  | 614.97   | -7531272 | 2058.32  | -13907292 | 7386.76  |    | MMHC   | 1934 | 2.22     | 2890 | 2.61     | 2368  | 2.84     |
|             | RSMAX2 | -1735838                  | 423.78   | -8344201 | 2018.29  | -16595132 | 3293.99  |    | RSMAX2 | 1684 | 2.73     | 1982 | 2.56     | 2262  | 1.96     |

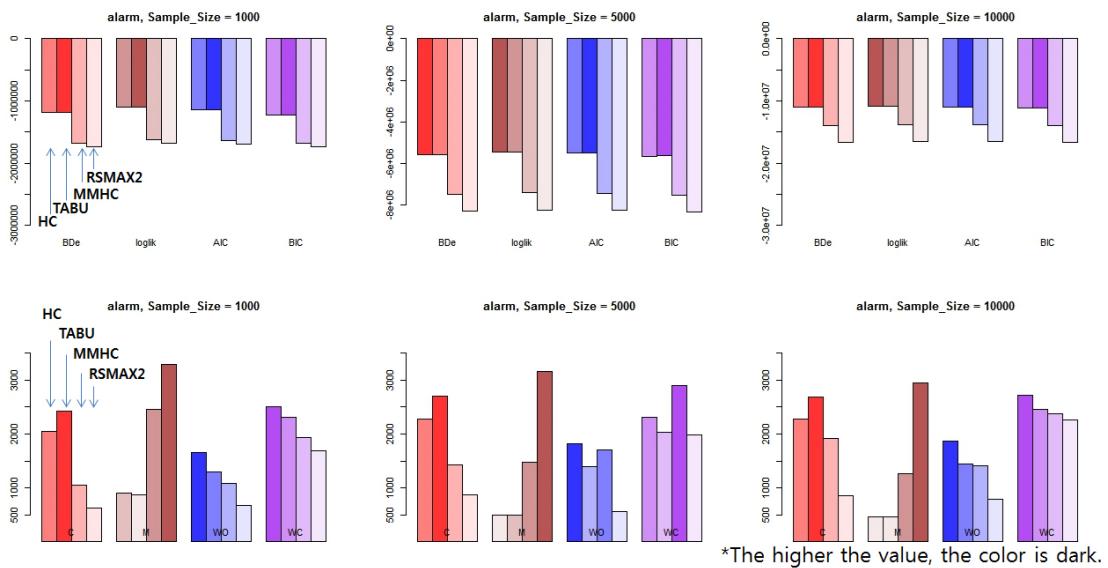


Figure 5.5: Comparison of scores and correct arcs via Hailfinder data set

### 5.1.4 The HailFinder Weather Forecast System Data Set

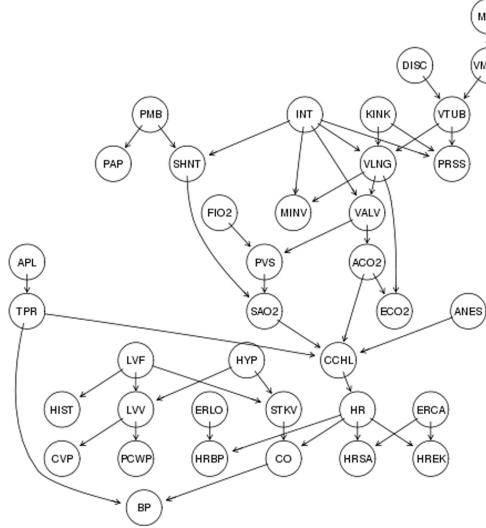


Figure 5.6: Bayesian network model of The HailFinder Weather Forecast System Data Set

**Description** Hailfinder is a Bayesian network designed to forecast severe summer hail in northeastern Colorado.

**Number of nodes** 56

**Number of arcs** 66

**Number of parameters** 2656

Abramson B. *et al.* (1988) motivate this example. Hailfinder is a Bayesian system that combines meteorological data and models with expert judgement, based on both experience and physical understanding, to forecast severe weather in North-eastern Colorado.

Table 5.4: Comparison of scores and correct arcs via Hailfinder data set

|             |        | Hailfinder (Num of Nodes = 56) |          |           |          |           |          |    |        |      |          |      |          |       |          |
|-------------|--------|--------------------------------|----------|-----------|----------|-----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                           |          | 5000      |          | 10000     |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                           | Std.Dev. | Sum.      | Std.Dev. | Sum.      | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -11006608                      | 381.65   | -24975425 | 256.04   | -49602871 | 297.73   | C  | HC     | 2270 | 0.96     | 5301 | 0.1      | 5586  | 0.6      |
|             | TABU   | -11005176                      | 380.57   | -24975425 | 256.04   | -49595842 | 296.18   |    | TABU   | 2686 | 1.26     | 5301 | 0.1      | 5473  | 0.8      |
|             | MMHC   | -13885378                      | 7417.41  | -28286426 | 1564.45  | -56127215 | 2369.94  |    | MMHC   | 1925 | 3.09     | 3247 | 0.78     | 3421  | 1.03     |
|             | RSMAX2 | -16586602                      | 3307.55  | -30165661 | 4750.83  | -60211607 | 10590.78 |    | RSMAX2 | 858  | 1.19     | 2513 | 1.45     | 2599  | 1.27     |
| loglik      | HC     | -10860805                      | 370.7    | -24580625 | 264.37   | -49103216 | 318.52   | M  | HC     | 468  | 0.69     | 1299 | 0.1      | 974   | 0.52     |
|             | TABU   | -10861471                      | 371.47   | -24580625 | 264.37   | -49101011 | 318.75   |    | TABU   | 468  | 0.69     | 1299 | 0.1      | 975   | 0.52     |
|             | MMHC   | -13790813                      | 7449.01  | -28061042 | 1631.98  | -55832193 | 2457.02  |    | MMHC   | 1268 | 1.56     | 3350 | 0.78     | 3179  | 1.03     |
|             | RSMAX2 | -16508716                      | 3325.33  | -30028409 | 4793.11  | -60050894 | 10637.32 |    | RSMAX2 | 2949 | 1.05     | 4086 | 1.44     | 4001  | 1.27     |
| AIC         | HC     | -10923175                      | 374.81   | -24722989 | 261.22   | -49264998 | 310.85   | WO | HC     | 1862 | 0.9      | 0    | 0        | 40    | 0.49     |
|             | TABU   | -10921249                      | 372.87   | -24722989 | 261.22   | -49262249 | 308.38   |    | TABU   | 1446 | 1.27     | 0    | 0        | 152   | 0.58     |
|             | MMHC   | -13816106                      | 7435.45  | -28135869 | 1609.27  | -55920440 | 2432.1   |    | MMHC   | 1407 | 1.98     | 3    | 0.17     | 0     | 0        |
|             | RSMAX2 | -16527481                      | 3318.41  | -30070905 | 4758.8   | -60096326 | 10600.54 |    | RSMAX2 | 793  | 0.71     | 1    | 0.1      | 0     | 0        |
| BIC         | HC     | -11148029                      | 416.51   | -25186896 | 253.21   | -49848249 | 291.84   | WC | HC     | 2714 | 1.56     | 1016 | 0.55     | 1028  | 0.75     |
|             | TABU   | -11136759                      | 427.99   | -25186896 | 253.21   | -49843539 | 288.2    |    | TABU   | 2452 | 2.25     | 1016 | 0.55     | 1112  | 1.08     |
|             | MMHC   | -13907292                      | 7386.76  | -28379700 | 1538.4   | -56238585 | 2345.93  |    | MMHC   | 2368 | 2.84     | 1424 | 2.67     | 1662  | 2.16     |
|             | RSMAX2 | -16595132                      | 3293.99  | -30209383 | 4647.37  | -60260116 | 10468.63 |    | RSMAX2 | 2262 | 1.96     | 166  | 1.36     | 132   | 1.07     |

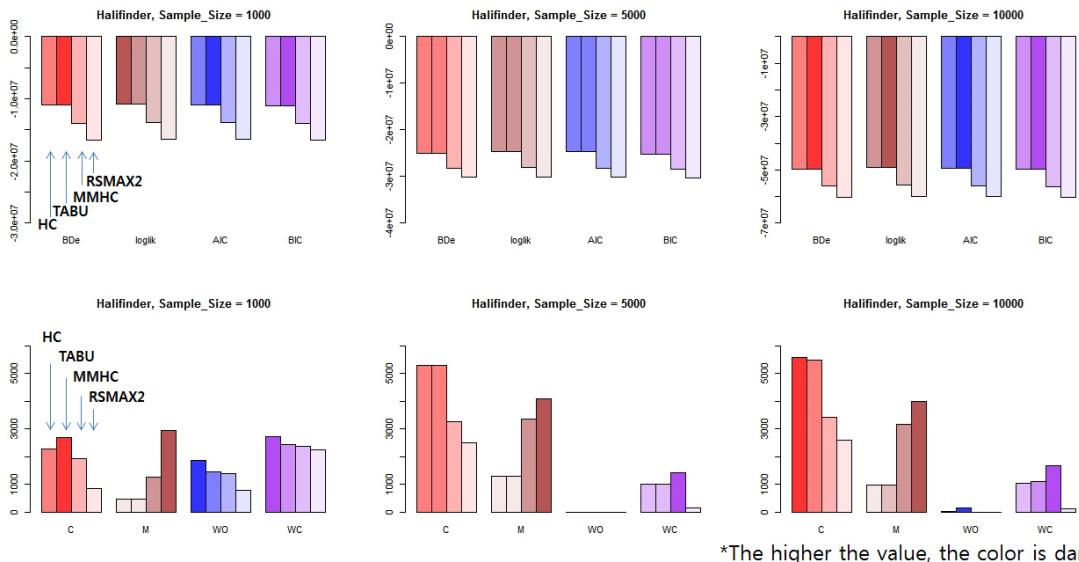


Figure 5.7: Comparison of scores and correct arcs via Hailfinder data set

### 5.1.5 Summary

| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|-------------|-------|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| Asia        | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 1    | 4    | 4      | 3  | 2    | 1    | 4      |
| Insurance   | 2     | 1    | 3    | 4      | 2  | 1    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 3      | 1  | 2    | 4    | 3      |
| Alarm       | 2     | 1    | 3    | 4      | 2  | 1    | 3    | 4      | 3  | 4    | 2    | 1      | 1  | 2    | 3    | 4      | 1  | 2    | 3    | 4      |
| HallFinder  | 2     | 1    | 3    | 4      | 2  | 1    | 3    | 4      | 4  | 4    | 2    | 1      | 1  | 2    | 3    | 4      | 1  | 2    | 3    | 4      |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| Asia        | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 1    | 4    | 4      | 3  | 2    | 1    | 4      |
| Insurance   | 2     | 1    | 3    | 4      | 2  | 1    | 3    | 4      | 3  | 4    | 2    | 1      | 1  | 2    | 3    | 4      | 1  | 2    | 3    | 4      |
| Alarm       | 1     | 2    | 3    | 4      | 2  | 1    | 3    | 4      | 4  | 3    | 2    | 1      | 1  | 3    | 2    | 4      | 2  | 3    | 1    | 4      |
| HallFinder  | 1     | 1    | 3    | 4      | 1  | 1    | 3    | 4      | 4  | 4    | 2    | 1      | 1  | 1    | 1    | 1      | 2  | 2    | 1    | 4      |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| Asia        | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 1    | 4    | 4      | 3  | 1    | 2    | 4      |
| Insurance   | 2     | 1    | 3    | 4      | 2  | 1    | 3    | 4      | 3  | 4    | 2    | 1      | 1  | 3    | 2    | 4      | 1  | 2    | 3    | 4      |
| Alarm       | 2     | 1    | 3    | 4      | 2  | 1    | 3    | 4      | 4  | 4    | 2    | 1      | 1  | 2    | 3    | 4      | 1  | 2    | 3    | 4      |
| HallFinder  | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 3    | 2    | 1      | 2  | 1    | 4    | 4      | 3  | 2    | 1    | 4      |

Figure 5.8: Summary for Comparison of scores and correct arcs via real data sets

When compared to the Score criteria were found to show good performance most TABU search algorithm, the order of Hill-Climbing algorithm.

However, as a result of comparing the target network and learning network directly, was a little different.

Result of comparing the target network and learning network directly, when C is large, M, WO, it can be said that performance is better when the WC is small.

TABU search algorithm, but were still many number of C, when it is a score criterion, considering that was overwhelming performance than other algorithms, it has been somewhat disappointing. Because rather WO, also the number of WC large, or shift the direction of the arrow, that unreasonable arrow is drawn is evaluated as disadvantages.

MMHC, RSMAX2 but C is also less M many, WO, I found that WC is also

small. Overall the number of arrows Hill-climbing, will be drawn smaller than the TABU search. This Hybrid algorithms such as MMHC, RSMAX2 is, in the learning process in comparison with the Score-Based Algorithm, is the result that it can be confirmed that that will conservatively subsequently arrow.

Since the shape of the model is different, by using them, performance comparison of the algorithm according to the node number is difficult. Also, it is difficult to discover that the sample size is also clear changes in accordance with the increase.

## 5.2 Synthetic Data According to Topologies

### 5.2.1 Bayesian Network Topologies

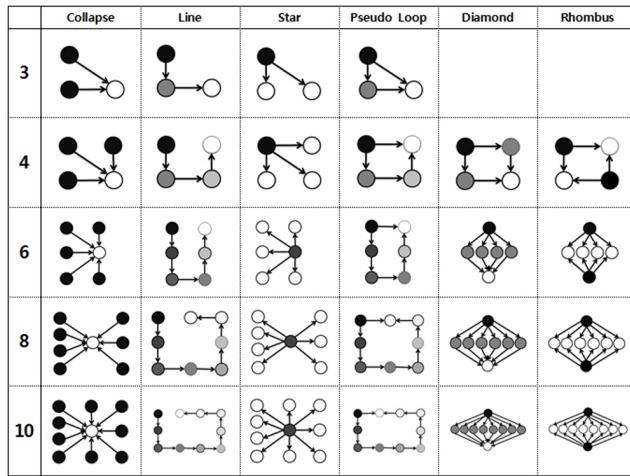


Figure 5.9: Bayesian Networks with varying topologies and number of nodes

Bayesian network model, as the number of nodes increases, difficult speaking any rules of the model accurately. Instead, it can be viewed separately with a certain unchanged properties called topology. Eitel J. M. L. (2008) was attempted to distinguish topology of Bayesian network.

In this paper, depending on the topology, after creating a set of models of the number of nodes to 3, 4, 6, 8, 10 pieces, and simulating. Cardinality was limited to two. In other words, all of the variable is the binary data. The probability value, which is imparted optionally under  $U(0, 1)$  distribution. And in order to avoid the influence of chance, all experiments are repeated 100 times, and overall results are reported.

### 5.2.2 Collapse

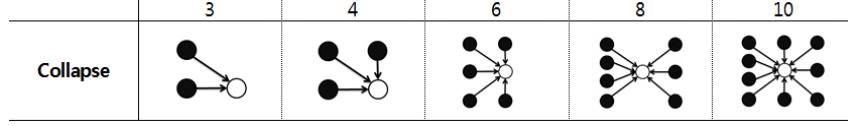


Figure 5.10: Bayesian Network Topology : Collapse

If one node has plurality of parent nodes, then this form called Collapse.

| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |   |
|-------------|-------|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|---|
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |   |
| 3           | 2     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 3    | 1    | 1      | 4  | 1    | 4    | 4      | 4  | 1    | 4    | 4      |   |
| 4           | 2     | 1    | 4    | 3      | 1  | 2    | 4    | 3      | 3  | 4    | 1    | 2      | 4  | 1    | 2    | 4      | 4  | 1    | 2    | 4      |   |
| 6           | 2     | 1    | 4    | 3      | 1  | 1    | 4    | 3      | 3  | 4    | 1    | 2      | 4  | 1    | 2    | 3      | 2  | 1    | 3    | 4      |   |
| 8           | 2     | 1    | 3    | 4      | 2  | 1    | 4    | 4      | 3  | 4    | 1    | 2      | 4  | 1    | 3    | 2      | 2  | 1    | 3    | 4      |   |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 4    | 3      | 3  | 4    | 1    | 2      | 4  | 1    | 2    | 1      | 4  | 4    | 1    | 3      | 4 |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |   |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |   |
| 3           | 2     | 1    | 4    | 4      | 1  | 2    | 4    | 4      | 4  | 3    | 1    | 1      | 4  | 1    | 4    | 4      | 4  | 1    | 4    | 4      |   |
| 4           | 2     | 1    | 4    | 4      | 1  | 2    | 4    | 4      | 1  | 3    | 4    | 1      | 4  | 1    | 2    | 2      | 4  | 1    | 4    | 4      |   |
| 6           | 2     | 1    | 4    | 3      | 2  | 1    | 4    | 3      | 3  | 4    | 1    | 2      | 2  | 1    | 4    | 2      | 2  | 1    | 3    | 4      |   |
| 8           | 2     | 1    | 4    | 3      | 2  | 1    | 4    | 3      | 3  | 4    | 1    | 2      | 2  | 1    | 4    | 2      | 2  | 1    | 3    | 4      |   |
| 10          | 2     | 1    | 3    | 4      | 2  | 1    | 4    | 4      | 3  | 4    | 1    | 2      | 4  | 1    | 3    | 2      | 2  | 1    | 3    | 4      |   |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |   |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |   |
| 3           | 2     | 1    | 4    | 4      | 1  | 2    | 4    | 4      | 4  | 3    | 1    | 1      | 4  | 1    | 4    | 4      | 4  | 1    | 4    | 4      |   |
| 4           | 2     | 1    | 4    | 4      | 1  | 2    | 4    | 4      | 1  | 3    | 4    | 1      | 4  | 1    | 2    | 2      | 4  | 1    | 4    | 4      |   |
| 6           | 1     | 1    | 3    | 4      | 1  | 1    | 4    | 4      | 4  | 4    | 1    | 1      | 1  | 1    | 1    | 1      | 2  | 1    | 3    | 4      |   |
| 8           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 3    | 4      |   |
| 10          | 2     | 1    | 3    | 4      | 2  | 1    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 3    | 4      | 2  | 1    | 3    | 4      |   |

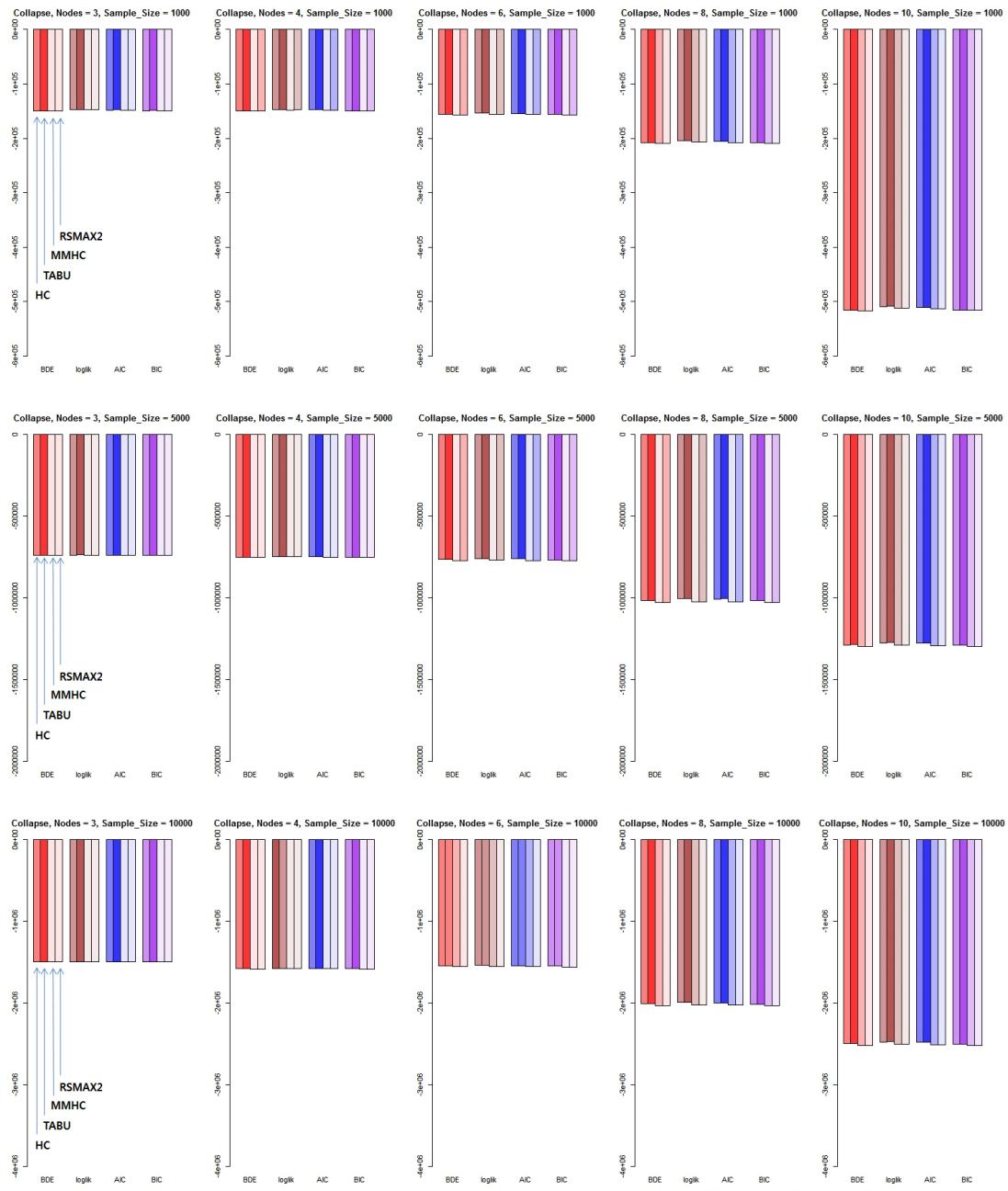
Figure 5.11: Summary for Comparison via Collapse

Like when using the Real data set, the TABU search is better than others according to score.

However, when compared by the number of C, TABU search and Hill-climbing has engaged in a conflict with each other. Rather the case of TABU search has a lot of WO and WC.

Not as much as TABU search case of Hill-climbing, as more the number of nodes, and as much sample size, appeared that the number WO is larger.

While sample size increases, then M is decreasing when using MMHC, but M is increasing when using RSMAX2.



\*The higher the value, the color is dark.

