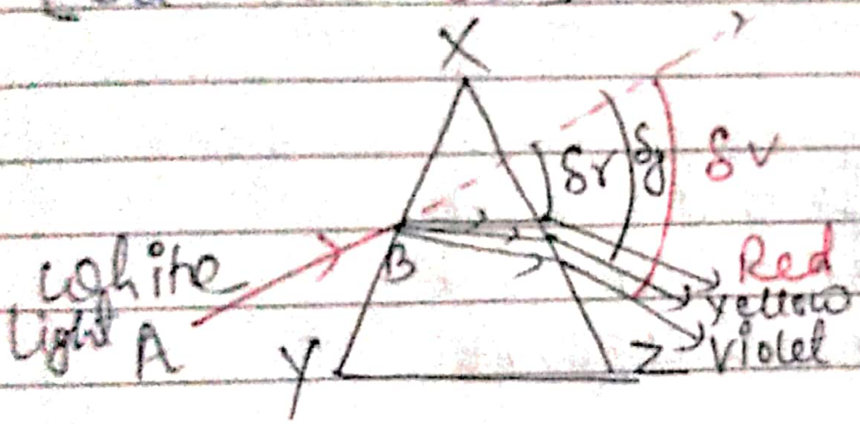
Dispersion

Date..........
Page..........

Angular Dispersion:-

Angular dispersion is defined as the deviation angle between the two extreme colours (violet and red) of light.

$$\left[\delta_a = \delta_v - \delta_r\right]$$



Let's consider a beam of white light passes through a glass prism, light of different colours are deviated by different angles.

The deviation produced by violet light is more than the deviation produced by red light ie $\delta_v > \delta_r$.

As we know for small angled prism; $\delta = (\mu - 1)A$

$\therefore$ $\delta_v = A(\mu_v - 1)$
$\delta_r = A(\mu_r - 1)$

where μ_v & μ_r are the refractive indices of red & violet light

$$\therefore \left[\delta_a = \delta_v - \delta_r = A(\mu_v - \mu_r)\right]$$

Date.........
Page.........

Dispersive power:

The ratio of angular dispersion between violet and red colours and the deviation suffered by the mean colour (yellow) is called the dispersive power (ω)

The value of $\omega = \dfrac{\delta v - \delta r}{\delta}$

where, $\delta v = (\mu v - 1) A$
$\delta r = (\mu r - 1) A$
$\delta = (\mu - 1) A$

$\therefore \omega = \dfrac{A(\mu v - \mu r)}{A(\mu - 1)}$

$$\boxed{\omega = \frac{(\mu v - \mu r)}{(\mu - 1)}}$$

Here, $\mu = \dfrac{\mu r + \mu v}{2}$ is the refractive index of mean colour of light.

Aberration in Lenses:

Date..........
Page..........

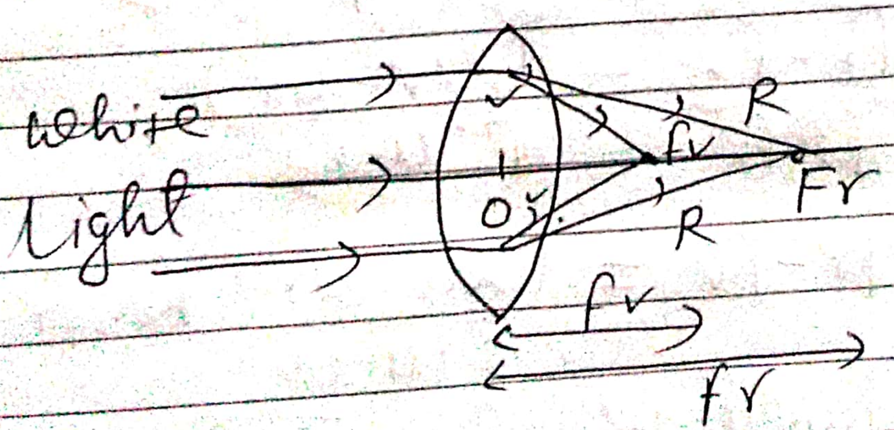
Chromatic aberration | Spherical aberration

Aberration is the deviation in the shape, size and position of the actual image formed compared to the image predicted by lens formula.

