Dynamic Auctions in Cloud Using Bank Account Mechanisms

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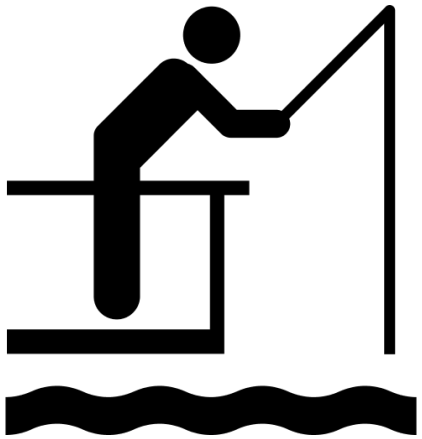
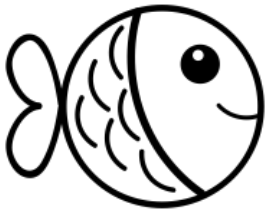
Goal Statement

- Implementation of research paper “Dynamic Auctions with Bank” Accounts by Vahab Mirrokni et al.
- Compare the results of the paper to that of our implementation.
- Proving this type of Dynamic Auction is better than the static Myerson Auctions.

Auction Goal: maximize revenue

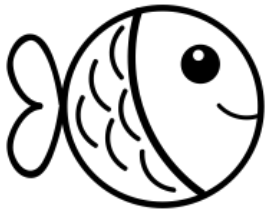




information	$v \sim F$	private value $v = 1$
actions	allocate $x(\hat{v})$ charge $p(\hat{v})$	my value is $\hat{v}$
Requirements:	IC: $v = \operatorname{argmax}_{\hat{v}} x(\hat{v}) \cdot v - p(\hat{v})$ IR: $x(v) \cdot v - p(v) \geq 0$	

Dynamic Setting

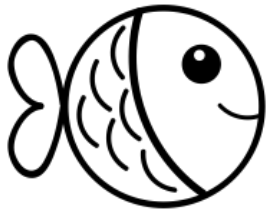
Day 1 | Day 2 | Day 3 | Day 4



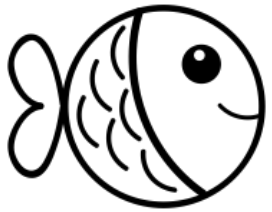
v~U[0, 2] | v~U[0, 4] | v~U[0, 6] | v~U[0, 4]

Same type, different items

Dynamic Auction (Clairvoyant)

	Day 1	Day 2	Day 3	Day 4
				
	$v \sim U[0,2]$	$v \sim U[0,4]$	$v \sim U[0,6]$	$v \sim U[0,4]$
private	v_1	v_2	v_3	v_4
report	$\hat{v}_1$	$\hat{v}_2$	$\hat{v}_3$	$\hat{v}_4$
outcome	$x_1(\hat{v}_1)$ $p_1(\hat{v}_1)$	$x_2(\hat{v}_1, \hat{v}_2)$ $p_2(\hat{v}_1, \hat{v}_2)$	$x_3(\hat{v}_1, \hat{v}_2, \hat{v}_3)$ $p_3(\hat{v}_1, \hat{v}_2, \hat{v}_3)$	$x_4(\hat{v}_1, \hat{v}_2, \hat{v}_3, \hat{v}_4)$ $p_4(\hat{v}_1, \hat{v}_2, \hat{v}_3, \hat{v}_4)$

Dynamic Auction (Clairvoyant)

	Day 1	Day 2	Day 3	Day 4
			 	
	$v \sim U[0,2]$	$v \sim U[0,4]$	$v \sim U[0,6]$	$v \sim U[0,4]$
outcome	$x_1(\hat{v}_1)$ $p_1(\hat{v}_1)$	$x_2(\hat{v}_1, \hat{v}_2)$ $p_2(\hat{v}_1, \hat{v}_2)$	$x_3(\hat{v}_1, \hat{v}_2, \hat{v}_3)$ $p_3(\hat{v}_1, \hat{v}_2, \hat{v}_3)$	$x_4(\hat{v}_1, \hat{v}_2, \hat{v}_3, \hat{v}_4)$ $p_4(\hat{v}_1, \hat{v}_2, \hat{v}_3, \hat{v}_4)$

$$utility_t(\hat{v}_{1..t}; v_t) \triangleq x_t(\hat{v}_{1..t}) \cdot v_t - p_t(\hat{v}_{1..t})$$

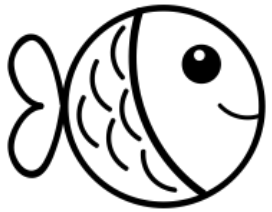
Requirements:

Dynamic IC: $v_t = \operatorname{argmax}_{\hat{v}_t}\left[uility_t(\hat{v}_{1..t}; v_t) + E[future\_utility(\hat{v}_{1..t})]\right]$