Figure 5.12: Comparison of scores via Collapse

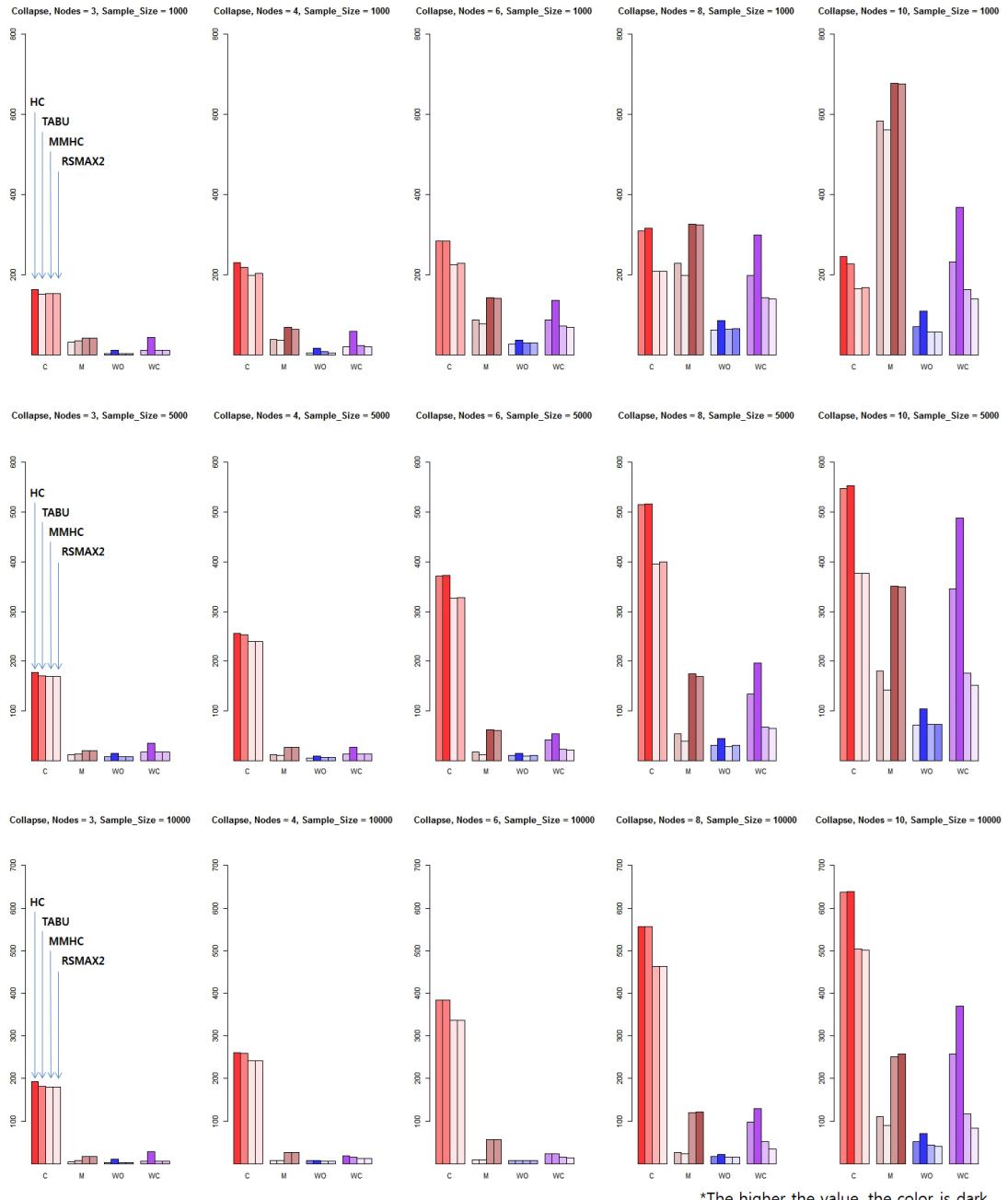


Figure 5.13: Comparison of correct arcs via Collapse

### 5.2.3 Line

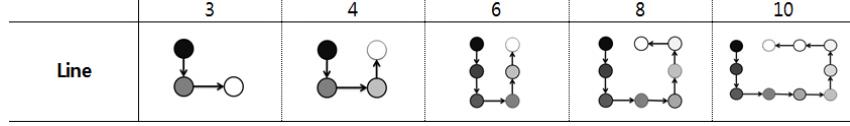


Figure 5.14: Bayesian Network Topology : Line

Multiple node bite the tail of the tail, then this form called Line. Though the figure is as of line.

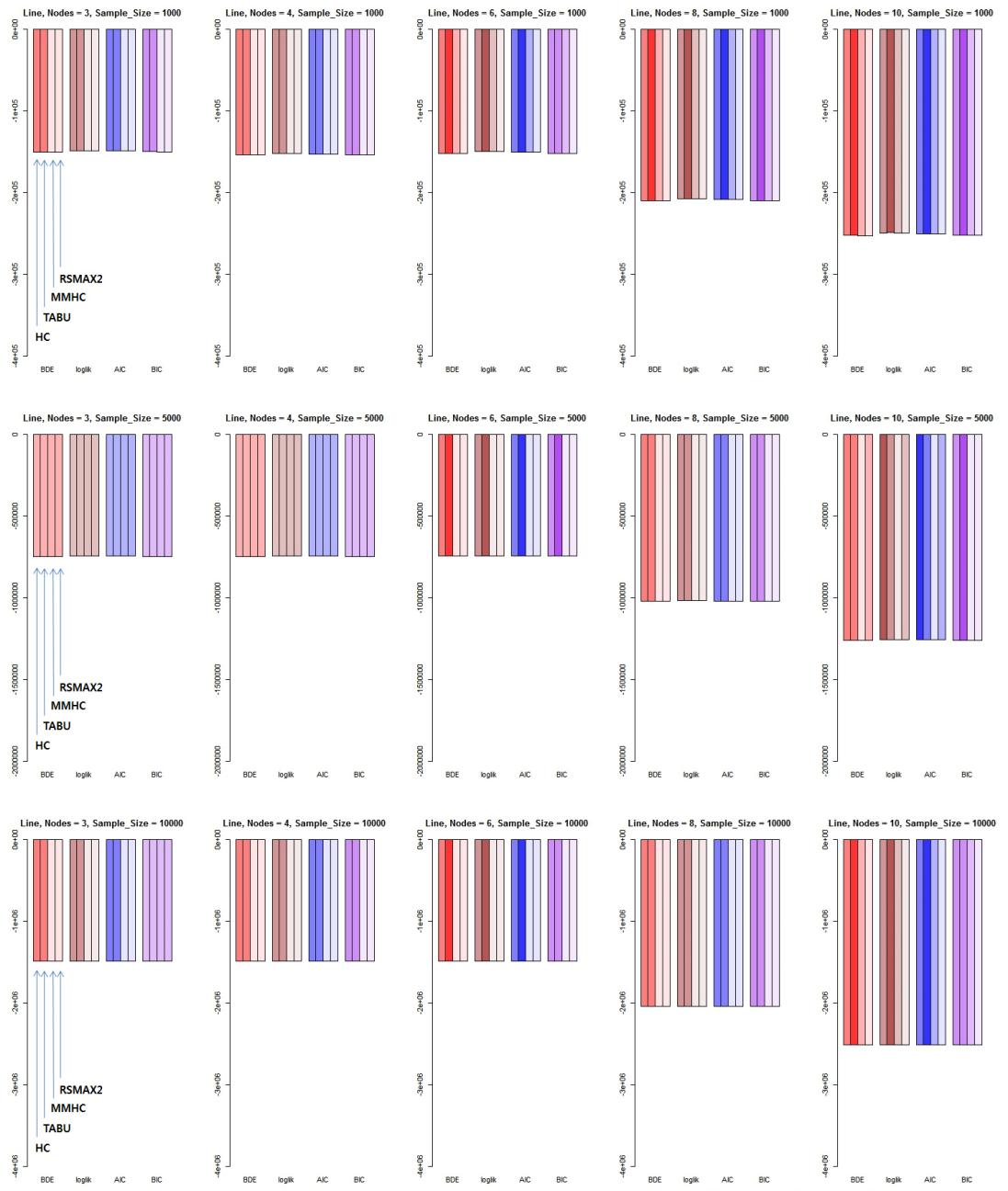
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |   |   |   |   |
|-------------|-------|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|---|---|---|---|
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |   |   |   |   |
| 3           | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 2    | 1      | 4 | 4 |   |   |
| 4           | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 2    | 1      | 4 | 4 |   |   |
| 6           | 2     | 1    | 3    | 4      | 1  | 4    | 2    | 2      | 3  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 2    | 1      | 3 | 4 |   |   |
| 8           | 2     | 1    | 3    | 4      | 1  | 4    | 2    | 3      | 3  | 4    | 2    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 2    | 1      | 4 | 4 |   |   |
| 10          | 2     | 1    | 3    | 4      | 1  | 4    | 2    | 3      | 4  | 4    | 4    | 2      | 1  | 4    | 4    | 4      | 1  | 4    | 4    | 2      | 1 | 4 | 4 |   |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |   |   |   |   |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |   |   |   |   |
| 3           | 1     | 1    | 1    | 1      | 1  | 4    | 1    | 1      | 1  | 1    | 1    | 1      | 4  | 4    | 4    | 4      | 1  | 4    | 4    | 4      | 1 | 4 | 4 |   |
| 4           | 1     | 1    | 1    | 1      | 1  | 4    | 1    | 1      | 1  | 1    | 1    | 1      | 4  | 4    | 4    | 4      | 1  | 4    | 4    | 4      | 1 | 4 | 4 |   |
| 6           | 2     | 1    | 4    | 4      | 1  | 4    | 1    | 1      | 1  | 1    | 1    | 1      | 4  | 4    | 4    | 4      | 1  | 4    | 4    | 4      | 1 | 4 | 4 |   |
| 8           | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 4      | 1  | 4    | 4    | 2      | 1 | 4 | 4 |   |
| 10          | 1     | 1    | 4    | 3      | 1  | 4    | 3    | 2      | 4  | 3    | 1    | 2      | 4  | 4    | 4    | 4      | 1  | 4    | 4    | 2      | 1 | 4 | 4 |   |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |   |   |   |   |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |   |   |   |   |
| 3           | 1     | 1    | 4    | 4      | 1  | 4    | 1    | 1      | 1  | 1    | 1    | 1      | 4  | 4    | 4    | 4      | 2  | 1    | 4    | 4      | 2 | 1 | 4 | 4 |
| 4           | 1     | 1    | 4    | 4      | 1  | 4    | 1    | 1      | 1  | 1    | 1    | 1      | 4  | 4    | 4    | 4      | 2  | 1    | 4    | 4      | 2 | 1 | 4 | 4 |
| 6           | 2     | 1    | 4    | 4      | 1  | 4    | 1    | 1      | 1  | 1    | 1    | 1      | 4  | 4    | 4    | 4      | 2  | 1    | 4    | 4      | 2 | 1 | 4 | 4 |
| 8           | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 4      | 1  | 4    | 4    | 2      | 1 | 4 | 4 |   |
| 10          | 2     | 1    | 3    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 4      | 1  | 4    | 4    | 2      | 1 | 3 | 4 |   |

Figure 5.15: Summary for Comparison via Line

Performance of each algorithm is compared to other topology were not different significantly occurs.

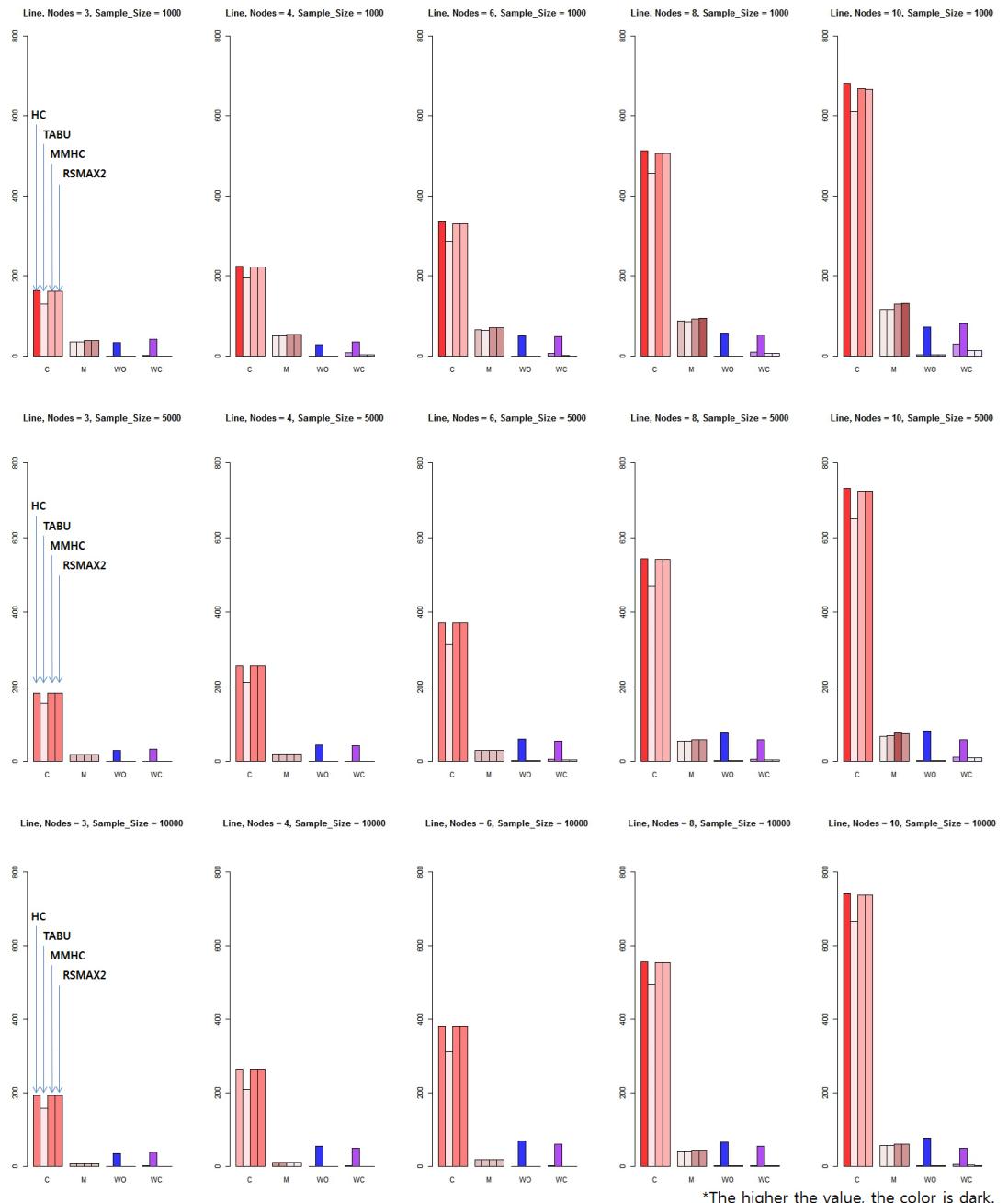
However, TABU search, despite good performance by score, the number of C is overwhelmingly smaller than the other algorithms, and M, WO, and WC is larger than the other algorithms.

Relatively, Hill-climbing has showed good performance for line form.



\*The higher the value, the color is dark.

Figure 5.16: Comparison of scores via Line



\*The higher the value, the color is dark.

Figure 5.17: Comparison of correct arcs via Line

### 5.2.4 Star

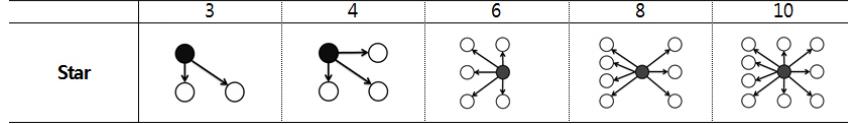


Figure 5.18: Bayesian Network Topologies : Star

If one node has plurality of child node, then this form called Star.

| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|-------------|-------|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 1000        | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 2    | 1      | 4  | 4    | 2    | 1      |
| 3           | 2     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 2    | 1      | 4  | 4    | 2    | 1      |
| 4           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 4    | 2    | 1      | 4  | 4    | 2    | 1      |
| 6           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 4  | 4    | 2    | 1      | 4  | 4    | 2    | 1      |
| 8           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 4  | 4    | 2    | 1      | 4  | 4    | 2    | 1      |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 4  | 4    | 2    | 1      | 3  | 2    | 1    | 2      |
| 5000        | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 3           | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 4           | 1     | 1    | 3    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 6           | 1     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 8           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 4    | 4    | 1      | 3  | 2    | 1    | 2      |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 4    | 4    | 1      | 3  | 2    | 1    | 2      |
| 10000       | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 3           | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 4           | 1     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 6           | 1     | 1    | 4    | 3      | 1  | 4    | 3    | 2      | 4  | 4    | 1    | 2      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 8           | 1     | 1    | 4    | 3      | 1  | 2    | 4    | 3      | 4  | 4    | 1    | 2      | 4  | 4    | 4    | 1      | 4  | 4    | 4    | 1      |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 4    | 4    | 1      | 2  | 3    | 2    | 1      |

Figure 5.19: Summary for Comparison via Star

Star also, the performance of each algorithm was not different significantly occurs.

When compared by score, TABU search is good then the other algorithms. But when compared to between the target network and learning network, Hill-climbing showed good performance relatively for the line form.

Specific point, when the node number is small, TABU search despite the good performance by score, but the number of C is overwhelmingly smaller than the other algorithms. And M, WO and WC is very large. However, as

the number of node increases, becoming the number of C is increased, and M, WO, WC is decreases.

And yet, did not win to the performance of Hill-climbing.

All algorithms while sample size increases, WO and WC is greatly reduced.

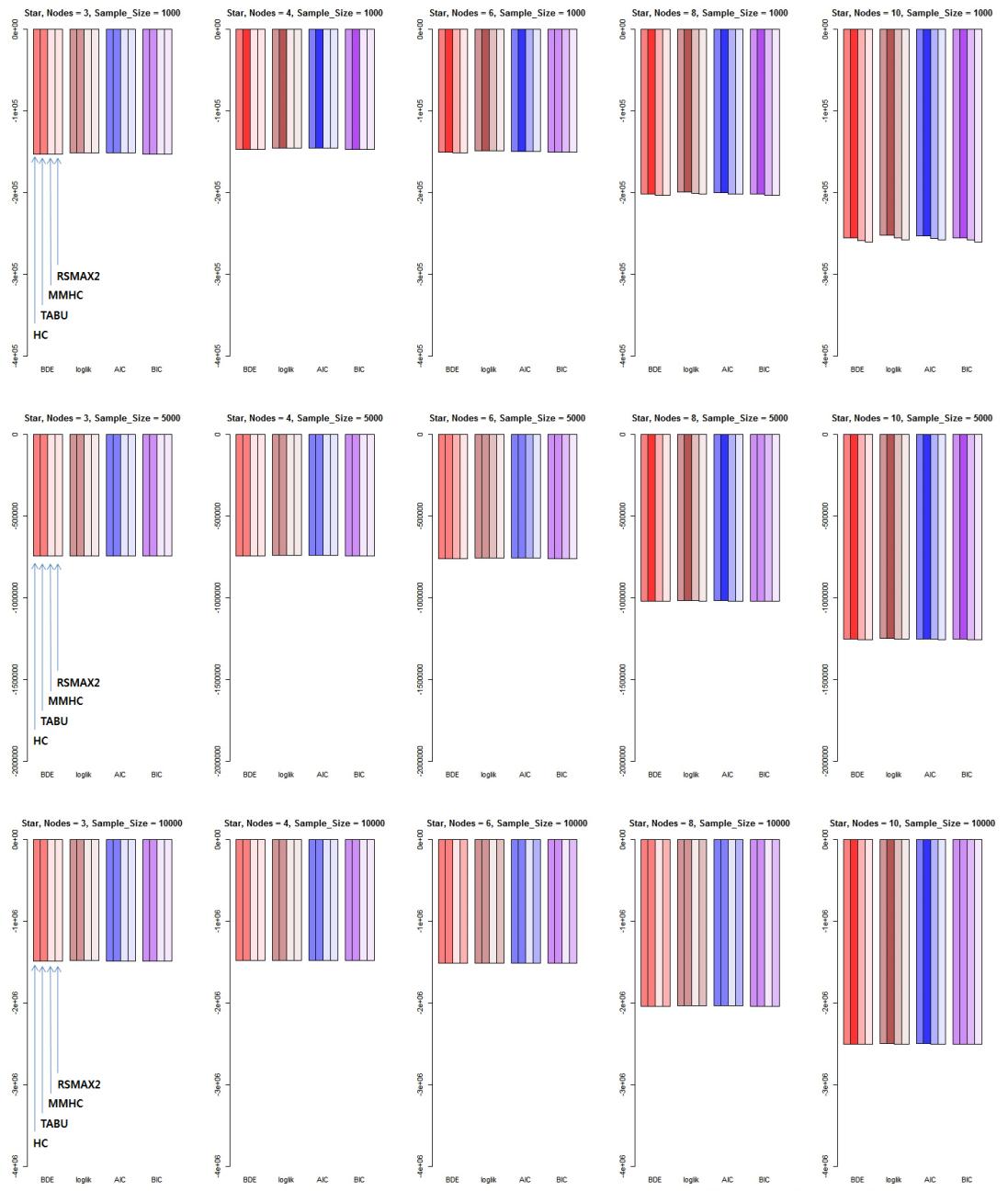
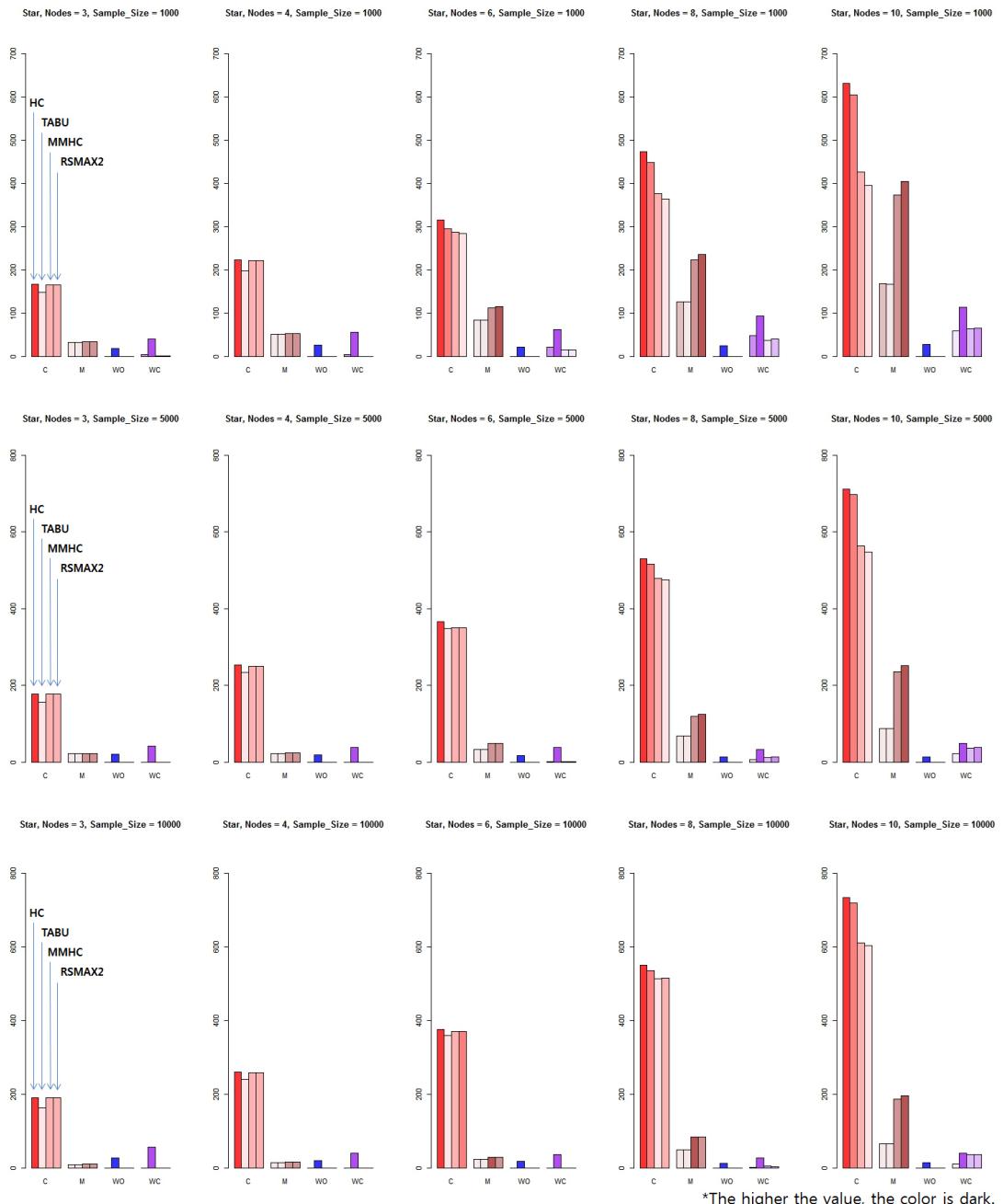


Figure 5.20: Comparison of scores via Star



\*The higher the value, the color is dark.