→ Chromatic aberration is an inability of a lens to focus all colours of light at a single point on the principle axis of a lens.

It is measured by difference in focal lengths between red and violet colours.

$$\text{Chromatic aberration} = f_r - f_v$$



Date.........
Page.........

Now,
from lens maker's formula

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

or $$\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{1}{f(\mu-1)} \quad \text{--- (1)}$$

For red colour & violet colour

$$\frac{1}{f_r} = (\mu_r-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

or $$\frac{1}{f_r} = \frac{(\mu_r-1)}{f(\mu-1)} \quad \text{--- (2)} \quad (\because \text{from (1)})$$

& $$\frac{1}{f_v} = \frac{(\mu_v-1)}{f(\mu-1)} \quad \text{--- (3)}$$

Now,

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{(\mu_v-1)-(\mu_r-1)}{f(\mu-1)}$$

or, $$\frac{f_r-f_v}{f_v \cdot f_r} = \frac{(\mu_v-\mu_r)}{f(\mu-1)}$$

or, $$f_r-f_v = \frac{(\mu_v-\mu_r)}{f(\mu-1)} f_v \cdot f_r$$

mean focal length $f = \sqrt{f_v \cdot f_r}$

$$f^2 = f_v \cdot f_r$$

Scanned with CamScanner

Date.........................

Page.........................

$$\therefore f_r - f_v = \frac{(\mu_v - \mu_r)}{f(\mu - 1)} f^2$$

$$\therefore \boxed{f_r - f_v = \frac{(\mu_v - \mu_r)}{(\mu - 1)} \cdot f = \omega f}$$

Numerical.

Date........
Page........

1. Given, $\mu_r = 1.54$
$\mu_v = 1.66$
$A = 5^\circ$
Angular dispersion,
$\delta = A(\mu_v - \mu_r) = ?$
find out.

2. Given, for

crocon glass
$\mu'_v = 1.523$
$\mu'_r = 1.513$

flint glass
$\mu_v = 1.773$
$\mu_r = 1.743$

Dispersive power = ? $\omega = \frac{\mu_v - \mu_r}{(\mu - 1)}$

find μ' & μ' for crown & flint glass by using formula,

$\mu' = \frac{\mu'_v + \mu'_r}{2} = \frac{1.523 + 1.513}{2}$

& $\mu = \frac{\mu_v + \mu_r}{2} = \frac{1.773 + 1.743}{2}$

Then
Calculate, ω for each glass.

3. Solve as in Q. 2.

Scanned with CamScanner

Date........
Page........

4. Given,

$A_c = 6^\circ$

$A_f = ?$

For crownglass

$\mu_r = 1.520$

$\mu_b = 1.531$

For flint glass

$\mu_r^f = 1.662$

$\mu_b^f = 1.684$

For achromatic combination,

$$(\mu_b^c - \mu_r^c) A_c = (\mu_b^f - \mu_r^f) A_f$$

$$\text{or, } (1.531 - 1.520) \times 6 = (1.684 - 1.662) A_f$$

or, $A_f = ?$ findout

5. Given,

Dispersive power $(\omega) = 0.026$

Chromatic aberration $(f_r - f_v) = 0.351$ cm

mean focal length $(f) = ?$

Now, Chromatic aberration $= \omega f$

$$\text{or, } f = \frac{0.351}{0.026}$$

$$\text{or } f = 13.5 \text{ cm}$$

6. Given, $R_1 = 10cm$, $R_2 = 20cm$.

$\mu_v = 1.523$

$\mu_r = 1.515$

$\Rightarrow \mu = \frac{\mu_v + \mu_r}{2}$

$= \frac{1.523 + 1.515}{2}$

$=$

Chromatic aberration = ?

