Classical AI: Search

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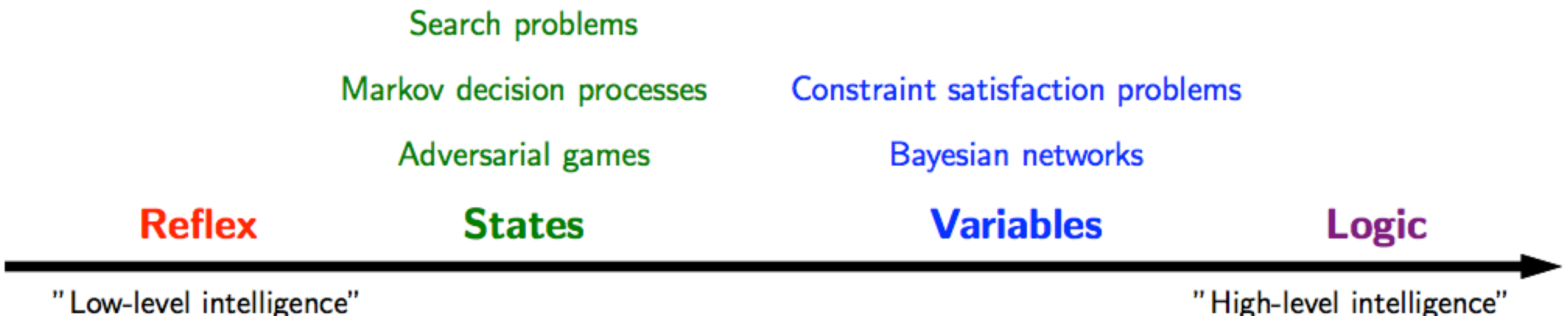
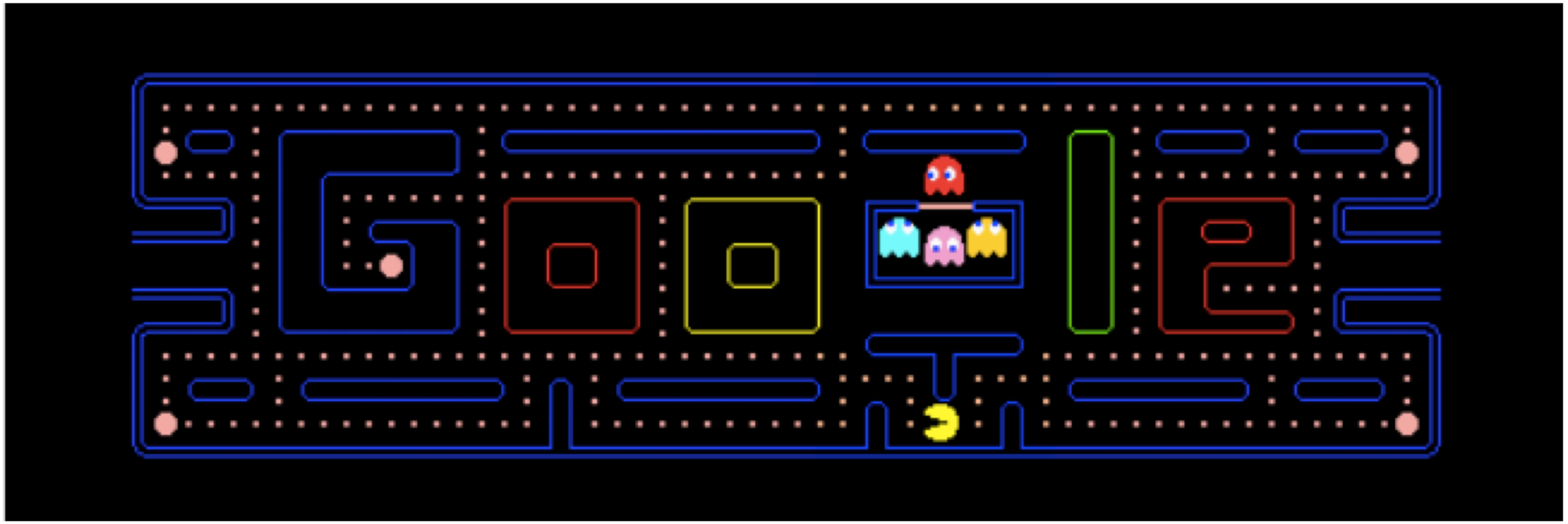
Today

- Recap – SVM
- Search
  - Uninformed vs Informed Search
- Uninformed Search
  - Breadth-First Search (BFS)
  - Depth-First Search (DFS)
  - Uniform-Cost Search (UCS)
- Informed Search (Heuristic)
  - Greedy
  - A*



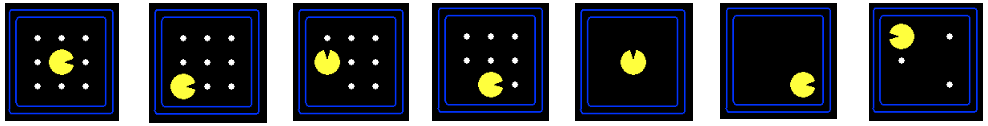
Pacman

- Reflex Agent vs Planning Agent (video)

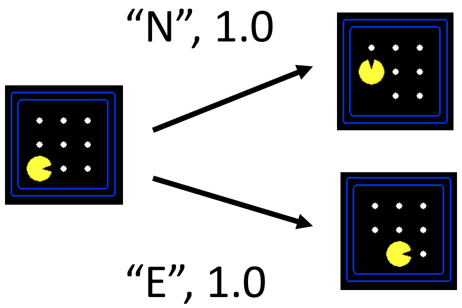


Search Problems

- A **search problem** consists of:
  - A state space



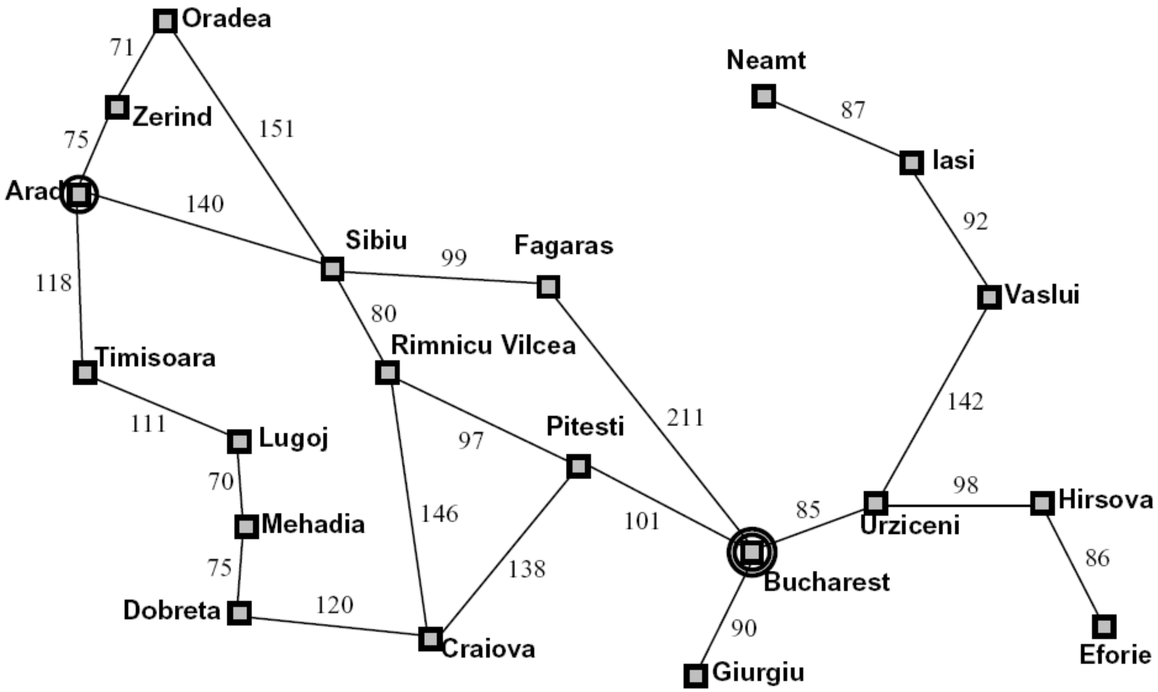
  - A successor function (with actions, costs)



  - A start state and a goal test

- A **solution** is a sequence of actions (a plan) which transforms the start state to a goal state

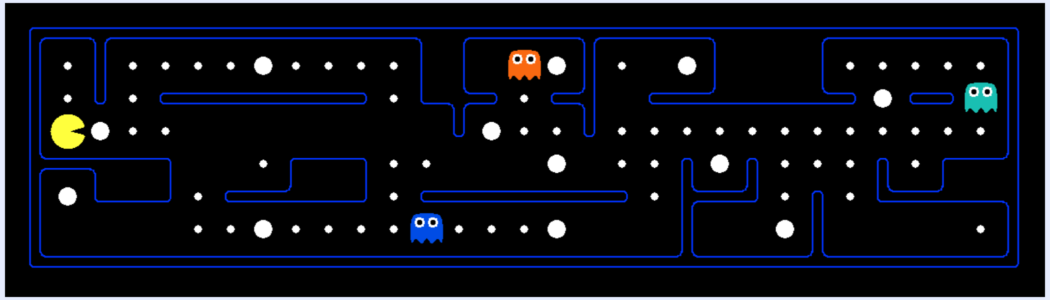
Example: Traveling in Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

What's in a State Space?

The world state includes every last detail of the environment



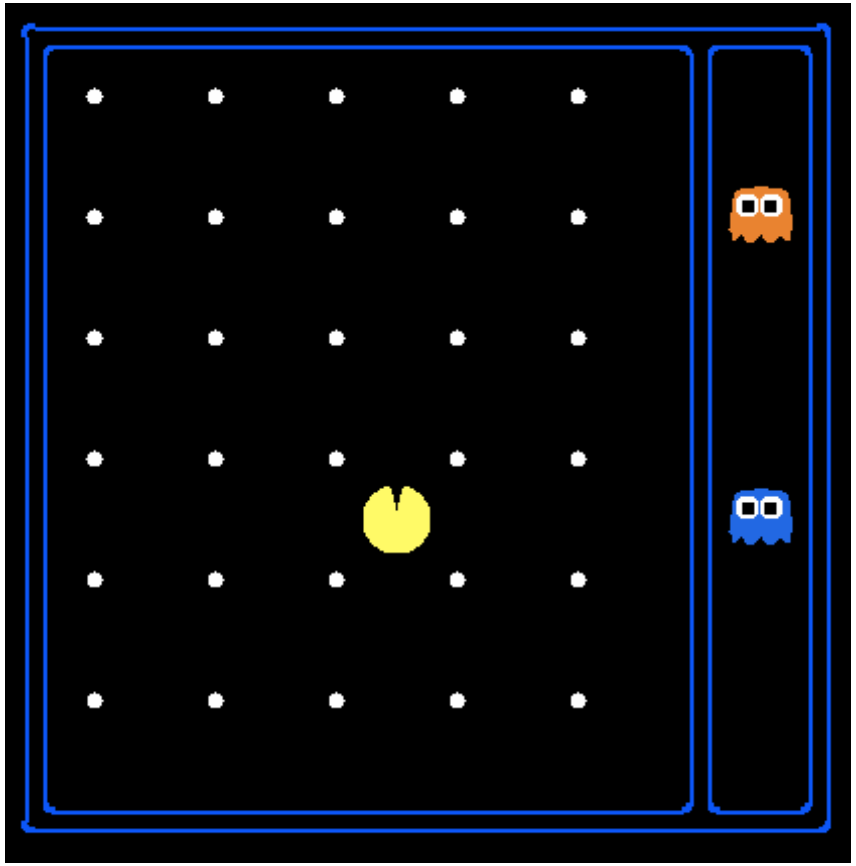
A search state keeps only the details needed for planning (abstraction)

- Problem: Pathing
  - States: (x,y) location
  - Actions: NSEW
  - Successor: update location only
  - Goal test: is (x,y)=END
- Problem: Eat-All-Dots
  - States: {(x,y), dot booleans}
  - Actions: NSEW
  - Successor: update location and possibly a dot boolean
  - Goal test: dots all false

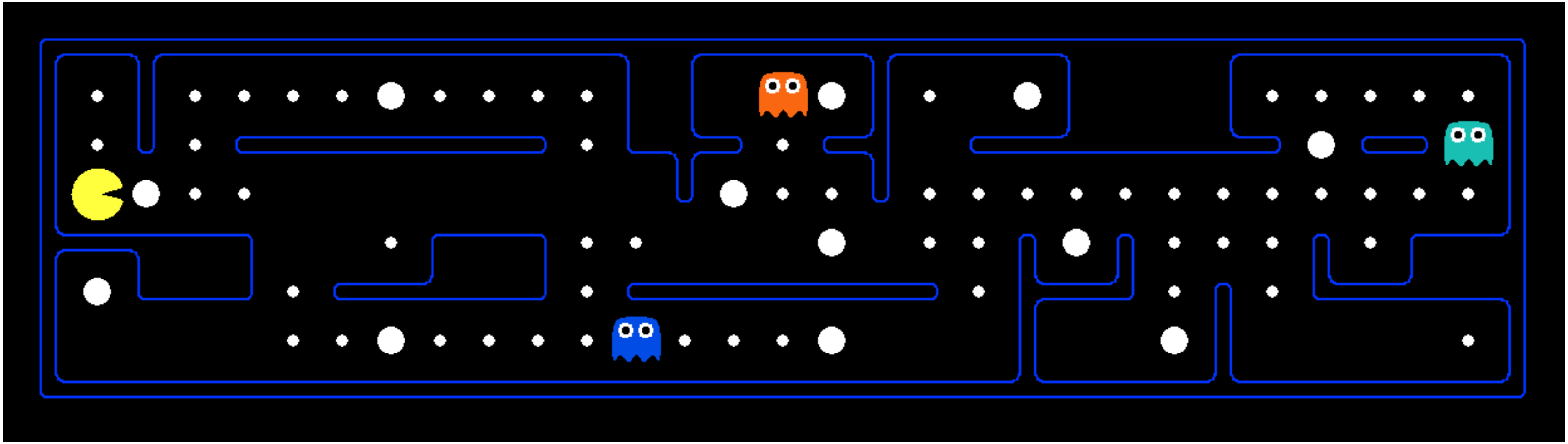
State Space Sizes?

- World state:
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW

- How many
  - World states?
    $120x(2^{30})x(12^2)x4$
  - States for pathing?
    120
  - States for eat-all-dots?
    $120x(2^{30})$

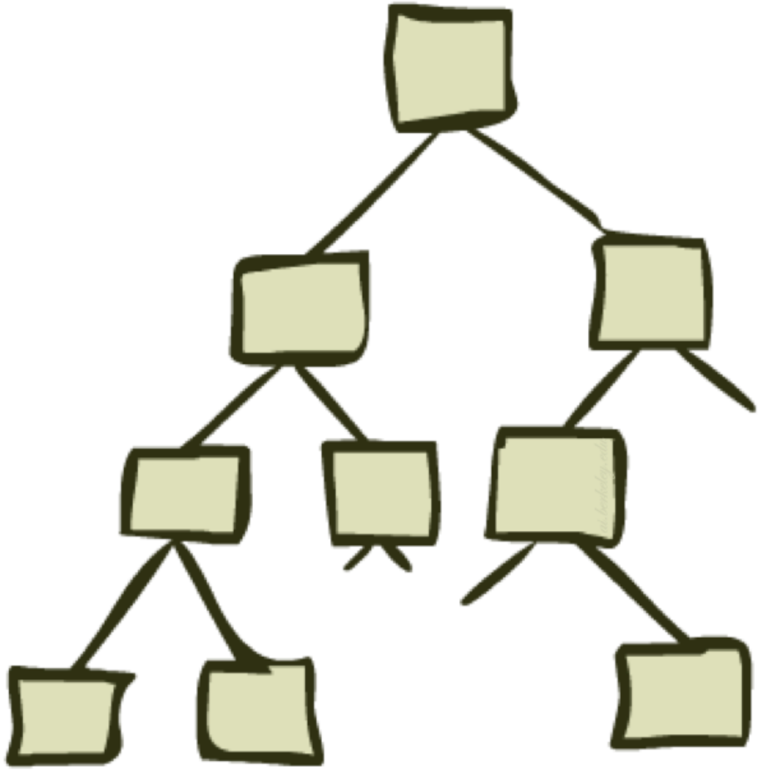


Quiz: Safe Passage



- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?
  - (agent position, dot booleans, power pellet booleans, remaining scared time)

State Space Graphs and Search Trees

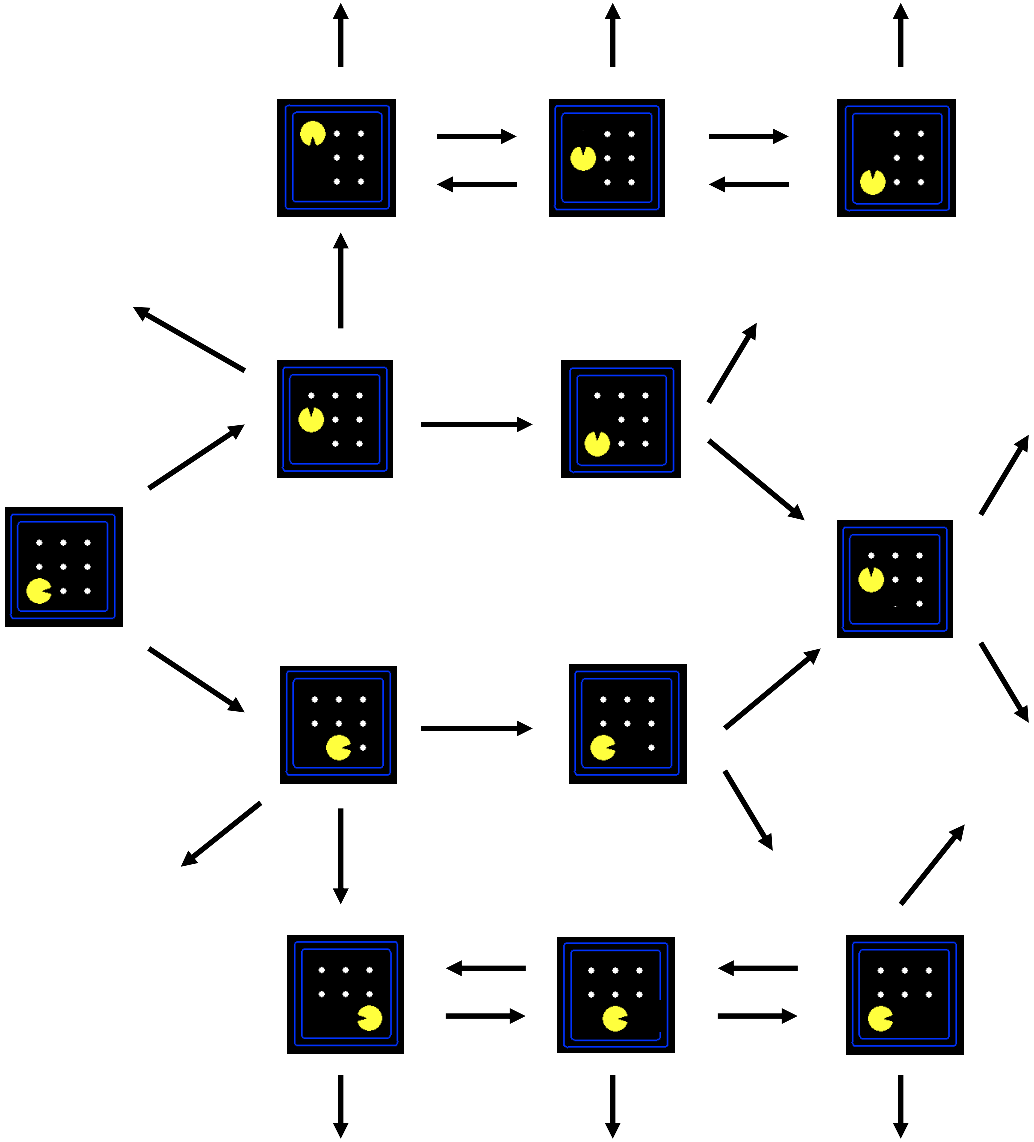


State Space Graphs

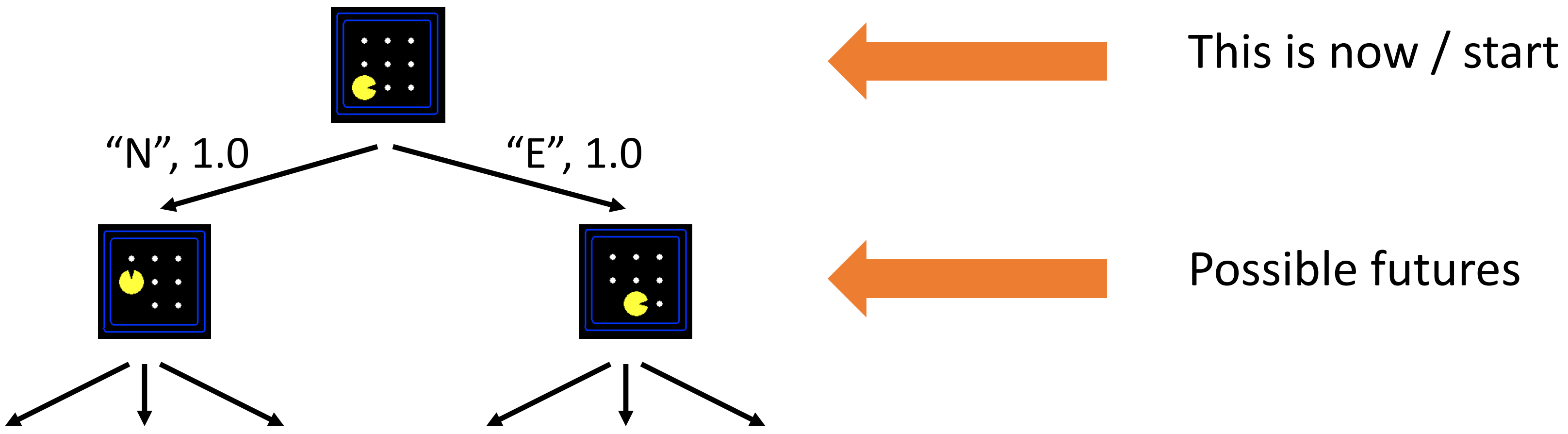
- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)

- In a state space graph, each state occurs only once!

