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A Study on Comparison of Bayesian Network Structure Learning Algorithms for Selecting Appropriate Models

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A Study on Comparison of Bayesian Network Structure Learning Algorithms for Selecting Appropriate Models

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국 문 초 록

본 논문에서는 R의 bnlearn 패키지에서 제공하는 베이지안 네트워크 구조학습 알고리즘 간의 성능을 비교하였다.

베이지안 네트워크 구조 학습 결과에 대한 성능 평가는 score를 이용하는 방법과, 목표 네트워크와 학습된 네트워크를 서로 비교하는 방법이 있다. 본 논문에서는 이 두 가지 방법으로 알고리즘별 성능을 비교했을 때, 결과가 서로 다를 수 있음을 확인하였다.

Topology에 따른 Synthetic Data를 생성, 이에 대하여 알고리즘별 성능을 비교하여, 목표 네트워크의 형태에 따라 적합한 알고리즘 선택을 할 수 있도록 객관적인 방향을 제시하고자 하였다.

그동안 베이지안 네트워크 데이터 생성기가 매우 고가이거나, 공개된 툴도 매우 사용하기 까다로웠기 때문에, 베이지안 네트워크 관련 실증 연구는 사례 데이터를 이용한 경우가 대부분이었다. 이에 따라 Bayesian Network 모델을 바탕으로 R에서 데이터를 생성할 수 있는 생성기를 제작하여 공개하였다.

Abstract

In this paper, we compare the performance between the Bayesian network structure learning algorithm provided by **bnlearn** package in **R**.

The performance of the study results is evaluated by using a score or comparing between the target network and the learning network. In this paper, it was confirmed that algorithm specific performance test results using fore-mentioned methods are different.

Unlike most previous studies which generally used real data, synthetic data generated based on topology was used to compare performance of contrast-specific algorithm. The aim of this paper is to provide objective guidance of selecting suitable algorithm in accordance to target network.

Previous tools suffer from serious trade-off between cost and complexity, restricting most studies relevant to Bayesian network to using only real data. To address such problem, a data generator based on Bayesian network model using **R** is built and introduced.

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Notations

r_i : Number of states of the finite random variable X_i,

x_{ik} : k-th value of X_i.

$q_i = \prod_{X_j \in \prod_{X_i}} r_j$: Number of possible configurations of the parent set $\prod_{X_i}$ of X_i.

ω_{ij} : j-th configuration of $\prod_{X_i}$ $(1 \leq j \leq q_i)$.

N_{ijk} : Number of instances in the data T where the variable X_i takes its k-th value x_{ik} and the variables in $\prod_{X_i}$ take their j-th configuration ω_{ij}.

$N_{ij} = \sum_{k=1}^{r_i} N_{ijk}$: Number of instances in the data T where the variables in $\prod_{X_i}$ take their j-th configuration ω_{ij}.

$N_{ik} = \sum_{j=1}^{q_i} N_{ijk}$: Number of instances in the data T where the variable X_i takes its k-th value x_{ik}.

$\mathbf{N}$: Total number of instances in the data T.

$\Theta_G = \{\Theta_i\}_{i=1,\cdots,n}$: Encodes parameters of a BN B with underlying DAG G

$\Theta_i = \{\Theta_{ij}\}_{j=1,\cdots,q_i}$: Encodes parameters concerning only the variable X_i of X in B

$\Theta_{ij} = \{\Theta_{ijk}\}_{k=1,\cdots,r_i}$: Encodes parameters for variable X_i of X in B given that its parents take their j-th configuration

Chapter 1

Introduction

1.1 Bayesian Network

Bayesian networks (BN) are graphical models where nodes represent random variables and arrows represent probabilistic dependencies between them (Kevin B. K. and Ann E. N., 2010).

A n-dimensional Bayesian network is a triple $B = (X, G, \Theta)$ where:

- X is a n-dimensional finite random vector each random variable X_i ranges over by a finite domain D_i. Henceforward, we denote the joint domain by $D = \prod_{i=1}^{n} D_i$
- $G = (N, E)$ is a directed acyclic graph (DAG) with nodes $N = \{X_1, \cdots, X_n\}$ and edges E representing direct dependencies between the variables.
- Θ encodes the parameters $\{\theta_{ijk}\}_{i\in 1,\cdots,n, j\in D_{\prod_{X_i}}, k\in D_i}$ of the network, where

$$\theta_{ijk} = P_B(X_i = x_{ik} | \prod_{X_i} = \omega_{ij}),$$

$\prod_{X_i}$ denotes the set of parents of X_i in G, $D_{\prod_{X_i}}$ denotes the joint domain of the variables in $\prod_{X_i, x_{ik}}$ is the k-th value of X_i and ω_{ij} is the j-th configuration of $\prod_{X_i}$.

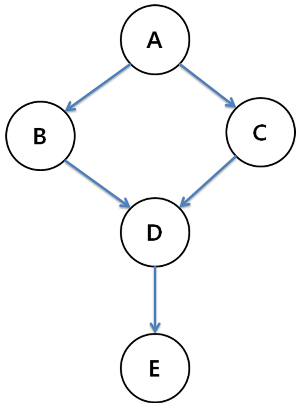


Figure 1.1: $P(A, B, C, D, E) = P(A)P(B|A)P(C|A)P(D|B, C)P(E|D)$

A BN defines a unique joint probability distribution over X given by

$$P_B(X_1, \cdots, X_n) = \prod_{i=1}^{n} P_B(X_i|\prod_{X_i}).$$

- A BN encodes the independence assumptions over the component random variables of X.
- An edge (j, i) in E represents a direct dependency of X_i from X_j.
- The set of all Bayesian networks with n variables is denotes by B_n.

1.2 Bayesian Network Structure Learning

The problem of learning a BN given data T consists of finding the BN that best fits the data T. In order to quantify the fitting of a BN a scoring function

ϕ is considered.

Learning a Bayesian network is as follows:

Given a data $T = \{y_1, \cdots, y_n\}$ and a scoring function ϕ, the problem of learning a Bayesian network is to find a Bayesian network $B \in B_n$ that maximizes the value $\phi(B, T)$.

Bayesian network structure learning algorithms can be grouped into two categories by Marco S. (2010).

Constraint-based algorithms These algorithms learn the network structure by analyzing the probabilistic relations entailed by the Markov property of Bayesian networks with conditional independence tests and then constructing a graph which satisfies the corresponding d-separation statements. The resulting models are often interpreted as causal models even when learned from observational data (Pearl J. 1988).

Score-based algorithms The main idea behind score-based learning is to optimize the degree of match between the generated network and the observations. (Benjamin B. P., 2003) These algorithms assign a score to each candidate Bayesian network and try to maximize it with some heuristic search algorithm. The search problem of identifying a Bayesian network that has a relative posterior probability greater than a given constant is NP-complete. (D.M. Chickering, 1996) Greedy search algorithms (such as hill-climbing or TABU search) are a common choice, but almost any kind of search procedure can be used.

Traditionally, in searching for a Bayesian network structure, the set of states was the set of all possible Bayesian network structures, the representation was a Directed Acyclic Graph(DAG) and the set of operators were various small local changes to a DAG, e.g. adding, removing or reversing an arc, as illustrated in Figure 1.2. (R. Daly and Q. Shen., 2007)

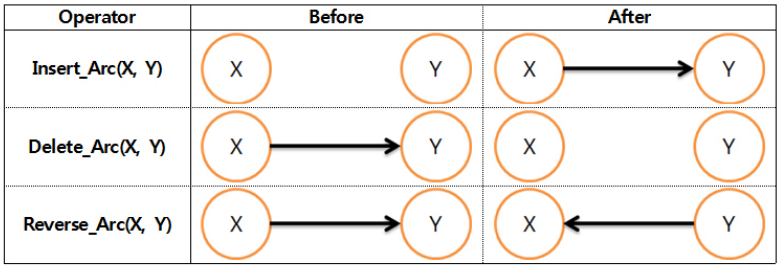


Figure 1.2: Basic Modification Operators in Searching for a Bayesian Network Structure

Chapter 2

Bayesian Network Structure Learning Algorithms in bnlearn Package

bnlearn is an R package which includes several algorithms for learning the structure of Bayesian networks with either discrete or continuous variables. Both constraint-based and score-based algorithms are implemented. (Marco S., 2010)

2.1 Constraint-based Algorithms

2.1.1 Grow-Shrink (GS) Markov Blanket Algorithm

Based on the Grow-Shrink Markov blanket, the simplest Markov blanket detection algorithm (Margaritis D., 2003) used in a structure learning algorithm.

The definition of a Markov blanket is as follows: for any variable $X \in U$, the Markov blanket $BL(X) \subseteq U$ is any set of variables such that for any $Y \in U - BL(X) - \{X\}, X \perp Y | BL(X)$. In other words, $BL(X)$ completely "shields" (d-separates) variable X from any other variable outside $BL(X) \cup \{X\}$.

In a Bayesian network graph, the Markov blanket of a node includes its parents, children and other parents of all of its children.

Algorithm. The GS Markov Blanket Algorithm

1. $S \leftarrow NULL$
2. **While** $\exists\, Y \in U - \{X\}$

   such that $Y \not\perp X | S$,

   do $S \leftarrow S \cup \{Y\}$. (Growing phase)

   End While
3. **While** $\exists\, Y \in S$

   such that $Y \perp X | S - \{Y\}$,

   do $S \leftarrow S - \{Y\}$. (Shrinking phase)

   End While
4. $B(X) \leftarrow S$.

GS, for the recovery of the Markov blanket of X is based on pairwise independent tests. It consists of two phases, a growing and a shrinking one. Starting from an empty set S, the growing phase adds variables to S as long as they are dependent with X given the current contents of S.

2.1.2 Incremental Association (IAMB) Algorithm

Based on the Incremental Association Markov blanket (IAMB) algorithm (Tsamardinos I. *et al.* 2003), which is based on a two-phase selection scheme (a forward selection followed by an attempt to remove false positives).

Algorithm. The IAMB Algorithm

1. (Forward phase)

    $S \leftarrow NULL$

    While S has changed

    Find the feature X in $V - S - \{T\}$ that maximizes $f(X;T|S)$

    If not $I(X;T|S)$

    Add X to S

    End If

    End While

2. (Backward phase)

    Remove from S all variables X, for which $I(X;T|S - \{X\})$

3. **Return** S

IAMB consists of two phases, a forward and a backward one.

The Markov blanket of a variable of interest T, will be denotes as $MB(T)$. An estimate of the $MB(T)$ is kept in the set S. In the forward phase all variables that belong in $MB(T)$ and possibly more (false positives) enter S while in the backward phase the false positives are identified and removed so that $S = MB(T)$ in the end.

The heuristic used in IAMB to identify potential Markov blanket members in 'forward phase' is the following:

Start with an empty candidate set for the S and admit into it (in the next iteration) the variable that maximizes a heuristic function $f(X;T|S)$. Function f should return a non-zero value for every variable that is a member of the Markov blanket for the algorithm to be sound, and is typically a measure of association between X and T given S. In our experiments we used f as the Mutual Information similar to what was suggested in Margaritis D. and Thrun S. (1999), J. Cheng *et al.* (2002): $f(X;T|S)$ is the Mutual Information between S and T given S. It is important that f is an informative and effective heuristic so that the set of candidate variables after 'forward phase' is as small as possible for two reasons: one is time efficiency (i.e. do not spend time considering irrelevant variables) and another is sample efficiency (do not require sample larger than what is absolutely necessary to perform conditional tests of independence).

In backward conditioning we remove features that do not belong to the $MB(T)$ one-by-one by testing whether a feature X from S is independent of T given the remaining S.

2.2 Score-Based Algorithms

2.2.1 Hill-Climbing (HC) Algorithm

A Hill-climbing is a greedy search on the space of the directed graphs. The optimized implementation uses score caching, score decomposability and score equivalence to reduce the number of duplicated tests.

Algorithm. The Hill-climbing(HC) Algorithm

1. **Current**: Make_Node(Initial State)
2. **While**

    Neighbor: a highest-valued successor of Current.State

    If Neighbor.Value < Current.Value **Then**

        Return Current.State

    End If

    Current ← Neighbor

   End While

It is simply a loop that continually moves in the direction of increasing value. The algorithm does not maintain a search tree, so the data structure for the current node only needs to record the state and the value of the objective function. Hill-climbing does not look beyond the immediate neighbors of the current state. This resembles trying to find the top of Mount Everest in a thick fog while suffering from amnesia. (Russell S. J. and Norvig P., 2009)

2.2.2 TABU Search Algorithm

A modified Hill-climbing is able to escape local optima by selecting a network that minimally decreases the score function.

A variant of Hill-climbing called TABU search has gained popularity (Fred W. G. and Manuel L., 1997). This algorithm maintains a TABU list of k previously visited states that cannot be revisited, as well as improving efficiency when searching graphs. This list allows the algorithm to escape from some local minima.

Algorithm. The TABU Search Algorithm

1. Choose $x \in X$ to start the precess.
2. Find $x' \in N(x)$ such that $f(x') < f(x)$.
3. If no such x' can be found, x is the local optimum and the method stops.
4. Otherwise, designate x' to be the new x and go to 2.

TABU search begins in the same way as ordinary local or neighborhood search, proceeding iteratively from one point solution to another until a chosen termination criterion is satisfied. Each $x \in X$ has an associated neighborhood $N(x) \subset X$, and each solution $x' \in N(x)$ is reached from x by an operation called 'move'.

As an initial point of departure, we may contrast TABU search with a simple descent method where the goal is to $\min f(x)$ (or a corresponding ascent method where the goal is to $\max f(x)$). Such method only permits moves to neighbor solutions that improve the current objective function value and ends when no improving solutions can be found. A pseudo-code of a generic descent method is presented in 'Algorithm'. The final x obtained by a descent method is called a local optimum, since it is at least as good or better than all solutions in its neighborhood. The evident shortcoming of a descent method is that such a local optimum in most cases will not be a global optimum, i.e., it usually will not minimize $f(x)$ over all $x \in X$.

2.3 Hybrid Algorithms

2.3.1 Max-Min Hill-Climbing (MMHC) Algorithm

A hybrid algorithm which combines the Max-Min Parents and Children algorithm (to restrict the search space) and the Hill-Climbing algorithm (to find the optimal network structure in the restricted space). (Tsamardinos I. *et al.*, 2006)

The algorithm first identifies the parents and children set of each variable, then performs a greedy hill-climbing search in the space of Bayesian networks. The search begins with an empty graph. The edge addition, deletion, or direction reversal that leads to the largest increase in score is taken and the search continues in a similar fashion recursively.

2.3.2 More general 2-phase Restricted Maximization (RS-MAX2)

A more general method is which Max-Min Hill-Climbing, uses any combination of constraint-based and score-based algorithms.

Chapter 3

The Comparison Methodology

3.1 The Number of Graphical Errors in the Learnt Structure

The comparison methodology used in this paper is similar to the method used in X.-w. Chen *et al.* (2006). The existence of the known network structures allows us to define three important terms which indicate the performance of the algorithm (in terms of the number of graphical errors in the learnt structure).

C (Correct Arcs) Edges present in the original network and in the learnt network structure.

M (Missing Arcs) Edges present in the original network but not in the learnt network structure.

WO (Wrongly Oriented Arcs) Edges present in the learnt network structure, but having opposite orientation when compared with the corresponding edge in the original network structure.

WC (Wrongly Corrected Arcs) Edges not present in the original network but included in the learnt network structure.

3.2 Network Scores

The values of the BDe, the Log-likelihood (LL), the AIC, and the BIC are metrics for the learned networks. (Alexandra M. C., 2009) These measures can offer an idea of the quality of the networks from different points of view. In all four cases, the higher the value of the metric, the better the network. (D. Heckerman *et al.*, 1995, Silvia A. *et al.*, 2004).

3.2.1 Bayesian Scoring Functions

Compute the posterior probability distribution, starting from a prior probability distribution on the possilbe networks, conditioned to data T, that is, $P(B|T)$.

The best network is the one that maximizes the posterior probability.

Since the term $P(T)$ is the same for all possible networks, in practice, for comparative purposes, computing $P(B, T)$ is sufficient.

As it is easier to work in the logarithmic space, the scoring functions use the value $\log(P(B, T))$ instead of $P(B, T)$.

BDe

D. Heckerman *et al.* (1995) proposed the Bayesian Dirichlet (BDe) score.

Given a directed acyclic graph (DAG) G such that $P(G) > 0$ then Θ_{ij} is Dirichlet for all Θ_{ij} in Θ_G. And given a Bayesian network B, data T can be seen as a multinomial sample of the joint space D with parameters

$$\Theta_D = \{\theta_{x_1,\cdots,x_n}\}_{x_i=1,\cdots,r_i, i\in 1,\cdots,n}$$

where $\theta_{x_1,\cdots,x_n} = \prod_{i=1}^{n} \theta_{x_I|\prod_{x_i}}$.

For any complete G, we have that $P(G) > 0$. Then $\rho(\Theta_G|G) = \prod_{i=1}^{n} \rho(\Theta_i|G)$ (global parameter independence) and $\rho(\Theta_i|G) = \prod_{i=1}^{q_i} \rho(\Theta_{ij}|G)$ for all $i = 1,\cdots,n$ (local parameter independence).

Given two DAGs G and G', such that $P(G) > 0$ and $P(G') > 0$, if X_i has the same parents in G and G_0, then $\rho(\Theta_{ij}|G) = \rho(\Theta_{ij}|G')$ for all $j = 1,\cdots,q_i$.

Suppose that $\rho(\Theta_D|G)$ is Dirichlet with equivalent sample size N' for some complete G in D. Then, for any Bayesian network B in D,

$$BDe(B,T) = P(B,T) = P(B) \times \prod_{i=1}^{n}\prod_{j=1}^{q_i}\left(\frac{\Gamma(N'_{ij})}{\Gamma(N_{ij}+N'_{ij})} \times \prod_{k=1}^{r_i}\frac{\Gamma(N_{ijk}+N'_{ijk})}{\Gamma(N'_{ijk})}\right)$$

where $N'_{ijk} = N' \times P(X_i = x_{ik}, \prod_{X_i} = \omega_{ij}|G)$.

The equivalent sample size N' expresses the strength of out belief in the prior distribution.

3.2.2 Information-theoretic Scoring Functions

log-**likelihood (LL)**

The log-**likelihood (LL) Score** is defined in the following way:

$$LL(B|T) = \sum_{i=1}^{n}\sum_{j=1}^{q_i}\sum_{k=1}^{r_i} N_{ijk}\log\left(\frac{N_{ijk}}{N_{ij}}\right).$$

The LL score tends to favor complete network structures and it does not provide an useful representation of the independence assumptions of the learned network.

This phenomenon of overfitting is usually avoided in two different ways:

- By limiting the number of parents per network variable.
- By using some penalization factor over the LL score : AIC, BIC

AIC and BIC

The measure of the quality of a BN can be computed in several different ways:

$$\phi(B|T) = LL(B|T) - f(N)|B|,$$

where $f(N)$ is a non-negative penalization function.

- If f(N) = 1, we have the **Akaike Information Criterion (AIC)** scoring function:

$$AIC(B|T) = LL(B|T) - |B|.$$

- If $f(N) = \frac{1}{2}\log(N)$, we have the **Bayesian Information Criterion (BIC)** score.
- If $f(N) = 0$, we have the LL score.

Chapter 4

Data Generation with BN_Data_Generator in R

If given a Bayesian network model, then we can make a data set based on the model. However, it does not provide data by **bnlearn**, and in addition, it was difficult to find other functions. This makes it very hard work to create data in **R**. Other tools suffer from the trade-off between cost and complexity, restricting most studies relevant to Bayesian network to using only real data.

To address such problems, a data generator based on the Bayesian network model using R is built and introduced. At present, it exists as R functional form, but here are plans to make an R package. An update on the current status at https://github.com/praster1/BN_Data_Generator. This generator was declared the GNU 2.0 license.

4.1 BN_Data_Generator Function in R

Description It based on a Bayesian network model to generates synthetic data.

Usage BN_Data_Generator (arcs, input_Probs, n, node_names)

Arguments

Table 4.1: Argunemts of BN_Data_Generator

Argument	Type	Description
arcs	matrix	A matrix that determines the arcs
input_Probs	list	The conditional probabilities.
n	constant	sample size
node_names	vector	node names

4.2 A Simple Example

Suppose we generate a data based on the model as show in Figure 4.1. This model is the "Bayesian network model of Asia Data Set by Lauritzen and Spiegelhalter" to be introduced in the next chapter.