Figure 5.21: Comparison of correct arcs via Star

### 5.2.5 Pseudo Loop

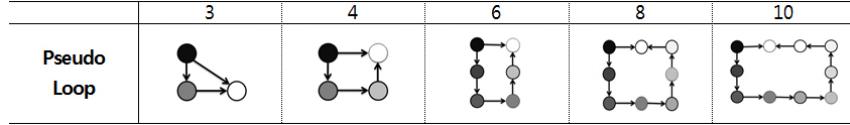


Figure 5.22: Bayesian Network Topologies : Pseudo Loop

At first, drew a line form. And next, root node has depended on the very last child node. Then it called Pseudo Loop. Actually loop does not have, it looks like a loop at first glance. (In fact, actually when loop is created, no longer Bayesian Network is not it.)

| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|-------------|-------|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 1000        | 2     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 3    | 1    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 3           | 2     | 1    | 4    | 3      | 1  | 4    | 2    | 2      | 4  | 3    | 1    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 4           | 2     | 1    | 4    | 3      | 1  | 4    | 2    | 2      | 4  | 3    | 1    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 6           | 2     | 1    | 4    | 3      | 1  | 4    | 3    | 2      | 3  | 4    | 1    | 2      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 8           | 2     | 1    | 3    | 4      | 1  | 4    | 2    | 2      | 3  | 4    | 1    | 1      | 4  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 10          | 2     | 1    | 4    | 3      | 1  | 4    | 2    | 3      | 4  | 3    | 2    | 1      | 4  | 1    | 4    | 4      | 2  | 1    | 3    | 4      |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 5000        | 2     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 3    | 1    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 3           | 2     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 4           | 2     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 6           | 1     | 1    | 3    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 4  | 1    | 4    | 4      | 4  | 1    | 4    | 4      |
| 8           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 1    | 4    | 4      | 4  | 1    | 3    | 2      |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 1    | 4    | 4      | 4  | 1    | 3    | 2      |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 10000       | 2     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 3    | 1    | 1      | 4  | 1    | 4    | 4      | 4  | 1    | 4    | 4      |
| 3           | 2     | 1    | 4    | 4      | 1  | 4    | 2    | 2      | 4  | 4    | 1    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 4           | 4     | 1    | 1    | 3      | 3  | 4    | 1    | 2      | 4  | 4    | 4    | 1      | 2  | 1    | 4    | 4      | 4  | 1    | 4    | 4      |
| 6           | 1     | 1    | 4    | 3      | 1  | 4    | 3    | 2      | 4  | 4    | 1    | 2      | 4  | 1    | 4    | 4      | 4  | 1    | 4    | 4      |
| 8           | 1     | 1    | 4    | 3      | 1  | 2    | 4    | 3      | 4  | 4    | 4    | 1      | 2  | 4    | 1    | 4      | 4  | 1    | 2    | 3      |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 4  | 4    | 2    | 1      | 4  | 1    | 4    | 4      | 4  | 1    | 2    | 2      |

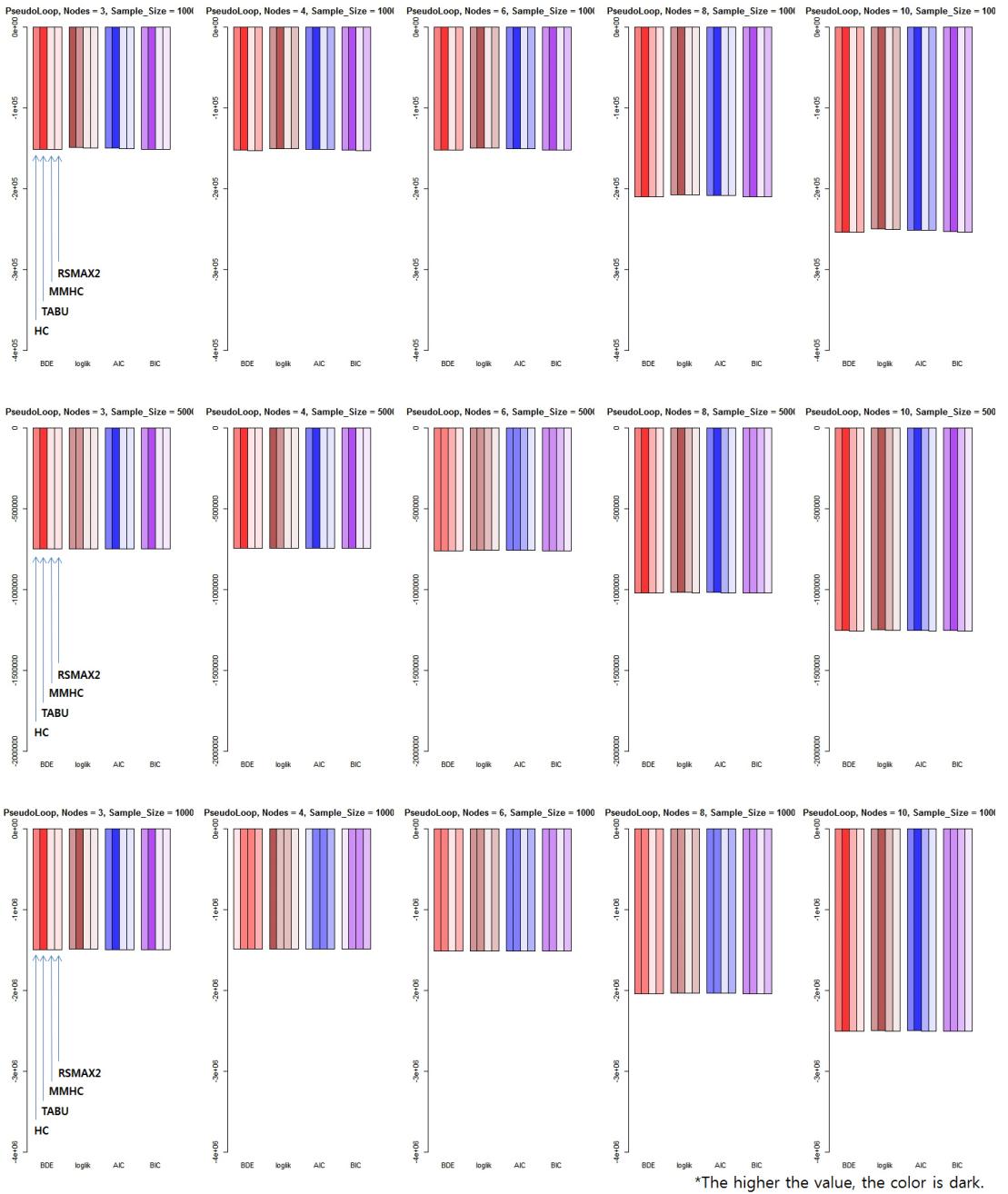
Figure 5.23: Summary for Comparison via Pseudo Loop

Although TABU search when compared on the basis of exhibited good performance Score, when compared to the network and learning network objectives, relatively Hill-climbing showed good performance in the line form.

When the sample size is 1000, the number of C by TABU search has not been improved. I shows how the number of C when sample size is larger, then

greatly improved. In particular M, WO, and WC with increasing sample size, reduced noticeable. And yet, did not win to the performance of Hill-climbing.

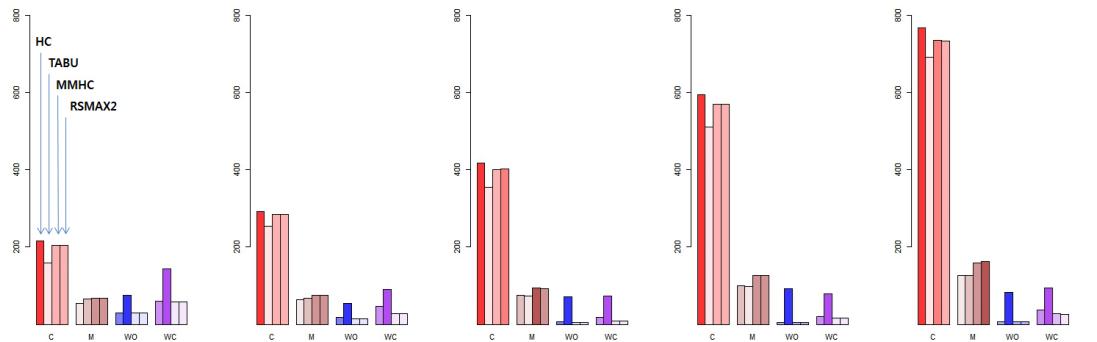
All algorithms while sample size increases, WO and WC is greatly reduced.



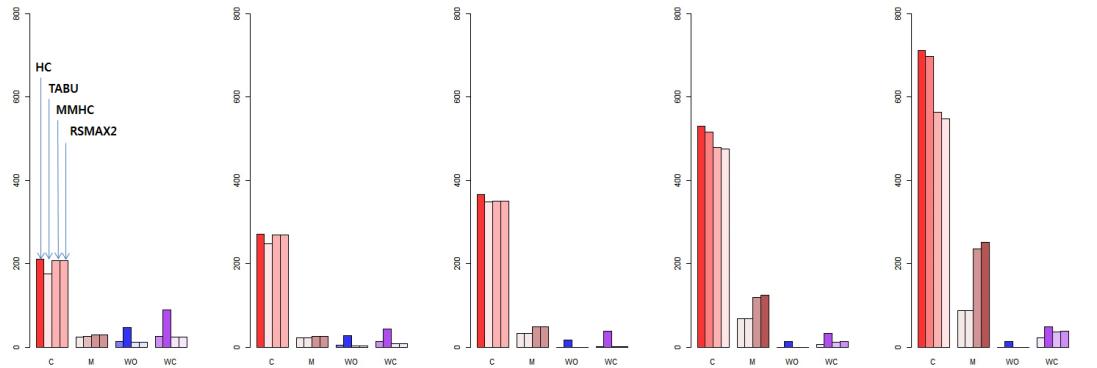
\*The higher the value, the color is dark.

Figure 5.24: Comparison of scores via Pseudo Loop

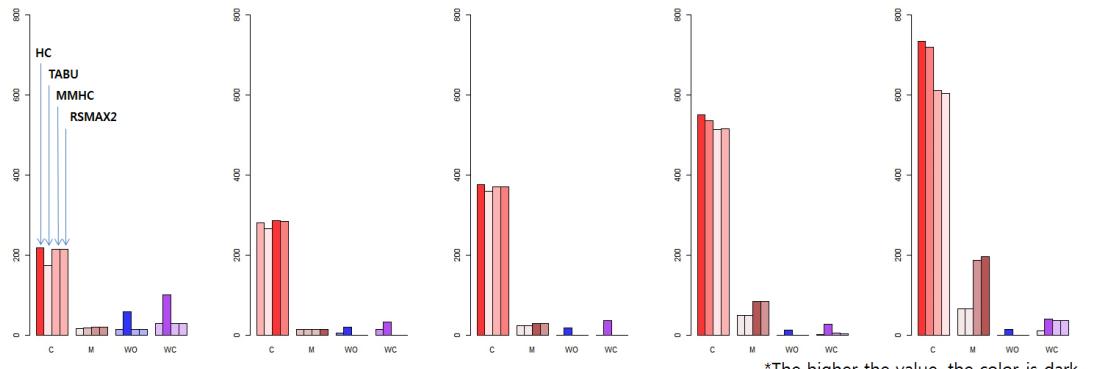
PseudoLoop, Nodes = 3, Sample\_Size = 100 PseudoLoop, Nodes = 4, Sample\_Size = 100 PseudoLoop, Nodes = 6, Sample\_Size = 100 PseudoLoop, Nodes = 8, Sample\_Size = 100 PseudoLoop, Nodes = 10, Sample\_Size = 100



PseudoLoop, Nodes = 3, Sample\_Size = 500 PseudoLoop, Nodes = 4, Sample\_Size = 500 PseudoLoop, Nodes = 6, Sample\_Size = 500 PseudoLoop, Nodes = 8, Sample\_Size = 500 PseudoLoop, Nodes = 10, Sample\_Size = 500



PseudoLoop, Nodes = 3, Sample\_Size = 1000 PseudoLoop, Nodes = 4, Sample\_Size = 1000 PseudoLoop, Nodes = 6, Sample\_Size = 1000 PseudoLoop, Nodes = 8, Sample\_Size = 1000 PseudoLoop, Nodes = 10, Sample\_Size = 1000



\*The higher the value, the color is dark.

Figure 5.25: Comparison of correct arcs via Pseudo Loop

### 5.2.6 Diamond

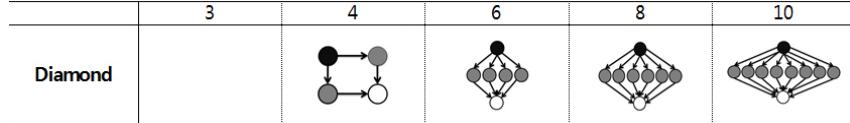


Figure 5.26: Bayesian Network Topologies : Diamond

A part of the top, one node has plurality of child node like Star form. And the bottom part, one node has plurality of parent node like Collapse form. If it connected, then it called Diamond.

| Sample Size<br>1000  | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|----------------------|-------|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|
|                      | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 4                    | 2     | 1    | 4    | 3      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 3      |
| 6                    | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 3      | 2  | 1    | 4    | 3      |
| 8                    | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 4  | 4    | 3    | 1      | 4  | 2    | 1    | 4      |
| 10                   | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 3    | 4      | 3  | 1    | 4    | 2      |
| Sample Size<br>5000  | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|                      | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 4                    | 2     | 1    | 4    | 3      | 2  | 1    | 3    | 4      | 3  | 4    | 1    | 1      | 2  | 1    | 4    | 3      | 2  | 1    | 4    | 3      |
| 6                    | 2     | 1    | 4    | 3      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 3      |
| 8                    | 2     | 1    | 3    | 4      | 2  | 1    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 3      | 1  | 2    | 4    | 3      |
| 10                   | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 3      | 2  | 1    | 4    | 3      |
| Sample Size<br>10000 | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|                      | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 4                    | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 4      | 1  | 1    | 4    | 4      |
| 6                    | 2     | 1    | 4    | 3      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 4      | 2  | 1    | 4    | 3      |
| 8                    | 2     | 1    | 4    | 3      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 4    | 3      | 1  | 2    | 4    | 3      |
| 10                   | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 3    | 4      | 2  | 3    | 4    | 1      |

Figure 5.27: Summary for Comparison via Diamond

Respectively, when compared to the score and C, TABU search and Hill-climbing showed a good performance. However, WO, WC also showed that many.

when compared to the WO and WC, MMHC showed that advantageous. RSMAX2 showed a lot of WC.

This phenomenon was stood out when the larger the sample size or the node.

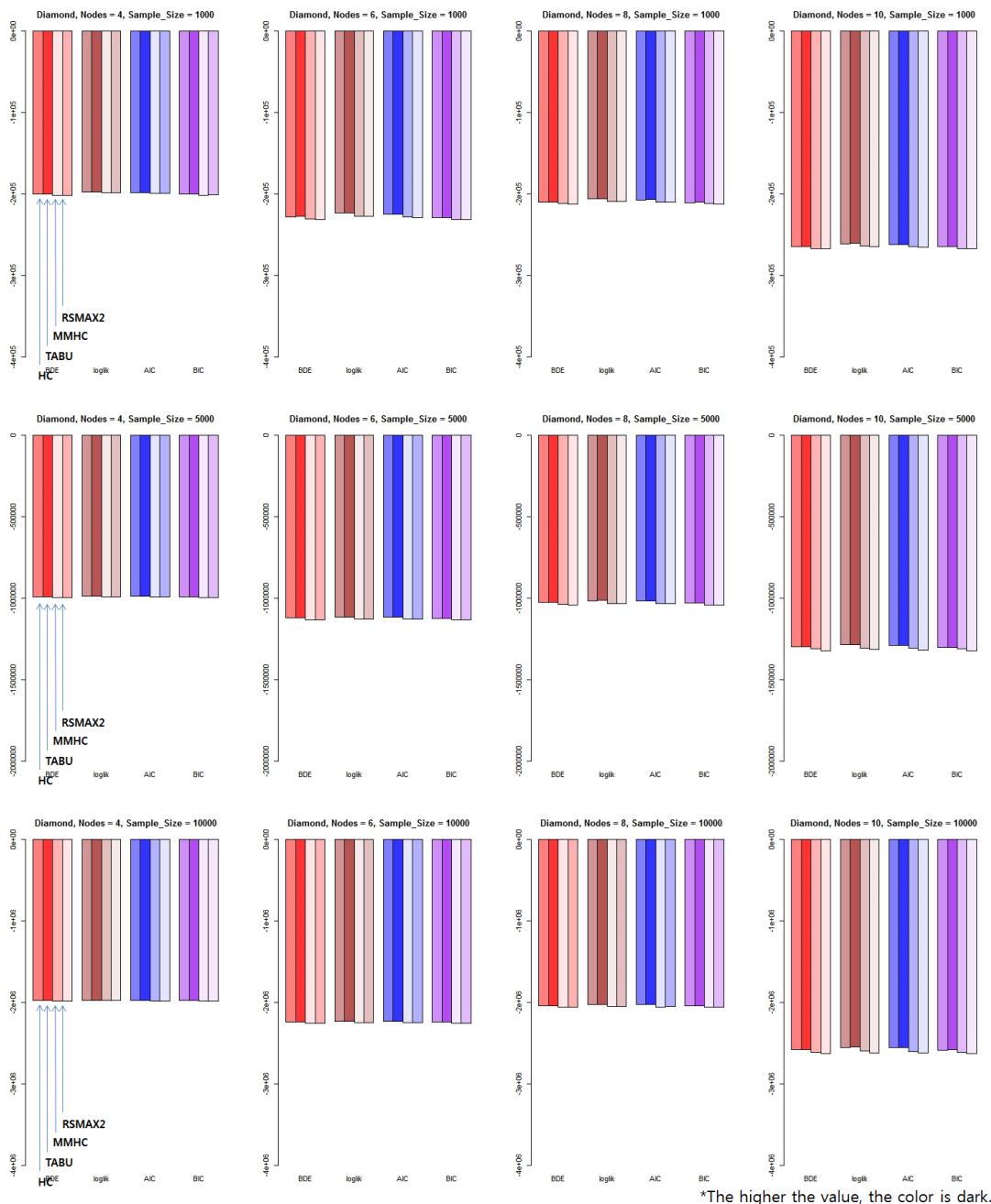


Figure 5.28: Comparison of scores via Diamond

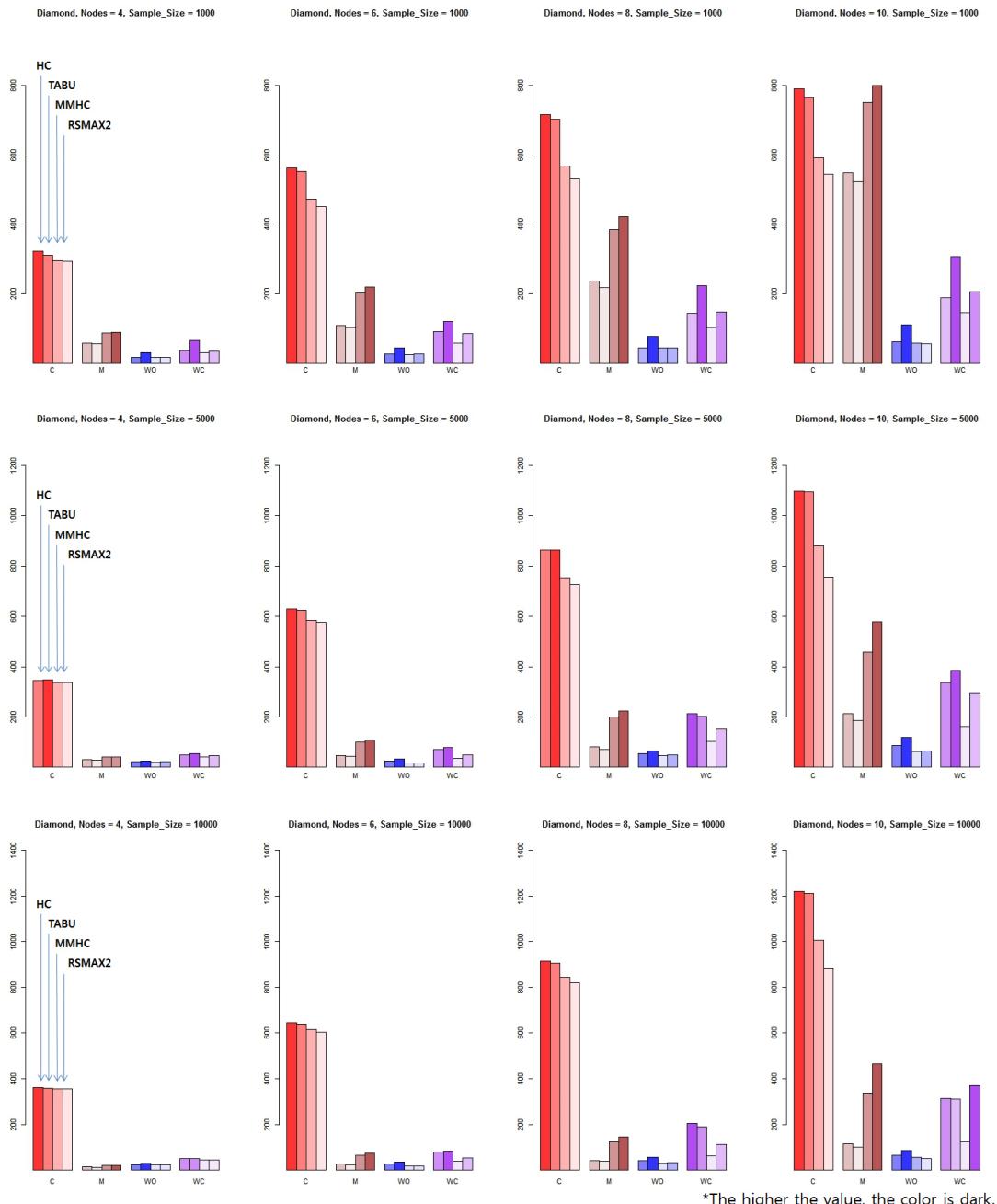


Figure 5.29: Comparison of correct arcs via Diamond

### 5.2.7 Rhombus

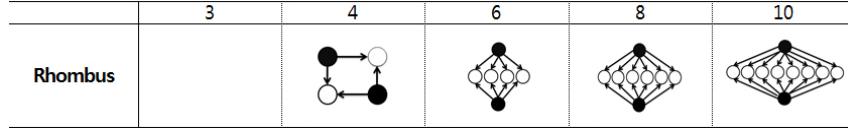


Figure 5.30: Bayesian Network Topologies : Rhombus

If two nodes has plurality of child node together, then it called Rhombus.

| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|-------------|-------|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|----|------|------|--------|
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 4           | 2     | 1    | 4    | 3      | 1  | 2    | 4    | 3      | 4  | 3    | 1    | 2      | 4  | 1    | 4    | 4      | 2  | 1    | 4    | 4      |
| 6           | 2     | 1    | 3    | 4      | 1  | 2    | 4    | 3      | 4  | 3    | 2    | 1      | 2  | 1    | 3    | 4      | 2  | 1    | 3    | 4      |
| 8           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 3    | 4      | 2  | 1    | 3    | 4      |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 3  | 1    | 2    | 4      | 3  | 1    | 2    | 4      |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 4           | 2     | 1    | 4    | 3      | 1  | 2    | 4    | 3      | 3  | 4    | 1    | 2      | 2  | 1    | 4    | 4      | 4  | 1    | 2    | 4      |
| 6           | 2     | 1    | 3    | 4      | 1  | 2    | 4    | 3      | 4  | 3    | 1    | 2      | 3  | 1    | 2    | 4      | 3  | 1    | 2    | 4      |
| 8           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 2    | 4      | 3  | 1    | 2    | 4      |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 3  | 1    | 2    | 4      | 3  | 2    | 1    | 4      |
| Sample Size | Score |      |      |        | C  |      |      |        | M  |      |      |        | WO |      |      |        | WC |      |      |        |
|             | HC    | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 | HC | TABU | MMHC | RSMAX2 |
| 4           | 2     | 1    | 4    | 3      | 1  | 2    | 4    | 3      | 3  | 4    | 1    | 2      | 4  | 1    | 4    | 4      | 4  | 1    | 2    | 4      |
| 6           | 2     | 1    | 3    | 4      | 1  | 2    | 4    | 3      | 4  | 3    | 1    | 2      | 3  | 1    | 2    | 4      | 3  | 1    | 2    | 4      |
| 8           | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 2  | 1    | 2    | 4      | 3  | 1    | 2    | 4      |
| 10          | 2     | 1    | 3    | 4      | 1  | 2    | 3    | 4      | 3  | 4    | 2    | 1      | 3  | 1    | 2    | 4      | 3  | 2    | 1    | 4      |

Figure 5.31: Summary for Comparison via Rhombus

Respectively, when compared to the score and C, TABU search and Hill-climbing showed a good performance. However, WO, WC also showed that many.

when compared to the WO and WC, RS MAX2 showed that advantageous. MMHC showed a lot of WC.

