CHAPTER 23

MATHEMATICS FOR ECONOMICS AND FINANCE

Introduction

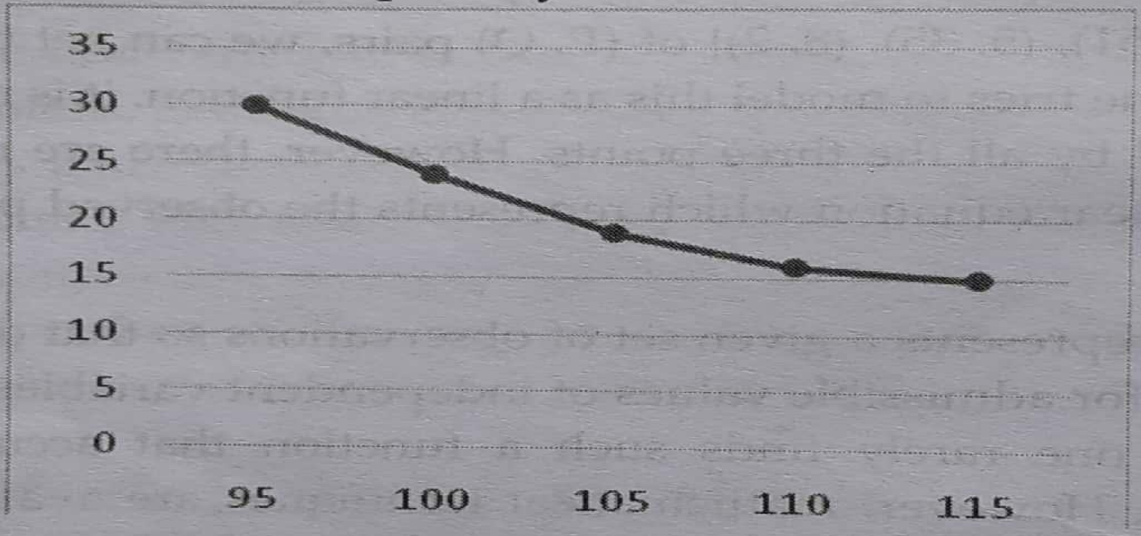
Human beings make use of a variety of resources to fulfill their needs. The goods, services and resources needed by humans, generally, exceed what is available. **Economics** is the study of how individuals and societies make decisions in the face of scarcity. Economics is divided into two categories – microeconomics and macroeconomics. **Microeconomics** focuses on the actions of individual agents e.g. households, workers, and businesses.**Macroeconomics** focuses on broad issues such as growth ofproduction, the number of unemployed people, the inflationary increase in prices, government deficits, and levels of exports and imports.

Demand and Supply

The table below shows the quantity of the petrol demanded per month in a certain city at different prices. When the price rises, people look for different alternatives to reduce their petrol consumption.

Price (per liter)	Quantity demanded (lakh liters)
Rs. 95	30
Rs. 100	24
Rs. 105	19
Rs. 110	16
Rs. 115	15

In economics, the term **demand** is used to refer to the amount of some good or service consumers are willing and able to purchase at each price. Price is what the buyer pays per unit of certain good or service. The total number of units purchased at this price is called the quantity demanded. As the price rises, the quantity demanded falls, and vice versa. This inverse relationship between the price and the quantity demanded is called the **law of demand**.

