MGM's College of Engineering and Technology

Kamothe, Navi Mumbai

Approved by AICTE , Recognized by Govt. of Maharashtra & Affiliated to University of Mumbai

MINI-PROJECT:

INVERSE LAPLACE TRANSFORM

SECOND YEAR ENGINEERING(SEMIII)
ENGINEERING MATHEMATICS-III

SUBMITTED BY:

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>TOPIC TO BE COVERED

1. Linearity Property
2. First Shifting Theorem (Time Shifting Property)
3. Multiplication by power of t
4. Second Shifting Theorem (Frequency Shifting Property)
5. Convolution Theorem
6. Method of Convolution Corrollary

First property

$L^{-1}(kf(t)) = kL^{-1}(f(t))$, where k is a constant.

2^{nd} **property - Linearity property**

If c_1 and c_2 are any two constants while $f_1(s)$ & $f_2(s)$ are the functions with Inverse Laplace transforms $F_1(t)$ & $F_2(t)$ respectively, then

$$\begin{aligned} L^{-1}(c_1 f_1(s) + c_2 f_2(s)) &= c_1 L^{-1}(f_1(s)) + c_2 L^{-1}(f_2(s)) \\ &= c_1 F_1(t) + c_2 F_2(t) \end{aligned}$$

3^{rd} property -Translation or Shifting property

If $L^{-1}(f(s)) = F(t)$ then $L^{-1}(f(s-a)) = e^{at}F(t)$

4^{th} property - Multiplication by power of t

If $L^{-1}(f(s)) = F(t)$, and $F(0) = F'(0) = \ldots F^(n-1)(0) = 0$
then $L^{-1}\left(\frac{d^n}{ds^n}f(s)\right) = (-1)^n t^n F(t)$, where $n = 1, 2, 3...$

Convolution Property

If $F(t)$ and$G(t)$ are the inverse transforms of $f(s)$ and $g(s)$, respectively, the inverse transform of the product $f(s)g(s)$ is the convolution of $F(t)$ and $G(t)$, written $(F * G)(t)$ and defined by

$$(F * G)(t) = \int_0^1 F(t-u)\ G(u)\ du$$

i.e.,

$$L^{-1}(f(S)g(s)) = (F * G)(t) = \int_0^1 F(t-u)\ G(u)\ du$$

Corrollary

Putting $t - u = v$ in the above integral, we obtain,

$$\begin{aligned}(F * G)(t) &= -\int_t^0 F(v)\ G(t-v)\ du \\ &= \int_0^t G(t-v)F(v)\ du \\ &= (G * F)(t)\end{aligned}$$

STANDARD FORMULAE

$L(1) = (\frac{1}{s})$	$L^{-1}(\frac{1}{s}) = 1$	$L(\text{Sin } at) = \frac{a}{s^2+a^2}$	$L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{1}{a}\text{Sin } at$
$L(e^{-at}) = \frac{1}{s+a}$	$L^{-1}(\frac{1}{s+a}) = e^{-at}$	$L(\text{Cos } at) = \frac{s}{s^2+a^2}$	$L^{-1}\left(\frac{s}{s^2+a^2}\right) = \text{Cos } at$
$L(e^{at}) = \frac{1}{s-a}$	$L^{-1}(\frac{1}{s-a}) = e^{at}$	$L(\text{Sin h } at) = \frac{a}{s^2-a^2}$	$L^{-1}\left(\frac{1}{s^2-a^2}\right) = \frac{1}{a}\text{Sin h } at$
$L(t^{n-1}) = \frac{(n-1)!}{s^n}$	$L^{-1}(\frac{1}{s^n}) = \frac{t^{n-1}}{(n-1)!}$	$L(\text{Cos h } at = \frac{s}{s^2-a^2}$	$L^{-1}\left(\frac{s}{s^2-a^2}\right) = \text{Cos h } at$
$L(t^{n-1}) = \frac{\Gamma n}{s^n}$	$L^{-1}(\frac{1}{s^n}) = \frac{t^{n-1}}{\Gamma n}$		

PROBLEMS:-

INVERSE LAPLACE TRANSFORMS BY USING THE STANDARD FORMULAE

Q.1. Find $L^{-1}\left(\frac{1-\sqrt{s}}{s^2}\right)^2$

$$L^{-1}\left(\frac{1-\sqrt{s}}{s^2}\right)^2 = L^{-1}\{\frac{1-2\sqrt{s}+s}{s^4}\}$$

$$= L^{-1}(\frac{1}{s^4}) \;-\; 2L^{-1}(\frac{1}{s^{\frac{7}{2}}}) + L^{-1}(\frac{1}{s^3})$$

$$= \frac{t^3}{3!} - 2\,\frac{t^{5/2}}{\Gamma 7/2} + \frac{t^2}{2!}$$

$$= \frac{t^3}{6} - \frac{16}{15\sqrt{\pi}}\,t^{5/2} + \frac{t^2}{2}$$ Ans.