However as the sample size increases, how the performance of all algorithms is overall improvement.

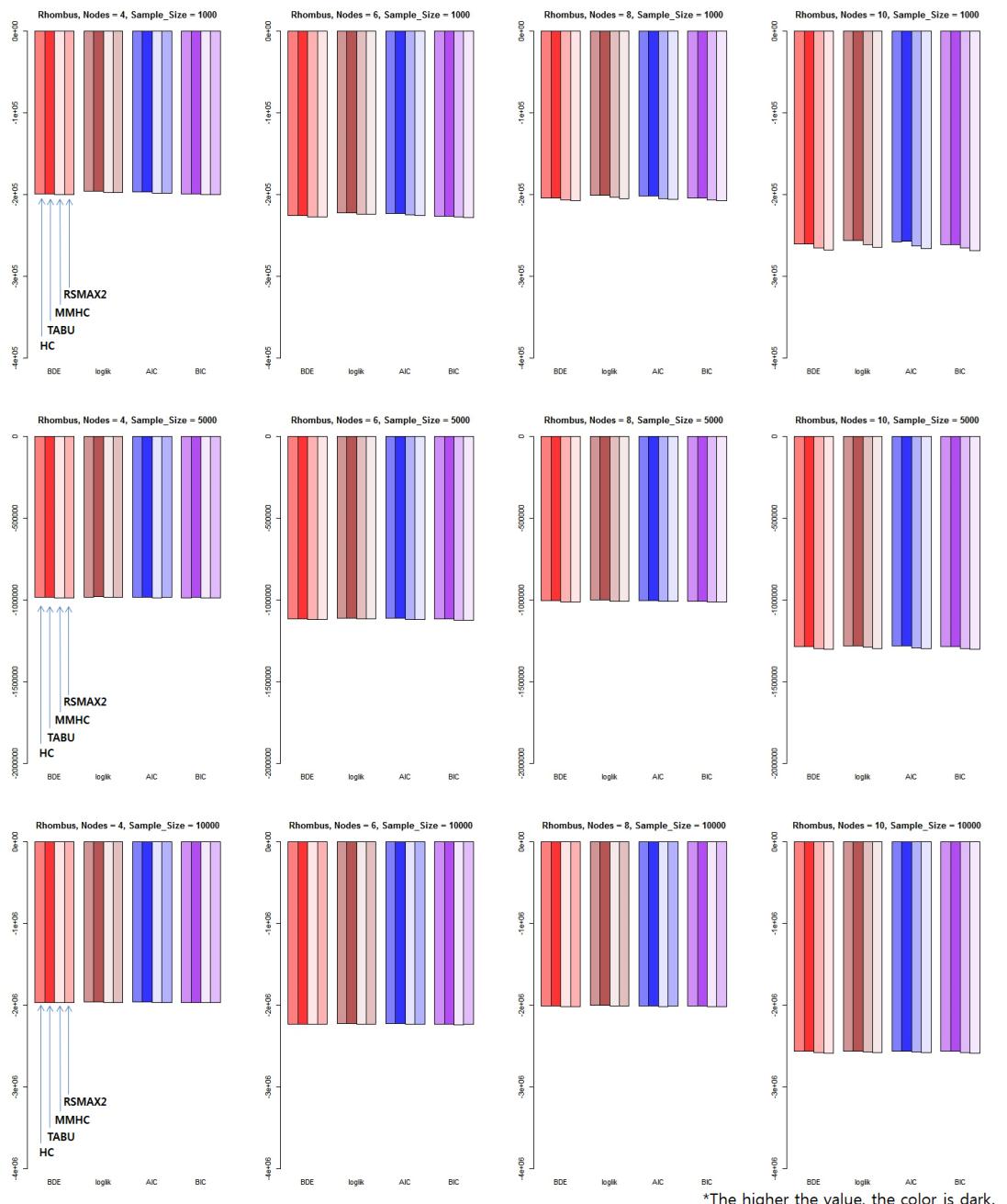


Figure 5.32: Comparison of scores via Rhombus

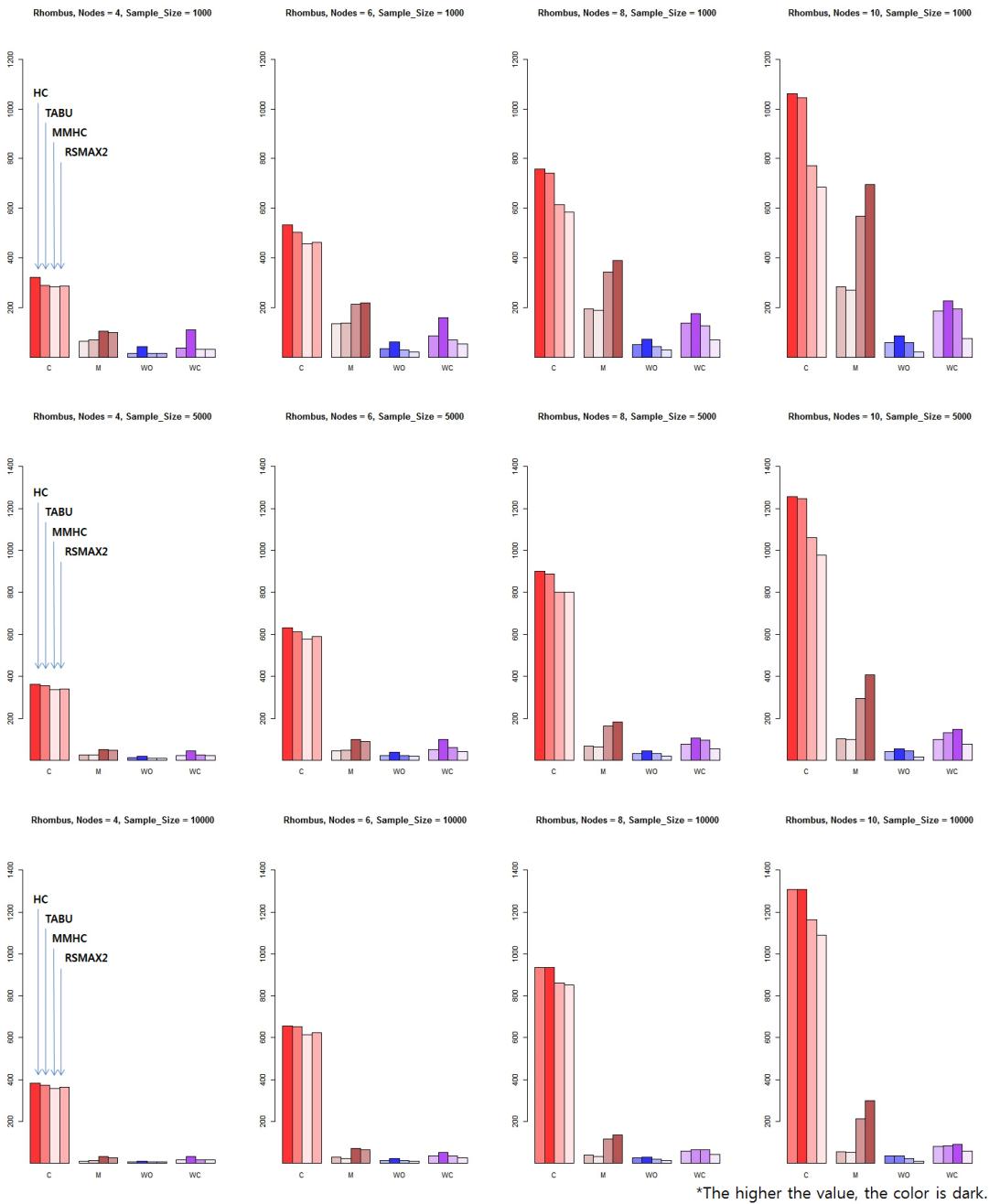


Figure 5.33: Comparison of correct arcs via Rhombus

# Chapter 6

## Discussion

Result of comparing the performance of each algorithm according to the Topology, TABU search has been found most best thing by score criteria. However, as a result of comparing between the target network and learning network directly, when a reference to "C is what is most" is, Hill-climbing was often appears that the performance is good. Rather TABU search has a lot of WO and WC.

Hybrid algorithm compared to Score-based algorithm is found to be that draw the arc more conservative. This makes not only C is often less missing arcs, but also WO and WC is drawn very small. It seems to use when WO and WC are fatal. Especially MMHC for Diamond form, RSMAX2 for Rhombus seems to be advantageous.

About Line and Star form, the performance difference due to relatively algorithm was not out compared to other topology.

In most of the topology, when the sample size is small, the number of C has not been improved. However, I shows how the number of C when sample

size is larger, then greatly improved. In addition, Sample size is larger, then the M, WO and WC is decreases.

On the basis of these results, algorithm users will be able to try to consider whether to choose what algorithms depending on whether their target network is any way. Especially, when the many M, WO, WC is fatal, it will be able to try to consider the selection of hybrid algorithm.

In future study, it can be to increase the number of node topology, to less sample size, or to increase the cardinality. Or by applying another algorithms, in addition it is possible to compare analyzed by combining two or more mutually topology.

In this paper, the probability when defining the relationship between the probability gave arbitrarily value between  $U(0, 1)$ . But it is possible to confirm the relationship between those given this probability "sequential" in future study.

It is desirable to complete the R package of Bayesian network data generator more than anything else.

In addition, analysis using the continuous data. Also we will be able to control the missing value of using BN. In this case it is possible to actively utilize Bayesian network data generator.

# Appendix

## A.1 Table for Collapse

Table 6.1: Comparison via Collapse (Num of Nodes = 3)

|             |        | Collapse (Num of Nodes = 3) |          |         |          |          |          |      |          |      |          |       |          |
|-------------|--------|-----------------------------|----------|---------|----------|----------|----------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                        |          | 5000    |          | 10000    |          | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                        | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -149310                     | 277.03   | -739492 | 1456.67  | -1493172 | 3026.14  | C    | HC       | 164  | 0.56     | 178   | 0.44     |
|             | TABU   | -149073                     | 273.72   | -738889 | 1448.5   | -1493171 | 3026.14  |      | TABU     | 152  | 0.67     | 171   | 0.57     |
|             | MMHC   | -149516                     | 279.61   | -740618 | 1469.62  | -1494739 | 3034.04  |      | MMHC     | 154  | 0.58     | 170   | 0.48     |
|             | RSMAX2 | -149516                     | 279.61   | -740618 | 1469.62  | -1494739 | 3034.04  |      | RSMAX2   | 154  | 0.58     | 170   | 0.48     |
| loglik      | HC     | -147279                     | 281.43   | -736966 | 1461.14  | -1490429 | 3030.09  | M    | HC       | 32   | 0.55     | 13    | 0.37     |
|             | TABU   | -147026                     | 277.87   | -736352 | 1452.81  | -1490428 | 3030.09  |      | TABU     | 36   | 0.52     | 14    | 0.35     |
|             | MMHC   | -147531                     | 284.24   | -738143 | 1474.42  | -1492072 | 3038.37  |      | MMHC     | 42   | 0.57     | 21    | 0.43     |
|             | RSMAX2 | -147531                     | 284.24   | -738143 | 1474.42  | -1492072 | 3038.37  |      | RSMAX2   | 42   | 0.57     | 21    | 0.43     |
| AIC         | HC     | -147817                     | 281.67   | -737532 | 1461.32  | -1491018 | 3030.17  | WO   | HC       | 4    | 0.2      | 9     | 0.29     |
|             | TABU   | -147570                     | 278.21   | -736921 | 1453.04  | -1491017 | 3030.17  |      | TABU     | 12   | 0.33     | 15    | 0.41     |
|             | MMHC   | -148049                     | 284.4    | -738693 | 1474.51  | -1492635 | 3038.37  |      | MMHC     | 4    | 0.2      | 9     | 0.29     |
|             | RSMAX2 | -148049                     | 284.4    | -738693 | 1474.51  | -1492635 | 3038.37  |      | RSMAX2   | 4    | 0.2      | 9     | 0.29     |
| BIC         | HC     | -149137                     | 282.27   | -739376 | 1461.93  | -1493141 | 3030.43  | WC   | HC       | 12   | 0.48     | 18    | 0.58     |
|             | TABU   | -148904                     | 279.05   | -738775 | 1453.8   | -1493140 | 3030.43  |      | TABU     | 44   | 1.16     | 36    | 1        |
|             | MMHC   | -149320                     | 284.82   | -740485 | 1474.84  | -1494664 | 3038.36  |      | MMHC     | 12   | 0.48     | 18    | 0.58     |
|             | RSMAX2 | -149320                     | 284.82   | -740485 | 1474.84  | -1494664 | 3038.36  |      | RSMAX2   | 12   | 0.48     | 18    | 0.58     |

Table 6.2: Comparison via Collapse (Num of Nodes = 4)

|             |        | Collapse (Num of Nodes = 4) |          |         |          |          |          |      |          |      |          |       |          |     |      |
|-------------|--------|-----------------------------|----------|---------|----------|----------|----------|------|----------|------|----------|-------|----------|-----|------|
| Sample Size |        | 1000                        |          | 5000    |          | 10000    |          | 1000 |          | 5000 |          | 10000 |          |     |      |
|             |        | Sum.                        | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |     |      |
| BDe         | HC     | -149071                     | 917.34   | -749900 | 4601.22  | -1577564 | 9508.45  | C    | HC       | 230  | 0.78     | 256   | 0.66     | 260 | 0.62 |
|             | TABU   | -149032                     | 917.06   | -749097 | 4595.92  | -1577556 | 9508.39  |      | TABU     | 219  | 0.92     | 254   | 0.72     | 259 | 0.68 |
|             | MMHC   | -149952                     | 924.62   | -751640 | 4613.92  | -1582394 | 9544.08  |      | MMHC     | 198  | 0.84     | 240   | 0.74     | 242 | 0.7  |
|             | RSMAX2 | -149735                     | 923.06   | -751640 | 4613.92  | -1582394 | 9544.08  |      | RSMAX2   | 204  | 0.83     | 240   | 0.74     | 242 | 0.7  |
| loglik      | HC     | -146796                     | 905.56   | -746922 | 4585.58  | -1574342 | 9490.79  | M    | HC       | 39   | 0.65     | 13    | 0.39     | 8   | 0.34 |
|             | TABU   | -146749                     | 905.21   | -746106 | 4580.17  | -1574344 | 9490.81  |      | TABU     | 38   | 0.58     | 11    | 0.31     | 8   | 0.34 |
|             | MMHC   | -147865                     | 914.28   | -748779 | 4599.09  | -1579382 | 9527.98  |      | MMHC     | 69   | 0.73     | 28    | 0.51     | 27  | 0.55 |
|             | RSMAX2 | -147612                     | 912.5    | -748779 | 4599.09  | -1579382 | 9527.98  |      | RSMAX2   | 65   | 0.72     | 28    | 0.51     | 27  | 0.55 |
| AIC         | HC     | -147508                     | 909.68   | -747706 | 4589.96  | -1575139 | 9495.35  | WO   | HC       | 6    | 0.24     | 6     | 0.24     | 7   | 0.29 |
|             | TABU   | -147464                     | 909.37   | -746895 | 4584.59  | -1575138 | 9495.35  |      | TABU     | 18   | 0.39     | 10    | 0.3      | 8   | 0.37 |
|             | MMHC   | -148493                     | 917.79   | -749518 | 4603.18  | -1580114 | 9532.07  |      | MMHC     | 8    | 0.27     | 7     | 0.26     | 6   | 0.28 |
|             | RSMAX2 | -148254                     | 916.09   | -749518 | 4603.18  | -1580114 | 9532.07  |      | RSMAX2   | 6    | 0.24     | 7     | 0.26     | 6   | 0.28 |
| BIC         | HC     | -149255                     | 919.82   | -750261 | 4604.23  | -1578012 | 9511.79  | WC   | HC       | 20   | 0.67     | 14    | 0.59     | 18  | 0.81 |
|             | TABU   | -149218                     | 919.58   | -749466 | 4599     | -1578001 | 9511.7   |      | TABU     | 60   | 1.26     | 28    | 0.9      | 16  | 0.73 |
|             | MMHC   | -150034                     | 926.42   | -751926 | 4616.5   | -1582753 | 9546.82  |      | MMHC     | 24   | 0.71     | 14    | 0.51     | 12  | 0.56 |
|             | RSMAX2 | -149830                     | 924.93   | -751926 | 4616.5   | -1582753 | 9546.82  |      | RSMAX2   | 20   | 0.67     | 14    | 0.51     | 12  | 0.56 |

Table 6.3: Comparison via Collapse (Num of Nodes = 6)

|             |        | Collapse (Num of Nodes = 6) |          |         |          |          |          |      |          |      |          |       |          |     |      |
|-------------|--------|-----------------------------|----------|---------|----------|----------|----------|------|----------|------|----------|-------|----------|-----|------|
| Sample Size |        | 1000                        |          | 5000    |          | 10000    |          | 1000 |          | 5000 |          | 10000 |          |     |      |
|             |        | Sum.                        | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |     |      |
| BDe         | HC     | -155997                     | 1596.19  | -764954 | 7828.59  | -1544669 | 15723.4  | C    | HC       | 284  | 0.97     | 371   | 1.04     | 384 | 1.1  |
|             | TABU   | -155793                     | 1593.85  | -764558 | 7823     | -1544669 | 15723.4  |      | TABU     | 284  | 0.99     | 372   | 1.01     | 384 | 1.1  |
|             | MMHC   | -157648                     | 1613.35  | -772886 | 7916.61  | -1555104 | 15832.76 |      | MMHC     | 225  | 0.88     | 327   | 0.98     | 337 | 1.08 |
|             | RSMAX2 | -157613                     | 1612.99  | -772786 | 7916.76  | -1555105 | 15832.78 |      | RSMAX2   | 229  | 0.88     | 328   | 0.99     | 337 | 1.08 |
| loglik      | HC     | -153357                     | 1570.64  | -759643 | 7774.48  | -1538497 | 15660.96 | M    | HC       | 88   | 1        | 18    | 0.5      | 9   | 0.35 |
|             | TABU   | -153059                     | 1567.21  | -759326 | 7769.87  | -1538497 | 15660.96 |      | TABU     | 78   | 0.92     | 13    | 0.37     | 9   | 0.35 |
|             | MMHC   | -155647                     | 1594.77  | -768924 | 7878.02  | -1550300 | 15784.66 |      | MMHC     | 144  | 1.2      | 63    | 0.85     | 56  | 0.7  |
|             | RSMAX2 | -155594                     | 1594.24  | -768820 | 7878.3   | -1550304 | 15784.72 |      | RSMAX2   | 141  | 1.21     | 61    | 0.84     | 56  | 0.7  |
| AIC         | HC     | -154234                     | 1579.42  | -761260 | 7791.09  | -1540234 | 15678.54 | WO   | HC       | 28   | 0.55     | 11    | 0.31     | 7   | 0.29 |
|             | TABU   | -153974                     | 1576.44  | -760910 | 7786.08  | -1540234 | 15678.54 |      | TABU     | 38   | 0.56     | 15    | 0.39     | 7   | 0.29 |
|             | MMHC   | -156234                     | 1600.46  | -770049 | 7889.11  | -1551579 | 15797.54 |      | MMHC     | 31   | 0.56     | 10    | 0.3      | 7   | 0.33 |
|             | RSMAX2 | -156189                     | 1600.01  | -769950 | 7889.37  | -1551582 | 15797.59 |      | RSMAX2   | 30   | 0.56     | 11    | 0.31     | 7   | 0.33 |
| BIC         | HC     | -156386                     | 1601.03  | -766529 | 7845.25  | -1546496 | 15741.92 | WC   | HC       | 88   | 1.74     | 42    | 1.56     | 24  | 0.95 |
|             | TABU   | -156219                     | 1599.16  | -766072 | 7838.94  | -1546496 | 15741.92 |      | TABU     | 136  | 2.16     | 54    | 1.53     | 24  | 0.95 |
|             | MMHC   | -157675                     | 1614.46  | -773714 | 7925.28  | -1556190 | 15843.99 |      | MMHC     | 72   | 1.19     | 24    | 0.77     | 16  | 0.68 |
|             | RSMAX2 | -157649                     | 1614.2   | -773632 | 7925.46  | -1556190 | 15844    |      | RSMAX2   | 70   | 1.18     | 22    | 0.63     | 14  | 0.65 |