- We can rarely build this full graph in memory (it's too big), but it's a useful idea



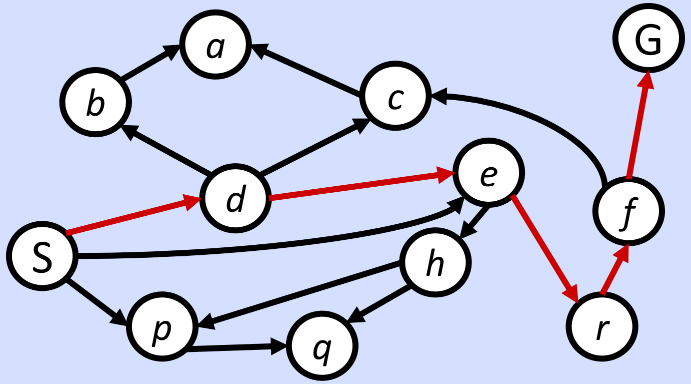
Search Trees



- A search tree:
  - A "what if" tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree

State Space Graphs vs. Search Trees

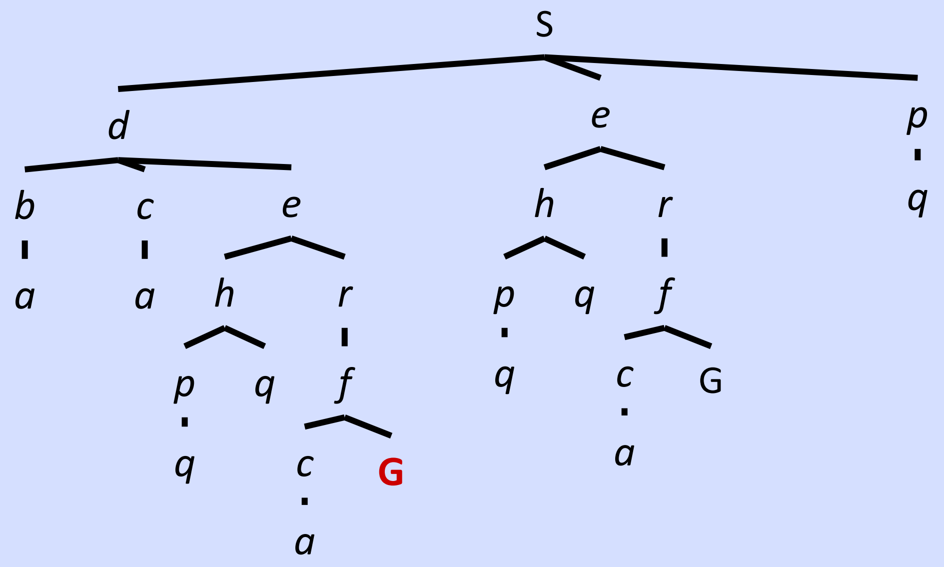
State Space Graph



Each NODE in in the search tree is an entire PATH in the state space graph.

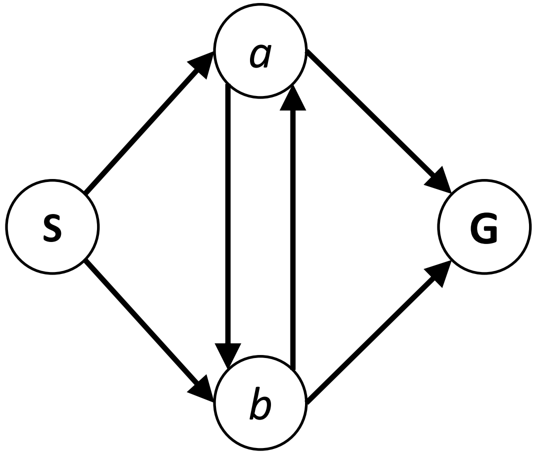
We construct both on demand – and we construct as little as possible.

Search Tree



Quiz: State Space Graphs vs. Search Trees

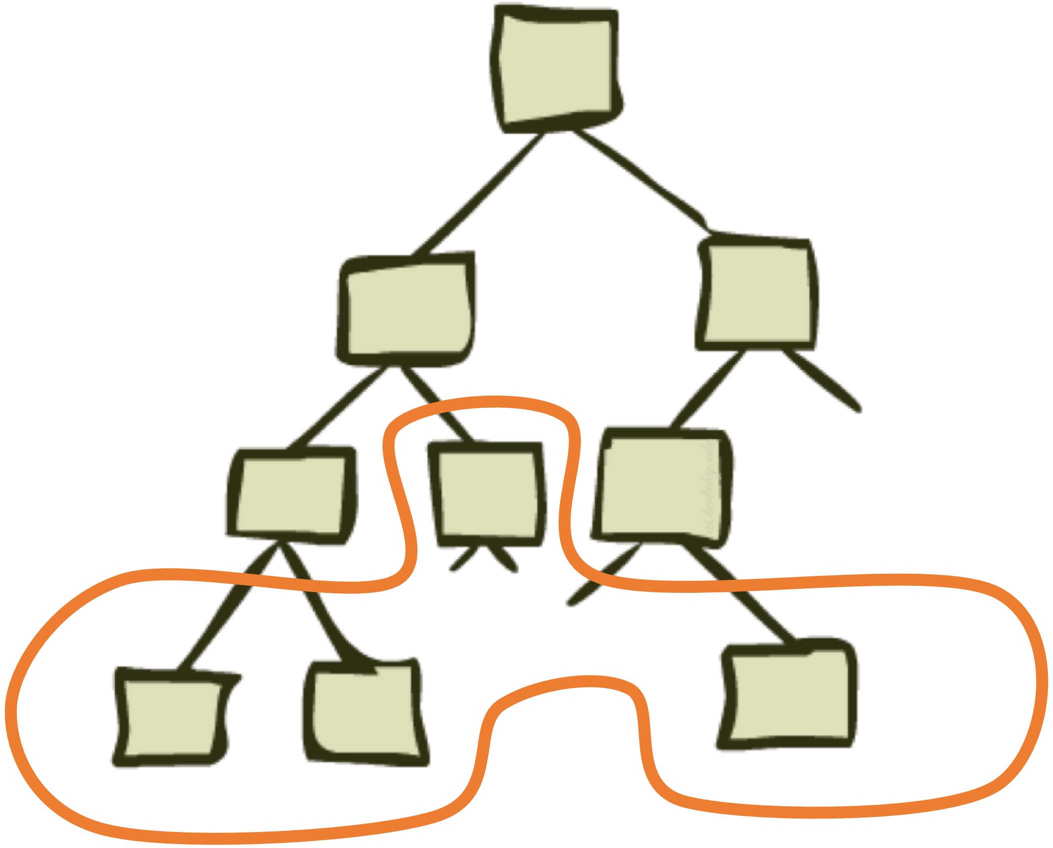
Consider this 4-state graph:



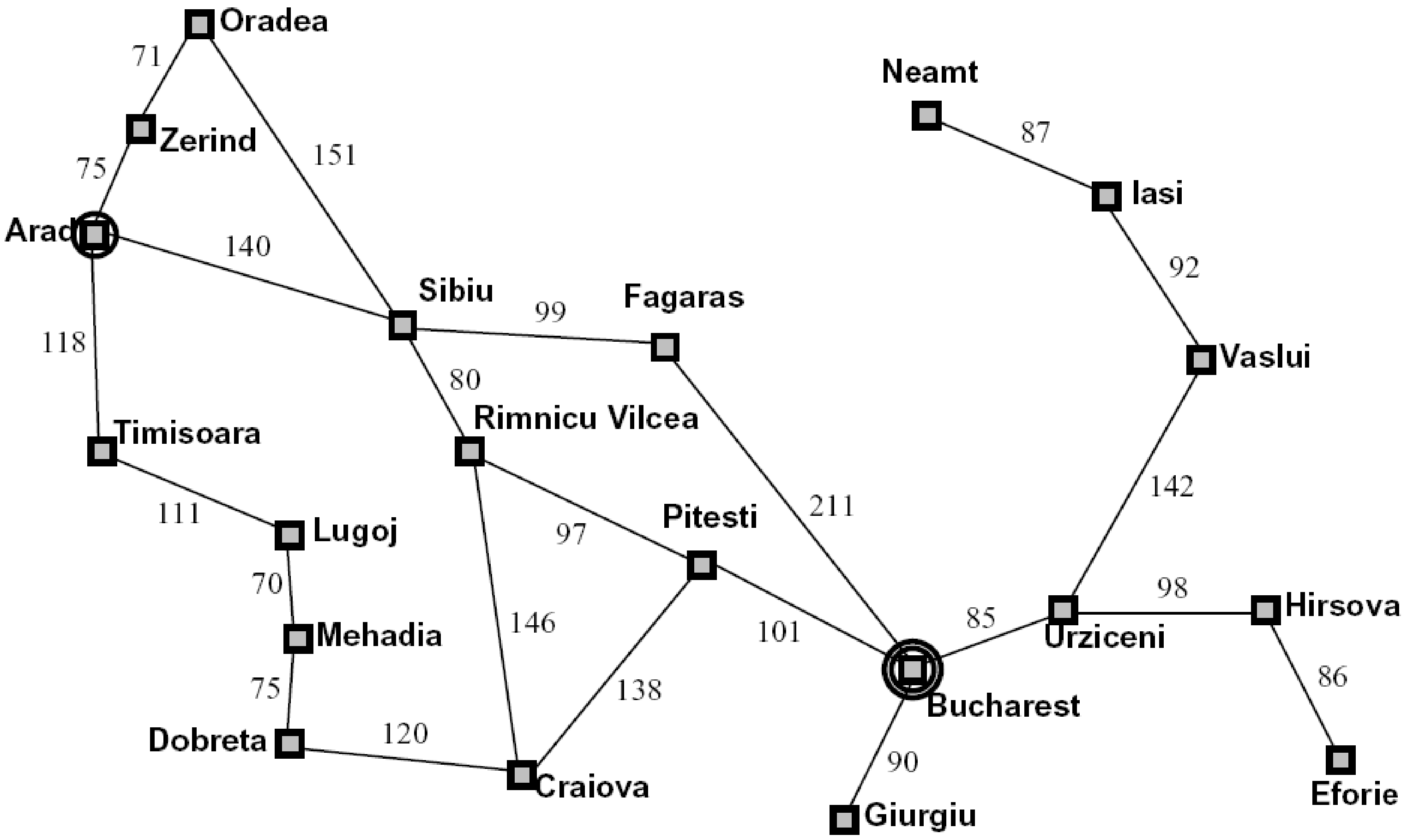
How big is its search tree (from S)?

Important: Lots of repeated structure in the search tree!

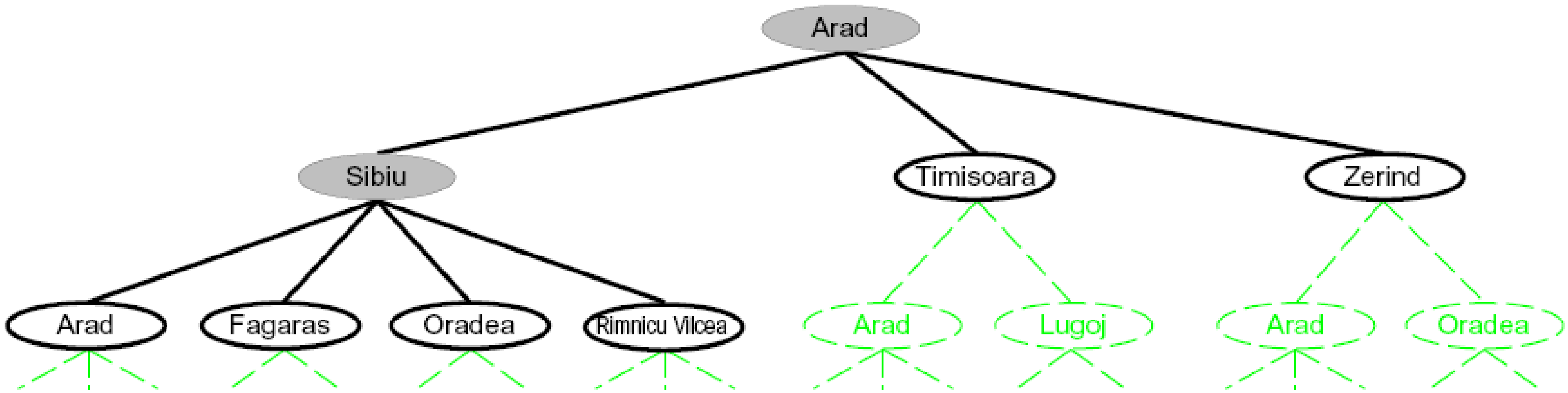
Tree Search



Search Example: Romania



Searching with a Search Tree



- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a fringe of partial plans under consideration
  - Try to expand as few tree nodes as possible

General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

More detailed pseudocode

function GRAPH-SEARCH (*state space graph*, *initial state*)

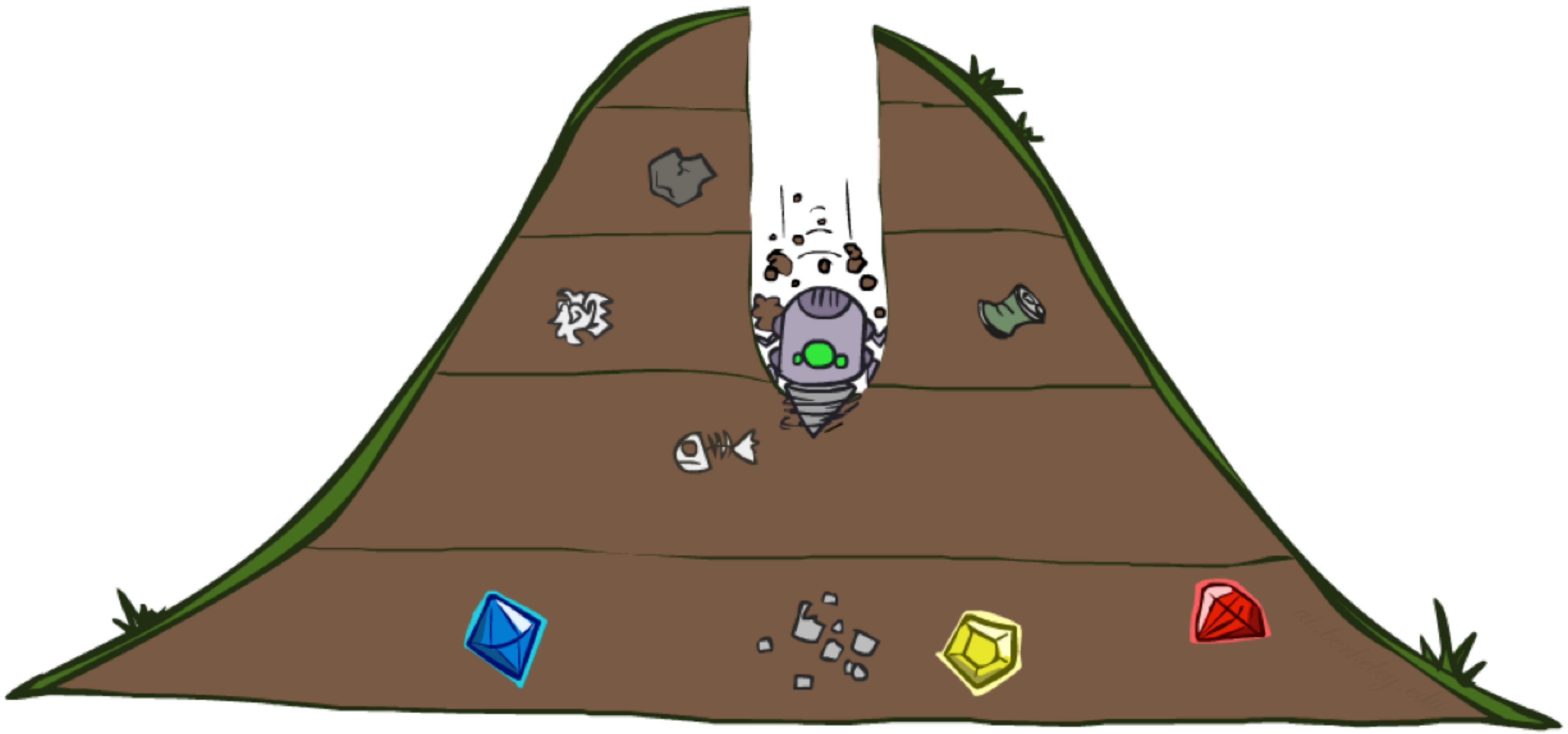
add the *initial state* to the *fringe* (A priority queue)
while the *fringe* is NOT empty **do**
    choose a *node* and remove it from the *fringe*
    if the *node* has NOT been expanded before **then**
        if the node is a Goal State **then return** success
        expand the *node*, adding its *successor nodes* to the *fringe*
        (only if the *successor node* has NOT been expanded before)
return failure

i.e., not in “visited” list

(To make sure a state is only expanded once)

