Lab Report 3: Trajectory estimation using EKF

Prateek Kumar Behera (20103082)
Department of Civil Engineering
Indian Institute of Technology Kanpur
April 19, 2021

Abstract: In this lab we discuss about how we can implement a EKF filter for a 2-d case and we also discuss about the assumptions made during the process and the state space and the measurement model.

1. Introduction

In this lab we will discuss on how to implement a Extended Kalman filter for a 2D navigation problem. We estimate the trajectory of the moving car. We are given the control points from where we can measure the car distances. We are given the measurements at 1hz frequency. We further discuss about the state space model and the measurement model and the assumptions made during the process. We first assume the value initial state and the state error covariance matrix and then we input these to the prediction stage where we have our state space model using which we estimate the states and state covariance matrix. The result of this stage is input to the corrector stage where we have our measurement model using which we update our states and state covariance matrix.

2. Methodology

2.1. Assumption Made

Following are the assumptions made while implementing the EKF filter for a 2D case:

1. Measurement noise and the process noise are uncorrelated.
   - $E[v_k * \omega_k^T] = 0$
2. Measurement noise are white.
   - $E[v_k * v_{k-1}^T] = 0$
3. Process noise are white.
   - $E[\omega_k * \omega_{k-1}^T] = 0$
4. Measurement noise are normally distributed with variance- covariance matrix R_k.
   - $E[v_k * v_k^T] = R_k$
5. Process noise are normally distributed with variance- covariance matrix Q_k.
   - $E[\omega_k * \omega_k^T] = Q_k$
6. Estimation error is normally distributed with Variance-covariance matrix P_k
   - $E[\epsilon_k * \epsilon_k^T] = P_k$
7. We assume a constant velocity model as nothing is given in question.

2.2. State Space Model

State space model consists of the state vector and the state transition model.
The state vector consists of 4 elements

$$\begin{bmatrix} X \\ Y \\ Vx \\ Vy \end{bmatrix}$$

where X and Y are the coordinates and Vx and Vy are the velocity in x and y direction.

The state transition model is as follows:

$$\begin{gathered} X_k = X_{k-1} + Vx_k * \Delta t \\ Vx_k = Vx_{k-1} \\ Y_k = Y_{k-1} + Vy_k * \Delta t \\ Vy_k = Vy_{k-1} \end{gathered}$$

where X and Y are the coordinates and Vx and Vy are the velocity in x and y direction and ΔT is the time interval.
Using the above equations we get the state transition model as follows

$$\begin{bmatrix} X_k \\ V_{xk} \\ Y_k \\ V_{yk} \end{bmatrix} = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{k-1} \\ V_{x_k-1} \\ Y_k \\ V_{y_k-1} \end{bmatrix} \tag{1}$$

where the state transition matrix is given by :

$$F_k = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

2.3. Measurement Model

As there are four control points we get the distance values from these 4 control points.:

$$\begin{bmatrix} -10 & 0 \\ 0 & -10 \\ 10 & 0 \\ 0 & 10 \end{bmatrix}$$

The measurement model is as given below:

$$\begin{aligned} \rho_1 &= \sqrt{(-10 - X_k)^2 + (0 - Y_k)^2} \\ \rho_2 &= \sqrt{(0 - X_k)^2 + (-10 - Y_k)^2} \\ \rho_3 &= \sqrt{(10 - X_k)^2 + (0 - Y_k)^2} \\ \rho_4 &= \sqrt{(0 - X_k)^2 + (10 - Y_k)^2} \end{aligned}$$

where ρ_1, ρ_2, ρ_3 ,ρ_4 are the distances observed and X_k, Y_k are the coordinates of the car.

2.4. Linearization Steps

Since the measurement model is non-linear we need to linearize it using taylor series expansion:

We linearize it about the nominal point of $\hat{X_k^-}$. Using the linearization method we get:

$$h(X_n)|_{X_n=\hat{X_k^-}} + \left.\frac{\partial h}{\partial X}\right|_{X_n=\hat{X_k^-}} (X_n - \hat{X_k^-}) + v_k$$

where v_k is the measurement error.

If $\hat{X_k^-}$ is close to X_n we can ignore the higher order terms as they will be further small. After solving the above equation we get:

$$h(\hat{X_k^-}) + H_k * \Delta X + v_k$$

where $h(\hat{X_k^-})$ is the measurement model evaluated at $\hat{X_k^-}$ and H_k is the measurement matrix.

