MGM's College of Engineering and Technology

Kamothe, Navi Mumbai

Approved by AICTE , Recognized by Govt. of Maharashtra & Affiliated to University of Mumbai

MINI-PROJECT:

INVERSE LAPLACE TRANSFORM

SECOND YEAR ENGINEERING(SEMIII)

ENGINEERING MATHEMATICS-III

SUBMITTED BY:

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>TOPIC TO BE COVERED

1. Linearity Property
2. First Shifting Theorem (Time Shifting Property)
3. Multiplication by power of t
4. Second Shifting Theorem (Frequency Shifting Property)
5. Convolution Theorem
6. Method of Convolution Corrollary

First property

$L^{-1}(kf(t)) = kL^{-1}(f(t))$, where k is a constant.

2^{nd} **property - Linearity property**

If c_1 and c_2 are any two constants while $f_1(s)$ & $f_2(s)$ are the functions with Inverse Laplace transforms $F_1(t)$ & $F_2(t)$ respectively, then

$$\begin{aligned} L^{-1}(c_1 f_1(s) + c_2 f_2(s)) &= c_1 L^{-1}(f_1(s)) + c_2 L^{-1}(f_2(s)) \\ &= c_1 F_1(t) + c_2 F_2(t) \end{aligned}$$

3^{rd} property -Translation or Shifting property

If $L^{-1}(f(s)) = F(t)$ then $L^{-1}(f(s-a)) = e^{at}F(t)$

4^{th} property - Multiplication by power of t

If $L^{-1}(f(s)) = F(t)$, and $F(0) = F'(0) = \ldots F^(n-1)(0) = 0$
then $L^{-1}\left(\frac{d^n}{ds^n}f(s)\right) = (-1)^n t^n F(t)$, where $n = 1, 2, 3...$

Convolution Property

If $F(t)$ and$G(t)$ are the inverse transforms of $f(s)$ and $g(s)$, respectively, the inverse transform of the product $f(s)g(s)$ is the convolution of $F(t)$ and $G(t)$, written $(F * G)(t)$ and defined by

$$(F * G)(t) = \int_0^1 F(t-u)\ G(u)\ du$$

i.e.,

$$L^{-1}(f(S)g(s)) = (F * G)(t) = \int_0^1 F(t-u)\ G(u)\ du$$

Corrollary

Putting $t - u = v$ in the above integral, we obtain,

$$
\begin{aligned}
(F * G)(t) &= -\int_t^0 F(v)\ G(t-v)\ du \\
&= \int_0^t G(t-v)F(v)\ du \\
&= (G * F)(t)
\end{aligned}
$$

STANDARD FORMULAE

L(1) = $\left(\frac{1}{s}\right)$	$L^{-1}\left(\frac{1}{s}\right)$ **= 1**	**L(Sin at)=$\frac{a}{s^2+a^2}$**	$L^{-1}\left(\frac{1}{s^2+a^2}\right)$ = $\frac{1}{a}$**Sin at**
L(e^{-at}) =$\frac{1}{s+a}$	$L^{-1}\left(\frac{1}{s+a}\right)$ = e^{-at}	L(Cos at)= $\frac{s}{s^2+a^2}$	$L^{-1}\left(\frac{s}{s^2+a^2}\right)$ = **Cos at**
L(e^{at}) =$\frac{1}{s-a}$	$L^{-1}\left(\frac{1}{s-a}\right)$ = e^{at}	L(Sin h at)= $\frac{a}{s^2-a^2}$	$L^{-1}\left(\frac{1}{s^2-a^2}\right)$ = $\frac{1}{a}$**Sin h at**
L(t^{n-1}) $= \frac{(n-1)!}{s^n}$	$L^{-1}\left(\frac{1}{s^n}\right)$ = $\frac{t^{n-1}}{(n-1)!}$	L(Cos h at = $\frac{s}{s^2-a^2}$	$L^{-1}\left(\frac{s}{s^2-a^2}\right)$ = **Cos h at**
L(t^{n-1}) $= \frac{\Gamma n}{s^n}$	$L^{-1}\left(\frac{1}{s^n}\right)$ = $\frac{t^{n-1}}{\Gamma n}$		