Table 6.4: Comparison via Collapse (Num of Nodes = 8)

|             |        | Collapse (Num of Nodes = 8) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|-----------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                        |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                        | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -207806                     | 2118.77  | -1014209 | 10346.61 | -2008048 | 20599.86 | C  | HC     | 309  | 1.2      | 514  | 1.05     | 557   | 0.92     |
|             | TABU   | -207509                     | 2115.9   | -1014025 | 10345.07 | -2007946 | 20598.88 |    | TABU   | 316  | 1.15     | 516  | 0.98     | 556   | 0.92     |
|             | MMHC   | -209178                     | 2132.1   | -1029358 | 10508.2  | -2032117 | 20868.83 |    | MMHC   | 209  | 1.02     | 396  | 1.15     | 464   | 1.03     |
|             | RSMAX2 | -209194                     | 2132.26  | -1029043 | 10506.55 | -2032628 | 20875.32 |    | RSMAX2 | 209  | 1.03     | 399  | 1.18     | 463   | 1.04     |
| loglik      | HC     | -204583                     | 2088.08  | -1002152 | 10217.44 | -1991302 | 20420.06 | M  | HC     | 229  | 1.49     | 55   | 0.73     | 26    | 0.48     |
|             | TABU   | -204078                     | 2083.15  | -1001816 | 10214.49 | -1991074 | 20417.87 |    | TABU   | 198  | 1.36     | 39   | 0.58     | 23    | 0.47     |
|             | MMHC   | -206772                     | 2109.57  | -1023648 | 10452.84 | -2022890 | 20778.21 |    | MMHC   | 326  | 1.54     | 175  | 1.32     | 120   | 0.97     |
|             | RSMAX2 | -206802                     | 2109.89  | -1023214 | 10450.49 | -2023458 | 20785.37 |    | RSMAX2 | 325  | 1.59     | 170  | 1.33     | 121   | 1        |
| AIC         | HC     | -205579                     | 2097.77  | -1005799 | 10256.47 | -1996126 | 20471.38 | WO | HC     | 62   | 0.75     | 31   | 0.6      | 17    | 0.4      |
|             | TABU   | -205163                     | 2093.71  | -1005524 | 10254.08 | -1995938 | 20469.59 |    | TABU   | 86   | 0.89     | 45   | 0.7      | 21    | 0.46     |
|             | MMHC   | -207409                     | 2115.72  | -1025215 | 10468.1  | -2025414 | 20803.07 |    | MMHC   | 65   | 0.73     | 29   | 0.59     | 16    | 0.44     |
|             | RSMAX2 | -207432                     | 2115.97  | -1024831 | 10466.06 | -2025965 | 20810.03 |    | RSMAX2 | 66   | 0.78     | 31   | 0.63     | 16    | 0.44     |
| BIC         | HC     | -208023                     | 2121.59  | -1017684 | 10383.91 | -2013517 | 20656.54 | WC | HC     | 198  | 2.34     | 134  | 2.94     | 98    | 2.17     |
|             | TABU   | -207826                     | 2119.67  | -1017607 | 10383.29 | -2013474 | 20656.17 |    | TABU   | 300  | 3.11     | 196  | 3.41     | 130   | 2.66     |
|             | MMHC   | -208972                     | 2130.83  | -1030321 | 10517.99 | -2034514 | 20892.87 |    | MMHC   | 144  | 1.48     | 68   | 1.28     | 52    | 1.16     |
|             | RSMAX2 | -208978                     | 2130.9   | -1030100 | 10516.95 | -2035003 | 20899.12 |    | RSMAX2 | 140  | 1.54     | 66   | 1.27     | 34    | 0.9      |

Table 6.5: Comparison via Collapse (Num of Nodes = 10)

|             |        | Collapse (Num of Nodes = 10) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|------------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                         |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                         | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -515936                      | 615.03   | -1287094 | 13077.37 | -2497620 | 25373.92 | C  | HC     | 246  | 1.26     | 547  | 1.23     | 638   | 0.81     |
|             | TABU   | -515720                      | 615.33   | -1286580 | 13072.14 | -2497070 | 25368.38 |    | TABU   | 228  | 1.42     | 553  | 1.09     | 639   | 0.78     |
|             | MMHC   | -517253                      | 607.4    | -1295331 | 13155.05 | -2516578 | 25568.05 |    | MMHC   | 165  | 0.98     | 376  | 1.36     | 504   | 1.33     |
|             | RSMAX2 | -517319                      | 607.92   | -1295470 | 13156.68 | -2517682 | 25579.82 |    | RSMAX2 | 168  | 0.97     | 376  | 1.42     | 501   | 1.31     |
| loglik      | HC     | -509066                      | 635.43   | -1274640 | 12955.38 | -2475008 | 25137.78 | M  | HC     | 583  | 1.52     | 181  | 1.35     | 110   | 1.02     |
|             | TABU   | -508662                      | 635.37   | -1273760 | 12946.24 | -2473823 | 25125.45 |    | TABU   | 562  | 1.6      | 142  | 1.16     | 90    | 0.86     |
|             | MMHC   | -511438                      | 622.4    | -1289913 | 13103.45 | -2505030 | 25455.77 |    | MMHC   | 678  | 1.19     | 351  | 1.61     | 252   | 1.59     |
|             | RSMAX2 | -511528                      | 623.55   | -1290159 | 13106.27 | -2506562 | 25472.12 |    | RSMAX2 | 675  | 1.23     | 350  | 1.66     | 258   | 1.62     |
| AIC         | HC     | -510914                      | 631.01   | -1278070 | 12988.78 | -2480768 | 25197.31 | WO | HC     | 71   | 0.9      | 72   | 0.89     | 52    | 0.72     |
|             | TABU   | -510592                      | 631.08   | -1277310 | 12980.91 | -2479777 | 25187.03 |    | TABU   | 110  | 1.05     | 105  | 1.09     | 71    | 0.89     |
|             | MMHC   | -512845                      | 620.3    | -1291245 | 13116.22 | -2507905 | 25483.56 |    | MMHC   | 57   | 0.78     | 73   | 0.85     | 44    | 0.66     |
|             | RSMAX2 | -512923                      | 621.19   | -1291461 | 13118.7  | -2509323 | 25498.71 |    | RSMAX2 | 57   | 0.73     | 74   | 0.85     | 41    | 0.64     |
| BIC         | HC     | -515449                      | 620.36   | -1289247 | 13097.94 | -2501533 | 25412.08 | WC | HC     | 232  | 2.74     | 346  | 4.52     | 258   | 3.79     |
|             | TABU   | -515328                      | 620.74   | -1288878 | 13094.17 | -2501242 | 25409.18 |    | TABU   | 368  | 3.11     | 488  | 5.27     | 370   | 4.55     |
|             | MMHC   | -516297                      | 615.28   | -1295586 | 13157.91 | -2518270 | 25584.02 |    | MMHC   | 164  | 1.83     | 176  | 2.02     | 116   | 1.59     |
|             | RSMAX2 | -516346                      | 615.51   | -1295704 | 13159.26 | -2519277 | 25594.78 |    | RSMAX2 | 140  | 1.52     | 152  | 1.68     | 84    | 1.28     |

## A.2 Table for Line

Table 6.6: Comparison via Line (Num of Nodes = 3)

|             |        | Line (Num of Nodes = 3) |          |         |          |          |          |      |          |      |          |       |          |     |      |
|-------------|--------|-------------------------|----------|---------|----------|----------|----------|------|----------|------|----------|-------|----------|-----|------|
| Sample Size |        | 1000                    |          | 5000    |          | 10000    |          | 1000 |          | 5000 |          | 10000 |          |     |      |
|             |        | Sum.                    | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |     |      |
| BDe         | HC     | -150015                 | 305.62   | -745538 | 1338.43  | -1487782 | 2685.73  | C    | HC       | 164  | 0.5      | 183   | 0.38     | 192 | 0.27 |
|             | TABU   | -150015                 | 305.62   | -745538 | 1338.43  | -1487782 | 2685.73  |      | TABU     | 130  | 0.77     | 155   | 0.76     | 157 | 0.78 |
|             | MMHC   | -150030                 | 305.5    | -745538 | 1338.43  | -1487783 | 2685.72  |      | MMHC     | 162  | 0.51     | 183   | 0.38     | 192 | 0.27 |
|             | RSMAX2 | -150030                 | 305.5    | -745538 | 1338.43  | -1487783 | 2685.72  |      | RSMAX2   | 162  | 0.51     | 183   | 0.38     | 192 | 0.27 |
| loglik      | HC     | -148189                 | 309.68   | -743251 | 1342.56  | -1485301 | 2689.93  | M    | HC       | 36   | 0.5      | 17    | 0.38     | 8   | 0.27 |
|             | TABU   | -148189                 | 309.68   | -743251 | 1342.56  | -1485301 | 2689.93  |      | TABU     | 36   | 0.5      | 17    | 0.38     | 8   | 0.27 |
|             | MMHC   | -148209                 | 309.51   | -743251 | 1342.56  | -1485306 | 2689.9   |      | MMHC     | 38   | 0.51     | 17    | 0.38     | 8   | 0.27 |
|             | RSMAX2 | -148209                 | 309.51   | -743251 | 1342.56  | -1485306 | 2689.9   |      | RSMAX2   | 38   | 0.51     | 17    | 0.38     | 8   | 0.27 |
| AIC         | HC     | -148654                 | 309.67   | -743734 | 1342.56  | -1485794 | 2689.88  | WO   | HC       | 0    | 0        | 0     | 0        | 0   | 0    |
|             | TABU   | -148654                 | 309.67   | -743734 | 1342.56  | -1485794 | 2689.88  |      | TABU     | 34   | 0.71     | 28    | 0.67     | 35  | 0.76 |
|             | MMHC   | -148671                 | 309.52   | -743734 | 1342.56  | -1485798 | 2689.86  |      | MMHC     | 0    | 0        | 0     | 0        | 0   | 0    |
|             | RSMAX2 | -148671                 | 309.52   | -743734 | 1342.56  | -1485798 | 2689.86  |      | RSMAX2   | 0    | 0        | 0     | 0        | 0   | 0    |
| BIC         | HC     | -149795                 | 309.64   | -745308 | 1342.57  | -1487571 | 2689.71  | WC   | HC       | 2    | 0.2      | 0     | 0        | 2   | 0.2  |
|             | TABU   | -149795                 | 309.64   | -745308 | 1342.57  | -1487571 | 2689.71  |      | TABU     | 42   | 0.82     | 32    | 0.74     | 38  | 0.79 |
|             | MMHC   | -149805                 | 309.56   | -745308 | 1342.57  | -1487571 | 2689.71  |      | MMHC     | 0    | 0        | 0     | 0        | 0   | 0    |
|             | RSMAX2 | -149805                 | 309.56   | -745308 | 1342.57  | -1487571 | 2689.71  |      | RSMAX2   | 0    | 0        | 0     | 0        | 0   | 0    |

Table 6.7: Comparison via Line (Num of Nodes = 4)

|             |        | Line (Num of Nodes = 4) |          |         |          |          |          |    |        |      |          |      |          |      |          |
|-------------|--------|-------------------------|----------|---------|----------|----------|----------|----|--------|------|----------|------|----------|------|----------|
| Sample Size |        | 1000                    |          | 5000    |          | 10000    |          |    | 1000   |      | 5000     |      | 10000    |      |          |
|             |        | Sum.                    | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |
| BDe         | HC     | -153818                 | 927.25   | -746004 | 4568.49  | -1488727 | 9127.48  | C  | HC     | 225  | 0.72     | 256  | 0.57     | 264  | 0.52     |
|             | TABU   | -153818                 | 927.25   | -746004 | 4568.49  | -1488727 | 9127.48  |    | TABU   | 197  | 0.88     | 212  | 0.95     | 210  | 1.03     |
|             | MMHC   | -153837                 | 927.32   | -746004 | 4568.49  | -1488728 | 9127.5   |    | MMHC   | 222  | 0.72     | 256  | 0.57     | 265  | 0.52     |
|             | RSMAX2 | -153837                 | 927.32   | -746004 | 4568.49  | -1488728 | 9127.5   |    | RSMAX2 | 222  | 0.72     | 256  | 0.57     | 265  | 0.52     |
| loglik      | HC     | -151947                 | 918.03   | -743641 | 4556.87  | -1486155 | 9114.79  | M  | HC     | 50   | 0.61     | 19   | 0.42     | 11   | 0.31     |
|             | TABU   | -151947                 | 918.03   | -743641 | 4556.87  | -1486155 | 9114.79  |    | TABU   | 50   | 0.61     | 19   | 0.42     | 11   | 0.31     |
|             | MMHC   | -151977                 | 918.17   | -743641 | 4556.87  | -1486157 | 9114.8   |    | MMHC   | 53   | 0.63     | 19   | 0.42     | 10   | 0.3      |
|             | RSMAX2 | -151977                 | 918.17   | -743641 | 4556.87  | -1486157 | 9114.8   |    | RSMAX2 | 53   | 0.63     | 19   | 0.42     | 10   | 0.3      |
| AIC         | HC     | -152436                 | 920.74   | -744150 | 4559.65  | -1486672 | 9117.62  | WO | HC     | 0    | 0        | 0    | 0        | 0    | 0        |
|             | TABU   | -152436                 | 920.74   | -744150 | 4559.65  | -1486672 | 9117.62  |    | TABU   | 28   | 0.73     | 44   | 0.89     | 54   | 1.01     |
|             | MMHC   | -152461                 | 920.85   | -744150 | 4559.65  | -1486674 | 9117.63  |    | MMHC   | 0    | 0        | 0    | 0        | 0    | 0        |
|             | RSMAX2 | -152461                 | 920.85   | -744150 | 4559.65  | -1486674 | 9117.63  |    | RSMAX2 | 0    | 0        | 0    | 0        | 0    | 0        |
| BIC         | HC     | -153636                 | 927.38   | -745808 | 4568.73  | -1488536 | 9127.83  | WC | HC     | 8    | 0.39     | 0    | 0        | 2    | 0.2      |
|             | TABU   | -153636                 | 927.38   | -745808 | 4568.73  | -1488536 | 9127.83  |    | TABU   | 36   | 0.77     | 42   | 0.82     | 50   | 0.87     |
|             | MMHC   | -153649                 | 927.43   | -745808 | 4568.73  | -1488538 | 9127.84  |    | MMHC   | 4    | 0.28     | 0    | 0        | 0    | 0        |
|             | RSMAX2 | -153649                 | 927.43   | -745808 | 4568.73  | -1488538 | 9127.84  |    | RSMAX2 | 4    | 0.28     | 0    | 0        | 0    | 0        |

Table 6.8: Comparison via Line (Num of Nodes = 6)

|             |        | Line (Num of Nodes = 6) |          |         |          |          |          |    |        |      |          |      |          |      |          |
|-------------|--------|-------------------------|----------|---------|----------|----------|----------|----|--------|------|----------|------|----------|------|----------|
| Sample Size |        | 1000                    |          | 5000    |          | 10000    |          |    | 1000   |      | 5000     |      | 10000    |      |          |
|             |        | Sum.                    | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |
| BDe         | HC     | -151628                 | 1546.55  | -743984 | 7648.25  | -1486614 | 15279.84 | C  | HC     | 335  | 1.13     | 371  | 1        | 381  | 0.98     |
|             | TABU   | -151627                 | 1546.54  | -743980 | 7648.24  | -1486613 | 15279.82 |    | TABU   | 286  | 1.41     | 313  | 1.43     | 312  | 1.55     |
|             | MMHC   | -151632                 | 1546.59  | -743988 | 7648.28  | -1486619 | 15279.86 |    | MMHC   | 330  | 1.17     | 371  | 1        | 381  | 0.96     |
|             | RSMAX2 | -151635                 | 1546.62  | -743988 | 7648.28  | -1486619 | 15279.86 |    | RSMAX2 | 330  | 1.17     | 371  | 1        | 381  | 0.96     |
| loglik      | HC     | -149731                 | 1528.3   | -741596 | 7625.59  | -1484031 | 15255.22 | M  | HC     | 65   | 0.73     | 28   | 0.53     | 19   | 0.42     |
|             | TABU   | -149727                 | 1528.27  | -741591 | 7625.58  | -1484026 | 15255.16 |    | TABU   | 64   | 0.72     | 28   | 0.53     | 19   | 0.42     |
|             | MMHC   | -149739                 | 1528.39  | -741603 | 7625.64  | -1484039 | 15255.26 |    | MMHC   | 70   | 0.77     | 28   | 0.53     | 19   | 0.42     |
|             | RSMAX2 | -149744                 | 1528.43  | -741603 | 7625.64  | -1484039 | 15255.26 |    | RSMAX2 | 70   | 0.77     | 28   | 0.53     | 19   | 0.42     |
| AIC         | HC     | -150243                 | 1533.39  | -742128 | 7630.8   | -1484566 | 15260.47 | WO | HC     | 0    | 0        | 1    | 0.1      | 0    | 0        |
|             | TABU   | -150240                 | 1533.36  | -742124 | 7630.79  | -1484562 | 15260.42 |    | TABU   | 50   | 1.04     | 59   | 1.12     | 69   | 1.24     |
|             | MMHC   | -150249                 | 1533.46  | -742134 | 7630.84  | -1484573 | 15260.51 |    | MMHC   | 0    | 0        | 1    | 0.1      | 0    | 0        |
|             | RSMAX2 | -150253                 | 1533.49  | -742134 | 7630.84  | -1484573 | 15260.51 |    | RSMAX2 | 0    | 0        | 1    | 0.1      | 0    | 0        |
| BIC         | HC     | -151499                 | 1545.88  | -743862 | 7647.79  | -1486495 | 15279.39 | WC | HC     | 6    | 0.34     | 6    | 0.34     | 2    | 0.2      |
|             | TABU   | -151499                 | 1545.88  | -743860 | 7647.79  | -1486495 | 15279.39 |    | TABU   | 48   | 0.86     | 54   | 0.94     | 60   | 1.01     |
|             | MMHC   | -151501                 | 1545.9   | -743864 | 7647.81  | -1486498 | 15279.41 |    | MMHC   | 2    | 0.2      | 4    | 0.28     | 0    | 0        |
|             | RSMAX2 | -151502                 | 1545.91  | -743864 | 7647.81  | -1486498 | 15279.41 |    | RSMAX2 | 0    | 0        | 4    | 0.28     | 0    | 0        |

Table 6.9: Comparison via Line (Num of Nodes = 8)

|             |        | Line (Num of Nodes = 8) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|-------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                    |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                    | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -209953                 | 2122.64  | -1020788 | 10373.59 | -2039890 | 20730.72 | C  | HC     | 513  | 1.25     | 544  | 1.13     | 557   | 1.04     |
|             | TABU   | -209950                 | 2122.62  | -1020788 | 10373.59 | -2039890 | 20730.72 |    | TABU   | 457  | 1.8      | 469  | 1.77     | 493   | 1.62     |
|             | MMHC   | -209969                 | 2122.8   | -1020795 | 10373.66 | -2039895 | 20730.77 |    | MMHC   | 507  | 1.28     | 541  | 1.11     | 555   | 1.04     |
|             | RSMAX2 | -209975                 | 2122.86  | -1020795 | 10373.66 | -2039895 | 20730.77 |    | RSMAX2 | 506  | 1.28     | 541  | 1.11     | 555   | 1.04     |
| loglik      | HC     | -207466                 | 2098.2   | -1017613 | 10342.8  | -2036443 | 20697.21 | M  | HC     | 87   | 0.86     | 55   | 0.72     | 42    | 0.57     |
|             | TABU   | -207461                 | 2098.16  | -1017613 | 10342.8  | -2036443 | 20697.21 |    | TABU   | 86   | 0.85     | 55   | 0.72     | 42    | 0.57     |
|             | MMHC   | -207497                 | 2098.5   | -1017628 | 10342.94 | -2036451 | 20697.29 |    | MMHC   | 93   | 0.92     | 58   | 0.73     | 44    | 0.57     |
|             | RSMAX2 | -207502                 | 2098.56  | -1017628 | 10342.94 | -2036451 | 20697.29 |    | RSMAX2 | 94   | 0.93     | 58   | 0.73     | 44    | 0.57     |
| AIC         | HC     | -208173                 | 2105.26  | -1018329 | 10349.9  | -2037167 | 20704.39 | WO | HC     | 0    | 0        | 1    | 0.1      | 1     | 0.1      |
|             | TABU   | -208169                 | 2105.23  | -1018329 | 10349.9  | -2037167 | 20704.39 |    | TABU   | 57   | 1.27     | 76   | 1.52     | 65    | 1.23     |
|             | MMHC   | -208198                 | 2105.5   | -1018341 | 10350.02 | -2037174 | 20704.47 |    | MMHC   | 0    | 0        | 1    | 0.1      | 1     | 0.1      |
|             | RSMAX2 | -208203                 | 2105.56  | -1018341 | 10350.02 | -2037174 | 20704.47 |    | RSMAX2 | 0    | 0        | 1    | 0.1      | 1     | 0.1      |
| BIC         | HC     | -209908                 | 2122.58  | -1020663 | 10373.07 | -2039777 | 20730.29 | WC | HC     | 10   | 0.44     | 6    | 0.34     | 2     | 0.2      |
|             | TABU   | -209907                 | 2122.57  | -1020663 | 10373.07 | -2039777 | 20730.29 |    | TABU   | 52   | 0.97     | 58   | 1.04     | 54    | 0.98     |
|             | MMHC   | -209918                 | 2122.68  | -1020665 | 10373.09 | -2039781 | 20730.33 |    | MMHC   | 6    | 0.34     | 4    | 0.28     | 2     | 0.2      |
|             | RSMAX2 | -209924                 | 2122.74  | -1020665 | 10373.09 | -2039781 | 20730.33 |    | RSMAX2 | 6    | 0.34     | 4    | 0.28     | 2     | 0.2      |

Table 6.10: Comparison via Line (Num of Nodes = 10)

|             |        | Line (Num of Nodes = 10) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|--------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                     |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                     | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -252395                  | 2557.7   | -1259678 | 12795.95 | -2515016 | 25551.17 | C  | HC     | 681  | 1.22     | 732  | 1.25     | 742   | 1.21     |
|             | TABU   | -252382                  | 2557.58  | -1259678 | 12795.95 | -2515011 | 25551.11 |    | TABU   | 611  | 1.92     | 650  | 2.08     | 667   | 2.04     |
|             | MMHC   | -252487                  | 2558.57  | -1259700 | 12796.2  | -2515043 | 25551.46 |    | MMHC   | 668  | 1.21     | 724  | 1.25     | 738   | 1.19     |
|             | RSMAX2 | -252499                  | 2558.67  | -1259698 | 12796.18 | -2515048 | 25551.52 |    | RSMAX2 | 666  | 1.22     | 725  | 1.26     | 738   | 1.19     |
| loglik      | HC     | -249115                  | 2525.52  | -1255615 | 12756.22 | -2510623 | 25508.18 | M  | HC     | 116  | 0.97     | 67   | 0.73     | 57    | 0.7      |
|             | TABU   | -249089                  | 2525.28  | -1255617 | 12756.24 | -2510609 | 25508.03 |    | TABU   | 116  | 0.97     | 68   | 0.72     | 57    | 0.7      |
|             | MMHC   | -249242                  | 2526.74  | -1255655 | 12756.66 | -2510664 | 25508.63 |    | MMHC   | 129  | 1.02     | 75   | 0.76     | 61    | 0.69     |
|             | RSMAX2 | -249259                  | 2526.89  | -1255651 | 12756.62 | -2510672 | 25508.73 |    | RSMAX2 | 131  | 1.02     | 74   | 0.76     | 61    | 0.69     |
| AIC         | HC     | -250010                  | 2534.45  | -1256538 | 12765.4  | -2511545 | 25517.35 | WO | HC     | 3    | 0.17     | 1    | 0.1      | 1     | 0.1      |
|             | TABU   | -249990                  | 2534.26  | -1256539 | 12765.41 | -2511534 | 25517.23 |    | TABU   | 73   | 1.67     | 82   | 1.79     | 76    | 1.86     |
|             | MMHC   | -250120                  | 2535.5   | -1256572 | 12765.78 | -2511582 | 25517.75 |    | MMHC   | 3    | 0.17     | 1    | 0.1      | 1     | 0.1      |
|             | RSMAX2 | -250135                  | 2535.63  | -1256569 | 12765.75 | -2511589 | 25517.84 |    | RSMAX2 | 3    | 0.17     | 1    | 0.1      | 1     | 0.1      |
| BIC         | HC     | -252206                  | 2556.35  | -1259546 | 12795.32 | -2514869 | 25550.4  | WC | HC     | 30   | 0.77     | 10   | 0.44     | 6     | 0.34     |
|             | TABU   | -252200                  | 2556.3   | -1259543 | 12795.3  | -2514869 | 25550.4  |    | TABU   | 80   | 1.45     | 58   | 1.07     | 50    | 0.92     |
|             | MMHC   | -252274                  | 2556.99  | -1259560 | 12795.49 | -2514891 | 25550.64 |    | MMHC   | 14   | 0.59     | 8    | 0.39     | 4     | 0.28     |
|             | RSMAX2 | -252285                  | 2557.09  | -1259560 | 12795.49 | -2514895 | 25550.69 |    | RSMAX2 | 14   | 0.51     | 8    | 0.39     | 2     | 0.2      |

