# Project-2: CE605

Prateek Kumar Behera 20103082 Geoinformatics Email: prateekbeh20@iitk.ac.in.com

### 1 Introduction

Analyze discharge data (X) collected independently at four sites. For each site we need to find:

- the magnitude of discharge  $X_k$  such that  $P(X >= X_k) = 0.01$  along with 90 percent confidence interval for  $X_k$ .
- Test the hypothesis that the mean discharge is equal to 2500 units.
- Perform Goodness of Fit Test for Selected Distribution for 5 percent significance level.

## 2 Exective Summary

- Plot the histogram of the data sites
- Select the distribution based on the histogram.
- Get the Value of  $X_k$  for which the value of  $P(X >= X_k) = 0.01$  or  $P(X < X_k) < 0.99$ .
- Get the CI for the estimated parameters for the selected distribution.
- Perform the Hypothesis testing for the mean discharge is equal to 2500 units.
- Perform the Chi-Square Test for goodness-of-fit test at 5 percent significance level.

## 3 Methodology

- We Start with plotting the histograms of the Data.
- We selected the distribution for the data by fitting the histogram of the data to different distributions.
- After getting the distribution we need to determine the value of  $x_k$  such that  $P(X >= x_k) = 0.01$  or  $P(X <= x_k) = 0.99$ .
- We can determine the estimated values of the point estimates using Maximum Likelihood estimated
  - Normal Distribution
    - \* Estimated Mean( $\overline{X}$ ) = 1/n \*  $\sum X_i$
    - \* Estimated Std Dev = 1/(n-1) \*  $\sum (X_i \overline{X})^2$
  - Exponential Distribution
    - \* Estimated Lambda = n /  $\sum X_i$
- We determine the value of  $x_k$  for different distribution
  - For Normal Distribution: We can use the formula  $x_k = 2.33 * \sigma + \mu$  for  $P(X \le x_k) = 0.99$  to get the value of  $X_k$  using the standard normal distribution table where z value is 2.33.

- For Exponential Distribution: by integrating the distribution function of distribution in interval 0 to  $x_k$  and the equating it to 0.99, so that we get the value of  $x_k$
- We can obtain the Confidence Interval for different parameters by:-
  - For Normal Distribution:-
    - \* CI for  $\mu = \bar{X}$   $t_{(n-1,\alpha/2)}$  \* S/ $\sqrt(n)$  ,  $\bar{X}$  +  $t_{(n-1,\alpha/2)}$  \* S /  $\sqrt(n)$  \* CI for  $\sigma^2 = \text{(n-1)}$  \*  $S^2$  /  $C_u$  , (n-1) \*  $S^2$  /  $C_l$

    - \* where  $C_u = \text{Upper Limit of Chi-Square Distribution}$ ,  $C_l = \text{Lower Limit of Chi-Square Distribution}$ ,  $\bar{X} = \text{Estimated Mean, S} = \text{Estimated Std Deviation.}$
  - For Exponential Distribution:-
    - \*  $\lambda = 1 / \mu$
    - \* CI for  $\lambda = [1/\bar{X} (1 Z_{(\alpha/2)}/\sqrt(n)), 1/\bar{X} (1 + Z_{(\alpha/2)}/\sqrt(n))$
    - \* where  $\lambda = \text{Parameter for Exponential Distribution}$ .
- After Obtaining the CI we perform the hypothesis Testing for dof = n 1 = 99 for which the population variance is not known so we use the T- Distribution to calculate the Rejection Region and calculate the Statistic using the formula:
  - $T = (\bar{X} \mu_0)/(S/\sqrt{(n)})$
  - where  $\bar{X}$ .= Estimated Mean ,  $\mu_0 = 2500$ (given in question).
- Goodness of Fit Test
  - Perform the Chi-Square Test with 5 percent significance level.

### Result

### Data Site 1

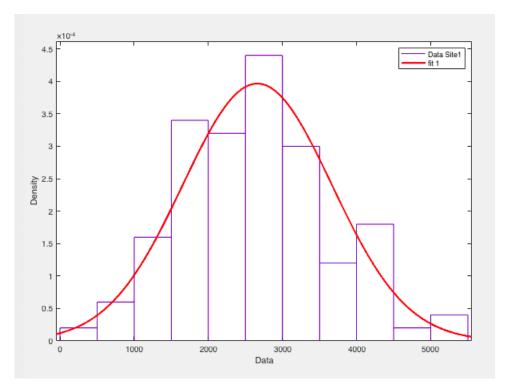


Figure 1: Fitted Data for Data Site 1.(Normal Distibution)

- Selected Distribution which fits the data is Normal Distribution.
- Value of Estimated Mean and Standard Deviation

- Estimated Mean( $\overline{X}$ ) = 1/n \*  $\sum X_i = 2661.1202$
- Estimated Std Dev =  $1/(n-1) * \sum (X_i \overline{X})^2 = 1005.77166728$
- Value of  $X_k$ 
  - -2.33\*1005.77166728+2661.1202=5004.56818476.
- 90 percent CI for the estimated Parameters
  - $-\text{ CI for }\mu = \bar{X} t_{(n-1,\alpha/2)} * \text{S}/\sqrt(n) \text{ , } \\ \bar{X} + t_{(n-1,\alpha/2)} * \text{S}/\sqrt(n) = [2494.1621032315356, 2828.078296768462]$
  - CI for  $\sigma = (\text{n-1}) * S^2 / C_u$ , (n-1) \*  $S^2 / C_l = [902.3280099776418, 1147.9162358469366]$
- Hypothesis Testing that the mean discharge is equal to 2500 units.
  - Significance level = 5 percent
  - Test Statistic =  $(\bar{X} \mu_0)/(S/\sqrt(n)) = 1.60195604272$
  - where  $\bar{X}$ .= Estimated Mean ,  $\mu_0 = 2500$ (given in question).
  - Critical Points for Rejection = [-1.987, 1.987] for dof = 99 and alpha = 5 percent
  - $-\mu = \mu_0 = 2500 \text{ H}_0 \text{ cannot be rejected}$
- Goodness of Fit Test
  - H0:- Assumed Distribution is Normal Distribution
  - HA:- Data does not follow normal Distribution
  - As the Test Statistic is 5.07 which does not lie in the Rejection Region (Critical Value = 11.070) the Null Hypothesis cannot be rejected.
  - Please look at end of this report for Goodness of fit Test Calculations.