2.5. Step by Step process for trajectory estimation

Steps to implement the Extended Kalman Filter are as follows:

1. We first start with selecting a initial estimate for the states and the the state error covariance matrix.
2. We pass these values to the prediction phase along with the value of process noise.
3. We calculate the value of Estimated parameters using the state transition model.

   $$X_k^- = F_{k-1} * X_{k-1}^+ + \omega_{k-1}$$

   After taking the expectation value of both sides of the equation we get the Expected value of the $\hat{X_k^-}$ as:

   $$\hat{X_k^-} = F_{k-1} * \hat{X_{k-1}^+}$$

   because the Expectation of the process noise is 0 as it is normally distributed which we assumed.
4. We also find out the value of the state error covariance matrix using the formula :

   $$P_k^- = F_{k-1} * P_{k-1}^+ * F_{k-1}^T + Q_{k-1}$$

5. The values of $\hat{X_k^-}$ and P_k^- are being input to the corrector module which has the measurement model.
6. We define the measurement model as defined in section 2.3 and linearize it about the predicted state $\hat{X_k^-}$ as mentioned in section 2.4.
7. We calculate the value of innovation by subtracting the h($\hat{X_k^-}$) from the observed values.

   $$\text{innovation} = Z - Z_{predicted}$$

   where Z is the observed value and $Z_{predicted}$ is h($\hat{X_k^-}$).
8. We get the value of gain factor (K_k) using the values of the P_k^-, the measurement error covariance matrix (R_k), and the value of H_k using the below equation:

   $$K_k = P_k^- * H_k^T * (H_k * (P_k^-) * H_k^T + R_k)^{-1};$$

9. We get the value of $\hat{X_k^+}$ AND P_k^+ which are the output of the corrector phase.

   $$\begin{gathered} \hat{X_k^+} = \hat{X_k^-} + K_k * innovation; \\ P_k^+ = (I - K_k * H_k) * P_k^- * (I - K_k * H_k)^T + \\ \mathrm{K}_k * R_k * K_k^T; \end{gathered}$$

3. Results and Discussion

1. How did you choose the initial state and covariance?

- We assumed the values of the coordinates(X,Y) which are the coordinates of the car and we have two (Vx,Vy) velocities in x and y direction which we can get by observing difference between two distances and dividing it by ΔT.
  We also assumed the initial covariance values as:-

$$P_0 = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.8 \end{bmatrix} \tag{3}$$

  It can also be taken into account that if we take the correlation to be 0 then the confidence on the estimated states may be overestimated.

2. How will you go about choosing the process noise covariance and measurement noise covariance?
   - We assumed the process noise covariance and the measurement noise covariance as nothing was mentioned in question.
3. Generate the results for different values of process noise and measurement noise covariances. Do you observe any differences? If yes, try to explain the reasons for these differences.
   - By using different values for the Process noise covariance(Q) and the measurement noise covariance(R) the value of prior estimation error and posterior estimation error is changing as both are dependent of the value of these matrices. For large values of Q_k and R_k we have large estimation errors and vice-versa.
4. Plot a graph of trace of posterior estimation error covariance vs. time. What can you understand from this graph?
   - Case 1: When Q and R as follows:

$$Q_k = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \tag{4}$$

$$R_k = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \tag{5}$$

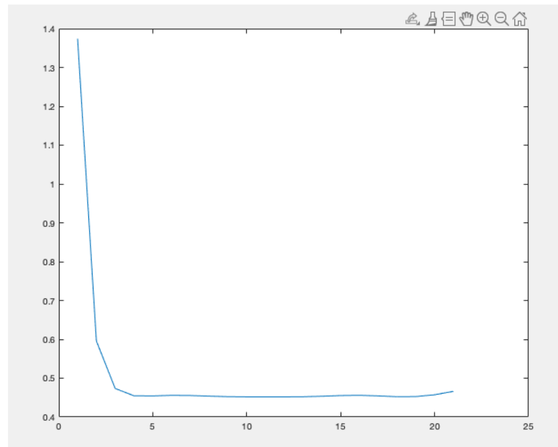

Figure 1: The plot of trace of posterior estimation error covariance for case 1.