Uninformed Search

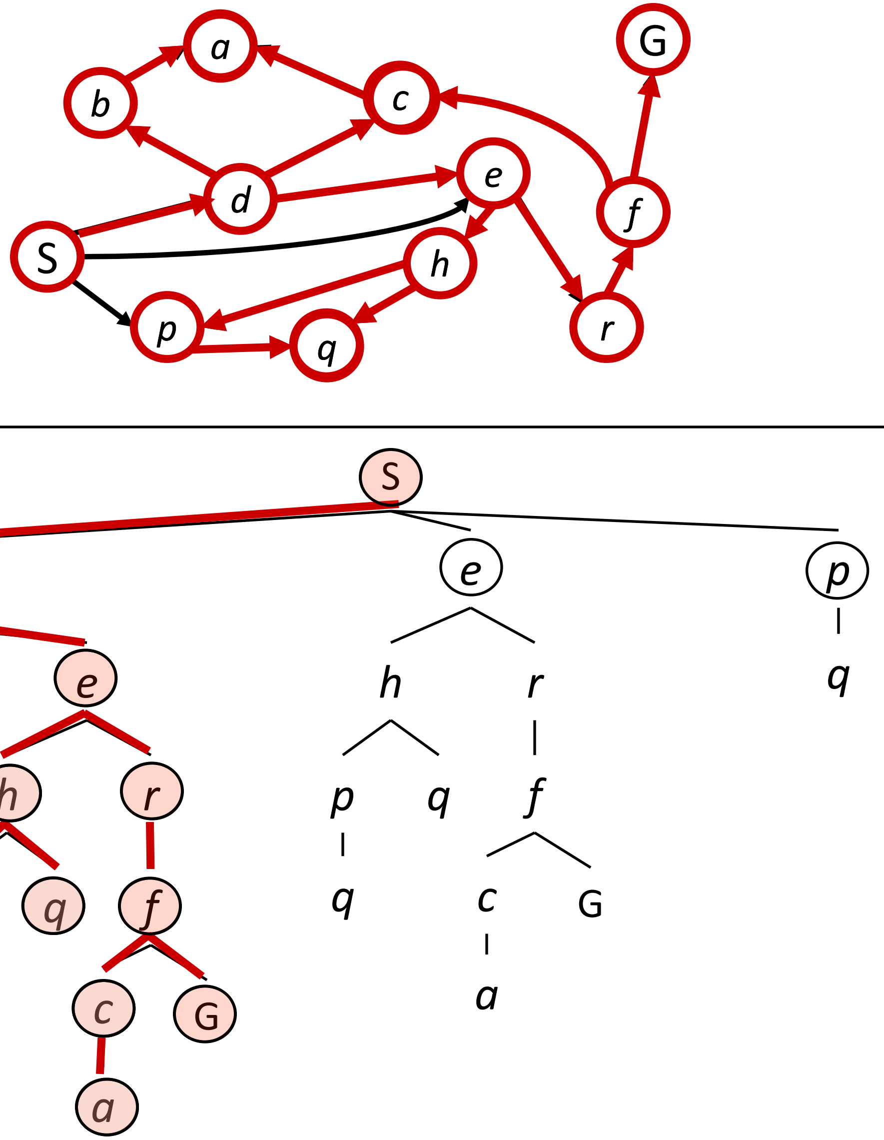
Depth-First Search



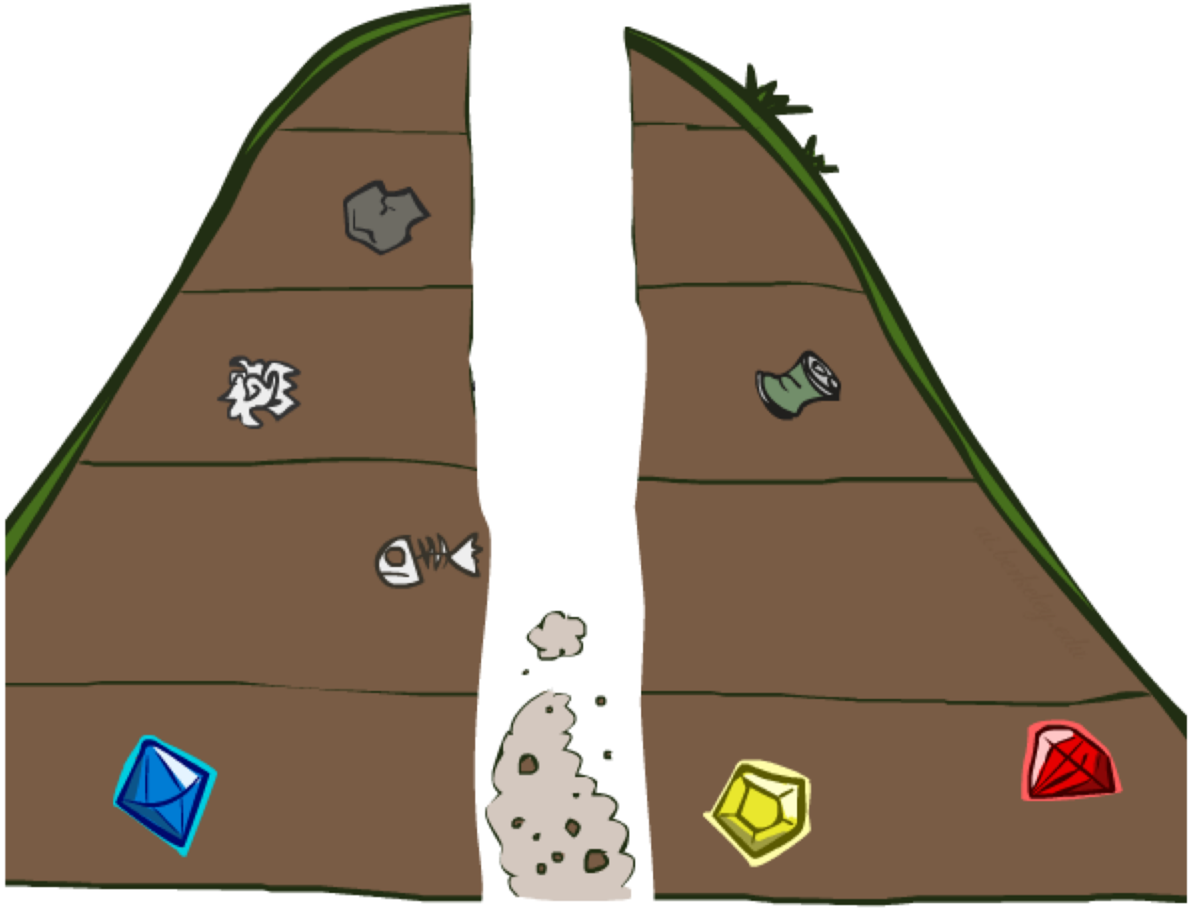
Depth-First Search

Strategy: expand a deepest node first

Implementation: Fringe is a LIFO stack



Search Algorithm Properties

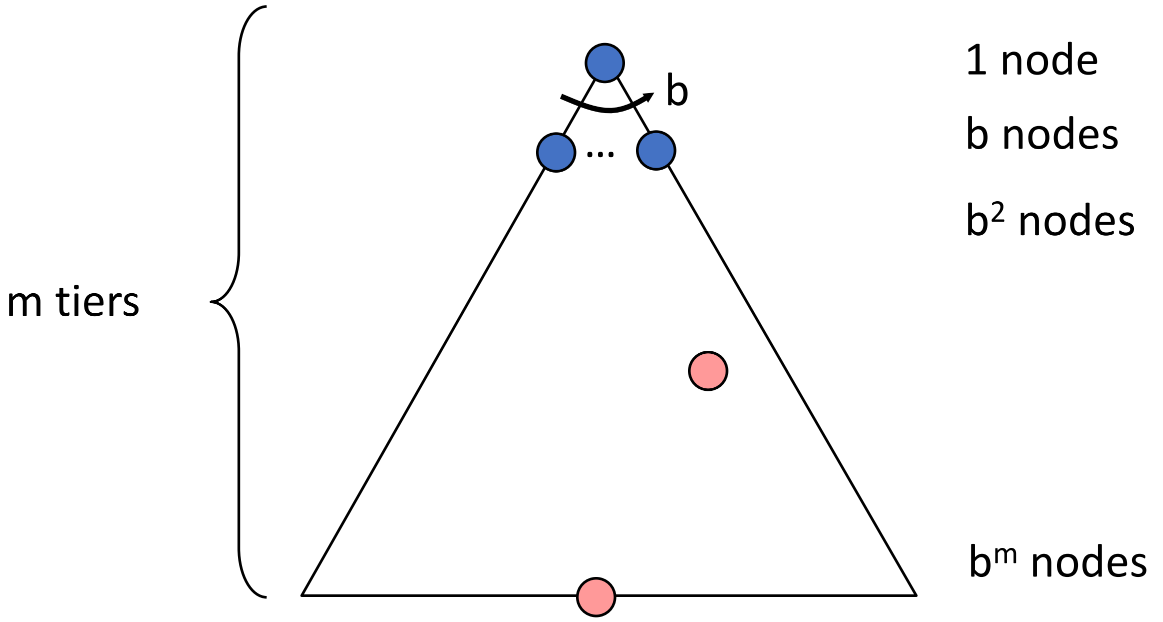


Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

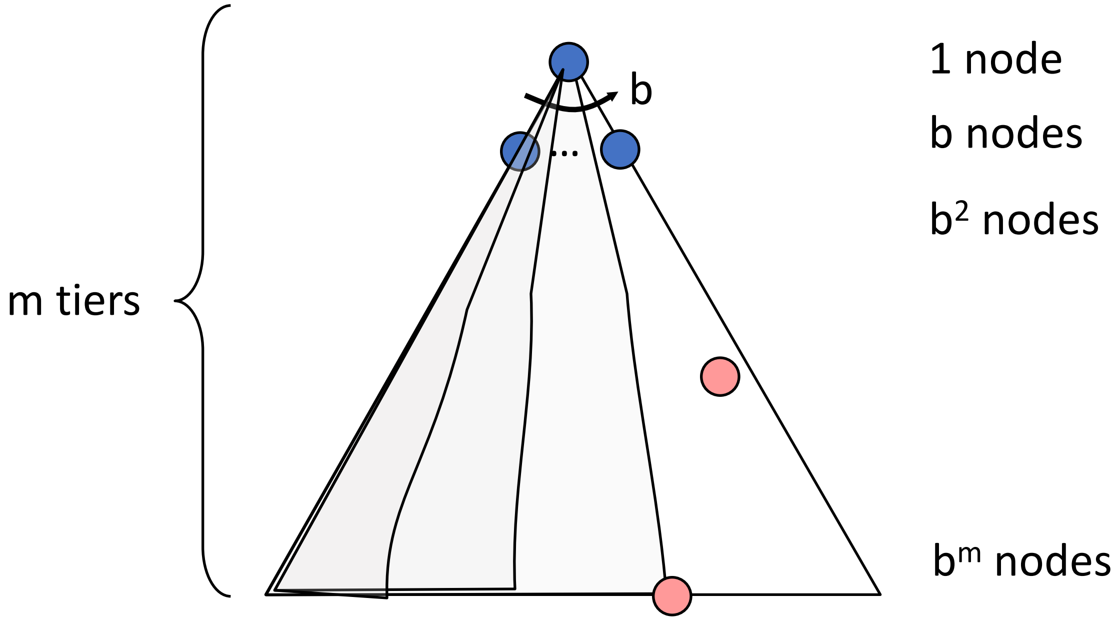
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths

- Number of nodes in entire tree?
  - $1 + b + b^2 + \dots . b^m = O(b^m)$

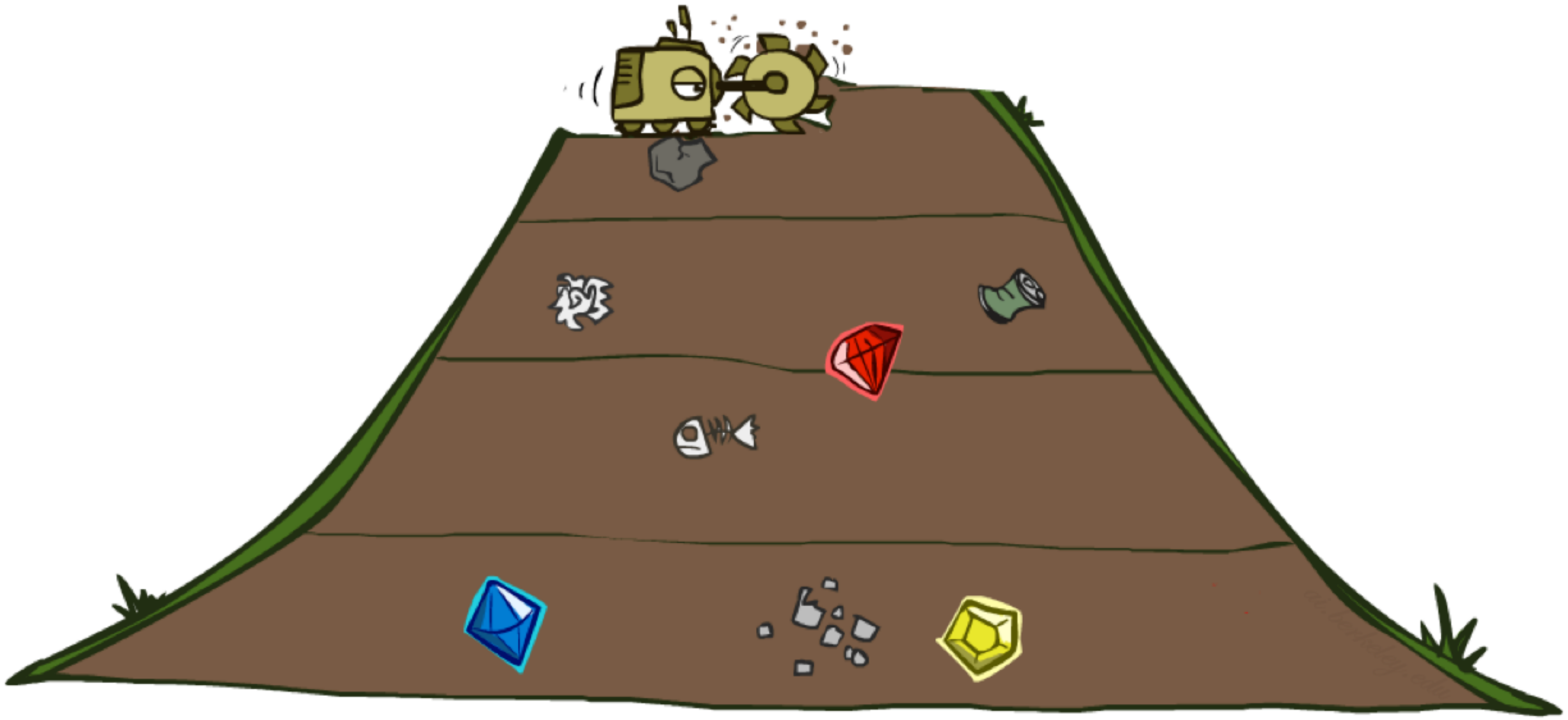


Depth-First Search (DFS) Properties

- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$
- How much space does the fringe take?
  - Only has siblings on path to root, so O(bm)
- Is it complete?
  - m could be infinite, so only if we prevent cycles (more later)
- Is it optimal?
  - No, it finds the "leftmost" solution, regardless of depth or cost



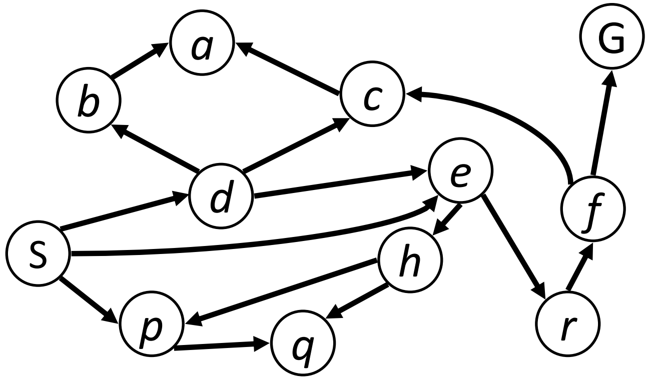
Breadth-First Search



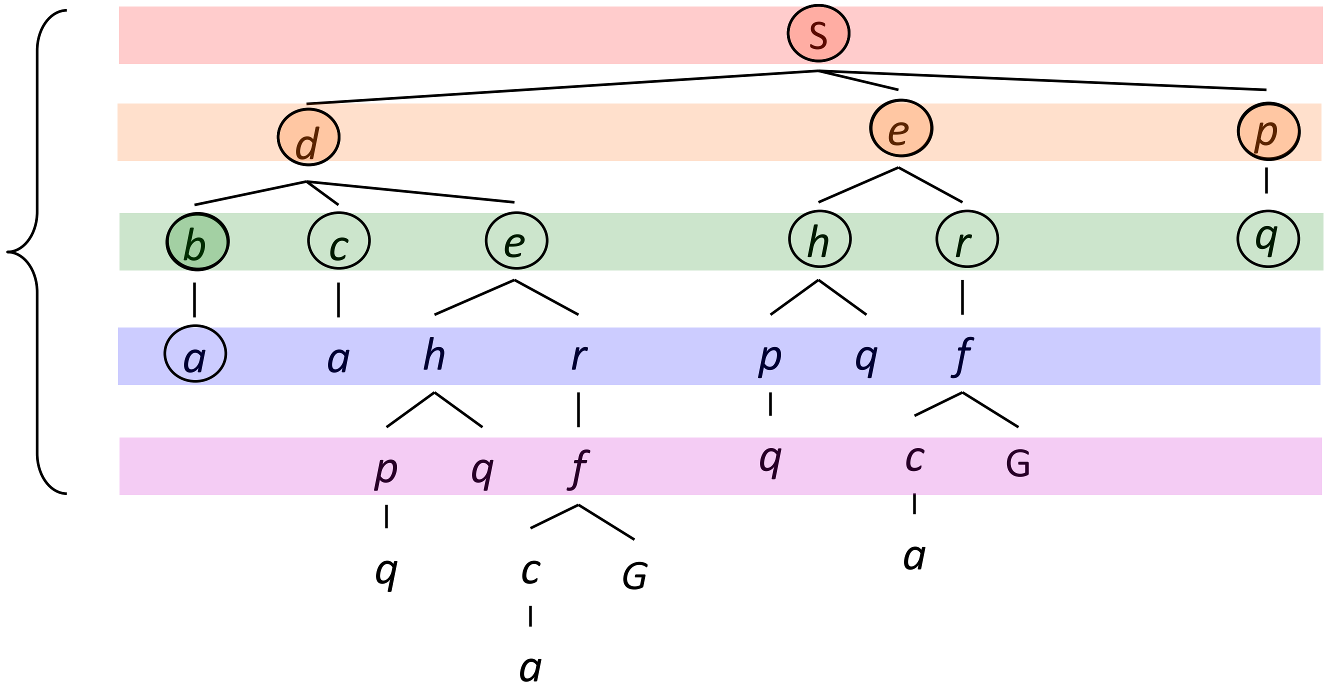
Breadth-First Search

Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue

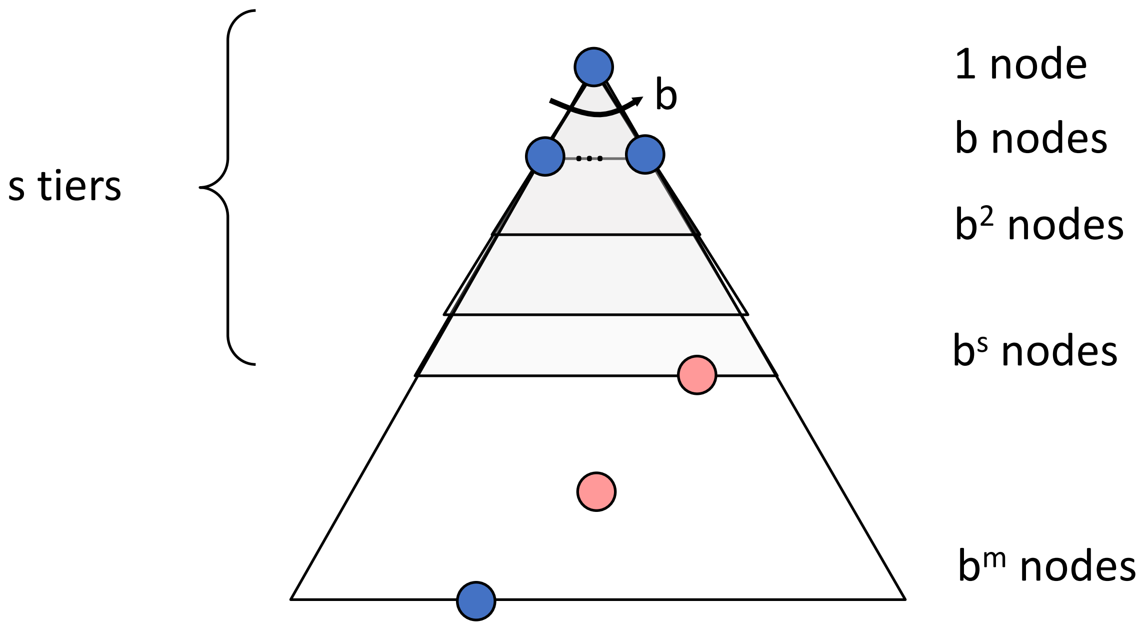


Search Tiers

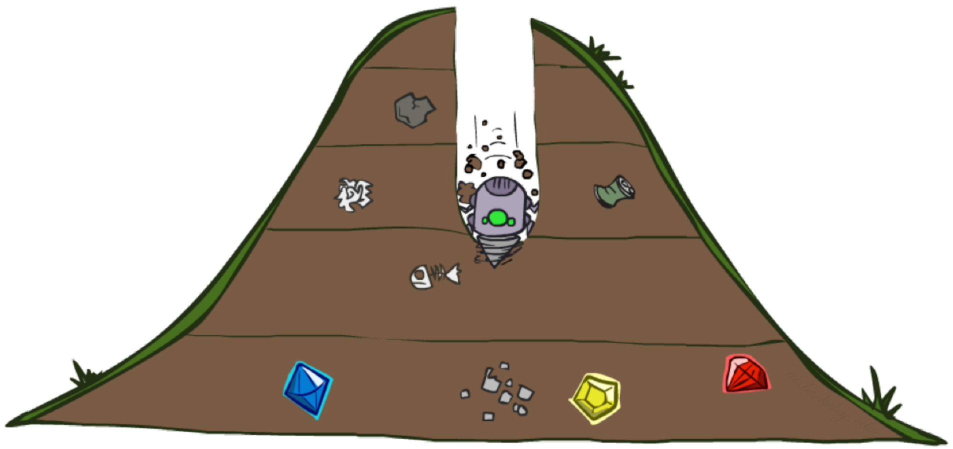


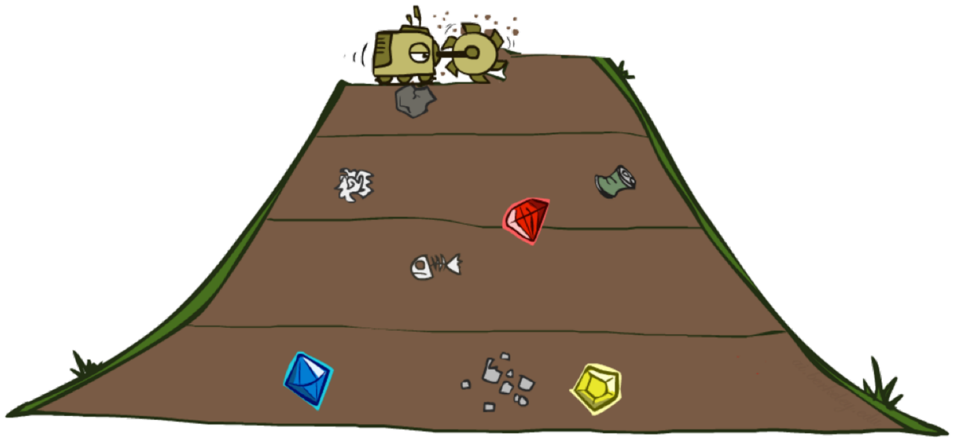
Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be s
  - Search takes time $O(b^s)$
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^s)$
- Is it complete?
  - s must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



Quiz: DFS vs BFS





Quiz: DFS vs BFS

- When will BFS outperform DFS?

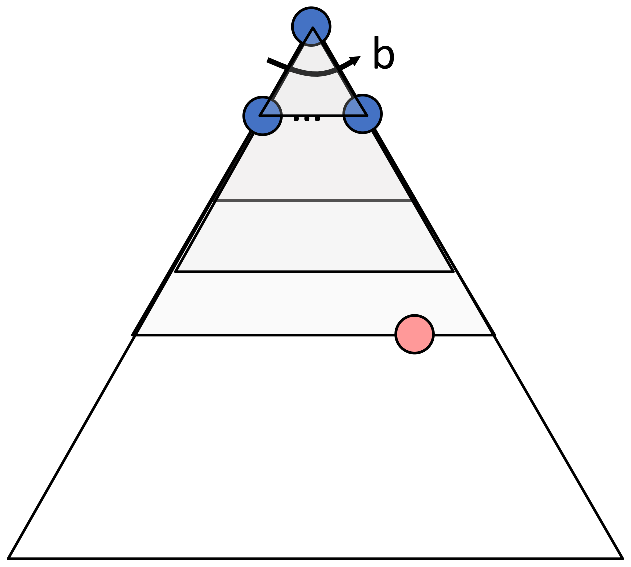
- When will DFS outperform BFS?

[Demo: dfs/bfs maze water (L2D6)]

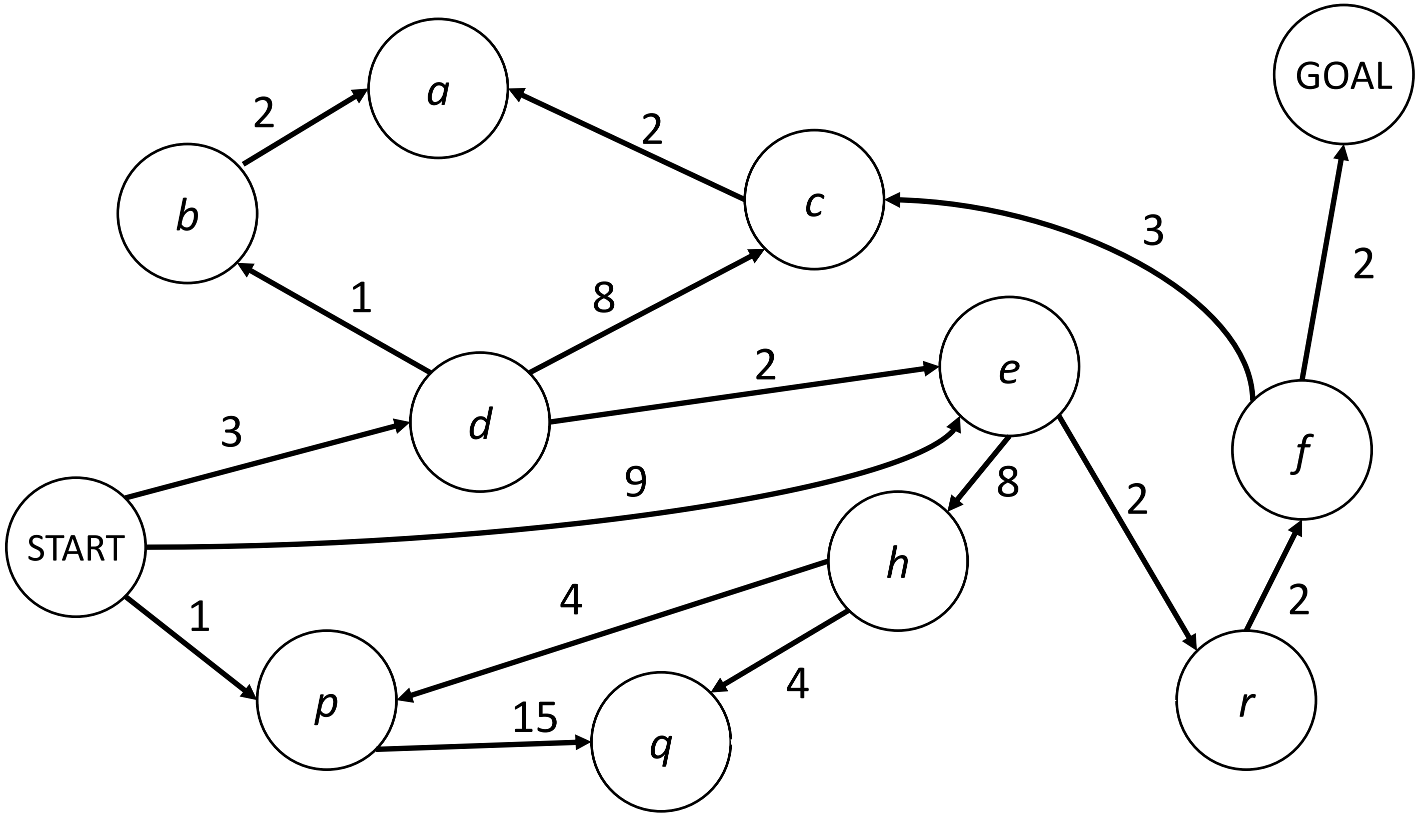
Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution…
  - Run a DFS with depth limit 2. If no solution…
  - Run a DFS with depth limit 3. …..

- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!

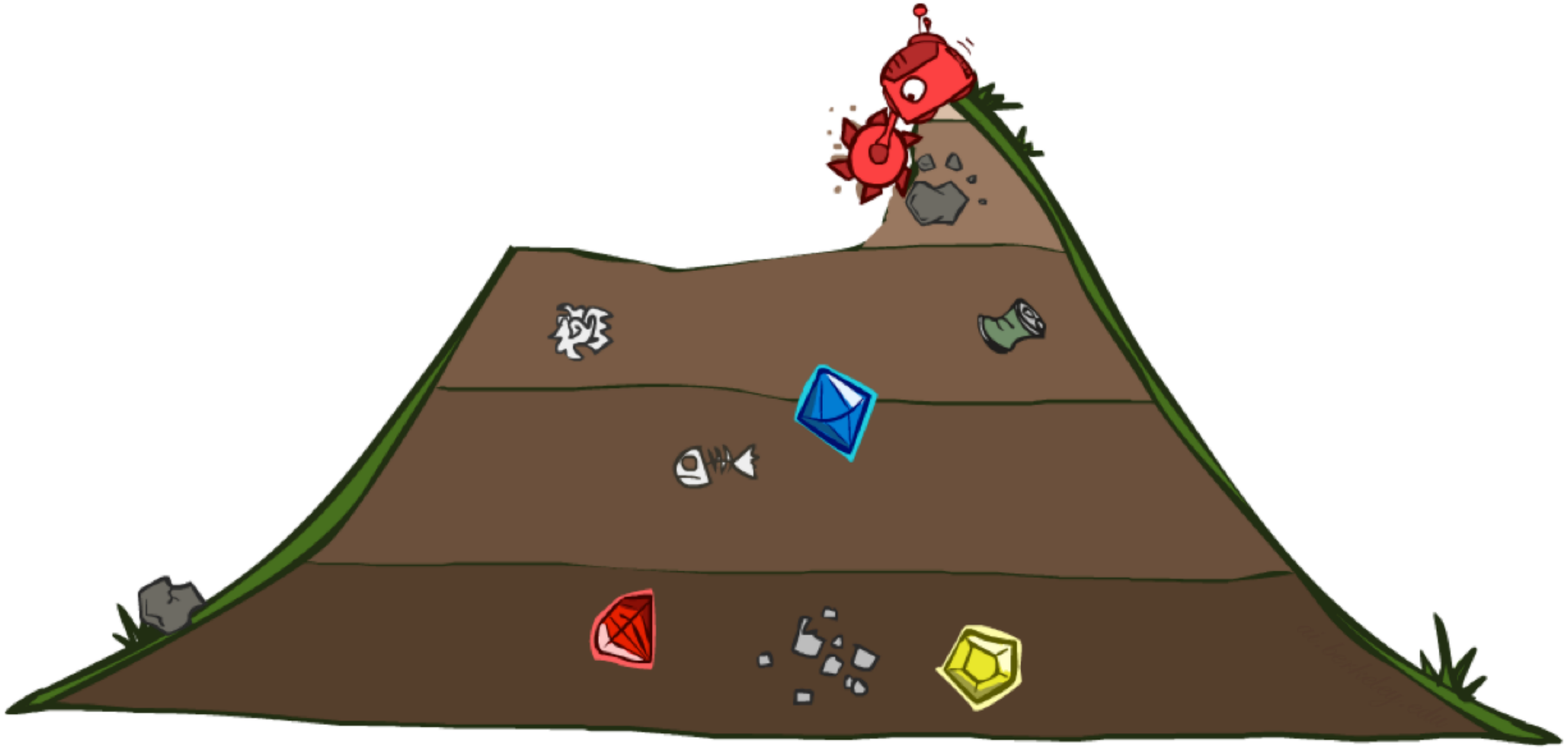


Cost-Sensitive Search



BFS finds the shortest path in terms of number of actions.
It does not find the least-cost path. We will now cover
a similar algorithm which does find the least-cost path.

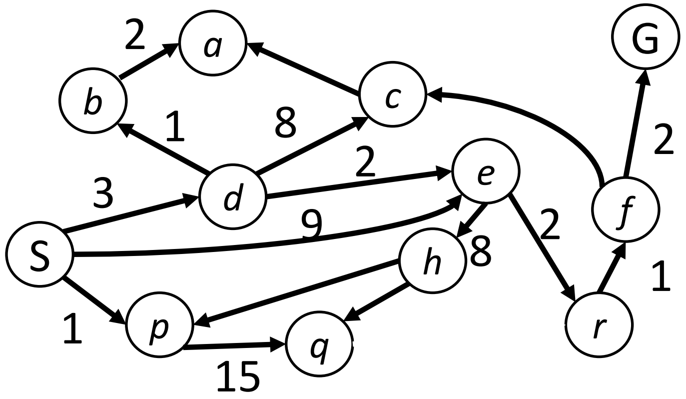
Uniform Cost Search

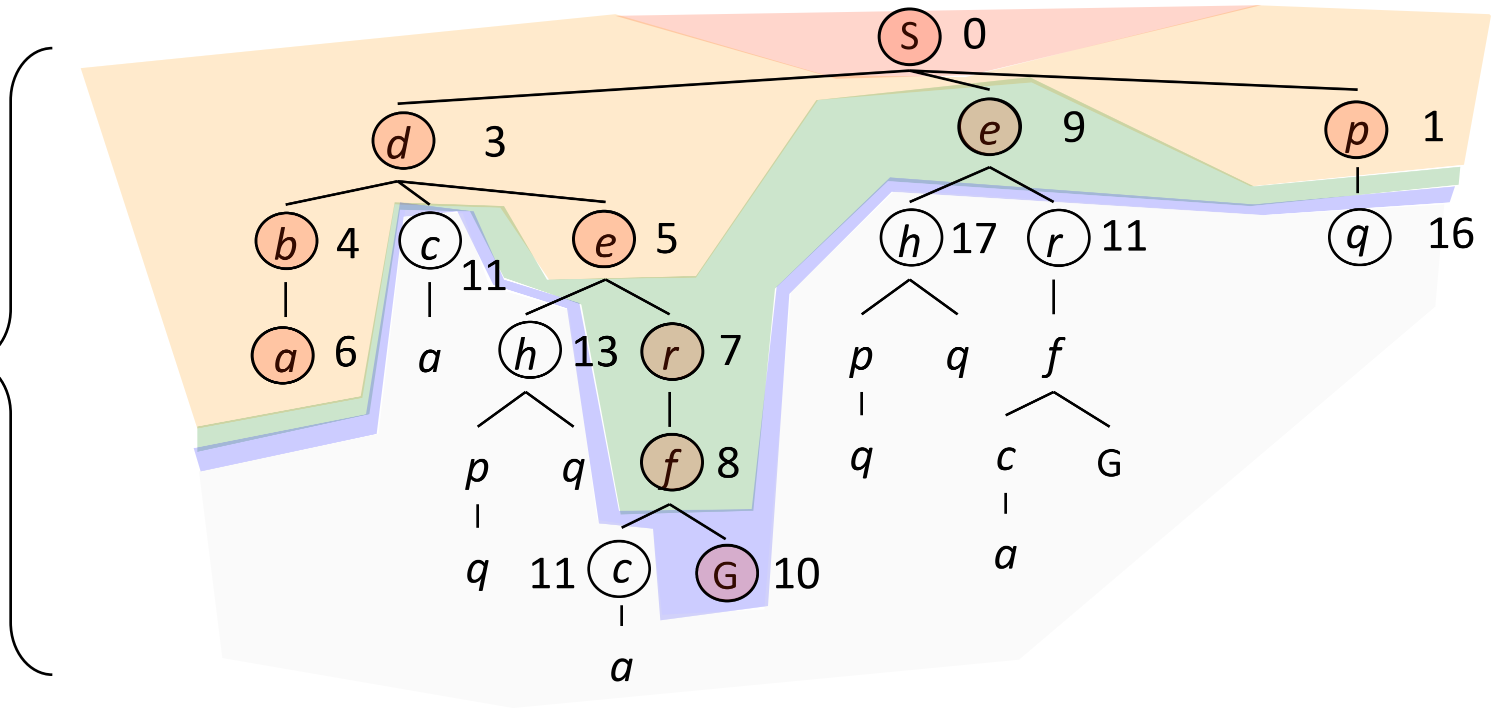


Uniform Cost Search

Strategy: expand a cheapest node first:

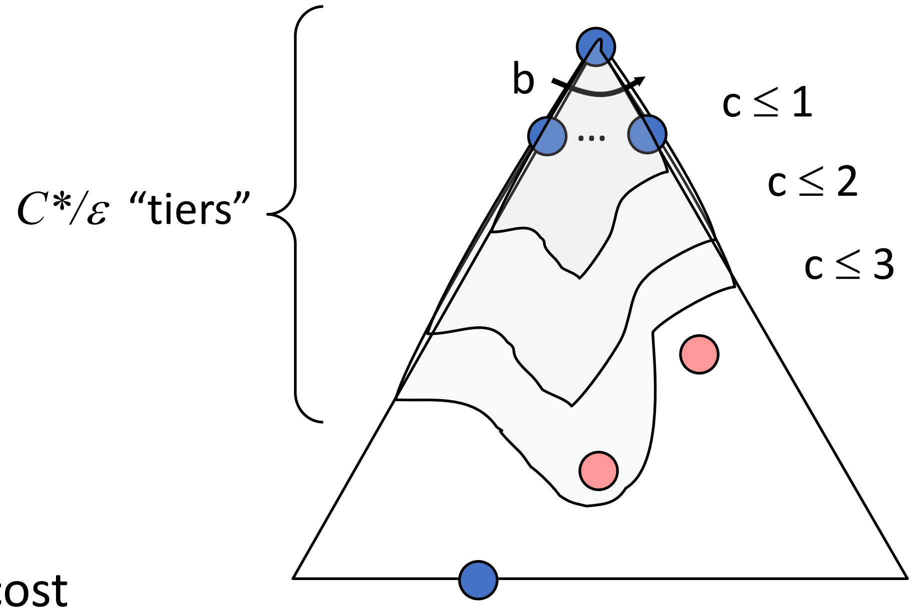
Fringe is a priority queue (priority: cumulative cost)





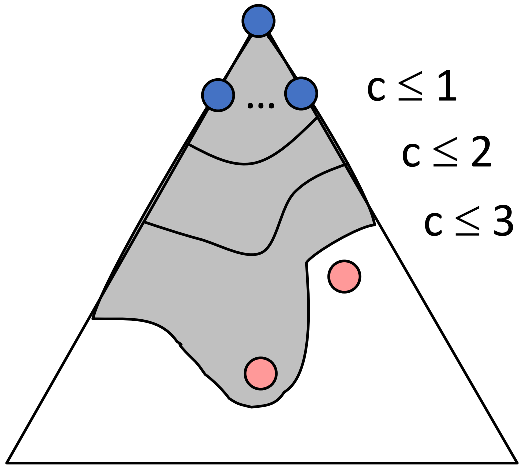
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε, then the "effective depth" is roughly C^*/ε
  - Takes time O(b$^{C^*/\varepsilon}$) (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so O(b$^{C^*/\varepsilon}$)
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes! (Proof next lecture via A*)



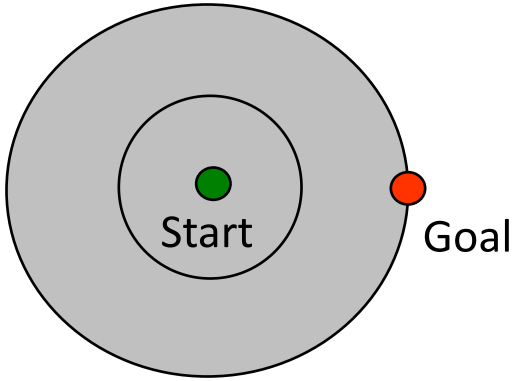
Uniform Cost Issues

- Remember: UCS explores increasing cost contours



- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location

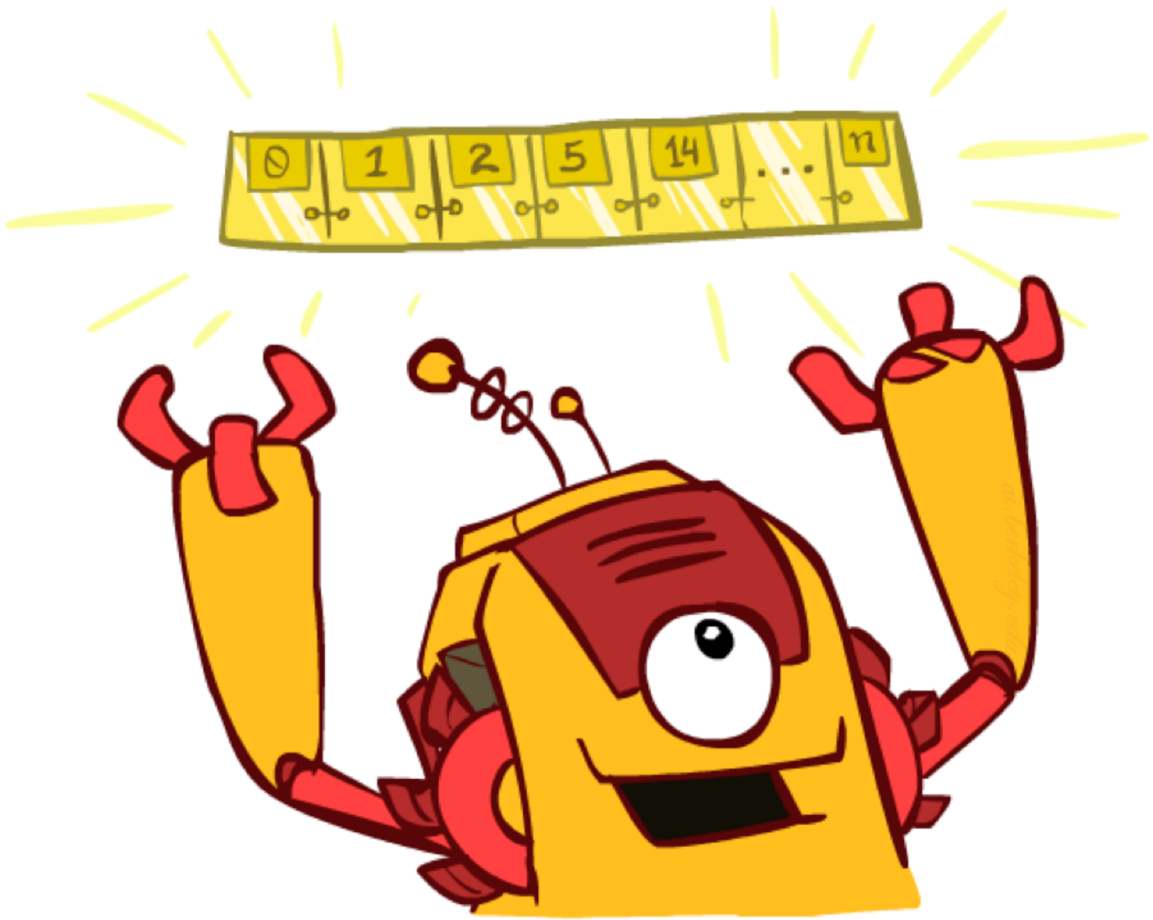


- We’ll fix that soon!

[Demo: empty grid UCS (L2D5)]
[Demo: maze with deep/shallow water DFS/BFS/UCS (L2D7)]

The One Queue

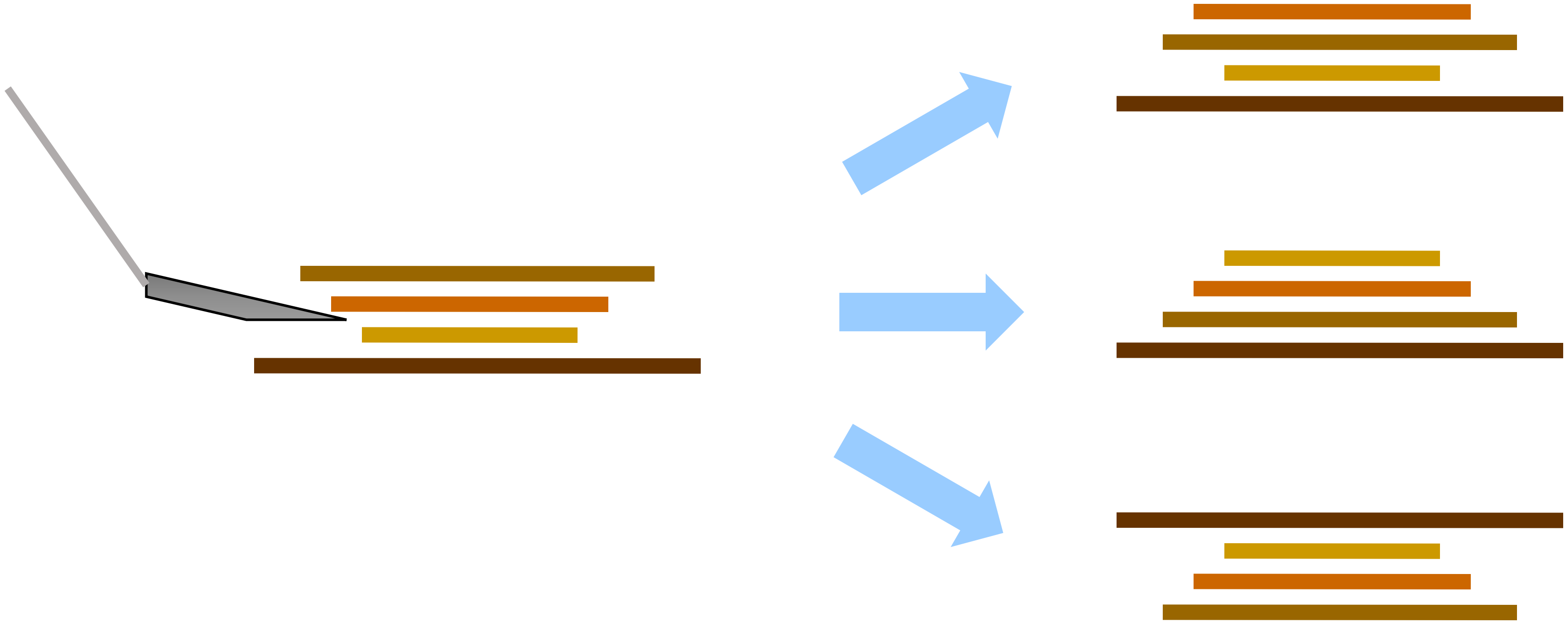
- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object



DFS vs BFS vs UCS

- (video) guess which search strategy?
- Pay attention to the area with deep water

Example: Pancake Problem



Cost: Number of pancakes flipped

Example: Pancake Problem

BOUNDS FOR SORTING BY PREFIX REVERSAL

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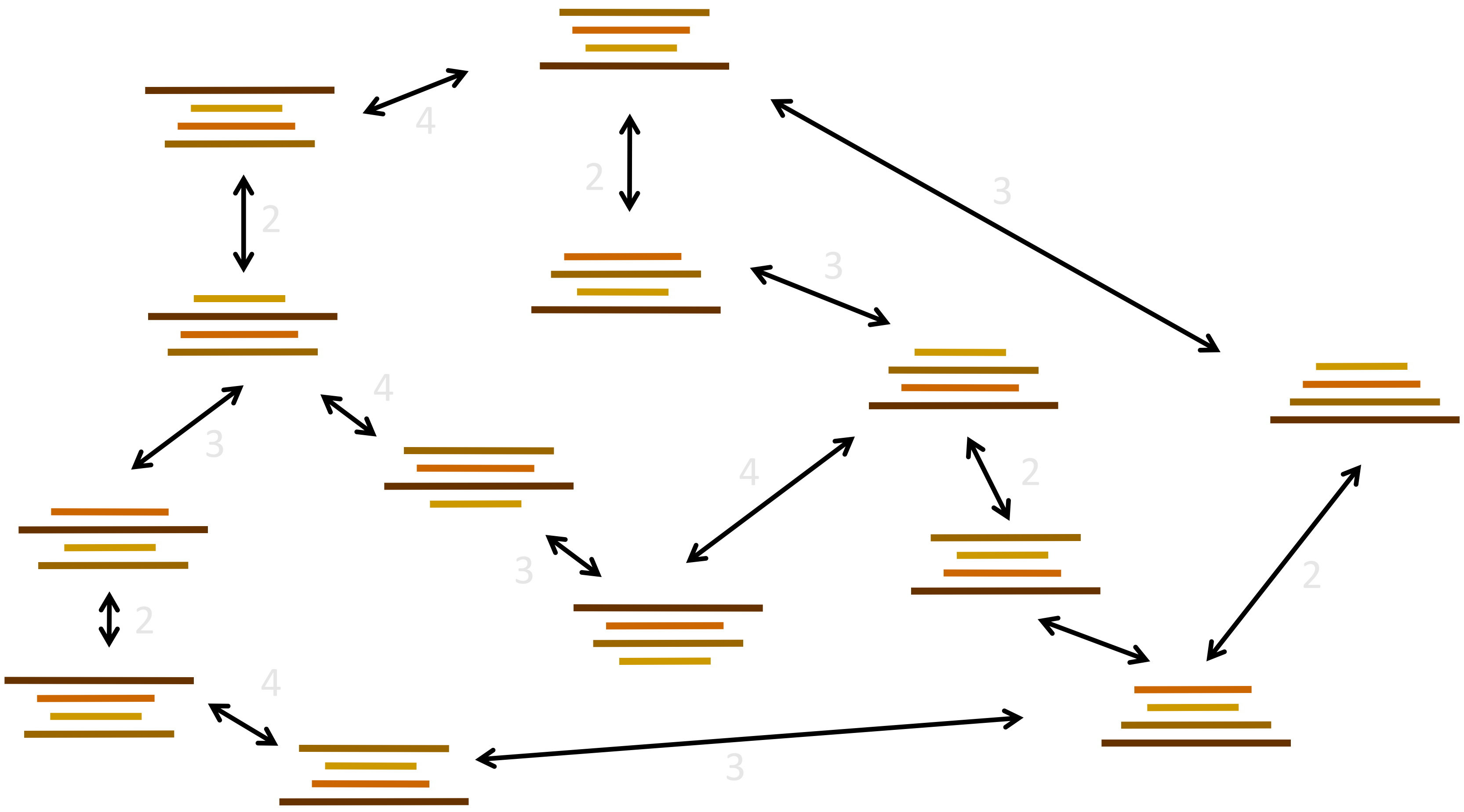
Received 18 January 1978
Revised 28 August 1978

For a permutation σ of the integers from 1 to n, let $f(\sigma)$ be the smallest number of prefix reversals that will transform σ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all σ in (the symmetric group) S_n. We show that $f(n) \leqslant (5n+5)/3$, and that $f(n) \geqslant 17n/16$ for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leqslant g(n) \leqslant 2n+3$.

