Estimating Lighthouse Intrinsic Parameters Using Levenberg-Marquardt Optimization

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Abstract

In this project we modify and implement the Levenberg-Marquardt Optimization to solve for the intrinsic parameters matrix of the lighthouse. This estimation process is a one-time calibration which helps to estimate the intrinsic parameters which we can later use for pose estimation. The aim of the calibration step is to hopefully improve pose estimation in VR. Results on simulated data as well as real data are mentioned at the end of the paper

1. Introduction

As a part of the class, we had assumed many things about the intrinsic parameters of the lighthouse itself such as (focus = 1 and the principal points of the lighthouse being at the origin). While these assumptions might be reasonable it leads to errors in the estimated pose of the user of the head mounted display using the photodiodes. The aim of the project is to get rid of this assumption and find a way to estimate these intrinsic parameters which we can then integrate in our existing homography based pose estimation method which will hopefully improve the estimated pose. In our current system if we keep our pose fixed to a point we observe that there is a large amount of jitteriness in the visual scene which is due to the wrongly estimated pose of the head mounted display (due to our assumptions about the intrinsic parameters).

To estimate the intrinsic parameters we modify the Levenberg-Marquardt algorithm that we had studied in class to estimate the pose and modify it such that it can estimate the intrinsic parameters of the lighthouse as well along with the pose. The parameters that we try to estimate are the focal length and the principal points of the lighthouse.

This is a one-time estimation technique that we do before hand with a bunch of recorded photodiode data. Once we have these intrinsic parameters estimated , we feed them into the pose estimation system and study the results.

2. Intrinsic Parameters Matrix

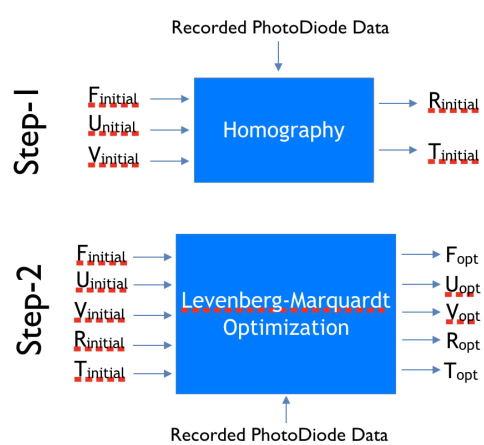
For our system we estimate the α parameters and the principal point u_0 and v_0. x^d and y^d are the distorted parameters that we retrieve from the recorded photodiode data.

$$K = \begin{pmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x^d \\ y^d \\ 1 \end{pmatrix} = \mathrm{K} \begin{pmatrix} x^n \\ y^n \\ 1 \end{pmatrix}$$

3. Overall System

The overall system includes two steps,



We first select some initial guess for the focal length and the principal points and run homography method to get an initial guess for the pose parameters. Once we have that we use this set of initial guesses to run the Levenberg-Marquardt optimization on the recorded photodiode data and get the optimal intrinsic parameters.

A graphical representation of the system is shown in Fig.1

4. Testing System Correctness

To test if the intrinsic parameter values estimated were in fact the correct values, we simulate lighthouse timestamps along with user-defined intrinsic parameter values and then we run the LM optimization and see if we get the same values back. This way we can test for the correctness of the implementation.

Here are the results for the simulation,

Mean Relative Error in F : 6%

Mean Relative Error in U_0 : 14%

Mean Relative Error in V_0 : 10%

Also the average residual when optimal parameters were used was $2.5 * 10^{-6}$ and when the optimal parameters were given default values of ($U_0 = V_0 = 0$ and $F = 1$) we got a residual of **0.067**. This clearly shows an improvement in the pose estimation system.

4.1. Results on Real Data

Here are the results for the real data,

Estimated U_0 : -0.00439392

Estimated V_0 : 0.426495

Estimated F : 0.979

Also the average residual when optimal parameters were used was $5.6 * 10^{-5}$ and when the optimal parameters were given default values of ($U_0 = V_0 = 0$ and $F = 1$) we got a residual of **0.092**. This clearly shows an improvement in the pose estimation system for real data as well.