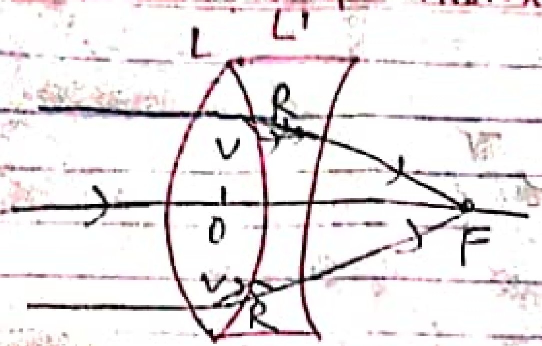
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{20}\right)$$

Then, find $\omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)}$

find chromatic aberration $= \omega \times f = ?$

Removal of chromatic Aberration in lens :-



For L, $\frac{1}{f_r} = \frac{(\mu_r - 1)}{f(\mu - 1)}$ for L' $\frac{1}{f_r'} = \frac{(\mu_r' - 1)}{f'(\mu' - 1)}$

Combined focal length of L & L'

for red colour, $\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f_r'} = \frac{(\mu_r - 1)}{f(\mu - 1)} + \frac{(\mu_r' - 1)}{f'(\mu' - 1)}$

for violet colour $\frac{1}{F_v} = \frac{(\mu_v - 1)}{f(\mu - 1)} + \frac{(\mu_v' - 1)}{f'(\mu' - 1)}$

for achromatic combination of lenses

$F_r = F_v$

$\frac{1}{F_r} = \frac{1}{F_v}$

or, $\frac{(\mu_r - 1)}{f(\mu - 1)} + \frac{(\mu_r' - 1)}{f'(\mu' - 1)} = \frac{(\mu_v - 1)}{f(\mu - 1)} + \frac{(\mu_v' - 1)}{f'(\mu' - 1)}$

or, $\frac{(\mu_r - \mu_v)}{f(\mu - 1)} = \frac{(\mu_v' - \mu_r')}{f'(\mu' - 1)}$

or, $\frac{-\omega}{f} = \frac{\omega'}{f'}$ $\left(\because \omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)} \right)$