Example: Pancake Problem

State space graph with costs as weights



General Tree Search

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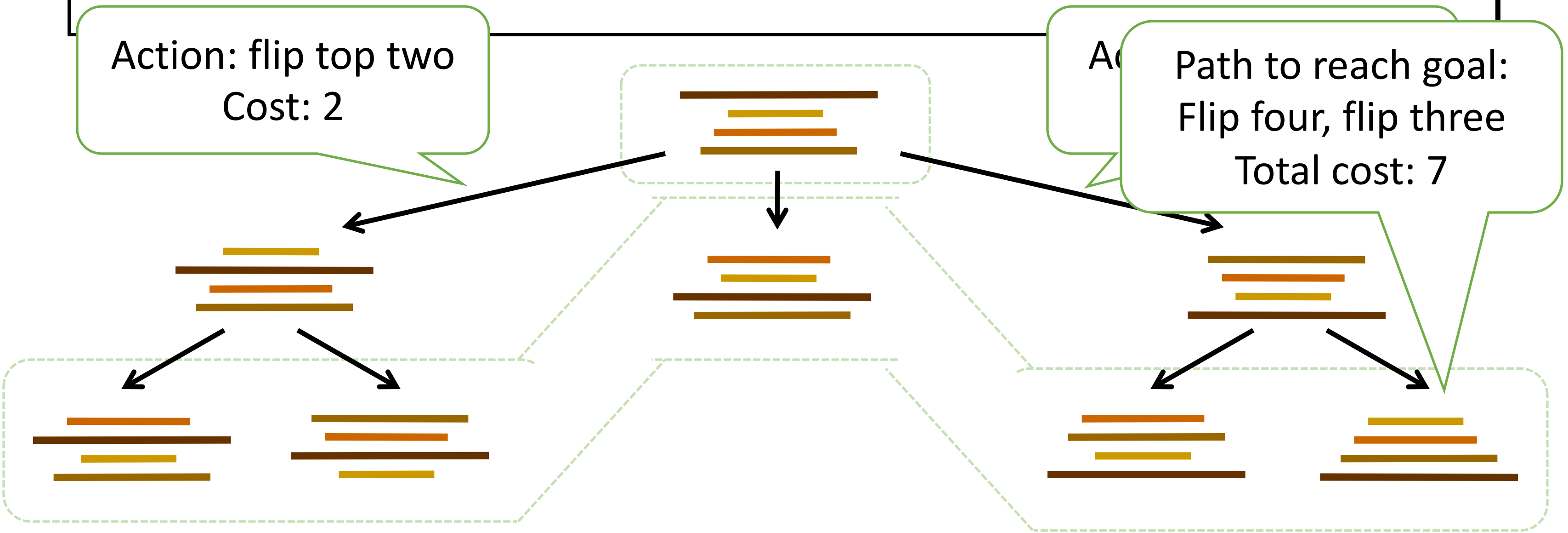
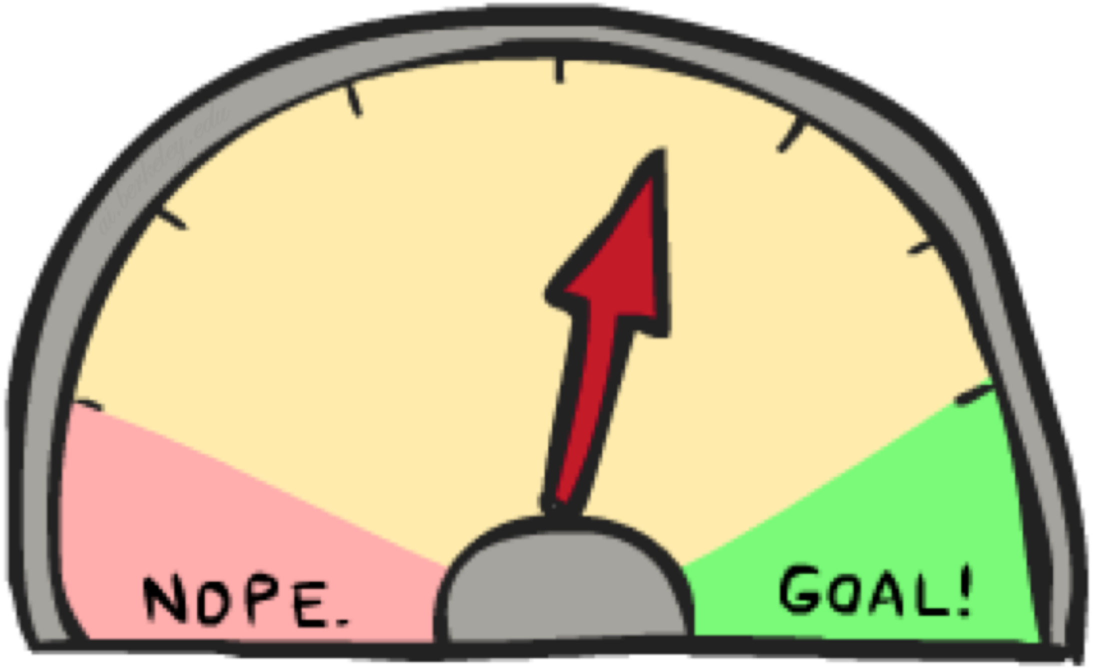
```



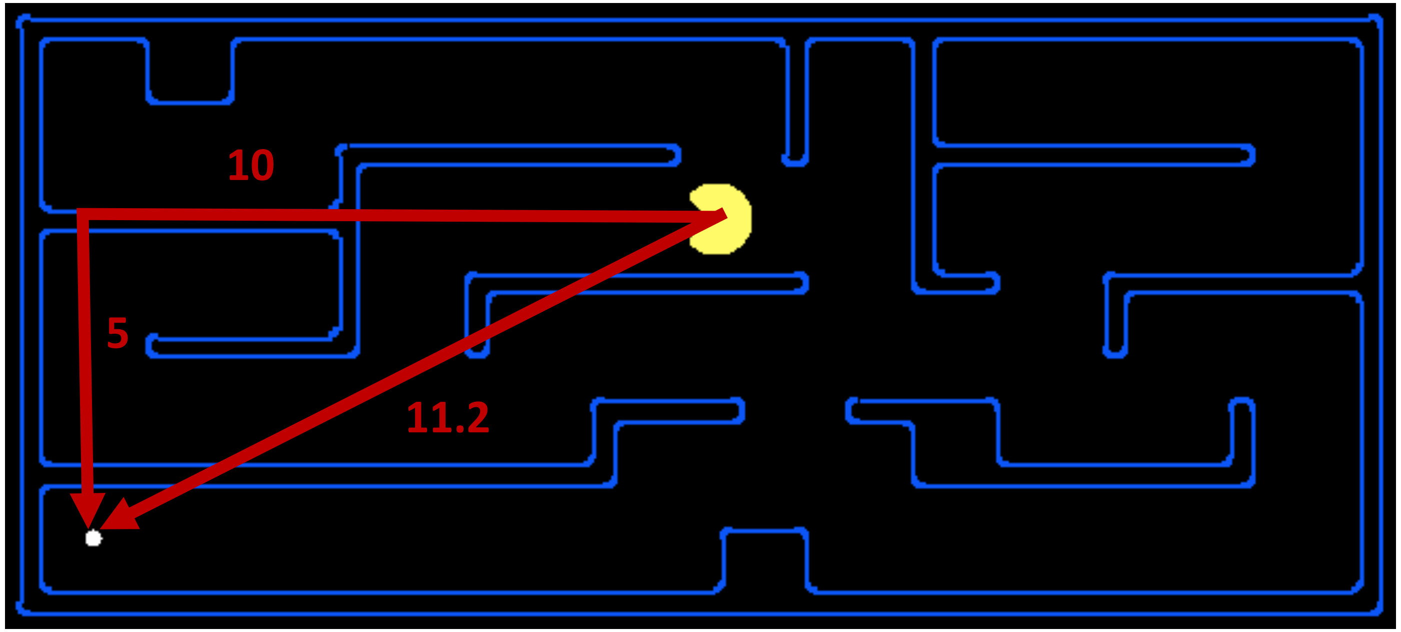
Informed Search

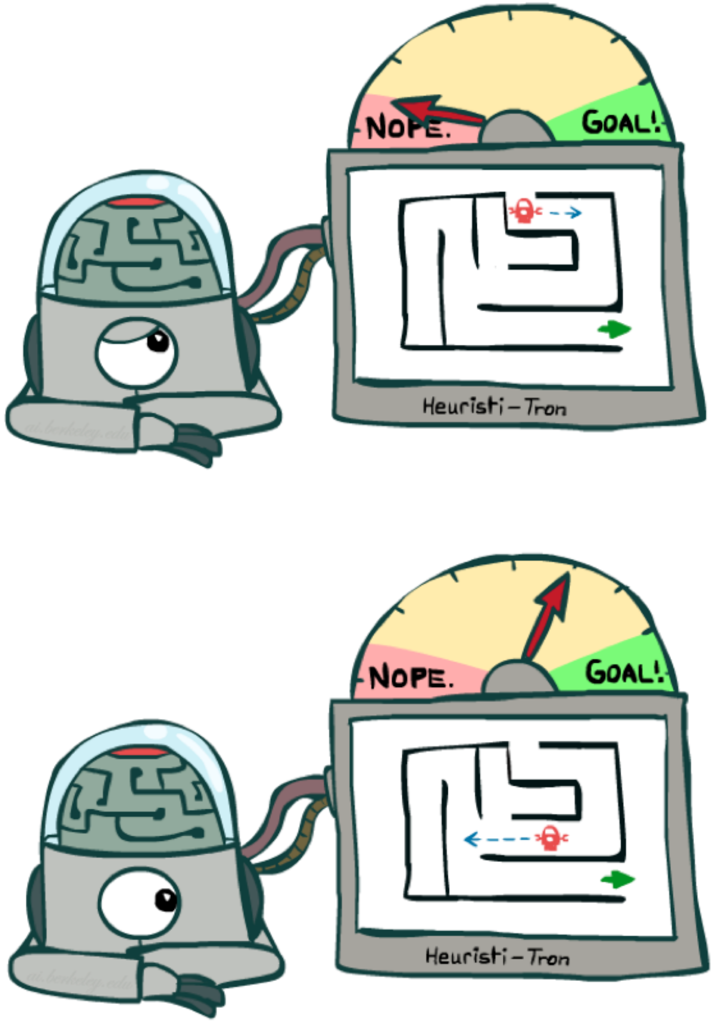
Informed Search



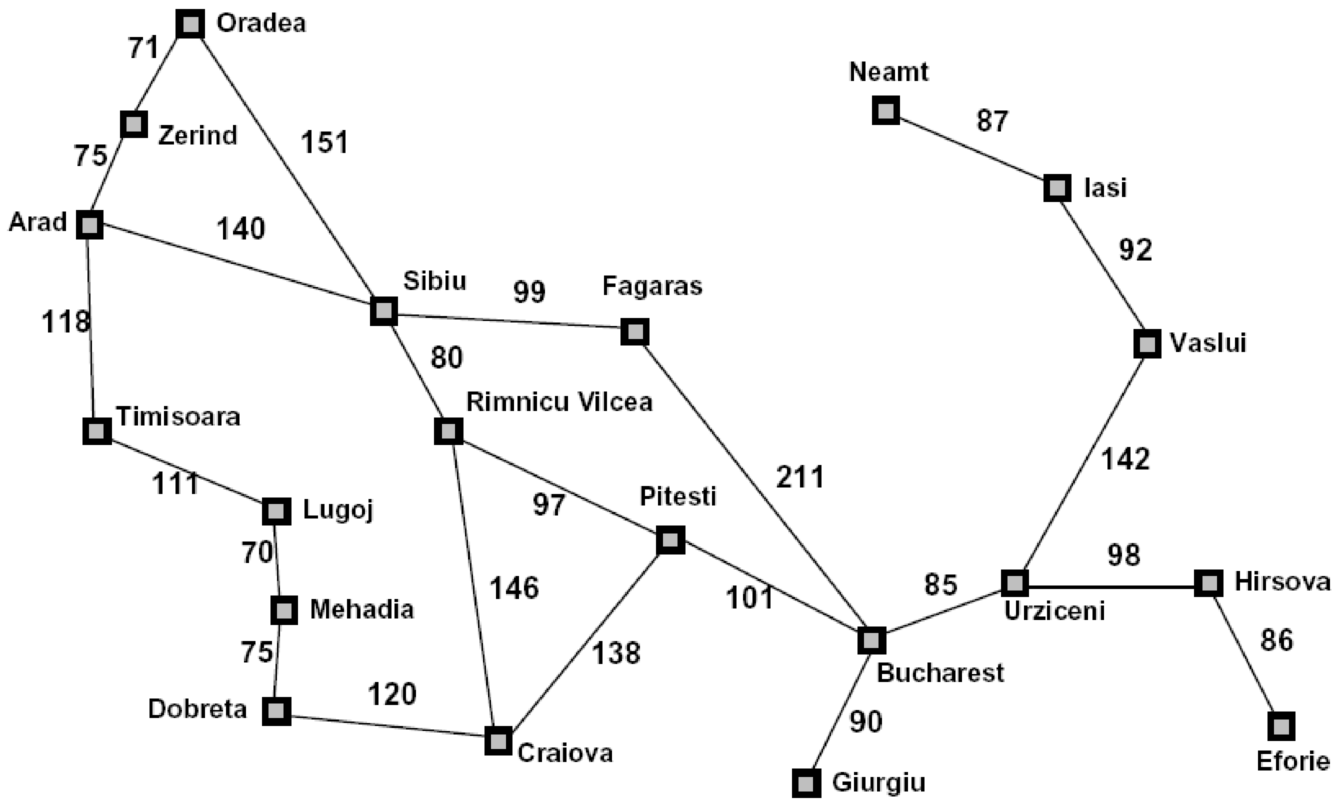
Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing





Example: Heuristic Function



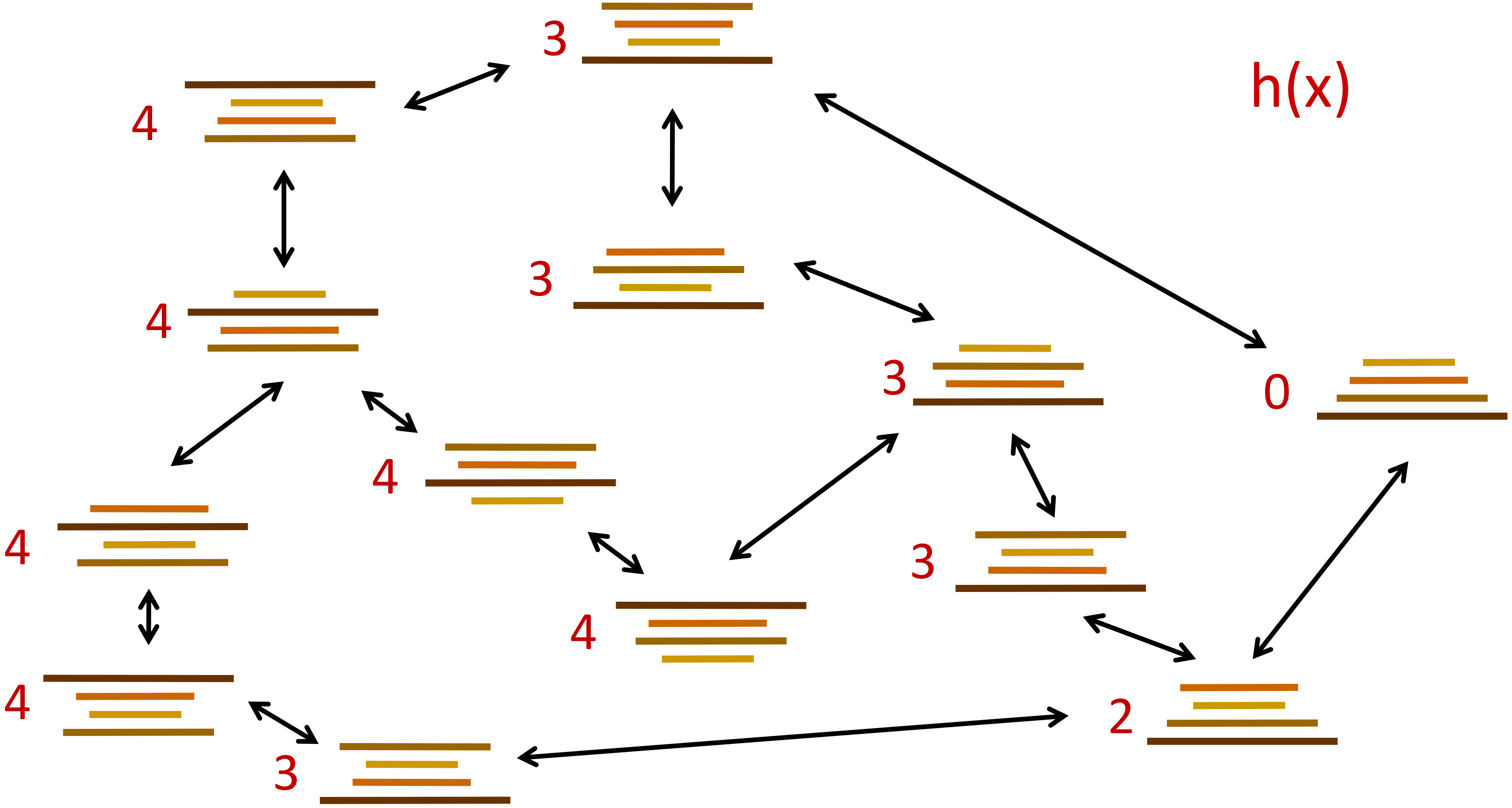
Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

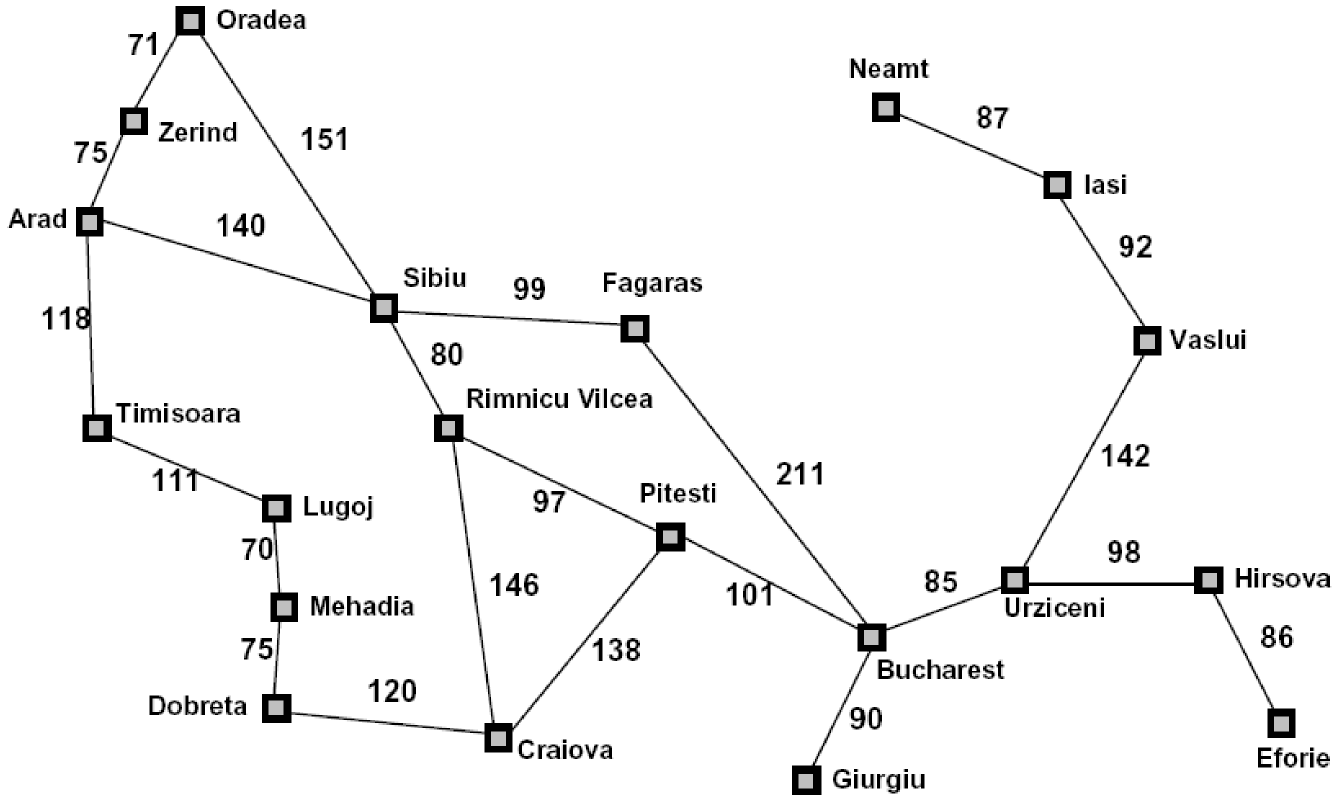
Example: Heuristic Function

Heuristic: the number of the largest pancake that is still out of place



Greedy Search

Example: Heuristic Function



Straight−line distance to Bucharest

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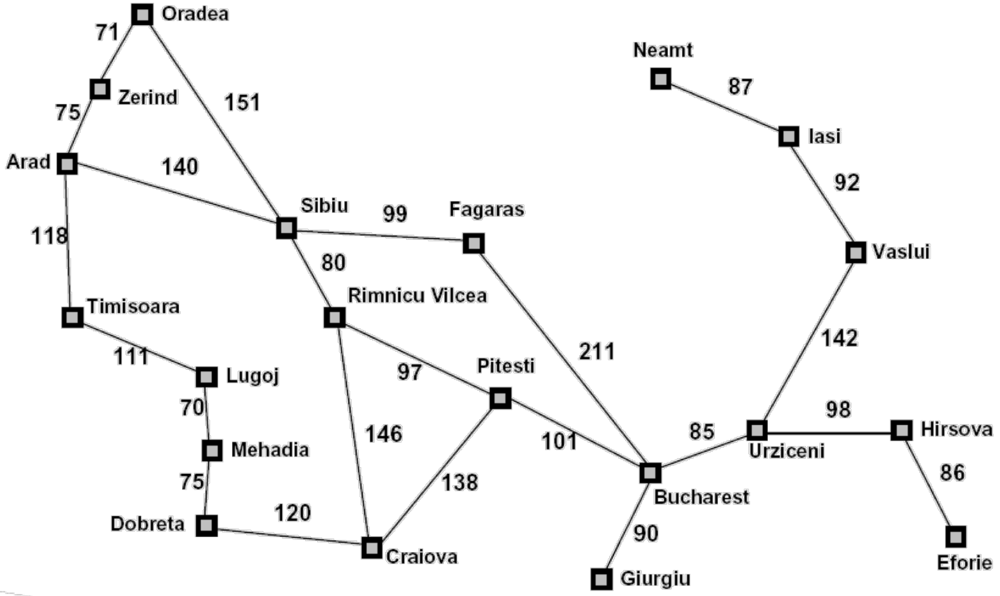
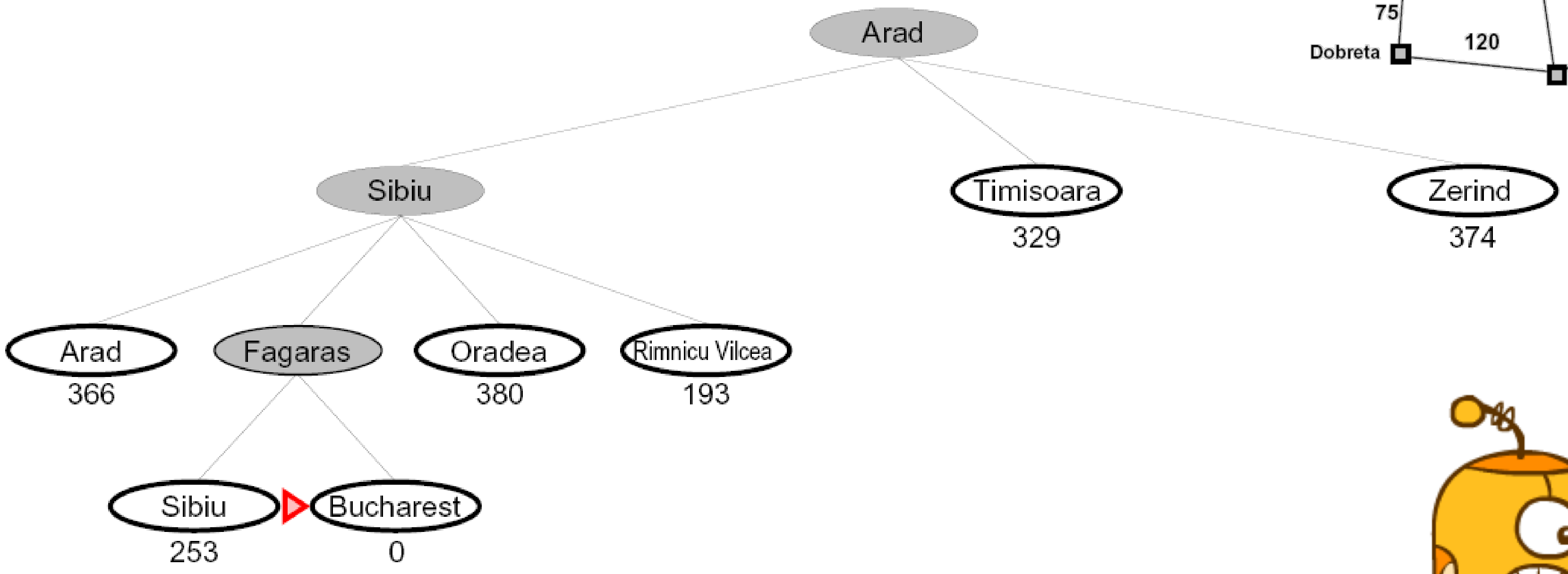
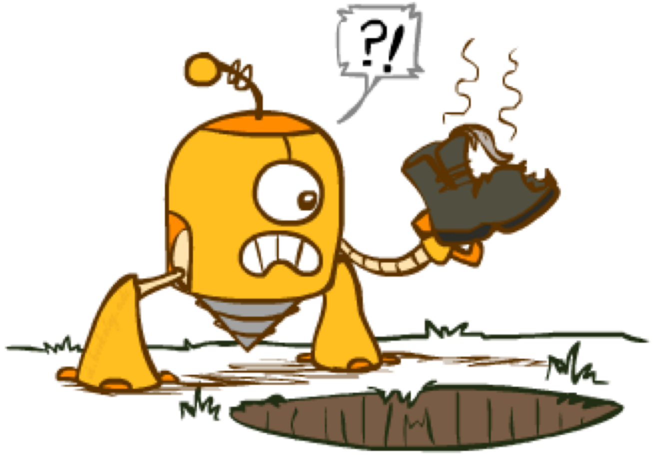
h(x)

Greedy Search

- Expand the node that seems closest…



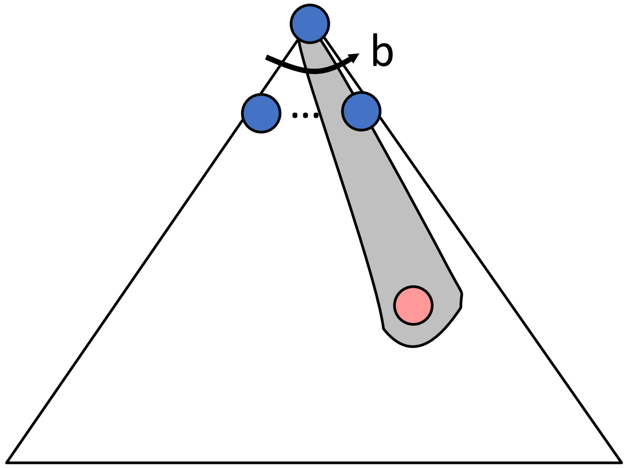




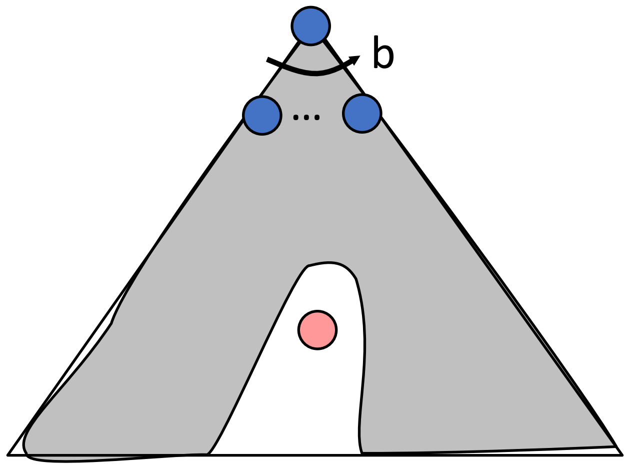
- What can go wrong?

Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state



- A common case:
  - Best-first takes you straight to the (wrong) goal