5. Takeaways

- Importance of lighthouse calibration for improving pose estimation

- Usage and implementation of an optimization technique such as Levenberg-Marquardt for calibrating the lighthouse base station and pose estimation

6. Conclusions and Future Work

- In VR , such a parameter estimation improves pose estimation and reduces jitteriness in the visual scene

- Future work could include modifying the LM algorithm to calculate a more generic intrinsic parameter matrix

7. Implementation

All the code to read serial port and record photodiode data into text files has been written in python.

The Levenberg-Marquardt algorithm has been implemented in C++ as a C++ class which makes it also usable for real-time applications.

8. References

[1] Zhang, Zhengyou. "A flexible new technique for camera calibration." IEEE Transactions on pattern analysis and machine intelligence 22.11 (2000): 1330-1334. [2] Shao-xiong, Tian, Lu Shan, and Liu Zong-ming. "Levenberg-Marquardt algorithm based nonlinear optimization of camera calibration for relative measurement." Control Conference (CCC), 2015 34th Chinese. IEEE, 2015.

Solving LM

$$R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}$$

$$R_y(\theta_y) = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix}$$

$$R_z(\theta_z) = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = R_z(\theta_z) R_x(\theta_x) R_y(\theta_y)$$

R is given in the next page

$$\underbrace{\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}}_{\mathbf{R}} = \underbrace{\begin{pmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{R}_z(\theta_z)} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{pmatrix}}_{\mathbf{R}_x(\theta_x)} \underbrace{\begin{pmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{pmatrix}}_{\mathbf{R}_y(\theta_y)}$$

$$= \begin{pmatrix} \cos(\theta_y)\cos(\theta_z) - \sin(\theta_x)\sin(\theta_y)\sin(\theta_z) & -\cos(\theta_x)\sin(\theta_z) & \sin(\theta_y)\cos(\theta_z) + \sin(\theta_x)\cos(\theta_y)\sin(\theta_z) \\ \cos(\theta_y)\sin(\theta_z) + \sin(\theta_x)\sin(\theta_y)\cos(\theta_z) & \cos(\theta_x)\cos(\theta_z) & \sin(\theta_y)\sin(\theta_z) - \sin(\theta_x)\cos(\theta_y)\cos(\theta_z) \\ -\cos(\theta_x)\sin(\theta_y) & \sin(\theta_x) & \cos(\theta_x)\cos(\theta_y) \end{pmatrix} \tag{36}$$

R matrix

$$H = K \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix}$$

where K is the calibration matrix

Our K matrix is given by

$$K = \begin{bmatrix} f & 0 & x_p \\ 0 & f & y_p \\ 0 & 0 & -1 \end{bmatrix}$$

f = focal length

$(x_p, y_p) \Rightarrow$ principal point

New H is as follows

$$H = \begin{bmatrix} f & 0 & x_p \\ 0 & f & y_p \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix}$$

$$H = \begin{bmatrix} f r_{11} + x_p r_{31} & f r_{12} + x_p r_{32} & f t_x + x_p t_z \\ f r_{21} + y_p r_{31} & f r_{22} + y_p r_{32} & f t_y + y_p t_z \\ -r_{31} & -r_{32} & -t_z \end{bmatrix}$$