It makes Arcs, input_Probs, node_names as follows:

```
R> arcs = rbind(
    # A S T L B E X D
```

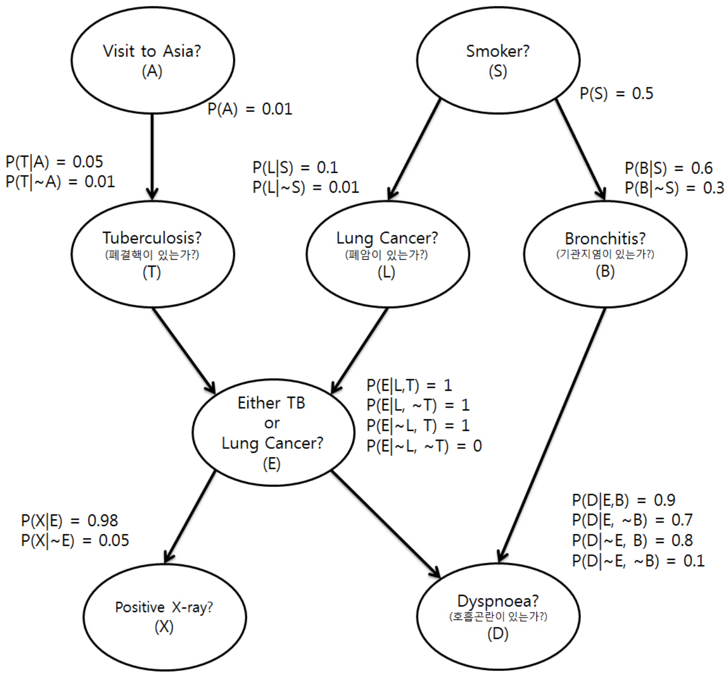


Figure 4.1: BN model of Asia Data Set by Lauritzen and Spiegelhalter

```
    c(0, 0, 1, 0, 0, 0, 0, 0), #A
    c(0, 0, 0, 1, 1, 0, 0, 0), #S
    c(0, 0, 0, 0, 0, 1, 0, 0), #T
    c(0, 0, 0, 0, 0, 1, 0, 0), #L
    c(0, 0, 0, 0, 0, 0, 0, 1), #B
    c(0, 0, 0, 0, 0, 0, 1, 1), #E
    c(0, 0, 0, 0, 0, 0, 0, 0), #X
    c(0, 0, 0, 0, 0, 0, 0, 0)) #D
R> arc_name = c("A", "S", "T", "L", "B", "E", "X", "D")
R> Probs = list(
 c(0.01), #P(A)
```

c(0.5), $\#P(S)$

c(0.05, 0.01), $\#P(T|A), P(T|\sim A)$

c(0.1, 0.01), $\#P(L|S), P(L|\sim S)$

c(0.6, 0.3), $\#P(B|S), P(B|\sim S)$

c(1, 1, 1, 0), $\#P(E|T, L), P(E|\sim T, L), P(E|T, \sim L), P(E|\sim T, \sim L)$

c(0.98, 0.05), $\#P(X|E), P(X|\sim E)$

$\#P(D|B, E), P(D|\sim B, E), P(D|B, \sim E), P(D|\sim B, \sim E)$

c(0.9, 0.7, 0.8, 0.1))

Suppose the sample size is 1000. If you type objects and sample size into BN_Data_Generator, then the data is generated.

```
R> n = 1000
R> res = BN_Data_Generator(arcs, Probs, n, arc_name)
R> data = res$data
R> head(data)
  A S T L B E X D
1 N N N N N N N N
2 N Y N N Y N N Y
3 N N N N N N N N
4 N Y N N N N N N
5 N N N N Y N N N
6 N Y N N Y N N Y
```

R> dim(data)

[1] 1000 8

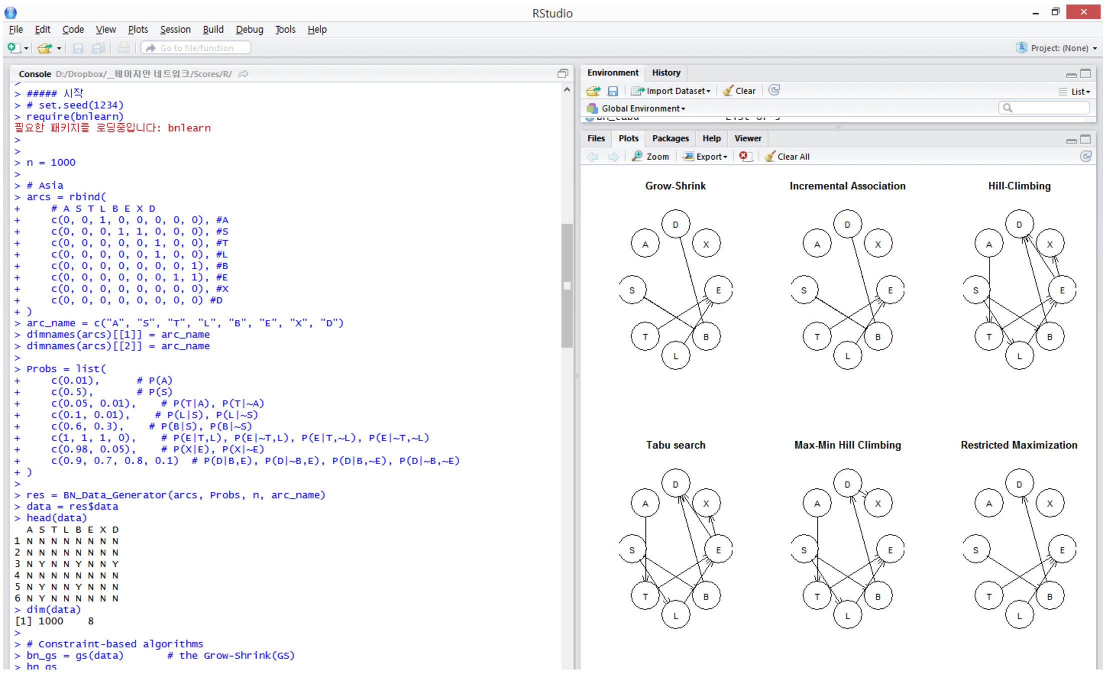


Figure 4.2: After make a data, execution results by bnlearn

Chapter 5

Simulation

Previous tools suffer from a serious trade-off between cost and complexity, restricting most studies relevant to Bayesian network to using only real data.

Therefore, in this paper, have been widely used so far, for the BN demonstration model, I tried to first apply the algorithm.

However, in order to measure the objective performance of the algorithm, it is necessary to try to analyze the synthetic data. Therefore, in this paper, by using the data generator BN that introduced, after generating the synthetic data in accordance with the topology, and algorithms were attempted to be applied to this.

In order to avoid the influence of chance, all experiments are repeated 100 times, and overall results are reported.

5.1 Real Data

5.1.1 Asia Data Set by Lauritzen and Spiegelhalter

Description Small synthetic data set from Lauritzen S. and Spiegelhalter D. (1988) about lung diseases (tuberculosis, lung cancer or bronchitis) and visits to Asia.

Number of nodes 8

Number of arcs 8

Number of parameters 18

Lauritzen S. and Spiegelhalter D. (1988) motivate this example as follows:

"Shortness-of-breath (dyspnoea) may be due to tuberculosis, lung cancer or bronchitis, or none of them, or more than one of them. A recent visit to Asia increases the chances of tuberculosis, while smoking is known to be a risk factor for both lung cancer and bronchitis. The results of a single chest X-ray do not discriminate between lung cancer and tuberculosis, as neither does the presence or absence of dyspnoea."

Table 5.1: Comparison of scores and correct arcs via Asia data set

		Asia (Num of Nodes = 8)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-229814	35.25	-1113883	85.65	-2218508	126.41	C	HC	677	0.55	716	0.37	735	0.48
	TABU	-229806	35.29	-1113857	85.9	-2218431	126.4		TABU	655	0.73	677	0.72	703	0.76
	MMHC	-249829	41.68	-1213021	117.02	-2417769	191.84		MMHC	461	0.55	503	0.48	514	0.59
	RSMAX2	-252800	43.81	-1233095	121.09	-2457976	173.35		RSMAX2	400	0	400	0	400	0
loglik	HC	-220520	36.46	-1102564	85.92	-2206160	126.05	M	HC	122	0.52	83	0.38	65	0.48
	TABU	-220505	36.54	-1102521	86.31	-2206030	126.02		TABU	122	0.52	83	0.38	65	0.48
	MMHC	-241431	43.03	-1202710	117.97	-2406616	192.19		MMHC	339	0.55	297	0.48	286	0.59
	RSMAX2	-244901	44.97	-1223631	121.63	-2447783	173.8		RSMAX2	400	0	400	0	400	0
AIC	HC	-222238	36.46	-1104349	85.87	-2208092	126.21	WO	HC	1	0.1	1	0.1	0	0
	TABU	-222226	36.53	-1104315	86.16	-2207985	126.19		TABU	23	0.51	40	0.62	32	0.66
	MMHC	-242973	43	-1204336	117.81	-2408295	192.1		MMHC	0	0	0	0	0	0
	RSMAX2	-246201	44.97	-1224936	121.62	-2449114	173.8		RSMAX2	0	0	0	0	0	0
BIC	HC	-226454	36.49	-1110166	85.76	-2215057	126.96	WC	HC	72	1.22	96	1.12	224	1.69
	TABU	-226449	36.52	-1110161	85.76	-2215033	126.99		TABU	112	1.43	170	1.46	292	2.02
	MMHC	-246757	42.97	-1209634	117.34	-2414348	191.83		MMHC	202	0.2	218	0.58	244	0.83
	RSMAX2	-249391	44.97	-1229189	121.6	-2453913	173.82		RSMAX2	0	0	8	0.39	50	0.87

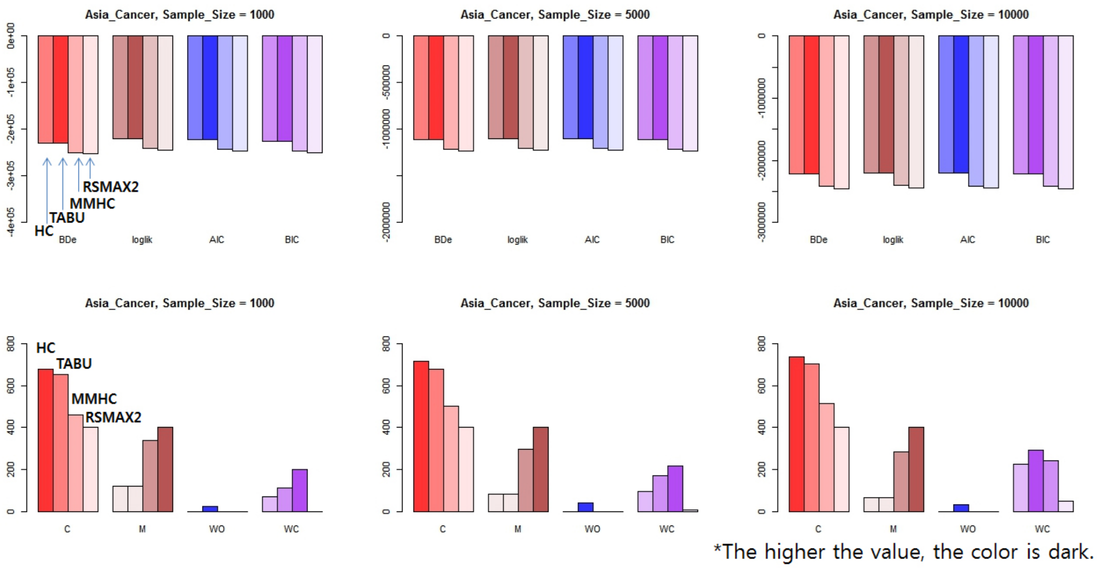


Figure 5.1: Comparison of scores and correct arcs via Asia data set

5.1.2 Insurance Evaluation Network Data Set

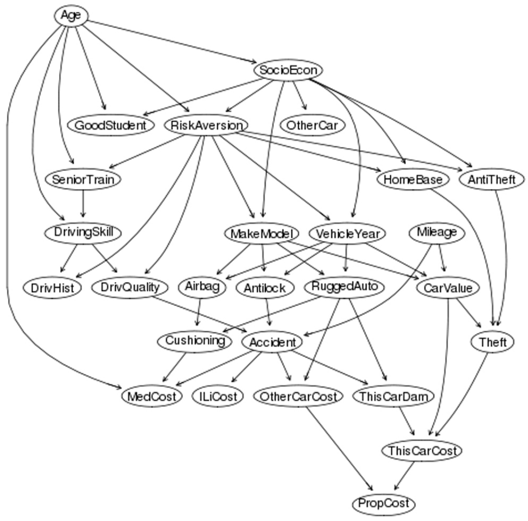


Figure 5.2: Bayesian network model of Insurance Evaluation Network Data Set

Description Insurance is a network for evaluating car insurance risks.

Number of nodes 27

Number of arcs 52

Number of parameters 984

Binder J. *et al.* (1997) motivate this example. This network for estimating the expected claim costs for a car insurance policyholder.

Table 5.2: Comparison of scores and correct arcs via Insurance data set

Sample Size		1000 Sum.	1000 Std.Dev.	5000 Sum.	5000 Std.Dev.	10000 Sum.	10000 Std.Dev.			1000 Sum.	1000 Std.Dev.	5000 Sum.	5000 Std.Dev.	10000 Sum.	10000 Std.Dev.
Insurance (Num of Nodes = 27)															
BDe	HC	-1427953	113.01	-6730331	189.16	-13298470	246.4	C	HC	1864	1.57	2226	0.85	2414	0.4
	TABU	-1422716	120.52	-6708058	190.97	-13294810	244.2		TABU	1961	1.98	2543	0.79	2826	0.68
	MMHC	-1644392	548.92	-7281377	826.67	-14112976	1680.81		MMHC	1146	1.82	1457	1.17	1731	1.88
	RSMAX2	-1735658	397.81	-8293044	1714.16	-16408869	2342.35		RSMAX2	862	1.42	977	1.77	848	0.61
loglik	HC	-1347816	118.57	-6603841	195.33	-13134593	257.01	M	HC	2253	1.11	1738	0.79	1413	0.53
	TABU	-1341071	128.83	-6580889	194.64	-13143085	241.01		TABU	2155	1.39	1642	0.77	1402	0.43
	MMHC	-1590326	585.75	-7192400	852.4	-14000007	1736.44		MMHC	3526	2.2	2849	1.34	2334	2.5
	RSMAX2	-1687201	417.78	-8223919	1734.08	-16330085	2362.44		RSMAX2	3761	1.31	3497	1.05	3469	0.77
AIC	HC	-1383888	112.68	-6652052	190.99	-13198494	248.77	WO	HC	1083	1.16	1236	0.73	1373	0.49
	TABU	-1378131	120.12	-6629457	191.37	-13197529	243.3		TABU	1084	1.56	1015	0.72	972	0.55
	MMHC	-1607916	566.08	-7218618	841.94	-14033715	1714.51		MMHC	528	1.39	894	0.76	1135	0.76
	RSMAX2	-1701145	409.22	-8241628	1727.25	-16350189	2356.64		RSMAX2	577	0.99	726	1.67	883	1.02
BIC	HC	-1472404	105.57	-6809152	185.18	-13428868	242.07	WC	HC	1810	2.28	2096	1.49	2240	1.58
	TABU	-1469072	108.51	-6787720	191.17	-13393809	256.04		TABU	1756	2.49	1906	1.52	1696	1.46
	MMHC	-1651080	519.05	-7304052	808.56	-14155238	1635.76		MMHC	1098	2.35	1494	2.45	1630	1.43
	RSMAX2	-1735362	389.69	-8299334	1705.3	-16422667	2335.81		RSMAX2	1120	2.03	1220	1.69	1596	1.56

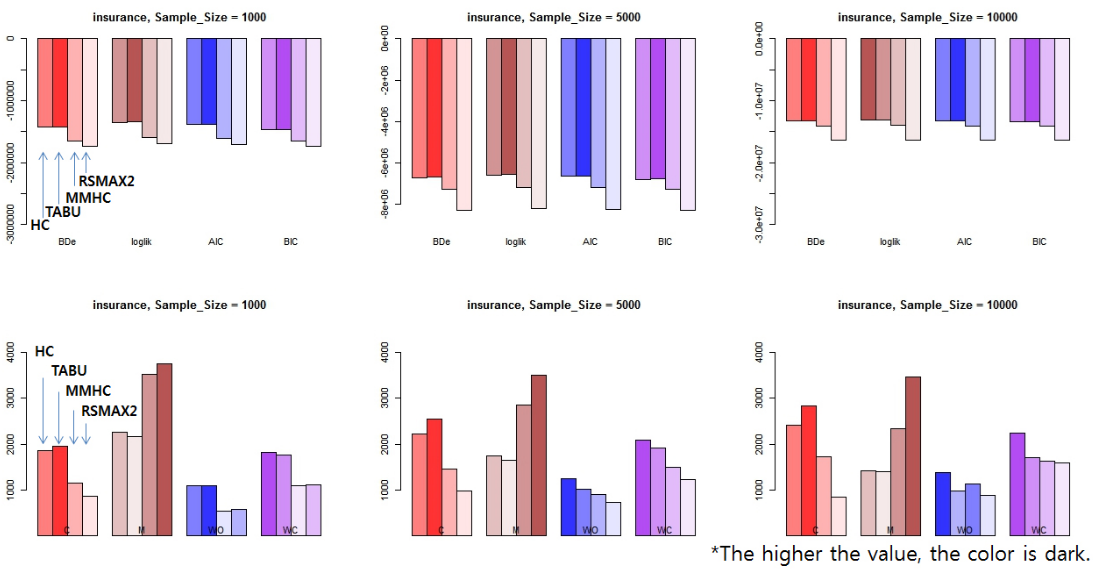


Figure 5.3: Comparison of scores and correct arcs via Insurance data set

5.1.3 ALARM Monitoring System Data Set

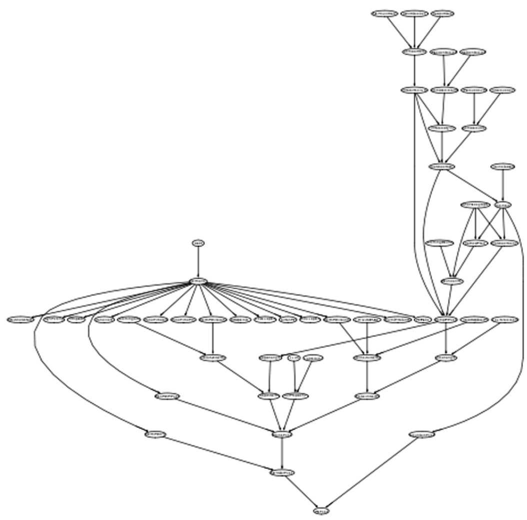


Figure 5.4: Bayesian network model of ALARM Monitoring System Data Set

Description The ALARM ("A Logical Alarm Reduction Mechanism") is a Bayesian network designed to provide an alarm message system for patient monitoring.

Number of nodes 37

Number of arcs 46

Number of parameters 509

Beinlich I. *et al.* (1989) motivate this example. ALARM (A Logical Alarm Reduction Mechanism) is a diagnostic application used to explore probabilistic reasoning techniques in belief networks. ALARM implements an alarm message system for patient monitoring: it calculates probabilities for a differential diagnosis based on available evidence.

Table 5.3: Comparison of scores and correct arcs via ALARM data set

		ALARM (Num of Nodes = 37)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-1178527	132.62	-5580032	259.61	-11006608	381.65	C	HC	2048	1.73	2283	0.43	2270	0.96
	TABU	-1177619	133.2	-5580099	246.64	-11005176	380.57		TABU	2426	1.8	2709	1.05	2686	1.26
	MMHC	-1673651	649.54	-7510018	2077.76	-13885378	7417.41		MMHC	1051	1.77	1421	1.75	1925	3.09
	RSMAX2	-1735940	439.39	-8318508	2058.09	-16586602	3307.55		RSMAX2	633	1.65	867	0.92	858	1.19
loglik	HC	-1099997	130.61	-5464607	260.53	-10860805	370.7	M	HC	900	1.06	498	0.4	468	0.69
	TABU	-1099325	130.77	-5465405	244.34	-10861471	371.47		TABU	878	0.97	501	0.36	468	0.69
	MMHC	-1617451	672.21	-7426574	2093.3	-13790813	7449.01		MMHC	2460	1.68	1480	1.04	1268	1.56
	RSMAX2	-1682617	453.09	-8242927	2079.53	-16508716	3325.33		RSMAX2	3286	1.6	3162	1.21	2949	1.05
AIC	HC	-1135950	129.48	-5509035	253.54	-10923175	374.81	WO	HC	1652	1.37	1819	0.46	1862	0.9
	TABU	-1134574	130.77	-5508543	244.04	-10921249	372.87		TABU	1296	1.52	1390	0.96	1446	1.27
	MMHC	-1634699	655.42	-7451159	2085	-13816106	7435.45		MMHC	1089	1.38	1699	2.05	1407	1.98
	RSMAX2	-1698026	443.88	-8266708	2064.51	-16527481	3318.41		RSMAX2	681	1.6	571	0.98	793	0.71
BIC	HC	-1224175	130.11	-5653808	251.23	-11148029	416.51	WC	HC	2498	2.58	2306	1.9	2714	1.56
	TABU	-1221071	133.9	-5649112	244.7	-11136759	427.99		TABU	2314	2.48	2032	2.08	2452	2.25
	MMHC	-1677023	614.97	-7531272	2058.32	-13907292	7386.76		MMHC	1934	2.22	2890	2.61	2368	2.84
	RSMAX2	-1735838	423.78	-8344201	2018.29	-16595132	3293.99		RSMAX2	1684	2.73	1982	2.56	2262	1.96

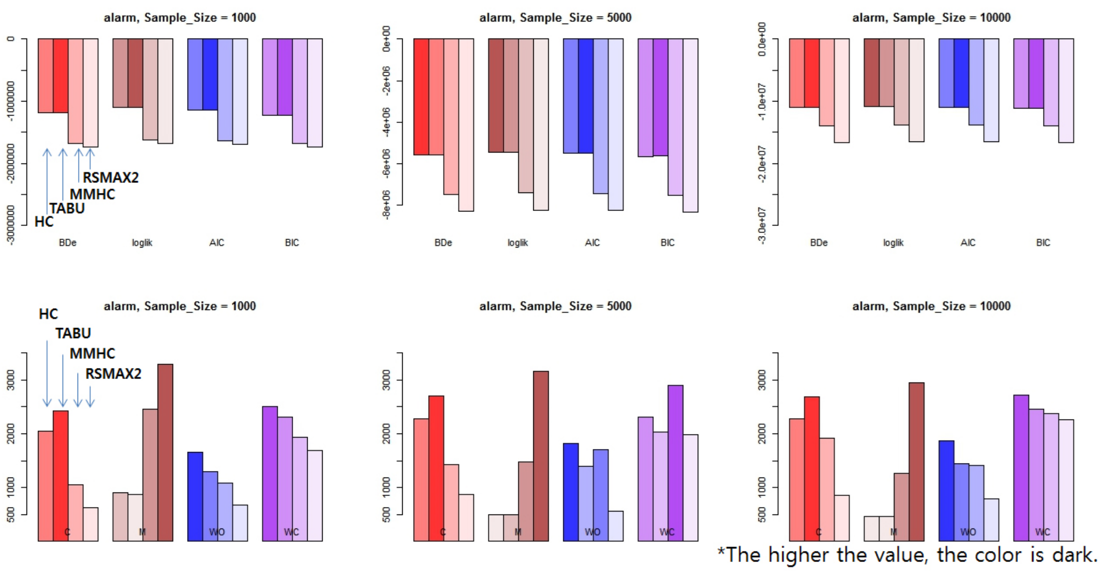


Figure 5.5: Comparison of scores and correct arcs via Hailfinder data set

5.1.4 The HaliFinder Weather Forecast System Data Set

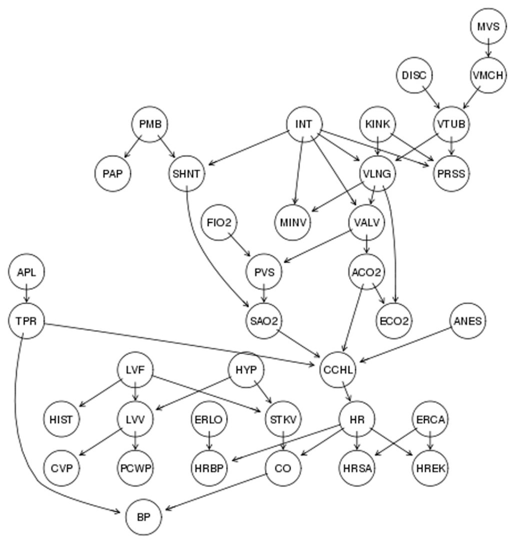


Figure 5.6: Bayesian network model of The HailFinder Weather Forecast System Data Set

Description Hailfinder is a Bayesian network designed to forecast severe summer hail in northeastern Colorado.