PROBLEMS:-

INVERSE LAPLACE TRANSFORMS BY USING THE STANDARD FORMULAE

Q.1. Find $L^{-1}\left(\frac{1-\sqrt{s}}{s^2}\right)^2$

$$L^{-1}\left(\frac{1-\sqrt{s}}{s^2}\right)^2 = L^{-1}\{\frac{1-2\sqrt{s}+s}{s^4}\}$$

$$= L^{-1}(\frac{1}{s^4}) \; - 2L^{-1}(\frac{1}{s^{\frac{7}{2}}}) + L^{-1}(\frac{1}{s^3})$$

$$= \frac{t^3}{3!} - 2\,\frac{t^{5/2}}{\Gamma 7/2} + \frac{t^2}{2!}$$

$$= \frac{t^3}{6} - \frac{16}{15\sqrt{\pi}}\,t^{5/2} + \frac{t^2}{2}$$ Ans.

Q.2. Find $L^{-1}\left[\frac{3s+4}{s^2+16}\right]$

$$L^{-1}\left[\frac{3s+4}{s^2+16}\right] = L^{-1}\left[\frac{3s}{s^2+16}\right] + L^{-1}\left[\frac{4}{s^2+16}\right] \quad = 3L^{-1}\left[\frac{s}{s^2+4^2}\right] + 4\,L^{-1}\left[\frac{1}{s^2+4^2}\right]$$

$$= 3\cos 4t + 4 \text{x} \frac{1}{4}\sin 4t$$

$$= 3\cos 4t + \sin 4t$$ Ans.

Q.3. Find $L^{-1}\left[\frac{s}{(s-2)^6}\right]$

$$L^{-1}\left[\frac{s}{(s-2)^6}\right] = L^{-1}\left[\frac{(s-2)+2}{(s-2)^6}\right] = L^{-1}\left[\frac{s-2}{(s-2)^6}\right] + L^{-1}\left[\frac{2}{(s-2)^6}\right]$$

$$= L^{-1}\left[\frac{1}{(s-2)^5}\right] + 2\,L^{-1}\left[\frac{1}{(s-2)^6}\right] = e^{2t}L^{-1}\left[\frac{1}{s^5}\right] + 2\,e^{2t}L^{-1}\left[\frac{1}{s^6}\right]$$

$$= e^{2t}\frac{t^4}{4!} + 2e^{2t}\frac{t^5}{5!} = e^{2t}\frac{t^4}{24} + e^{2t}\frac{t^5}{60}$$ Ans.

[redacted] **Find $L^{-1}\{\frac{s+1}{s^2-4}\}$**

$\frac{s+1}{s^2-4}=\frac{s+1}{(s+2)(s-2)}=\frac{A}{s+2}+\frac{B}{s-2}$

S+1 = A(s-2) +B(s+2)

Let s=2, 3= Bx4 $\Longrightarrow B=\frac{3}{4}$

Let s=-2, -1= Ax-4 $\Longrightarrow A=\frac{1}{4}$

$L^{-1}\frac{s+1}{s^2-4}=L^{-1}\frac{1/4}{s+2}+L^{-1}\frac{3/4}{s-2}=\frac{1}{4}L^{-1}\frac{1}{s+2}+\frac{3}{4}L^{-1}\frac{1}{s-2}$

$=\frac{1}{4}e^{-2t}+\frac{3}{4}e^{2t}=\frac{1}{4}(e^{-2t}+3e^{2t})$ Ans.

Find $L^{-1}\,\frac{3s+1}{(s+1)(s^2+2)}$

$$\frac{3s+1}{(s+1)(s^2+2)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+2}$$

3s+1 = A$(s^2+2)+(Bs+C)(s+1)$

Let s=-1, -2= 3A$\Longrightarrow A=-2/3$

Let s= 0, 1= 2A +C $\Longrightarrow C = 1-2A \Longrightarrow C = 7/3$

Let s=1, 4 = 3A+(B+C)2$\Longrightarrow B = 2/3$ (substitute the values of A & C)

$$L^{-1}\,\frac{3s+1}{(s+1)(s^2+2)} = L^{-1}\,\frac{-(\frac{2}{3})}{s+1} + L^{-1}\,\frac{(\frac{2}{3})s+(\frac{7}{3})}{s^2+2}$$

$L^{-1}\,F(s) = \frac{-1}{t} L^{-1}[F'(s)]$ & $L^{-1}[F'(s)] = -t\,L^{-1}[F(s)]$

Find $L^{-1}\log\{\frac{s+a}{s+b}\}$

$L^{-1}\log\{\frac{s+a}{s+b}\} = \frac{-1}{t} L^{-1}\{\frac{d}{ds}\left[\log\left(\frac{s+a}{s+b}\right)\right]\}$

$= \frac{-1}{t} L^{-1}\{\frac{d}{ds}[\log(s+a) - \log(s+b)]\}$

$= \frac{-1}{t} L^{-1}\{\frac{1}{s+a} - \frac{1}{s+b}\}$

$= \frac{-1}{t}\{e^{-at} - e^{-bt}\}$ Ans.