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

$$\tilde{h} = [h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9]$$

Lets write each element of $\tilde{h}$ in terms of θ_x, θ_y, θ_z, f, x_p and y_p

$$h_1 = f r_{11} + x_p r_{31}$$

$$h_1 = f\left(\cos(\theta_y)\cos(\theta_z) - \sin(\theta_x)\sin(\theta_y)\sin(\theta_z)\right) -$$
$$x_p \cos(\theta_x)\sin(\theta_y)$$

$$h_2 = -f\cos(\theta_x)\sin(\theta_z) + x_p \sin(\theta_x)$$

$$h_3 = f t_x + x_p t_z$$

$$h_4 = f\left(\cos(\theta_y)\sin(\theta_z) + \sin(\theta_x)\sin(\theta_y)\cos(\theta_z)\right)$$
$$- y_p \cos(\theta_x)\sin(\theta_y)$$

$$h_5 = f\cos(\theta_x)\cos(\theta_z) + y_p \sin(\theta_x)$$

$$h_6 = f t_y + y_p t_z$$

$$h_7 = \cos\theta_x \sin\theta_y$$

$$h_8 = -\sin\theta_x$$

$$h_9 = -t_z$$

Let $P = [\theta_x, \theta_y, \theta_z, t_x, t_y, t_z, f, x_p, y_p]$

$$g(P) = \begin{bmatrix} g_1(P) \\ g_2(P) \\ g_3(P) \\ g_4(P) \\ g_5(P) \\ g_6(P) \\ g_7(P) \\ g_8(P) \\ g_9(P) \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix}$$

$$f(\tilde{h}) = \begin{bmatrix} f_1(\tilde{h}) \\ f_2(\tilde{h}) \\ f_3(\tilde{h}) \\ f_4(\tilde{h}) \\ f_5(\tilde{h}) \\ f_6(\tilde{h}) \\ f_7(\tilde{h}) \\ f_8(\tilde{h}) \end{bmatrix} = \begin{bmatrix} x_1^n \\ y_1^n \\ x_2^n \\ y_2^n \\ x_3^n \\ y_3^n \\ x_4^n \\ y_4^n \end{bmatrix}$$

Let calculate gradients nows

$$\frac{\partial g_1}{\partial p_1} = -f\cos(\theta_x)\sin(\theta_y)\sin(\theta_z) + x_p\sin(\theta_x)\sin(\theta_y)$$

$$\frac{\partial g_1}{\partial p_2} = -f\left(\sin(\theta_y)\cos(\theta_z) + \sin(\theta_x)\cos(\theta_y)\sin(\theta_z)\right) - x_p\cos(\theta_x)\cos(\theta_y)$$

$$\frac{\partial g_1}{\partial p_3} = -f\left(\cos(\theta_y)\sin(\theta_z) + \sin(\theta_x)\sin(\theta_y)\cos(\theta_z)\right)$$

$$\frac{\partial g_1}{\partial p_4} = \frac{\partial g_1}{\partial p_5} = \frac{\partial g_1}{\partial p_6} = 0$$

$$\frac{\partial g_1}{\partial p_7} = \cos(\theta_y)\cos(\theta_z) - \sin(\theta_x)\sin(\theta_y)\sin(\theta_z)$$

$$\frac{\partial g_1}{\partial p_8} = \cos(\theta_x)\sin(\theta_y)$$

$$\frac{\partial g_1}{\partial p_9} = 0$$

$$\frac{\partial g_2}{\partial P_1} = f \sin(\theta_x) \sin(\theta_z) + x_p \cos(\theta_x)$$

$$\frac{\partial g_2}{\partial P_2} = 0$$

$$\frac{\partial g_2}{\partial P_3} = -f \cos(\theta_x) \cos(\theta_z)$$

$$\frac{\partial g_2}{\partial P_4} = \frac{\partial g_2}{\partial P_5} = \frac{\partial g_2}{\partial P_6} = 0$$

$$\frac{\partial g_2}{\partial P_7} = -\cos(\theta_x) \sin(\theta_z)$$

$$\frac{\partial g_2}{\partial P_8} = \sin(\theta_x)$$

$$\frac{\partial g_2}{\partial P_9} = 0$$

$$\frac{\partial g_3}{\partial p_1} = \frac{\partial g_3}{\partial p_2} = \frac{\partial g_3}{\partial p_3} = 0$$