Number of nodes 56

Number of arcs 66

Number of parameters 2656

Abramson B. *et al.* (1988) motivate this example. Hailfinder is a Bayesian system that combines meteorological data and models with expert judgement, based on both experience and physical understanding, to forecast severe weather in North-eastern Colorado.

Table 5.4: Comparison of scores and correct arcs via Hailfinder data set

		Hailfinder (Num of Nodes = 56)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-11006608	381.65	-24975425	256.04	-49602871	297.73	C	HC	2270	0.96	5301	0.1	5586	0.6
	TABU	-11005176	380.57	-24975425	256.04	-49595842	296.18		TABU	2686	1.26	5301	0.1	5473	0.8
	MMHC	-13885378	7417.41	-28286426	1564.45	-56127215	2369.94		MMHC	1925	3.09	3247	0.78	3421	1.03
	RSMAX2	-16586602	3307.55	-30165661	4750.83	-60211607	10590.78		RSMAX2	858	1.19	2513	1.45	2599	1.27
loglik	HC	-10860805	370.7	-24580625	264.37	-49103216	318.52	M	HC	468	0.69	1299	0.1	974	0.52
	TABU	-10861471	371.47	-24580625	264.37	-49101011	318.75		TABU	468	0.69	1299	0.1	975	0.52
	MMHC	-13790813	7449.01	-28061042	1631.98	-55832193	2457.02		MMHC	1268	1.56	3350	0.78	3179	1.03
	RSMAX2	-16508716	3325.33	-30028409	4793.11	-60050894	10637.32		RSMAX2	2949	1.05	4086	1.44	4001	1.27
AIC	HC	-10923175	374.81	-24722989	261.22	-49264998	310.85	WO	HC	1862	0.9	0	0	40	0.49
	TABU	-10921249	372.87	-24722989	261.22	-49262249	308.38		TABU	1446	1.27	0	0	152	0.58
	MMHC	-13816106	7435.45	-28135869	1609.27	-55920440	2432.1		MMHC	1407	1.98	3	0.17	0	0
	RSMAX2	-16527481	3318.41	-30070905	4758.8	-60096326	10600.54		RSMAX2	793	0.71	1	0.1	0	0
BIC	HC	-11148029	416.51	-25186896	253.21	-49848249	291.84	WC	HC	2714	1.56	1016	0.55	1028	0.75
	TABU	-11136759	427.99	-25186896	253.21	-49843539	288.2		TABU	2452	2.25	1016	0.55	1112	1.08
	MMHC	-13907292	7386.76	-28379700	1538.4	-56238585	2345.93		MMHC	2368	2.84	1424	2.67	1662	2.16
	RSMAX2	-16595132	3293.99	-30209383	4647.37	-60260116	10468.63		RSMAX2	2262	1.96	166	1.36	132	1.07

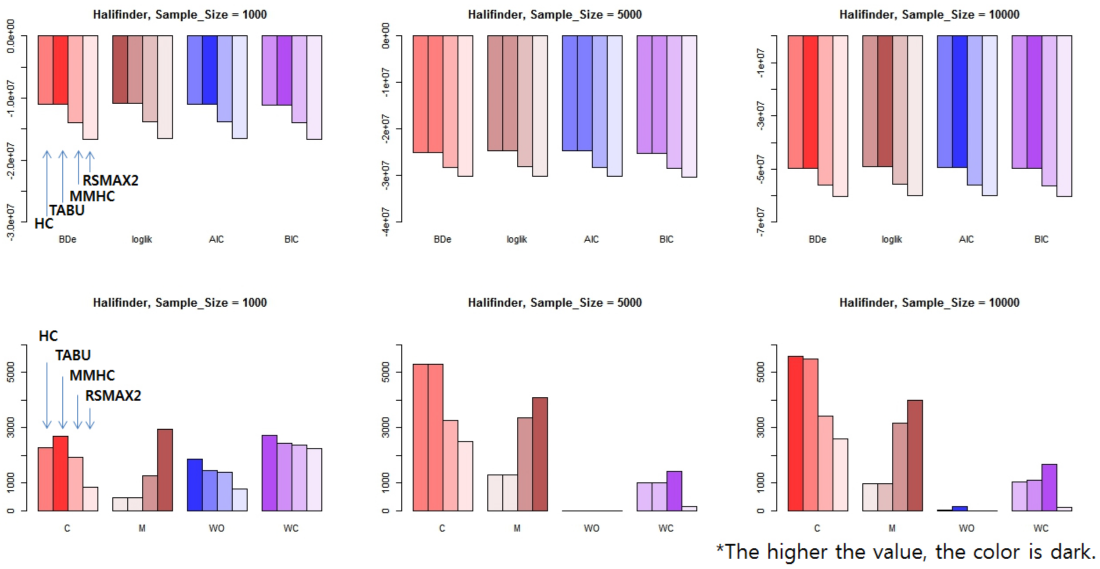


Figure 5.7: Comparison of scores and correct arcs via Hailfinder data set

5.1.5 Summary

Sample Size 1000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
Asia	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	3	2	1	4
Insurance	2	1	3	4	2	1	3	4	3	4	2	1	2	1	4	3	1	2	4	3
Alarm	2	1	3	4	2	1	3	4	3	4	2	1	1	2	3	4	1	2	3	4
HaliFinder	2	1	3	4	2	1	3	4	4	4	2	1	1	2	3	4	1	2	3	4

Sample Size 5000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
Asia	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	3	2	1	4
Insurance	2	1	3	4	2	1	3	4	3	4	2	1	1	2	3	4	1	2	3	4
Alarm	1	2	3	4	2	1	3	4	4	3	2	1	1	3	2	4	2	3	1	4
HaliFinder	1	1	3	4	1	1	3	4	4	4	2	1	1	1	1	1	2	2	1	4

Sample Size 10000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
Asia	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	3	1	2	4
Insurance	2	1	3	4	2	1	3	4	3	4	2	1	1	3	2	4	1	2	3	4
Alarm	2	1	3	4	2	1	3	4	4	4	2	1	1	2	3	4	1	2	3	4
HaliFinder	2	1	3	4	1	2	3	4	4	3	2	1	2	1	4	4	3	2	1	4

Figure 5.8: Summary for Comparison of scores and correct arcs via real data sets

When compared to the Score criteria were found to show good performance most TABU search algorithm, the order of Hill-Climbing algorithm.

However, as a result of comparing the target network and learning network directly, was a little different.

Result of comparing the target network and learning network directly, when C is large, M, WO, it can be said that performance is better when the WC is small.

TABU search algorithm, but were still many number of C, when it is a score criterion, considering that was overwhelming performance than other algorithms, it has been somewhat disappointing. Because rather WO, also the number of WC large, or shift the direction of the arrow, that unreasonable arrow is drawn is evaluated as disadvantages.

MMHC, RSMAX2 but C is also less M many, WO, I found that WC is also small. Overall the number of arrows Hill-climbing, will be drawn smaller than the TABU search. This Hybrid algorithms such as MMHC, RSMAX2 is, in the learning process in comparison with the Score-Based Algorithm, is the result that it can be confirmed that that will conservatively subsequently arrow.

Since the shape of the model is different, by using them, performance comparison of the algorithm according to the node number is difficult. Also, it is difficult to discover that the sample size is also clear changes in accordance with the increase.

5.2 Synthetic Data According to Topologies

5.2.1 Bayesian Network Topologies

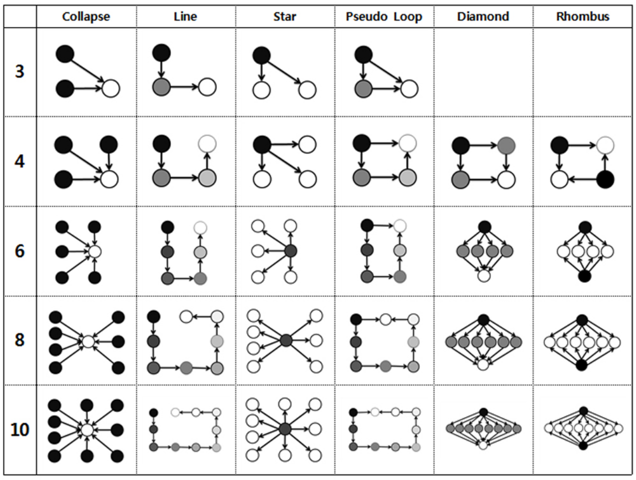


Figure 5.9: Bayesian Networks with varying topologies and number of nodes

Bayesian network model, as the number of nodes increases, difficult speaking any rules of the model accurately. Instead, it can be viewed separately with a certain unchanged properties called topology. Eitel J. M. L. (2008) was attempted to distinguish topology of Bayesian network.

In this paper, depending on the topology, after creating a set of models of the number of nodes to 3, 4, 6, 8, 10 pieces, and simulating. Cardinality was limited to two. In other words, all of the variable is the binary data. The probability value, which is imparted optionally under $U(0,1)$ distribution. And in order to avoid the influence of chance, all experiments are repeated 100 times, and overall results are reported. Constraint-based Learning Algorithms often makes undirected arcs. So, this has been excluded from comparison.

5.2.2 Collapse

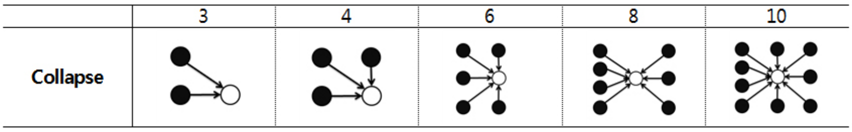


Figure 5.10: Bayesian Network Topology : Collapse

If one node has plurality of parent nodes, then this form called Collapse.

Sample Size 1000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	2	1	4	4	1	4	2	2	4	3	1	1	4	1	4	4	4	1	4	4
4	2	1	4	3	1	2	4	3	3	4	1	2	4	1	2	4	4	1	2	4
6	2	1	4	3	1	1	4	3	3	4	1	2	4	1	2	3	2	1	3	4
8	2	1	3	4	2	1	4	4	3	4	1	2	4	1	3	2	2	1	3	4
10	2	1	3	4	1	2	4	3	3	4	1	2	2	1	4	4	2	1	3	4

Sample Size 5000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	2	1	4	4	1	2	4	4	4	3	1	1	4	1	4	4	4	1	4	4
4	2	1	4	4	1	2	4	4	3	4	1	1	4	1	2	2	4	1	4	4
6	2	1	4	3	2	1	4	3	3	4	1	2	2	1	4	2	2	1	3	4
8	2	1	4	3	2	1	4	3	3	4	1	2	2	1	4	2	2	1	3	4
10	2	1	3	4	2	1	4	4	3	4	1	2	4	1	3	2	2	1	3	4

Sample Size 10000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	2	1	4	4	1	2	4	4	4	3	1	1	4	1	4	4	4	1	4	4
4	2	1	4	4	1	2	4	4	4	4	1	1	2	1	4	4	1	2	4	4
6	1	1	3	4	1	1	4	4	4	4	1	1	1	1	1	1	1	1	3	4
8	2	1	3	4	1	2	3	4	3	4	2	1	2	1	4	4	2	1	3	4
10	2	1	3	4	2	1	3	4	3	4	2	1	2	1	3	4	2	1	3	4

Figure 5.11: Summary for Comparison via Collapse

Like when using the Real data set, the TABU search is better than others according to score.

However, when compared by the number of C, TABU search and Hill-climbing has engaged in a conflict with each other. Rather the case of TABU search has a lot of WO and WC.

Not as much as TABU search case of Hill-climbing, as more the number of nodes, and as much sample size, appeared that the number WO is larger.

While sample size increases, then M is decreasing when using MMHC, but M is increasing when using RSMAX2.

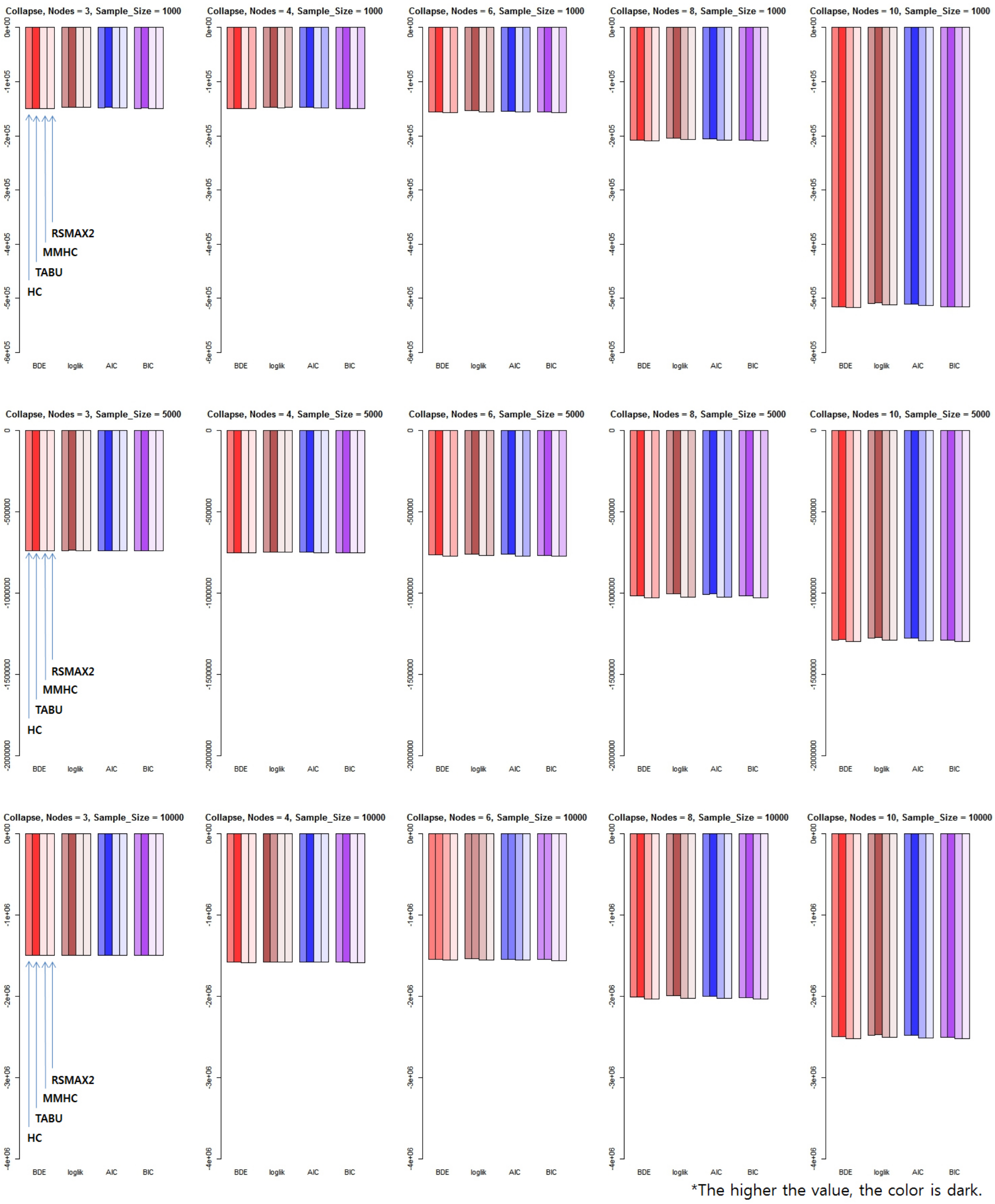


Figure 5.12: Comparison of scores via Collapse

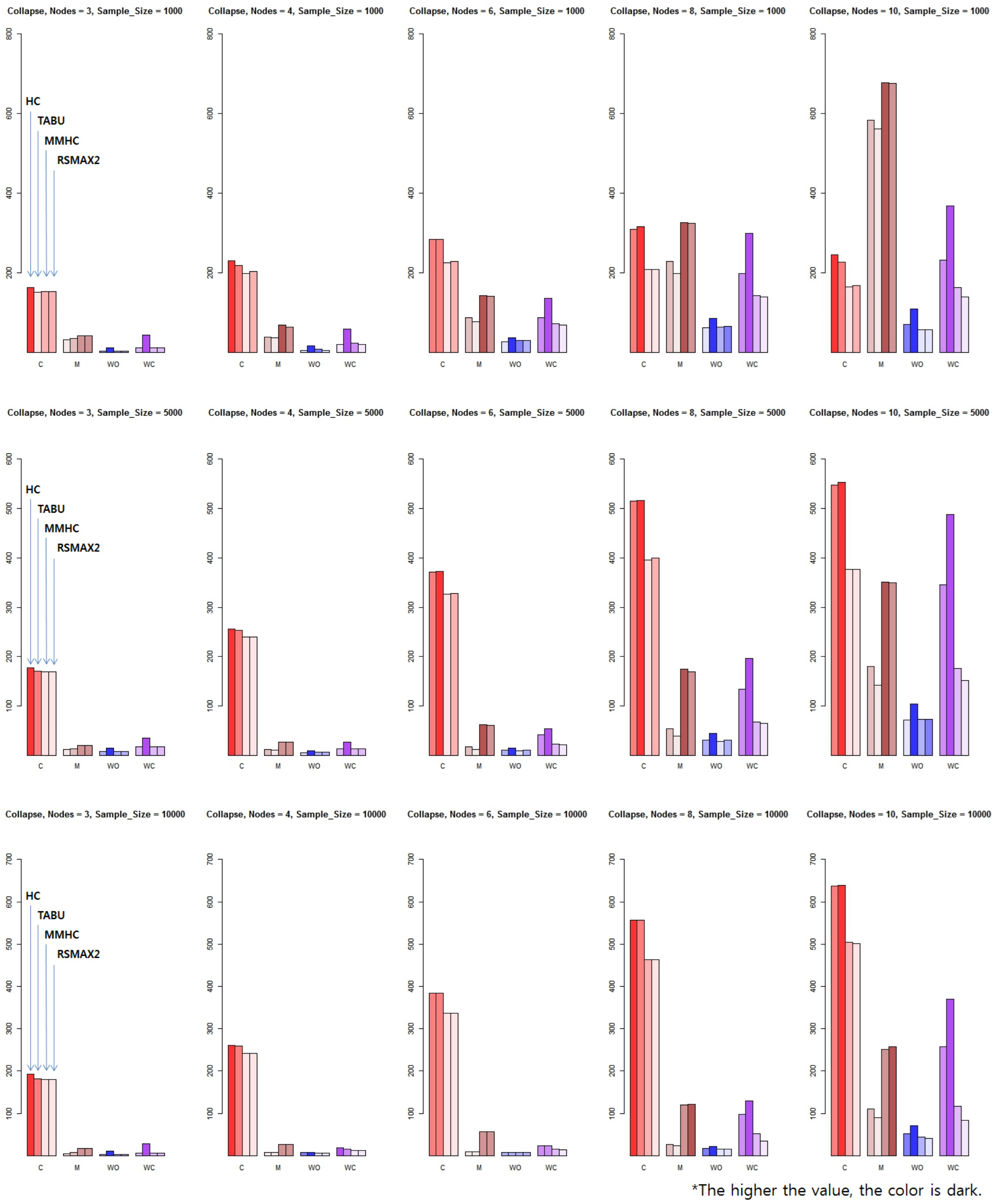


Figure 5.13: Comparison of correct arcs via Collapse

5.2.3 Line

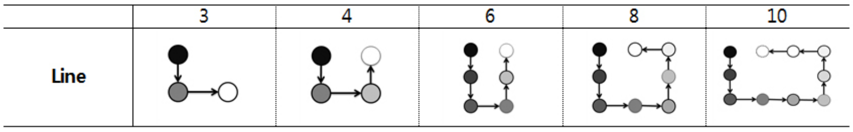


Figure 5.14: Bayesian Network Topology : Line

Multiple node bite the tail of the tail, then this form called Line. Though the figure is as of line.

Sample Size 1000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	2	1	4	4
4	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	2	1	4	4
6	2	1	3	4	1	4	2	2	3	4	1	1	4	1	4	4	2	1	3	4
8	2	1	3	4	1	4	2	3	3	4	2	1	4	1	4	4	2	1	4	4
10	2	1	3	4	1	4	2	3	4	4	2	1	4	1	4	4	2	1	4	4

Sample Size 5000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	1	1	1	1	1	4	1	1	1	1	1	1	4	1	4	4	4	1	4	4
4	1	1	1	1	1	4	1	1	1	1	1	1	4	1	4	4	4	1	4	4
6	2	1	4	4	1	4	1	1	1	1	1	1	4	1	4	4	4	1	4	4
8	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	2	1	4	4
10	1	1	4	3	1	4	3	2	4	3	1	2	4	1	4	4	2	1	4	4

Sample Size 10000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	1	1	4	4	1	4	1	1	1	1	1	1	4	1	4	4	2	1	4	4
4	1	1	4	4	3	4	1	1	1	1	4	4	4	1	4	4	2	1	4	4
6	2	1	4	4	1	4	1	1	1	1	1	1	4	1	4	4	2	1	4	4
8	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	4	1	4	4
10	2	1	3	4	1	4	2	2	4	4	1	1	4	1	4	4	2	1	3	4

Figure 5.15: Summary for Comparison via Line

Performance of each algorithm is compared to other topology were not different significantly occurs.

However, TABU search, despite good performance by score, the number of C is overwhelmingly smaller than the other algorithms, and M, WO, and WC is larger than the other algorithms.

Relatively, Hill-climbing has showed good performance for line form.