### A.3 Table for Star

Table 6.11: Comparison via Star (Num of Nodes = 3)

|             |        | Star (Num of Nodes = 3) |          |         |          |          |          |      |          |      |          |       |          |     |      |
|-------------|--------|-------------------------|----------|---------|----------|----------|----------|------|----------|------|----------|-------|----------|-----|------|
| Sample Size |        | 1000                    |          | 5000    |          | 10000    |          | 1000 |          | 5000 |          | 10000 |          |     |      |
|             |        | Sum.                    | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |     |      |
| BDe         | HC     | -152540                 | 292.09   | -743506 | 1366.63  | -1483710 | 2748.14  | C    | HC       | 167  | 0.49     | 178   | 0.44     | 191 | 0.29 |
|             | TABU   | -152540                 | 292.09   | -743506 | 1366.63  | -1483710 | 2748.14  |      | TABU     | 149  | 0.58     | 157   | 0.56     | 163 | 0.49 |
|             | MMHC   | -152544                 | 292.08   | -743522 | 1366.47  | -1483713 | 2748.1   |      | MMHC     | 166  | 0.5      | 177   | 0.45     | 190 | 0.3  |
|             | RSMAX2 | -152544                 | 292.08   | -743522 | 1366.47  | -1483713 | 2748.1   |      | RSMAX2   | 166  | 0.5      | 177   | 0.45     | 190 | 0.3  |
| loglik      | HC     | -150745                 | 295.8    | -741242 | 1370.3   | -1481255 | 2751.93  | M    | HC       | 33   | 0.49     | 22    | 0.44     | 9   | 0.29 |
|             | TABU   | -150745                 | 295.8    | -741242 | 1370.3   | -1481255 | 2751.93  |      | TABU     | 33   | 0.49     | 22    | 0.44     | 9   | 0.29 |
|             | MMHC   | -150754                 | 295.8    | -741262 | 1370.13  | -1481261 | 2751.85  |      | MMHC     | 34   | 0.5      | 23    | 0.45     | 10  | 0.3  |
|             | RSMAX2 | -150754                 | 295.8    | -741262 | 1370.13  | -1481261 | 2751.85  |      | RSMAX2   | 34   | 0.5      | 23    | 0.45     | 10  | 0.3  |
| AIC         | HC     | -151214                 | 295.89   | -741720 | 1370.41  | -1481746 | 2751.92  | WO   | HC       | 0    | 0        | 0     | 0        | 0   | 0    |
|             | TABU   | -151214                 | 295.89   | -741720 | 1370.41  | -1481746 | 2751.92  |      | TABU     | 18   | 0.39     | 21    | 0.41     | 28  | 0.45 |
|             | MMHC   | -151221                 | 295.89   | -741739 | 1370.24  | -1481751 | 2751.85  |      | MMHC     | 0    | 0        | 0     | 0        | 0   | 0    |
|             | RSMAX2 | -151221                 | 295.89   | -741739 | 1370.24  | -1481751 | 2751.85  |      | RSMAX2   | 0    | 0        | 0     | 0        | 0   | 0    |
| BIC         | HC     | -152365                 | 296.12   | -743278 | 1370.76  | -1483516 | 2751.89  | WC   | HC       | 4    | 0.28     | 0     | 0        | 0   | 0    |
|             | TABU   | -152365                 | 296.12   | -743278 | 1370.76  | -1483516 | 2751.89  |      | TABU     | 40   | 0.8      | 42    | 0.82     | 56  | 0.9  |
|             | MMHC   | -152367                 | 296.13   | -743293 | 1370.62  | -1483517 | 2751.88  |      | MMHC     | 2    | 0.2      | 0     | 0        | 0   | 0    |
|             | RSMAX2 | -152367                 | 296.13   | -743293 | 1370.62  | -1483517 | 2751.88  |      | RSMAX2   | 2    | 0.2      | 0     | 0        | 0   | 0    |

Table 6.12: Comparison via Star (Num of Nodes = 4)

|             |        | Star (Num of Nodes = 4) |          |         |          |          |          |    |        |      |          |      |          |      |          |
|-------------|--------|-------------------------|----------|---------|----------|----------|----------|----|--------|------|----------|------|----------|------|----------|
| Sample Size |        | 1000                    |          | 5000    |          | 10000    |          |    | 1000   |      | 5000     |      | 10000    |      |          |
|             |        | Sum.                    | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |
| BDe         | HC     | -146919                 | 893.21   | -741016 | 4566.75  | -1478923 | 9126.62  | C  | HC     | 224  | 0.77     | 253  | 0.59     | 260  | 0.57     |
|             | TABU   | -146911                 | 893.18   | -741016 | 4566.75  | -1478923 | 9126.62  |    | TABU   | 198  | 0.77     | 234  | 0.7      | 240  | 0.67     |
|             | MMHC   | -146934                 | 893.3    | -741028 | 4566.78  | -1478925 | 9126.65  |    | MMHC   | 222  | 0.76     | 250  | 0.61     | 259  | 0.57     |
|             | RSMAX2 | -146934                 | 893.3    | -741028 | 4566.78  | -1478925 | 9126.65  |    | RSMAX2 | 222  | 0.76     | 250  | 0.61     | 259  | 0.57     |
| loglik      | HC     | -145054                 | 883.82   | -738685 | 4555.34  | -1476383 | 9114.18  | M  | HC     | 51   | 0.69     | 22   | 0.46     | 15   | 0.39     |
|             | TABU   | -145043                 | 883.79   | -738685 | 4555.34  | -1476383 | 9114.18  |    | TABU   | 51   | 0.69     | 22   | 0.46     | 15   | 0.39     |
|             | MMHC   | -145074                 | 883.96   | -738703 | 4555.37  | -1476389 | 9114.23  |    | MMHC   | 53   | 0.69     | 25   | 0.48     | 16   | 0.39     |
|             | RSMAX2 | -145074                 | 883.96   | -738703 | 4555.37  | -1476389 | 9114.23  |    | RSMAX2 | 53   | 0.69     | 25   | 0.48     | 16   | 0.39     |
| AIC         | HC     | -145538                 | 886.51   | -739191 | 4558.08  | -1476896 | 9116.96  | WO | HC     | 0    | 0        | 0    | 0        | 0    | 0        |
|             | TABU   | -145529                 | 886.48   | -739191 | 4558.08  | -1476896 | 9116.96  |    | TABU   | 26   | 0.48     | 19   | 0.39     | 20   | 0.4      |
|             | MMHC   | -145555                 | 886.63   | -739207 | 4558.11  | -1476901 | 9117     |    | MMHC   | 0    | 0        | 0    | 0        | 0    | 0        |
|             | RSMAX2 | -145555                 | 886.63   | -739207 | 4558.11  | -1476901 | 9117     |    | RSMAX2 | 0    | 0        | 0    | 0        | 0    | 0        |
| BIC         | HC     | -146725                 | 893.12   | -740840 | 4567.01  | -1478745 | 9126.99  | WC | HC     | 4    | 0.28     | 0    | 0        | 0    | 0        |
|             | TABU   | -146722                 | 893.11   | -740840 | 4567.01  | -1478745 | 9126.99  |    | TABU   | 56   | 1.03     | 38   | 0.79     | 40   | 0.8      |
|             | MMHC   | -146736                 | 893.18   | -740849 | 4567.04  | -1478746 | 9127     |    | MMHC   | 0    | 0        | 0    | 0        | 0    | 0        |
|             | RSMAX2 | -146736                 | 893.18   | -740849 | 4567.04  | -1478746 | 9127     |    | RSMAX2 | 0    | 0        | 0    | 0        | 0    | 0        |

Table 6.13: Comparison via Star (Num of Nodes = 6)

|             |        | Star (Num of Nodes = 6) |          |         |          |          |          |    |        |      |          |      |          |      |          |
|-------------|--------|-------------------------|----------|---------|----------|----------|----------|----|--------|------|----------|------|----------|------|----------|
| Sample Size |        | 1000                    |          | 5000    |          | 10000    |          |    | 1000   |      | 5000     |      | 10000    |      |          |
|             |        | Sum.                    | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |
| BDe         | HC     | -150411                 | 1531.16  | -757733 | 7732.81  | -1513789 | 15448.13 | C  | HC     | 316  | 1.09     | 367  | 1.05     | 377  | 1        |
|             | TABU   | -150410                 | 1531.16  | -757733 | 7732.81  | -1513789 | 15448.13 |    | TABU   | 295  | 1.13     | 349  | 1.11     | 359  | 1.13     |
|             | MMHC   | -150735                 | 1534.21  | -757847 | 7733.72  | -1513842 | 15448.62 |    | MMHC   | 287  | 0.93     | 351  | 0.97     | 370  | 0.96     |
|             | RSMAX2 | -150780                 | 1534.64  | -757849 | 7733.73  | -1513831 | 15448.51 |    | RSMAX2 | 285  | 0.94     | 351  | 0.98     | 371  | 0.96     |
| loglik      | HC     | -148517                 | 1512.88  | -755376 | 7710.07  | -1511230 | 15423.37 | M  | HC     | 84   | 0.91     | 33   | 0.6      | 23   | 0.47     |
|             | TABU   | -148514                 | 1512.85  | -755376 | 7710.07  | -1511230 | 15423.37 |    | TABU   | 84   | 0.91     | 33   | 0.6      | 23   | 0.47     |
|             | MMHC   | -148908                 | 1516.55  | -755527 | 7711.32  | -1511302 | 15424.05 |    | MMHC   | 113  | 1.02     | 49   | 0.66     | 30   | 0.52     |
|             | RSMAX2 | -148958                 | 1517.02  | -755529 | 7711.32  | -1511288 | 15423.91 |    | RSMAX2 | 115  | 1.05     | 49   | 0.67     | 29   | 0.5      |
| AIC         | HC     | -149020                 | 1517.88  | -755904 | 7715.29  | -1511763 | 15428.65 | WO | HC     | 0    | 0        | 0    | 0        | 0    | 0        |
|             | TABU   | -149018                 | 1517.86  | -755904 | 7715.29  | -1511763 | 15428.65 |    | TABU   | 21   | 0.46     | 18   | 0.39     | 18   | 0.39     |
|             | MMHC   | -149381                 | 1521.28  | -756041 | 7716.42  | -1511828 | 15429.26 |    | MMHC   | 0    | 0        | 0    | 0        | 0    | 0        |
|             | RSMAX2 | -149429                 | 1521.73  | -756043 | 7716.42  | -1511815 | 15429.13 |    | RSMAX2 | 0    | 0        | 0    | 0        | 0    | 0        |
| BIC         | HC     | -150255                 | 1530.15  | -757624 | 7732.31  | -1513684 | 15447.68 | WC | HC     | 22   | 0.63     | 2    | 0.2      | 0    | 0        |
|             | TABU   | -150255                 | 1530.15  | -757624 | 7732.31  | -1513684 | 15447.68 |    | TABU   | 62   | 1.05     | 38   | 0.84     | 36   | 0.77     |
|             | MMHC   | -150542                 | 1532.88  | -757716 | 7733.04  | -1513724 | 15448.05 |    | MMHC   | 16   | 0.55     | 2    | 0.2      | 0    | 0        |
|             | RSMAX2 | -150585                 | 1533.29  | -757718 | 7733.05  | -1513715 | 15447.95 |    | RSMAX2 | 16   | 0.61     | 2    | 0.2      | 0    | 0        |

Table 6.14: Comparison via Star (Num of Nodes = 8)

|             |        | Star (Num of Nodes = 8) |          |          |          |          |          |    |        |          |      |          |       |          |      |
|-------------|--------|-------------------------|----------|----------|----------|----------|----------|----|--------|----------|------|----------|-------|----------|------|
| Sample Size |        | 1000                    |          | 5000     |          | 10000    |          |    | 1000   |          | 5000 |          | 10000 |          |      |
|             |        | Sum.                    | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    | Sum.   | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |      |
| BDe         | HC     | -201803                 | 2040.6   | -1019596 | 10374.74 | -2036502 | 20724.35 | C  | HC     | 473      | 1.43 | 531      | 1.16  | 550      | 1.08 |
|             | TABU   | -201791                 | 2040.48  | -1019592 | 10374.7  | -2036502 | 20724.35 |    | TABU   | 449      | 1.45 | 517      | 1.22  | 537      | 1.19 |
|             | MMHC   | -203068                 | 2052.25  | -1020595 | 10383.47 | -2037573 | 20733.29 |    | MMHC   | 376      | 1.06 | 480      | 0.94  | 515      | 0.97 |
|             | RSMAX2 | -203559                 | 2056.74  | -1020999 | 10387.34 | -2037277 | 20730.78 |    | RSMAX2 | 364      | 1.07 | 475      | 0.96  | 516      | 0.95 |
| loglik      | HC     | -199240                 | 2015.34  | -1016514 | 10344.81 | -2033143 | 20691.63 | M  | HC     | 127      | 1.12 | 69       | 0.9   | 50       | 0.69 |
|             | TABU   | -199212                 | 2015.06  | -1016505 | 10344.72 | -2033143 | 20691.63 |    | TABU   | 126      | 1.11 | 69       | 0.9   | 50       | 0.69 |
|             | MMHC   | -200686                 | 2028.73  | -1017613 | 10354.51 | -2034291 | 20701.33 |    | MMHC   | 224      | 1.07 | 120      | 0.9   | 85       | 0.82 |
|             | RSMAX2 | -201207                 | 2033.49  | -1018029 | 10358.48 | -2034000 | 20698.85 |    | RSMAX2 | 236      | 1.16 | 125      | 0.97  | 84       | 0.81 |
| AIC         | HC     | -199938                 | 2022.32  | -1017220 | 10351.82 | -2033861 | 20698.77 | WO | HC     | 0        | 0    | 0        | 0     | 0        | 0    |
|             | TABU   | -199917                 | 2022.11  | -1017213 | 10351.75 | -2033861 | 20698.77 |    | TABU   | 25       | 0.58 | 14       | 0.4   | 13       | 0.34 |
|             | MMHC   | -201311                 | 2034.99  | -1018285 | 10361.2  | -2034983 | 20708.21 |    | MMHC   | 0        | 0    | 0        | 0     | 0        | 0    |
|             | RSMAX2 | -201822                 | 2039.66  | -1018697 | 10365.13 | -2034691 | 20705.73 |    | RSMAX2 | 0        | 0    | 0        | 0     | 0        | 0    |
| BIC         | HC     | -201651                 | 2039.45  | -1019520 | 10374.66 | -2036450 | 20724.5  | WC | HC     | 48       | 0.86 | 6        | 0.34  | 2        | 0.2  |
|             | TABU   | -201647                 | 2039.42  | -1019520 | 10374.66 | -2036450 | 20724.5  |    | TABU   | 94       | 1.35 | 34       | 0.9   | 28       | 0.7  |
|             | MMHC   | -202845                 | 2050.36  | -1020474 | 10382.97 | -2037477 | 20733.04 |    | MMHC   | 38       | 0.79 | 12       | 0.48  | 6        | 0.34 |
|             | RSMAX2 | -203331                 | 2054.8   | -1020874 | 10386.81 | -2037182 | 20730.53 |    | RSMAX2 | 40       | 0.85 | 14       | 0.51  | 4        | 0.28 |

Table 6.15: Comparison via Star (Num of Nodes = 10)

|             |        | Star (Num of Nodes = 10) |          |          |          |          |          |    |        |          |      |          |       |          |      |
|-------------|--------|--------------------------|----------|----------|----------|----------|----------|----|--------|----------|------|----------|-------|----------|------|
| Sample Size |        | 1000                     |          | 5000     |          | 10000    |          |    | 1000   |          | 5000 |          | 10000 |          |      |
|             |        | Sum.                     | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    | Sum.   | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |      |
| BDe         | HC     | -255441                  | 2602.56  | -1253002 | 12766.69 | -2501901 | 25495.03 | C  | HC     | 632      | 1.66 | 712      | 1.43  | 735      | 1.27 |
|             | TABU   | -255433                  | 2602.49  | -1252992 | 12766.59 | -2501898 | 25495.01 |    | TABU   | 605      | 1.78 | 698      | 1.46  | 720      | 1.33 |
|             | MMHC   | -258398                  | 2631.34  | -1256122 | 12796.2  | -2505279 | 25526.79 |    | MMHC   | 426      | 1.01 | 564      | 0.96  | 612      | 0.98 |
|             | RSMAX2 | -260229                  | 2650.33  | -1257416 | 12808.52 | -2506828 | 25541.93 |    | RSMAX2 | 395      | 1.07 | 548      | 0.99  | 604      | 0.97 |
| loglik      | HC     | -252250                  | 2571.69  | -1249006 | 12727.66 | -2497561 | 25452.62 | M  | HC     | 168      | 1.53 | 88       | 0.98  | 65       | 0.76 |
|             | TABU   | -252230                  | 2571.48  | -1248989 | 12727.48 | -2497556 | 25452.59 |    | TABU   | 167      | 1.52 | 88       | 0.98  | 65       | 0.76 |
|             | MMHC   | -255543                  | 2603.78  | -1252426 | 12760.08 | -2501207 | 25486.93 |    | MMHC   | 374      | 1.41 | 236      | 1.15  | 188      | 1.13 |
|             | RSMAX2 | -257446                  | 2623.52  | -1253747 | 12772.68 | -2502780 | 25502.3  |    | RSMAX2 | 405      | 1.53 | 252      | 1.23  | 196      | 1.2  |
| AIC         | HC     | -253119                  | 2580.36  | -1249924 | 12736.79 | -2498483 | 25461.77 | WO | HC     | 0        | 0    | 0        | 0     | 0        | 0    |
|             | TABU   | -253104                  | 2580.21  | -1249910 | 12736.64 | -2498479 | 25461.75 |    | TABU   | 28       | 0.65 | 14       | 0.47  | 15       | 0.39 |
|             | MMHC   | -256278                  | 2611.14  | -1253237 | 12768.19 | -2502045 | 25495.29 |    | MMHC   | 0        | 0    | 0        | 0     | 0        | 0    |
|             | RSMAX2 | -258160                  | 2630.66  | -1254547 | 12780.69 | -2503611 | 25510.6  |    | RSMAX2 | 0        | 0    | 0        | 0     | 0        | 0    |
| BIC         | HC     | -255251                  | 2601.64  | -1252916 | 12766.53 | -2501807 | 25494.78 | WC | HC     | 60       | 0.96 | 22       | 0.69  | 10       | 0.44 |
|             | TABU   | -255249                  | 2601.62  | -1252911 | 12766.49 | -2501807 | 25494.78 |    | TABU   | 114      | 1.56 | 50       | 1.11  | 40       | 0.85 |
|             | MMHC   | -258081                  | 2629.2   | -1255880 | 12794.6  | -2505066 | 25525.44 |    | MMHC   | 64       | 1.06 | 36       | 0.82  | 36       | 0.82 |
|             | RSMAX2 | -259912                  | 2648.18  | -1257154 | 12806.79 | -2506607 | 25540.51 |    | RSMAX2 | 66       | 1.07 | 38       | 0.79  | 36       | 0.82 |

## A.4 Table for PseudoLoop

Table 6.16: Comparison via Pseudo Loop (Num of Nodes = 3)

|             |        | Pseudo Loop (Num of Nodes = 3) |          |         |          |          |          |      |          |      |          |       |          |
|-------------|--------|--------------------------------|----------|---------|----------|----------|----------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                           |          | 5000    |          | 10000    |          | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                           | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -151048                        | 268.73   | -747628 | 1328.06  | -1491992 | 2667.08  | C    | HC       | 215  | 0.81     | 212   | 0.57     |
|             | TABU   | -151037                        | 268.65   | -747621 | 1328.09  | -1491981 | 2667.1   |      | TABU     | 159  | 1.04     | 176   | 0.79     |
|             | MMHC   | -151376                        | 271.53   | -747934 | 1327.92  | -1492459 | 2668.62  |      | MMHC     | 204  | 0.8      | 208   | 0.6      |
|             | RSMAX2 | -151376                        | 271.53   | -747934 | 1327.92  | -1492459 | 2668.62  |      | RSMAX2   | 204  | 0.8      | 208   | 0.6      |
| loglik      | HC     | -148954                        | 271.26   | -745139 | 1331.38  | -1489279 | 2670.43  | M    | HC       | 55   | 0.59     | 25    | 0.46     |
|             | TABU   | -148962                        | 271.19   | -745139 | 1331.4   | -1489276 | 2670.4   |      | TABU     | 66   | 0.57     | 27    | 0.47     |
|             | MMHC   | -149324                        | 274.4    | -745471 | 1331.16  | -1489766 | 2671.94  |      | MMHC     | 67   | 0.64     | 30    | 0.54     |
|             | RSMAX2 | -149324                        | 274.4    | -745471 | 1331.16  | -1489766 | 2671.94  |      | RSMAX2   | 67   | 0.64     | 30    | 0.54     |
| AIC         | HC     | -149574                        | 271.37   | -745707 | 1331.5   | -1489858 | 2670.5   | WO   | HC       | 30   | 0.52     | 13    | 0.34     |
|             | TABU   | -149572                        | 271.31   | -745705 | 1331.52  | -1489853 | 2670.48  |      | TABU     | 75   | 0.87     | 47    | 0.73     |
|             | MMHC   | -149922                        | 274.44   | -746030 | 1331.31  | -1490337 | 2672.04  |      | MMHC     | 29   | 0.52     | 12    | 0.33     |
|             | RSMAX2 | -149922                        | 274.44   | -746030 | 1331.31  | -1490337 | 2672.04  |      | RSMAX2   | 29   | 0.52     | 12    | 0.33     |
| BIC         | HC     | -151096                        | 271.66   | -747558 | 1331.89  | -1491946 | 2670.74  | WC   | HC       | 60   | 1.04     | 26    | 0.68     |
|             | TABU   | -151069                        | 271.62   | -747549 | 1331.94  | -1491933 | 2670.77  |      | TABU     | 144  | 1.63     | 90    | 1.34     |
|             | MMHC   | -151390                        | 274.56   | -747851 | 1331.8   | -1492396 | 2672.39  |      | MMHC     | 58   | 1.04     | 24    | 0.65     |
|             | RSMAX2 | -151390                        | 274.56   | -747851 | 1331.8   | -1492396 | 2672.39  |      | RSMAX2   | 58   | 1.04     | 24    | 0.65     |