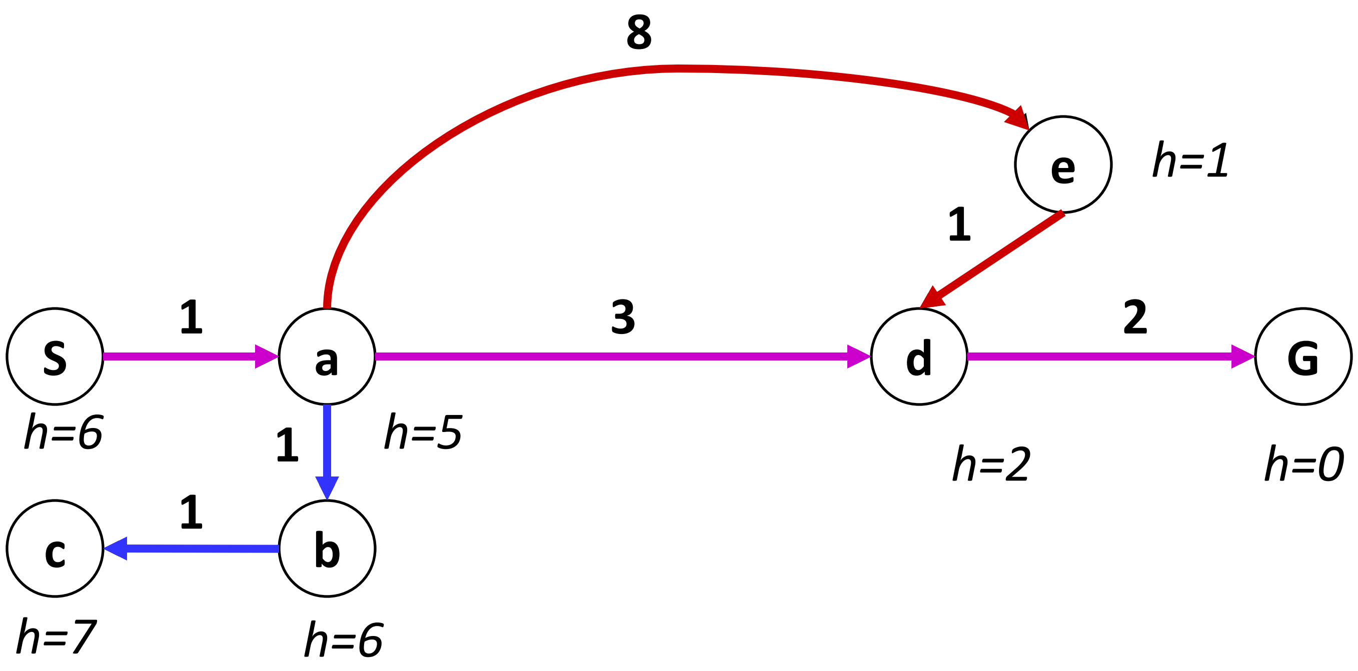
- Worst-case: like a badly-guided DFS

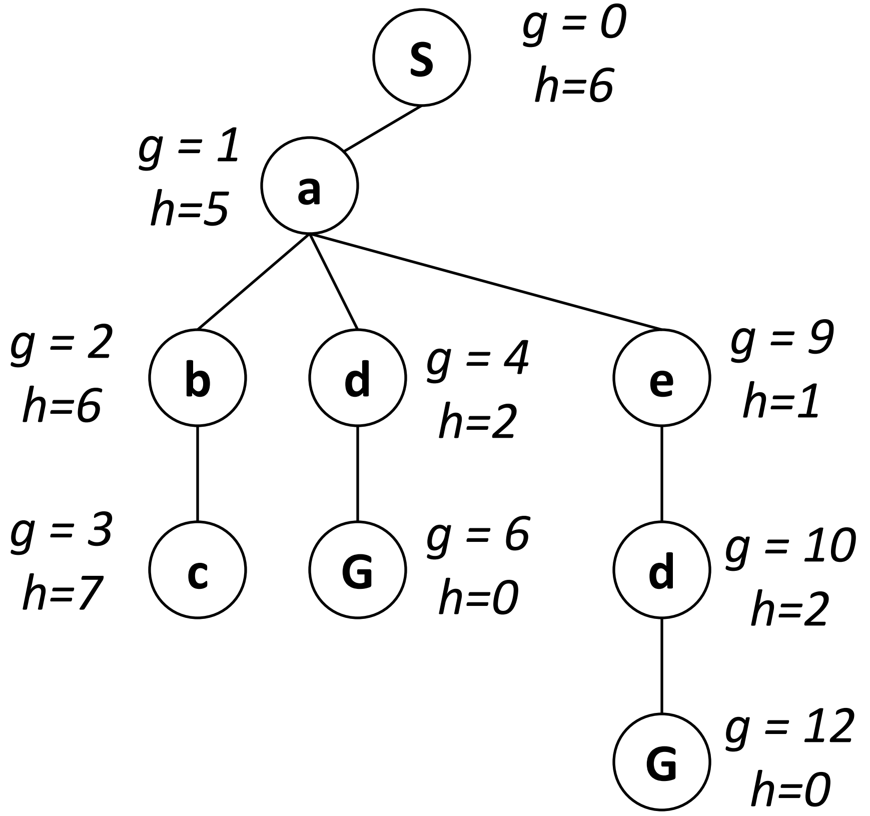
[Demo: contours greedy empty (L3D1)]
[Demo: contours greedy pacman small maze (L3D4)]

A* Search

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)



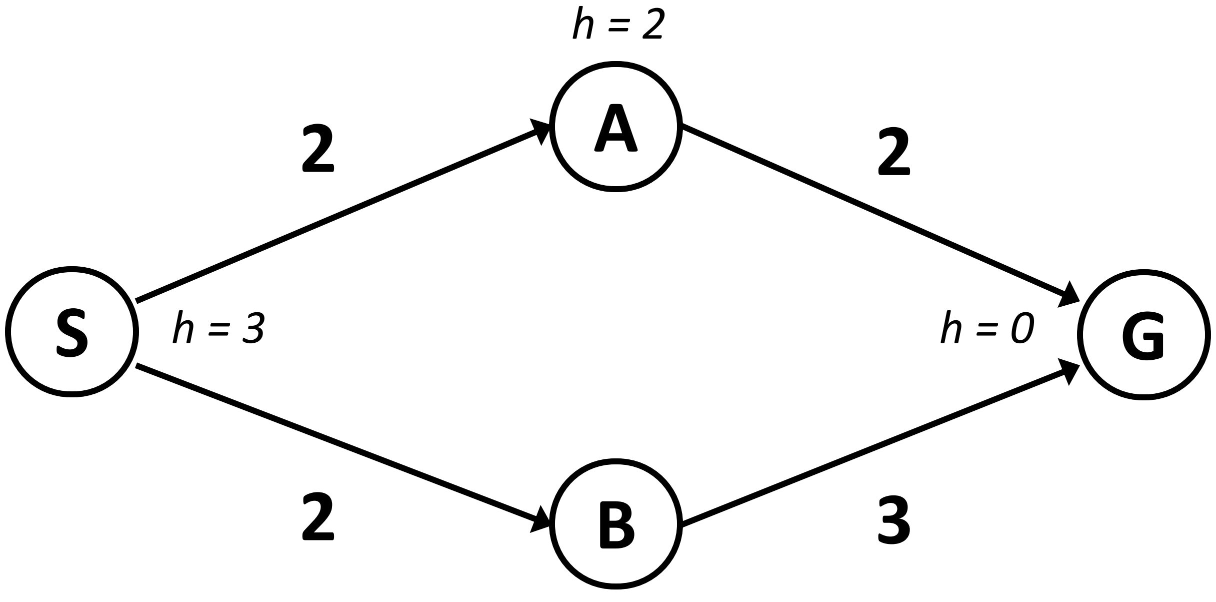


- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

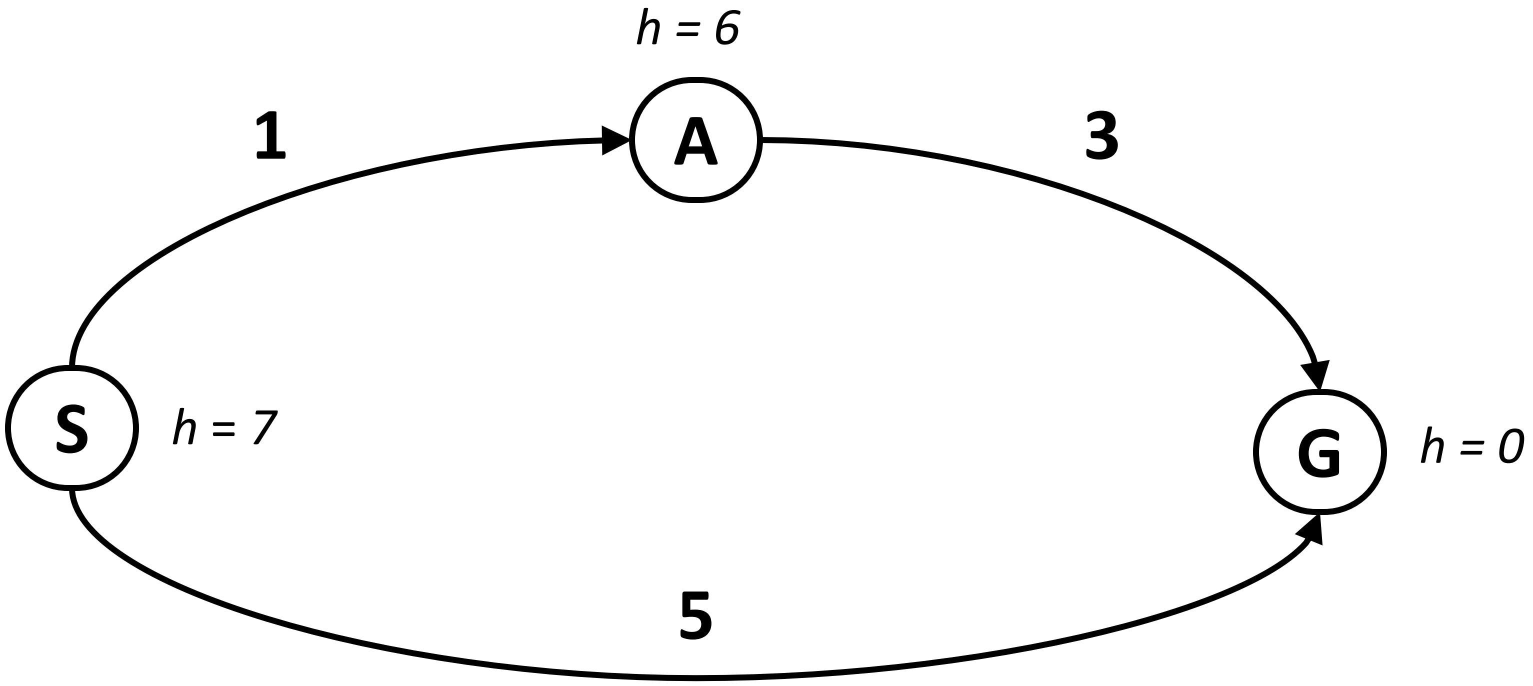
When should A* terminate?

- Should we stop when we enqueue a goal?



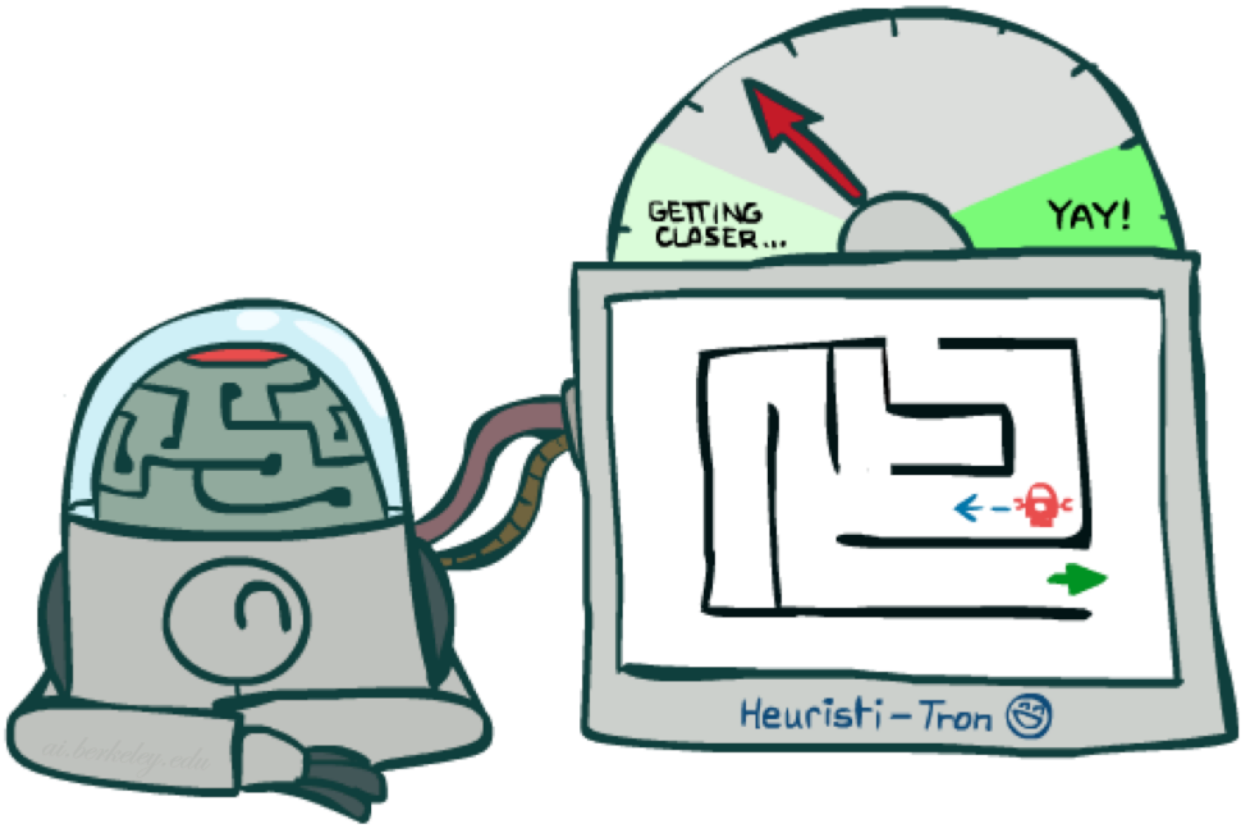
- No: only stop when we dequeue a goal

Is A* Optimal?

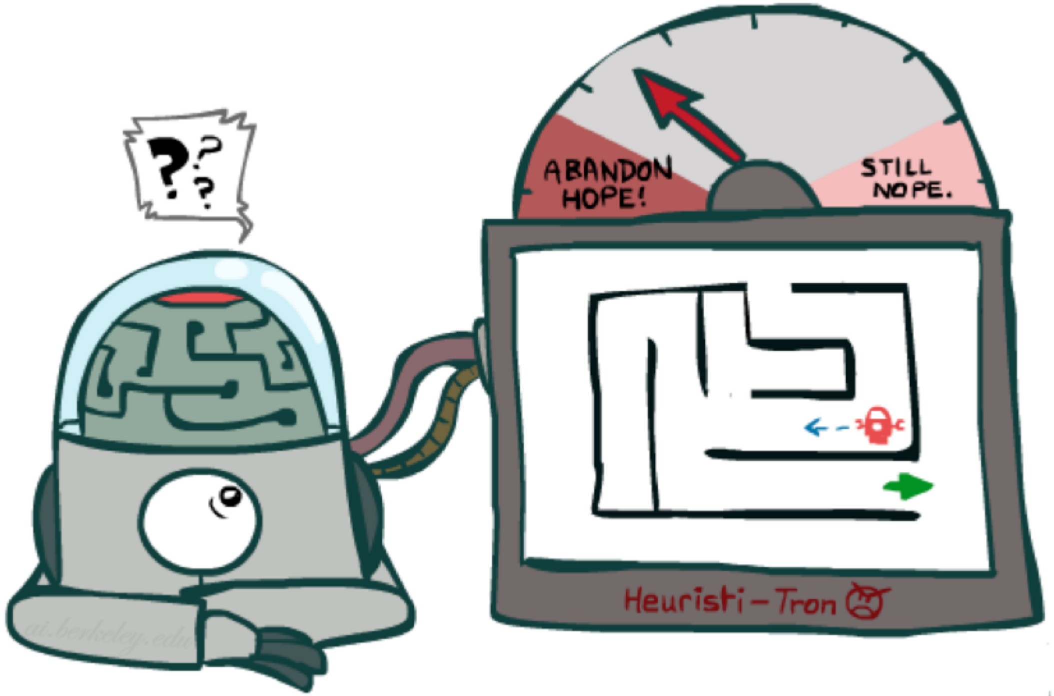


- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

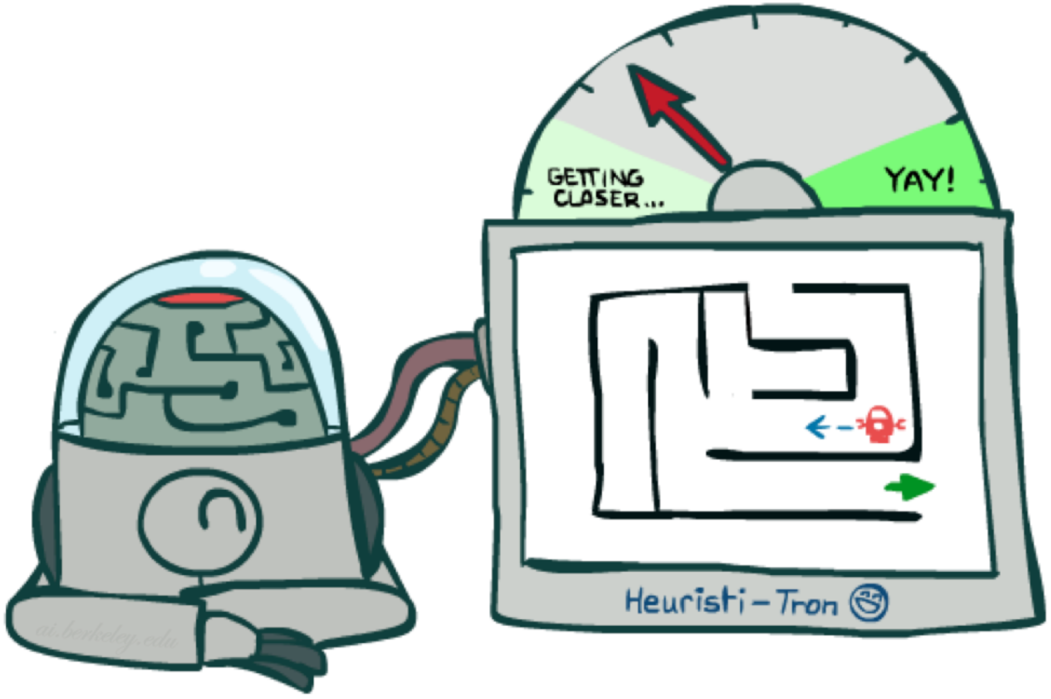
Admissible Heuristics



Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

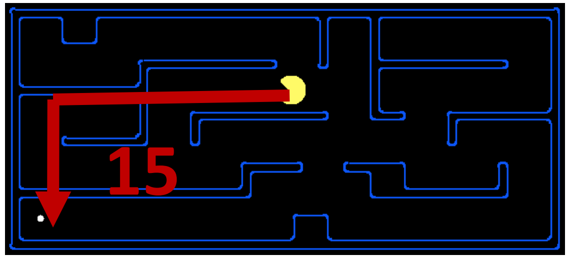
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:



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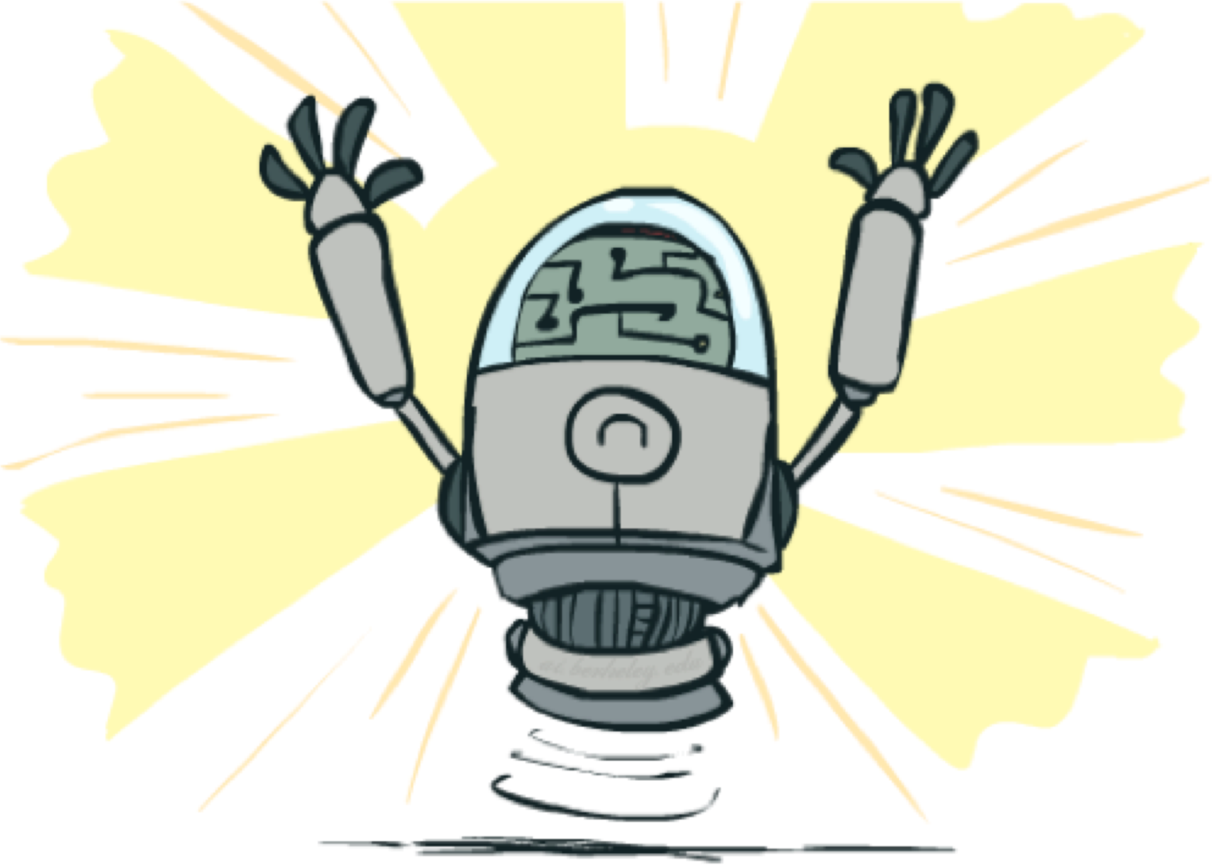
- Coming up with admissible heuristics is most of what's involved in using A* in practice.

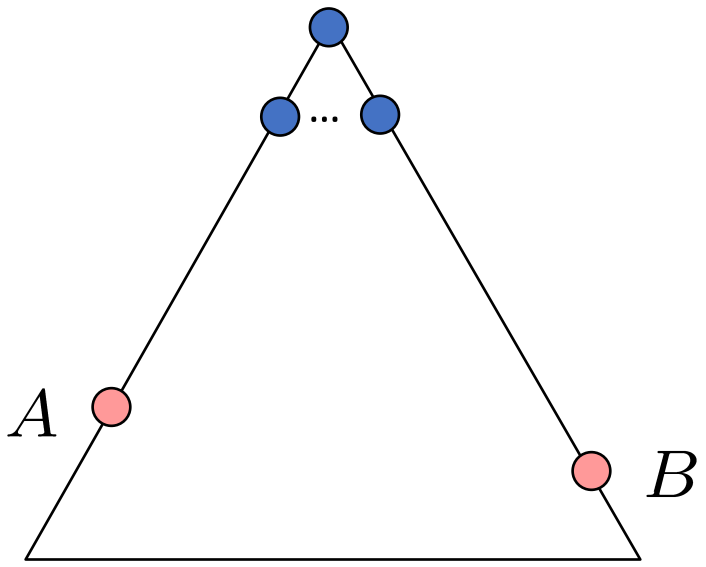
Optimality of A* Tree Search



Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible



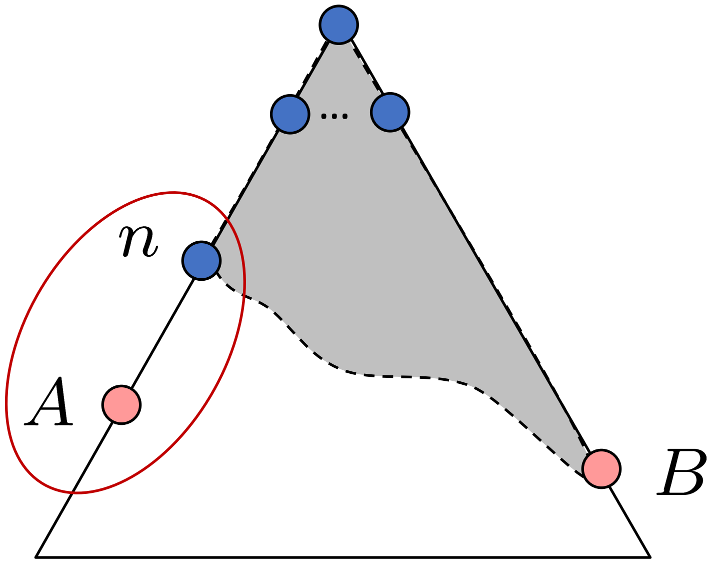
Claim:

- A will exit the fringe before B