or, $\frac{\omega}{\omega'} + \frac{f}{f'} = 0$

or $\left[\frac{\omega}{f} + \frac{\omega'}{f'} = 0 \right]$

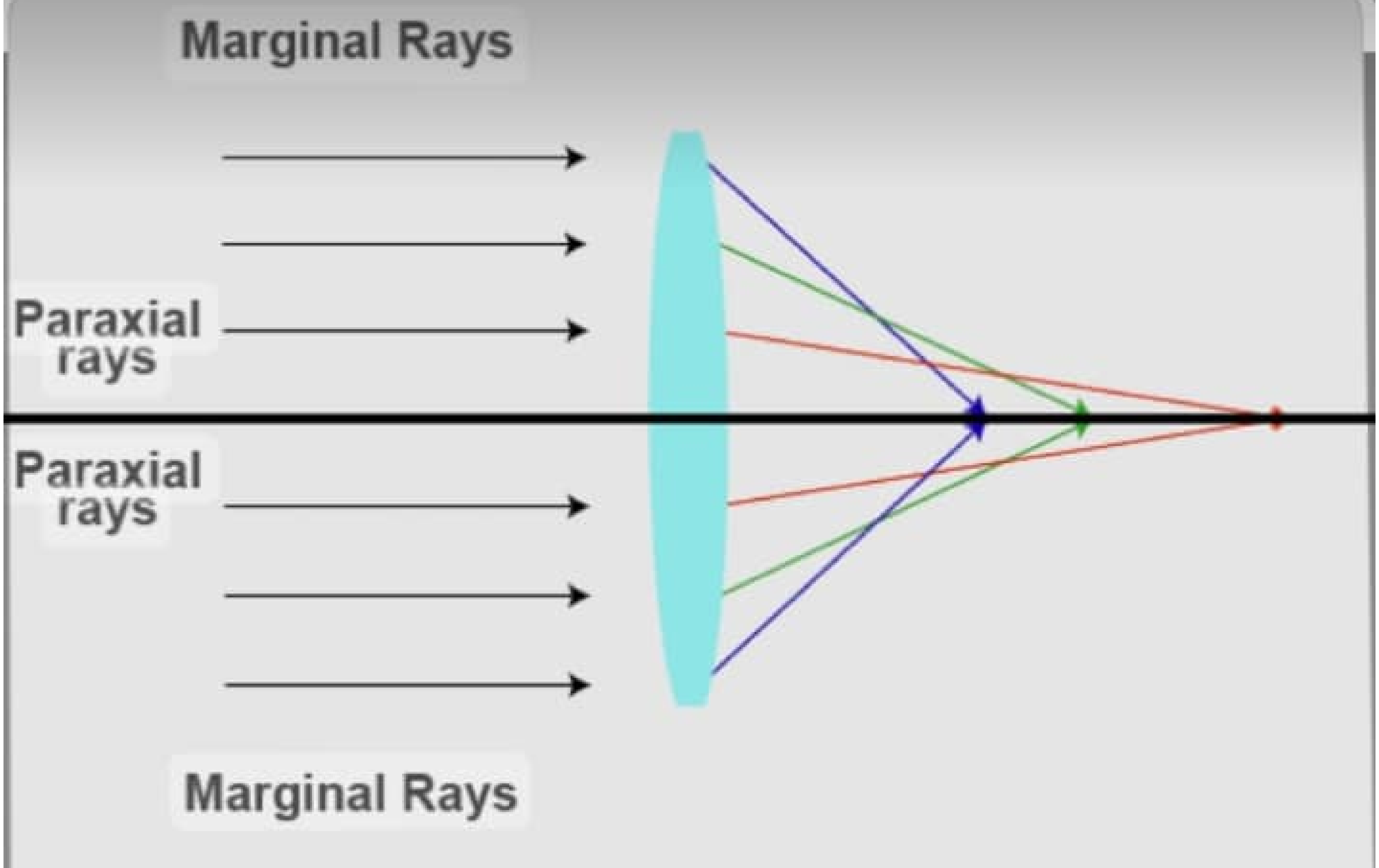
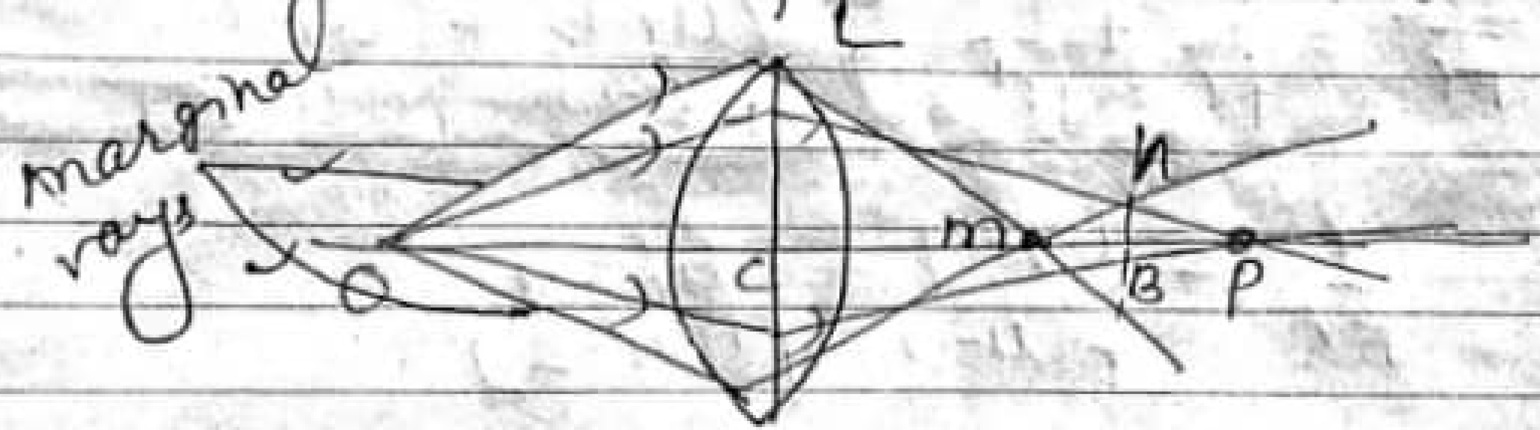