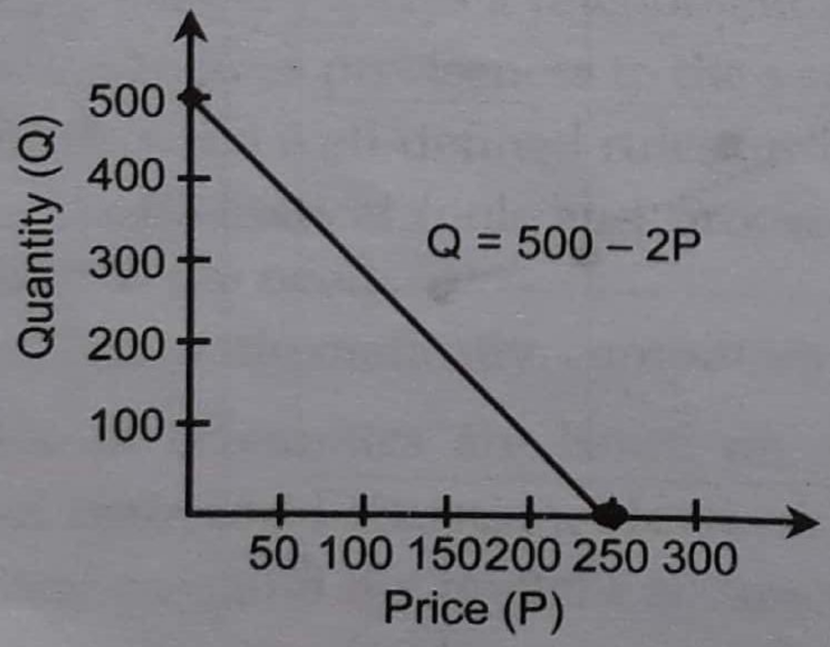


If we plot graph taking the price in the horizontal axis and the demand in the vertical axis, we get a downward sloping curve, known as the **demand curve**.

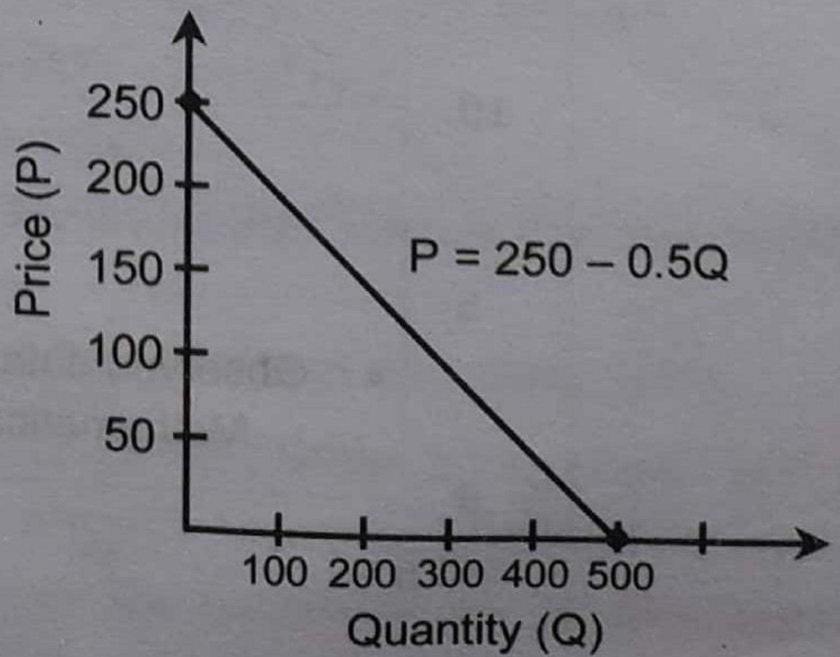
If P is the price and Q_D is the quantity demanded, we model the demand function by the relationship $Q_D = f(P)$ or simply $Q = f(P)$.

However, in economics, there is a tradition of plotting Q on the horizontal axis and P on the vertical one. The relationship expressed as $P = g(Q)$ is also known as an inverse demand function.

For example, the demand function $Q = 500 - 2P$my be written as $P = 250 - 0.5Q$ in terms of an inverse demand function.



(a)



(b)