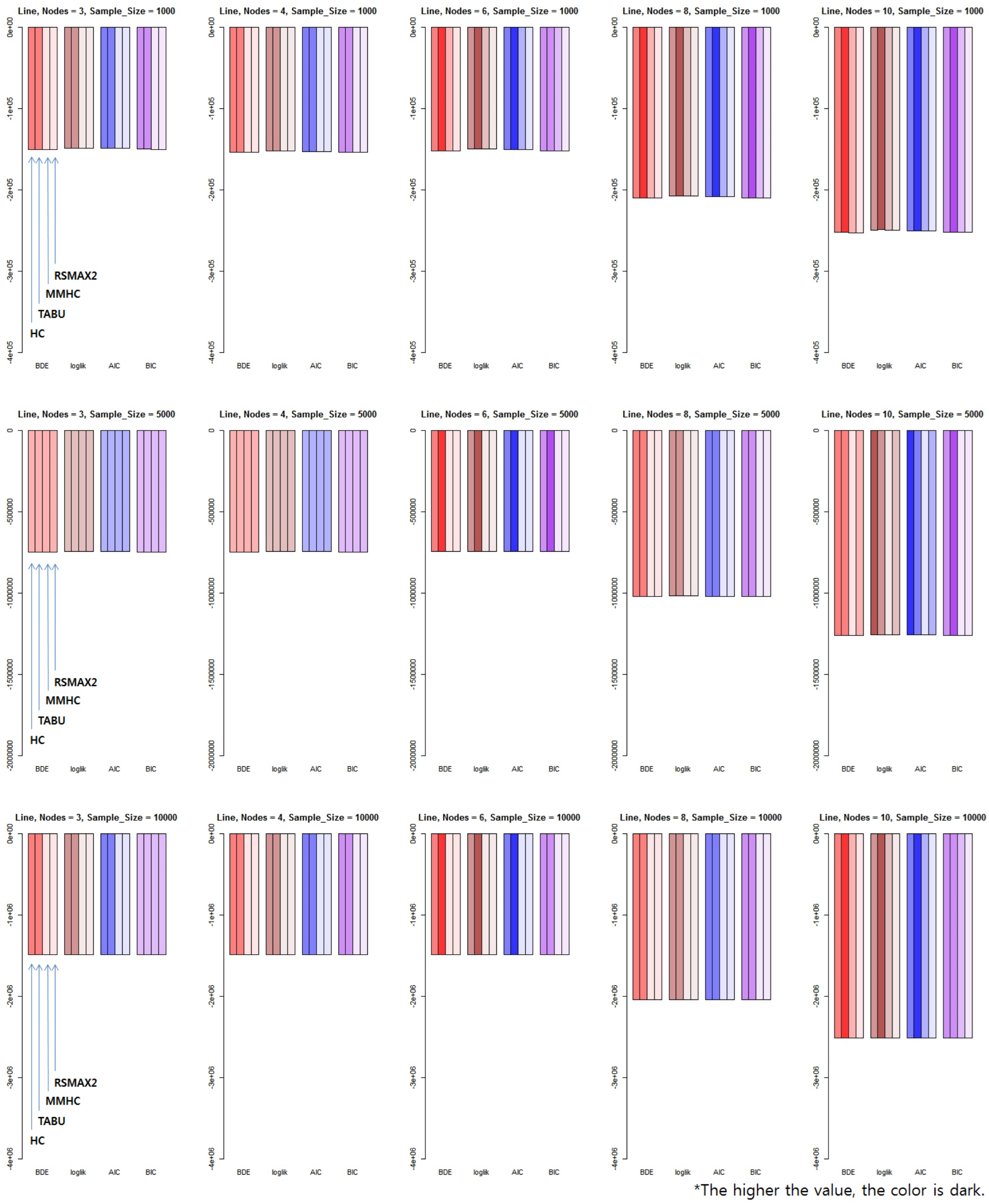


Figure 5.16: Comparison of scores via Line

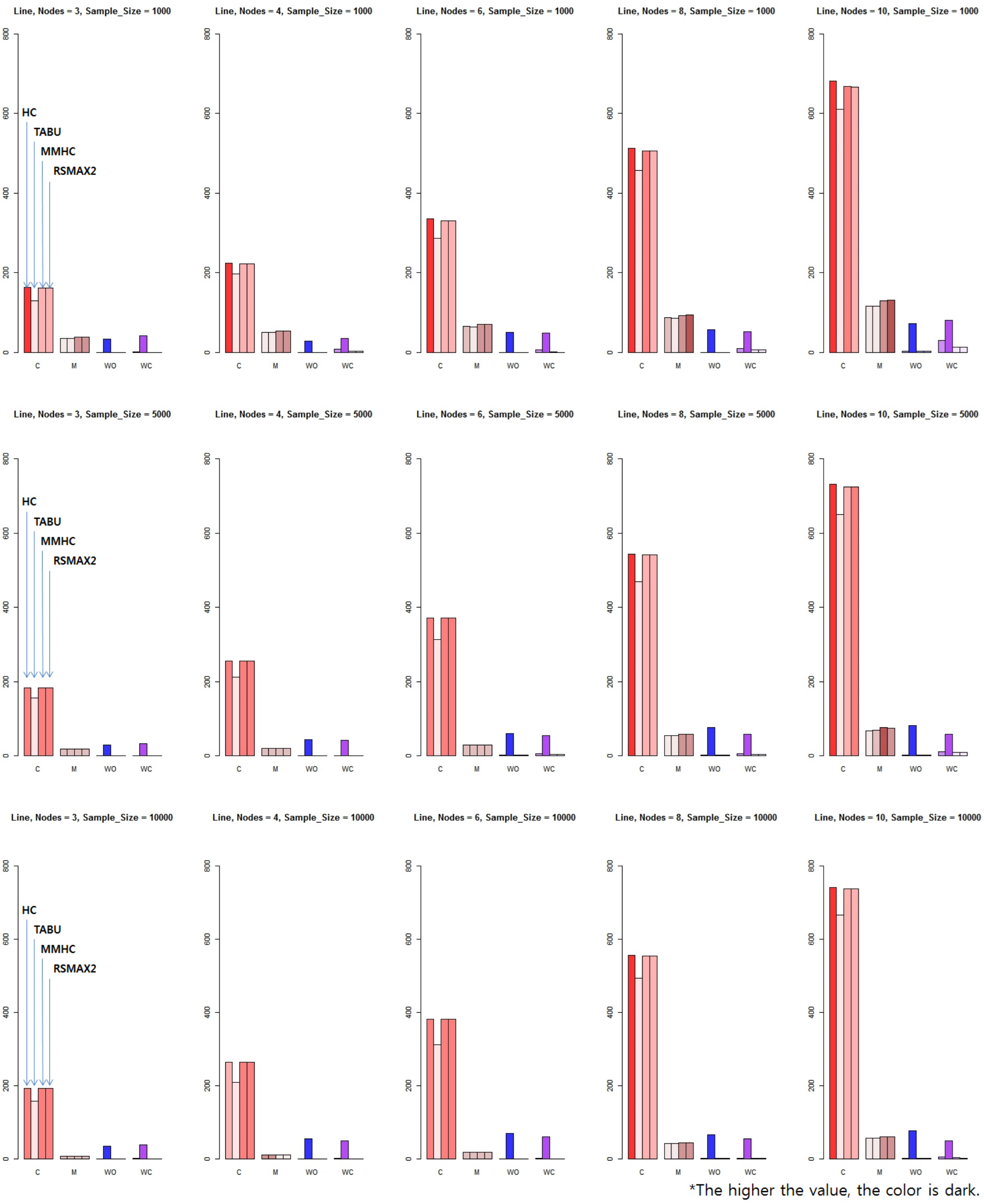


Figure 5.17: Comparison of correct arcs via Line

5.2.4 Star

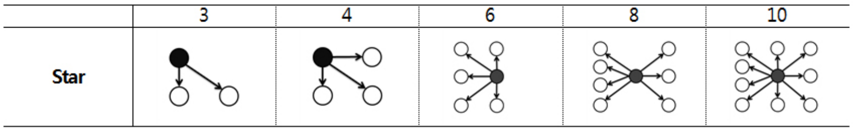


Figure 5.18: Bayesian Network Topologies : Star

If one node has plurality of child node, then this form called Star.

Sample Size 1000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	2	1	4	4
4	2	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	2	1	4	4
6	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	2	1	4	4
8	2	1	3	4	1	2	3	4	3	4	2	1	4	1	4	4	2	1	4	3
10	2	1	3	4	1	2	3	4	3	4	2	1	4	1	4	4	4	1	3	2

Sample Size 5000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	4	1	4	4
4	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	4	1	4	4
6	1	1	3	4	1	4	2	2	4	4	1	1	4	1	4	4	4	1	4	4
8	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	4	1	3	2
10	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	4	1	3	2

Sample Size 10000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	4	1	4	4
4	1	1	4	4	1	4	2	2	4	4	1	1	4	1	4	4	4	1	4	4
6	1	1	4	3	1	4	3	2	4	4	1	2	4	1	4	4	4	1	4	4
8	1	1	4	3	1	2	4	3	4	4	1	2	4	1	4	4	4	1	2	3
10	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	4	1	2	2

Figure 5.19: Summary for Comparison via Star

Star also, the performance of each algorithm was not different significantly occurs.

When compared by score, TABU search is good then the other algorithms. But when compared to between the target network and learning network, Hill-climbing showed good performance relatively for the line form.

Specific point, when the node number is small, TABU search despite the good performance by score, but the number of C is overwhelmingly smaller than the other algorithms. And M, WO and WC is very large. However, as the number of node increases, becoming the number of C is increased, and M, WO, WC is decreases.

And yet, did not win to the performance of Hill-climbing.

All algorithms while sample size increases, WO and WC is greatly reduced.

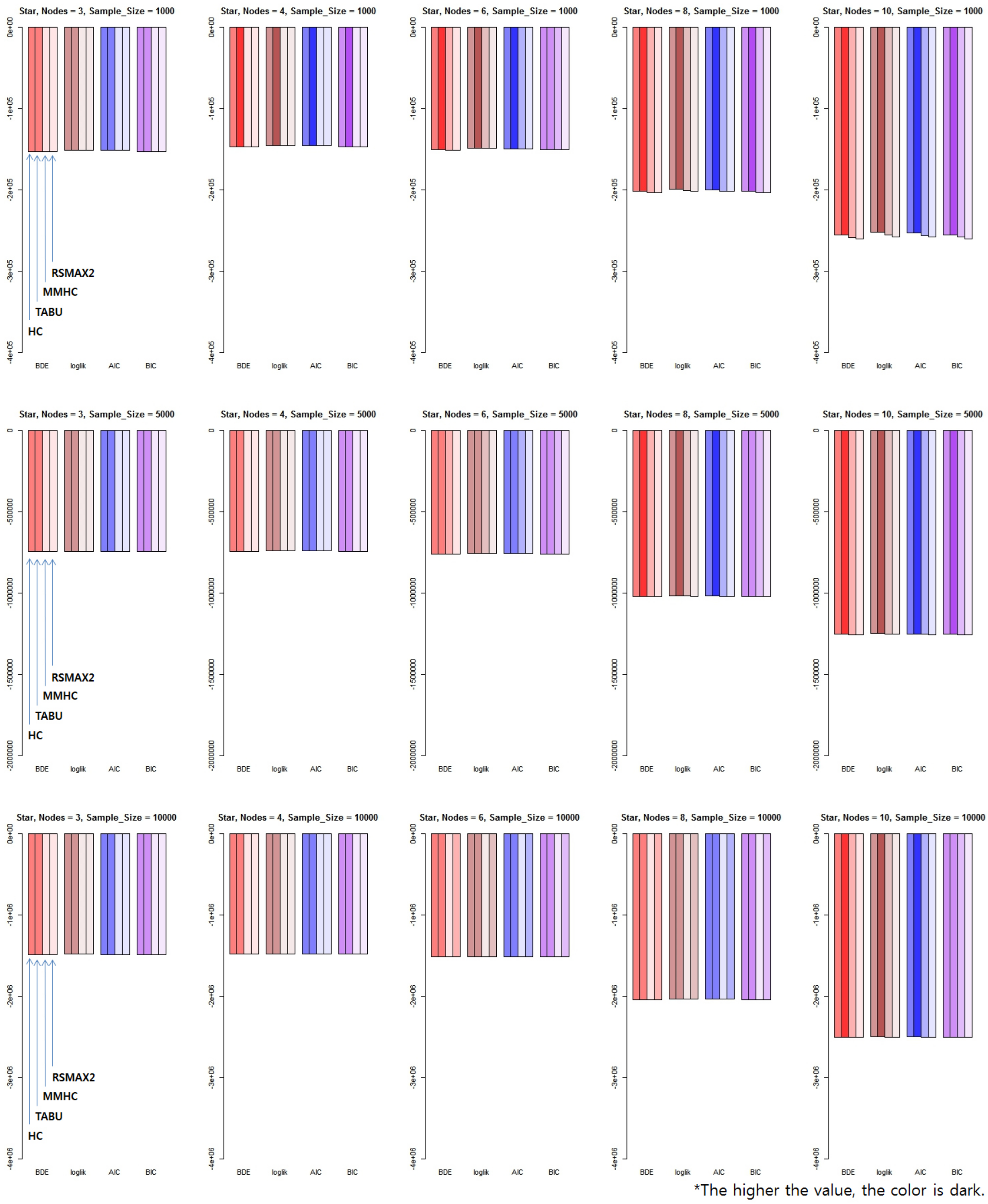


Figure 5.20: Comparison of scores via Star

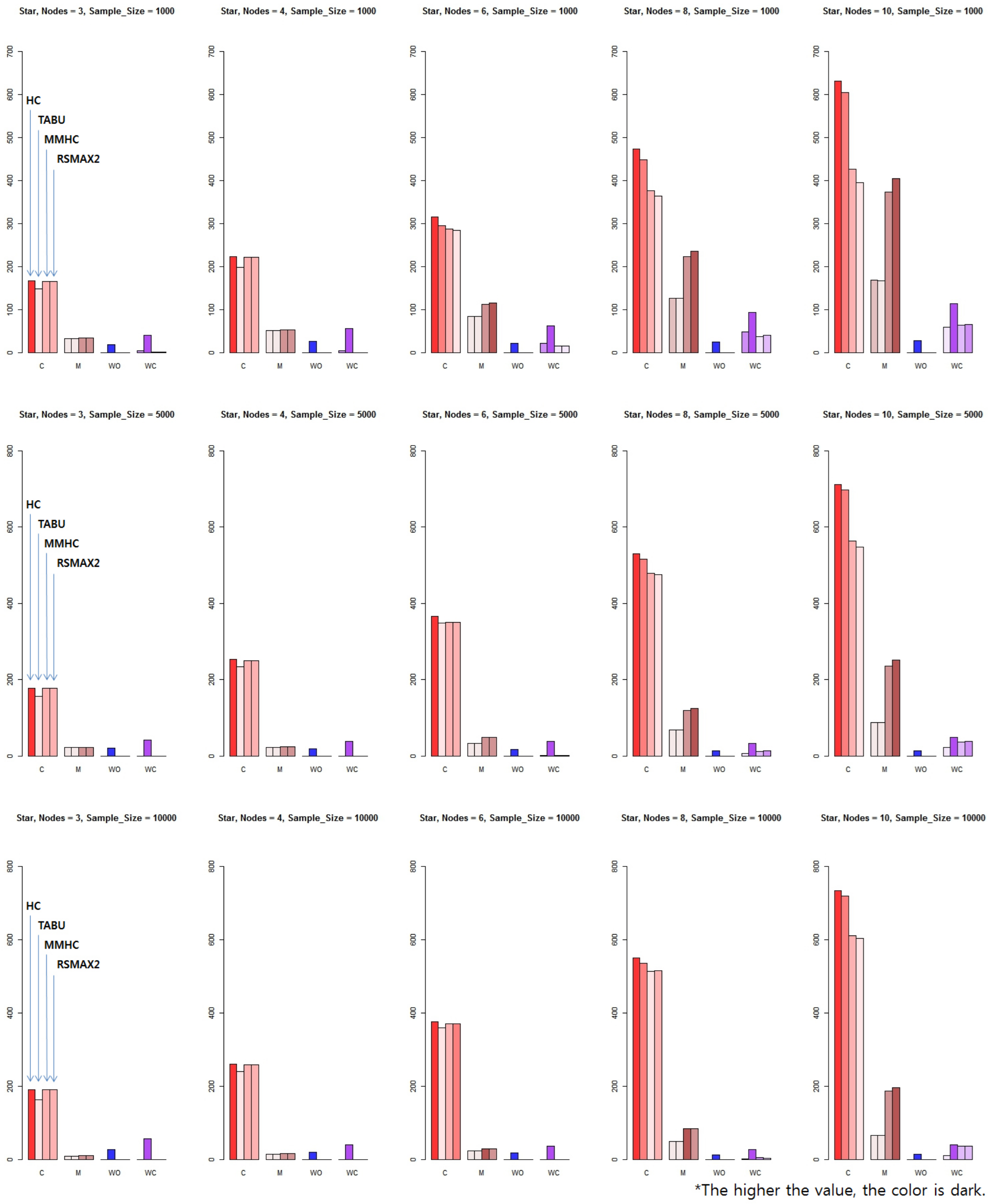


Figure 5.21: Comparison of correct arcs via Star

5.2.5 Pseudo Loop

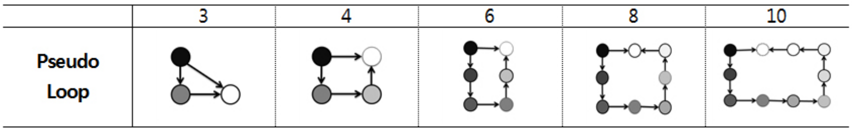


Figure 5.22: Bayesian Network Topologies : Pseudo Loop

At first, drew a line form. And next, root node has depended on the very last child node. Then it called Pseudo Loop. Actually loop does not have, it looks like a loop at first glance. (In fact, actually when loop is created, no longer Bayesian Network is not it.)

Sample Size 1000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	2	1	4	4	1	4	2	2	4	3	1	1	2	1	4	4	2	1	4	4
4	2	1	4	3	1	4	2	2	4	3	1	1	2	1	4	4	2	1	4	4
6	2	1	4	3	1	4	3	2	3	4	1	2	2	1	4	4	2	1	4	4
8	2	1	3	4	1	4	2	2	3	4	1	1	4	1	4	4	2	1	4	4
10	2	1	4	3	1	4	2	3	4	3	2	1	4	1	4	4	2	1	3	4

Sample Size 5000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	2	1	4	4	1	4	2	2	4	3	1	1	2	1	4	4	2	1	4	4
4	2	1	4	4	1	4	2	2	4	4	1	1	2	1	4	4	2	1	4	4
6	1	1	3	4	1	4	2	2	4	4	1	1	4	1	4	4	4	1	4	4
8	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	4	1	3	2
10	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	4	1	3	2

Sample Size 10000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
3	2	1	4	4	1	4	2	2	4	3	1	1	4	1	4	4	4	1	4	4
4	4	1	1	3	3	4	1	2	4	4	4	1	2	1	4	4	2	1	4	4
6	1	1	4	3	1	4	3	2	4	4	1	2	4	1	4	4	4	1	4	4
8	1	1	4	3	1	2	4	3	4	4	1	2	4	1	4	4	4	1	2	3
10	2	1	3	4	1	2	3	4	4	4	2	1	4	1	4	4	4	1	2	2

Figure 5.23: Summary for Comparison via Pseudo Loop

Although TABU search when compared on the basis of exhibited good performance Score, when compared to the network and learning network objectives, relatively Hill-climbing showed good performance in the line form.

When the sample size is 1000, the number of C by TABU search has not been improved. I shows how the number of C when sample size is larger, then greatly improved. In particular M, WO, and WC with increasing sample size, reduced noticeable. And yet, did not win to the performance of Hill-climbing.

All algorithms while sample size increases, WO and WC is greatly reduced.