Q.2. Find $L^{-1}\left[\frac{3s+4}{s^2+16}\right]$

$$L^{-1}\left[\frac{3s+4}{s^2+16}\right]= L^{-1}\left[\frac{3s}{s^2+16}\right]+L^{-1}\left[\frac{4}{s^2+16}\right] \quad =3L^{-1}\left[\frac{s}{s^2+4^2}\right]+4\,L^{-1}\left[\frac{1}{s^2+4^2}\right]$$

$$= 3\cos 4t+ 4\text{x}\frac{1}{4}\sin 4t$$

$$= 3\cos 4t+ \sin 4t$$ Ans.

Q.3. Find $L^{-1}\left[\frac{s}{(s-2)^6}\right]$

$$L^{-1}\left[\frac{s}{(s-2)^6}\right]= L^{-1}\left[\frac{(s-2)+2}{(s-2)^6}\right]=L^{-1}\left[\frac{s-2}{(s-2)^6}\right]+L^{-1}\left[\frac{2}{(s-2)^6}\right]$$

$$=L^{-1}\left[\frac{1}{(s-2)^5}\right]+2\,L^{-1}\left[\frac{1}{(s-2)^6}\right]= e^{2t}L^{-1}\left[\frac{1}{s^5}\right]+2\,e^{2t}L^{-1}\left[\frac{1}{s^6}\right]$$

$$= e^{2t}\frac{t^4}{4!} + 2e^{2t}\frac{t^5}{5!} = e^{2t}\frac{t^4}{24} + e^{2t}\frac{t^5}{60}$$ Ans.

[redacted] **Find $L^{-1}\{\frac{s+1}{s^2-4}\}$**

$\frac{s+1}{s^2-4}=\frac{s+1}{(s+2)(s-2)} = \frac{A}{s+2}+\frac{B}{s-2}$

S+1 = A(s-2) +B(s+2)

Let s=2, 3= Bx4 $\Longrightarrow B=\frac{3}{4}$

Let s=-2, -1= Ax-4 $\Longrightarrow A=\frac{1}{4}$

$L^{-1}\frac{s+1}{s^2-4}=L^{-1}\frac{1/4}{s+2}+L^{-1}\frac{3/4}{s-2}=\frac{1}{4}L^{-1}\frac{1}{s+2}+\frac{3}{4}L^{-1}\frac{1}{s-2}$

$=\frac{1}{4}e^{-2t}+\frac{3}{4}e^{2t}=\frac{1}{4}(e^{-2t}+3e^{2t})$ Ans.

Find $L^{-1}\,\frac{3s+1}{(s+1)(s^2+2)}$

$$\frac{3s+1}{(s+1)(s^2+2)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+2}$$

3s+1 = A$(s^2+2)+(Bs+C)(s+1)$

Let s=-1, -2= 3A$\Longrightarrow A = -2/3$

Let s= 0, 1= 2A +C $\Longrightarrow C = 1-2A \Longrightarrow C = 7/3$

Let s=1, 4 = 3A+(B+C)2$\Longrightarrow B = 2/3$ (substitute the values of A & C)

$$L^{-1}\,\frac{3s+1}{(s+1)(s^2+2)} = L^{-1}\,\frac{-(\frac{2}{3})}{s+1} + L^{-1}\,\frac{(\frac{2}{3})s+(\frac{7}{3})}{s^2+2}$$

$L^{-1}\,F(s) = \frac{-1}{t} L^{-1}[F'(s)]$ & $L^{-1}[F'(s)] = -t\,L^{-1}[F(s)]$

Find $L^{-1}\log\{\frac{s+a}{s+b}\}$

$$L^{-1}\log\{\frac{s+a}{s+b}\} = \frac{-1}{t} L^{-1}\{\frac{d}{ds}\left[\log\left(\frac{s+a}{s+b}\right)\right]\}$$

$$= \frac{-1}{t} L^{-1}\{\frac{d}{ds}[\log(s+a) - \log(s+b)]\}$$

$$= \frac{-1}{t} L^{-1}\{\frac{1}{s+a} - \frac{1}{s+b}\}$$

$$= \frac{-1}{t}\{e^{-at} - e^{-bt}\}$$ Ans.