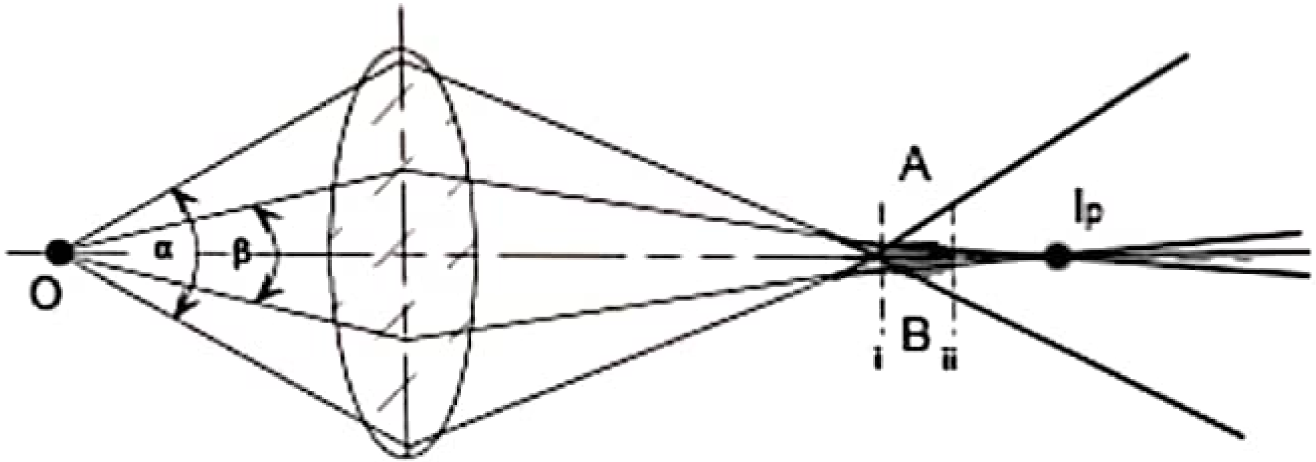
Figure 1 – Spherical aberration diagram showing light near the edge of a lens focuses nearer to the lens

Spherical Aberration in lens:

The inability of a lens to focus the paraxial rays and marginal rays at the same point on the principle axis is called spherical abberration.