Furthermore, when the demand is modeled asa linear equation

$$P = a - bQ$$

for constants $a > 0$, $b > 0$,

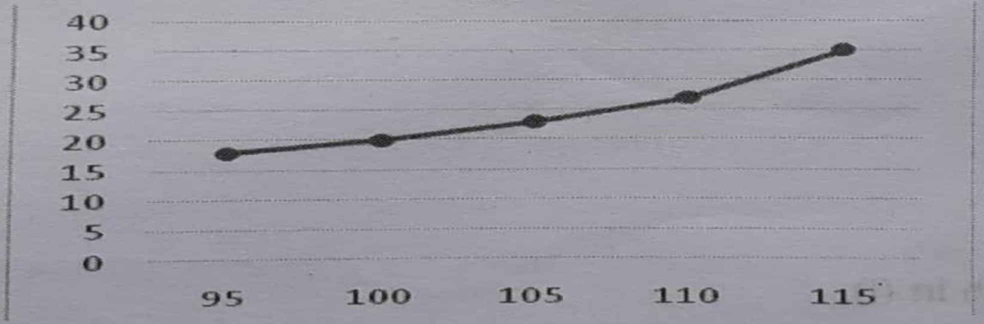
- the vertical intercept a represents the the price when $Q = 0$, i.e. there is no quantity demanded,
- the slope $-b$ represents the rate of change of the price with respect to the quantity demanded. If the quantity demanded increases by 1 unit, the price is decreased by b units.
- the horizontal intercept $\frac{a}{b}$ represents the quantity demanded when the price is zero.

In economics, **supply** is the amount of some good or service a producer is willing to supply ateach price. Here, price is what the producer receives for selling one unit of a good or service. When the price rises, a producer is willing to increase the quantity of goods or services to be supplied. So, a rise in price almost alwaysleads to an increase in the quantity supplied of that good or service, and vice versa.

The following table shows the prices of petrol and the amount supplied at each price in a month.

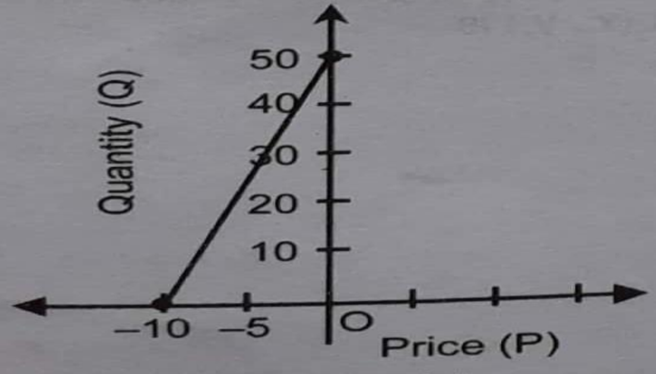
Price (per liter)	Supply (lakh liters)
Rs. 95	18
Rs. 100	20
Rs. 105	23
Rs. 110	27
Rs. 115	35

Again, if we plot a graph with price in the horizontal axis and the supply in the vertical axis, we get an upward sloping curve, known as the **supplycurve**.

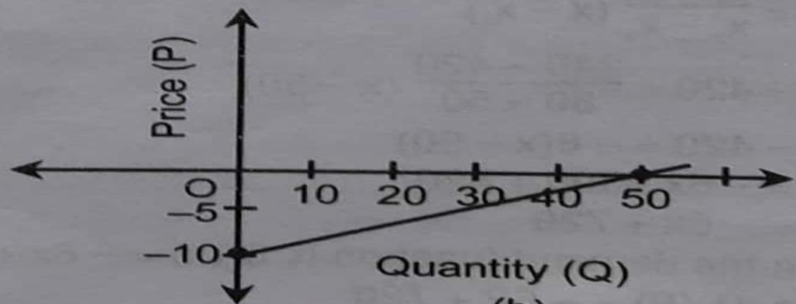


If P is the price and Qs is the quantity supplied, we model the supply function by the relationship Qs = f(P) or simply Q = f(P).If Q is plotted on the horizontal axis and P on the vertical one,the relationship expressed as P = g(Q) is also known as an inverse supply function.

For example, the supply function Q = 50 + 5P, expressed as an inverse supply function is P = 0.2Q – 10.



(a)



(b)

Furthermore, when the supply is modeled as a linear equation

$$P = c + dQ$$

for a constant$d > 0$,

- the vertical intercept c represents the the price at which there is no quantity supplied.
- the slope $d$means that if the price is increased by d units, the quantity supplied will be increased by 1 unit.
- the horizontal intercept$-\frac{c}{d}$ represents the quantity supplied when the price is zero.

WORKED OUT EXAMPLES

EXAMPLE 1

The demand for a good priced at Rs. 50 is 420 units, and when the price is Rs. 80, the demand becomes 240 units. Assuming that the demand function is linear, find the formula defining it.