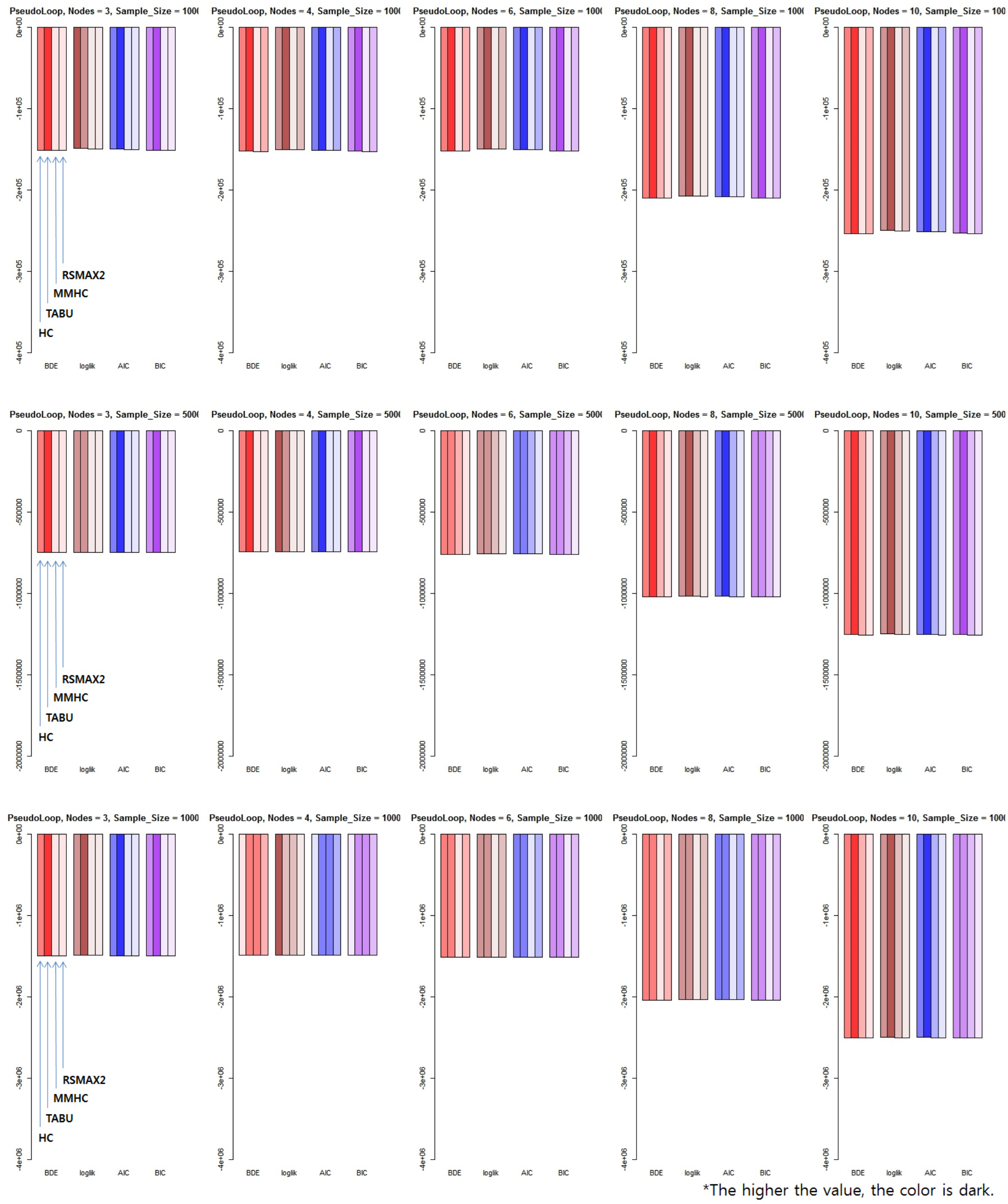


Figure 5.24: Comparison of scores via Pseudo Loop

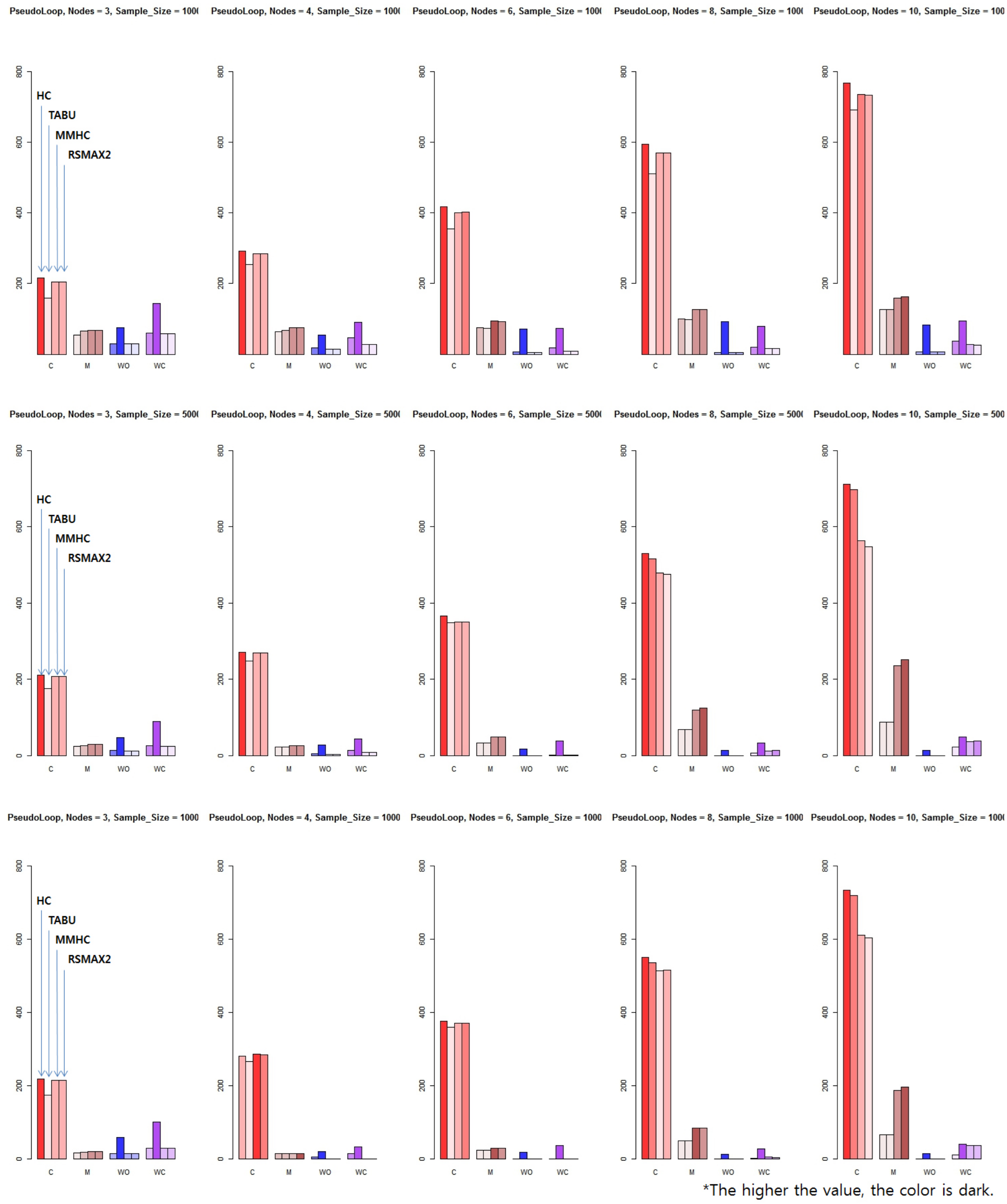


Figure 5.25: Comparison of correct arcs via Pseudo Loop

5.2.6 Diamond

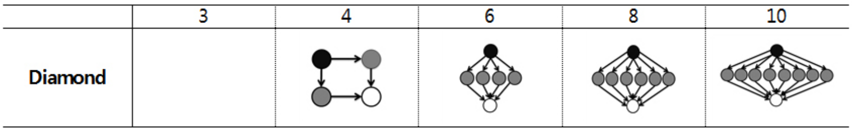


Figure 5.26: Bayesian Network Topologies : Diamond

A part of the top, one node has plurality of child node like Star form. And the bottom part, one node has plurality of parent node like Collapse form. If it connected, then it called Diamond.

Sample Size 1000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
4	2	1	4	3	1	2	3	4	3	4	2	1	2	1	4	4	2	1	4	3
6	2	1	3	4	1	2	3	4	3	4	2	1	2	1	4	3	2	1	4	3
8	2	1	3	4	1	2	3	4	3	4	2	1	4	1	4	4	3	1	4	2
10	2	1	3	4	1	2	3	4	3	4	2	1	2	1	3	4	3	1	4	2

Sample Size 5000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
4	2	1	4	3	2	1	3	4	3	4	1	1	2	1	4	3	2	1	4	3
6	2	1	4	3	1	2	3	4	3	4	2	1	2	1	4	4	2	1	4	3
8	2	1	3	4	2	1	3	4	3	4	2	1	2	1	4	3	1	2	4	3
10	2	1	3	4	1	2	3	4	3	4	2	1	2	1	4	3	2	1	4	3

Sample Size 10000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
4	2	1	3	4	1	2	3	4	3	4	2	1	2	1	4	4	1	1	4	4
6	2	1	4	3	1	2	3	4	3	4	2	1	2	1	4	4	2	1	4	3
8	2	1	4	3	1	2	3	4	3	4	2	1	2	1	4	3	1	2	4	3
10	2	1	3	4	1	2	3	4	3	4	2	1	2	1	3	4	2	3	4	1

Figure 5.27: Summary for Comparison via Diamond

Respectively, when compared to the score and C, TABU search and Hill-climbing showed a good performance. However, WO, WC also showed that many.

when compared to the WO and WC, MMHC showed that advantageous. RSMAX2 showed a lot of WC.

This phenomenon was stood out when the larger the sample size or the node.

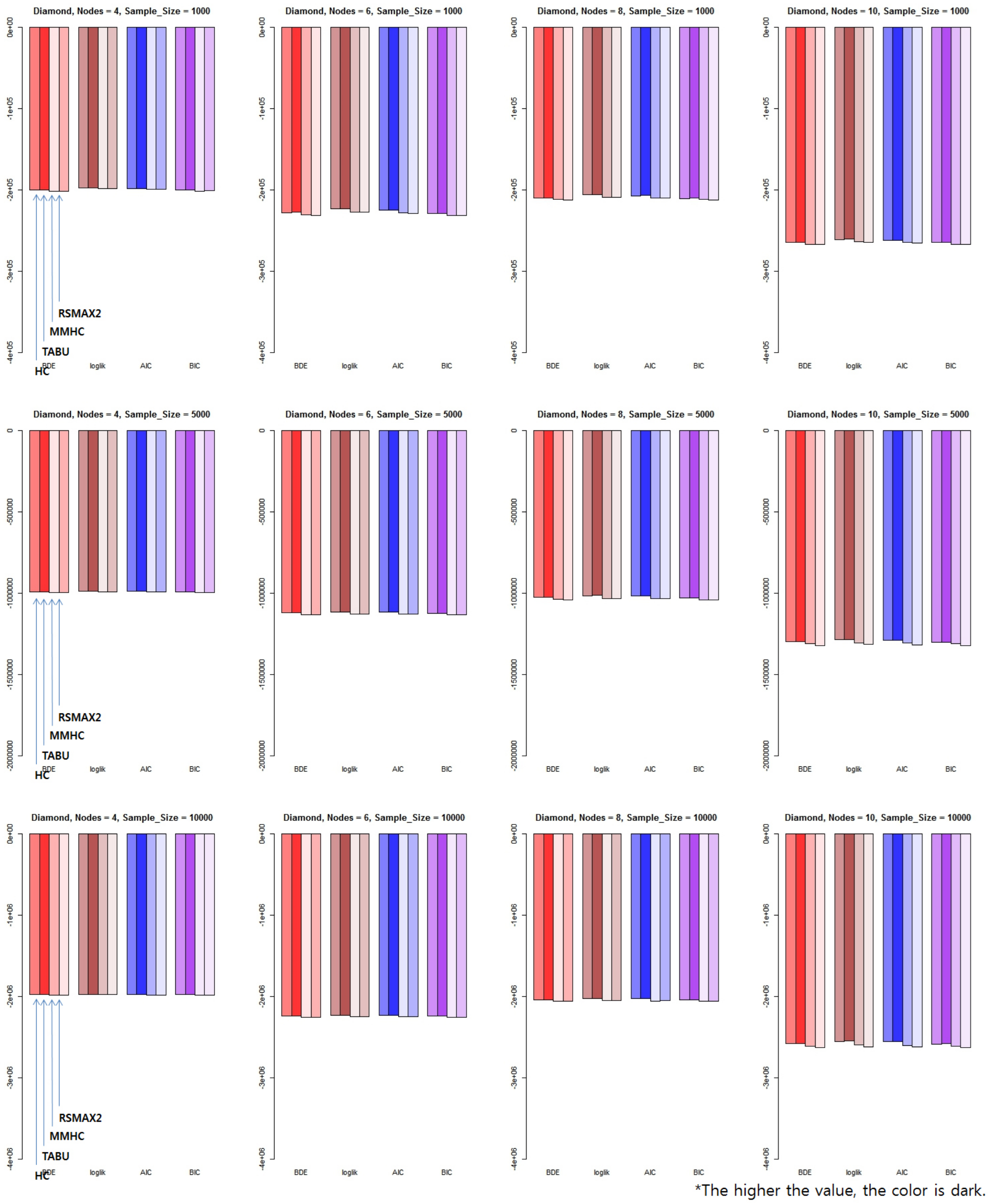


Figure 5.28: Comparison of scores via Diamond

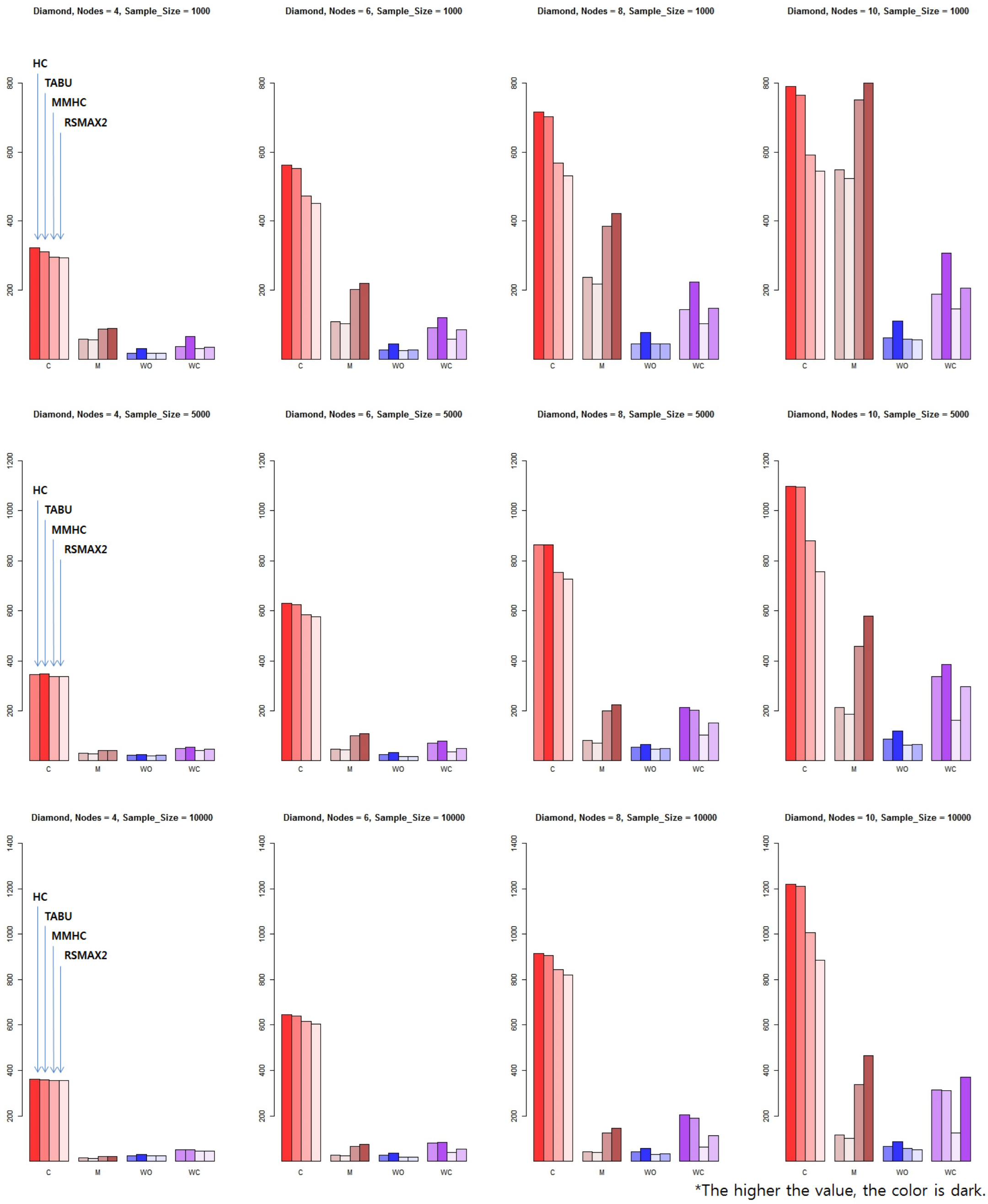


Figure 5.29: Comparison of correct arcs via Diamond

5.2.7 Rhombus

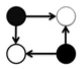
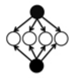
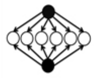
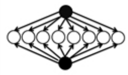
	3	4	6	8	10
Rhombus					

Figure 5.30: Bayesian Network Topologies : Rhombus

If two nodes has plurality of child node together, then it called Rhombus.

Sample Size 1000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
4	2	1	4	3	1	2	4	3	4	3	1	2	4	1	4	4	2	1	4	4
6	2	1	3	4	1	2	4	3	4	3	2	1	2	1	3	4	2	1	3	4
8	2	1	3	4	1	2	3	4	3	4	2	1	2	1	3	4	2	1	3	4
10	2	1	3	4	1	2	3	4	3	4	2	1	3	1	2	4	3	1	2	4
Sample Size 5000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
4	2	1	4	3	1	2	4	3	3	4	1	2	2	1	4	4	4	1	2	4
6	2	1	3	4	1	2	4	3	4	3	1	2	3	1	2	4	3	1	2	4
8	2	1	3	4	1	2	3	4	3	4	2	1	2	1	2	4	3	1	2	4
10	2	1	3	4	1	2	3	4	3	4	2	1	3	1	2	4	3	2	1	4
Sample Size 10000	Score				C				M				WO				WC			
	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2	HC	TABU	MMHC	RSMAX2
4	2	1	4	3	1	2	4	3	4	3	1	2	4	1	4	4	4	1	2	4
6	2	1	4	3	1	2	4	3	3	4	1	2	2	1	2	4	2	1	2	4
8	2	1	4	3	2	1	3	4	3	4	2	1	2	1	3	4	3	1	1	4
10	2	1	3	4	2	1	3	4	3	4	2	1	1	1	3	4	3	2	1	4

Figure 5.31: Summary for Comparison via Rhombus

Respectively, when compared to the score and C, TABU search and Hill-climbing showed a good performance. However, WO, WC also showed that many.

when compared to the WO and WC, RSMAX2 showed that advantageous. MMHC showed a lot of WC.

However as the sample size increases, how the performance of all algorithms is overall improvement.

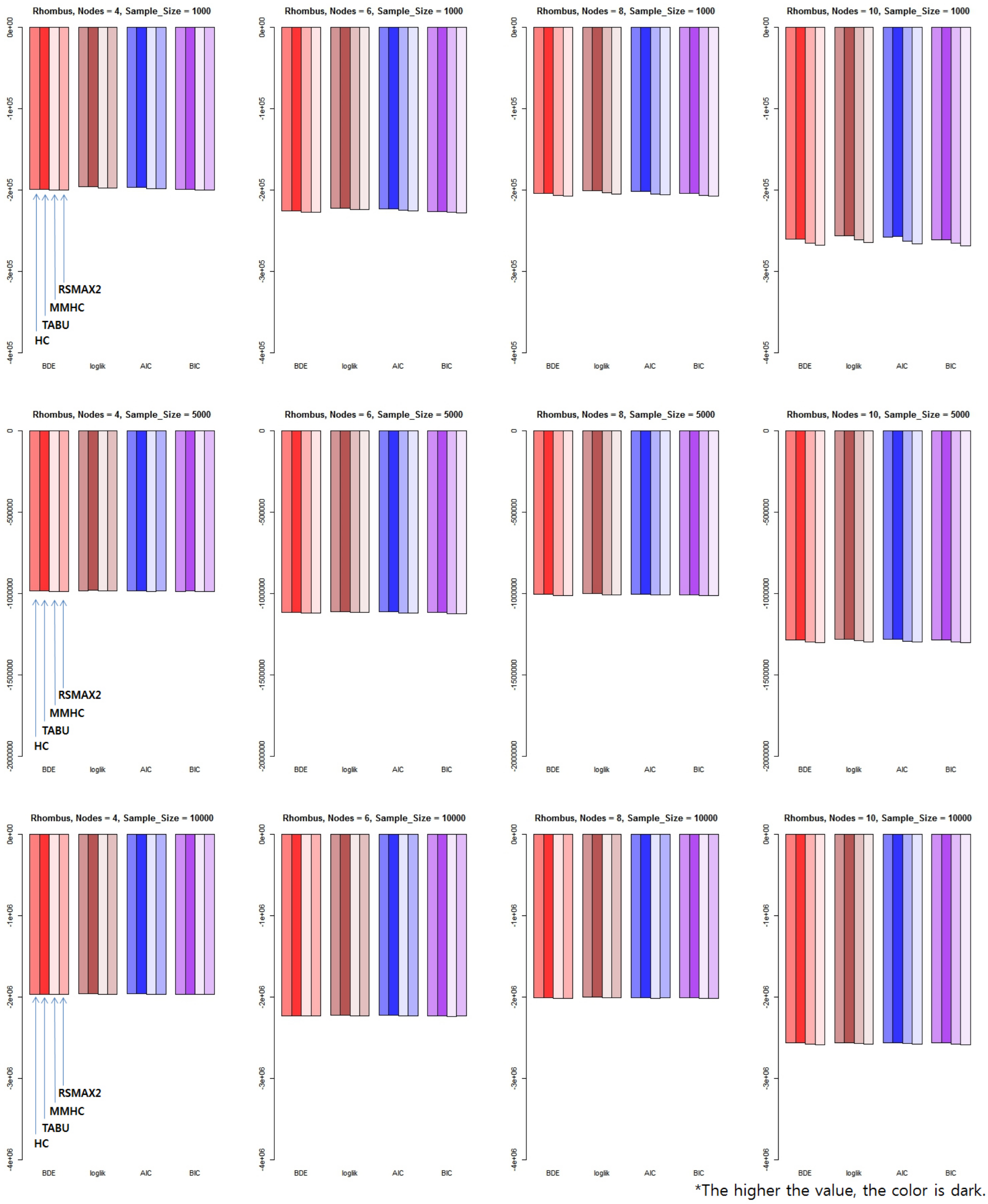


Figure 5.32: Comparison of scores via Rhombus

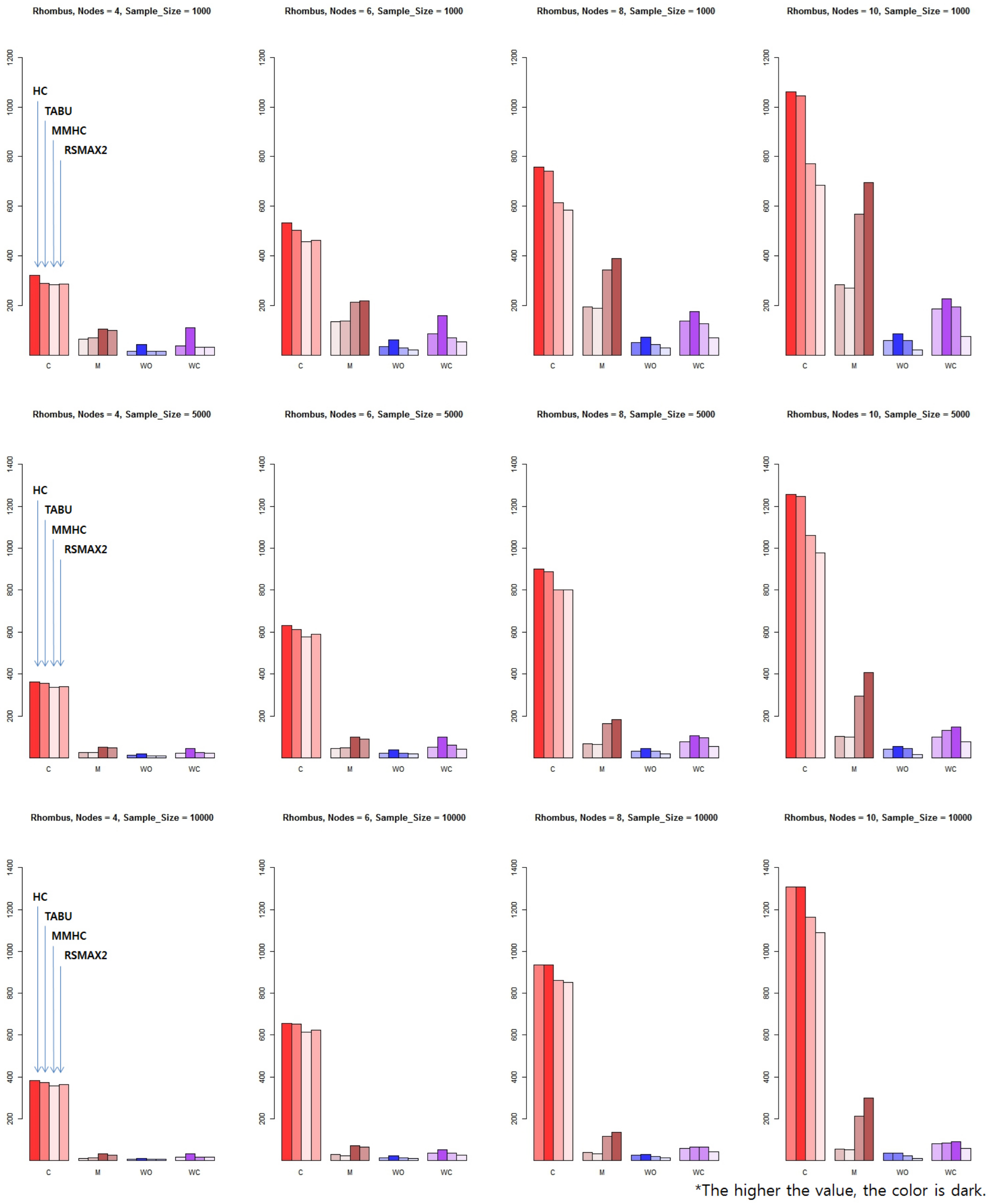


Figure 5.33: Comparison of correct arcs via Rhombus

Chapter 6

Discussion

Result of comparing the performance of each algorithm according to the Topology, TABU search has been found most best thing by score criteria. However, as a result of comparing between the target network and learning network directly, when a reference to "What C is a lot?" is, Hill-climbing was often appears that the performance is good. Rather TABU search has a lot of WO and WC.