Find $L^{-1}[2\tanh^{-1} s]$

$$L^{-1}[2\tanh^{-1} s]= L^{-1}\{2 * \tfrac{1}{2}\log[\tfrac{1+s}{1-s}]\} =L^{-1}\{\log[\tfrac{1+s}{1-s}]\}$$

$$=\frac{-1}{t}L^{-1}\left\{\frac{d}{ds}\log\left(\frac{1+s}{1-s}\right)\right\}$$

$$=\frac{-1}{t}L^{-1}\left\{\frac{d}{ds}[\log(1+s)-\log(1-s)]\right\} = \frac{-1}{t}L^{-1}\{\frac{1}{1+s}-\frac{1*(-1)}{(1-s)}\}$$

$$=\frac{-1}{t}L^{-1}\{\frac{1}{(s+1)}-\frac{1}{(s-1)}\}$$

$$=\frac{-1}{t}\{e^{-t}-e^{t}\}$$

$$=\frac{2}{t}\{\frac{e^{t}-e^{-t}}{2}\}=\frac{2}{t}\sinh t$$ Ans.

INVERSE LAPLACE TRANSFORMS BY USING INTEGRATION OF F(s)

- $L^{-1}\frac{1}{s}F(s) = \int_0^t L^{-1}F(s)du$
- --

Find $L^{-1}\frac{1}{s^2(s+1)}$

$L^{-1}\frac{1}{s^2(s+1)}$=$L^{-1}\frac{1}{s}(\frac{1}{s\,(s+1)})$

Consider L^{-1} $(\frac{1}{s\,(s+1)})$. Here F(s) = $(\frac{1}{(s+1)})$

$L^{-1}\frac{1}{s}(\frac{1}{(s+1)})$= $\int_0^t L^{-1}[\frac{1}{s+1}]$du =$\int_0^t e^{-u}du = \{-e^{-u}\}_0^t$= $1 - e^{-t}$ -------(1)

$L^{-1}\frac{1}{s}(\frac{1}{s\,(s+1)})$=$\int_0^t L^{-1}[\frac{1}{s(s+1)}]$du=$\int_0^t[1 - e^{-u}]du$ {Using (!), The variables t & u can be interchanged}

= $\{u - (-e^{-u})\}_0^t$

= $\{t + e^{-t} - 1\}$ Ans.

Find $L^{-1}\frac{1}{s(s+1)}$

Let $F_1(s)=\frac{1}{s+1}$ & $F_2(s)=\frac{1}{s}$

$L^{-1}F_1(s)= e^{-t} = f_1(t)$ & $L^{-1}F_2(s)= 1=f_2(t)$

By convolution theorem, $L^{-1}\{\frac{1}{(s+1)}\ \frac{1}{s}\}= \int_0^t e^{-u} * 1\, du$

$= \left[\frac{e^{-u}}{-1}\right]_0^t = \{-[e^{-t} - 1]\}= (1 - e^{-t})$ Ans.

Find $L^{-1}[\frac{1}{s(s^2+4)}]$

Let $F_1(s)= \frac{1}{s^2+4}$ & $F_2(s)=\frac{1}{s}$

$L^{-1}F_1(s)=\frac{1}{2}Sin\ 2t\ = f_1(t)$ & $L^{-1}F_2(s)= 1= f_2(t)$

By convolution theorem , $L^{-1}[\frac{1}{s(s^2+4)}] = \frac{1}{2}\int_0^t Sin\ 2u\ * 1\ du$

$$= \frac{1}{2}(\frac{-Cos\ 2u}{2})_0^t = \frac{-1}{4}(Cos\ 2t - 1)$$

$$= \frac{1}{4}(1 - Cos\ 2t)$$ Ans.