Solution:

Method I:

Let the demand corresponding to the price Rs. P be Q units. Since the demand function is linear, for some constants a, b,

$Q = aP + b$ ... (i)

Since the demand corresponding to the price Rs. 50 is 420 units,

$420 = a(50) + b$ ... (ii)

Similarly, $240 = a(80) + b$ ... (iii)

Subtracting (ii) from (i),

$420 - 240 = -30a$

or, $-30a = 180$

or, $a = -6$

Putting the value of a in (i),

$420 = -300 + b$

or, $b = 720$

Putting the values of a and b in (i),

$Q = -6P + 720$

Hence, the demand function is:

$Q_D(P) = -6P + 720$

Method II:

Since the demand function is linear, the straight line corresponding to the demand function passes through (50, 420) and (80, 240). The equation of the line passing through (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or, } y - 420 = \frac{240 - 420}{80 - 50}(x - 50)$$

$$\text{or, } y - 420 = -6(x - 50)$$

$$\text{or, } y = -6x + 300 + 420$$

$$\text{or, } y = -6x + 720$$

Hence the demand function is: $Q_D(x) = -6x + 720$

Hence, $Q_D(P) = -6P + 720$

EXAMPLE 2

The supply equation for an item is given by $4P = 120 + 5Q$, where P is measured in rupees and Q is measured in 1000 units.

a. Sketch the corresponding curve.

b. How many units will be marketed when the unit price is Rs.70.

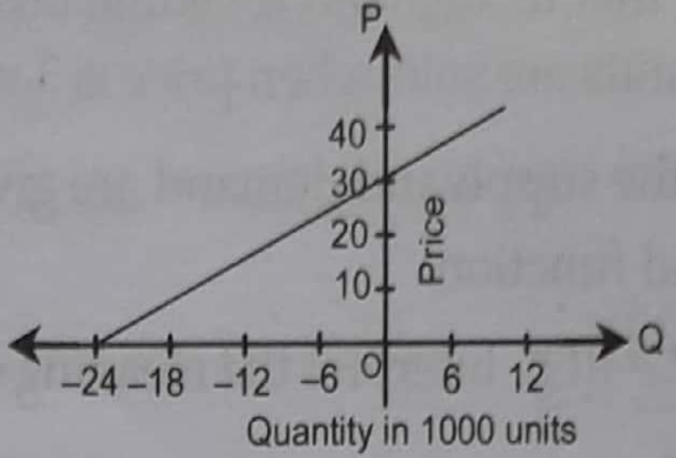
Solution:

a. We have

$P = 0 \Rightarrow Q = -24$

and $Q = 20 \Rightarrow P = 30$

The supply curve is now sketching as



b. if P = Rs.70, then

$4 \times 70 = 120 + 5Q$

or, $280 - 120 = 5Q$

or, $Q = 32$

Hence the total quantity supplied is 32000 units when price is Rs.70 per unit.

EXAMPLE 3

From the set of (P, Q) pairs {(1, 51), (5, 35), (8, 2)}, model the demand as a quadratic function of price. Hence, calculate the demand corresponding to the price $P = 3$.

Solution:

Let the quadratic demand function be $Q_D(P) = aP^2 + bP + c$

Corresponding to (1, 51),

$$51 = a(1)^2 + b(1) + c$$

or, $a + b + c = 51$ ... (1)

Corresponding to (5, 35),

$$35 = a(5)^2 + b(5) + c$$

or, $25a + 5b + c = 35$ ... (2)

Corresponding to (8, 2),

$$2 = a(8)^2 + b(8) + c$$

or, $64a + 8b + c = 2$ ... (3)

Solving (1), (2), (3),

$$a = -1,\ b = 2,\ c = 50$$

Hence, the required demand function is given by $Q_D(P) = -P^2 + 2P + 50$

For $P = 3$,

$$Q_D(P) = -(3)^2 + 2(3) + 50 = 35.$$

Exercise 23.1

1. If the demand is given by the equation $P = Q^2 + 8Q + 25$, find the quantity demanded if the price is 45.
2. From the following information, determine the linear demand function and supply function

Price (P)	Demand (Q_D)	Supply (Q_S)
4	7	1
7	1	7

3. If the demand function of a good is described by the equation$2P + 3Q = 60$,where P and Q denote the price and quantity demanded, respectively, find the largest and smallest values of P, Q for which the corresponding function is economically meaningful.
4. A monopolist sells 30 units of output when price is Rs. 12 and 40 units whenprice is Rs. 10. Find the demand function assuming it be linear. Use this function to predict quantity demandedwhen price is Rs. 8. What domain restrictions would you put on this demandfunction?