Hybrid algorithm compared to Score-based algorithm is found to be that draw the arc more conservative. This makes not only C is often less missing arcs, but also WO and WC is drawn very small. It seems to use when WO and WC are fatal. Especially MMHC for Diamond form, RSMAX2 for Rhombus seems to be advantageous.

About Line and Star form, the performance difference due to relatively algorithm was not large compared to other topology.

In most of the topology, when the sample size is small, the number of C has not been improved. However, sample size is larger, then C was increased.

In addition, M, WO and WC was decreased.

On the basis of these results, algorithm users will be able to try to consider whether to choose what algorithms depending on whether their target network is any way. Especially, when the many M, WO, WC is fatal, it will be able to try to consider the selection of hybrid algorithm.

In future study, it can be to increase the number of node topology, to less sample size, or to increase the cardinality. Or by applying another algorithms, in addition it is possible to compare analyzed by combining two or more mutually topology.

In this paper, the probability when defining the relationship between the probability gave arbitrarily value between $U(0, 1)$. But it is possible to confirm the relationship when given "sequential" probability in future study.

It is desirable to complete the R package of Bayesian network data generator more than anything else.

In addition, analysis using the continuous data. Also we will be able to control the missing value of using BN. In this case it is possible to actively utilize Bayesian network data generator.

Appendix

A.1 Table for Collapse

Table 6.1: Comparison via Collapse (Num of Nodes = 3)

		Collapse (Num of Nodes = 3)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-149310	277.03	-739492	1456.67	-1493172	3026.14	C	HC	164	0.56	178	0.44	193	0.26
	TABU	-149073	273.72	-738889	1448.5	-1493171	3026.14		TABU	152	0.67	171	0.57	182	0.54
	MMHC	-149516	279.61	-740618	1469.62	-1494739	3034.04		MMHC	154	0.58	170	0.48	180	0.4
	RSMAX2	-149516	279.61	-740618	1469.62	-1494739	3034.04		RSMAX2	154	0.58	170	0.48	180	0.4
loglik	HC	-147279	281.43	-736966	1461.14	-1490429	3030.09	M	HC	32	0.55	13	0.37	4	0.2
	TABU	-147026	277.87	-736352	1452.81	-1490428	3030.09		TABU	36	0.52	14	0.35	8	0.27
	MMHC	-147531	284.24	-738143	1474.42	-1492072	3038.37		MMHC	42	0.57	21	0.43	17	0.38
	RSMAX2	-147531	284.24	-738143	1474.42	-1492072	3038.37		RSMAX2	42	0.57	21	0.43	17	0.38
AIC	HC	-147817	281.67	-737532	1461.32	-1491018	3030.17	WO	HC	4	0.2	9	0.29	3	0.17
	TABU	-147570	278.21	-736921	1453.04	-1491017	3030.17		TABU	12	0.33	15	0.41	10	0.36
	MMHC	-148049	284.4	-738693	1474.51	-1492635	3038.37		MMHC	4	0.2	9	0.29	3	0.17
	RSMAX2	-148049	284.4	-738693	1474.51	-1492635	3038.37		RSMAX2	4	0.2	9	0.29	3	0.17
BIC	HC	-149137	282.27	-739376	1461.93	-1493141	3030.43	WC	HC	12	0.48	18	0.58	6	0.34
	TABU	-148904	279.05	-738775	1453.8	-1493140	3030.43		TABU	44	1.16	36	1	28	0.99
	MMHC	-149320	284.82	-740485	1474.84	-1494664	3038.36		MMHC	12	0.48	18	0.58	6	0.34
	RSMAX2	-149320	284.82	-740485	1474.84	-1494664	3038.36		RSMAX2	12	0.48	18	0.58	6	0.34

Table 6.2: Comparison via Collapse (Num of Nodes = 4)

		Collapse (Num of Nodes = 4)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-149071	917.34	-749900	4601.22	-1577564	9508.45	C	HC	230	0.78	256	0.66	260	0.62
	TABU	-149032	917.06	-749097	4595.92	-1577556	9508.39		TABU	219	0.92	254	0.72	259	0.68
	MMHC	-149952	924.62	-751640	4613.92	-1582394	9544.08		MMHC	198	0.84	240	0.74	242	0.7
	RSMAX2	-149735	923.06	-751640	4613.92	-1582394	9544.08		RSMAX2	204	0.83	240	0.74	242	0.7
loglik	HC	-146796	905.56	-746922	4585.58	-1574342	9490.79	M	HC	39	0.65	13	0.39	8	0.34
	TABU	-146749	905.21	-746106	4580.17	-1574344	9490.81		TABU	38	0.58	11	0.31	8	0.34
	MMHC	-147865	914.28	-748779	4599.09	-1579382	9527.98		MMHC	69	0.73	28	0.51	27	0.55
	RSMAX2	-147612	912.5	-748779	4599.09	-1579382	9527.98		RSMAX2	65	0.72	28	0.51	27	0.55
AIC	HC	-147508	909.68	-747706	4589.96	-1575139	9495.35	WO	HC	6	0.24	6	0.24	7	0.29
	TABU	-147464	909.37	-746895	4584.59	-1575138	9495.35		TABU	18	0.39	10	0.3	8	0.37
	MMHC	-148493	917.79	-749518	4603.18	-1580114	9532.07		MMHC	8	0.27	7	0.26	6	0.28
	RSMAX2	-148254	916.09	-749518	4603.18	-1580114	9532.07		RSMAX2	6	0.24	7	0.26	6	0.28
BIC	HC	-149255	919.82	-750261	4604.23	-1578012	9511.79	WC	HC	20	0.67	14	0.59	18	0.81
	TABU	-149218	919.58	-749466	4599	-1578001	9511.7		TABU	60	1.26	28	0.9	16	0.73
	MMHC	-150034	926.42	-751926	4616.5	-1582753	9546.82		MMHC	24	0.71	14	0.51	12	0.56
	RSMAX2	-149830	924.93	-751926	4616.5	-1582753	9546.82		RSMAX2	20	0.67	14	0.51	12	0.56

Table 6.3: Comparison via Collapse (Num of Nodes = 6)

		Collapse (Num of Nodes = 6)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-155997	1596.19	-764954	7828.59	-1544669	15723.4	C	HC	284	0.97	371	1.04	384	1.1
	TABU	-155793	1593.85	-764558	7823	-1544669	15723.4		TABU	284	0.99	372	1.01	384	1.1
	MMHC	-157648	1613.35	-772886	7916.61	-1555104	15832.76		MMHC	225	0.88	327	0.98	337	1.08
	RSMAX2	-157613	1612.99	-772786	7916.76	-1555105	15832.78		RSMAX2	229	0.88	328	0.99	337	1.08
loglik	HC	-153357	1570.64	-759643	7774.48	-1538497	15660.96	M	HC	88	1	18	0.5	9	0.35
	TABU	-153059	1567.21	-759326	7769.87	-1538497	15660.96		TABU	78	0.92	13	0.37	9	0.35
	MMHC	-155647	1594.77	-768924	7878.02	-1550300	15784.66		MMHC	144	1.2	63	0.85	56	0.7
	RSMAX2	-155594	1594.24	-768820	7878.3	-1550304	15784.72		RSMAX2	141	1.21	61	0.84	56	0.7
AIC	HC	-154234	1579.42	-761260	7791.09	-1540234	15678.54	WO	HC	28	0.55	11	0.31	7	0.29
	TABU	-153974	1576.44	-760910	7786.08	-1540234	15678.54		TABU	38	0.56	15	0.39	7	0.29
	MMHC	-156234	1600.46	-770049	7889.11	-1551579	15797.54		MMHC	31	0.56	10	0.3	7	0.33
	RSMAX2	-156189	1600.01	-769950	7889.37	-1551582	15797.59		RSMAX2	30	0.56	11	0.31	7	0.33
BIC	HC	-156386	1601.03	-766529	7845.25	-1546496	15741.92	WC	HC	88	1.74	42	1.56	24	0.95
	TABU	-156219	1599.16	-766072	7838.94	-1546496	15741.92		TABU	136	2.16	54	1.53	24	0.95
	MMHC	-157675	1614.46	-773714	7925.28	-1556190	15843.99		MMHC	72	1.19	24	0.77	16	0.68
	RSMAX2	-157649	1614.2	-773632	7925.46	-1556190	15844		RSMAX2	70	1.18	22	0.63	14	0.65

Table 6.4: Comparison via Collapse (Num of Nodes = 8)

		Collapse (Num of Nodes = 8)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-207806	2118.77	-1014209	10346.61	-2008048	20599.86	C	HC	309	1.2	514	1.05	557	0.92
	TABU	-207509	2115.9	-1014025	10345.07	-2007946	20598.88		TABU	316	1.15	516	0.98	556	0.92
	MMHC	-209178	2132.1	-1029358	10508.2	-2032117	20868.83		MMHC	209	1.02	396	1.15	464	1.03
	RSMAX2	-209194	2132.26	-1029043	10506.55	-2032628	20875.32		RSMAX2	209	1.03	399	1.18	463	1.04
loglik	HC	-204583	2088.08	-1002152	10217.44	-1991302	20420.06	M	HC	229	1.49	55	0.73	26	0.48
	TABU	-204078	2083.15	-1001816	10214.49	-1991074	20417.87		TABU	198	1.36	39	0.58	23	0.47
	MMHC	-206772	2109.57	-1023648	10452.84	-2022890	20778.21		MMHC	326	1.54	175	1.32	120	0.97
	RSMAX2	-206802	2109.89	-1023214	10450.49	-2023458	20785.37		RSMAX2	325	1.59	170	1.33	121	1
AIC	HC	-205579	2097.77	-1005799	10256.47	-1996126	20471.38	WO	HC	62	0.75	31	0.6	17	0.4
	TABU	-205163	2093.71	-1005524	10254.08	-1995938	20469.59		TABU	86	0.89	45	0.7	21	0.46
	MMHC	-207409	2115.72	-1025215	10468.1	-2025414	20803.07		MMHC	65	0.73	29	0.59	16	0.44
	RSMAX2	-207432	2115.97	-1024831	10466.06	-2025965	20810.03		RSMAX2	66	0.78	31	0.63	16	0.44
BIC	HC	-208023	2121.59	-1017684	10383.91	-2013517	20656.54	WC	HC	198	2.34	134	2.94	98	2.17
	TABU	-207826	2119.67	-1017607	10383.29	-2013474	20656.17		TABU	300	3.11	196	3.41	130	2.66
	MMHC	-208972	2130.83	-1030321	10517.99	-2034514	20892.87		MMHC	144	1.48	68	1.28	52	1.16
	RSMAX2	-208978	2130.9	-1030100	10516.95	-2035003	20899.12		RSMAX2	140	1.54	66	1.27	34	0.9

Table 6.5: Comparison via Collapse (Num of Nodes = 10)

		Collapse (Num of Nodes = 10)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-515936	615.03	-1287094	13077.37	-2497620	25373.92	C	HC	246	1.26	547	1.23	638	0.81
	TABU	-515720	615.33	-1286580	13072.14	-2497070	25368.38		TABU	228	1.42	553	1.09	639	0.78
	MMHC	-517253	607.4	-1295331	13155.05	-2516578	25568.05		MMHC	165	0.98	376	1.36	504	1.33
	RSMAX2	-517319	607.92	-1295470	13156.68	-2517682	25579.82		RSMAX2	168	0.97	376	1.42	501	1.31
loglik	HC	-509066	635.43	-1274640	12955.38	-2475008	25137.78	M	HC	583	1.52	181	1.35	110	1.02
	TABU	-508662	635.37	-1273760	12946.24	-2473823	25125.45		TABU	562	1.6	142	1.16	90	0.86
	MMHC	-511438	622.4	-1289913	13103.45	-2505030	25455.77		MMHC	678	1.19	351	1.61	252	1.59
	RSMAX2	-511528	623.55	-1290159	13106.27	-2506562	25472.12		RSMAX2	675	1.23	350	1.66	258	1.62
AIC	HC	-510914	631.01	-1278070	12988.78	-2480768	25197.31	WO	HC	71	0.9	72	0.89	52	0.72
	TABU	-510592	631.08	-1277310	12980.91	-2479777	25187.03		TABU	110	1.05	105	1.09	71	0.89
	MMHC	-512845	620.3	-1291245	13116.22	-2507905	25483.56		MMHC	57	0.78	73	0.85	44	0.66
	RSMAX2	-512923	621.19	-1291461	13118.7	-2509323	25498.71		RSMAX2	57	0.73	74	0.85	41	0.64
BIC	HC	-515449	620.36	-1289247	13097.94	-2501533	25412.08	WC	HC	232	2.74	346	4.52	258	3.79
	TABU	-515328	620.74	-1288878	13094.17	-2501242	25409.18		TABU	368	3.11	488	5.27	370	4.55
	MMHC	-516297	615.28	-1295586	13157.91	-2518270	25584.02		MMHC	164	1.83	176	2.02	116	1.59
	RSMAX2	-516346	615.51	-1295704	13159.26	-2519277	25594.78		RSMAX2	140	1.52	152	1.68	84	1.28

A.2 Table for Line

Table 6.6: Comparison via Line (Num of Nodes = 3)

		Line (Num of Nodes = 3)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-150015	305.62	-745538	1338.43	-1487782	2685.73	C	HC	164	0.5	183	0.38	192	0.27
	TABU	-150015	305.62	-745538	1338.43	-1487782	2685.73		TABU	130	0.77	155	0.76	157	0.78
	MMHC	-150030	305.5	-745538	1338.43	-1487783	2685.72		MMHC	162	0.51	183	0.38	192	0.27
	RSMAX2	-150030	305.5	-745538	1338.43	-1487783	2685.72		RSMAX2	162	0.51	183	0.38	192	0.27
loglik	HC	-148189	309.68	-743251	1342.56	-1485301	2689.93	M	HC	36	0.5	17	0.38	8	0.27
	TABU	-148189	309.68	-743251	1342.56	-1485301	2689.93		TABU	36	0.5	17	0.38	8	0.27
	MMHC	-148209	309.51	-743251	1342.56	-1485306	2689.9		MMHC	38	0.51	17	0.38	8	0.27
	RSMAX2	-148209	309.51	-743251	1342.56	-1485306	2689.9		RSMAX2	38	0.51	17	0.38	8	0.27
AIC	HC	-148654	309.67	-743734	1342.56	-1485794	2689.88	WO	HC	0	0	0	0	0	0
	TABU	-148654	309.67	-743734	1342.56	-1485794	2689.88		TABU	34	0.71	28	0.67	35	0.76
	MMHC	-148671	309.52	-743734	1342.56	-1485798	2689.86		MMHC	0	0	0	0	0	0
	RSMAX2	-148671	309.52	-743734	1342.56	-1485798	2689.86		RSMAX2	0	0	0	0	0	0
BIC	HC	-149795	309.64	-745308	1342.57	-1487571	2689.71	WC	HC	2	0.2	0	0	2	0.2
	TABU	-149795	309.64	-745308	1342.57	-1487571	2689.71		TABU	42	0.82	32	0.74	38	0.79
	MMHC	-149805	309.56	-745308	1342.57	-1487571	2689.71		MMHC	0	0	0	0	0	0
	RSMAX2	-149805	309.56	-745308	1342.57	-1487571	2689.71		RSMAX2	0	0	0	0	0	0

Table 6.7: Comparison via Line (Num of Nodes = 4)

		Line (Num of Nodes = 4)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-153818	927.25	-746004	4568.49	-1488727	9127.48	C	HC	225	0.72	256	0.57	264	0.52
	TABU	-153818	927.25	-746004	4568.49	-1488727	9127.48		TABU	197	0.88	212	0.95	210	1.03
	MMHC	-153837	927.32	-746004	4568.49	-1488728	9127.5		MMHC	222	0.72	256	0.57	265	0.52
	RSMAX2	-153837	927.32	-746004	4568.49	-1488728	9127.5		RSMAX2	222	0.72	256	0.57	265	0.52
loglik	HC	-151947	918.03	-743641	4556.87	-1486155	9114.79	M	HC	50	0.61	19	0.42	11	0.31
	TABU	-151947	918.03	-743641	4556.87	-1486155	9114.79		TABU	50	0.61	19	0.42	11	0.31
	MMHC	-151977	918.17	-743641	4556.87	-1486157	9114.8		MMHC	53	0.63	19	0.42	10	0.3
	RSMAX2	-151977	918.17	-743641	4556.87	-1486157	9114.8		RSMAX2	53	0.63	19	0.42	10	0.3
AIC	HC	-152436	920.74	-744150	4559.65	-1486672	9117.62	WO	HC	0	0	0	0	0	0
	TABU	-152436	920.74	-744150	4559.65	-1486672	9117.62		TABU	28	0.73	44	0.89	54	1.01
	MMHC	-152461	920.85	-744150	4559.65	-1486674	9117.63		MMHC	0	0	0	0	0	0
	RSMAX2	-152461	920.85	-744150	4559.65	-1486674	9117.63		RSMAX2	0	0	0	0	0	0
BIC	HC	-153636	927.38	-745808	4568.73	-1488536	9127.83	WC	HC	8	0.39	0	0	2	0.2
	TABU	-153636	927.38	-745808	4568.73	-1488536	9127.83		TABU	36	0.77	42	0.82	50	0.87
	MMHC	-153649	927.43	-745808	4568.73	-1488538	9127.84		MMHC	4	0.28	0	0	0	0
	RSMAX2	-153649	927.43	-745808	4568.73	-1488538	9127.84		RSMAX2	4	0.28	0	0	0	0

Table 6.8: Comparison via Line (Num of Nodes = 6)

		Line (Num of Nodes = 6)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-151628	1546.55	-743984	7648.25	-1486614	15279.84	C	HC	335	1.13	371	1	381	0.98
	TABU	-151627	1546.54	-743980	7648.24	-1486613	15279.82		TABU	286	1.41	313	1.43	312	1.55
	MMHC	-151632	1546.59	-743988	7648.28	-1486619	15279.86		MMHC	330	1.17	371	1	381	0.96
	RSMAX2	-151635	1546.62	-743988	7648.28	-1486619	15279.86		RSMAX2	330	1.17	371	1	381	0.96
loglik	HC	-149731	1528.3	-741596	7625.59	-1484031	15255.22	M	HC	65	0.73	28	0.53	19	0.42
	TABU	-149727	1528.27	-741591	7625.58	-1484026	15255.16		TABU	64	0.72	28	0.53	19	0.42
	MMHC	-149739	1528.39	-741603	7625.64	-1484039	15255.26		MMHC	70	0.77	28	0.53	19	0.42
	RSMAX2	-149744	1528.43	-741603	7625.64	-1484039	15255.26		RSMAX2	70	0.77	28	0.53	19	0.42
AIC	HC	-150243	1533.39	-742128	7630.8	-1484566	15260.47	WO	HC	0	0	1	0.1	0	0
	TABU	-150240	1533.36	-742124	7630.79	-1484562	15260.42		TABU	50	1.04	59	1.12	69	1.24
	MMHC	-150249	1533.46	-742134	7630.84	-1484573	15260.51		MMHC	0	0	1	0.1	0	0
	RSMAX2	-150253	1533.49	-742134	7630.84	-1484573	15260.51		RSMAX2	0	0	1	0.1	0	0
BIC	HC	-151499	1545.88	-743862	7647.79	-1486495	15279.39	WC	HC	6	0.34	6	0.34	2	0.2
	TABU	-151499	1545.88	-743860	7647.79	-1486495	15279.39		TABU	48	0.86	54	0.94	60	1.01
	MMHC	-151501	1545.9	-743864	7647.81	-1486498	15279.41		MMHC	2	0.2	4	0.28	0	0
	RSMAX2	-151502	1545.91	-743864	7647.81	-1486498	15279.41		RSMAX2	0	0	4	0.28	0	0

Table 6.9: Comparison via Line (Num of Nodes = 8)

		Line (Num of Nodes = 8)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-209953	2122.64	-1020788	10373.59	-2039890	20730.72	C	HC	513	1.25	544	1.13	557	1.04
	TABU	-209950	2122.62	-1020788	10373.59	-2039890	20730.72		TABU	457	1.8	469	1.77	493	1.62
	MMHC	-209969	2122.8	-1020795	10373.66	-2039895	20730.77		MMHC	507	1.28	541	1.11	555	1.04
	RSMAX2	-209975	2122.86	-1020795	10373.66	-2039895	20730.77		RSMAX2	506	1.28	541	1.11	555	1.04
loglik	HC	-207466	2098.2	-1017613	10342.8	-2036443	20697.21	M	HC	87	0.86	55	0.72	42	0.57
	TABU	-207461	2098.16	-1017613	10342.8	-2036443	20697.21		TABU	86	0.85	55	0.72	42	0.57
	MMHC	-207497	2098.5	-1017628	10342.94	-2036451	20697.29		MMHC	93	0.92	58	0.73	44	0.57
	RSMAX2	-207502	2098.56	-1017628	10342.94	-2036451	20697.29		RSMAX2	94	0.93	58	0.73	44	0.57
AIC	HC	-208173	2105.26	-1018329	10349.9	-2037167	20704.39	WO	HC	0	0	1	0.1	1	0.1
	TABU	-208169	2105.23	-1018329	10349.9	-2037167	20704.39		TABU	57	1.27	76	1.52	65	1.23
	MMHC	-208198	2105.5	-1018341	10350.02	-2037174	20704.47		MMHC	0	0	1	0.1	1	0.1
	RSMAX2	-208203	2105.56	-1018341	10350.02	-2037174	20704.47		RSMAX2	0	0	1	0.1	1	0.1
BIC	HC	-209908	2122.58	-1020663	10373.07	-2039777	20730.29	WC	HC	10	0.44	6	0.34	2	0.2
	TABU	-209907	2122.57	-1020663	10373.07	-2039777	20730.29		TABU	52	0.97	58	1.04	54	0.98
	MMHC	-209918	2122.68	-1020665	10373.09	-2039781	20730.33		MMHC	6	0.34	4	0.28	2	0.2
	RSMAX2	-209924	2122.74	-1020665	10373.09	-2039781	20730.33		RSMAX2	6	0.34	4	0.28	2	0.2