#### 4.2 Data Site 2

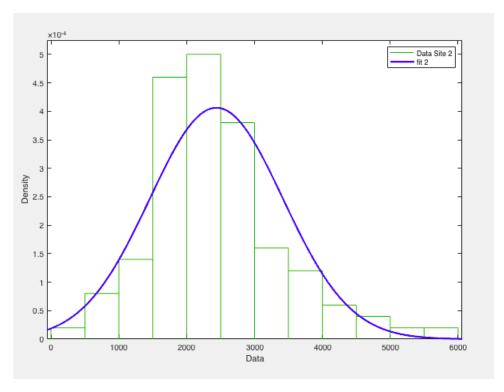


Figure 2: Caption of the sample figure.

- Selected Distribution which fits the data is Normal Distribution.
- Value of Estimated Mean and Standard Deviation
  - Estimated Mean( $\overline{X}$ ) = 1/n \*  $\sum X_i = 2437.4885$

- Estimated Std Dev =  $1/(n-1) * \sum (X_i \overline{X})^2 = 981.933137767$
- Value of  $X_k$ 
  - -2.33\*981.933137767+2437.4885=4725.392711.
- 90 percent CI for the estimated Parameters
  - $-\text{ CI for }\mu = \bar{X} t_{(n-1,\alpha/2)} * \text{S}/\sqrt(n) \;, \\ \bar{X} + t_{(n-1,\alpha/2)} * \text{S}/\sqrt(n) = [2274.4875991306562, 2600.4894008693436]$
  - CI for  $\sigma$  = (n-1) \*  $S^2$  /  $C_u$  , (n-1) \*  $S^2$  /  $C_l$  = [880.9412742046759, 1120.7086340058231]
- Hypothesis Testing that the mean discharge is equal to 2500 units.
  - Significance level = 5 percent
  - Test Statistic =  $(\bar{X} \mu_0)/(S/\sqrt{n}) = -0.636616665592$
  - where  $\bar{X}$ .= Estimated Mean ,  $\mu_0 = 2500$  (given in question).
  - Critical Points for Rejection = [-1.987 , 1.987] for  $\mathrm{dof} = 99$  and  $\mathrm{alpha} = 5$  percent.
  - $-\mu = \mu_0 = 2500 \text{ H}0 \text{ cannot be rejected.}$
- Goodness of Fit Test
  - H0:- Assumed Distribution is Normal Distribution
  - HA:- Data does not follow normal Distribution
  - As the Test Statistic is 8.75 which does not lie in the Rejection Region (Critical Value = 11.070) the Null Hypothesis cannot be rejected.
  - Please look at end of this report for Goodness of fit Test Calculations.

#### 4.3 Data Site 3

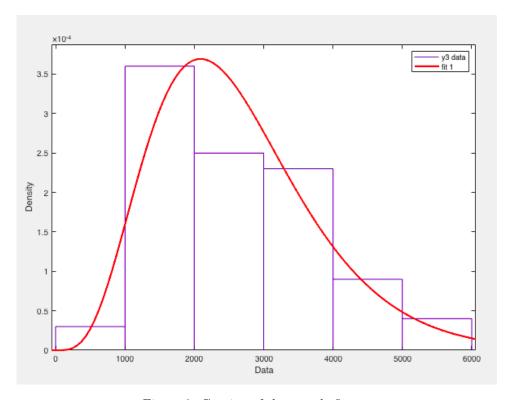


Figure 3: Caption of the sample figure.

- Selected Distribution which fits the data is Chi-Square Distribution.
- Value of Estimated Mean and Standard Deviation
- 90 percent CI for the estimated Parameters
  - CI for Std Deviation=  $[(n-1)*S^2/\chi^2(\alpha/2), (n-1)*S^2/\chi^2(\alpha/2)]$ ) = [2253.9488488326087, 4020.0981716345786].

### 4.4 Data Site 4

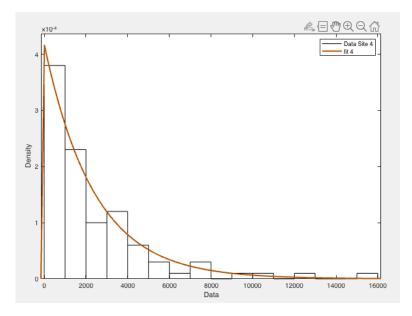


Figure 4: Caption of the sample figure.