Table 6.17: Comparison via Pseudo Loop (Num of Nodes = 4)

|             |        | Pseudo Loop (Num of Nodes = 4) |          |         |          |          |          |    |        |      |          |      |          |      |          |
|-------------|--------|--------------------------------|----------|---------|----------|----------|----------|----|--------|------|----------|------|----------|------|----------|
| Sample Size |        | 1000                           |          | 5000    |          | 10000    |          |    | 1000   |      | 5000     |      | 10000    |      |          |
|             |        | Sum.                           | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |
| BDe         | HC     | -152222                        | 917.31   | -744364 | 4566.24  | -1484836 | 9118.22  | C  | HC     | 292  | 1.01     | 272  | 0.74     | 281  | 0.75     |
|             | TABU   | -152199                        | 917.16   | -744349 | 4566.12  | -1484812 | 9118.06  |    | TABU   | 254  | 1.2      | 249  | 0.87     | 266  | 0.81     |
|             | MMHC   | -152470                        | 919.25   | -744582 | 4567.56  | -1484812 | 9118.06  |    | MMHC   | 284  | 1.03     | 270  | 0.76     | 286  | 0.77     |
|             | RSMAX2 | -152467                        | 919.23   | -744582 | 4567.56  | -1484814 | 9118.08  |    | RSMAX2 | 284  | 1.03     | 270  | 0.76     | 285  | 0.76     |
| loglik      | HC     | -150106                        | 906.24   | -741897 | 4553.74  | -1482129 | 9104.5   | M  | HC     | 64   | 0.72     | 23   | 0.47     | 14   | 0.35     |
|             | TABU   | -150101                        | 906.2    | -741906 | 4553.78  | -1482132 | 9104.51  |    | TABU   | 67   | 0.75     | 23   | 0.47     | 14   | 0.35     |
|             | MMHC   | -150417                        | 908.65   | -742140 | 4555.17  | -1482132 | 9104.51  |    | MMHC   | 76   | 0.78     | 27   | 0.49     | 14   | 0.35     |
|             | RSMAX2 | -150412                        | 908.62   | -742140 | 4555.17  | -1482137 | 9104.56  |    | RSMAX2 | 76   | 0.78     | 27   | 0.49     | 15   | 0.36     |
| AIC         | HC     | -150724                        | 909.75   | -742460 | 4556.85  | -1482703 | 9107.67  | WO | HC     | 19   | 0.49     | 5    | 0.3      | 5    | 0.3      |
|             | TABU   | -150709                        | 909.66   | -742459 | 4556.83  | -1482696 | 9107.62  |    | TABU   | 54   | 0.85     | 28   | 0.59     | 20   | 0.49     |
|             | MMHC   | -151003                        | 911.94   | -742693 | 4558.24  | -1482696 | 9107.62  |    | MMHC   | 15   | 0.44     | 3    | 0.22     | 0    | 0        |
|             | RSMAX2 | -150999                        | 911.92   | -742693 | 4558.24  | -1482700 | 9107.66  |    | RSMAX2 | 15   | 0.44     | 3    | 0.22     | 0    | 0        |
| BIC         | HC     | -152241                        | 918.39   | -744295 | 4567.01  | -1484773 | 9119.1   | WC | HC     | 46   | 1.13     | 14   | 0.82     | 14   | 0.82     |
|             | TABU   | -152201                        | 918.14   | -744261 | 4566.77  | -1484729 | 9118.81  |    | TABU   | 90   | 1.34     | 44   | 0.83     | 32   | 0.74     |
|             | MMHC   | -152441                        | 920.03   | -744495 | 4568.24  | -1484729 | 9118.81  |    | MMHC   | 28   | 0.81     | 8    | 0.56     | 0    | 0        |
|             | RSMAX2 | -152440                        | 920.03   | -744495 | 4568.24  | -1484730 | 9118.82  |    | RSMAX2 | 28   | 0.81     | 8    | 0.56     | 0    | 0        |

Table 6.18: Comparison via Pseudo Loop (Num of Nodes = 6)

|             |        | Pseudo Loop (Num of Nodes = 6) |          |         |          |          |          |    |        |      |          |      |          |      |          |
|-------------|--------|--------------------------------|----------|---------|----------|----------|----------|----|--------|------|----------|------|----------|------|----------|
| Sample Size |        | 1000                           |          | 5000    |          | 10000    |          |    | 1000   |      | 5000     |      | 10000    |      |          |
|             |        | Sum.                           | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |
| BDe         | HC     | -151573                        | 1547.91  | -757733 | 7732.81  | -1513789 | 15448.13 | C  | HC     | 418  | 1.23     | 367  | 1.05     | 377  | 1        |
|             | TABU   | -151527                        | 1547.33  | -757733 | 7732.81  | -1513789 | 15448.13 |    | TABU   | 355  | 1.46     | 349  | 1.11     | 359  | 1.13     |
|             | MMHC   | -151714                        | 1549.34  | -757847 | 7733.72  | -1513842 | 15448.62 |    | MMHC   | 401  | 1.19     | 351  | 0.97     | 370  | 0.96     |
|             | RSMAX2 | -151695                        | 1549.12  | -757849 | 7733.73  | -1513831 | 15448.51 |    | RSMAX2 | 402  | 1.25     | 351  | 0.98     | 371  | 0.96     |
| loglik      | HC     | -149508                        | 1528.02  | -755376 | 7710.07  | -1511230 | 15423.37 | M  | HC     | 75   | 0.78     | 33   | 0.6      | 23   | 0.47     |
|             | TABU   | -149453                        | 1527.33  | -755376 | 7710.07  | -1511230 | 15423.37 |    | TABU   | 74   | 0.79     | 33   | 0.6      | 23   | 0.47     |
|             | MMHC   | -149698                        | 1529.94  | -755527 | 7711.32  | -1511302 | 15424.05 |    | MMHC   | 94   | 0.81     | 49   | 0.66     | 30   | 0.52     |
|             | RSMAX2 | -149669                        | 1529.62  | -755529 | 7711.32  | -1511288 | 15423.91 |    | RSMAX2 | 93   | 0.88     | 49   | 0.67     | 29   | 0.5      |
| AIC         | HC     | -150106                        | 1533.95  | -755904 | 7715.29  | -1511763 | 15428.65 | WO | HC     | 7    | 0.33     | 0    | 0        | 0    | 0        |
|             | TABU   | -150056                        | 1533.31  | -755904 | 7715.29  | -1511763 | 15428.65 |    | TABU   | 71   | 1.18     | 18   | 0.39     | 18   | 0.39     |
|             | MMHC   | -150277                        | 1535.68  | -756041 | 7716.42  | -1511828 | 15429.26 |    | MMHC   | 5    | 0.26     | 0    | 0        | 0    | 0        |
|             | RSMAX2 | -150251                        | 1535.39  | -756043 | 7716.42  | -1511815 | 15429.13 |    | RSMAX2 | 5    | 0.26     | 0    | 0        | 0    | 0        |
| BIC         | HC     | -151574                        | 1548.51  | -757624 | 7732.31  | -1513684 | 15447.68 | WC | HC     | 18   | 0.81     | 2    | 0.2      | 0    | 0        |
|             | TABU   | -151535                        | 1547.99  | -757624 | 7732.31  | -1513684 | 15447.68 |    | TABU   | 74   | 1.12     | 38   | 0.84     | 36   | 0.77     |
|             | MMHC   | -151698                        | 1549.76  | -757716 | 7733.04  | -1513724 | 15448.05 |    | MMHC   | 8    | 0.39     | 2    | 0.2      | 0    | 0        |
|             | RSMAX2 | -151679                        | 1549.56  | -757718 | 7733.05  | -1513715 | 15447.95 |    | RSMAX2 | 8    | 0.39     | 2    | 0.2      | 0    | 0        |

Table 6.19: Comparison via Pseudo Loop (Num of Nodes = 8)

|             |        | Pseudo Loop (Num of Nodes = 8) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|--------------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                           |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                           | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -209665                        | 2121.1   | -1019596 | 10374.74 | -2036502 | 20724.35 | C  | HC     | 595  | 1.25     | 531  | 1.16     | 550   | 1.08     |
|             | TABU   | -209658                        | 2121.03  | -1019592 | 10374.7  | -2036502 | 20724.35 |    | TABU   | 510  | 1.92     | 517  | 1.22     | 537   | 1.19     |
|             | MMHC   | -209955                        | 2123.8   | -1020595 | 10383.47 | -2037573 | 20733.29 |    | MMHC   | 569  | 1.2      | 480  | 0.94     | 515   | 0.97     |
|             | RSMAX2 | -209957                        | 2123.79  | -1020999 | 10387.34 | -2037277 | 20730.78 |    | RSMAX2 | 569  | 1.25     | 475  | 0.96     | 516   | 0.95     |
| loglik      | HC     | -207015                        | 2095.04  | -1016514 | 10344.81 | -2033143 | 20691.63 | M  | HC     | 100  | 0.93     | 69   | 0.9      | 50    | 0.69     |
|             | TABU   | -207005                        | 2094.94  | -1016505 | 10344.72 | -2033143 | 20691.63 |    | TABU   | 97   | 0.92     | 69   | 0.9      | 50    | 0.69     |
|             | MMHC   | -207366                        | 2098.34  | -1017613 | 10354.51 | -2034291 | 20701.33 |    | MMHC   | 126  | 0.91     | 120  | 0.9      | 85    | 0.82     |
|             | RSMAX2 | -207366                        | 2098.29  | -1018029 | 10358.48 | -2034000 | 20698.85 |    | RSMAX2 | 126  | 0.99     | 125  | 0.97     | 84    | 0.81     |
| AIC         | HC     | -207802                        | 2102.89  | -1017220 | 10351.82 | -2033861 | 20698.77 | WO | HC     | 5    | 0.26     | 0    | 0        | 0     | 0        |
|             | TABU   | -207794                        | 2102.81  | -1017213 | 10351.75 | -2033861 | 20698.77 |    | TABU   | 93   | 1.61     | 14   | 0.4      | 13    | 0.34     |
|             | MMHC   | -208127                        | 2105.95  | -1018285 | 10361.2  | -2034983 | 20708.21 |    | MMHC   | 5    | 0.26     | 0    | 0        | 0     | 0        |
|             | RSMAX2 | -208127                        | 2105.91  | -1018697 | 10365.13 | -2034691 | 20705.73 |    | RSMAX2 | 5    | 0.26     | 0    | 0        | 0     | 0        |
| BIC         | HC     | -209733                        | 2122.17  | -1019520 | 10374.66 | -2036450 | 20724.5  | WC | HC     | 20   | 0.72     | 6    | 0.34     | 2     | 0.2      |
|             | TABU   | -209730                        | 2122.14  | -1019520 | 10374.66 | -2036450 | 20724.5  |    | TABU   | 78   | 1.24     | 34   | 0.9      | 28    | 0.7      |
|             | MMHC   | -209995                        | 2124.61  | -1020474 | 10382.97 | -2037477 | 20733.04 |    | MMHC   | 16   | 0.68     | 12   | 0.48     | 6     | 0.34     |
|             | RSMAX2 | -209994                        | 2124.59  | -1020874 | 10386.81 | -2037182 | 20730.53 |    | RSMAX2 | 16   | 0.68     | 14   | 0.51     | 4     | 0.28     |

Table 6.20: Comparison via Pseudo Loop (Num of Nodes = 10)

|             |        | Pseudo Loop (Num of Nodes = 10) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|---------------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                            |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                            | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -253245                         | 2567.31  | -1253002 | 12766.69 | -2501901 | 25495.03 | C  | HC     | 768  | 1.37     | 712  | 1.43     | 735   | 1.27     |
|             | TABU   | -253238                         | 2567.24  | -1252992 | 12766.59 | -2501898 | 25495.01 |    | TABU   | 691  | 2.09     | 698  | 1.46     | 720   | 1.33     |
|             | MMHC   | -253926                         | 2574.34  | -1256122 | 12796.2  | -2505279 | 25526.79 |    | MMHC   | 735  | 1.35     | 564  | 0.96     | 612   | 0.98     |
|             | RSMAX2 | -253822                         | 2572.93  | -1257416 | 12808.52 | -2506828 | 25541.93 |    | RSMAX2 | 732  | 1.35     | 548  | 0.99     | 604   | 0.97     |
| loglik      | HC     | -249796                         | 2533.4   | -1249006 | 12727.66 | -2497561 | 25452.62 | M  | HC     | 126  | 1.03     | 88   | 0.98     | 65    | 0.76     |
|             | TABU   | -249784                         | 2533.29  | -1248989 | 12727.48 | -2497556 | 25452.59 |    | TABU   | 127  | 1.03     | 88   | 0.98     | 65    | 0.76     |
|             | MMHC   | -250566                         | 2541.35  | -1252426 | 12760.08 | -2501207 | 25486.93 |    | MMHC   | 159  | 1.07     | 236  | 1.15     | 188   | 1.13     |
|             | RSMAX2 | -250468                         | 2539.94  | -1253747 | 12772.68 | -2502780 | 25502.3  |    | RSMAX2 | 162  | 1.09     | 252  | 1.23     | 196   | 1.2      |
| AIC         | HC     | -250785                         | 2543.27  | -1249924 | 12736.79 | -2498483 | 25461.77 | WO | HC     | 6    | 0.28     | 0    | 0        | 0     | 0        |
|             | TABU   | -250775                         | 2543.17  | -1249910 | 12736.64 | -2498479 | 25461.75 |    | TABU   | 82   | 1.68     | 14   | 0.47     | 15    | 0.39     |
|             | MMHC   | -251519                         | 2550.85  | -1253237 | 12768.19 | -2502045 | 25495.29 |    | MMHC   | 6    | 0.28     | 0    | 0        | 0     | 0        |
|             | RSMAX2 | -251418                         | 2549.43  | -1254547 | 12780.69 | -2503611 | 25510.6  |    | RSMAX2 | 6    | 0.28     | 0    | 0        | 0     | 0        |
| BIC         | HC     | -253211                         | 2567.48  | -1252916 | 12766.53 | -2501807 | 25494.78 | WC | HC     | 38   | 0.89     | 22   | 0.69     | 10    | 0.44     |
|             | TABU   | -253207                         | 2567.44  | -1252911 | 12766.49 | -2501807 | 25494.78 |    | TABU   | 94   | 1.59     | 50   | 1.11     | 40    | 0.85     |
|             | MMHC   | -253858                         | 2574.19  | -1255880 | 12794.6  | -2505066 | 25525.44 |    | MMHC   | 28   | 0.81     | 36   | 0.82     | 36    | 0.82     |
|             | RSMAX2 | -253749                         | 2572.73  | -1257154 | 12806.79 | -2506607 | 25540.51 |    | RSMAX2 | 26   | 0.79     | 38   | 0.79     | 36    | 0.82     |

## A.5 Table for Diamond

Table 6.21: Comparison via Diamond (Num of Nodes = 4)

|             |        | Diamond (Num of Nodes = 4) |          |         |          |          |          |      |          |      |          |     |      |     |      |
|-------------|--------|----------------------------|----------|---------|----------|----------|----------|------|----------|------|----------|-----|------|-----|------|
| Sample Size |        | 1000                       |          | 5000    |          | 10000    |          | 1000 |          | 5000 |          |     |      |     |      |
|             |        | Sum.                       | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |     |      |     |      |
| BDe         | HC     | -200073                    | 296.07   | -990529 | 1488.95  | -1975845 | 2993.22  | C    | HC       | 324  | 0.9      | 346 | 0.82 | 361 | 0.75 |
|             | TABU   | -199991                    | 296.24   | -990330 | 1491.16  | -1975702 | 2993.15  |      | TABU     | 312  | 1.06     | 347 | 0.74 | 359 | 0.65 |
|             | MMHC   | -201258                    | 300.03   | -993837 | 1498.75  | -1981603 | 2991.61  |      | MMHC     | 296  | 0.92     | 338 | 0.81 | 357 | 0.74 |
|             | RSMAX2 | -201155                    | 300.7    | -993753 | 1498.49  | -1981605 | 2991.62  |      | RSMAX2   | 294  | 0.95     | 337 | 0.85 | 356 | 0.74 |
| loglik      | HC     | -197171                    | 299.61   | -986793 | 1492.52  | -1971733 | 2996.83  | M    | HC       | 58   | 0.74     | 31  | 0.58 | 15  | 0.39 |
|             | TABU   | -197103                    | 299.77   | -986678 | 1494.75  | -1971710 | 2996.88  |      | TABU     | 56   | 0.74     | 27  | 0.53 | 12  | 0.33 |
|             | MMHC   | -198517                    | 303.8    | -990244 | 1502.38  | -1977631 | 2995     |      | MMHC     | 87   | 0.84     | 41  | 0.62 | 20  | 0.43 |
|             | RSMAX2 | -198408                    | 304.54   | -990154 | 1502.07  | -1977635 | 2995.03  |      | RSMAX2   | 89   | 0.86     | 41  | 0.64 | 21  | 0.43 |
| AIC         | HC     | -198021                    | 299.8    | -987703 | 1492.56  | -1972665 | 2996.82  | WO   | HC       | 18   | 0.54     | 23  | 0.62 | 24  | 0.65 |
|             | TABU   | -197941                    | 299.98   | -987550 | 1494.83  | -1972596 | 2996.89  |      | TABU     | 32   | 0.6      | 26  | 0.52 | 29  | 0.59 |
|             | MMHC   | -199294                    | 303.99   | -991099 | 1502.5   | -1978516 | 2995.11  |      | MMHC     | 17   | 0.51     | 21  | 0.57 | 23  | 0.65 |
|             | RSMAX2 | -199188                    | 304.69   | -991011 | 1502.21  | -1978519 | 2995.13  |      | RSMAX2   | 17   | 0.51     | 22  | 0.58 | 23  | 0.65 |
| BIC         | HC     | -200107                    | 300.29   | -990669 | 1492.72  | -1976025 | 2996.79  | WC   | HC       | 38   | 1.09     | 50  | 1.28 | 52  | 1.35 |
|             | TABU   | -199997                    | 300.49   | -990392 | 1495.11  | -1975790 | 2996.89  |      | TABU     | 66   | 1.24     | 54  | 1.1  | 52  | 0.93 |
|             | MMHC   | -201201                    | 304.49   | -993885 | 1502.91  | -1981706 | 2995.5   |      | MMHC     | 32   | 0.93     | 42  | 1.11 | 46  | 1.23 |
|             | RSMAX2 | -201102                    | 305.07   | -993804 | 1502.66  | -1981706 | 2995.5   |      | RSMAX2   | 36   | 1        | 46  | 1.17 | 46  | 1.23 |

Table 6.22: Comparison via Diamond (Num of Nodes = 6)

|             |        | Diamond (Num of Nodes = 6) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|----------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                       |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                       | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -227675                    | 1369.41  | -1121009 | 6747.41  | -2235141 | 13460.08 | C  | HC     | 563  | 1.94     | 629  | 1.93     | 646   | 1.97     |
|             | TABU   | -227492                    | 1367.9   | -1120796 | 6745.95  | -2234852 | 13458.77 |    | TABU   | 552  | 2.04     | 624  | 1.91     | 640   | 1.86     |
|             | MMHC   | -230771                    | 1388.94  | -1132149 | 6825.51  | -2253172 | 13584.78 |    | MMHC   | 472  | 1.56     | 585  | 1.68     | 615   | 1.83     |
|             | RSMAX2 | -230917                    | 1389.62  | -1131904 | 6826.26  | -2252879 | 13583.61 |    | RSMAX2 | 452  | 1.62     | 576  | 1.72     | 605   | 1.78     |
| loglik      | HC     | -223207                    | 1344.59  | -1114502 | 6710.88  | -2227675 | 13418.27 | M  | HC     | 109  | 1.16     | 47   | 0.67     | 27    | 0.51     |
|             | TABU   | -223007                    | 1342.88  | -1114367 | 6709.99  | -2227544 | 13417.96 |    | TABU   | 103  | 1.1      | 44   | 0.64     | 24    | 0.47     |
|             | MMHC   | -227292                    | 1370.51  | -1126733 | 6796.22  | -2246828 | 13549.96 |    | MMHC   | 202  | 1.32     | 99   | 0.96     | 66    | 0.83     |
|             | RSMAX2 | -227411                    | 1370.96  | -1126502 | 6797.25  | -2246548 | 13548.9  |    | RSMAX2 | 221  | 1.51     | 108  | 1.06     | 76    | 0.87     |
| AIC         | HC     | -224783                    | 1353.67  | -1116357 | 6721.4   | -2229646 | 13429.37 | WO | HC     | 28   | 0.6      | 24   | 0.74     | 27    | 0.84     |
|             | TABU   | -224583                    | 1352     | -1116194 | 6720.3   | -2229460 | 13428.72 |    | TABU   | 45   | 0.73     | 32   | 0.63     | 36    | 0.72     |
|             | MMHC   | -228390                    | 1376.69  | -1128204 | 6804.33  | -2248431 | 13558.88 |    | MMHC   | 26   | 0.5      | 16   | 0.56     | 19    | 0.71     |
|             | RSMAX2 | -228524                    | 1377.25  | -1127969 | 6805.26  | -2248148 | 13557.78 |    | RSMAX2 | 27   | 0.53     | 16   | 0.56     | 19    | 0.71     |
| BIC         | HC     | -228650                    | 1376.01  | -1122401 | 6755.7   | -2236752 | 13469.41 | WC | HC     | 92   | 1.83     | 72   | 2.12     | 80    | 2.49     |
|             | TABU   | -228450                    | 1374.43  | -1122147 | 6753.91  | -2236367 | 13467.54 |    | TABU   | 120  | 1.89     | 80   | 1.84     | 84    | 2.03     |
|             | MMHC   | -231084                    | 1391.89  | -1132998 | 6830.78  | -2254210 | 13591.02 |    | MMHC   | 58   | 1.07     | 36   | 1.25     | 40    | 1.42     |
|             | RSMAX2 | -231256                    | 1392.74  | -1132749 | 6831.39  | -2253917 | 13589.82 |    | RSMAX2 | 86   | 1.46     | 50   | 1.34     | 54    | 1.5      |