Table 6.10: Comparison via Line (Num of Nodes = 10)

		Line (Num of Nodes = 10)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-252395	2557.7	-1259678	12795.95	-2515016	25551.17	C	HC	681	1.22	732	1.25	742	1.21
	TABU	-252382	2557.58	-1259678	12795.95	-2515011	25551.11		TABU	611	1.92	650	2.08	667	2.04
	MMHC	-252487	2558.57	-1259700	12796.2	-2515043	25551.46		MMHC	668	1.21	724	1.25	738	1.19
	RSMAX2	-252499	2558.67	-1259698	12796.18	-2515048	25551.52		RSMAX2	666	1.22	725	1.26	738	1.19
loglik	HC	-249115	2525.52	-1255615	12756.22	-2510623	25508.18	M	HC	116	0.97	67	0.73	57	0.7
	TABU	-249089	2525.28	-1255617	12756.24	-2510609	25508.03		TABU	116	0.97	68	0.72	57	0.7
	MMHC	-249242	2526.74	-1255655	12756.66	-2510664	25508.63		MMHC	129	1.02	75	0.76	61	0.69
	RSMAX2	-249259	2526.89	-1255651	12756.62	-2510672	25508.73		RSMAX2	131	1.02	74	0.76	61	0.69
AIC	HC	-250010	2534.45	-1256538	12765.4	-2511545	25517.35	WO	HC	3	0.17	1	0.1	1	0.1
	TABU	-249990	2534.26	-1256539	12765.41	-2511534	25517.23		TABU	73	1.67	82	1.79	76	1.86
	MMHC	-250120	2535.5	-1256572	12765.78	-2511582	25517.75		MMHC	3	0.17	1	0.1	1	0.1
	RSMAX2	-250135	2535.63	-1256569	12765.75	-2511589	25517.84		RSMAX2	3	0.17	1	0.1	1	0.1
BIC	HC	-252206	2556.35	-1259546	12795.32	-2514869	25550.4	WC	HC	30	0.77	10	0.44	6	0.34
	TABU	-252200	2556.3	-1259543	12795.3	-2514869	25550.4		TABU	80	1.45	58	1.07	50	0.92
	MMHC	-252274	2556.99	-1259560	12795.49	-2514891	25550.64		MMHC	14	0.59	8	0.39	4	0.28
	RSMAX2	-252285	2557.09	-1259560	12795.49	-2514895	25550.69		RSMAX2	14	0.51	8	0.39	2	0.2

A.3 Table for Star

Table 6.11: Comparison via Star (Num of Nodes = 3)

		Star (Num of Nodes = 3)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-152540	292.09	-743506	1366.63	-1483710	2748.14	C	HC	167	0.49	178	0.44	191	0.29
	TABU	-152540	292.09	-743506	1366.63	-1483710	2748.14		TABU	149	0.58	157	0.56	163	0.49
	MMHC	-152544	292.08	-743522	1366.47	-1483713	2748.1		MMHC	166	0.5	177	0.45	190	0.3
	RSMAX2	-152544	292.08	-743522	1366.47	-1483713	2748.1		RSMAX2	166	0.5	177	0.45	190	0.3
loglik	HC	-150745	295.8	-741242	1370.3	-1481255	2751.93	M	HC	33	0.49	22	0.44	9	0.29
	TABU	-150745	295.8	-741242	1370.3	-1481255	2751.93		TABU	33	0.49	22	0.44	9	0.29
	MMHC	-150754	295.8	-741262	1370.13	-1481261	2751.85		MMHC	34	0.5	23	0.45	10	0.3
	RSMAX2	-150754	295.8	-741262	1370.13	-1481261	2751.85		RSMAX2	34	0.5	23	0.45	10	0.3
AIC	HC	-151214	295.89	-741720	1370.41	-1481746	2751.92	WO	HC	0	0	0	0	0	0
	TABU	-151214	295.89	-741720	1370.41	-1481746	2751.92		TABU	18	0.39	21	0.41	28	0.45
	MMHC	-151221	295.89	-741739	1370.24	-1481751	2751.85		MMHC	0	0	0	0	0	0
	RSMAX2	-151221	295.89	-741739	1370.24	-1481751	2751.85		RSMAX2	0	0	0	0	0	0
BIC	HC	-152365	296.12	-743278	1370.76	-1483516	2751.89	WC	HC	4	0.28	0	0	0	0
	TABU	-152365	296.12	-743278	1370.76	-1483516	2751.89		TABU	40	0.8	42	0.82	56	0.9
	MMHC	-152367	296.13	-743293	1370.62	-1483517	2751.88		MMHC	2	0.2	0	0	0	0
	RSMAX2	-152367	296.13	-743293	1370.62	-1483517	2751.88		RSMAX2	2	0.2	0	0	0	0

Table 6.12: Comparison via Star (Num of Nodes = 4)

		Star (Num of Nodes = 4)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-146919	893.21	-741016	4566.75	-1478923	9126.62	C	HC	224	0.77	253	0.59	260	0.57
	TABU	-146911	893.18	-741016	4566.75	-1478923	9126.62		TABU	198	0.77	234	0.7	240	0.67
	MMHC	-146934	893.3	-741028	4566.78	-1478925	9126.65		MMHC	222	0.76	250	0.61	259	0.57
	RSMAX2	-146934	893.3	-741028	4566.78	-1478925	9126.65		RSMAX2	222	0.76	250	0.61	259	0.57
loglik	HC	-145054	883.82	-738685	4555.34	-1476383	9114.18	M	HC	51	0.69	22	0.46	15	0.39
	TABU	-145043	883.79	-738685	4555.34	-1476383	9114.18		TABU	51	0.69	22	0.46	15	0.39
	MMHC	-145074	883.96	-738703	4555.37	-1476389	9114.23		MMHC	53	0.69	25	0.48	16	0.39
	RSMAX2	-145074	883.96	-738703	4555.37	-1476389	9114.23		RSMAX2	53	0.69	25	0.48	16	0.39
AIC	HC	-145538	886.51	-739191	4558.08	-1476896	9116.96	WO	HC	0	0	0	0	0	0
	TABU	-145529	886.48	-739191	4558.08	-1476896	9116.96		TABU	26	0.48	19	0.39	20	0.4
	MMHC	-145555	886.63	-739207	4558.11	-1476901	9117		MMHC	0	0	0	0	0	0
	RSMAX2	-145555	886.63	-739207	4558.11	-1476901	9117		RSMAX2	0	0	0	0	0	0
BIC	HC	-146725	893.12	-740840	4567.01	-1478745	9126.99	WC	HC	4	0.28	0	0	0	0
	TABU	-146722	893.11	-740840	4567.01	-1478745	9126.99		TABU	56	1.03	38	0.79	40	0.8
	MMHC	-146736	893.18	-740849	4567.04	-1478746	9127		MMHC	0	0	0	0	0	0
	RSMAX2	-146736	893.18	-740849	4567.04	-1478746	9127		RSMAX2	0	0	0	0	0	0

Table 6.13: Comparison via Star (Num of Nodes = 6)

		Star (Num of Nodes = 6)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-150411	1531.16	-757733	7732.81	-1513789	15448.13	C	HC	316	1.09	367	1.05	377	1
	TABU	-150410	1531.16	-757733	7732.81	-1513789	15448.13		TABU	295	1.13	349	1.11	359	1.13
	MMHC	-150735	1534.21	-757847	7733.72	-1513842	15448.62		MMHC	287	0.93	351	0.97	370	0.96
	RSMAX2	-150780	1534.64	-757849	7733.73	-1513831	15448.51		RSMAX2	285	0.94	351	0.98	371	0.96
loglik	HC	-148517	1512.88	-755376	7710.07	-1511230	15423.37	M	HC	84	0.91	33	0.6	23	0.47
	TABU	-148514	1512.85	-755376	7710.07	-1511230	15423.37		TABU	84	0.91	33	0.6	23	0.47
	MMHC	-148908	1516.55	-755527	7711.32	-1511302	15424.05		MMHC	113	1.02	49	0.66	30	0.52
	RSMAX2	-148958	1517.02	-755529	7711.32	-1511288	15423.91		RSMAX2	115	1.05	49	0.67	29	0.5
AIC	HC	-149020	1517.88	-755904	7715.29	-1511763	15428.65	WO	HC	0	0	0	0	0	0
	TABU	-149018	1517.86	-755904	7715.29	-1511763	15428.65		TABU	21	0.46	18	0.39	18	0.39
	MMHC	-149381	1521.28	-756041	7716.42	-1511828	15429.26		MMHC	0	0	0	0	0	0
	RSMAX2	-149429	1521.73	-756043	7716.42	-1511815	15429.13		RSMAX2	0	0	0	0	0	0
BIC	HC	-150255	1530.15	-757624	7732.31	-1513684	15447.68	WC	HC	22	0.63	2	0.2	0	0
	TABU	-150255	1530.15	-757624	7732.31	-1513684	15447.68		TABU	62	1.05	38	0.84	36	0.77
	MMHC	-150542	1532.88	-757716	7733.04	-1513724	15448.05		MMHC	16	0.55	2	0.2	0	0
	RSMAX2	-150585	1533.29	-757718	7733.05	-1513715	15447.95		RSMAX2	16	0.61	2	0.2	0	0

Table 6.14: Comparison via Star (Num of Nodes = 8)

		Star (Num of Nodes = 8)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-201803	2040.6	-1019596	10374.74	-2036502	20724.35	C	HC	473	1.43	531	1.16	550	1.08
	TABU	-201791	2040.48	-1019592	10374.7	-2036502	20724.35		TABU	449	1.45	517	1.22	537	1.19
	MMHC	-203068	2052.25	-1020595	10383.47	-2037573	20733.29		MMHC	376	1.06	480	0.94	515	0.97
	RSMAX2	-203559	2056.74	-1020999	10387.34	-2037277	20730.78		RSMAX2	364	1.07	475	0.96	516	0.95
loglik	HC	-199240	2015.34	-1016514	10344.81	-2033143	20691.63	M	HC	127	1.12	69	0.9	50	0.69
	TABU	-199212	2015.06	-1016505	10344.72	-2033143	20691.63		TABU	126	1.11	69	0.9	50	0.69
	MMHC	-200686	2028.73	-1017613	10354.51	-2034291	20701.33		MMHC	224	1.07	120	0.9	85	0.82
	RSMAX2	-201207	2033.49	-1018029	10358.48	-2034000	20698.85		RSMAX2	236	1.16	125	0.97	84	0.81
AIC	HC	-199938	2022.32	-1017220	10351.82	-2033861	20698.77	WO	HC	0	0	0	0	0	0
	TABU	-199917	2022.11	-1017213	10351.75	-2033861	20698.77		TABU	25	0.58	14	0.4	13	0.34
	MMHC	-201311	2034.99	-1018285	10361.2	-2034983	20708.21		MMHC	0	0	0	0	0	0
	RSMAX2	-201822	2039.66	-1018697	10365.13	-2034691	20705.73		RSMAX2	0	0	0	0	0	0
BIC	HC	-201651	2039.45	-1019520	10374.66	-2036450	20724.5	WC	HC	48	0.86	6	0.34	2	0.2
	TABU	-201647	2039.42	-1019520	10374.66	-2036450	20724.5		TABU	94	1.35	34	0.9	28	0.7
	MMHC	-202845	2050.36	-1020474	10382.97	-2037477	20733.04		MMHC	38	0.79	12	0.48	6	0.34
	RSMAX2	-203331	2054.8	-1020874	10386.81	-2037182	20730.53		RSMAX2	40	0.85	14	0.51	4	0.28

Table 6.15: Comparison via Star (Num of Nodes = 10)

		Star (Num of Nodes = 10)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-255441	2602.56	-1253002	12766.69	-2501901	25495.03	C	HC	632	1.66	712	1.43	735	1.27
	TABU	-255433	2602.49	-1252992	12766.59	-2501898	25495.01		TABU	605	1.78	698	1.46	720	1.33
	MMHC	-258398	2631.34	-1256122	12796.2	-2505279	25526.79		MMHC	426	1.01	564	0.96	612	0.98
	RSMAX2	-260229	2650.33	-1257416	12808.52	-2506828	25541.93		RSMAX2	395	1.07	548	0.99	604	0.97
loglik	HC	-252250	2571.69	-1249006	12727.66	-2497561	25452.62	M	HC	168	1.53	88	0.98	65	0.76
	TABU	-252230	2571.48	-1248989	12727.48	-2497556	25452.59		TABU	167	1.52	88	0.98	65	0.76
	MMHC	-255543	2603.78	-1252426	12760.08	-2501207	25486.93		MMHC	374	1.41	236	1.15	188	1.13
	RSMAX2	-257446	2623.52	-1253747	12772.68	-2502780	25502.3		RSMAX2	405	1.53	252	1.23	196	1.2
AIC	HC	-253119	2580.36	-1249924	12736.79	-2498483	25461.77	WO	HC	0	0	0	0	0	0
	TABU	-253104	2580.21	-1249910	12736.64	-2498479	25461.75		TABU	28	0.65	14	0.47	15	0.39
	MMHC	-256278	2611.14	-1253237	12768.19	-2502045	25495.29		MMHC	0	0	0	0	0	0
	RSMAX2	-258160	2630.66	-1254547	12780.69	-2503611	25510.6		RSMAX2	0	0	0	0	0	0
BIC	HC	-255251	2601.64	-1252916	12766.53	-2501807	25494.78	WC	HC	60	0.96	22	0.69	10	0.44
	TABU	-255249	2601.62	-1252911	12766.49	-2501807	25494.78		TABU	114	1.56	50	1.11	40	0.85
	MMHC	-258081	2629.2	-1255880	12794.6	-2505066	25525.44		MMHC	64	1.06	36	0.82	36	0.82
	RSMAX2	-259912	2648.18	-1257154	12806.79	-2506607	25540.51		RSMAX2	66	1.07	38	0.79	36	0.82

A.4 Table for PseudoLoop

Table 6.16: Comparison via Pseudo Loop (Num of Nodes = 3)

		Pseudo Loop (Num of Nodes = 3)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-151048	268.73	-747628	1328.06	-1491992	2667.08	C	HC	215	0.81	212	0.57	219	0.54
	TABU	-151037	268.65	-747621	1328.09	-1491981	2667.1		TABU	159	1.04	176	0.79	174	0.87
	MMHC	-151376	271.53	-747934	1327.92	-1492459	2668.62		MMHC	204	0.8	208	0.6	215	0.56
	RSMAX2	-151376	271.53	-747934	1327.92	-1492459	2668.62		RSMAX2	204	0.8	208	0.6	215	0.56
loglik	HC	-148954	271.26	-745139	1331.38	-1489279	2670.43	M	HC	55	0.59	25	0.46	16	0.37
	TABU	-148962	271.19	-745139	1331.4	-1489276	2670.4		TABU	66	0.57	27	0.47	18	0.39
	MMHC	-149324	274.4	-745471	1331.16	-1489766	2671.94		MMHC	67	0.64	30	0.54	20	0.45
	RSMAX2	-149324	274.4	-745471	1331.16	-1489766	2671.94		RSMAX2	67	0.64	30	0.54	20	0.45
AIC	HC	-149574	271.37	-745707	1331.5	-1489858	2670.5	WO	HC	30	0.52	13	0.34	15	0.36
	TABU	-149572	271.31	-745705	1331.52	-1489853	2670.48		TABU	75	0.87	47	0.73	58	0.93
	MMHC	-149922	274.44	-746030	1331.31	-1490337	2672.04		MMHC	29	0.52	12	0.33	15	0.36
	RSMAX2	-149922	274.44	-746030	1331.31	-1490337	2672.04		RSMAX2	29	0.52	12	0.33	15	0.36
BIC	HC	-151096	271.66	-747558	1331.89	-1491946	2670.74	WC	HC	60	1.04	26	0.68	30	0.72
	TABU	-151069	271.62	-747549	1331.94	-1491933	2670.77		TABU	144	1.63	90	1.34	100	1.52
	MMHC	-151390	274.56	-747851	1331.8	-1492396	2672.39		MMHC	58	1.04	24	0.65	30	0.72
	RSMAX2	-151390	274.56	-747851	1331.8	-1492396	2672.39		RSMAX2	58	1.04	24	0.65	30	0.72

Table 6.17: Comparison via Pseudo Loop (Num of Nodes = 4)

		Pseudo Loop (Num of Nodes = 4)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-152222	917.31	-744364	4566.24	-1484836	9118.22	C	HC	292	1.01	272	0.74	281	0.75
	TABU	-152199	917.16	-744349	4566.12	-1484812	9118.06		TABU	254	1.2	249	0.87	266	0.81
	MMHC	-152470	919.25	-744582	4567.56	-1484812	9118.06		MMHC	284	1.03	270	0.76	286	0.77
	RSMAX2	-152467	919.23	-744582	4567.56	-1484814	9118.08		RSMAX2	284	1.03	270	0.76	285	0.76
loglik	HC	-150106	906.24	-741897	4553.74	-1482129	9104.5	M	HC	64	0.72	23	0.47	14	0.35
	TABU	-150101	906.2	-741906	4553.78	-1482132	9104.51		TABU	67	0.75	23	0.47	14	0.35
	MMHC	-150417	908.65	-742140	4555.17	-1482132	9104.51		MMHC	76	0.78	27	0.49	14	0.35
	RSMAX2	-150412	908.62	-742140	4555.17	-1482137	9104.56		RSMAX2	76	0.78	27	0.49	15	0.36
AIC	HC	-150724	909.75	-742460	4556.85	-1482703	9107.67	WO	HC	19	0.49	5	0.3	5	0.3
	TABU	-150709	909.66	-742459	4556.83	-1482696	9107.62		TABU	54	0.85	28	0.59	20	0.49
	MMHC	-151003	911.94	-742693	4558.24	-1482696	9107.62		MMHC	15	0.44	3	0.22	0	0
	RSMAX2	-150999	911.92	-742693	4558.24	-1482700	9107.66		RSMAX2	15	0.44	3	0.22	0	0
BIC	HC	-152241	918.39	-744295	4567.01	-1484773	9119.1	WC	HC	46	1.13	14	0.82	14	0.82
	TABU	-152201	918.14	-744261	4566.77	-1484729	9118.81		TABU	90	1.34	44	0.83	32	0.74
	MMHC	-152441	920.03	-744495	4568.24	-1484729	9118.81		MMHC	28	0.81	8	0.56	0	0
	RSMAX2	-152440	920.03	-744495	4568.24	-1484730	9118.82		RSMAX2	28	0.81	8	0.56	0	0

Table 6.18: Comparison via Pseudo Loop (Num of Nodes = 6)

		Pseudo Loop (Num of Nodes = 6)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-151573	1547.91	-757733	7732.81	-1513789	15448.13	C	HC	418	1.23	367	1.05	377	1
	TABU	-151527	1547.33	-757733	7732.81	-1513789	15448.13		TABU	355	1.46	349	1.11	359	1.13
	MMHC	-151714	1549.34	-757847	7733.72	-1513842	15448.62		MMHC	401	1.19	351	0.97	370	0.96
	RSMAX2	-151695	1549.12	-757849	7733.73	-1513831	15448.51		RSMAX2	402	1.25	351	0.98	371	0.96
loglik	HC	-149508	1528.02	-755376	7710.07	-1511230	15423.37	M	HC	75	0.78	33	0.6	23	0.47
	TABU	-149453	1527.33	-755376	7710.07	-1511230	15423.37		TABU	74	0.79	33	0.6	23	0.47
	MMHC	-149698	1529.94	-755527	7711.32	-1511302	15424.05		MMHC	94	0.81	49	0.66	30	0.52
	RSMAX2	-149669	1529.62	-755529	7711.32	-1511288	15423.91		RSMAX2	93	0.88	49	0.67	29	0.5
AIC	HC	-150106	1533.95	-755904	7715.29	-1511763	15428.65	WO	HC	7	0.33	0	0	0	0
	TABU	-150056	1533.31	-755904	7715.29	-1511763	15428.65		TABU	71	1.18	18	0.39	18	0.39
	MMHC	-150277	1535.68	-756041	7716.42	-1511828	15429.26		MMHC	5	0.26	0	0	0	0
	RSMAX2	-150251	1535.39	-756043	7716.42	-1511815	15429.13		RSMAX2	5	0.26	0	0	0	0
BIC	HC	-151574	1548.51	-757624	7732.31	-1513684	15447.68	WC	HC	18	0.81	2	0.2	0	0
	TABU	-151535	1547.99	-757624	7732.31	-1513684	15447.68		TABU	74	1.12	38	0.84	36	0.77
	MMHC	-151698	1549.76	-757716	7733.04	-1513724	15448.05		MMHC	8	0.39	2	0.2	0	0
	RSMAX2	-151679	1549.56	-757718	7733.05	-1513715	15447.95		RSMAX2	8	0.39	2	0.2	0	0

Table 6.19: Comparison via Pseudo Loop (Num of Nodes = 8)

		Pseudo Loop (Num of Nodes = 8)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-209665	2121.1	-1019596	10374.74	-2036502	20724.35	C	HC	595	1.25	531	1.16	550	1.08
	TABU	-209658	2121.03	-1019592	10374.7	-2036502	20724.35		TABU	510	1.92	517	1.22	537	1.19
	MMHC	-209955	2123.8	-1020595	10383.47	-2037573	20733.29		MMHC	569	1.2	480	0.94	515	0.97
	RSMAX2	-209957	2123.79	-1020999	10387.34	-2037277	20730.78		RSMAX2	569	1.25	475	0.96	516	0.95
loglik	HC	-207015	2095.04	-1016514	10344.81	-2033143	20691.63	M	HC	100	0.93	69	0.9	50	0.69
	TABU	-207005	2094.94	-1016505	10344.72	-2033143	20691.63		TABU	97	0.92	69	0.9	50	0.69
	MMHC	-207366	2098.34	-1017613	10354.51	-2034291	20701.33		MMHC	126	0.91	120	0.9	85	0.82
	RSMAX2	-207366	2098.29	-1018029	10358.48	-2034000	20698.85		RSMAX2	126	0.99	125	0.97	84	0.81
AIC	HC	-207802	2102.89	-1017220	10351.82	-2033861	20698.77	WO	HC	5	0.26	0	0	0	0
	TABU	-207794	2102.81	-1017213	10351.75	-2033861	20698.77		TABU	93	1.61	14	0.4	13	0.34
	MMHC	-208127	2105.95	-1018285	10361.2	-2034983	20708.21		MMHC	5	0.26	0	0	0	0
	RSMAX2	-208127	2105.91	-1018697	10365.13	-2034691	20705.73		RSMAX2	5	0.26	0	0	0	0
BIC	HC	-209733	2122.17	-1019520	10374.66	-2036450	20724.5	WC	HC	20	0.72	6	0.34	2	0.2
	TABU	-209730	2122.14	-1019520	10374.66	-2036450	20724.5		TABU	78	1.24	34	0.9	28	0.7
	MMHC	-209995	2124.61	-1020474	10382.97	-2037477	20733.04		MMHC	16	0.68	12	0.48	6	0.34
	RSMAX2	-209994	2124.59	-1020874	10386.81	-2037182	20730.53		RSMAX2	16	0.68	14	0.51	4	0.28