Find $L^{-1}[2\tanh^{-1} s]$

$$L^{-1}[2\tanh^{-1} s] = L^{-1}\{2 * \tfrac{1}{2}\log[\tfrac{1+s}{1-s}]\} = L^{-1}\{\log[\tfrac{1+s}{1-s}]\}$$

$$= \frac{-1}{t} L^{-1}\left\{\frac{d}{ds}\log\left(\frac{1+s}{1-s}\right)\right\}$$

$$= \frac{-1}{t} L^{-1}\left\{\frac{d}{ds}[\log(1+s) - \log(1-s)]\right\} = \frac{-1}{t} L^{-1}\{\frac{1}{1+s} - \frac{1*(-1)}{(1-s)}\}$$

$$= \frac{-1}{t} L^{-1}\{\frac{1}{(s+1)} - \frac{1}{(s-1)}\}$$

$$= \frac{-1}{t}\{e^{-t} - e^{t}\}$$

$$= \frac{2}{t}\{\frac{e^{t}-e^{-t}}{2}\} = \frac{2}{t}\sinh t$$ Ans.

INVERSE LAPLACE TRANSFORMS BY USING INTEGRATION OF F(s)

- $L^{-1}\frac{1}{s}F(s) = \int_0^t L^{-1}F(s)du$
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Find $L^{-1}\frac{1}{s^2(s+1)}$

$L^{-1}\frac{1}{s^2(s+1)}$=$L^{-1}\frac{1}{s}(\frac{1}{s\,(s+1)})$

Consider $L^{-1}\,(\frac{1}{s\,(s+1)})$. Here F(s) = $(\frac{1}{(s+1)})$

$L^{-1}\frac{1}{s}(\frac{1}{(s+1)})$= $\int_0^t L^{-1}[\frac{1}{s+1}]$du =$\int_0^t e^{-u}du = \{-e^{-u}\}_0^t$= $1 - e^{-t}$ -------(1)

$L^{-1}\frac{1}{s}(\frac{1}{s\,(s+1)})$=$\int_0^t L^{-1}[\frac{1}{s(s+1)}]$du=$\int_0^t[1 - e^{-u}]du$ {Using (!), The variables t & u can be interchanged}

= $\{u - (-e^{-u})\}_0^t$

= $\{t + e^{-t} - 1\}$ Ans.

Find $L^{-1}\frac{1}{s(s+1)}$

Let $F_1(s)=\frac{1}{s+1}$ & $F_2(s)=\frac{1}{s}$

$L^{-1}F_1(s)= e^{-t} = f_1(t)$ & $L^{-1}F_2(s)= 1=f_2(t)$

By convolution theorem, $L^{-1}\{\frac{1}{(s+1)}\ \frac{1}{s}\}= \int_0^t e^{-u} * 1\, du$

$= \left[\frac{e^{-u}}{-1}\right]_0^t = \{-[e^{-t} - 1]\}= (1 - e^{-t})$ Ans.

Find $L^{-1}[\frac{1}{s(s^2+4)}]$

Let $F_1(s)= \frac{1}{s^2+4}$ & $F_2(s)=\frac{1}{s}$

$L^{-1}F_1(s)=\frac{1}{2}Sin\ 2t\ = f_1(t)$ & $L^{-1}F_2(s)= 1= f_2(t)$

By convolution theorem , $L^{-1}[\frac{1}{s(s^2+4)}] = \frac{1}{2}\int_0^t Sin\ 2u\ * 1\ du$

$= \frac{1}{2}(\frac{-Cos\ 2u}{2})_0^t = \frac{-1}{4}(Cos\ 2t - 1)$

$= \frac{1}{4}(1 - Cos\ 2t)$ Ans.

Sample 1 (General Text):
"The quick brown fox jumps over the lazy dog. This sentence contains every letter of the English alphabet. Testing text extraction and conversion features requires diverse content, including punctuation, numbers (1234567890), and special characters (!@#$%^&*()). Let's see how well this works!"

Sample 2 (Longer Paragraph):
"Artificial intelligence is rapidly transforming various industries, from healthcare to finance. Machine learning models analyze vast amounts of data to generate insights, automate tasks, and improve efficiency. Speech-to-text technology enables seamless communication, while text-to-speech tools enhance accessibility. Ensuring accurate extraction of text from documents remains a crucial challenge in modern computing."

Sample 3 (Dialogue):
Alice: "Hey Bob, have you tried the new text extraction tool?"
Bob: "Yes! It works surprisingly well. I extracted text from a complex PDF in seconds!"
Alice: "That's awesome! Can it convert text to speech too?"
Bob: "Absolutely. It even supports multiple languages!"

Sample 4 (Multilingual Text for Translation Tests):

- English: "Hello, how are you?"
- Spanish: "Hola, ¿cómo estás?"
- French: "Bonjour, comment ça va?"
- German: "Hallo, wie geht es dir?"

Sample 5 (Numbers, Dates, and Special Characters):
"Invoice No: 2025-0319
Total Amount: $1,250.75
Due Date: 25th March 2025
Contact: support@example.com
Reference ID: #ABCD1234XYZ"