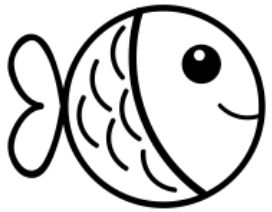
Ex-post IR: $\sum_t utility_t(v_{1..t}; v_t) \geq 0$

Objective: maximize expected revenue = $E[\sum_t p_t(v_{1..t})]$, subject to DIC & ep-IR

Dynamic Auction (Clairvoyant)

	Day 1	Day 2	Day 3	Day 4
				
	$v \sim U[0,2]$	$v \sim U[0,4]$	$v \sim U[0,6]$	$v \sim U[0,4]$
outcome	$x_1(\hat{v}_1)$ $p_1(\hat{v}_1)$	$x_2(\hat{v}_1, \hat{v}_2)$ $p_2(\hat{v}_1, \hat{v}_2)$	$x_3(\hat{v}_1, \hat{v}_2, \hat{v}_3)$ $p_3(\hat{v}_1, \hat{v}_2, \hat{v}_3)$	$x_4(\hat{v}_1, \hat{v}_2, \hat{v}_3, \hat{v}_4)$ $p_4(\hat{v}_1, \hat{v}_2, \hat{v}_3, \hat{v}_4)$
	$x_1(\hat{v}_1; F_{1..4})$ $p_1(\hat{v}_1; F_{1..4})$	$x_2(\hat{v}_{1..2}; F_{1..4})$ $p_2(\hat{v}_{1..2}; F_{1..4})$	$x_3(\hat{v}_{1..3}; F_{1..4})$ $p_3(\hat{v}_{1..3}; F_{1..4})$	$x_4(\hat{v}_{1..4}; F_{1..4})$ $p_4(\hat{v}_{1..4}; F_{1..4})$

Dynamic Auction (Non-clairvoyant)



$v \sim U[0,2]$

	Day 1	Day 2	Day 3	Day 4
private	v_1	v_2	v_3	v_4
report	$\hat{v}_1$	$\hat{v}_2$	$\hat{v}_3$	$\hat{v}_4$
outcome	$x_1(\hat{v}_1; F_1)$ $p_1(\hat{v}_1; F_1)$	$x_2(\hat{v}_{1..2}; F_{1..2})$ $p_2(\hat{v}_{1..2}; F_{1..2})$	$x_3(\hat{v}_{1..3}; F_{1..3})$ $p_3(\hat{v}_{1..3}; F_{1..3})$	$x_4(\hat{v}_{1..4}; F_{1..4})$ $p_4(\hat{v}_{1..4}; F_{1..4})$
clairvoyant outcome	$x_1(\hat{v}_1; F_{1..4})$ $p_1(\hat{v}_1; F_{1..4})$	$x_2(\hat{v}_{1..2}; F_{1..4})$ $p_2(\hat{v}_{1..2}; F_{1..4})$	$x_3(\hat{v}_{1..3}; F_{1..4})$ $p_3(\hat{v}_{1..3}; F_{1..4})$	$x_4(\hat{v}_{1..4}; F_{1..4})$ $p_4(\hat{v}_{1..4}; F_{1..4})$

Results overview

	Simple Format	High Revenue	Optimality	Forecast-free
Static Auctions	**YES**	NO	Arbitrarily large gap	**YES**
Dynamic Auctions	NO	**YES**	**YES**	NO
Bank Account Mechanisms	**YES**	**YES**	**YES**	NO

Dynamic Environment: Model

- Seller has a sequence of items to sell to buyers.
- Items arrive over time.
  - Each period, one item for sale.
  - The item will be destroyed, if not sold.
- Unknown actual value until the t-th period.
  - Stage-wise independent, commonly known priors.
  - Additive valuation.
- Seller's allocation rule and payment rule could depend on past periods.

Dynamic Environment: Applica

- Google sells ad impressions to advertisers.
- Impressions may come from users' searches on search engines.
  - Arrive over time, destroyed immediately if not sold.
- The value of each impression varies with (at least) the user's information (location, time, age, gender, cookies, etc.).
- Currently, the auctions are rarely conducted dynamically.

Non-clairvoyance: importance

1. In practice, the seller (Google) cannot predict the type of the next item (impression).

2. More importantly, even if the seller could predict the types, the buyers might not trust the seller's prediction. (Common knowledge assumption violated)

Bank Account Mechanism

- Maintains a state variable b_t for each buyer
  - Instead of keeping all bidding histories
- The outcome for each item only depends on the bids on this item and the bank account balances
  - Easy to implement

Every dynamic auction is "isomorphic" to a bank account mechanism.