- Selected Distribution which fits the data is Exponential Distribution.
- Value of Estimated Mean and Standard Deviation
  - Estimated lambda( $\lambda$ ) = n /  $\sum X_i = 0.000416769052931$
- Value of  $X_k$ 
  - By integrating  $\lambda * exp^{-\lambda *x}$  over 0 to  $X_k = 0.99$  we get the value of  $X_k = 11050$ .

```
syms x x_k
       %Got the Value of estimated lambda from the Python Code.
       estimated_lamda = 0.000416769052931;
       Distribution function for exponential RV.
       expr = lamda * exp(-lamda * x);
       %integrating from 0 to x_k
       eqns = int(expr, 0, x_k);
       %Solving the equation for F(X < x_k) = 0.99
10
11
       S = solve(eqns == 0.99 , [x_k]);
       disp(eval(S));
Command Window
     aues2
     1.1050e+04
fx >>
```

Figure 5: Value of  $X_k$  obtained using Matlab Code for Integration.

- 90 percent CI for the estimated Parameters
  - CI for  $\lambda = [1/\bar{X} (1 Z_{(\alpha/2)}/\sqrt(n)), 1/\bar{X} (1 + Z_{(\alpha/2)}/\sqrt(n)) = [0.0003482105, 0.00048532756]$
  - where  $\lambda$  = Parameter for Exponential Distribution.
- Hypothesis Testing that the mean discharge is equal to 2500 units.
  - Significance level = 5 percent
  - Test Statistic =  $(\bar{X} \mu_0)/(S/\sqrt(n)) = -0.374701564649$
  - where  $\bar{X}$ .= Estimated Mean ,  $\mu_0 = 2500$ (given in question).