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
    1. f(n) is less or equal to f(A)



$$f(n) = g(n) + h(n)$$ Definition of f-cost

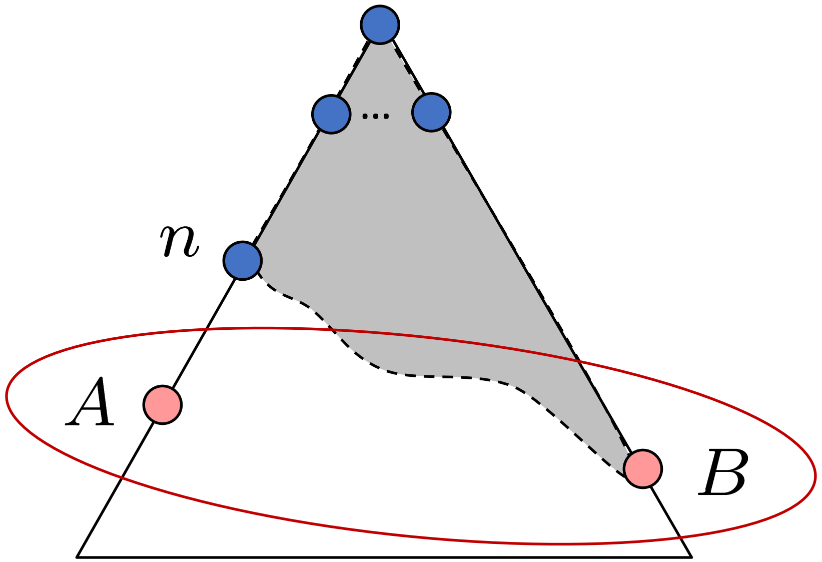
$$f(n) \leq g(A)$$ Admissibility of h

$$g(A) = f(A)$$ h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)



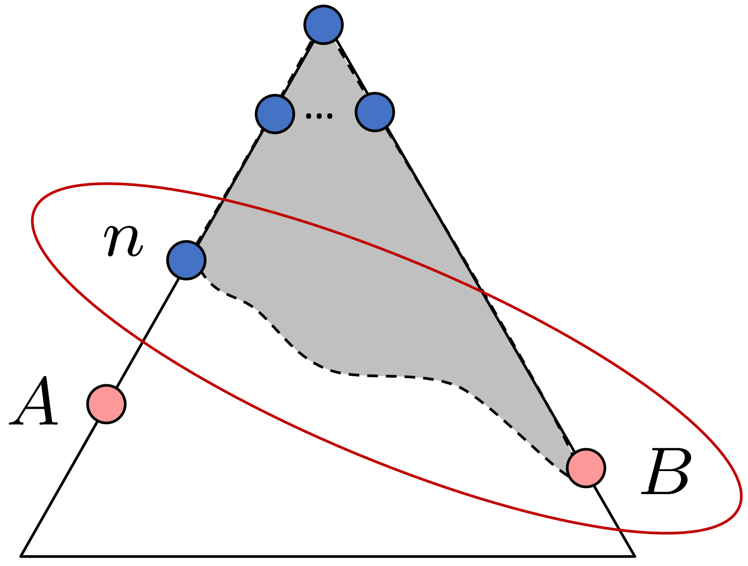
$g(A) < g(B)$ B is suboptimal

$f(A) < f(B)$ h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

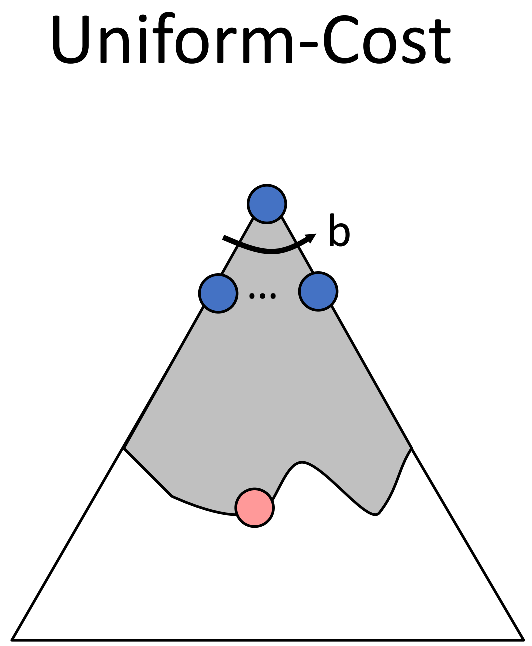
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal

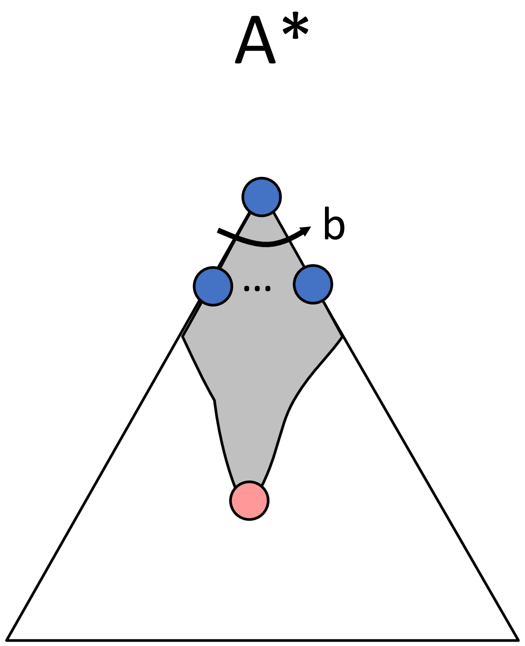


$$f(n) \leq f(A) < f(B)$$

Properties of A*

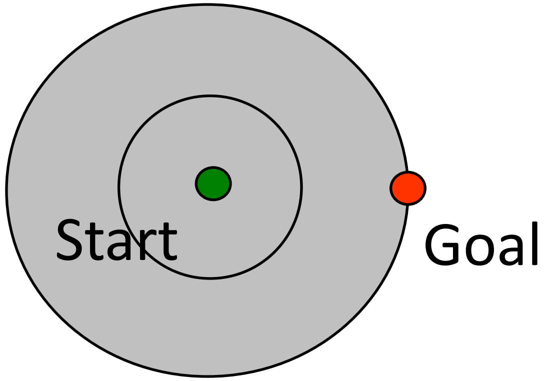
Properties of A*



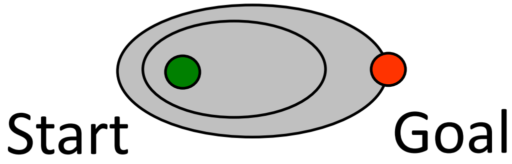


UCS vs A* Contours

- Uniform-cost expands equally in all "directions"

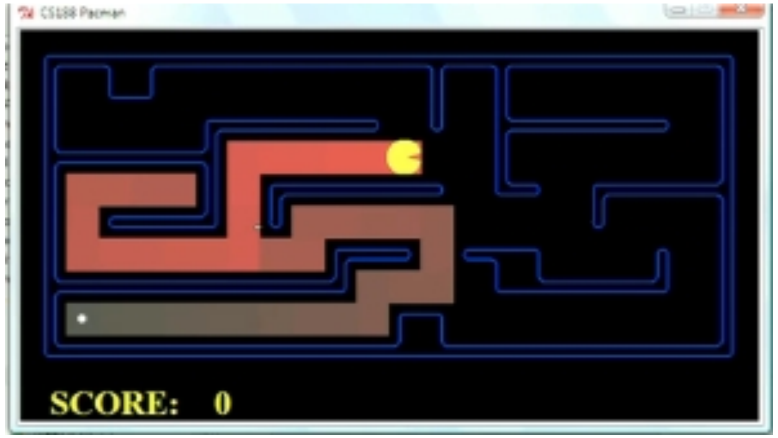


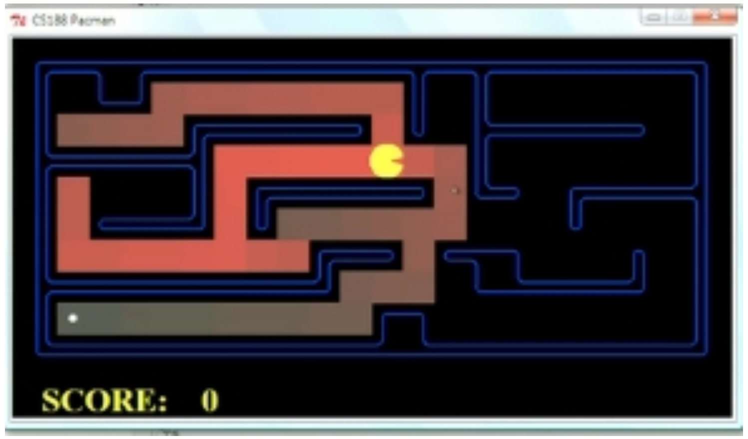
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



[Demo: contours UCS / greedy / A* empty (L3D1)]
[Demo: contours A* pacman small maze (L3D5)]

Comparison



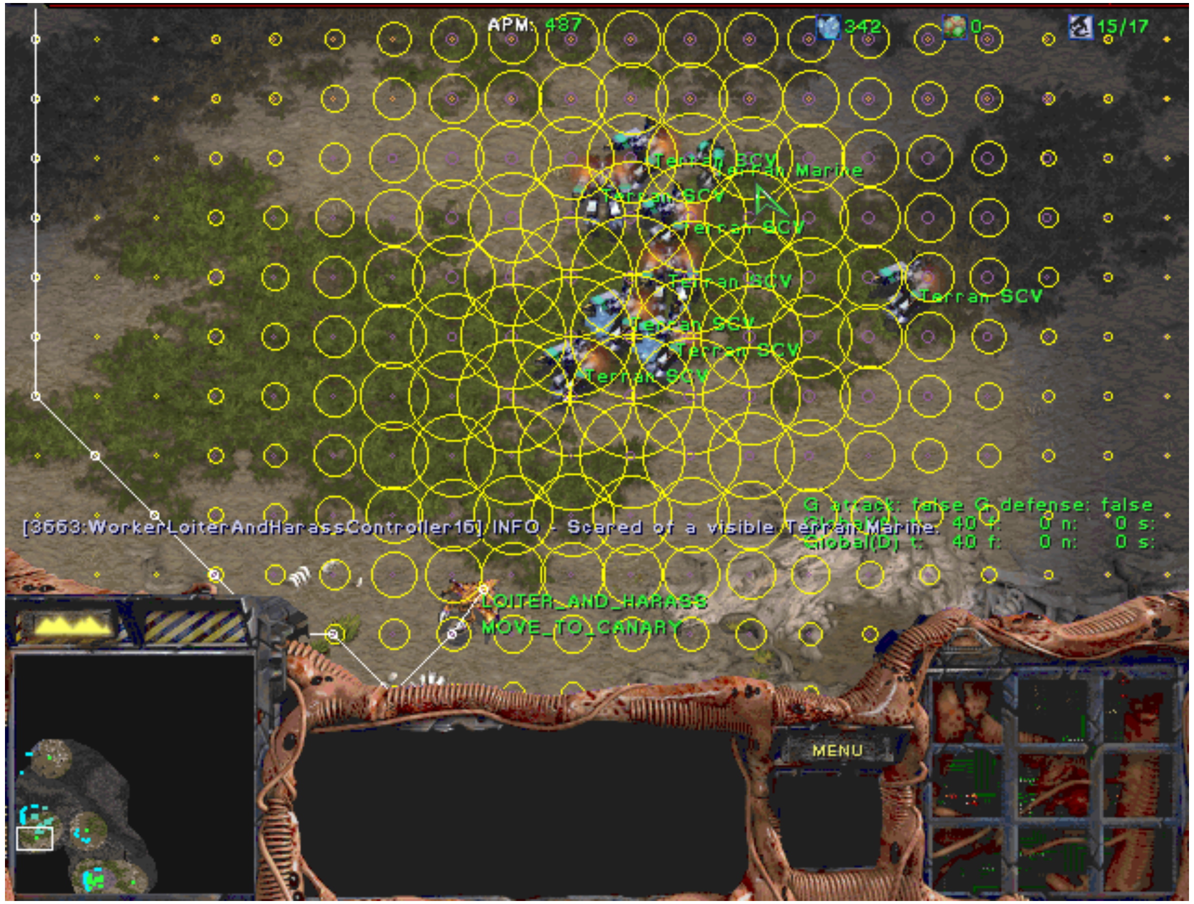


Greedy

Uniform Cost

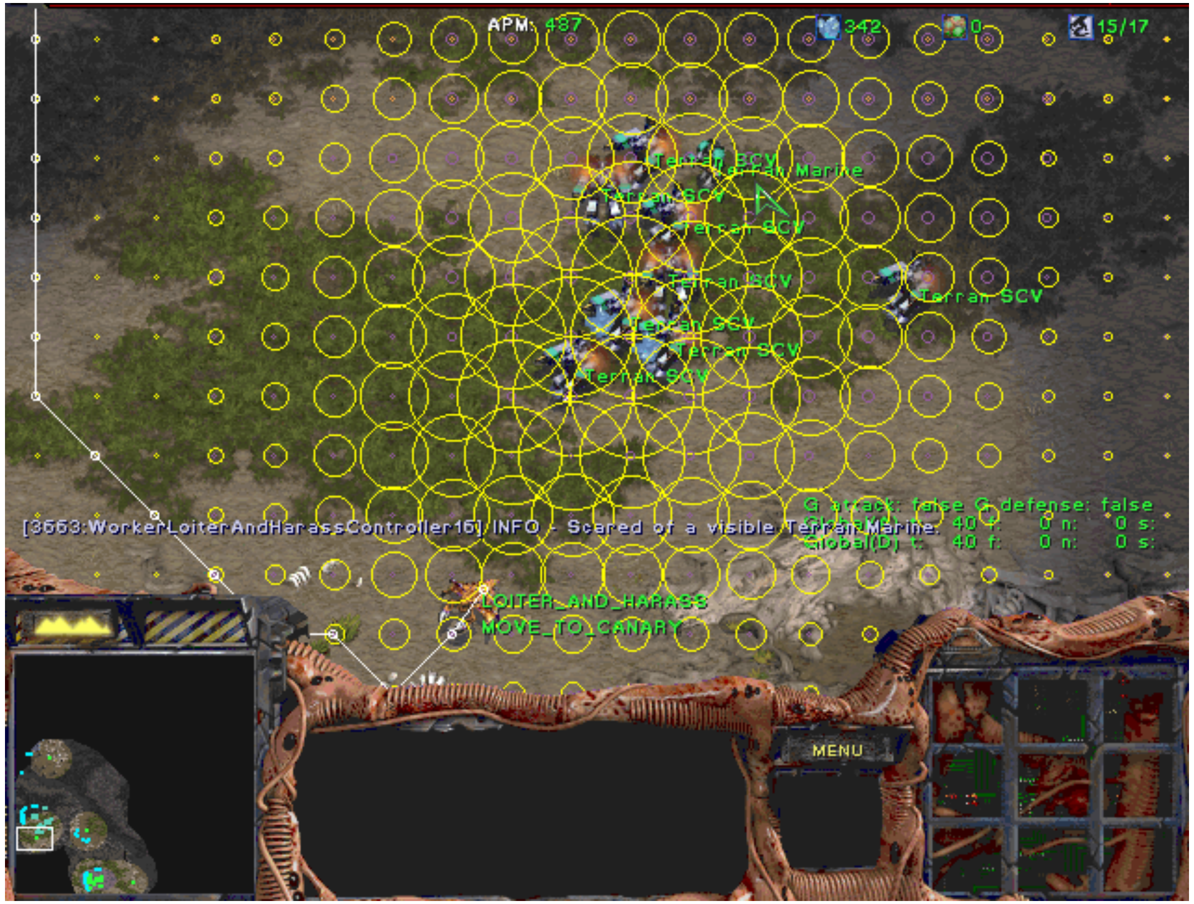
A*

A* Applications



A* Applications

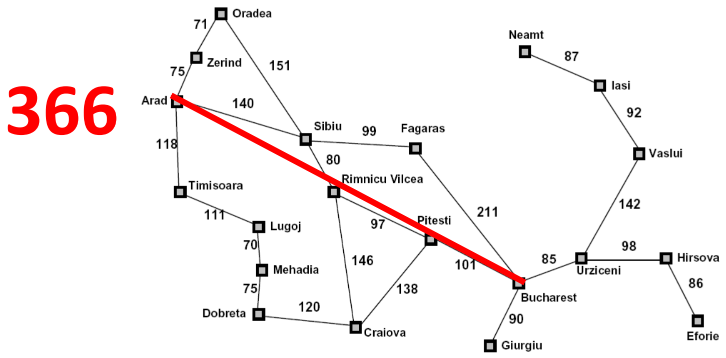
- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

