

## Q/A Assignment:

### Solution 1

In the given scenario where a certain feature, let's call it ' $n$ ', is copied to create a new feature denoted as ' $n + 1$ '. If we then retrain a logistic regression model, the weights assigned to the new features,  $w_{\text{new}_n}$  and  $w_{\text{new}_{n+1}}$ , are likely to be adjusted so that their sum closely matches the original weight assigned to ' $n$ ', which we'll call  $w_n$ .

This adjustment occurs because the logistic regression model distributes the importance originally attributed to feature ' $n$ ' between both the original and duplicated features, ensuring a balanced allocation of significance.

### Solution 2

True statement will be : (option b) "E is better than A with over 95% confidence, B is worse than A with over 95% confidence. You need to run the test for longer to tell where C and D compare to A with 95% confidence."

We have a 95% confidence level indicating that template E surpasses template A, while template B falls short in comparison to template A. However, a more extended test is required to establish a 95% confidence comparison between templates C and D with template A.

Upon examination, the z-scores for templates B, C, and D are all below 1.96, indicating that we cannot assert with 95% confidence that they differ significantly from template A. Conversely, the z-score for template E exceeds 1.96, providing us with 95% confidence that it outperforms template A.

### Solution 3

In logistic regression with sparse feature vectors, the computational cost for a single gradient descent iteration can be outlined as follows, given  $m$  training examples,  $n$  features, and  $k$  average non-zero entries in each sparse vector (where  $k \ll n$ ):

1. **Forward Propagation:** - Cost per example:  $O(k)$  - Total cost for all examples:  $O(m \cdot k)$
2. **Compute Gradient:** - Cost per example:  $O(k)$  - Total cost for all examples:  $O(m \cdot k)$
3. **Update Weights/ Backward Propagation:** - Cost:  $O(k)$

Total computational cost for one iteration:

$$O(m \cdot k) \text{ (prediction)} + O(m \cdot k) \text{ (gradient)} + O(k) \text{ (weight update)}$$

Approximating the dominant term:

$$O(m \cdot k)$$

### Solution 4

These approaches carry implications for the nature of the training data for V2:

a. Opting for stories near the decision boundary can serve as a strategy to enhance the classifier's performance in instances where uncertainty is high. This approach may yield a model more adept at handling ambiguous cases, thereby improving overall accuracy.

b. Randomly selecting labeled stories offers a broader and potentially more diverse training dataset.

While this approach does not specifically target the model's weaknesses, it can contribute to an overall performance boost by encompassing a wide array of examples.

c. Selecting stories where  $V1$ 's predictions are both incorrect and farthest from the decision boundary aims to rectify  $V1$ 's most confidently made errors. This targeted approach has the potential to significantly enhance  $V2$ 's accuracy by focusing on correcting cases where  $V1$  went wrong.

In terms of enhancing the pure accuracy of classifier  $V2$ , one might anticipate the following ranking:

1. Method c: By concentrating on stories where  $V1$  made the most confident errors,  $V2$  can learn from these mistakes, leading to potentially substantial accuracy improvements, assuming these errors are indicative of common mistakes that require correction.

2. Method a: Training on stories close to the decision boundary aids  $V2$  in handling ambiguous cases, which are often challenging for classifiers. Improvements in this area can be highly beneficial; however, this method may not directly address confidently wrong predictions.

3. Method b: While this method can yield a general improvement, it does not specifically target  $V1$ 's weaknesses and may include many 'easy' examples that  $V1$  already handles effectively.

### Solution 5

**(a) Maximum Likelihood Estimate (MLE):** The MLE for  $p$  is obtained by maximizing the likelihood function. The likelihood function is the probability of observing the given sequence of heads and tails, given the parameter  $p$ .

$$\text{Likelihood}(\theta) = \binom{n}{k} p^k (1-p)^{n-k}$$

Taking the logarithm (log-likelihood) and maximizing with respect to  $p$ , we get:

$$\frac{d}{dp}(\log(\text{Likelihood})) = \frac{k}{p} - \frac{n-k}{1-p}$$

Setting the derivative to zero:

$$\frac{k}{\hat{p}_{\text{MLE}}} - \frac{n-k}{1-\hat{p}_{\text{MLE}}} = 0$$

Solving for  $\hat{p}_{\text{MLE}}$ , we find:

$$\hat{p}_{\text{MLE}} = \frac{k}{n}$$

### **(b) Bayesian Estimate:**

Assume a uniform prior distribution  $P(p) = 1$  for  $0 \leq p \leq 1$ , and  $P(p) = 0$  otherwise.

For the binomial distribution the likelihood will be:  $P(k|p) = \binom{n}{k} p^k (1-p)^{n-k}$ , where  $k$  is the number of heads observed in  $n$  tosses.

Then applying Bayes' theorem to obtain the posterior distribution:

$$P(p|k, n) \propto P(k|p)P(p)$$

This results in a beta distribution:  $P(p|k, n) \propto p^k (1-p)^{n-k}$ .

So, The Bayesian estimate is the mean of the posterior distribution:

$$\text{Bayesian Estimate} = \frac{k+1}{n+2}$$

**(c) Maximum a Posteriori (MAP) Estimate:**

The MAP estimate is the mode of the posterior distribution:

$$\text{MAP Estimate} = \frac{k}{n}$$

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## Coding Assignment: Implementation and Optimization of GPT-2 Model:

The implementation and optimization of the GPT-2 model can be found in the following GitHub repository:

<https://github.com/pratapdey/GPT-2-small/blob/main/Implementation%20and%20Optimization%20of%20GPT-2%20Model.ipynb>

Alternatively, the full assignment is in the following repository:

<https://github.com/pratapdey/GPT-2-small>

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