- Critical Points for Rejection = [-1.987, 1.987] for dof = 99 and alpha = 5 percent.
- $-\mu = \mu_0 = 2500 \text{ H}_0 \text{ cannot be rejected}.$
- Goodness of Fit Test
  - H0:- Assumed Distribution is Normal Distribution
  - HA:- Data does not follow normal Distribution
  - As the Test Statistic is 116.172 which lies in the Rejection Region (Critical Value = 7.8154) the Null Hypothesis is rejected.
  - Please look at end of this report for Goodness of fit Test Calculations.
- 5 Python Code to Evaluate Estimation of Parmeters and their 90 percent Confidence Interval and Perform Hypothesis Testing that that the mean discharge is equal to 2500 units.

```
import math
from scipy.integrate import quad
from matplotlib import pyplot as plt
import numpy as np
f = open("20103082.txt", "r")
f_out = open("20103082_output","w+")
def estimate_parameters(pdf , data):
   n = len(data)
   if pdf == "Normal":
 # by method of Maximum Likelihood
 # calculate estimated paramters
 sum_x = 0
 for i in range(0,n):
     sum_x += data[i]
 estimated_mean = (1.0 * sum_x/n)
 squared_sum = 0
       for i in range(0,n):
      squared_sum += math.pow((data[i] - estimated_mean),2)
 estimated_variance = (1.0*squared_sum/(n-1))
 estimated_stddev = math.sqrt(estimated_variance)
 return [estimated_mean,estimated_stddev]
   if pdf == "Chi-Square":
 \#Dof = k - 1 - m
 \# k = 2 * sqrt(n) . To get the no. of bins
 # m = 1 for nu(Parameter for Chi-Square.
 k = 2 * math.sqrt(n)
 return k - 1 - 1
   if pdf == "Exponential":
 #By using method of Max. Likelihood
       lamda = 1.0 * n / sum(data)
 return lamda
def confidence_interval(distribution, data, *params):
   #we have to calculate for 90% CI.
   n = len(data)
   if distribution == "Normal":
 #Population variance is not known
 #1. Calculate sample mean
 #2. Calculate S^2
 X_bar = 1.0 * sum(data)/n
 squared_sum = 0
```

```
for i in range(0,n):
     squared_sum += math.pow((data[i] - X_bar),2)
 S_square = 1.0 * squared_sum/(n-1)
 sample_stddev = math.sqrt(S_square)
 ##Calculate the CI using formula [ X_bar - t(n-1,alpha/2) * S/sqrt(n) , X_bar - t(n-1,alpha/2)
   /2) * S/sqrt(n) ]
 ## Value of t(100,5\%) from T-distribution table = 1.66
 low_mean = X_bar - 1.66 * sample_stddev/math.sqrt(n)
 high_mean = X_bar + 1.66 * sample_stddev/math.sqrt(n)
 ## CI for Variance
 ## Population mean is not known
 ## 1.We get the CI using formula [ (n-1) * S^2 / Cu , (n-1) * S^2 / Cl ]
 ## For the Interval Estimation we use the Chi square dist. table.
 ## Cu = 123 and Cl = 76 for dof = 99 and alpha/2 = 0.05 and 0.95 respectively.
 Cu = 123
 C1 = 76
 low_stddev = math.sqrt((n - 1) * S_square/Cu)
 high_stddev = math.sqrt((n - 1) * S_square/Cl)
 return [[low_mean,high_mean],[low_stddev,high_stddev]]
   if distribution == "Chi-Square":
 ## 2*dof = std. deviation
 ## Estimating the CI for std deviation and from that we can estimate CI for dof.
       ## CI for VARIANCE = [(n-1)*S^2/Chi-Square(alpha/2),(n-1)*S^2/Chi-Square(alpha/2)]
       ## alpha/2 = 0.05
 dof = params[0]
 sample_mean = sum(data)/n
 print "Value of Chi-Square dist for dof = {} for alpha = {} and {} \n is {} {} respectively".
   format(dof,0.05,0.95, 27.587, 8.672)
        squared_sum = 0
        for i in range(0,n):
            squared_sum += math.pow((data[i] - sample_mean),2)
        S_square = 1.0 * squared_sum/(n-1)
        low_variance = 1.0 * (n - 1) * S_square / 27.587
 high_variance = 1.0 * (n - 1) * S_square / 8.672
 return [low variance, high variance]
   if distribution == "Exponential":
 ## To Calculate the CI for the Exponential Distribution
 ## As mean = 1/lamda
 ## we use formula [1/X_bar (1 - Z(alpha/2)/sqrt(n)), 1/X_bar (1 + Z(alpha/2)/sqrt(n)) ]
 ## Z(alpha/2) = 1.645 for alpha = 90%
 sample_mean = sum(data)/n
 low_lambda = 1/sample_mean * (1 - 1.645/ math.sqrt(n))
high_lambda = 1/sample_mean * (1 + 1.645/ math.sqrt(n))
 return [low_lambda,high_lambda]
def hypothesis_testing(data, mean = -1 , variance = 0):
   n = len(data)
   #H0 => mu = mu_0 = 2500
   #HA => mu != mu_0
   mu_0 = 2500
   #(5% significance level)
   significance_level = 5
   if variance == 0:
     #Compute Test Statistic using T-Distribution
      #Population Variance is not known
     sample_mean = sum(data)/n
 squared_sum = 0
      for i in range(0,n):
          squared_sum += math.pow((data[i] - sample_mean),2)
      S_square = 1.0 * squared_sum/(n-1)
```

```
sample_stddev = math.sqrt(S_square)
 T = 1.0 * (sample_mean - mu_0)/(1.0 * sample_stddev/math.sqrt(n))
 print "Test Statistic for Hypothesis Testing that mean discharge = 2500 is {}".format(T)
 f_out.write("Test Statistic for Hypothesis Testing that mean discharge = 2500 is {}".format(T
      #For significance level 5 we have to check between
 # range -1.987 to 1.987 for dof = n - 1 = 99
 if T < 1.987 and T > -1.987:
    return ["HO cannot be rejected",1]
         return ["HO can be rejected",0]
data1 = []
data2 = []
data3 = []
data4 = []
header = f.readline()
while True:
   line = f.readline()
   if len(line) != 0:
      val = line.split()
 data1.append(float(val[0]))
 data2.append(float(val[1]))
 data3.append(float(val[2]))
 data4.append(float(val[3]))
   else:
      break
print("####################### For DATA SITE 1 ###########################")
f_out.write("###############################")