5. On a linear supply function, quantity suppliedrises from 30 to 90 when price rises from 40 to 80. How much further will price have to rise for quantitysupplied to reach to120?

6. A firm knows that its demand schedule takes the form $P = a - bQ$. If 400 units are sold when price is Rs 6 and 600 units are sold when price is 3,what are the values of the parameters a and b?

7. Suppose that the supplyand demand are given by the equations$Q - 3P = -1$, $Q + P = 2$. Find

   a. the demand function,

   b. the supply function,

   expressed as $P = f(Q)$. Interpret the meaning of slope and intercepts in each of the functions so formed.

8. The demand function of a certain good is$Q_D(P) = 100 - P + 2Y + 0.5A$,where Q, P, Y and A denote quantity demanded, price, income, and advertising expenditure, respectively.Calculate the demand when $P = 10$, $Y = 40$ and $A = 6$. Assuming that the price and income are fixed, calculate the additional advertising expenditure needed to raise the demand to 179 units.

9. The price-quantity pairs (P, Q) for the demand of a certain good are (1, 13), (2, 21), (3, 31). Find the quadratic equation that describes the demand function.

☑ Answers:

1. 2
2. $Q_D = 15 - 2p$, $Q_S = 2P - 7$
3. $0 \leq P \leq 30$, $0 \leq Q \leq 20$
4. $Q_D(P) = -5P + 90$, 50, ($0 \leq P \leq 18$) or ($0 \leq Q \leq 90$)
5. by Rs.20 (to reach 100)
6. $a = 12$, $b = \frac{3}{200}$
7. a. $P = 2 - Q$, b. $P = \frac{1}{3}Q + \frac{1}{3}$
8. $Q_D(P) = 173$, by 12
9. $Q_D(P) = P^2 + 5P + 7$

MATHEMATICS FOR ECONOMICS AND FINANCE

Exercise 23.1

1. If the demand is given by the equation $p = q^2 + 8q + 25$, find the quantity demanded if the price is 45.

Solution:

Here, demand equation is $p = q^2 + 8q + 25$
when $p = 45$, $q = ?$
Now, $45 = q^2 + 89 + 25$
or, $q^2 + 89 - 20 = 0$
or, $q^2 + (10 -2(q - 20) = 0$
or, $q^2 + 10q - 2q - 20 = 0$
or, $q (q + 10) - 2 (q + 10) = 0$
or, $(q + 10) (q - 2) = 0$
or, $q = 2$ or -10 (neglected)
So, $q = 2$

2. From the following information, determine the linear demand function and supply function

Price (p)	Demand (qd)	Supply (qs)
4	7	1
7	1	7

Solution:

For demand function

Let the demand function be $P = a - bQ$.

Then from above talbe, $4 = a - b \,.\, 7$ ... (i)

$7 = a - b.1$ ... (ii)

i.e.
$$\begin{array}{rcl} a - 7b &=& 4 \\ a - b &=& 7 \\ -\quad + & & - \\ \hline -6b &=& -3 \end{array}$$

$\therefore b = \frac{1}{2}$

Put $b = \frac{1}{2}$ in equation (i), we get

$a - b \,.\, 7 = 4$

or, $a = 4 + 7\,\frac{1}{2} = \frac{15}{2}$

$\therefore$ The demand function is

$p = a - bQ$

or, $\frac{15}{2} - \frac{1}{2}Q = p$

or, $15 - Q = 2p$

or, $Q = 15 - 2p$

Again, let the supply function be $p = a + bQ$.