$$\frac{\partial g_3}{\partial p_4} = f$$

$$\frac{\partial g_3}{\partial p_5} = 0$$

$$\frac{\partial g_3}{\partial p_6} = x_p$$

$$\frac{\partial g_3}{\partial p_7} = t_x$$

$$\frac{\partial g_3}{\partial p_8} = t_z$$

$$\frac{\partial g_3}{\partial p_9} = 0$$

$$\frac{\partial g_4}{\partial p_1} = f\cos(\theta_x)\sin(\theta_y)\cos(\theta_z) + y_p\sin(\theta_x)\sin(\theta_y)$$

$$\frac{\partial g_4}{\partial p_2} = f\left(-\sin(\theta_y)\sin\theta_z + \sin(\theta_x)\cos(\theta_y)\cos(\theta_z)\right)$$
$$- y_p\cos(\theta_x)\cos(\theta_y)$$

$$\frac{\partial g_4}{\partial p_3} = f\left(\cos(\theta_y)\cos(\theta_z) - \sin(\theta_x)\sin(\theta_y)\sin(\theta_z)\right)$$

$$\frac{\partial g_4}{\partial p_4} = 0 = \frac{\partial g_4}{\partial p_5} = \frac{\partial g_4}{\partial p_6}$$

$$\frac{\partial g_4}{\partial p_7} = \cos(\theta_y)\sin(\theta_z) + \sin(\theta_x)\sin(\theta_y)\cos(\theta_z)$$

$$\frac{\partial g_4}{\partial p_8} = 0$$

$$\frac{\partial g_4}{\partial p_9} = -\cos(\theta_x)\sin(\theta_y)$$

$$\frac{\partial g_5}{\partial P_1} = -f \sin(\theta_x)\cos(\theta_z) + y_p \cos(\theta_x)$$

$$\frac{\partial g_5}{\partial P_2} = 0$$

$$\frac{\partial g_5}{\partial P_3} = -f \cos(\theta_x)\sin(\theta_z)$$

$$\frac{\partial g_5}{\partial P_4} = \frac{\partial g_5}{\partial P_5} = \frac{\partial g_5}{\partial P_6} = 0$$

$$\frac{\partial g_5}{\partial P_7} = \cos(\theta_x)\cos(\theta_z)$$

$$\frac{\partial g_5}{\partial P_8} = 0$$

$$\frac{\partial g_5}{\partial P_9} = \sin(\theta_x)$$

$$\frac{\partial g_6}{\partial p_1} = \frac{\partial g_6}{\partial p_2} = \frac{\partial g_6}{\partial p_3} = 0$$

$$\frac{\partial g_6}{\partial p_4} = 0$$

$$\frac{\partial g_6}{\partial p_5} = b$$

$$\frac{\partial g_6}{\partial p_6} = \partial p$$

$$\frac{\partial g_6}{\partial p_7} = ty$$

$$\frac{\partial g_6}{\partial p_8} = 0$$

$$\frac{\partial g_6}{\partial p_9} = tz$$

$$\frac{\partial g_7}{\partial P_1} = -\sin\theta_x \sin\theta_y$$

$$\frac{\partial g_7}{\partial P_2} = \cos\theta_x \cos\theta_y$$

$$\frac{\partial g_7}{\partial P_3} = \frac{\partial g_7}{\partial P_4} = \frac{\partial g_7}{\partial P_5} = \frac{\partial g_7}{\partial P_6} = \frac{\partial g_7}{\partial P_7} =$$

$$\frac{\partial g_7}{\partial P_8} = \frac{\partial g_7}{\partial P_9} = 0$$

$$\frac{\partial g_8}{\partial p_1} = -\cos(\theta_x)$$

$$\frac{\partial g_8}{\partial p_i} = 0, \; i \in [2, 9]$$

$$\frac{\partial g_9}{\partial p_i} = 0, \; i \in [1, 9], \; i \neq 6$$

$$\frac{\partial g_9}{\partial p_6} = -1$$