#Assuming the above distribution to be Normal using Histogram
#1. P(X > x_k) = 0.01, so P(X < x_k) = 0.99
#2. P(Z < (x_k - mean)/variance) = 0.99
#3a. According to Std. Normal Distribution table
#3b. (x_k - mean)/std_dev = 2.33
  Estimate the Paramters.
[est_mean,est_stddev] = estimate_parameters("Normal",data1)
print "est_mean = {}".format(est_mean)
print "est_stddev = {}".format(est_stddev)
f_out.write("est_mean = {}\n".format(est_mean))
f_out.write("est_stddev = {}\n".format(est_stddev))
#Rearranging the eqn (x_k - mean)/stddev = 2.33 from #3b
# we get x_k = 2.33 * std_dev + mean
x_k = 2.33 * est_stddev + est_mean
print "Value of x_k = {}".format(x_k)
f_{out.write("Value of x_k = {}\n".format(x_k))
[mean_interval, variance_interval] = confidence_interval("Normal", data1)
```

```
print "mean_interval = {}".format(mean_interval)
print "variance_interval = {}".format(variance_interval)
f_out.write("90 % CI for mean = {}\n".format(mean_interval))
f_out.write("90 % CI for variance = {}\n".format(variance_interval))
HO = "mu = mu_0 = 2500"
HA = "mu != mu_0"
res = hypothesis_testing(data1)
if res[1] == 1:
  f_out.write("{} {}\n".format(H0,res[0]))
else:
   f_out.write("{} {}\n".format(HA,res[0]))
f_out.write('\n')
print("###################### For DATA SITE 2 ##########################")
f_out.write("#################################")
f_out.write("##############################")
Estimate the Paramters.
[est_mean,est_stddev] = estimate_parameters("Normal",data2)
print "est_mean = {}".format(est_mean)
print "est_stddev = {}".format(est_stddev)
f_out.write("estimated mean = {}\n".format(est_mean))
f_out.write("estimated standard deviation = {}\n".format(est_stddev))
#Rearranging the eqn (x_k - mean)/variance = 2.33 from #3b
# we get x k = 2.33 * est stddev + est mean
x_k = 2.33 * est_stddev + est_mean
print "Value of x_k = {}".format(x_k)
f_{out.write("Value of x_k = {}\n".format(x_k))}
[mean_interval, variance_interval] = confidence_interval("Normal", data2)
print "mean_interval = {}".format(mean_interval)
print "variance_interval = {}".format(variance_interval)
f_out.write("90 % CI for mean = {}\n".format(mean_interval))
f_out.write("90 % CI for variance = {}\n".format(variance_interval))
res = hypothesis_testing(data2)
if res[1] == 1:
   f_out.write("{} {}\n".format(H0,res[0]))
else:
   f_out.write("{} {}\n".format(HA,res[0]))
f out.write('\n')
print("####################### For DATA SITE 3 ################################")
print("###############################")
f_out.write("###################### For DATA SITE3 ############################")
dof = estimate_parameters("Chi-Square",data3)
print "Degree of freedom = {}".format(dof)
f_out.write("Degree of freedom = {}".format(dof))
variance_interval = confidence_interval("Chi-Square", data3,dof)
stddev_interval = [math.sqrt(variance_interval[0]), math.sqrt(variance_interval[1])]
print "Confidence Interval for Std dev = {} ".format(stddev_interval)
f_out.write("Confidence Interval for Std dev = {} ".format(stddev_interval))
f_out.write('\n')
```

```
print("####################### For DATA SITE 4 ##############################")
f_out.write("##################### For DATA SITE 4 ############################")
f_out.write("###############################")
#Estimate the Paramters.
lamda = estimate_parameters("Exponential",data4)
print "lambda = {}\n".format(lamda)
f_out.write("lambda = {}\n".format(lamda))
lambda_interval = confidence_interval("Exponential", data4)
print "Confidence Interval for Lambda = {}".format(lambda_interval)
f_out.write("90 % Confidence Interval for Lambda = {}\n".format(lambda_interval))
####### Calculate x_k #############
***************
## Getting the Value of x_k by integrating the distibution
## function in interval 0 to x_k and equating with 0.99
## Got the Value of x_k from Matlab Code(Snippet Pasted in Report)
x_k = 11050
print "x_k value obtained by using matlab code(Snipped Pasted) for integration"
print "Value of x_k = {}".format(x_k)
f_{out.write("Value of x_k = {}\n".format(x_k))}
res = hypothesis_testing(data4)
if res[1] == 1:
  f_out.write("{} {}\n".format(H0,res[0]))
f_out.write("{} {}\n".format(HA,res[0]))
```

```
#################### For DATA SITE 1
##
est mean = 2661.1202
est stddev = 1005.77166728
Value of x k = 5004.56818476
90 % CI for mean = [2494.1621032315356, 2828.078296768462]
90 % CI for variance = [902.3280099776418, 1147.9162358469366]
Test Statistic for Hypothesis Testing that mean discharge = 2500 is
1.60195604272mu = mu 0 = 2500 H0 cannot be rejected
##
##################### For DATA SITE 2
estimated mean = 2437.4885
estimated standard deviation = 981.933137767
Value of x k = 4725.392711
90 % CI for mean = [2274.4875991306562, 2600.4894008693436]
90 % CI for variance = [880.9412742046759, 1120.7086340058231]
Test Statistic for Hypothesis Testing that mean discharge = 2500 is -
0.636616665592mu = mu 0 = 2500 HO cannot be rejected
Degree of freedom = 18.0
Confidence Interval for Std dev = [2253.9488488326087,
4020.0981716345786]
########################## For DATA SITE 4
lambda = 0.000416769052931
90 % Confidence Interval for Lambda = [0.0003482105437235746,
0.000485327562137764961
Value of x k = 11050
Test Statistic for Hypothesis Testing that mean discharge = 2500 is -
0.374701564649mu = mu 0 = 2500 HO cannot be rejected
```