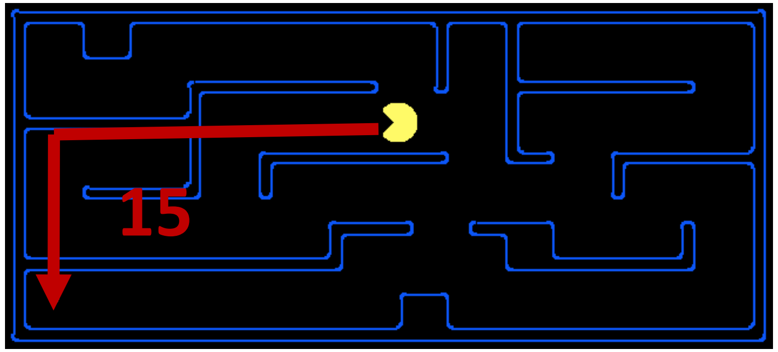


[Demo: UCS / A* pacman tiny maze (L3D6,L3D7)]
[Demo: guess algorithm Empty Shallow/Deep (L3D8)]

Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available

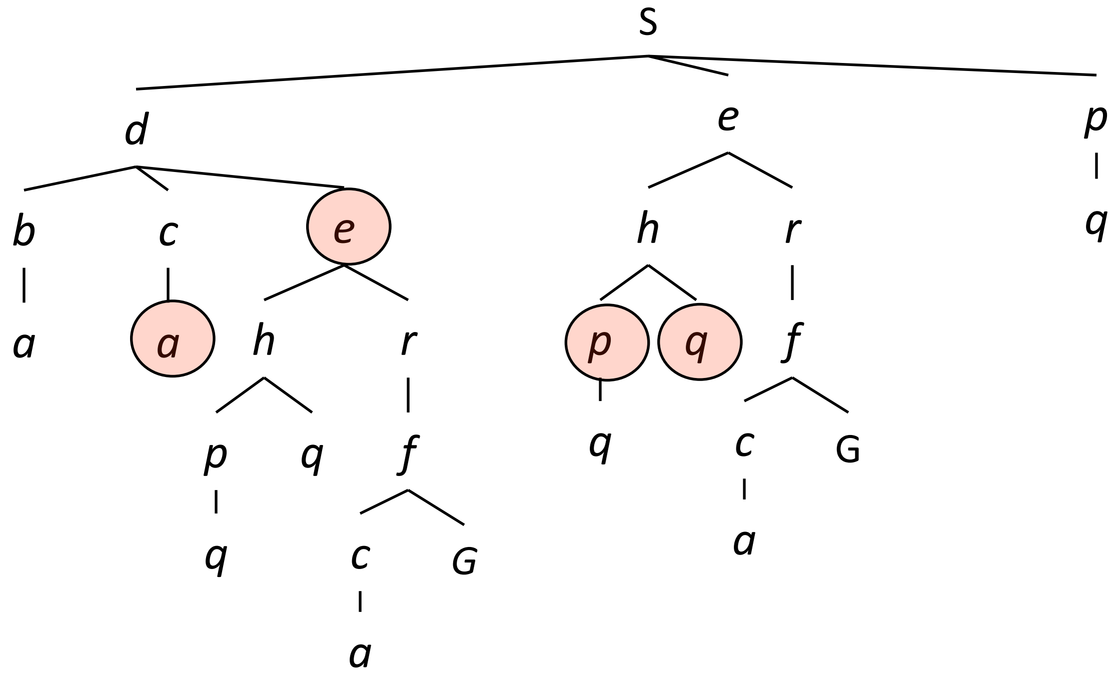




- Inadmissible heuristics are often useful too

Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

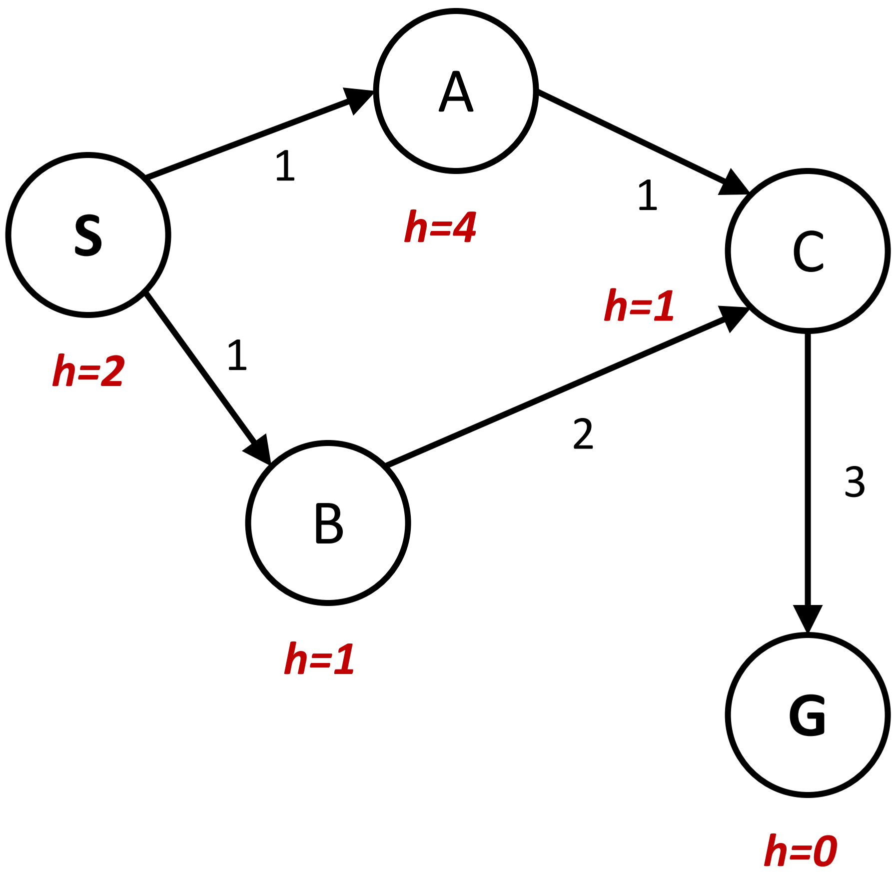


Graph Search

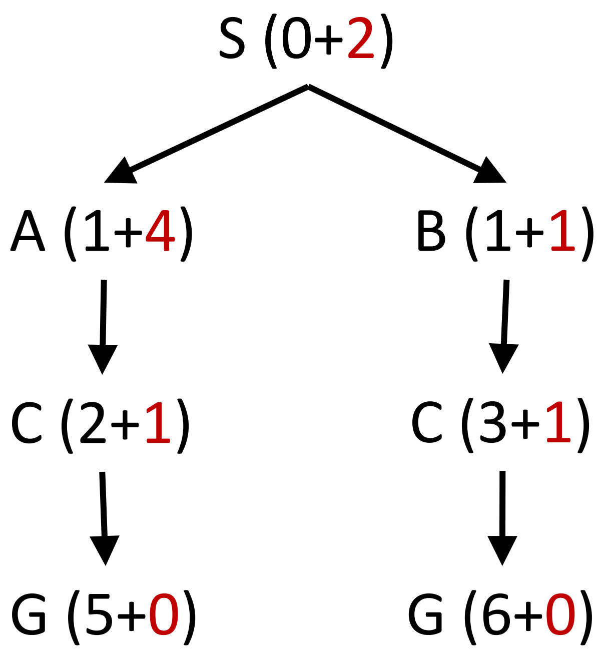
- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong?

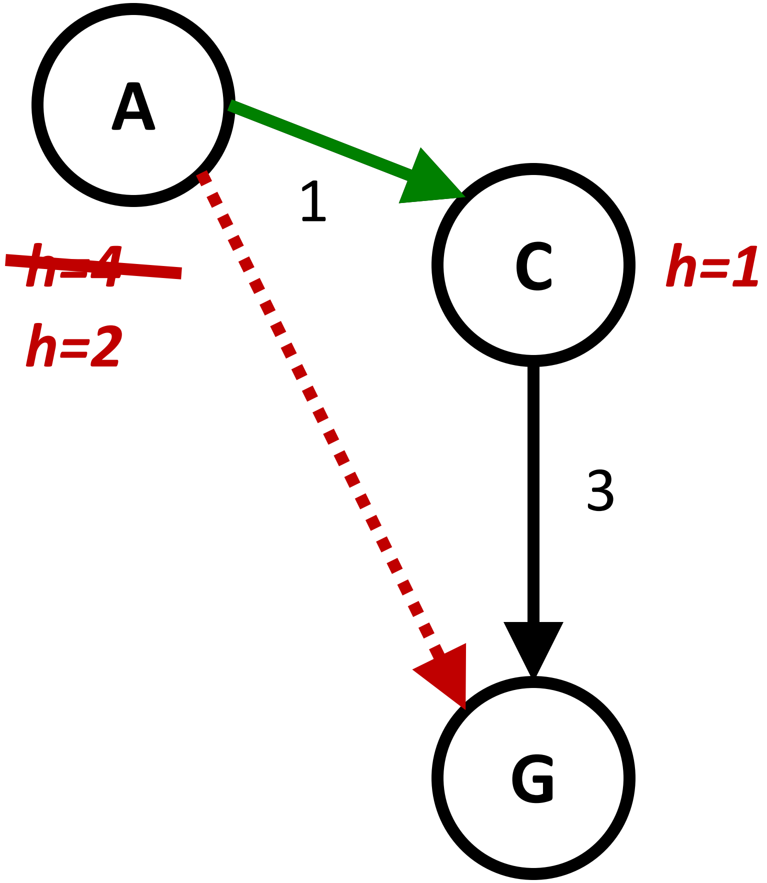
State space graph



Search tree



Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G

  - Consistency: heuristic “arc” cost ≤ actual cost for each arc

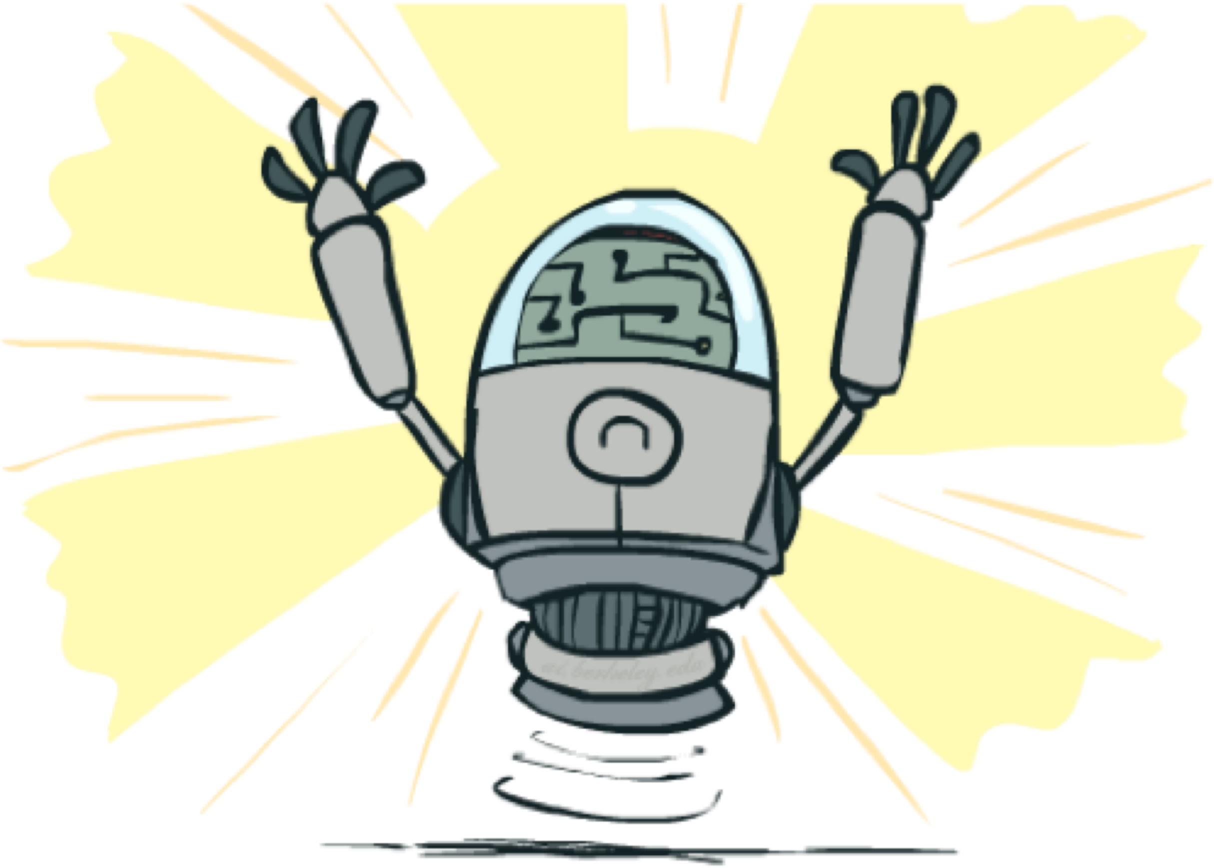
    h(A) – h(C) ≤ cost(A to C)

- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)

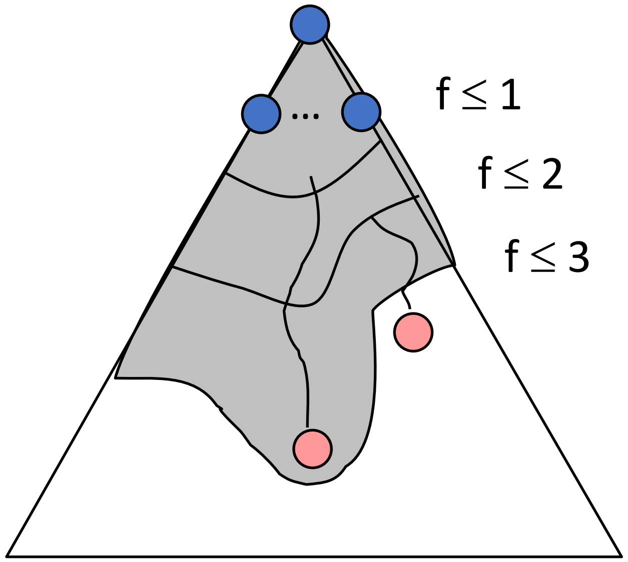
  - A* graph search is optimal

Optimality of A* Graph Search



Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



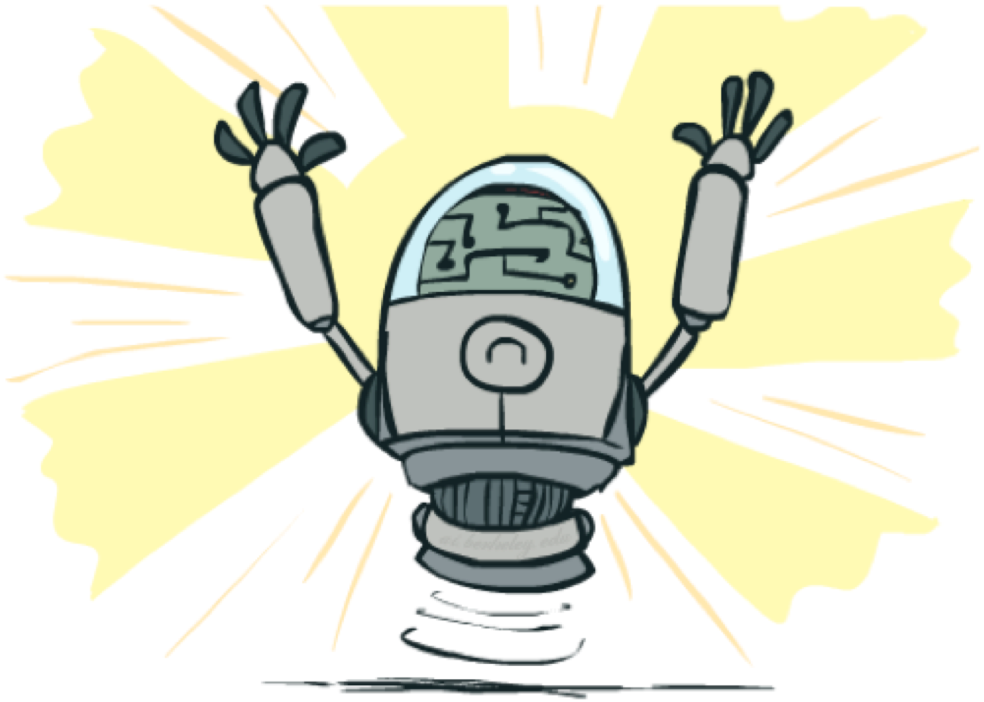
Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)

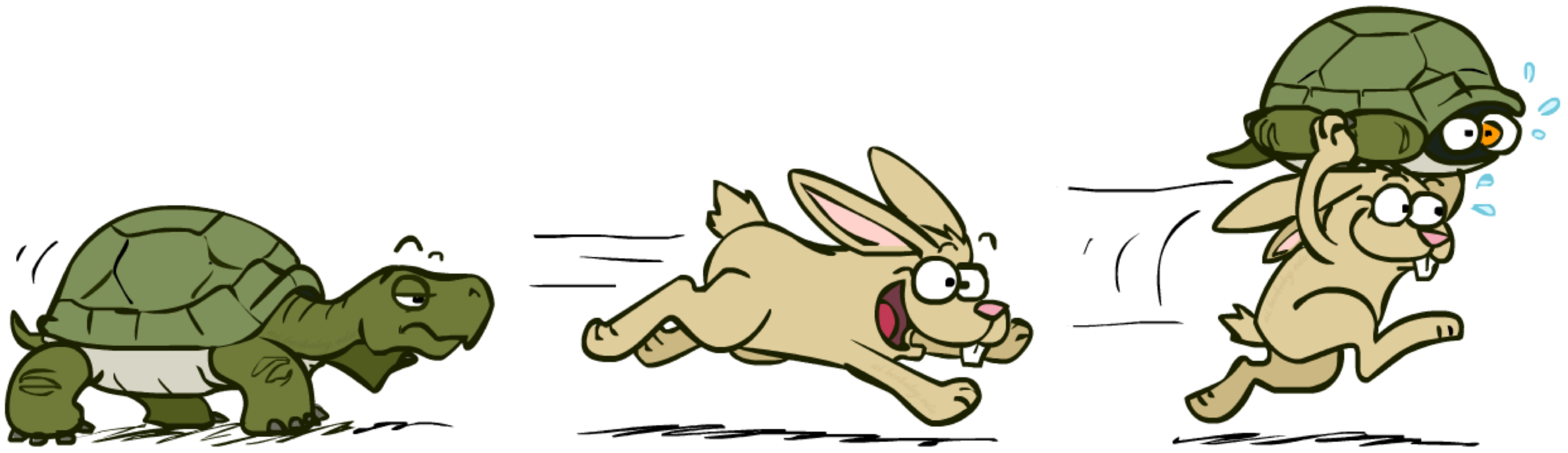
- Consistency implies admissibility

- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems



A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```