Then from above table,

$$a + b1 = 4 \quad \text{... (iii)}$$

$$a + b.7 = 7 \quad \text{... (iv)}$$

$$\underline{- \quad - \quad -}$$

$$-6b = -3$$

$$\therefore b = \frac{1}{2}$$

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put $b = \frac{1}{2}$ equation (iii), we get

$$a + \frac{1}{2} = 4 \Rightarrow a = 4 - \frac{1}{2} = \frac{7}{2}$$

$\therefore$ The supply function $p = \frac{7}{2} + \frac{1}{2}.Q$

or, $2p = 7 + Q$

$\therefore Q = 2p - 7$

3. If the demand function of a good is described by the equation $2p + 3q = 60$ where p and q denote the price and quantity demanded, respectively, find the largest and smallest values of p, q for which the corresponding function is economically meaningful.

Solution:

Here, the demand equation is $2p + 3q = 60$, p and q are the price and quantity demanded.

Now, when $p = 0$, $3q = 60 \Rightarrow q = 20$

and when $q = 0$, $2p = 60 \Rightarrow p = 30$

i.e. the limit for quantity (q) is $0 \le q \le 20$

and the limit for price (p) is $0 \le p \le 30$

4. A monopolist sells 30 units of output when price is Rs. 12 and 40 units whenprice is Rs. 10. Find the demand function assuming it be linear. Use this function to predict quantity demanded when price is Rs. 8. What domain restrictions would you put on this demandfunction?

Solution:

Here, $q_1 = 30$ units, $p_1 = $ Rs.12

$q_2 = 40$ units, $p_2 = $ Rs.10

Now, the demand equation is

$$p - p_1 = \frac{p_2 - p_1}{q_2 - q_1} (q - q_1)$$

$$\text{or, } p - 12 = \frac{10 - 12}{40 - 30}(q - 30)$$

$$\text{or, } p - 12 = -\frac{1}{5}(q - 30)$$

or, $5p - 60 = -q + 30 \Rightarrow q = -5p + 90.$

When price is Rs.8 only, then, $q = -5 \times 8 + 90 = 50$

for domain restriction: if $q = 0$ then $p = 18$

if $p = 0$, $q = 90$

$\therefore$ Domain $= 0 \leq p \leq 18$ or $0 \leq q \leq 90$

5. On a linear supply function, quantity supplyrises from 30 to 90 when price rises from 40 to 80. How much further will price have to rise for quantitysupply to reach to120?

Solution:

Here, $q_1 = 30$, $q_2 = 90$, $p_1 = 40$, $p_2 = 80$

Now, supply function is $p - p_1 = \frac{p_2 - p_1}{q_2 - q_1}(q - q_1)$

or, $p - 40 = \frac{80 - 40}{90 - 30}(q - 30)$

or, $p - 40 = \frac{2}{3}(q - 30)$

or, $3p - 120 = 2q - 60$

or, $3p = 2q + 60$

when $q = 120$, then $3p = 2 \times 120 + 60$

or, $p = \frac{240 + 60}{3}$ $\therefore p = 100$

So, price should to rise by Rs.20 to 100.

6. A firm knows that its demand schedule takes the form p = a – bq. If 400 units are sold when price is Rs 6 and 600 units are sold when price is 3,what are the values of the parameters a and b?

Solution:

Here, the demand schedule is p = a – bq ...(i)

When p = Rs.60, q = 400 and

when p = Rs.3, q = 600

Now,

$$\begin{array}{rl} a - b \times 400 = 6 & \quad \ldots \text{(ii)} \\ a - b \times 600 = 3 & \quad \ldots \text{(iii)} \\ - \quad + \qquad\quad - & \\ \hline 200b = 3 & \end{array}$$

$$\therefore b = \frac{3}{200}$$

Put $b = \frac{2}{200}$ in equation (ii), we get

$a - 400\,b = 6$

or, $a = 6 + 400 \times \frac{3}{200} = 12.$

So, the parameters are a = 12 and $b = \frac{3}{200}$

7. Suppose that the supplyand demand are given by the equations$Q - 3P = -1$, $Q + P = 2$. Find

a. the demand function,

b. the supply function,

expressed as $P = f(Q)$. Interpret the meaning of slope and intercepts in each of the functions so formed