					Goodness of Fit Test				
					Chi Square Test For Data Site 1				
	interval	observation	Z(lower range)	Z(Higher range)	Z(interval)	Cumulative Area	P	E	Oi^2/Ei
1	476	0			-inf ,-2.17258078068554	0.015	0.015	1.5	
2	1442	12	-1.212124219	-2.172580781	-2.17258078068554 ,-1.21212421900887	0.1131	0.0981	9.81	14.6788990
3	1925	14	-0.7318959382	-1.212124219	-1.21212421900887 ,-0.731895938170539	0.2327	0.1196	11.96	16.3879598
4	2408	16	-0.2516676573	-0.7318959382	-0.731895938170539 ,-0.251667657332205	0.4013	0.1686	16.86	15.183867
5	2891	16	0.2285606235	-0.2516676573	-0.251667657332205 ,0.228560623506129	0.5871	0.1858	18.58	13.778256
6	3374	19	0.7087889043	0.2285606235	0.228560623506129 ,0.708788904344462	0.758	0.1709	17.09	21.123464
7	3857	10	1.189017185	0.7087889043	0.708788904344462 ,1.1890171851828	0.881	0.123	12.3	8.1300813
8		13	2.629702028	1.189017185	1.1890171851828 ,2.6297020276978	0.988	0.107	10.7	15.794392
					,				105.076920
ormula Used			(X-mu)/sigma			Value from Std.		E = n*P	100.07002
Excel			(B(i) - 2661.1202)/1005.77167	D(i-1)	CONCATENATE(E2 & " ," & D2)	Normal Dist. Table	G(i)-G(i-1)	E = 100 * H(i)	POWER(C(i),2)/
LAUGI			(B(i) 2001:1202)/1000:77107	D((1))	CONCATENATE (EZ a , a DZ)	140111ai Dist. Table	O(1) O(1 1)	L - 100 TI(I)	1 0 11 2 11 (0(1),2)//
			Dof = 5	T = 105.07 - 100	Critical Value of Dejection - 44 070				
			D0I = 5		Critical Value of Rejection = 11.070	est_mean	est_stddev		
				5.07	(Obtained from Chi- Square Distribution Table)	2661.1202	1005.771667		
				T = Test Statistic					
					H0:- Assumed Distribution is Normal Distribution				
					HA :- Data does not follow normal Distribution				
s the Test Sta	atistic is 5.	07 which does	s not lie in the Rejection Region	(Critical Value = 11.	070) the Null Hypothesis cannot be rejected.				
					ted using formula or the distribution table				
			ed to obtain the Z interval Column						
J		,		3 010					
					Chi Square Test For Data Site 2				
					o oqualo rost ror bata one 2				
	lustr 1	alaan	7(10,000,000,000)	7(11) mb an	7/internal\	Cumulative Area	<b>.</b>	-	O:42 /F:
				Z(Higher range)	Z(interval)		P	E	Oi^2/Ei
1					-inf ,-2.04014758112907	0.0207	0.0207	2.07	
2	956.7	3		-2.040147581	-2.04014758112907 ,-1.50803395339151	0.0668	0.0461	4.61	1.9522776
3	1479.2	9	-0.9759203257	-1.508033953	-1.50803395339151 ,-0.975920325653947	0.166	0.0992	9.92	8.1653225
4	2001.7	23	-0.4438066979	-0.9759203257	-0.975920325653947 ,-0.443806697916385	0.33	0.164	16.4	32.256097
5	2524.3	25	0.08840876975		-0.443806697916385 ,0.0884087697499322	0.5319	0.2019	20.19	30.955918
6			0.6205223975		0.0884087697499322 ,0.620522397487495	0.7324	0.2005		
7					0.620522397487495 ,1.15263602522506	0.8749	0.1425	14.25	
8	5659.3	12	3.281090536	1.152636025	1.15263602522506 ,3.28109053617531	0.9995	0.1246	12.46	11.5569823
									108.57579
ormula Used			(X-mu)/sigma			Value from Std.		E = n*P	
Excel			(B(i) - 2437.4885)/981.9331	D(i-1)	CONCATENATE(E2 & " ," & D2)	Normal Dist. Table	G(i)-G(i-1)	E = 100 * H(i)	POWER(C(i),2)/
			Dof = 5	T = 108.57 - 100	Critical Value of Rejection = 11.070				
				8.57	(Obtained from Chi- Square Distribution Table)				
					H0:- Assumed Distribution is Normal Distribution				
					HA :- Data does not follow normal Distribution				
e the Teet Sta	atietic ie 8	75 which does	not lie in the Dejection Degion	(Critical Value = 11)				est mean	ast stdday
				`	070) the Null Hypothesis cannot be rejected.			est_mean	est_stddev
tervals & Free	quency O	otained using	Matlab Code. For rest of the colu	umns values calcula	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table			est_mean 2437.4885	_
tervals & Free	quency O	otained using		umns values calcula	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table			_	_
tervals & Free	quency O	otained using	Matlab Code. For rest of the colu	umns values calcula	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table			_	_
itervals & Free	quency O	otained using	Matlab Code. For rest of the colu	umns values calcula	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table			_	_
itervals & Free	quency O	otained using	Matlab Code. For rest of the colu	umns values calcula	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table			_	_
itervals & Free	quency O	otained using	Matlab Code. For rest of the colu	umns values calcula	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table			_	_
tervals & Free	quency O e Column	otained using is just include	Matlab Code. For rest of the column dt to obtain the Z interval Column	umns values calcula	170) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4	Cumulative Area	P	_	981.93313
tervals & Free Higher Range	e Column	otained using is just include	Matlab Code. For rest of the colud to obtain the Z interval Column	umns values calcula n in which we get the Z(Higher range)	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4 Z(interval)	Cumulative Area		2437.4885 E	981.93313 Oi^2 /Ei
ntervals & Frei Higher Rangi	e Column  interval	observation	Matlab Code. For rest of the colude to obtain the Z interval Column  Z(lower range)  -0.8888806109	umns values calcula in in which we get the Z(Higher range)	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4 Z(interval) -inf,~0.88880610914509	0.2177	0.2177	2437.4885 E 21.77	981.93313' Oi^2 /Ei
ttervals & Fred Higher Range 1 1 2	interval 13.2 1563.5	observation  0 54	Matlab Code. For rest of the column to obtain the Z interval Column Z (lower range)  -0.8888806109 -0.3113801453	zmns values calcula in which we get the Z(Higher range) -inf -0.8888806109	O70) the Null Hypothesis cannot be rejected. ed using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval) -inf _0.888880610914509 -0.888880610914509 _0.31138014527845	0.2177 0.3783	0.2177 0.1606	2437.4885 E 21.77 16.06	981.93313 Oi^2 /Ei 181.56911