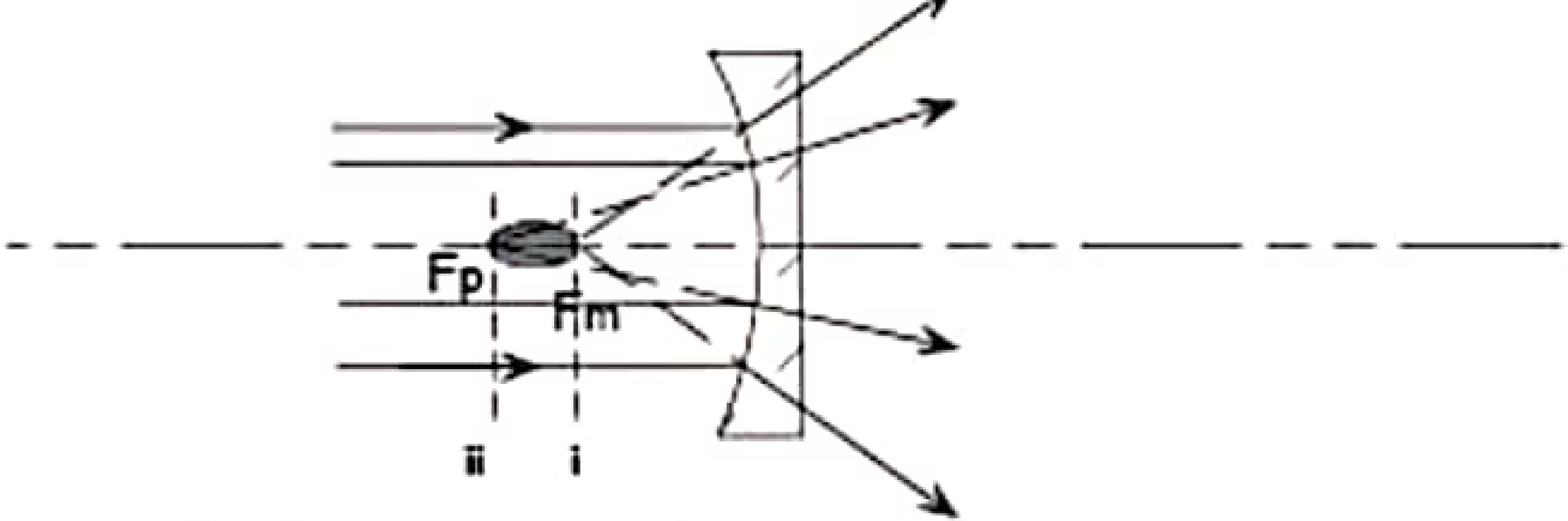


Scanned with CamScanner



Spherical aberration in convex Lens

The spherical aberration produced by a concave lens is shown in the figure. It is positive for the convex lens and negative for the concave lens, both paraxial and marginal rays cannot focus at a single point. The rays incident on the lens near the principal axis are called paraxial rays and the rays falling near the edge are called marginal rays.



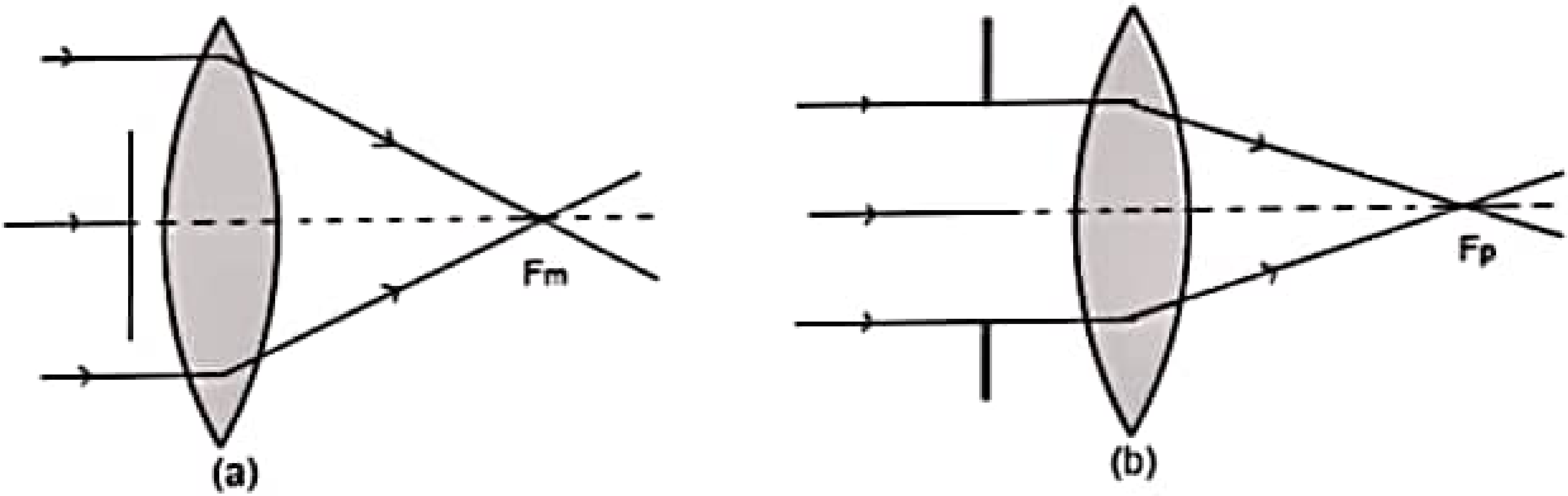
Spherical aberration in concave Lens

Removal of Spherical Aberration in lenses

For a single lens, spherical aberration cannot be entirely eliminated. However, it can be reduced by following methods:

1. By using stop

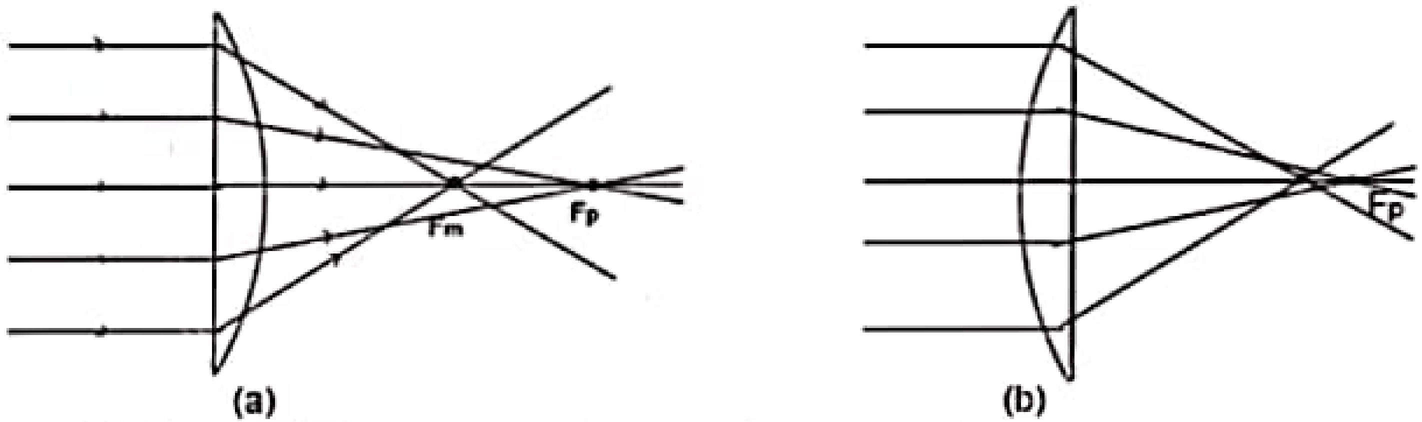
Spherical aberration can be reduced either by cutting off the paraxial rays or by cutting off the marginal rays.



Reduction of spherical aberration by using stops in different positions.

2. By using plano-convex lenses

If parallel rays of light incident on the plane surface of the plano-convex lens, the spherical aberration will be maximum because incident rays entire deviation at the convex surface. Similarly if parallel rays of light incident on the convex surface, spherical aberration will be minimum.



Reduction of spherical aberration by using plano - convex lens in different positions.

A telescope objective receives parallel rays of light from distant object. To reduce spherical aberration, the convex surface of the plano-convex lens is always towards the distant object. In a microscope objective, the rays fall on it from a very near point object and hence the incident rays are bound to be much oblique than the emergent rays. If the convex surface is towards the object, spherical aberration will be maximum and will be minimum if the plane surface faces the object.

2. By using Plano-convex lenses:

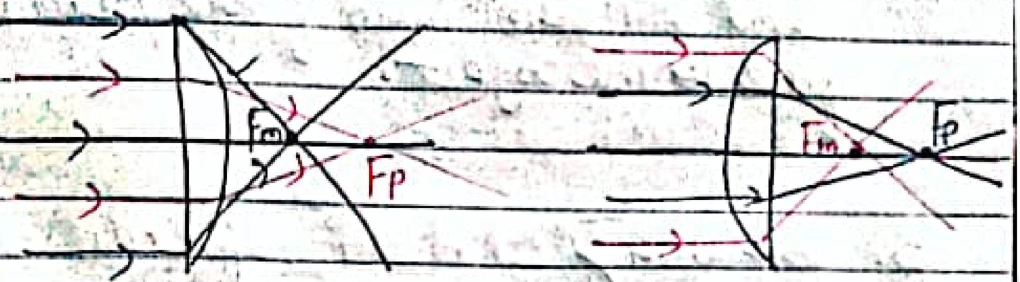


Fig (a) Fig (b)

If the parallel rays of light is incident on the plane surface of the lens, the spherical aberration will be maximum, because the incident light suffers entire deviation at convex surface.

But, if parallel rays of light is incident on the convex surface, spherical aberration will be minimum.

For example: In Telescope objective lens receives parallel light rays from distance, to reduce the spherical aberration, the convex surface of the plano convex lens

Date.........
Page.........

is kept towards the distant object.
But, in
a microscope, if the convex surface is placed towards object, the spherical aberration will be maximum. and will be minimum if the plane surface face to the object.

3. By using combination of convex & concave lenses:

As the focal length of marginal ray fm lies to the left of paraxial fp rays in convex lens, while in a concave lens the marginal rays focus fm lies on the right of fp. Hence by using combination of these two lenses spherical aberration can be removed.