Solution:

Here, demand functions $p + q = 2$ and

Supply function is $q - 3p = -1$

Now,

a. demand function is $p = 2 - q$

b. supply function is $3p = q + 1 \therefore p = \frac{1}{3}q + \frac{1}{3}$

8. The demand function of a certain good is $Q_D(p) = 100 - p + 2Y + 0.5A$ where q, p, Y and A denote quantity demanded, price, income, and advertising expenditure, respectively. Calculate the demand when p = 10, Y = 40 and A = 6. Assuming that the price and income are fixed, calculate the additional advertising expenditure needed to raise the demand to 179 units.

Solution:

Here, the demand equation is $Q_D(P) = 100 - P + 2y + 0.5A$

When $P = 10$, $y = 40$, $A = 6$

demand $(Q_D(p)) = 100 - 10 + 2 \times 40 + 0.5 \times 6 = 100 - 10 + 80 + 3 = 173$

When price and income remain constant, then

$Q_D(p) = 179$ units $\Rightarrow 179 = 100 - 10 + 2 \times 40 + 0.5A$

or, $179 = 170 + 0.5A$

or, $0.5A = 9$

or, $A = \dfrac{9}{0.5} = 18$

So, the advertising expenditure should be raised by $18 - 6 = 12$.

9. The price-quantity pairs (p, q) for the demand of a certain good are (1, 13), (2, 21), (3, 31). Find the quadratic equation that describes the demand function.

Solution:

Here, price-quantity pairs (p, q) are (1, 13), (2, 21) and (3, 31)

Let the quadratioc equation for demand function be $q_D(p) = ap^2 + bp + c$.

for (1, 13), $13 = a.1^2 + b.1 + c \Rightarrow a + b + c = 13$ ... (i)

for (2, 21), $21 = 4a + 2b + c \Rightarrow 4a + 2b + c = 21$ ... (ii)

for (3,31), $9a + 3b + c = 31$ ... (iii)

Soliving (i) and (ii)

$$\begin{array}{l} a + b + c = 13 \\ 4a + 2b + c = 24 \\ -\quad -\quad -\quad - \\ \hline -3a - b = -8 \end{array}$$

or, $3a + b = 8$ ... (iv)

Soliving (ii) and (iii)

$$\begin{array}{l} 4a + 2b + c = 21 \\ 9a + 3b + c = 31 \\ -\quad -\quad -\quad - \\ \hline 5a + b = 10 \text{ ... (v)} \end{array}$$

Again, solving (iv) and (v), we get

$$\begin{array}{l} 4a + b = 10 \\ 9a + b = 8 \\ -\quad -\quad - \\ \hline 2a = 2 \qquad \therefore a = 1 \end{array}$$

put a = 1 in equation (iv), we get

$3.1 + b = 8 \Rightarrow b = 5$

Also, put a = 1 and b = 5 in equation (i), then

$1 + 5 + c = 13$

$\therefore c = 13 - 6 = 7$

So, the required quadratic equation is

$q_D(p) = p^2 + 5p + 7$

WORKED OUT EXAMPLES

EXAMPLE 1

Find the maximum value of the utility function, $U = x_1x_2$, subject to the budgetary constraint, $x_1 + 4x_2 = 360$.

Solution:

We have

$$U = x_1x_2 \quad \text{... (1)}$$

$$x_1 + 4x_2 = 360 \quad \text{.... (2)}$$

From (2), $x_1 = 360 - 4x_2$

Putting the value of x_1 in (1),

$$U = (360 - 4x_2)x_2$$

$$U = 360x_2 - 4x_2^2$$

Differentiating w.r.t. x_2,

$$U' = 360 - 8x_2$$

$$U'' = -8$$

Since $U'' < 0$, the value of U will be maximum if

$$U' = 0$$

$$360 - 8x_2 = 0$$

$$x_2 = 45$$

$$x_1 = 360 - 4(45) = 180$$

Hence, the maximum value of the utility function $U = x_1x_2 = (180)(45) = 8100$.