tervals & Fred Higher Range 1 1 2 3	interval 13.2 1563.5 3 3113.9	observation  0 54	Matlab Code. For rest of the column to obtain the Z interval Column Z(lower range)  -0.8888806109 -0.3113801453 0.2661575712	Z(Higher range) -inf -0.8888806109 -0.3113801453	270) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval) -0.888880610914509 -0.888880610914509 -0.888880610914509,-0.31138014527845 -0.31138014527845,0.266157571242317	0.2177 0.3783 0.6026	0.2177 0.1606 0.2243	E 21.77 16.06 22.43	981.93313 Oi^2 /Ei 181.56911: 12.884529(
tervals & Free Higher Range 1 1 2 3 4	interval 13.2 2 1563.5 3 3113.9 4 4664.2	observation  Observation  0  54  17	Matlab Code. For rest of the colid to obtain the Z interval Column  Z(lower range) -0.8888806109 -0.3113801453 0.2661575712 0.8436580369	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712	0.70) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval)  -0.88880610914509  -0.88880610914509  -0.88880610914509  -0.31138014527845  -0.31138014527845  -0.266157571242317  0.266157571242317	0.2177 0.3783 0.6026 0.7995	0.2177 0.1606 0.2243 0.1969	E 21.77 16.06 22.43 19.69	981.93313  Oi^2 /Ei  181.56911 12.884529 11.427120
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the column to obtain the Z interval Column Z (lower range)  -0.8888806109 -0.3113801453 -0.2661575712 -0.8436580369 -1.421158503	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z[interval] -inf -0.88880610914509 -0.88880610914509 -0.31138014527845 -0.31138014527845 -0.266157571242317 0.266157571242317 ,0.843658036878376 0.843658036878376 ,1.42115850251443	0.2177 0.3783 0.6026 0.7995 0.9222	0.2177 0.1606 0.2243 0.1969 0.1227	E 21.77 16.06 22.43 19.69 12.27	981.93313  Oi^2 /Ei  181.56911 12.884529 11.427120 3.9934800
tervals & Free Higher Range 1 1 2 3 4	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17	Matlab Code. For rest of the column to obtain the Z interval Column 2 (lower range)  -0.8888806109 -0.3113801453 -0.2661575712 -0.8436580369 1.421158503	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369	0.70) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval)  -0.88880610914509  -0.88880610914509  -0.88880610914509  -0.31138014527845  -0.31138014527845  -0.266157571242317  0.266157571242317	0.2177 0.3783 0.6026 0.7995	0.2177 0.1606 0.2243 0.1969 0.1227	E 21.77 16.06 22.43 19.69	981.93313 Oi^2 /Ei 181.56911: 12.884529 11.427120: 3.9934800:
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the column to obtain the Z interval Column Z (lower range)  -0.8888806109 -0.3113801453 -0.2661575712 -0.8436580369 -1.421158503	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z[interval] -inf -0.88880610914509 -0.88880610914509 -0.31138014527845 -0.31138014527845 -0.266157571242317 0.266157571242317 ,0.843658036878376 0.843658036878376 ,1.42115850251443	0.2177 0.3783 0.6026 0.7995 0.9222	0.2177 0.1606 0.2243 0.1969 0.1227	E 21.77 16.06 22.43 19.69 12.27	981.933133 Oi^2 /Ei 181.569111 12.8845291 11.4271203 3.99348003 6.2982005
tervals & Free Higher Range 1 2 3 4 5	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the column to obtain the Z interval Column Z (lower range)  -0.8888806109 -0.3113801453 -0.2661575712 -0.8436580369 -1.421158503	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z[interval] -inf -0.88880610914509 -0.88880610914509 -0.31138014527845 -0.31138014527845 -0.266157571242317 0.266157571242317 ,0.843658036878376 0.843658036878376 ,1.42115850251443	0.2177 0.3783 0.6026 0.7995 0.9222	0.2177 0.1606 0.2243 0.1969 0.1227	E 21.77 16.06 22.43 19.69 12.27	981.93313  Oi^2 /Ei  181.56911 12.884529 11.427120 3.9934800 6.2982005
tervals & Frei Higher Range 1 1 2 3 4 5 6	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the column to obtain the Z interval Column Z (lower range)  -0.8888806109 -0.3113801453 -0.2661575712 -0.8436580369 -1.421158503	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z[interval] -inf -0.88880610914509 -0.88880610914509 -0.31138014527845 -0.31138014527845 -0.266157571242317 0.266157571242317 ,0.843658036878376 0.843658036878376 ,1.42115850251443	0.2177 0.3783 0.6026 0.7995 0.9222	0.2177 0.1606 0.2243 0.1969 0.1227	E 21.77 16.06 22.43 19.69 12.27	981.93313  Oi^2 /Ei  181.56911 12.884529 11.427120 3.9934800 6.2982005
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the column to obtain the Z interval Column to obtain the Z interval Column -0.8888806109 -0.3113801453 -0.2661575712 -0.8436580369 -1.421158503 -4.886422053	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369 1.421158503	270) the Null Hypothesis cannot be rejected.  170 the Null Hypothe	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.933133 Oi^2 /Ei 181.569111 12.8845291 11.4271203 3.99348003 6.2982005 216.172444
tervals & Frei Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the column to obtain the Z interval Column to obtain the Z interval Column -0.8888806109 -0.3113801453 -0.2661575712 -0.8436580369 -1.421158503 -4.886422053	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369	070) the Null Hypothesis cannot be rejected. ted using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z[interval] -inf -0.88880610914509 -0.88880610914509 -0.31138014527845 -0.31138014527845 -0.266157571242317 0.266157571242317 ,0.843658036878376 0.843658036878376 ,1.42115850251443	0.2177 0.3783 0.6026 0.7995 0.9222	0.2177 0.1606 0.2243 0.1969 0.1227	E 21.77 16.06 22.43 19.69 12.27 7.78	981.93313 Oi^2 /Ei 181.56911: 12.884529 11.427120: 3.9934800: 6.2982005 216.172444
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the colud to obtain the Z interval Column  Z(lower range)  -0.8888806109  -0.3113801453  0.2661575712  0.8436580369  1.421158503  4.886422053  (X-mu/sigma (B(i) - 2624.1)/1189.8	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369 1.421158503	270) the Null Hypothesis cannot be rejected. ed using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval) -inf -0.888880610914509 -0.888880610914509 -0.31138014527845 -0.31138014527845 0.266157571242317 0.266157571242317 0.843658036878376 0.843658036878376 1.42115850251443 1.42115850251443 .4.88642205252375  CONCATENATE(E2 & " , " & D2)	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.93313 Oi^2 /Ei 181.56911 12.884529 11.427120 3.9934800 6.2982005 216.17244