Table 6.23: Comparison via Diamond (Num of Nodes = 8)

|             |        | Diamond (Num of Nodes = 8) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|----------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                       |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                       | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -209595                    | 2127.11  | -1025068 | 10403.36 | -2037346 | 20674.17 | C  | HC     | 716  | 1.69     | 863  | 2.24     | 915   | 2.25     |
|             | TABU   | -209421                    | 2125.27  | -1023800 | 10387.05 | -2036857 | 20669.02 |    | TABU   | 703  | 1.72     | 864  | 2.25     | 905   | 2.23     |
|             | MMHC   | -211560                    | 2146.17  | -1037778 | 10529.39 | -2058839 | 20894.21 |    | MMHC   | 569  | 1.35     | 753  | 1.75     | 844   | 1.89     |
|             | RSMAX2 | -211926                    | 2150.08  | -1038825 | 10539.5  | -2057404 | 20878.41 |    | RSMAX2 | 531  | 1.37     | 726  | 1.74     | 821   | 1.97     |
| loglik      | HC     | -205811                    | 2090.16  | -1014574 | 10296.95 | -2023359 | 20531.31 | M  | HC     | 238  | 1.77     | 82   | 1.1      | 43    | 0.74     |
|             | TABU   | -205506                    | 2086.8   | -1012976 | 10276.38 | -2023235 | 20529.95 |    | TABU   | 218  | 1.62     | 71   | 0.88     | 39    | 0.74     |
|             | MMHC   | -208703                    | 2118.59  | -1031757 | 10468.59 | -2050455 | 20809.76 |    | MMHC   | 385  | 2.04     | 200  | 1.37     | 126   | 1.1      |
|             | RSMAX2 | -209127                    | 2123.07  | -1032608 | 10476.78 | -2048320 | 20787.44 |    | RSMAX2 | 423  | 2.16     | 224  | 1.58     | 147   | 1.28     |
| AIC         | HC     | -207088                    | 2102.72  | -1017796 | 10329.53 | -2027288 | 20571.13 | WO | HC     | 46   | 0.8      | 55   | 1.17     | 42    | 1.07     |
|             | TABU   | -206840                    | 2100.01  | -1016298 | 10310.24 | -2027058 | 20568.68 |    | TABU   | 79   | 0.97     | 65   | 1.28     | 56    | 1.08     |
|             | MMHC   | -209578                    | 2127.12  | -1033414 | 10485.41 | -2052653 | 20831.89 |    | MMHC   | 46   | 0.7      | 47   | 1.02     | 30    | 0.96     |
|             | RSMAX2 | -209981                    | 2131.39  | -1034333 | 10494.26 | -2050736 | 20811.57 |    | RSMAX2 | 46   | 0.67     | 50   | 1.1      | 32    | 1.04     |
| BIC         | HC     | -210222                    | 2133.64  | -1028295 | 10435.78 | -2041453 | 20714.73 | WC | HC     | 144  | 2.2      | 214  | 4.31     | 204   | 4.92     |
|             | TABU   | -210114                    | 2132.51  | -1027123 | 10420.65 | -2040841 | 20708.34 |    | TABU   | 224  | 2.73     | 202  | 4.28     | 190   | 4.72     |
|             | MMHC   | -211725                    | 2148.09  | -1038814 | 10540.32 | -2060578 | 20911.77 |    | MMHC   | 104  | 1.46     | 104  | 2.19     | 64    | 1.99     |
|             | RSMAX2 | -212077                    | 2151.82  | -1039955 | 10551.31 | -2059446 | 20898.67 |    | RSMAX2 | 148  | 1.67     | 152  | 2.51     | 112   | 2.42     |

Table 6.24: Comparison via Diamond (Num of Nodes = 10)

|             |        | Diamond (Num of Nodes = 10) |          |          |          |          |          |       |          |      |          |      |          |      |          |      |          |  |  |
|-------------|--------|-----------------------------|----------|----------|----------|----------|----------|-------|----------|------|----------|------|----------|------|----------|------|----------|--|--|
| Sample Size |        | 1000                        |          |          | 5000     |          |          | 10000 |          |      | 1000     |      |          | 5000 |          |      | 10000    |  |  |
|             |        | Sum.                        | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. | Sum.  | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |  |  |
| BDe         | HC     | -264207                     | 2682.62  | -1299179 | 13200.46 | -2576611 | 26172.29 | C     | HC       | 790  | 1.53     | 1099 | 2.3      | 1219 | 2.29     |      |          |  |  |
|             | TABU   | -264111                     | 2681.69  | -1298587 | 13194.17 | -2575401 | 26159.7  |       | TABU     | 765  | 1.66     | 1096 | 2.29     | 1211 | 2.31     |      |          |  |  |
|             | MMHC   | -266497                     | 2704.09  | -1310181 | 13306.04 | -2608702 | 26498.01 |       | MMHC     | 592  | 1.26     | 880  | 1.7      | 1007 | 1.85     |      |          |  |  |
|             | RSMAX2 | -267200                     | 2711.32  | -1322040 | 13429.81 | -2626080 | 26666.54 |       | RSMAX2   | 545  | 1.23     | 757  | 2.11     | 884  | 2.17     |      |          |  |  |
| loglik      | HC     | -260692                     | 2648.92  | -1285989 | 13069.45 | -2548967 | 25882.88 | M     | HC       | 548  | 2.5      | 214  | 1.72     | 116  | 1.22     |      |          |  |  |
|             | TABU   | -260503                     | 2647.1   | -1284563 | 13053.86 | -2547010 | 25862.35 |       | TABU     | 523  | 2.51     | 186  | 1.54     | 102  | 1.19     |      |          |  |  |
|             | MMHC   | -263463                     | 2674.73  | -1303889 | 13245.86 | -2598362 | 26395.52 |       | MMHC     | 750  | 2.55     | 457  | 2.18     | 337  | 2.04     |      |          |  |  |
|             | RSMAX2 | -264162                     | 2682     | -1315306 | 13365.27 | -2615253 | 26559.62 |       | RSMAX2   | 799  | 2.52     | 578  | 3.03     | 464  | 2.99     |      |          |  |  |
| AIC         | HC     | -261787                     | 2659.52  | -1289764 | 13106.64 | -2556311 | 25958.98 | WO    | HC       | 62   | 0.84     | 87   | 1.27     | 65   | 1.27     |      |          |  |  |
|             | TABU   | -261637                     | 2658.08  | -1288575 | 13093.68 | -2554569 | 25940.72 |       | TABU     | 112  | 1.11     | 118  | 1.53     | 87   | 1.41     |      |          |  |  |
|             | MMHC   | -264352                     | 2683.47  | -1305573 | 13261.94 | -2600972 | 26421.28 |       | MMHC     | 58   | 0.73     | 63   | 1.09     | 56   | 1.08     |      |          |  |  |
|             | RSMAX2 | -265057                     | 2690.78  | -1317118 | 13382.59 | -2618011 | 26586.69 |       | RSMAX2   | 56   | 0.67     | 65   | 1.06     | 52   | 1.09     |      |          |  |  |
| BIC         | HC     | -264474                     | 2685.57  | -1302065 | 13228.16 | -2582788 | 26233.71 | WC    | HC       | 188  | 2.25     | 338  | 4.84     | 314  | 5.94     |      |          |  |  |
|             | TABU   | -264420                     | 2685.05  | -1301648 | 13223.79 | -2581820 | 26223.64 |       | TABU     | 308  | 2.93     | 386  | 5.21     | 312  | 5.68     |      |          |  |  |
|             | MMHC   | -266534                     | 2704.94  | -1311060 | 13314.44 | -2610382 | 26514.31 |       | MMHC     | 146  | 1.63     | 162  | 2.37     | 124  | 2.18     |      |          |  |  |
|             | RSMAX2 | -267253                     | 2712.32  | -1323022 | 13439.23 | -2627954 | 26684.5  |       | RSMAX2   | 206  | 1.94     | 296  | 3.22     | 370  | 5.02     |      |          |  |  |

## A.6 Table for Rhombus

Table 6.25: Comparison via Rhombus (Num of Nodes = 4)

|             |        | Rhombus (Num of Nodes = 4) |          |         |          |          |          |       |          |      |          |      |          |      |          |      |          |  |  |
|-------------|--------|----------------------------|----------|---------|----------|----------|----------|-------|----------|------|----------|------|----------|------|----------|------|----------|--|--|
| Sample Size |        | 1000                       |          |         | 5000     |          |          | 10000 |          |      | 1000     |      |          | 5000 |          |      | 10000    |  |  |
|             |        | Sum.                       | Std.Dev. | Sum.    | Std.Dev. | Sum.     | Std.Dev. | Sum.  | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |  |  |
| BDe         | HC     | -198861                    | 327.32   | -984754 | 1633.13  | -1963993 | 3258.75  | C     | HC       | 322  | 0.91     | 361  | 0.63     | 382  | 0.41     |      |          |  |  |
|             | TABU   | -198677                    | 325.57   | -984051 | 1625.9   | -1963989 | 3258.82  |       | TABU     | 290  | 1.18     | 356  | 0.72     | 375  | 0.58     |      |          |  |  |
|             | MMHC   | -199920                    | 332.08   | -987749 | 1644.31  | -1969244 | 3289.06  |       | MMHC     | 282  | 0.85     | 337  | 0.73     | 359  | 0.64     |      |          |  |  |
|             | RSMAX2 | -199857                    | 331.74   | -987360 | 1644.92  | -1968685 | 3292.72  |       | RSMAX2   | 287  | 0.86     | 341  | 0.74     | 365  | 0.61     |      |          |  |  |
| loglik      | HC     | -195894                    | 330.88   | -980937 | 1636.6   | -1959765 | 3261.63  | M     | HC       | 64   | 0.89     | 27   | 0.58     | 10   | 0.3      |      |          |  |  |
|             | TABU   | -195681                    | 328.87   | -980207 | 1629.08  | -1959755 | 3261.83  |       | TABU     | 69   | 0.85     | 26   | 0.54     | 13   | 0.37     |      |          |  |  |
|             | MMHC   | -197126                    | 336.4    | -984056 | 1648.11  | -1965156 | 3292.42  |       | MMHC     | 104  | 0.85     | 52   | 0.69     | 33   | 0.6      |      |          |  |  |
|             | RSMAX2 | -197045                    | 335.99   | -983652 | 1648.78  | -1964568 | 3296.19  |       | RSMAX2   | 99   | 0.86     | 48   | 0.69     | 27   | 0.57     |      |          |  |  |
| AIC         | HC     | -196765                    | 331.51   | -981875 | 1636.94  | -1960737 | 3261.99  | WO    | HC       | 14   | 0.35     | 12   | 0.36     | 8    | 0.27     |      |          |  |  |
|             | TABU   | -196564                    | 329.6    | -981153 | 1629.5   | -1960729 | 3262.15  |       | TABU     | 41   | 0.6      | 18   | 0.41     | 12   | 0.33     |      |          |  |  |
|             | MMHC   | -197915                    | 336.83   | -984949 | 1648.39  | -1966084 | 3292.68  |       | MMHC     | 14   | 0.35     | 11   | 0.35     | 8    | 0.27     |      |          |  |  |
|             | RSMAX2 | -197843                    | 336.45   | -984551 | 1649.04  | -1965507 | 3296.4   |       | RSMAX2   | 14   | 0.35     | 11   | 0.35     | 8    | 0.27     |      |          |  |  |
| BIC         | HC     | -198902                    | 333.09   | -984932 | 1638.02  | -1964241 | 3263.3   | WC    | HC       | 36   | 0.77     | 24   | 0.71     | 16   | 0.55     |      |          |  |  |
|             | TABU   | -198730                    | 331.41   | -984236 | 1630.88  | -1964240 | 3263.3   |       | TABU     | 110  | 1.54     | 44   | 1.05     | 32   | 0.93     |      |          |  |  |
|             | MMHC   | -199851                    | 337.91   | -987859 | 1649.33  | -1969430 | 3293.61  |       | MMHC     | 32   | 0.74     | 26   | 0.73     | 18   | 0.58     |      |          |  |  |
|             | RSMAX2 | -199801                    | 337.59   | -987480 | 1649.87  | -1968892 | 3297.15  |       | RSMAX2   | 32   | 0.74     | 24   | 0.71     | 16   | 0.55     |      |          |  |  |

Table 6.26: Comparison via Rhombus (Num of Nodes = 6)

|             |        | Rhombus (Num of Nodes = 6) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|----------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                       |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                       | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -225795                    | 1354.42  | -1116344 | 6693.89  | -2227143 | 13353.16 | C  | HC     | 533  | 1.9      | 631  | 1.82     | 656   | 1.8      |
|             | TABU   | -225655                    | 1353.18  | -1116283 | 6693.87  | -2226948 | 13352.84 |    | TABU   | 503  | 2.26     | 613  | 1.87     | 652   | 1.91     |
|             | MMHC   | -227218                    | 1364.21  | -1120926 | 6727.09  | -2233878 | 13399.99 |    | MMHC   | 457  | 1.75     | 578  | 1.85     | 614   | 1.88     |
|             | RSMAX2 | -227388                    | 1364.66  | -1121223 | 6730.44  | -2233720 | 13399.35 |    | RSMAX2 | 462  | 1.78     | 590  | 1.86     | 623   | 1.85     |
| loglik      | HC     | -222302                    | 1335.09  | -1111605 | 6667.57  | -2221896 | 13323.91 | M  | HC     | 133  | 1.41     | 46   | 0.74     | 29    | 0.62     |
|             | TABU   | -222134                    | 1333.64  | -1111527 | 6667.5   | -2221679 | 13323.57 |    | TABU   | 136  | 1.41     | 48   | 0.76     | 25    | 0.54     |
|             | MMHC   | -223985                    | 1346.6   | -1116412 | 6702.13  | -2228828 | 13371.81 |    | MMHC   | 214  | 1.33     | 98   | 0.95     | 71    | 0.86     |
|             | RSMAX2 | -224166                    | 1347     | -1116689 | 6705.34  | -2228649 | 13371.1  |    | RSMAX2 | 217  | 1.41     | 91   | 0.94     | 65    | 0.89     |
| AIC         | HC     | -223428                    | 1341.66  | -1112869 | 6674.78  | -2223192 | 13331.28 | WO | HC     | 34   | 0.52     | 23   | 0.42     | 15    | 0.36     |
|             | TABU   | -223273                    | 1340.29  | -1112797 | 6674.72  | -2222982 | 13330.94 |    | TABU   | 61   | 0.68     | 39   | 0.63     | 23    | 0.51     |
|             | MMHC   | -224993                    | 1352.44  | -1117596 | 6708.9   | -2230058 | 13378.85 |    | MMHC   | 29   | 0.54     | 24   | 0.47     | 15    | 0.36     |
|             | RSMAX2 | -225171                    | 1352.88  | -1117880 | 6712.16  | -2229887 | 13378.16 |    | RSMAX2 | 21   | 0.46     | 19   | 0.39     | 12    | 0.33     |
| BIC         | HC     | -226192                    | 1357.8   | -1116988 | 6698.28  | -2227864 | 13357.83 | WC | HC     | 86   | 1.21     | 52   | 0.93     | 36    | 0.82     |
|             | TABU   | -226068                    | 1356.63  | -1116935 | 6698.27  | -2227680 | 13357.52 |    | TABU   | 160  | 1.75     | 98   | 1.54     | 54    | 1.2      |
|             | MMHC   | -227467                    | 1366.8   | -1121455 | 6730.95  | -2234493 | 13404.24 |    | MMHC   | 70   | 1.11     | 60   | 1.04     | 36    | 0.82     |
|             | RSMAX2 | -227637                    | 1367.32  | -1121761 | 6734.38  | -2234350 | 13403.62 |    | RSMAX2 | 52   | 0.97     | 42   | 0.82     | 26    | 0.68     |

Table 6.27: Comparison via Rhombus (Num of Nodes = 8)

|             |        | Rhombus (Num of Nodes = 8) |          |          |          |          |          |    |        |      |          |      |          |       |          |
|-------------|--------|----------------------------|----------|----------|----------|----------|----------|----|--------|------|----------|------|----------|-------|----------|
| Sample Size |        | 1000                       |          | 5000     |          | 10000    |          |    |        | 1000 |          | 5000 |          | 10000 |          |
|             |        | Sum.                       | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. |    |        | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. |
| BDe         | HC     | -203646                    | 2070.26  | -1004851 | 10212.09 | -2006720 | 20396.87 | C  | HC     | 757  | 2.39     | 901  | 2.1      | 934   | 2.1      |
|             | TABU   | -203604                    | 2069.82  | -1004781 | 10211.26 | -2006576 | 20395.19 |    | TABU   | 740  | 2.59     | 889  | 2.28     | 937   | 2.08     |
|             | MMHC   | -206444                    | 2100.29  | -1011247 | 10281.04 | -2014821 | 20484.37 |    | MMHC   | 615  | 1.78     | 803  | 1.99     | 860   | 1.97     |
|             | RSMAX2 | -207282                    | 2107.49  | -1011363 | 10281.33 | -2013892 | 20473.37 |    | RSMAX2 | 584  | 1.48     | 800  | 1.84     | 851   | 1.9      |
| loglik      | HC     | -200394                    | 2038.1   | -1000444 | 10168.22 | -2001783 | 20347.75 | M  | HC     | 194  | 1.82     | 66   | 1        | 40    | 0.79     |
|             | TABU   | -200344                    | 2037.54  | -1000377 | 10167.39 | -2001664 | 20346.26 |    | TABU   | 189  | 1.79     | 65   | 1.01     | 34    | 0.67     |
|             | MMHC   | -203577                    | 2072.15  | -1007145 | 10240.36 | -2010170 | 20438.09 |    | MMHC   | 342  | 1.78     | 164  | 1.37     | 118   | 1.17     |
|             | RSMAX2 | -204608                    | 2081.19  | -1007359 | 10241.53 | -2009346 | 20428.13 |    | RSMAX2 | 388  | 1.95     | 182  | 1.48     | 135   | 1.32     |
| AIC         | HC     | -201494                    | 2049.13  | -1001662 | 10180.42 | -2003044 | 20360.34 | WO | HC     | 49   | 0.8      | 33   | 0.62     | 26    | 0.71     |
|             | TABU   | -201445                    | 2048.6   | -1001593 | 10179.58 | -2002914 | 20358.76 |    | TABU   | 71   | 0.91     | 46   | 0.72     | 29    | 0.54     |
|             | MMHC   | -204491                    | 2081.31  | -1008252 | 10251.44 | -2011336 | 20449.76 |    | MMHC   | 43   | 0.66     | 33   | 0.62     | 22    | 0.5      |
|             | RSMAX2 | -205438                    | 2089.54  | -1008428 | 10252.25 | -2010475 | 20439.45 |    | RSMAX2 | 28   | 0.59     | 18   | 0.44     | 14    | 0.38     |
| BIC         | HC     | -204193                    | 2076.21  | -1005631 | 10220.16 | -2007590 | 20405.74 | WC | HC     | 136  | 1.82     | 76   | 1.3      | 58    | 1.43     |
|             | TABU   | -204147                    | 2075.75  | -1005556 | 10219.29 | -2007421 | 20403.85 |    | TABU   | 176  | 2.06     | 106  | 1.54     | 66    | 1.21     |
|             | MMHC   | -206734                    | 2103.81  | -1011860 | 10287.52 | -2015539 | 20491.85 |    | MMHC   | 126  | 1.57     | 96   | 1.41     | 66    | 1.21     |
|             | RSMAX2 | -207475                    | 2110.05  | -1011912 | 10287.17 | -2014545 | 20480.27 |    | RSMAX2 | 70   | 1.25     | 54   | 1.06     | 44    | 0.92     |

Table 6.28: Comparison via Rhombus (Num of Nodes = 10)

|             |        | Rhombus (Num of Nodes = 10) |          |          |          |          |          |      |          |       |          |      |          | 1000 |          |      | 5000     |      |          |
|-------------|--------|-----------------------------|----------|----------|----------|----------|----------|------|----------|-------|----------|------|----------|------|----------|------|----------|------|----------|
| Sample Size |        | 1000                        |          |          |          | 5000     |          |      |          | 10000 |          |      |          | 1000 |          |      | 5000     |      |          |
|             |        | Sum.                        | Std.Dev. | Sum.     | Std.Dev. | Sum.     | Std.Dev. | Sum. | Std.Dev. | Sum.  | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. | Sum. | Std.Dev. |
| BDe         | HC     | -260099                     | 2631.01  | -1284605 | 12999.26 | -2564149 | 25946.15 | C    | HC       | 1060  | 2.85     | 1257 | 2.45     | 1308 | 2.36     | M    | HC       | 282  | 2.28     |
|             | TABU   | -260011                     | 2630.06  | -1284511 | 12998.24 | -2564128 | 25945.98 |      | TABU     | 1045  | 2.98     | 1248 | 2.48     | 1309 | 2.27     |      | TABU     | 270  | 2.25     |
|             | MMHC   | -264881                     | 2678.69  | -1296057 | 13113.29 | -2576754 | 26078.33 |      | MMHC     | 772   | 2.01     | 1060 | 2.25     | 1162 | 2.19     |      | MMHC     | 569  | 2.09     |
|             | RSMAX2 | -267848                     | 2707.82  | -1302660 | 13177.71 | -2583552 | 26138.92 |      | RSMAX2   | 684   | 1.53     | 978  | 1.66     | 1089 | 1.85     |      | RSMAX2   | 695  | 2.17     |
| loglik      | HC     | -256003                     | 2590.39  | -1279003 | 12943.36 | -2557878 | 25883.58 | M    | HC       | 282   | 2.28     | 102  | 1.44     | 55   | 1.09     | WO   | HC       | 58   | 0.79     |
|             | TABU   | -255877                     | 2589.05  | -1278879 | 12942.02 | -2557857 | 25883.42 |      | TABU     | 270   | 2.25     | 99   | 1.38     | 54   | 1.04     |      | TABU     | 85   | 0.93     |
|             | MMHC   | -261402                     | 2644.17  | -1290922 | 13062.19 | -2570958 | 26020.6  |      | MMHC     | 569   | 2.09     | 296  | 1.81     | 213  | 1.57     |      | MMHC     | 59   | 0.75     |
|             | RSMAX2 | -264733                     | 2676.94  | -1298011 | 13131.38 | -2578147 | 26085.06 |      | RSMAX2   | 695   | 2.17     | 407  | 2.09     | 300  | 1.89     |      | RSMAX2   | 21   | 0.54     |
| AIC         | HC     | -257421                     | 2604.54  | -1280577 | 12959.12 | -2559500 | 25899.79 | WC   | HC       | 58    | 0.79     | 41   | 0.64     | 37   | 0.73     | WC   | TABU     | 85   | 0.93     |
|             | TABU   | -257311                     | 2603.36  | -1280465 | 12957.9  | -2559479 | 25899.63 |      | TABU     | 53    | 0.72     | 37   | 0.51     | 53   | 0.72     |      | MMHC     | 59   | 0.75     |
|             | MMHC   | -262541                     | 2655.62  | -1292328 | 13076.25 | -2572422 | 26035.24 |      | MMHC     | 44    | 0.76     | 25   | 0.52     | 44   | 0.76     |      | RSMAX2   | 21   | 0.54     |
|             | RSMAX2 | -265720                     | 2686.86  | -1299247 | 13143.78 | -2579487 | 26098.48 |      | RSMAX2   | 15    | 0.41     | 11   | 0.35     | 15   | 0.41     |      | RSMAX2   | 76   | 1.33     |
| BIC         | HC     | -260900                     | 2639.27  | -1285706 | 13010.48 | -2565348 | 25958.23 | WC   | HC       | 186   | 2.05     | 100  | 1.35     | 82   | 1.51     | WC   | TABU     | 226  | 2.08     |
|             | TABU   | -260830                     | 2638.5   | -1285633 | 13009.66 | -2565327 | 25958.07 |      | TABU     | 130   | 1.59     | 84   | 1.14     | 84   | 1.14     |      | MMHC     | 194  | 1.85     |
|             | MMHC   | -265336                     | 2683.71  | -1296909 | 13122.09 | -2577700 | 26088.01 |      | MMHC     | 148   | 1.77     | 92   | 1.32     | 148  | 1.77     |      | RSMAX2   | 78   | 1.36     |
|             | RSMAX2 | -268141                     | 2711.22  | -1303275 | 13184.2  | -2584318 | 26146.85 |      | RSMAX2   | 60    | 1.08     | 60   | 1.08     | 60   | 1.08     |      | RSMAX2   | 76   | 1.33     |

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