   - Case 2: When Q and R as follows:

$$Q_k = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.8 \end{bmatrix} \tag{6}$$

$$R_k = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.7 \end{bmatrix} \tag{7}$$

   - The graph in the 1st case where we have taken the value of process noise covariance and measurement noise covariance are very small the graph is not changing at every instant but for case2 when the value of Q_k and R_k is large we have varying estimation errors.
5. Did you need to linearize either the state transition or measurement model? If yes, why? How did you choose the nominal value about which the model is linearized?
   - Yes as the measurement model is nonlinear we had to linearize it.The best nominal point at which we could linearize the measurement model is $\hat{X_k^-}$.

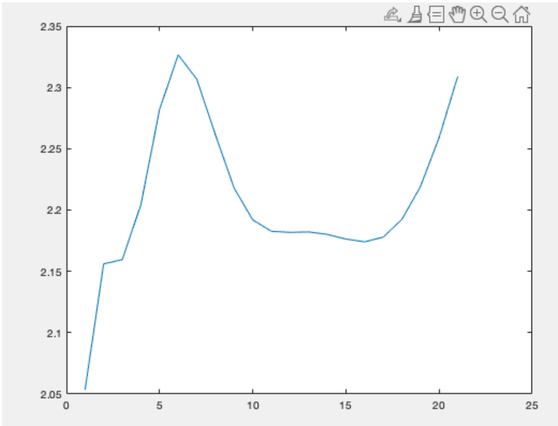

Figure 2: The plot of trace of posterior estimation error covariance for case2.

As this is the predicted state it will be near to the true value.
For the state transition model we assumed that it was a constant velocity model as nothing was mentioned.

Table 1: State vector containing X,Y ,VelocityX, VelocityY

X	Y
-9.688106038	-9.691977672
-8.931586322	-8.935238332
-7.94877049	-7.950104494
-6.953033262	-6.953563914
-5.979058189	-5.979259161
-4.987443847	-4.987512545
-3.993730165	-3.993753533
-2.993010141	-2.993018248
-1.99303051	-1.993033338
-0.9989767956	-0.9989777649
-8.85E-05	-8.88E-05
0.999089941	0.9990898411
1.993143621	1.993143592
2.992667803	2.992667794
3.993496968	3.993496966
4.987086749	4.987086748
5.979059193	5.979059193
6.949339258	6.949339258
7.927644718	7.927644718
8.896992827	8.896992827
9.862837366	9.862837366

Table 2: Continued from above table

VelocityX	VelocityY
0.4594233009	0.5651122605
0.6804139131	0.8488508611
0.8962656334	0.9628592105
0.9700479104	1.014272731
0.9733955495	1.016970019
0.9882807628	0.9671547845
0.992849751	0.9084970481
0.9995429397	0.8933759679
0.9999228765	0.9114603258
0.9948588683	0.9329873364
0.9983480441	0.9536993815
0.999067722	0.9620138448
0.9947281548	0.9652707837
0.9988647137	0.9698335839
1.000547703	0.9730390724
0.9946580953	0.977651393
0.9924299484	0.987889977
0.9745973487	0.9934561898
0.9774662076	1.003905931
0.9714440075	1.002046593
0.9674506599	1.006586246

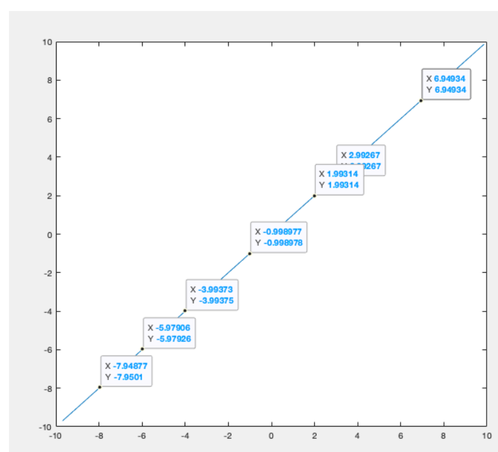

Figure 3: Trajectory of the path obtained using the X and Y coordinates.

4. Conclusion

In this lab we discussed about how we can implement the EKF by various assumptions made during the process. Also for different values of the process noise covariance and measurement noise covariance we will get different values of $P_k^+ and P_k^+$.