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the colud to obtain the Z interval Column  Z(lower range)  -0.8888806109  -0.3113801453  0.2661575712  0.8436580369  1.421158503  4.886422053  (X-mu/sigma (B(i) - 2624.1)/1189.8	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369 1.421158503 D(i-1) T = 216.172-100	270) the Null Hypothesis cannot be rejected. ed using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval) -inf _0.888880610914509 -0.888880610914509 _0.31138014527845 -0.31138014527845 _0.266157571242317 0.266157571242317 _0.843658036878376 0.843658036878376 _1.42115850251443 1.42115850251443 _4.88642205252375  CONCATENATE(E2 & " , " & D2)  Critical Value of Rejection = 7.815	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.93313 Oi^2 /Ei 181.56911 12.884529 11.427120 3.9934800 6.2982005 216.17244
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the colud to obtain the Z interval Column  Z(lower range)  -0.8888806109  -0.3113801453  0.2661575712  0.8436580369  1.421158503  4.886422053  (X-mu/sigma (B(i) - 2624.1)/1189.8	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369 1.421158503	270) the Null Hypothesis cannot be rejected. ed using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval) -inf -0.888880610914509 -0.888880610914509 -0.31138014527845 -0.31138014527845 0.266157571242317 0.266157571242317 0.843658036878376 0.843658036878376 1.42115850251443 1.42115850251443 .4.88642205252375  CONCATENATE(E2 & " , " & D2)	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.93313 Oi^2 /Ei 181.56911 12.884529 11.427120 3.9934800 6.2982005 216.17244
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the colud to obtain the Z interval Column  Z(lower range)  -0.8888806109  -0.3113801453  0.2661575712  0.8436580369  1.421158503  4.886422053  (X-mu/sigma (B(i) - 2624.1)/1189.8	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369 1.421158503 D(i-1) T = 216.172-100	270) the Null Hypothesis cannot be rejected.  12 ted using formula or the distribution table interval of Z.  13 ted using formula or the distribution table interval of Z.  14 Telephone	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.93313  Oi^2 /Ei  181.56911 12.884529 11.427120 3.9934800 6.2982005 216.17244
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the colud to obtain the Z interval Column  Z(lower range)  -0.8888806109  -0.3113801453  0.2661575712  0.8436580369  1.421158503  4.886422053  (X-mu/sigma (B(i) - 2624.1)/1189.8	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369 1.421158503 D(i-1) T = 216.172-100	270) the Null Hypothesis cannot be rejected. ed using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval) -inf _0.888880610914509 -0.888880610914509 _0.31138014527845 -0.31138014527845 _0.266157571242317 0.266157571242317 _0.843658036878376 0.843658036878376 _1.42115850251443 1.42115850251443 _4.88642205252375  CONCATENATE(E2 & " , " & D2)  Critical Value of Rejection = 7.815	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.93313  Oi^2 /Ei  181.56911 12.884529 11.427120 3.9934800 6.2982005 216.17244
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the colud to obtain the Z interval Column  Z(lower range)  -0.8888806109  -0.3113801453  0.2661575712  0.8436580369  1.421158503  4.886422053  (X-mu/sigma (B(i) - 2624.1)/1189.8	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369 1.421158503 D(i-1) T = 216.172-100	270) the Null Hypothesis cannot be rejected.  12 ted using formula or the distribution table interval of Z.  13 ted using formula or the distribution table interval of Z.  14 Telephone	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.933133 Oi^2 /Ei 181.569111 12.8845291 11.4271203 3.99348003
tervals & Free Higher Range	interval 13.2 2 1563.5 3 3113.9 4 4664.2 6 6214.5	observation  Observation  0  54  17  15	Matlab Code. For rest of the colud to obtain the Z interval Column  Z(lower range)  -0.8888806109  -0.3113801453  0.2661575712  0.8436580369  1.421158503  4.886422053  (X-mu/sigma (B(i) - 2624.1)/1189.8	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8436580369 1.421158503 D(i-1) T = 216.172-100	270) the Null Hypothesis cannot be rejected. ed using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval) -inf ,-0.888880610914509 -0.888880610914509 ,-0.31138014527845 -0.31138014527845 ,0.266157571242317 0.266157571242317 ,0.843658036878376 0.843658036878376 ,1.42115850251443 1.42115850251443 ,4.88642205252375  CONCATENATE(E2 & " ," & D2)  Critical Value of Rejection = 7.815 (Obtained from Chi- Square Distribution Table) H0:- Assumed Distribution is Normal Distribution	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.93313 Oi^2 /Ei 181.56911: 12.884529 11.427120: 3.9934800: 6.2982005 216.172444
tervals & Free Higher Range 1 1 2 3 4 5 6 6 ormula Used Excel	interval 13.2 1563.5 3 3113.9 4664.2 6 6214.5 15517	observation  Observation  Observation  O  544  17  15  7	Matlab Code. For rest of the colud to obtain the Z interval Column  Z(lower range)  -0.8888806109  -0.3113801453  0.2661575712  0.8436580369  1.421158503  4.886422053  (X-mu/sigma (B(i) - 2624.1)/1189.8	Z(Higher range) -inf -0.8888806109 -0.3113801453 0.2661575712 0.8438580369 1.421158503  D(i-1)  T = 216.172-100 116.172	270) the Null Hypothesis cannot be rejected. ed using formula or the distribution table interval of Z.  Chi Square Test For Data Site 4  Z(interval) -inf ,-0.888880610914509 -0.888880610914509 ,-0.31138014527845 -0.31138014527845 ,0.266157571242317 0.266157571242317 ,0.843658036878376 0.843658036878376 ,1.42115850251443 1.42115850251443 ,4.88642205252375  CONCATENATE(E2 & " , " & D2)  Critical Value of Rejection = 7.815 (Obtained from Chi- Square Distribution Table) H0:- Assumed Distribution is Normal Distribution HA:- Data does not follow normal Distribution	0.2177 0.3783 0.6026 0.7995 0.9222 1	0.2177 0.1606 0.2243 0.1969 0.1227 0.0778	E 21.77 16.06 22.43 19.69 12.27 7.78	981.93313  Oi^2 /Ei  181.56911 12.884529 11.427120 3.9934800 6.2982005 216.17244