Table 6.20: Comparison via Pseudo Loop (Num of Nodes = 10)

		Pseudo Loop (Num of Nodes = 10)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-253245	2567.31	-1253002	12766.69	-2501901	25495.03	C	HC	768	1.37	712	1.43	735	1.27
	TABU	-253238	2567.24	-1252992	12766.59	-2501898	25495.01		TABU	691	2.09	698	1.46	720	1.33
	MMHC	-253926	2574.34	-1256122	12796.2	-2505279	25526.79		MMHC	735	1.35	564	0.96	612	0.98
	RSMAX2	-253822	2572.93	-1257416	12808.52	-2506828	25541.93		RSMAX2	732	1.35	548	0.99	604	0.97
loglik	HC	-249796	2533.4	-1249006	12727.66	-2497561	25452.62	M	HC	126	1.03	88	0.98	65	0.76
	TABU	-249784	2533.29	-1248989	12727.48	-2497556	25452.59		TABU	127	1.03	88	0.98	65	0.76
	MMHC	-250566	2541.35	-1252426	12760.08	-2501207	25486.93		MMHC	159	1.07	236	1.15	188	1.13
	RSMAX2	-250468	2539.94	-1253747	12772.68	-2502780	25502.3		RSMAX2	162	1.09	252	1.23	196	1.2
AIC	HC	-250785	2543.27	-1249924	12736.79	-2498483	25461.77	WO	HC	6	0.28	0	0	0	0
	TABU	-250775	2543.17	-1249910	12736.64	-2498479	25461.75		TABU	82	1.68	14	0.47	15	0.39
	MMHC	-251519	2550.85	-1253237	12768.19	-2502045	25495.29		MMHC	6	0.28	0	0	0	0
	RSMAX2	-251418	2549.43	-1254547	12780.69	-2503611	25510.6		RSMAX2	6	0.28	0	0	0	0
BIC	HC	-253211	2567.48	-1252916	12766.53	-2501807	25494.78	WC	HC	38	0.89	22	0.69	10	0.44
	TABU	-253207	2567.44	-1252911	12766.49	-2501807	25494.78		TABU	94	1.59	50	1.11	40	0.85
	MMHC	-253858	2574.19	-1255880	12794.6	-2505066	25525.44		MMHC	28	0.81	36	0.82	36	0.82
	RSMAX2	-253749	2572.73	-1257154	12806.79	-2506607	25540.51		RSMAX2	26	0.79	38	0.79	36	0.82

A.5 Table for Diamond

Table 6.21: Comparison via Diamond (Num of Nodes = 4)

		Diamond (Num of Nodes = 4)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-200073	296.07	-990529	1488.95	-1975845	2993.22	C	HC	324	0.9	346	0.82	361	0.75
	TABU	-199991	296.24	-990330	1491.16	-1975702	2993.15		TABU	312	1.06	347	0.74	359	0.65
	MMHC	-201258	300.03	-993837	1498.75	-1981603	2991.61		MMHC	296	0.92	338	0.81	357	0.74
	RSMAX2	-201155	300.7	-993753	1498.49	-1981605	2991.62		RSMAX2	294	0.95	337	0.85	356	0.74
loglik	HC	-197171	299.61	-986793	1492.52	-1971733	2996.83	M	HC	58	0.74	31	0.58	15	0.39
	TABU	-197103	299.77	-986678	1494.75	-1971710	2996.88		TABU	56	0.74	27	0.53	12	0.33
	MMHC	-198517	303.8	-990244	1502.38	-1977631	2995		MMHC	87	0.84	41	0.62	20	0.43
	RSMAX2	-198408	304.54	-990154	1502.07	-1977635	2995.03		RSMAX2	89	0.86	41	0.64	21	0.43
AIC	HC	-198021	299.8	-987703	1492.56	-1972665	2996.82	WO	HC	18	0.54	23	0.62	24	0.65
	TABU	-197941	299.98	-987550	1494.83	-1972596	2996.89		TABU	32	0.6	26	0.52	29	0.59
	MMHC	-199294	303.99	-991099	1502.5	-1978516	2995.11		MMHC	17	0.51	21	0.57	23	0.65
	RSMAX2	-199188	304.69	-991011	1502.21	-1978519	2995.13		RSMAX2	17	0.51	22	0.58	23	0.65
BIC	HC	-200107	300.29	-990669	1492.72	-1976025	2996.79	WC	HC	38	1.09	50	1.28	52	1.35
	TABU	-199997	300.49	-990392	1495.11	-1975790	2996.89		TABU	66	1.24	54	1.1	52	0.93
	MMHC	-201201	304.49	-993885	1502.91	-1981706	2995.5		MMHC	32	0.93	42	1.11	46	1.23
	RSMAX2	-201102	305.07	-993804	1502.66	-1981706	2995.5		RSMAX2	36	1	46	1.17	46	1.23

Table 6.22: Comparison via Diamond (Num of Nodes = 6)

		Diamond (Num of Nodes = 6)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-227675	1369.41	-1121009	6747.41	-2235141	13460.08	C	HC	563	1.94	629	1.93	646	1.97
	TABU	-227492	1367.9	-1120796	6745.95	-2234852	13458.77		TABU	552	2.04	624	1.91	640	1.86
	MMHC	-230771	1388.94	-1132149	6825.51	-2253172	13584.78		MMHC	472	1.56	585	1.68	615	1.83
	RSMAX2	-230917	1389.62	-1131904	6826.26	-2252879	13583.61		RSMAX2	452	1.62	576	1.72	605	1.78
loglik	HC	-223207	1344.59	-1114502	6710.88	-2227675	13418.27	M	HC	109	1.16	47	0.67	27	0.51
	TABU	-223007	1342.88	-1114367	6709.99	-2227544	13417.96		TABU	103	1.1	44	0.64	24	0.47
	MMHC	-227292	1370.51	-1126733	6796.22	-2246828	13549.96		MMHC	202	1.32	99	0.96	66	0.83
	RSMAX2	-227411	1370.96	-1126502	6797.25	-2246548	13548.9		RSMAX2	221	1.51	108	1.06	76	0.87
AIC	HC	-224783	1353.67	-1116357	6721.4	-2229646	13429.37	WO	HC	28	0.6	24	0.74	27	0.84
	TABU	-224583	1352	-1116194	6720.3	-2229460	13428.72		TABU	45	0.73	32	0.63	36	0.72
	MMHC	-228390	1376.69	-1128204	6804.33	-2248431	13558.88		MMHC	26	0.5	16	0.56	19	0.71
	RSMAX2	-228524	1377.25	-1127969	6805.26	-2248148	13557.78		RSMAX2	27	0.53	16	0.56	19	0.71
BIC	HC	-228650	1376.01	-1122401	6755.7	-2236752	13469.41	WC	HC	92	1.83	72	2.12	80	2.49
	TABU	-228450	1374.43	-1122147	6753.91	-2236367	13467.54		TABU	120	1.89	80	1.84	84	2.03
	MMHC	-231084	1391.89	-1132998	6830.78	-2254210	13591.02		MMHC	58	1.07	36	1.25	40	1.42
	RSMAX2	-231256	1392.74	-1132749	6831.39	-2253917	13589.82		RSMAX2	86	1.46	50	1.34	54	1.5

Table 6.23: Comparison via Diamond (Num of Nodes = 8)

		Diamond (Num of Nodes = 8)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-209595	2127.11	-1025068	10403.36	-2037346	20674.17	C	HC	716	1.69	863	2.24	915	2.25
	TABU	-209421	2125.27	-1023800	10387.05	-2036857	20669.02		TABU	703	1.72	864	2.25	905	2.23
	MMHC	-211560	2146.17	-1037778	10529.39	-2058839	20894.21		MMHC	569	1.35	753	1.75	844	1.89
	RSMAX2	-211926	2150.08	-1038825	10539.5	-2057404	20878.41		RSMAX2	531	1.37	726	1.74	821	1.97
loglik	HC	-205811	2090.16	-1014574	10296.95	-2023359	20531.31	M	HC	238	1.77	82	1.1	43	0.74
	TABU	-205506	2086.8	-1012976	10276.38	-2023235	20529.95		TABU	218	1.62	71	0.88	39	0.74
	MMHC	-208703	2118.59	-1031757	10468.59	-2050455	20809.76		MMHC	385	2.04	200	1.37	126	1.1
	RSMAX2	-209127	2123.07	-1032608	10476.78	-2048320	20787.44		RSMAX2	423	2.16	224	1.58	147	1.28
AIC	HC	-207088	2102.72	-1017796	10329.53	-2027288	20571.13	WO	HC	46	0.8	55	1.17	42	1.07
	TABU	-206840	2100.01	-1016298	10310.24	-2027058	20568.68		TABU	79	0.97	65	1.28	56	1.08
	MMHC	-209578	2127.12	-1033414	10485.41	-2052653	20831.89		MMHC	46	0.7	47	1.02	30	0.96
	RSMAX2	-209981	2131.39	-1034333	10494.26	-2050736	20811.57		RSMAX2	46	0.67	50	1.1	32	1.04
BIC	HC	-210222	2133.64	-1028295	10435.78	-2041453	20714.73	WC	HC	144	2.2	214	4.31	204	4.92
	TABU	-210114	2132.51	-1027123	10420.65	-2040841	20708.34		TABU	224	2.73	202	4.28	190	4.72
	MMHC	-211725	2148.09	-1038814	10540.32	-2060578	20911.77		MMHC	104	1.46	104	2.19	64	1.99
	RSMAX2	-212077	2151.82	-1039955	10551.31	-2059446	20898.67		RSMAX2	148	1.67	152	2.51	112	2.42

Table 6.24: Comparison via Diamond (Num of Nodes = 10)

		Diamond (Num of Nodes = 10)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-264207	2682.62	-1299179	13200.46	-2576611	26172.29	C	HC	790	1.53	1099	2.3	1219	2.29
	TABU	-264111	2681.69	-1298587	13194.17	-2575401	26159.7		TABU	765	1.66	1096	2.29	1211	2.31
	MMHC	-266497	2704.09	-1310181	13306.04	-2608702	26498.01		MMHC	592	1.26	880	1.7	1007	1.85
	RSMAX2	-267200	2711.32	-1322040	13429.81	-2626080	26666.54		RSMAX2	545	1.23	757	2.11	884	2.17
loglik	HC	-260692	2648.92	-1285989	13069.45	-2548967	25882.88	M	HC	548	2.5	214	1.72	116	1.22
	TABU	-260503	2647.1	-1284563	13053.86	-2547010	25862.35		TABU	523	2.51	186	1.54	102	1.19
	MMHC	-263463	2674.73	-1303889	13245.86	-2598362	26395.52		MMHC	750	2.55	457	2.18	337	2.04
	RSMAX2	-264162	2682	-1315306	13365.27	-2615253	26559.62		RSMAX2	799	2.52	578	3.03	464	2.99
AIC	HC	-261787	2659.52	-1289764	13106.64	-2556311	25958.98	WO	HC	62	0.84	87	1.27	65	1.27
	TABU	-261637	2658.08	-1288575	13093.68	-2554569	25940.72		TABU	112	1.11	118	1.53	87	1.41
	MMHC	-264352	2683.47	-1305573	13261.94	-2600972	26421.28		MMHC	58	0.73	63	1.09	56	1.08
	RSMAX2	-265057	2690.78	-1317118	13382.59	-2618011	26586.69		RSMAX2	56	0.67	65	1.06	52	1.09
BIC	HC	-264474	2685.57	-1302065	13228.16	-2582788	26233.71	WC	HC	188	2.25	338	4.84	314	5.94
	TABU	-264420	2685.05	-1301648	13223.79	-2581820	26223.64		TABU	308	2.93	386	5.21	312	5.68
	MMHC	-266534	2704.94	-1311060	13314.44	-2610382	26514.31		MMHC	146	1.63	162	2.37	124	2.18
	RSMAX2	-267253	2712.32	-1323022	13439.23	-2627954	26684.5		RSMAX2	206	1.94	296	3.22	370	5.02

A.6 Table for Rhombus

Table 6.25: Comparison via Rhombus (Num of Nodes = 4)

		Rhombus (Num of Nodes = 4)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-198861	327.32	-984754	1633.13	-1963993	3258.75	C	HC	322	0.91	361	0.63	382	0.41
	TABU	-198677	325.57	-984051	1625.9	-1963989	3258.82		TABU	290	1.18	356	0.72	375	0.58
	MMHC	-199920	332.08	-987749	1644.31	-1969244	3289.06		MMHC	282	0.85	337	0.73	359	0.64
	RSMAX2	-199857	331.74	-987360	1644.92	-1968685	3292.72		RSMAX2	287	0.86	341	0.74	365	0.61
loglik	HC	-195894	330.88	-980937	1636.6	-1959765	3261.63	M	HC	64	0.89	27	0.58	10	0.3
	TABU	-195681	328.87	-980207	1629.08	-1959755	3261.83		TABU	69	0.85	26	0.54	13	0.37
	MMHC	-197126	336.4	-984056	1648.11	-1965156	3292.42		MMHC	104	0.85	52	0.69	33	0.6
	RSMAX2	-197045	335.99	-983652	1648.78	-1964568	3296.19		RSMAX2	99	0.86	48	0.69	27	0.57
AIC	HC	-196765	331.51	-981875	1636.94	-1960737	3261.99	WO	HC	14	0.35	12	0.36	8	0.27
	TABU	-196564	329.6	-981153	1629.5	-1960729	3262.15		TABU	41	0.6	18	0.41	12	0.33
	MMHC	-197915	336.83	-984949	1648.39	-1966084	3292.68		MMHC	14	0.35	11	0.35	8	0.27
	RSMAX2	-197843	336.45	-984551	1649.04	-1965507	3296.4		RSMAX2	14	0.35	11	0.35	8	0.27
BIC	HC	-198902	333.09	-984932	1638.02	-1964241	3263.3	WC	HC	36	0.77	24	0.71	16	0.55
	TABU	-198730	331.41	-984236	1630.88	-1964240	3263.3		TABU	110	1.54	44	1.05	32	0.93
	MMHC	-199851	337.91	-987859	1649.33	-1969430	3293.61		MMHC	32	0.74	26	0.73	18	0.58
	RSMAX2	-199801	337.59	-987480	1649.87	-1968892	3297.15		RSMAX2	32	0.74	24	0.71	16	0.55

Table 6.26: Comparison via Rhombus (Num of Nodes = 6)

		Rhombus (Num of Nodes = 6)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-225795	1354.42	-1116344	6693.89	-2227143	13353.16	C	HC	533	1.9	631	1.82	656	1.8
	TABU	-225655	1353.18	-1116283	6693.87	-2226948	13352.84		TABU	503	2.26	613	1.87	652	1.91
	MMHC	-227218	1364.21	-1120926	6727.09	-2233878	13399.99		MMHC	457	1.75	578	1.85	614	1.88
	RSMAX2	-227388	1364.66	-1121223	6730.44	-2233720	13399.35		RSMAX2	462	1.78	590	1.86	623	1.85
loglik	HC	-222302	1335.09	-1111605	6667.57	-2221896	13323.91	M	HC	133	1.41	46	0.74	29	0.62
	TABU	-222134	1333.64	-1111527	6667.5	-2221679	13323.57		TABU	136	1.41	48	0.76	25	0.54
	MMHC	-223985	1346.6	-1116412	6702.13	-2228828	13371.81		MMHC	214	1.33	98	0.95	71	0.86
	RSMAX2	-224166	1347	-1116689	6705.34	-2228649	13371.1		RSMAX2	217	1.41	91	0.94	65	0.89
AIC	HC	-223428	1341.66	-1112869	6674.78	-2223192	13331.28	WO	HC	34	0.52	23	0.42	15	0.36
	TABU	-223273	1340.29	-1112797	6674.72	-2222982	13330.94		TABU	61	0.68	39	0.63	23	0.51
	MMHC	-224993	1352.44	-1117596	6708.9	-2230058	13378.85		MMHC	29	0.54	24	0.47	15	0.36
	RSMAX2	-225171	1352.88	-1117880	6712.16	-2229887	13378.16		RSMAX2	21	0.46	19	0.39	12	0.33
BIC	HC	-226192	1357.8	-1116988	6698.28	-2227864	13357.83	WC	HC	86	1.21	52	0.93	36	0.82
	TABU	-226068	1356.63	-1116935	6698.27	-2227680	13357.52		TABU	160	1.75	98	1.54	54	1.2
	MMHC	-227467	1366.8	-1121455	6730.95	-2234493	13404.24		MMHC	70	1.11	60	1.04	36	0.82
	RSMAX2	-227637	1367.32	-1121761	6734.38	-2234350	13403.62		RSMAX2	52	0.97	42	0.82	26	0.68

Table 6.27: Comparison via Rhombus (Num of Nodes = 8)

		Rhombus (Num of Nodes = 8)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-203646	2070.26	-1004851	10212.09	-2006720	20396.87	C	HC	757	2.39	901	2.1	934	2.1
	TABU	-203604	2069.82	-1004781	10211.26	-2006576	20395.19		TABU	740	2.59	889	2.28	937	2.08
	MMHC	-206444	2100.29	-1011247	10281.04	-2014821	20484.37		MMHC	615	1.78	803	1.99	860	1.97
	RSMAX2	-207282	2107.49	-1011363	10281.33	-2013892	20473.37		RSMAX2	584	1.48	800	1.84	851	1.9
loglik	HC	-200394	2038.1	-1000444	10168.22	-2001783	20347.75	M	HC	194	1.82	66	1	40	0.79
	TABU	-200344	2037.54	-1000377	10167.39	-2001664	20346.26		TABU	189	1.79	65	1.01	34	0.67
	MMHC	-203577	2072.15	-1007145	10240.36	-2010170	20438.09		MMHC	342	1.78	164	1.37	118	1.17
	RSMAX2	-204608	2081.19	-1007359	10241.53	-2009346	20428.13		RSMAX2	388	1.95	182	1.48	135	1.32
AIC	HC	-201494	2049.13	-1001662	10180.42	-2003044	20360.34	WO	HC	49	0.8	33	0.62	26	0.71
	TABU	-201445	2048.6	-1001593	10179.58	-2002914	20358.76		TABU	71	0.91	46	0.72	29	0.54
	MMHC	-204491	2081.31	-1008252	10251.44	-2011336	20449.76		MMHC	43	0.66	33	0.62	22	0.5
	RSMAX2	-205438	2089.54	-1008428	10252.25	-2010475	20439.45		RSMAX2	28	0.59	18	0.44	14	0.38
BIC	HC	-204193	2076.21	-1005631	10220.16	-2007590	20405.74	WC	HC	136	1.82	76	1.3	58	1.43
	TABU	-204147	2075.75	-1005556	10219.29	-2007421	20403.85		TABU	176	2.06	106	1.54	66	1.21
	MMHC	-206734	2103.81	-1011860	10287.52	-2015539	20491.85		MMHC	126	1.57	96	1.41	66	1.21
	RSMAX2	-207475	2110.05	-1011912	10287.17	-2014545	20480.27		RSMAX2	70	1.25	54	1.06	44	0.92

Table 6.28: Comparison via Rhombus (Num of Nodes = 10)

		Rhombus (Num of Nodes = 10)													
Sample Size		1000		5000		10000				1000		5000		10000	
		Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.			Sum.	Std.Dev.	Sum.	Std.Dev.	Sum.	Std.Dev.
BDe	HC	-260099	2631.01	-1284605	12999.26	-2564149	25946.15	C	HC	1060	2.85	1257	2.45	1308	2.36
	TABU	-260011	2630.06	-1284511	12998.24	-2564128	25945.98		TABU	1045	2.98	1248	2.48	1309	2.27
	MMHC	-264881	2678.69	-1296057	13113.29	-2576754	26078.33		MMHC	772	2.01	1060	2.25	1162	2.19
	RSMAX2	-267848	2707.82	-1302660	13177.71	-2583552	26138.92		RSMAX2	684	1.53	978	1.66	1089	1.85
loglik	HC	-256003	2590.39	-1279003	12943.36	-2557878	25883.58	M	HC	282	2.28	102	1.44	55	1.09
	TABU	-255877	2589.05	-1278879	12942.02	-2557857	25883.42		TABU	270	2.25	99	1.38	54	1.04
	MMHC	-261402	2644.17	-1290922	13062.19	-2570958	26020.6		MMHC	569	2.09	296	1.81	213	1.57
	RSMAX2	-264733	2676.94	-1298011	13131.38	-2578147	26085.06		RSMAX2	695	2.17	407	2.09	300	1.89
AIC	HC	-257421	2604.54	-1280577	12959.12	-2559500	25899.79	WO	HC	58	0.79	41	0.64	37	0.73
	TABU	-257311	2603.36	-1280465	12957.9	-2559479	25899.63		TABU	85	0.93	53	0.72	37	0.51
	MMHC	-262541	2655.62	-1292328	13076.25	-2572422	26035.24		MMHC	59	0.75	44	0.76	25	0.52
	RSMAX2	-265720	2686.86	-1299247	13143.78	-2579487	26098.48		RSMAX2	21	0.54	15	0.41	11	0.35
BIC	HC	-260900	2639.27	-1285706	13010.48	-2565348	25958.23	WC	HC	186	2.05	100	1.35	82	1.51
	TABU	-260830	2638.5	-1285633	13009.66	-2565327	25958.07		TABU	226	2.08	130	1.59	84	1.14
	MMHC	-265336	2683.71	-1296909	13122.09	-2577700	26088.01		MMHC	194	1.85	148	1.77	92	1.32
	RSMAX2	-268141	2711.22	-1303275	13184.2	-2584318	26146.85		RSMAX2	76	1.33	78	1.36	60	1.08

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