Bank Account Mechanism

- Keeps a state variable b_t for each buyer
- Based on the balance:
  - Charges a upfront fee $s_t \leq b_t$
  - Chooses a per-period IC-IR mechanism $\langle x_t(\hat{v}_t, b_t), p'_t(\hat{v}_t, b_t)\rangle$

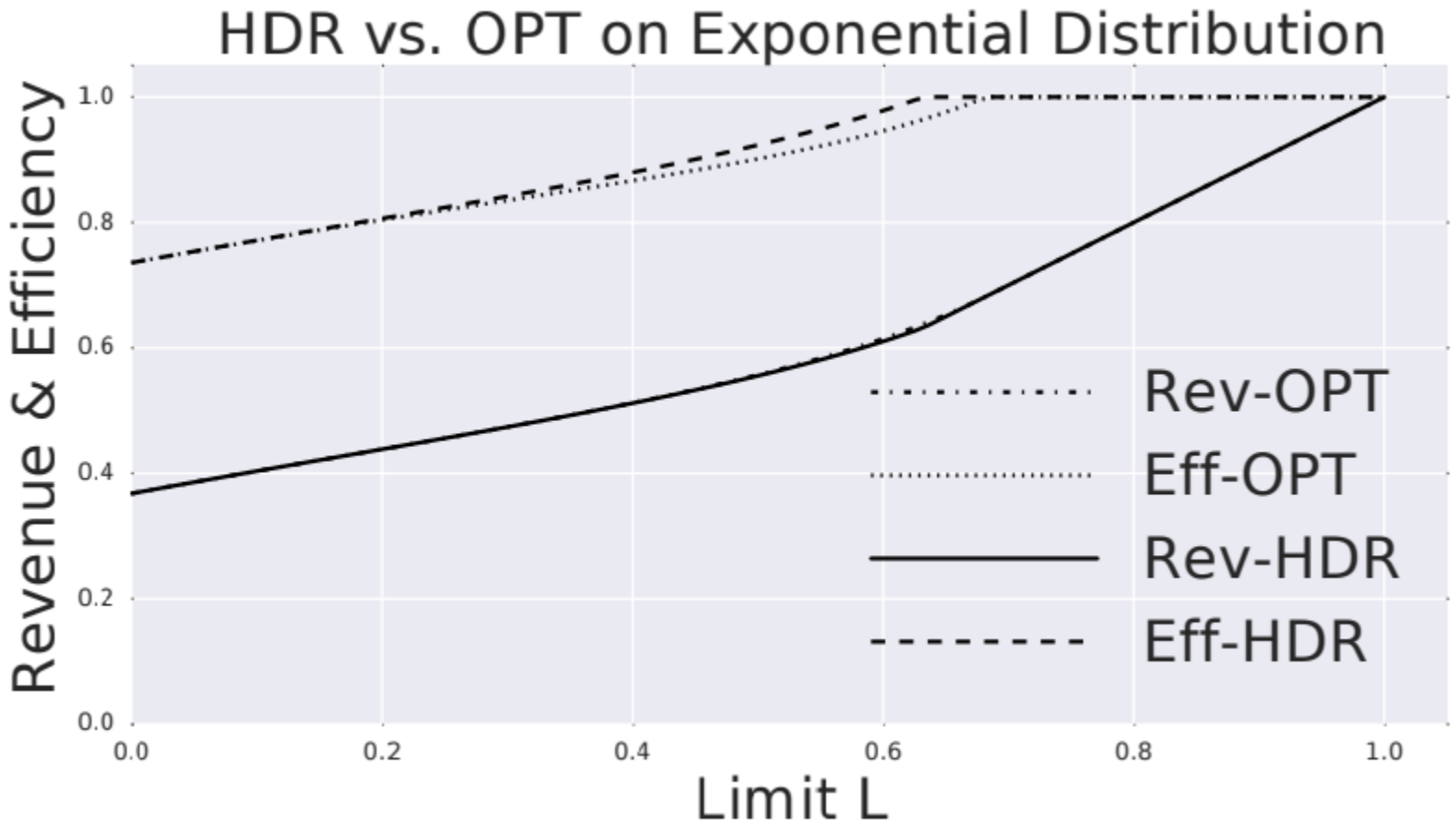
    satisfying the balance-independent property:

$$E_{v_t}[x_t \cdot v_t - (p'_t + s_t)] = const \geq 0$$

- Update balance:

$$0 \leq b_{t+1} \leq b_t + x_t \cdot v_t - (p'_t + s_t)$$

Results


HDR vs. OPT on Exponential Distribution
Revenue & Efficiency
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Limit L
Rev-OPT
Eff-OPT
Rev-HDR
Eff-HDR

Conclusion

- Dynamic auctions gives better revenue as compared to static auctions
- Bank Account Mechanism is simplest way to implement Dynamic Auctions and generate optimal revenue.

Thanks!!